Section 5-2: Source Transformations

P 5.2-1 The circuit shown in Figure P 5.2-1*a* has been divided into two parts. The circuit shown in Figure P 5.2-1*b* was obtained by simplifying the part to the right of the terminals using source transformations. The part of the circuit to the left of the terminals was not changed.

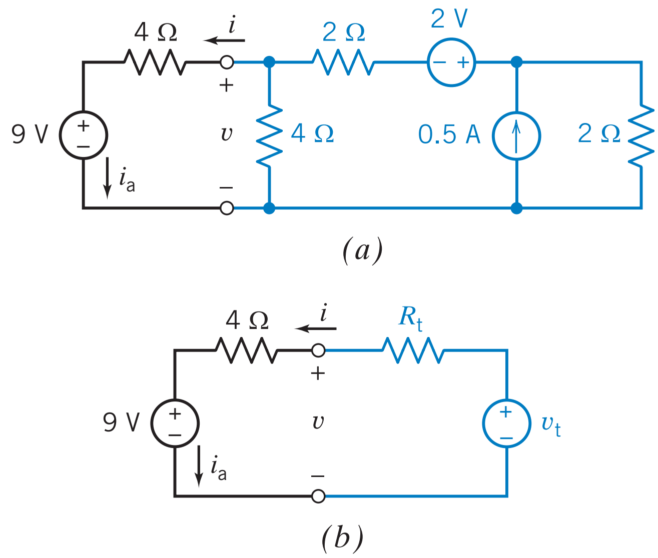


Figure P 5.2-1

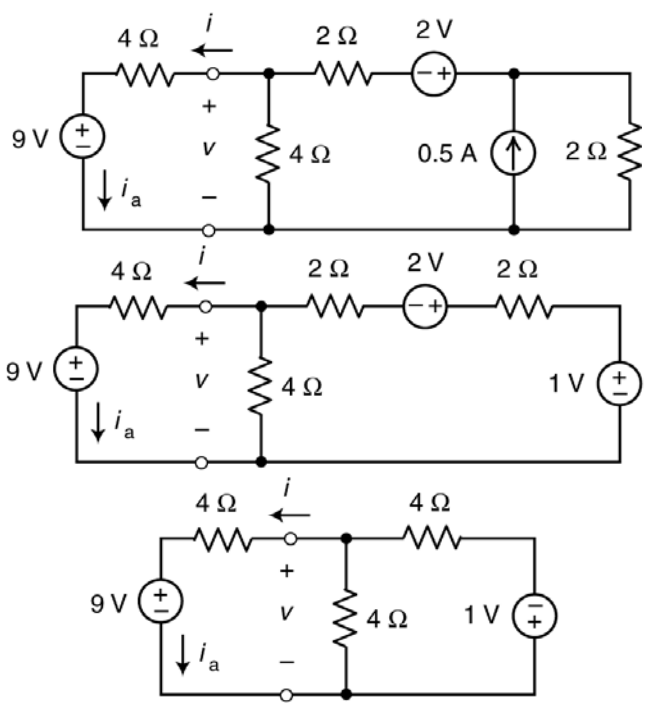
(a) Determine the values of R_t and v_t in Figure P 5.2-1*b*.

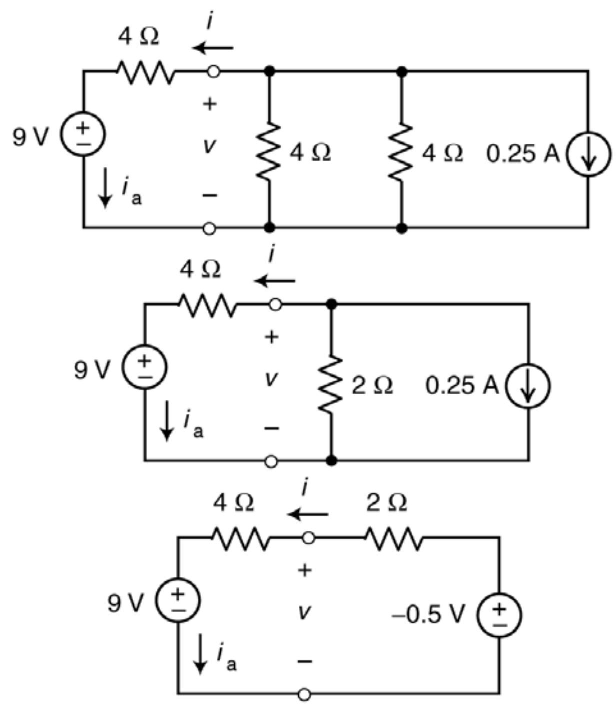
(b) Determine the values of the current *i* and the voltage *v* in Figure P 5.2-1*b*. The circuit in Figure P 5.2-1*b* is equivalent to the circuit in Figure P 5.2-1*a*. Consequently, the current *i* and the voltage *v* in Figure P 5.2-1*a* have the same values as do the current *i* and the voltage *v* in Figure P 5.2-1*b*.

(c) Determine the value of the current i_a in Figure P 5.2-1*a*.

Solution:

(a)





$$\therefore\ R_t = 2\ \Omega$$
$$v_t = -0.5\ \text{V}$$

(b) $$-9-4i-2i+(-0.5)=0$$

$$i = \frac{-9+(-0.5)}{4+2} = -1.58\ \text{A}$$

$$v = 9+4\,i = 9+4(-1.58) = 2.67\ \text{V}$$

(c) $i_a = i = -1.58$ A

(checked using LNAP 8/15/02)

P 5.2-2 Consider the circuit of Figure P 5.2-2. Find i_a by simplifying the circuit (using source transformations) to a single-loop circuit so that you need to write only one KVL equation to find i_a.

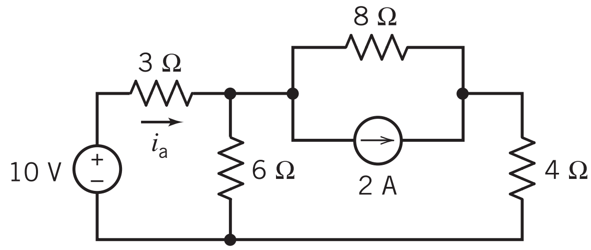
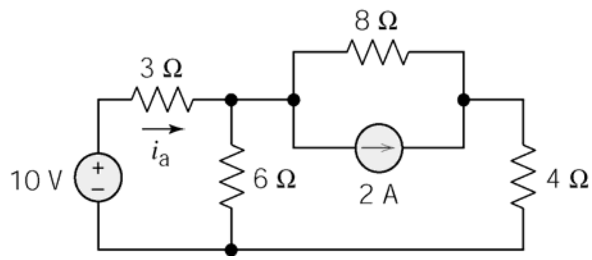
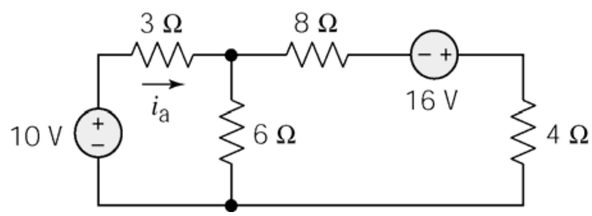
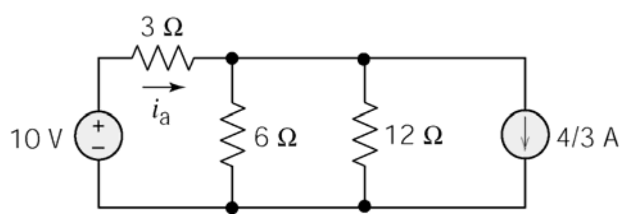


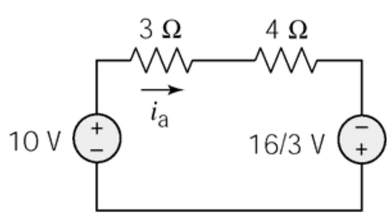
Figure P 5.2-2

Solution:









Finally, apply KVL: $-10+3\,i_a+4\,i_a-\dfrac{16}{3}=0 \quad \therefore\ \underline{i_a = 2.19\text{ A}}$

(checked using LNAP 8/15/02)

P 5.2-3 Find v_o using source transformations if $i = 5/2$ A in the circuit shown in Figure P 5.2-3.

Hint: Reduce the circuit to a single mesh that contains the voltage source labeled v_o.

Answer: $v_o = 28$ V

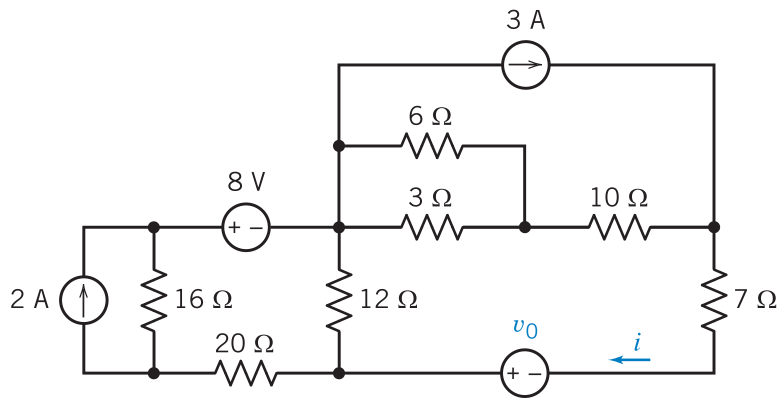
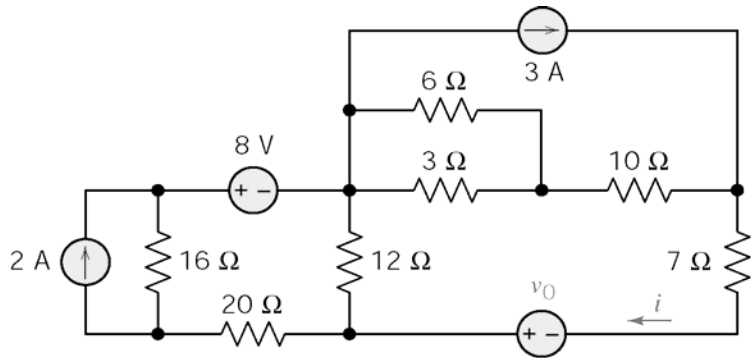
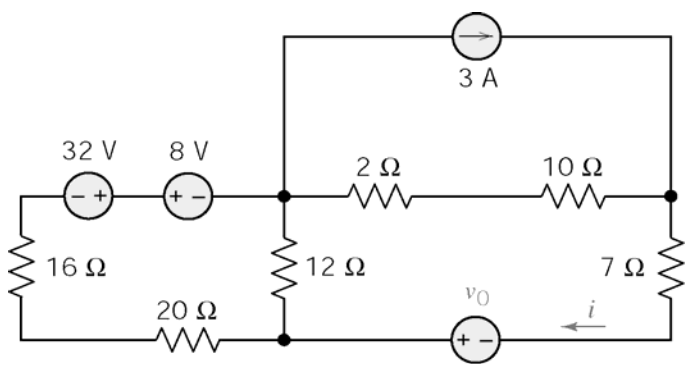


Figure P 5.2-3

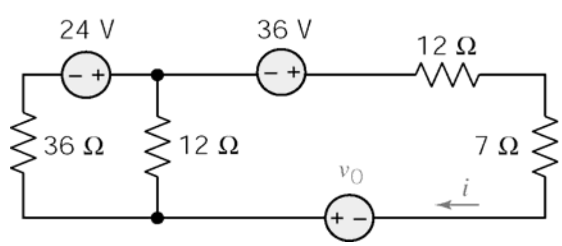
Solution:



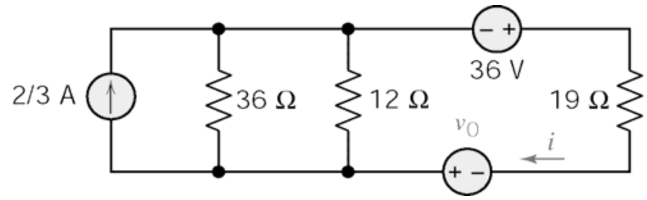
Source transformation at left; equivalent resistor for parallel 6 and 3 Ω resistors:



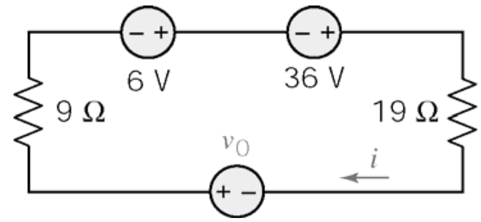
Equivalents for series resistors, series voltage source at left; series resistors, then source transformation at top:



Source transformation at left; series resistors at right:



Parallel resistors, then source transformation at left:



Finally, apply KVL to loop

$$-6 + i\ (9+19) - 36 - v_o = 0$$

$$i = 5/2\ \Rightarrow v_o = -42 + 28\ (5/2) = 28\text{ V}$$

(checked using LNAP 8/15/02)

P 5.2-4 Determine the value of the current i_a in the circuit shown in Figure P 5.2-4.

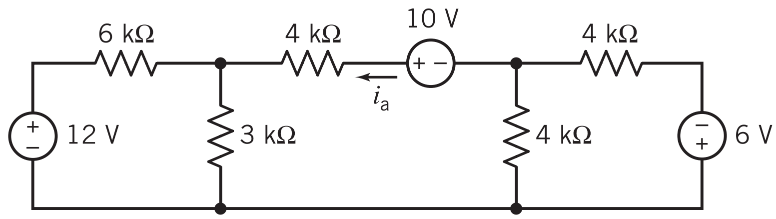
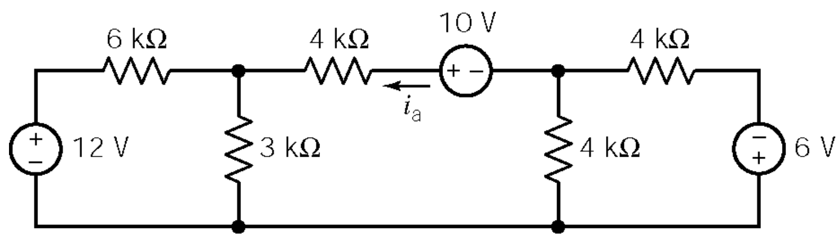
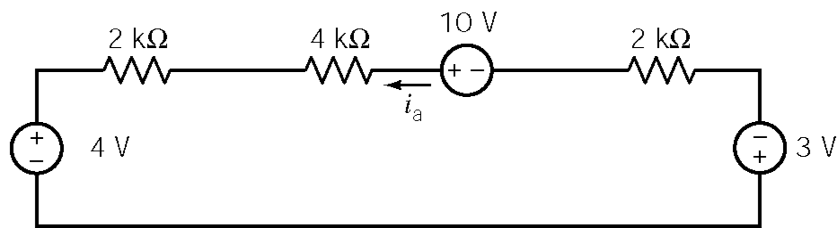


Figure P 5.2-4

Solution:





$$-4 - 2000\, i_a - 4000\, i_a + 10 - 2000\, i_a - 3 = 0$$

$$\therefore\ i_a = 375\ \mu\text{A}$$

(checked using LNAP 8/15/02)

P 5.2-5 Use source transformations to find the current i_a in the circuit shown in Figure P 5.2-5.

Answer: $i_a = 1$ A

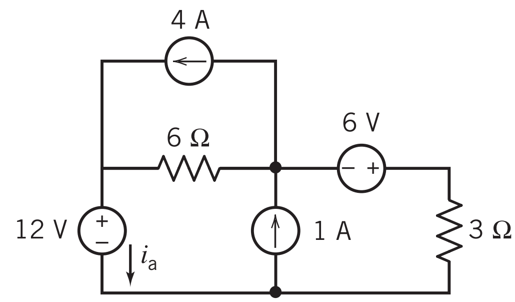
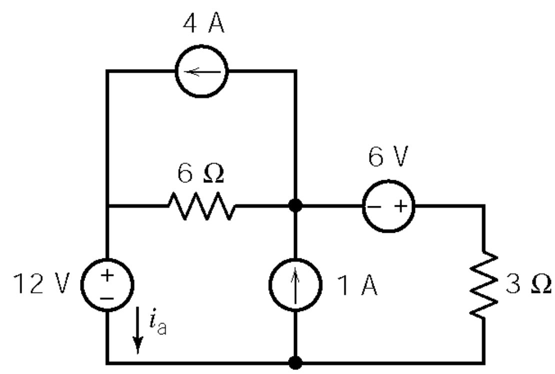
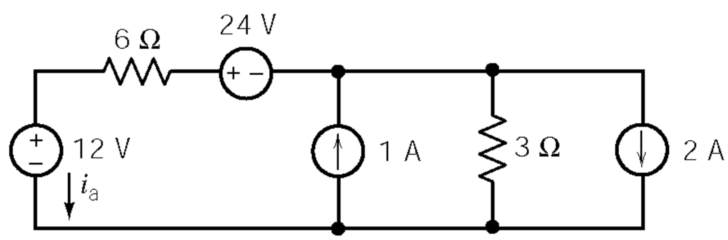
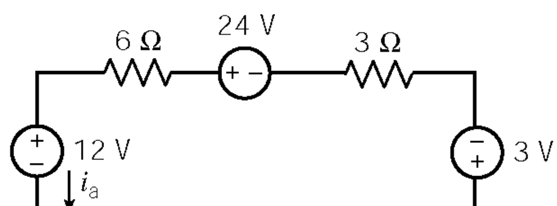


Figure P 5.2-5.

Solution:







$$-12-6\,i_a + 24 - 3\,i_a - 3 = 0 \Rightarrow i_a = 1 \text{ A}$$

(checked using LNAP 8/15/02)

P 5.2-6 Use source transformations to find the value of the voltage v_a in Figure P 5.2-6.

Answer: $v_a = 7$ V

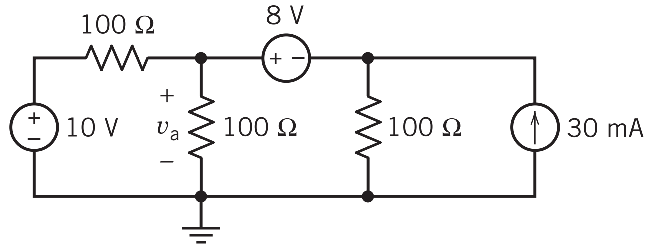
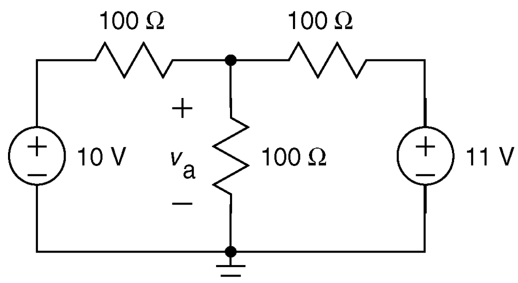


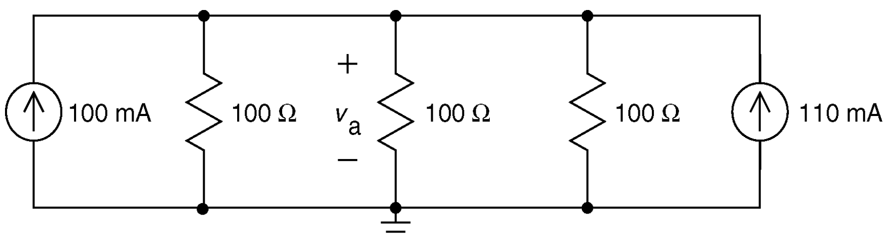
Figure P 5.2-6

Solution:

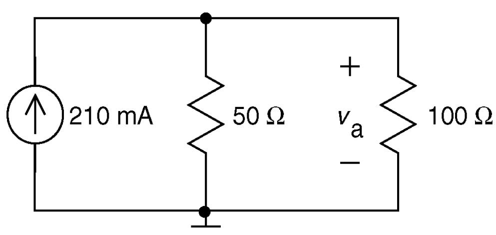
A source transformation on the right side of the circuit, followed by replacing series resistors with an equivalent resistor:



Source transformations on both the right side and the left side of the circuit:



Replacing parallel resistors with an equivalent resistor and also replacing parallel current sources with an equivalent current source:



Finally,

$$v_a = \frac{50(100)}{50+100}(0.21) = \frac{100}{3}(0.21) = 7 \text{ V}$$

(checked using LNAP 8/15/02)

P5.2-7

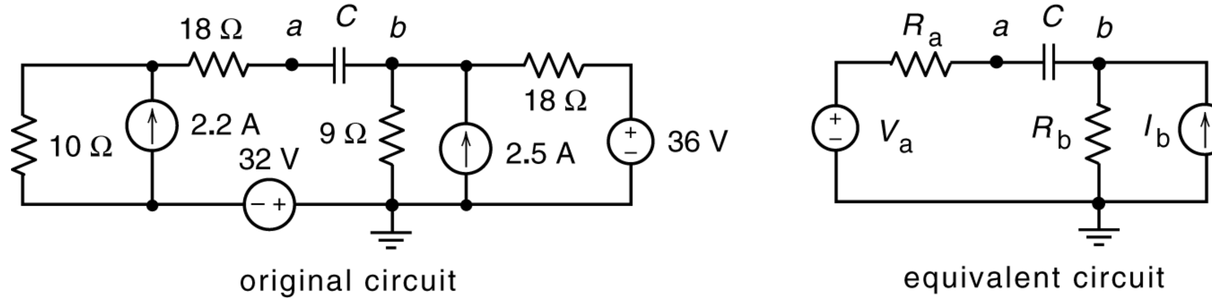
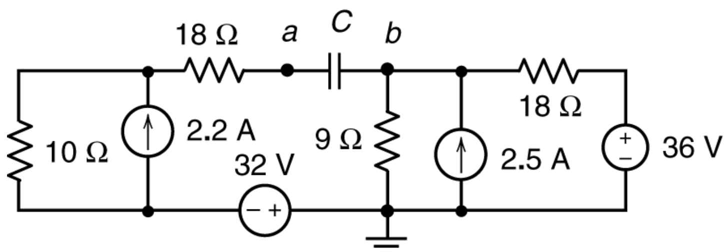


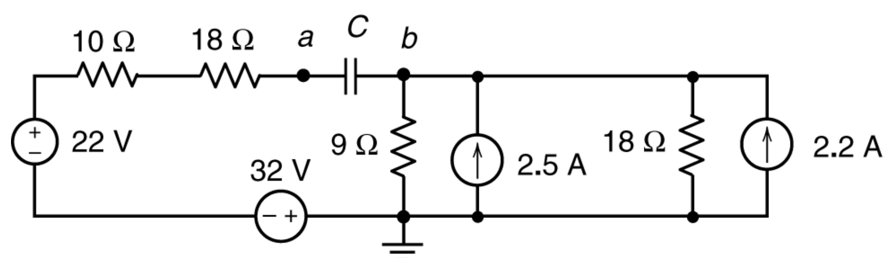
Figure P5.2-7

The equivalent circuit in Figure P5.2-7 is obtained from the original circuit using source transformations and equivalent resistances. (The lower case letters *a* and *b* identify the nodes of the capacitor in both the original and equivalent circuits.) Determine the values of R_a, V_a, R_b and I_b in the equivalent circuit.

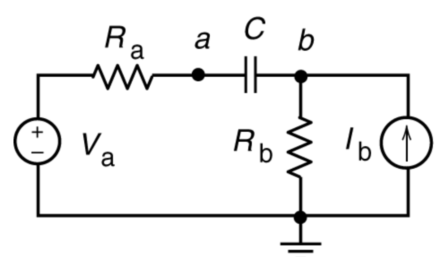
Solution



Performing a source transformation at each end of the circuit yields



Thenx



where

$V_a = 2.2(10) - 32 = -10\text{ V}$, $R_a = 18 + 10 = 28\ \Omega$, $R_b = 18 \| 9 = 6\ \Omega$ and $I_b = 2.5 + \frac{36}{18} = 4.5\text{ A}$

P 5.2-8 The circuit shown in Figure P 5.2-8 contains an unspecified resistance *R*.

(a) Determine the value of the current i when $R = 4\ \Omega$.

(b) Determine the value of the voltage v when $R = 8\ \Omega$.

(c) Determine the value of R that will cause $i = 1$ A.

(d) Determine the value of R that will cause $v = 16$ V.

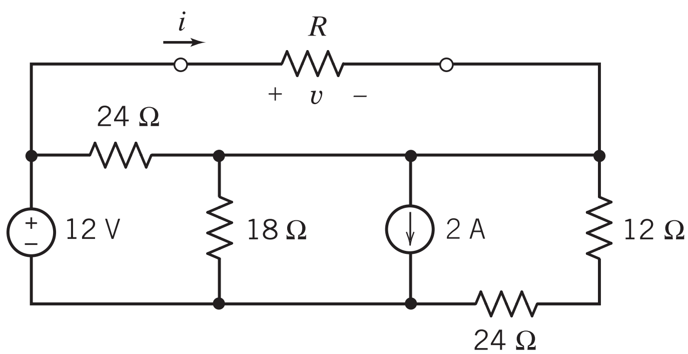
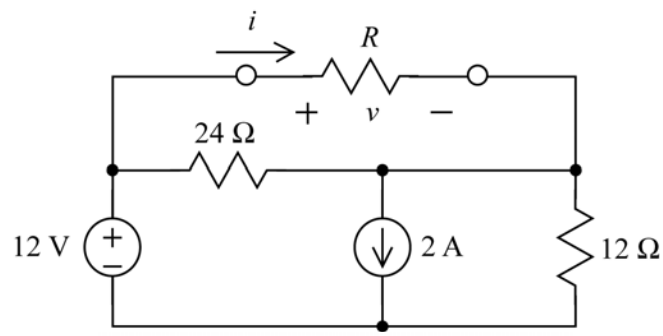


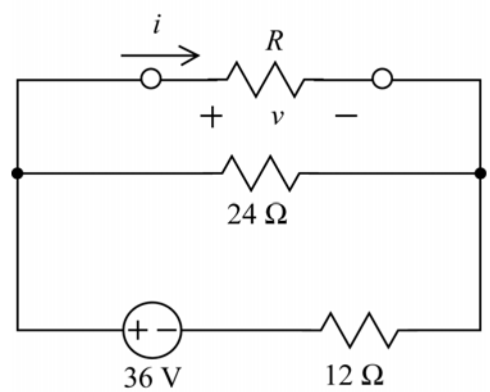
Figure P 5.2-8

Solution:



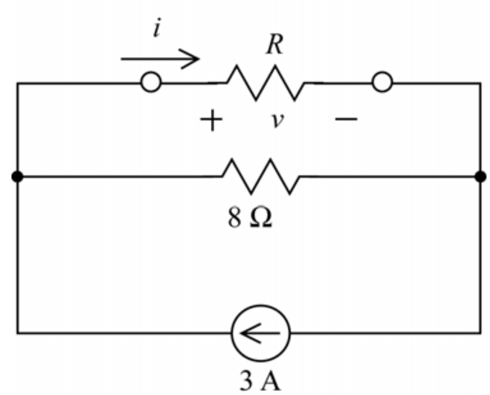
Replace series and parallel resistors by an equivalent resistor.

$$18 \,||\, (12 + 24) = 12\ \Omega$$



Do a source transformation, then replace series voltage sources by an equivalent voltage source.

Do two more source transformations



Now current division gives

$$i = \left(\frac{8}{8+R}\right)3 = \frac{24}{8+R}$$

Then Ohm's Law gives

$$v = Ri = \frac{24R}{8+R}$$

(a) $i = \dfrac{24}{8+4} = 2\ \text{A}$

(b) $v = \dfrac{24(8)}{8+8} = 12\ \text{V}$

(c) $1 = \dfrac{24}{8+R} \quad \Rightarrow \quad R = 16\ \Omega$

(d) $16 = \dfrac{24R}{8+R} \quad \Rightarrow \quad R = 16\ \Omega$

(checked: LNAP 6/9/04)

P 5.2-9 Determine the value of the power supplied by the current source in the circuit shown in Figure P 5.2-9.

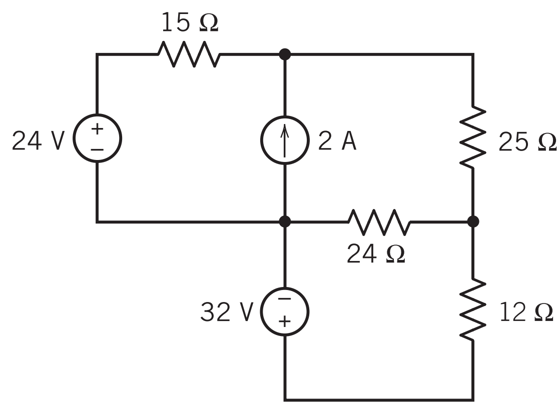
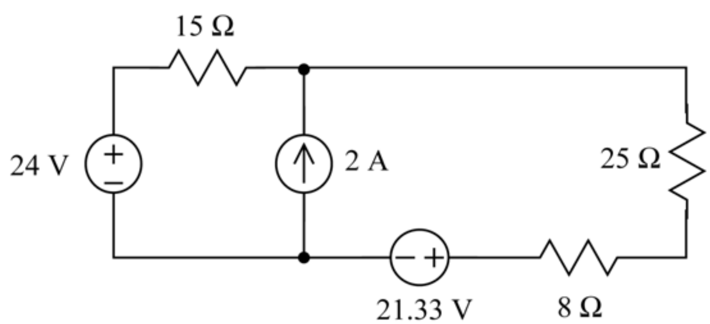
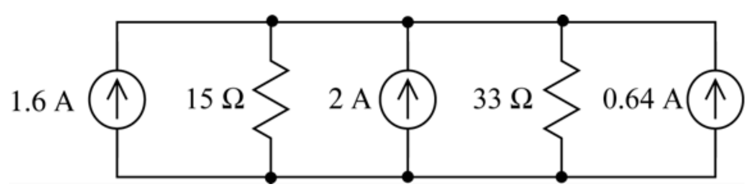


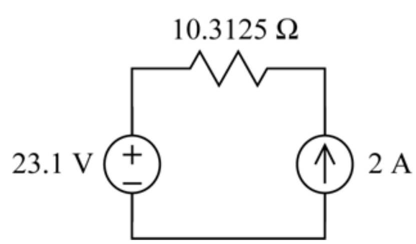
Figure P 5.2-9

Solution:

Use source transformations and equivalent resistances to reduce the circuit as follows







The power supplied by the current source is given by

$$p = \left[23.1 + 2(10.3125)\right]2 = 87.45 \text{ W}$$

Section 5-3 Superposition

P5.3-1

The inputs to the circuit shown in Figure P5.3-1 are the voltage source voltages v_1 and v_2.. The output of the circuit is the voltage v_o. The output is related to the inputs by

$$v_o = a\, v_1 + b\, v_2$$

where *a* and *b* are constants. Determine the values of *a* and *b*.

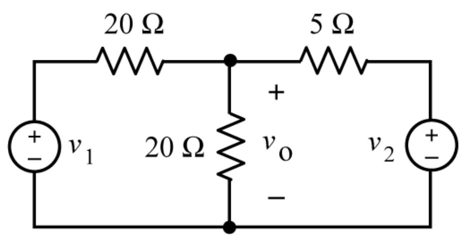
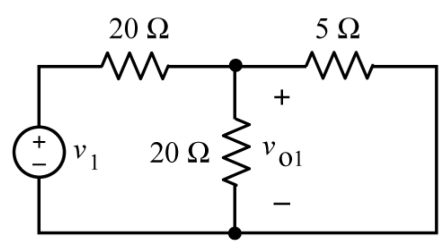


Figure P5.3-1

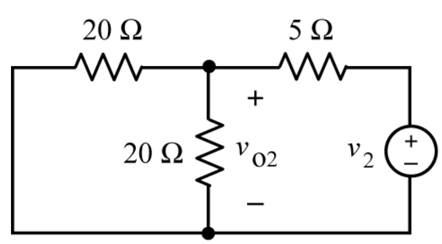
Solution:

Let $v_{o1} = a\, v_1$ be the output when $v_2 = 0$. In this case, the right voltage source acts like a short circuit so we have the circuit show to the right. Then



$$v_{o1} = \frac{20 \| 5}{20 + (20 \| 5)} v_1 = \frac{4}{20+4} v_1 = \frac{1}{6} v_1 \quad \Rightarrow \quad a = \frac{1}{6}$$

Let $v_{o2} = b\, v_2$ be the output when $v_1 = 0$. In this case, the left voltage source acts like a short circuit so we have the circuit show to the right. Then



$$v_{o2} = \frac{20 \| 20}{5 + (20 \| 20)} v_2 = \frac{10}{5+10} v_2 = \frac{2}{3} v_2 \quad \Rightarrow \quad b = \frac{2}{3}$$

P5.3-2

A particular linear circuit has two inputs, v_1 and v_2, and one output, v_o. Three measurements are made. The first measurement shows that the output is $v_o = 4$ V when the inputs are $v_1 = 2$ V and $v_2 = 0$. The second measurement shows that the output is $v_o = 10$ V when the inputs are $v_1 = 0$ and $v_2 = -2.5$ V. In the third measurement the inputs are $v_1 = 3$ V and $v_2 = 3$ V. What is the value of the output in the third measurement?

Solution:

The output of a linear circuit is a linear combination of the inputs:

$$v_o = a_1 v_1 + a_2 v_2$$

From the first two measurements we have:

$$\begin{pmatrix} 4 \\ 10 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -2.5 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \quad \Rightarrow \quad \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

Now the output of the third measurement can be determine to be

$$v_o = a_1(3) + a_2(3) = (2)(3) + (-4)(3) = -6 \text{ V}$$

P5.3-3
The circuit shown in Figure P5.3-3 has two inputs, v_s and i_s, and one output i_o. The output is related to the inputs by the equation

$$i_o = a\,i_s + b\,v_s$$

Given the following two facts:

The output is i_o = 0.45 A when the inputs are i_s = 0.25 A and v_s = 15 V.

and

The output is i_o = 0.30 A when the inputs are i_s = 0.50 A and v_s = 0 V.

Determine the values of the constants a and b and the values of the resistances are R_1 and R_2.
Answers: a = 0.6 A/A, b = 0.02 A/V, R_1 = 30 Ω and R_2 = 20 Ω.

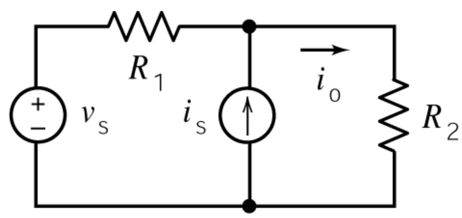
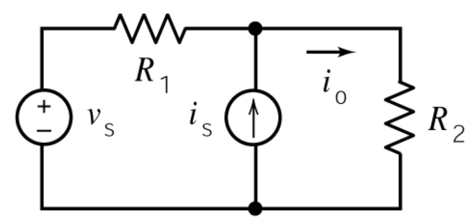


Figure P5.3-3

Solution:



From the 1st fact:

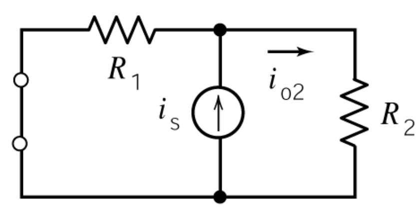
$$0.45 = a(0.25) + b(15)$$

From the 2nd fact:

$$0.30 = a(0.50) + b(0) \quad \Rightarrow \quad a = \frac{0.30}{0.50} = 0.60$$

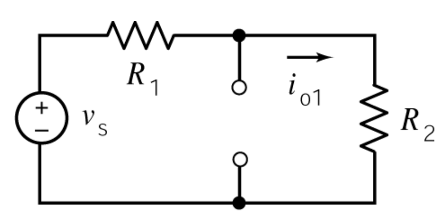
Substituting gives $\quad 0.45 = (0.60)(0.25) + b(15) \quad \Rightarrow \quad b = \dfrac{0.45 - (0.60)(0.25)}{15} = 0.02$

Next, consider the circuit:



$$a\,i_s = i_{o1} = i_o\Big|_{v_s=0} = \left(\frac{R_1}{R_1+R_2}\right) i_s$$

so $\quad 0.60 = \dfrac{R_1}{R_1+R_2} \quad \Rightarrow \quad 2\,R_1 = 3\,R_2$



and $\quad b\,v_s = i_{o2} = i_o\Big|_{i_s=0} = \dfrac{v_s}{R_1+R_2}$

so $\quad 0.02 = \dfrac{1}{R_1+R_2} \quad \Rightarrow \quad R_1 + R_2 = \dfrac{1}{0.02} = 50\ \Omega$

Solving these equations gives R_1 = 30 Ω and R_2 = 20 Ω.

P 5.3-4 Use superposition to find the value of the voltage v in Figure P 5.3-4.

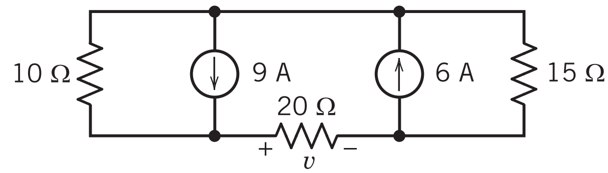
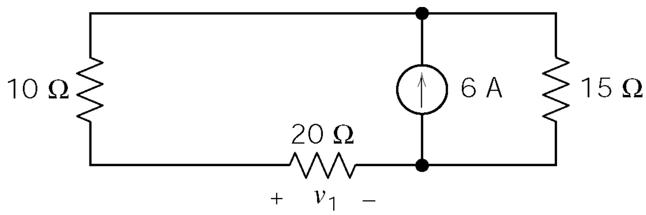


Figure P 5.3-4

Solution:

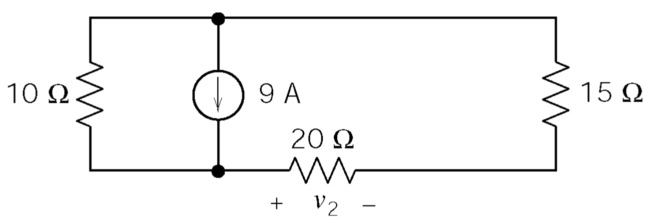
Consider 6 A source only (open 9 A source)



Use current division:

$$\frac{v_1}{20} = 6\left[\frac{15}{15+30}\right] \Rightarrow \underline{v_1 = 40 \text{ V}}$$

Consider 9 A source only (open 6 A source)



Use current division:

$$\frac{v_2}{20} = 9\left[\frac{10}{10+35}\right] \Rightarrow \underline{v_2 = 40 \text{ V}}$$

$$\underline{\therefore v = v_1 + v_2 = 40 + 40 = 80 \text{ V}}$$

(checked using LNAP 8/15/02)

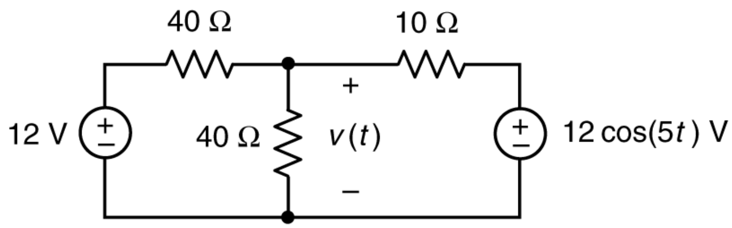
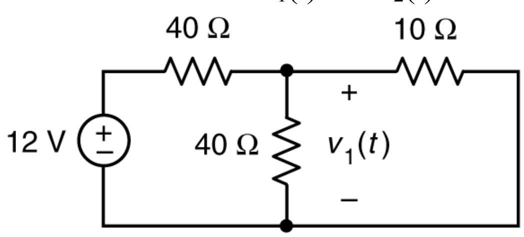


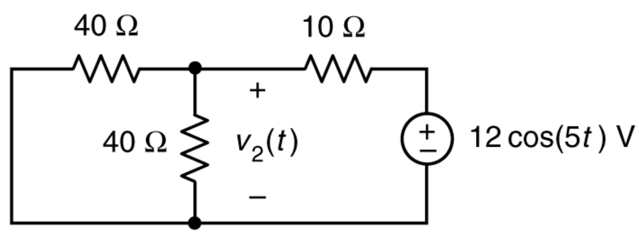
Figure P5.3-5

P5.3-5 Determine $v(t)$, the voltage across the vertical resistor in the circuit in Figure P5.3-5.

Solution;

We'll use superposition. Let $v_1(t)$ the be the part of $v(t)$ due to the voltage source acting alone. Similarly, let $v_2(t)$ the be the part of $v(t)$ due to the voltage source acting alone. We can use these circuits to calculate $v_1(t)$ and $v_2(t)$.





Notice that $v_1(t)$ is the voltage across parallel resistors. Using equivalent resistance, we calculate $40||10 = 8\ \Omega$. Next, using voltage division we calculate

$$v_1(t) = \frac{8}{8+40}(12) = 2\text{ V}$$

Similarly $v_2(t)$ is the voltage across parallel resistors Using equivalent resistance we first determine $40||40 = 20\ \Omega$ and then calculate

$$v_2(t) = \frac{20}{10+20}(12\cos(5t)) = 8\cos(5t)\text{ V}$$

Using superposition $\quad v(t) = v_1(t) + v_2(t) = 2 + 8\cos(5t)\text{ V}$

P 5.3-6 Use superposition to find the value of the current i in Figure P 5.3-6.

Answer: i = 3.5 mA

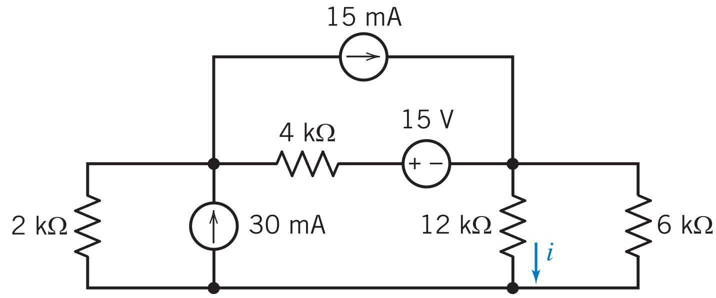
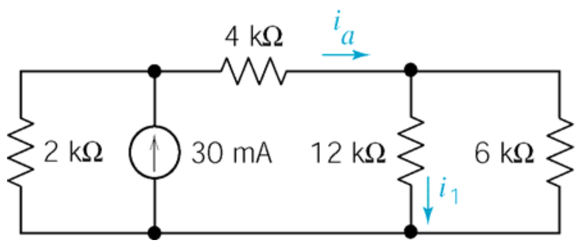
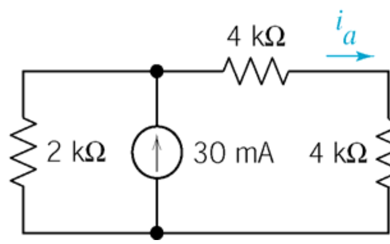


Figure P5.3-6

Solution:

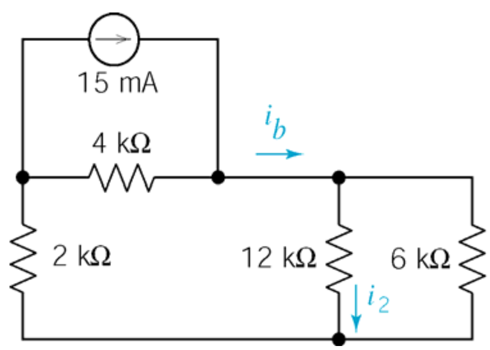
Consider 30 mA source only (open 15 mA and short 15 V sources). Let i_1 be the part of i due to the 30 mA current source.

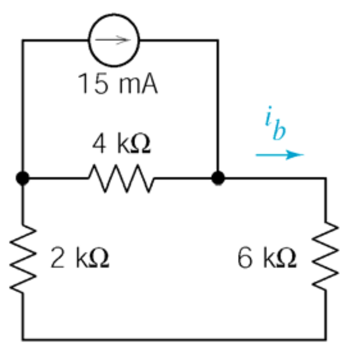




$$i_a = 30\left(\frac{2}{2+8}\right) = 6 \text{ mA} \Rightarrow i_1 = i_a\left(\frac{6}{6+12}\right) = \underline{2 \text{ mA}}$$

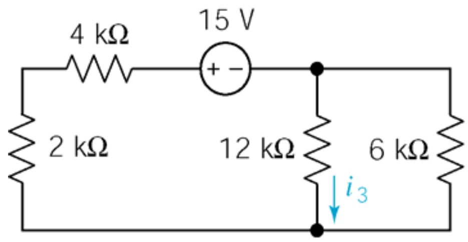
Consider 15 mA source only (open 30 mA source and short 15 V source) Let i_2 be the part of i due to the 15 mA current source.

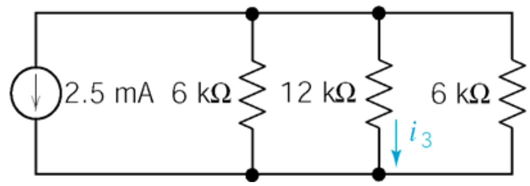




$$i_b = 15\left(\frac{4}{4+6}\right) = 6 \text{ mA} \Rightarrow i_2 = i_b\left(\frac{6}{6+12}\right) = \underline{2 \text{ mA}}$$

Consider 15 V source only (open both current sources). Let i_3 be the part of i due to the 15 V voltage source.





$$i_3 = -2.5\left(\frac{6 \| 6}{(6 \| 6)+12}\right) = -10\left(\frac{3}{3+12}\right) = \underline{-0.5 \text{ mA}}$$

Finally, $\underline{i = i_1 + i_2 + i_3 = 2 + 2 - 0.5 = 3.5 \text{ mA}}$

(checked using LNAP 8/15/02)

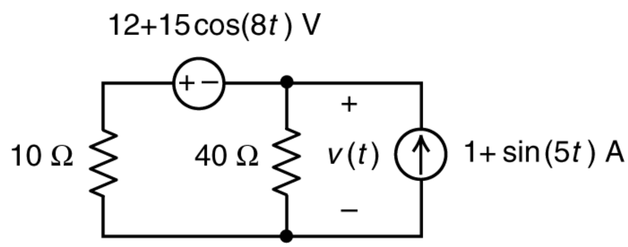
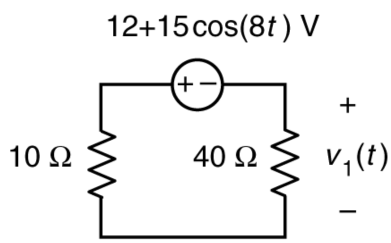
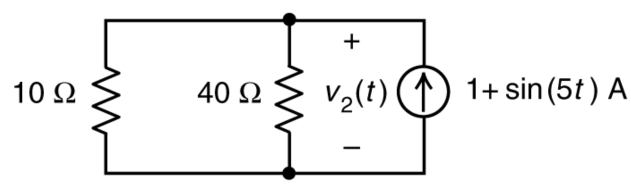


Figure P5.3-7

P5.3-7 Determine $v(t)$, the voltage across the 40 **Ω** resistor in the circuit in Figure P5.3-7.

Solution:
We'll use superposition. Let $v_1(t)$ the be the part of $v(t)$ due to the voltage source acting alone. Similarly, let $v_2(t)$ the be the part of $v(t)$ due to the voltage source acting alone. We can use these circuits to calculate $v_1(t)$ and $v_2(t)$.





Using voltage division we calculate

$$v_1(t) = -\frac{40}{10+40}\left(12+15\cos(8t)\right) = -9.6 - 12\cos(8t)$$

Using equivalent resistance we first determine 10||40 = 8 **Ω** and then calculate

$$v_2(t) = 8\left(1+\sin(5t)\right) = 8 + 8\sin(5t)$$

Using superposition $\quad v(t) = v_1(t) + v_2(t) = -1.6 + 8\sin(5t) - 12\cos(8t) \text{ V}$

P 5.3-8 Use superposition to find the value of the current i_x in Figure P 5.3-8.

Answer: $i = 3.5$ mA

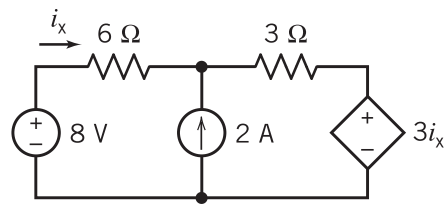
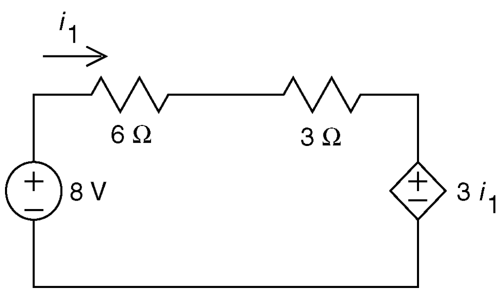


Figure P5.3-8

Solution:

Consider 8 V source only (open the 2 A source)



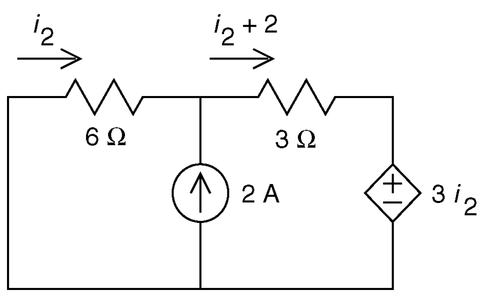
Let i_1 be the part of i_x due to the 8 V voltage source.

Apply KVL to the supermesh:

$$6\left(i_1\right)+3\left(i_1\right)+3\left(i_1\right)-8=0$$

$$i_1=\frac{8}{12}=\frac{2}{3}\text{ A}$$

Consider 2 A source only (short the 8 V source)



Let i_2 be the part of i_x due to the 2 A current source.

Apply KVL to the supermesh:

$$6\left(i_2\right)+3\left(i_2+2\right)+3\,i_2=0$$

$$i_2=\frac{-6}{12}=-\frac{1}{2}\text{ A}$$

Finally, $$i_x=i_1+i_2=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}\text{ A}$$

P 5.3-9 The input to the circuit shown in Figure P 5.3-9 is the voltage source voltage, v_s. The output is the voltage v_o. The current source current, i_a, is used to adjust the relationship between the input and output. Design the circuit so that input and output are related by the equation $v_o = 2v_s + 9$.
Hint: Determine the required values of A and i_a.

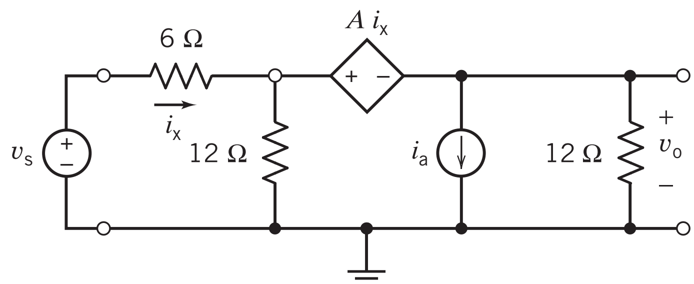
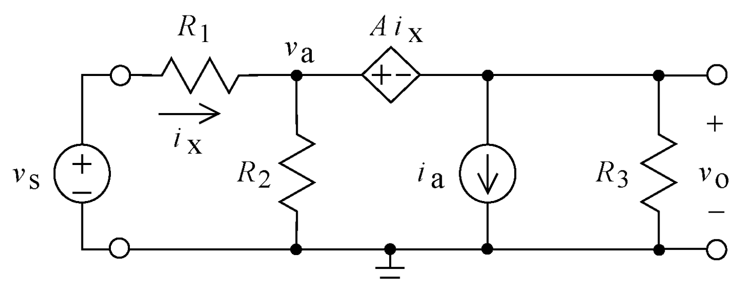


Figure P 5.3-9

Soluton:



$$i_x = \frac{v_s - v_a}{R_1}$$

$$v_a - v_o = A\,i_x = A\frac{v_s - v_a}{R_1}$$

$$v_a = \frac{R_1\,v_o + A\,v_s}{R_1 + A}$$

Apply KCL to the supernode corresponding to the CCVS to get

$$\frac{v_a - v_s}{R_1} + \frac{v_a}{R_2} + i_a + \frac{v_o}{R_3} = 0$$

$$\frac{R_1 + R_2}{R_1\,R_2}v_a - \frac{v_s}{R_1} + i_a + \frac{v_o}{R_3} = 0$$

$$\frac{R_1 + R_2}{R_1\,R_2}\left(\frac{R_1\,v_o + A\,v_s}{R_1 + A}\right) - \frac{v_s}{R_1} + i_a + \frac{v_o}{R_3} = 0$$

$$\left(\frac{R_1 + R_2}{R_2\left(R_1 + A\right)} + \frac{1}{R_3}\right)v_o + \left(\frac{\left(R_1 + R_2\right)A}{R_1\,R_2\left(R_1 + A\right)} - \frac{1}{R_1}\right)v_s + i_a = 0$$

$$\frac{R_3\left(R_1 + R_2\right) + R_2\left(R_1 + A\right)}{R_2\,R_3\left(R_1 + A\right)}v_o + \frac{A - R_2}{R_2\left(R_1 + A\right)}v_s + i_a = 0$$

$$v_o = \frac{R_3\left(R_2 - A\right)}{R_3\left(R_1 + R_2\right) + R_2\left(R_1 + A\right)} v_s - \frac{R_2\, R_3\left(R_1 + A\right)}{R_3\left(R_1 + R_2\right) + R_2\left(R_1 + A\right)} i_a$$

When $R_1 = 6\ \Omega$, $R_2 = 12\ \Omega$ and $R_3 = 12\ \Omega$

$$v_o = \frac{12 - A}{24 + A} v_s - \frac{12\left(6 + A\right)}{24 + A} i_a$$

Comparing this equation to $v_o = 2 v_s + 9$, we requires

$$\frac{12 - A}{24 + A} = 2 \quad \Leftrightarrow \quad A = -12\ \ \frac{\text{V}}{\text{A}}$$

Then $2 v_s + 9 = v_o = 2 v_s + 6 i_a$ so we require

$$9 = 6 i_a \quad \Rightarrow \quad i_a = 1.5\ \text{A}$$

(checked: LNAP 6/22/04)

P 5.3-10 The circuit shown in Figure P 5.3-10 has three inputs: v_1, v_2, and i_3. The output of the circuit is v_o. The output is related to the inputs by

$$v_o = av_1 + bv_2 + ci_3$$

where a, b, and c are constants. Determine the values of a, b, and c.

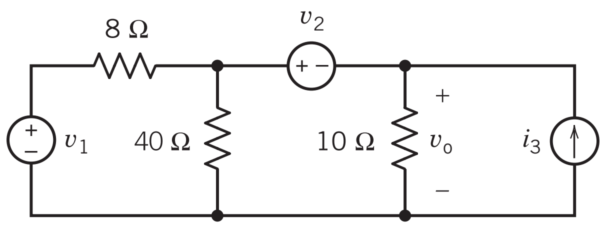
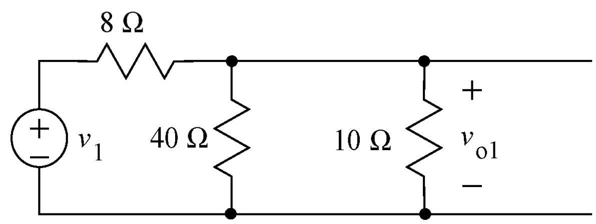


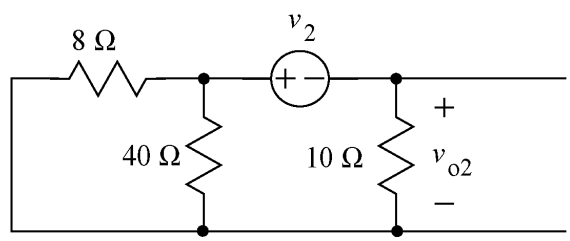
Figure P 5.3-10

Solution:

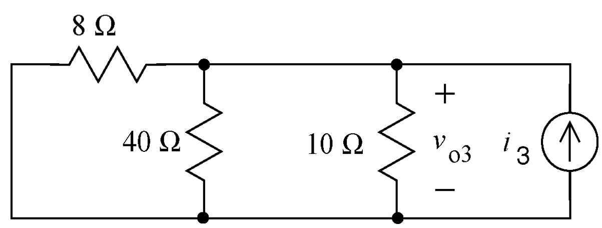
$$v_{o1} = \frac{40\,||\,10}{8+40\,||\,10} v_1 = \frac{1}{2} v_1 \quad \Rightarrow \quad a = \frac{1}{2}$$



$$v_{o2} = -\frac{10}{8\,||\,40+10} v_1 = -\frac{3}{5} v_2 \quad \Rightarrow \quad b = -\frac{3}{5}$$



$$v_{o3} = \left(8\,||\,10\,||\,40\right) i_3 = 4\,i_3 \quad \Rightarrow \quad c = 4$$



(checked: LNAP 6/22/04)

P 5.3-11 Determine the voltage $v_o(t)$ for the circuit shown in Figure P 5.3-11.

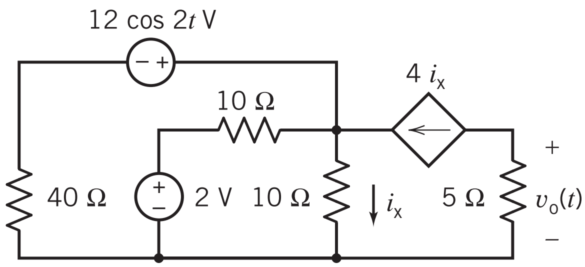
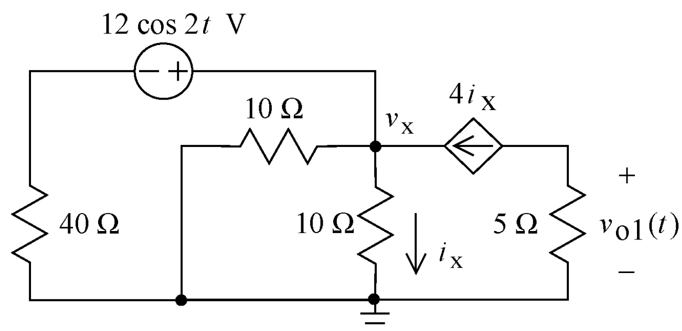


Figure P 5.3-11

Solution: Using superposition:



$$v_x = 10\,i_x$$

and

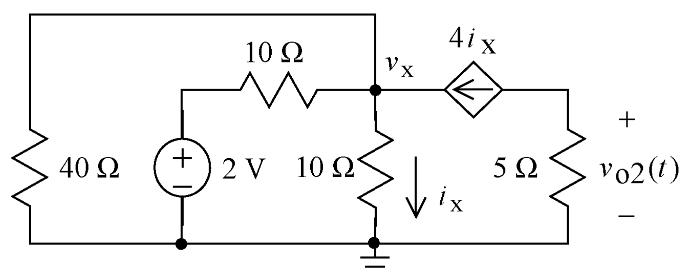
$$\frac{v_x - 12\cos 2t}{40} + \frac{v_x}{10} + \frac{v_x}{10} = 4\,i_x$$

so

$$\frac{10\,i_x - 12\cos 2t}{40} = 2\,i_x \quad \Rightarrow \quad i_x = -\frac{12}{70}\cos 2t$$

Finally,

$$v_{o1} = -5\left(4\,i_x\right) = 3.429\cos 2t \text{ V}$$



$$v_x = 10\,i_x$$

and

$$\frac{v_x}{40} + \frac{v_x - 2}{10} + \frac{v_x}{10} = 4\,i_x$$

so

$$-0.2 = 1.75\,i_x \quad \Rightarrow \quad i_x = -0.11429 \text{ A}$$

Finally,

$$v_{o1} = -5\left(4\,i_x\right) = 2.286 \text{ V}$$

$$v_o = v_{o1} + v_{o2} = 3.429\cos 2t + 2.286 \text{ V}$$

(checked: LNAP 6/22/04)

P 5.3-12 Determine the value of the voltage v_o in the circuit shown in Figure P 5.3-12.

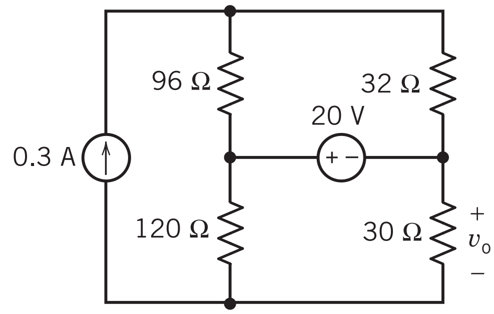
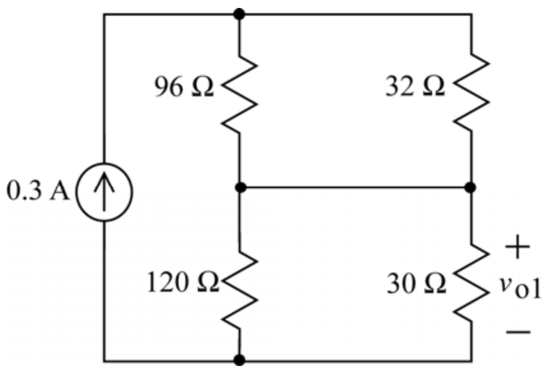
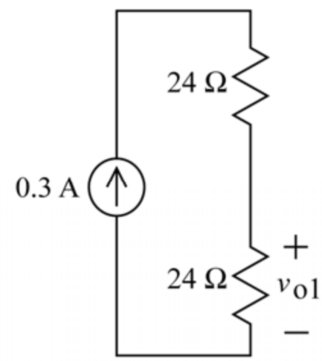
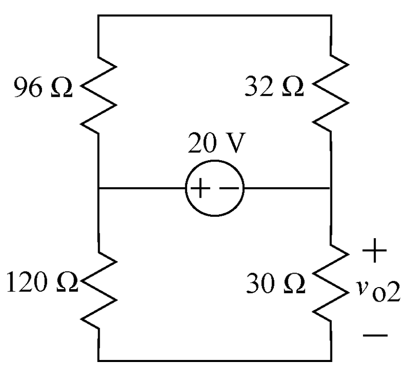


Figure P 5.3-12

Solution: Using superposition:







$$v_{o1} = 24(0.3) = 7.2 \text{ V}$$

$$v_{o2} = -\frac{30}{120+30}20 = -4 \text{ V}$$

$$v_o = v_{o1} + v_{o2} = 3.2 \text{ V}$$

(checked: LNAP 5/24/04)

P 5.3-13 Determine the value of the voltage v_o in the circuit shown in Figure P 5.3-13.

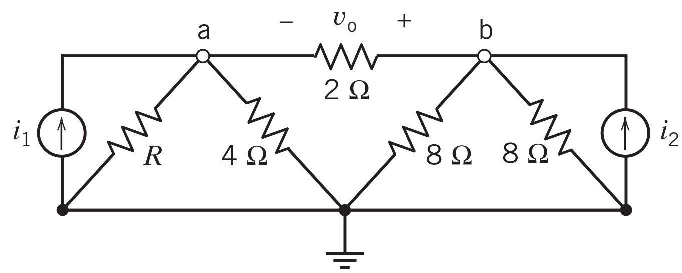


Figure P 5.3-13

Solution:
Using superposition

$$v_o = -2\left(\frac{R\|4}{6+(R\|4)}\right)i_1 + 2\left(\frac{4}{2+(R\|4)+4}\right)i_2$$

Comparing to $v_o = -0.5\,i_1 + 4$, we require

$$-2\left(\frac{R\|4}{6+(R\|4)}\right) = -0.5 \quad\Rightarrow\quad 4(R\|4) = 6+(R\|4) \quad\Rightarrow\quad R\|4 = 2 \quad\Rightarrow\quad R = 4\ \Omega$$

and

$$2\left(\frac{4}{2+(R\|4)+4}\right)i_2 = 4 \quad\Rightarrow\quad 2\left(\frac{4}{2+(4\|4)+4}\right)i_2 = 4 \quad\Rightarrow\quad i_2 = 4\ \text{A}$$

(checked LNAP 6/12/04)

P 5.3-14 Determine values of the current, i_a, and the resistance, R, for the circuit shown in Figure P 5.3-14.

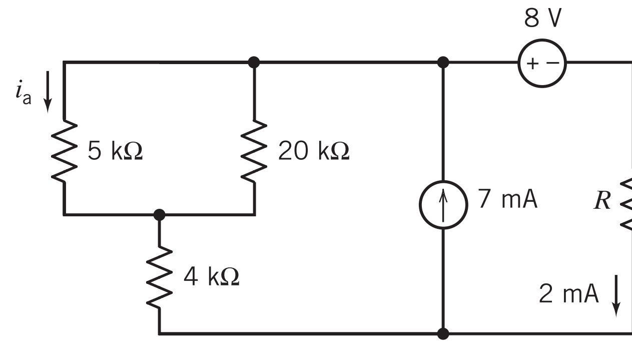


Figure P 5.3-14

Solution:

Use units of mA, kΩ and V.

$$4 + (5||20) = 8 \text{ k}\Omega$$

(a) Using superposition

$$2 = \left(\frac{8}{R+8}\right)7 - \frac{8}{R+8} \quad \Rightarrow \quad 2\left(R+8\right) = 48 \quad \Rightarrow \quad R = 16 \text{ k}\Omega$$

(b) Using superposition again

$$i_a = \left(\frac{5}{5+20}\right)\left[\left(\frac{16}{8+16}\right)7 + \frac{8}{8+16}\right] = \frac{4}{5}\left(\frac{2}{3}\times 7 + \frac{1}{3}\right) = 4 \text{ mA}$$

P 5.3-15 The circuit shown in Figure P 5.3-15 has three inputs: v_1, i_2, and v_3. The output of the circuit is the current i_o. The output of the circuit is related to the inputs by

$$i_1 = av_o + bv_2 + ci_3$$

where a, b, and c are constants. Determine the values of a, b, and c.

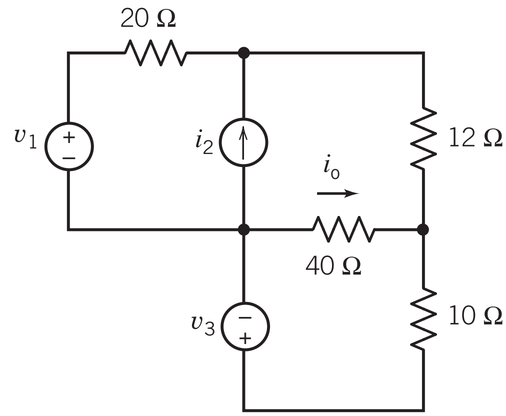


Figure P 5.3-15

Solution:

$$i_o = \left(-\frac{10}{10+40}\right)\left(\frac{v_1}{20+12+(40\,\|\,10)}\right) + \left(-\frac{10}{10+40}\right)\left(\frac{20}{20+\left[12+(40\,\|\,10)\right]}\right)i_2$$
$$+\left(-\frac{20+12}{40+(20+12)}\right)\left(\frac{v_3}{10+\left[40\,\|\,(20+12)\right]}\right)$$

$$i_o = \left(-\frac{1}{200}\right)v_1 + \left(-\frac{1}{10}\right)i_2 + \left(-\frac{1}{62.5}\right)v_3$$

So

$$a = -0.05,\ b = -0.1 \text{ and } c = -0.016$$

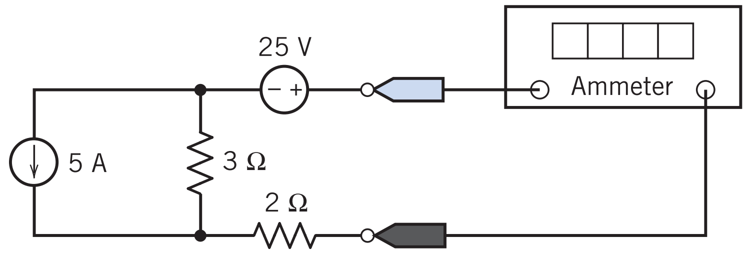
(checked: LNAP 6/19/04)

P 5.3-16 Using the superposition principle, find the value of the current measured by the ammeter in Figure P 5.3-16*a*.

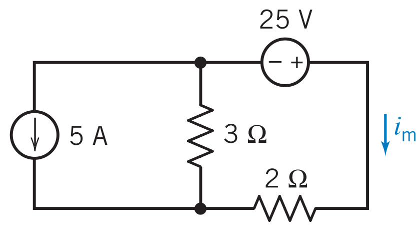
Hint: Figure P 5.3-16*b* shows the circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m.

Answer:

$$i_m = \frac{25}{3+2} - \frac{3}{2+3} 5 = 5 - 3 = 2 \text{ A}$$



(a)



(b)

Figure P 5.3-16

Solution:

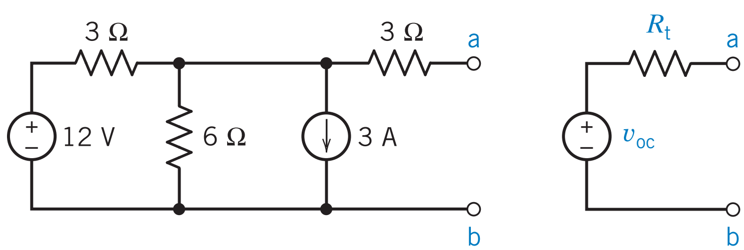
$$i_m = \frac{25}{3+2} - \frac{3}{2+3}(5) = 5 - 3 = 2 \text{ A}$$

Section 5-4: Thèvenin's Theorem

P 5.4-1 Determine values of R_t and v_{oc} that cause the circuit shown in Figure P 5.4-1*b* to be the Thévenin equivalent circuit of the circuit in Figure P 5.4-1*a*.

Hint: Use source transformations and equivalent resistances to reduce the circuit in Figure P 5.4-1*a* until it is the circuit in Figure P 5.4-1*b*.

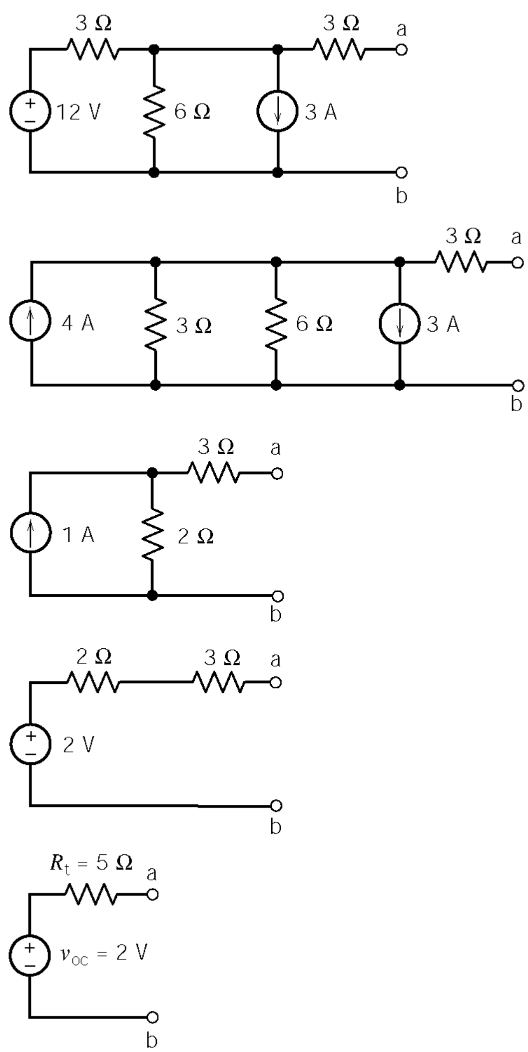
Answer: $R_t = 5\ \Omega$ and $v_{oc} = 2$ V



(a) *(b)*

Figure P 5.4-1

Solution:



(checked using LNAP 8/15/02)

P 5.4-2 The circuit shown in Figure P 5.4-2*b* is the Thévenin equivalent circuit of the circuit shown in Figure P 5.4-2*a*. Find the value of the open-circuit voltage, v_{oc}, and Thévenin resistance, R_t.

Answer: $v_{oc} = -12$ V and $R_t = 16\ \Omega$

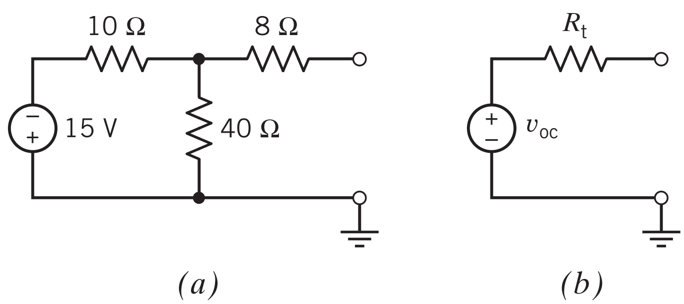
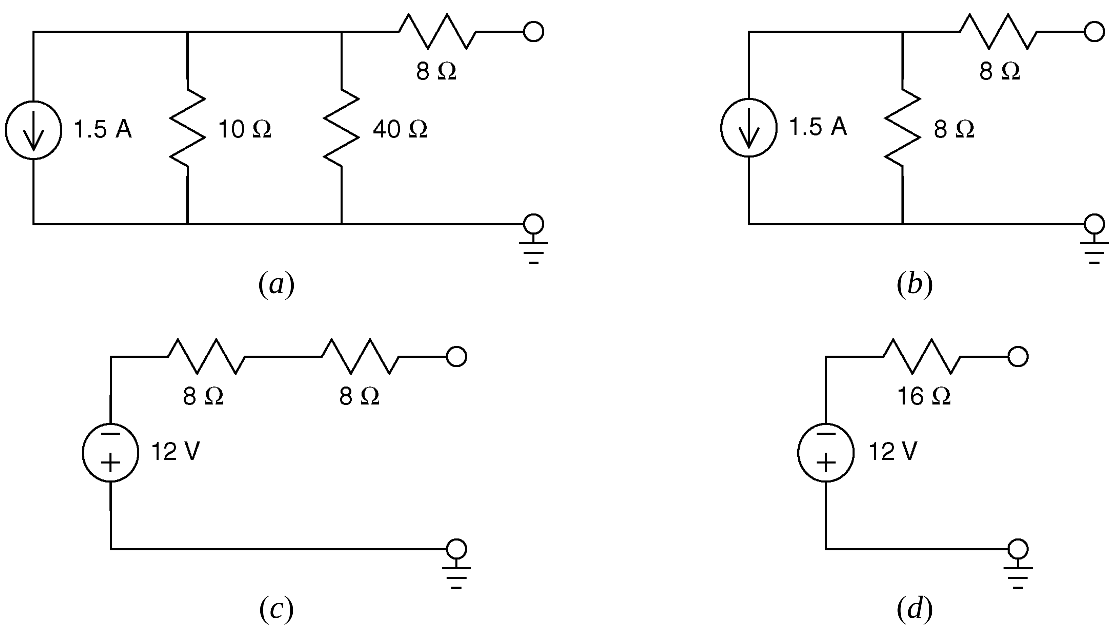


Figure P 5.4-2

Solution:

The circuit from Figure P5.4-2*a* can be reduced to its Thevenin equivalent circuit in four steps:



Comparing (*d*) to Figure P5.4-2*b* shows that the Thevenin resistance is $R_t = 16\ \Omega$ and the open circuit voltage, $v_{oc} = -12$ V.

P 5.4-3 The circuit shown in Figure P 5.4-3*b* is the Thévenin equivalent circuit of the circuit shown in Figure P 5.4-3*a*. Find the value of the open-circuit voltage, v_{oc}, and Thévenin resistance, R_t.

Answer: v_{oc} = 2 V and R_t = 4 Ω

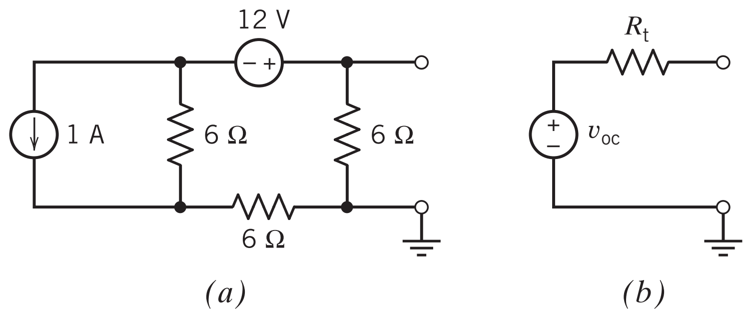
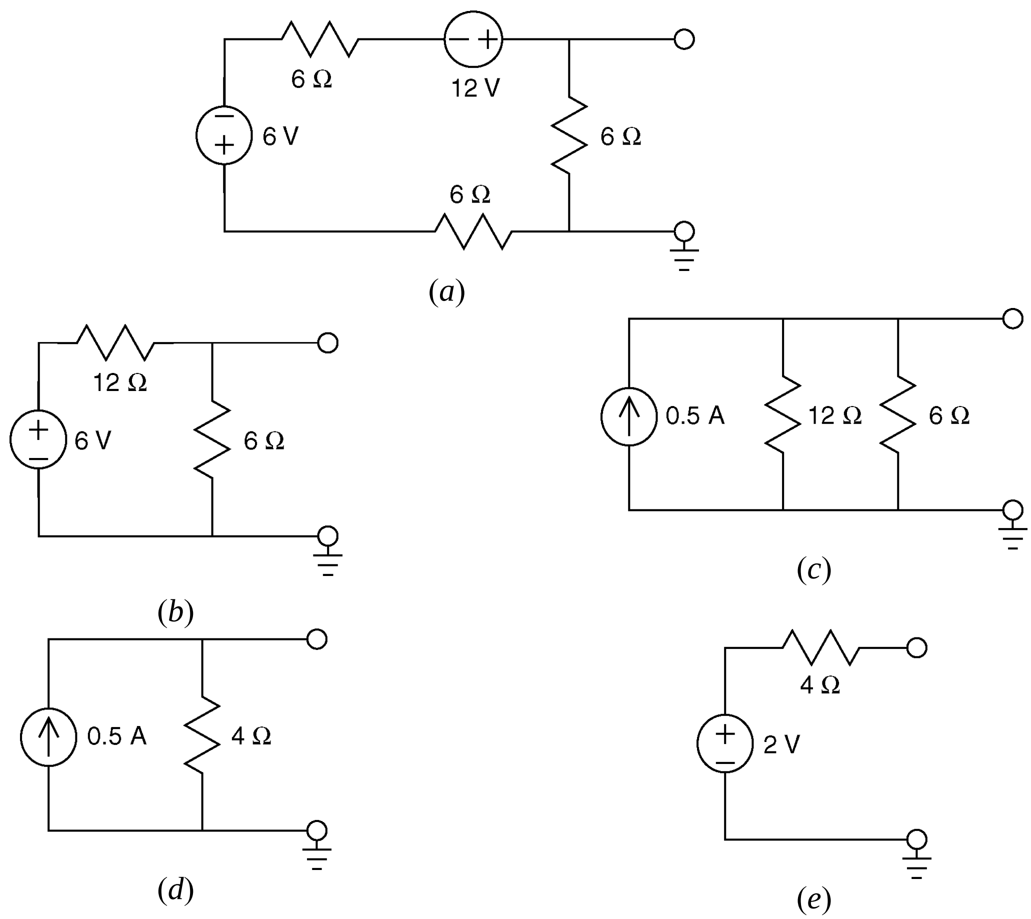


Figure P 5.4-3

Solution:

The circuit from Figure P5.4-3*a* can be reduced to its Thevenin equivalent circuit in five steps:



Comparing (*e*) to Figure P5.4-3*b* shows that the Thevenin resistance is R_t = 4 Ω and the open circuit voltage, v_{oc} = 2 V.

(checked using LNAP 8/15/02)

P 5.4-4 Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-4.

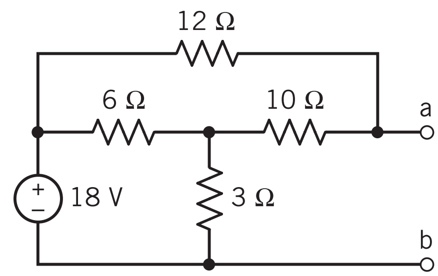
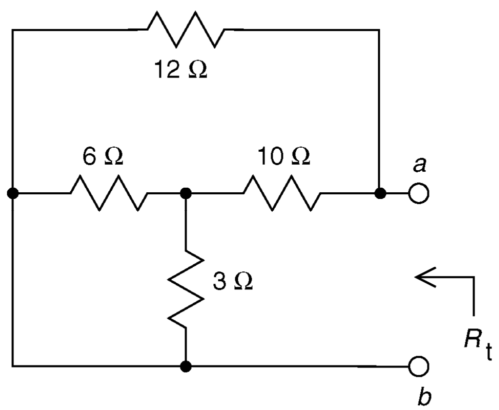
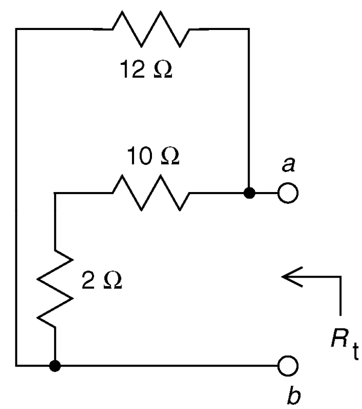


Figure P 5.4-4

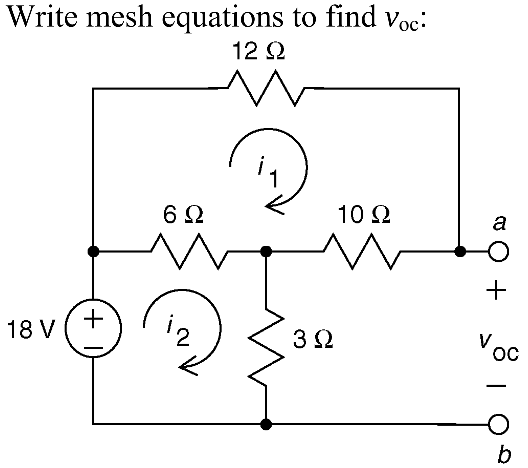
Find R_t:





$$R_t = \frac{12(10+2)}{12+(10+2)} = 6\ \Omega$$

Write mesh equations to find v_{oc}:



Mesh equations:

$$12\,i_1 + 10\,i_1 - 6\left(i_2 - i_1\right) = 0$$

$$6\left(i_2 - i_1\right) + 3\,i_2 - 18 = 0$$

$$28\,i_1 = 6\,i_2$$

$$9\,i_2 - 6\,i_1 = 18$$

$$36\,i_1 = 18 \quad \Rightarrow \quad i_1 = \frac{1}{2}\ \text{A}$$

$$i_2 = \frac{14}{3}\left(\frac{1}{2}\right) = \frac{7}{3}\ \text{A}$$

Finally, $$v_{oc} = 3\,i_2 + 10\,i_1 = 3\left(\frac{7}{3}\right) + 10\left(\frac{1}{2}\right) = 12\ \text{V}$$

(checked using LNAP 8/15/02)

P 5.4-5 Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-5.

Answer: $v_{oc} = -2$ V and $R_t = -8/3\ \Omega$

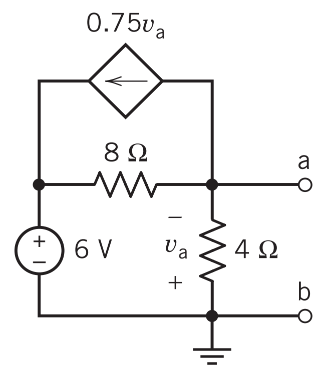
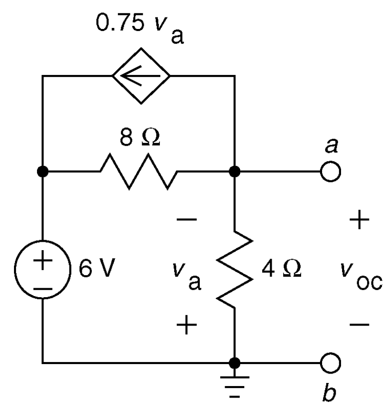


Figure P 5.4-4

Solution:

Find v_{oc}:

Notice that v_{oc} is the node voltage at node a. Express the controlling voltage of the dependent source as a function of the node voltage:



$$v_a = -v_{oc}$$

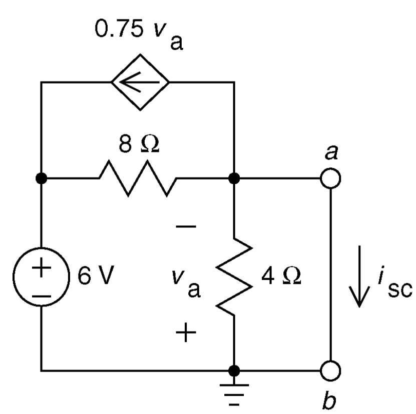
Apply KCL at node a:

$$-\left(\frac{6-v_{oc}}{8}\right)+\frac{v_{oc}}{4}+\left(-\frac{3}{4}v_{oc}\right)=0$$

$$-6+v_{oc}+2\,v_{oc}-6\,v_{oc}=0 \quad\Rightarrow\quad v_{oc}=-2\text{ V}$$

Find R_t:

We'll find i_{sc} and use it to calculate R_t. Notice that the short circuit forces



$$v_a = 0$$

Apply KCL at node a:

$$-\left(\frac{6-0}{8}\right)+\frac{0}{4}+\left(-\frac{3}{4}0\right)+i_{sc}=0$$

$$i_{sc}=\frac{6}{8}=\frac{3}{4}\text{ A}$$

$$R_t=\frac{v_{oc}}{i_{sc}}=\frac{-2}{3/4}=-\frac{8}{3}\ \Omega$$

(checked using LNAP 8/15/02)

P 5.4-6 Find the Thévenin equivalent circuit for the circuit shown in Figure P 5.4-6.

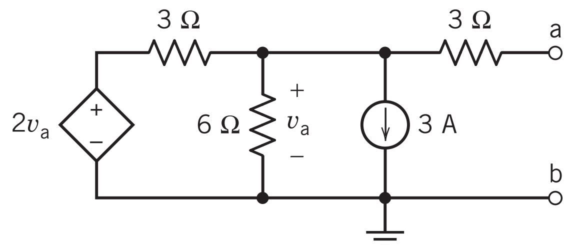
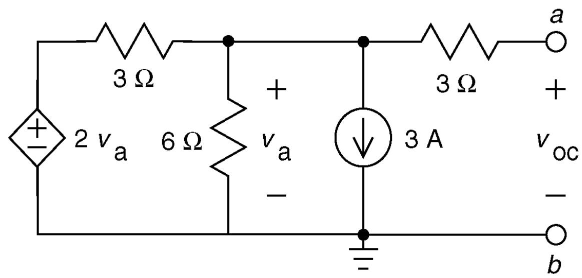


Figure P 5.4-6

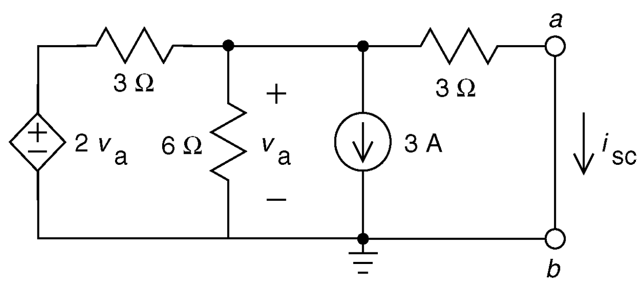
Solution:
Find v_{oc}:



Apply KCL at the top, middle node: $$\frac{2\,v_a - v_a}{3} = \frac{v_a}{6} + 3 + 0 \quad \Rightarrow \quad v_a = 18\text{ V}$$

The voltage across the right-hand 3 Ω resistor is zero so: $v_a = v_{oc} = 18$ V

Find i_{sc}:



Apply KCL at the top, middle node: $$\frac{2\,v_a - v_a}{3} = \frac{v_a}{6} + 3 + \frac{v_a}{3} \quad \Rightarrow \quad v_a = -18\text{ V}$$

Apply Ohm's law to the right-hand 3 Ω resistor : $$i_{sc} = \frac{v_a}{3} = \frac{-18}{3} = -6\text{ V}$$

Finally: $$R_t = \frac{v_{oc}}{i_{sc}} = \frac{18}{-6} = -3\,\Omega$$

(checked using LNAP 8/15/02)

P5.4-7 The equivalent circuit in Figure P5.4-7 is obtained by replacing part of the original circuit by its Thevenin equivalent circuit. The values of the parameters of the Thevenin equivalent circuit are

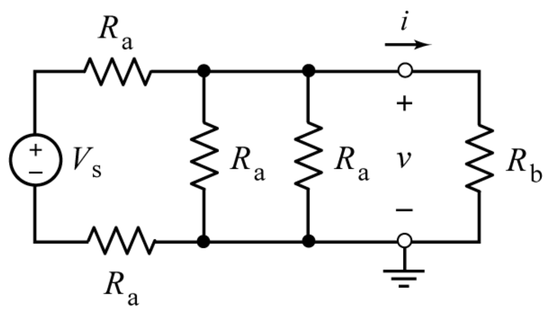
$$v_{oc} = 15 \text{ V and } R_t = 60\ \Omega$$

Determine the following:

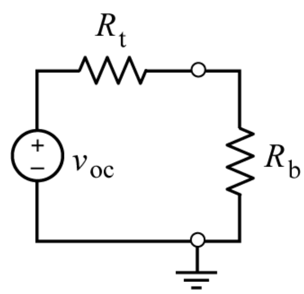
a.) The values of V_s and R_a. (Three resistors in the original circuit have equal resistance, R_a.)

b.) The value of R_b required to cause i = 0.2 A.

c.) The value of R_b required to cause v = 12 V.



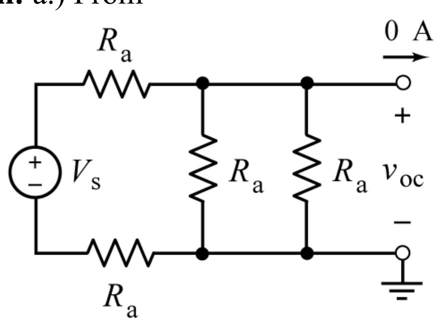
original circuit

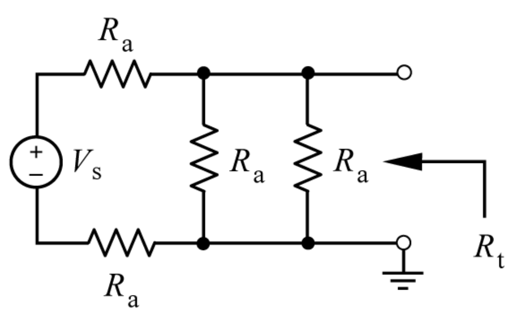


equivalent circuit

Figure P5.4-7

Solution: a.) From



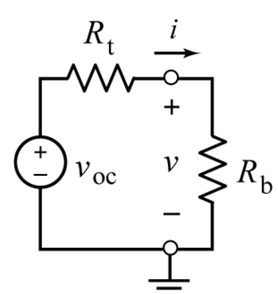


We see that $v_{oc} = \dfrac{V_s}{5}$ and $R_t = \dfrac{2}{5} R_a$. With the given values of v_{oc} and R_t we calculate

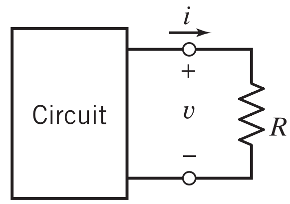
$$15 = \frac{V_s}{5} \quad \Rightarrow \quad V_s = 75 \text{ V} \quad \text{and} \quad 60 = \frac{2}{5} R_a \quad \Rightarrow \quad R_a = 150\ \Omega .$$

b.) $\quad i = \dfrac{v_{oc}}{R_t + R_b} \quad \Rightarrow \quad 0.2 = \dfrac{15}{60 + R_b} \quad \Rightarrow \quad R_b = 15\ \Omega$

c.) $\quad v = \dfrac{R_b}{R_t + R_b} v_{oc} \quad \Rightarrow \quad 12 = \dfrac{15\, R_b}{60 + R_b} \quad \Rightarrow \quad R_b = 240\ \Omega$



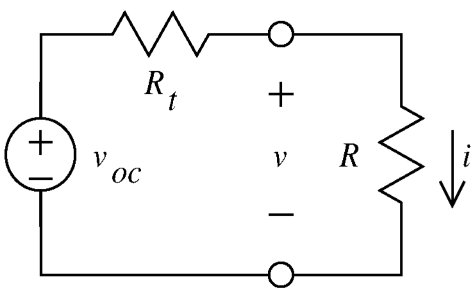
P 5.4-8 A resistor, R, was connected to a circuit box as shown in Figure P 5.4-8. The voltage, v, was measured. The resistance was changed, and the voltage was measured again. The results are shown in the table. Determine the Thévenin equivalent of the circuit within the box and predict the voltage, v, when $R = 8$ kΩ.



R	v
2 kΩ	6 V
4 kΩ	2 V

Figure P 5.4-8

Solution:



$$v = \frac{R}{R_t + R} v_{oc}$$

From the given data:

$$\left. \begin{array}{l} 6 = \dfrac{2000}{R_t + 2000} v_{oc} \\ 2 = \dfrac{4000}{R_t + 4000} v_{oc} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} v_{oc} = 1.2 \text{ V} \\ R_t = -1600\ \Omega \end{array} \right.$$

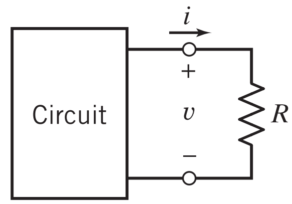
When $R = 8000\ \Omega$,

$$v = \frac{8000}{-1600 + 8000}(1.2) = 1.5 \text{ V}$$

P 5.4-9 A resistor, R, was connected to a circuit box as shown in Figure P 5.4-9. The current, i, was measured. The resistance was changed, and the current was measured again. The results are shown in the table.

(a) Specify the value of R required to cause i = 2 mA.

(b) Given that $R > 0$, determine the maximum possible value of the current i.

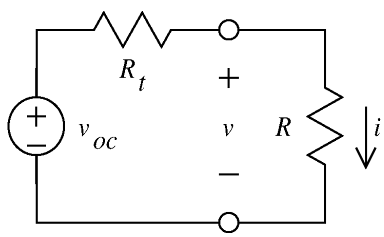


R	i
2 kΩ	4 mA
4 kΩ	3 mA

Figure P 5.4-9

Hint: Use the data in the table to represent the circuit by a Thévenin equivalent.

Solution:



$$i = \frac{v_{oc}}{R_t + R}$$

From the given data:

$$\left.\begin{array}{l} 0.004 = \dfrac{v_{oc}}{R_t + 2000} \\ 0.003 = \dfrac{v_{oc}}{R_t + 4000} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} v_{oc} = 24 \text{ V} \\ R_t = 4000\ \Omega \end{array}\right.$$

(a) When $i = 0.002$ A:

$$0.002 = \frac{24}{4000 + R} \quad \Rightarrow \quad R = 8000\ \Omega$$

(b) Maximum i occurs when R = 0:

$$\frac{24}{4000} = 0.006 = 6 \text{ mA} \quad \Rightarrow \quad i \le 6 \text{ mA}$$

P 5.4-10 For the circuit of Figure P 5.4-10, specify the resistance R that will cause current i_b to be 2 mA. The current i_a has units of amps.

Hint: Find the Thévenin equivalent circuit of the circuit connected to R.

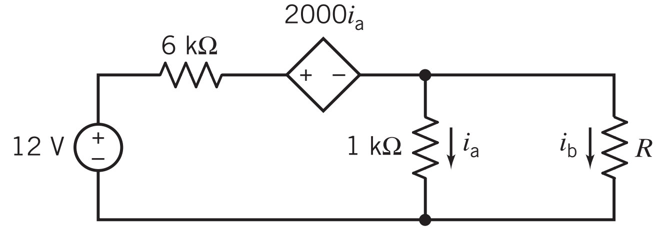
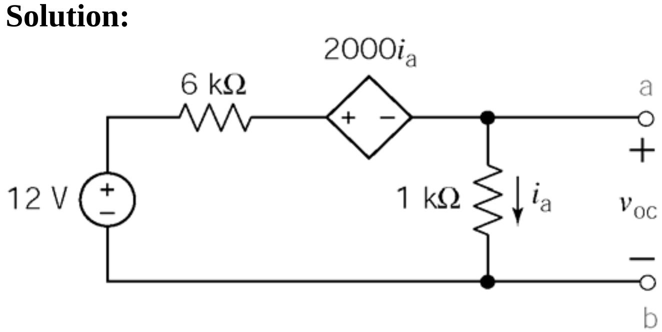


Figure P 5.4-10

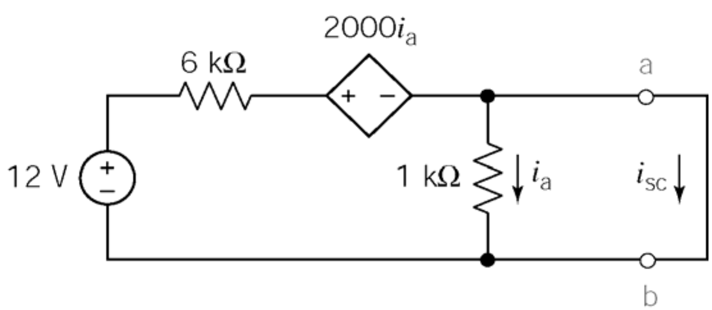
Solution:



$$-12+6000\,i_a+2000\,i_a+1000\,i_a=0$$

$$i_a = 4/3000\text{ A}$$

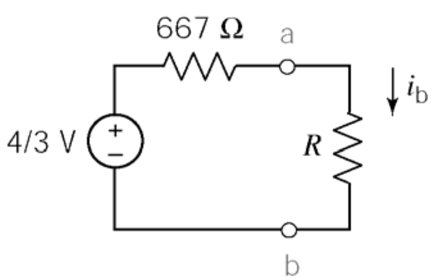
$$v_{oc}=1000\ i_a=\frac{4}{3}\text{ V}$$



$i_a = 0$ due to the short circuit

$$-12+6000\,i_{sc}=0 \;\Rightarrow\; i_{sc}=2\text{ mA}$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{\frac{4}{3}}{.002} = 667\ \Omega$$



$$i_b = \frac{\frac{4}{3}}{667+R}$$

$i_b = 0.002$ A requires

$$R = \frac{\frac{4}{3}}{0.002} - 667 = 0$$

(checked using LNAP 8/15/02)

P 5.4-11 For the circuit of Figure P 5.4-11, specify the value of the resistance R_L that will cause current i_L to be –2 A.

Answer: $R_L = 12\ \Omega$

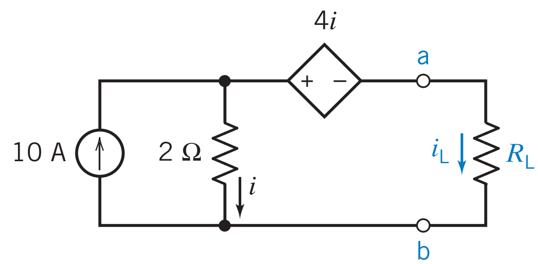
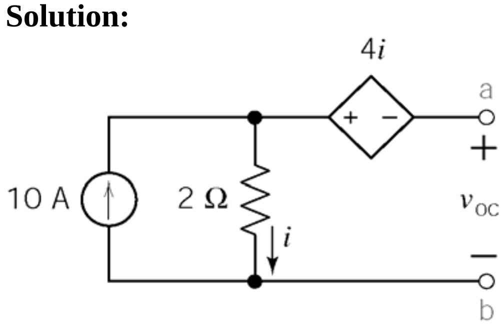


Figure P 5.4-11

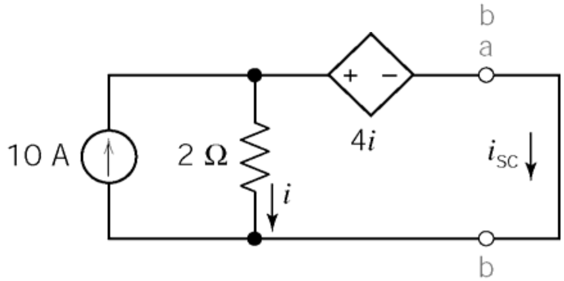
Solution:



$$10 = i + 0 \quad \Rightarrow \quad i = 10 \text{ A}$$

$$v_{oc} + 4\,i - 2\,i = 0$$

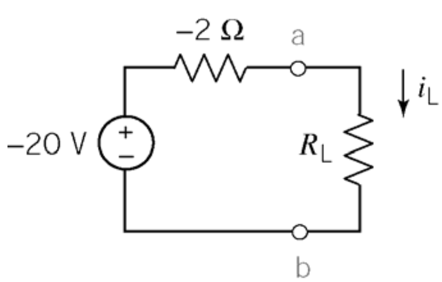
$$\Rightarrow \quad v_{oc} = -2\,i = \underline{-20 \text{ V}}$$



$$i + i_{sc} = 10 \quad \Rightarrow \quad i = 10 - i_{sc}$$

$$4\,i + 0 - 2\,i = 0 \quad \Rightarrow \quad i = 0 \quad \Rightarrow \quad i_{sc} = 10 \text{ A}$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{-20}{10} = -2\,\Omega$$



$$-2 = i_L = \frac{-20}{R_L - 2} \quad \Rightarrow \quad R_L = 12\,\Omega$$

(checked using LNAP 8/15/02)

P 5.4-12 The circuit shown in Figure P 5.4-12 contains an adjustable resistor. The resistance R can be set to any value in the range $0 \le R \le 100$ kΩ.

(a) Determine the maximum value of the current i_a that can be obtained by adjusting R. Determine the corresponding value of R.

(b) Determine the maximum value of the voltage v_a that can be obtained by adjusting R. Determine the corresponding value of R.

(c) Determine the maximum value of the power supplied to the adjustable resistor that can be obtained by adjusting R. Determine the corresponding value of R.

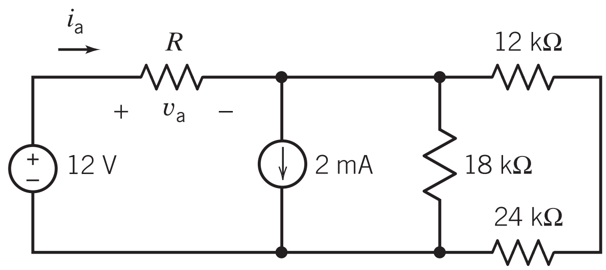
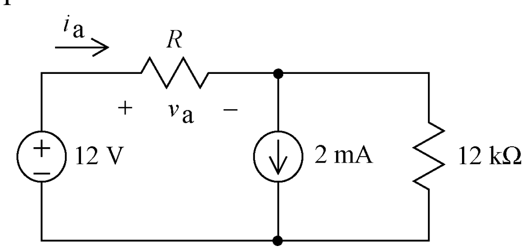
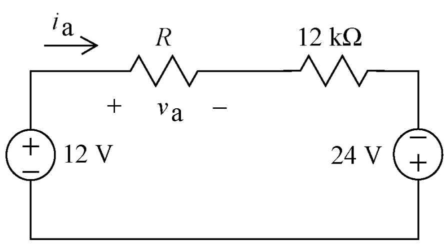


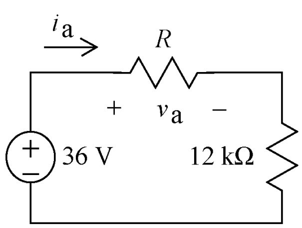
Figure P 5.4-12

Solution: Replace the part of the circuit that is connected to the variable resistor by its Thevenin equivalent circuit:



$$18\text{ k}\Omega \,||\, (12\text{ k}\Omega + 24\text{ k}\Omega) = 18\text{ k}\Omega \,||\, 36\text{ k}\Omega = 12\text{ k}\Omega$$





$$i_a = \frac{36}{R+12000} \quad \text{and} \quad v_a = \frac{R}{R+12000} 36$$

$$p = i_a\, v_a = \left(\frac{36}{R+12000}\right)^2 R$$

(a) $i_a = \dfrac{36}{0+12000} = 3$ mA when $R = 0\ \Omega$ (a short circuit).

(b) $v_a = \dfrac{10^5}{10^5 + 12000} 36 = 32.14$ V when R is as large as possible, i.e. $R = 100$ kΩ.

(c) Maximum power is delivered to the adjustable resistor when $R = R_t = 12\text{ k}\Omega$. Then

$$p = i_a \, v_a = \left(\frac{36}{12000+12000}\right)^2 12000 = 0.027 = 27 \text{ mW}$$

(checked: LNAP 6/22/04)

P 5.4-13 The circuit shown in Figure P 5.4-13 consists of two parts, the source (to the left of the terminals) and the load. The load consists of a single adjustable resistor having resistance $0 \le R_L \le 20\ \Omega$. The resistance R is fixed, but unspecified. When $R_L = 4\ \Omega$, the load current is measured to be $i_o = 0.375$ A. When $R_L = 8\ \Omega$, the value of the load current is $i_o = 0.300$ A.

(a) Determine the value of the load current when $R_L = 10\ \Omega$.

(b) Determine the value of R.

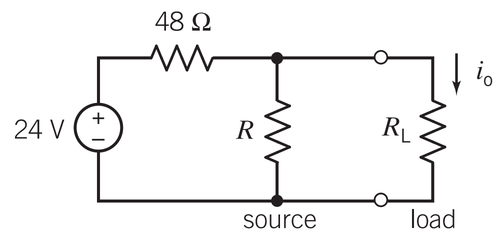
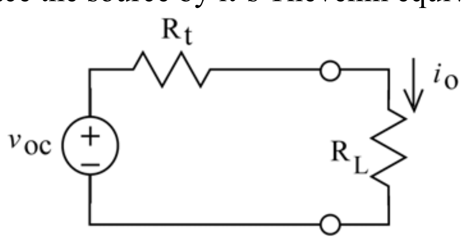


Figure P 5.4-13

Solution:

Replace the source by it's Thevenin equivalent circuit to get



$$i_o = \frac{v_{oc}}{R_t + R_L}$$

Using the given formation

$$\left.\begin{array}{l} 0.375 = \dfrac{v_{oc}}{R_t + 4} \\[2ex] 0.300 = \dfrac{v_{oc}}{R_t + 8} \end{array}\right\} \Rightarrow \quad 0.375(R_t + 4) = 0.300(R_t + 8)$$

So

$$R_t = \frac{(0.300)8 - (0.375)4}{0.075} = 12\ \Omega \text{ and } v_{oc} = 0.3(12+8) = 6\ \text{V}$$

(a) When $R_L = 10\ \Omega,\ i_o = \dfrac{6}{12+10} = 0.27\overline{27}$ A.

(b) $12\ \Omega = R_t = 48 \parallel R \quad \Rightarrow \quad R = 16\ \Omega$.

(checked: LNAP 5/24/04)

P 5.4-14 The circuit shown in Figure P 5.4-14 contains an unspecified resistance, R. Determine the value of R in each of the following two ways.

(a) Write and solve mesh equations.

(b) Replace the part of the circuit connected to the resistor R by a Thévenin equivalent circuit. Analyze the resulting circuit.

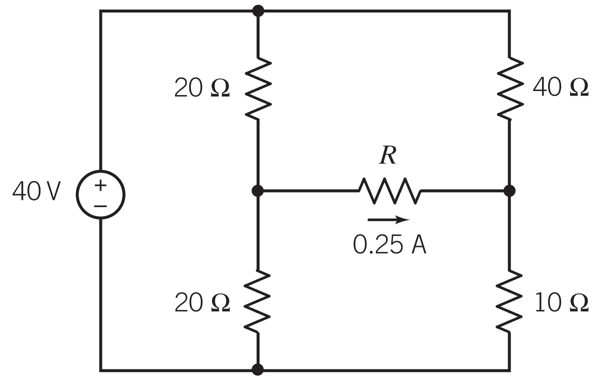
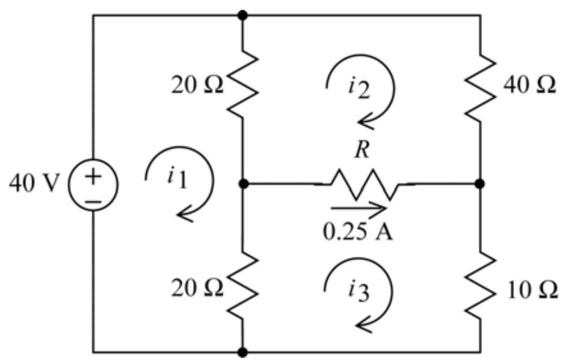


Figure P 5.4-14

Solution:

(a)



$$i_3 - i_2 = 0.25 \text{ A}$$

Apply KVL to mesh 1 to get

$$20(i_1 - i_2) + 20(i_1 - i_3) - 40 = 0$$

Apply KVL to the supermesh corresponding to the unspecified resistance to get

$$40i_2 + 10i_3 - 20(i_1 - i_3) - 20(i_1 - i_2) = 0$$

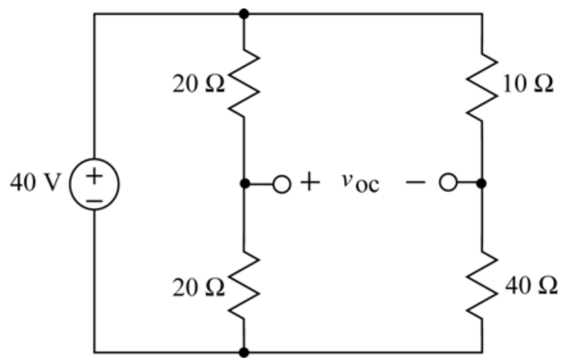
Solving, for example using MATLAB, gives

$$\begin{bmatrix} 0 & -1 & 1 \\ 40 & -20 & -20 \\ -40 & 60 & 30 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 40 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1.875 \\ 0.750 \\ 1.000 \end{bmatrix}$$

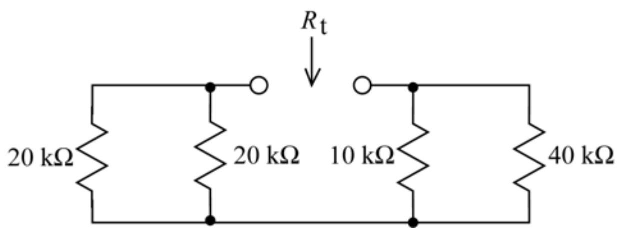
Apply KVL to mesh 2 to get

$$40i_2 + R(i_2 - i_3) - 20(i_1 - i_2) = 0 \quad \Rightarrow \quad R = \frac{20(i_1 - i_2) - 40i_2}{i_2 - i_3} = 30 \ \Omega$$

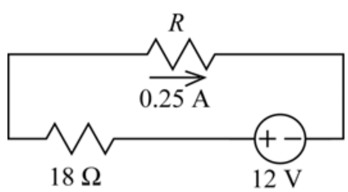
(b)



$$v_{oc} = \left(\frac{20}{20+20}\right)40 - \left(\frac{40}{10+40}\right)40 = -12 \text{ V}$$



$$R_t = 18\ \Omega$$



$$0.25 = \frac{12}{18+R} \quad \Rightarrow \quad R = 30\ \Omega$$

(checked: LNAP 5/25/04)

P 5.4-15 Consider the circuit shown in Figure P 5.4-15. Replace the part of the circuit to the left of terminals a–b by its Thévenin equivalent circuit. Determine the value of the current i_o.

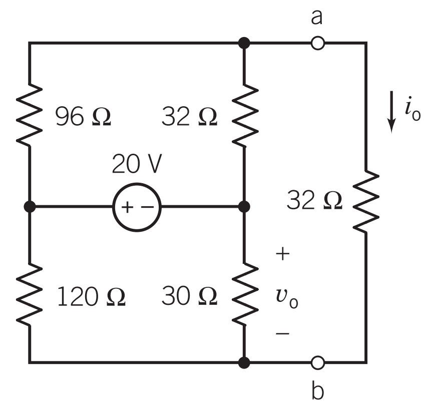
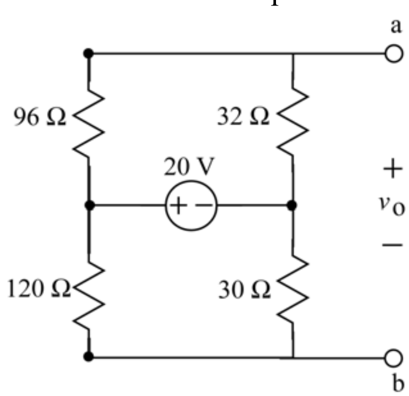


Figure P 5.4-15

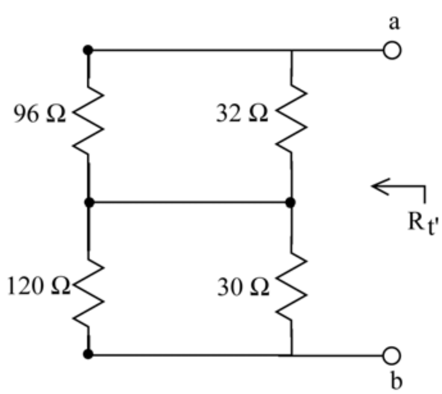
Solution:

Find the Thevenin equivalent circuit for the part of the circuit to the left of the terminals a-b.



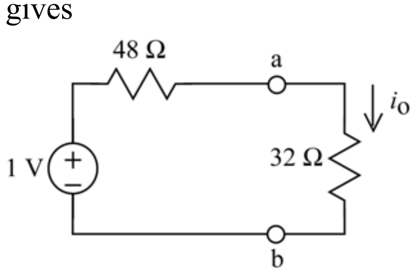
Using voltage division twice

$$v_{oc} = \frac{32}{32+96}20 - \frac{30}{120+30}20 = 5-4 = 1\text{ V}$$



$$R_t = (96 \,||\, 32) + (120 \,||\, 30) = 24 + 24 = 48\ \Omega$$

Replacing the part of the circuit to the left of terminals a-b by its Thevenin equivalent circuit gives



$$i_o = \frac{1}{48+32} = 0.0125\text{ A} = 12.5\text{ mA}$$

(checked: LNAP 5/24/04)

P 5.4-16 An ideal voltmeter is modeled as an open circuit. A more realistic model of a voltmeter is a large resistance. Figure P 5.4-16*a* shows a circuit with a voltmeter that measures the voltage v_m. In Figure P 5.4-16*b* the voltmeter is replaced by the model of an ideal voltmeter, an open circuit. The voltmeter measures v_{mi}, the ideal value of v_m.

As $R_m \to \infty$, the voltmeter becomes an ideal voltmeter and $v_m \to v_{mi}$. When $R_m < \infty$, the voltmeter is not ideal and $v_m > v_{mi}$. The difference between v_m and v_{mi} is a measurement error caused by the fact that the voltmeter is not ideal.

(a) Determine the value of v_{mi}.

(b) Express the measurement error that occurs when $R_m = 1000\ \Omega$ as a percentage of v_{mi}.

(c) Determine the minimum value of R_m required to ensure that the measurement error is smaller than 2 percent of v_{mi}.

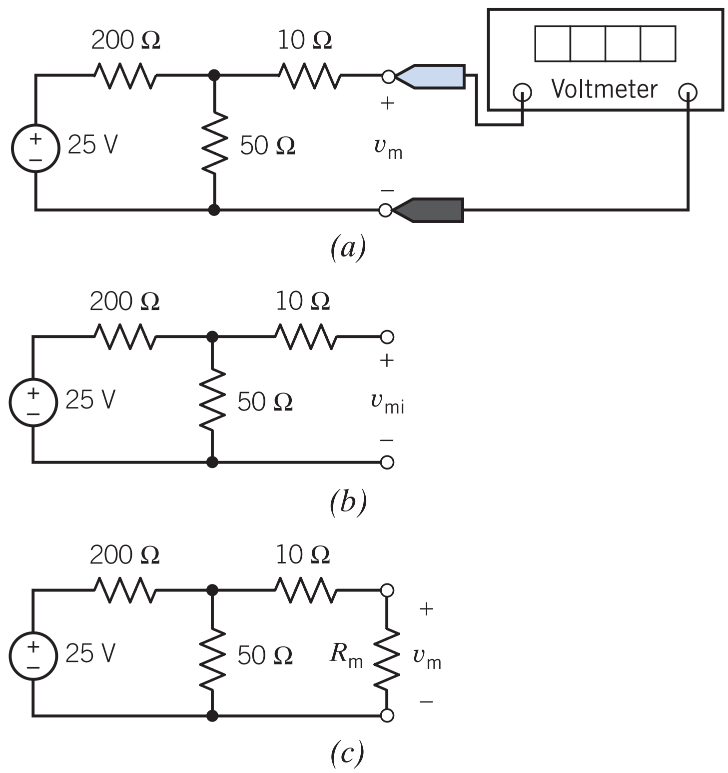
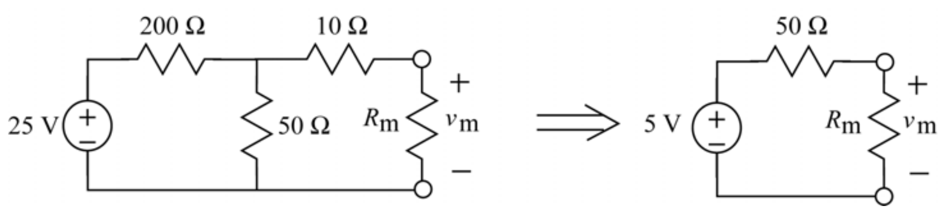


Figure P 5.4-16

Solution: Replace the circuit by its Thevenin equivalent circuit:



$$v_m = \left(\frac{R_m}{R_m + 50}\right) 5$$

(a)
$$v_{mi} = \lim_{R_m \to \infty} v_m = 5\ \text{V}$$

(b) When $R_m = 1000\ \Omega$, $v_m = 4.763$ V so

$$\%\ \text{error} = \frac{5 - 4.762}{5} \times 100 = 4.76\%$$

(c) $$0.02 \geq \frac{5-\left(\dfrac{R_{\mathrm{m}}}{R_{\mathrm{m}}+50}\right)5}{5} \quad \Rightarrow \quad \frac{R_{\mathrm{m}}}{R_{\mathrm{m}}+50} \geq 0.98 \quad \Rightarrow \quad R_{\mathrm{m}} \geq 2450\ \Omega$$

(checked: LNAP 6/16/04)

P5.4-17

Given that $0 \le R \le \infty$ in the circuit shown in Figure P5.4-17, consider these two observations:

Observation 1: When $R = 2\ \Omega$ then $v_R = 4$ V and $i_R = 2$ A.

Observation 1: When $R = 6\ \Omega$ then $v_R = 6$ V and $i_R = 1$ A.

Determine the following

a) The maximum value of i_R and the value of R that causes i_R to be maximal.

b) The maximum value of v_R and the value of R that causes v_R to be maximal.

c) The maximum value of $p_R = i_R v_R$ and the value of R that causes p_R to be maximal.

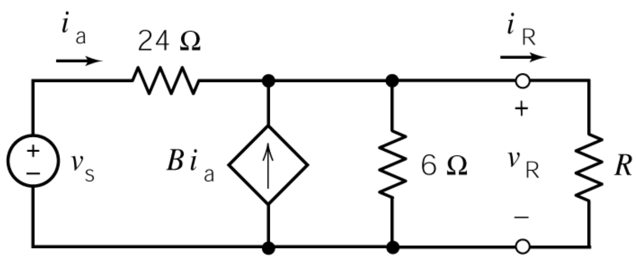
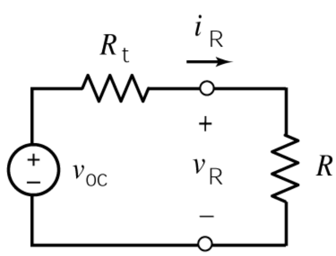


Figure P5.4-17

Solution:

We can replace the part of the circuit to the left of the terminals by its Thevenin equivalent circuit:



Using voltage division $v_R = \dfrac{R}{R+R_t} v_{oc}$ and using Ohm's law

$i_R = \dfrac{v_{oc}}{R+R_t}$.

By inspection , $v_R = \dfrac{R}{R+R_t} v_{oc} = \dfrac{v_{oc}}{1+\dfrac{R_t}{R}}$ will be maximum when

$R = \infty$. The maximum value of v_R will be v_{oc}. Similarly,

$i_R = \dfrac{v_{oc}}{R+R_t}$ will be maximum when $R = 0$. The maximum value

of i_R will be $\dfrac{v_{oc}}{R_t} = i_{sc}$.

The maximum power transfer theorem tells use that $p_R = i_R v_R$ will be maximum when $R = R_t$.

Then $p_R = i_R v_R = \left(\dfrac{v_{oc}}{R+R_t}\right)\left(\dfrac{R}{R+R_t} v_{oc}\right) = R\left(\dfrac{v_{oc}}{R+R_t}\right)^2$.

Let's substitute the given data into the equation $i_{\mathrm{R}} = \dfrac{v_{\mathrm{oc}}}{R+R_{\mathrm{t}}}$.

When R = 2 Ω we get $2 = \dfrac{v_{\mathrm{oc}}}{2+R_{\mathrm{t}}} \quad \Rightarrow \quad 4+2\,R_{\mathrm{t}} = v_{\mathrm{oc}}$. When R = 6 Ω we get

$1 = \dfrac{v_{\mathrm{oc}}}{6+R_{\mathrm{t}}} \quad \Rightarrow \quad 6+R_{\mathrm{t}} = v_{\mathrm{oc}}$.

So $6+R_{\mathrm{t}} = 4+2\,R_{\mathrm{t}} \quad \Rightarrow \quad R_{\mathrm{t}} = 2\ \Omega$ and $v_{\mathrm{oc}} = 4+2\,R_{\mathrm{t}} = 8\ \mathrm{V}$. Also $i_{\mathrm{sc}} = \dfrac{v_{\mathrm{oc}}}{R_{\mathrm{t}}} = \dfrac{8}{2} = 4$ A.

P5.4-18 Consider the circuit shown in Figure P5.4-18. Determine

a) The value of v_R that occurs when $R = 9\ \Omega$.

b) The value of R that causes $v_R = 5.4$ V.

c) The value of R that causes $i_R = 300$ mA.

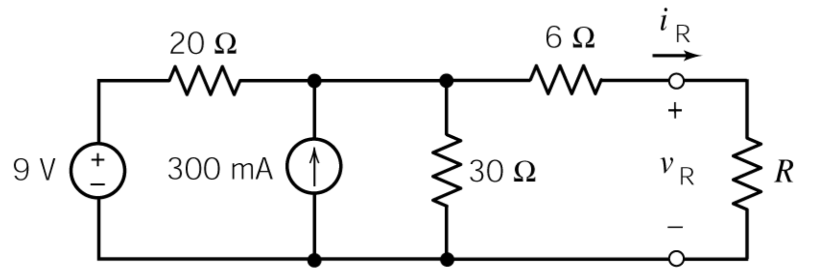
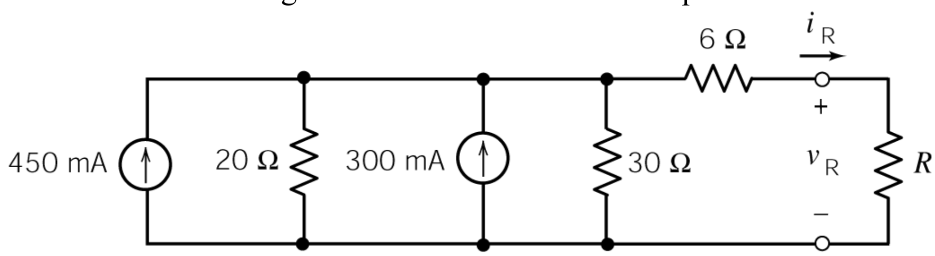
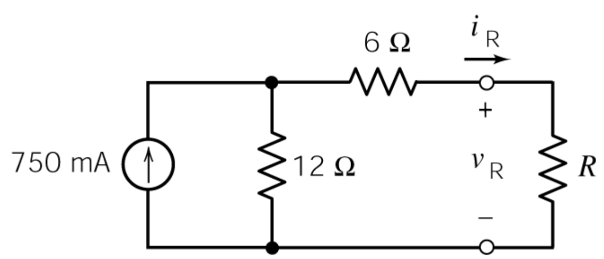
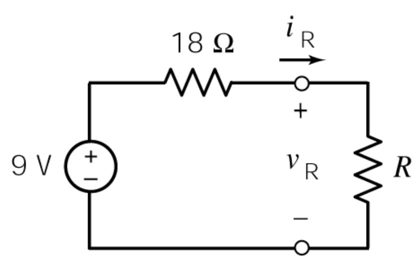


Figure P5.4-18

Solution: Reduce this circuit using source transformations and equivalent resistance:







Now $v_R = \left(\dfrac{R}{R+18}\right)9$ and $i_R = \dfrac{9}{R+18}$ so the questions can be easily answered:

a) When $R = 9\ \Omega$ then $v_R = 3$ V.

b) When $R = 27\ \Omega$ then $v_R = 5.4$ V.

c) When $R = 12\ \Omega$ then $i_R = 300$ mA.

P5.4-19 The circuit shown in Figure P5.4-19*a* can be reduced to the circuit shown in Figure P5.4-19*b* using source transformations and equivalent resistances. Determine the values of the source voltage v_{oc} and the resistance *R*.

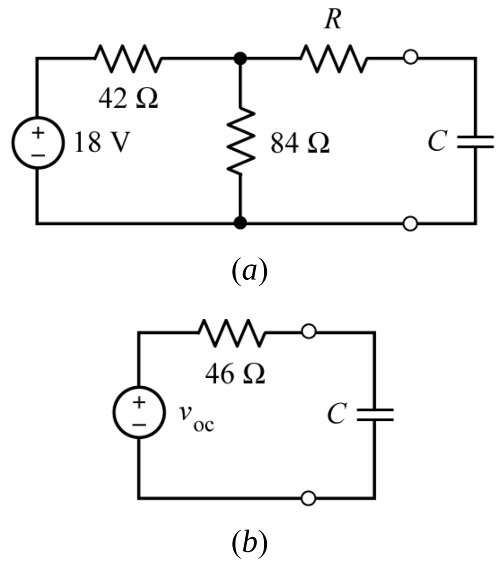
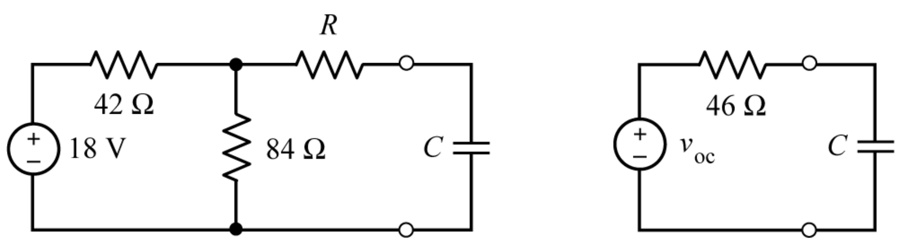


Figure P5.4-19

Solution



$$46 = R_t = R + (42 \,||\, 84) = R + 28 \quad \Rightarrow \quad R = 18\ \Omega$$

$$v_{oc} = \frac{84}{42+84}(18) = 12\ \text{V}$$

P5.4-20 The equivalent circuit in Figure P5.4-20 is obtained by replacing part of the original circuit by its Thevenin equivalent circuit. The values of the parameters of the Thevenin equivalent circuit are

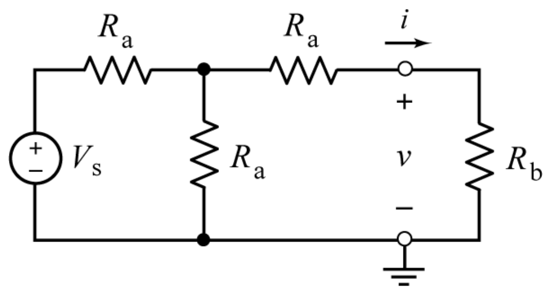
$$v_{oc} = 15 \text{ V and } R_t = 60\ \Omega$$

Determine the following:

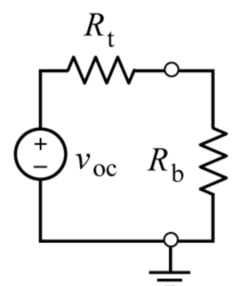
a.) The values of V_s and R_a. (Three resistors in the original circuit have equal resistance, R_a.)

b.) The value of R_b required to cause $i = 0.2$ A.

c.) The value of R_b required to cause $v = 5$ V.



original circuit

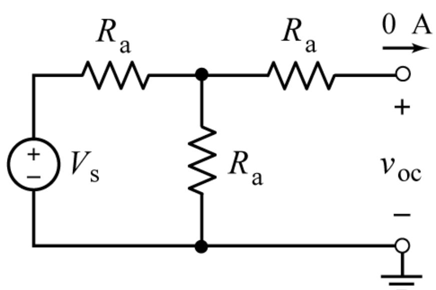


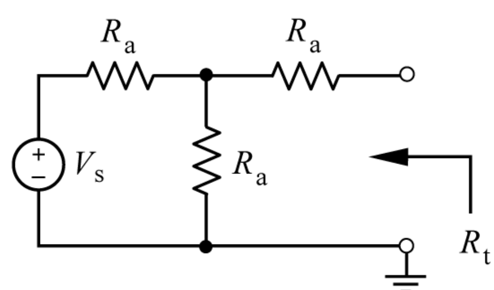
equivalent circuit

Figure P5.4-20

Solution

a.) From



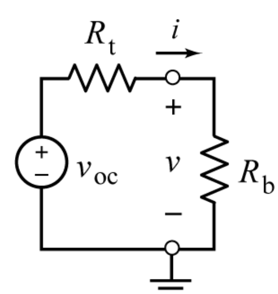


We see that $v_{oc} = \frac{V_s}{2}$ and $R_t = \frac{3}{2} R_a$. With the given values of v_{oc} and R_t we calculate

$$15 = \frac{V_s}{2} \quad \Rightarrow \quad V_s = 30 \text{ V} \quad \text{and} \quad 60 = \frac{3}{2} R_a \quad \Rightarrow \quad R_a = 40\ \Omega .$$

b.) $i = \dfrac{v_{oc}}{R_t + R_b} \quad \Rightarrow \quad 0.2 = \dfrac{15}{60 + R_b} \quad \Rightarrow \quad R_b = 15\ \Omega$

c.) $v = \dfrac{R_b}{R_t + R_b} v_{oc} \quad \Rightarrow \quad 5 = \dfrac{15\, R_b}{60 + R_b} \quad \Rightarrow \quad R_b = 30\ \Omega$



Section 5-5: Norton's Theorem

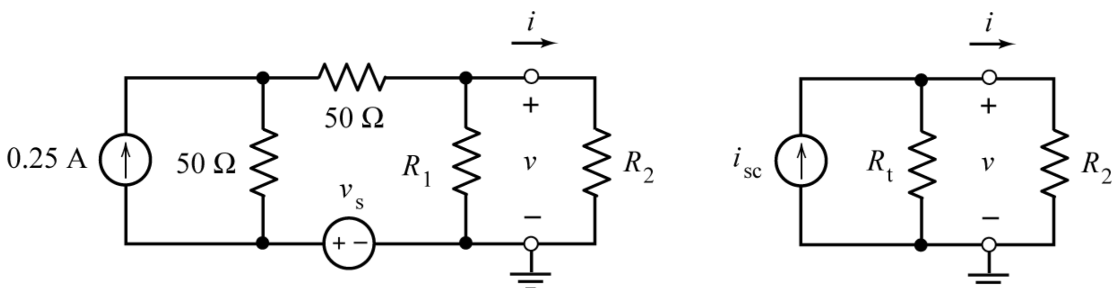
P5.5-1 The part of the circuit shown in Figure P5.3-1a to the left of the terminals can be reduced to its Norton equivalent circuit using source transformations and equivalent resistance. The resulting Norton equivalent circuit, shown in Figure P5.3-1b, will be characterized by the parameters:

$$i_{sc} = 0.5 \text{ A} \quad \text{and} \quad R_t = 20\ \Omega$$

a) Determine the values of v_s and R_1.

b) Given that $0 \le R_2 \le \infty$, determine the maximum values of the voltage, v, and of the power, $p = vi$.

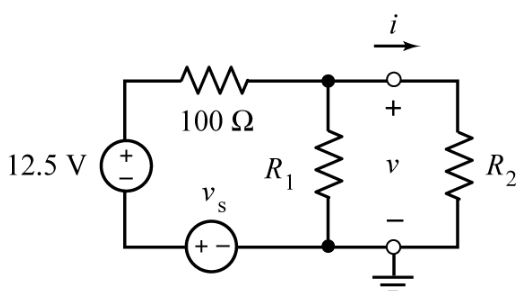
Answers: $v_s = 37.5$ V, $R_1 = 25\ \Omega$, max $v = 10$ V and max $p = 1.25$ W

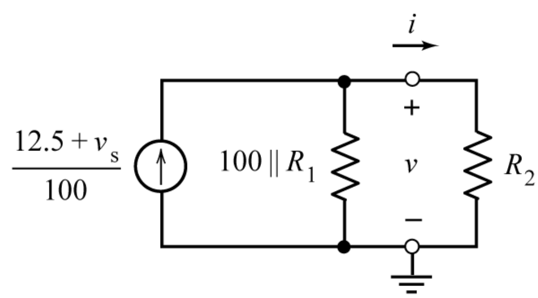


(*a*) (*b*)

Figure P5.5-1

Solution: Two source transformations reduce the circuit as follows:





Recognizing the parameters of the Norton equivalent circuit gives:

$$0.5 = i_{sc} = \frac{12.5 + v_s}{100} \quad \Rightarrow \quad v_s = 37.5 \text{ V} \ \text{ and } \ 20 = R_t = 100 \,||\, R_1 = \frac{100\, R_1}{100 + R_1} \quad \Rightarrow \quad R_1 = 25\ \Omega$$

Next, the voltage across resistor R_2 is given by $v = i_{sc}\left(R_t \,||\, R_2\right) = \dfrac{R_t\, R_2\, i_{sc}}{R_t + R_2} = \dfrac{R_t\, i_{sc}}{\dfrac{R_t}{R_2} + 1}$ so this voltage is maximum when $R_2 = \infty$ and max $v = R_t\, i_{sc} = 10$ V. The power $p = vi$ will be maximum when $R_2 = R_t = 20\ \Omega$. Then $v = \dfrac{R_t\, i_{sc}}{2} = \dfrac{20(0.5)}{2} = 5\ \text{V}$, $i = \dfrac{v}{R_2} = \dfrac{5}{20} = 0.25\ \text{A}$ and $p = v\,i = 5(0.25) = 1.25\ \text{W}$.

P 5.5-2 Two black boxes are shown in Figure P 5.5-2. Box A contains the Thévenin equivalent of some linear circuit, and box B contains the Norton equivalent of the same circuit. With access to just the outsides of the boxes and their terminals, how can you determine which is which, using only one shorting wire?

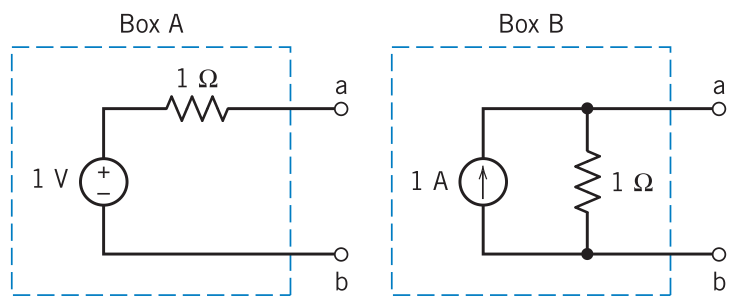


Figure P 5.5-2

Solution:

When the terminals of the boxes are open-circuited, no current flows in Box A, but the resistor in Box B dissipates 1 watt. Box B is therefore warmer than Box A. If you short the terminals of each box, the resistor in Box A will draw 1 amp and dissipate 1 watt. The resistor in Box B will be shorted, draw no current, and dissipate no power. Then Box A will warm up and Box B will cool off.

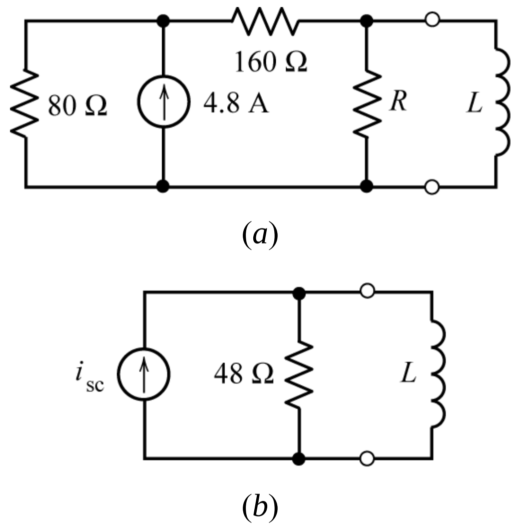
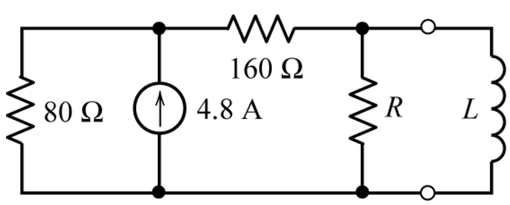
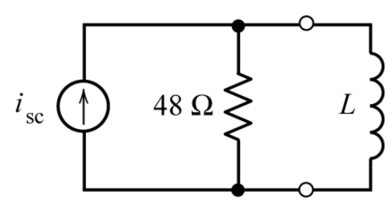


Figure P5.5-3

P5.5-3 The circuit shown in Figure P5.5-3*a* can be reduced to the circuit shown in Figure P5.5-3*b* using source transformations and equivalent resistances. Determine the values of the source current i_{sc} and the resistance *R*.

Solution:





$$48 = R_t = R \| (80+160) = \frac{240\,R}{R+240} \quad \Rightarrow \quad R = 60\ \Omega$$

$$i_{sc} = \frac{80}{80+160}(4.8) = 1.6\ \text{A}$$

P 5.5-4 Find the Norton equivalent circuit for the circuit shown in Figure P 5.5-4.

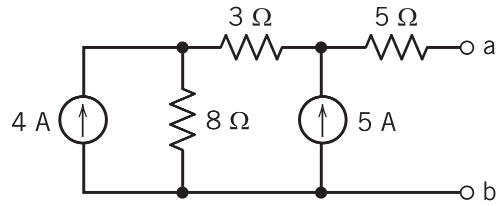
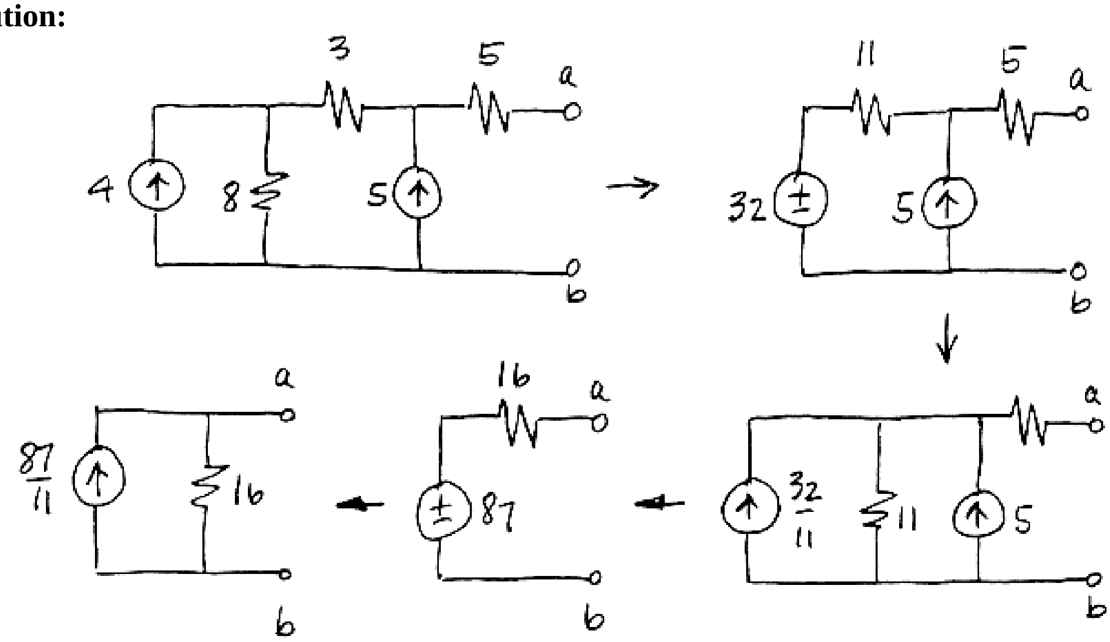


Figure P 5.5-4

Solution:



P 5.5-5 The circuit shown in Figure P 5.5-5*b* is the Norton equivalent circuit of the circuit shown in Figure P 5.5-5*a*. Find the value of the short-circuit current, i_{sc}, and Thévenin resistance, R_t.

Answer: i_{sc} = 1.13 A and R_t = 7.57 Ω

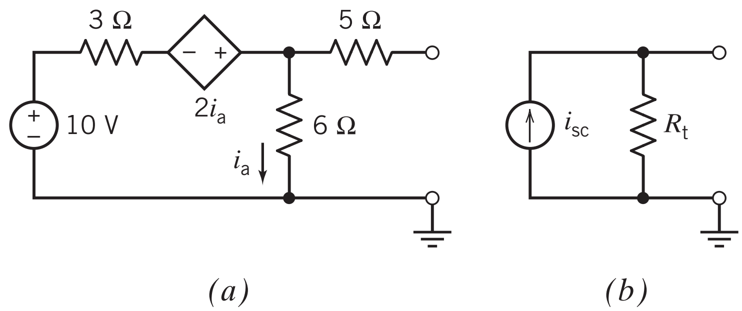


Figure P 5.5-5

Solution:

To determine the value of the short circuit current, i_{sc}, we connect a short circuit across the terminals of the circuit and then calculate the value of the current in that short circuit. Figure (a) shows the circuit from Figure 5.6-4*a* after adding the short circuit and labeling the short circuit current. Also, the meshes have been identified and labeled in anticipation of writing mesh equations. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

In Figure (a), mesh current i_2 is equal to the current in the short circuit. Consequently, $i_2 = i_{sc}$. The controlling current of the CCVS is expressed in terms of the mesh currents as

$$i_a = i_1 - i_2 = i_1 - i_{sc}$$

Apply KVL to mesh 1 to get

$$3\,i_1 - 2\left(i_1 - i_2\right) + 6\left(i_1 - i_2\right) - 10 = 0 \quad \Rightarrow \quad 7\,i_1 - 4\,i_2 = 10 \qquad \textbf{(1)}$$

Apply KVL to mesh 2 to get

$$5\,i_2 - 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad -6\,i_1 + 11\,i_2 = 0 \quad \Rightarrow \quad i_1 = \frac{11}{6}\,i_2$$

Substituting into equation 1 gives

$$7\left(\frac{11}{6}\,i_2\right) - 4\,i_2 = 10 \quad \Rightarrow \quad i_2 = 1.13 \text{ A} \quad \Rightarrow \quad i_{sc} = 1.13 \text{ A}$$

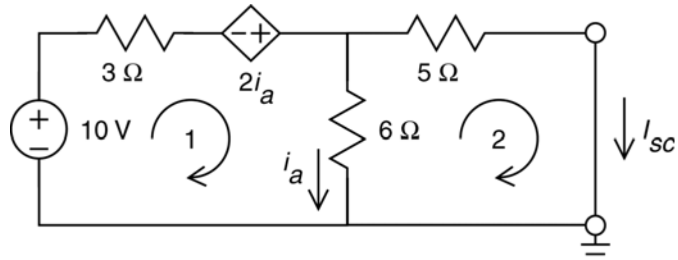


Figure (a) Calculating the short circuit current, i_{sc}, using mesh equations.

To determine the value of the Thevenin resistance, R_t, first replace the 10 V voltage source by a 0 V voltage source, i.e. a short circuit. Next, connect a current source across the terminals of the circuit and then label the voltage across that current source as shown in Figure (b). The Thevenin resistance will be calculated from the current and voltage of the current source as

$$R_t = \frac{v_T}{i_T}$$

In Figure (b), the meshes have been identified and labeled in anticipation of writing mesh equations. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

In Figure (b), mesh current i_2 is equal to the negative of the current source current. Consequently, $i_2 = i_T$. The controlling current of the CCVS is expressed in terms of the mesh currents as

$$i_a = i_1 - i_2 = i_1 + i_T$$

Apply KVL to mesh 1 to get

$$3\,i_1 - 2\left(i_1 - i_2\right) + 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad 7\,i_1 - 4\,i_2 = 0 \quad \Rightarrow \quad i_1 = \frac{4}{7}\,i_2 \qquad \textbf{(2)}$$

Apply KVL to mesh 2 to get

$$5\,i_2 + v_T - 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad -6\,i_1 + 11\,i_2 = -v_T$$

Substituting for i_1 using equation 2 gives

$$-6\left(\frac{4}{7}\,i_2\right) + 11\,i_2 = -v_T \quad \Rightarrow \quad 7.57\,i_2 = -v_T$$

Finally,

$$R_t = \frac{v_T}{i_T} = \frac{-v_T}{-i_T} = \frac{-v_T}{i_2} = 7.57\ \Omega$$

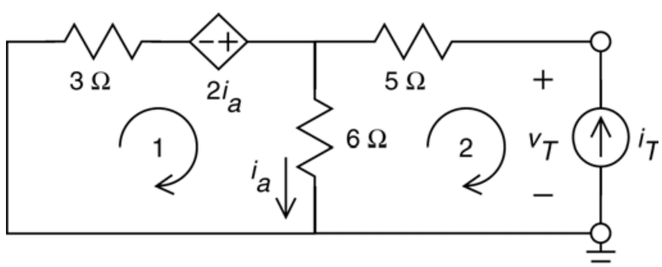


Figure (b) Calculating the Thevenin resistance, $R_t = \frac{v_T}{i_T}$, using mesh equations.

To determine the value of the open circuit voltage, v_{oc}, we connect an open circuit across the terminals of the circuit and then calculate the value of the voltage across that open circuit. Figure (c) shows the circuit from Figure 4.6-4*a* after adding the open circuit and labeling the open circuit voltage. Also, the meshes have been identified and labeled in anticipation of writing mesh equations. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

In Figure (c), mesh current i_2 is equal to the current in the open circuit. Consequently, $i_2 = 0$ A. The controlling current of the CCVS is expressed in terms of the mesh currents as

$$i_a = i_1 - i_2 = i_1 - 0 = i_1$$

Apply KVL to mesh 1 to get

$$3\,i_1 - 2\left(i_1 - i_2\right) + 6\left(i_1 - i_2\right) - 10 = 0 \quad \Rightarrow \quad 3\,i_1 - 2\left(i_1 - 0\right) + 6\left(i_1 - 0\right) - 10 = 0$$

$$\Rightarrow \quad i_1 = \frac{10}{7} = 1.43 \text{ A}$$

Apply KVL to mesh 2 to get

$$5\,i_2 + v_{oc} - 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad v_{oc} = 6\left(i_1\right) = 6\left(1.43\right) = 8.58 \text{ V}$$

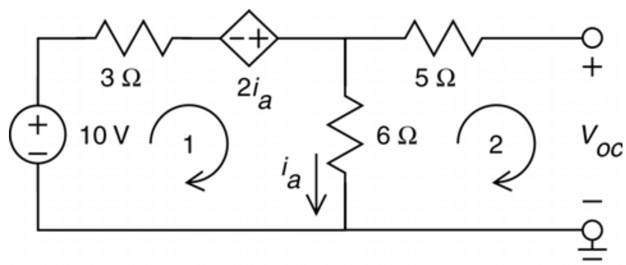


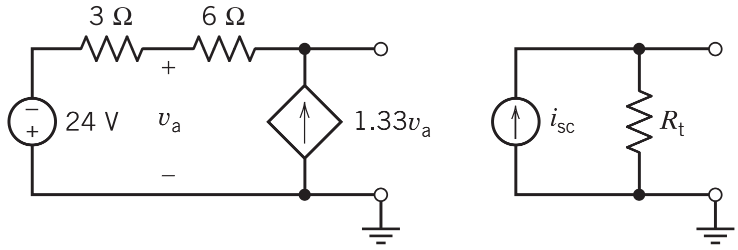
Figure (c) Calculating the open circuit voltage, v_{oc}, using mesh equations.

As a check, notice that $R_t\ i_{sc} = \left(7.57\right)\left(1.13\right) = 8.55 \approx v_{oc}$

(checked using LNAP 8/16/02)

P 5.5-6 The circuit shown in Figure P 5.5-6*b* is the Norton equivalent circuit of the circuit shown in Figure P 5.5-6*a*. Find the value of the short-circuit current, i_{sc}, and Thévenin resistance, R_t.

Answer: $i_{sc} = -24$ A and $R_t = -3\ \Omega$



(*a*) (*b*)

Figure P 5.5-6

Solution:

To determine the value of the short circuit current, I_{sc}, we connect a short circuit across the terminals of the circuit and then calculate the value of the current in that short circuit. Figure (a) shows the circuit from Figure 4.6-5a after adding the short circuit and labeling the short circuit current. Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure (a), node voltage v_1 is equal to the negative of the voltage source voltage. Consequently, $v_1 = -24$ V . The voltage at node 3 is equal to the voltage across a short, $v_3 = 0$. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the voltage across a short, i.e. $v_3 = 0$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad -48 = 3\,v_a \quad \Rightarrow \quad v_a = -16 \text{ V}$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3}v_2 = i_{sc} \quad \Rightarrow \quad \frac{9}{6}v_a = i_{sc} \quad \Rightarrow \quad i_{sc} = \frac{9}{6}(-16) = -24 \text{ A}$$

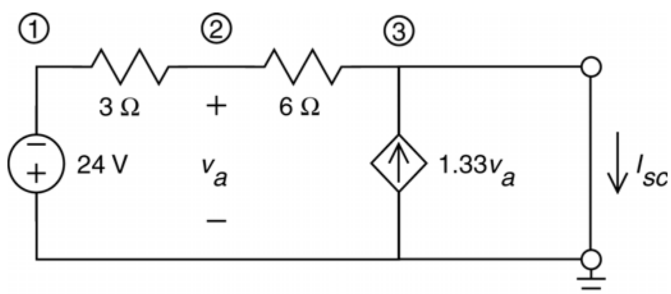


Figure (a) Calculating the short circuit current, I_{sc}, using mesh equations.

To determine the value of the Thevenin resistance, R_{th}, first replace the 24 V voltage source by a 0 V voltage source, i.e. a short circuit. Next, connect a current source circuit across the terminals of the circuit and then label the voltage across that current source as shown in Figure (b). The Thevenin resistance will be calculated from the current and voltage of the current source as

$$R_{th} = \frac{v_T}{i_T}$$

Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure (b), node voltage v_1 is equal to the across a short circuit, i.e. $v_1 = 0$. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the voltage across the current source, i.e. $v_3 = v_T$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad v_T = 3\,v_a$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3}v_2 + i_T = 0 \quad \Rightarrow \quad 9\,v_2 - v_3 + 6\,i_T = 0$$

$$\Rightarrow \quad 9\,v_a - v_T + 6\,i_T = 0$$

$$\Rightarrow \quad 3\,v_T - v_T + 6\,i_T = 0 \quad \Rightarrow \quad 2\,v_T = -6\,i_T$$

Finally,

$$R_t = \frac{v_T}{i_T} = -3\,\Omega$$

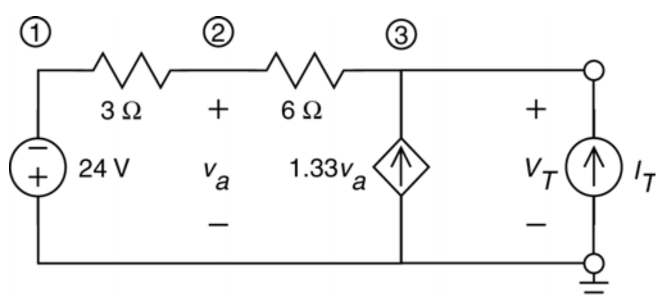


Figure (b) Calculating the Thevenin resistance, $R_{th} = \frac{v_T}{i_T}$, using mesh equations.

To determine the value of the open circuit voltage, v_{oc}, we connect an open circuit across the terminals of the circuit and then calculate the value of the voltage across that open circuit. Figure (c) shows the circuit from Figure P 4.6-5a after adding the open circuit and labeling the open circuit voltage. Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure (c), node voltage v_1 is equal to the negative of the voltage source voltage. Consequently, $v_1 = -24$ V. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the open circuit voltage, i.e. $v_3 = v_{oc}$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad -48 + v_{oc} = 3\,v_a$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3} v_2 = 0 \quad \Rightarrow \quad 9\,v_2 - v_3 = 0 \quad \Rightarrow \quad 9\,v_a = v_{oc}$$

Combining these equations gives

$$3\left(-48 + v_{oc}\right) = 9\,v_a = v_{oc} \quad \Rightarrow \quad v_{oc} = 72 \text{ V}$$

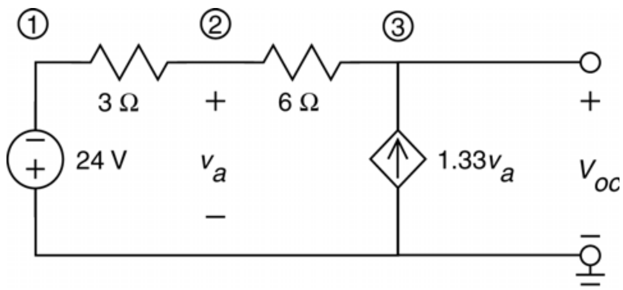


Figure (c) Calculating the open circuit voltage, v_{oc}, using node equations.

As a check, notice that

$$R_{th}\, I_{sc} = (-3)(-24) = 72 = V_{oc}$$

(checked using LNAP 8/16/02)

P 5.5-7 Determine the value of the resistance R in the circuit shown in Figure P 5.5-7 by each of the following methods:

(a) Replace the part of the circuit to the left of terminals a–b by its the Norton equivalent circuit. Use current division to determine the value of R.

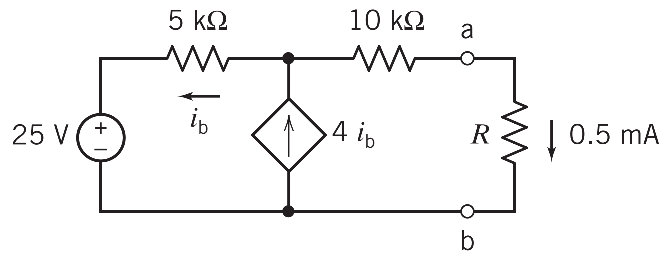
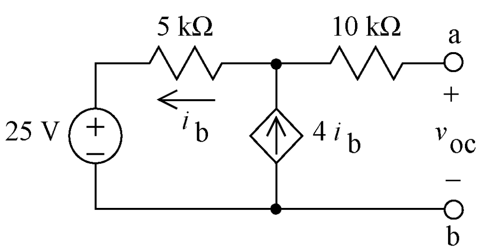


Figure P 5.5-7

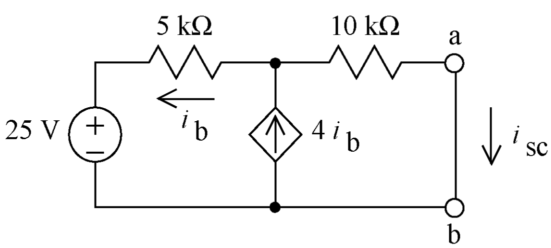
(b) Analyze the circuit shown Figure P 5.5-6 using mesh equations. Solve the mesh equations to determine the value of R.

Solution: (a) Replace the part of the circuit that is connected to the left of terminals a-b by its Norton equivalent circuit:



Apply KCL at the top node of the dependent source to see that $i_b = 0$ A . Then

$$v_{oc} = 25 + 5000\left(i_b\right) = 25 \text{ V}$$



Apply KVL to the supermesh corresponding to the dependent source to get

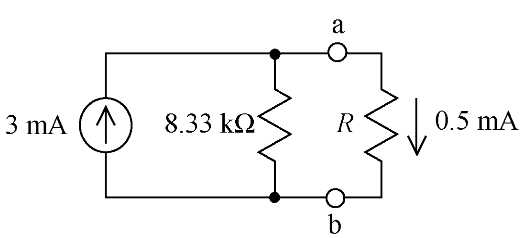
$$-5000\, i_b + 10000\left(3\, i_b\right) - 25 = 0 \quad \Rightarrow \quad i_b = 1 \text{ mA}$$

Apply KCL to get

$$i_{sc} = 3\, i_b = 3 \text{ mA}$$

Then

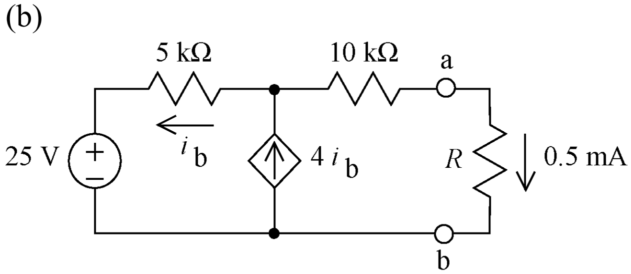
$$R_t = \frac{v_{oc}}{i_{sc}} = 8.3\bar{3} \text{ k}\Omega$$



Current division gives

$$0.5 = \frac{8333}{R + 8333} 3 \quad \Rightarrow \quad R = 41.67 \text{ k}\Omega$$

(b)



Notice that i_b and 0.5 mA are the mesh currents.

Apply KCL at the top node of the dependent source to get

$$i_b + 0.5 \times 10^{-3} = 4\, i_b \quad \Rightarrow \quad i_b = \frac{1}{6} \text{ mA}$$

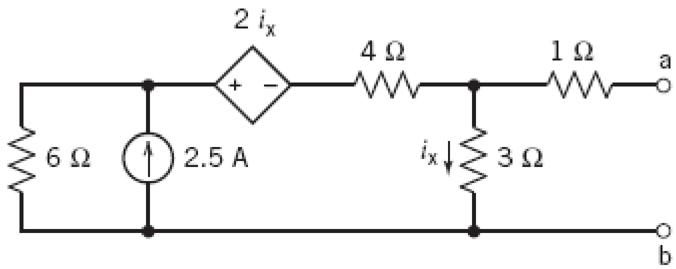
Apply KVL to the supermesh corresponding to the dependent source to get

$$-5000\, i_{\text{b}} + (10000 + R)\left(0.5\times10^{-3}\right) - 25 = 0$$

$$-5000\left(\frac{1}{6}\times10^{-3}\right) + (10000 + R)\left(0.5\times10^{-3}\right) = 25$$

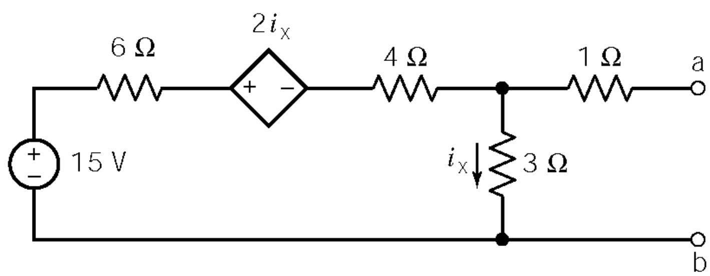
$$R = \frac{\frac{125}{6}}{0.5\times10^{-3}} = 41.67\ \text{k}\Omega$$

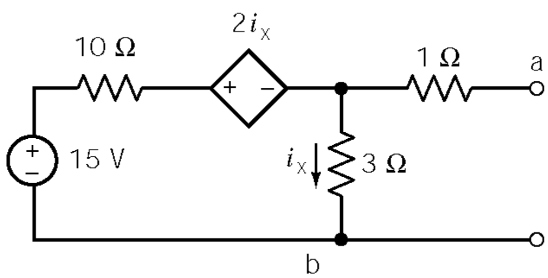
P5.5-8 Find the Norton equivalent circuit of this circuit:



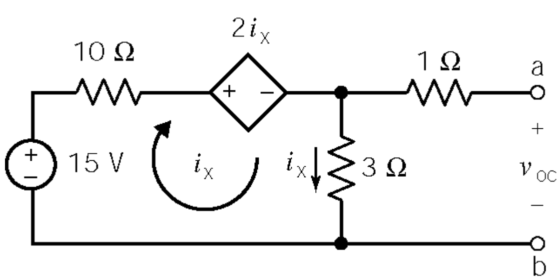
Solution

Simplify the circuit using a source transformation:





Identify the open circuit voltage and short circuit current.

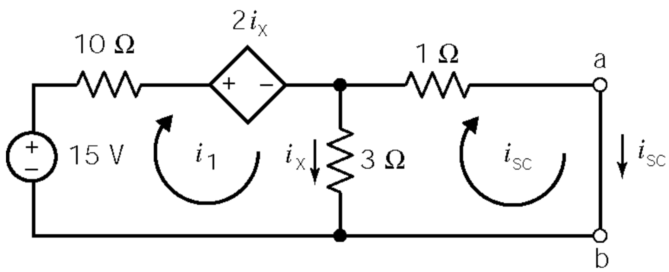


Apply KVL to the mesh to get:

$$(10+2+3)i_x - 15 = 0 \quad \Rightarrow \quad i_x = 1 \text{ A}$$

Then

$$v_{oc} = 3\, i_x = 3 \text{ V}$$



Express the controlling current of the CCVS in terms of the mesh currents:

$$i_x = i_1 - i_{sc}$$

The mesh equations are

$$10\,i_1 + 2\left(i_1 - i_{sc}\right) + 3\left(i_1 - i_{sc}\right) - 15 = 0 \quad \Rightarrow \quad 15\,i_1 - 5\,i_{sc} = 15$$

and

$$i_{sc} - 3\left(i_1 - i_{sc}\right) = 0 \quad \Rightarrow \quad i_1 = \frac{4}{3}\,i_{sc}$$

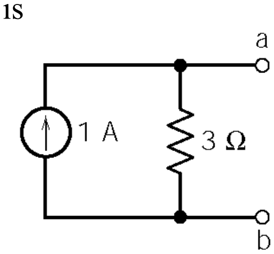
so

$$15\left(\frac{4}{3}i_{sc}\right) - 5\,i_{sc} = 15 \quad \Rightarrow \quad i_{sc} = 1\text{ A}$$

The Thevenin resistance is

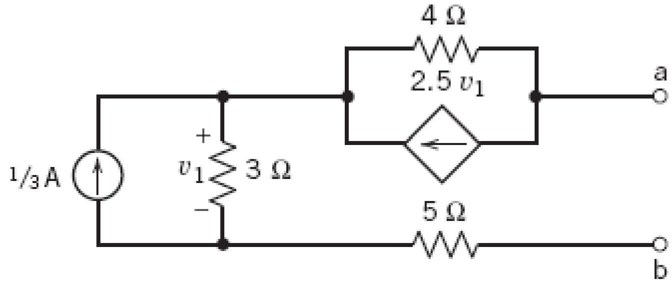
$$R_t = \frac{3}{1} = 3\ \Omega$$

Finally, the Norton equivalent circuit is



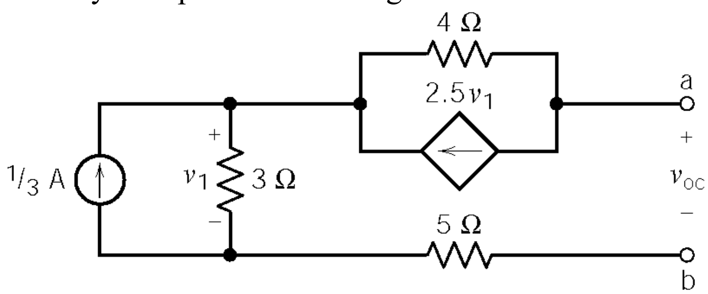
(checked: LNAP 6/21/04)

P5.5-9 Find the Norton equivalent circuit of this circuit:



Solution

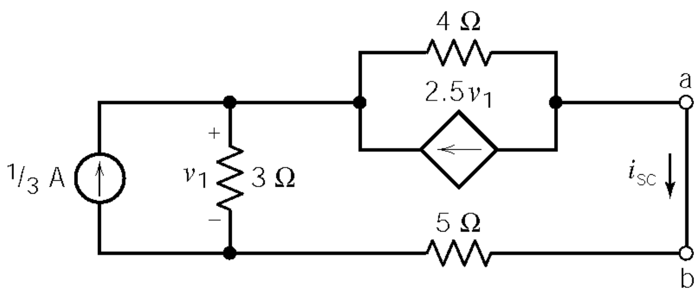
Identify the open circuit voltage and short circuit current.



$$v_1 = \left(\frac{1}{3}\right)3 = 1 \text{ V}$$

Then

$$v_{oc} = v_1 - 4\left(2.5\,v_1\right) = -9 \text{ V}$$



$$v_1 = 3\left(\frac{1}{3} - i_{sc}\right) = 1 - 3\,i_{sc}$$

$$4\left(2.5\,v_1 + i_{sc}\right) + 5\,i_{sc} - v_1 = 0$$

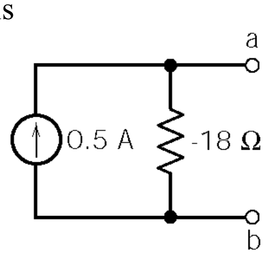
$$\Rightarrow \quad 9\,v_1 + 9\,i_{sc} = 0$$

$$9\left(1 - 3\,i_{sc}\right) + 9\,i_{sc} = 0 \quad \Rightarrow \quad i_{sc} = \frac{1}{2} \text{ A}$$

The Thevenin resistance is

$$R_t = \frac{-9}{0.5} = -18\ \Omega$$

Finally, the Norton equivalent circuit is



(checked: LNAP 6/21/04)

P 5.5-10 An ideal ammeter is modeled as a short circuit. A more realistic model of an ammeter is a small resistance. Figure P 5.5-10*a* shows a circuit with an ammeter that measures the current i_m. In Figure P 5.5-10*b* the ammeter is replaced by the model of an ideal ammeter, a short circuit. The ammeter measures i_{mi}, the ideal value of i_m.

As $R_m \to 0$, the ammeter becomes an ideal ammeter and $i_m \to i_{mi}$. When $R_m > 0$, the ammeter is not ideal and $i_m < i_{mi}$. The difference between i_m and i_{mi} is a measurement error caused by the fact that the ammeter is not ideal.

(a) Determine the value of i_{mi}.

(b) Express the measurement error that occurs when $R_m = 20\ \Omega$ as a percentage of i_{mi}.

(c) Determine the maximum value of R_m required to ensure that the measurement error is smaller than 2 percent of i_{mi}.

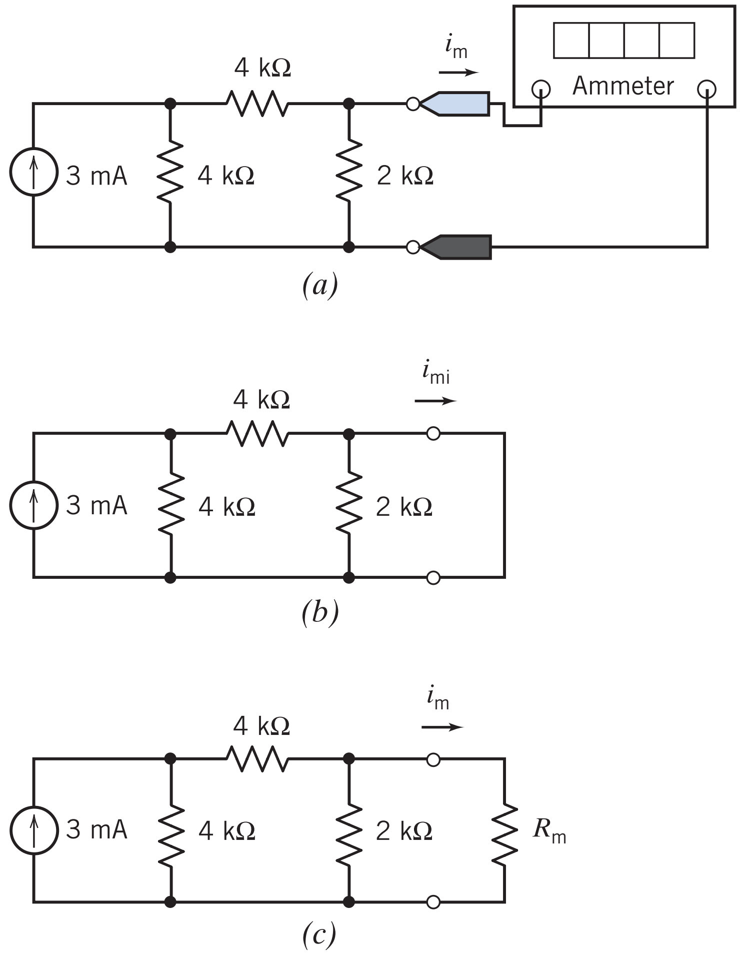
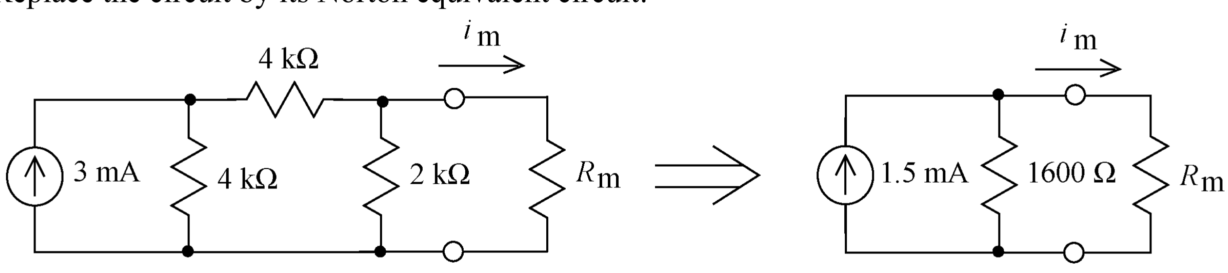


Figure P 5.5-10

Solution:
Replace the circuit by its Norton equivalent circuit:



$$i_m = \left(\frac{1600}{1600 + R_m}\right)\left(1.5\times10^{-3}\right)$$

(a)

$$i_{\text{mi}} = \lim_{R_{\text{m}} \to 0} \quad i_{\text{m}} = 1.5 \text{ mA}$$

(b) When $R_{\text{m}} = 20\ \Omega$ then $i_{\text{m}} = 1.48$ mA so

$$\%\ \text{error} = \frac{1.5 - 1.48}{1.5} \times 100 = 1.23\%$$

(c)

$$0.02 \geq \frac{0.015 - \left(\dfrac{1600}{1600 + R_{\text{m}}}\right)(0.015)}{0.015} \quad \Rightarrow \quad \frac{1600}{1600 + R_{\text{m}}} \geq 0.98 \quad \Rightarrow \quad R_{\text{m}} \leq 32.65\ \Omega$$

(checked: LNAP 6/18/04)

P 5.5-11 Determine values of R_t and i_{sc} that cause the circuit shown in Figure P 5.5-11*b* to be the Norton equivalent circuit of the circuit in Figure P 5.5-11*a*.

Answer: $R_t = 3\ \Omega$ and $i_{sc} = -2$ A

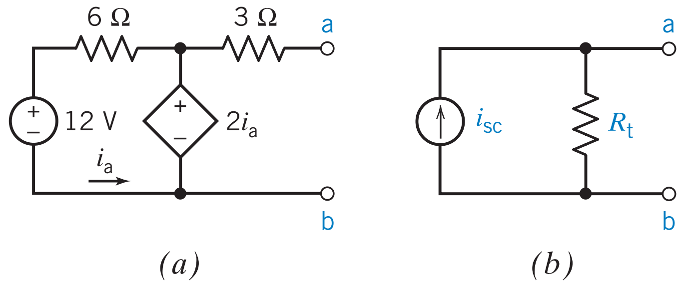
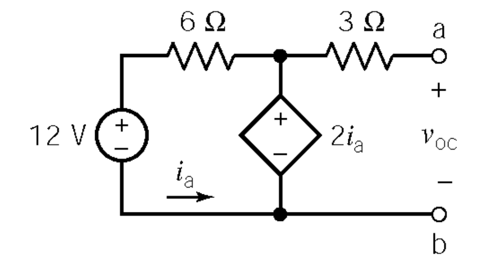


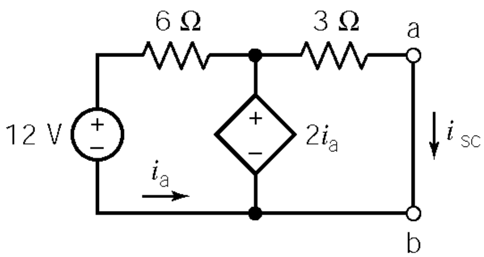
Figure P 5.5-11

Solution:



$$i_a = \frac{2\,i_a - 12}{6} \quad \Rightarrow \quad i_a = -3 \text{ A}$$

$$v_{oc} = 2\,i_a = -6 \text{ V}$$



$$12 + 6\,i_a = 2\,i_a \quad \Rightarrow \quad i_a = -3 \text{ A}$$

$$3\,i_{sc} = 2\,i_a \quad \Rightarrow \quad i_{sc} = \frac{2}{3}(-3) = -2 \text{ A}$$

$$R_t = \frac{-6}{-2} = 3\,\Omega$$

P 5.5-12 Use Norton's theorem to formulate a general expression for the current i in terms of the variable resistance R shown in Figure P 5.5-12.

Answer: $i = 20/(8 + R)$ A

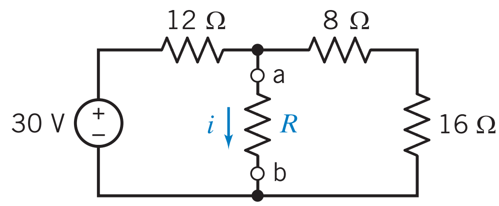
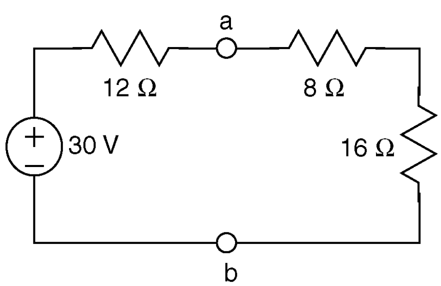


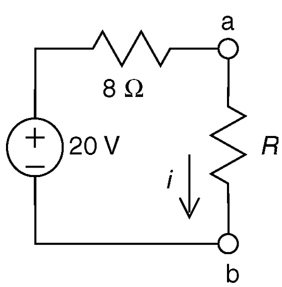
Figure P 5.5-12

Solution:



$$R_t = \frac{12\times 24}{12+24} = \frac{12\times 24}{36} = 8\ \Omega$$

$$v_{oc} = \frac{24}{12+24}(30) = 20\ \text{V}$$



$$i = \frac{20}{8+R}$$

Section 5-6: Maximum Power Transfer

P 5.6-1 The circuit model for a photovoltaic cell is given in Figure P 5.6-1 (Edelson, 1992). The current i_s is proportional to the solar insolation (kW/m^2).

(a) Find the load resistance, R_L, for maximum power transfer.

(b) Find the maximum power transferred when $i_s = 1$ A.

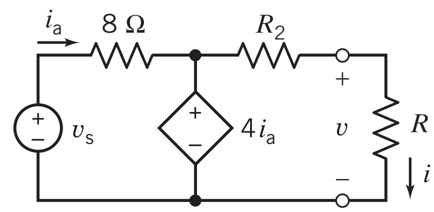
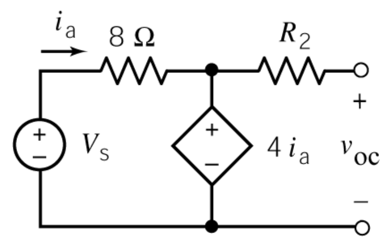


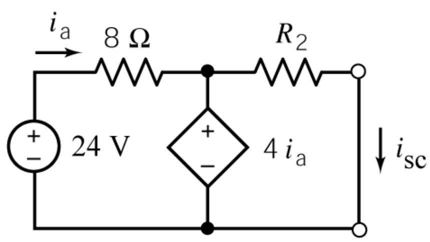
Figure P 5.6-1

Solution:

(a) The value of the current in R_2 is 0 A so $v_{oc} = 4i_a$. Then KVL gives



$$8i_a + 4i_a - V_s = 0 \quad \Rightarrow \quad V_s = 12i_a = 3(4i_a) = 3(v_{oc}) = 24 \text{ V}$$



Next, KVL gives

$$8i_a + 4i_a - 24 = 0 \quad \Rightarrow \quad i_a = 2 \text{ A}$$

and

$$4i_a = R_2\, i_{sc} \quad \Rightarrow \quad 4(2) = R_2(2) \quad \Rightarrow \quad R_2 = 4\ \Omega$$

(b) The power delivered to the resistor to the right of the terminals is maximized by setting R equal to the Thevenin resistance of the part of the circuit to the left of the terminals:

$$R = R_t = \frac{v_{oc}}{i_{sc}} = \frac{8}{2} = 4\ \Omega$$

Then

$$p_{max} = \frac{{v_{oc}}^2}{4R_t} = \frac{8^2}{4(4)} = 4 \text{ W}$$

P 5.6-2 For the circuit in Figure P 5.6-2, (a) find *R* such that maximum power is dissipated in *R* and (b) calculate the value of maximum power.

Answer: $R = 60\ \Omega$ and $P_{max} = 54$ mW

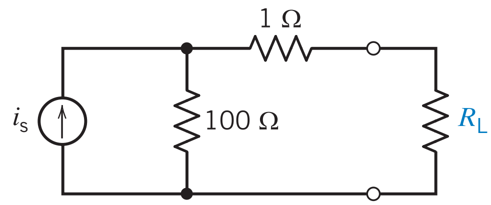
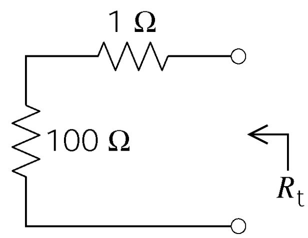


Figure P 5.6-2

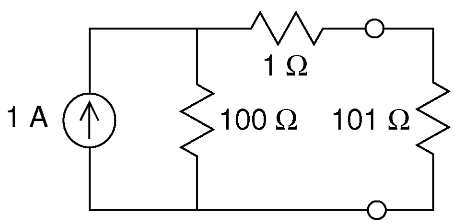
Solution:

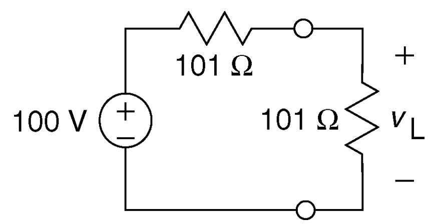
a) For maximum power transfer, set R_L equal to the Thevenin resistance:

$$R_L = R_t = 100 + 1 = 101\,\Omega$$



b) To calculate the maximum power, first replace the circuit connected to R_L be its Thevenin equivalent circuit:





The voltage across R_L is $$v_L = \frac{101}{101+101}(100) = 50\text{ V}$$

Then $$p_{\max} = \frac{{v_L}^2}{R_L} = \frac{50^2}{101} = 24.75\text{ W}$$

P 5.6-3 For the circuit in Figure P 5.6-3, prove that for R_s variable and R_L fixed, the power dissipated in R_L is maximum when $R_s = 0$.

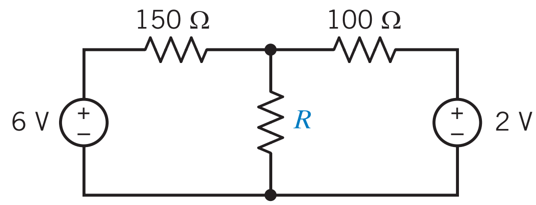
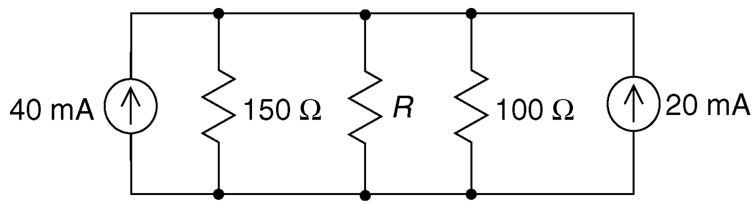
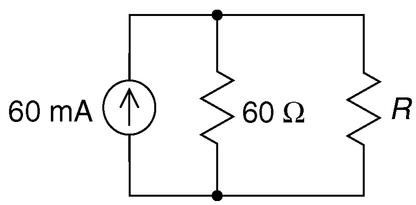


Figure P 5.6-3

Solution:
Reduce the circuit using source transformations:





Then (a) maximum power will be dissipated in resistor R when: $R = R_t = 60\ \Omega$ and (b) the value of that maximum power is

$$P_{max} = i_R^2(R) = (0.03)^2(60) = \underline{54\text{ mW}}$$

P 5.6-4 Find the maximum power to the load R_L if the maximum power transfer condition is met for the circuit of Figure P 5.6-4.

Answer: max $p_L = 0.75$ W

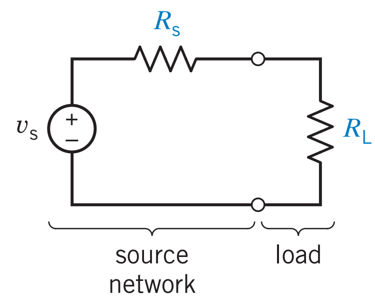
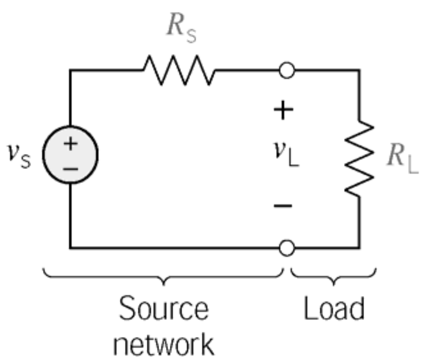


Figure P 5.6-4

Solution:



$$v_L = v_S\left[\frac{R_L}{R_S + R_L}\right]$$

$$\therefore\ p_L = \frac{v_L^2}{R_L} = \frac{v_S^2 R_L}{(R_S + R_L)^2}$$

By inspection, p_L is max when you reduce R_S to get the smallest denominator.

$\underline{\therefore \text{ set } R_S = 0}$

P 5.6-5 Determine the maximum power that can be absorbed by a resistor, R, connected to terminals a–b of the circuit shown in Figure P 5.6-5. Specify the required value of R.

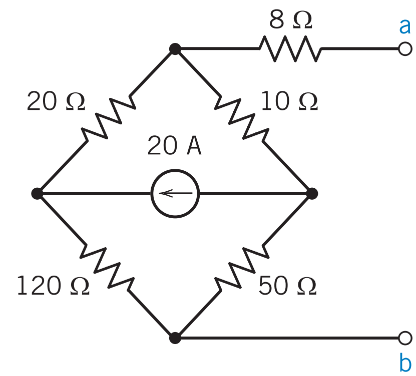
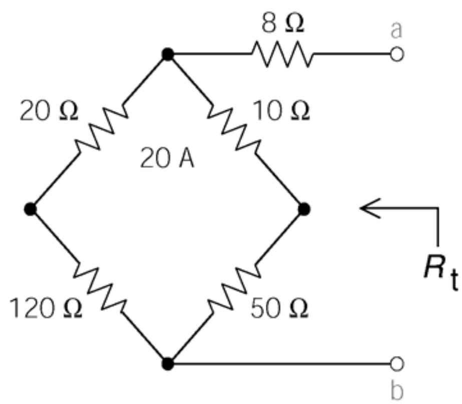
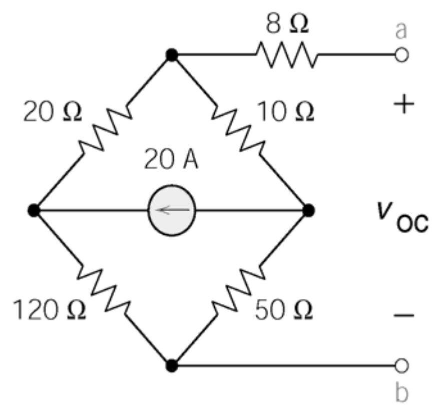


Figure P 5.6-5

Solution:





The required value of R is

$$R = R_t = 8 + \frac{(20+120)(10+50)}{(20+120)+(10+50)} = 50\ \Omega$$

$$v_{oc} = \left[\frac{170}{170+30}(20)\right]10 - \left[\frac{30}{170+30}(20)\right]50$$
$$= \frac{170(20)(10) - 30(20)(50)}{200} = \frac{4000}{200} = 20\ \text{V}$$

The maximum power is given by

$$p_{\max} = \frac{{v_{oc}}^2}{4\,R_t} = \frac{20^2}{4(50)} = 2\ \text{W}$$

P 5.6-6 Figure P 5.6-6 shows a source connected to a load through an amplifier. The load can safely receive up to 15 W of power. Consider three cases:

(a) $A = 20$ V/V and $R_o = 10\ \Omega$. Determine the value of R_L that maximizes the power delivered to the load and the corresponding maximum load power.

(b) $A = 20$ V/V and $R_L = 8\ \Omega$. Determine the value of R_o that maximizes the power delivered to the load and the corresponding maximum load power.

(c) $R_o = 10\ \Omega$ and $R_L = 8\ \Omega$. Determine the value of A that maximizes the power delivered to the load and the corresponding maximum load power.

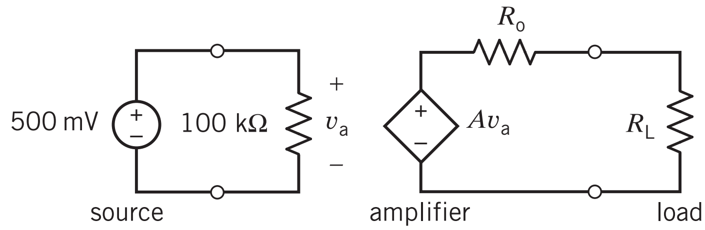
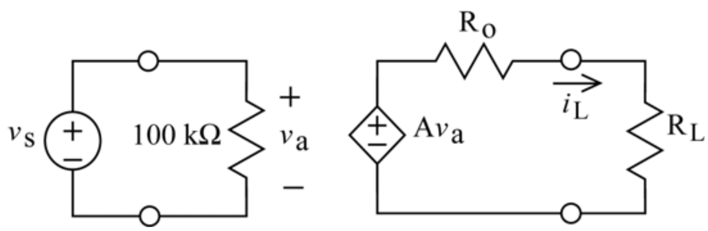


Figure P 5.6-6

Solution:



$$i_L = \frac{A}{R_o + R_L} v_s$$

$$P_L = i_L^{\,2} R_L = \frac{A^2 v_s^{\,2} R_L}{(R_o + R_L)^2}$$

(a) $R_t = R_o$ so $R_L = R_o = 10\ \Omega$ maximizes the power delivered to the load. The corresponding load power is

$$P_L = \frac{20^2 \left(\frac{1}{2}\right)^2 10}{(10+10)^2} = 2.5 \text{ W}.$$

(b) $R_o = 0$ maximizes P_L (The numerator of P_L does not depend on R_o so P_L can be maximized by making the denominator as small as possible.) The corresponding load power is

$$P_L = \frac{A^2 v_s{}^2 R_L}{R_L{}^2} = \frac{A^2 v_s{}^2}{R_L} = \frac{20^2\left(\frac{1}{2}\right)^2}{8} = 12.5 \text{ W.}$$

(c) P_L is proportional to A^2 so the load power continues to increase as A increases. The load can safely receive 15 W. This limit corresponds to

$$15 = \frac{A^2\left(\frac{1}{2}\right)^2 8}{(18)^2} \quad \Rightarrow \quad A = 36\sqrt{\frac{15}{8}} = 49.3 \text{ V.}$$

(checked: LNAP 6/9/04)

P 5.6-7 The circuit in Figure P 5.6-7 contains a variable resistance, R, implemented using a potentiometer. The resistance of the variable resistor varies over the range $0 \le R \le 1000\ \Omega$. The variable resistor can safely receive 1/4 W power. Determine the maximum power received by the variable resistor. Is the circuit safe?

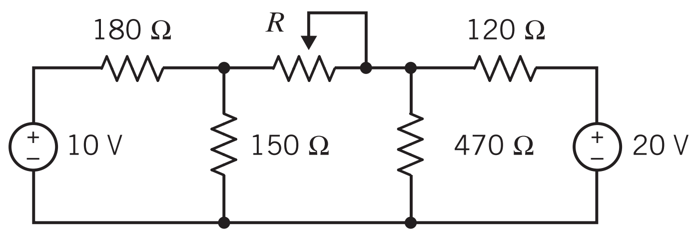
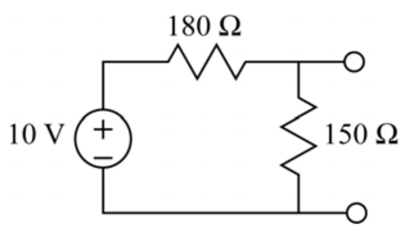


Figure P 5.6-7

Solution:

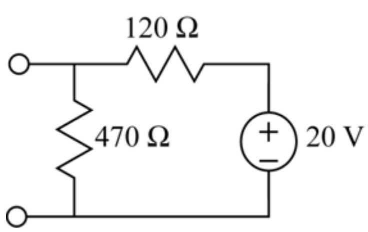
Replace the part of the circuit connected to the variable resistor by its Thevenin equivalent circuit. First, replace the left part of the circuit by its Thevenin equivalent:



$$v_{\text{oc1}} = \left(\frac{150}{150+180}\right)10 = 4.545\ \text{V}$$

$$R_{\text{t1}} = 180 \,||\, 150 = 81.8\ \Omega$$

Next, replace the right part of the circuit by its Thevenin equivalent:



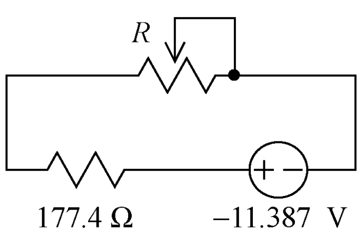
$$v_{\text{oc2}} = \left(\frac{470}{470+120}\right)20 = 15.932\ \text{V}$$

$$R_{\text{t2}} = 120 \,||\, 470 = 95.6\ \Omega$$

Now, combine the two partial Thevenin equivalents:

$$v_{\text{oc}} = v_{\text{oc1}} - v_{\text{oc2}} = -10.387\ \text{V and } R_{\text{t}} = R_{\text{t1}} + R_{\text{t2}} = 177.4\ \Omega$$

So



The power received by the adjustable resistor will be maximum when $R = R_t = 177.4\ \Omega$. The maximum power received by the adjustable resistor will be $p = \dfrac{(-11.387)^2}{4(177.4\ \Omega)} = 0.183\ \text{W}$.

(checked LNAPDC 7/24/04)

P 5.6-8 For the circuit of Figure P 5.6-8, find the power delivered to the load when R_L is fixed and R_t may be varied between 1 Ω and 5 Ω. Select R_t so that maximum power is delivered to R_L.

Answer: 13.9 W

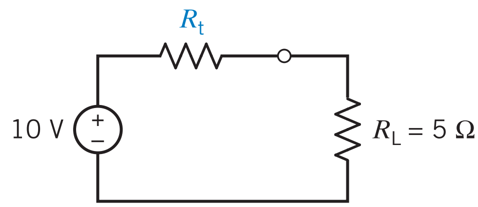


Figure P 5.6-8

Solution:

$$p = i\,v = \left(\frac{10}{R_t + R_L}\right)\left[\frac{R_L}{R_t + R_L}(10)\right] = \frac{100\,R_L}{\left(R_t + R_L\right)^2}$$

The power increases as R_t decreases so choose $R_t = 1\ \Omega$. Then

$$p_{\max} = i\,v = \frac{100\,(5)}{(1+5)^2} = 13.9\ \text{W}$$

P 5.6-9 A resistive circuit was connected to a variable resistor, and the power delivered to the resistor was measured as shown in Figure P 5.6-9. Determine the Thévenin equivalent circuit.

Answer: $R_t = 20\ \Omega$ and $v_{oc} = 20$ V

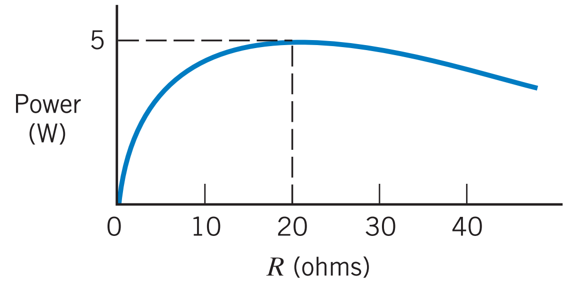


Figure P 5.6-9

Solution:

From the plot, the maximum power is 5 W when $R = 20\ \Omega$. Therefore:

$$R_t = 20\ \Omega$$

and

$$p_{\max} = \frac{v_{oc}^{\ 2}}{4\,R_t} \quad \Rightarrow \quad v_{oc} = \sqrt{p_{\max}\,4\,R_t} = \sqrt{5(4)20} = 20\ \text{V}$$

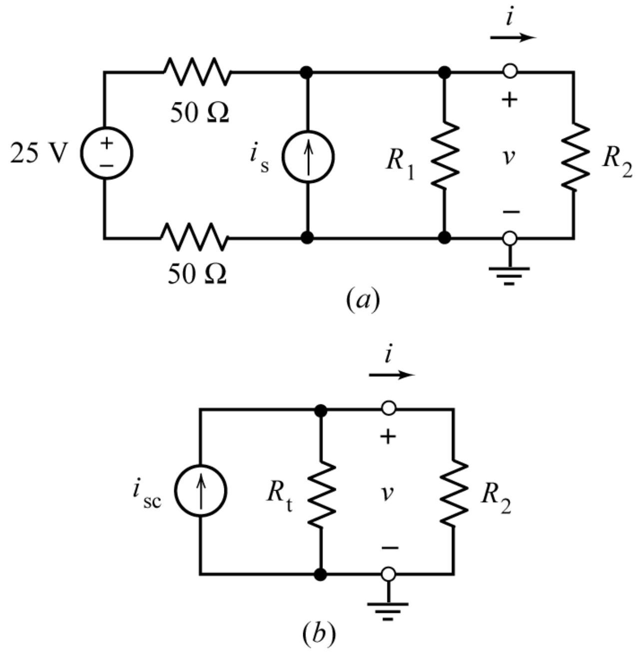


Figure P5.6-10

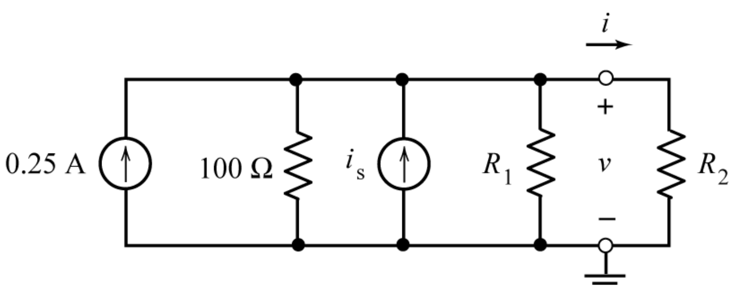
P5.6-10 The part circuit shown in Figure P5.6-10*a* to left of the terminals can be reduced to its Norton equivalent circuit using source transformations and equivalent resistance. The resulting Norton equivalent circuit, shown in Figure P5.6-10*b*, will be characterized by the parameters:

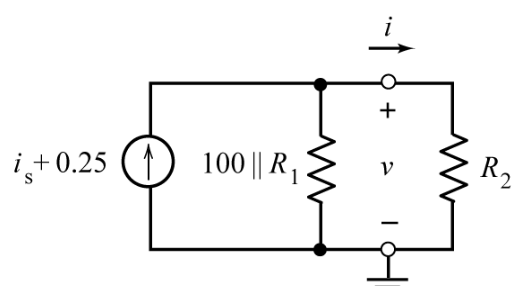
$$i_{sc} = 1.5 \text{ A} \quad \text{and} \quad R_t = 80\ \Omega$$

(*a*) Determine the values of i_s and R_1.

(*b*) Given that $0 \le R_2 \le \infty$, determine the maximum value of $p = vi$, the power delivered to R_2.

Solution: Two source transformations reduce the circuit as follows:





(*a*) Recognizing the parameters of the Norton equivalent circuit gives:

$$1.5 = i_{sc} = i_s + 0.25 \quad \Rightarrow \quad i_s = 1.25 \text{ A and } 80 = R_t = 100 \,||\, R_1 = \frac{100\,R_1}{100 + R_1} \quad \Rightarrow \quad R_1 = 400\ \Omega$$

(*b*) The maximum value of the power delivered to R_2 occurs when $R_2 = R_t = 80\ \Omega$. Then

$$i = \frac{1}{2} i_{sc} = 0.75 \text{ A} \quad \text{and} \quad p = \left(\frac{1}{2} i_{sc}\right)^2 R_t = \left(0.625^2\right) 80 = 45 \text{ W}$$

P5.6-11. Given that $0 \le R \le \infty$ in the circuit shown in Figure P5.6-12, determine (*a*) maximum value of i_a, (*b*) the maximum value of v_a, and (*c*) the maximum value of $p_a = i_a v_a$.

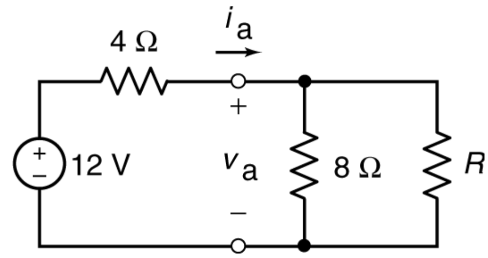
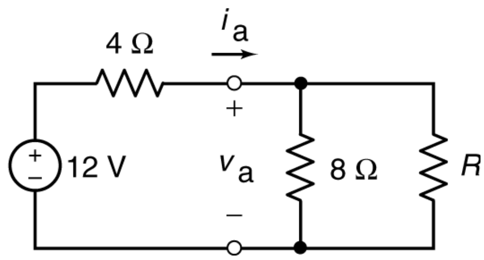
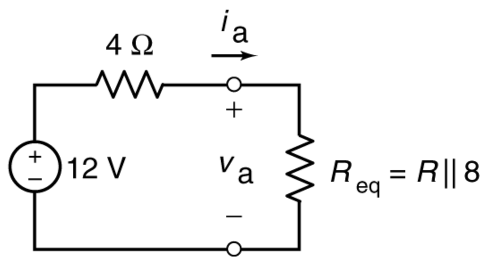


Figure P5.6-11

Solution:



Replace the parallel combination of resistor *R* and the 8 Ω resistor by an equivalent resistance.



Using voltage division

$$v_a = \frac{R_{eq}}{4+R_{eq}}(12) = \frac{1}{\dfrac{4}{R_{eq}}+1}(12)$$

Consequently, the maximum value of v_a corresponds to the is obtained by maximizing R_{eq}. The maximum of R_{eq} is obtained by maximizing *R*. Given that $0 \le R \le \infty$, the maximum value of R_{eq} is 8 Ω and the maximum value of v_a is

$$v_{a\,max} = \frac{1}{\dfrac{4}{8}+1}(12) = 8 \text{ V}$$

Using Ohm's law

$$i_a = \frac{12}{4+R_{eq}}$$

Consequently, the maximum value of i_a corresponds to the is obtained by minimizing R_{eq}. The minimum of R_{eq} is obtained by maximizing *R*. Given that $0 \le R \le \infty$, the minimum value of R_{eq} is 0 Ω and the maximum value of i_a is

$$i_{\text{a max}} = \frac{12}{4+0} = 3 \text{ A}$$

The maximum power theorem indicates that the maximum value of $p_a = i_a v_a$ occurs when R_{eq} = R_t. In this case, R_t = 4 Ω. We require R_{eq} = 4 Ω which is accomplished by making R = 8 Ω, an acceptable value since
$0 \le 8 \le \infty$. Then

$$p_a = \frac{\left(\frac{12}{2}\right)^2}{R_{eq}} = \frac{\left(\frac{12}{2}\right)^2}{4} = 9 \text{ W}$$

P5.6-12. Given that $0 \le R \le \infty$ in the circuit shown in Figure P5.6-12, determine value of R that maximizes the power $p_a = i_a v_a$ and the corresponding maximum value of p_a.

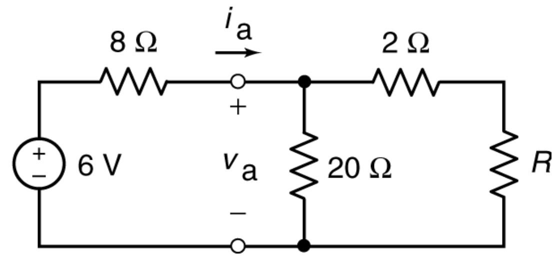
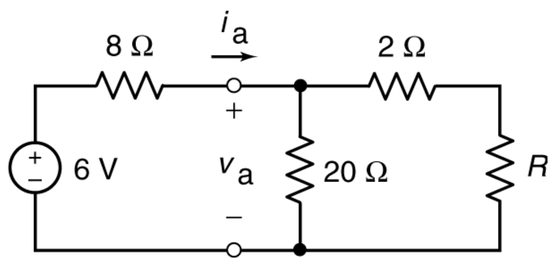
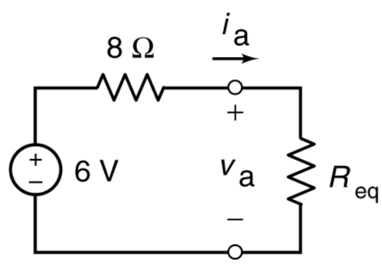


Figure P5.6-12

Solution:



Replace the combination of resistor R and the 20 Ω and 2 Ω resistors by an equivalent resistance.



The maximum power theorem indicates that the maximum value of $p_a = i_a v_a$ occurs when $R_{eq} = R_t$. In this case, $R_t = 8\ \Omega$. We require

$$8 = R_{eq} = \frac{20(R+2)}{20+(R+2)} = \frac{20R+40}{R+22}$$

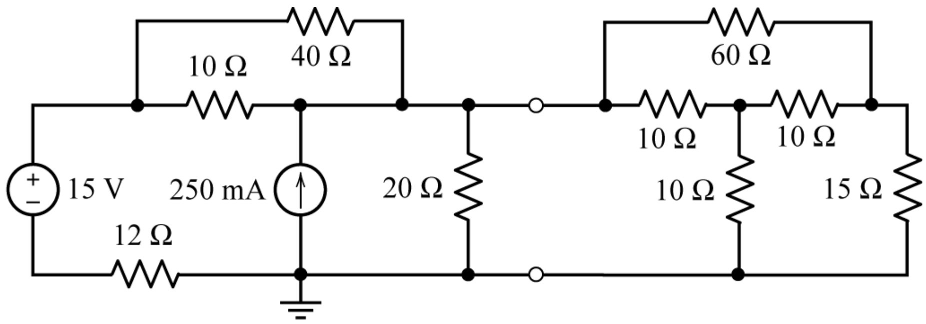
$$8(R+22) = 20R+40 \quad \Rightarrow \quad R = \frac{8(22)-40}{20-8} = 11.333\ \Omega$$

This isn't a standard resistance value but it is an acceptable value for this problem since $0 \le 11.333 \le \infty$. Then

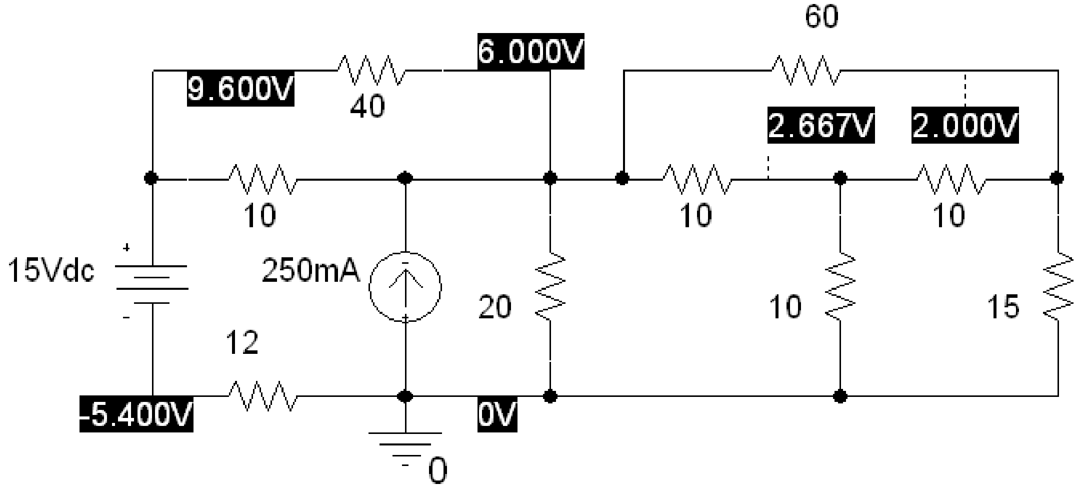
$$p_a = \frac{\left(\frac{6}{2}\right)^2}{R_{eq}} = \frac{\left(\frac{6}{2}\right)^2}{8} = 1.125\ \text{W}$$

Section 5.8 Using PSpice to Determine the Thevenin Equivalent Circuit

P5.8-1



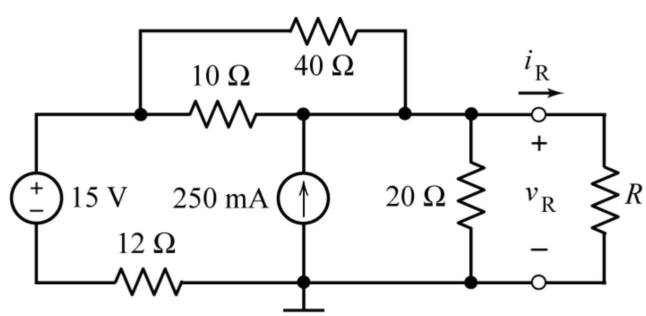
a) Here are the results of simulating the circuit in PSpice. The numbers shown in white on a black background are the node voltages.



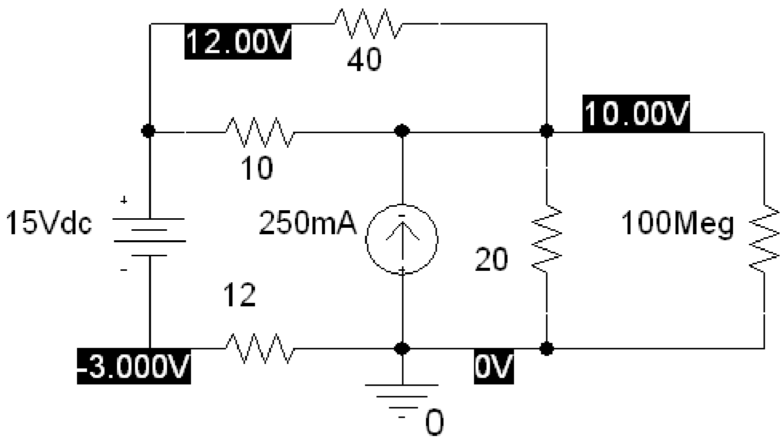
b) Add a resistor across the terminals of Circuit A. Then

$$v_{oc} = v_R \quad \text{when} \quad R \approx \infty$$

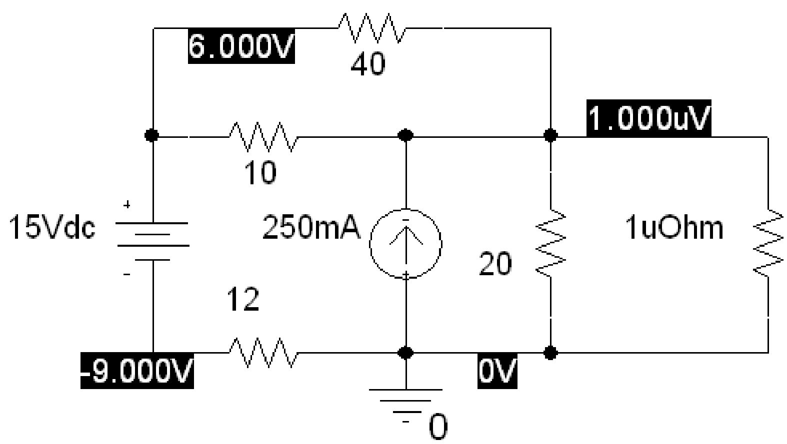
$$i_{sc} = \frac{v_R}{R} \quad \text{when} \quad R \approx 0$$



Here are the PSpice simulation results:



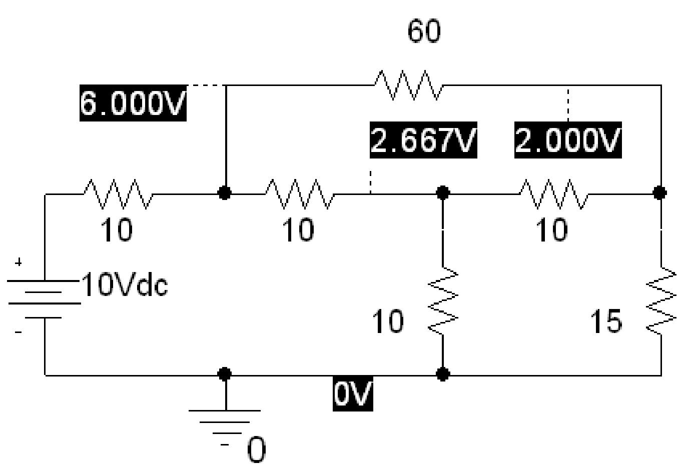
$$v_{oc} = 10 \text{ V}$$



$$i_{sc} = \frac{1\times10^{-6}}{1\times10^{-6}} = 1 \text{ A}$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{10}{1} = 10\ \Omega$$

c) Here is the result of simulation the circuit after replacing Circuit A by its Thevenin equivalent:



d) The node voltages of Circuit B are the same before and after replacing Circuit A by its Thevenin equivalent circuit.

Section 5-9 How Can We Check…?

P 5.9-1 For the circuit of Figure P 5.9-1, the current *i* has been measured for three different values of *R* and is listed in the table. Are the data consistent?

$R(\Omega)$	i(mA)
5000	16.5
500	43.8
0	97.2

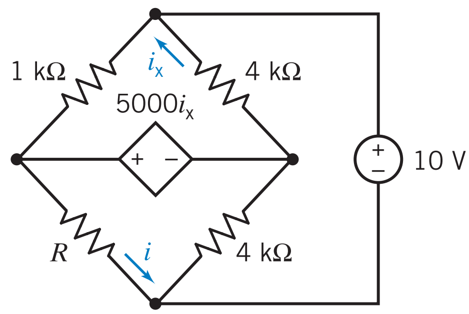
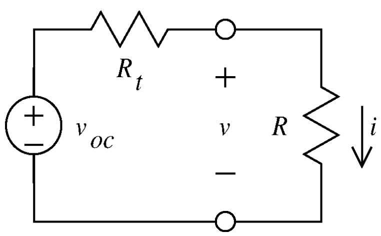


Figure P 5.9-1

Solution:



Use the data in the first two lines of the table to determine v_{oc} and R_t:

$$\left.\begin{array}{l} 0.0972 = \dfrac{v_{oc}}{R_t + 0} \\ 0.0438 = \dfrac{v_{oc}}{R_t + 500} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} v_{oc} = 39.9 \text{ V} \\ R_t = 410\,\Omega \end{array}\right.$$

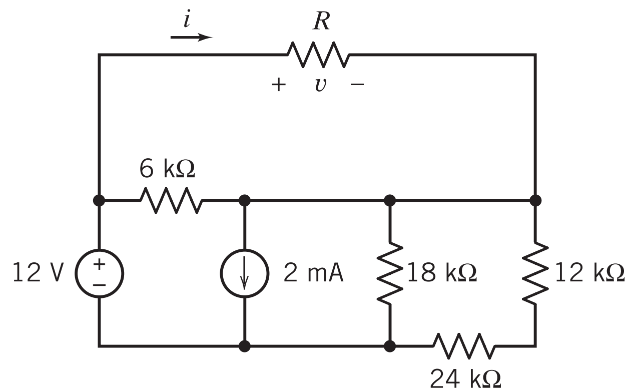
Now check the third line of the table. When R= 5000 Ω:

$$i = \frac{v_{oc}}{R_t + R} = \frac{39.9}{410 + 5000} = 7.37 \text{ mA}$$

which disagree with the data in the table.

<u>The data is not consistent.</u>

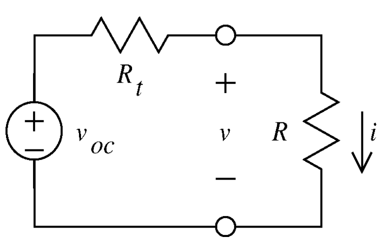
P 5.9-2 Your lab partner built the circuit shown in Figure P 5.9-2 and measured the current i and voltage v corresponding to several values of the resistance R. The results are shown in the table in Figure P 5.9-2. Your lab partner says that $R_L = 8000\ \Omega$ is required to cause $i = 1$ mA. Do you agree? Justify your answer.



R	i	v
open	0 mA	12 V
10 kΩ	0.857 mA	8.57 V
short	3 mA	0 V

Figure P 5.9-2

Solution:



Use the data in the table to determine v_{oc} and i_{sc}:

$$v_{oc} = 12\text{ V} \qquad \text{(line 1 of the table)}$$

$$i_{sc} = 3\text{ mA} \qquad \text{(line 3 of the table)}$$

so $\quad R_t = \dfrac{v_{oc}}{i_{sc}} = 4\text{ k}\Omega$

Next, check line 2 of the table. When R = 10 kΩ:

$$i = \frac{v_{oc}}{R_t + R} = \frac{12}{10(10^3) + 5(10^3)} = 0.857\text{ mA}$$

which agrees with the data in the table.

To cause $i = 1$ mA requires $\quad 0.001 = i = \dfrac{v_{oc}}{R_t + R} = \dfrac{12}{10(10^3) + R} \quad \Rightarrow \quad R = 8000\ \Omega$

I agree with my lab partner's claim that $R = 8000$ causes $i = 1$ mA.

P 5.9-3 In preparation for lab, your lab partner determined the Thévenin equivalent of the circuit connected to R_L in Figure P 5.9-3. She says that the Thévenin resistance is $R_t = \frac{6}{11}R$ and the open-circuit voltage is $v_{oc} = \frac{60}{11}$ V. In lab, you built the circuit using $R = 110\ \Omega$ and $R_L = 40\ \Omega$ and measured that $i = 54.5$ mA. Is this measurement consistent with the prelab calculations? Justify your answers.

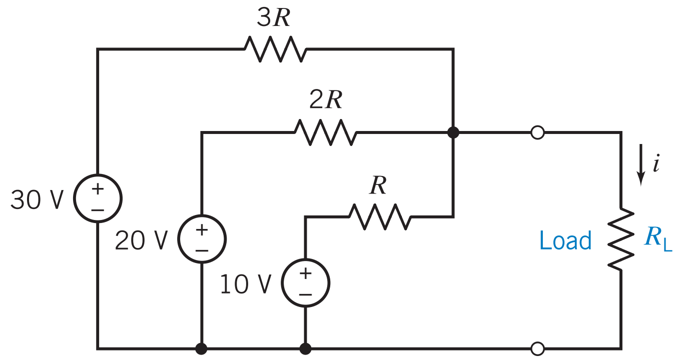
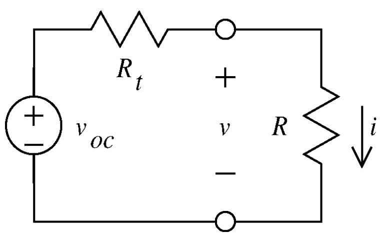


Figure P 5.9-3

Solution:



$$\frac{1}{R_t} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{3R} = \frac{11}{6R} \quad \Rightarrow \quad R_t = \frac{6R}{11}$$

and

$$v_{oc} = \left(\frac{2/3}{3+2/3}\right)30 + \left(\frac{3/4}{2+3/4}\right)20 + \left(\frac{6/5}{1+6/5}\right)10 = \frac{180}{11}$$

so the prelab calculation isn't correct.

But then

$$i = \frac{v_{oc}}{R_t + R} = \frac{\frac{180}{11}}{\frac{6}{11}(110)+40} = \frac{\frac{180}{11}}{60+40} = 163 \text{ mA} \neq 54.5 \text{ mA}$$

so the measurement does not agree with the corrected prelab calculation.

P 5.9-4 Your lab partner claims that the current i in Figure P 5.9-4 will be no greater than 12.0 mA, regardless of the value of the resistance R. Do you agree?

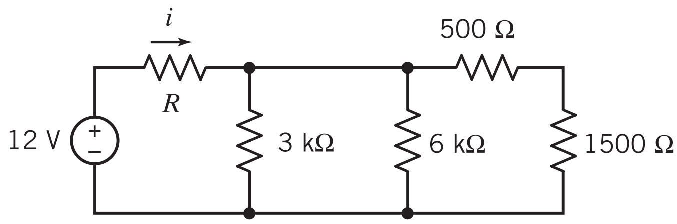


Figure P 5.9-4

Solution:

$$6000 \,||\, 3000 \,||\, (500+1500) = 2000 \,||\, 2000 = 1000\ \Omega$$

$$i = \frac{12}{R+1000} \le \frac{12}{1000} = 12\ \text{mA}$$

How about that?! Your lab partner is right.

(checked using LNAP 6/21/05)

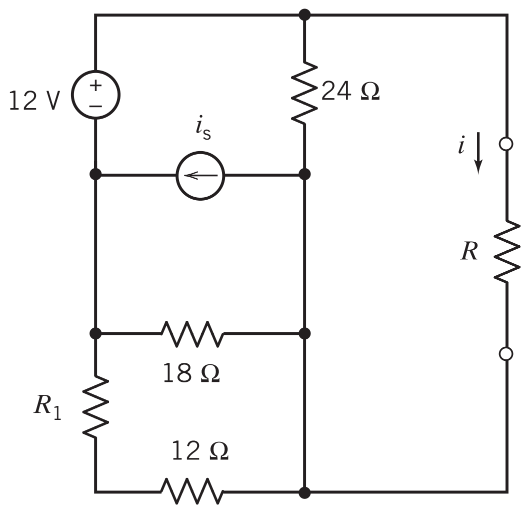
P 5.9-5 Figure P 5.9-5 shows a circuit and some corresponding data. Two resistances, R_1 and R, and the current source current are unspecified. The tabulated data provide values of the current, i, and voltage, v, corresponding to several values of the resistance R.

(a) Consider replacing the part of the circuit connected to the resistor R by a Thévenin equivalent circuit. Use the data in rows 2 and 3 of the table to find the values of R_t and v_{oc}, the Thévenin resistance and the open-circuit voltage.

(b) Use the results of part (a) to verify that the tabulated data are consistent.

(c) Fill in the blanks in the table.

(d) Determine the values of R_1 and i_s.



(a)

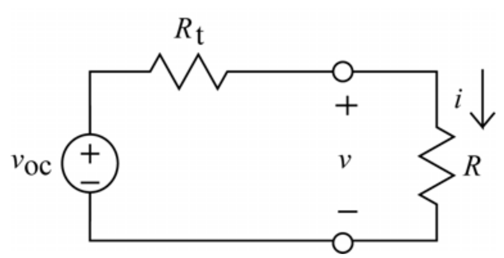
R, Ω	i, A	v, V
0	3	0
10	1.333	13.33
20	0.857	17.14
40	0.5	?
80	?	21.82

(b)

Figure P5.9-5

Solution:

(a)



KVL gives

$$v_{oc} = (R_t + R)i$$

from row 2

$$v_{oc} = (R_t + 10)(1.333)$$

from row 3

$$v_{oc} = (R_t + 20)(0.857)$$

So

$$(R_t + 10)(1.333) = (R_t + 20)(0.857)$$

$$28(R_t + 10) = 18(R_t + 20)$$

Solving gives

$$10R_t = 360 - 280 = 80 \quad \Rightarrow \quad R_t = 8\ \Omega$$

and

$$v_{oc} = (8 + 10)(1.333) = 24\ \text{V}$$

(b)

$$i = \frac{v_{\text{oc}}}{R_{\text{t}} + R} = \frac{24}{8+R} \quad \text{and} \quad v = \frac{R}{R+R_{\text{t}}} v_{\text{oc}} = \frac{24R}{R+8}$$

When $R = 0$, $i = 3$ A, and $v = 0$ V.

When $R = 40\ \Omega$, $i = \frac{1}{2}$ A.

When $R = 80\ \Omega$, $v = \frac{24(80)}{88} = \frac{240}{11} = 21.82$.

These are the values given in the tabulated data so the data is consistent.

(c) When $R = 40\ \Omega$, $v = \frac{24(40)}{48} = 20$ V.

When $R = 80\ \Omega$, $i = \frac{24}{88} = 0.2727$ A.

(d) First

$$8 = R_{\text{t}} = 24 \,||\, 18 \,||\, \left(R_1 + 12\right) \quad \Rightarrow \quad R_1 = 24\ \Omega$$

the, using superposition,

$$24 = v_{\text{oc}} = \frac{24}{24 + \left(18 \,||\, \left(R_1 + 12\right)\right)} 12 + \left(24 \,||\, 18\left(R_1 + 12\right)\right) i_{\text{s}} = 8 + 8 i_{\text{s}} \quad \Rightarrow \quad i_{\text{s}} = 2 \text{ A}$$

(checked using LNAP 6/21/05)

Design Problems

DP 5-1 The circuit shown in Figure DP 5-1*a* has four unspecified circuit parameters: v_s, R_1, R_2, and R_3. To design this circuit, we must specify the values of these four parameters. The graph shown in Figure DP 5-1*b* describes a relationship between the current *i* and the voltage *v*.

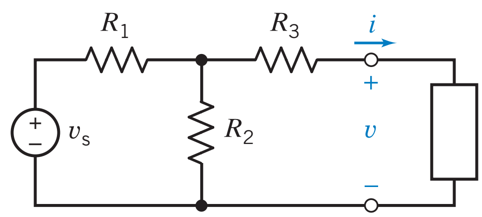
Specify values of v_s, R_1, R_2, and R_3 that cause the current *i* and the voltage *v* in Figure DP 5-1*a* to satisfy the relationship described by the graph in Figure DP 5-1*b*.

First Hint: The equation representing the straight line in Figure DP 5-1*b* is

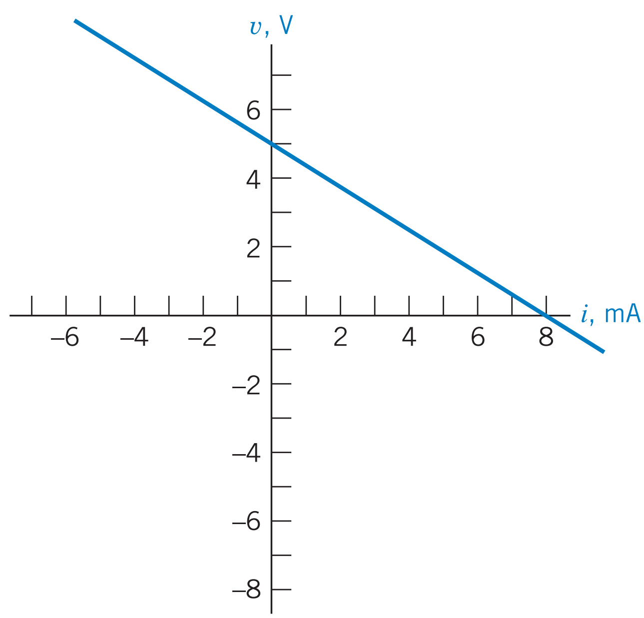
$$v = -R_t i + v_{oc}$$

That is, the slope of the line is equal to –1 times the Thévenin resistance and the "*v*-intercept" is equal to the open-circuit voltage.

Second Hint: There is more than one correct answer to this problem. Try setting $R_1 = R_2$.



(*a*)



(*b*)

Figure DP 5-1

Solution:

The equation of representing the straight line in Figure DP 5-1b is $v = -R_t\, i + v_{oc}$. That is, the slope of the line is equal to -1 times the Thevenin resistance and the "*v* - intercept" is equal to the open circuit voltage. Therefore: $R_t = -\dfrac{0-5}{0.008-0} = 625\ \Omega$ and v_{oc} = 5 V.

Try $R_1 = R_2 = 1\ \text{k}\Omega$. ($R_1 \parallel R_2$ must be smaller than R_t = 625 Ω.) Then

$$5 = \frac{R_2}{R_1 + R_2} v_s = \frac{1}{2} v_s \quad \Rightarrow v_s = 10\ \text{V}$$

and

$$625 = R_3 + \frac{R_1 R_2}{R_1 + R_2} = R_3 + 500 \quad \Rightarrow \quad R_3 = 125\ \Omega$$

Now v_s, R_1, R_2 and R_3 have all been specified so the design is complete.

DP 5-2 The circuit shown in Figure DP 5.2*a* has four unspecified circuit parameters: i_s, R_1, R_2, and R_3. To design this circuit, we must specify the values of these four parameters. The graph shown in Figure DP 5-2*b* describes a relationship between the current *i* and the voltage *v*.

Specify values of i_s, R_1, R_2, and R_3 that cause the current *i* and the voltage *v* in Figure DP 5-2*a* to satisfy the relationship described by the graph in Figure DP 5-2*b*.

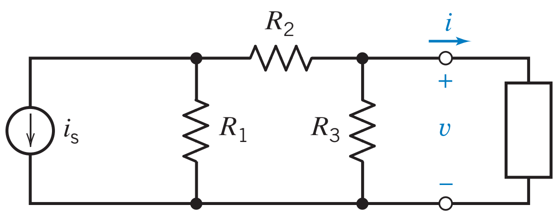
First Hint: Calculate the open-circuit voltage, v_{oc}, and the Thévenin resistance, R_t, of the part of the circuit to the left of the terminals in Figure DP 5-2*a*.

Second Hint: The equation representing the straight line in Figure DP 5-2*b* is

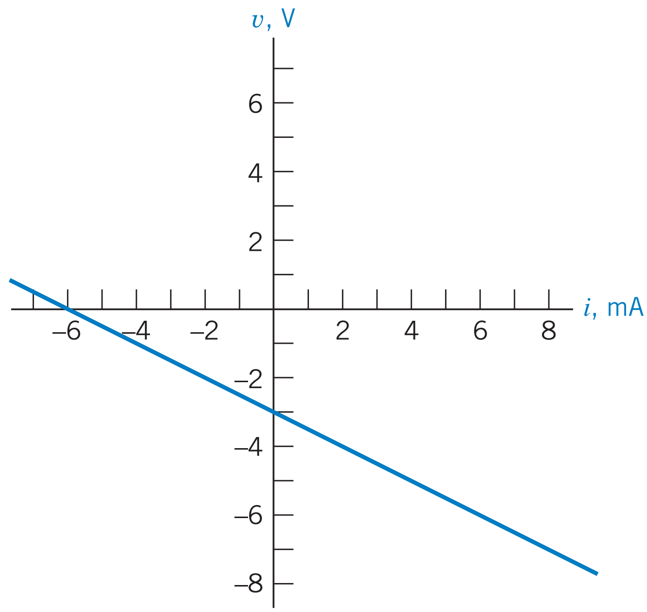
$$v = -R_t i + v_{oc}$$

That is, the slope of the line is equal to –1 times the Thévenin resistance and the "*v*-intercept" is equal to the open-circuit voltage.

Third Hint: There is more than one correct answer to this problem. Try setting both R_3 and $R_1 + R_2$ equal to twice the slope of the graph in Figure DP 5-2*b*.



(*a*)



(*b*)

Figure DP 5-2

Solution:

The equation of representing the straight line in Figure DP 5-2b is $v = -R_t\, i + v_{oc}$. That is, the slope of the line is equal to -1 times the Thevenin resistance and the "*v* - intercept" is equal to the open circuit voltage. Therefore: $R_t = -\dfrac{0-(-3)}{-0.006-0} = 500\ \Omega$ and $v_{oc} = -3$ V.

From the circuit we calculate

$$R_t = \frac{R_3\left(R_1 + R_2\right)}{R_1 + R_2 + R_3} \text{ and } v_{oc} = -\frac{R_1 R_3}{R_1 + R_2 + R_3} i_s$$

so

$$500\ \Omega = \frac{R_3\left(R_1 + R_2\right)}{R_1 + R_2 + R_3} \text{ and } -3\ \text{V} = -\frac{R_1 R_3}{R_1 + R_2 + R_3} i_\text{s}$$

Try $R_3 = 1\text{k}\Omega$ and $R_1 + R_2 = 1\text{k}\Omega$. Then $R_t = 500\ \Omega$ and

$$-3 = -\frac{1000 R_1}{2000} i_\text{s} = -\frac{R_1}{2} i_s \quad \Rightarrow 6 = R_1\, i_\text{s}$$

This equation can be satisfied by taking R_1 = 600 Ω and i_s = 10 mA. Finally, R_2 = 1 kΩ - 400 Ω = 400 Ω. Now i_s, R_1, R_2 and R_3 have all been specified so the design is complete.

DP 5-3 The circuit shown in Figure DP 5-3*a* has four unspecified circuit parameters: v_s, R_1, R_2, and R_3. To design this circuit, we must specify the values of these four parameters. The graph shown in Figure DP 5-3*b* describes a relationship between the current *i* and the voltage *v*.

Is it possible to specify values of v_s, R_1, R_2, and R_3 that cause the current *i* and the voltage *v* in Figure DP 5-1*a* to satisfy the relationship described by the graph in Figure DP 5-3*b*? Justify your answer.

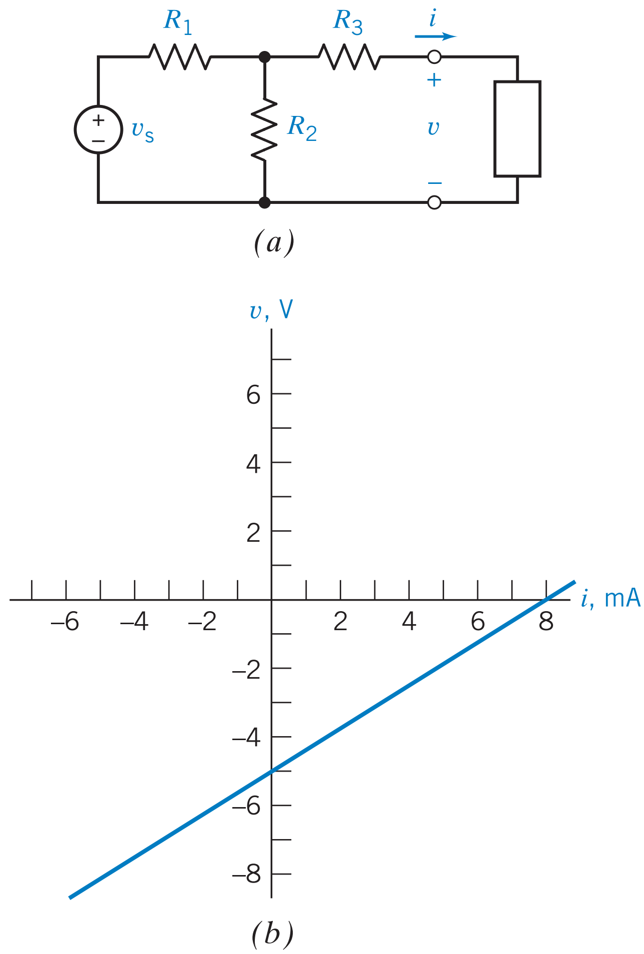


Figure DP 5-3

Solution:

The slope of the graph is positive so the Thevenin resistance is negative. This would require

$R_3 + \dfrac{R_1 R_2}{R_1 + R_2} < 0$, which is not possible since R_1, R_2 and R_3 will all be non-negative.

Is it not possible to specify values of v_s, R_1, R_2 and R_3 that cause the current *i* and the voltage *v* in Figure DP 5-3*a* to satisfy the relationship described by the graph in Figure DP 5-3*b*.

DP 5-4 The circuit shown in Figure DP 5-4*a* has four unspecified circuit parameters: v_s, R_1, R_2, and d, where d is the gain of the CCCS. To design this circuit, we must specify the values of these four parameters. The graph shown in Figure DP 5-4*b* describes a relationship between the current i and the voltage v.

Specify values of v_s, R_1, R_2, and d that cause the current i and the voltage v in Figure DP 5-4*a* to satisfy the relationship described by the graph in Figure DP 5-4*b*.

First Hint: The equation representing the straight line in Figure DP 5-4b is

$$v = -R_t i + v_{oc}$$

That is, the slope of the line is equal to –1 times the Thévenin resistance and the "v-intercept" is equal to the open-circuit voltage.

Second Hint: There is more than one correct answer to this problem. Try setting $R_1 = R_2$.

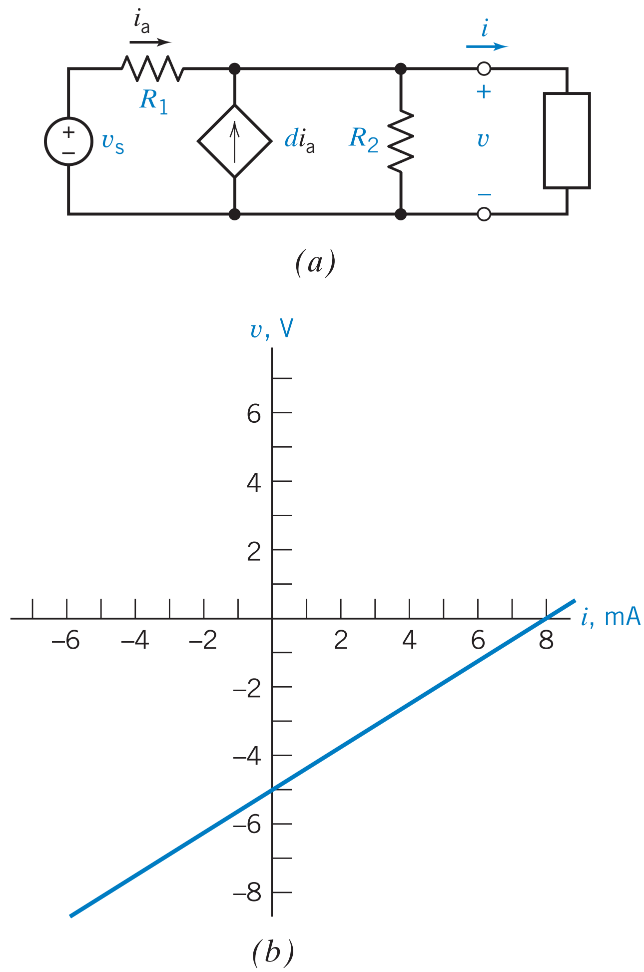


Figure DP 5-4

Solution:

The equation of representing the straight line in Figure DP 5-4b is $v = -R_t\, i + v_{oc}$. That is, the slope of the line is equal to the Thevenin impedance and the "v - intercept" is equal to the open circuit voltage. Therefore: $R_t = -\dfrac{-5-0}{0-0.008} = -625\ \Omega$ and $v_{oc} = -5$ V.

The open circuit voltage, v_{oc}, the short circuit current, i_{sc}, and the Thevenin resistance, R_t, of this circuit are given by

$$v_{oc} = \frac{R_2\left(d+1\right)}{R_1+\left(d+1\right)R_2}v_s \quad ,$$

$$i_{sc} = \frac{\left(d+1\right)}{R_1}v_s$$

and

$$R_t = \frac{R_1 R_2}{R_1 + (d+1) R_2}$$

Let $R_1 = R_2 = 1$ kΩ. Then

$$-625\ \Omega = R_t = \frac{1000}{d+2} \quad \Rightarrow \quad d = \frac{1000}{-625} - 2 = -3.6 \text{ A/A}$$

and

$$-5 = \frac{(d+1) v_s}{d+2} \quad \Rightarrow \quad v_s = \frac{-3.6+2}{-3.6+1}(-5) = -3.077 \text{ V}$$

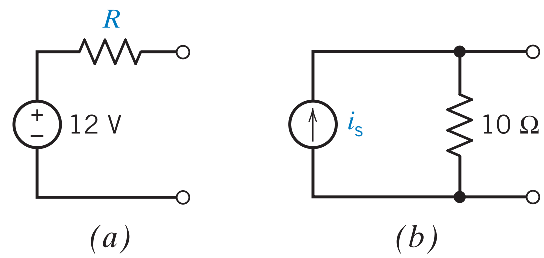
Now v_s, R_1, R_2 and d have all been specified so the design is complete.

Chapter 5 Circuit Theorems

Exercises

Exercise 5.2-1 Determine values of R and i_s so that the circuits shown in Figures E 5.2-1*a,b* are equivalent to each other due to a source transformation.

Answer: $R = 10\ \Omega$ and $i_s = 1.2$ A

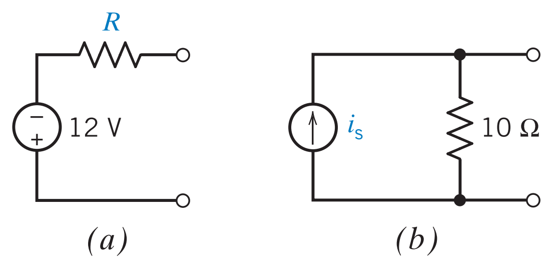


Figures E 5.2-1

Exercise 5.2-2 Determine values of R and i_s so that the circuits shown in Figures E 5.2-2*a,b* are equivalent to each other due to a source transformation.

Hint: Notice that the polarity of the voltage source in Figure E 5.2-2*a* is not the same as in Figure E 5.2-1*a*.

Answer: $R = 10\ \Omega$ and $i_s = -1.2$ A



Figures E 5.2-2

Exercise 5.2-3 Determine values of R and v_s so that the circuits shown in Figures E 5.2-3*a,b* are equivalent to each other due to a source transformation.

Answer: $R = 8\ \Omega$ and $v_s = 24$ V

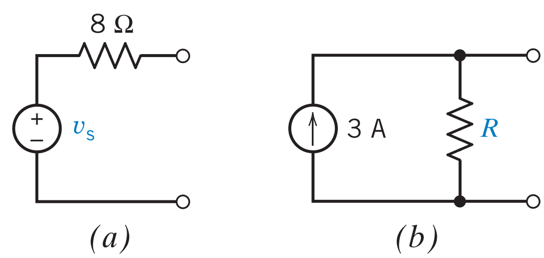


Figure E 5.2-3

Exercise 5.2-4 Determine values of R and v_s so that the circuits shown in Figures E 5.2-4*a,b* are equivalent to each other due to a source transformation.

Hint: Notice that the reference direction of the current source in Figure E 5.2-4*b* is not the same as in Figure E 5.2-3*b*.

Answer: $R = 8\ \Omega$ and $v_s = -24$ V

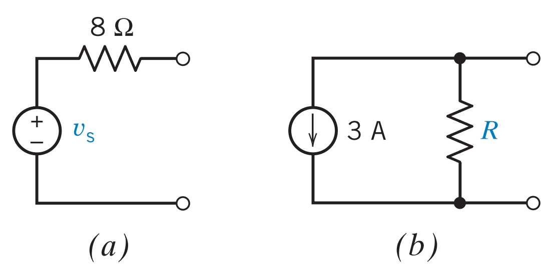
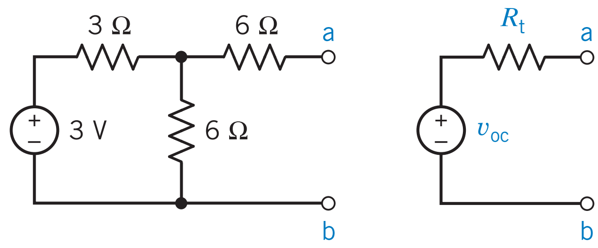


Figure E 5.2-4

Exercise 5.4-1 Determine values of R_t and v_{oc} that cause the circuit shown in Figure E 5.4-1*b* to be the Thévenin equivalent circuit of the circuit in Figure E 5.4-1*a*.

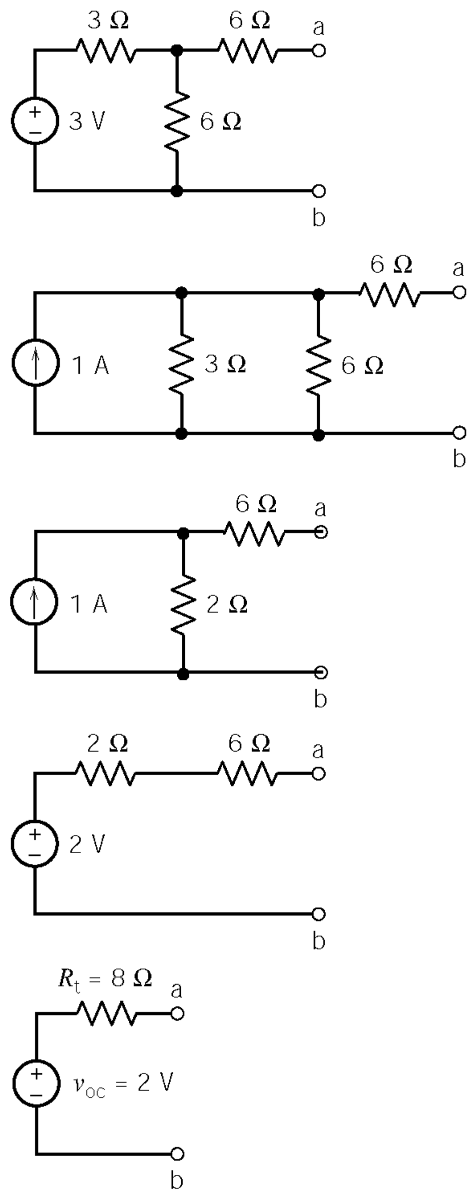
Answer: $R_t = 8\ \Omega$ and $v_{oc} = 2$ V



(a) *(b)*

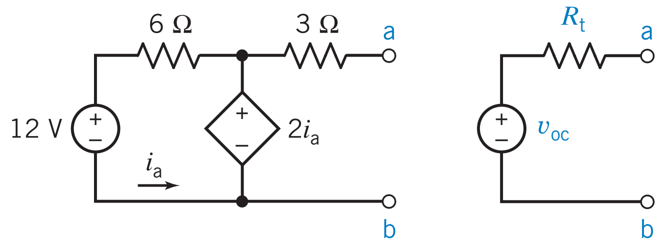
Figure E 5.2-1

Solution:



Exercise 5.4-2 Determine values of R_t and v_{oc} that cause the circuit shown in Figure E 5.4-2*b* to be the Thévenin equivalent circuit of the circuit in Figure E 5.4-2*a*.

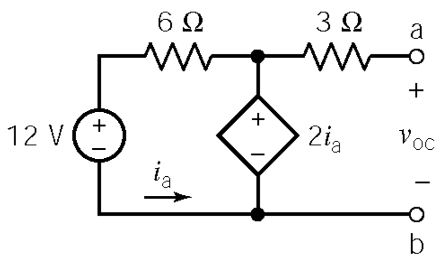
Answer: $R_t = 3\ \Omega$ and $v_{oc} = -6$ V



(a) *(b)*

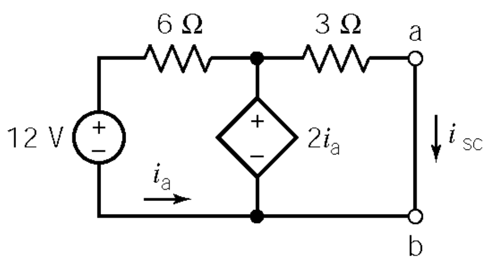
Figure E 5.2-2

Solution:



$$i_a = \frac{2\,i_a - 12}{6} \quad \Rightarrow \quad i_a = -3 \text{ A}$$

$$v_{oc} = 2\,i_a = -6 \text{ V}$$



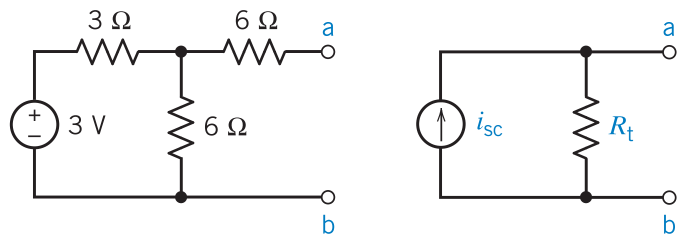
$$12 + 6\,i_a = 2\,i_a \quad \Rightarrow \quad i_a = -3 \text{ A}$$

$$3\,i_{sc} = 2\,i_a \quad \Rightarrow \quad i_{sc} = \frac{2}{3}(-3) = -2 \text{ A}$$

$$R_t = \frac{-6}{-2} = 3\,\Omega$$

Exercise 5.5-1 Determine values of R_t and i_{sc} that cause the circuit shown in Figure E 5.5-1*b* to be the Norton equivalent circuit of the circuit in Figure E 5.5-1*a*.

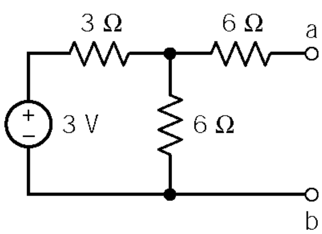
Answer: $R_t = 8\ \Omega$ and $i_{sc} = 0.25$ A

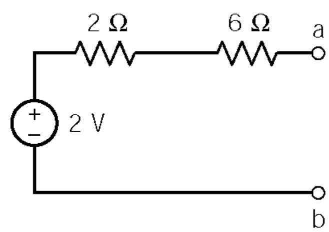


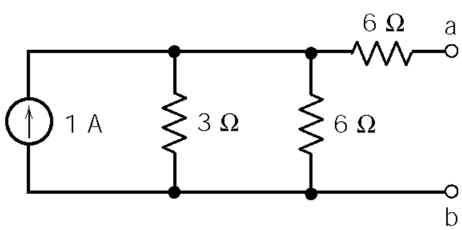
(a) *(b)*

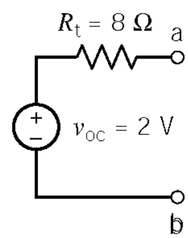
Figure E 5.5-1

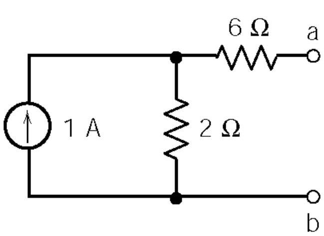
Solution:

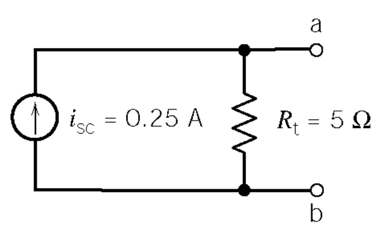












Exercise 5.6-1 Find the maximum power that can be delivered to R_L for the circuit of Figure E 5.6-1 using a Thévenin equivalent circuit.

Answer: 9 W when $R_L = 4\ \Omega$

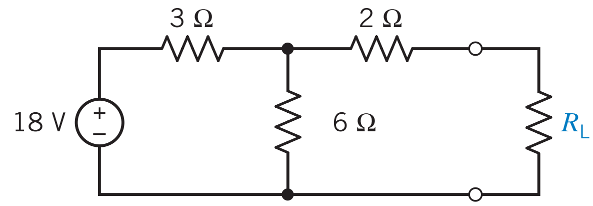
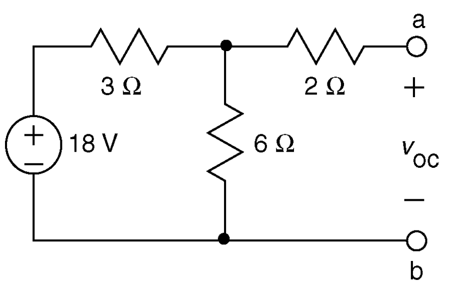
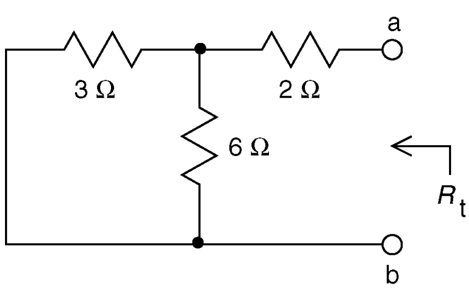


Figure E 5.6-1

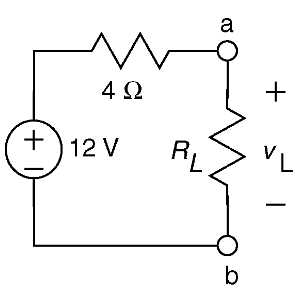
Solution:



$$v_{oc} = \frac{6}{6+3}(18) = 12\ \text{V}$$



$$R_t = 2 + \frac{(3)(6)}{3+6} = 4\ \Omega$$



For maximum power, we require

$$R_L = R_t = 4\ \Omega$$

Then

$$p_{\max} = \frac{{v_{oc}}^2}{4\,R_t} = \frac{12^2}{4(4)} = 9\ \text{W}$$

PSpice Problems

SP 5-1 The circuit in Figure SP 5.1 has three inputs: v_1, v_2, and i_3. The circuit has one output, v_o. The equation

$$v_o = av_1 + bv_2 + ci_3$$

expresses the output as a function of the inputs. The coefficients *a*, *b*, and *c* are real constants.

(a) Use PSpice, and the principle of superposition, to determine the values of *a*, *b*, and *c*.

(b) Suppose $v_1 = 10$ V, $v_2 = 8$ V, and we want the output to be $v_o = 7$ V. What is the required value of i_3?

Hint: The output is given by $v_o = a$ when $v_1 = 1$ V, $v_2 = 0$ V, and $i_3 = 0$ A.

Answer: (a) $v_o = 0.3333v_1 + 0.3333v_2 + 33.33i_3$, (b) $i_3 = 30$ mA

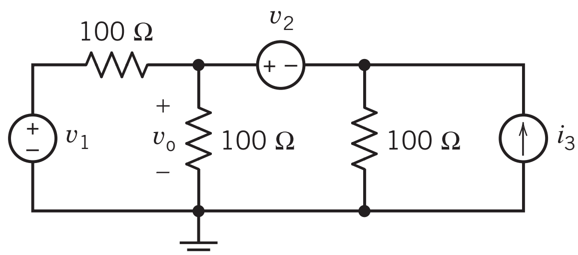
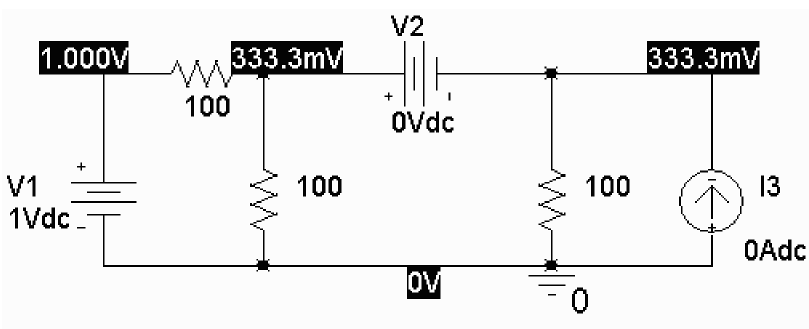
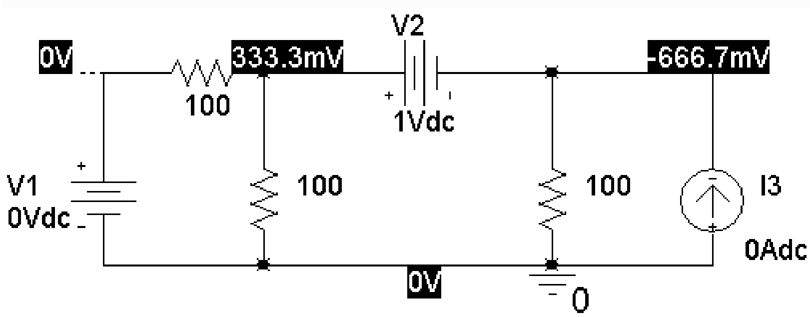


Figure SP 5.1

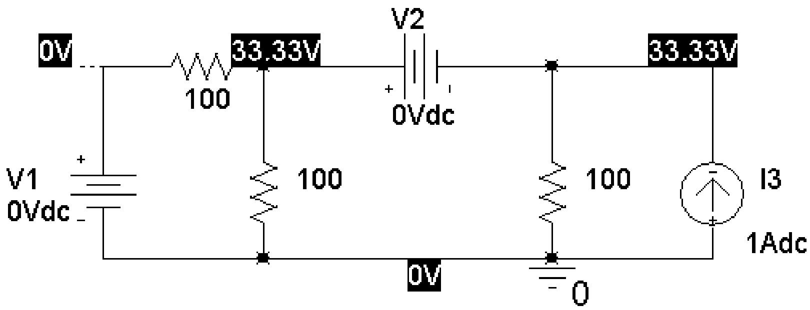
Solution:



a = 0.3333



b = 0.3333



c =33.33 V/A

(a) $$v_o = 0.3333\,v_1 + 0.3333\,v_2 + 33.33\,i_3$$

(b) $$7 = 0.3333\left(10\right) + 0.3333\left(8\right) + 33.33\,i_3 \quad \Rightarrow \quad i_3 = \frac{7 - \frac{18}{3}}{\frac{100}{3}} = \frac{3}{100} = 30\text{ mA}$$

SP 5-2 The pair of terminals a–b partitions the circuit in Figure SP 5.2 into two parts. Denote the node voltages at nodes 1 and 2 as v_1 and v_2. Use PSpice to demonstrate that performing a source transformation on the part of the circuit to the left of the terminal does not change anything to the right of the terminals. In particular, show that the current, i_o, and the node voltages, v_1 and v_2, have the same values after the source transformation as before the source transformation.

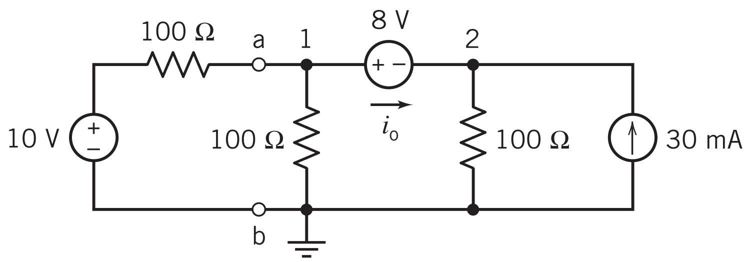
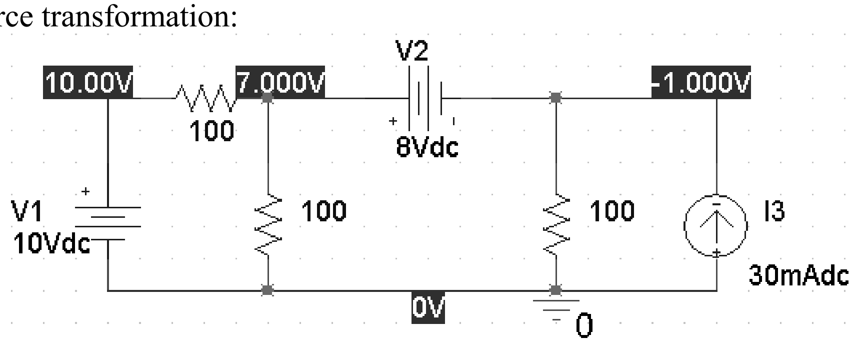


Figure SP 5.2

Solution:

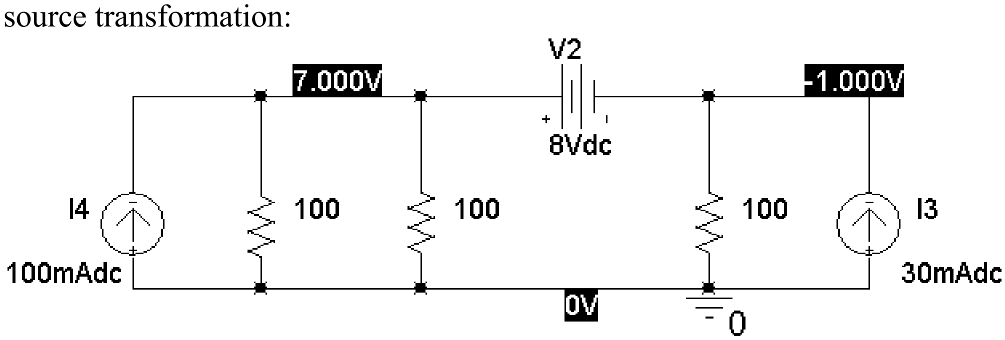
Before the source transformation:



VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V_V1	-3.000E-02
V_V2	-4.000E-02

After the source transformation:



VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V_V2	-4.000E-02

SP 5-3 Use PSpice to find the Thévenin equivalent circuit for the circuit shown in Figure SP 5.3.

Answer: $v_{oc} = -2$ V and $R_t = -8/3\ \Omega$

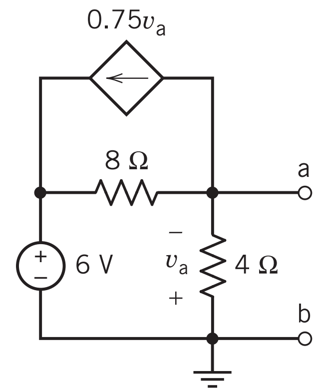
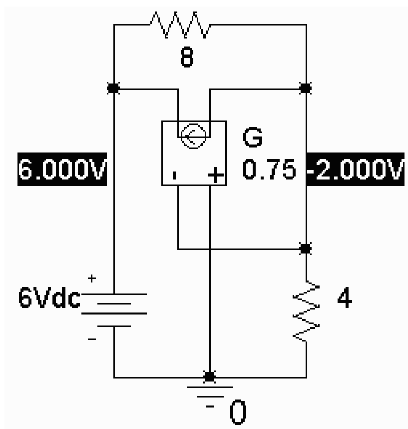
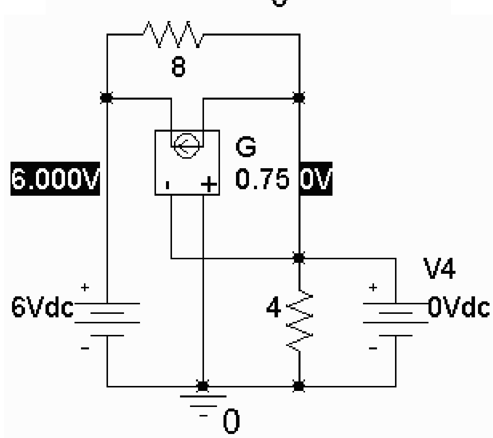


Figure SP 5.3

Solution:



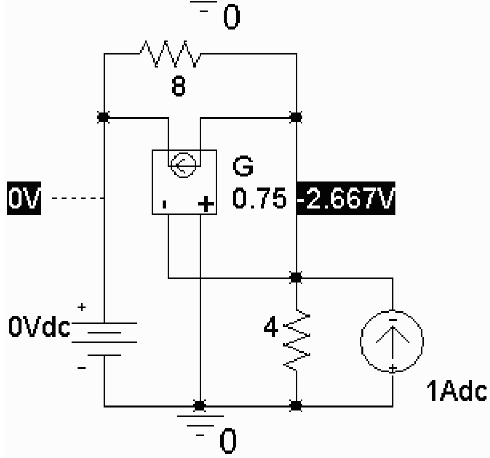
$v_{oc} = -2$ V



VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V_V3	-7.500E-01
V_V4	7.500E-01

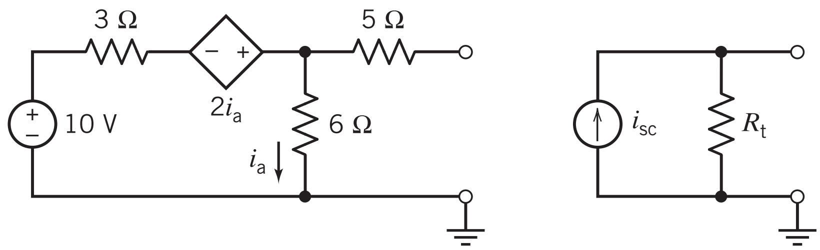
$i_{sc} = 0.75$ A



$R_t = -2.66\ \Omega$

SP 5-4 The circuit shown in Figure SP 5-4*b* is the Norton equivalent circuit of the circuit shown in Figure SP 5-4*a*. Find the value of the short-circuit current, i_{sc}, and Thévenin resistance, R_t.

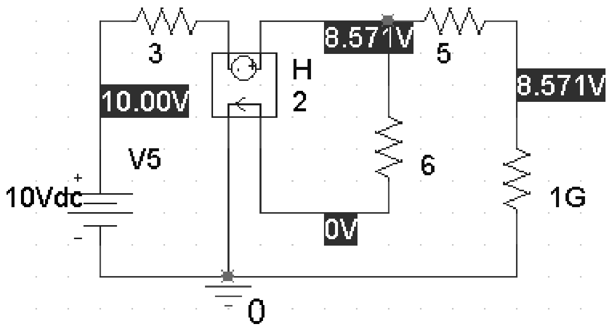
Answer: i_{sc} = 1.13 V and R_t = 7.57 Ω



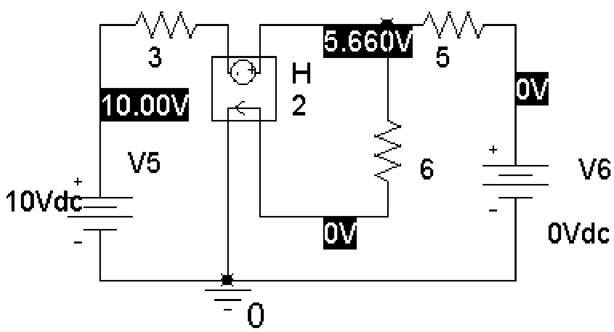
(*a*) (*b*)

Figure SP 5-4

Solution:



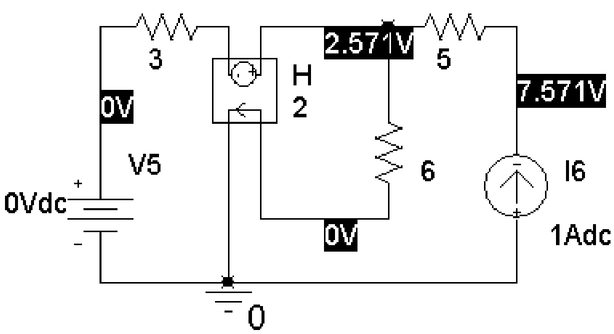
v_{oc} = 8.571 V



VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V_V5	-2.075E+00
V_V6	1.132E+00
X_H1.VH_H1	9.434E-01

i_{sc} = 1.132 A



R_t = 7.571 Ω