Today's Outline

3. Analysis Methods

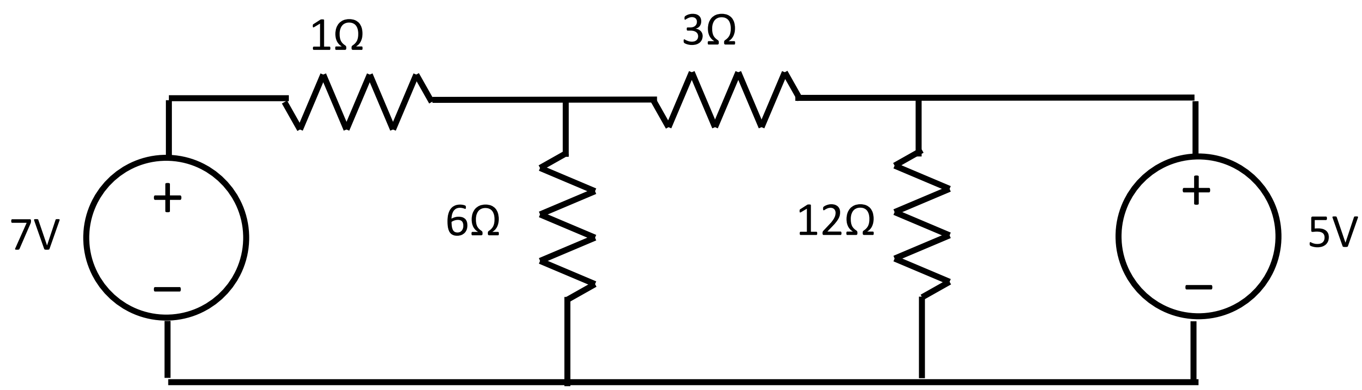
- Mesh Analysis

Mesh Current Method

- A *systematic* way to apply KVL, KCL, Ohm's law (or other terminal laws) to solve for the variables in a circuit
- Rather than solving for node voltage variables, we solve for mesh current variables
- Also known as **mesh analysis**
- *Not* necessarily the most efficient way to solve a circuit
- Ideally suited for solutions on computer because the procedure is systematic, like the node voltage method

Mesh Current Method

We illustrate the procedure by finding the power delivered by the 7V source in the following circuit.



Mesh Current Method

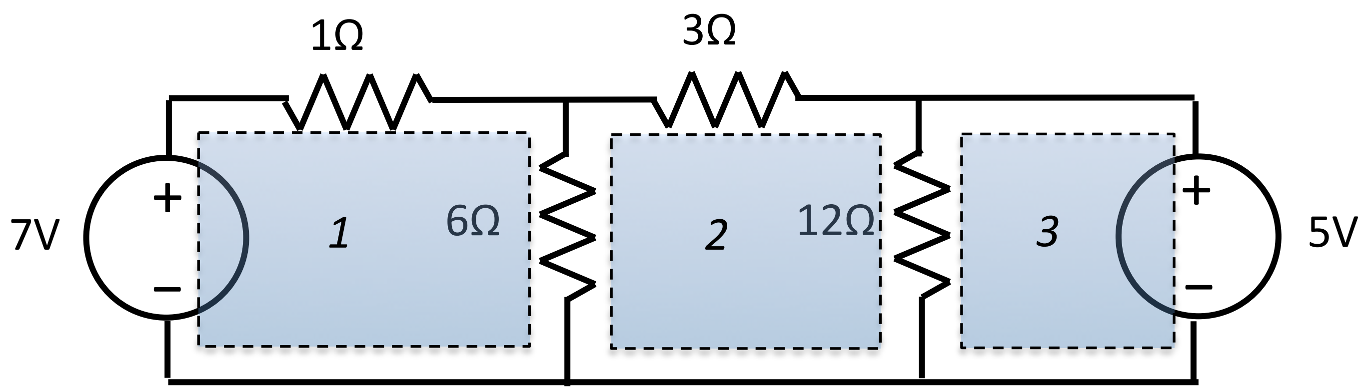
Step #1:

Identify the ***meshes*** of the circuit. A mesh is a loop that cannot be sub-divided into smaller loops.

Label each mesh with numbers 1,2,3… or letters A,B,C…, and write down corresponding circulating current variables i_1, i_2, i_3… or i_A, i_B, i_C…

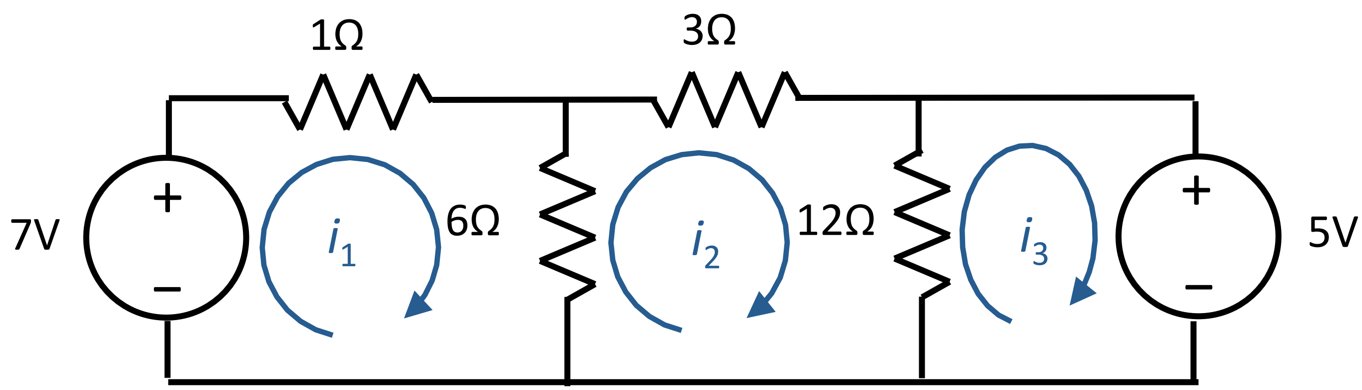
Mesh Current Method

Meshes are identified below, and can also be thought of as the smallest loops that can be used to tile the circuit.



Mesh Current Method

Mesh current variables are defined. The specific direction chosen is not important, but the variable definition must be applied consistently throughout the problem.



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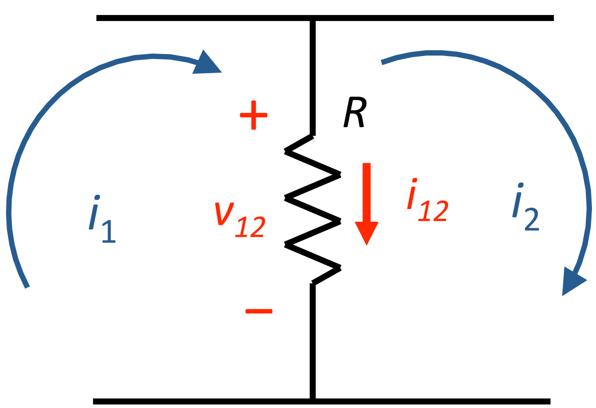
Mesh Current Method

Step #2:

Apply KVL around each mesh. Express each element voltage using the current variables defined earlier (by KCL and terminal laws of elements). For example:



v_{12} and i_{12} = temporary variables

KCL: $-i_1 + i_{12} + i_2 = 0$

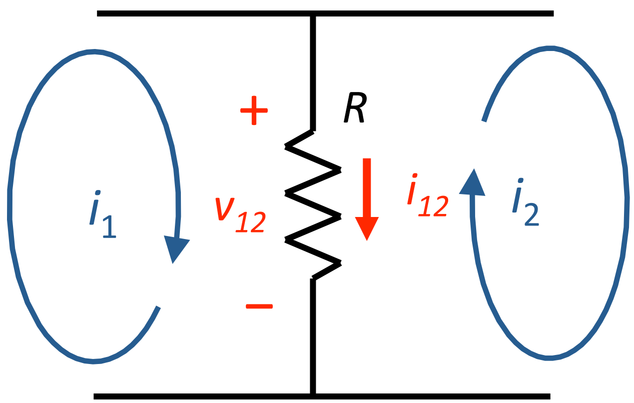
$$i_{12} = i_1 - i_2$$

Ohm's Law: $v_{12} = i_{12} R$

$$= \underbrace{(i_1 - i_2)\, R}_{\textit{voltage term in KVL equation}}$$

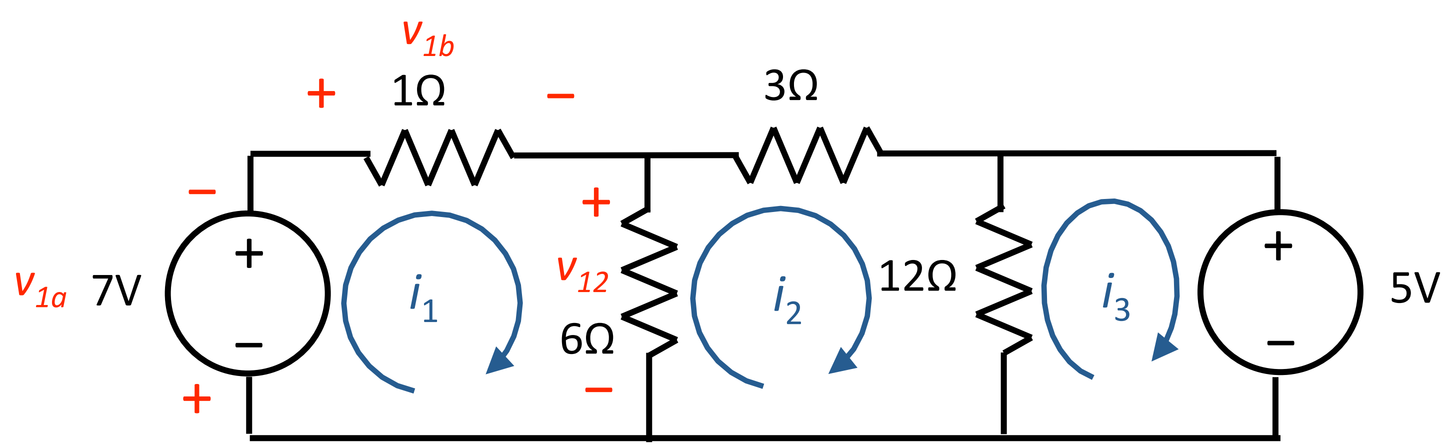
Mesh Current Method

Note: An equivalent way of understanding why the current i_{12} through R is given by $i_{12} = i_1 - i_2$ is to imagine i_{12} as the net current in the presence of two circulating currents i_1 and i_2.



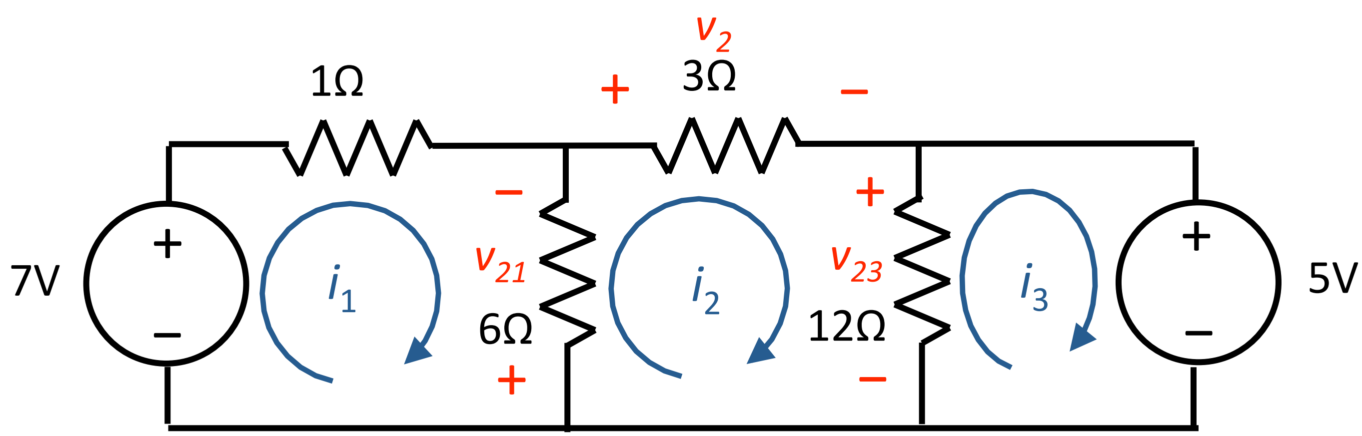
Mesh Current Method

KVL on mesh 1: $0 = \underbrace{-7\text{V}}_{v_{1a}} + \underbrace{i_1 1\Omega}_{v_{1b}} + \underbrace{(i_1 - i_2)6\Omega}_{v_{12}}$



Mesh Current Method

KVL on mesh 2: $$0 = \underbrace{(i_2 - i_1)6\Omega}_{v_{21}} + \underbrace{i_2 3\Omega}_{v_2} + \underbrace{(i_2 - i_3)12\Omega}_{v_{23}}$$



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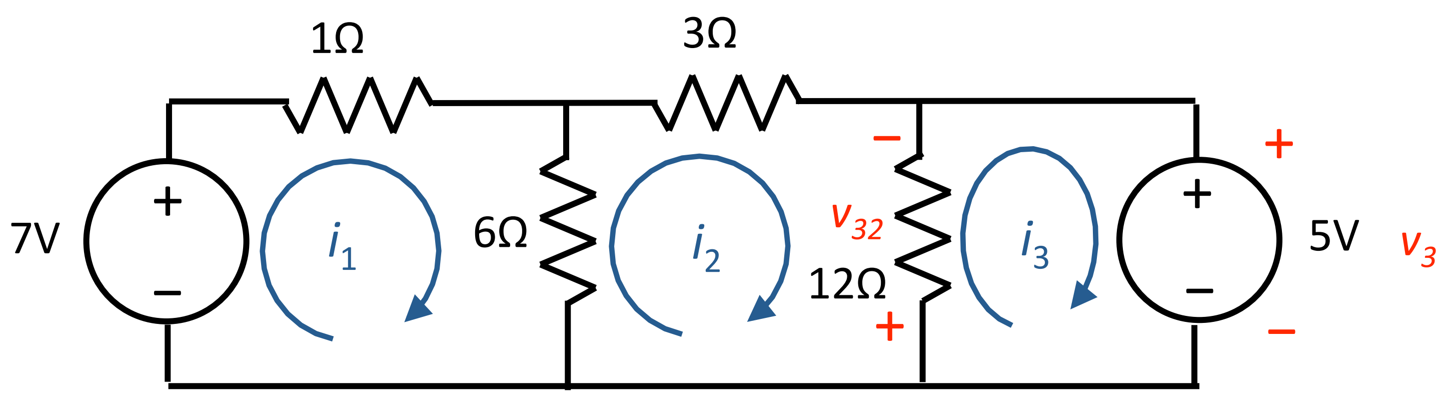
Mesh Current Method

KVL on mesh 3: $0 = \underbrace{(i_3 - i_2)12\Omega}_{v_{32}} + \underbrace{5\text{V}}_{v_3}$

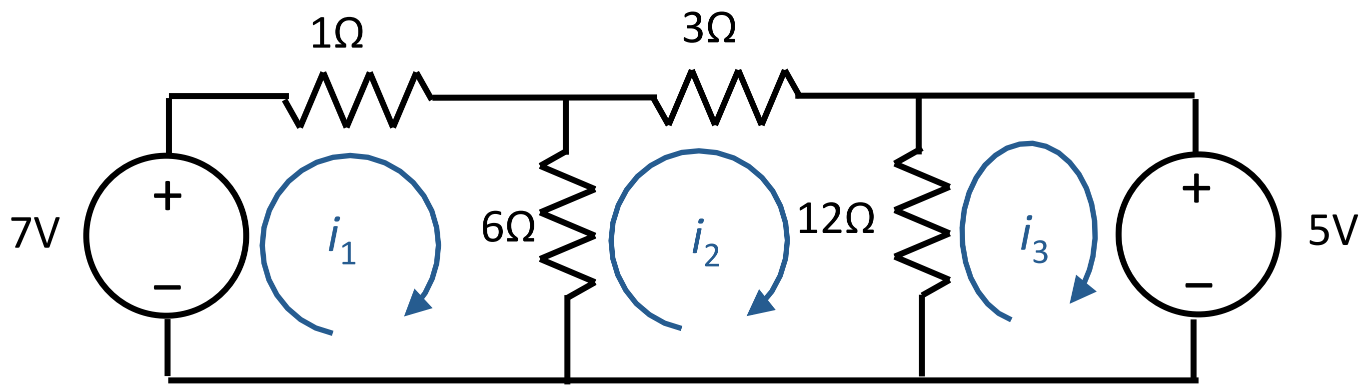


Mesh Current Method

KVL on mesh 1: $0 = -7\text{V} + i_1 1\Omega + (i_1 - i_2)6\Omega$

KVL on mesh 2: $0 = (i_2 - i_1)6\Omega + i_2 3\Omega + (i_2 - i_3)12\Omega$

KVL on mesh 3: $0 = (i_3 - i_2)12\Omega + 5\text{V}$



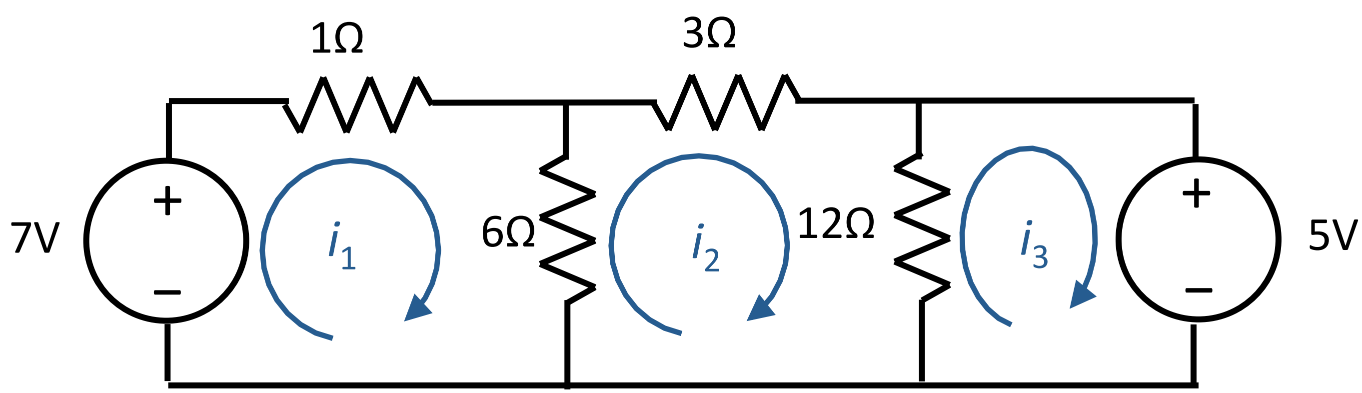
Mesh Current Method

KVL on mesh 1: $0 = -7\text{V} + i_1 1\Omega + \boxed{(i_1 - i_2)6\Omega}$

KVL on mesh 2: $0 = \boxed{(i_2 - i_1)6\Omega} + i_2 3\Omega + \boxed{(i_2 - i_3)12\Omega}$

KVL on mesh 3: $0 = \boxed{(i_3 - i_2)12\Omega} + 5\text{V}$

We see branch voltages with opposite signs in different KVL equations. We do not know in advance the physical voltage drops, (or equivalently, which way the current flows in each branch).



Mesh Current Method

Step #3:

Solve for the mesh current variables, using any linear algebra technique of your preference. The simplest method is repeated substitution (but there are many techniques developed for solution by computer).

Any quantity can be found in terms of the solved mesh currents.

Mesh Current Method

Use repeated substitution to find the value of i_1, and then i_2 and i_3.

mesh 3:
$$0 = (i_3 - i_2)12\Omega + 5\text{V}$$
$$i_3 = i_2 - \frac{5}{12}\text{A}$$

mesh 2:
$$\begin{aligned} 0 &= (i_2 - i_1)6\Omega + i_2 3\Omega + (i_2 - i_3)12\Omega \\ &= (i_2 - i_1)6\Omega + i_2 3\Omega + (i_2 - (i_2 - \tfrac{5}{12}\text{A}))12\Omega \end{aligned}$$

substitution of mesh 3 equation

$$= (i_2 - i_1)6\Omega + i_2 3\Omega + 5\text{V}$$
$$i_2 = \tfrac{6}{9} i_1 - \tfrac{5}{9}\text{A}$$

mesh 1:
$$\begin{aligned} 0 &= -7\text{V} + i_1 1\Omega + (i_1 - i_2)6\Omega \\ &= -7\text{V} + i_1 1\Omega + (i_1 - (\tfrac{6}{9} i_1 - \tfrac{5}{9}\text{A}))6\Omega \end{aligned}$$

substitution of mesh 2,1 equations

$$= -7\text{V} + i_1 1\Omega + i_1 6\Omega - i_1 4\Omega + \tfrac{10}{3}\text{V}$$
$$i_1 = \frac{(7 - \frac{10}{3})\text{V}}{(1+6-4)\Omega}$$
$$= \tfrac{11}{9}\text{A} = 1.222\text{A}$$

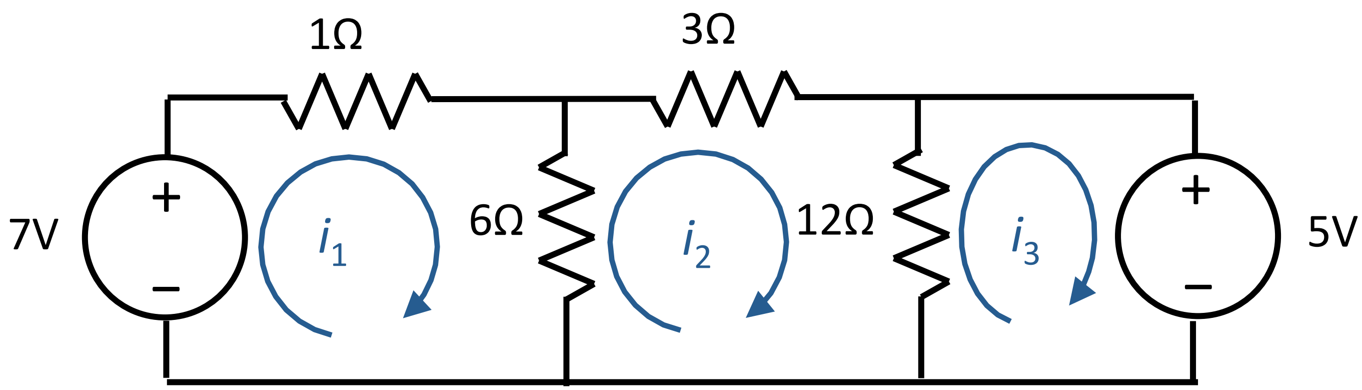
Mesh Current Method

mesh 2: $$i_2 = \frac{6}{9} i_1 - \frac{5}{9}\text{A}$$
$$= \frac{6}{9}\left(\frac{11}{9}\text{A}\right) - \frac{5}{9}\text{A}$$
$$= \frac{7}{27}\text{A} = 0.2592\text{A}$$

mesh 3: $$i_3 = i_2 - \frac{5}{12}\text{A}$$
$$= \frac{7}{27}\text{A} - \frac{5}{12}\text{A}$$
$$= -\frac{17}{108}\text{A}$$
$$= -0.1574\text{A}$$

Mesh Current Method

We can now easily calculate the power delivered by the 7V voltage source (the original question).



i_1=1.222A i_2=0.2592A i_3=−0.1574A

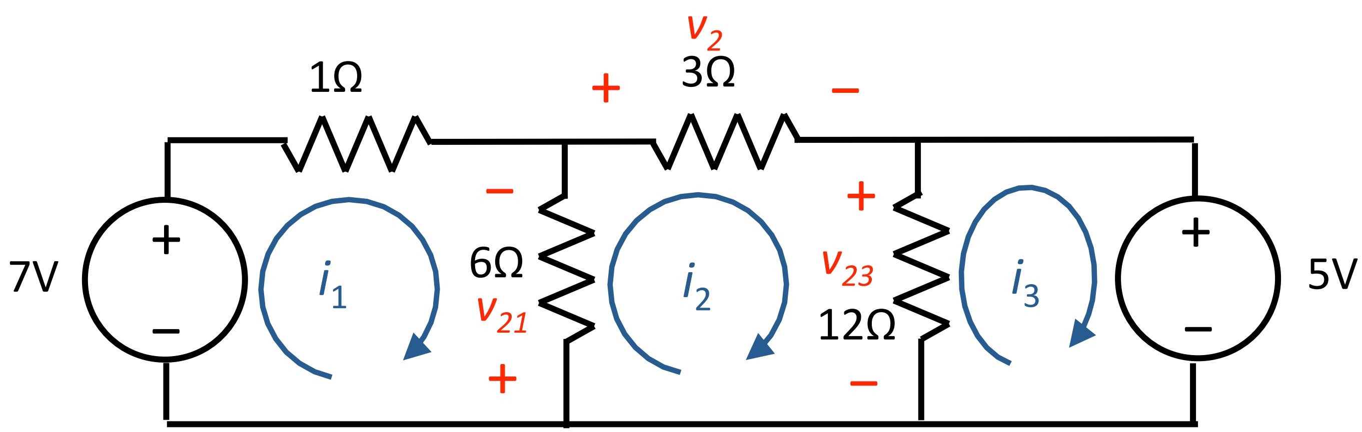
P_{7V}= power delivered by 7V voltage source

= (7V)(i_1)

= 7V×1.222A = +8.554W

Mesh Current Method

We can check that our answer obeys KVL. For example:



i_1=1.222A i_2=0.2592A i_3=−0.1574A

$$v_{23} = (i_2 - i_3)12\Omega = \left[0.2592\text{A} - (-0.1574\text{A})\right]12\Omega = 4.999\text{V}$$

$$v_2 = i_2 3\Omega = 0.2592\text{A} \cdot 3\Omega = 0.778\text{V}$$

$$v_{21} = (i_2 - i_1)6\Omega = \left[0.2592\text{A} - (1.222\text{A})\right]6\Omega = -5.777\text{V}$$

accurate to within round-off errors!

Summary of Mesh Current Method

Step #1: Label meshes, and define mesh current variables circulating in each mesh.

Step #2: Write KVL equations for each mesh using mesh current variables only, by intrinsically using KCL and terminal laws (such as Ohm's law).

Step #3: Solve the linear system of equations, and use the mesh currents to calculate the desired quantity.