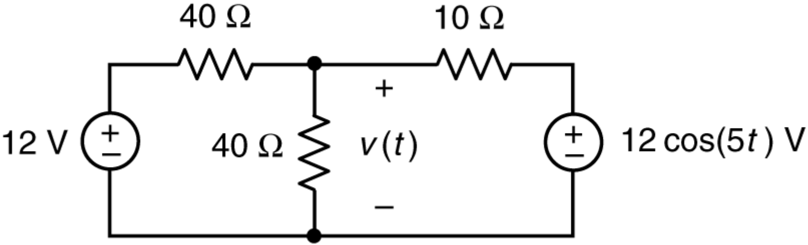
ECSE 200 - Electric Circuits 1
Tutorial 6

ECE Dept., McGill University

Problem P 5.3-5

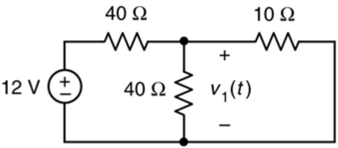
Determine v(t), the voltage across the vertical resistor in the circuit in the figure.

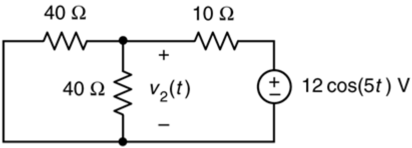


Problem P 5.3-5 Solution

Solution;

We'll use superposition. Let $v_1(t)$ the be the part of $v(t)$ due to the voltage source acting alone. Similarly, let $v_2(t)$ the be the part of $v(t)$ due to the voltage source acting alone. We can use these circuits to calculate $v_1(t)$ and $v_2(t)$.





Notice that $v_1(t)$ is the voltage across parallel resistors. Using equivalent resistance, we calculate $40 \| 10 = 8\ \Omega$. Next, using voltage division we calculate

$$v_1(t) = \frac{8}{8+40}(12) = 2 \text{ V}$$

Similarly $v_2(t)$ is the voltage across parallel resistors Using equivalent resistance we first determine $40 \| 40 = 20\ \Omega$ and then calculate

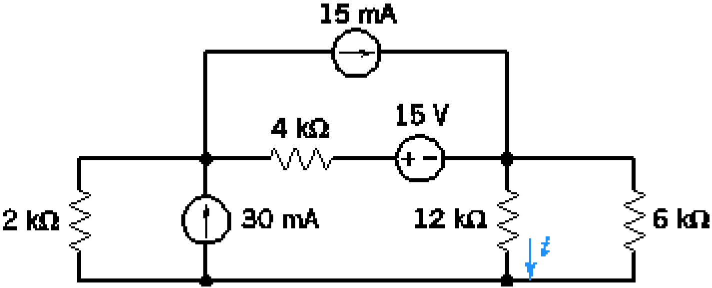
$$v_2(t) = \frac{20}{10+20}(12\cos(5t)) = 8\cos(5t) \text{ V}$$

Using superposition

$$v(t) = v_1(t) + v_2(t) = 2 + 8\cos(5t) \text{ V}$$

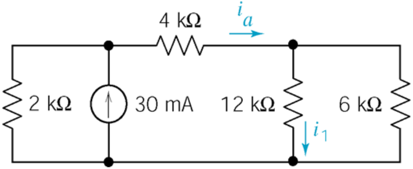
Problem P 5.3-6

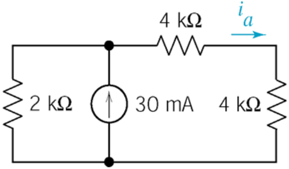
Use superposition to find the value of the current i in the figure.



Problem P 5.3-6 Solution

Solution:
Consider 30 mA source only (open 15 mA and short 15 V sources). Let i_1 be the part of i due to the 30 mA current source.

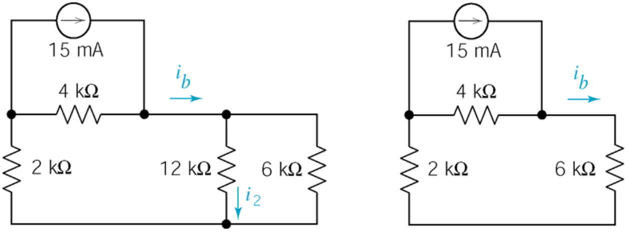




$$i_a = 30\left(\frac{2}{2+8}\right) = 6 \text{ mA} \Rightarrow i_1 = i_a\left(\frac{6}{6+12}\right) = \underline{2 \text{ mA}}$$

Consider 15 mA source only (open 30 mA source and short 15 V source) Let i_2 be the part of i due to the 15 mA current source.

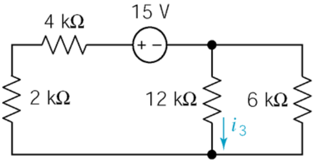
Problem P 5.3-6 Solution

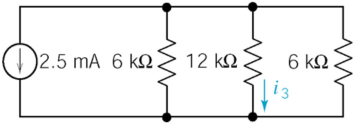


$$i_b = 15\left(\frac{4}{4+6}\right) = 6 \text{ mA} \Rightarrow i_2 = i_b\left(\frac{6}{6+12}\right) = \underline{2 \text{ mA}}$$

Consider 15 V source only (open both current sources). Let i_3 be the part of i due to the 15 V voltage source.

Problem P 5.3-6 Solution



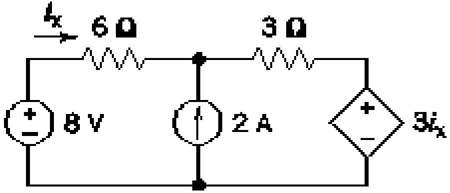


$$i_3 = -2.5\left(\frac{6\|6}{(6\|6)+12}\right) = -10\left(\frac{3}{3+12}\right) = \underline{-0.5\text{ mA}}$$

Finally, $\underline{i = i_1 + i_2 + i_3 = 2 + 2 - 0.5 = 3.5\text{ mA}}$

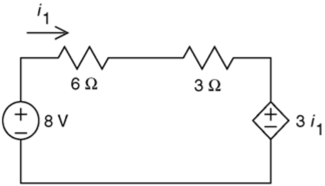
Problem P 5.3-8

Use superposition to find the value of the current i_x in the figure.



Problem P 5.3-8 Solution

Consider 8 V source only (open the 2 A source)



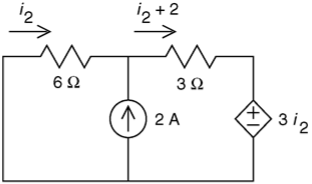
Let i_1 be the part of i_x due to the 8 V voltage source.

Apply KVL to the supermesh:

$$6(i_1)+3(i_1)+3(i_1)-8=0$$

$$i_1=\frac{8}{12}=\frac{2}{3}\text{ A}$$

Consider 2 A source only (short the 8 V source)



Let i_2 be the part of i_x due to the 2 A current source.

Apply KVL to the supermesh:

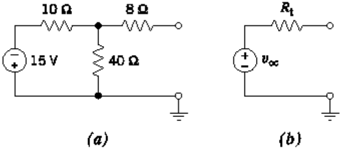
$$6(i_2)+3(i_2+2)+3\,i_2=0$$

$$i_2=\frac{-6}{12}=-\frac{1}{2}\text{ A}$$

Finally, $\quad i_x=i_1+i_2=\frac{2}{3}-\frac{1}{2}=\frac{1}{6}\text{ A}$

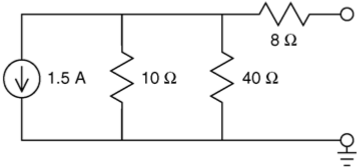
Problem P 5.4-2

The circuit shown in Figure (b) is the Thvenin equivalent circuit of the circuit shown in Figure (a). Find the value of the open-circuit voltage, $v_o c$, and Thvenin resistance, R_t.

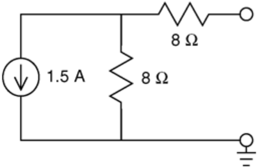


Problem P 5.4-2 Solution

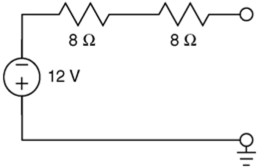
The circuit from Figure P5.4-2*a* can be reduced to its Thevenin equivalent circuit in four steps:



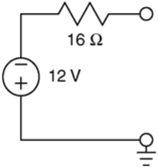
(*a*)



(*b*)



(*c*)

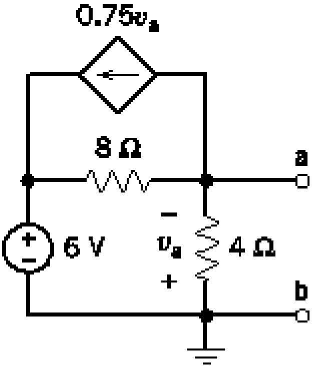


(*d*)

Comparing (*d*) to Figure P5.4-2*b* shows that the Thevenin resistance is $R_t = 16\ \Omega$ and the open circuit voltage, $v_{oc} = -12$ V.

Problem P 5.4-5

Find the Thvenin equivalent circuit for the circuit shown in the figure.



Problem P 5.4-5 Solution

Solution:
Find v_{oc}:

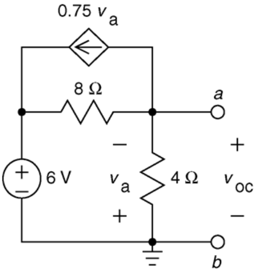
Notice that v_{oc} is the node voltage at node a. Express the controlling voltage of the dependent source as a function of the node voltage:

$$v_a = -v_{oc}$$

Apply KCL at node a:

$$-\left(\frac{6-v_{oc}}{8}\right)+\frac{v_{oc}}{4}+\left(-\frac{3}{4}v_{oc}\right)=0$$

$$-6+v_{oc}+2\,v_{oc}-6\,v_{oc}=0 \quad\Rightarrow\quad v_{oc}=-2\text{ V}$$



Problem P 5.4-5 Solution

Find R_t:

We'll find i_{sc} and use it to calculate R_t. Notice that the short circuit forces

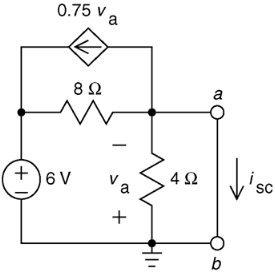
$$v_a = 0$$

Apply KCL at node a:

$$-\left(\frac{6-0}{8}\right)+\frac{0}{4}+\left(-\frac{3}{4}0\right)+i_{sc}=0$$

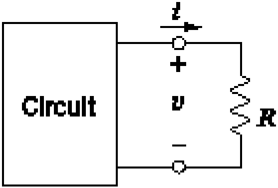
$$i_{sc}=\frac{6}{8}=\frac{3}{4}\text{ A}$$

$$R_t=\frac{v_{oc}}{i_{sc}}=\frac{-2}{3/4}=-\frac{8}{3}\ \Omega$$



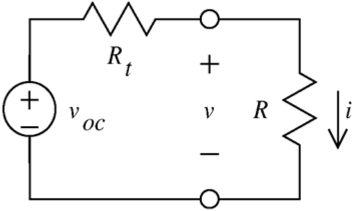
Problem P 5.4-8

A resistor, R, was connected to a circuit box as shown in the figure. The voltage, v, was measured. The resistance was changed, and the voltage was measured again. The results are shown in the table. Determine the Thvenin equivalent of the circuit within the box and predict the voltage, v, when R $= 8k\Omega$. Assume the circuit in the box is a linear circuit.



R	v
2 kΩ	6 V
4 kΩ	2 V

Problem P 5.4-8 Solution



$$v = \frac{R}{R_t + R} v_{oc}$$

From the given data:

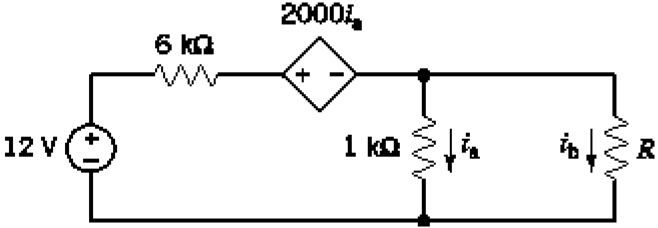
$$\left. \begin{array}{l} 6 = \dfrac{2000}{R_t + 2000} v_{oc} \\ 2 = \dfrac{4000}{R_t + 4000} v_{oc} \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} v_{oc} = 1.2 \text{ V} \\ R_t = -1600\ \Omega \end{array} \right.$$

When $R = 8000\ \Omega$,

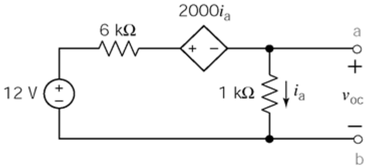
$$v = \frac{8000}{-1600 + 8000}(1.2) = 1.5 \text{ V}$$

Problem P 5.4-10

For the circuit of the figure, specify the resistance R that will cause current ib to be 2 mA. The current ia has units of amps.



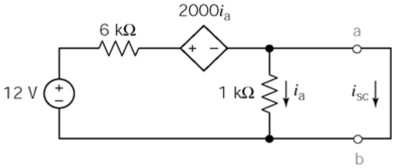
Problem P 5.4-10 Solution



$$-12+6000\,i_a+2000\,i_a+1000\,i_a=0$$

$$i_a = 4/3000 \text{ A}$$

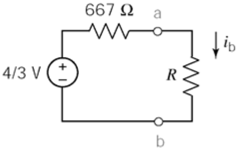
$$v_{oc}=1000\ i_a=\frac{4}{3}\text{ V}$$



$i_a = 0$ due to the short circuit

$$-12+6000\,i_{sc}=0 \Rightarrow i_{sc}=2 \text{ mA}$$

$$R_t=\frac{v_{oc}}{i_{sc}}=\frac{\frac{4}{3}}{.002}=667\ \Omega$$



$$i_b=\frac{\frac{4}{3}}{667+R}$$

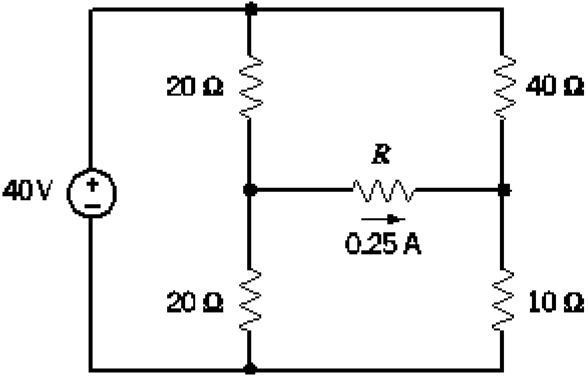
$i_b = 0.002$ A requires

$$R = \frac{\frac{4}{3}}{0.002}-667 = 0$$

Problem P 5.4-14

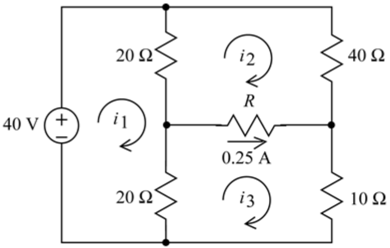
The circuit shown in the figure contains an unspecified resistance, R. Determine the value of R in each of the following two ways.

- Write and solve mesh equations.
- Replace the part of the circuit connected to the resistor R by a Thvenin equivalent circuit. Analyze the resulting circuit.



Problem P 5.4-14 Solution

(a)



$$i_3 - i_2 = 0.25 \text{ A}$$

Apply KVL to mesh 1 to get

$$20(i_1 - i_2) + 20(i_1 - i_3) - 40 = 0$$

Apply KVL to the supermesh corresponding to the unspecified resistance to get

$$40i_2 + 10i_3 - 20(i_1 - i_3) - 20(i_1 - i_2) = 0$$

Problem P 5.4-14 Solution

Solving, for example using MATLAB, gives

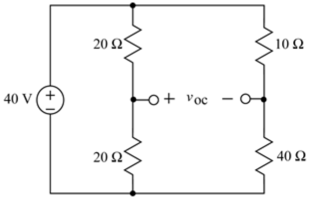
$$\begin{bmatrix} 0 & -1 & 1 \\ 40 & -20 & -20 \\ -40 & 60 & 30 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0.25 \\ 40 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1.875 \\ 0.750 \\ 1.000 \end{bmatrix}$$

Apply KVL to mesh 2 to get

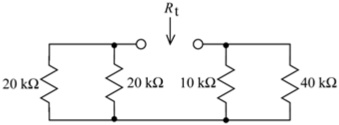
$$40 i_2 + R\left(i_2 - i_3\right) - 20\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad R = \frac{20\left(i_1 - i_2\right) - 40 i_2}{i_2 - i_3} = 30\ \Omega$$

Problem P 5.4-14 Solution

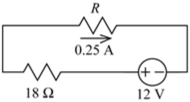
(b)



$$v_{oc} = \left(\frac{20}{20+20}\right)40 - \left(\frac{40}{10+40}\right)40 = -12 \text{ V}$$



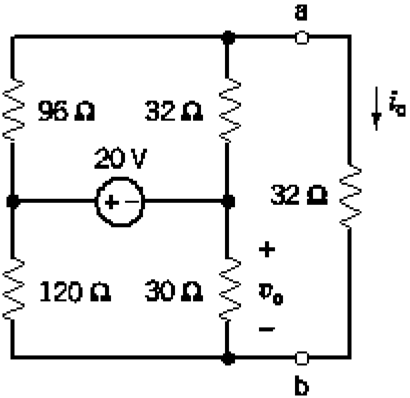
$$R_t = 18\ \Omega$$



$$0.25 = \frac{12}{18+R} \quad \Rightarrow \quad R = 30\ \Omega$$

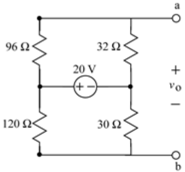
Problem P 5.4-15

Consider the circuit shown in the figure. Replace the part of the circuit to the left of terminals ab by its Thvenin equivalent circuit. Determine the value of the current i_o.



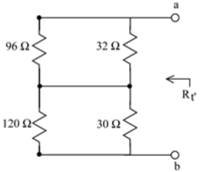
Problem P 5.4-15 Solution

Find the Thevenin equivalent circuit for the part of the circuit to the left of the terminals a-b.



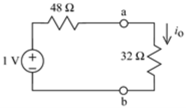
Using voltage division twice

$$v_{oc} = \frac{32}{32+96}20 - \frac{30}{120+30}20 = 5 - 4 = 1 \text{ V}$$



$$R_t = (96 \| 32) + (120 \| 30) = 24 + 24 = 48\ \Omega$$

Replacing the part of the circuit to the left of terminals a-b by its Thevenin equivalent circuit gives



$$i_o = \frac{1}{48+32} = 0.0125 \text{ A } = 12.5 \text{ mA}$$

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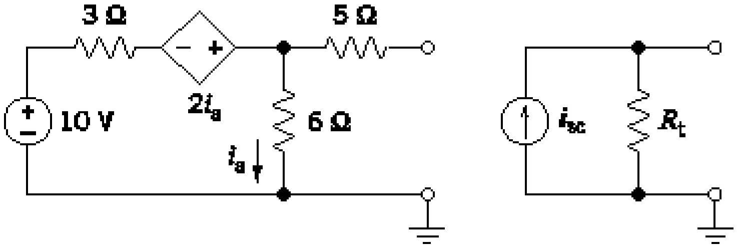
Problem P 5.5-5

The circuit shown in Figure (b) is the Norton equivalent circuit of the circuit shown in Figure (a). Find the value of the short-circuit current, i_{sc}, and Thvenin resistance, R_t.



(a) (b)

Problem P 5.5-5 Solution

To determine the value of the short circuit current, i_{sc}, we connect a short circuit across the terminals of the circuit and then calculate the value of the current in that short circuit. Figure (a) shows the circuit from Figure 5.6-4*a* after adding the short circuit and labeling the short circuit current. Also, the meshes have been identified and labeled in anticipation of writing mesh equations. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

In Figure (a), mesh current i_2 is equal to the current in the short circuit. Consequently, $i_2 = i_{sc}$. The controlling current of the CCVS is expressed in terms of the mesh currents as

$$i_a = i_1 - i_2 = i_1 - i_{sc}$$

Apply KVL to mesh 1 to get

$$3\,i_1 - 2\left(i_1 - i_2\right) + 6\left(i_1 - i_2\right) - 10 = 0 \quad \Rightarrow \quad 7\,i_1 - 4\,i_2 = 10 \qquad \textbf{(1)}$$

Apply KVL to mesh 2 to get

$$5\,i_2 - 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad -6\,i_1 + 11\,i_2 = 0 \quad \Rightarrow \quad i_1 = \frac{11}{6}\,i_2$$

Substituting into equation 1 gives

$$7\left(\frac{11}{6}\,i_2\right) - 4\,i_2 = 10 \quad \Rightarrow \quad i_2 = 1.13\text{ A} \quad \Rightarrow \quad i_{sc} = 1.13\text{ A}$$

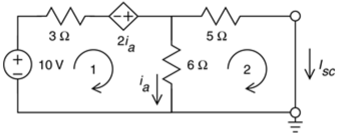


Figure (a) Calculating the short circuit current, i_{sc}, using mesh equations.

Problem P 5.5-5 Solution

To determine the value of the Thevenin resistance, R_t, first replace the 10 V voltage source by a 0 V voltage source, i.e. a short circuit. Next, connect a current source across the terminals of the circuit and then label the voltage across that current source as shown in Figure (b). The Thevenin resistance will be calculated from the current and voltage of the current source as

$$R_t = \frac{v_T}{i_T}$$

In Figure (b), the meshes have been identified and labeled in anticipation of writing mesh equations. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

In Figure (b), mesh current i_2 is equal to the negative of the current source current. Consequently, $i_2 = i_T$. The controlling current of the CCVS is expressed in terms of the mesh currents as

$$i_a = i_1 - i_2 = i_1 + i_T$$

Problem P 5.5-5 Solution

Apply KVL to mesh 1 to get

$$3\,i_1 - 2\left(i_1 - i_2\right) + 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad 7\,i_1 - 4\,i_2 = 0 \quad \Rightarrow \quad i_1 = \frac{4}{7}\,i_2$$

Apply KVL to mesh 2 to get

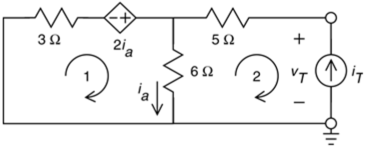
$$5\,i_2 + v_T - 6\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad -6\,i_1 + 11\,i_2 = -v_T$$

Substituting for i_1 using equation 2 gives

$$-6\left(\frac{4}{7}\,i_2\right) + 11\,i_2 = -v_T \quad \Rightarrow \quad 7.57\,i_2 = -v_T$$

Finally,

$$R_t = \frac{v_T}{i_T} = \frac{-v_T}{-i_T} = \frac{-v_T}{i_2} = 7.57\ \Omega$$



Problem P 5.5-5 Solution

To determine the value of the open circuit voltage, v_{oc}, we connect an open circuit across the terminals of the circuit and then calculate the value of the voltage across that open circuit. Figure (c) shows the circuit from Figure 4.6-4*a* after adding the open circuit and labeling the open circuit voltage. Also, the meshes have been identified and labeled in anticipation of writing mesh equations. Let i_1 and i_2 denote the mesh currents in meshes 1 and 2, respectively.

In Figure (c), mesh current i_2 is equal to the current in the open circuit. Consequently, $i_2 = 0$ A. The controlling current of the CCVS is expressed in terms of the mesh currents as

$$i_a = i_1 - i_2 = i_1 - 0 = i_1$$

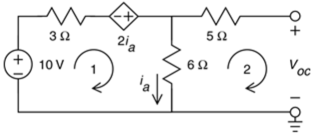
Apply KVL to mesh 1 to get

$$3\,i_1 - 2\left(i_1 - i_2\right) + 6\left(i_1 - i_2\right) - 10 = 0 \quad\Rightarrow\quad 3\,i_1 - 2\left(i_1 - 0\right) + 6\left(i_1 - 0\right) - 10 = 0$$

$$\Rightarrow\quad i_1 = \frac{10}{7} = 1.43 \text{ A}$$

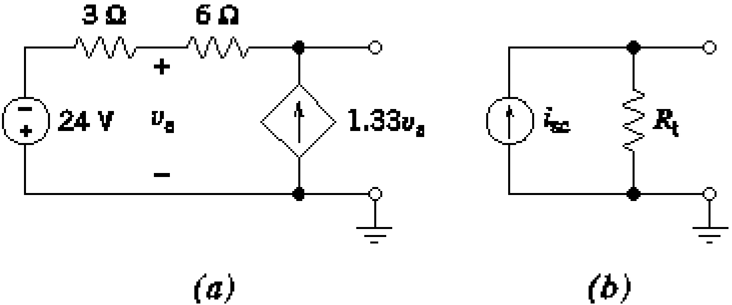
Apply KVL to mesh 2 to get

$$5\,i_2 + v_{oc} - 6\left(i_1 - i_2\right) = 0 \quad\Rightarrow\quad v_{oc} = 6\left(i_1\right) = 6\left(1.43\right) = 8.58 \text{ V}$$



Problem P 5.5-6

The circuit shown in Figure (b) is the Norton equivalent circuit of the circuit shown in Figure (a). Find the value of the short-circuit current, i_{sc}, and Thvenin resistance, R_t.



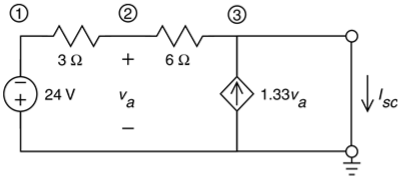
Problem P 5.5-6 Solution

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad -48 = 3\,v_a \quad \Rightarrow \quad v_a = -16 \text{ V}$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3}\,v_2 = i_{sc} \quad \Rightarrow \quad \frac{9}{6}\,v_a = i_{sc} \quad \Rightarrow \quad i_{sc} = \frac{9}{6}(-16) = -24 \text{ A}$$



Problem P 5.5-6 Solution

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad v_T = 3\,v_a$$

Apply KCL at node 3 to get

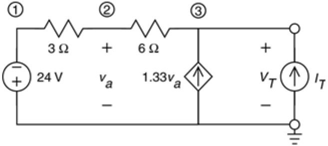
$$\frac{v_2 - v_3}{6} + \frac{4}{3}\,v_2 + i_T = 0 \quad \Rightarrow \quad 9\,v_2 - v_3 + 6\,i_T = 0$$

$$\Rightarrow \quad 9\,v_a - v_T + 6\,i_T = 0$$

$$\Rightarrow \quad 3\,v_T - v_T + 6\,i_T = 0 \quad \Rightarrow \quad 2\,v_T = -6\,i_T$$

Finally,

$$R_t = \frac{v_T}{i_T} = -3\,\Omega$$



Problem P 5.5-6 Solution

Apply KCL at node 2 to get

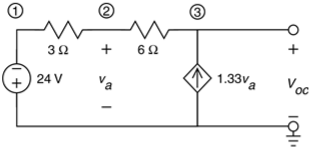
$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \;\Rightarrow\; 2\,v_1 + v_3 = 3\,v_2 \;\Rightarrow\; -48 + v_{oc} = 3\,v_a$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3}\,v_2 = 0 \;\Rightarrow\; 9\,v_2 - v_3 = 0 \;\Rightarrow\; 9\,v_a = v_{oc}$$

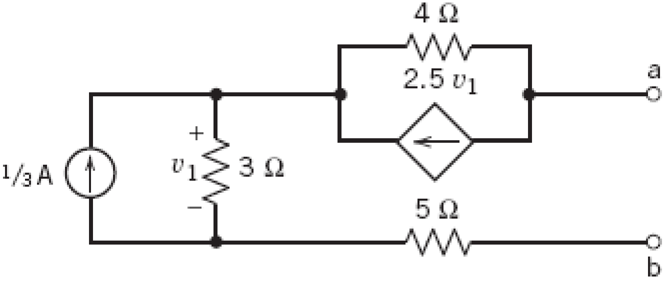
Combining these equations gives

$$3\left(-48 + v_{oc}\right) = 9\,v_a = v_{oc} \;\Rightarrow\; v_{oc} = 72\text{ V}$$



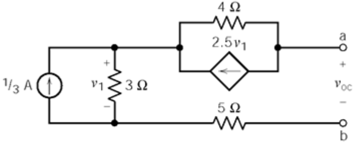
Problem P 5.5-9

Find the Norton equivalent circuit of this circuit.



Problem P 5.5-9 Solution

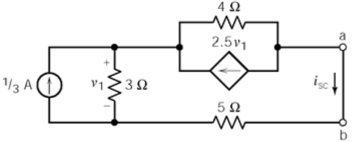
Identify the open circuit voltage and short circuit current.



$$v_1 = \left(\frac{1}{3}\right)3 = 1 \text{ V}$$

Then

$$v_{oc} = v_1 - 4\left(2.5\,v_1\right) = -9 \text{ V}$$



$$v_1 = 3\left(\frac{1}{3} - i_{sc}\right) = 1 - 3\,i_{sc}$$

$$4\left(2.5\,v_1 + i_{sc}\right) + 5 i_{sc} - v_1 = 0$$

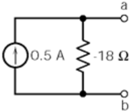
$$\Rightarrow \quad 9\,v_1 + 9\,i_{sc} = 0$$

$$9\left(1 - 3\,i_{sc}\right) + 9\,i_{sc} = 0 \quad \Rightarrow \quad i_{sc} = \frac{1}{2} \text{ A}$$

The Thevenin resistance is

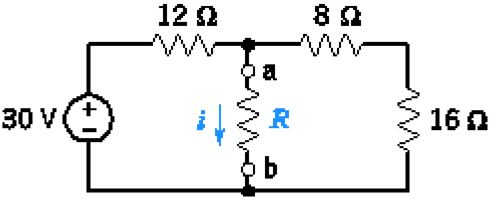
$$R_t = \frac{-9}{0.5} = -18\ \Omega$$

Finally, the Norton equivalent circuit is

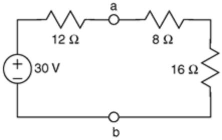


Problem P 5.5-12

Use Nortons theorem to formulate a general expression for the current i in terms of the variable resistance R shown in the figure.

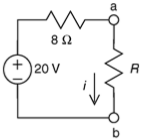


Problem P 5.5-12 Solution



$$R_t = \frac{12\times 24}{12+24} = \frac{12\times 24}{36} = 8\ \Omega$$

$$v_{oc} = \frac{24}{12+24}(30) = 20\ \text{V}$$



$$i = \frac{20}{8+R}$$

Thank you !