

数值代数作业 C5

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1 第一题

用两种方法解差分方程的解如下。图中取松弛因子为 1.73，迭代 70 次，经测试为最佳松弛因子。

SOR 迭代法需要确定松弛因子，且只有系数矩阵有良好性质时才能找到最佳松弛因子，而共轭梯度法不需要提前确定参数。二者在计算 PDE 时都不需要生成矩阵 A，从而在求解稀疏线性方程组时速度较快。

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共轭梯度法解差分方程的解为 (C 从小到大, I 从小到大)
0.0151899 0.027955 0.0428829 0.0616269 0.0856209 0.116993 0.152124 0.190461 0.232909 0.283013 0.335483 0.392939 0.456058 0.524443 0.596509 0.671382 0.749908 0.831787 0.91601
0.027955 0.0440659 0.0606318 0.0799115 0.105947 0.13891 0.179124 0.217213 0.260133 0.310588 0.365948 0.422761 0.4885 0.555895 0.628014 0.699729 0.77486 0.852641 0.931651
0.0428829 0.0606318 0.077544 0.0971519 0.122712 0.160832 0.201288 0.241857 0.284133 0.339204 0.392191 0.449904 0.515006 0.586492 0.659362 0.730834 0.802612 0.874955 0.94985
0.0616269 0.0799115 0.0971519 0.115363 0.14276 0.179877 0.227977 0.266926 0.30953 0.363714 0.423537 0.479719 0.548206 0.616548 0.691313 0.761864 0.833257 0.902143 0.972085
0.0856209 0.105947 0.122712 0.14276 0.168008 0.21157 0.25762 0.302558 0.344217 0.402772 0.458464 0.518448 0.587309 0.660771 0.736411 0.803533 0.873674 0.936995 1.00152
0.116993 0.13891 0.160832 0.179877 0.21157 0.252803 0.312857 0.354449 0.397974 0.457337 0.523819 0.5893 0.64631 0.717373 0.790929 0.858584 0.919672 0.980856 1.03582
0.152124 0.179124 0.201288 0.227977 0.25762 0.312857 0.369272 0.431793 0.473222 0.534143 0.599194 0.657306 0.727637 0.784088 0.854895 0.914609 0.970693 1.02468 1.0746
0.190461 0.217213 0.241857 0.266926 0.302558 0.354449 0.431793 0.48645 0.551757 0.614642 0.669439 0.723606 0.781352 0.840317 0.8977 0.957833 1.00964 1.06353 1.11235
0.232909 0.260133 0.284133 0.30953 0.344217 0.397974 0.473222 0.551757 0.604203 0.671183 0.736173 0.776256 0.828972 0.887362 0.943393 1.0013 1.05491 1.10637 1.15594
0.283013 0.310588 0.339204 0.363714 0.402772 0.457337 0.534143 0.614642 0.671183 0.726118 0.79354 0.843161 0.890844 0.948803 1.00464 1.0605 1.1116 1.16054 1.20658
0.335483 0.365948 0.392191 0.423537 0.458454 0.523819 0.599194 0.669439 0.736173 0.79354 0.848334 0.909058 0.958157 1.00191 1.05996 1.10945 1.16198 1.2099 1.25766
0.392939 0.422761 0.449904 0.479719 0.518448 0.5803 0.657306 0.723606 0.776256 0.843161 0.909058 0.955332 1.01445 1.06009 1.10967 1.16436 1.21179 1.26469 1.31199
0.456058 0.4885 0.515006 0.548206 0.587309 0.64631 0.727637 0.781352 0.828972 0.890844 0.958157 1.01445 1.07249 1.12395 1.17449 1.22563 1.27482 1.32523 1.37391
0.524443 0.555895 0.586492 0.616548 0.660771 0.717373 0.784088 0.840317 0.887362 0.948803 1.00191 1.06009 1.12395 1.1725 1.22877 1.2794 1.33035 1.38209 1.43518
0.596509 0.628014 0.659362 0.691313 0.736411 0.790929 0.854895 0.8977 0.943393 1.00464 1.05996 1.10967 1.17449 1.22877 1.28524 1.3361 1.38269 1.43483 1.49513
0.671382 0.699729 0.730834 0.761864 0.803533 0.85584 0.914069 0.957833 1.0013 1.0605 1.10945 1.16436 1.22563 1.2794 1.33136 1.37661 1.42566 1.47808 1.5476
0.749908 0.77486 0.802612 0.833257 0.873674 0.919672 0.970693 1.00964 1.05491 1.1116 1.16198 1.21179 1.27482 1.33035 1.38269 1.42566 1.46579 1.5109 1.57942
0.831787 0.852641 0.874955 0.902143 0.936995 0.980856 1.02468 1.06353 1.10627 1.16054 1.2099 1.26469 1.32523 1.38209 1.43483 1.47808 1.5109 1.52317 1.53859
0.91601 0.931651 0.94985 0.972085 1.00152 1.03582 1.0746 1.11235 1.15594 1.20658 1.25766 1.31199 1.37391 1.43518 1.49513 1.5476 1.57942 1.53859 1.2441

SOR迭代法解差分方程的解为 (J 从小到大, I 从小到大)
0.0213807 0.0402851 0.0611972 0.0850485 0.112345 0.143392 0.178389 0.217467 0.260715 0.308188 0.359912 0.415892 0.476105 0.540497 0.60898 0.681413 0.757568 0.837067 0.919208
0.0402851 0.0686505 0.0970893 0.126839 0.158688 0.193157 0.230599 0.271258 0.315301 0.362831 0.413901 0.468517 0.526638 0.588166 0.65294 0.720706 0.79108 0.863471 0.936946
0.0611972 0.0970893 0.131876 0.166798 0.202747 0.240355 0.28008 0.322245 0.367075 0.414715 0.46524 0.518659 0.574918 0.633887 0.695353 0.758993 0.824339 0.890725 0.957209
0.0850485 0.126839 0.166798 0.206074 0.245558 0.285926 0.32769 0.371222 0.416789 0.464565 0.514642 0.567034 0.62168 0.678433 0.737049 0.797171 0.858303 0.919772 0.980703
0.112345 0.158688 0.202747 0.245558 0.287972 0.330668 0.374175 0.418392 0.465111 0.513025 0.56274 0.614275 0.667567 0.722462 0.778707 0.835936 0.893652 0.95191 1.00781
0.143392 0.193157 0.240355 0.285926 0.330668 0.375239 0.420166 0.465859 0.512621 0.560557 0.610081 0.66092 0.713115 0.766512 0.820863 0.875807 0.930864 0.985427 1.03876
0.178389 0.230599 0.28008 0.32769 0.374175 0.420166 0.466179 0.512623 0.559802 0.607928 0.657121 0.707415 0.758758 0.811005 0.86392 0.917159 0.970269 1.02267 1.07368
0.217467 0.271258 0.322245 0.371222 0.418392 0.465859 0.512623 0.559593 0.607043 0.655216 0.704225 0.75411 0.804528 0.856232 0.908162 0.960244 1.01207 1.0631 1.11267
0.260715 0.315301 0.367075 0.416789 0.465111 0.512621 0.559802 0.607043 0.654643 0.702909 0.751667 0.801263 0.851564 0.902461 0.953766 1.0052 1.05657 1.10676 1.15574
0.308188 0.362831 0.414715 0.464565 0.513025 0.560557 0.607928 0.655216 0.702909 0.75091 0.799639 0.849046 0.89911 0.949745 1.0008 1.05204 1.10315 1.15364 1.20287
0.359912 0.413901 0.46524 0.514642 0.56274 0.610081 0.657121 0.704225 0.751667 0.799639 0.848254 0.897557 0.947535 0.998127 1.04923 1.10069 1.15226 1.20357 1.25396
0.415892 0.468517 0.518659 0.567034 0.614275 0.66092 0.707415 0.75411 0.801263 0.849046 0.897557 0.946827 0.99684 1.04755 1.09892 1.1509 1.20341 1.25623 1.3088
0.476105 0.526638 0.574918 0.62168 0.667567 0.713115 0.758758 0.804528 0.851564 0.89911 0.947535 0.99684 1.04699 1.09793 1.14967 1.2023 1.25602 1.31099 1.36693
0.540497 0.588166 0.633887 0.678433 0.722462 0.766512 0.811005 0.856232 0.902461 0.949745 0.998127 1.04755 1.09793 1.14915 1.20122 1.25438 1.30922 1.36669 1.4274
0.60898 0.65294 0.695353 0.737049 0.778707 0.820863 0.86392 0.908162 0.953766 1.0008 1.04923 1.09892 1.14967 1.20122 1.25345 1.30657 1.36166 1.42109 1.48801
0.681413 0.720706 0.758993 0.797171 0.835936 0.875807 0.917159 0.960244 1.0052 1.05204 1.10069 1.1509 1.2023 1.25438 1.30657 1.35865 1.41168 1.46999 1.54315
0.757568 0.79108 0.824339 0.858303 0.893652 0.930864 0.970269 1.01207 1.05657 1.10315 1.15226 1.20341 1.25602 1.30922 1.36166 1.41168 1.45837 1.50607 1.57673
0.837067 0.863471 0.890725 0.919772 0.95121 0.985427 1.02267 1.0631 1.10676 1.15364 1.20357 1.25623 1.31099 1.36669 1.42109 1.46999 1.50607 1.52121 1.53734
0.919208 0.936946 0.957209 0.980703 1.00781 1.03876 1.07368 1.11267 1.15574 1.20287 1.25396 1.3088 1.36693 1.4274 1.48801 1.54315 1.57673 1.53734 1.24338
SOR迭代次数为70
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图 1: 共轭梯度法和 SOR 迭代

2 第二题

测试结果如下，这里取 $n=20$ 。

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第二题方程的解为  
0.333317 0.333637 0.332238 0.333954 0.334261 0.333702 0.333039 0.332627 0.332537 0.332708  
0.333037 0.333422 0.333776 0.334034 0.334151 0.334099 0.33386 0.33343 0.332808 0.332002
```

图 2: 求解希尔伯特矩阵

3 第三题

Jacobi 迭代法，GS 迭代法和共轭梯度法求解结果相同。GS 迭代法只迭代 3 次，收敛速度非常快，Jacobi 迭代法迭代 82 次，收敛速度较慢，共轭梯度法理论上最多迭代 n 次，这里限制设为 $2n$ ，实际迭代 $n=5$ 次。说明在矩阵性质较好时，GS 迭代法的速度可能快于共轭梯度法。

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Jacobi迭代法求得矩阵的解为  
1 -2 3 -2 1  
迭代次数为82  
  
GS迭代法求得矩阵的解为  
1 -2 3 -2 1  
迭代次数为3  
  
共轭梯度法在限制条件下计算了5步  
共轭梯度法求得矩阵的解为  
1 -2 3 -2 1
```

图 3: 第三题