Distributed Adaptive Consensus Disturbance Rejection: a Directed-spanning-tree Perspective

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Outline

Background

Preliminaries

Main Results

Examples

Conclusions

Multi-Agent Systems

- Agents (Abilities to sense, compute, communicate, $\cdots$)
- Local interactions (Cooperative, competitive, $\cdots$)
- Global Behaviors (Consensus, formation, $\cdots$)

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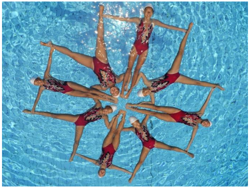
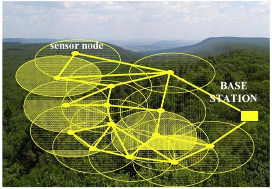


(d) Synchronised swim

(e) Formation fighters



(f) Sensor network

(The pictures were downloaded from un-copyrighted websites with thanks)

Graph Notations

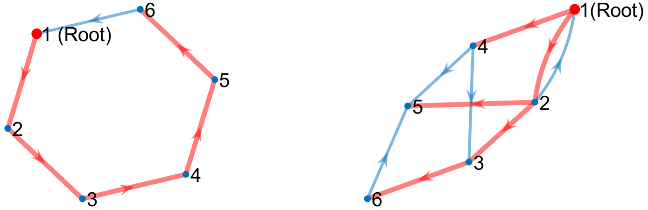


Figure 1: Examples of a digraph with a directed spanning tree (DST).

Problem Setup

$$
\begin{aligned}
\dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + D\omega_i(t), \\
\dot{\omega}_i(t) &= E\omega_i(t), \\
x_i \in \mathbb{R}^n&, \quad u_i \in \mathbb{R}^m, \quad \omega_i \in \mathbb{R}^s, \ i \in \mathcal{I}_N.
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Assumption 1

There exist a matrix F such that $D = BF$.

Assumption 2

The eigenvalues of the matrix E are simple with zero real parts.

Assumption 3

The pair (A, B) is stabilizable; the pair (E, D) is observable.

Problem Setup

Lemma 1

Under the observability of (E, D) *of Assumption 3, the pair* (A_H, H) *is also observable, where* $A_H = \begin{pmatrix} A & D \\ 0_{s\times n} & E \end{pmatrix}$ *and* $H = (\boldsymbol{I}_n, 0_{n\times s})$.

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Lemma 2

Under Assumption 4, the following statements hold for $\mathcal{L}$ *of* $\mathcal{G}$:

1. $0 = \lambda_1(\mathcal{L}) < \Re(\lambda_2(\mathcal{L})) \le \Re(\lambda_3(\mathcal{L})) \le \cdots \le \Re(\lambda_N(\mathcal{L}))$. *Moreover,* $null(\mathcal{L}) = span(\boldsymbol{1}_N)$.
2. $\Xi\mathcal{L} = \Pi\Xi$. *Moreover,* $null(\Xi) = span(\boldsymbol{1}_N)$.
3. $\lambda_i(\Pi) = \lambda_{i+1}(\mathcal{L}), i = 1, \cdots, N-1$.

Note: Ξ and Π are defined based on $\bar{\mathcal{G}}$; their specific forms are omitted here.

Static coupling case: a DST perspective

The control law is proposed for agent $i \in \mathcal{V}$ as:

$$u_i = -K\chi_i - Fz_i \tag{2a}$$

$$\dot{\chi}_i = (A - BK)\chi_i + c\Gamma_x \sum_{j=1}^{N} a_{ij}(\rho_i - \rho_j) \tag{2b}$$

$$\dot{z}_i = Ez_i + c\Gamma_\omega \sum_{j=1}^{N} a_{ij}(\rho_i - \rho_j) \tag{2c}$$

where $\rho_i = x_i - \chi_i$.

Static coupling case: a DST perspective

Since (A, B) is stabilizable, there exists a $P > 0$ such that

$$AP + PA^T - 2BB^T < 0. \tag{3}$$

Moreover, since (A_H, H) is observable by Lemma 1, there exists a $Q > 0$ such that

$$QA_H + A_H^T Q - 2H^T H < 0. \tag{4}$$

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Lemma 3

Under Assumptions 1-4, the consensus disturbance rejection problem of the multiagent system (1) can be solved by the static scheme (2). The parameters are designed as $K = B^T P^{-1}$, $\Gamma := (\Gamma_x^T, \Gamma_\omega^T)^T = Q^{-1} H^T$, *and* $c \geq \frac{1}{\Re(\lambda_2(\mathcal{L}))}$.

Static coupling case: a DST perspective

Proof of Lemma 3.

(main idea) Denote $e_i = \begin{pmatrix} x_i - \chi_i \\ \omega_i - z_i \end{pmatrix}$ as the composite observer error system. It is clear that $\rho_i = He_i$.

$$\Rightarrow \qquad \dot{e} = (\mathbf{I}_N \otimes A_H - c\mathcal{L} \otimes \Gamma H)e.$$

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Consider the transformation $\xi = (\Xi \otimes \mathbf{I}_{n+s})e$, $\delta = (\Xi \otimes \mathbf{I}_n)x$.

$$\Rightarrow \qquad \begin{aligned} \dot{\xi} &= (\mathbf{I}_{N-1} \otimes A_H - c\Pi \otimes \Gamma H)\xi \\ \dot{\delta} &= \big(\mathbf{I}_{N-1} \otimes (A - BK)\big)\delta + \big(\mathbf{I}_{N-1} \otimes B(K, F)\big)\xi. \end{aligned}$$

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The design of c, Γ are such that $(\mathbf{I}_{N-1} \otimes A_H - c\Pi \otimes \Gamma H)$ is Hurwitz;
The design of K is such that $A - BK$ is Hurwitz. $\square$

Adaptive coupling case: a DST perspective

The control law is proposed for agent $i \in \mathcal{V}$ as:

$$u_i = -K\chi_i - Fz_i \tag{5a}$$

$$\dot{\chi}_i = (A - BK)\chi_i + \Gamma_x \sum_{j=1}^{N} c_{ij} a_{ij}(\rho_i - \rho_j) \tag{5b}$$

$$\dot{z}_i = Ez_i + \Gamma_\omega \sum_{j=1}^{N} c_{ij} a_{ij}(\rho_i - \rho_j) \tag{5c}$$

$$\dot{c}_{ij} = \begin{cases} \gamma\Bigg((\rho_{i_k} - \rho_{k+1}) - \sum_{j \in \bar{\mathcal{N}}_{\text{out}}(k+1)} (\rho_{k+1} \\ \quad -\rho_j)\Bigg)^T (\rho_{i_k} - \rho_{k+1}) \triangleq \dot{\bar{c}}_{k+1,i_k}, & \text{if } e_{ji} \in \bar{\mathcal{E}} \\ 0, & \text{if } e_{ji} \in \mathcal{E} \setminus \bar{\mathcal{E}} \end{cases} \tag{5d}$$

where $\rho_i = x_i - \chi_i$.

Adaptive coupling case: a DST perspective

Theorem 1

Under Assumptions 1-4, the consensus disturbance rejection problem of the multiagent system (1) can be solved by the adaptive scheme (5). The parameters are designed as $K = B^T P^{-1}$, $\Gamma = Q^{-1} H^T$, *and* $\gamma \in \mathbb{R}^+$. *Moreover, the gains* $\bar{c}_{k+1,i_k}$, $k \in \mathcal{I}_{N-1}$, *in* $\bar{\mathcal{G}}$ *converge to some finite constant values.*

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Proof of Theorem 1.

(main idea) Define $\mathcal{L}^c$ as the gain-dependent Laplacian matrix:

$$\mathcal{L}^c_{ij} = -c_{ij} a_{ij},\ i \neq j;$$

$$\mathcal{L}^c_{ii} = \sum_{j=1, j\neq i}^{N} c_{ij} a_{ij},\ i \in \mathcal{I}_N.$$

$$\Rightarrow \qquad \dot{e} = (\mathbf{I}_N \otimes A_H - \mathcal{L}^c \otimes \Gamma H) e.$$

Adaptive coupling case: a DST perspective

Proof of Theorem 1 Cont.

$$\Rightarrow \qquad \dot{\xi} = (\mathbf{I}_{N-1} \otimes A_H - \Pi^c \otimes \Gamma H)\xi.$$

Here, Π^c is defined based on the DST $\bar{\mathcal{G}}$ and the gain-dependent Laplacian matrix. Consider the candidate Lyapunov function

$$V = \frac{1}{2}\xi^T(\mathbf{I}_{N-1} \otimes Q)\xi + \sum_{k=1}^{N-1} \frac{a_{k+1,i_k}}{2\gamma}(\bar{c}_{k+1,i_k}(t) - \phi_{k+1,i_k})^2$$

where Q is a solution to (4) and $\phi_{k+1,i_k} \in \mathbb{R}^+$, $k \in \mathcal{I}_{N-1}$.

Adaptive coupling case: a DST perspective

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where Q is a solution to (4) and $\phi_{k+1,i_k} \in \mathbb{R}^+, k \in \mathcal{I}_{N-1}$. With appropriate selection of ϕ_{k+1,i_k}, if can be guaranteed that $\dot{V} \leq 0$ and $\dot{V} = 0$ iff $\xi = 0$.

$$\Rightarrow \xi \to 0. \qquad \Rightarrow \delta \to 0 \quad (i.e., consensus).$$

□

Example 1

Second-order MAS over a Ring (Figure 1, left):

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$
$$D = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \; E = \begin{pmatrix} 0 & 1.5 \\ -0.8 & 0 \end{pmatrix}.$$

$\Rightarrow$ Assumptions 1-4 hold; $F = (0, 1)$.

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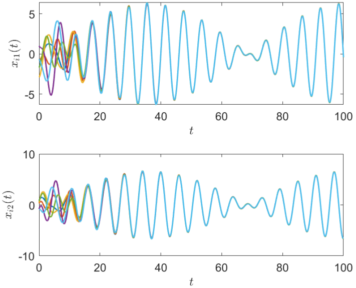
Parameters: $K = (0.1251, 0.5732)$, $\gamma = 0.01$ in (5d),

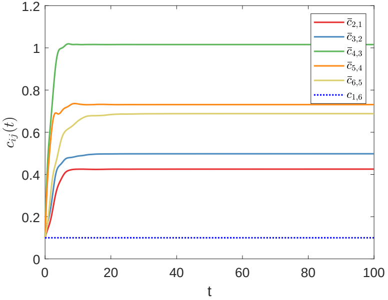
$$\Gamma_x = \begin{pmatrix} 0.6906 & 0.0951 \\ 0.0951 & 0.8973 \end{pmatrix}, \; \Gamma_\omega = \begin{pmatrix} 0.2850 & 0.1484 \\ 0.0341 & 0.3348 \end{pmatrix}.$$

Example 1

Results:



(a) The states x_i of the agents.



(b) The adaptive coupling gains c_{ij}.

Figure 2: The states x_i of the second-order agents and adaptive gains c_{ij} under DST-based adaptive scheme (5) ($\gamma = 0.01$).

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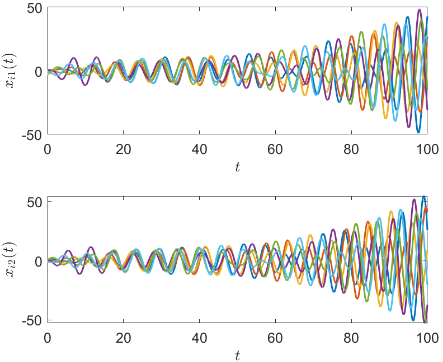

Figure 3: The states x_i of the second-order agents under non-adaptive scheme $\big((5)$ with $\gamma = 0\big)$.

Example 2

YF-22 UAVs over a digraph (Figure 1, right):

$$A = \begin{pmatrix} -0.2840 & -23.0960 & 2.4200 & 9.9130 \\ 0 & -4.1170 & 0.8430 & 0.2720 \\ 0 & -33.8840 & -8.2630 & -19.5430 \\ 0 & 0 & 1.0000 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} 20.1680 \\ 0.5440 \\ -39.0850 \\ 0 \end{pmatrix}, \ E = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}.$$

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$$B = \begin{pmatrix} 20.1680 \\ 0.5440 \\ -39.0850 \\ 0 \end{pmatrix}, \; E = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}.$$

Here, the four states of $x_i = (V_i, \alpha_i, q_i, \theta_i)$ are **speed in meters per second**, **angle of attack in degrees**, **pitch rate in degrees per second**, and **pitch in degrees**, respectively.
The vibrations, as disturbance ω_i to be rejected has frequency 2 radians per second and satisfies Assumption 1 with $F = (1, 0)$.

Example 2

Results:

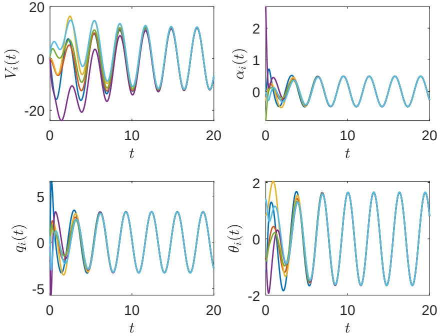

Figure 4: The states x_i of the UAVs under the proposed adaptive scheme (5).

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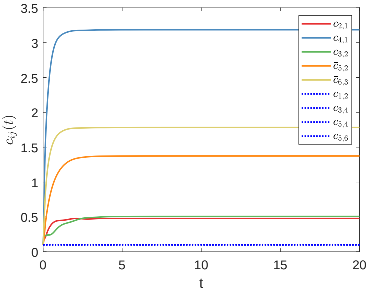
Results:



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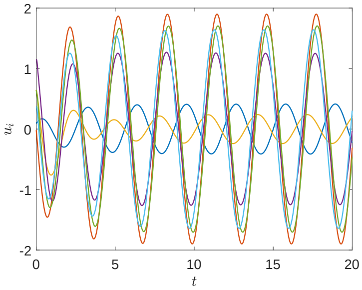

(b) The control inputs u_i.

Concluding Remarks

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- We have reproduced the lower bound for a static homogeneous coupling gain in the literature by exploring the structure of a DST;
- We have proposed a DST-based adaptive consensus disturbance rejection scheme, which eliminates the requirement for the global information of the Laplacian eigenvalues;
- Future works?

Further reading

[1]. **Dongdong Yue**, Simone Baldi, Jinde Cao, and Bart De Schutter. Distributed adaptive optimization with weight-balancing. ***IEEE Transactions on Automatic Control***, 67(4): 2068-2075, 2022.

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Thank you for listening!

Question?