1. Fundamentals

- Definitions
- Units
- Charge, Current
- Algebraic Variables
- Voltage, Energy, Power
- Linear Circuit Elements
- Active and Passive Elements
- the Resistor
- Independent Sources, Dependent Sources
- Voltmeters, Ammeters, Switches, Transducers

Today's Outline

1. Fundamentals

- Units
- Definitions
- Charge
- Current
- Algebraic Variables

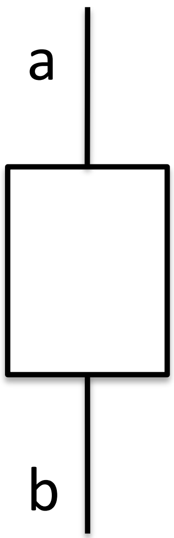
Circuit Analysis Units (SI)

Quantity	Unit	Symbol/Abbreviation
length	meter	m
mass	kilogram	kg
time	second	s
charge	Coulomb	C
current	Ampere	A
voltage	Volt	V
resistance	Ohm	Ω
capacitance	Farad	F
inductance	Henry	H
energy	Joule	J
power	Watt	W

SI Unit Multipliers

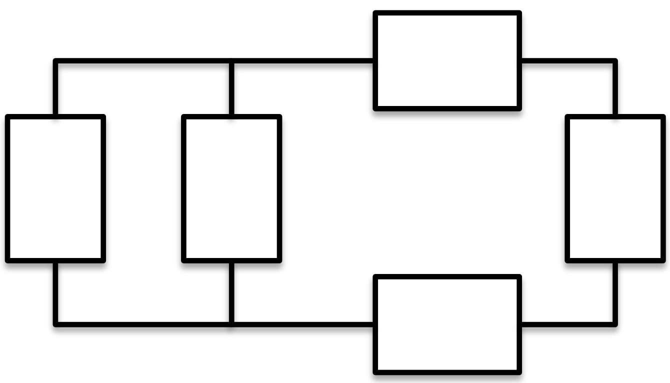
Prefix	Symbol	Multiplier
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
-	-	-
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

Definitions



- an **element** is an *idealized mathematical model* of a physical component (eg. a resistor)

- an electrical connection is denoted with a line, representing an *idealized mathematical model for a wire* that carries charge from one point to another



- a **circuit** is an *idealized mathematical model* for a physical assembly of components, or for a single component itself

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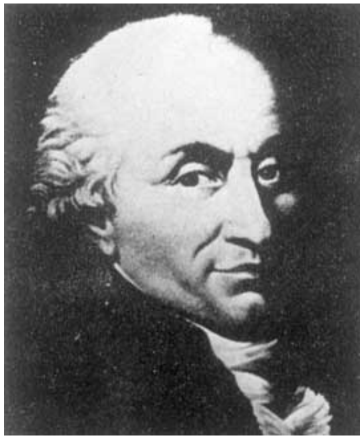
Electric Charge

Electric charge = fundamental property of particles

- signed quantity (positive or negative)
- SI unit is the Coulomb (abbreviated C)
- charge variables usually given the symbol *q*, or *Q*
- quantized in units of the electron charge 1.602×10^{-19}C
- charge is conserved, never destroyed or created, but redistributed

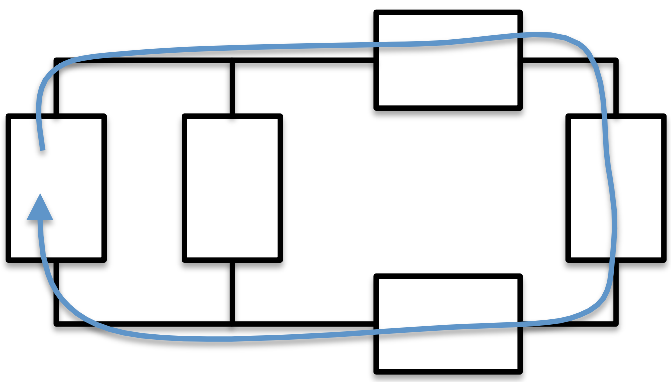


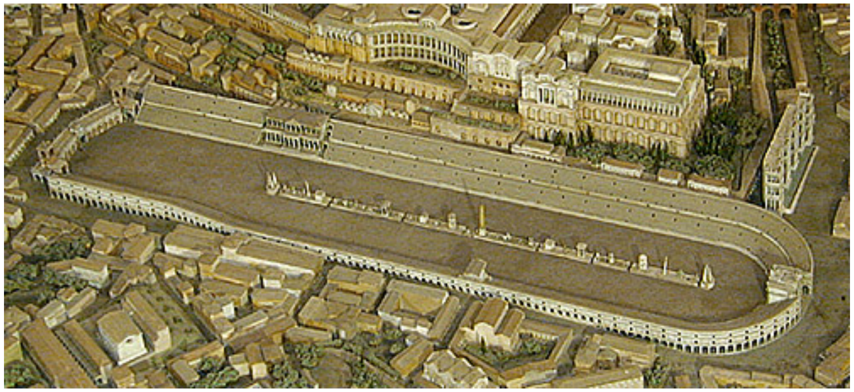
Charles Augustin Coulomb
(1736-1806)

Electric Charge in a Circuit

Charge *circulates* through the elements of a circuit, a result of the conservation of charge. *Charge is neither produced nor consumed by the elements within a circuit.*

circuitus: latin for going around



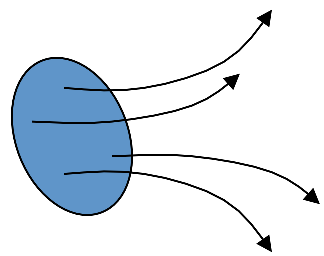


circus maximus, Rome

Electric Current

Electric current = measure of the flow of electric charge

- *direction* and *magnitude* of flow must be indicated (ie. like water flowing through a hose)



- SI unit is the Ampere (abbreviated A), where 1 A = 1 C / s
- current variables usually given the symbol *i*, or *I*

André-Marie Ampère
(1775 –1836)

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Algebraic Variables

- "al-jabr", commonly ascribed to the *Compendious Book on Calculation by Completion and Balancing*
- new idea: variables and equations are abstract mathematical entities separate from the physical world

Muhammad ibn Mūsā al-Khwārizmī
(~780-850)
latin: *Algoritmi*

الكتاب المختصر

في حساب الجبر و المقابلة

تصنيف

الشيخ الاجل ابي عبد الله محمد بن موسي

الخوارزمي

طبع في مدينة لندن
سنة ١٨٣٠ المسيحية

Algebraic Variables

- physical situations can be described by ***algebraic variables***
- the algebraic variable has both a *definition* connecting it to the real world, and a *value*
- there is often more than one set of algebraic variables that describes the exact same physical situation
- *without a definition, there is no way to interpret the meaning of an algebraic variable!*

Algebraic Variables

- Example: the following ***algebraic*** descriptions of ***identical*** water flow are ***equally valid***

S_1=+1100 m^3/s

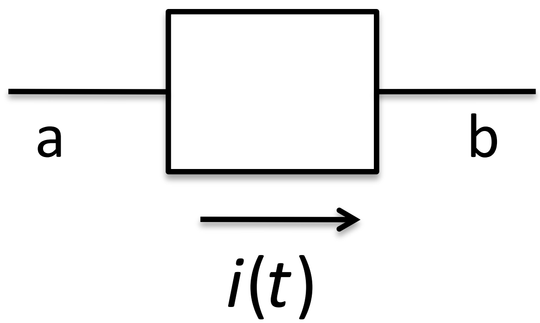
S_2=−1100 m^3/s

- in both cases, the variable is *defined* with a diagram and assigned a *value* that is consistent with the physical situation

Current and Charge Variables

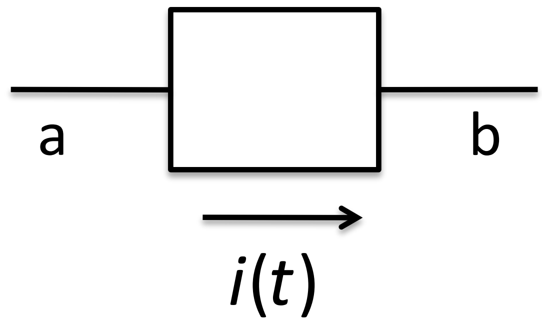
In this example, let us define the variables:

- $q(t)$ = charge that has passed through the element from node "a" to node "b"
- $i(t)$ = current flow through element from node "a" to node "b"



With the above definitions, it follows that: $$i(t) = \frac{d}{dt} q(t)$$

Current and Charge Variables



$q(t)$ = charge passed from "a" to "b"

The ***fundamental theorem of calculus*** implies:

$$i(t) = \frac{d}{dt}q(t) \quad \rightarrow \quad q(t_2) - q(t_1) = \int_{t_1}^{t_2} i(t')dt'$$

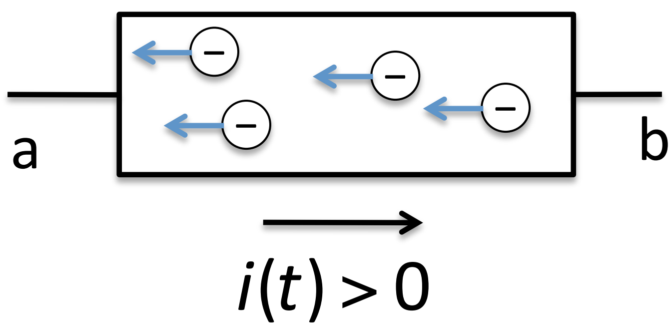
$$q(t_2) = \int_{t_1}^{t_2} i(t')dt' + q(t_1)$$

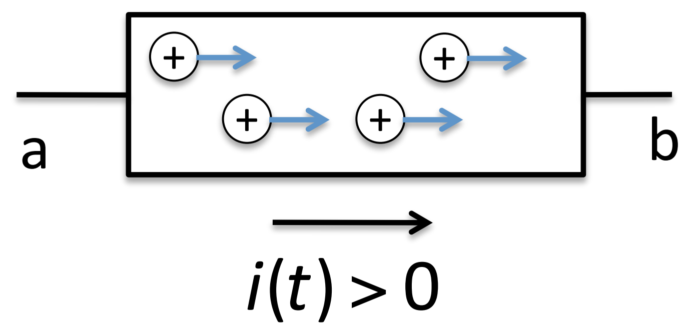
Note: the bounds of integration cannot be ignored.

Current and Charge Variables

Note: A current produced by the flow of negatively charged electrons is equivalent to a current resulting from the flow of fictitious positive charges in the opposite direction.

We *often* imagine fictitious positive charges instead of electrons because it is simpler to think of particle flow in the same direction as current flow.





Example

A 1.5V battery is rated at 1500 mA-hrs (typical rating).

1) How much charge can the battery deliver to a circuit, in SI units?

2) If the battery is being recharged, from a completely discharged state, with a constant current of 100mA, how long will it take to fully recharge the battery?

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Example

A 1.5V battery is rated at 1500 mA-hrs (typical rating).

1) How much charge can the battery deliver to a circuit, in SI units?

$$
\begin{aligned}
Q &= 1500 \text{ mA-hrs} \times (60\text{s / min}) \times (60\text{min / hr}) \\
&= 1.5 \text{ A-hrs} \times (3600\text{s / hr}) \\
&= 5400 \text{ A s} \\
&= 5400 \text{ C} \\
&= 5.4 \text{ kC}
\end{aligned}
$$

Note that 1 mole = 6.02×10^{23} of electrons corresponds to a charge of 96 kC. It will take approximately 18 batteries rated at 1500 mA-hrs to deliver 1 mole of electrons to a circuit.

Example

A 1.5V battery is rated at 1500 mA-hrs (typical rating).

2) If the battery is being recharged, from a completely discharged state, with a constant current of 100mA, how long will it take to fully recharge the battery?

$I = dQ / dt$

$= \Delta Q / \Delta t$

thus,

$\Delta t = \Delta Q / I = 5.4 \text{ kC} / 100 \text{ mA}$

$= (5.4 \times 10^3 \text{ C}) / (0.1 \text{ C/s})$

$= 54 \text{ ks}$

Note that this is equal to Δt = 1500mA-hrs / 100 mA = 15 hrs