A Directed Spanning Tree based framework for Distributed Adaptive Control (DST-DAC)

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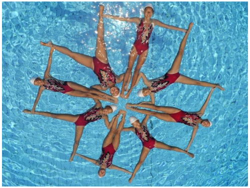
Outline

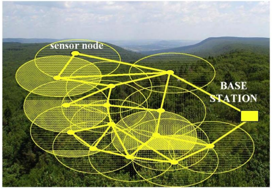
Multiagent Systems

Local interactions lead to Global behaviors!



(a) Synchronised swim

(b) Formation fighters



(c) Sensor network

(The pictures were downloaded from un-copyrighted websites with thanks)

Adaptive Multiagent Systems

- Dynamics uncertainties (unmodeled dynamics, $\cdots$)
- Networked uncertainties (unknown graph Laplacian, $\cdots$)
- Co-existence of both

[1]. Das, Abhijit, and Frank L. Lewis. "Distributed adaptive control for synchronization of unknown nonlinear networked systems." Automatica 2010.

[2]. Yu, Wenwu, et al. "Distributed adaptive control of synchronization in complex networks." IEEE TAC, 2012.

[3]. Li, Zhongkui, et al. "Distributed consensus of linear multi-agent systems with adaptive dynamic protocols." Automatica, 2013.

[4]. Ghapani, Sheida, et al. "Fully distributed flocking with a moving leader for Lagrange networks with parametric uncertainties." Automatica, 2016.

Adaptive Multiagent Systems

Adaptive Mutiagent Systems (Yue et al., 2019-)

I. Dynamics uncertainties mainly: Adaptive feedback + Static Coupling

NN against time delay (IET CTA, 2019)	NN-MRACon (CCDC 2022, EI)	MRASC (submitted, IEEE TAC)	Decentralized MRACO (submitted, IFAC 2023, EI)

II. Networked uncertainties mainly: Static feedback + Adaptive Coupling

DST-based formation (IEEE TCNS, 2021)	Distributed optimization (IEEE TAC, 2022)	Anti-disturbance (CCC 2022, EI)	Distributed Resource Allocation (revised, IEEE/CAA JAS)

III. Co-existence of both: Adaptive feedback + Adaptive Coupling

NN for formation (IEEE TNSE, 2020)	NN for consensus (IEEE TNNLS, 2021)	NN for average tracking (AJC, 2021)	Edge-based NN (IJCNN 2019, EI)

Graph Notations

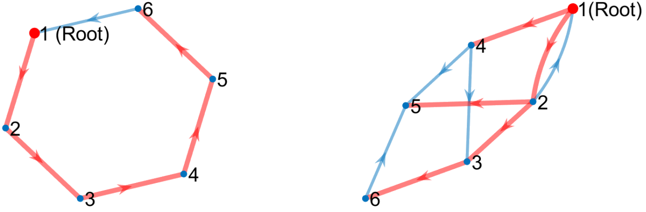


Figure 1: Examples of a digraph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$ with a DST $\bar{\mathcal{G}}(\mathcal{V}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$.

Note: we always assume that node 1 is the root of $\bar{\mathcal{G}}$, WLOG.

Suppose $\mathcal{G}$ contains a DST $\bar{\mathcal{G}}$. Let $\tilde{\mathcal{L}} = \mathcal{L} - \bar{\mathcal{L}}$.

- Define $\Xi \in \mathbb{R}^{(N-1)\times N}$ as

$$\Xi_{kj} = \begin{cases} -1, & \text{if } j = k+1, \\ 1, & \text{if } j = i_k, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

- Define $Q \in \mathbb{R}^{(N-1)\times(N-1)} := \tilde{Q} + \bar{Q}$ with

$$Q_{kj} = \underbrace{\sum_{c\in\bar{\mathcal{V}}_{j+1}} \left(\tilde{\mathcal{L}}_{k+1,c} - \tilde{\mathcal{L}}_{i_k,c}\right)}_{\tilde{Q}_{kj}} + \underbrace{\sum_{c\in\bar{\mathcal{V}}_{j+1}} \left(\bar{\mathcal{L}}_{k+1,c} - \bar{\mathcal{L}}_{i_k,c}\right)}_{\bar{Q}_{kj}}. \tag{2}$$

$\bar{\mathcal{V}}_{j+1}$: the vertex set of the subtree of $\bar{\mathcal{G}}$ rooting at node $j+1$;
i_k: the in-neighbor of node $k+1$ in $\bar{\mathcal{G}}$, $k \in \mathcal{I}_{N-1} \triangleq \{1, \cdots, N-1\}$.

Lemma 0

The following statements hold for $\mathcal{L}$ of $\mathcal{G}$, Ξ and Q defined as above:

1. *$\mathcal{L}$ has a simple zero eigenvalue corresponding to the right eigenvector $\mathbf{1}_N$, and the other eigenvalues have positive real parts.*
2. $\Xi\mathcal{L} = Q\Xi$.
3. *$\bar{Q}$ can be explicitly written as*

$$\bar{Q}_{kj} = \begin{cases} \bar{a}_{j+1,i_j}, & \text{if } j = k, \\ -\bar{a}_{j+1,i_j}, & \text{if } j = i_k - 1, \\ 0, & \text{otherwise.} \end{cases}$$

4. *The eigenvalues of Q are exactly the nonzero eigenvalues of $\mathcal{L}$.*

Proof of Lemma 0.

- Statement 1 is sufficient and necessary for the existence of a DST, the proof can be found in [1, Lemma 2.4];
- Statements 2-3 can be proved by showing that (see [3] for details): $\mathcal{L} = \mathcal{L}J\Xi$ and $Q = \Xi\mathcal{L}J$, where $J \in \mathbb{R}^{N\times(N-1)}$ is defined with $J_{ik} = \begin{cases} 0, & \text{if} \quad i \in \bar{\mathcal{V}}_{k+1}, \\ 1, & \text{otherwise.} \end{cases}$
- Statement 4 is a direct application of [2, Lemma 10].

[1]. Ren, Wei, and Randal W. Beard. Distributed consensus in multi-vehicle cooperative control. Springer London, 2008.

[2]. Wu, Chai Wah, and Leon O. Chua. "Synchronization in an array of linearly coupled dynamical systems." IEEE TCS-I, 1995.

[3]. Yue, Dongdong, et al. "A directed spanning tree adaptive control solution to time-varying formations." IEEE TCNS, 2021.

□

DST-DAC for TVF

$$\dot{x}_i = Ax_i + Bu_i \quad i \in \mathcal{I}_N \triangleq \{1, 2, \cdots, N\} \tag{3}$$

Assumption 1 (DST)

The digraph $\mathcal{G}$ has at least one DST $\bar{\mathcal{G}}(\mathcal{V}, \bar{\mathcal{E}}, \bar{\mathcal{A}})$.

Definition 1 ([1],TVF)

The linear multiagent system (3) is said to achieve the time-varying formation defined by the time-varying vector $h(t) = \mathrm{col}(h_1(t), \cdots, h_N(t))$ if, for any initial states, there holds

$$\lim_{t\to\infty} ((x_i - h_i) - (x_j - h_j)) = 0, \ \forall i, j \in \mathcal{I}_N.$$

[1]. Dong, Xiwang, et al. ”Formation control for high-order linear time-invariant multiagent systems with time delays.” IEEE TCNS, 2014.

DST-DAC for TVF

Lemma 1 (N-S condition for TVF)

Under Assumption 1, and for any DST $\bar{\mathcal{G}}$, define Ξ as in (1). Then, the TVF for multi-agent system (3) can be achieved if and only if $\lim_{t\to\infty} \|(\Xi \otimes \boldsymbol{I}_n)d(t)\| = 0$ $(d = col(x_i - h_i))$.

DST-DAC for TVF

Lemma 1 (N-S condition for TVF)

Under Assumption 1, and for any DST $\bar{\mathcal{G}}$, define Ξ as in (1). Then, the TVF for multi-agent system (3) can be achieved if and only if $\lim_{t\to\infty} \|(\Xi \otimes \boldsymbol{I}_n)d(t)\| = 0$ $(d = col(x_i - h_i))$.

The control law is proposed for agent $i \in \mathcal{V}$ as:

$$u_i = K_0 x_i + K_1 d_i + K_2 \sum_{j\in\mathcal{N}_{\text{in}}(i)} \alpha_{ij}(t)(d_i - d_j) \tag{4}$$

$$\alpha_{ij}(t) = \begin{cases} a_{ij}, & \text{if} \quad e_{ji} \in \mathcal{E} \setminus \bar{\mathcal{E}}, \\ \bar{a}_{k+1,i_k}(t), & \text{if} \quad e_{ji} \in \bar{\mathcal{E}} \end{cases}$$

$$\dot{\bar{a}}_{k+1,i_k} = \rho_{k+1,i_k}\left((d_{i_k} - d_{k+1}) - \sum_{j\in\bar{\mathcal{N}}_{\text{out}}(k+1)} (d_{k+1} - d_j)\right)^T \Gamma(d_{i_k} - d_{k+1})$$

DST-DAC for TVF

Algorithm 1 Feasibility check and control parameters design

1: Find a matrix K_0 s.t. the formation feasibility condition

$$(A + BK_0)(h_{i_k}(t) - h_{k+1}(t)) - (\dot{h}_{i_k}(t) - \dot{h}_{k+1}(t)) = 0 \quad (5)$$

holds $\forall k \in \mathcal{I}_{N-1}$ for any DST $\bar{\mathcal{G}}$. If such K_0 exists, continue; else, the algorithm terminates without solutions;

2: Choose K_1 s.t. $(A + BK_0 + BK_1, B)$ is stabilizable. For some η, $\theta \in \mathbb{R}^+$, solve the following LMI:

$$(A + BK_0 + BK_1)P + P(A + BK_0 + BK_1)^T - \eta BB^T + \theta P \leq 0 \quad (6)$$

to get a $P > 0$;

3: Set $K_2 = -B^T P^{-1}$, $\Gamma = P^{-1}BB^T P^{-1}$ and choose $\rho_{k+1,i_k} \in \mathbb{R}^+$.

DST-DAC for TVF

Theorem 1 (Main result for TVF)

Under Assumption 1, and feasibility condition (5), the TVF problem in Definition 1 is solved by DST-DAC (4), along with the parameters designed in Algorithm 1.

DST-DAC for TVF

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Proof of Theorem 1.

(main idea) Define $\bar{d} = (\Xi \otimes \mathbf{I}_n)d$. The feasibility condition (5) guarantees that

$$\dot{\bar{d}} = (\mathbf{I}_{N-1} \otimes (A + BK_0 + BK_1) + Q(t) \otimes BK_2)\bar{d},$$

where $Q(t) = \tilde{Q} + \bar{Q}(t)$ with fixed $\tilde{Q}$ defined as in (2), and

$$\bar{Q}_{kj}(t) = \begin{cases} \bar{a}_{j+1,i_j}(t), & \text{if} \quad j = k, \\ -\bar{a}_{j+1,i_j}(t), & \text{if} \quad j = i_k - 1, \\ 0, & \text{otherwise.} \end{cases}$$

DST-DAC for TVF

Proof of Theorem 1 Cont.

Consider the Lyapunov candidate

$$V(t) = \frac{1}{2}\bar{d}^T(\mathbf{I}_{N-1} \otimes P^{-1})\bar{d} + \sum_{k=1}^{N-1} \frac{1}{2\rho_{k+1,i_k}}(\bar{a}_{k+1,i_k}(t) - \phi_{k+1,i_k})^2$$

where P is a solution to (6) and $\phi_{k+1,i_k} \in \mathbb{R}^+$, $k \in \mathcal{I}_{N-1}$.

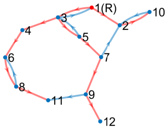
Note the adaptive gains cancel out the time-varying part in $\dot{\bar{d}}$:

$$\sum_{k=1}^{N-1} \bar{a}_{k+1,i_k}(\bar{d}_k - \sum_{j+1\in\bar{\mathcal{N}}_{\text{out}}(k+1)} \bar{d}_j)^T\Gamma\bar{d}_k = \sum_{k=1}^{N-1}\sum_{j=1}^{N-1} \bar{Q}_{jk}(t)\bar{d}_j^T\Gamma\bar{d}_k$$

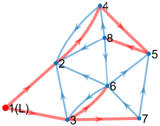
Then, with appropriate selection of ϕ_{k+1,i_k}, it can be guaranteed that $\dot{V} \leq 0$ and $\dot{V} = 0$ iff $\bar{d} = 0$. $\Rightarrow \bar{d} \to 0$. $\Rightarrow$ TVF.

□

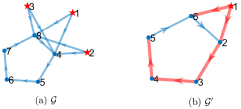
DST-DAC for TVF



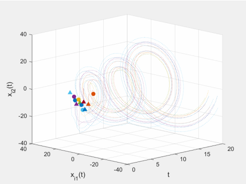
(a) Leaderless



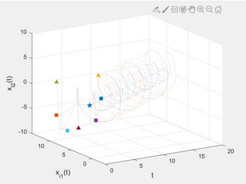
(b) Single leader



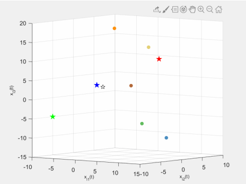
(c) Multiple leaders



(a) TVF



(b) TVF tracking



(c) TVF containment

DST-DAC for DO

$$\min_{z\in\mathbb{R}^n} F(z) \triangleq \sum_{i=1}^{N} f_i(z). \quad (7)$$

Assumption 1

The global cost function $F(\cdot)$ is differentiable and strictly convex over $\mathbb{R}^n$. Each local cost function $f_i(\cdot)$ is differentiable; and $\nabla f_i(x) = \Upsilon x + \psi_i(x)$, where $\Upsilon \in \mathbb{R}^{n\times n}$ with $\Upsilon \geq 0$, and $\|\psi_i(x)\| \leq K$ for some $K \in \mathbb{R}^+$ (could be unknown), for all $x \in \mathbb{R}^n$ and all $i \in \mathcal{V}$.

Assumption 2

The digraph $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{W})$ is strongly connected.

DST-DAC for DO

weight-balanced case:

$$\dot{x}_i = -\gamma_1 \nabla f_i(x_i) - \sum_{j \in \mathcal{N}_{\text{in}}(i)} \alpha_{ij}(t)(x_i - x_j) - \sum_{j \in \mathcal{N}_{\text{in}}(i)} w_{ij}(y_i - y_j) \quad (8)$$

$$\dot{y}_i = \sum_{j \in \mathcal{N}_{\text{in}}(i)} \alpha_{ij}(t)(x_i - x_j)$$

$$\alpha_{ij}(t) = \begin{cases} w_{ij}, & \text{if } e_{ji} \in \mathcal{E} \setminus \bar{\mathcal{E}}, \\ \bar{a}_{k+1,i_k}(t), & \text{if } e_{ji} \in \bar{\mathcal{E}} \end{cases}$$

$$\dot{\bar{a}}_{k+1,i_k} = \gamma_2 \Big((x_{i_k} - x_{k+1}) - \sum_{j \in \bar{\mathcal{N}}_{\text{out}}(k+1)} (x_{k+1} - x_j) \Big)^T (x_{i_k} - x_{k+1})$$

In a compact form:

$$\begin{aligned} \dot{x} &= -\gamma_1 \nabla f(x) - (\mathcal{L}^{\mathcal{A}}(t) \otimes \mathbf{I}_n)x - (\mathcal{L} \otimes \mathbf{I}_n)y \\ \dot{y} &= (\mathcal{L}^{\mathcal{A}}(t) \otimes \mathbf{I}_n)x \end{aligned} \quad (9)$$

DST-DAC for DO

Let us transfer any equilibrium $(\tilde{x}, \tilde{y})$ of (9) to the origin and apply a change of coordinates:

$$\begin{aligned} \mu &= x - \tilde{x}, \qquad \nu = y - \tilde{y} \\ \bar{\mu} &= (\Xi \otimes \mathbf{I}_n)\mu, \qquad \bar{\nu} = (\Xi \otimes \mathbf{I}_n)\nu \end{aligned} \tag{10}$$

where Ξ is defined as in (1). In these new coordinates, the algorithm (8) read

$$\begin{aligned} \dot{\bar{\mu}} &= -\gamma_1(\Xi \otimes \mathbf{I}_n)h - (Q^{\mathcal{A}}(t) \otimes \mathbf{I}_n)\bar{\mu} - (Q \otimes \mathbf{I}_n)\bar{\nu} \\ \dot{\bar{\nu}} &= (Q^{\mathcal{A}}(t) \otimes \mathbf{I}_n)\bar{\mu} \\ \dot{\bar{a}}_{k+1,i_k} &= \gamma_2\left(\bar{\mu}_k - \sum_{j \in \bar{\mathcal{N}}_{\text{out}}(k+1)} \bar{\mu}_{j-1}\right)^T \bar{\mu}_k \end{aligned} \tag{11}$$

where $h = \nabla f(\mu + \tilde{x}) - \nabla f(\tilde{x})$, and Q as well as $Q^{\mathcal{A}}(t)$, $\forall t$, are defined as in (2) based on the DST $\bar{\mathcal{G}}$.

DST-DAC for DO

Lemma 1

Suppose that $\mathcal{G}$ is weight-balanced and Assumptions 1-2 hold. If $(\tilde{x}, \tilde{y})$ is an equilibrium point of (9), then $\tilde{x} = \mathbf{1}_N \otimes z^*$, i.e., the global minimizer of (7).

Lemma 2

For system (11) with arbitrary initial conditions, $(\bar{\mu}, \bar{\nu})$ asymptotically converges to the origin, and the weights $\bar{a}_{k+1,i_k}$, $k \in \mathcal{I}_{N-1}$, converge to some finite constant values.

Theorem 1

Suppose that $\mathcal{G}$ is weight-balanced and Assumptions 1-2 hold. Then, algorithm (8) drives x_i to z^* asymptotically, for all $i \in \mathcal{V}$, and for any $x_i(0)$, $y_i(0) \in \mathbb{R}^n$. Moreover, the weights $\bar{a}_{k+1,i_k}$, $k \in \mathcal{I}_{N-1}$, in $\bar{\mathcal{G}}^{\mathcal{A}}$ converge to some finite constant values.

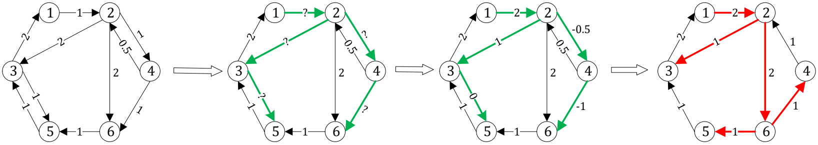
DST-DAC for DO

weight-unbalanced case: weight-balancing based on a DST $\bar{\mathcal{G}}$!

$$\beta_{ij}(t) = \begin{cases} w_{ij}, & \text{if } e_{ji} \in \mathcal{E} \setminus \bar{\mathcal{E}}, \\ \bar{b}_{k+1,i_k}(t), & \text{if } e_{ji} \in \bar{\mathcal{E}} \end{cases} \tag{12}$$

$$\dot{\bar{b}}_{k+1,i_k} = -\gamma_3 \mathrm{sig}^q\Big(\sum_{p\in\mathcal{N}_{\text{in}}(k+1)} \beta_{k+1,p} - \sum_{c\in\mathcal{N}_{\text{out}}(k+1)} \beta_{c,k+1} \Big)$$

for $k \in \mathcal{I}_{N-1}$, where $q \in (0,1)$, $\gamma_3 \in \mathbb{R}^+$; $\beta_{ij}(0) = w_{ij}$, $\forall i,j \in \mathcal{V}$.



(a) (b) (c) (d)

DST-DAC for DO

Lemma 3

The digraph $\mathcal{G}$ is weight-balanced iff $\mathbf{1}_N^T \mathcal{L}_{2:N} = 0$, where $\mathcal{L}_{2:N}$ is the block submatrix of $\mathcal{L}$ containing the second to the last columns.

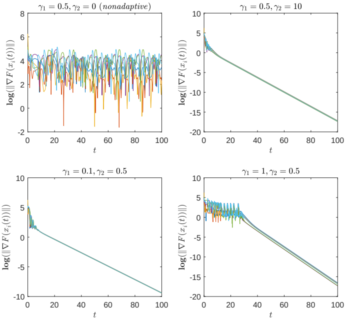
Lemma 4

Suppose $\bar{\mathcal{G}}$ is a DST of $\mathcal{G}$. Let us take the weights in $\bar{\mathcal{G}}$ as independent variables $\bar{b}_{k+1,i_k}$, $k \in \mathcal{I}_{N-1}$, and denote the according Laplacian matrix with independent variables as $\mathcal{L}^{\mathcal{B}}$. Then, there exists a unique solution $\bar{b}^* := (\bar{b}^*_{2,i_1}, \bar{b}^*_{3,i_2}, \cdots, \bar{b}^*_{N,i_{N-1}})^T \in \mathbb{R}^{N-1}$ to the system of implicit linear equations $\mathbf{1}_N^T \mathcal{L}^{\mathcal{B}}_{2:N} = 0$.

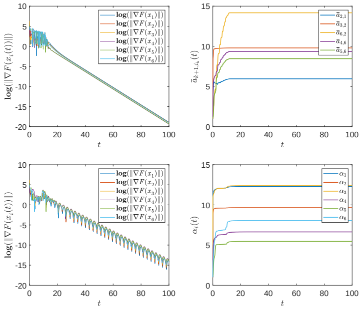
Theorem 2

Let Assumption 2 hold. Then, the balancing law (12) drives $\bar{b}$ to $\bar{b}^*$ in finite time.

DST-DAC for DO



(a) Need for adaptation



(b) DST (top) vs Node (down)

DST-DAC for CDR

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) + D\omega_i(t), \tag{13}$$
$$\dot{\omega}_i(t) = E\omega_i(t),$$
$$x_i \in \mathbb{R}^n, \quad u_i \in \mathbb{R}^m, \quad \omega_i \in \mathbb{R}^s, \ i \in \mathcal{I}_N.$$

Assumption 1

There exist a matrix F such that $D = BF$.

Assumption 2

The eigenvalues of the matrix E are simple with zero real parts.

Assumption 3

The pair (A, B) is stabilizable; the pair (E, D) is observable.

Assumption 4

The communication digraph $\mathcal{G}$ contains a directed spanning tree $\bar{\mathcal{G}}$.

DST-DAC for CDR

Lemma 1

Under the observability of (E, D) *of Assumption 3, the pair* (A_H, H) *is also observable, where* $A_H = \begin{pmatrix} A & D \\ 0_{s\times n} & E \end{pmatrix}$ *and* $H = (\boldsymbol{I}_n, 0_{n\times s})$.

DST-DAC for CDR

Lemma 1

Under the observability of (E, D) of Assumption 3, the pair (A_H, H) is also observable, where $A_H = \begin{pmatrix} A & D \\ 0_{s\times n} & E \end{pmatrix}$ *and* $H = (\boldsymbol{I}_n, 0_{n\times s})$.

Since (A, B) is stabilizable, there exists a $P > 0$ such that

$$AP + PA^T - 2BB^T < 0. \tag{14}$$

Moreover, since (A_H, H) is observable by Lemma 1, there exists a $Q > 0$ such that

$$QA_H + A_H^T Q - 2H^T H < 0. \tag{15}$$

DST-DAC for CDR

Let $\rho_i = x_i - \chi_i$. Consider the control law for agent $i \in \mathcal{V}$ as:

$$u_i = -K\chi_i - Fz_i \tag{16}$$

$$\dot{\chi}_i = (A - BK)\chi_i + c\Gamma_x \sum_{j=1}^{N} a_{ij}(\rho_i - \rho_j)$$

$$\dot{z}_i = Ez_i + c\Gamma_\omega \sum_{j=1}^{N} a_{ij}(\rho_i - \rho_j)$$

Lemma 2 ([1])

Under Assumptions 1-4, the consensus disturbance rejection problem of the multiagent system (13) can be solved by (16) with parameters $K = B^T P^{-1}$, $\Gamma := (\Gamma_x^T, \Gamma_\omega^T)^T = Q^{-1}H^T$, *and* $c \geq \frac{1}{\Re(\lambda_2(\mathcal{L}))}$.

[1]. Ding, Zhengtao. "Consensus disturbance rejection with disturbance observers." IEEE TIE, 2015.

DST-DAC for CDR

The control law is proposed for agent $i \in \mathcal{V}$ as:

$$u_i = -K\chi_i - Fz_i \tag{17}$$

$$\dot{\chi}_i = (A - BK)\chi_i + \Gamma_x \sum_{j=1}^{N} c_{ij} a_{ij}(\rho_i - \rho_j)$$

$$\dot{z}_i = Ez_i + \Gamma_\omega \sum_{j=1}^{N} c_{ij} a_{ij}(\rho_i - \rho_j)$$

$$\dot{c}_{ij} = \begin{cases} \gamma\Big((\rho_{i_k} - \rho_{k+1}) - \sum_{j \in \bar{\mathcal{N}}_{\text{out}}(k+1)} (\rho_{k+1} \\ \quad -\rho_j)\Big)^T (\rho_{i_k} - \rho_{k+1}) \triangleq \dot{\bar{c}}_{k+1,i_k}, & \text{if } e_{ji} \in \bar{\mathcal{E}} \\ 0, & \text{if } e_{ji} \in \mathcal{E} \setminus \bar{\mathcal{E}} \end{cases}$$

DST-DAC for CDR

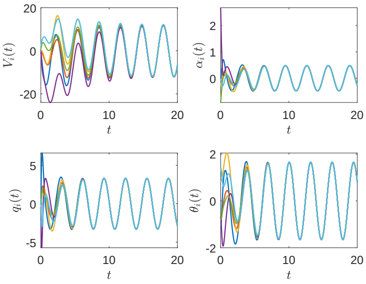
Theorem 1

Under Assumptions 1-4, the consensus disturbance rejection problem of the multiagent system (13) can be solved by the adaptive scheme (17) with parameters $K = B^T P^{-1}$, $\Gamma = Q^{-1} H^T$, *and* $\gamma \in \mathbb{R}^+$. *Moreover, the gains* $\bar{c}_{k+1,i_k}$, $k \in \mathcal{I}_{N-1}$, *in* $\bar{\mathcal{G}}$ *converge to some finite values.*

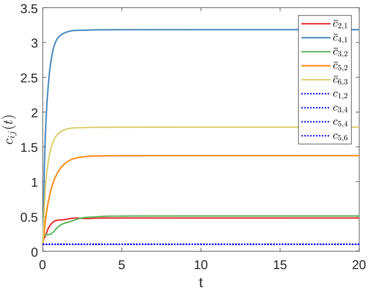
DST-DAC for CDR

Theorem 1

Under Assumptions 1-4, the consensus disturbance rejection problem of the multiagent system (13) can be solved by the adaptive scheme (17) with parameters $K = B^T P^{-1}$, $\Gamma = Q^{-1} H^T$, and $\gamma \in \mathbb{R}^+$. Moreover, the gains $\bar{c}_{k+1,i_k}$, $k \in \mathcal{I}_{N-1}$, in $\bar{\mathcal{G}}$ converge to some finite values.



(e) longitudinal control of UAVs



(f) The adaptive coupling gains c_{ij}

Conclusions

- We have presented a framework of distributed adaptive control based on a directed spanning tree (DST-DAC);
- We have explained how DST-DAC can accommodate to solve TVF, DO, and CDR problems;
- We have discussed the superiority (eliminates $\lambda_2(\mathcal{L})$, faster convergence) and also the deficiency (not fully distributed) of DST-DAC;
- Future works?

Acknowledgements

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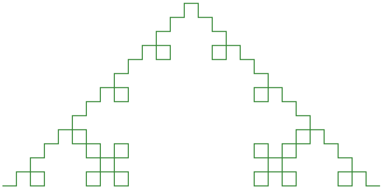
Thank you for listening!

Question?



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