Experimental Results

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Chapter 1

Simulation study

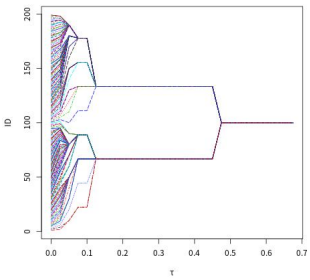
We simulate data from

$$P(Y=1|x) = \frac{exp(\beta^T x+\mu)}{1+exp(\beta^T x+\mu)}$$
$$P(Y=0|x) = \frac{exp(\beta^T x+\mu)}{1+exp(\beta^T x+\mu)} \tag{1.1}$$

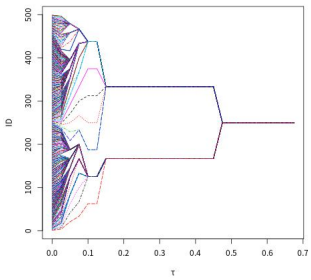
,where the predictors $\boldsymbol{x_i} = (x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5})^T$ are simulated from multivariate normal distribution with mean 0 and diagonal variance matrix with elements 1 and μ_i are generated from distribution $P(\mu_i = \alpha) = P(\mu_i = -\alpha) = 0.5$.The coefficient vector was set to be $\beta = (-0.3, 1, -1, 2, 0.5)^T$. We compare the performance of our our estimators using the two concave penalty functions, namely SCAD and MCP.

Next, we investigate the performance of parameter estimation of the ADMM algorithm with τ selected via the modified BIC criterion in (5.1) with c = 1, 2, 5, 10, 20, 50, 100, 150, 200, 250, 300, 350, 400, 450, 500. Tables reports the Monte Carlo mean, median and standard error (s.e.) of the estimated groups $\hat{K}$ by the SCAD and MCP based on 100 simulation replications. In addition, to study the estimation accuracy, Table 5.2 reports the mean and s.e. of the root mean squared errors (RMSE) $\|\hat{\mu} - \mu\|/\sqrt{n}$ and $\|\hat{\beta} - \beta\|/\sqrt{n}$ for the estimated values of μ and β, respectively. The Jaccard coefficient is defined as $\frac{n_{11}}{n_{10}+n_{01}+n_{11}}$, where n_{11} is the number of pairs that are comembers in both set A and set B; n_{10} is the number of pairs that are comembers in set A but not set B, and n_{01} is defined similarly.

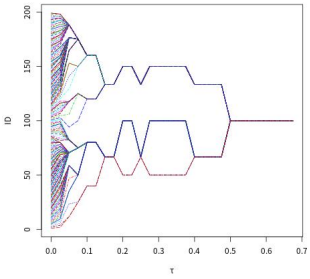
The solution paths of the sublogistic model based on a particular sample with $\alpha = 0.8$ are shown in Fig. 1.1. It can be seen that for a wide range of tuning parameter $\tau \in (0, 0.65)$, two subgroups are correctly identified by fitting the model. Tables 1.1 presents the mean,median and standard error for $\hat{K}$ in 20 replications for the model. Tables 1.2 presents the mean and standard error of the RMSE for the estimated values of μ and β for the sublogistic model with MCP penalties. The resulting Jaccard coefficients are summarized in Table 1.3.



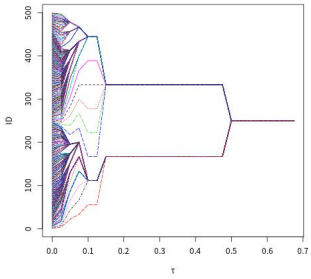
(a) MCP, n = 200, $\alpha = 0.8$



(b) MCP, n = 500, $\alpha = 0.8$



(c) SCAD, n = 200, $\alpha = 0.8$



(d) SCAD, n = 500, $\alpha = 0.8$

Figure 1.1: Shown are solution paths for the data cluster against τ by using SCAD and MCP penalties in Example 2.

Table 1.1: *The mean, median and standard error (s.e.) of $\hat{K}$ by the SCAD methods in Example 2.*

c	n	$\mu = 0.8$			$\mu = 1$			$\mu = 1.2$			$\mu = 1.4$			$\mu = 1.5$		
		mean	median	s.e.	mean	median	s.e.	mean	median	s.e.	mean	median	s.e.	mean	median	s.e.
1	200	4.30	4.00	2.60	4.40	4.00	2.70	4.20	3.50	2.50	4.40	2.50	4.10	4.80	4.50	3.20
	500	5.00	2.00	5.50	4.40	2.50	3.40	6.40	6.00	4.90	5.30	3.50	4.10	6.00	3.00	8.10
2	200	3.50	2.00	2.30	3.60	2.50	2.10	3.40	2.50	1.80	3.40	2.50	1.90	4.50	4.00	3.20
	500	4.00	2.00	3.00	3.80	2.00	3.20	6.20	4.50	5.00	5.00	3.00	4.10	5.80	3.00	8.00
5	200	2.00	2.00	0.00	2.30	2.00	0.80	2.40	2.00	1.00	2.40	2.00	0.81	2.60	2.00	0.94
	500	2.60	2.00	1.80	3.20	2.00	3.00	3.60	2.00	3.80	4.10	2.00	4.10	3.80	2.00	3.20
10	200	2.00	2.00	0.00	2.00	2.00	0.22	2.00	2.00	0.00	2.20	2.00	0.52	2.20	2.00	0.62
	500	2.00	2.00	0.22	2.20	2.00	0.79	3.00	2.00	3.40	3.40	2.00	2.60	2.60	2.00	1.60
20	200	1.90	2.00	0.31	2.00	2.00	0.22	2.00	2.00	0.00	2.00	2.00	0.00	2.10	2.00	0.45
	500	2.00	2.00	0.22	2.20	2.00	0.49	2.00	2.00	0.00	2.30	2.00	0.57	2.20	2.00	0.64
50	200	1.90	2.00	0.31	2.00	2.00	0.22	2.00	2.00	0.00	2.00	2.00	0.00	2.10	2.00	0.45
	500	2.00	2.00	0.22	2.20	2.00	0.49	2.00	2.00	0.00	2.30	2.00	0.57	2.20	2.00	0.64
100	200	1.70	2.00	0.47	2.00	2.00	0.32	2.00	2.00	0.00	2.00	2.00	0.00	2.10	2.00	0.45
	500	2.00	2.00	0.22	2.20	2.00	0.49	2.00	2.00	0.00	2.30	2.00	0.57	2.20	2.00	0.64
200	200	1.40	1.00	0.49	1.60	2.00	0.49	2.00	2.00	0.00	2.00	2.00	0.00	2.10	2.00	0.45
	500	2.00	2.00	0.22	2.20	2.00	0.49	2.00	2.00	0.00	2.30	2.00	0.57	2.20	2.00	0.64
250	200	1.20	1.00	0.37	1.60	2.00	0.51	2.00	2.00	0.22	2.00	2.00	0.00	2.10	2.00	0.45
	500	2.00	2.00	0.22	2.20	2.00	0.49	2.00	2.00	0.00	2.30	2.00	0.57	2.20	2.00	0.64
300	200	1.00	1.00	0.00	1.40	1.00	0.50	1.80	2.00	0.37	2.00	2.00	0.00	2.10	2.00	0.45
	500	2.00	2.00	0.22	2.20	2.00	0.49	2.00	2.00	0.00	2.30	2.00	0.57	2.20	2.00	0.64
350	200	1.00	1.00	0.00	1.20	1.00	0.41	1.80	2.00	0.44	2.00	2.00	0.00	2.10	2.00	0.45
	500	2.00	2.00	0.22	2.20	2.00	0.49	2.00	2.00	0.00	2.30	2.00	0.57	2.20	2.00	0.64
400	200	1.00	1.00	0.00	1.00	1.00	0.22	1.40	1.00	0.51	2.00	2.00	0.00	2.10	2.00	0.45
	500	2.00	2.00	0.32	2.20	2.00	0.49	2.00	2.00	0.00	2.30	2.00	0.57	2.20	2.00	0.64
450	200	1.00	1.00	0.00	1.00	1.00	0.22	1.40	1.00	0.50	2.00	2.00	0.00	2.10	2.00	0.45
	500	1.80	2.00	0.37	2.20	2.00	0.49	2.00	2.00	0.00	2.30	2.00	0.57	2.20	2.00	0.64
500	200	1.00	1.00	0.00	1.00	1.00	0.00	1.40	1.00	0.49	2.00	2.00	0.00	2.10	2.00	0.45
	500	1.80	2.00	0.37	2.00	2.00	0.32	2.00	2.00	0.00	2.30	2.00	0.57	2.20	2.00	0.64

Table 1.2: *The mean and standard error (s.e.) (shown in parentheses) of the RMSE for the estimated values of μ and β for the MCP methods.*

		μ					β				
c	n	$\mu = 0.8$	$\mu = 1$	$\mu = 1.2$	$\mu = 1.4$	$\mu = 1.5$	$\mu = 0.8$	$\mu = 1$	$\mu = 1.2$	$\mu = 1.4$	$\mu = 1.5$
1	200	$0.2255_{(0.1283)}$	$0.2702_{(0.1015)}$	$0.2628_{(0.1587)}$	$0.2495_{(0.1432)}$	$0.2722_{(0.1221)}$	$0.2255_{(0.1283)}$	$0.2702_{(0.1015)}$	$0.2628_{(0.1587)}$	$0.2495_{(0.1432)}$	$0.2722_{(0.1221)}$
	500	$0.1255_{(0.1052)}$	$0.1350_{(0.1024)}$	$0.1470_{(0.0917)}$	$0.1413_{(0.1035)}$	$0.1347_{(0.1100)}$	$0.1290_{(0.0420}$	$0.1315_{(0.0493)}$	$0.1280_{(0.0495)}$	$0.1388_{(0.0453)}$	$0.1374_{(0.0328)}$
2	200	$0.2199_{(0.1309)}$	$0.2647_{(0.1045)}$	$0.2549_{(0.1602)}$	$0.2459_{(0.1435)}$	$0.2672_{(0.1247)}$	$0.2145_{(0.0766)}$	$0.2278_{(0.1036)}$	$0.1966_{(0.0794)}$	$0.2214_{(0.0548)}$	$0.2234_{(0.0560)}$
	500	$0.1222_{(0.1044)}$	$0.1263_{(0.1052)}$	$0.1412_{(0.0970)}$	$0.1349_{(0.1020)}$	$0.1304_{(0.1085)}$	$0.1287_{(0.0421)}$	$0.1316_{(0.0493)}$	$0.1283_{(0.0491)}$	$0.1389_{(0.0454)}$	$0.1375_{(0.0328)}$
5	200	$0.2015_{(0.1449)}$	$0.2490_{(0.1155)}$	$0.2395_{(0.1635)}$	$0.2120_{(0.1649)}$	$0.2233_{(0.1340)}$	$0.2144_{(0.0764)}$	$0.2249_{(0.1033)}$	$0.1949_{(0.0789)}$	$0.2190_{(0.0537)}$	$0.2225_{(0.0561)}$
	500	$0.1090_{(0.1115)}$	$0.1202_{(0.1079)}$	$0.1068_{(0.0950)}$	$0.1183_{(0.1028)}$	$0.1089_{(0.1002)}$	$0.1290_{(0.0420)}$	$0.1320_{(0.0492)}$	$0.1284_{(0.0493)}$	$0.1387_{(0.0454)}$	$0.1371_{(0.0331)}$
10	200	$0.2015_{(0.1449)}$	$0.2457_{(0.1184)}$	$0.2278_{(0.1713)}$	$0.2058_{(0.1653)}$	$0.2137_{(0.1296)}$	$0.2144_{(0.0764)}$	$0.2250_{(0.1035)}$	$0.1936_{(0.0797)}$	$0.2191_{(0.0537)}$	$0.2229_{(0.0559)}$
	500	$0.1076_{(0.1085)}$	$0.1136_{(0.1031)}$	$0.0971_{(0.0906)}$	$0.1080_{(0.1008)}$	$0.0892_{(0.0787)}$	$0.1289_{(0.0420)}$	$0.1317_{(0.0492)}$	$0.1283_{(0.0494)}$	$0.1387_{(0.0452)}$	$0.1372_{(0.0330)}$
20	200	$0.2559_{(0.2560)}$	$0.2457_{(0.1184)}$	$0.2278_{(0.1713)}$	$0.1978_{(0.1650)}$	$0.2098_{(0.1287)}$	$0.2130_{(0.0733)}$	$0.2250_{(0.1035)}$	$0.1936_{(0.0797)}$	$0.2184_{(0.0550)}$	$0.2220_{(0.0552)}$
	500	$0.1076_{(0.1085)}$	$0.1130_{(0.1021)}$	$0.0913_{(0.0748)}$	$0.0916_{(0.0770)}$	$0.0852_{(0.0701)}$	$0.1289_{(0.0420)}$	$0.1317_{(0.0492)}$	$0.1278_{(0.0491)}$	$0.1390_{(0.0452)}$	$0.1372_{(0.0330)}$
50	200	$0.2559_{(0.2560)}$	$0.2457_{(0.1184)}$	$0.2278_{(0.1713)}$	$0.1978_{(0.1650)}$	$0.2098_{(0.1287)}$	$0.2130_{(0.0733)}$	$00.2250_{(0.1035)}$	$0.1936_{(0.0797)}$	$0.2184_{(0.0550)}$	$0.2220_{(0.0552)}$
	500	$0.1076_{(0.1085)}$	$0.1130_{(0.1021)}$	$0.0913_{(0.0748)}$	$0.0916_{(0.0770)}$	$0.0852_{(0.0701)}$	$0.1289_{(0.0420)}$	$0.1317_{(0.0492)}$	$0.1278_{(0.0491)}$	$0.1390_{(0.0452)}$	$0.1372_{(0.0330)}$
100	200	$0.3951_{(0.3238)}$	$0.2848_{(0.2177)}$	$0.2278_{(0.1713)}$	$0.1978_{(0.1650)}$	$0.2098_{(0.1287)}$	$0.2147_{(0.0701)}$	$0.2251_{(0.1035)}$	$0.1936_{(0.0797)}$	$0.2184_{(0.0550)}$	$0.2220_{(0.0552)}$
	500	$0.1076_{(0.1085)}$	$0.1130_{(0.1021)}$	$0.0913_{(0.0748)}$	$0.0916_{(0.0770)}$	$0.0852_{(0.0701)}$	$0.1289_{(0.0420)}$	$0.1317_{(0.0492)}$	$0.1278_{(0.0491)}$	$0.1390_{(0.0452)}$	$0.1372_{(0.0330)}$
200	200	$0.6075_{(0.3318)}$	$0.5350_{(0.3818)}$	$0.2278_{(0.1713)}$	$0.1978_{(0.1650)}$	$0.2098_{(0.1287)}$	$0.2286_{(0.0663)}$	$0.2267_{(0.1043)}$	$0.1936_{(0.0797)}$	$0.2184_{(0.0550)}$	$0.2220_{(0.0552)}$
	500	$0.1076_{(0.1085)}$	$0.1130_{(0.1021)}$	$0.0913_{(0.0748)}$	$0.0916_{(0.0770)}$	$0.0852_{(0.0701)}$	$0.1289_{(0.0420)}$	$0.1317_{(0.0492)}$	$0.1278_{(0.0491)}$	$0.1390_{(0.0452)}$	$0.1372_{(0.0330)}$
250	200	$0.7297_{(0.2649)}$	$0.6158_{(0.3902)}$	$0.2785_{(0.2777)}$	$0.1978_{(0.1650)}$	$0.2098_{(0.1287)}$	$0.2349_{(0.0649)}$	$0.2302_{(0.1044)}$	$0.2020_{(0.0871)}$	$0.2184_{(0.0550)}$	$0.2220_{(0.0552)}$
	500	$0.1076_{(0.1085)}$	$0.1130_{(0.1021)}$	$0.0913_{(0.0748)}$	$0.0916_{(0.0770)}$	$0.0852_{(0.0701)}$	$0.1289_{(0.0420)}$	$0.1317_{(0.0492)}$	$0.1278_{(0.0491)}$	$0.1390_{(0.0452)}$	$0.1372_{(0.0330)}$
300	200	$0.8325_{(0.0417)}$	$0.7195_{(0.3963)}$	$0.3892_{(0.3879)}$	$0.1978_{(0.1650)}$	$0.2098_{(0.1287)}$	$0.2393_{(0.0647)}$	$0.2286_{(0.1051)}$	$0.2060_{(0.0896)}$	$0.2184_{(0.0550)}$	$0.2220_{(0.0552)}$
	500	$0.1076_{(0.1085)}$	$0.1130_{(0.1021)}$	$0.0913_{(0.0748)}$	$0.0916_{(0.0770)}$	$0.0852_{(0.0701)}$	$0.1289_{(0.0420)}$	$0.1317_{(0.0492)}$	$0.1278_{(0.0491)}$	$0.1390_{(0.0452)}$	$0.1372_{(0.0330)}$
350	200	$0.8325_{(0.0417)}$	$0.8675_{(0.3446)}$	$0.5019_{(0.4444)}$	$0.1978_{(0.1650)}$	$0.2098_{(0.1287)}$	$0.2393_{(0.0647)}$	$0.2339_{(0.0935)}$	$0.2164_{(0.0816)}$	$0.2184_{(0.0550)}$	$0.2220_{(0.0552)}$
	500	$0.1076_{(0.1085)}$	$0.1130_{(0.1021)}$	$0.0913_{(0.0748)}$	$0.0916_{(0.0770)}$	$0.0852_{(0.0701)}$	$0.1289_{(0.0420)}$	$0.1317_{(0.0492)}$	$0.1278_{(0.0491)}$	$0.1390_{(0.0452)}$	$0.1372_{(0.0330)}$
400	200	$0.8325_{(0.0417)}$	$0.9917_{(0.1813)}$	$0.7881_{(0.5028)}$	$0.1978_{(0.1650)}$	$0.2098_{(0.1287)}$	$0.2393_{(0.0647)}$	$0.2484_{(0.0877)}$	$0.2385_{(0.0795)}$	$0.2184_{(0.0550)}$	$0.2220_{(0.0552)}$
	500	$0.1457_{(0.1879)}$	$.1130_{(0.1021)}$	$0.0913_{(0.0748)}$	$0.0916_{(0.0770)}$	$0.0852_{(0.0701)}$	$0.1294_{(0.0423)}$	$0.1317_{(0.0492)}$	$0.1278_{(0.0491)}$	$0.1390_{(0.0452)}$	$0.1372_{(0.0330)}$
450	200	$0.8325_{(0.0417)}$	$0.9917_{(0.1813)}$	$0.8444_{(0.4813)}$	$0.1978_{(0.1650)}$	$0.2098_{(0.1287)}$	$0.2393_{(0.0647)}$	$0.2484_{(0.0877)}$	$0.2436_{(0.0789)}$	$0.2184_{(0.0550)}$	$0.2220_{(0.0552)}$
	500	$0.2196_{(0.2725)}$	$0.1130_{(0.1021)}$	$0.0913_{(0.0748)}$	$0.0916_{(0.0770)}$	$0.0852_{(0.0701)}$	$0.1272_{(0.0308)}$	$0.1317_{(0.0492)}$	$0.1278_{(0.0491)}$	$0.1390_{(0.0452)}$	$0.1372_{(0.0330)}$
500	200	$0.8325_{(0.0417)}$	$1.0310_{(0.0255)}$	$0.8929_{(0.4699)}$	$0.1978_{(0.1650)}$	$0.2098_{(0.1287)}$	$0.2393_{(0.0647)}$	$0.2508_{(0.0922)}$	$0.2479_{(0.0799)}$	$0.2184_{(0.0550)}$	$0.2220_{(0.0552)}$
	500	$0.2196_{(0.2725)}$	$0.1603_{(0.2223)}$	$0.0913_{(0.0748)}$	$0.0916_{(0.0770)}$	$0.0852_{(0.0701)}$	$0.1272_{(0.0308)}$	$0.1395_{(0.0644)}$	$0.1278_{(0.0491)}$	$0.1390_{(0.0452)}$	$0.1372_{(0.0330)}$

Table 1.3: *Jaccard*

c	n	$\mu = 0.8$	$\mu = 1$	$\mu = 1.2$	$\mu = 1.4$	$\mu = 1.5$
1	200	0.646	0.806	0.708	0.843	0.860
	500	0.726	0.645	0.477	0.671	0.606
2	200	0.682	0.861	0.775	0.867	0.860
	500	0.772	0.724	0.510	0.734	0.635
5	200	0.875	0.975	0.867	0.958	0.967
	500	0.960	0.803	0.739	0.807	0.775
10	200	0.875	0.975	0.875	0.975	1.000
	500	1.000	0.915	0.817	0.849	0.928
20	200	0.850	0.975	0.875	1.000	1.000
	500	1.000	0.945	0.875	0.967	0.967
50	200	0.850	0.975	0.875	1.000	1.000
	500	1.000	0.945	0.875	0.967	0.967
100	200	0.775	0.950	0.875	1.000	1.000
	500	1.000	0.945	0.875	0.967	0.967
200	200	0.600	0.800	0.875	1.000	1.000
	500	1.000	0.945	0.875	0.967	0.967
250	200	0.550	0.750	0.850	1.000	1.000
	500	1.000	0.945	0.875	0.967	0.967
300	200	0.500	0.675	0.800	1.000	1.000
	500	1.000	0.945	0.875	0.967	0.967
350	200	0.500	0.575	0.750	1.000	1.000
	500	1.000	0.945	0.875	0.967	0.967
400	200	0.500	0.525	0.600	1.000	1.000
	500	0.975	0.945	0.875	0.967	0.967
450	200	0.500	0.525	0.575	1.000	1.000
	500	0.925	0.945	0.875	0.967	0.967
500	200	0.500	0.500	0.550	1.000	1.000
	500	0.925	0.950	0.875	0.967	0.967

Chapter 2

Empirical example

Nowadays, telecom industry faces fierce com-petition in satisfying its customers. The role of churn prediction system is not only restricted to accurately predict churners but also to interpret customer churn behavior. Experiments are conducted on the Cell2cell churn datasets which

First, missing value imputation is applied. Missing values are treated differently based on the percentage of missing values in an attribute. Imputation procedures are used for attributes with more than 5 of the values missing. Depending on the variable, zero imputation, median imputation or modus imputation is used. Dummy variables are created flagging variables where missing variables are imputed. For attributes with less than 5% of the values missing, the instances containing the missing value are removed from the data in order to limit the impact of imputation procedures. Categorical variables are transformed into binary variables using dummy encoding. This technique creates $v1$ dummy variables, where v equals the number of distinct values of the categorical variables. These newly created variables indicate the presence or absence of a particular characteristic.

Second, outlier detection and treatment is applied. Outliers are unusual values that are typically defined as being more than three standard deviations away from a variable's mean value.

A last preprocessing step involves undersampling. Typically, the class variable in a churn prediction setting is heavily skewed, i.e. the number of churners is often much lower than the number of non-churners.

Considering that a classifier trained on a small set of well-chosen and highly predictive variables will have better predictive performance, We use fisher score which is simple but effective to reduce e the number of variables. The fisher score is defined as

$$Fisher\ score = \frac{\|\overline{X}_c - \overline{X}_{nc}\|}{\sqrt{S_c^2 + S_{nc}^2}} \tag{2.1}$$

where $\overline{X}_c$ and $\overline{X}_{nc}$ the mean value, and S_c^2 and S_{nc}^2 the variance of an independent variable for respectively churners and non-churners. Table 9 gives an overview of

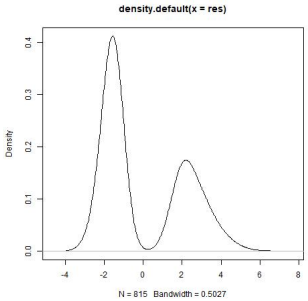
the selected variables through applying the fisher selection.

Table 2.1: *Overview of selected variables in cell2cell dataset.*

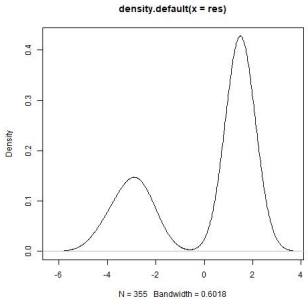
Variable	Definition
Callwait	Mean number of call waiting calls
changem	% change in minutes of use
creditde	Low credit rating – de
custcare	Mean number of customer care calls
directas	Mean number of director assisted calls
eqpdays	Number of days of the current equipment
incalls	Mean number of inbound voice calls
mou	Mean monthly minutes of use
opeakvce	Mean number of in and out off-peak voice call
outcalls	Mean number of outbound voice calls
phones	# handsets issued
recchrge	Mean total recurring charge
retcalls	Number of calls previously made to the retention team
revenue	Mean monthly revenue
price	Handset price
webcap	Handset is web capable

We fit the data with two LLM models that constructs a decision tree in the first step to identify homogenous customer segments and in a second step applies classifier to each of these segments. One is the ordinary logistic regressions and the other model is the sublogistic regressions. Then, we plot the kernel density estimates of the Pearson residuals of the first model in . The Pearson residuals are defined as$r_i = \frac{Y_i - \widehat{E(Y_i)}}{\sqrt{\widehat{Var(Y_i)}}}$. It is clearly seen that after adjusting for the effects of covariates, the distribution of residuals still shows multiple modes.

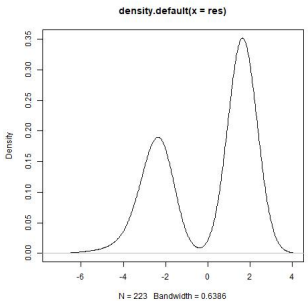
Next, we use sublogistic regressions and identify subgroups by our proposed ADMM algorithm and plot the kernel density estimates of the Pearson residuals. As shown in , after incorporating the heterogeneous latent intercepts, the distribution of fitted residuals mostly has a single mode, i.e., the residuals follow a homogeneous distribution.



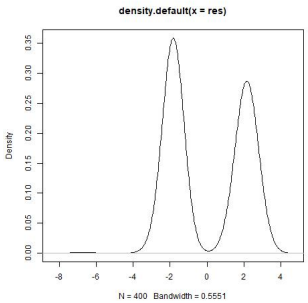
(a) Pearson residual of Group1



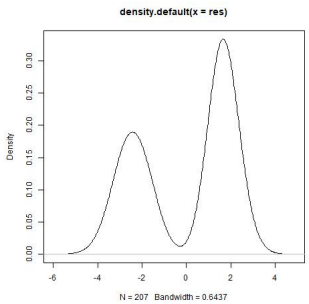
(b) Pearson residual of Group2



(c) Pearson residual of Group3

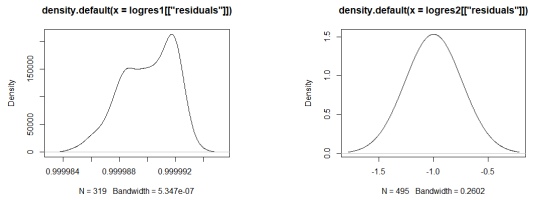


(d) Pearson residual of Group4



(e) Pearson residual of Group5

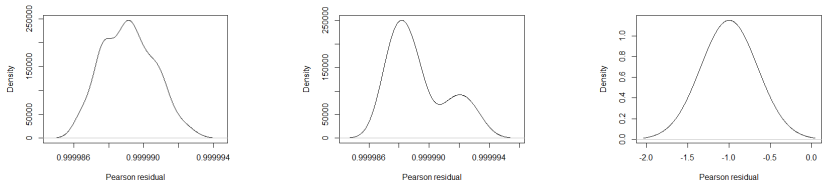
Figure 2.1: Kernel density plot of the Pearson residuals of each group identifed by the LLM model with ordinary logistic regressions.



(a) Pearson residual of Group1 subgroup1

(b) Pearson residual of Group1 subgroup2

Figure 2.2: Kernel density plot of the Pearson residuals of subgroups in Group1 identifed by the LLM model with sublogistic regressions.

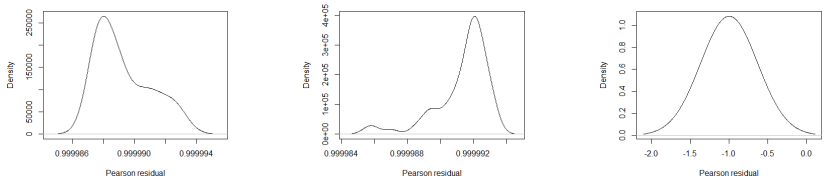


(a) Pearson residual of Group2 subgroup1

(b) Pearson residual of Group2 subgroup2

(c) Pearson residual of Group2 subgroup3

Figure 2.3: Kernel density plot of the Pearson residuals of subgroups in Group2 identifed by the LLM model with sublogistic regressions.

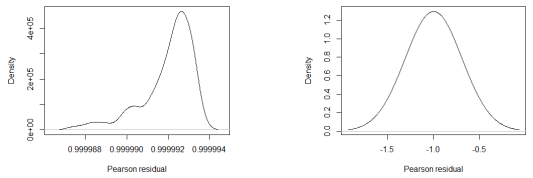


(a) Pearson residual of Group3 subgroup1

(b) Pearson residual of Group3 subgroup2

(c) Pearson residual of Group3 subgroup3

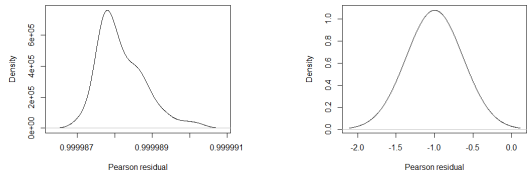
Figure 2.4: Kernel density plot of the Pearson residuals of subgroups in Group3 identifed by the LLM model with sublogistic regressions.



(a) Pearson residual of Group4 subgroup1

(b) Pearson residual of Group4 subgroup2

Figure 2.5: Kernel density plot of the Pearson residuals of subgroups in Group4 identifed by the LLM model with sublogistic regressions.



(a) Pearson residual of Group5 subgroup1

(b) Pearson residual of Group5 subgroup2

Figure 2.6: Kernel density plot of the Pearson residuals of subgroups in Group5 identifed by the LLM model with sublogistic regressions.