McGill

December 2016
Final Examination

Electric Circuits 1

ECSE-200 Section: 1

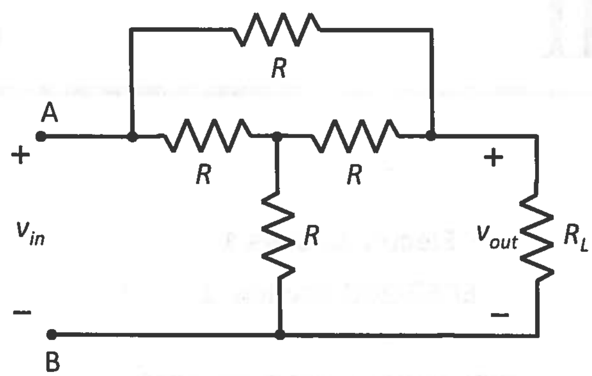
12 December 2016, 2:00PM

Examiner: Thomas Szkopek   Assoc Examiner: Michael Rabbat

INSTRUCTIONS:

- This is a **CLOSED BOOK** examination.
- **NO CRIB SHEETS** are permitted.
- Provide your answers in an **EXAM BOOKLET**.
- **STANDARD CALCULATOR** permitted ONLY.
- This examination consists of 4 questions, with a total of 6 pages, including the cover page.
- This examination is **PRINTED ON BOTH SIDES** of the paper

1. Consider the circuit below. Answer the questions. [12 pts]



a) What is the physical law at the origin of Kirchoff's current law? [1pt]

b) What is the physical law at the origin of Kirchoff's voltage law? [1pt]

c) What is the equivalent resistance between nodes A and B if R_L is an open circuit? [2pts]

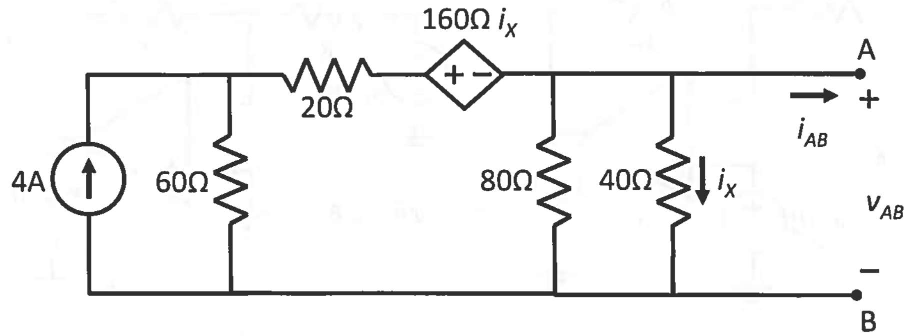
d) What is the equivalent resistance between nodes A and B if R_L is a short circuit? [2pts]

e) What is the equivalent resistance between nodes A and B if $R_L = R$? [2pts]

f) What is the ratio v_{out}/v_{in} if $R_L = R$? [2pts]

g) What is the equivalent resistance between nodes A and B if $R_L = 3R$? [2pts]

2. Consider the circuit below. Answer the questions. [12 pts]



a) What is Thévenin's theorem? [1pt]

b) Write the equation that relates the quantities v_{oc}, i_{sc} and R_T. [1pt]

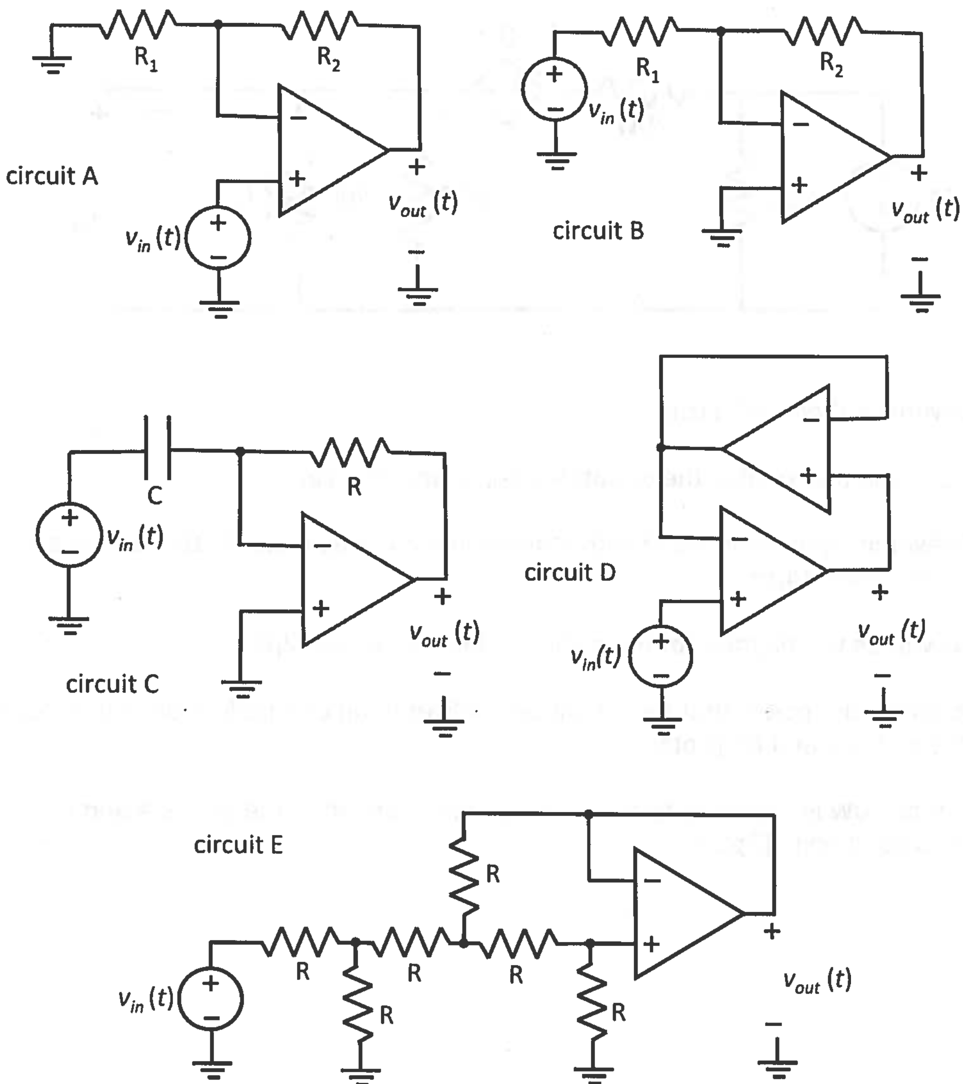
c) Draw the Thévenin equivalent circuit with respect to the nodes A and B. Be sure to label the nodes A and B in your diagram. [4pts]

d) Draw the i_{AB} versus v_{AB} diagram for the circuit. Label your axes. [2pts]

e) What is the maximum power that the circuit can deliver to an optimally chosen load resistor attached to the nodes A and B? [2pts]

f) It is desired that 10W is delivered to a load resistor R attached to the nodes A and B. What value(s) of R achieve this condition? [2pts]

3. Consider the circuits below. Assume ideal op-amp behaviour. Answer the questions. [12 pts]



a) Give two reasons why negative feedback is useful for op-amp circuits. [2pts]

b) Express v_{out} in terms of v_{in} for circuit A ? [2pts]

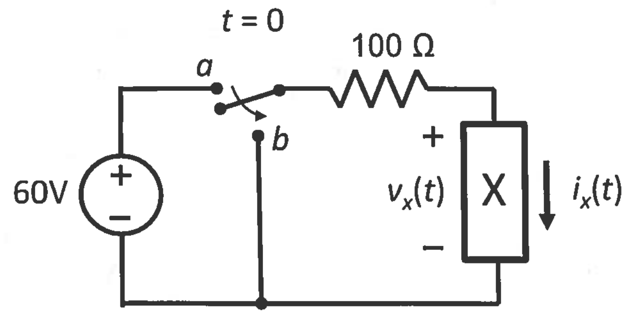
c) Express v_{out} in terms of v_{in} for circuit B ? [2pts]

d) Express v_{out} in terms of v_{in} for circuit C ? [2pts]

e) Express v_{out} in terms of v_{in} for circuit D ? [2pts]

f) Express v_{out} in terms of v_{in} for circuit E ? [2pts]

4. Consider the circuit below. The circuit is in dc steady state for $t < 0$ with the switch in position a. The switch instantaneously moves to position b at $t = 0$s. The circuit reaches a new dc steady state as $t \rightarrow \infty$. Answer the questions. [12 pts]



Assume X is a 50 μH inductor.

a) What is $i_x(t)$ for $t > 0$? [2pts]

b) What is $v_x(t)$ for $t > 0$? [2pts]

Assume X is a 500 pF capacitor.

c) What is $v_x(t)$ for $t > 0$? [2pts]

d) What is $i_x(t)$ for $t > 0$? [2pts]

e) At what time t does the capacitor deliver the maximum power to the circuit? [2pts]

Assume X is a parallel combination of a 10 nF capacitor and a 100 μH inductor.

f) What are the roots of the characteristic equation describing the natural response of the circuit for the time $t > 0$? [2pts]

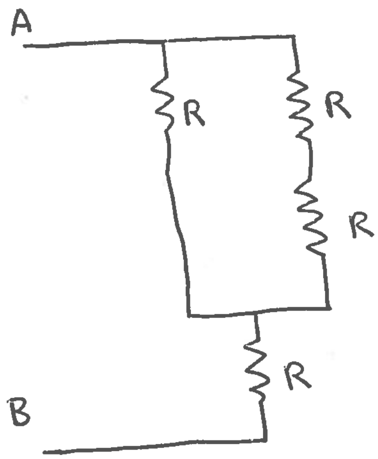
end

1.

a) conservation of charge [+1]

b) conservation of energy [+1]

c)

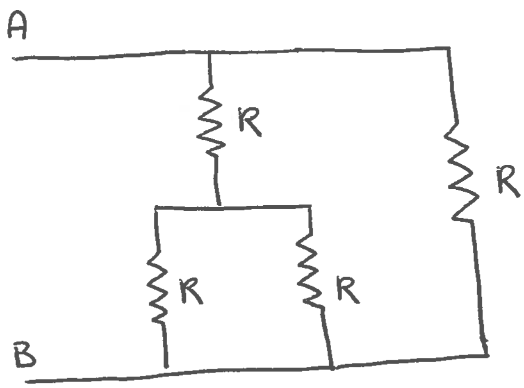


$$R_{AB} = R // 2R + R$$

$$= \frac{R \cdot 2R}{R + 2R} + R$$

$$= \frac{5}{3} R \quad [+2]$$

d)

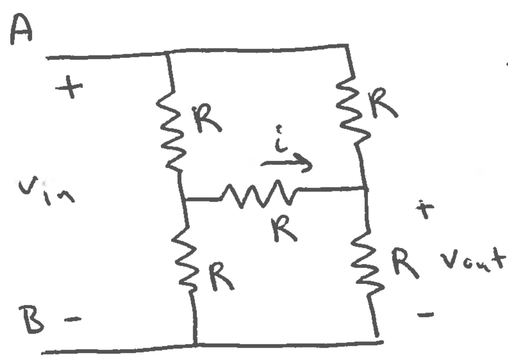


$$R_{AB} = (R + R//R) // R$$

$$= \frac{\frac{3}{2} R \cdot R}{\frac{3}{2} R + R}$$

$$= \frac{3}{5} R \quad [+2]$$

e)



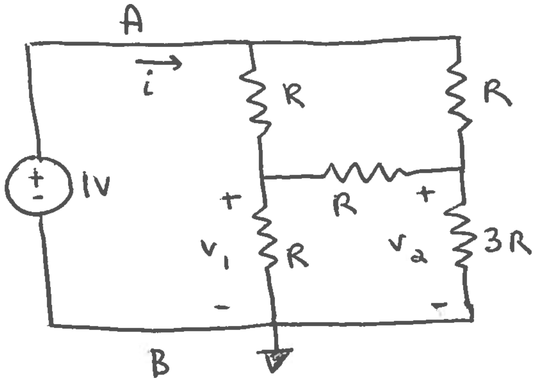
This is a balanced Wheatstone bridge, and it can be shown that $i = 0$.

$$R_{AB} = (R + R) // (R + R)$$

$$= 2R // 2R$$

$$= R \quad [+2]$$

f) $\frac{V_{out}}{v_{in}} = \frac{R}{R + R} = \frac{1}{2}$ [+2] g)



$$0 = \frac{v_1}{R} + \frac{v_1 - v_2}{R} + \frac{v_1 - 1V}{R} \longrightarrow 1 = 3v_1 - v_2$$

$$0 = \frac{v_2}{3R} + \frac{v_2 - v_1}{R} + \frac{v_2 - 1V}{R} \longrightarrow 3 = -3 + 7v_2$$

$$v_1 = \frac{\begin{vmatrix} 1 & -1 \\ 3 & 7 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ -3 & 7 \end{vmatrix}} = \frac{5}{9} V \qquad v_2 = \frac{\begin{vmatrix} 3 & 1 \\ -3 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ -3 & 7 \end{vmatrix}} = \frac{6}{9} V$$

$$i = \frac{v_1}{R} + \frac{v_2}{3R} = \frac{7}{9} \cdot \frac{1V}{R}$$

$$R_{AB} = \frac{1V}{i} = \frac{9}{7} \cdot R \quad [+2]$$

$$= 1.286 \cdot R$$

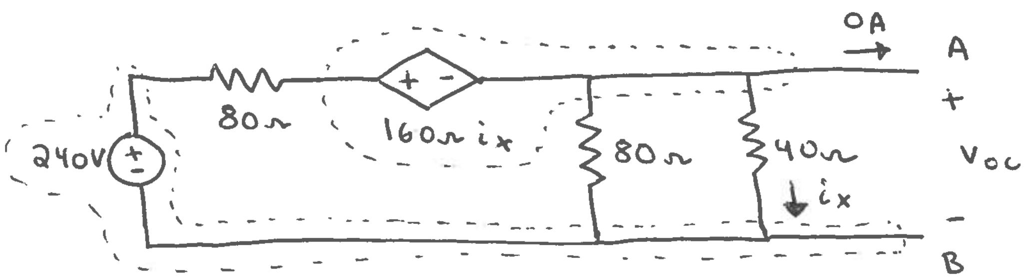
2.

a) Any two terminal circuit composed of resistors, dependent sources and independent sources is equivalent to a series combination of a resistance and independent voltage source. [+1]

b) $V_{oc} = i_{sc} \cdot R_T$ [+1]

c) First simplify the circuit by source transformation.

Find V_{oc}.



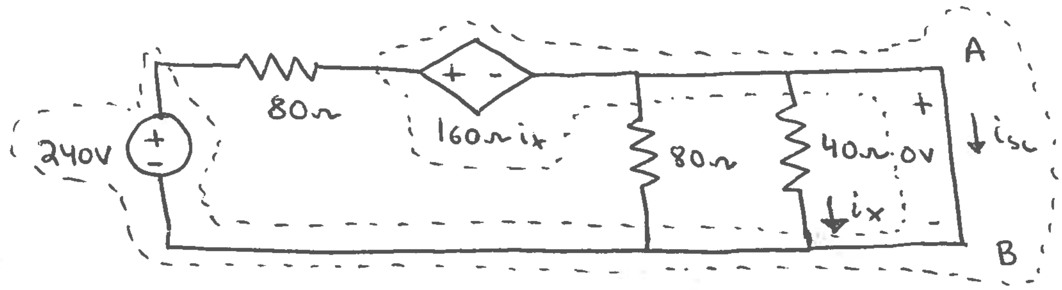
$$0 = \frac{V_{oc}}{40\Omega} + \frac{V_{oc}}{80\Omega} + \frac{V_{oc} + 160\Omega i_x - 240V}{80\Omega}$$

$$i_x = V_{oc} / 40\Omega$$

$$\therefore \quad 0 = \frac{V_{oc}}{40} + \frac{V_{oc}}{80} + \frac{V_{oc} + 4V_{oc} - 240V}{80}$$

$$V_{oc} = \frac{240V/2}{1 + \frac{1}{2} + \frac{1}{2}(1+4)} = 30V \quad [+1]$$

Find i_{sc}.

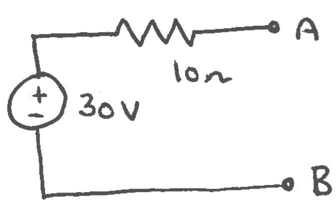


$$i_x = \frac{0V}{40\Omega} = 0\,A$$

$$0 = i_{sc} + \frac{0V}{40\Omega} + \frac{0V}{80\Omega} + \frac{0 - 240V}{80\Omega}$$

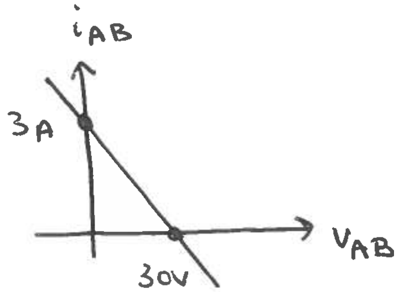
$i_{sc} = 3\,A$ [+1]

$$R_T = \frac{V_{oc}}{i_{sc}} = \frac{30V}{3A} = 10\Omega$$ [+1]



[+1]

d)

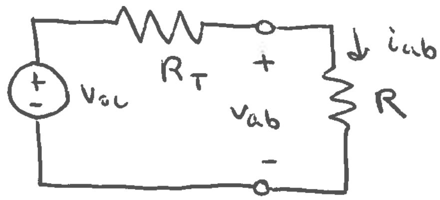


[+2]

e) $P_{max} = \frac{v_{oc}}{2} \cdot \frac{i_{sc}}{2}$ [+1]

$= 22.5\,W$ [+1]

f)



$v_{ab} = v_{oc} - i_{ab} \cdot R_T$

$p_{abs} = v_{ab} \cdot i_{ab}$

$\therefore\ v_{ab} = v_{oc} - \frac{p_{abs} \cdot R_T}{v_{ab}}$

$$0 = v_{ab}^2 - v_{ab} \cdot v_{oc} + p_{abs} \cdot R_T$$

$$= v_{ab}^2 - v_{ab} \cdot 30V + 10W \cdot 10\Omega$$

$$v_{ab} = \frac{30V \pm \sqrt{(30V)^2 - 4 \cdot 1 \cdot 10W \cdot 10\Omega}}{2}$$

$$= 26.18\,V,\ 3.820\,V$$

$$i_{ab} = \frac{p_{abs}}{v_{ab}} = 0.3820\,A,\ 2.618\,A$$

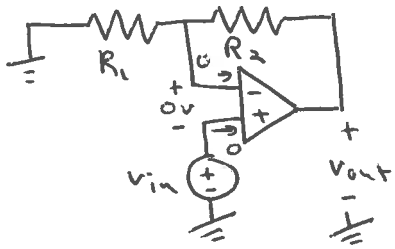
$$R = \frac{v_{ab}}{i_{ab}} = 68.5\,\Omega,\ 1.459\,\Omega \quad [+2]$$

3.

a) stability [+1]

programmability [+1]

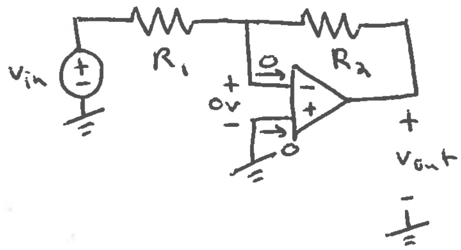
b)



$$0 = \frac{v_{in}}{R_1} + \frac{v_{in} - v_{out}}{R_2}$$

$$v_{out} = \left(1 + \frac{R_2}{R_1}\right) v_{in} \quad [+2]$$

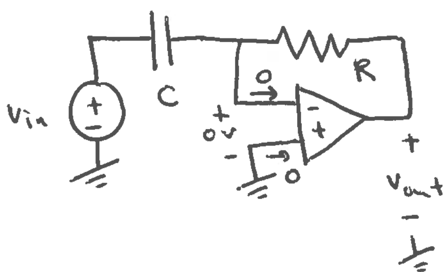
c)



$$0 = \frac{0 - v_{in}}{R_1} + \frac{0 - v_{out}}{R_2}$$

$$v_{out} = -\frac{R_2}{R_1} \cdot v_{in} \quad [+2]$$

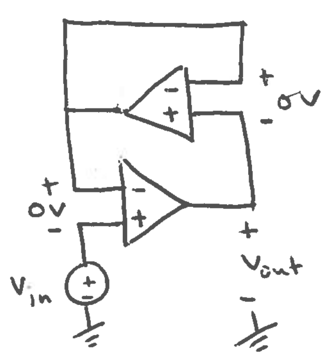
d)



$$0 = C \frac{d}{dt}(0 - v_{in}) + \frac{0 - v_{out}}{R}$$

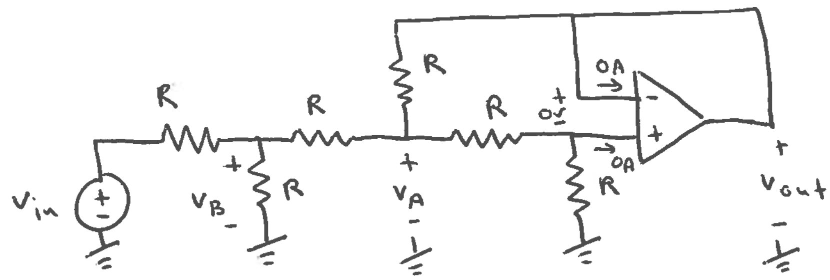
$$v_{out} = -RC \frac{dv_{in}}{dt} \quad [+2]$$

e)



$$v_{out} = v_{in} \quad [+2]$$

f)



$$0 = \frac{V_{out}}{R} + \frac{V_{out} - V_A}{R} \rightarrow V_A = 2 V_{out}$$

$$0 = \frac{V_A - V_{out}}{R} + \frac{V_A - V_{out}}{R} + \frac{V_A - V_B}{R}$$

$$\longrightarrow V_B = 3 V_A - 2 V_{out} = 4 V_{out}$$

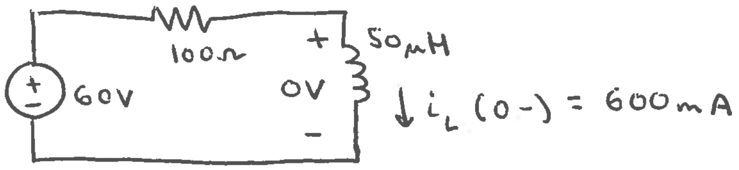
$$0 = \frac{V_B - V_A}{R} + \frac{V_B}{R} + \frac{V_B - V_{in}}{R}$$

$$\longrightarrow V_{in} = 3 V_B - V_A = 10 V_{out}$$

$$V_{out} = \frac{1}{10} V_{in} \quad [+2]$$

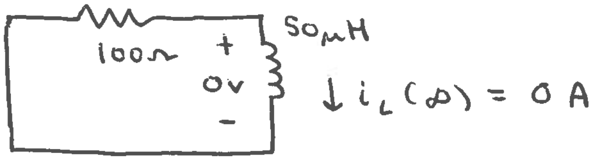
4.

a) $t < 0$

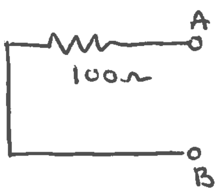


$i_L(0+) = i_L(0-) = 600\,mA$

$t \to \infty$



$t > 0$



$\tau = L/R_T = 50\mu H / 100\Omega = 500ns$ [+1]

$i_L(t) = [i_L(0+) - i_L(\infty)]\exp(-t/\tau) + i_L(\infty)$

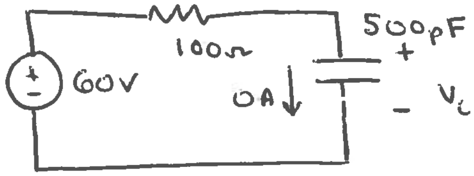
$= 600mA \exp(-t/500ns)$ [+1]

b) $v_L(t) = L \frac{di_L}{dt}$ [+1] or $V_L = -R \cdot i_L$

$= 50\mu H \cdot 600mA \cdot \frac{-1}{500ns} \exp(-t/500ns)$

$= -60V \exp(-t/500ns)$ [+1]

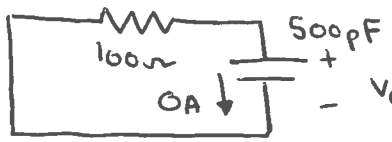
c) $t < 0$



$V_c(0-) = 60\text{V}$

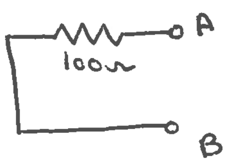
$$V_c(0+) = V_c(0-) = 60\text{V}$$

$t \to \infty$



$V_c(\infty) = 0\text{V}$

$t > 0$



$$\tau = R_T \cdot C = 100\Omega \cdot 500\text{pF} = 50\text{ns} \quad [+1]$$

$$V_c(t) = [V_c(0+) - V_c(\infty)] \exp(-t/\tau) + V_c(\infty)$$

$$= 60\text{V} \exp(-t/50\text{ns}) \quad [+1]$$

d) $$i_c(t) = C \frac{dv_c}{dt} \quad [+1] \quad \text{or} \quad i_c = -V_c/R$$

$$= 500\text{pF} \cdot 60\text{V} \cdot \frac{-1}{50\text{ns}} \exp(-t/50\text{ns})$$

$$= -600\text{mA} \exp(-t/50\text{ns}) \quad [+1]$$

e)

$$P_{del} = -v_c(t) \cdot i_c(t)$$

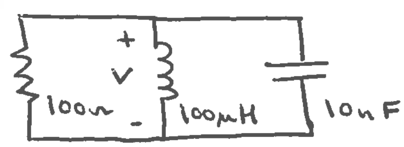
$t<0$: $\quad P_{del} = 0\,W$

$t>0$: $\quad P_{del} = 60V\exp(-t/50ns) \cdot 600mA\exp(-t/50ns)$

$$= 36W\exp(-t/25ns)$$

P_{del} is maximum at $t = 0+$. [+2]

f) $t>0$



$$0 = \frac{v}{R} + \frac{v}{sL} + \frac{v}{1/sC} = \left(\frac{1}{R} + \frac{1}{sL} + sC\right) \cdot v$$

$$\therefore \quad 0 = \frac{1}{R} + \frac{1}{sL} + sC$$

$$0 = s^2 + s \cdot \frac{1}{RC} + \frac{1}{LC}$$

$$= s^2 + 10^6 \cdot s + 10^{12}$$

$$s = \frac{-10^6 \pm \sqrt{(10^6)^2 - 4 \cdot 1 \cdot 10^{12}}}{2 \cdot 1}$$

$$= -500{,}000 \pm i \cdot 866{,}025 \quad [s^{-1}]$$

[+2]