HW1

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1

P = 1/4
$C^5_5(\frac{1}{4})^5 + c^5_4(\frac{1}{4})^4(\frac{3}{4}) = 0.015625$

2

Define X represents that how many games needed until the series ends
P(X=4) $= 2(\frac{1}{2})^4 = \frac{2}{16}$
P(X=5) $= 2C^4_3(\frac{1}{2})^4(\frac{1}{2}) = \frac{2}{8}$
P(X=6) $= 2C^5_3(\frac{1}{2})^5(\frac{1}{2}) = \frac{5}{16}$
P(X=7) $= 2C^6_3(\frac{1}{2})^6(\frac{1}{2}) = \frac{5}{16}$
E(X) $= 4\frac{1}{8} + 5\frac{1}{4} + 6\frac{5}{16} + 7\frac{5}{16} = 5.8125$

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$X \sim U(0, 80)$
$f(X) = \frac{1}{80}, 0 < x < 80$
$P(X < 20) = \int_0^{20} \frac{1}{80}dx = \frac{1}{4}$

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$X1 \sim Exp(\frac{1}{\mu_1})$
$X2 \sim Exp(\frac{1}{\mu_2})$
$f_{X1}(x_1) = \mu_1 e^{-\mu_1 x_1}$
$f_{X2}(x_2) = \mu_2 e^{-\mu_2 x_2}$
$f_{X1X2}(x_1, x_2) = \mu_1\mu_2 e^{-\mu_1 x_1 - \mu_2 x_2}, x_1 > 0, x_2 > 0$

(a)

$P(X_1 > 10) = 1 - P(X_1 <= 10) = 1 - \int_0^{10} \mu_1 e^{-\mu_1 x_1} dx_1 = e^{-10\mu_1}$
(b)
$P(X_2 < X_1) = \int_0^{\infty} \int_0^{x_2} \mu_1 \mu_2 e^{-\mu_1 x_1 - \mu_2 x_2} dx_1 dx_2$
$= \mu_1 \mu_2 \int_0^{\infty} \int_0^{x_2} e^{-\mu_1 x_1 - \mu_2 x_2} dx_1 dx_2$
$= \mu_2 \int_0^{\infty} e^{-x_2(\mu_1+\mu_2)} dx_2$
$= \frac{\mu_2}{\mu_1+\mu_2}$

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$Define\ X\ is\ how\ much\ time\ that\ train\ delayed$
$Define\ Y\ is\ the\ arrival\ time\ that\ bus\ arrived\ station$
$Y \sim N(\mu = 20, \sigma = 2)$
$P(late\ for\ work)$
$= P(Train\ delayed\ 12\ minutes) + P(The\ bus\ arrival\ time\ over\ 8:20)$
$= \frac{15}{16} \times P(Y > 20) + P(X = 12)$
$= \frac{15}{16} \times P(Z > 0) + \frac{1}{16} = 0.53125$

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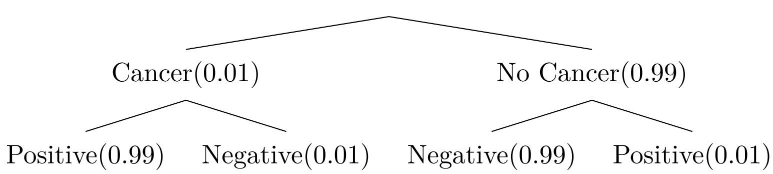
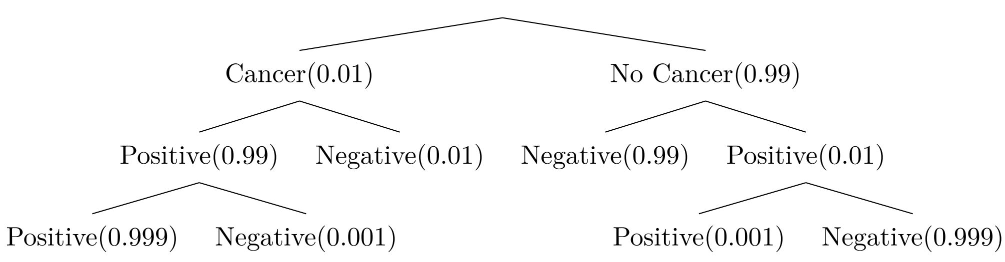


(a)
$P(Cancer|Positive) = \frac{0.01 \times 0.99}{0.01 \times 0.99 + 0.99 \times 0.01} = 0.5$



(b)
$P(Cancer|Postive) = \frac{0.0099 \times 0.999}{0.0099 \times 0.999 + 0.0099 \times 0.001} = 0.999$

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$X_1, X_2, \ldots, X_{49} \overset{iid}{\sim} (\mu = 8.5, \sigma = 3.5)$
$\bar{X} \xrightarrow[CLT]{a} \mathrm{N}(\mu = 8.5, \frac{\sigma^2}{n} = \frac{3.5^2}{49})$
(a)
$P(\bar{X} < 10) = P(Z < \frac{10-8.5}{\sqrt{\frac{3.5^2}{49}}})$
$= P(Z < 3) \simeq 0.99865$
(b)
$P(7 < \bar{X} < 10) = P(-3 < X < 3) \simeq 0.9973$
(c)
$P(X < 7.5) = P(Z < -2) = 0.02275$

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$\{^{H_0:p=0.4}_{H_1:p\neq 0.4}$
$Z_{\frac{\alpha}{2}}\sqrt{\frac{0.4\times 0.6}{600}} = 0.04$
$\Longrightarrow Z_{\frac{\alpha}{2}} = 2$
$\Longrightarrow \frac{\alpha}{2} = 0.0228$
$\alpha = 0.0456$

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X	0	1	2	3
O_i	39	23	12	1
p_i	0.5086	0.3488	0.1146	0.0280
E_i	38	26	9	2

H_0 ： 資料抽自二項分配 H_1 ： 資料非抽自二項分配
$\chi^2 = \sum_{i=1}^{4} \frac{(O_i - E_i)^2}{E_i} \overset{H_0}{\sim} \chi^2(2)$
$RR : \{\chi^2 >= \chi^2_{0.05}(2) = 5.991\}$
$\chi^2_0 = \sum_{i=1}^{3} \frac{O_i^2}{E_i} - 75 = 1.87$
$\chi^2_0 \notin RR \Longrightarrow do\ not\ reject\ H_0$

$There\,is\,no\,significant\,proof\,to\,reject\,that\,the\,data\,follow\,Binomial\,distribution$

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$\{H_0 : OR\ and\ Prob\ are\ independent\ ; H_1 : OR\ and\ Prob\ are\ not\ independent$

$$R_1 = 24 + 11 + 10 = 45$$
$$\mathrm{R}_2 = 7 + 13 + 5 = 25$$
$$\mathrm{R}_3 = 4 + 6 + 20 = 30$$

$$\mathrm{C}_1 = 24 + 7 + 4 = 35$$
$$\mathrm{C}_2 = 11 + 13 + 6 = 30$$
$$\mathrm{C}_3 = 10 + 5 + 20 = 35$$

E_{ij}	C_1	C_2	C_3
R_1	15.75	13.5	15.75
R_3	8.75	7.5	8.75
R_3	10.5	9	10.5

$\chi^2 = \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{(O_{ij}-E_{ij})^2}{E_ij} \overset{H_0}{\sim} \chi^2(4)$
$RR : \{\chi^2 >= \chi^2_{0.01}(4) = 13.277\}$
$\chi^2_0 = 26.49 \in RR \Longrightarrow reject\ H_0$
$The\ grades\ in\ Probability\ and\ OR\ exist\ significant\ relationship\ .At\ \alpha = 0.01\ significant\ level.$