Distributed Adaptive Consensus via Event-triggered Sampling: An Edge-based Method

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[1]Presenting author.

Outline

Multi-Agent Systems

- Agents (Abilities to sense, compute, communicate, $\cdots$)
- Local interactions (Cooperative, competitive, $\cdots$)
- Global Behaviors (Consensus, formation, $\cdots$)

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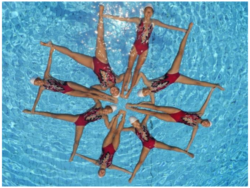
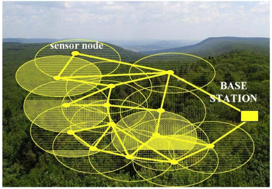


(d) Synchronised swim

(e) Formation fighters



(f) Sensor network

(The pictures were downloaded from un-copyrighted websites with thanks)

Event-trigger plus Adaptive

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) \quad i \in \mathcal{V} \triangleq \{1, 2, \cdots, N\}.$$

$$u_i(t) = cK \sum_{j \in \mathcal{N}_i} (x_i(t) - x_j(t)).$$

Fact: Let $\mathcal{G}$ be undirected connected; K stabilize (A, B); and c be sufficiently large ($c \geq \lambda_2(\mathcal{L})$). Then, the agents reach consensus.

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Save comm. and control energy.

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Adaptive coupling:

$$u_i(t) = c_i(t)K \sum_{j \in \mathcal{N}_i} (x_i(t) - x_j(t)).$$

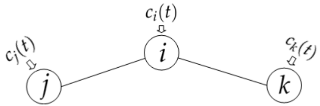
Remove global knowledge of $\lambda_2(\mathcal{L})$.

Node-based versus Edge-based

Node-based:

$$u_i(t) = c_i(t)K \sum_{j \in \mathcal{N}_i} (x_i(t) - x_j(t))$$

$$\dot{c}_i(t) = \rho_i \| K \sum_{j \in \mathcal{N}_i} (x_i(t) - x_j(t)) \|^2.$$



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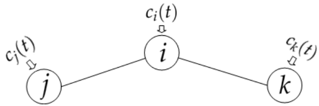
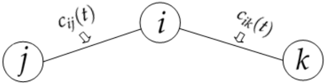
Edge-based:

$$u_i(t) = K \sum_{j \in \mathcal{N}_i} c_{ij}(t)(x_i(t) - x_j(t))$$

$$\dot{c}_{ij}(t) = \rho_{ij} \| K(x_i(t) - x_j(t)) \|^2.$$





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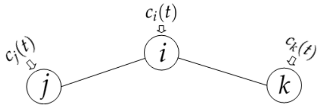

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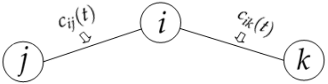

Fact: Let $\mathcal{G}$ be undirected connected; K stabilize (A,B); $\rho_i, \rho_{ij} > 0$. Then, both algorithms guarantee consensus.

Control Law

Denote $x_{ij}(t) \triangleq x_i(t) - x_j(t)$. For $i \in \mathcal{V}$ and $j \in \mathcal{N}_i$, consider

$$\left.\begin{aligned}
&\tilde{x}_{ij}(t) \triangleq x_{ij}(t_k^{ij}), \qquad t \in [t_k^{ij}, t_{k+1}^{ij}) \\
&\epsilon_{ij}(t) \triangleq \tilde{x}_{ij} - x_{ij} \\
&f_{ij}(t) = \epsilon_{ij}^T \Gamma \epsilon_{ij} - \omega x_{ij}^T \Gamma x_{ij} - \frac{\theta}{c_{ij}} \mathrm{e}^{-\eta t} \\
&t_{k+1}^{ij} = \inf_{t > t_k^{ij}} \{ f_{ij}(\epsilon_{ij}, x_{ij}, c_{ij}, t) = 0 \}, \qquad t_0^{ij} = 0
\end{aligned}\right\} \text{E-T sampling}$$

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$$\dot{c}_{ij}(t) = \rho_{ij} \tilde{x}_{ij}^T \Gamma \tilde{x}_{ij} \qquad \leftarrow \text{Adaptive coupling}$$

$$u_i = K \sum_{j \in \mathcal{N}_i} c_{ij}(t) \tilde{x}_{ij}(t) \qquad \leftarrow \text{Distributed control}$$

Control Parameters

Algorithm 1

1. Choose $\kappa_1, \kappa_2 \in \mathbb{R}^+$, then solve the following Linear Matrix Inequality (LMI):

$$AP + PA^T - \kappa_1 BB^T + \kappa_2 P \leq 0$$

   to get a $P > 0$;
2. Let $K = -B^T P^{-1}$, $\Gamma = P^{-1} BB^T P^{-1}$;
3. Initialize $c_{ij}(0) = c_{ji}(0) \in \mathbb{R}^+$; select $\rho_{ij} = \rho_{ji}, \theta, \eta \in \mathbb{R}^+$ and $\omega \in (0, 1)$.

Note: The first two steps are for local stabilization; the final step is for global stability.

Convergence Results

Theorem 1

Let $\mathcal{G}$ be undirected connected. Under the proposed control design,

- *the agents reach consensus;*
- *the adaptive coupling gains c_{ij} converge to some finite constants;*
- *the closed-loop system is Zeno-free.*

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Proof: Omitted. Hint on the candidate Lyapunov function:

$$V = \frac{1}{2}\sum_{i\in\mathcal{V}} e_i^T P^{-1} e_i + \frac{1-\omega}{16(1+\omega)} \sum_{i\in\mathcal{V}} \sum_{j\in\mathcal{N}_i} \frac{(c_{ij}-\bar{c})^2}{\rho_{ij}}$$

where $e_i = x_i - \mathbf{ave}(x)$ is the consensus error.

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Corollary 1

The proposed method applies to switching connected graphs.

Example

MAS: $N = 6, \quad A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \end{pmatrix};$

$\mathcal{G}$ is randomly switching between Star and Ring.

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Parameters: $\kappa_1 = \kappa_2 = 1, K = (-6, -6), \Gamma = 36 * \text{ones}(2);$
$c_{ij}(0) = 1, \rho_{ij} = 0.5, \theta = \eta = 0.5, \omega = 0.9.$

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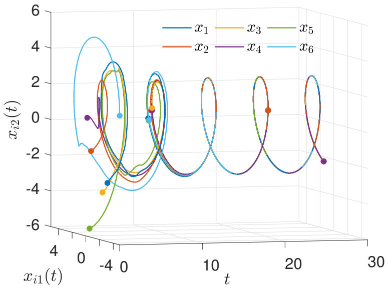
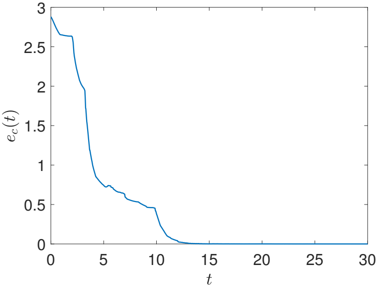
Results:

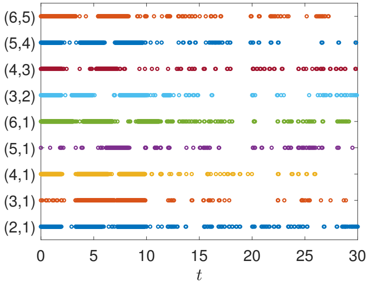


(a) States $x_i(t), i \in \mathcal{V}$

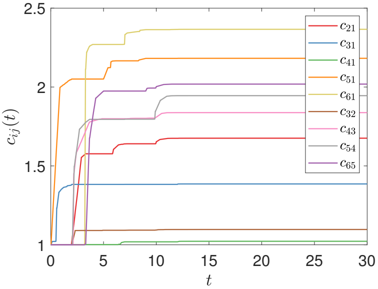


(b) Global consensus error

Example

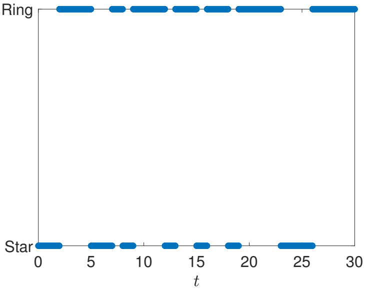


(a) Triggering instants

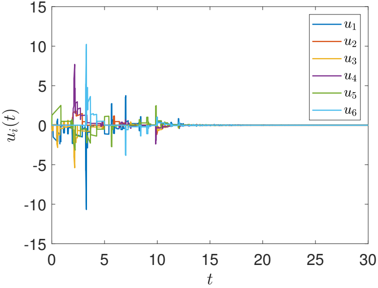


(b) Adaptive coupling gains

Example



(a) Graph switching signals



(b) Control inputs

Concluding Remarks

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- The method is fully distributed, and able to save comm. and control energy;
- Future works?

Further reading

[1]. **Dongdong Yue**, Simone Baldi, Jinde Cao, and Bart De Schutter. Distributed adaptive optimization with weight-balancing. ***IEEE Transactions on Automatic Control***, doi: 10.1109/TAC.2021.3071651, to appear.

[2]. **Dongdong Yue**, Simone Baldi, Jinde Cao, Qi Li, and Bart De Schutter. A directed spanning tree adaptive control solution to time-varying formations. ***IEEE Transactions on Control of Network Systems***, 8(2): 690-701, 2021.

[3]. **Dongdong Yue**, Jinde Cao, Qi Li, and Qingshan Liu. Neural-network-based fully distributed adaptive consensus for a class of uncertain multiagent systems. ***IEEE Transactions on Neural Networks and Learning Systems***, 32(7): 2965-2977, 2021.

[4]. **Dongdong Yue**, Qi Li, Kil To Chong, and Jinde Cao. Neural-network-embedded distributed average tracking of agents with matching unknown nonlinearities. ***Asian Journal of Control***, 23(6): 2628-2641, 2021.

[5]. **Dongdong Yue**, Jinde Cao, Qi Li, and Mahmoud Abdel-Aty. Distributed neuro-adaptive formation control for uncertain multi-agent systems: node- and edge-based designs. ***IEEE Transactions on Network Science and Engineering***, 7(4): 2656–2666, 2020.

[6]. **Dongdong Yue**, Jinde Cao, Qi Li, and Xinli Shi. Neuro-adaptive consensus strategy for a class of nonlinear time-delay multi-agent systems with an unmeasurable high-dimensional leader. ***IET Control Theory & Applications***, 13(2): 230–238, 2019.

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(b) Wenying Xu, SEU

(c) Jinde Cao, IEEE Fellow, SEU

(d) Bart De Schutter, IEEE Fellow, TUD

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Thank you for listening!

Question?