

# 数值代数作业 C4

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## 1 第一题

在 SOR 中选取  $\omega = 1.7$ , 当  $\epsilon = 0.0001$  时选为 0.9, 迭代次数 Jacobi>G-S>SOR。随着  $\epsilon$  的减小, 迭代次数减少, 虽然计算所得误差减小, 但与理论值误差增大, 尤其可观察到在  $\epsilon = 0.0001$  时,  $y$  在 0 附近的值接近 0.5。

### 1.1 $\epsilon=1$

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epsilon为1时, Jacobi迭代法求得矩阵的解为
0.0128542 0.0256307 0.0383301 0.0509533 0.0635011 0.0759741 0.0883731 0.100699 0.112952 0.125134
0.137244 0.149284 0.161254 0.173155 0.184988 0.196753 0.208452 0.220084 0.23165 0.243151
0.254588 0.265961 0.277271 0.288519 0.299705 0.310829 0.321893 0.332897 0.343841 0.354727
0.365554 0.376324 0.387036 0.397692 0.408292 0.418836 0.429326 0.439761 0.450142 0.460471
0.470746 0.480969 0.49114 0.501261 0.51133 0.52135 0.531319 0.54124 0.551111 0.560935
0.570711 0.580439 0.590121 0.599756 0.609346 0.61889 0.628388 0.637843 0.647253 0.65662
0.665943 0.675224 0.684462 0.693659 0.702813 0.711927 0.721 0.730033 0.739025 0.747978
0.756892 0.765763 0.774604 0.783403 0.792165 0.800889 0.809576 0.818226 0.826841 0.83542
0.843963 0.852471 0.860945 0.869384 0.877789 0.88616 0.894498 0.902803 0.911075 0.919315
0.927323 0.935699 0.943843 0.951957 0.960039 0.968091 0.976113 0.984105 0.992067
每一项与精确解的误差为
-1.62512e-05 -3.19543e-05 -4.71187e-05 -6.17529e-05 -7.58667e-05 -8.94676e-05 -0.000102566 -0.000115168 -0.000127285 -0.000138923
-0.000150091 -0.000160797 -0.00017105 -0.000180855 -0.000190223 -0.000199159 -0.000207674 -0.00021577 -0.000223461 -0.000230747
-0.000237642 -0.000244148 -0.000250275 -0.000256027 -0.000261414 -0.000266439 -0.000271113 -0.000275437 -0.000279424 -0.000283073
-0.000286396 -0.000289394 -0.000292079 -0.000294451 -0.000296521 -0.000298229 -0.000299769 -0.000300957 -0.000301566 -0.000302495
-0.000302857 -0.000303249 -0.000303678 -0.000304141 -0.000304639 -0.000305169 -0.000305737 -0.000306341 -0.000306981 -0.000307647
-0.000292668 -0.000290376 -0.000287875 -0.000285162 -0.000282249 -0.000279133 -0.000275825 -0.000272323 -0.000268637 -0.000264766
-0.000260718 -0.000256493 -0.000252099 -0.000247536 -0.000242812 -0.000237925 -0.000232886 -0.000227691 -0.00022235 -0.000216861
-0.000211233 -0.000205464 -0.000199562 -0.000193525 -0.000187363 -0.000181073 -0.000174662 -0.000168131 -0.000161485 -0.000154725
-0.000147855 -0.000140878 -0.000133797 -0.000126613 -0.000119331 -0.000111953 -0.000104482 -9.6919e-05 -8.92691e-05 -8.15327e-05
-7.3714e-05 -6.58141e-05 -5.78366e-05 -4.97828e-05 -4.1656e-05 -3.34578e-05 -2.51911e-05 -1.68576e-05 -8.45995e-06
迭代次数为24630
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图 1: Jacobi,epsilon=1

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epsilon为1时，G-S迭代法求得矩阵的解为
0.0128543 0.0256308 0.0383303 0.0509535 0.0635013 0.0759744 0.0883735 0.100699 0.112953 0.125134
0.137244 0.149284 0.161255 0.173156 0.184989 0.196754 0.208452 0.220084 0.231651 0.243152
0.254589 0.265962 0.277272 0.28852 0.299706 0.31083 0.321894 0.332898 0.343842 0.354728
0.365555 0.376325 0.387037 0.397693 0.408293 0.418837 0.429327 0.439762 0.450144 0.460472
0.470747 0.48097 0.491142 0.501262 0.511332 0.521351 0.531321 0.541241 0.551113 0.560936
0.570712 0.58044 0.590122 0.599757 0.609347 0.618891 0.62839 0.637844 0.647254 0.656621
0.665945 0.675225 0.684463 0.69366 0.702814 0.711928 0.721001 0.730034 0.739026 0.747979
0.756893 0.765768 0.774605 0.783404 0.792165 0.800889 0.809577 0.818227 0.826842 0.83542
0.843964 0.852472 0.860945 0.869384 0.877789 0.886161 0.894499 0.902803 0.911076 0.919315
0.927523 0.935699 0.943843 0.951957 0.960039 0.968091 0.976113 0.984105 0.992067
每一项与精确解的误差为
-1.62006e-05 -3.18537e-05 -4.69683e-05 -6.15538e-05 -7.56188e-05 -8.91724e-05 -0.000102223 -0.00011478 -0.000126851 -0.000138444
-0.000149568 -0.000160232 -0.000170442 -0.000180207 -0.000189535 -0.000198432 -0.000206908 -0.000214969 -0.000222623 -0.000229876
-0.000236737 -0.000243211 -0.000249307 -0.00025503 -0.000260388 -0.000265387 -0.000270033 -0.000274334 -0.000278296 -0.000281924
-0.000285226 -0.000288206 -0.000290872 -0.000293229 -0.000295283 -0.000297039 -0.000298504 -0.000299682 -0.00030058 -0.000301203
-0.000301566 -0.000301645 -0.000301474 -0.000301049 -0.000300375 -0.000299457 -0.0002983 -0.000296908 -0.000295286 -0.000293499
-0.000291372 -0.00028909 -0.000286595 -0.000283894 -0.000280991 -0.000277889 -0.000274593 -0.000271107 -0.000267435 -0.000263582
-0.000259551 -0.000255345 -0.00025097 -0.000246429 -0.000241725 -0.000236863 -0.000231845 -0.000226676 -0.000221359 -0.000215897
-0.000210294 -0.000204554 -0.000198679 -0.000192672 -0.000186538 -0.000180279 -0.000173898 -0.000167398 -0.000160783 -0.000154055
-0.000147217 -0.000140272 -0.000133223 -0.000126073 -0.000118824 -0.00011148 -0.000104042 -9.65138e-05 -8.88974e-05 -8.11955e-05
-7.34106e-05 -6.55451e-05 -5.76014e-05 -4.95819e-05 -4.14889e-05 -3.33246e-05 -2.50914e-05 -1.67915e-05 -8.42697e-06
迭代次数为12947

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图 2: GS,epsilon=1

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epsilon为1时，SOR迭代法求得矩阵的解为
0.0128543 0.0256309 0.0383304 0.0509537 0.0635016 0.0759747 0.0883738 0.1007 0.112953 0.125135
0.137245 0.149285 0.161255 0.173157 0.18499 0.196755 0.208453 0.220085 0.231652 0.243153
0.25459 0.265963 0.277273 0.288521 0.299707 0.310831 0.321895 0.332899 0.343843 0.354729
0.365556 0.376326 0.387038 0.397694 0.408294 0.418839 0.429328 0.439764 0.450145 0.460473
0.470748 0.480972 0.491143 0.501263 0.511333 0.521352 0.531322 0.541242 0.551114 0.560937
0.570713 0.580442 0.590123 0.599759 0.609348 0.618892 0.628391 0.637845 0.647255 0.656622
0.665946 0.675226 0.684465 0.693661 0.702816 0.711929 0.721002 0.730035 0.739027 0.74798
0.756894 0.765769 0.774606 0.783405 0.792166 0.800889 0.809577 0.818228 0.826842 0.835421
0.843964 0.852472 0.860946 0.869385 0.87779 0.886161 0.894499 0.902804 0.911076 0.919316
0.927523 0.935699 0.943844 0.951957 0.960039 0.968091 0.976113 0.984105 0.992067
每一项与精确解的误差为
-1.61497e-05 -3.17523e-05 -4.68172e-05 -6.13534e-05 -7.537e-05 -8.88758e-05 -0.00010188 -0.00011439 -0.000126416 -0.000137965
-0.000149045 -0.000159666 -0.000169835 -0.000179559 -0.000188847 -0.000197707 -0.000206145 -0.00021417 -0.000221788 -0.000229003
-0.000235836 -0.000242279 -0.000248344 -0.000254038 -0.000259368 -0.000264341 -0.000268963 -0.00027324 -0.000277179 -0.000280787
-0.000284069 -0.000287031 -0.00028968 -0.000292021 -0.000294061 -0.000295804 -0.000297257 -0.000298426 -0.000299315 -0.000299931
-0.000300078 -0.0003000362 -0.0003000188 -0.000299761 -0.000299036 -0.000298169 -0.000297013 -0.000295624 -0.000294007 -0.000292166
-0.000290106 -0.000287832 -0.000285347 -0.000282656 -0.000279764 -0.000276675 -0.000273393 -0.000269922 -0.000266266 -0.000262343
-0.000258417 -0.000254231 -0.000249875 -0.000245355 -0.000240673 -0.000235833 -0.000230839 -0.000225694 -0.000220401 -0.000214965
-0.000209389 -0.000203675 -0.000197827 -0.000191849 -0.000185743 -0.000179513 -0.000173162 -0.000166692 -0.000160108 -0.00015341
-0.000146604 -0.000139691 -0.000132674 -0.000125556 -0.000118339 -0.000111027 -0.000103622 -9.61264e-05 -8.85427e-05 -8.08736e-05
-7.31214e-05 -6.52886e-05 -5.73775e-05 -4.93905e-05 -4.13299e-05 -3.31979e-05 -2.49968e-05 -1.67287e-05 -8.39569e-06
迭代次数为2593

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图 3: SOR,epsilon=1

## 1.2 $\epsilon=0.1$

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epsilon为0.1时，Jacobi迭代法求得矩阵的解为
0.0504577 0.0967829 0.139351 0.178504 0.214552 0.247778 0.278438 0.306765 0.332971 0.35725
0.379776 0.400708 0.420192 0.43836 0.45533 0.471213 0.486106 0.500099 0.513275 0.525708
0.537465 0.548608 0.559192 0.569269 0.578884 0.588079 0.596894 0.605361 0.613513 0.621379
0.628984 0.636352 0.643505 0.650463 0.657242 0.66386 0.67033 0.676667 0.682882 0.688987
0.694992 0.700905 0.706735 0.71249 0.718176 0.723799 0.729366 0.734882 0.74035 0.745776
0.751163 0.756516 0.761836 0.767127 0.772391 0.777631 0.78285 0.788049 0.793229 0.798394
0.803543 0.808679 0.813802 0.818914 0.824016 0.829109 0.834193 0.83927 0.84434 0.849403
0.854461 0.859513 0.86456 0.869604 0.874643 0.879679 0.884711 0.889741 0.894768 0.899792
0.904814 0.909834 0.914853 0.91987 0.924885 0.929898 0.934911 0.939922 0.944933 0.949942
0.954951 0.959958 0.964966 0.969972 0.974978 0.979983 0.984988 0.989992 0.994996
每一项与精确解的误差为
-0.00212577 -0.00385589 -0.00524559 -0.00634324 -0.00719117 -0.00782634 -0.00828101 -0.00858326 -0.00875753 -0.00882501
-0.00888406 -0.00871056 -0.00855818 -0.00835869 -0.00812218 -0.00785724 -0.00757124 -0.00727036 -0.00695986 -0.00664413
-0.00632682 -0.00601096 -0.00569904 -0.00539303 -0.00509456 -0.00480484 -0.00452486 -0.00425528 -0.00399661 -0.00374915
-0.00351304 -0.00328831 -0.0030749 -0.00287263 -0.00268128 -0.00250057 -0.00233018 -0.00216974 -0.00201888 -0.0018772
-0.00174491 -0.00161979 -0.00150323 -0.00139423 -0.00129239 -0.00119732 -0.00110855 -0.001026 -0.000949018 -0.000877369
-0.000810726 -0.000748775 -0.000691223 -0.000637732 -0.000588119 -0.00054219 -0.000499544 -0.000460025 -0.000423421 -0.000389953
-0.000358166 -0.000329149 -0.000302317 -0.000277511 -0.000254589 -0.000233413 -0.000213858 -0.000195804 -0.000179142 -0.000163768
-0.000149589 -0.000136512 -0.000124458 -0.000113347 -0.00010311 -9.3679e-05 -8.49935e-05 -7.69957e-05 -6.96334e-05 -6.28569e-05
-5.66215e-05 -5.08846e-05 -4.56078e-05 -4.07348e-05 -3.62927e-05 -3.21904e-05 -2.842e-05 -2.49548e-05 -2.17709e-05 -1.88458e-05
-1.6159e-05 -1.36913e-05 -1.14254e-05 -9.34496e-06 -7.43513e-06 -5.68212e-06 -4.07333e-06 -2.59705e-06 -1.24257e-06
迭代次数为9268

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图 4: Jacobi,epsilon=0.1

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epsilon=0.1时，GS迭代法求得矩阵的解为
0.0504577 0.096783 0.139351 0.178504 0.214553 0.247778 0.278438 0.306765 0.332971 0.35725
0.379776 0.400709 0.420193 0.43836 0.455331 0.471213 0.486106 0.5001 0.513276 0.525708
0.537465 0.548608 0.559192 0.569269 0.578884 0.58808 0.596894 0.605361 0.613514 0.621379
0.628984 0.636353 0.643506 0.650463 0.657242 0.66386 0.670331 0.676667 0.682883 0.688988
0.694992 0.700905 0.706735 0.71249 0.718176 0.723799 0.729366 0.734882 0.74035 0.745776
0.751164 0.756516 0.761836 0.767127 0.772391 0.777632 0.78285 0.788049 0.79323 0.798394
0.803543 0.808679 0.813802 0.818914 0.824016 0.829109 0.834193 0.83927 0.84434 0.849403
0.854461 0.859513 0.864561 0.869604 0.874643 0.879679 0.884711 0.889741 0.894768 0.899792
0.904814 0.909835 0.914853 0.91987 0.924885 0.929898 0.934911 0.939922 0.944933 0.949942
0.954951 0.959958 0.964966 0.969972 0.974978 0.979983 0.984988 0.989992 0.994996
每一项与精确解的误差为
-0.00212571 -0.00385577 -0.00524542 -0.00634303 -0.00719091 -0.00782605 -0.00828068 -0.00858291 -0.00875715 -0.0088246
0.00880364 -0.00871012 -0.00855772 -0.00835822 -0.0081217 -0.00785676 -0.00757075 -0.00726987 -0.00695937 -0.00664363
-0.00632633 -0.00601048 -0.00569855 -0.00539256 -0.00509408 -0.00480438 -0.0045244 -0.00425484 -0.00399617 -0.00374872
0.00351262 -0.0032379 -0.0030745 -0.00287224 -0.00268091 -0.00250021 -0.00232982 -0.0021694 -0.00201855 -0.00187688
0.001744 -0.00161949 -0.00150294 -0.00139395 -0.00129212 -0.00119707 -0.0011084 -0.00102576 -0.000948794 -0.000877155
-0.000810521 -0.000748581 -0.000691036 -0.000637605 -0.000588021 -0.00054203 -0.000499392 -0.000459881 -0.000423284 -0.000389401
0.000358043 -0.000329034 -0.000302208 -0.000277409 -0.000254492 -0.000233322 -0.000213773 -0.000195724 -0.000179067 -0.000163699
-0.000149523 -0.000136452 -0.000124401 -0.000113295 -0.000103061 -9.36343e-05 -8.49521e-05 -7.69577e-05 -6.95983e-05 -6.28249e-05
-5.65921e-05 -5.0858e-05 -4.55836e-05 -4.0733e-05 -3.6273e-05 -3.21728e-05 -2.84042e-05 -2.49409e-05 -2.17586e-05 -1.88351e-05
-1.61498e-05 -1.36835e-05 -1.14188e-05 -9.33958e-06 -7.43083e-06 -5.67884e-06 -4.07097e-06 -2.59555e-06 -1.24185e-06
迭代次数为4835

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图 5: GS,epsilon=0.1

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epsilon=0.1时，SOR迭代法求得矩阵的解为
0.0504578 0.0967831 0.139352 0.178505 0.214553 0.247779 0.278439 0.306766 0.332972 0.357251
0.379776 0.400709 0.420193 0.438361 0.455331 0.471214 0.486107 0.5001 0.513276 0.525709
0.537466 0.548609 0.559193 0.56927 0.578885 0.58808 0.596895 0.605362 0.613514 0.62138
0.628985 0.636353 0.643506 0.650464 0.657243 0.66386 0.670331 0.676668 0.682883 0.688988
0.694992 0.700905 0.706736 0.71249 0.718176 0.7238 0.729367 0.734882 0.740351 0.745777
0.751164 0.756516 0.761836 0.767127 0.772391 0.777632 0.78285 0.788049 0.79323 0.798394
0.803543 0.808679 0.813802 0.818915 0.824017 0.829109 0.834194 0.83927 0.84434 0.849403
0.854461 0.859513 0.864561 0.869604 0.874643 0.879679 0.884711 0.889741 0.894768 0.899792
0.904814 0.909835 0.914853 0.91987 0.924885 0.929898 0.934911 0.939922 0.944933 0.949942
0.954951 0.959959 0.964966 0.969972 0.974978 0.979983 0.984988 0.989992 0.994996
每一项与精确解的误差为
-0.00212563 -0.00385562 -0.00524521 -0.00634276 -0.00719059 -0.00782568 -0.00828027 -0.00858246 -0.00875667 -0.0088241
0.00880311 -0.00870957 -0.00855717 -0.00835765 -0.00812112 -0.00785617 -0.00757015 -0.00726927 -0.00695957 -0.00664304
0.00632574 -0.00600989 -0.00569797 -0.00539198 -0.00509352 -0.00480393 -0.00452386 -0.0042543 -0.00399565 -0.00374821
0.00351212 -0.00323742 -0.00307403 -0.00287179 -0.00268047 -0.00249978 -0.00232941 -0.002169 -0.00201816 -0.00187651
0.00174364 -0.00161915 -0.00150261 -0.00139364 -0.00129182 -0.00119678 -0.00110813 -0.0010255 -0.000948541 -0.000876914
-0.000810292 -0.000748362 -0.000690828 -0.000637408 -0.000587834 -0.000541852 -0.000499224 -0.000459722 -0.000423134 -0.000389259
-0.000357909 -0.000328908 -0.000302089 -0.000277297 -0.000254388 -0.000233224 -0.000213681 -0.000195639 -0.000178987 -0.000163624
0.000149454 -0.000136387 -0.000124341 -0.00011324 -0.00010301 -9.35871e-05 -8.49086e-05 -7.69178e-05 -6.95617e-05 -6.27915e-05
-5.65617e-05 -5.08303e-05 -4.55585e-05 -4.07104e-05 -3.62527e-05 -3.21547e-05 -2.83881e-05 -2.49267e-05 -2.17462e-05 -1.88243e-05
-1.61404e-05 -1.36750e-05 -1.14122e-05 -9.33417e-06 -7.42652e-06 -5.67555e-06 -4.06861e-06 -2.59405e-06 -1.24114e-06
迭代次数为957

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图 6: SOR,epsilon=0.1

### 1.3 $\epsilon=0.01$

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epsilon=0.01时，Jacobi迭代法求得矩阵的解为
0.255 0.385 0.4525 0.48875 0.509375 0.522187 0.531094 0.538047 0.544023 0.549512
0.554756 0.559878 0.564999 0.56999 0.574985 0.579992 0.584996 0.589998 0.594999 0.6
0.605 0.61 0.615 0.62 0.625 0.63 0.635 0.64 0.645 0.65
0.655 0.66 0.665 0.67 0.675 0.68 0.685 0.69 0.695 0.7
0.705 0.71 0.715 0.72 0.725 0.73 0.735 0.74 0.745 0.75
0.755 0.76 0.765 0.77 0.775 0.78 0.785 0.79 0.795 0.8
0.805 0.81 0.815 0.82 0.825 0.83 0.835 0.84 0.845 0.85
0.855 0.86 0.865 0.87 0.875 0.88 0.885 0.89 0.895 0.9
0.905 0.91 0.915 0.92 0.925 0.93 0.935 0.94 0.945 0.95
0.955 0.96 0.965 0.97 0.975 0.98 0.985 0.99 0.995
每一项与精确解的误差为
-0.0606004 -0.0573325 -0.0376067 -0.0220924 -0.0122562 -0.00657327 -0.00345044 -0.0017855 -0.00091495 -0.000465653
-0.000235851 -0.000119045 -5.99452e-05 -3.01323e-05 -1.51316e-05 -7.59254e-06 -3.8103e-06 -1.91197e-06 -9.61104e-07 -4.83449e-07
-2.44407e-07 -1.23807e-07 -6.34868e-08 -3.26977e-08 -1.73064e-08 -9.2283e-09 -5.19325e-09 -2.94224e-09 -1.81945e-09 -1.11618e-09
-7.66221e-10 -5.05606e-10 -3.76302e-10 -2.6032e-10 -1.62926e-10 -1.4368e-10 -1.14409e-10 -8.17223e-11 -6.58555e-11 -4.69278e-11
-3.77193e-11 -2.60482e-11 -1.6324e-11 -1.54097e-11 -1.23384e-11 -8.75722e-12 -6.98974e-12 -4.94094e-12 -3.92986e-12 -2.76579e-12
-1.9169e-12 -1.53522e-12 -1.21136e-12 -8.44214e-13 -6.63136e-13 -4.5941e-13 -3.58935e-13 -2.46914e-13 -1.91624e-13 -1.30784e-13
-1.00386e-13 -6.79436e-14 -5.17364e-14 -3.4639e-14 -2.59792e-14 -1.70974e-14 -1.26365e-14 -8.43769e-15 -6.43929e-15 -4.66294e-15
-3.77476e-15 -3.21965e-15 -2.88658e-15 -2.77556e-15 -2.66454e-15 -2.55351e-15 -2.44249e-15 -2.33147e-15 -2.22045e-15 -2.10942e-15
-1.9984e-15 -1.88738e-15 -1.77636e-15 -1.55431e-15 -1.55431e-15 -1.33227e-15 -1.33227e-15 -1.11022e-15 -1.11022e-15 -8.8178e-16
-8.8178e-16 -6.66134e-16 -6.66134e-16 -4.44089e-16 -4.44089e-16 -2.22045e-16 -1.11022e-16 0 1.11022e-16
迭代次数为646

```

图 7: Jacobi,epsilon=0.01

```

epsilon为0.01时，G-S迭代法求得矩阵的解为
0.255 0.385 0.4525 0.48875 0.509375 0.522187 0.531094 0.538047 0.544023 0.549512
0.554756 0.559878 0.564939 0.569969 0.574985 0.579992 0.584996 0.589998 0.594999 0.6
0.605 0.61 0.615 0.62 0.625 0.63 0.635 0.64 0.645 0.65
0.655 0.66 0.665 0.67 0.675 0.68 0.685 0.69 0.695 0.7
0.705 0.71 0.715 0.72 0.725 0.73 0.735 0.74 0.745 0.75
0.755 0.76 0.765 0.77 0.775 0.78 0.785 0.79 0.795 0.8
0.805 0.81 0.815 0.82 0.825 0.83 0.835 0.84 0.845 0.85
0.855 0.86 0.865 0.87 0.875 0.88 0.885 0.89 0.895 0.9
0.905 0.91 0.915 0.92 0.925 0.93 0.935 0.94 0.945 0.95
0.955 0.96 0.965 0.97 0.975 0.98 0.985 0.99 0.995
每一项与精确解的误差为
-0.0660604 -0.0573325 -0.0376066 -0.0220923 -0.0122561 -0.0065732 -0.00345037 -0.00178544 -0.000914894 -0.00046561
-0.000235811 -0.00019015 -5.99176e-05 -3.01114e-05 -1.51131e-05 -7.57856e-06 -3.79807e-06 -1.90278e-06 -9.53146e-07 -4.77498e-07
-2.39296e-07 -1.20001e-07 -6.02419e-08 -3.02915e-08 -1.52684e-08 -7.72299e-09 -3.92602e-09 -2.00992e-09 -1.03903e-09 -5.44236e-10
-2.90022e-10 -1.57955e-10 -8.83175e-11 -5.08847e-11 -3.02763e-11 -1.86052e-11 -1.1784e-11 -7.6642e-12 -5.09515e-12 -3.44536e-12
-2.35956e-12 -1.63025e-12 -1.13298e-12 -7.89924e-13 -5.5167e-13 -3.85247e-13 -2.68785e-13 -1.87073e-13 -1.29785e-13 -8.9706e-14
-6.16174e-14 -4.21855e-14 -2.87548e-14 -1.96509e-14 -1.35447e-14 -9.34792e-15 -6.99441e-15 -5.55112e-15 -4.66294e-15 -4.44089e-15
-4.21855e-15 -3.9968e-15 -3.9968e-15 -3.9968e-15 -3.77476e-15 -3.77476e-15 -3.55271e-15 -3.55271e-15 -3.33067e-15 -3.33067e-15
-3.10862e-15 -2.9976e-15 -2.88658e-15 -2.77556e-15 -2.66454e-15 -2.55351e-15 -2.44249e-15 -2.33147e-15 -2.22045e-15 -2.10942e-15
-1.9984e-15 -1.88738e-15 -1.77636e-15 -1.55431e-15 -1.55431e-15 -1.33227e-15 -1.33227e-15 -1.11022e-15 -1.11022e-15 -8.88178e-16
-8.88178e-16 -6.66134e-16 -6.66134e-16 -4.44089e-16 -4.44089e-16 -2.22045e-16 -1.11022e-16 0 1.11022e-16
迭代次数为377

```

图 8: GS,epsilon=0.01

```

epsilon为0.01时，SOR迭代法求得矩阵的解为
0.255 0.385 0.4525 0.48875 0.509375 0.522187 0.531094 0.538047 0.544023 0.549512
0.554756 0.559878 0.564939 0.569969 0.574985 0.579992 0.584996 0.589998 0.594999 0.6
0.605 0.61 0.615 0.62 0.625 0.63 0.635 0.64 0.645 0.65
0.655 0.66 0.665 0.67 0.675 0.68 0.685 0.69 0.695 0.7
0.705 0.71 0.715 0.72 0.725 0.73 0.735 0.74 0.745 0.75
0.755 0.76 0.765 0.77 0.775 0.78 0.785 0.79 0.795 0.8
0.805 0.81 0.815 0.82 0.825 0.83 0.835 0.84 0.845 0.85
0.855 0.86 0.865 0.87 0.875 0.88 0.885 0.89 0.895 0.9
0.905 0.91 0.915 0.92 0.925 0.93 0.935 0.94 0.945 0.95
0.955 0.96 0.965 0.97 0.975 0.98 0.985 0.99 0.995
每一项与精确解的误差为
-0.0660603 -0.0573324 -0.0376065 -0.0220922 -0.012256 -0.00657314 -0.00345032 -0.0017854 -0.00091486 -0.000465582
-0.00023579 -0.00018998 -5.99047e-05 -3.01017e-05 -1.51057e-05 -7.573e-06 -3.79399e-06 -1.89968e-06 -9.5085e-07 -4.75861e-07
-2.37984e-07 -1.1909e-07 -5.95356e-08 -2.98004e-08 -1.489e-08 -7.44114e-09 -3.73513e-09 -1.85402e-09 -9.34448e-10 -4.65459e-10
-2.34367e-10 -1.14767e-10 -5.7479e-11 -2.87708e-11 -1.52494e-11 -9.04743e-12 -2.48734e-12 -7.76086e-13 -2.21712e-12 -9.67892e-13
-4.72955e-13 6.65912e-13 -4.67515e-13 -2.89324e-13 -1.11022e-13 4.82947e-13 -1.84297e-13 -3.72147e-13 7.88258e-13 1.8674e-13
1.52101e-13 -1.70419e-13 -2.72005e-14 1.16129e-13 4.86278e-14 -8.80407e-14 -6.31717e-14 9.88098e-14 5.71765e-14 -2.4869e-14
-2.67564e-14 2.17604e-14 4.27436e-14 -1.22125e-14 -6.21275e-15 2.45359e-14 1.9984e-14 -9.54792e-15 2.55351e-15 1.52101e-14
9.21485e-15 1.11022e-15 3.44169e-15 1.11022e-14 1.15463e-14 3.21965e-15 2.77556e-15 5.44009e-15 5.88418e-15 4.10783e-15
3.21965e-15 5.44009e-15 3.21965e-15 2.10942e-15 3.33067e-15 4.55191e-15 1.44329e-15 1.11022e-15 3.21965e-15
1.77636e-15 2.10942e-15 1.44329e-15 1.88738e-15 1.66533e-15 1.33227e-15 9.99201e-16 5.51112e-16 3.33067e-16
迭代次数为180

```

图 9: SOR,epsilon=0.01

## 1.4 $\epsilon=0.0001$

```

epsilon为0.0001时，Jacobi迭代法求得矩阵的解为
0.500049 0.509951 0.515 0.52 0.525 0.53 0.535 0.54 0.545 0.55
0.555 0.56 0.565 0.57 0.575 0.58 0.585 0.59 0.595 0.6
0.605 0.61 0.615 0.62 0.625 0.63 0.635 0.64 0.645 0.65
0.655 0.66 0.665 0.67 0.675 0.68 0.685 0.69 0.695 0.7
0.705 0.71 0.715 0.72 0.725 0.73 0.735 0.74 0.745 0.75
0.755 0.76 0.765 0.77 0.775 0.78 0.785 0.79 0.795 0.8
0.805 0.81 0.815 0.82 0.825 0.83 0.835 0.84 0.845 0.85
0.855 0.86 0.865 0.87 0.875 0.88 0.885 0.89 0.895 0.9
0.905 0.91 0.915 0.92 0.925 0.93 0.935 0.94 0.945 0.95
0.955 0.96 0.965 0.97 0.975 0.98 0.985 0.99 0.995
每一项与精确解的误差为
-0.00495052 -4.90178e-05 -4.88083e-07 -5.05415e-09 -2.72936e-10 -1.91805e-11 -1.67613e-11 -1.30262e-12 -1.15574e-12 -8.75966e-14
-7.72715e-14 -8.88178e-15 -8.10463e-15 -4.10783e-15 -3.9968e-15 -3.77476e-15 -3.77476e-15 -3.77476e-15 -3.77476e-15 -3.77476e-15
-3.66374e-15 -3.66374e-15 -3.66374e-15 -3.66374e-15 -3.55271e-15 -3.55271e-15 -3.44169e-15 -3.44169e-15 -3.44169e-15 -3.44169e-15
-3.33067e-15 -3.33067e-15 -3.21965e-15 -3.21965e-15 -3.21965e-15 -3.10862e-15 -3.21965e-15 -2.9976e-15 -3.10862e-15 -2.9976e-15
-3.10862e-15 -2.9976e-15 -2.88658e-15 -2.88658e-15 -2.88658e-15 -2.88658e-15 -2.88658e-15 -2.77556e-15 -2.77556e-15 -2.66454e-15
-2.66454e-15 -2.66454e-15 -2.55351e-15 -2.55351e-15 -2.44249e-15 -2.44249e-15 -2.44249e-15 -2.33147e-15 -2.22045e-15 -2.22045e-15
-2.10942e-15 -2.22045e-15 -1.9984e-15 -2.10942e-15 -1.88738e-15 -1.9984e-15 -1.88738e-15 -1.88738e-15 -1.77636e-15 -1.77636e-15
-1.66533e-15 -1.55431e-15 -1.55431e-15 -1.44329e-15 -1.44329e-15 -1.33227e-15 -1.33227e-15 -1.33227e-15 -1.33227e-15 -1.22125e-15
-1.22125e-15 -1.11022e-15 -1.11022e-15 -9.99201e-16 -9.99201e-16 -8.88178e-16 -8.88178e-16 -7.77156e-16 -8.88178e-16 -6.66134e-16
-7.77156e-16 -5.51112e-16 -6.66134e-16 -4.44089e-16 -4.44089e-16 -3.33067e-16 -2.22045e-16 -2.22045e-16 -1.11022e-16
迭代次数为120

```

图 10: Jacobi,epsilon=0.0001

```

epsilon为0.0001时，G-S迭代法求得矩阵的解为
0.50005 0.509951 0.515 0.52 0.525 0.53 0.535 0.54 0.545 0.55
0.555 0.56 0.565 0.57 0.575 0.58 0.585 0.59 0.595 0.6
0.605 0.61 0.615 0.62 0.625 0.63 0.635 0.64 0.645 0.65
0.655 0.66 0.665 0.67 0.675 0.68 0.685 0.69 0.695 0.7
0.705 0.71 0.715 0.72 0.725 0.73 0.735 0.74 0.745 0.75
0.755 0.76 0.765 0.77 0.775 0.78 0.785 0.79 0.795 0.8
0.805 0.81 0.815 0.82 0.825 0.83 0.835 0.84 0.845 0.85
0.855 0.86 0.865 0.87 0.875 0.88 0.885 0.89 0.895 0.9
0.905 0.91 0.915 0.92 0.925 0.93 0.935 0.94 0.945 0.95
0.955 0.96 0.965 0.97 0.975 0.98 0.985 0.99 0.995
每一项与精确解的误差为
-0.0049505 -4.90151e-05 -4.85321e-07 -4.80703e-09 -4.77387e-11 -4.86833e-13 -9.4369e-15 -4.21885e-15 -4.10783e-15 -4.10783e-15
-3.9968e-15 -3.9968e-15 -3.88578e-15 -3.9968e-15 -3.88578e-15 -3.77476e-15 -3.77476e-15 -3.77476e-15 -3.77476e-15 -3.77476e-15
-3.66374e-15 -3.66374e-15 -3.66374e-15 -3.66374e-15 -3.66374e-15 -3.55271e-15 -3.55271e-15 -3.44169e-15 -3.44169e-15 -3.44169e-15
-3.33067e-15 -3.33067e-15 -3.21965e-15 -3.21965e-15 -3.21965e-15 -3.10862e-15 -3.21965e-15 -2.9976e-15 -3.10862e-15 -2.9976e-15
-3.10862e-15 -2.9976e-15 -2.88658e-15 -2.88658e-15 -2.88658e-15 -2.88658e-15 -2.88658e-15 -2.77556e-15 -2.77556e-15 -2.66454e-15
-2.66454e-15 -2.66454e-15 -2.55351e-15 -2.55351e-15 -2.44249e-15 -2.44249e-15 -2.44249e-15 -2.33147e-15 -2.22045e-15 -2.22045e-15
-2.10942e-15 -2.22045e-15 -1.9984e-15 -2.10942e-15 -1.88738e-15 -1.9984e-15 -1.88738e-15 -1.88738e-15 -1.77636e-15 -1.77636e-15
-1.66533e-15 -1.55431e-15 -1.55431e-15 -1.55431e-15 -1.44329e-15 -1.44329e-15 -1.33227e-15 -1.33227e-15 -1.33227e-15 -1.22125e-15
-1.22125e-15 -1.11022e-15 -1.11022e-15 -9.99201e-16 -9.99201e-16 -8.88178e-16 -8.88178e-16 -7.77156e-16 -8.88178e-16 -6.66134e-16
-7.77156e-16 -5.55112e-16 -6.66134e-16 -4.44089e-16 -4.44089e-16 -3.33067e-16 -2.22045e-16 -2.22045e-16 -1.11022e-16
迭代次数为110

```

图 11: GS,epsilon=0.0001

```

epsilon为0.0001时，SOR迭代法求得矩阵的解为
0.500049 0.509951 0.515 0.52 0.525 0.53 0.535 0.54 0.545 0.55
0.555 0.56 0.565 0.57 0.575 0.58 0.585 0.59 0.595 0.6
0.605 0.61 0.615 0.62 0.625 0.63 0.635 0.64 0.645 0.65
0.655 0.66 0.665 0.67 0.675 0.68 0.685 0.69 0.695 0.7
0.705 0.71 0.715 0.72 0.725 0.73 0.735 0.74 0.745 0.75
0.755 0.76 0.765 0.77 0.775 0.78 0.785 0.79 0.795 0.8
0.805 0.81 0.815 0.82 0.825 0.83 0.835 0.84 0.845 0.85
0.855 0.86 0.865 0.87 0.875 0.88 0.885 0.89 0.895 0.9
0.905 0.91 0.915 0.92 0.925 0.93 0.935 0.94 0.945 0.95
0.955 0.96 0.965 0.97 0.975 0.98 0.985 0.99 0.995
每一项与精确解的误差为
-0.00495052 -4.90228e-05 -4.87721e-07 -5.51589e-09 -2.49696e-10 -5.62265e-11 -1.49367e-11 -3.88589e-12 -9.84657e-13 -2.44915e-13
-6.18394e-14 -1.78746e-14 -7.43849e-15 -5.10703e-15 -4.66294e-15 -4.55191e-15 -4.55191e-15 -4.55191e-15 -4.55191e-15 -4.55191e-15
-4.44089e-15 -4.44089e-15 -4.44089e-15 -4.44089e-15 -4.44089e-15 -4.32987e-15 -4.32987e-15 -4.32987e-15 -4.32987e-15 -4.32987e-15
-4.10783e-15 -4.10783e-15 -3.9968e-15 -3.9968e-15 -3.9968e-15 -3.77476e-15 -3.77476e-15 -3.77476e-15 -3.77476e-15 -3.77476e-15
-3.55271e-15 -3.44169e-15 -3.33067e-15 -3.33067e-15 -3.33067e-15 -3.33067e-15 -3.33067e-15 -3.21965e-15 -3.21965e-15 -3.21965e-15
-3.21965e-15 -3.21965e-15 -3.10862e-15 -3.10862e-15 -3.10862e-15 -3.10862e-15 -3.10862e-15 -2.9976e-15 -2.9976e-15 -2.9976e-15
-2.88658e-15 -2.88658e-15 -2.88658e-15 -2.88658e-15 -2.88658e-15 -2.88658e-15 -2.88658e-15 -2.77556e-15 -2.77556e-15 -2.66454e-15
-2.66454e-15 -2.66454e-15 -2.55351e-15 -2.55351e-15 -2.44249e-15 -2.44249e-15 -2.44249e-15 -2.33147e-15 -2.22045e-15 -2.22045e-15
-2.22045e-15 -2.22045e-15 -2.10942e-15 -2.10942e-15 -1.9984e-15 -1.9984e-15 -1.88738e-15 -1.88738e-15 -1.88738e-15 -1.77636e-15
-1.77636e-15 -1.66533e-15 -1.66533e-15 -1.66533e-15 -1.66533e-15 -1.55431e-15 -1.55431e-15 -1.55431e-15 -1.55431e-15 -1.44329e-15
-1.44329e-15 -1.33227e-15 -1.33227e-15 -1.33227e-15 -1.33227e-15 -1.33227e-15 -1.33227e-15 -1.33227e-15 -1.33227e-15 -1.22125e-15
-1.22125e-15 -1.11022e-15 -1.11022e-15 -1.11022e-15 -1.11022e-15 -1.11022e-15 -1.11022e-15 -1.11022e-15 -1.11022e-15 -1.11022e-15
-9.99201e-16 -9.99201e-16 -9.99201e-16 -9.99201e-16 -9.99201e-16 -9.99201e-16 -9.99201e-16 -9.99201e-16 -9.99201e-16 -9.99201e-16
-8.88178e-16 -8.88178e-16 -8.88178e-16 -8.88178e-16 -8.88178e-16 -8.88178e-16 -8.88178e-16 -8.88178e-16 -8.88178e-16 -8.88178e-16
-7.77156e-16 -7.77156e-16 -7.77156e-16 -7.77156e-16 -7.77156e-16 -7.77156e-16 -7.77156e-16 -7.77156e-16 -7.77156e-16 -7.77156e-16
-6.66134e-16 -6.66134e-16 -6.66134e-16 -6.66134e-16 -6.66134e-16 -6.66134e-16 -6.66134e-16 -6.66134e-16 -6.66134e-16 -6.66134e-16
-5.55112e-16 -5.55112e-16 -5.55112e-16 -5.55112e-16 -5.55112e-16 -5.55112e-16 -5.55112e-16 -5.55112e-16 -5.55112e-16 -5.55112e-16
-4.44089e-16 -4.44089e-16 -4.44089e-16 -4.44089e-16 -4.44089e-16 -4.44089e-16 -4.44089e-16 -4.44089e-16 -4.44089e-16 -4.44089e-16
-3.33067e-16 -3.33067e-16 -3.33067e-16 -3.33067e-16 -3.33067e-16 -3.33067e-16 -3.33067e-16 -3.33067e-16 -3.33067e-16 -3.33067e-16
-2.22045e-16 -2.22045e-16 -2.22045e-16 -2.22045e-16 -2.22045e-16 -2.22045e-16 -2.22045e-16 -2.22045e-16 -2.22045e-16 -2.22045e-16
-1.11022e-16 -1.11022e-16 -1.11022e-16 -1.11022e-16 -1.11022e-16 -1.11022e-16 -1.11022e-16 -1.11022e-16 -1.11022e-16 -1.11022e-16
迭代次数为137

```

图 12: SOR,epsilon=0.0001

## 2 第二题

$$\omega = 1.7$$

N为20时，Jacobi迭代次数为1104 CPU用时为234ms  
 N为20时，G-S迭代次数为580 CPU用时为125ms  
 N为20时，SOR迭代次数为89 CPU用时为16ms

图 13: PDE,N=20

N为40时, Jacobi迭代次数为4216 CPU用时为3390ms  
N为40时, G-S迭代次数为2214 CPU用时为1750ms  
N为40时, SOR迭代次数为420 CPU用时为344ms

图 14: PDE,  $N=40$ 

N为80时, Jacobi迭代次数为16028 CPU用时为51750ms  
N为80时, G-S迭代次数为8437 CPU用时为27094ms  
N为80时, SOR迭代次数为1660 CPU用时为5797ms

图 15: PDE,  $N=80$