Today's Outline

7. First Order Circuits

- sequential switching

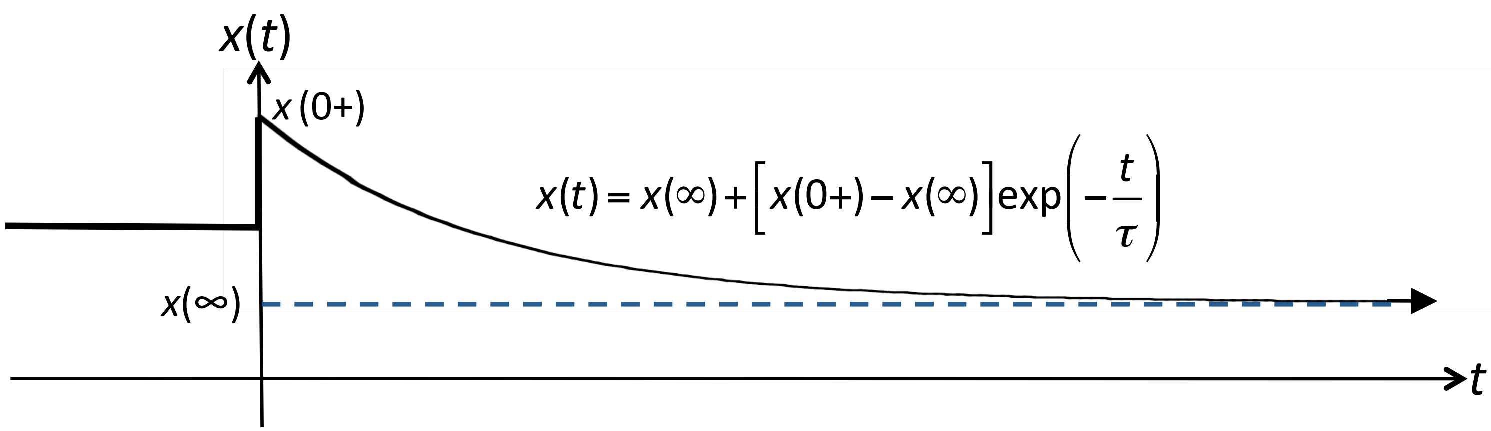
constant input: general procedure

step #1: Find the initial value of the circuit variable of interest, $x(0+)$, using circuit analysis and continuity of capacitor voltage or inductor current.

step #2: Find the final value of the variable of interest, $x(\infty)$, using dc steady state models for the capacitor or inductor.

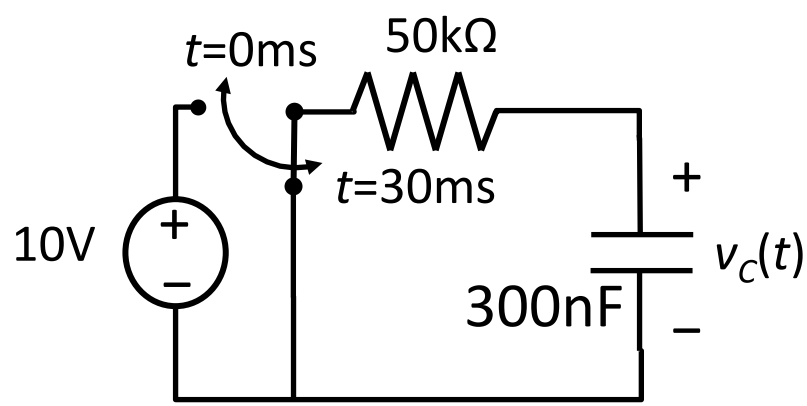
step #3: Find the Thévenin equivalent resistance R_T as seen from the terminals of the capacitor or inductor. The time constant $\tau = R_T C$ or $\tau = L/R_T$.

step #4: Construct the solution.



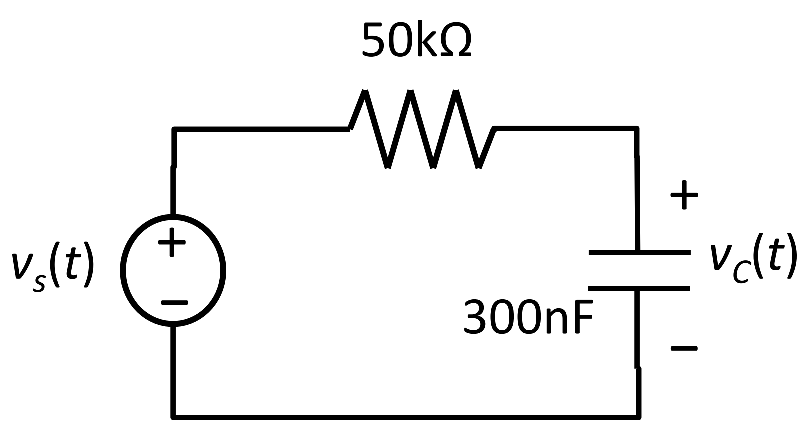
multiple switching events

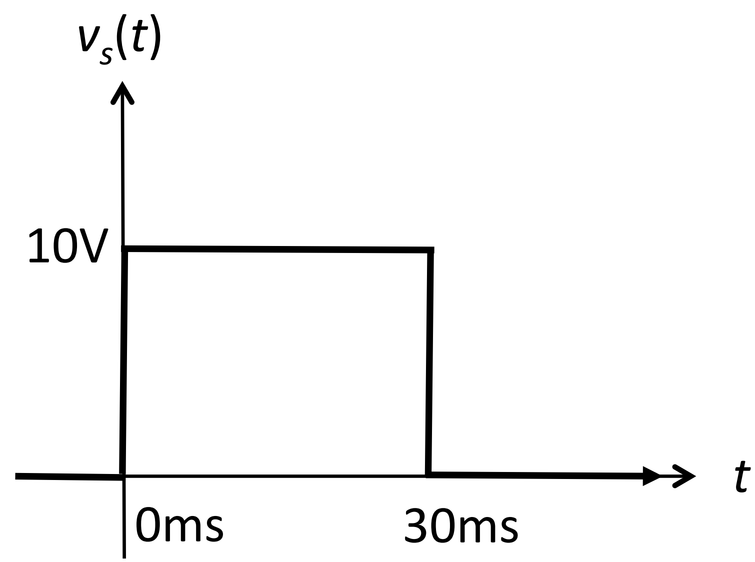
In many circuits, we are concerned with more than one switching event. Consider the RC circuit below where the switch moves twice (to the source at t=0ms, and back to the short at t=30ms).



sequential switching

An equivalent *RC* circuit with a time-dependent source instead of a switch can be used:

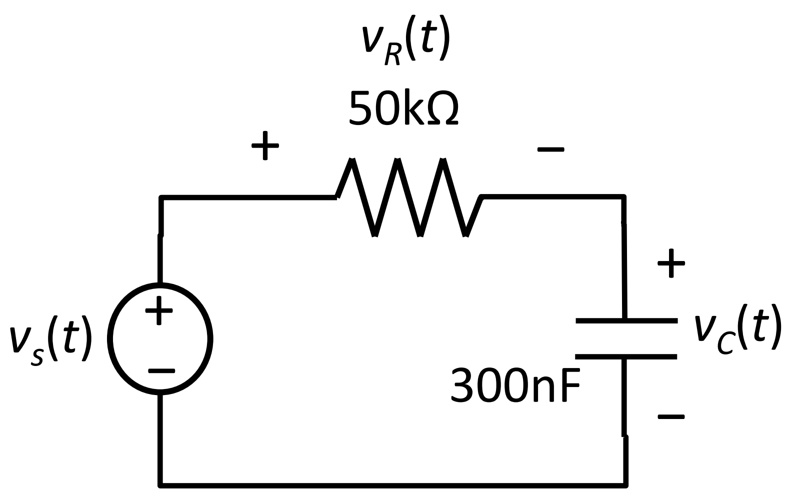


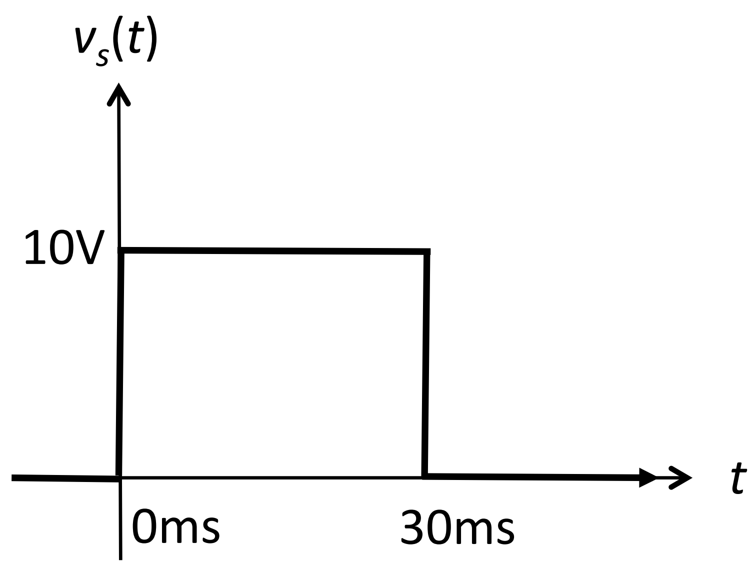


The time dependent source description is more natural when considering voltages or currents that carry information (such as audio signals, radio signals and radar signals).

sequential switching

For a circuit with sequential switching, we apply the general solution technique ***for each time interval***. We illustrate for the *RC* circuit below. Assuming dc steady state for t<0, we find the voltages $v_C(t)$ and $v_R(t)$.





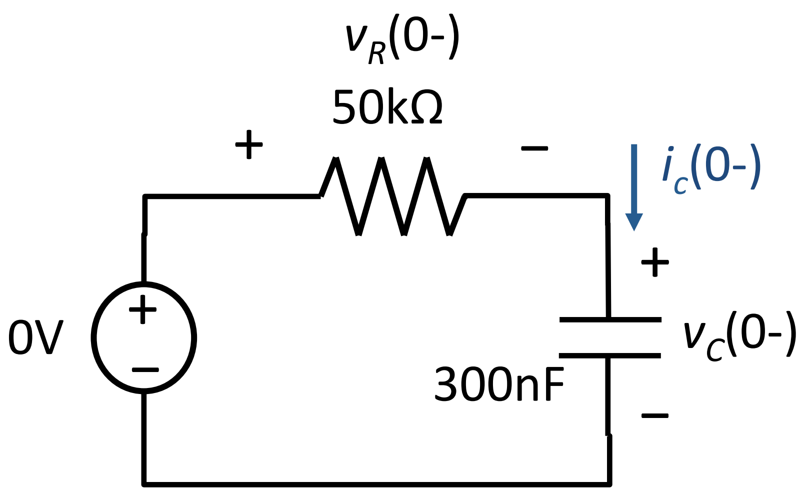
sequential switching

$t < 0:$ capacitor is an open at dc steady state: $i_c(0\text{-}) = 0$

Ohm's Law: $v_R(0\text{-}) = 50\text{k}\Omega\ i_C(0\text{-}) = 0\text{V}$

KVL: $0 = 0 + v_R(0-) + v_C(0-) = 0 + 0 + v_C(0-)$

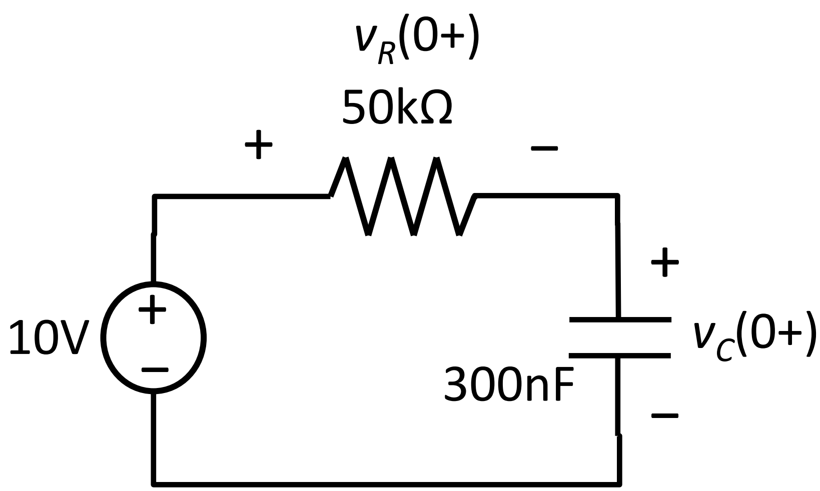
$\therefore v_C(0-) = 0\text{V}$



sequential switching

$t = 0+$: The source voltage instantaneously changes value from 0V to 10V.

Capacitor voltage is continuous: $v_C(0+) = v_C(0-) = 0\text{V}$



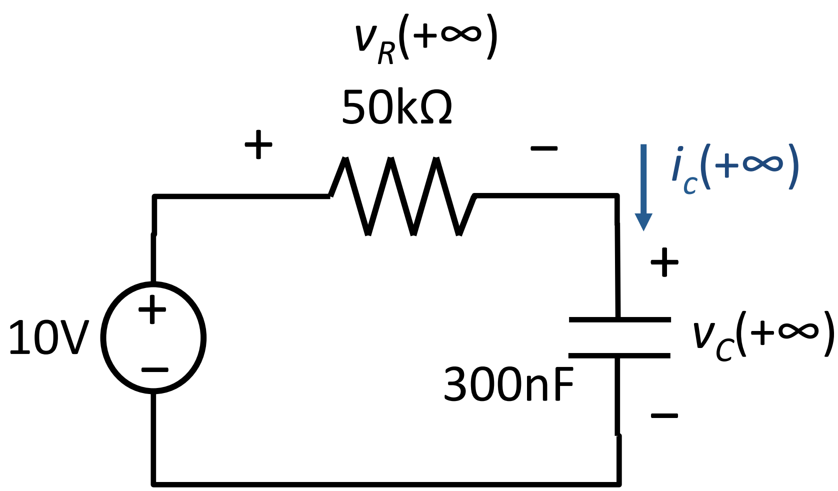
sequential switching

$0 < t < 30\text{ms}$: Find the anticipated dc steady state, ***assuming no further switching events*** (ie. the circuit cannot be used to detect future events, time travel, etc…).

Capacitor is an open at dc steady state: $i_c(+\infty) = 0$

Ohm's Law: $v_R(+\infty) = 50\text{k}\Omega\ i_C(+\infty) = 0\text{V}$

KVL: $$0 = -10\text{V} + v_R(+\infty) + v_C(+\infty) = -10\text{V} + 0\text{V} + v_C(+\infty)$$
$$\therefore v_C(+\infty) = 10\text{V}$$



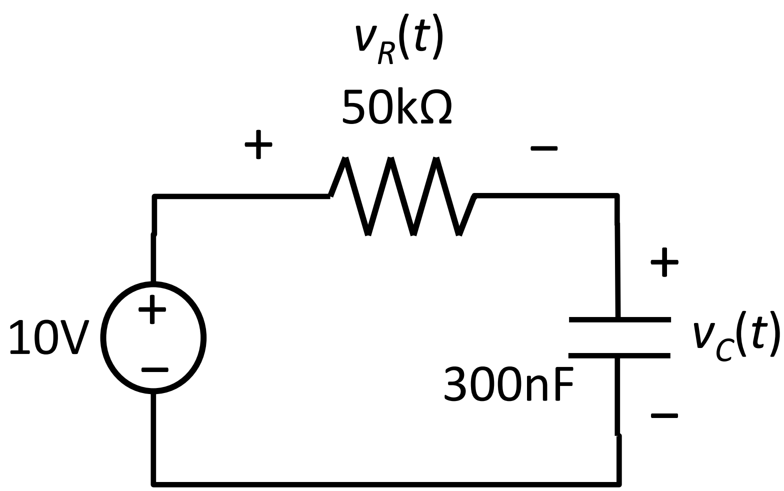
The time constant is:

$\tau = RC = 50\text{k}\Omega\ 300\text{nF} = 15\text{ms}$

sequential switching

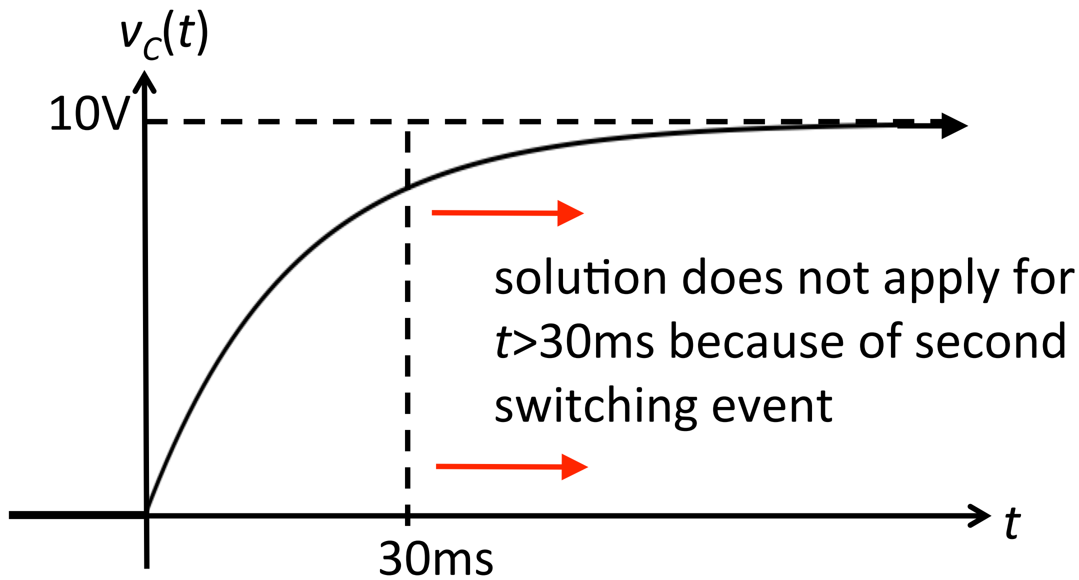
$0 < t < 30\text{ms}$: Construct the capacitor voltage using the values $v_C(0+) = 0\text{V}$, $v_C(+\infty) = 10\text{V}$, $\tau = 15\text{ms}$.

$$v_C(t) = v_C(+\infty) + \left[v_C(0+) - v_C(+\infty)\right]\exp(-t/\tau)$$
$$= 10\text{V} - 10\text{V}\exp(-t/15\text{ms})$$



sequential switching

$0 < t < 30\text{ms}$:

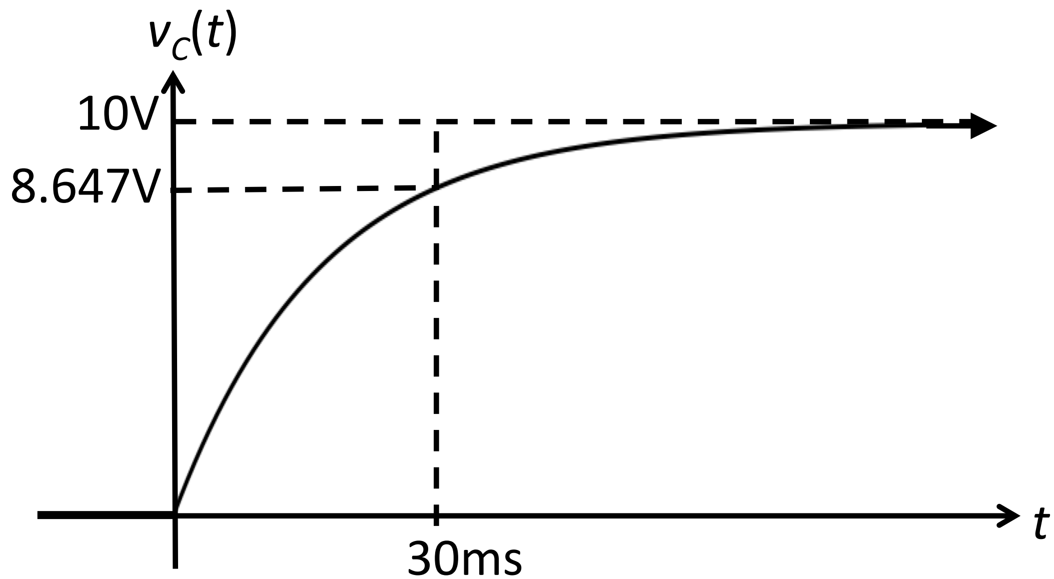


The solution found to this point applies for $0 < t < 30\text{ms}$.

The switching event at t=30ms disturbs the subsequent evolution of capacitor voltage, which we now solve.

sequential switching

t = 30ms− :



$$v_C(30\text{ms}-) = 10\text{V} - 10\text{V}\exp(-30\text{ms}/15\text{ms})$$
$$= 10\text{V} - 10\text{V}\exp(-2)$$
$$= 8.647\text{V}$$

sequential switching

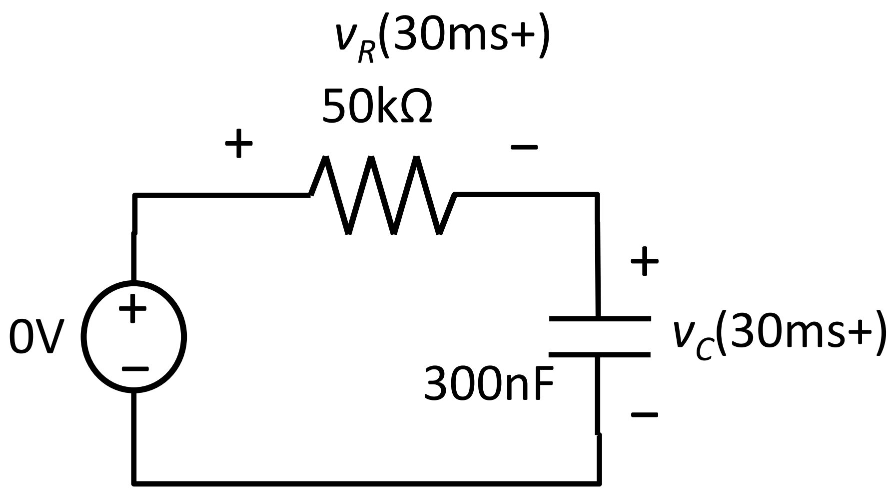
t = 30ms+ :

The source voltage instantaneously changes value from 10V back to 0V.

We consider the response for t>30ms using the same general solution method, but with different initial conditions and different switching time.

Capacitor voltage is continuous:

$$v_C(30\text{ms}+) = v_C(30\text{ms}-) = 8.647\text{V}$$



sequential switching

$t > 30\text{ms}$:

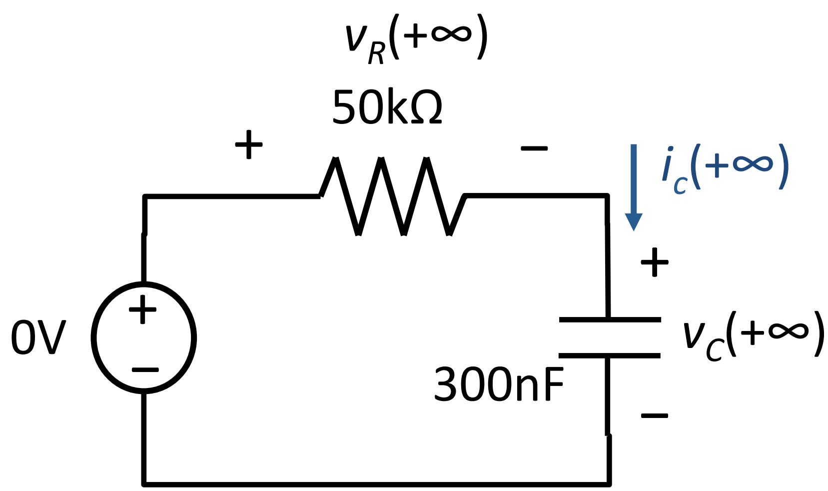
Consider the final dc steady state now anticipated.

Capacitor is an open at dc steady state: $i_c(+\infty) = 0$

Ohm's Law: $v_R(+\infty) = 50\text{k}\Omega\ i_C(+\infty) = 0\text{V}$

KVL: $0 = 0\text{V} + v_R(+\infty) + v_C(+\infty) = 0\text{V} + 0\text{V} + v_C(+\infty)$

$\therefore v_C(+\infty) = 0\text{V}$



The time constant is:

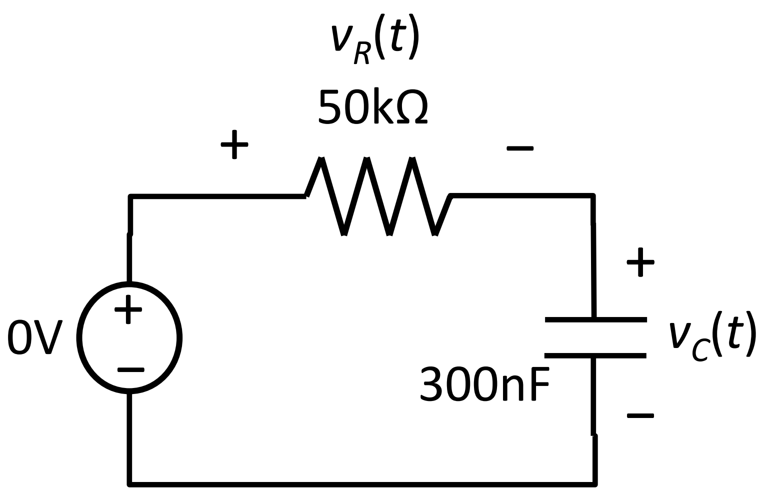
$\tau = RC = 50\text{k}\Omega\ 300\text{nF} = 15\text{ms}$

sequential switching

$t > 30\text{ms}$:

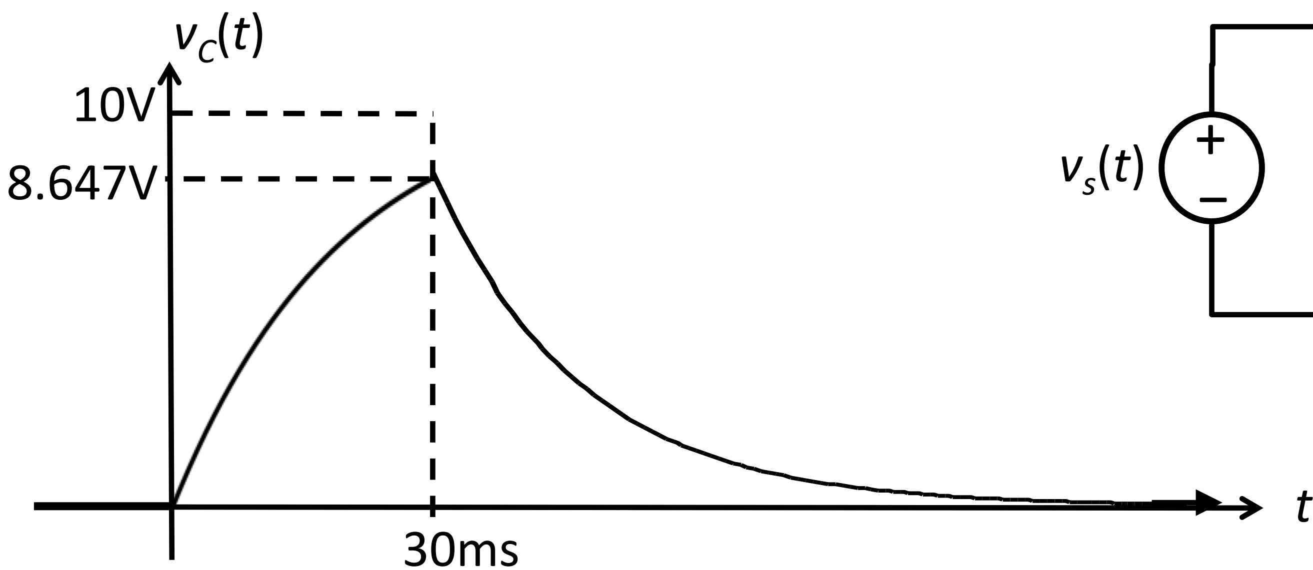
Construct the capacitor voltage using the values $v_C(30\text{ms}+) = 8.647\text{V}$, $v_C(+\infty) = 0\text{V}$, $\tau = 15\text{ms}$.

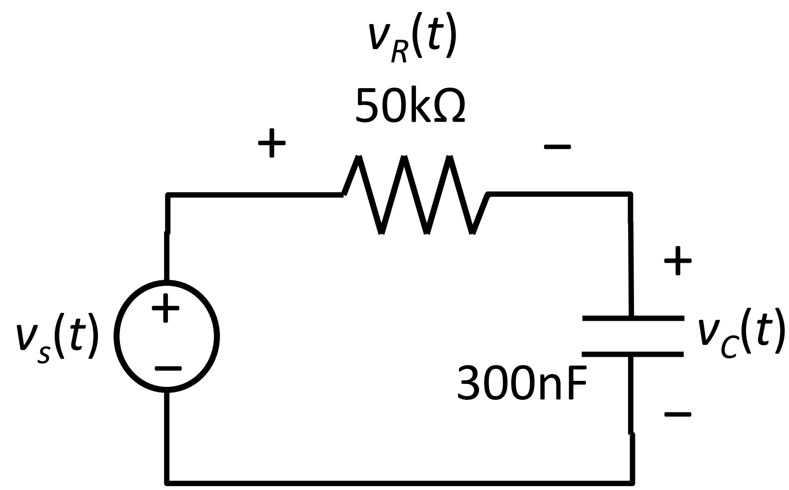
$$v_C(t) = v_C(+\infty) + \left[v_C(30\text{ms}+) - v_C(+\infty)\right]\exp(-(t - 30\text{ms})/\tau)$$
$$= 8.647\text{V}\exp(-(t - 30\text{ms})/15\text{ms})$$



IMPORTANT: note the time shift, because the switching event occurs at t=30ms≠0. Check the value $v_C(30\text{ms})$ to be sure.

sequential switching



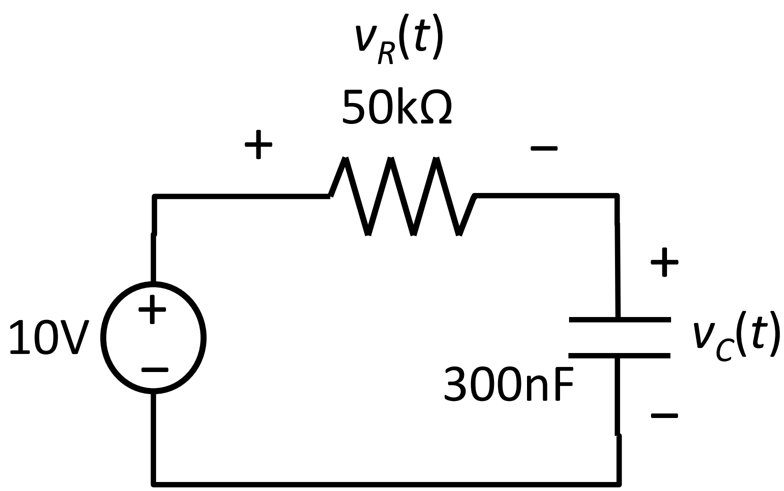


$$v_C(t) = \begin{cases} 0\text{V} & t < 0 \\ 10\text{V} - 10\text{V}\exp(-t/15\text{ms}) & 0 < t < 30\text{ms} \\ 8.647\text{V}\exp(-(t-30\text{ms})/15\text{ms}) & 30\text{ms} < t \end{cases}$$

sequential switching

$0 < t < 30\text{ms}$: We then construct the resistor voltage over the different time intervals, using KVL here:

$$
\begin{aligned}
v_R(t) &= 10\text{V} - v_C(t) \\
&= 10\text{V} - \left[10\text{V} - 10\text{V}\exp(-t/15\text{ms})\right] \\
&= 10\text{V}\exp(-t/15\text{ms})
\end{aligned}
$$

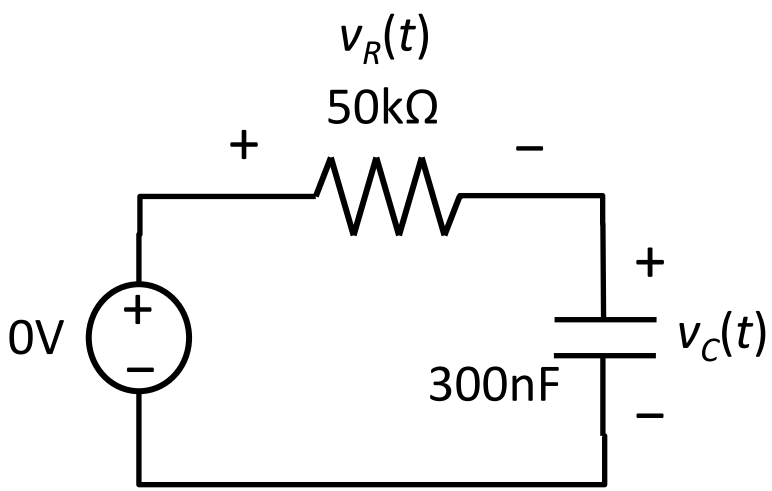


sequential switching

$t > 30\text{ms}$:

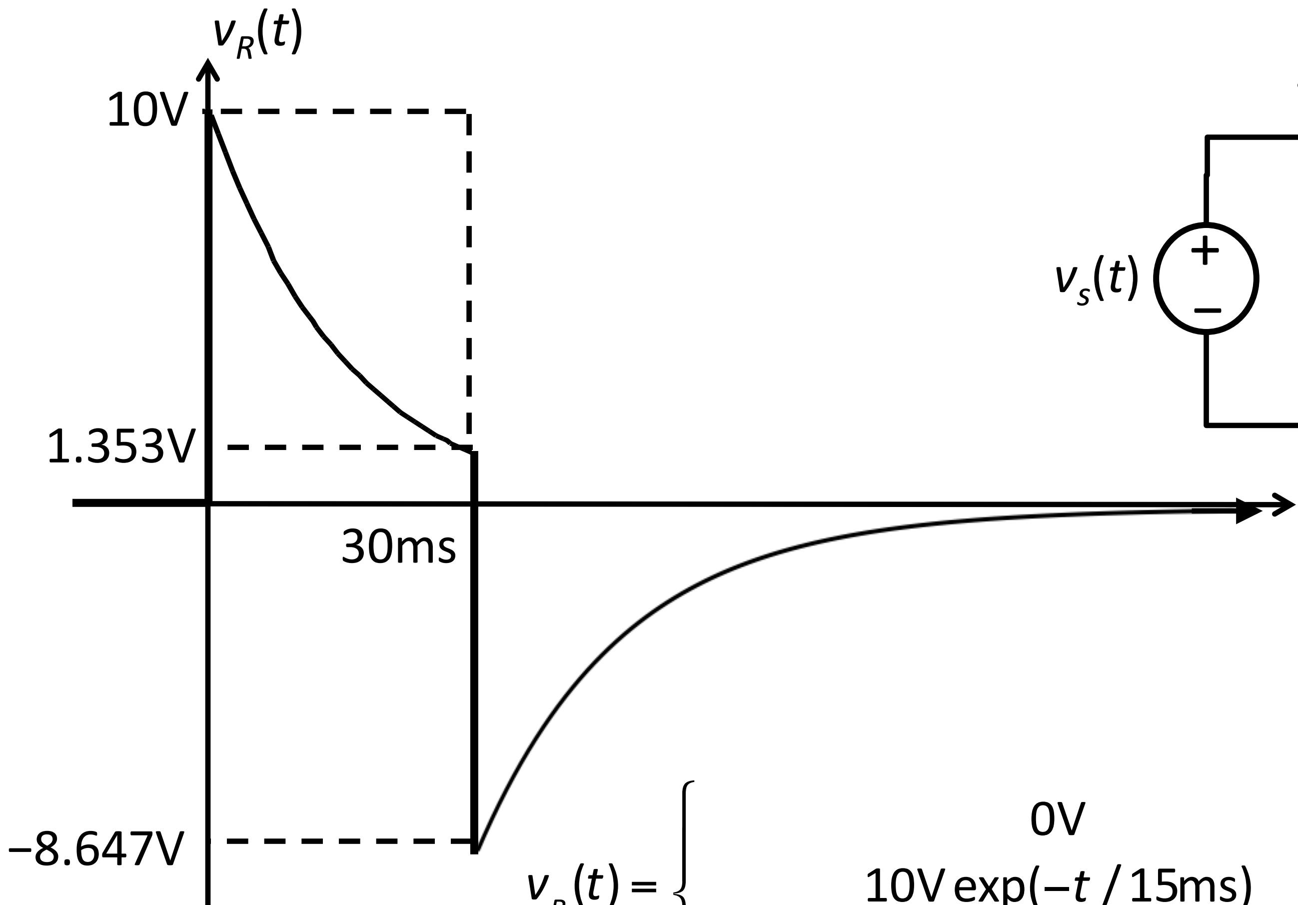
We construct the resistor voltage again using KVL here:

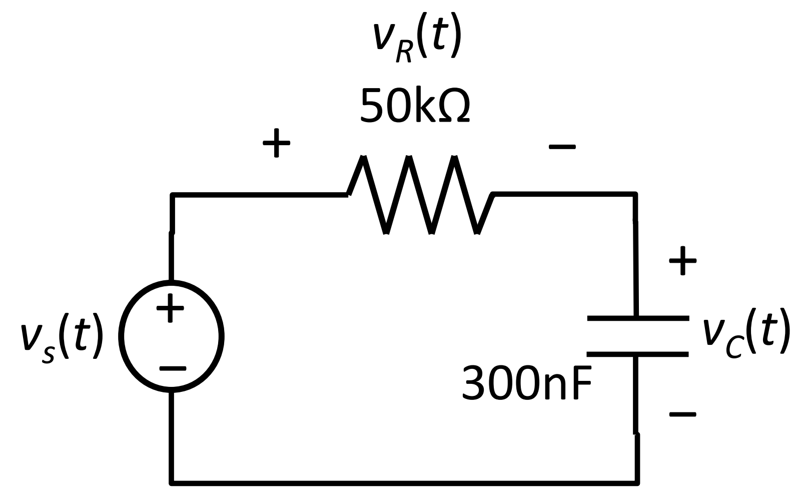
$$\begin{aligned} v_R(t) &= 0\text{V} - v_C(t) \\ &= 0\text{V} - \left[8.647\text{V}\exp(-(t-30\text{ms})/15\text{ms})\right] \\ &= -8.647\text{V}\exp(-(t-30\text{ms})/15\text{ms}) \end{aligned}$$



McGill

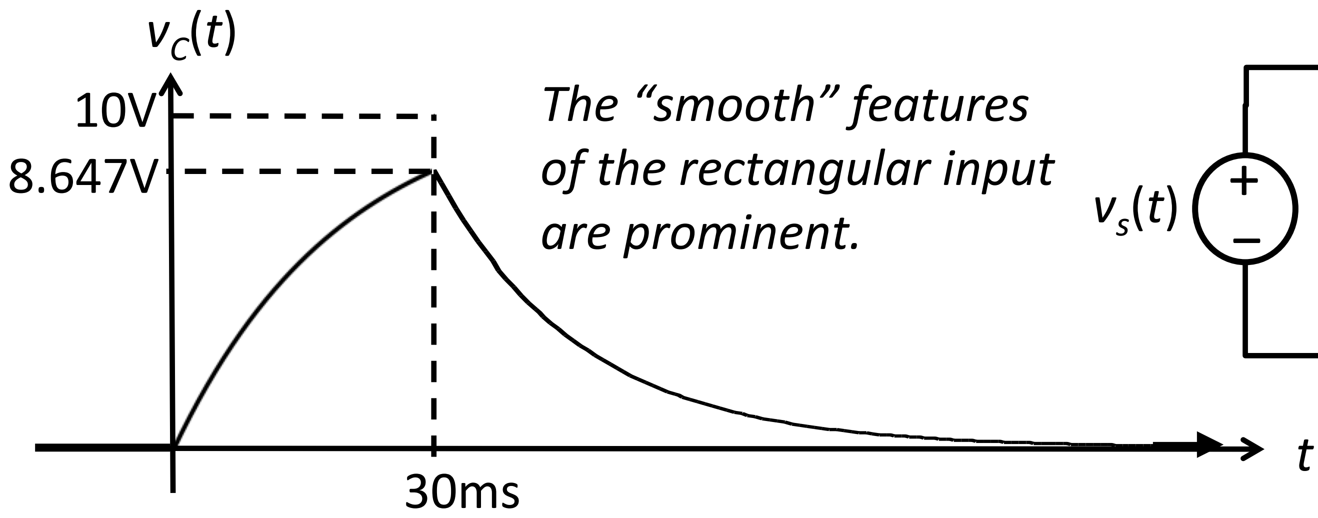
sequential switching



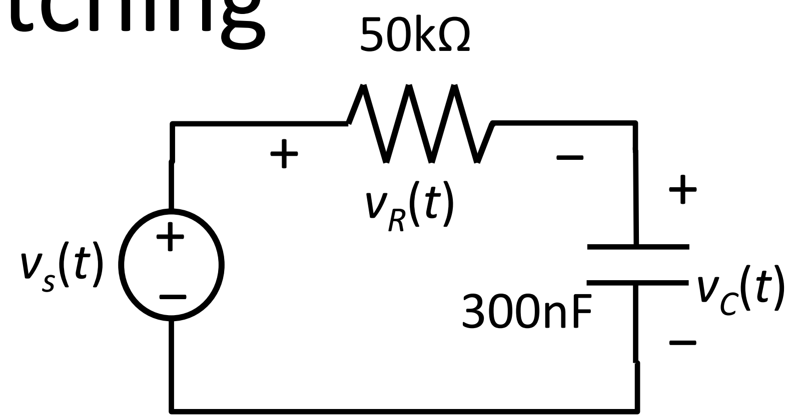


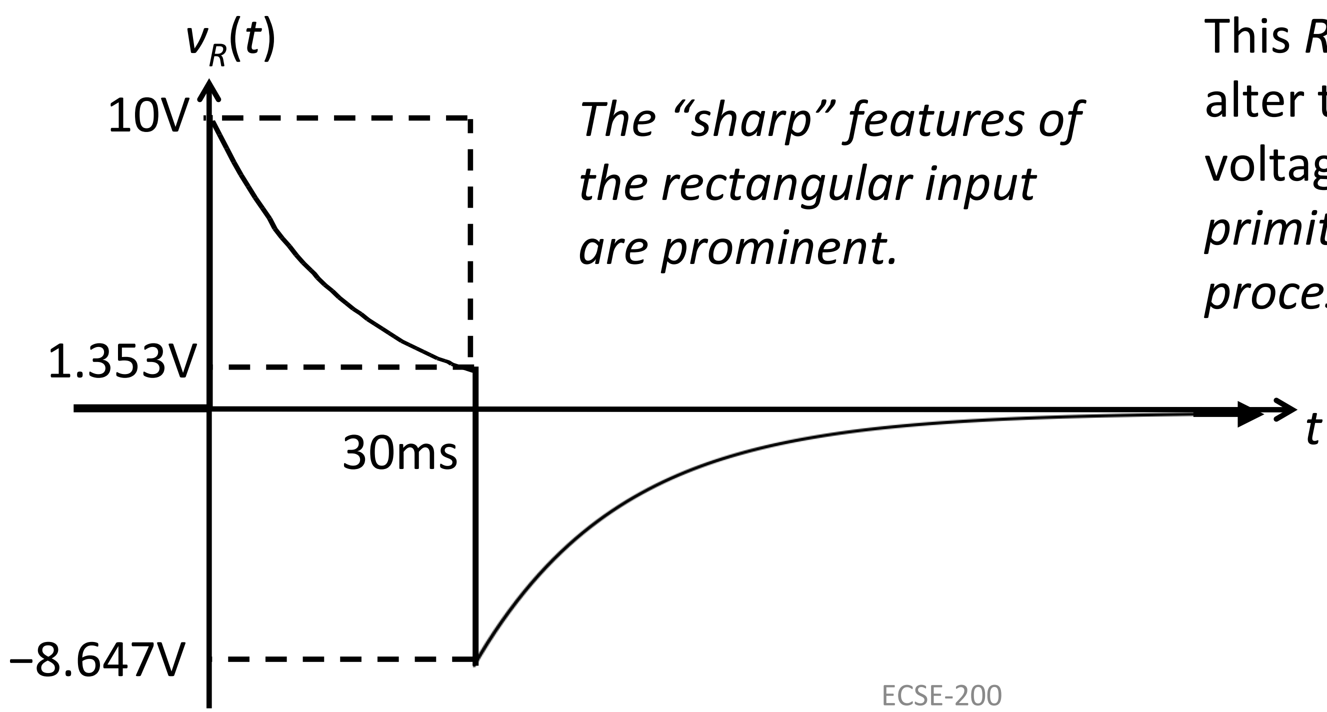
$$v_R(t) = \begin{cases} 0\text{V} & t < 0 \\ 10\text{V}\exp(-t/15\text{ms}) & 0 < t < 30\text{ms} \\ -8.647\text{V}\exp(-(t-30\text{ms})/15\text{ms}) & 30\text{ms} < t \end{cases}$$

sequential switching



The "smooth" features of the rectangular input are prominent.

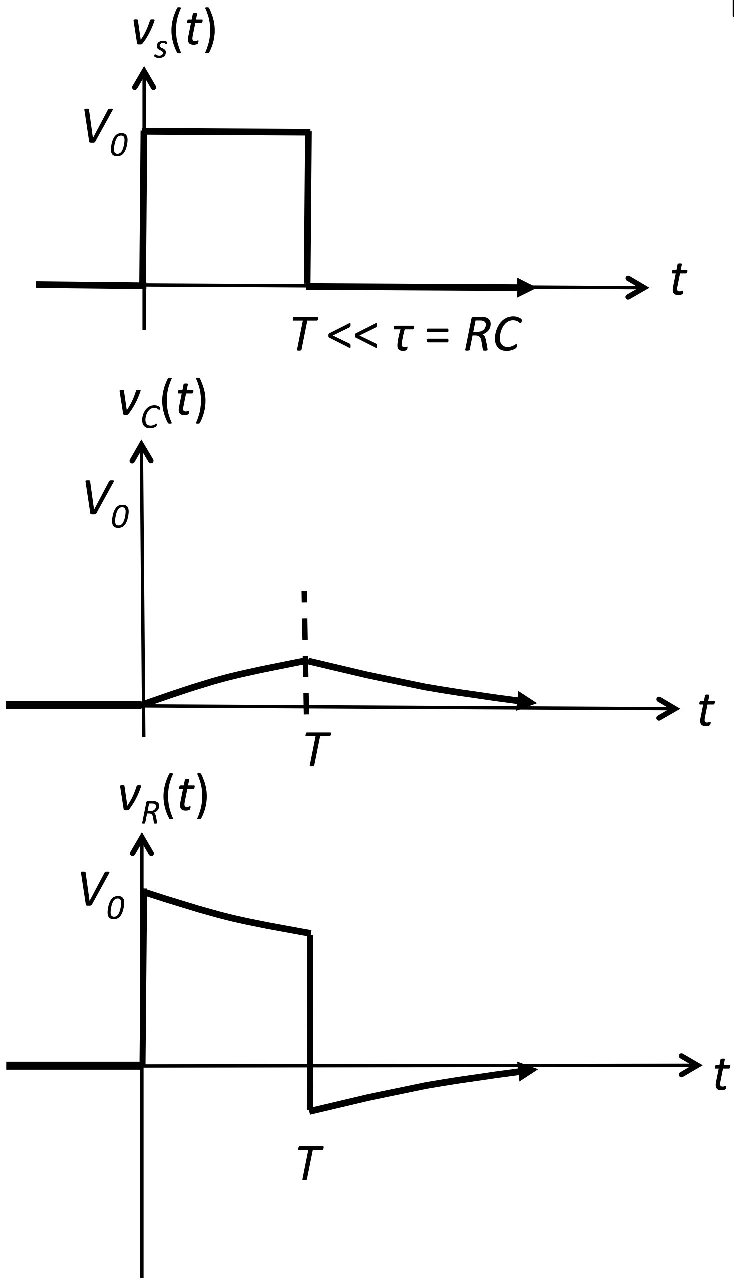


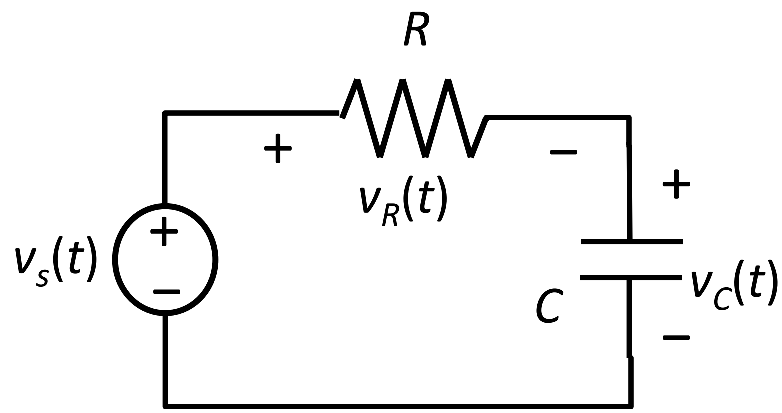


The "sharp" features of the rectangular input are prominent.

This *RC* circuit can be used to alter the shape of the input voltage versus time. *This is a primitive example of signal processing.*

“short” pulse RC response



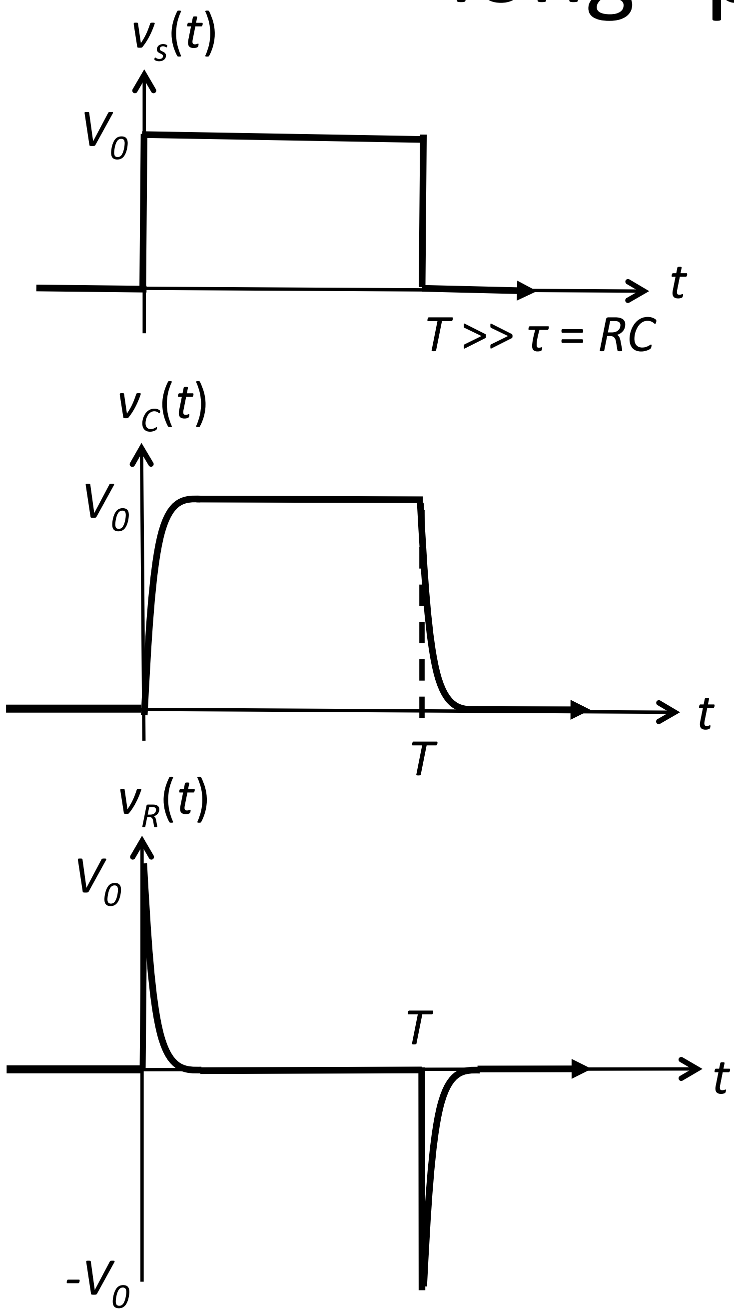


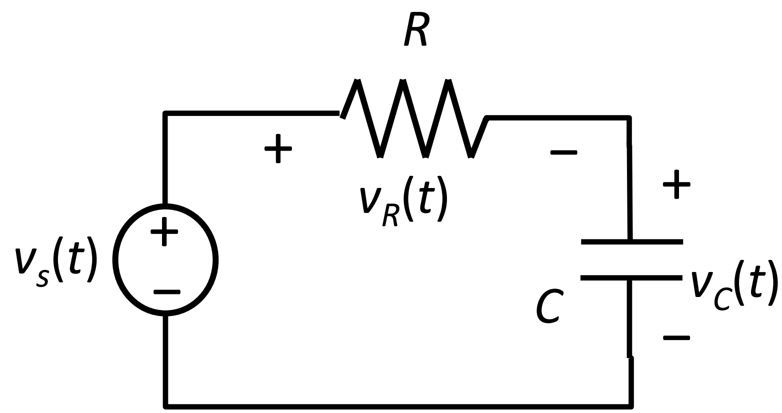
source pulse width $T << \tau = RC$

Capacitor voltage responds too slowly to reproduce the shape of the short pulse.

Resistor voltage (proportional to capacitor current) response reproduces the shape of the short pulse.

“long” pulse RC response



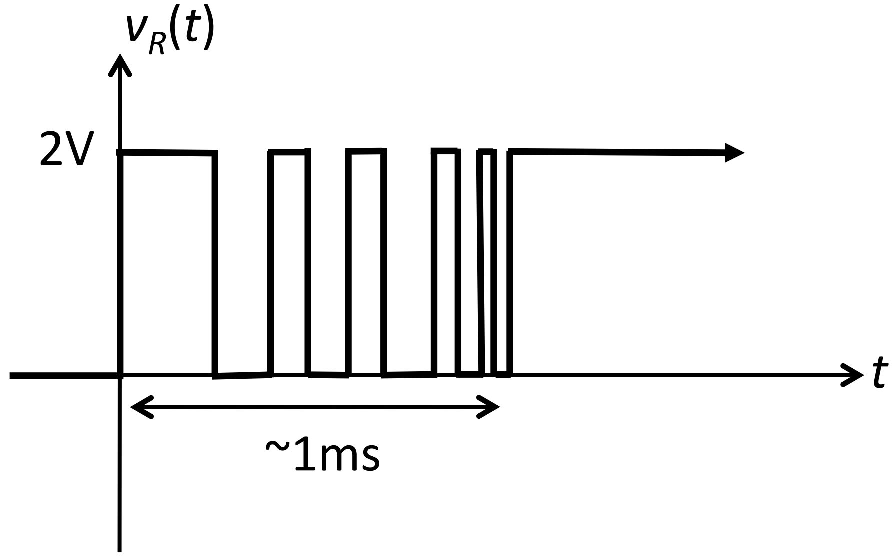


source pulse width $T >> \tau = RC$

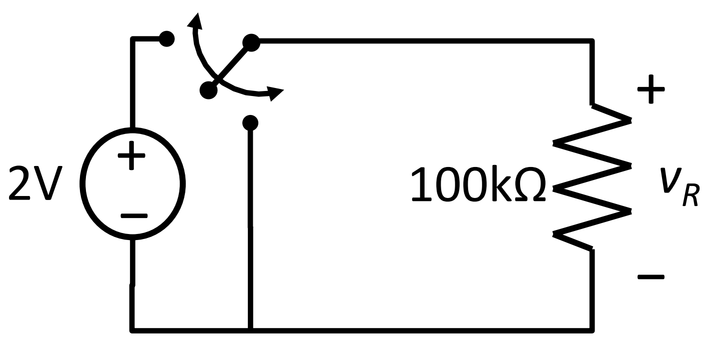
Capacitor voltage reproduces the shape of the long pulse.

Resistor voltage (proportional to capacitor current) decays too quickly, capturing only the rising and falling edges of the pulse.

application: “debouncer”



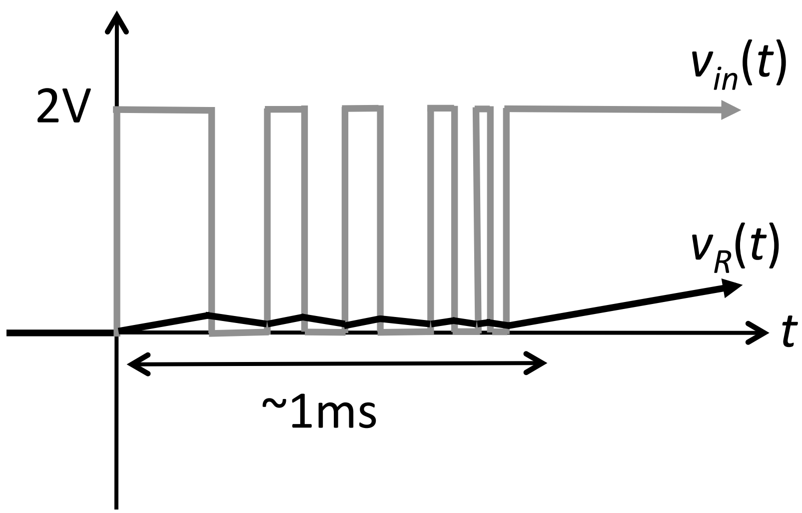
A contact switch “bounces” in the process of forming or breaking an electrical contact.

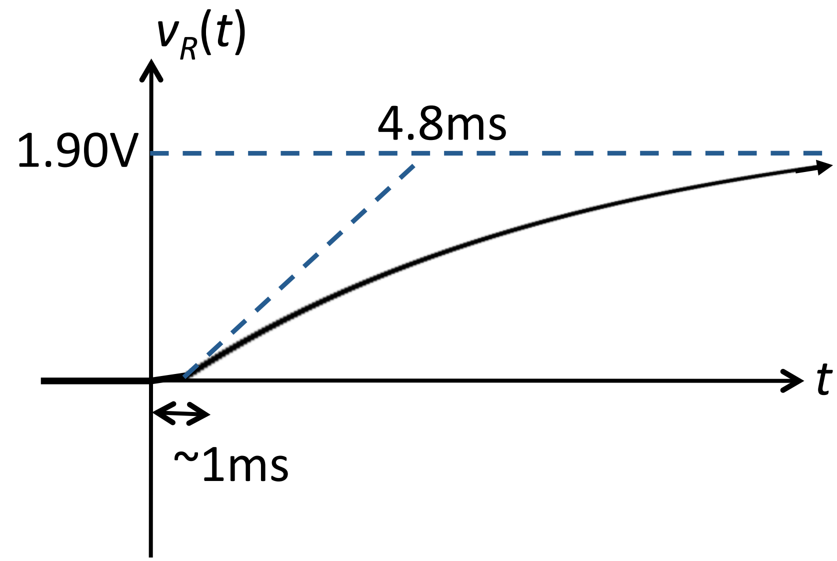


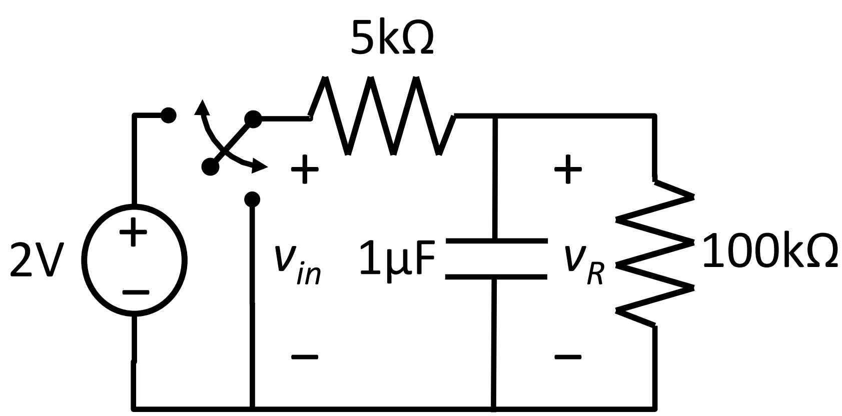
The multiple transitions can cause a single switching event to be counted several times by a digital counting circuit that acts on the transition.

An *RC* circuit can be added to smooth out the transitions, or “debounce” the switch.

application: "debouncer"







Analysis of the first order response:

$$v_R(0) = 0\text{V}$$

$$v_R(+\infty) = 2\text{V}\,\frac{100\text{k}\Omega}{100\text{k}\Omega + 5\text{k}\Omega} = 1.90\text{V}$$

$$\tau = \left(100\text{k}\Omega \,||\, 5\text{k}\Omega\right)\cdot 1\mu\text{F} = 4.8\text{ms}$$

The penalty is a slower response and voltage division. Can this be avoided?

sequential switching: general procedure

For each time interval of constant input:

step #1: Find the initial value of the circuit variable of interest, $x(t_0+)$, using circuit analysis and continuity of capacitor voltage or inductor current.

step #2: Find the anticipated final value of the variable of interest, $x(\infty)$, using dc steady state models for the capacitor or inductor.

step #3: Find the Thévenin equivalent resistance R_T as seen from the terminals of the capacitor or inductor. The time constant $\tau = R_T C$ or $\tau = L/R_T$.

step #4: Construct the solution.

$$x(t) = x(\infty) + \left[x(t_0+) - x(\infty)\right]\exp\left(-\frac{t-t_0}{\tau}\right)$$