2. Resistive Circuits

- Circuit Topology
- Kirchoff's Current and Voltage Laws
- Equivalent Circuits
- Series and Parallel Resistors
- Equivalent Resistance and Circuit Analysis
- Ammeters, Voltmeters, Ohmmeters

Today’s Outline

2. Resistive Circuits

- Circuit Topology
- Kirchoff’s Current Law
- Kirchoff’s Voltage Law

Circuit Topology

Precise language is needed to describe the parts of a circuit:

Element: a model for a physical component

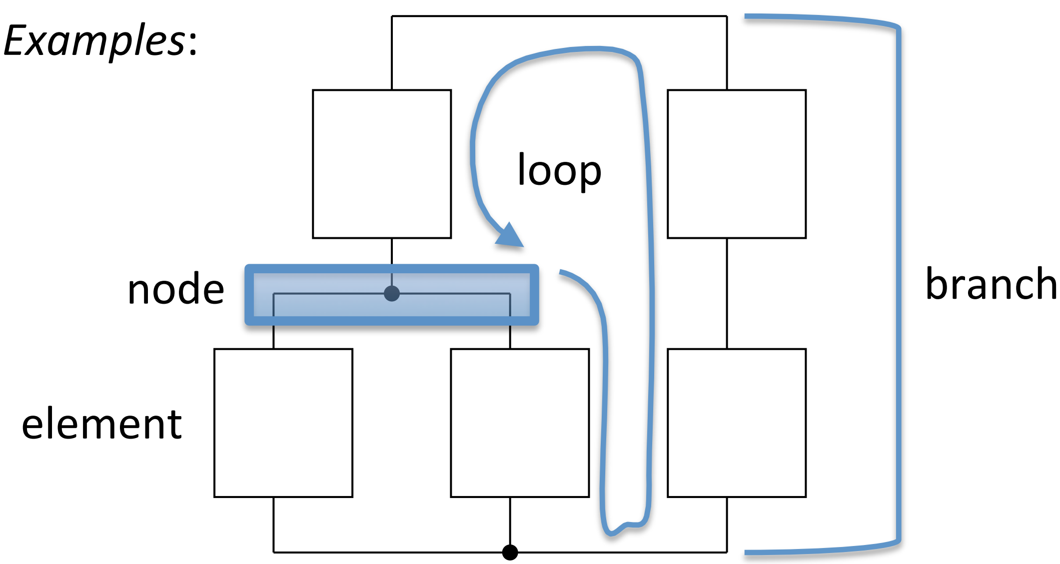
Node: a point where multiple elements meet

Branch: a path that connects two nodes

Loop: a closed path through a circuit with no node passed more than once

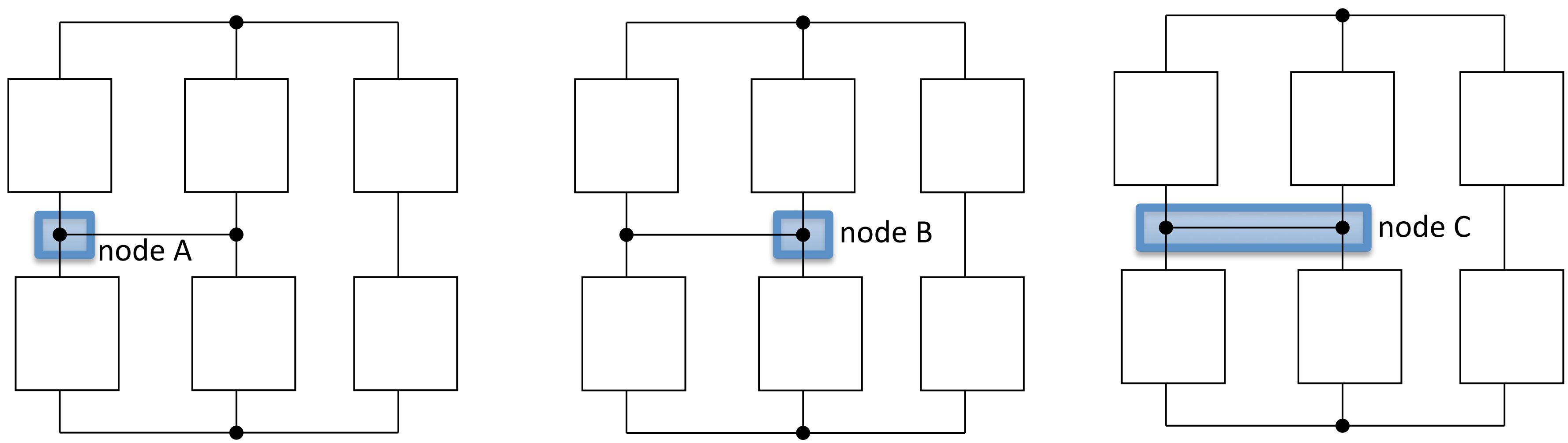
Mesh: a loop that cannot be broken up into smaller loops

Examples:



a note on nodes

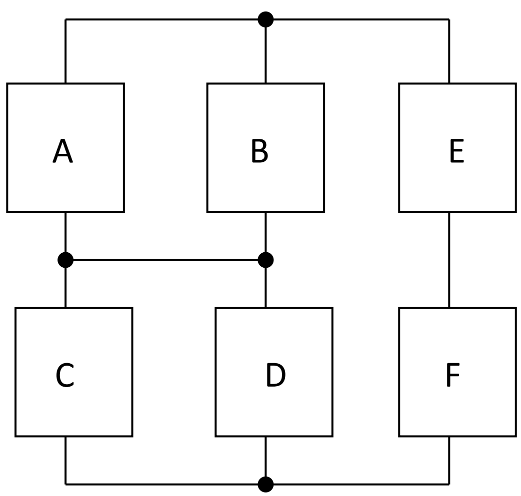
A **node**, being the point where multiple elements meet, can be identified in several ways on the same portion of a circuit. Consider the examples below:

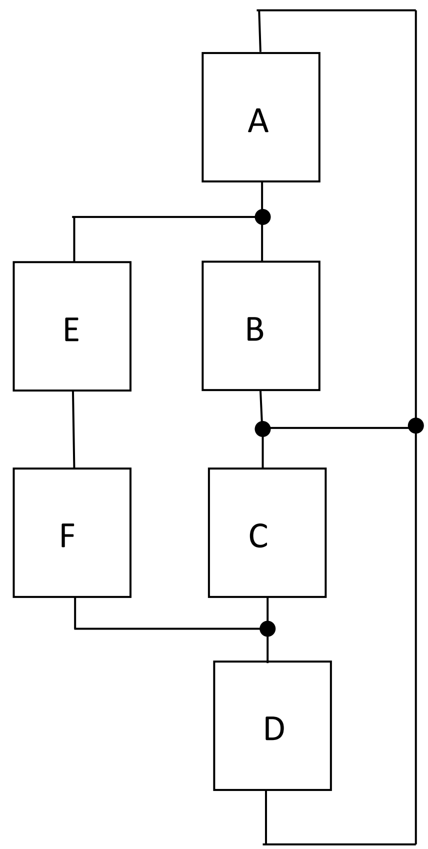


It is important to be clear about precisely which node one is discussing.

Circuit Topology

A circuit is defined by the connections between elements, meaning the ***topology*** of the circuit. The two circuits below are equivalent.

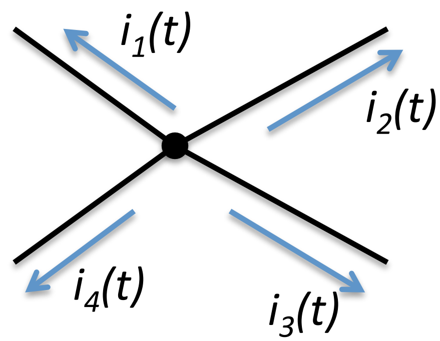




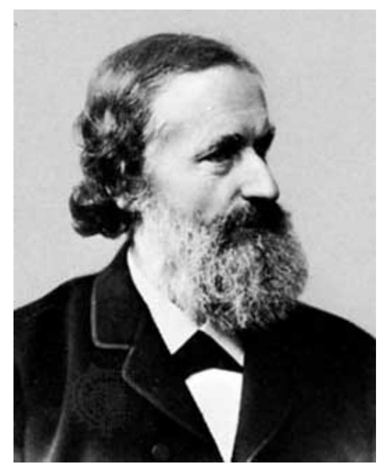
Kirchoff's Current Law (KCL)

Kirchoff's Current Law: The algebraic sum of currents leaving a *node* is zero,

$$\sum_m i_m(t) = 0$$



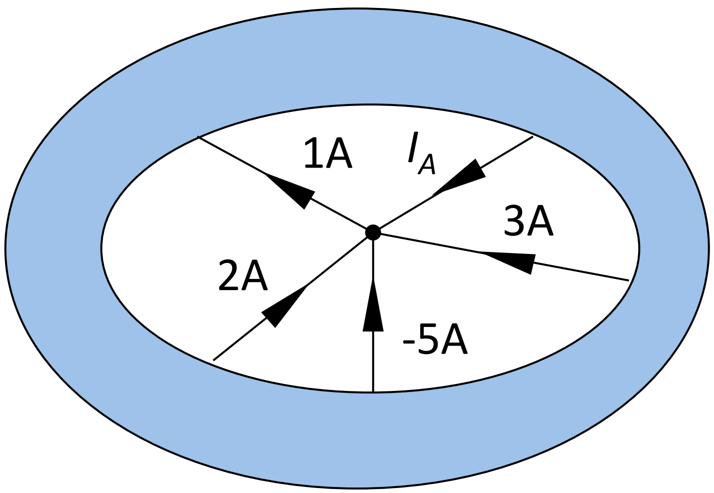
The physical basis for KCL is the ***conservation of charge*** (charge cannot be created or destroyed at a node).



Gustav Robert Kirchoff (1824-1887)

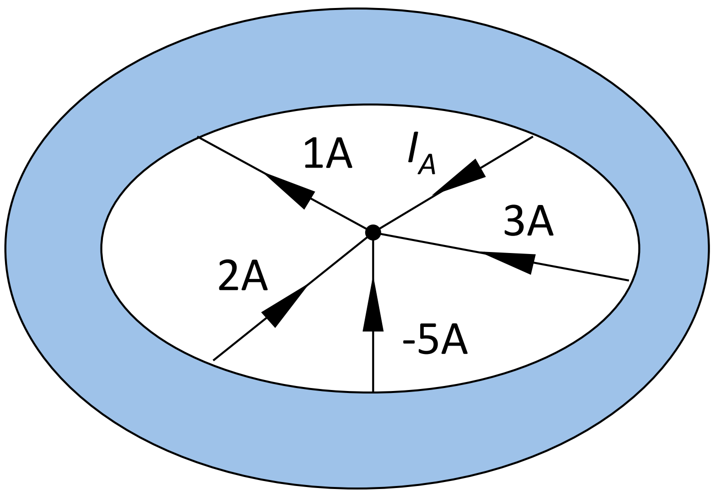
Example 1

Use KCL to find the unknown current I_A below.



Example 1

Use KCL to find the unknown current I_A below.



We sum the currents leaving the node:

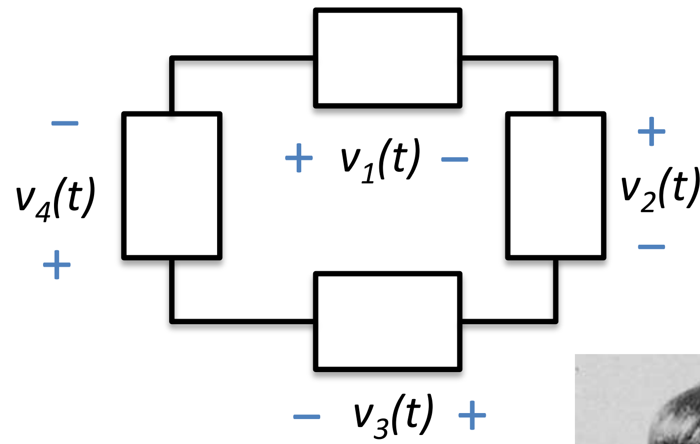
$0 = 1A + (-2A) + 5A + (-3A) + (-I_A)$

$I_A = 1A$

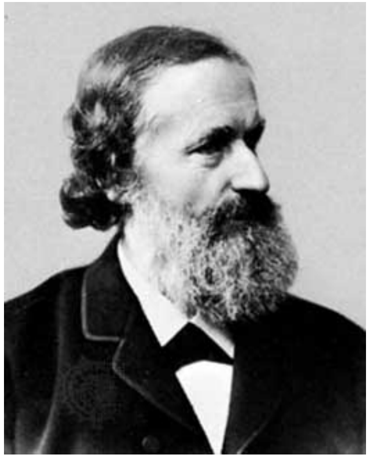
Kirchoff's Voltage Law (KVL)

Kirchoff's Voltage Law: The algebraic sum of voltage drops around a loop is zero,

$$\sum_m v_m(t) = 0$$



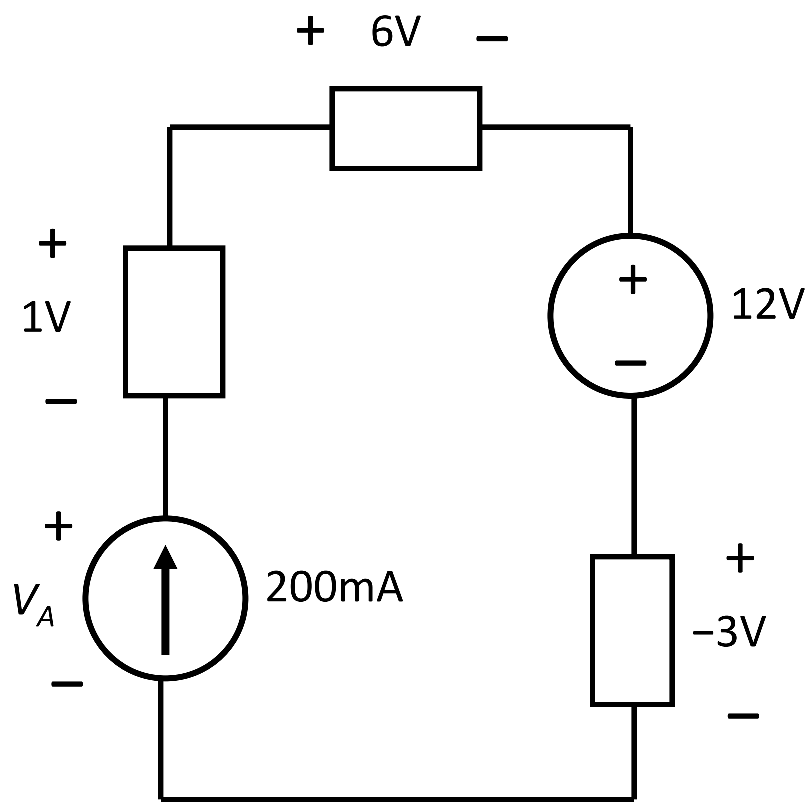
The physical basis for KVL is the ***conservation of energy*** (the potential energy of a particle cannot be increased by traversing a closed loop).



Gustav Robert Kirchoff (1824-1887)

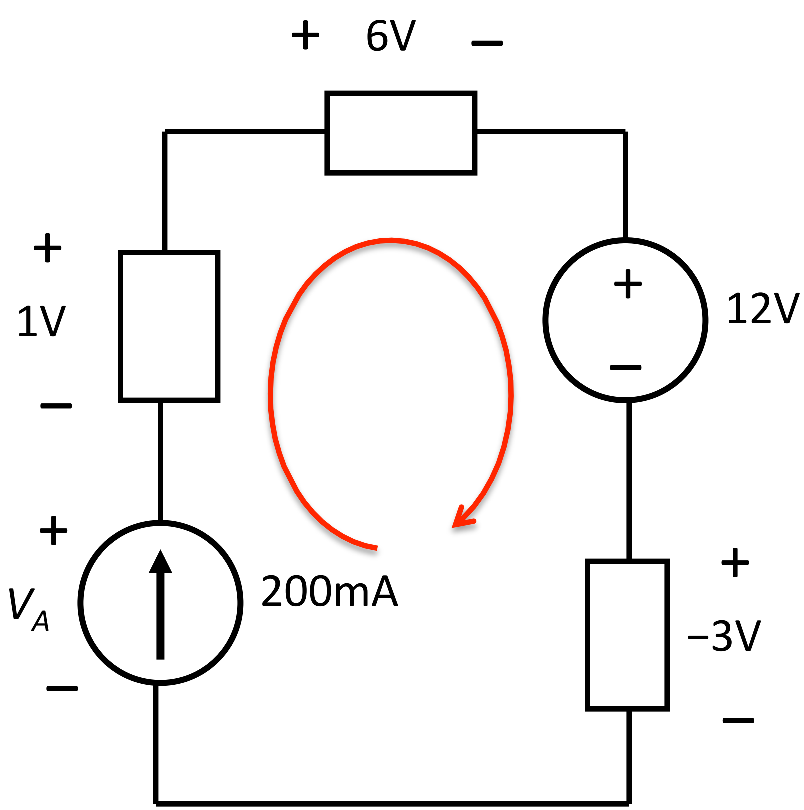
Example 2

Use KVL to find the unknown voltage V_A below.



Example 2

Use KVL to find the unknown voltage V_A below.



We sum the voltage drops across the loop, choosing a clockwise direction:

$$0 = (-V_A) + (-1V) + 6V + 12V + (-3V)$$

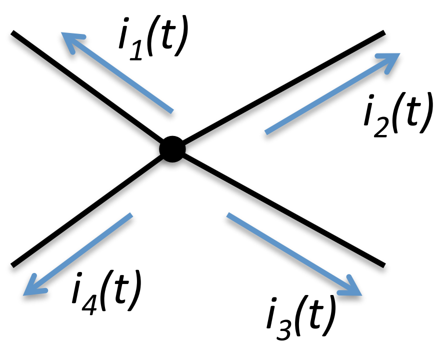
$$V_A = 14V$$

Note that the same answer will be found by taking the loop in counter-clockwise direction.

Equivalent Forms of KCL and KVL

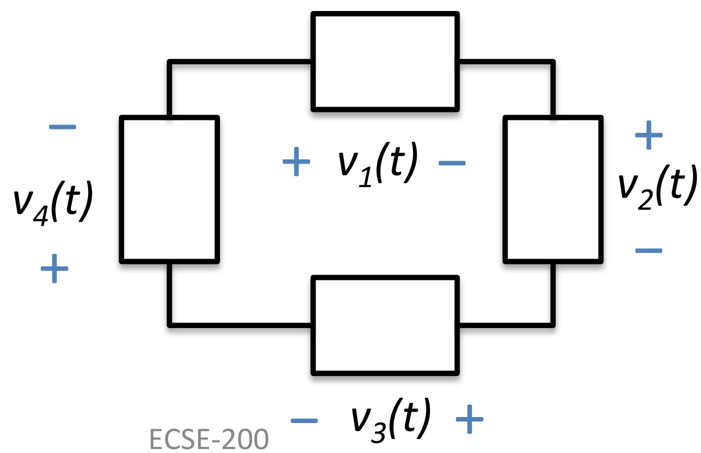
KCL: The algebraic sum of currents leaving a node is zero, as is the algebraic sum of currents entering the node. Also, currents leaving are equal to currents entering.

$$\sum_m i_m(t) = 0$$



KVL: The algebraic sum of voltage drops around a loop is zero, as is the sum of voltage rises around the loop. Also, voltage drops are equal to voltage rises.

$$\sum_m v_m(t) = 0$$



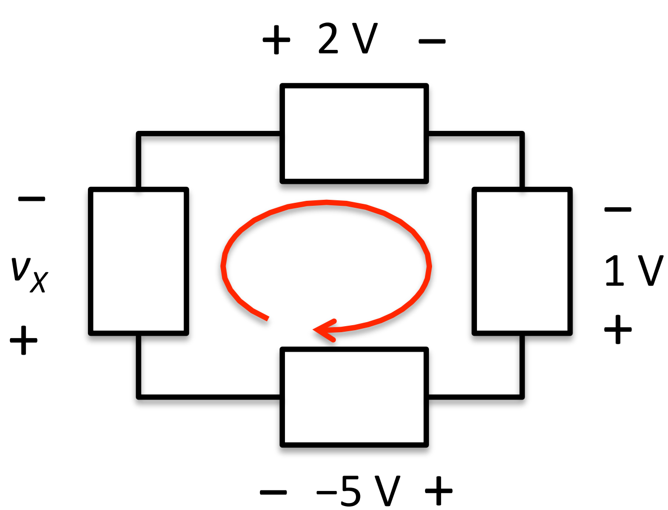
Keeping it Simple

To keep equations simple, we will use the following explicit forms:

KCL: The algebraic sum of currents leaving a node is zero.

KVL: The algebraic sum of voltage drops around a loop is zero.

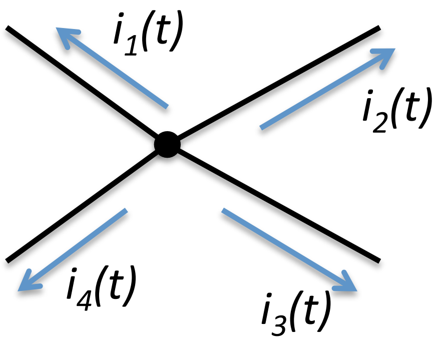
Example: KVL can be written in multiple ways for the following single-loop circuit. Note that it is very simple to express everything as a voltage drop.

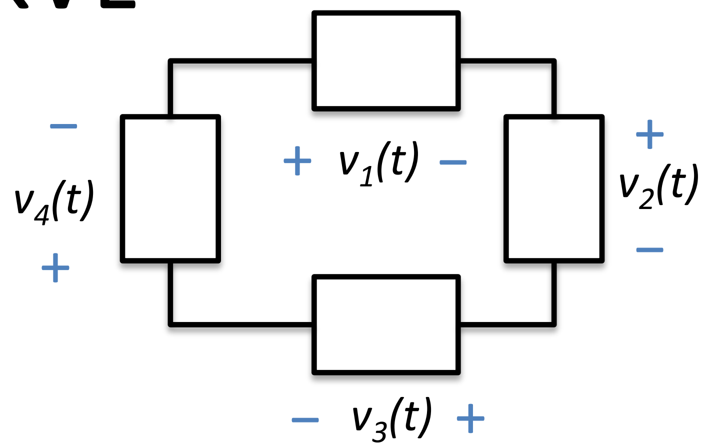


KVL: $0 = v_x + 2V - (\ 1V\ ) + (\ -5V\ )$

$v_x = 5V + 1V - 2V = 4V$

KCL and KVL





$$\sum_m i_m(t) = 0$$

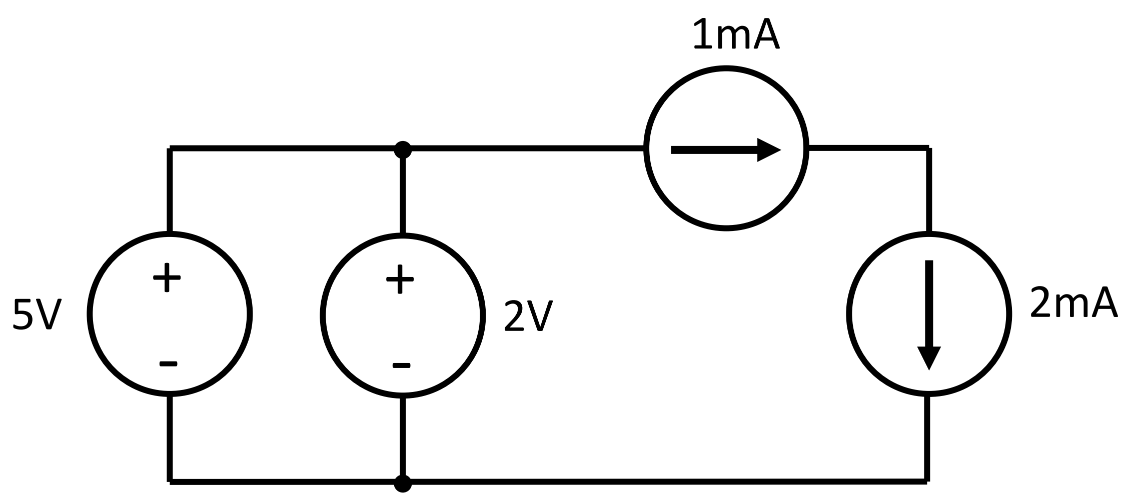
$$\sum_m v_m(t) = 0$$

- KCL and KVL are based on ***fundamental physical laws***
- KCL is satisfied at every node, and KVL is satisfied on every loop, at all time, in a physical circuit
- KCL and KVL *do not* give insight into how charge "knows" where to flow and exchange energy
- KCL and KVL, together with I-V relations for each element, uniquely determine the currents and voltages in a circuit

Consistency with KCL and KVL

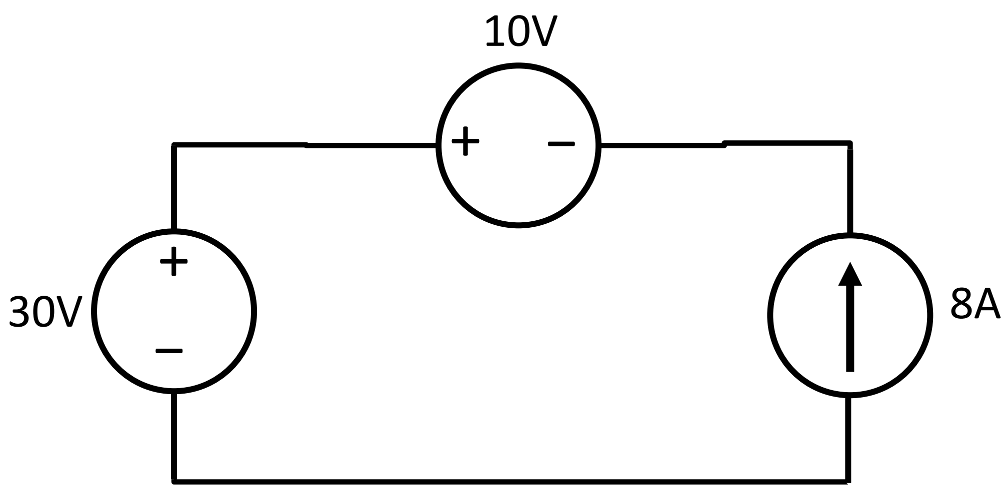
• all physical circuits obey KCL and KVL
• there are connections of elements that are inconsistent with KCL and KVL, and thus represent unphysical situations
• *circuit models incapable of predicting physical circuit behaviour are nonsense*!

Example: what is wrong with the circuit below?



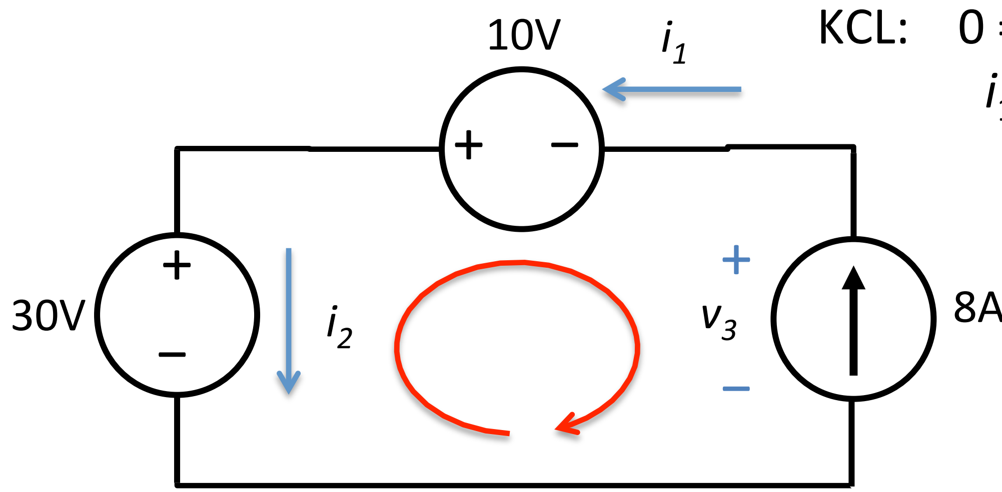
Example 3

For the circuit below, how much power is being delivered or absorbed by each source?



Example 3

Define the necessary variables, and apply KCL and KVL.



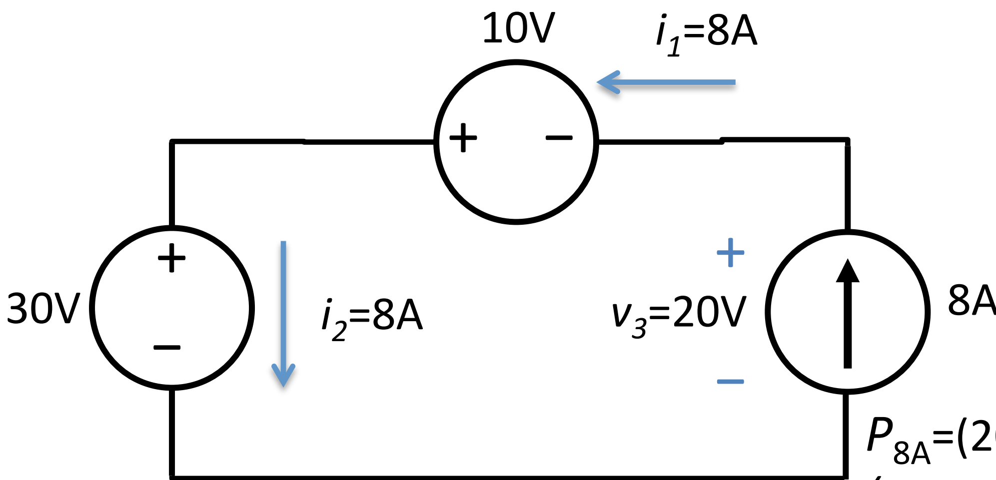
KCL: $0 = i_1 - 8A$
$i_1 = 8A$

KCL: $0 = -i_2 + 8A$
$i_2 = 8A$

KVL: $0 = -30V + 10V + v_3$
$v_3 = 20V$

Example 3

P_{10V}=(10V)(8A)=80W delivered
(opposite of passive sign convention)



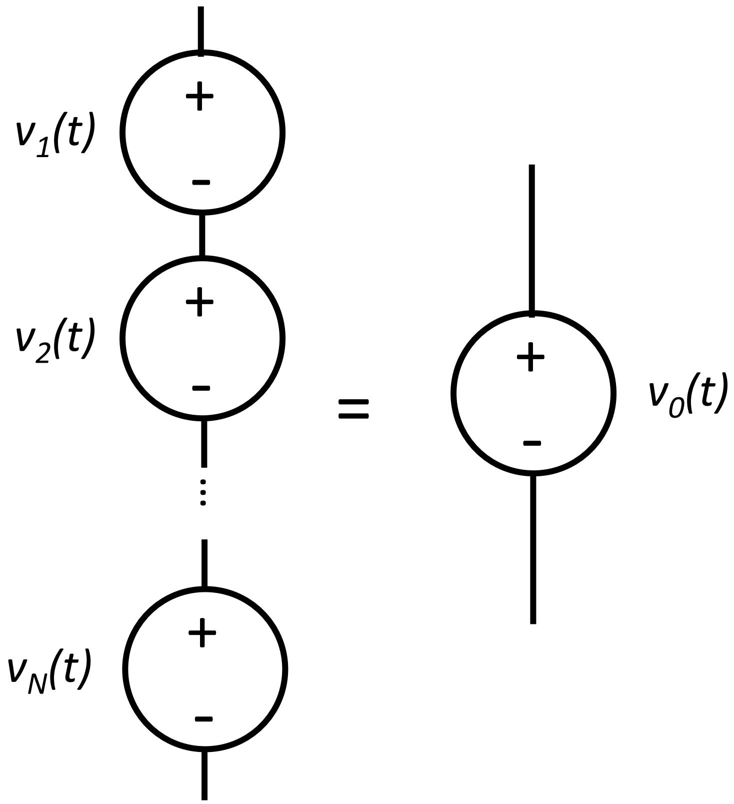
P_{8A}=(20V)(8A)=160W delivered
(opposite of passive sign convention)

P_{30V}=(30V)(8A)=240W absorbed
(passive sign convention)

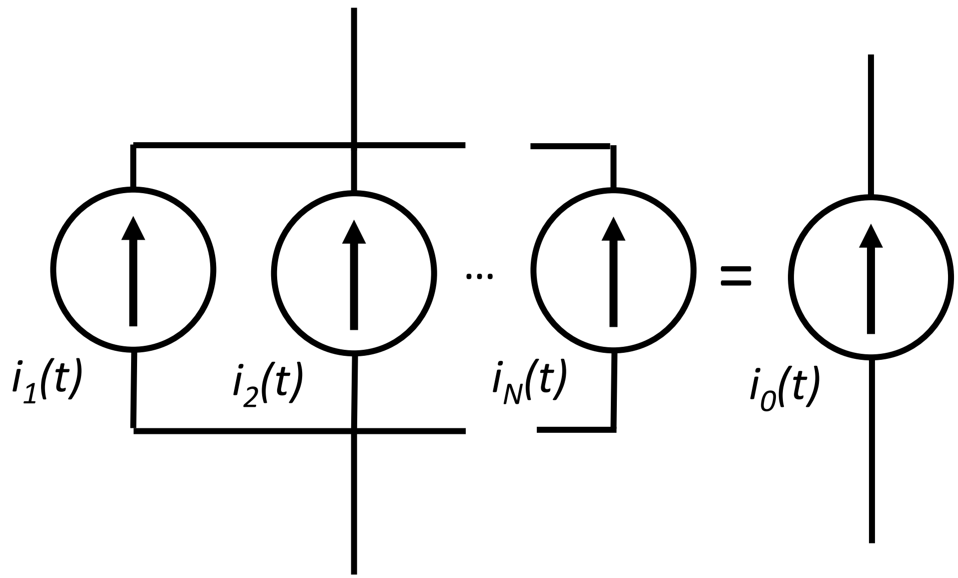
Note: The total power delivered by circuit elements to the flowing charges is equal to the total power absorbed by circuit elements from the flowing charges.

Exercise

Use KCL, KVL and the definitions of independent voltage sources and independent current sources to show the two following equivalences.



$v_0(t)=v_1(t)+v_2(t)+\ldots+v_N(t)$



$i_0(t)=i_1(t)+i_2(t)+\ldots+i_N(t)$