

Quantitative Strategy

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Background reading

If you have not previously used Principal Component Analysis, we would recommend reading through the following introductory materials:

Key Concepts

Rolling window	p. 2
Multivariate analysis	p. 3
Actionable insight	p. 5
Dimensionality reduction	p. 6
Stability testing	p. 8

These materials were taken from last year's Competition. Innovations put forward by competition participants included different kinds of noise reduction techniques (e.g. Multi-Scale PCA) as well as an application of the obscure "Procrustes Rotation" to **overcome factor instability**.

If you would like to see a sample application of PCA by market practitioners – various research reports can be found in the public domain. We would highlight one of Morgan Stanley's FX Pulse from a couple of years ago, available for download [here](#); please see pages 10-14.

Categorising factors

Your task in the Code in Finance programs is to characterise the factors driving currency markets right now. Generally speaking, there are two ways to do this:

- **By looking at the factors themselves**

If a particular factor attaches significant weights to emerging market currencies but no weight to currencies like the US dollar and Japanese yen, you can say that it represents emerging market risk.

- **By regressing factor moves against major benchmarks**

If a factor's moves are highly correlated with returns of the S&P 500 then it can be said to be tied to the health of the US stock market. In the Sample Analysis provided along with the Assignment, the first factor was found to be related to the MSCI Emerging Markets index.

If you are looking for an easy-to-access collection of benchmarks, we would recommend looking at 100-200 ETFs with the highest Volume (number of shares traded per day); Yahoo! Finance provides a convenient list [here](#).

The first approach is recommended for those who have macroeconomic research or trading experience.

Supplemental data

One way to stand out from other program participants is by enriching your analysis with additional time series. Yahoo! Finance is a convenient source of prices for ETFs; you can download historical prices for any US listed stock or ETF by using the following syntax:

`http://ichart.yahoo.com/table.csv?s={ticker}`

For instance, the following URL gives you a CSV with all historical prices for Apple's stock:

`http://ichart.yahoo.com/table.csv?s=AAPL`

Be sure to convert prices to returns before doing any analysis on them.

If your university gives you access to Bloomberg or Reuters, you can access a much greater variety of data there; we would particularly recommend seeing if you can access MSCI Index data. [Quandl](#) is another source of potentially useful time series, but be aware that a lot of their data sets require you to pay a subscription fee.

Word of caution

Please note that the currency data we have given you is comes from a daily reconciliation process which normally takes place at 14:15 Central European Time – right around the time US markets open. If you are evaluating relationships between daily FX returns and daily returns in US markets, you will very few significant R-squared values as the *time series are out of sync*.

The easiest way to overcome this issue is by looking at weekly rather than daily returns. For US markets in particular, you may also want to look at returns based on market open prices (9:30am for equities) rather than market close prices.

Submission instructions

Please use [this form](#) to submit a 2-3 page write-up identifying and explaining the key factors driving global currency markets right now.

Key concepts: Rolling window

Suppose we just opened up a new portfolio and invested roughly equal amounts of capital into the stocks of Google, Microsoft, and Apple. The most straightforward analysis involves looking at total portfolio value and regressing it against the S&P 500¹.

In regressing our new portfolio's returns against S&P returns over the past twelve months, we get a roughly 1:1 relationship (see Chart 1) – the Beta is roughly +1.

This regression provides a simple measure of the portfolio's exposure to macro risk (Beta vs S&P). Are we done? Nowhere near. It is important to consider the assumptions in our model and to test them.

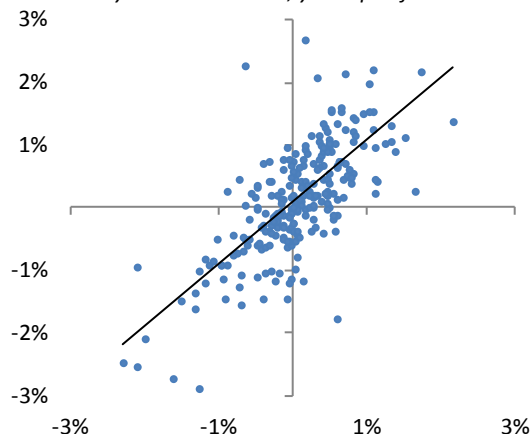
First off, consider that in running the original regression we used twelve months of returns. We have no reason to believe that these twelve months are more representative of the portfolio's likely future behavior than just the most recent three months. Perhaps the last few months have actually been anomalous – in that case, looking at a longer window (two years) would be best.

Testing model assumptions involves assessing each assumption's likely impact. To that end, let's look at what this portfolio's Beta would have been historically under 3-month, 6-month, and 12-month windows (Graph 2).

This approach is known as rolling window regression. It shows that Beta of 1 is actually a pretty good measure in the current risk environment, but realized Beta can drop as low as 0.5 and rise as high as 2.0 for several months at a time.

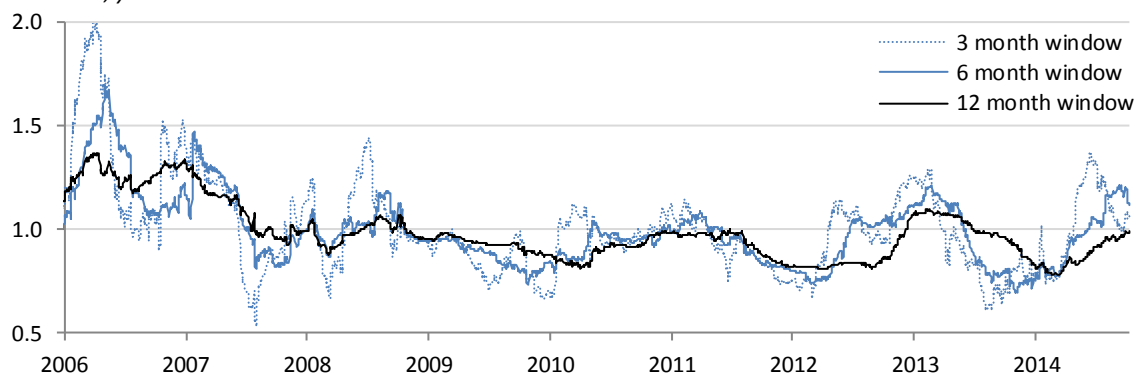
Graph 1. Basic regression

x-axis daily S&P 500 returns, y-axis portfolio returns



Graph 2. Rolling window Beta

x-axis date, y-axis Beta



Here are some other assumptions you might want to test:

- *Simple linear regression is optimal for capturing the relationship between the portfolio and the S&P 500.*
Ridge regression or LASSO regression might provide a better fit. You might also want to experiment with machine learning methods.
- *The patterns we are capturing are persistent.*
Betas for this particular portfolio are consistently quite high – but if there were periods during which it hovered around zero, we would want to look for other ways to capture systemic risk. Alternatively, we might try to come up with a regime-switching model.
- *Volatility is constant, there are no significant exogenous variables, distributions are roughly normal.*
If you don't have a strong statistics background, don't worry about these particular assumptions. But if you do, consider looking at volatility-adjusted returns, plots of residuals, and histograms of returns.

Even though the analysis we are looking at is relatively simple, this is by no means a comprehensive list.

¹ For this exercise, we first calculate what historical returns would have been if we had held the same positions we just put on today. Generating historical returns assuming fixed portfolio composition is usually the first step in doing any risk assessment – even for portfolios which have been around for a long time. If the portfolio we are looking at here had a large Tesla position a month ago which has since been unwound, the portfolio's dynamics versus the S&P a month ago has nothing to do with its likely risk profile now.

Key concepts: Multivariate analysis

It is common practice to measure a portfolio's overall risk in terms of a single Beta against a major benchmark like the S&P 500. Unfortunately a single benchmark provides limited insight into the different sources of risk in a portfolio.

Suppose you have one portfolio consisting of blue-chip stocks and another made up of small, risky stocks ("micro-caps"). The market's appetite for exposure to micro-caps would be completely ignored if all you were looking at was a single Beta measure against the S&P 500.

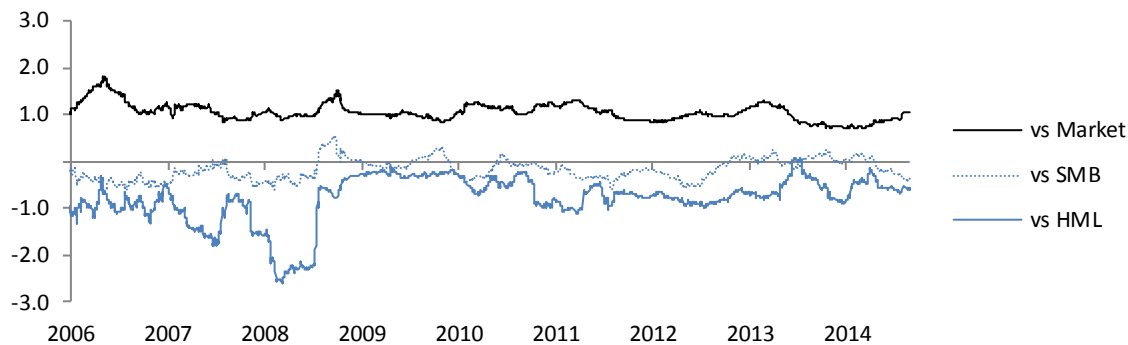
This problem was analysed in depth by Eugene Fama and Kenneth French, and they created what has come to be known as the Fama-French factor model. In addition to providing an overall market price factor (comparable to overall S&P 500 returns), they provide two additional benchmarks you can regress your portfolio against:

- **SMB (Small Minus Big)**
Performance of low market cap stocks relative to large market cap stocks. Long-term, the former tend to outperform the latter.
- **HML (High Minus Low)**
Performance of value stocks (high book-to-market) relative to growth stocks (low book-to-market). The former tend to do better.

Regressing our Google, Microsoft, and Apple portfolio against these three factors, we get the following:

Graph 3. Exposure to Fama-French factors

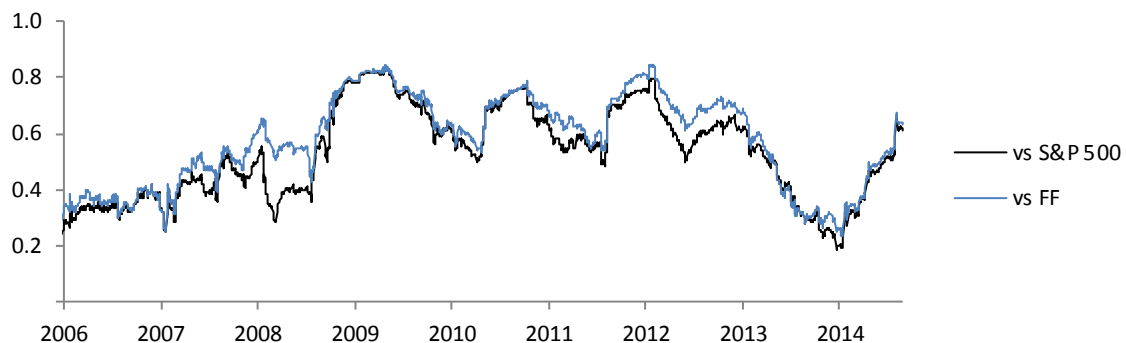
x-axis date, y-axis portfolio's 6-month Beta versus relevant Fama-French factor



Remember that we want to test every single assumption in our model. One such assumption here is that the Fama-French factor model does a better job explaining the dynamics of our portfolio than a single Beta against the S&P 500 in general, or the tech sector in particular. To test this assumption, let's look at adjusted R-squared² for these two models:

Graph 4. Fama-French performance versus univariate model

x-axis date, y-axis adjusted R-squared



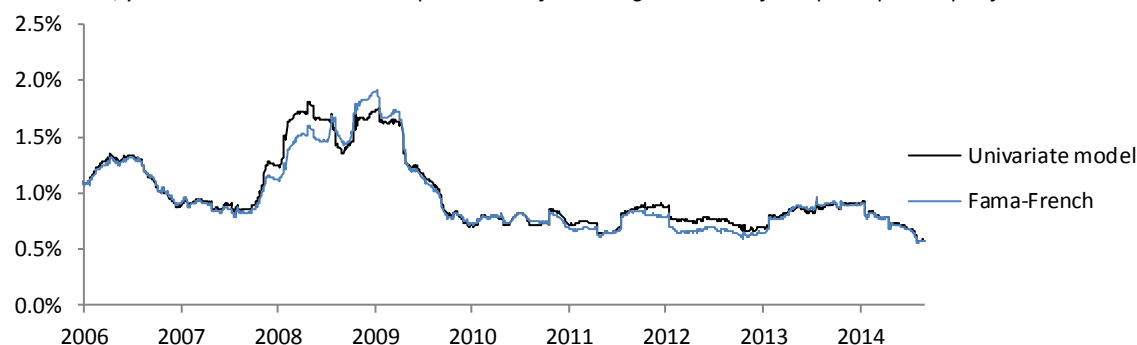
A multi-factor model seems to generally do a better job explaining our portfolio's performance than a model looking only at the S&P 500 – but only marginally. It may be tempting to use Fama-French in place of a univariate model, but be aware that the more complex a model gets, the more susceptible it becomes to "overfitting": it may be matching historical dynamics so closely that out-of-sample performance actually gets worse.

² If you are not sure what R-squared is or why we are using the adjusted R-squared – definitely look it up.

To illustrate this point, let's look at how well the Fama-French model performs versus a univariate model out-of-sample:

Graph 5. Out-of-sample performance of Fama-French

x-axis date, y-axis six month Root Mean Squared Error from using Betas out-of-sample to predict portfolio returns



In this particular case, Fama-French sometimes actually performs worse than a univariate model!

Out-of-sample testing is a very useful tool for assessing the relative value of multivariate models – but it too has limitations. If you had a dozen different models which you tested out of sample, the one that does best may not necessarily be optimal: you may be overfitting the out-of-sample data. One way to mitigate this effect is by partitioning your data into three subsets (training, validation, and testing) rather than two; another is to use recursive partitioning³. No single approach overcomes the simple fact that the past is not necessarily a good predictor of the future.

There are two takeaways from this:

- **When in doubt, go with the simpler model**
If a more complex model yields only a marginal improvement in performance, the improvement is unlikely to hold up consistently out-of-sample.
- **Model selection should be driven by intuition**
Blindly testing dozens of different models is likely to lead to overfitting. Instead of relying entirely on model performance in backtests, use your intuition as much as possible to guide model construction and selection. Which variables should you be including in your analysis? What statistical techniques should you use? Is the model likely to hold up well in a distressed environment? Answering these questions is more art than science.

Seasoned quants would tell you that failure to abide by these two maxims is a big part of the reason why so many widely-used financial models failed during the Credit Crisis of 2007-2009.

³ If you are unfamiliar with sample partitioning and recursive partitioning, Google them. Extensive documentation on both is freely available in the public domain.

Key concepts: Actionable insight

Let's try and use basic regression to analyse risk for a slightly more complex portfolio:

Table 1. Exposures breakdown for sample portfolio

Ticker	Weight in Portfolio	Betas vs	Betas vs Fama-French factors		
		S&P 500	Market	SMB	HML
ALJ	12.50%	1.31	1.01	0.63	-0.15
BBRY	12.50%	0.85	0.54	0.09	-1.13
GILD	12.50%	1.50	1.20	-0.82	-1.82
GY	12.50%	1.12	0.82	0.60	-0.19
HALO	12.50%	2.71	1.35	2.35	-2.02
KERX	12.50%	1.25	0.37	0.58	-2.56
NKTR	12.50%	2.53	1.33	1.12	-3.00
PBR	12.50%	0.53	0.61	0.19	0.77
Weighted average Beta		1.47	0.90	0.59	-1.26
Portfolio Beta		1.55	0.94	0.66	-1.33

(To make things simple, the above Betas are based on just the past six months of returns.)

Think about what you can do with these Betas:

- **Exposures versus the S&P 500**

In theory, you can put on a hedge for your overall Beta exposure by shorting an S&P 500 linked Exchange Traded Fund (ETF) like the SPY. You can also put on pair-trades – for example, to hedge out macro exposure for our MSFT holding, we can short INTL. Lastly, you can look for stocks with negative Betas and buy those to reduce overall Beta exposure.

In practice, many funds are long-only and have no interest in hedging out their Beta no matter how high it might be. Putting on pair trades requires extensive analysis on the short side of the trade; even though INTL may be a good hedge for MSFT, if it is deemed to have strong fundamentals it would be unwise to short it. Also, it can be hard to find stocks with consistently negative Betas.

- **Exposures versus Fama-French factors**

This table tells us that we are long large, growth firms. Unfortunately this yields zero actionable insight: a professional portfolio manager would not need a quantitative model to tell him something so general.

To be clear: both sets of measures do have their uses. If you are running a financial firm where dozens of traders each have complex portfolios, having basic Betas for each trader can still be very valuable; if you see that Beta versus the S&P 500 or versus one of the Fama-French factors suddenly rose for one of your traders, you would probably want to have a chat with them to find out how their market outlook has changed.

More broadly - risk can be measured versus different benchmarks (BMs), and each approach is useful in different ways:

- | | |
|------------------------|---|
| (1) Versus major BMs | Suitable for large-scale risk analysis performed for control and regulatory purposes at major institutions like bulge bracket banks. Most routine risk reports Jamie Dimon receives from JPMorgan's trading business come from forward-looking simulations referencing major global benchmarks like the S&P, FTSE, Nikkei, etc. |
| (2) Versus Fama-French | Allows for better assessment of fund performance than a single Beta versus a major benchmark. For non equity portfolios, Barra provides multiple other risk factors. These are more or less the industry standard. |
| (3) Granular factors | Bespoke risk assessment intended for use by portfolio managers at hedge funds. Factors are chosen and some cases created so as to guide position sizing, risk hedging, and even trade idea selection. |

The latter is by far the most difficult – not only because it tends to make use of far more sophisticated statistical concepts but also because it requires a practical mindset most quants lack, even among highly-paid professionals. This competition's primary purpose is to help you develop such a mindset so that you can generate actionable insight.

Key concepts: Dimensionality reduction

Put yourself into the position of a typical trader managing a reasonably diversified portfolio. As explained on the previous page, generic overall measures do not provide much actionable insight. For your risk manager's work to be useful to you, you would want them to be able to explain your risk to you in more detail.

One way to do this is by adding more benchmarks to our analysis. Suppose we have a portfolio with 20 positions and we assess its exposure to a basket of 10 different benchmarks such as sector ETFs, fixed income indices, and some commodity prices. Approaching this problem with basic regression would generate 200 Betas. We go from having too little information to having too much; we also have to contend with what is known as the "Curse of Dimensionality" – to put it simply, the Betas we would be getting are highly unreliable.

It is important to reduce the number of dimensions in our analysis. Fortunately there are quite a few dimensionality reduction techniques. Among these, the most widely used is Principal Component Analysis (PCA).⁴

Running PCA on the portfolio used on the previous page, we get the following Principal Components:

Table 2. PCA risk factors

Asset	Sector	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
ALJ	Petroleum products	0.15	0.11	-0.43	-0.29	0.28
BBRY	Phones	0.16	0.41	-0.73	0.29	-0.30
GILD	Pharma	0.18	0.11	-0.05	-0.05	0.02
GY	Aerospace, defense	0.12	-0.01	-0.03	-0.10	0.09
HALO	Pharma	0.70	-0.63	-0.12	0.25	0.11
KERX	Pharma	0.36	0.41	0.46	0.49	-0.27
NKTR	Pharma	0.53	0.36	0.20	-0.64	-0.02
PBR	Oil & gas	0.02	0.33	0.05	0.32	0.86
Variance explained		39%	14%	13%	11%	10%

The variance explained values show how significant each component is to this portfolio. Typically any Principal Component explaining over 10% of variance is deemed significant, and those explaining between 5 and 10% are worth looking at. Note that these are rules of thumb only; cutoffs should be raised for smaller portfolios and lowered for larger, more complex ones. Determining the right threshold in a given analysis requires practice.

Let's look at how these components explain the portfolio's moves during a particular day:

Table 3. Translating market moves into risk factor moves

		Moves by factor							
					Fact 1	Fact 2	Fact 3		
					-25.09%	+12.46%	+3.71%		
	Market move (4 Apr 2014)	PCA risk factors			Explained moves			Total	
		Fact1	Fact2	Fact3	Fact1	Fact2	Fact3	Explained	Residual
ALJ	-2.86%	0.15	0.11	-0.43	-3.72%	+1.32%	-1.59%	-3.98%	-1.12%
BBRY	-2.68%	0.16	0.41	-0.73	-4.02%	+5.16%	-2.72%	-1.58%	+1.10%
GILD	-2.43%	0.18	0.11	-0.05	-4.60%	+1.32%	-0.17%	-3.45%	-1.02%
GY	-2.93%	0.12	-0.01	-0.03	-3.04%	-0.11%	-0.11%	-3.26%	-0.33%
HALO	-27.26%	0.70	-0.63	-0.12	-17.62%	-7.89%	-0.44%	-25.96%	+1.31%
KERX	-4.10%	0.36	0.41	0.46	-9.08%	+5.11%	+1.72%	-2.25%	+1.85%
NKTR	-5.37%	0.53	0.36	0.20	-13.27%	+4.45%	+0.76%	-8.06%	-2.69%
PBR	+1.29%	0.02	0.33	0.05	-0.55%	+4.10%	+0.18%	+3.72%	+2.43%

On this particular day, Halozyme (HALO) fell almost 30%. Note that this fall is explained by factors 1 & 2 even though it was clearly an idiosyncratic event rather than the sum of two macro moves. The model seems to be working but is at odds with common sense.

⁴ There is a great deal of literature on PCA in the public domain, including multiple papers detailing its various applications finance. If you have not done much work with matrices, you may find some of the more technical details hard to understand. Don't worry, you don't necessarily need to understand how exactly PCA works to be able to use it.

The ability to find and address such gaps is critical to being an effective quant⁵. In this particular case, a practitioner might conclude that our model has been warped to fit an extreme outlier – at the cost of fit elsewhere in the sample. Indeed, when we eliminate this single day from our sample, we find that our risk factors are quite different from the output in Table 2:

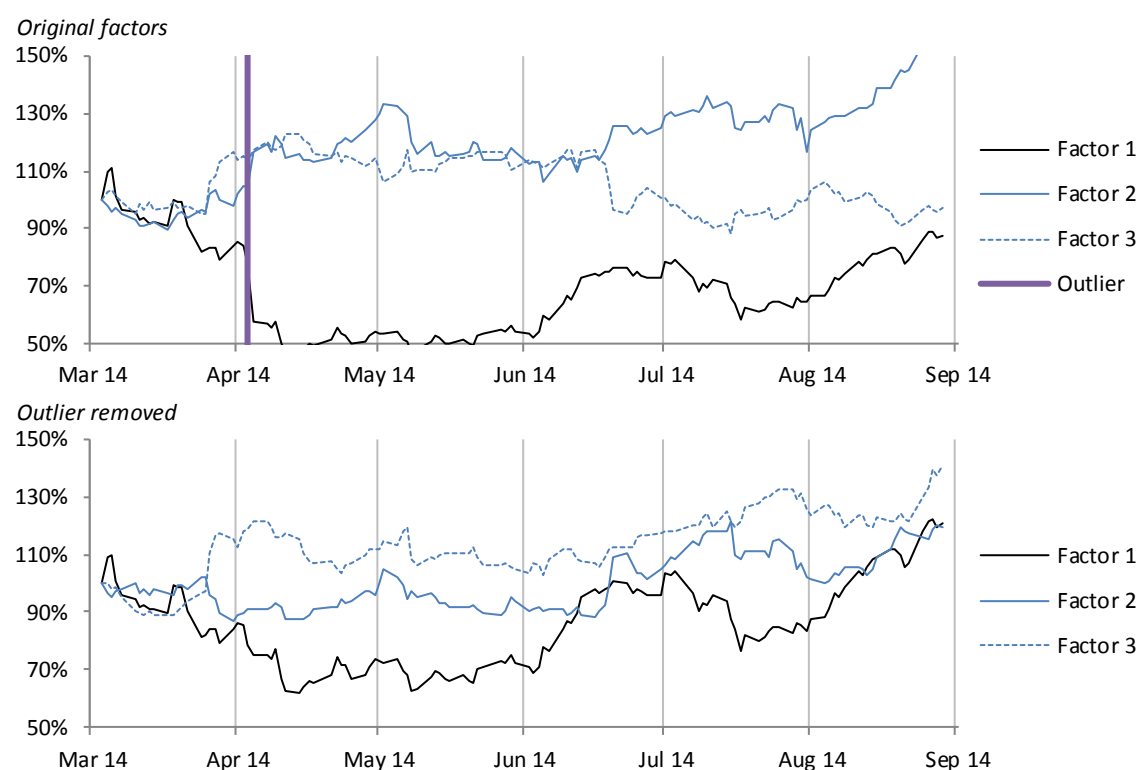
Table 4. PCA risk factors - Outlier removed

Asset	Sector	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
ALJ	Petroleum products	0.16	0.40	-0.27	-0.36	0.02
BBRY	Phones	0.18	0.86	0.11	0.37	-0.01
GILD	Pharma	0.21	0.08	-0.00	-0.05	-0.02
GY	Aerospace, defense	0.12	0.01	-0.09	-0.11	0.01
HALO	Pharma	0.54	-0.17	-0.28	0.16	0.75
KERX	Pharma	0.43	-0.20	0.68	0.34	-0.13
NKTR	Pharma	0.64	-0.10	-0.22	-0.28	-0.57
PBR	Oil & gas	0.05	0.14	0.56	-0.71	0.32
Variance explained		35%	14%	13%	12%	11%

In addition to looking at the factors themselves, you can look at how each set of factors would have explained market moves historically. Translating “moves by factor” from Table 3 into a cumulative time series, we see the following:

Graph 3. Risk factor moves

*x-axis date, y-axis cumulative moves by factor**



** Outlier detailed in Table 3 is marked in the upper chart.*

Though Factor 1 moves change little, you can see that Factors 2 and 3 behave very differently following the elimination of our outlier. This updated model can still be refined further – but we will pause here as this section is only meant to give you a general sense for how PCA works.

Be aware that it may not make sense to go through the work of building a solid PCA based model for portfolios as simple as the one being analysed here. That said, few hedge funds would ever hold so few positions; most funds have dozens and sometimes even hundreds of holdings – and in such cases, dimensionality reduction becomes essential to effective risk management.

⁵ See the Bloomberg piece entitled “Markets Rely on Too Much Mathematics” from shortly after the Financial Crisis for a few thoughts on this subject from Paul Willmott of willmott.com, home to arguably the largest professional quant community in the world.

Key concepts: Stability assessment

Think for a moment about the true risk factors underlying a portfolio rather than those we get from PCA. For PCA to do a good job retrieving those factors, each of the following requirements needs to be satisfied:

1. True risk factors in the portfolio should be consistent with PCA style framework, i.e. they must be orthogonal.
2. Our sample size must be large enough for the true risk factors to be inferred.
3. The true risk factors must be stable through time.

There is not much we can do about the first requirement. The problem with the second and third requirements is that trying to satisfy the former by growing the sample makes the latter more problematic. To keep our analysis in this section simple, we have chosen a sample size of six months so that we can test the third assumption.

To assess how stable the risk factors are, let's look at the top five principal components generated as of two separate dates (Table 5A). These look quite different. A common way to try and make them more similar is by carrying out a transformation known as a VARIMAX rotation. Without going into the nuts and bolts of how rotation works – look at how the factors it yields compare for these same two dates (Table 5B).

Table 5A – PCA factors: Raw

1 Mar '14 to 1 Sep '14

	1	2	3	4	5
ALJ	0.16	0.40	-0.27	-0.36	0.02
BBRY	0.18	0.86	0.11	0.37	-0.01
GILD	0.21	0.08	0.00	-0.05	-0.02
GY	0.12	0.01	-0.09	-0.11	0.01
HALO	0.54	-0.17	-0.28	0.16	0.75
KERX	0.43	-0.20	0.68	0.34	-0.13
NKTR	0.64	-0.10	-0.22	-0.28	-0.57
PBR	0.05	0.14	0.56	-0.71	0.32

1 Sep '10 to 1 Mar '11

	1	2	3	4	5
ALJ	0.57	-0.73	-0.32	0.10	0.09
BBRY	0.26	0.20	-0.11	-0.10	-0.80
GILD	0.17	-0.01	0.08	0.00	-0.09
GY	0.40	0.27	0.39	0.64	0.22
HALO	0.32	0.15	0.29	0.05	-0.02
KERX	0.34	0.57	-0.65	-0.16	0.33
NKTR	0.40	0.02	0.46	-0.71	0.12
PBR	0.16	0.05	-0.08	0.21	-0.41

Table 5B - PCA factors: Rotated

1 Mar '14 to 1 Sep '14

	1	2	3	4	5
ALJ	0.32	0.23	-0.45	-0.18	0.03
BBRY	0.00	0.96	0.01	0.04	-0.02
GILD	0.20	0.09	0.03	-0.05	0.06
GY	0.15	-0.02	-0.08	-0.04	0.07
HALO	0.09	-0.01	0.01	0.02	0.99
KERX	0.11	0.11	0.89	-0.09	0.02
NKTR	0.90	-0.11	0.06	0.13	-0.14
PBR	0.04	-0.03	0.01	-0.97	-0.01

1 Sep '10 to 1 Mar '11

	1	2	3	4	5
ALJ	-0.01	-0.99	0.00	-0.01	-0.03
BBRY	-0.09	0.07	-0.04	-0.12	-0.87
GILD	0.10	-0.07	0.03	-0.10	-0.13
GY	0.91	0.00	-0.01	0.11	0.03
HALO	0.38	0.02	0.00	-0.25	-0.11
KERX	0.00	0.00	-1.00	0.01	0.02
NKTR	0.03	0.00	0.00	-0.94	0.08
PBR	0.10	-0.05	0.03	0.14	-0.46

Instability is evident: it is hard to find two factors which are alike. This is the core problem affecting PCA-based risk management models. Intuition built through experience allows for the incorporation of tweaks and workarounds which can still make such models useful – and, indeed, standard for many hedge funds.