Introduction to Optimization

Term Project Final Report

Mixed integer linear programming problem for personnel multi-day shift scheduling: A case study in Taipei Veterans General Hospital

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Introduction

For company, labor cost accounts for big amounts of business costs. Especially for the companies in high developed countries, their personnel costs are way much higher than those in low developed countries. Therefore, efficiently reducing personnel cost is the top issue for companies to deal with. Over the years, the coronavirus has had a tremendous impact on the whole world, the demand for medical care has increased dramatically which results in the serious shortage of medical resource and thus the problem of personnel scheduling for every hospital becomes more complicated. Overall, it becomes the first and foremost issue for each hospital to maintain sufficient capability of medical care on the condition that the personnel cost is minimized.

When it comes to personnel scheduling, there is a lot of limitation that makes personnel scheduling not as trivial as we think. For instance, Ministry of Labor has established laws, such as maximum working hours per day are eight hours and if overtime work is inevitable, company must pay overtime and overtime working hours are at most four hours, that avoid exploitation of labors. Furthermore, with the rising awareness of labor right, workers now pay more attention to their working environments and their compensation package. Thus, leaders must take these things into account when performing personnel scheduling.

Under these conditions, the most important purpose of personnel scheduling is to allocate jobs to employees efficiently with the goal of minimizing the total personnel cost. As far as hospital management is concerned, improper staffing will lead to terrible hospital performance, for example, patients may have to wait for a long time until his or her turn for consultation. Additionally, inappropriate staffing will bring about unsatisfaction for staff which may result in workers going on strike if things get worse and it will subsequently generate extra cost because companies or institutions have to spend more time dealing with it. As a result, we have to do personnel scheduling in hospital more carefully. In this report, we tried to optimize schedule homogenous nurse’s shift, taking the number of staff required per shift into consideration to obey labor laws and minimize the costs of staff payroll.

In this term project, we considered a case study in Taipei Veterans General Hospital to solve the personnel scheduling problem in department of nursing in division of oncology. Differing from the original personnel schedule design, the model in this research was modified to cater to the circumstances in Taiwan, and then we tried to find out whether this model is helpful to reduce the total labor cost. Consequently, we exactly reduce the total cost by approximate6 percent. The detailed information was presented int he following sections.

The model formulation

In this model of personnel scheduling, we will first state the assumptions used. Then, we will define parameters and decision variables. Lastly, we will formulate the whole model.

Assumptions

- The total number of personnel to get scheduled is determined : 22
- The hourly wages of any staff member are known :180
- All staff are homogeneous, meaning they have relatively same skills :division of oncology
- The total number of staff required per shifts of each day is determined
- There are four types of shifts per day
  —Morning (D) from 8 to 14, 6h;
  —Evening(G) from 14 to 20, 6h;
  —Morning and evening(L) from 8 to 20, 12h;
  —Overnight(N) from 20 to 8, 12h;
- Personnel who work 12-hour shifts must rest the following day
- The overtime, holidays and overnight payroll costs is determined
- The overnight working hours is 12pm to 8am
- Each staff member must work 144 hours per month and can be assigned to one shift from the above four types
- The number of workdays in each period is 30 days and the holidays are predetermined from the calendar
- Total and per shift maximum overtime is specified
- No staff can work 24h a day

Parameters

The indices *i* and *j* represent i^{th}staff and j^{th}workday

m : Total number of staff

r : Total number of workdays

T_j: Set of off days

S_i: Monthly payroll of staff *i*, including base pay and all other benefits

C_i: Payroll of 1-hour work of staff I, yields by dividing the base pay minimum working time of staff (S_i/W)

W: Minimum working time of each staff

V: Maximum working time of each staff

F: Maximum day that L-type staff can work in the morning and/or evening shifts as overtime

B: Maximum day that D/G-type staff can work in the morning and/or evening shifts as overtime

x: Maximum day that D/G-type staff can work in the night shifts as overtime

f: Maximum day that N-type staff can work in the morning and/or evening shifts as overtime

Ej: Minimum number of staff needed in the morning for j day

Oj: Minimum number of staff needed in the evening for j day

Pj: Minimum number of staff needed in the night for j day

α: Percentage added to base salary for working on overtime or holiday

β: Percentage added to base salary for working on overnight

Decision variables

Di: Binary variable determines if staff i works on the morning shift or not

Gi: Binary variable determines if staff i works on the evening shift or not

Li: Binary variable determines if staff i works on the morning and evening shift or not

Ni: Binary variable determines if staff i works on the night shift or not

dij: Binary variable determines if staff i works on the morning shift of day j or not

gij: Binary variable determines if staff i works on the evening shift of day j or not

lij: Binary variable determines if staff i works on the morning and evening shift of day j or not

nij: Binary variable determines if staff i works on the night shift of day j or not

Ui: Total working hours of staff i

tij: Total off-day working hours of staff i on day j

ngij: Binary variable determines if G-type staff i works on the night shift of day j or not

gnij: Binary variable determines if N-type staff i works on the evening shift of day j or not

glij: Binary variable determines if L-type staff i works on the evening shift of day j or not

gdij: Binary variable determines if D-type staff i works on the evening shift of day j or not

ndij: Binary variable determines if D-type staff i works on the night shift of day j or not

dnij: Binary variable determines if N-type staff i works on the morning shift of day j or not

dlij: Binary variable determines if L-type staff i works on the morning shift of day j or not

dgij: Binary variable determines if G-type staff i works on the morning shift of day j or not

The proposed mathematical model with respect to the mentioned variables is as follows:

Min Z =

$$\sum_{i=1}^{m}\left\{\begin{matrix}[\mathrm{Si}(\mathrm{Di}+\mathrm{Gi}+\mathrm{Li}+\mathrm{Ni})]+\ [\alpha\mathrm{Ci}(\mathrm{Ui}-\mathrm{W})]+\ \left(\alpha\mathrm{Cij}\sum_{j=1}^{r}\mathrm{tij}\right)\\ +\left[\beta\mathrm{Ci}\sum_{j=1}^{r}(\mathrm{ndij}+\mathrm{ngij}+\mathrm{nij})\right]\end{matrix}\right\} \quad (1)$$

S.t:

$\mathrm{Di}+\mathrm{Gi}+\mathrm{Li}+\mathrm{Ni}=1;\ \forall\mathrm{i}$ (2)

$\mathrm{dij}\geq 24\mathrm{Di};\ \forall\mathrm{i},\mathrm{j}$ (3)

$\mathrm{dij}\leq\mathrm{Di};\ \forall\mathrm{i},\mathrm{j}$ (4)

$\mathrm{gij}\geq 24\mathrm{Gi};\ \forall\mathrm{i},\mathrm{j}$ (5)

$gij\leq Gi;\forall i,j$ (6)

$lij\geq 12Li;\forall i,j$ (7)

$lij\leq Li;\forall i,j$ (8)

$lij+li(j+1)\leq Li;\forall i,j=1.2\dots r$ (9)

$nij\geq 12Ni;\forall i,j$ (10)

$nij\leq Ni;\forall i,j$ (11)

$nij+ni(j+1)\leq Ni;\forall i,j=1.2\dots r$ (12)

$ngij\leq Gix;\forall i,j$ (13)

$gnij\leq Nif;\forall i,j$ (14)

$nij+gni(j+1)\leq Ni;\forall i,j=1.2\dots r$ (15)

$ndij\leq Dix;\forall i,j$ (16)

$dnij\leq Nf;\forall i,j$ (17)

$nij+dnij\leq Ni;\forall i,j$ (18)

$gdij\leq DiB;\forall i,j$ (19)

$dgij\leq GiB;\forall i,j$ (20)

$dlij\leq LiF;\forall i,j$ (21)

$dlij+lij\leq Li;\forall i,j$ (22)

$glij\leq LiF;\forall i,j$ (23)

$glij+lij\leq Li;\forall i,j$ (24)

$\sum_{i=1}^{m}(dij+dnij+dlij+dgij+lij)\geq Ej;\forall j$ (25)

$\sum_{i=1}^{m}(gij+gnij+glij+gdij+lij)\geq Oj;\forall j$ (26)

$\sum_{i=1}^{m}(nij+ngij+ndij)\geq Qj;\forall j$ (27)

$$Ui=\sum_{j=1}^{r}\left\{\begin{matrix}6(dij+dnij+dlij+dgij+gij+gnij+glij+gdij)\\ +12(lij+nij+ngij+ndij)\end{matrix}\right\};\forall i \quad (28)$$

$$W \leq Ui \leq V \tag{29}$$

$$tij = 6(dij + dnij + dlij + dgij + gij + gnij + glij + gdij)$$
$$+12(ngij + ndij); \forall i \in m.j \in Tj \tag{30}$$

$$\sum_{i=1}^{m} Di \ \geq P1 \tag{31}$$

$$\sum_{i=1}^{m} Gi \ \geq P2 \tag{32}$$

$$\sum_{i=1}^{m} Li \ \geq P3 \tag{33}$$

$$\sum_{i=1}^{m} Ni \ \geq P4 \tag{34}$$

$$\sum_{j=1}^{23} ((dij + di(j+1) + di(j+2) + di(j+3) + di(j+4) + di(j+5) + di(j+6)) \ \leq 6Di; \forall i \tag{35}$$

$$\sum_{j=1}^{23} ((gij + gi(j+1) + gi(j+2) + gi(j+3) + gi(j+4) + gi(j+5) + gi(j+6)) \ \leq 6Gi; \forall i \tag{36}$$

$$Di, Gi, Li, Ni \in \{0,1\}; \forall i \tag{37}$$

$$dij, gij, lij, nij, ngij, ndij, gnij, dnij, glij, dlij, gdij, dgij \in \{0,1\}; \forall i, j \tag{38}$$

$$Uij, tij \ \geq 0; \ \forall i, j \tag{39}$$

The objective function (1) is regarding minimization of the total cost of payrolls which consists of base pay, benefits, overtime, and overnight payments. Constraint (2) ensures that each staff member is assigned exactly one shift. Constraints (3) and (5) and (7) and (10) ensure that staff meets the minimum working time. If that staff work in morning or evening shifts, they must work at least 24daysin a month for this shift. If that staff work in morning and evening or night shifts, they must work at least 12 days. Constraints (4) and (6) and (8) and (11) ensure that staff who selected one of four type of shifts work at their selected shifts. Constraints (9) and (12) ensure that staff who worked at 12-hours shifts should be off the next day to obey the 12-hours work 24-hours off rule. Constraint (13) states that staff with evening shifts may work up to x night(s) as overtime. Constraint (14) states that staff worked in night shift may work up to f evening shift(s) as overtime. To meet staff's satisfaction, Constraints (15) and (18) ensure the overtime evening or morning shift for the night-shift staff must be the same shift before that night. Constraint (16) states that staff with morning shift may work up to x night(s) as overtime. Constraint (17) states that staff with night shift may work up to f morning shift(s) as overtime. Constraint (19) states that staff with morning shifts may work up to B evening shift(s) as overtime. Constraint (20) states that staff with evening shifts may work up to B morning shift(s) as overtime. Constraint (21) states that staff with morning and evening (L) shifts may work up to F days of morning shift(s) as overtime. Constraint (22) ensures that staff with morning and evening (L) shift just work at morning shift as overtime in the days which are not assigned to them as L shifts. Constraint (23) states that staff with morning and evening (L) shifts may work up to F days of evening shift(s) as overtime.

Constraint (24) ensures that staff with morning and evening (L) shift just work at evening shift as overtime in the days which are not assigned to them as L shifts. Constraint (25) and (26) and (27) ensure the number of required staff meet nurse-patient ratios in the morning, evening, and night, respectively. Equation (28) represents the total working hours of each staff during a month. Constraint (29) ensures compliance with the minimum and maximum limits for working hours for each staff. Equation (30) represents the total working hours of each staff during off days in a month. Constraint (31) and (32) and (33) and (34) ensure there are enough nurses in each shift. Constraints (35) and (36) ensure that staff with morning or evening shift won't work 7 consecutive days. Binary variables are stated in equations (37) and (38), and continuous variables are stated in (39).

Table 1 Model Parameters

α	β	X	f	B	F	P1	P2	P3	P4
0.33	0.53	4	7	7	4	10	3	3	6

Result

As the model we indicated previously, this section provides the results. The computational experiment is conducted on GUROBI 9.1.0 optimization software, running on Intel i5, 3.2 gigahertz, 64-bit processor with 16 gigabyte RAM and Windows 10 OS.

In this case study, a number of 22 nurses indivision of oncology with different salaries and benefits are analyzed for a 30 days' period (from January 1, 2022 to January 30,2022). In this period, there are 2 off days in the calendar in Taiwan, which are days 1 and 30. The required number of staff in workdays for morning, evening and night shifts are 5, 2 and 3, respectively.

Maximum possible overtime working hours of personnel are set as 46 hours which complied with Labor Standard Act in Taiwan. Other information for the model is illustrated in table 1, and we obtained detailed information of the division of Oncology of local hospital from the nurse who works there, such like salaries, benefits and overtime payment etc.

We obtained an optimal solution value of 1,673,573 NTD for the scheduled period. The schedule for each 22 nurses is illustrated in Table 2. This solution reduced the total cost of payment by 5.9 percent ($106,427), compared with the original scheduling conducted by the division which we research.

Table 2

欄1	Shift	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	D		D	D	D	D	D	D		D	D	D	D	D		D	D	D		D	D	D	D	D		D	D	D	D	D	
2	G		G	G	G	G	G		G	G	G	G	G		G	G	G	G	G		G	G	G	G		G	G	G	G	G	
3	D		D		D	D	D	D	D		D	D	D	D	D	D		D	D	D	D	D	D		D	D	D	D	D	D	
4	D		D	D	D	D		D	D	D	D		D	D	D	D		D	D	D	D	D	D		D	D	D	D	D	D	
5	D		D	D	D	D		D	D	D	D		D	D	D	D	D	D		D	D	D	D	D	D		D	D	D	D	
6	G		G	G	G		G	G	G		G	G	G	G	G	G		G	G	G	G	G	G		G	G	G	G	G	G	
7	D		D	D	D	D	D	D		D	D	D	D	D	D		D	D	D	D	D	D			D	D	D	D	D	D	
8	D		D	D	D	D	D		D	D	D	D	D	D		D	D	D	D	D		D	D		D	D	D	D	D	D	
9	L		L		L		L		L		L		L		L		L		L			L			L				L		
10	G		G	G	G	G	G		G	G		G	G	G	G	G		G	G	G	G	G	G		G	G	G	G	G	G	
11	D		D	D	D		D	D	D	D	D	D		D	D	D	D	D		D	D	D	D	D		D	D	D	D	D	
12	N		N		N		N		N		N			N		N			N			N			N			N		N	
13	N			N				N		N		N		N		N		N			N		N		N		N		N		
14	N		N				N		N		N		N		N		N		N		N		N			N			N		
15	L	L		L		L		L			L			L					L			L			L		L		L		L
16	N		N			N		N		N		N		N		N		N		N		N		N		N		N		N	
17	N			N		N		N		N		N			N		N			N		N		N		N		N		N	
18	L	L		L					L					L		L		L		L		L		L			L		L		L
19	D	D	D	D	N	D	D	D	D	D			D N	D	D	D	D			D N	D	D	D	D N	D		D	D	D	D	D
20	N	N		N		N			N		N		N		N		N		N		N		N		N		N		N		N
21	D	D N	D		D N	D	N	D	D	D	D		D	D	D	D	D		D		D	D	D		D	D	D	D	D	D	D N
22	D	D N	D		D	D		D	D		D	D	D	D	D	D		D N	D	D	D	D	D		D		D N	D	D	D	D N

Discussion and Analysis

There are three fixed shifts for every month, which are morning shift (8 a.m.-4 p.m.), night shift (4 p.m.-12 a.m.) and midnight shift (12 a.m.-8 a.m.), in Taipei Veterans General Hospital. In this research, we designed four fixed shifts, morning shift (8 a.m.-2 p.m.), evening shift (2 p.m.-8 p.m.), morning and evening shift (8 a.m.-8 p.m.), and night shift (8 p.m.-8 a.m.). In this four-fixed shift design, we added an additional shift and adjusted working hours of each shift. Furthermore, we made a special arrangement that nurses in each fixed shift can assist in other fixed shifts as overtime. Comparing with the shifts in Taipei Veterans General Hospital, we not only provided more off days for nurses who work in the night shift, but also allow nurses who work in shifts which working hours are six hours to provide assistance for night shift.

In terms of outcome, our design provided more flexibility in personnel scheduling. As far as total cost is concerned, we slightly decreased the cost of personnel cost. However, some factors are not taken into account in this design. According to the shift schedule, some nurses in the six-hour shifts need to work for consecutive days which is not permitted by Labor Standard Act. Additionally, some nurses have to work continuously over twelve hours which also violates the law.

Overall, with regard to the setting of parameters, although we make some adjustment to comply with Labor Standard Act, there are some constraints that we didn’t modify specifically to meet the rules. In the pre-assumption of the model, we assumed that each nurse is equipped with same skills, and didn’t adjust the base salaries and compensation with respect to job tenure. For one thing, we didn’t have sufficient information on hand. For another, we would like to maintain the simplicity with regard to calculation. In conclusion, we will increase the feasibility of the model if we improve these drawbacks and thus make it accessible for the division of hospital.

Conclusion

In recent years, with the increasing variable costs of the healthcare systems and medical centers, one of the most important difficulties for hospital managers is to optimally schedule staff shifts so that minimum costs are provided while following local labor laws. There are several constraints and limitations are taken into considerations, such as the required number of staff on each shift, available personnel, labor laws, personnel physical health, etc. Hence, it is important to have a comprehensive mathematical model which is able to formulate all these constraints and factors. In this study, a mixed-integer linear programming model is presented for homogeneous personnel shift scheduling. In this mathematical model, staff salaries and benefits, staff number, required number of personnel per shift, hospital and legislative rules are given as the input of the model. Then, we put this model into practice- case study problem in a division of Taipei Veterans General Hospital with 22 personnel. The results suggest a decrease in the costs by approximately 6 percent in comparison with the current schedule. Although we omit some factors in modeling this problem, we still believe that our result has certain reference value. Interestingly, we find that the nurses assigned to do overtime shift are all junior nurses. It really confirms to real workforce in Taiwan. Considering this result, we can conclude that if this model is practiced on a larger scale, with the higher number of staff, the resulting cost reduction would be much more significant. Additionally, it may be helpful for schedulers to set shift schedules efficiently and conveniently. For further researches, this problem can also be modeled by considering heterogeneous personnel (with different skills). On the other hand, we could also include other factors into the model, such as considering binary variables for hiring and firing staff in each period. Finally, we believe that this model is useful for hospital to reduce the labor cost. However, we can’t guarantee that the nurses will not be so exhausted as before.

Reference

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