Today's Outline

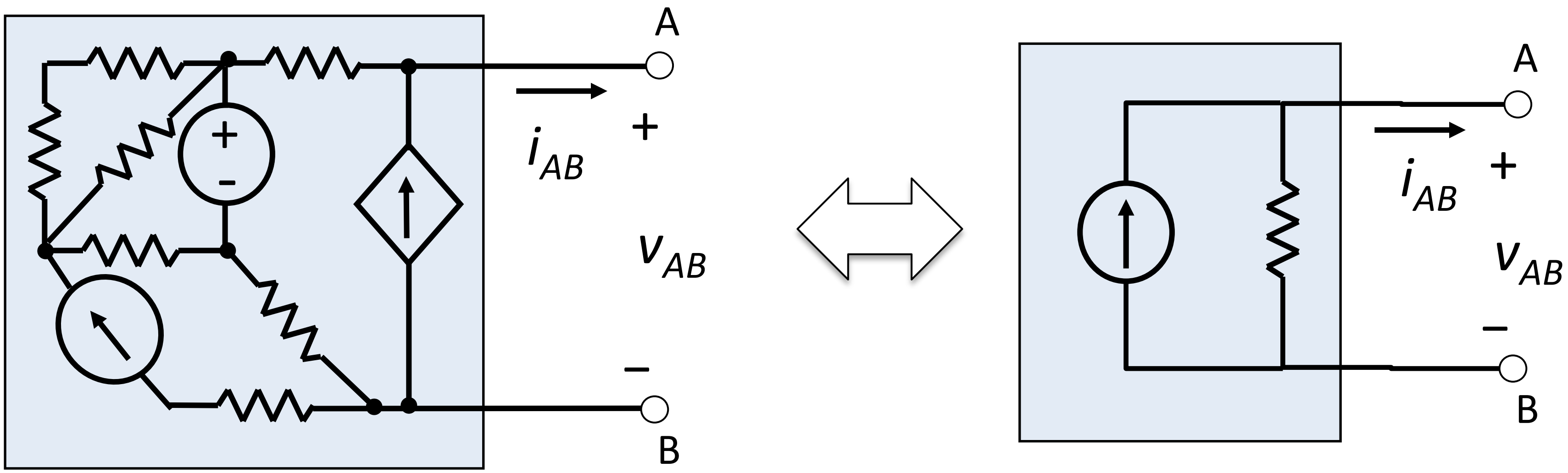
4. Circuit Theorems

- Norton's Theorem

Norton's Theorem

Norton's Theorem: any two terminal circuit composed of independent sources, dependent sources and ideal resistors is equivalent to a parallel combination of a current source and an ideal resistor, known as a **Norton equivalent circuit.** Norton's theorem follows from Thévenin's theorem by a source transformation.

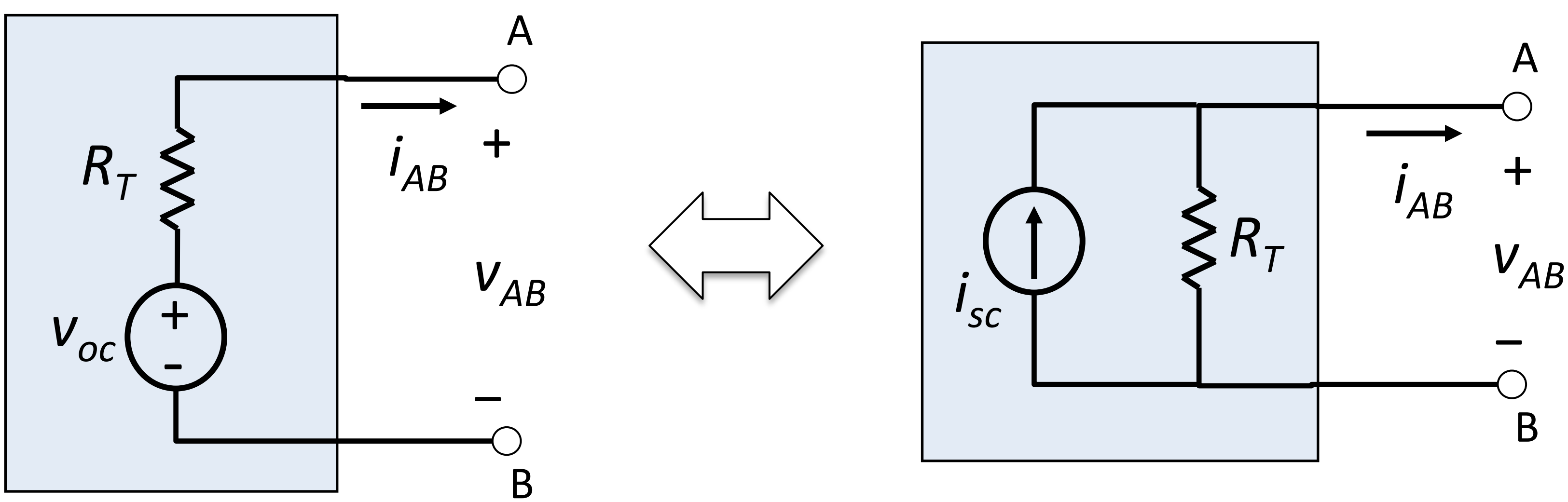
Edward Lawry Norton (1898-1983)



Thévenin and Norton Equivalent Circuits

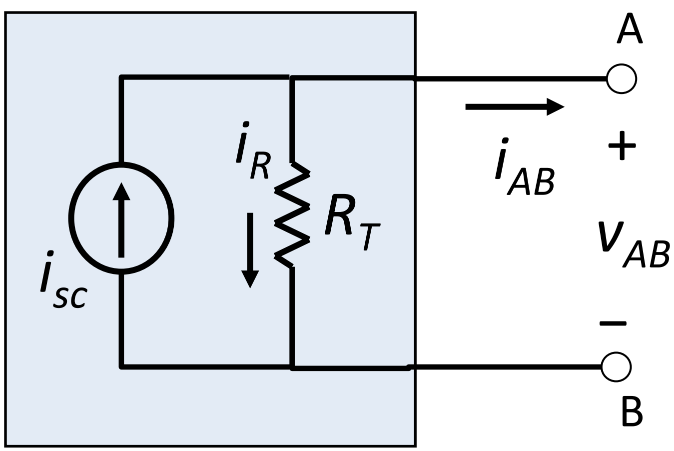
We can convert between a Thévenin equivalent circuit and a Norton equivalent circuit using the source transformation.

$$\boxed{v_{oc} = i_{sc} R_T}$$



Norton Equivalent Circuit

To understand the meaning of the current source, which we denote i_{sc}, we analyze the terminal law for the Norton equivalent circuit.



KCL: $0 = -i_{sc} + i_R + i_{AB}$

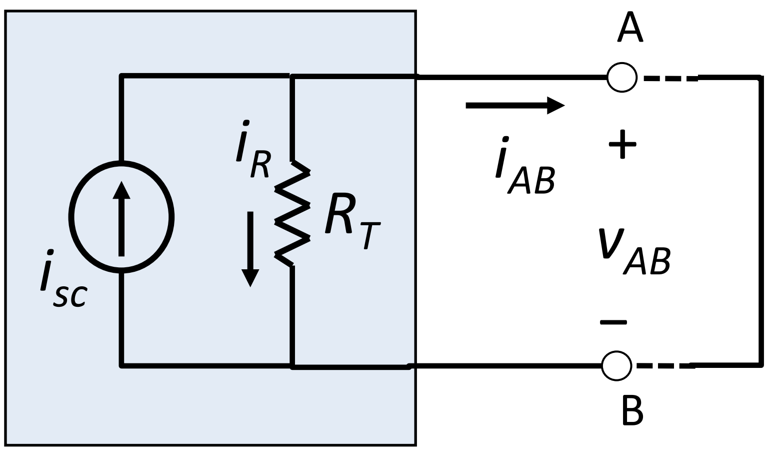
Ohm: $i_R = v_{AB} / R_T$

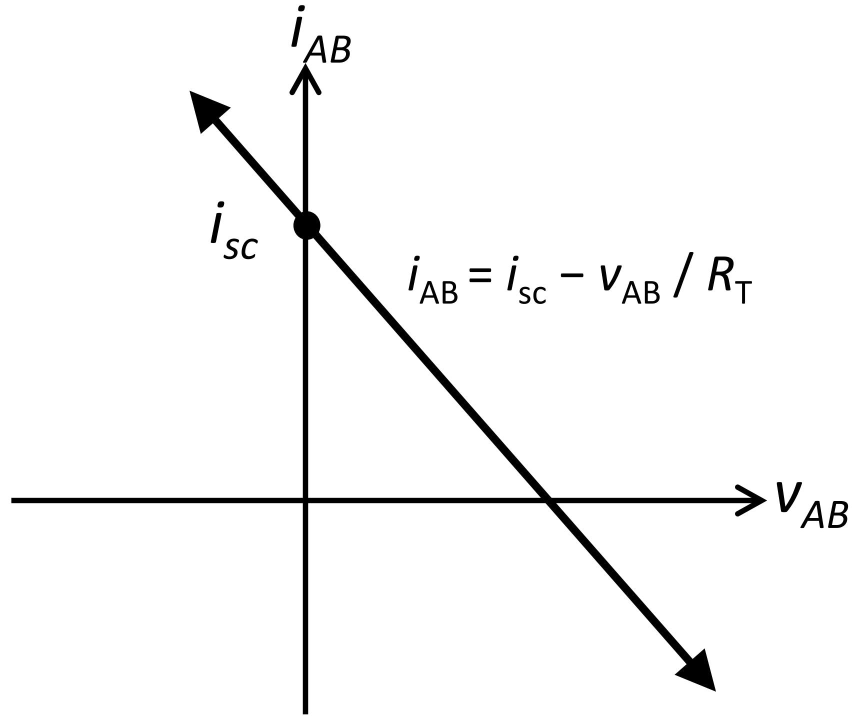
Combining the above:

$i_{AB} = i_{sc} - v_{AB} / R_T$

McGill

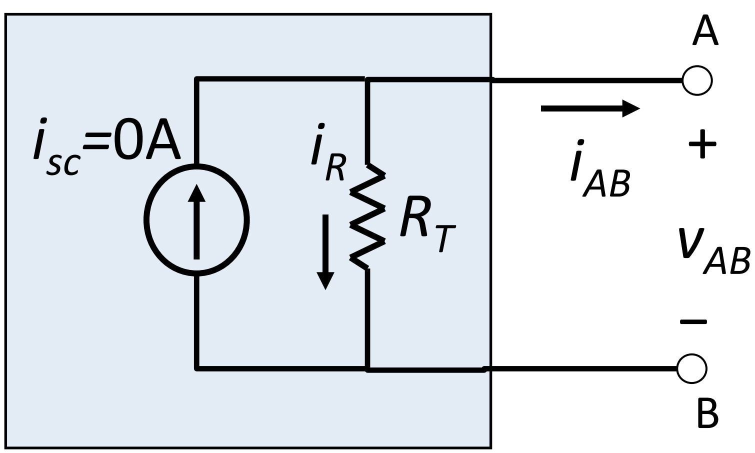
Short Circuit Current

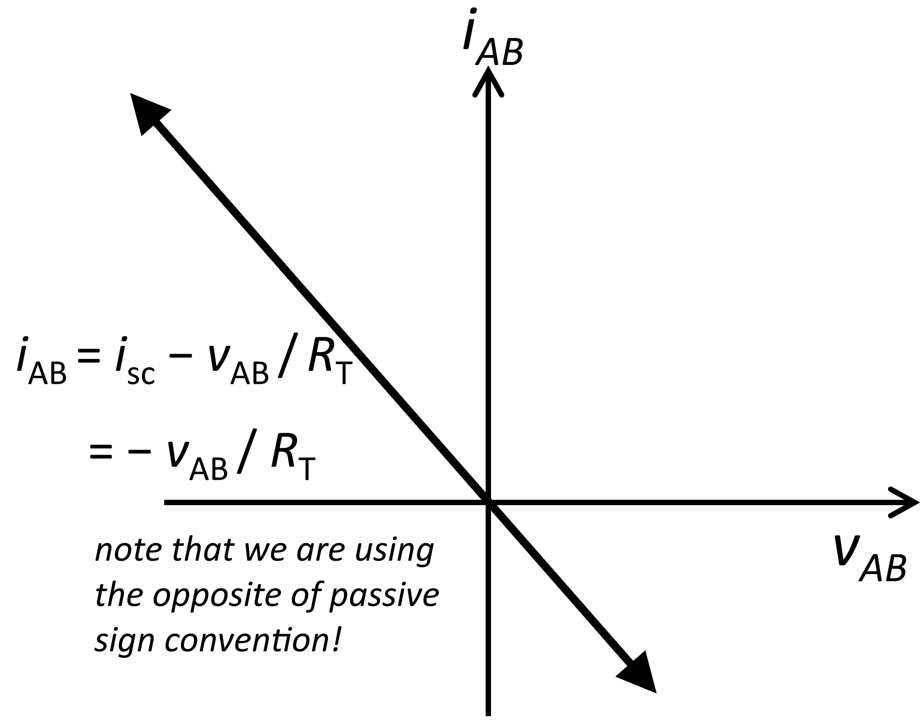




The current i_{sc} is the **short circuit current**, the current that flows through the terminals AB when there is zero terminal voltage v_{AB}=0V (the terminals AB are said to be *shorted*).

Thévenin Resistance





If the current source is *turned off*, i_{sc}=0A, the circuit behaves like an ideal resistor with value equal to the **Thévenin resistance**, R_T.

Finding a Norton Equivalent Circuit

Does the circuit contain **dependent** sources?

no

1. Find i_{sc} .
2. Find R_T as the equivalent resistance with all sources turned off.

or

1. Use source transformations.

yes

1. Find i_{sc} . Is $i_{sc} = 0$?

no

2. Find v_{oc}.
3. Then, $R_T = v_{oc}/i_{sc}$.

yes

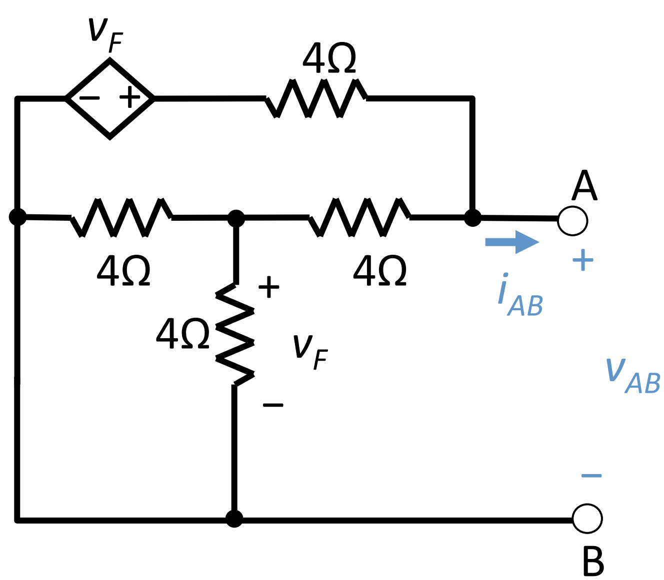
2. Apply a known voltage (eg. v_{ab} = 1V), and find i_{ab}.
3. Find R_T using:

$$i_{ab} = i_{sc} - v_{ab}/R_T$$

Here, we find i_{sc} first instead of v_{oc}. Alternatively, you can find the Thévenin circuit and apply a source transformation.

Example 1

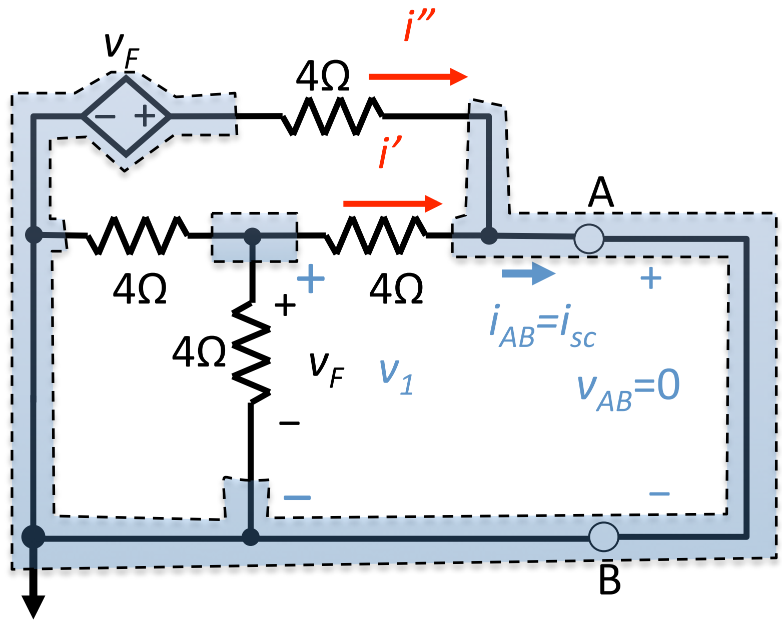
Find the Norton equivalent of the following circuit with respect to the terminals AB.



Strategy: Find i_{sc}, then v_{oc}, and then $R_T = v_{oc} / i_{sc}$. If this does not work, apply a test source or solve for i_{AB}-v_{AB} directly.

Example 1

First, apply a short to AB and find i_{sc}.



We can solve with a single node voltage equation and control variable equation:

$$0 = \frac{v_1}{4\Omega} + \frac{v_1}{4\Omega} + \frac{v_1}{4\Omega}$$

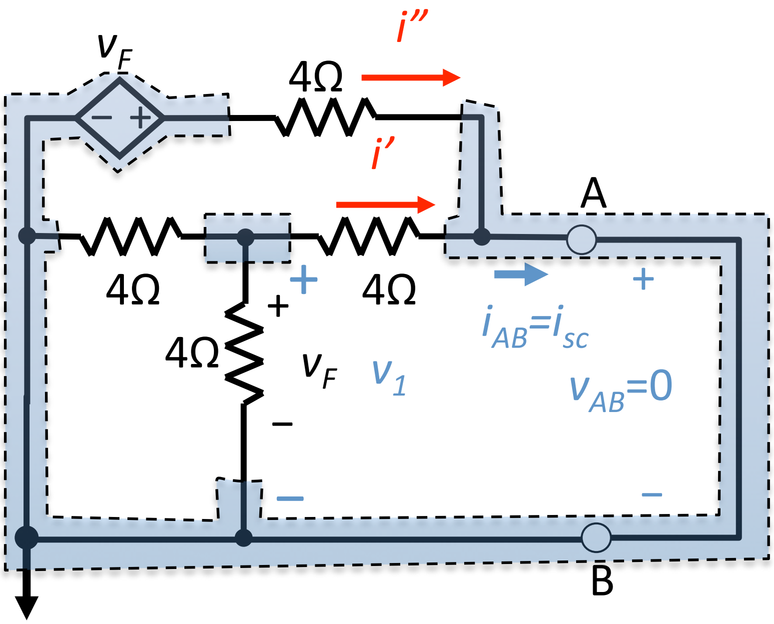
$$v_F = v_1$$

$$\rightarrow \quad v_F = v_1 = 0\text{V}$$

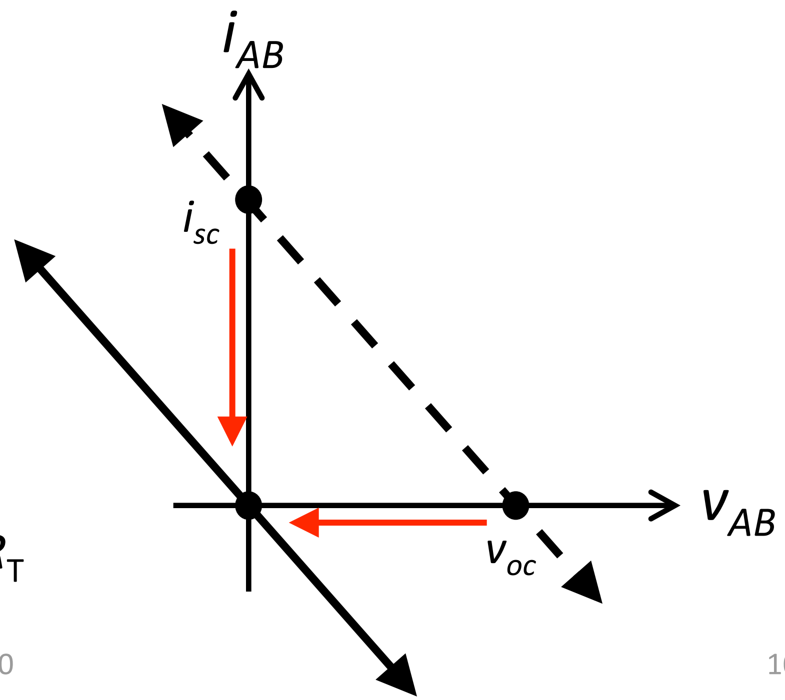
$$i_{sc} = i' + i'' = \frac{v_1}{4\Omega} + \frac{v_F}{4\Omega} = 0\text{A}$$

Example 1

Since i_{sc} =0, the i_{AB}-v_{AB} line passes through the origin. The two intercepts corresponding to open circuit and short circuit conditions are in fact the same intercept!



$R_T = v_{oc} / i_{sc}$ = 0V/0A =

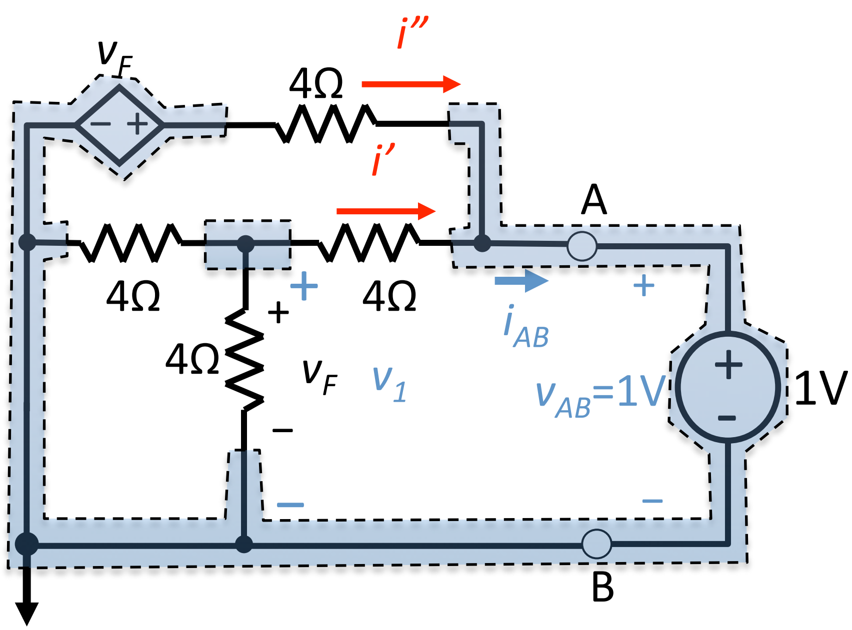


The equation of the line is simplified:

$$i_{AB} = i_{sc} - v_{AB} / R_T = - v_{AB} / R_T$$

Example 1

We need another point on the line. We therefore apply a test voltage of v_{AB}=1V and find the resulting i_{AB}. We could also apply a test current and find the resulting voltage.



We can solve with a single node voltage equation and control variable equation:

$$0=\frac{v_1}{4\Omega}+\frac{v_1}{4\Omega}+\frac{v_1-1\text{V}}{4\Omega}$$

$$v_F=v_1$$

$$\rightarrow \quad v_1=\frac{1/4}{3/4}\text{V}=\frac{1}{3}\text{V}$$

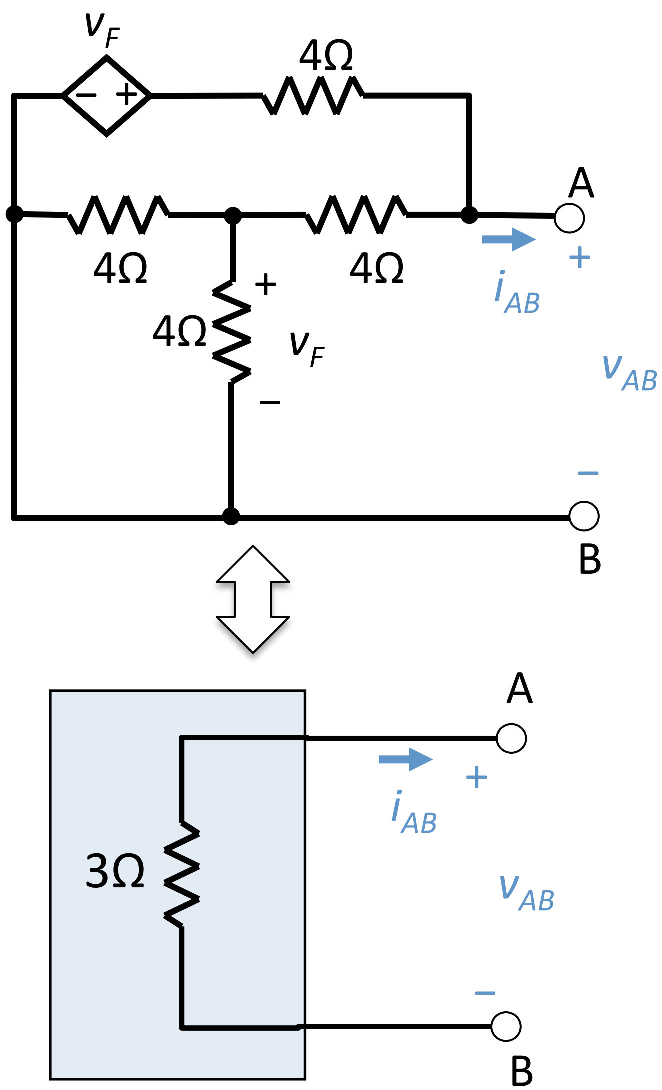
$$i_{AB}=i'+i''=\frac{v_1-1\text{V}}{4\Omega}+\frac{v_F-1\text{V}}{4\Omega}$$

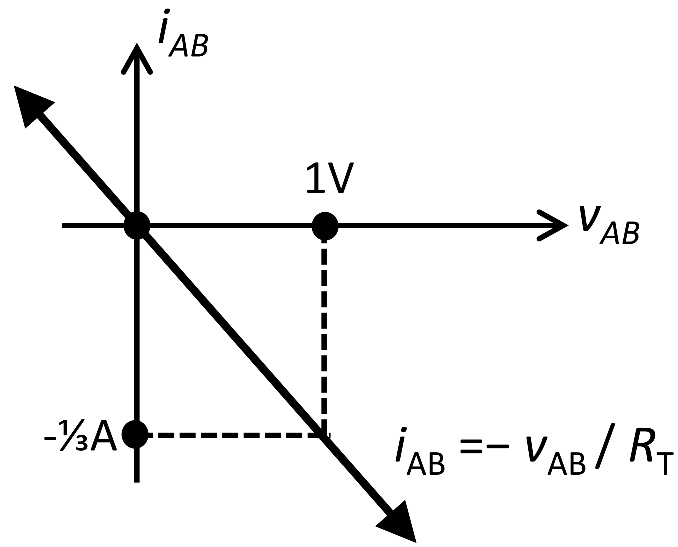
$$=\frac{-2/3\text{V}}{4\Omega}+\frac{-2/3\text{V}}{4\Omega}$$

$$=-\frac{1}{3}\text{A}$$

Example 1

We now have another point on the line:



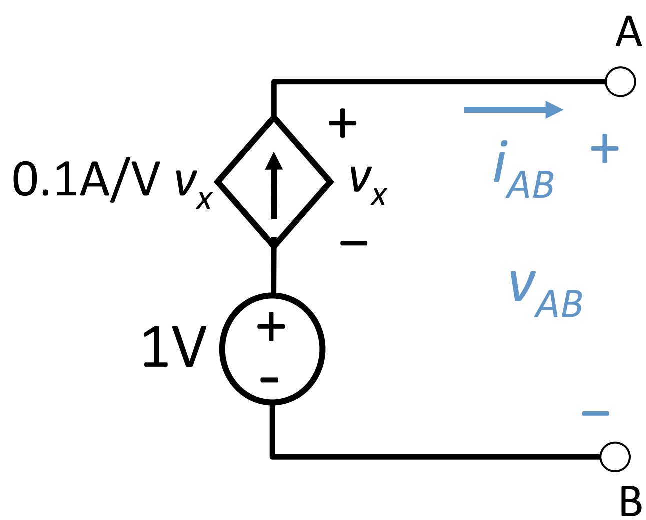


$$\therefore R_T = -\frac{v_{AB}}{i_{AB}} = -\frac{1\text{V}}{-\frac{1}{3}\text{A}} = 3\Omega$$

Using the fact that a 0A source is an open circuit, the Norton equivalent circuit becomes a Thévenin resistance.

Example 2

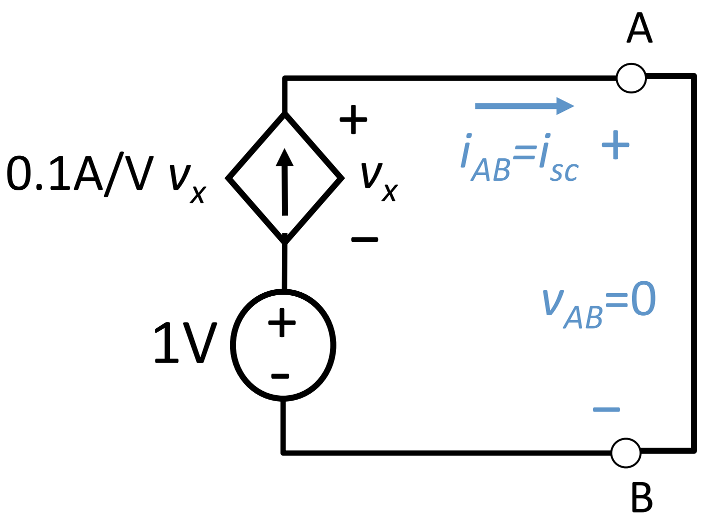
Find the Norton equivalent of the following circuit with respect to the terminals AB.



Strategy: Find $i_{sc,}$ then v_{oc}, and then $R_T = v_{oc} / i_{sc}$. If this does not work, apply a test source or solve for i_{AB}-v_{AB} directly.

Example 2

First, apply a short circuit to AB, and find i_{sc}.



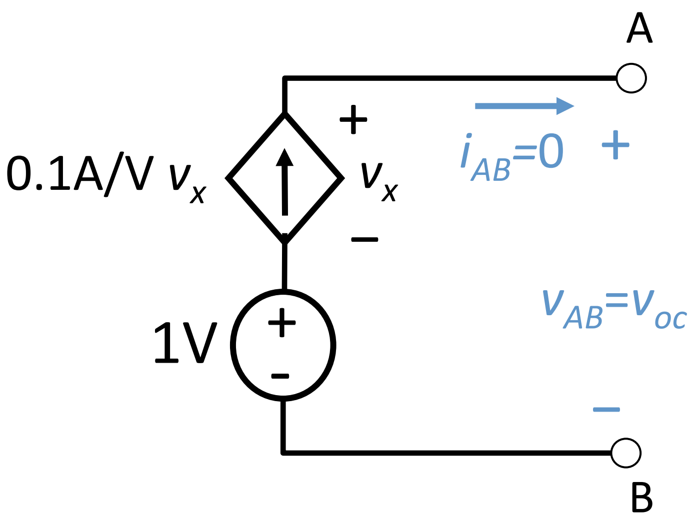
We can solve with a single KVL equation and control variable equation:

$$0 = -1\text{V} - v_x \quad \rightarrow \quad v_x = -1\text{V}$$

$$i_{sc} = 0.1\tfrac{\text{A}}{\text{V}} \cdot v_x = -100\text{mA}$$

Example 2

Second, apply an open circuit to AB, and find v_{oc}.



We can solve with a control variable equation and a single KVL equation:

$$0 = i_{AB} = 0.1\tfrac{\text{A}}{\text{V}} \cdot v_x \quad \rightarrow \quad v_x = 0\text{V}$$

$$0 = -1\text{V} - v_x + v_{oc} \quad \rightarrow \quad v_{oc} = 1\text{V} - v_x = 1\text{V}$$

Example 2

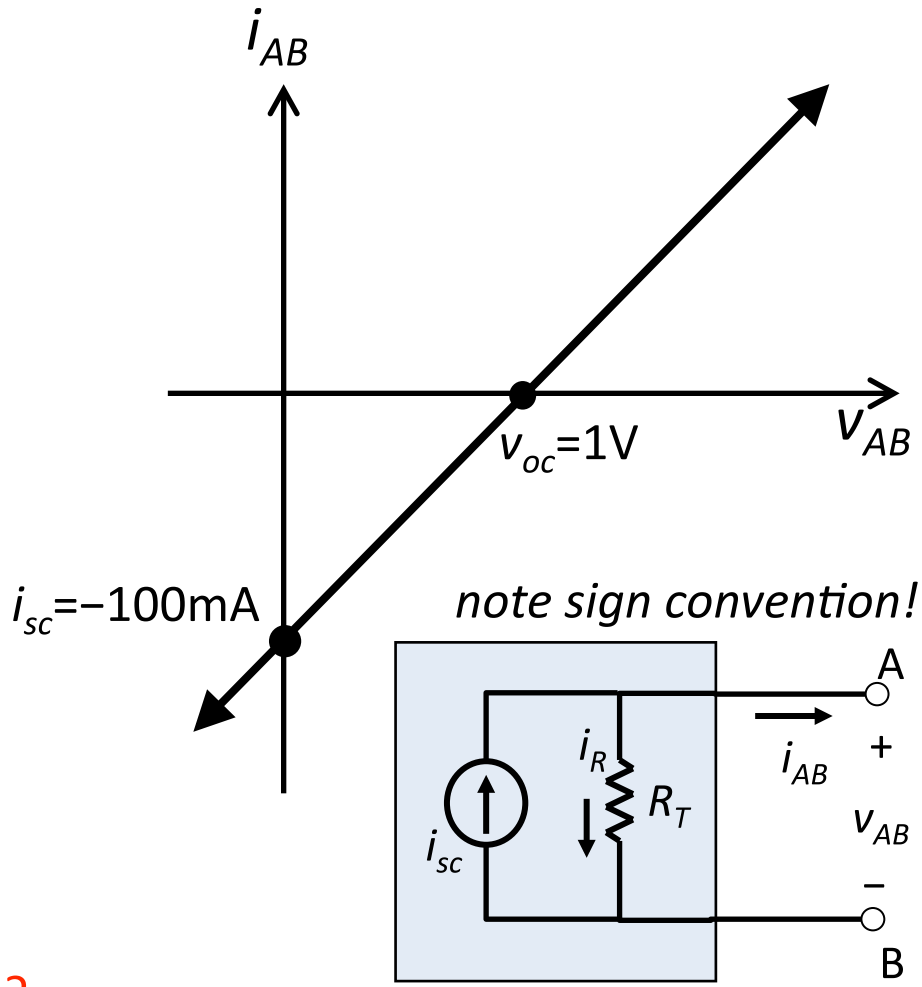
The i_{AB}-v_{AB} diagram has an opposite slope to what we usually observe. The Thévenin resistance is negative!

$$R_T = \frac{v_{oc}}{i_{sc}} = \frac{1\text{V}}{-100\text{mA}} = -10\Omega$$

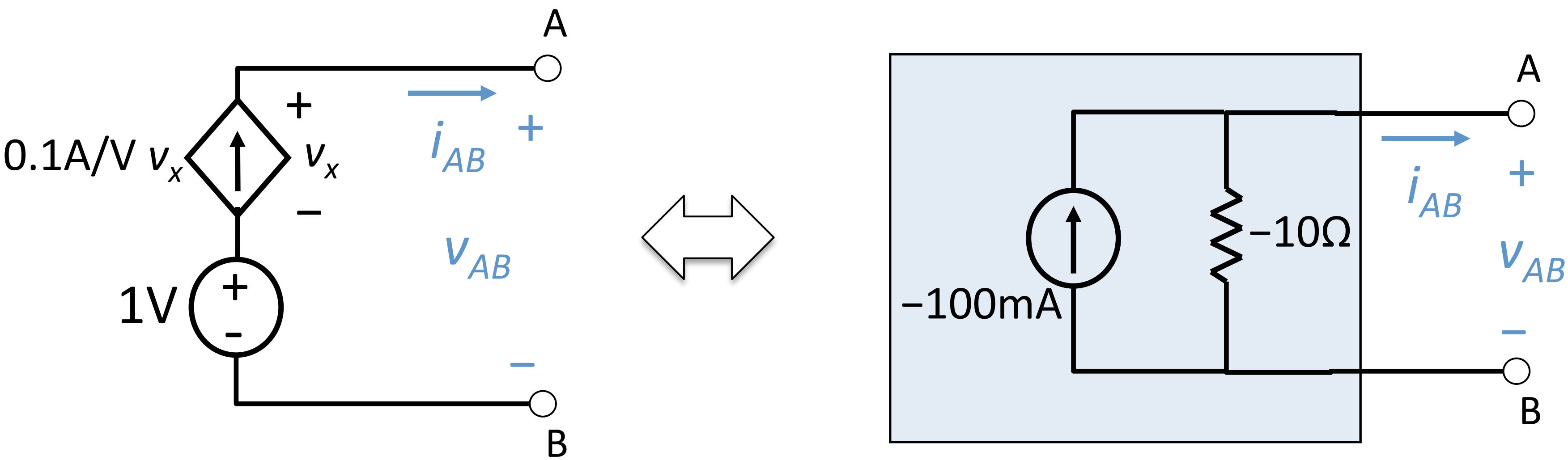
One consequence is that this circuit model can *deliver* an arbitrarily large amount of power $p_{DEL}=i_{AB}\,v_{AB}$.

Suggestion: Show that the Thévenin resistance is still negative if you swap the labels A and B.

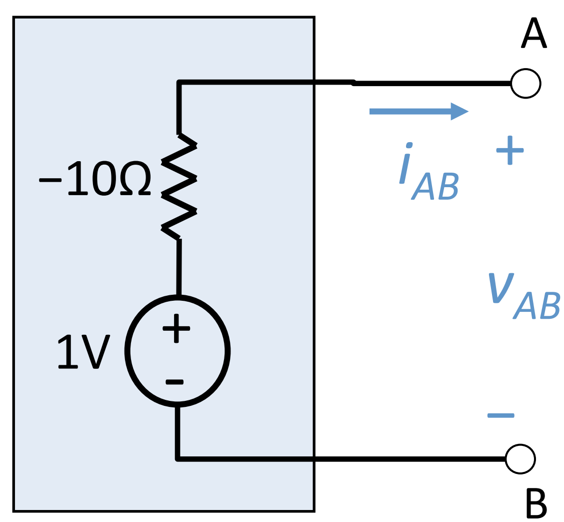
Question: What happens if a 10Ω load resistance is connected across A and B?



Example 2: Negative Thévenin Resistance



Remember that the Norton and Thévenin equivalent circuits are just compact models to represent linear two-terminal circuits.



Circuits with dependent sources can give rise to negative Thévenin resistance. *Does this mean that nature permits a "negative resistance"?*