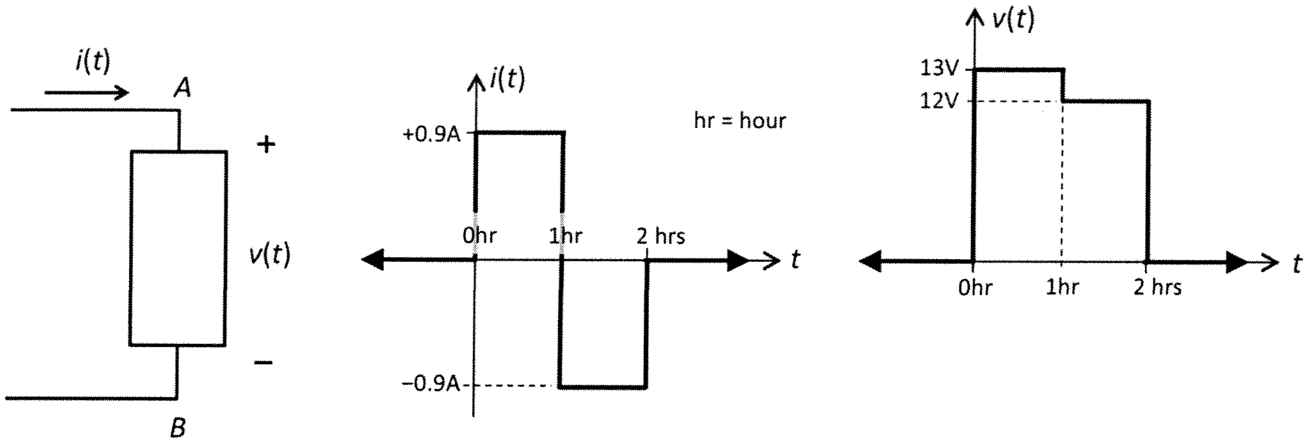
ECSE-200 Quiz #2 (21 Jan 2011)

NAME____ McGill ID#____________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram and plots. Answer the questions.



a) Plot the net charge $q(t)$ that has entered the element through terminal A versus time t. You may assume that $q(t) = 0$ for $t<0$. Label the axes of your plot with SI units. [3pts]

b) Plot the net instantaneous power $p(t)$ absorbed by the element versus time t. Label the axes of your plot with SI units. [3pts]

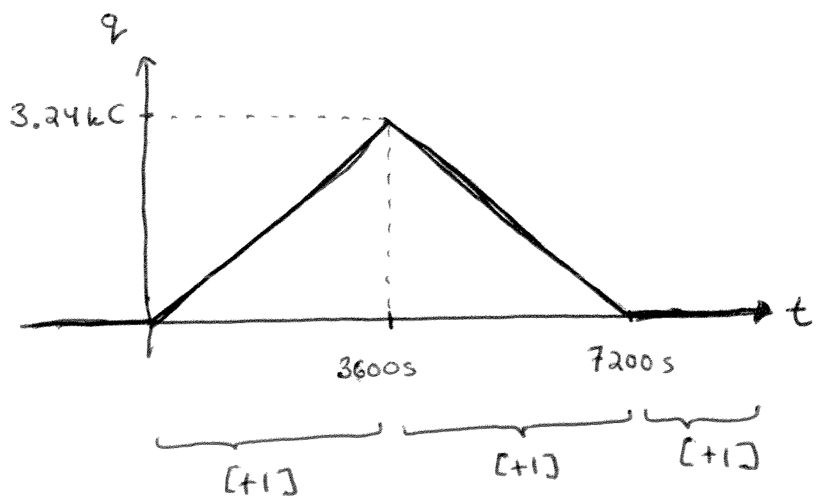
c) Plot the net energy $U(t)$ absorbed by the element versus time t. You may assume that $U(t)$=0 for t<0. Label the axes of your plot with SI units. [4pts]

a) $q(t) = q(0) + \int_0^t i(t')\,dt'$

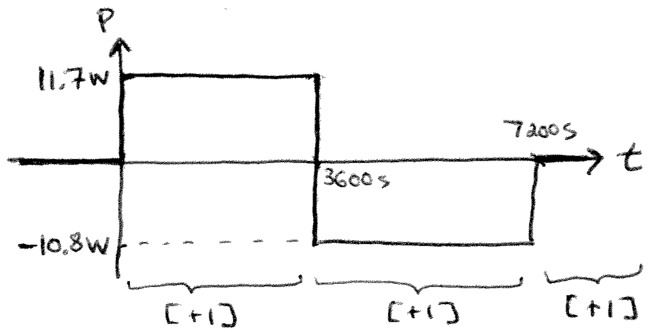
$q_{max} = 0.9A \cdot 1hr$

$= 0.9A \cdot 3600s$

$= 3.24\ kC$



b) $p(t) = i(t)\,v(t) = \begin{cases} 13V \cdot 0.9A = 11.7\,W & 0 < t < 3600s \\ 12V \cdot (-0.9A) = -10.8\,W & 3600s < t < 7200s \end{cases}$

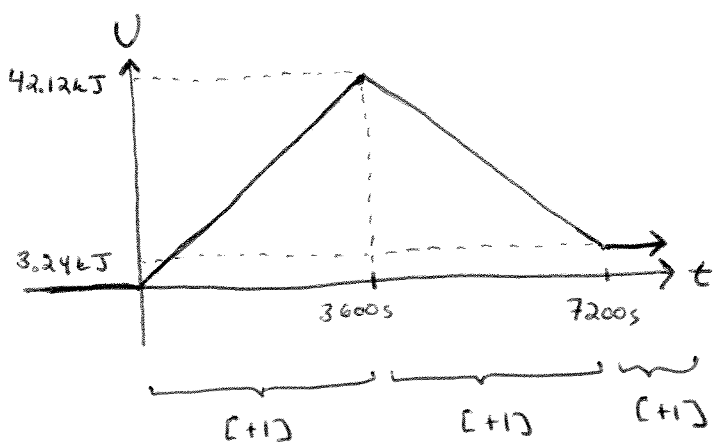


c) $U(t) = U(0) + \int_0^t p(t')\,dt'$ [+1]

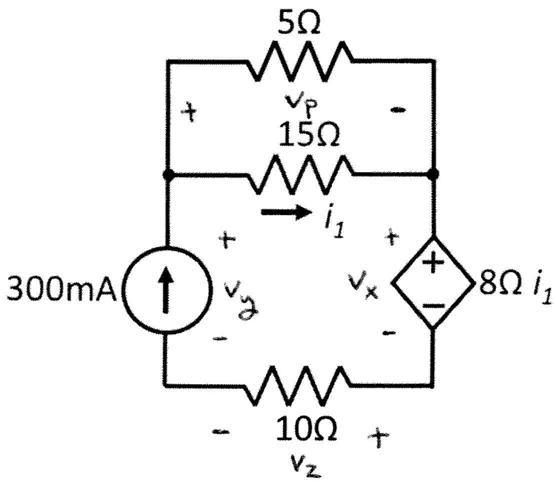
$U_{max} = 11.7W \cdot 3600s = 42.12\,kJ$

$U(2hrs) = 11.7W \cdot 3600s + (-10.8W) \cdot 3600s$

$= 3.24\,kJ$



2. Consider the circuit diagram. Answer the questions.



a) What is the power delivered or absorbed by the dependent source? [3pts]

b) What is the power delivered or absorbed by the independent source? [4pts]

c) How long must one wait for the 10Ω resistor to dissipate 1kW-hr of energy? Note that the local cost for 1kW-hr is about $0.05. [3pts]

a)

$$i_1 = 300\,mA \cdot \frac{5\Omega}{5\Omega + 15\Omega}$$

$$= 75\,mA \quad [+1]$$

$$P_{\substack{dep\\abs}} = V_x \cdot 300\,mA \quad [+1]$$

$$= (8\Omega \cdot 75\,mA) \cdot 300\,mA$$

$= 180\,mW$ absorbed by dependent source [+1]

b)

$V_p = 15\Omega \cdot i_1 = 15\Omega \cdot 75\,mA$ (Ohm)

$= 1.125V$ [+1]

$V_z = 10\Omega \cdot 300\,mA$ (Ohm)

$= 3.0\,V$ [+1]

KVL: $0 = -V_y + V_p + V_x + V_z$

$V_y = V_p + V_x + V_z$

$= 1.125\text{V} + 8\Omega \cdot 75\text{mA} + 3.0\text{V}$

$= 4.725\text{V}$ [+1]

$P_{\substack{ind \\ del}} = 300\text{mA} \cdot V_y$

$= 300\text{mA} \cdot 4.725\text{V}$

$= 1.4175$ W delivered by independent source [+1]

c) $P_{diss} = (300\text{mA})^2 \cdot 10\Omega$

$= 0.9$ W [+1]

$\frac{\Delta U_{diss}}{\Delta t} = P_{diss}$ [+1]

$\Delta t = \frac{\Delta U_{diss}}{P_{diss}} = \frac{1 \text{ kW-hr}}{0.9\text{W}} = 1111.1$ hrs

$= 4$ Ms

$= 4 \times 10^6$ s

$= 46$ days 7 hrs 6 min 40 s

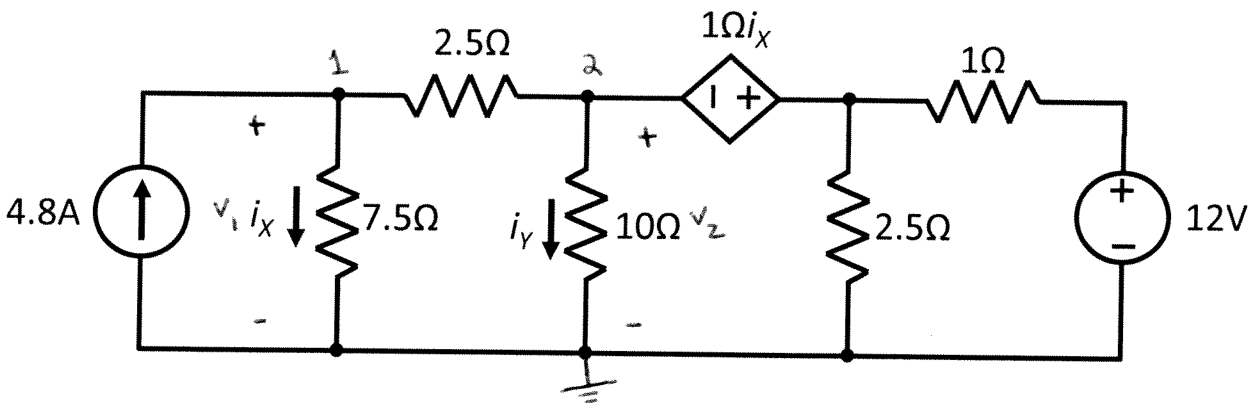
[+1] for any

ECSE-200 Quiz #3 (28 Jan 2011)

NAME____ McGill ID#____________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram. Answer the questions.



a) What is the value of the current i_X? [6pts]

b) What is the value of the current i_Y? [2pts]

c) How much power does the independent current source deliver or absorb? [2pts]

a)

$$-4.8 + \frac{v_1}{7.5} + \frac{v_1 - v_2}{2.5} = 0 \qquad [+2]$$

$$\frac{v_2 - v_1}{2.5} + \frac{v_2}{10} + \frac{v_2 + i_x}{2.5} + \frac{v_2 + i_x - 12}{1} = 0 \qquad [+2]$$

$$i_x = v_1 / 7.5 \qquad [+1]$$

$$4.8 = 0.5333\, v_1 - 0.4\, v_2$$

$$12 = -0.2133\, v_1 + 1.9\, v_2$$

work space

$$v_1 = \frac{\begin{vmatrix} 4.8 & -0.4 \\ 12 & 1.9 \end{vmatrix}}{\begin{vmatrix} 0.5333 & -0.4 \\ -0.2133 & 1.9 \end{vmatrix}} = \frac{13.92}{0.9279} = 15.00\,V$$

$$i_x = \frac{15.00V}{7.5\Omega} = 2.00\,A \quad [+1]$$

b) $$i_Y = \frac{v_2}{10\Omega} \quad [+1]$$

$$v_2 = \frac{\begin{vmatrix} 0.5333 & 4.8 \\ -0.2133 & 12 \end{vmatrix}}{\begin{vmatrix} 0.5333 & -0.4 \\ -0.2133 & 1.9 \end{vmatrix}} = \frac{7.423}{0.9279} = 8.00\,V$$

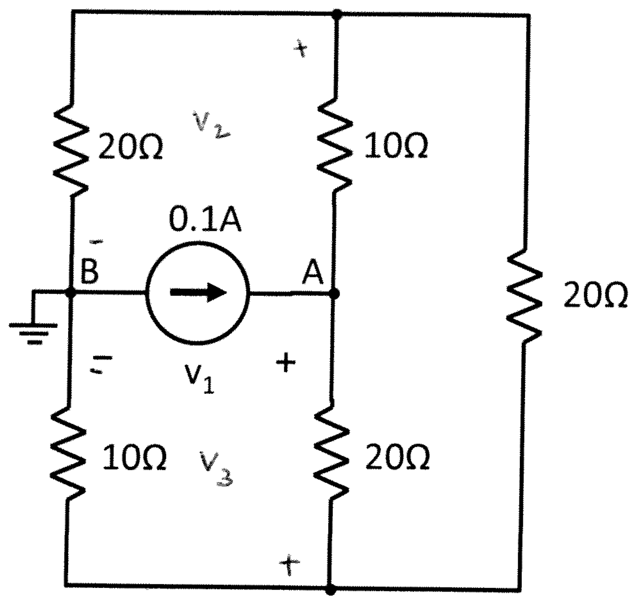
$$i_Y = \frac{8.00V}{10\Omega} = 0.8\,A \quad [+1]$$

c) $$P_{del} = 4.8A \cdot v_1 \quad [+1]$$

$$= 4.8A \cdot 15.00V$$

$= 72$ W delivered by independent current source [+1]

2. Consider the circuit diagram. Answer the questions.



a) What is the voltage v_1? [5pts]

b) Show by explicit calculation of currents that KCL is satisfied at node B. [2pts]

c) The current source is removed from the circuit, leaving a network of resistors alone. What is the equivalent resistance between nodes A and B? [2pts]

HINT: Consider your work in part a).

a)

$$0 = -0.1 + \frac{v_1 - v_2}{10} + \frac{v_1 - v_3}{20} \quad [+1]$$

$$0 = \frac{v_2}{20} + \frac{v_2 - v_1}{10} + \frac{v_2 - v_3}{20} \quad [+1]$$

$$0 = \frac{v_3}{10} + \frac{v_3 - v_1}{20} + \frac{v_3 - v_2}{20} \quad [+1]$$

$$0.1 = 0.15\,v_1 - 0.1\,v_2 - 0.05\,v_3$$

$$0 = -0.1\,v_1 + 0.2\,v_2 - 0.05\,v_3$$

$$0 = -0.05\,v_1 - 0.05\,v_2 + 0.2\,v_3$$

work space

$$V_1 = \frac{\begin{vmatrix} 0.1 & -0.1 & -0.05 \\ 0 & 0.2 & -0.05 \\ 0 & -0.05 & 0.2 \end{vmatrix}}{\begin{vmatrix} 0.15 & -0.1 & -0.05 \\ -0.1 & 0.2 & -0.05 \\ -0.05 & -0.05 & 0.2 \end{vmatrix}} = \frac{3.75\times10^{-3}}{2.625\times10^{-3}} = 1.429\text{ V} \quad [+2]$$

b)

$$V_2 = \frac{\begin{vmatrix} 0.15 & 0.1 & -0.05 \\ -0.1 & 0 & -0.05 \\ -0.05 & 0 & 0.2 \end{vmatrix}}{\begin{vmatrix} 0.15 & -0.1 & -0.05 \\ -0.1 & 0.2 & -0.05 \\ -0.05 & -0.05 & 0.2 \end{vmatrix}} = \frac{2.25\times10^{-3}}{2.625\times10^{-3}} = 0.857\text{ V}$$

$$V_3 = \frac{\begin{vmatrix} 0.15 & -0.1 & 0.1 \\ -0.1 & 0.2 & 0 \\ -0.05 & -0.05 & 0 \end{vmatrix}}{\begin{vmatrix} 0.15 & -0.1 & -0.05 \\ -0.1 & 0.2 & -0.05 \\ -0.05 & -0.05 & 0.2 \end{vmatrix}} = \frac{1.5\times10^{-3}}{2.625\times10^{-3}} = 0.571\text{ V}$$

KCL: at B

$$0 = 0.1\text{A} - \frac{V_2}{20\Omega} - \frac{V_3}{10\Omega} \quad [+1]$$

$$= 0.1\text{A} - \frac{0.857\text{V}}{20\Omega} - \frac{0.571\text{V}}{10\Omega}$$

$$= 0.1\text{A} - 0.04285\text{A} - 0.0571\text{A} \quad [+1]$$

$$= 0.000\text{ A}$$

c) Definition of equivalent resistance gives:

$$R_{eq} = \frac{V_1}{0.1\text{A}} \quad [+1]$$

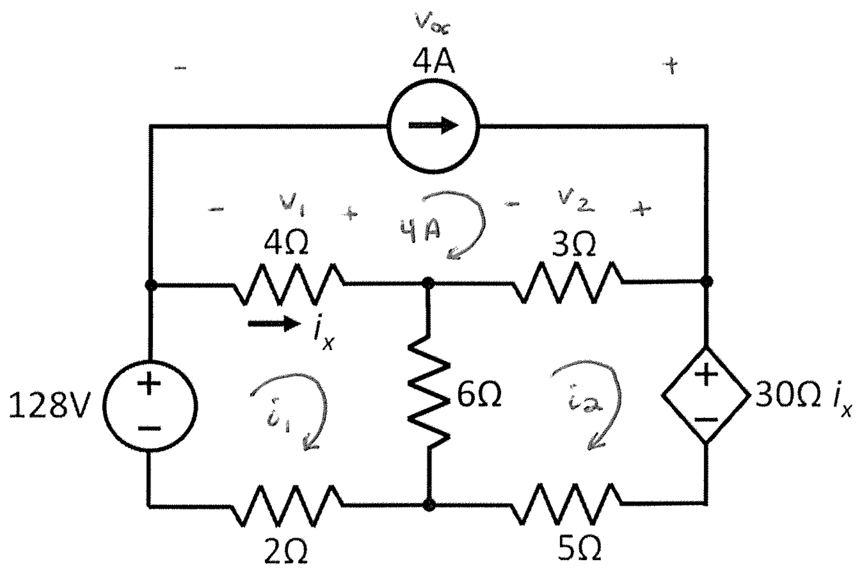
$$= \frac{1.429\text{V}}{0.1\text{A}} = 14.29\ \Omega \quad [+1]$$

ECSE-200 Quiz #4 (4 Feb 2011)

NAME____ McGill ID#____________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram. Answer the questions.



a) Which technique, node-voltage analysis or mesh-current analysis, gives a system of equations with fewer variables? [1pt]

b) What is the value of the current i_X? [5pts]

c) How much power does the 4A current source deliver or absorb? [~~2pts~~] 3pts

d) How much power does the 6Ω resistor absorb? [~~3pts~~] 2pts

a) node voltage → 3 variables

mesh current → 2 variables

∴ mesh current [+1]

b) $0 = -128 + 4(i_1 - 4) + 6(i_1 - i_2) + 2i_1$ [+1]

$0 = 30 i_x + 5 i_2 + 6(i_2 - i_1) + 3(i_2 - 4)$ [+1]

$i_x = i_1 - 4\,A$ [+1]

$$144 = 12i_1 - 6i_2$$
$$132 = 24i_1 + 14i_2$$

$$i_1 = \frac{\begin{vmatrix} 144 & -6 \\ 132 & 14 \end{vmatrix}}{\begin{vmatrix} 12 & -6 \\ 24 & 14 \end{vmatrix}} = 9A \quad [+1]$$

$$i_x = i_1 - 4A$$
$$= 5A \quad [+1]$$

c)

$$v_\alpha = v_1 + v_2$$
$$= 4\Omega(4A - i_1) + 3\Omega(4A - i_2) \quad [+1]$$

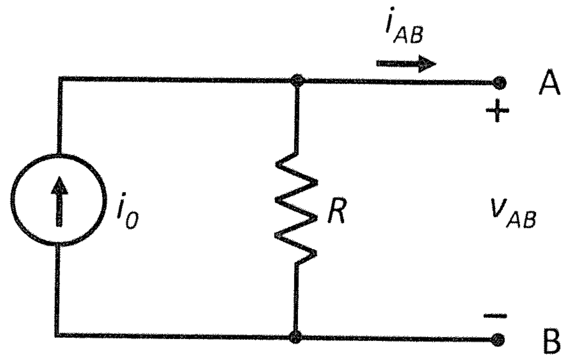
$$i_2 = \frac{\begin{vmatrix} 12 & 144 \\ 24 & 132 \end{vmatrix}}{\begin{vmatrix} 12 & -6 \\ 24 & 14 \end{vmatrix}} = -6A \quad [+1]$$

$$P_{del} = 4A \cdot v_\alpha$$
$$= 4A\left(4\Omega(4A - 9A) + 3\Omega(4A - (-6A))\right)$$
$$= 40W \quad [+1]$$ delivered by source

d)

$$P_{abs} = 6\Omega \cdot (i_1 - i_2)^2 \quad [+1]$$
$$= 6\Omega\,(9A - (-6A))^2$$
$$= 1.35\,kW \quad [+1]$$

2. Consider the circuit diagram and plot below. If you connect a 200kΩ resistor across the terminals A and B, you find that v_{AB}=1.24V. If you connect a 12.5kΩ resistor across the terminals A and B, you find that v_{AB}=0.31V. Answer the questions.

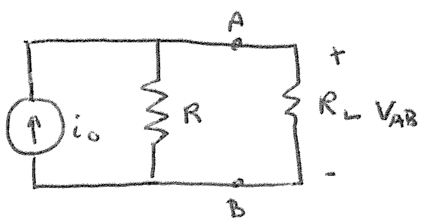


a) Determine the value of i_0 and R. [4pts]

b) If a resistor with conductance G_L = 0 S is connected across the terminals A and B, what is the resulting voltage v_{AB}? [2pts]

c) What resistance should you connect across the terminals A and B in order to give v_{AB} = 1.00V? [2pts]

a)



$1.24V = i_o \cdot R /\!/ 200k\Omega$ [+1]

$0.31V = i_o \cdot R /\!/ 12.5k\Omega$ [+1]

$$1.24V = i_o \frac{R \cdot 200k\Omega}{R + 200k\Omega} \qquad 0.31V = i_o \frac{R \cdot 12.5k\Omega}{R + 12.5k\Omega}$$

$$i_o = 1.24V\left(\frac{1}{R} + \frac{1}{200k\Omega}\right) \qquad i_o = 0.31V\left(\frac{1}{R} + \frac{1}{12.5k\Omega}\right)$$

$$\therefore \; 1.24V\left(\frac{1}{R} + \frac{1}{200k\Omega}\right) = 0.31V\left(\frac{1}{R} + \frac{1}{12.5k\Omega}\right)$$

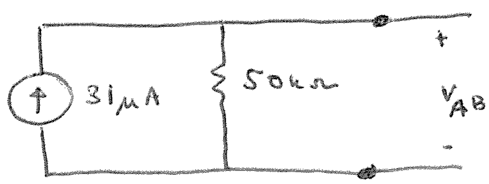
$$\frac{1}{R} = \frac{\frac{0.31}{1.24}\left(\frac{1}{12.5k\Omega}\right) - \frac{1}{200k\Omega}}{\left(1 - \frac{0.31}{1.24}\right)} = 20\,\mu S$$

$R = 50\,k\Omega$ [+1]

$i_o = 1.24V \left(\frac{1}{50k\Omega} + \frac{1}{200k\Omega} \right)$

$= 31\,\mu A$ [+1]

b)



$G_L = 0S$ is equivalent to an open,

$R_L = 1/G_L \to \infty$

$V_{AB} = i_o R$ [+1]

$= 31\mu A \cdot 50k\Omega$

$= 1.55V$ [+1]

c)

$1.00V = i_o \cdot R // R_L$ [+1]

$R // R_L = \frac{1.00V}{31\mu A} = 32.26\,k\Omega$

$\frac{1}{50k\Omega} + \frac{1}{R_L} = \frac{1}{32.26k\Omega}$

$R_L = \frac{1}{\frac{1}{32.26k\Omega} - \frac{1}{50k\Omega}}$

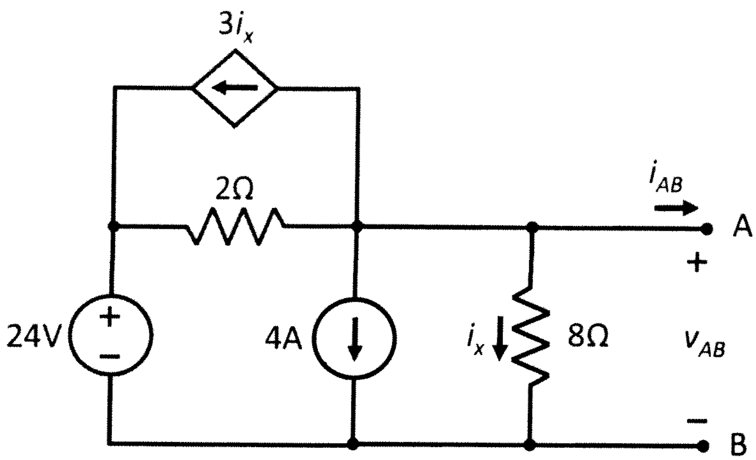
$R_L = 90.91\,k\Omega$ [+1]

ECSE-200 Quiz #5 (11 Feb 2011)

NAME_______________________________ McGill ID#_______________

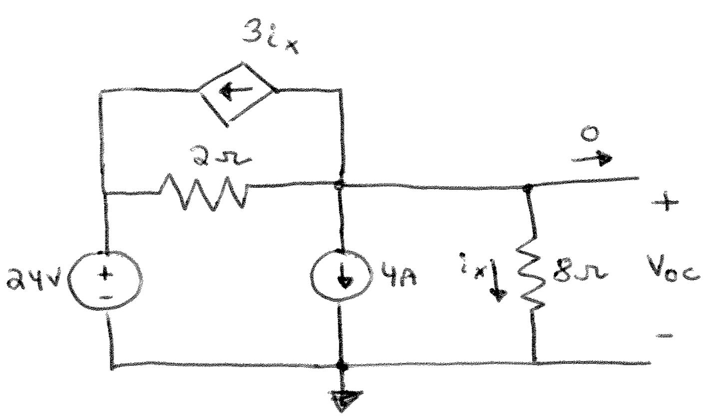
READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram. Answer the questions.



a) What is the Thévenin equivalent of the above circuit with respect to the terminals A and B? [8pts]

b) What is the current i_{AB} if a 3Ω resistor is connected across the terminals A and B? [2pts]



Use node-voltage method:

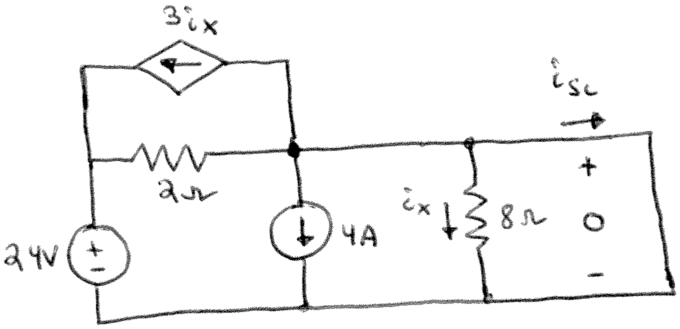
$$0 = 3i_x + \frac{V_{oc} - 24V}{2\Omega} + 4A + \frac{V_{oc}}{8\Omega} \quad [+1]$$

$$i_x = \frac{V_{oc}}{8\Omega} \quad [+1]$$

$$0 = \frac{3V_{oc}}{8\Omega} + \frac{V_{oc} - 24V}{2\Omega} + 4A + \frac{V_{oc}}{8\Omega}$$

$$V_{oc} = \frac{12A - 4A}{\frac{3}{8\Omega} + \frac{1}{2\Omega} + \frac{1}{8\Omega}} = 8V$$

work space



Ohm:

$$i_x = \frac{0V}{8\Omega} = 0A \quad [+1]$$

KCL:

$$\frac{0-24V}{2\Omega} + 4A + i_{sc} = 0 \quad [+1]$$

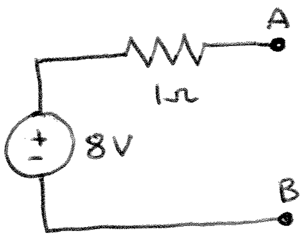
$$i_{sc} = 12A - 4A = 8A \quad [+1]$$

$$R_T = \frac{v_{oc}}{i_{sc}} \quad [+1]$$

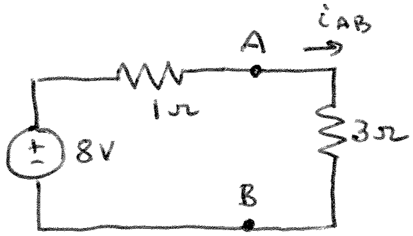
$$= \frac{8V}{8A} = 1\Omega$$

[+1]

[+1]



b)

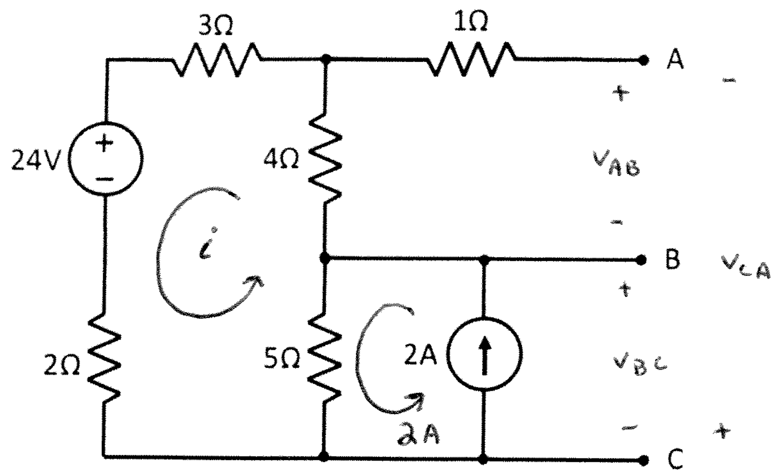


$$i_{AB} = \frac{v_{oc}}{R_T + R} \quad [+1]$$

$$= \frac{8V}{1\Omega + 3\Omega}$$

$$= 2A \quad [+1]$$

2. Consider the circuit below. Answer the questions.



a) What is the Thévenin equivalent of the circuit with respect to the terminals A and B. [4pts]

b) What is the Thévenin equivalent of the circuit with respect to the terminals B and C. [3pts]

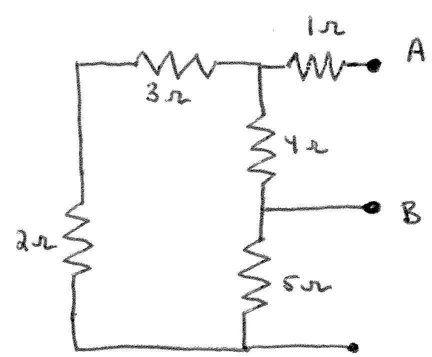
c) What is the Thévenin equivalent of the circuit with respect to the terminals C and A. [3pts]

a) Using mesh current with all terminals open:

$$0 = 4\Omega \cdot i + 3\Omega \cdot i + 24V + 2\Omega \cdot i + 5\Omega \cdot (i - 2A) \quad [+1]$$

$$i = \frac{-24V + 10V}{4\Omega + 3\Omega + 2\Omega + 5\Omega} = -1A$$

$$V_{AB} = -i \cdot 4\Omega + 0A \cdot 1\Omega = 4V \quad [+1]$$

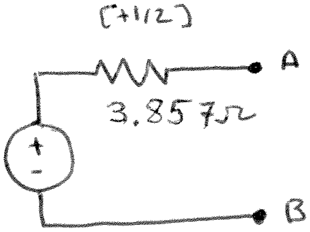


$$R_T = 1\Omega + 4\Omega \,//\, (3\Omega + 2\Omega + 5\Omega) \quad [+1]$$

$$= 3.857\Omega$$

[+1/2]

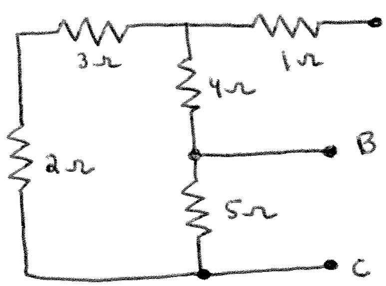
∴



[+1/2] 4V

b) From analysis with open terminals:

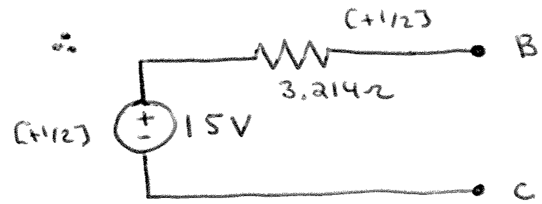
$$V_{BC} = 5\Omega\,(2A - i) = 15V \quad [+1]$$



$$R_T = 5\Omega \,//\, (4\Omega + 3\Omega + 2\Omega) \quad [+1]$$

$$= 3.214\Omega$$

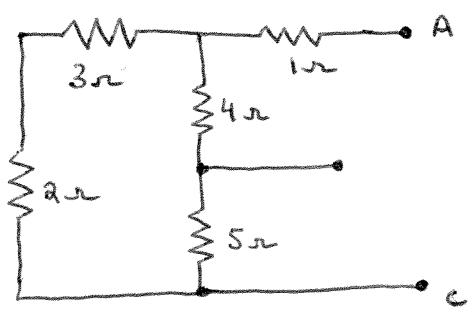
∴



c) From analysis with open terminals:

$$V_{CA} = -V_{AB} - V_{BC}$$

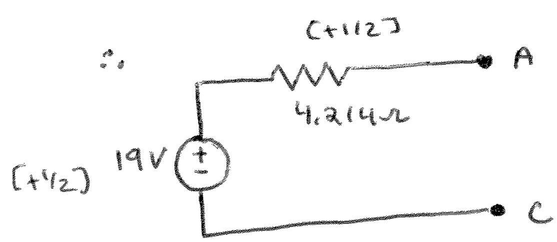
$$= -19V \quad [+1]$$



$$R_T = 1\Omega + (3\Omega + 2\Omega) // (4\Omega + 5\Omega) \quad [+1]$$

$$= 4.214\Omega$$

∴



or

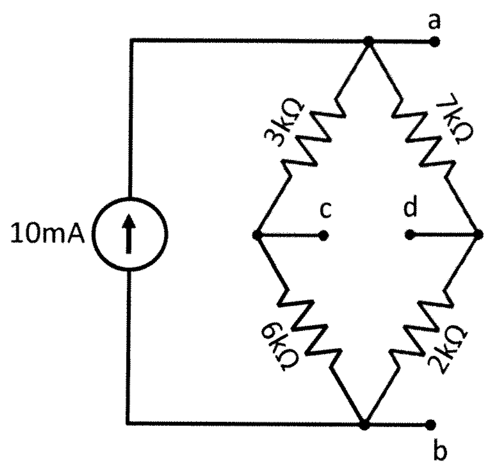
Take note of polarity of source and terminal labels.

ECSE-200 Quiz #6 (18 Feb 2011)

NAME______________________ McGill ID#______________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram. Answer the questions.

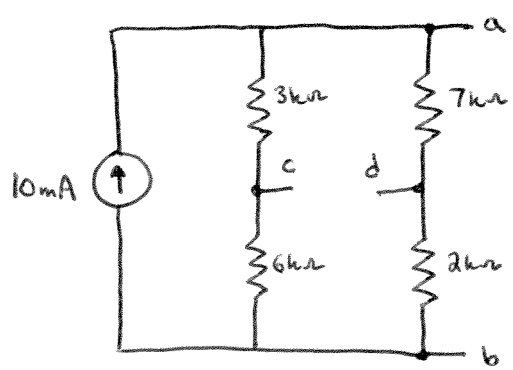


a) What is the maximum power that the network above can deliver to a load resistor connected to the terminals a and b? [3pts]

b) What is the maximum power that the network above can deliver to a load resistor connected to the terminals c and d? [4pts]

c) What is the maximum power that the network above can deliver to a load resistor connected to the terminals c and b? [3pts]

a)



$V_{oc,ab} = 10mA \cdot (3k\Omega + 6k\Omega) // (7k\Omega + 2k\Omega)$ [+½]

$= 45V$

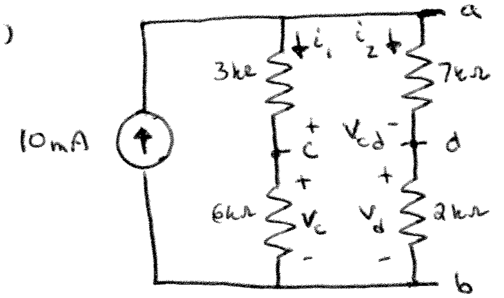
$i_{sc,ab} = 10mA$ [+½]

$P_{max,ab} = \frac{V_{oc,ab}}{2} \cdot \frac{i_{sc,ab}}{2}$ [+1]

$= \frac{45V}{2} \cdot \frac{10mA}{2}$

$= 112.5mW$ [+1]

b)



$$i_1 = 10\,mA \cdot \frac{9\,k\Omega}{18\,k\Omega} = 5\,mA$$

$$i_2 = 10\,mA \cdot \frac{9\,k\Omega}{18\,k\Omega} = 5\,mA$$

[+1]

$$V_c = 6\,k\Omega \cdot i_1$$
$$= 6\,k\Omega \cdot 5\,mA = 30\,V$$

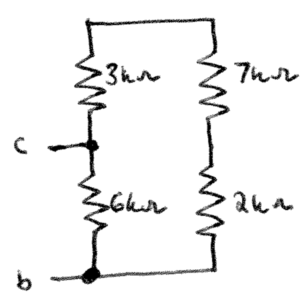
$$V_d = 2\,k\Omega \cdot i_2$$
$$= 2\,k\Omega \cdot 5\,mA = 10\,V$$

$$V_{cd} = V_c - V_d = 20\,V \quad [+1]$$

$$R_T = (6\,k\Omega + 2\,k\Omega) \,//\, (3\,k\Omega + 7\,k\Omega) \quad [+1]$$
$$= 4.235\,k\Omega$$

$$P_{max,cd} = \frac{V_{oc,cd}}{2} \cdot \frac{i_{sc,cd}}{2}$$
$$= \frac{V_{oc,cd}^2}{4\,R_{T,cd}}$$
$$= 23.61\,mW \quad [+1]$$

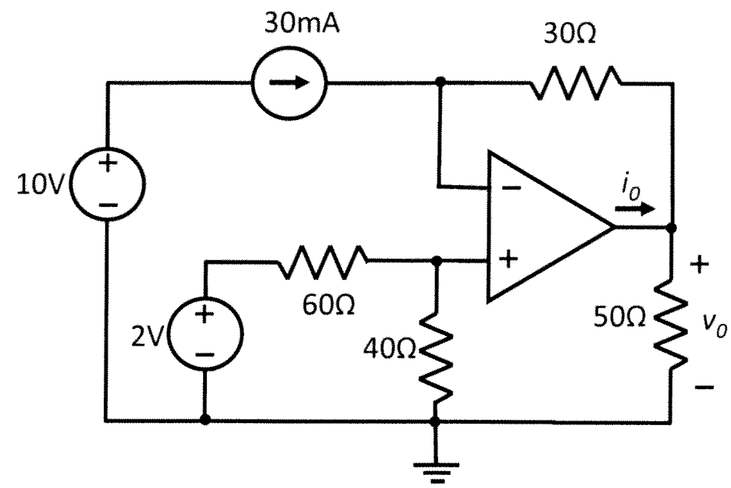
c) $V_{oc,cb} = V_c = 30\,V$ [+1]



$$R_{T,cb} = 6\,k\Omega \,//\, (3\,k\Omega + 7\,k\Omega + 2\,k\Omega) \quad [+1]$$
$$= 4\,k\Omega$$

$$P_{max,cb} = \frac{V_{oc,cb}}{2} \cdot \frac{i_{sc,cb}}{2}$$
$$= \frac{V_{oc,cb}^2}{4\,R_{T,cb}}$$
$$= 56.25\,mW \quad [+1]$$

2. Consider the circuit below. Assume ideal op-amp behaviour. Answer the questions.

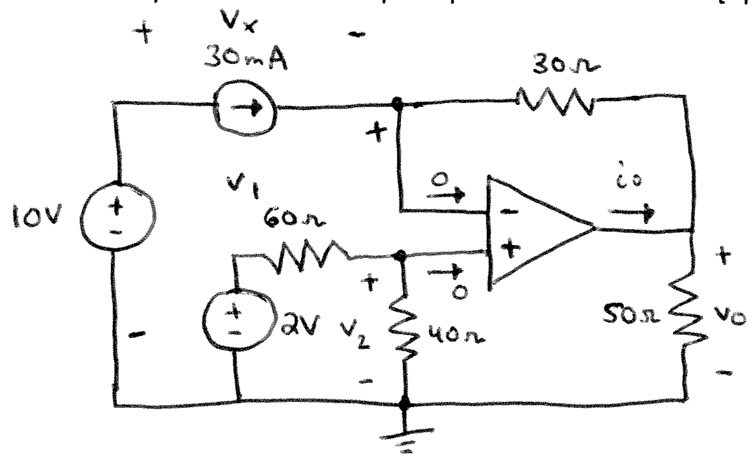


a) What is the voltage v_O? [4pts]

b) How much power does the 30mA current source deliver or absorb? [3pts]

c) How much power does the op-amp deliver or absorb? [3pts]

a)



ideal op-amp:

input currents = 0

virtual short → $V_2 = V_1$ [+1]

$$V_2 = \frac{40\Omega}{40\Omega + 60\Omega} \cdot 2V \quad [+1]$$

$$= 0.8V$$

node voltage equation at inverting input: $0 = -30mA + \frac{0.8V - V_o}{30\Omega}$ [+1]

$$V_o = 0.8V - 30mA \cdot 30\Omega$$

$$= -0.1\ V \quad [+1]$$

b) KVL: $0 = -10V + v_x + 0.8V$ [+1]

$v_x = 9.2V$

$P_{abs} = 30mA \cdot v_x$ [+1]

$= 276\,mW$ is absorbed by the current source. [+1]

c) KCL: $0 = -i_o + \frac{V_o}{50\Omega} + \frac{V_o - V_1}{30\Omega}$ [+1]

$= -i_o + \frac{(-0.1V)}{50\Omega} + \frac{(-0.1V - 0.8V)}{30\Omega}$

$i_o = -32mA$

$P_{del} = i_o \cdot V_o$ [+1]

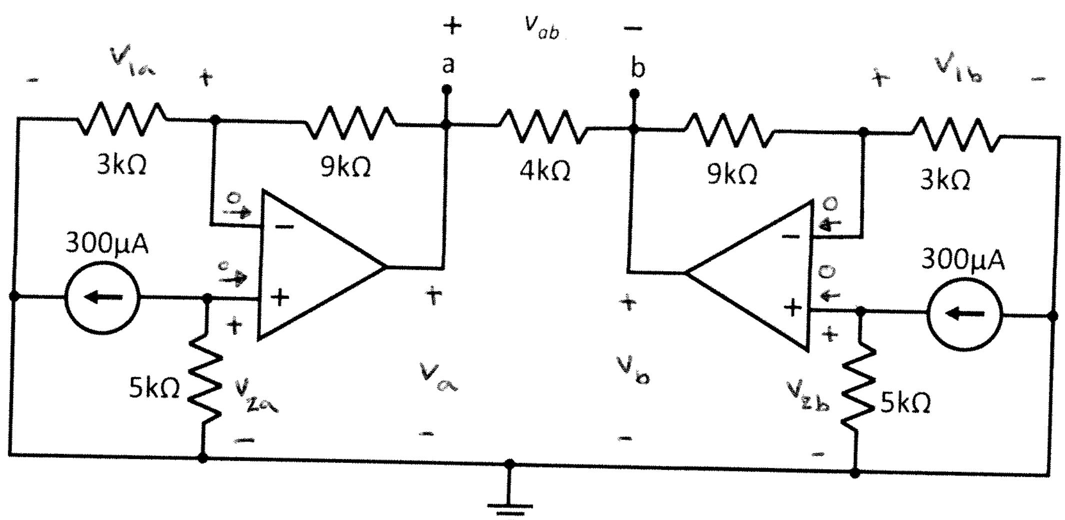
$= (-32mA) \cdot (-0.1V)$

$= 3.2\,mW$ is delivered by the op-amp. [+1]

NAME______________________________ McGill ID#________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram. Assume ideal op-amp behaviour. Answer the questions.



a) What is the voltage v_{ab}? [6pts]

b) How much power does the 4kΩ resistor absorb? [2pts]

c) Would your answer to part a) change if a 5kΩ resistor was connected between the non-inverting input terminals of the two op-amps? [2pts]

a)

$V_{2a} = -300\mu A \cdot 5k\Omega = -1.5V$ [+1]

$V_{1a} = V_{2a}$ [+1/2]

$$0 = \frac{-1.5V}{3k\Omega} + \frac{(-1.5V - V_a)}{9k\Omega} \quad [+1]$$

$$V_a = \frac{9k\Omega}{3k\Omega}(-1.5V) + (-1.5V) = -6V$$

$V_{2b} = +300\mu A \cdot 5k\Omega = +1.5V$ [+1]

$V_{1b} = V_{2b}$ [+1/2]

$$0 = \frac{1.5V}{3k\Omega} + \frac{(1.5V - V_b)}{9k\Omega} \quad [+1]$$

$$V_b = \frac{9k\Omega}{3k\Omega}(+1.5V) + (1.5V) = +6V$$

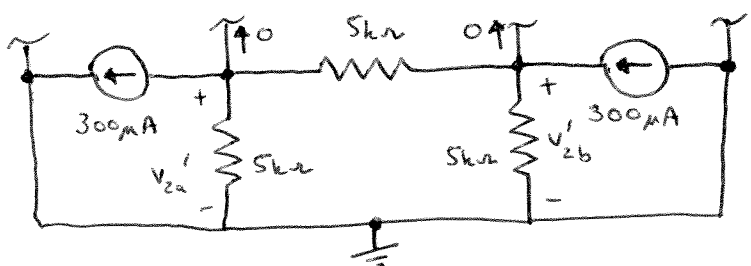
$$V_{ab} = V_a - V_b$$
$$= -12V \quad [+1]$$

b)

$$P_{abs} = \frac{V_{ab}^2}{4k\Omega} = \frac{(-12V)^2}{4k\Omega} = 36\,mW$$

[+1] [+1]

c)



$$0 = 300\mu A + \frac{V_{2a}'}{5k\Omega} + \frac{V_{2a}' - V_{2b}'}{5k\Omega}$$

$$0 = -300\mu A + \frac{V_{2b}'}{5k\Omega} + \frac{V_{2b}' - V_{2a}'}{5k\Omega}$$

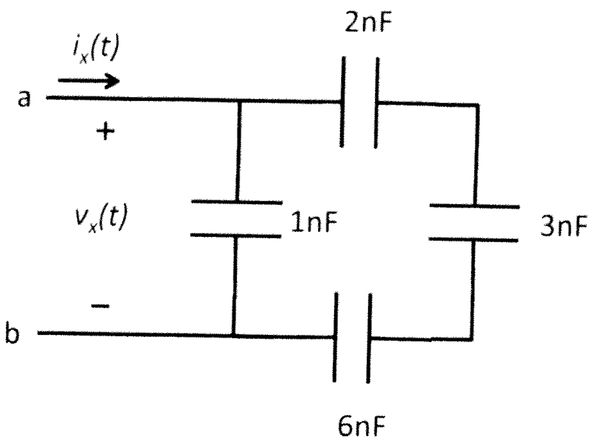
$\therefore$ $V_{2a}' \neq V_{2a} = -1.5V$ [+1]

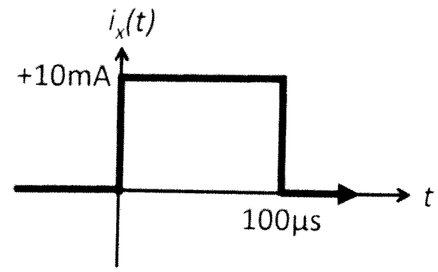
$V_{2b}' \neq V_{2b} = +1.5V$

$\therefore$ V_a and V_b will be different, as will V_{ab}.

The answer to part a) will be different. [+1]

2. Consider the circuit and plot below. There is zero charge separation on the capacitors for $t<0$. Answer the questions.





a) What is the equivalent capacitance between nodes a and b? [2 pts]

b) Plot the voltage $v_x(t)$ versus t. [3pts]

c) What is the energy stored on the capacitors at time t = 200μs? [2pts]

d) What is the maximum in instantaneous power absorbed by the capacitors? [3pts]

a) $$C_{eq} = 1\,nF + \frac{1}{\frac{1}{2nF} + \frac{1}{3nF} + \frac{1}{6nF}} \quad [+1]$$

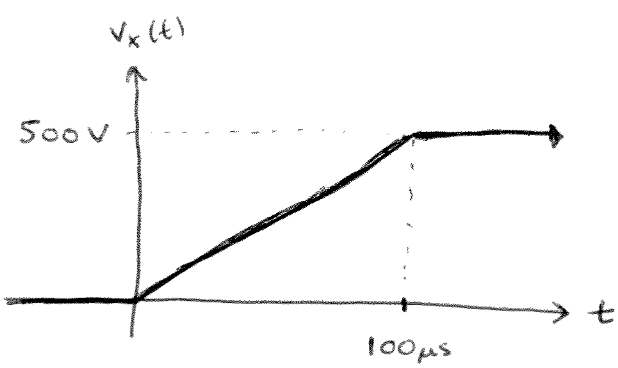
$$= 2nF \quad [+1]$$

b) $$i_x(t) = C_{eq}\frac{dv_x(t)}{dt} \quad \therefore \quad v_x(t) = v_x(0) + \frac{1}{C_{eq}}\int_0^t i_x(t')\,dt'$$

$v_x(0) = 0$ since $q_x(0) = 0$

$$v_x(100\mu s) = \frac{1}{2nF} \cdot 10mA \cdot 100\mu s$$

$$= 500\,V$$



[+1] [+1] [+1]

work space

c) $U = \frac{1}{2} C_{eq} v_x^2$ [+1]

$= \frac{1}{2} \cdot 2nF \cdot (500V)^2$

$= 250 \mu J$ [+1]

d) Maximum power absorption occurs at time of maximum $i_x \cdot v_x$, at the instant before $t = 100\mu s$. [+2]

$P_{\substack{max \\ abs}} = i_x(100\mu s - \Delta t) \cdot v_x(100\mu s - \Delta t)$ as $\Delta t \to 0$

$= 10mA \cdot 500V$

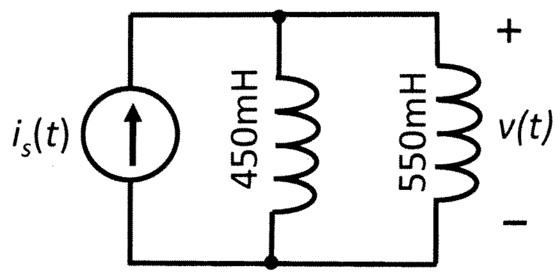
$= 5W$ [+1]

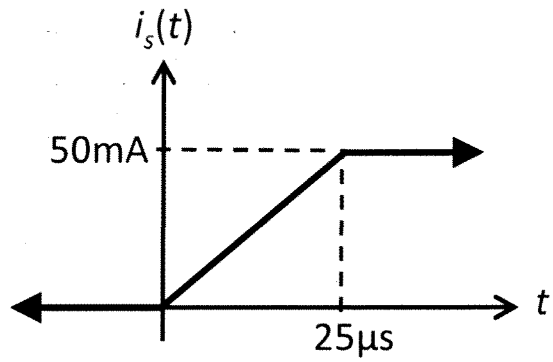
ECSE-200 Quiz #8(11 Mar 2011)

NAME____ McGill ID#____________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram and the plot. Assume there is zero stored flux linkage in the inductors for $t<0$. Answer the questions.





a) Plot the voltage *v(t)*. [5pts]

b) How much energy does the current source deliver to the inductors? [2pts]

c) What is the current in the 550mH inductor at t = 50μs? [3pts]

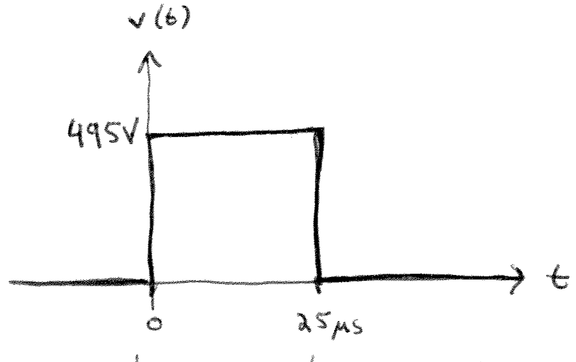
HINT: Consider your answer to part a).

a) $L_{eq} = \left(\frac{1}{450mH} + \frac{1}{550mH} \right)^{-1} = 247.5\ mH$ [+1]

$v = L_{eq} \frac{di_s}{dt}$ [+1]

$0 < t < 25\mu s$

$V = 247.5mH \cdot \frac{50mA}{25\mu s} = 495V$



[+1] [+1] [+1]

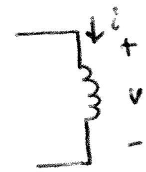
work space

b) $U = \frac{1}{2} L_{eq} i_s^2$ [+1]

$= \frac{1}{2} \cdot 247.5\,\text{mH} \cdot (50\,\text{mA})^2$

$= 309.4\ \mu\text{J}$ [+1]

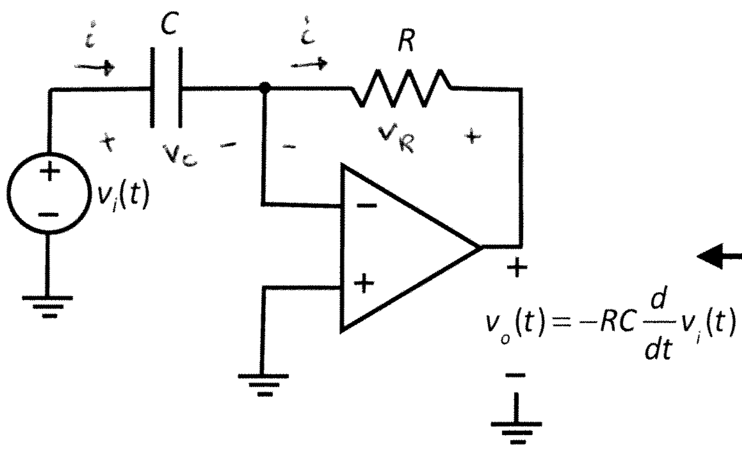
c) $v = L \frac{di}{dt} \rightarrow i(t) - i(0) = \frac{1}{L} \int_0^t v(t')\,dt'$ [+1]

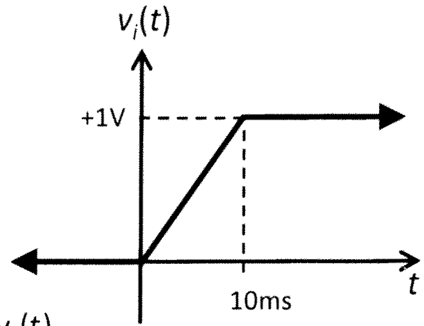


$i(50\mu s) - 0\,\text{mA} = \frac{1}{550\,\text{mH}} \cdot 495\,\text{V} \cdot 25\,\mu s$ [+1]

$i(50\mu s) = 22.5\ \text{mA}$ [+1]

2. Consider the circuit and the plot below. Assume that there is zero charge separation on the capacitor for $t<0$. Assume ideal op-amp behavior. Answer the questions.





a) Find the value of C such that a current of 7mA flows through the independent voltage source $v_i(t)$ during the time $0 < t < 10\text{ms}$? [3pts]

For the remainder of this question, assume C is equal to the value that you found in a).

b) How much energy does the capacitor C store at t = 20ms? [2pts]

c) Find the value of R such that $v_o(t) = -3.5\text{V}$ during the time $0 < t < 10\text{ms}$? [2pts]

For the remainder of this question, assume R is equal to the value that you found in c).

d) What energy does the resistor absorb over the time interval $0 < t < 10\text{ms}$? [3pts]

a) $i = C \frac{dv_i}{dt}$ [+1]

$C = \frac{i}{dv_i/dt} = \frac{7\text{mA}}{1\text{V}/10\text{ms}}$ [+1]

$= 70\,\mu\text{F}$ [+1]

b) $U = \frac{1}{2} C v_i^2$ [+1]

$= \frac{1}{2} \cdot 70\,\mu\text{F} \cdot 1\text{V}$

$= 35\,\mu\text{J}$ [+1]

work space

c) $v_o(t) = -v_R(t) = -i \cdot R$ [+1]

$$R = \frac{-v_o}{i}$$

$$= \frac{-(-3.5V)}{7mA}$$

$$= 500\Omega$$ [+1]

d) $P = i^2 R$ [+1]

$$= (7mA)^2 \cdot 500\Omega$$

$$= 24.5\ mW$$

$U = \int P\,dt' = P \cdot \Delta t$ [+1]

$$= 24.5mW \cdot 10ms$$

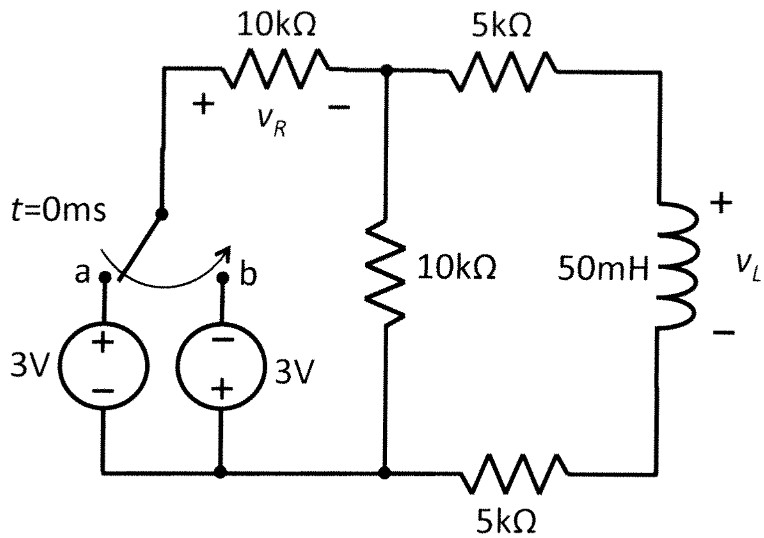
$$= 245\ \mu J$$ [+1]

ECSE-200 Quiz #9(18 Mar 2011)

NAME____ McGill ID#____________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

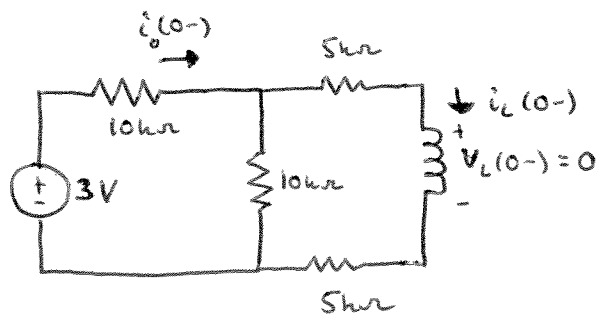
1. Consider the circuit diagram. Assume dc steady state for $t < 0$. Answer the questions.



a) Find the voltage $v_L(t)$ for $t > 0$. Plot $v_L(t)$ versus t, including the dc steady state value for $t < 0$. [5pts]

b) Find the voltage $v_R(t)$ for $t > 0$. Plot $v_R(t)$ versus t, including the dc steady state value for $t < 0$. [5pts]

$t < 0$



$$i_o(0-) = \frac{3V}{10k\Omega + 10k\Omega // (5k\Omega + 5k\Omega)}$$

$$= 200\mu A$$

$$i_L(0-) = \frac{10k\Omega}{10k\Omega + 10k\Omega} \cdot 200\mu A$$

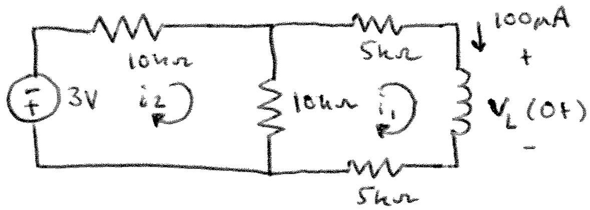
$$= 100\mu A$$

inductor current is continuous, thus:

$$i_L(0+) = i_L(0-) = 100\mu A \qquad [+1]$$

work space

$t = 0^+$



use mesh currents: $i_1 = 100\mu A$

$$0 = 3V + i_2 \cdot 10k\Omega + (i_2 - 100\mu A)\cdot 10k\Omega$$

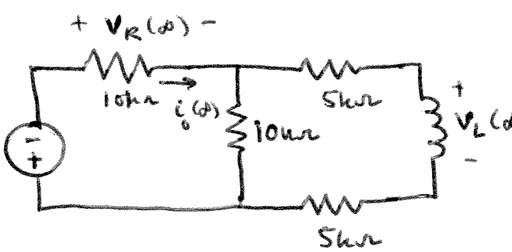
$$i_2 = \frac{-3V + 100\mu A \cdot 10k\Omega}{20k\Omega}$$

$$= -100\mu A$$

$V_R(0+) = 10k\Omega \cdot i_2 = -1V$ [+1]

$V_L(0+) = -5k\Omega \cdot i_1 - 10k\Omega(i_1 - i_2) - 5k\Omega \cdot i_1 = -3V$ [+1]

$t \to \infty$



$V_L(\infty) = 0$ [+1]

$$i_o(\infty) = \frac{-3V}{10k\Omega + 10k\Omega // (5k\Omega + 5k\Omega)}$$

$$= -200\mu A$$

$V_R(\infty) = i_o(\infty) \cdot 10k\Omega$

$= -2V$ [+1]

$R_{TH} = 5k\Omega + 10k\Omega // 10k\Omega + 5k\Omega$ [+1]

$= 15k\Omega$

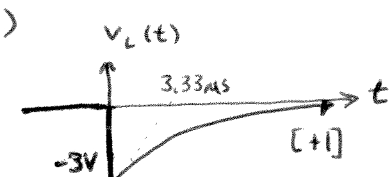
$\tau = \frac{L}{R_{TH}}$

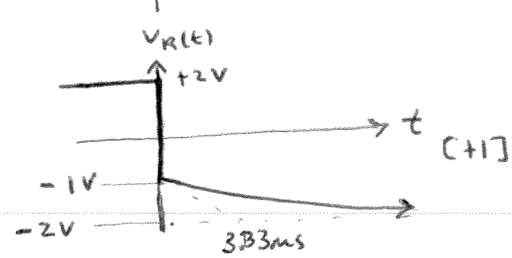
$= 3.33ms$ [+1]

$x(t) = x(\infty) + [x(0+) - x(\infty)]\exp(-t/\tau)$

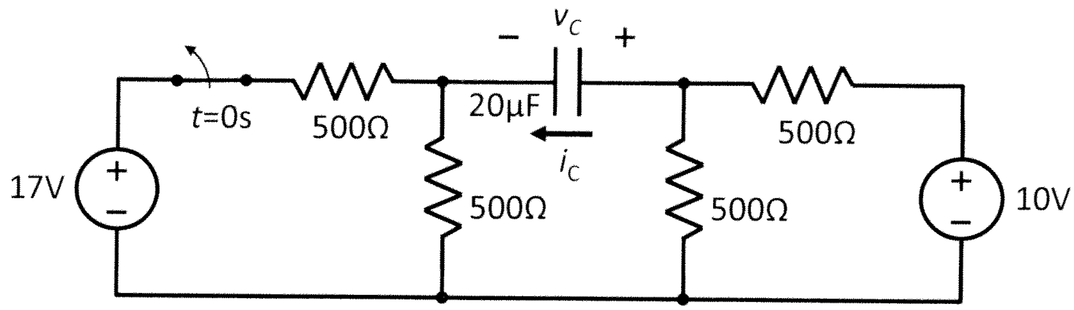
$V_L(t) = -3V\exp(-t/3.33ms)$ [+1/2]

$V_R(t) = -2V + 1V\exp(-t/3.33ms)$ [+1/2]





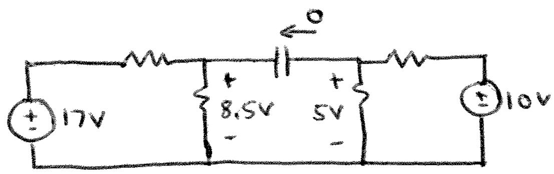
2. Consider the circuit below. Assume dc steady state for $t < 0$. Answer the questions.



a) Find the voltage $v_C(t)$ for $t > 0$. Plot $v_C(t)$ versus t, including the dc steady state value for $t < 0$. [5pts]

b) What is the power delivered by the 10V source at $t = 0+$? [5pts]

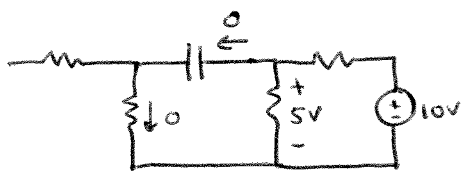
a) $t < 0$



$V_c(0-) = 5V - 8.5V$
$= -3.5V$

$V_c(0+) = v_c(0-) = -3.5V$ [+1]

$t \to \infty$



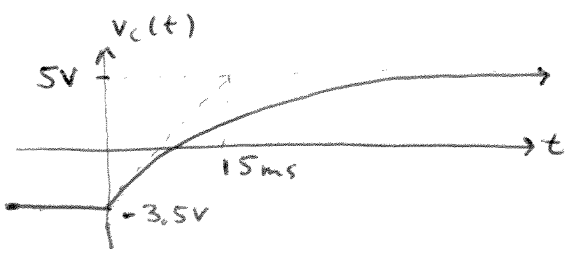
$V_c(\infty) = 5V$ [+1]

$R_{TH} = 500\Omega // 500\Omega + 500\Omega$
$= 250\Omega + 500\Omega$
$= 750\Omega$ [+1]

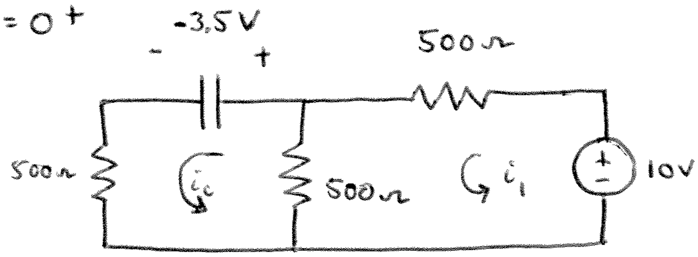
$\tau = R_{TH} \cdot C = 750\Omega \cdot 20\mu F = 15ms$ [+1]

$V_c(t) = V_c(\infty) + [V_c(0+) - V_c(\infty)] \exp(-t/\tau)$

$= 5V - 8.5V \exp(-t/15ms)$ [+1/2]



b) $t = 0^+$



[+1] for circuit diagram

$$i_c(0^+) = C \left.\frac{dv_c}{dt}\right|_{0^+} = 20\mu F \cdot \frac{-8.5V}{-15ms} \quad (+1]$$

$$= 11.33\,mA$$

KVL: $0 = -10V + 500\Omega \cdot i_1 + 500\Omega \cdot (i_1 - i_c)$ [+1]

$$i_1 = \frac{10V + 500\Omega \cdot 11.33\,mA}{500\Omega + 500\Omega}$$

$$= 15.67\,mA \quad [+1]$$

$$P_{del} = 10V \cdot i_1$$

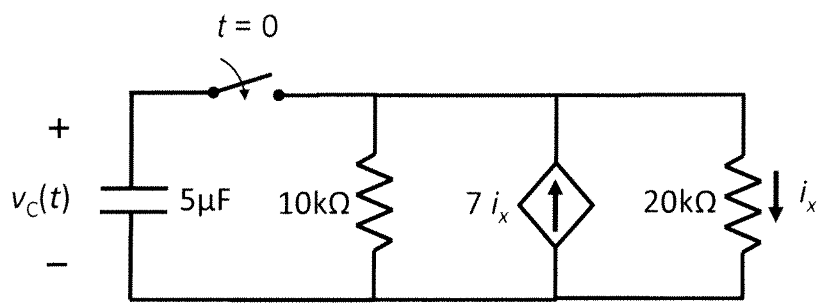
$$= 156.7\,mW \quad [+1]$$

ECSE-200 Quiz #10(25 Mar 2011)

NAME____ McGill ID#____________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram. Assume dc steady state for $t < 0$, where $v_C(t) = 1\text{V}$ for $t < 0$. Answer the questions.



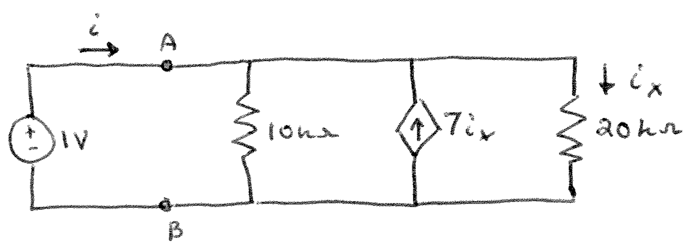
a) What is the Thévenin equivalent circuit that the capacitor is connected to at $t = 0$? [3pts]

b) What is the voltage $v_C(t)$ for $t > 0$? Plot $v_C(t)$ versus t. [5pts]

c) The capacitor is rated to a maximum voltage of 150V. At what time t will the capacitor voltage exceed its rated value? [2pts]

BONUS: The dependent current source 7 i_x is replaced by a dependent current source $A\, i_x$. What is the range of A values that results in ***stable*** circuit response? [2pts]

a)

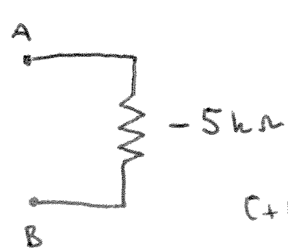


Apply test source since $V_{oc} = 0$. [+1]

$$i = \frac{1\text{V}}{10\text{k}\Omega} + \frac{1\text{V}}{20\text{k}\Omega} - 7\cdot\left(\frac{1\text{V}}{20\text{k}\Omega}\right)$$

$$= -0.2\text{mA}$$

$$\underbrace{R_T = \frac{1\text{V}}{i}}_{[+1]} = -5\text{k}\Omega$$



[+1]

b) $v_c(0+) = v_c(0-) = 1\,V$ [+1]

$v_c(\infty) = 0\,V$ [+1]

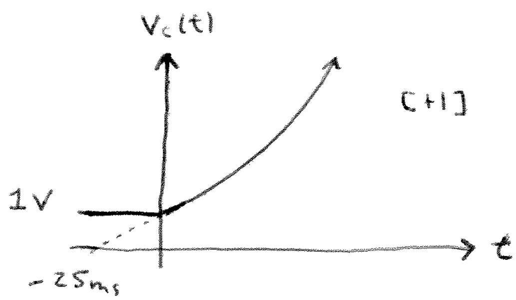
$\tau = R_{TH} \cdot C$

$= -5\,k\Omega \cdot 5\,\mu F$ [+1]

$= -25\,ms$

$v_c(t) = v_c(\infty) + [v_c(0+) - v_c(\infty)]\exp(-t/\tau)$

$= 1V \exp(t/25ms)$ [+1]



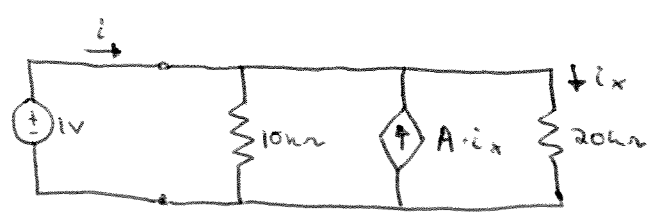
[+1]

c) $150V = 1V \exp(t/25ms)$ [+1]

$t = 25ms \cdot \ln(150)$

$= 125.3\ ms$ [+1]

BONUS



stable response → $\tau > 0$

→ $R_{TH} > 0$ [+1]

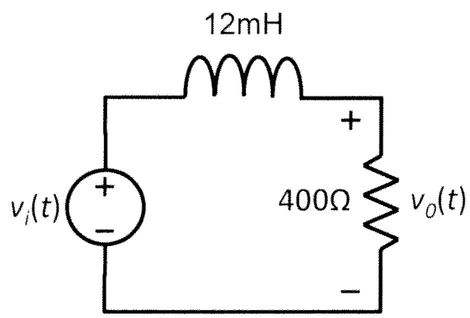
→ $i > 0$

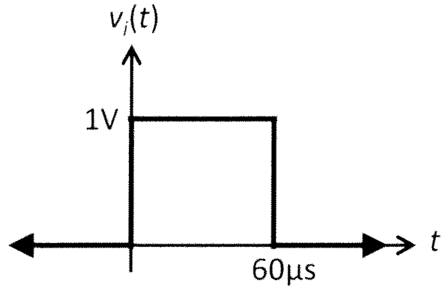
$$i = \frac{1V}{10k\Omega} + \frac{1V}{20k\Omega} - A \cdot \frac{1V}{20k\Omega} > 0$$

$$A < \frac{\frac{1}{10k\Omega} + \frac{1}{20k\Omega}}{\frac{1}{20k\Omega}}$$

$A < 3$ [+1]

2. Consider the circuit below. Assume dc steady state for $t < 0$. Answer the questions.





a) Find the voltage $v_o(t)$ for $0 < t < 60\mu s$. [5pts]

b) Find the voltage $v_o(t)$ for $60\mu s < t$. [4pts]

c) Plot $v_O(t)$ versus t, including the dc steady state value for $t < 0$. [1pt]

BONUS: What is the maximum energy that is stored in the inductor? [2pts]

a) $i_L(0+) = i_L(0-) = 0A$ [+1]

$\tau = \frac{L}{R_{TH}} = \frac{12mH}{400\Omega}$

$V_o(0+) = i_L(0+) \cdot 400\Omega = 0V$ [+1]

$= 30\mu s$ [+1]

$V_o(\infty) = 1V$ [+1]

$$V_o(t) = V_o(\infty) + [v_o(0+) - v_o(\infty)] \exp(-t/\tau)$$
$$= 1V - 1V \exp(-t/30\mu s) \quad [+1]$$

b) $i_L(60\mu s+) = i_L(60\mu s-) = \frac{V_o(60\mu s-)}{400\Omega} = \frac{1V - 1V\exp(-60\mu s/30\mu s)}{400\Omega}$ [+1]

$$= \frac{0.865V}{400\Omega}$$

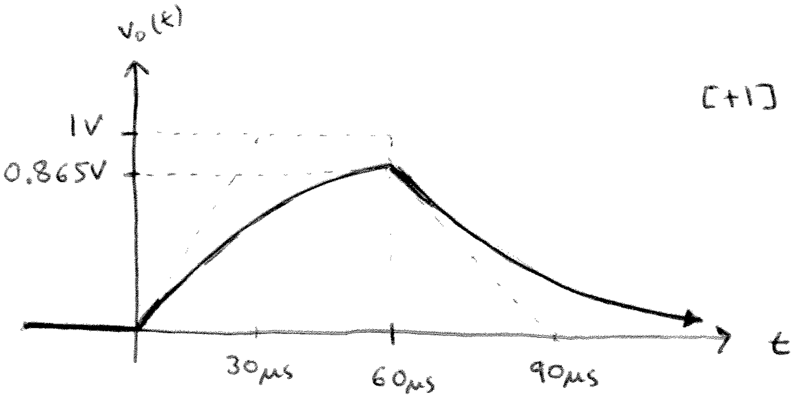
$V_o(60\mu s+) = i_L(60\mu s+) \cdot 400\Omega = 0.865V$ [+1]

$V_o(\infty) = 0V$ [+1]

τ is unchanged

$$V_o(t) = V_o(\infty) + [v_o(60\mu s+) - v_o(\infty)] \exp(-(t - 60\mu s)/30\mu s)$$
$$= 0.865V \exp(-(t-60\mu s)/30\mu s) \quad [+1]$$

c)



[+1]

BONUS

$v_o = i_L \cdot 400\,\Omega$ $\therefore$ maximum inductor current takes place at time of maximum v_o, which occurs at $t = 60\mu s$. [+1]

$$U_{max} = \frac{1}{2} L \, i_L^2(60\mu s)$$

$$= \frac{1}{2} \cdot 12\,mH \cdot \left(\frac{0.865V}{400\,\Omega} \right)^2$$

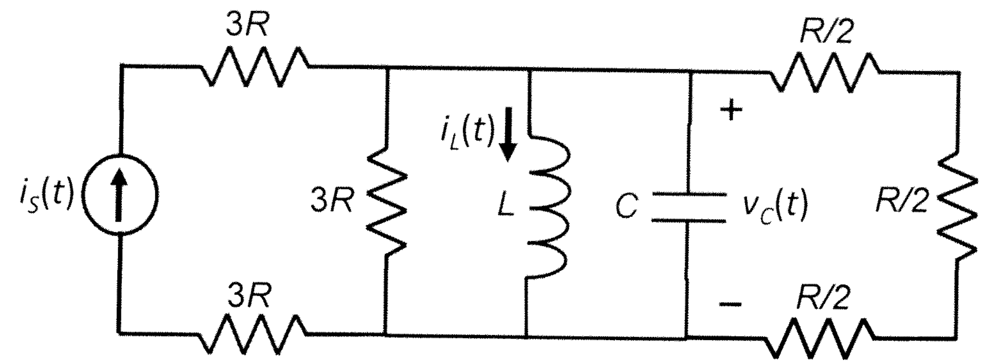
$$= 28.06\,nJ$$ [+1]

ECSE-200 Quiz #11(1 Apr 2011)

NAME_______________________________________ McGill ID#________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram. Answer the questions.



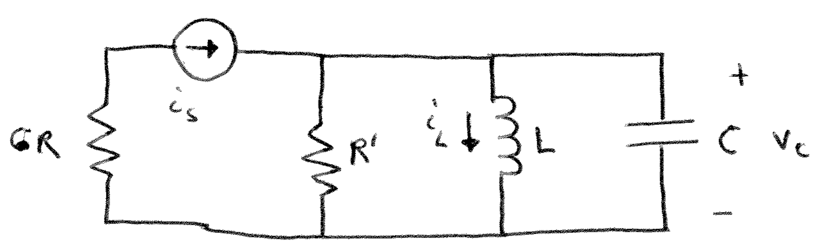
a) What is the differential equation satisfied by $v_C(t)$? [6pts]

HINT: You may find it useful to simplify the circuit.

b) What is the differential equation satisfied by $i_L(t)$? [4pts]

BONUS: Consider a capacitor C and an inductor L connected together. The total energy stored in the capacitor and the inductor is $U = \frac{1}{2}Cv^2 + \frac{1}{2}Li^2$. Prove that $dU/dt = 0$. [2pts]

a)



$$R' = \frac{3R \cdot 3R/2}{3R + 3R/2} = R \qquad [+1]$$

KCL:

$$0 = -i_s + \frac{v_c}{R} + \frac{1}{L}\int_0^t v_c(t')\,dt' + i_L(0) + C\frac{dv_c}{dt}$$

or

$$0 = -i_s + \frac{v_c}{R} + \frac{v_c}{sL} + \frac{v_c}{1/sC} \qquad [+1]$$

differentiation gives :
$$\frac{di_s}{dt} = \frac{1}{R}\frac{dv_c}{dt} + \frac{1}{L} v_c + C\frac{d^2 v_c}{dt}$$

[+1] [+1] [+1] [+1]

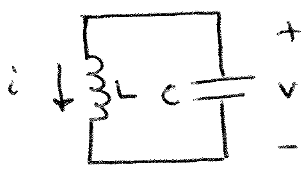
b) $v_c = L\dfrac{di_L}{dt}$ [+1]

Substitute into previous KCL equation: [+1]

$$0 = -i_s + \frac{L}{R}\frac{di_L}{dt} + i_L(t) + LC\frac{d^2 i_L}{dt^2}$$

[+1/2] [+1/2] [+1/2] [+1/2]

c)



$$U = \tfrac{1}{2}Cv^2 + \tfrac{1}{2}Li^2$$

$$\frac{dU}{dt} = Cv\frac{dv}{dt} + Li\frac{di}{dt}$$ [+1/2]

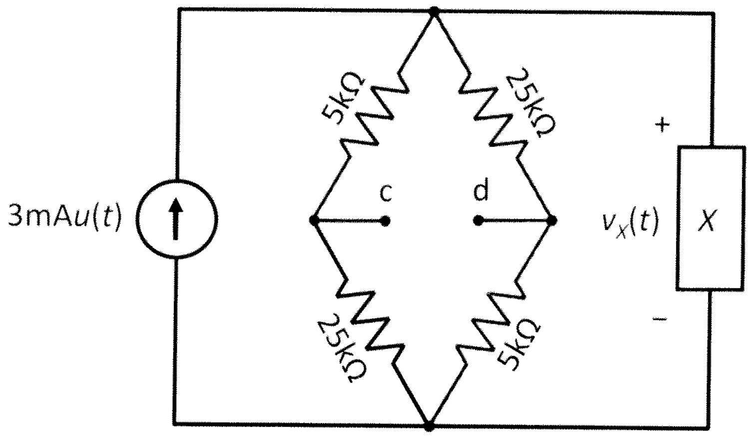
$$v = +L\frac{di}{dt}$$ [+1/2]

$$i = -C\frac{dv}{dt}$$ [+1/2]

$$\frac{dU}{dt} = CL\frac{di}{dt}\cdot\frac{dv}{dt} + LC\left(-\frac{dv}{dt}\right)\frac{di}{dt}$$ [+1/2]

$$= 0$$

2. Consider the circuit below. Assume dc steady state for $t < 0$. Answer the questions.



a) Assume that element X is a 10nF capacitor. Find the voltage $v_X(t)$ for $0 < t$. [5pts]

b) Assume that element X is a 10mH inductor. Find the voltage $v_X(t)$ for $0 < t$. [5pts]

BONUS: Assume again that element X is a 10mH inductor. If a 10kΩ resistor is attached across the terminals c and d, will the time constant be lengthened or shortened as compared to that of part b? [2pts]

a) $v_x(0+) = v_x(0-) = 0mA \cdot R_{TH} = 0V$ [+1]

$R_{TH} = (5k\Omega + 25k\Omega) // (5k\Omega + 25k\Omega) = 15k\Omega$ [+1]

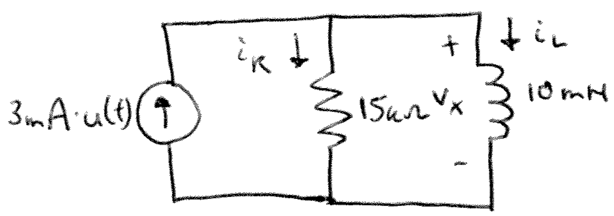
$v_x(\infty) = 3mA \cdot 15k\Omega = 45V$ [+1]

$\tau = R_{TH} \cdot C = 15k\Omega \cdot 10nF = 150\mu s$ [+1]

$v_x(t) = v_x(\infty) + [v_x(0+) - v_x(\infty)] \exp(-t/\tau)$ $t > 0$

$= 45V - 45V \exp(-t/150\mu s)$ [+1] $t > 0$

b)



$i_L(0+) = i_L(0-) = 0mA$ [+1] $\qquad i_L(\infty) = 3mA$ [+1]

$$\tau = \frac{L}{R_{TH}} = \frac{10mH}{15k\Omega} = 667ns \quad [+1]$$

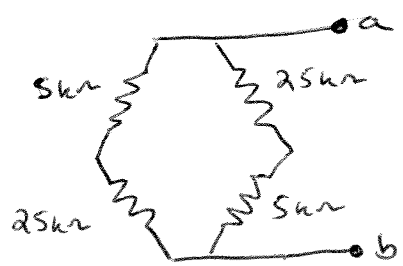
$i_L(t) = 3mA - 3mA\,\exp(-t/667ns)$ [+1] $\quad t>0$

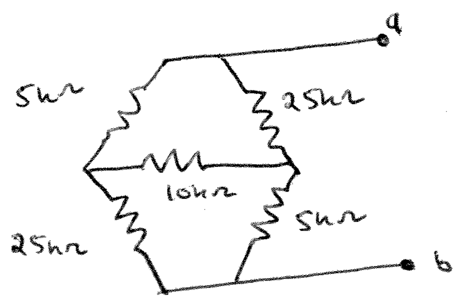
$$V_x(t) = 10mH \cdot \frac{di_L}{dt}$$

$= 45V\,\exp(-t/667ns)$ [+1] $\quad t>0$

* It is also acceptable to use $V_x(0+) = 45V$ and $V_x(\infty) = 0V$.

c)





$R_{TH} = 15k\Omega$ $\qquad R_{TH}' < 15k\Omega$ [+1]

$\tau = \frac{L}{R_{TH}}$ $\qquad \tau' = \frac{L}{R_{TH}'} > \tau$ [+1]

In other words, R_{TH} is <u>reduced</u> and τ is <u>lengthened</u>.