Electric Circuits 1

ECSE-200 Section: 1

26 April 2012, 2:00PM

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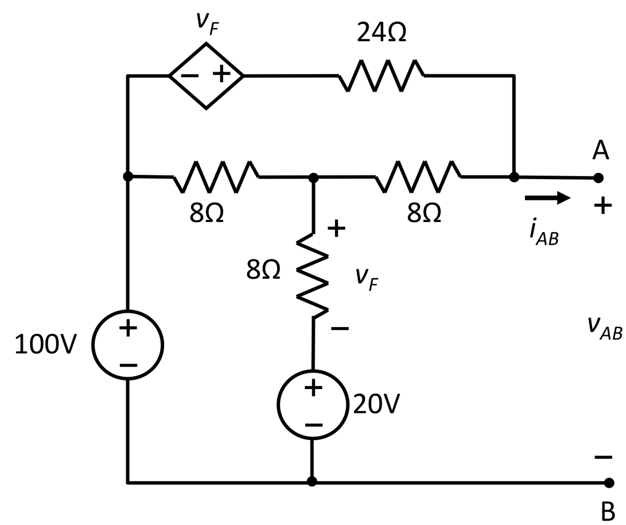
INSTRUCTIONS:

- This is a CLOSED BOOK examination.
- NO CRIB SHEETS are permitted.
- Provide your answers in an EXAM BOOKLET.
- STANDARD CALCULATOR permitted ONLY.
- This examination consists of 4 questions, with a total of 6 pages, including the cover page.
- This examination is PRINTED ON BOTH SIDES of the paper

1. Consider the circuit below. Answer the questions. [12 pts]



a) What is the physical basis of Kirchoff's Voltage Law? [1pt]

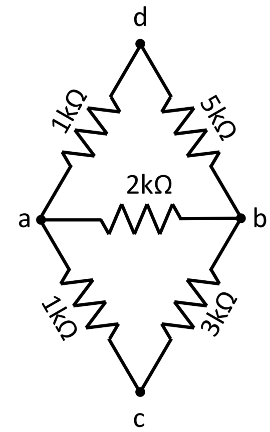
b) What is the value of v_F when an open circuit is applied to terminals A and B? [2pts]

c) What is the value of v_F when a short circuit is applied to terminals A and B? [2pts]

d) What is the Thévenin equivalent circuit with respect to the terminals A and B? Clearly label A and B on your circuit diagram. [5pts]

e) What is the maximum power that the circuit can deliver to an optimally chosen load resistor attached to the terminals A and B? What is the value of the optimally chosen load resistor? [2pts]

2. Consider the circuit below. Answer the questions. [12 pts]



a) What is the definition of a passive element? [1pt]

b) What is the equivalent resistance between terminals a and b? [2pts]

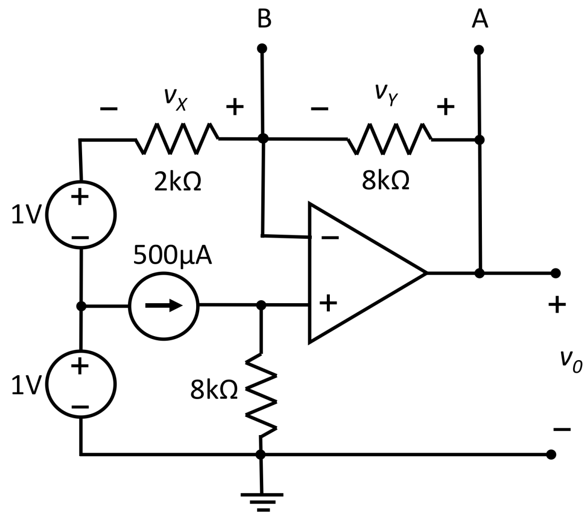
c) What is the equivalent resistance between terminals b and c? [2pts]

d) What is the equivalent resistance between terminals b and d? [2pts]

e) What is the equivalent resistance between terminals d and c? [3pts]

f) A capacitor is attached to the terminals d and c, resulting in a first order circuit. How should the capacitance be chosen to give a time constant of 10μs? [1pt]

3. Consider the circuit below. Assume ideal op-amp behaviour. Answer the questions. [12 pts]



a) Give one reason why negative feedback is used in op-amp circuits? [1pt]

b) How much power does the independent current source deliver or absorb? [2pts]

c) What is the voltage v_X? [2pts]

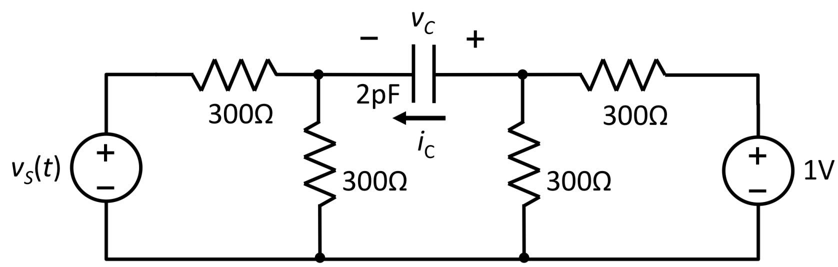
d) What is the voltage v_Y? [2pts]

e) What is the voltage v_O? [2pts]

f) What is the Thévenin resistance with respect to the terminals A and B? [2pts]

g) If a 10µH inductor is attached to the terminals A and B, what is the time constant for the resulting first order circuit? [1pt]

4. Consider the circuit and equation below. Assume dc steady state behaviour for $t<0$. The function $u(t)$ is the unit step function. Answer the questions. [12 pts]



$v_S(t) = 1\text{V} \times u(t) - 1\text{V} \times u(t-1.2\text{ns})$

a) Plot the voltage $v_S(t)$ versus t. [1pt]

b) What is $v_C(t)$ for $t < 0$? [1pt]

c) What is $v_C(t)$ for $0 < t < 1.2$ns ? [3pts]

d) What is $v_C(t)$ for $1.2\text{ns} < t$? [3pts]

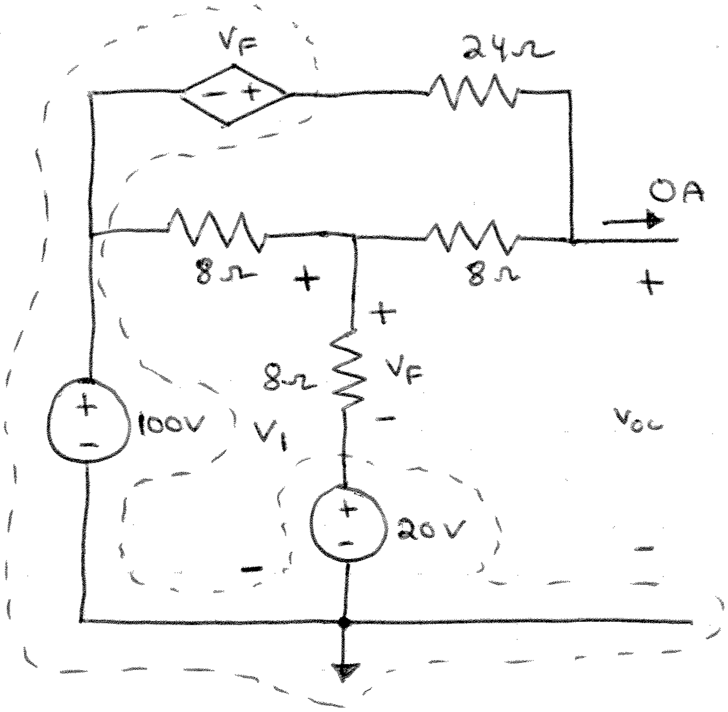
e) Plot $v_C(t)$ versus t. Be sure to label your axes. [2pts]

f) Plot $i_C(t)$ versus t. Be sure to label your axes. [2pts]

end

1. a) conservation of energy [+1]

b)



node voltage method [+1]

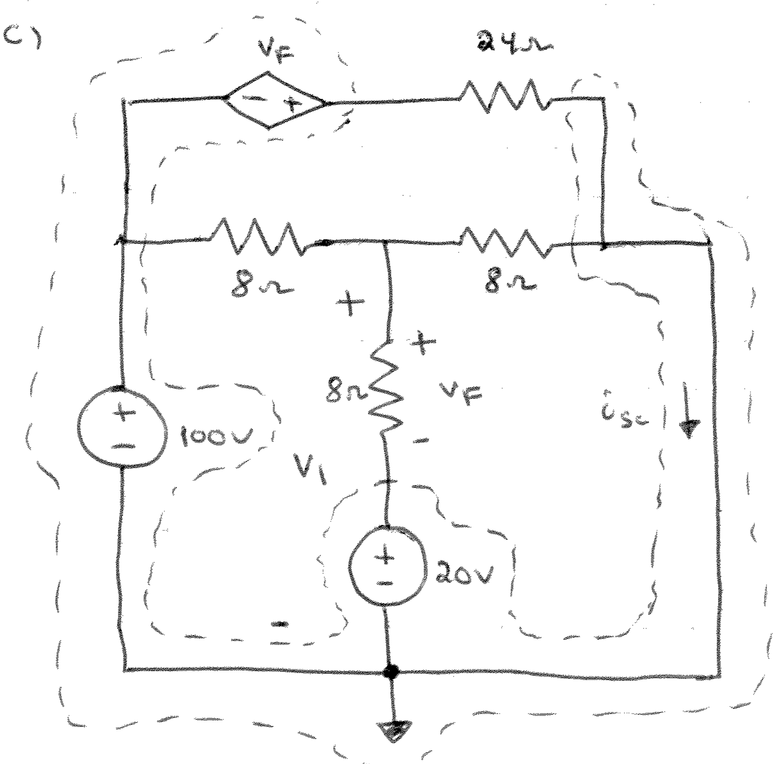
$$0 = \frac{V_1 - 20V}{8\Omega} + \frac{V_1 - 100V}{8\Omega} + \frac{V_1 - V_F - 100V}{32\Omega}$$

$$V_F = V_1 - 20V$$

$$\therefore \; 0 = \frac{V_F}{8\Omega} + \frac{V_F - 80V}{8\Omega} + \frac{-80V}{32\Omega}$$

$V_F = 50\ V$ [+1]

c)



node voltage method [+1]

$$0 = \frac{V_1 - 20V}{8\Omega} + \frac{V_1 - 100V}{8\Omega} + \frac{V_1}{8\Omega}$$

$$V_F = V_1 - 20V$$

$$\therefore \; 0 = \frac{V_F}{8\Omega} + \frac{V_F - 80V}{8\Omega} + \frac{V_F + 20V}{8\Omega}$$

$V_F = 20V$ [+1]

d) From b):

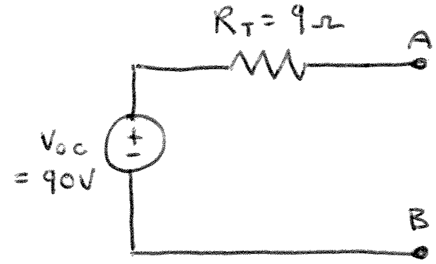
$$V_{oc} = V_1 - 8\Omega \cdot \left(\frac{V_1 - V_F - 100V}{32\Omega}\right)$$
$$= V_F + 20V - 8\Omega\left(\frac{-80V}{32\Omega}\right)$$
$$= 90V \quad [+1]$$

From c):

$$i_{sc} = \frac{V_1}{8\Omega} + \frac{V_F + 100V}{24\Omega}$$
$$= \frac{V_F + 20V}{8\Omega} + \frac{V_F + 100V}{24\Omega}$$
$$= 10A \quad [+1]$$

$$R_T = \frac{V_{oc}}{i_{sc}} = \frac{90V}{10A} = 9\Omega$$

[+1] [+1]



[+1 for final answer]

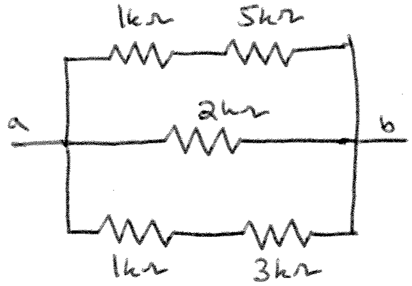
e) Maximum power occurs when $R_L = R_T$. [+1]

$$P_{max} = \frac{V_{oc}}{2} \cdot \frac{i_{sc}}{2} = \frac{90V \cdot 10A}{4} = 225W \quad [+1]$$

2.

a) an element that never delivers more energy to a circuit than it has received from a circuit. [+1]

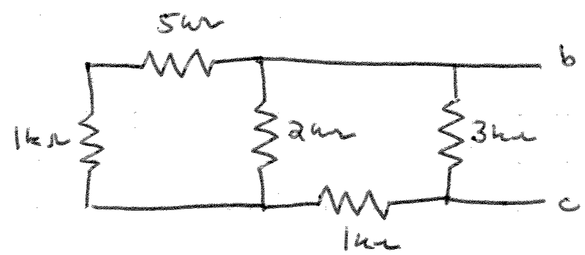
b)



$R_{ab} = (1k\Omega + 5k\Omega) \,//\, 2k\Omega \,//\, (1k\Omega + 3k\Omega)$ [+1]

$= 1.091\ k\Omega$ [+1]

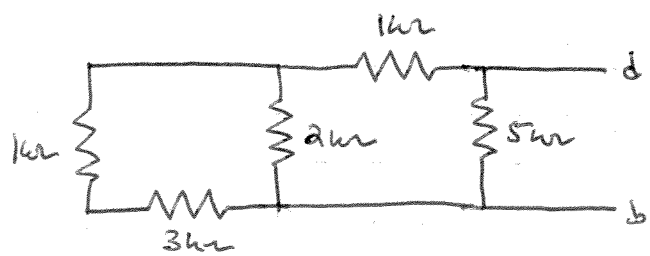
c)



$R_{bc} = 3k\Omega \,//\, [1k\Omega + 2k\Omega \,//\, (5k\Omega + 1k\Omega)]$ [+1]

$= 1.364\ k\Omega$ [+1]

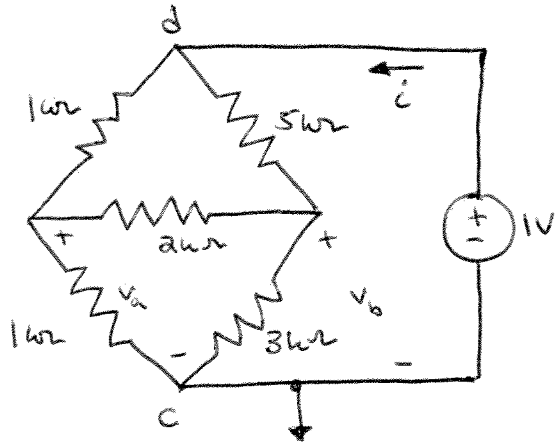
d)



$R_{bd} = 5k\Omega \,//\, [1k\Omega + 2k\Omega \,//\, (1k\Omega + 3k\Omega)]$ [+1]

$= 1.591\ k\Omega$ [+1]

e)



node voltage method:

$$\frac{v_a}{1k\Omega} + \frac{v_a - v_b}{2k\Omega} + \frac{v_a -}{1k\Omega} = 0 \quad [+\tfrac{1}{2}]$$

$$\frac{v_b}{3k\Omega} + \frac{v_b - v_a}{2k\Omega} + \frac{v_b - 1V}{5k\Omega} = 0 \quad [+\tfrac{1}{2}]$$

$$5v_a - v_b = 2V$$

$$-15v_a + 31v_b = 6V$$

$$v_a = 0.486V \quad v_b = 0.429V$$

$$i = \frac{v_a}{1k\Omega} + \frac{v_b}{3k\Omega} = 0.629 \text{ mA}$$

$$R_{dc} = \frac{1V}{i} \quad [+1]$$

$$= 1.590 \text{ k}\Omega$$

f)

$$\tau = R_{dc} \cdot C$$

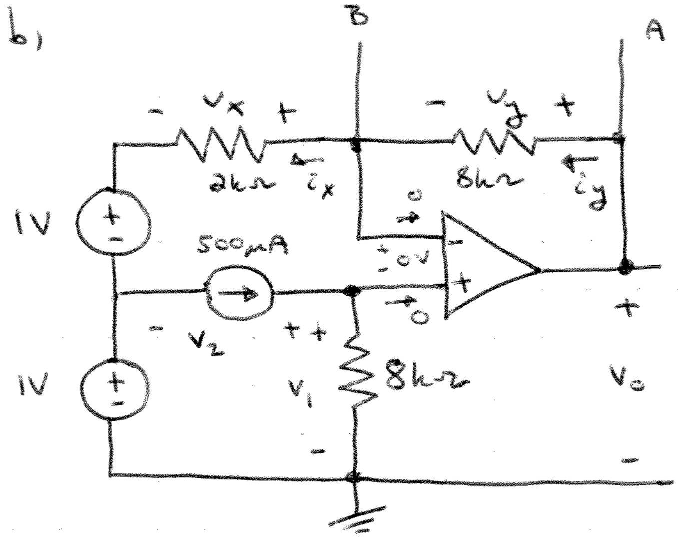
$$C = \frac{\tau}{R_{dc}} = \frac{10\mu s}{1.590 k\Omega} = 6.29 \text{ nF} \quad [+1]$$

3. a) stability

programmability

operation independent of open-loop gain

b)



Ohm: $v_1 = 8k\Omega \cdot 500\mu A = 4V$

KVL: $0 = -1V - v_2 + v_1$

$v_2 = v_1 - 1V = 3V$ [+1]

$P_{del} = v_2 \cdot 500\mu A = 1.5mW$ delivered [+1]

c) KVL: $0 = -1V - 1V - v_x + v_1$ [+1]

$v_x = -1V - 1V + v_1$

$= 2V$ [+1]

d) Ohm: $i_x = \frac{v_x}{2k\Omega} = \frac{2V}{2k\Omega} = 1mA$

KCL: $0 = +i_x - i_y$ [+1] $\quad i_y = 1mA$

Ohm: $v_y = 8k\Omega \cdot i_y$

$= 8V$ [+1]

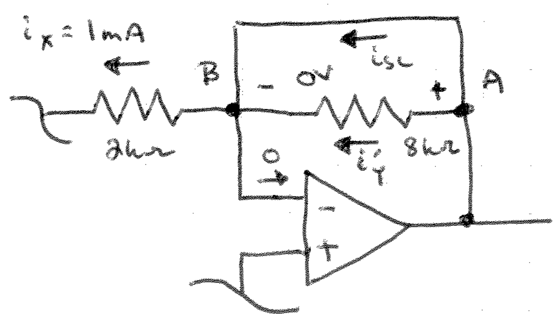
e) KVL: $0 = -1V - 1V - v_x - v_y + v_o$ [+1]

$v_o = 1V + 1V + v_x + v_y$

$= 1V + 1V + 2V + 8V = 12V$ [+1]

f) Find R_T by finding v_{oc} and i_{sc}.

$v_{oc} = 8V$ (from d))



$i_Y' = 0A$ (Ohm)

KCL: $0 = 1mA - i_{sc}$

$i_{sc} = 1mA$

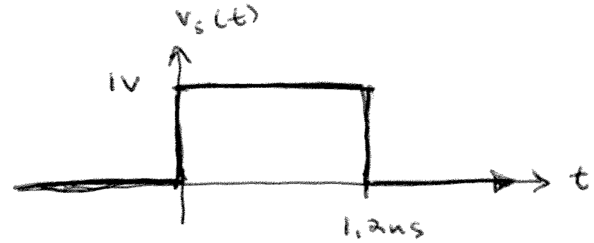
$R_T = \frac{v_{oc}}{i_{sc}}$ [+1]

$= \frac{8V}{1mA} = 8k\Omega$ [+1]

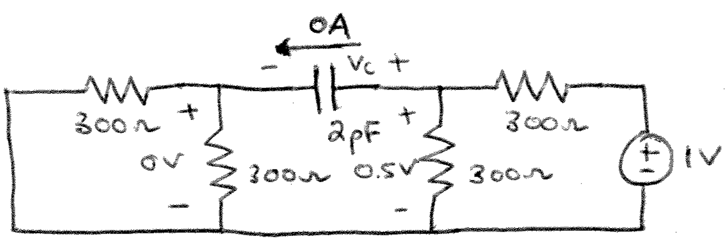
g) $\tau = \frac{L}{R_{TH}} = \frac{10\mu H}{8k\Omega} = 1.25\ ns$ [+1]

4.

a)



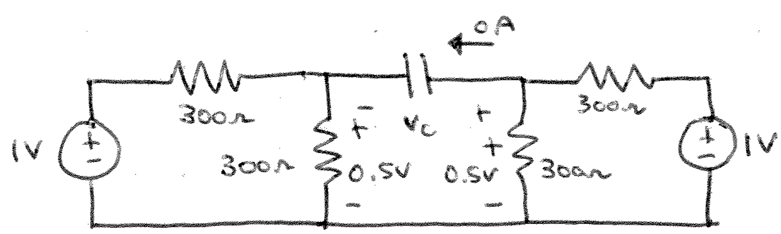
b) $t<0$



$V_c(t) = 0.5V$ (+1)
$t<0$

c) $V_c(0+) = V_c(0) = 0.5V$

as $t \to \infty$ neglecting 2^{nd} switching:



$V_c(\infty) = 0V$ (+1)

$R_{TH} = 300\Omega \parallel 300\Omega + 300\Omega \parallel 300\Omega = 300\Omega$

$\tau = R_{TH} C = 600ps$ (+1)

$V_c(t) = V_c(\infty) + (V_c(0) - V_c(\infty)) \exp(-t/\tau)$

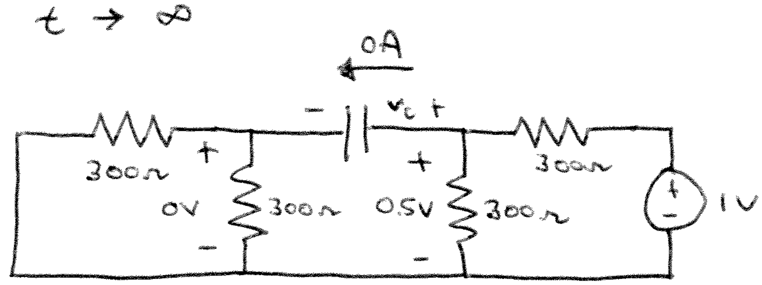
$= 0.5V \exp(-t/600ps)$ (+1)

$0 < t < 1.2ns$

d) $v_c(1.2ns+) = v_c(1.2ns-) = 0.5V \exp(-1.2ns/600ps)$

$= 0.0677 \text{ V}$ (+1)

as $t \to \infty$



$v_c(\infty) = 0.5V$ (+1)

$R_{TH} = 300\Omega // 300\Omega + 300\Omega // 300\Omega = 300\Omega$

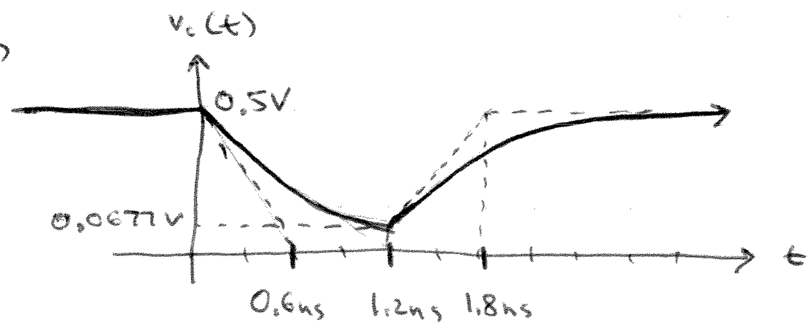
$\tau = R_{TH} \cdot C = 600ps$

$v_c(t) = v_c(\infty) + (v_c(1.2ns) - v_c(\infty)) \exp(-(t-1.2ns)/600ps)$

$= 0.5V - 0.432V \exp(-(t-1.2ns)/600ps)$ (+1)

$1.2ns < t$

e)



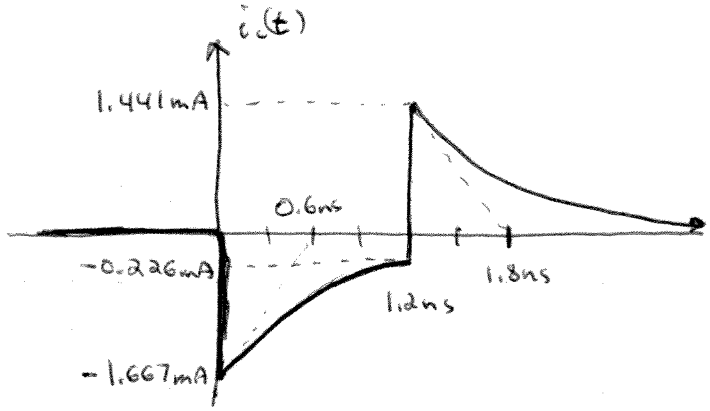
(+1) for shape
(+1) for values

f) $i_c = C \dfrac{dv_c}{dt}$

$i_c(0+) = 2pF \cdot \dfrac{-0.5V}{0.6ns} = -1.667 mA$

$i_c(1.2ns-) = 2pF \cdot \dfrac{-0.0677V}{0.6ns} = -0.226 mA$

$i_c(1.2ns+) = 2pF \cdot \dfrac{(0.5V - 0.0677V)}{0.6ns} = 1.441 mA$



[+1] for shape
[+1] for values