Design Problems

DP 3-1 The circuit shown in Figure DP 3.1 uses a potentiometer to produce a variable voltage. The voltage v_m varies as a knob connected to the wiper of the potentiometer is turned. Specify the resistances R_1 and R_2 so that the following three requirements are satisfied:

1. The voltage v_m varies from 8 V to 12 V as the wiper moves from one end of the potentiometer to the other end of the potentiometer.
2. The voltage source supplies less than 0.5 W of power.
3. Each of R_1, R_2, and R_P dissipates less than 0.25 W.

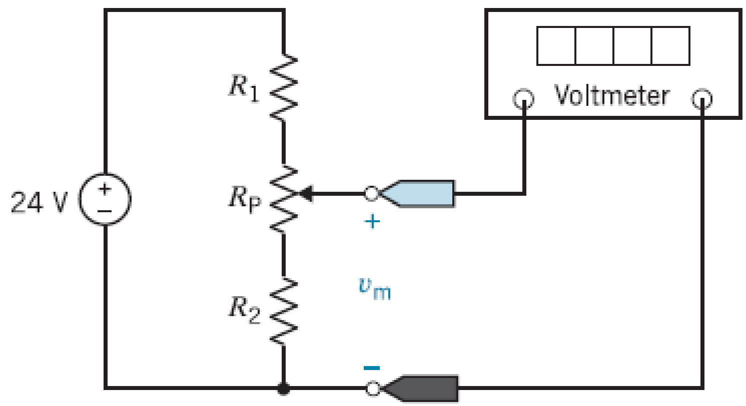
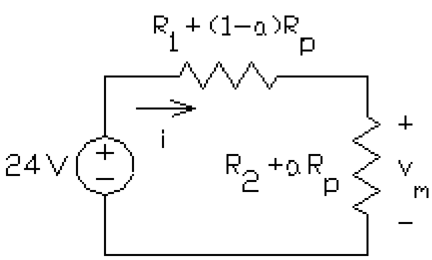


Figure DP 3.1

Solution:



Using voltage division:

$$v_m = \frac{R_2 + aR_p}{R_1 + (1-a)R_p + R_2 + aR_p} 24 = \frac{R_2 + aR_p}{R_1 + R_2 + R_p} 24$$

$$v_m = 8 \text{ V when a} = 0 \;\Rightarrow\; \frac{R_2}{R_1 + R_2 + R_p} = \frac{1}{3}$$

$$v_m = 12 \text{ V when a} = 1 \;\Rightarrow\; \frac{R_2 + R_p}{R_1 + R_2 + R_p} = \frac{1}{2}$$

The specification on the power of the voltage source indicates

$$\frac{24^2}{R_1 + R_2 + R_p} \le \frac{1}{2} \quad\Rightarrow\quad R_1 + R_2 + R_p \ge 1152\,\Omega$$

Try $R_p = 2000\ \Omega$. Substituting into the equations obtained above using voltage division gives $3R_2 = R_1 + R_2 + 2000$ and $2(R_2 + 2000) = R_1 + R_2 + 2000$. Solving these equations gives $R_1 = 6000\,\Omega$ and $R_2 = 4000\,\Omega$.

With these resistance values, the voltage source supplies 48 mW while R_1, R_2 and R_p dissipate 24 mW, 16 mW and 8 mW respectively. Therefore the design is complete.

DP 3-2 The resistance R_L in Figure DP 3.2 is the equivalent resistance of a pressure transducer. This resistance is specified to be 200 Ω ± 5 percent. That is, 190 Ω ≤ R_L ≤ 210 Ω. The voltage source is a 12 V ± 1 percent source capable of supplying 5 W. Design this circuit, using 5 percent, 1/8-watt resistors for R_1 and R_2, so that the voltage across R_L is

$$v_o = 4\text{V} \pm 10\%$$

(A 5 percent, 1/8-watt 100-Ω resistor has a resistance between 95 and 105 Ω and can safely dissipate 1/8-W continuously.)

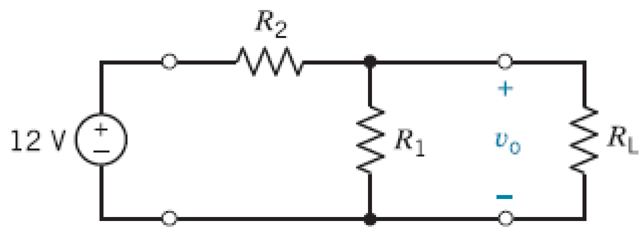


Figure DP 3.2

Solution:

Try $R_1 = \infty$. That is, R_1 is an open circuit. From KVL, 8 V will appear across R_2. Using voltage division, $\frac{200}{R_2+200}12 = 4 \quad \Rightarrow \quad R_2 = 400\ \Omega$. The power required to be dissipated by R_2 is $\frac{8^2}{400} = 0.16\text{ W} < \frac{1}{8}\text{ W}$. To reduce the voltage across any one resistor, let's implement R_2 as the series combination of two 200 Ω resistors. The power required to be dissipated by each of these resistors is $\frac{4^2}{200} = 0.08\text{ W} < \frac{1}{8}\text{ W}$.

Now let's check the voltage:

$$11.88\frac{190}{190+420} < v_0 < 12.12\frac{210}{210+380}$$

$$3.700 < v_0 < 4.314$$

$$4 - 7.5\% < v_0 < 4 + 7.85\%$$

Hence, v_o = 4 V ± 8% and the design is complete.

DP 3-3 A phonograph pickup, stereo amplifier, and speaker are shown in Figure DP 3.3*a* and redrawn as a circuit model as shown in Figure DP 3.3*b*. Determine the resistance *R* so that the voltage *v* across the speaker is 16 V. Determine the power delivered to the speaker.

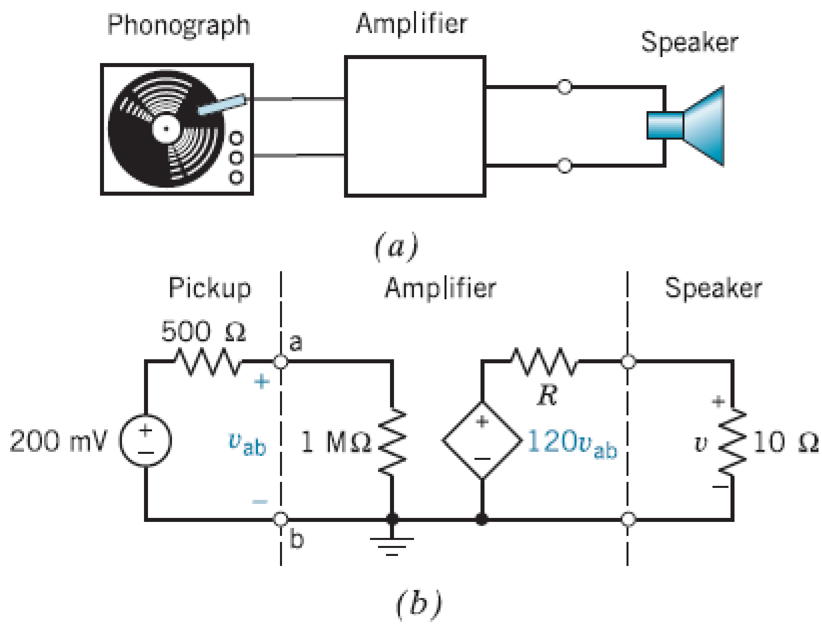


Figure DP 3.3

Solution:

$$V_{ab} \cong 200 \text{ mV}$$

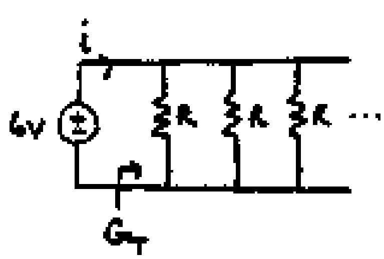
$$v = \frac{10}{10+R}\;120\,V_{ab} \;=\frac{10}{10+R}\;(120)\;(0.2)$$

$$\text{let } v = 16 \;=\; \frac{240}{10+R} \;\Rightarrow\; \underline{R = 5\,\Omega}$$

$$\therefore\; \text{P} = \frac{16^2}{10} = \underline{25.6\text{W}}$$

DP 3-4 A Christmas tree light set is required that will operate from a 6-V battery on a tree in a city park. The heavy-duty battery can provide 9A for the four-hour period of operation each night. Design a parallel set of lights (select the maximum number of lights) when the resistance of each bulb is 12 Ω.

Solution:



$$i = G_T v = \frac{N}{R} v \quad \text{where } G_T = \sum_{n=1}^{N} \frac{1}{R_n} = N\left(\frac{1}{R}\right)$$

$$\therefore\; N = \frac{iR}{v} = \frac{(9)(12)}{6} = \underline{18 \text{ bulbs}}$$

DP 3-5 The input to the circuit shown in Figure DP 3.5 is the voltage source voltage, v_s. The output is the voltage v_o. The output is related to the input by

$$v_o = \frac{R_2}{R_1 + R_2} v_s = g v_s$$

The output of the voltage divider is proportional to the input. The constant of proportionality, g, is called the gain of the voltage divider and is given by

$$g = \frac{R_2}{R_1 + R_2}$$

The power supplied by the voltage source is

$$p = v_s i_s = v_s \left(\frac{v_s}{R_1 + R_2} \right) = \frac{v_s^2}{R_1 + R_2} = \frac{v_s^2}{R_{in}}$$

where

$$R_{in} = R_1 + R_2$$

is called the input resistance of the voltage divider.

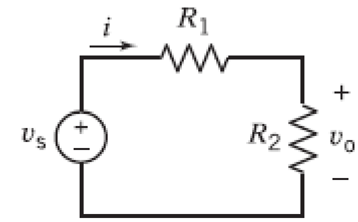


Figure DP 3.5

(a) Design a voltage divider to have a gain, $g = 0.65$.

(b) Design a voltage divider to have a gain, $g = 0.65$, and an input resistance, $R_{in} = 2500\ \Omega$.

Solution:

Notice that $$g = \frac{R_2}{R_1 + R_2} \quad \Rightarrow \quad g\, R_1 = (1 - g) R_2$$

Thus either resistance can be determined from the other resistance and the gain of the voltage divider. Also

$$g = \frac{R_2}{R_1 + R_2} = \frac{R_2}{R_{in}} \quad \Rightarrow \quad R_2 = g\, R_{in}$$

Consequently $$g\, R_1 = (1 - g) R_2 = (1 - g)\, g\, R_{in} \quad \Rightarrow \quad R_1 = (1 - g) R_{in}$$

(a) The solution of this problem is not unique. Given any value of R_1, we can determine a value of R_2 that will cause $g = 0.65$. Let's pick a convenient value for R_1, say

$$R_1 = 100\ \Omega$$

Then $$g\, R_1 = (1 - g) R_2 \quad \Rightarrow \quad R_2 = \frac{g\, R_1}{1 - g} = \frac{0.65 \times 100}{1 - 0.65} = 186\ \Omega$$

(b) $$R_2 = g\, R_{in} = 0.65 \times 2500 = 1625\ \Omega$$

and $$R_1 = (1 - g) R_{in} = (1 - 0.65) 2500 = 875\ \Omega$$

DP 3-6 The input to the circuit shown in Figure DP 3.6 is the current source current, i_s. The output is the current i_o. The output is related to the input by

$$i_o = \frac{R_1}{R_1 + R_2} i_s = g i_s$$

The output of the current divider is proportional to the input. The constant of proportionality, g, is called the gain of the current divider and is given by

$$g = \frac{R_1}{R_1 + R_2}$$

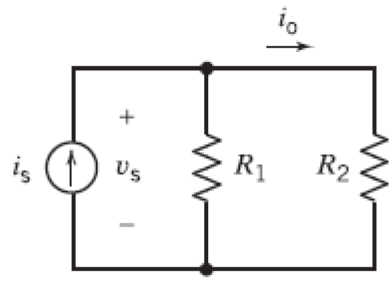


Figure DP 3.6

The power supplied by the current source is

$$p = v_s i_s = \left[i_s \left(\frac{R_1 R_2}{R_1 + R_2} \right) \right] i_s = \frac{R_1 R_2}{R_1 + R_2} i_s^2 = R_{in} i_s^2$$

where
$$R_{in} = \frac{R_1 R_2}{R_1 + R_2}$$

is called the input resistance of the current divider.

(a) Design a current divider to have a gain, $g = 0.65$.
(b) Design a current divider to have a gain, $g = 0.65$, and an input resistance, $R_{in} = 10000\ \Omega$.

Solution:

Notice that
$$g = \frac{R_1}{R_1 + R_2} \quad \Rightarrow \quad g\, R_2 = (1-g) R_1$$

Thus either resistance can be determined from the other resistance and the gain of the current divider. Also

$$g = \frac{R_1}{R_1 + R_2} = \frac{R_{in}}{R_2} \quad \Rightarrow \quad R_2 = \frac{R_{in}}{g}$$

Consequently
$$(1-g) R_1 = g\, R_2 = R_{in} \quad \Rightarrow \quad R_1 = \frac{R_{in}}{(1-g)}$$

Thus specified values of g and R_{in} uniquely determine the required values of R_1 and R_2.

(a) The solution of this problem is not unique. Given any value of R_1, we can determine a value of R_2 that will cause $g = 0.65$. Let's pick a convenient value for R_1, say

$$R_1 = 100\ \Omega$$

Then
$$g\, R_2 = (1-g) R_1 \quad \Rightarrow \quad R_2 = \frac{(1-g) R_1}{g} = \frac{(1-0.65) \times 100}{0.65} = 54\ \Omega$$

(b)
$$R_2 = \frac{R_{in}}{g} = \frac{10000}{0.65} = 15385\ \Omega$$

and
$$R_1 = \frac{R_{in}}{(1-g)} = \frac{10000}{(1-0.65)} = 28571\ \Omega$$

DP 3-7 Design the circuit shown in Figure DP 3-7 to have an output $v_o = 8.5$ V when the input is $v_s = 12$ V. The circuit should require no more than 1 mW from the voltage source.

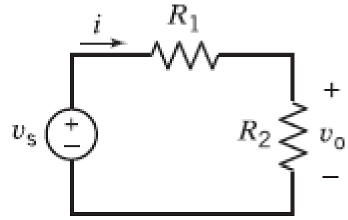


Figure DP 3.7

Solution:

The required gain is $$g = \frac{v_o}{v_s} = \frac{8.5}{12} = 0.7083$$

Consequently, $$0.7083\,R_1 = (1 - 0.7083)\,R_2 \quad \Rightarrow \quad R_2 = 2.428\,R_1$$

It is also required that

$$0.001 \ge \frac{12^2}{R_1 + R_2} \quad \Rightarrow \quad R_1 + R_2 \ge 144000 \quad \Rightarrow \quad 3.428\,R_1 \ge 144000 \quad \Rightarrow \quad R_1 \ge 42007\ \Omega$$

For example, $$R_1 = 45\ \text{k}\Omega \quad \text{and} \quad R_2 = 109.26\ \text{k}\Omega$$

DP 3-8 Design the circuit shown in Figure DP 3.8 to have an output $i_o = 1.8$ mA when the input is $i_s = 5$ mA. The circuit should require no more than 1 mW from the current source.

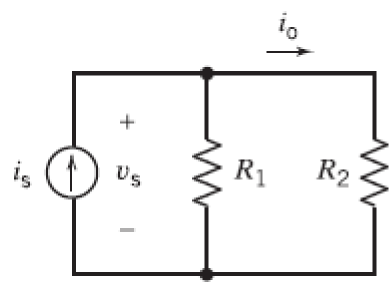


Figure DP 3.8

Solution:

The required gain is $$g = \frac{i_o}{i_s} = \frac{1.8}{5} = 0.36$$

Consequently, $$0.36\,R_2 = (1 - 0.36)\,R_1 \quad \Rightarrow \quad R_2 = 1.778\,R_1$$

It is also required that

$$0.001 \ge 0.005^2 \frac{R_1 R_2}{R_1 + R_2} \quad \Rightarrow \quad \frac{R_1 + R_2}{R_1 R_2} = \frac{2.778\,R_1}{1.778\,R_1^2} \ge \frac{0.005^2}{0.001} = 0.025$$

$$\Rightarrow \quad R_1 \le \frac{2.778}{1.778(0.025)} = 62.5\ \Omega$$

For example, $$R_1 = 60\ \Omega \quad \text{and} \quad R_2 = 106.7\ \Omega$$

DP 3.9 A thermistor is a temperature dependent resistor. The thermistor resistance, R_T, is related to the temperature by the equation

$$R_{\mathrm{T}} = R_{\mathrm{T}}\, e^{\beta\left(1/T - 1/T_{\mathrm{o}}\right)}$$

where T has units of °K and R is in Ohms. R_0 is resistance at temperature T_0 and the parameter β is in °K. For example, suppose that a particular thermistor has a resistance R_0 = 620 Ω at the temperature T_0 = 20 °C = 293 °K and β = 3330 °K. At T = 70 °C = 343 °K the resistance of this thermistor will be

$$R_{\mathrm{T}} = 620\, e^{3330\left(1/342 - 1/293\right)} = 121.68\ \ \Omega$$

In Figure DP 3-9 this particular thermistor in used in a voltage divider circuit. Specify the value of the resistor R that will cause the voltage v_T across the thermistor to be 4 V when the temperature is 100 °C.

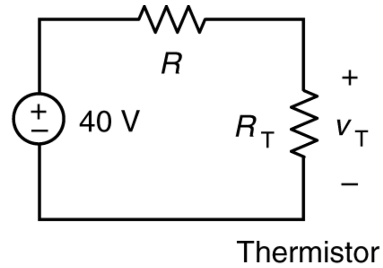


Figure DP 3-9

Solution

At $T = 373$°K $$R_{\mathrm{T}} = 620\, e^{3330\left(1/372 - 1/293\right)} = 54.17\ \ \Omega$$

Using voltage division

$$4 = v_{\mathrm{T}} = \frac{54.17}{R + 54.17}(40) \quad \Rightarrow \quad R + 54.17 = \frac{54.17}{4}(40)$$

$$R = 541.7 - 54.17 = 487.54\ \Omega$$

DP3-10 The circuit shown in Figure DP 3-10 contains a thermistor that has a resistance $R_0 = 620\ \Omega$ at the temperature $T_0 = 20$ °C = 293 °K and $\beta = 3330$ °K. (See problem DP 3-9.) Design this circuit (that is, specify the values of R and V_s) so that the thermistor voltage is $v_T = 4$ V when $T = 100$ °C and $v_T = 20$ V when $T = 0$ °C.

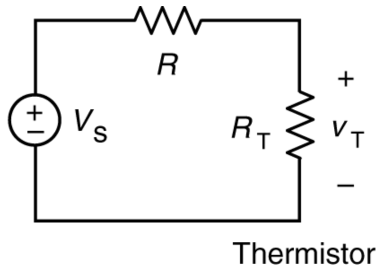


Figure DP 3-10

Solution

At $T = 0°C = 273°K$ $\qquad$ $R_T = 620\,e^{3330(1/273-1/293)} = 1425.6\ \ \Omega$

At $T = 100°C = 373°K$ $\qquad$ $R_T = 620\,e^{3330(1/373-1/293)} = 54.17\ \ \Omega$

Using voltage division

$$4 = v_T = \frac{54.17}{R+54.17}(V_s) \quad \Rightarrow \quad R + 54.17 = \frac{54.17}{4}(V_s)$$

$$20 = v_T = \frac{1425.6}{R+1425.6}(V_s) \quad \Rightarrow \quad R + 1425.6 = \frac{1425.6}{20}(V_s)$$

In matrix form:

$$\begin{bmatrix} \dfrac{54.17}{4} & -1 \\ \dfrac{1425.6}{20} & -1 \end{bmatrix} \begin{bmatrix} V_s \\ R \end{bmatrix} = \begin{bmatrix} 54.17 \\ 1425.6 \end{bmatrix}$$

Solving gives $\qquad$ $V_s = 23.7528$ V and $R = 267.5029\ \Omega$

DP3-11 The circuit shown in Figure DP 3-11 is designed help orange grower protect their crops against frost by sounding an alarm when the temperature falls below freezing. It contains a thermistor that has a resistance R_0 = 620 Ω at the temperature T_0 = 20 °C = 293 °K and β = 3330 °K. (See problem DP 3-9.)

The alarm will sound when the voltage at the – input of the comparator is less than the voltage at the + input. Using voltage division twice, we see that the alarm sounds whenever

$$\frac{R_2}{R_T + R_2} < \frac{R_4}{R_3 + R_4}$$

Determine values of R_2, R_3 and R_4 that cause the alarm to sound whenever the temperature is below freezing.

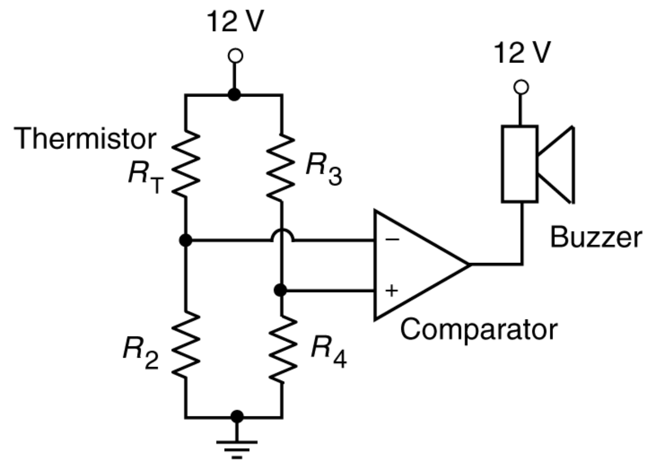


Figure DP 3-11

Solution:

The solution is not unique. For example, pick R_3 = 30 kΩ and R_4 = 10 kΩ. The alarm turns on and off when

$$\frac{R_2}{R_T + R_2} = \frac{10}{30+10} = 0.25$$

At freezing the temperature is T = 0°C = 273°K and the thermistor resistance is

$$R_T = 620\, e^{3330(1/273 - 1/293)} = 1425.6\ \ \Omega$$

At freezing, we have $$\frac{R_2}{1425.6 + R_2} = 0.25 \quad \Rightarrow \quad 0.75\, R_2 = 356.4$$

$$R_2 = 475.2\ \Omega$$

Let's check. At –2 °C $$R_T = 620\, e^{3330(1/271 - 1/293)} = 1560\ \ \Omega$$

$$\frac{475.2}{1560 + 475.2} = 0.2335 < 0.25$$

so the alarm is on. At 2 °C $$R_T = 620\, e^{3330(1/275 - 1/293)} = 1305\ \ \Omega$$

$$\frac{475.2}{1305 + 475.2} = 0.2669 > 0.25$$

so the alarm is off.

Chapter 3 Resistive Circuits

Exercises

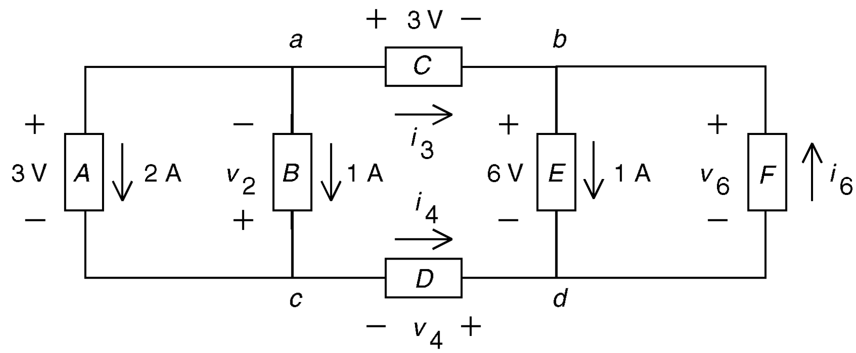


Figure E 3.2-1

Exercise 3.2-1 Determine the values of i_3, i_4, i_6, v_2, v_4, and v_6 in Figure E 3.2-1.

Answer: $i_3 = –3$ A, $i_4 = 3$ A, $i_6 = 4$ A, $v_2 = –3$ V, $v_4 = –6$ V, $v_6 = 6$ V

Solution:

Apply KCL at node *a* to get $2 + 1 + i_3 = 0 \Rightarrow i_3 = -3$ A

Apply KCL at node *c* to get $2 + 1 = i_4 \Rightarrow i_4 = 3$ A

Apply KCL at node *b* to get $i_3 + i_6 = 1 \Rightarrow -3 + i_6 = 1 \Rightarrow i_6 = 4$ A

Apply KVL to the loop consisting of elements *A* and *B* to get

$$-v_2 - 3 = 0 \Rightarrow v_2 = -3 \text{ V}$$

Apply KVL to the loop consisting of elements *C*, *E*, *D*, and *A* to get

$$3 + 6 + v_4 - 3 = 0 \Rightarrow v_4 = -6 \text{ V}$$

Apply KVL to the loop consisting of elements *E* and *F* to get

$$v_6 - 6 = 0 \Rightarrow v_6 = 6 \text{ V}$$

Check: The sum of the power supplied by all branches is

$$-(3)(2) + (-3)(1) - (3)(-3) + (-6)(3) - (6)(1) + (6)(4) = -6 - 3 + 9 - 18 - 6 + 24 = 0$$

Exercise 3.3-1 Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.3-1*a*.
Hint: Figure E3.3-1*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.
Answer : $v_m = 2$ V

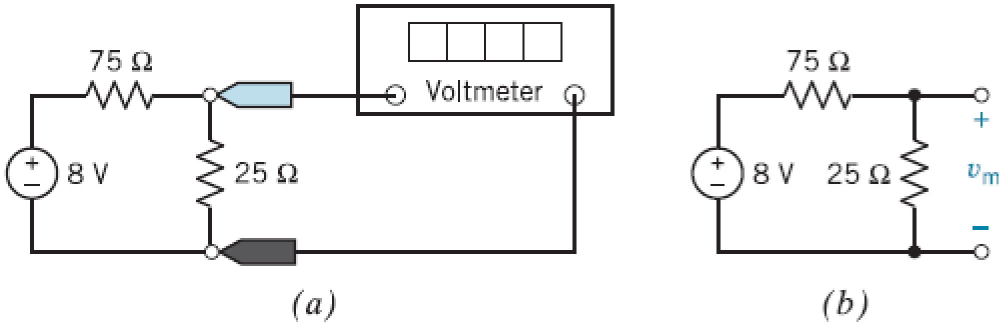


Figure E 3.3-1

Solution: From voltage division $\Rightarrow v_m = \frac{25}{25+75}(8) = 2 \quad \text{V}$

Exercise 3.3-2 Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.3-2*a*.
Hint: Figure E 3.3-2*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.
Answer: $v_m = -2$ V

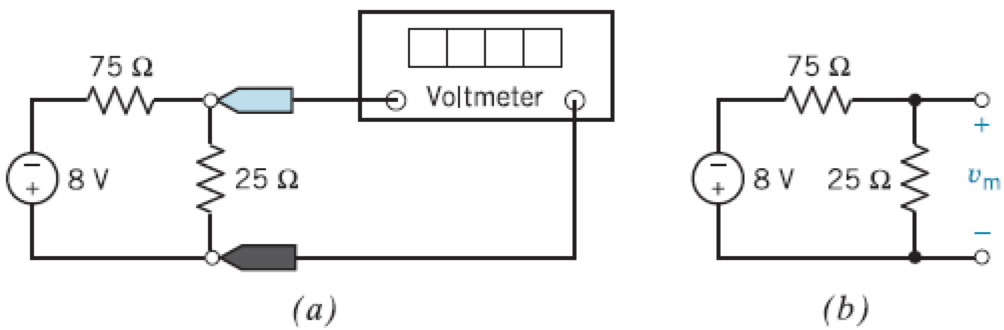
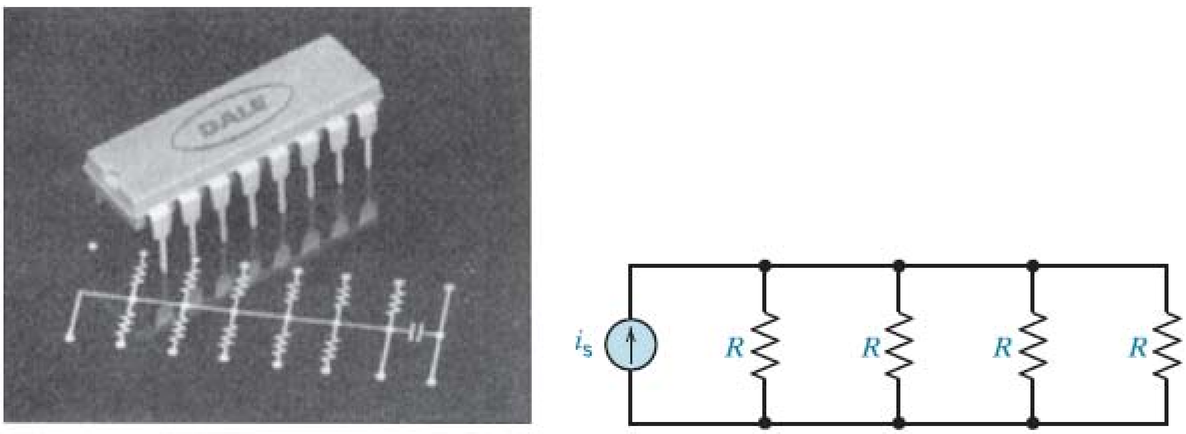


Figure E 3.3-2

Solution: From voltage division $\Rightarrow v_m = \frac{25}{25+75}(-8) = -2 \quad \text{V}$

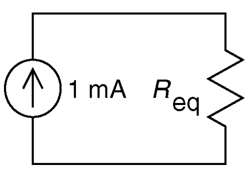
Exercise 3.4-1 A resistor network consisting of parallel resistors is shown in a package used for printed circuit board electronics in Figure E 3.4-1*a*. This package is only 2 cm × 0.7 cm, and each resistor is 1 kΩ. The circuit is connected to use four resistors as shown in Figure E 3.4-1*b*. Find the equivalent circuit for this network. Determine the current in each resistor when $i_s = 1$ mA.
Answer: $R_p = 250\ \Omega$



(a) (b)

Figure E 3.4-1

Solution:

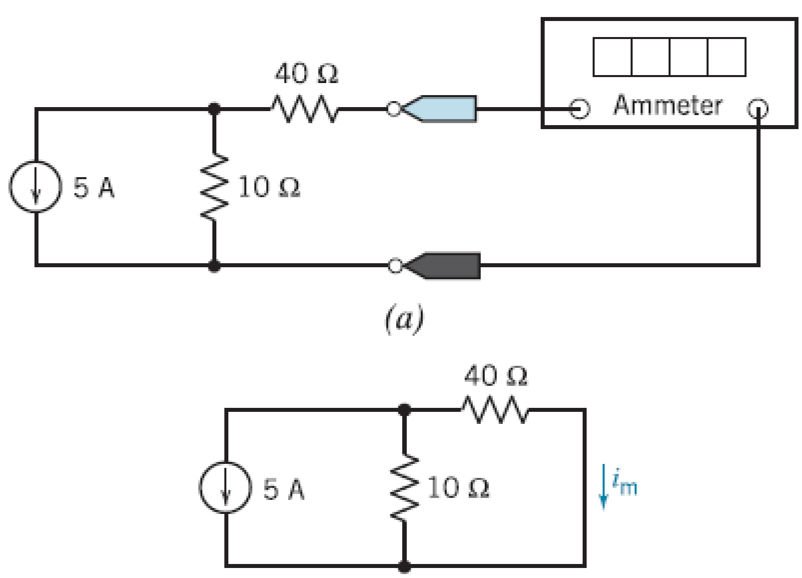


$$\frac{1}{R_{eq}} = \frac{1}{10^3}+\frac{1}{10^3}+\frac{1}{10^3}+\frac{1}{10^3} = \frac{4}{10^3} \Rightarrow R_{eq} = \frac{10^3}{4} = \underline{\frac{1}{4}\ \text{k}\Omega}$$

By current division, the current in each resistor $= \frac{1}{4}(10^{-3}) = \underline{\frac{1}{4}\ \text{mA}}$

Exercise 3.4-2 Determine the current measured by the ammeter in the circuit shown in Figure E 3.4-2*a*.

Hint: Figure E 3.4-2*b* shows the circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m.
Answer: $i_m = -1$ A



(b)

Figure E 3.4-2

Solution:
From current division

$$i_m = \frac{10}{10+40}(-5) = -1 \quad \text{A}$$

Exercise 3.6-1 Determine the resistance measured by the ohmmeter in Figure E 3.6-1.

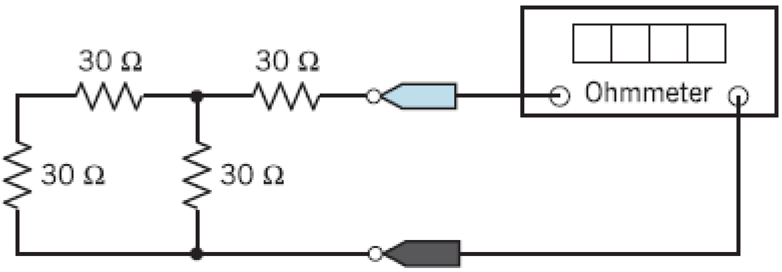


Figure E 3.6-1

Answer: $\dfrac{(30+30)\cdot 30}{(30+30)+30}+30=50\ \Omega$

Section 3-2 Kirchhoff's Laws

P 3.2-1 Consider the circuit shown in Figure P 3.2-1. Determine the values of the power supplied by branch *B* and the power supplied by branch *F*.

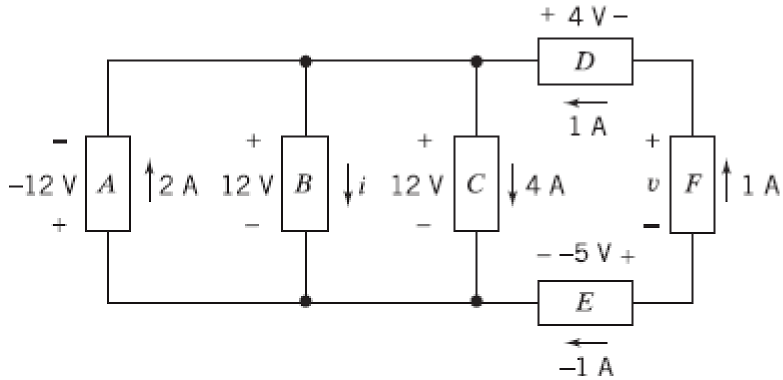
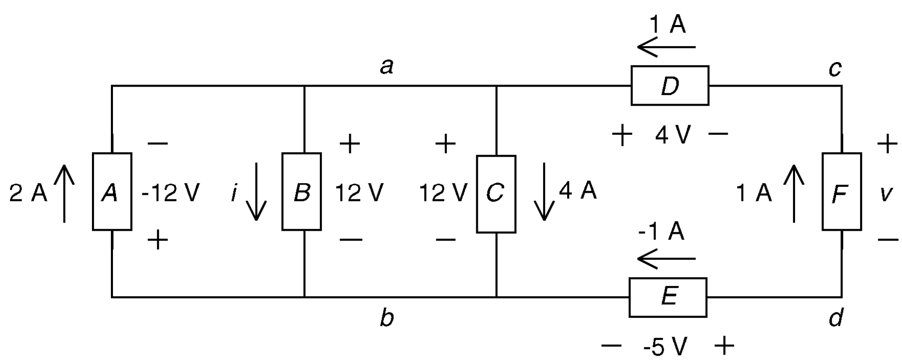


Figure P 3.2-1

Solution:



Apply KCL at node *a* to get $2 + 1 = i + 4 \Rightarrow \; i = \text{-}1$ A

The current and voltage of element *B* adhere to the passive convention so (12)(-1) = -12 W is power received by element *B*. The power supplied by element *B* is 12 W.

Apply KVL to the loop consisting of elements *D*, *F*, *E*, and *C* to get

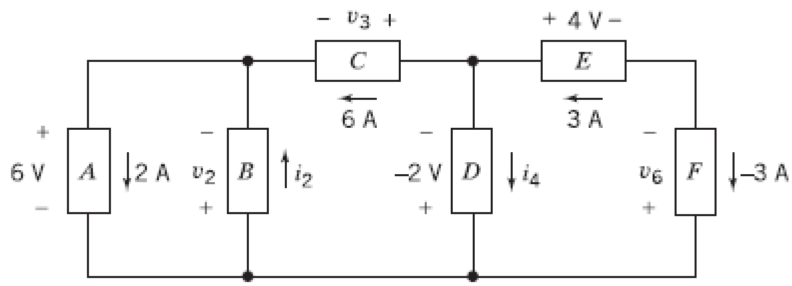
$$4 + v + (-5) - 12 = 0 \Rightarrow \; v = 13 \text{ V}$$

The current and voltage of element *F* do not adhere to the passive convention so (13)(1) = 13 W is the power supplied by element *F*.

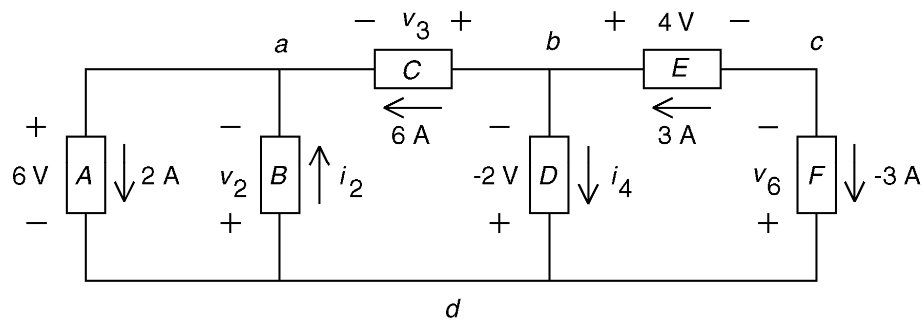
Check: The sum of the power supplied by all branches is

$$-(2)(-12) + \underline{12} - (4)(12) + (1)(4) + \underline{13} - (-1)(-5) = 24 + 12 - 48 + 4 + 13 - 5 = 0$$

P 3.2-2 Determine the values of i_2, i_4, v_2, v_3, and v_6 in Figure P 3.2-2.



Solution:



Apply KCL at node *a* to get $\quad 2 = i_2 + 6 = 0 \Rightarrow \; i_2 = -4$ A

Apply KCL at node *b* to get $\quad 3 = i_4 + 6 \Rightarrow \; i_4 = -3$ A

Apply KVL to the loop consisting of elements *A* and *B* to get

$$-v_2 - 6 = 0 \Rightarrow \; v_2 = -6 \text{ V}$$

Apply KVL to the loop consisting of elements *C*, *D*, and A to get

$$-v_3 - (-2) - 6 = 0 \Rightarrow \; v_4 = -4 \text{ V}$$

Apply KVL to the loop consisting of elements *E*, *F* and *D* to get

$$4 - v_6 + (-2) = 0 \Rightarrow \; v_6 = 2 \text{ V}$$

Check: The sum of the power supplied by all branches is

$$-(6)(2) - (-6)(-4) - (-4)(6) + (-2)(-3) + (4)(3) + (2)(-3) = -12 - 24 + 24 + 6 + 12 - 6 = 0$$

P 3.2-3 Consider the circuit shown in Figure P 3.2-3.

(a) Suppose that $R_1 = 6\ \Omega$ and $R_2 = 3\ \Omega$. Find the current i and the voltage v.

(b) Suppose, instead, that $i = 1.5$ A and $v = 2$ V. Determine the resistances R_1 and R_2.

(c) Suppose, instead, that the voltage source supplies 24 W of power and that the current source supplies 9 W of power. Determine the current i, the voltage v, and the resistances R_1 and R_2.

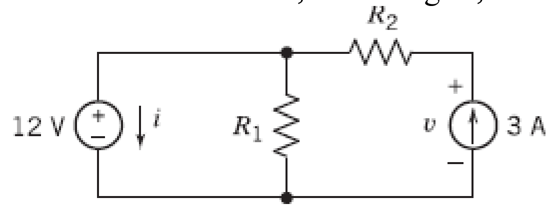
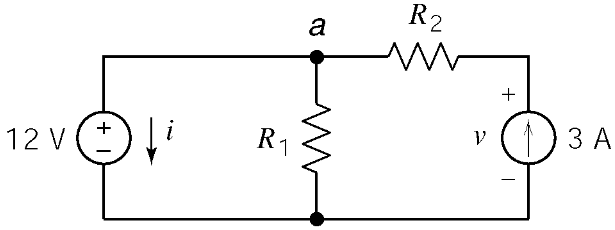


Figure P 3.2-3

Solution:



KVL : $-12 - R_2(3) + v = 0$ (outside loop)

$$v = 12 + 3R_2 \quad \text{or} \quad R_2 = \frac{v-12}{3}$$

KCL $\quad i + \dfrac{12}{R_1} - 3 = 0$ (top node)

$$i = 3 - \frac{12}{R_1} \quad \text{or} \quad R_1 = \frac{12}{3-i}$$

(a)
$$v = 12 + 3(4) = 24 \text{ V} \quad \text{and} \quad i = 3 - \frac{12}{8} = 1.5 \text{ A}$$

(b)
$$R_2 = \frac{42-12}{3} = \underline{10\ \Omega}\ ;\ R_1 = \frac{12}{3-2.25} = \underline{16\ \Omega}$$

(checked using LNAP 7/27/08)

(c) $24 = -12\, i$, because 12 and i adhere to the passive convention.

$$\therefore \underline{i = -2 \text{ A}} \quad \text{and} \quad R_1 = \frac{12}{3+2} = \underline{2.4\ \Omega}$$

$9 = 3v$, because 3 and v do not adhere to the passive convention

$$\therefore \underline{v = 3 \text{ V}} \quad \text{and} \quad R_2 = \frac{3-12}{3} = \underline{-3\ \Omega}$$

The situations described in (b) and (c) cannot occur if R_1 and R_2 are required to be nonnegative.

P 3.2-4 Determine the power absorbed by each of the resistors in the circuit shown in Figure P 3.2-4.

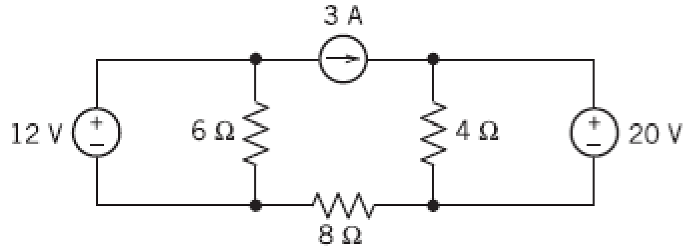
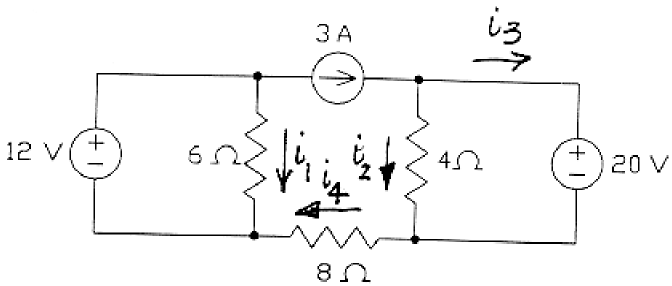


Figure P 3.2-4

Solution:



$$i_1 = \frac{12}{6} = 2 \text{ A}$$

$$i_2 = \frac{20}{4} = 5 \text{ A}$$

$$i_3 = 3 - i_2 = -2 \text{ A}$$

$$i_4 = i_2 + i_3 = 3 \text{ A}$$

Power absorbed by the 4 Ω resistor = $4 \cdot i_2^2 = \underline{100 \text{ W}}$

Power absorbed by the 6 Ω resistor = $6 \cdot i_1^2 = \underline{24 \text{ W}}$

Power absorbed by the 8 Ω resistor = $8 \cdot i_4^2 = \underline{72 \text{ W}}$

(checked using LNAP 8/16/02)

P 3.2-5 Determine the power absorbed by each of the resistors in the circuit shown in Figure P 3.2-5.
Answer: The 4-Ω resistor absorbs 16 W, the 6-Ω resistor absorbs 24 W, and the 8-Ω resistor absorbs 8 W.

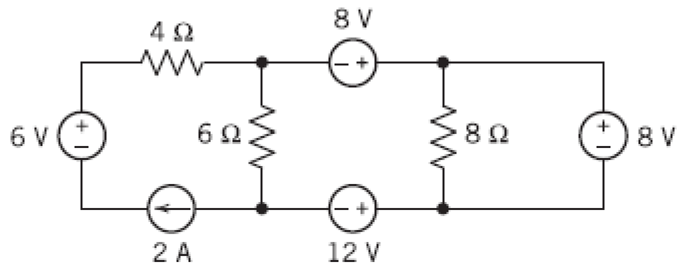
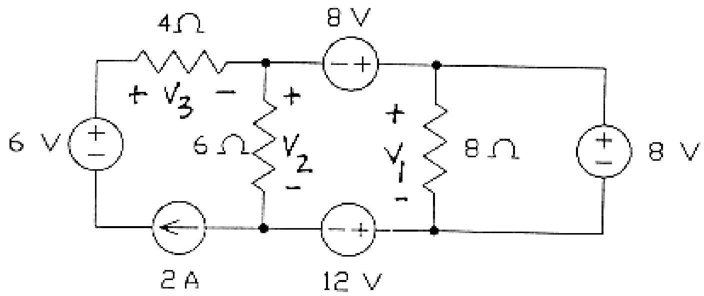


Figure P 3.2-5

Solution:



$$v_1 = 8 \text{ V}$$

$$v_2 = -8+8+12 = 12 \text{ V}$$

$$v_3 = 2 \cdot 4 = 8 \text{ V}$$

$$4\Omega: \quad P = \frac{v_3^2}{4} = \underline{16 \text{ W}}$$

$$6\Omega: \quad P = \frac{v_2^2}{6} = \underline{24 \text{ W}}$$

$$8\Omega: \quad P = \frac{v_1^2}{8} = \underline{8 \text{ W}}$$

(checked using LNAP 8/16/02)

P 3.2-6 Determine the power supplied by each voltage source in the circuit of Figure P 3.2-6.

Answer: The 2-V voltage source supplies 2 mW and the 3-V voltage source supplies –6 mW.

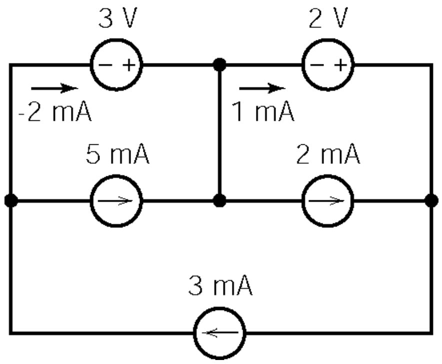
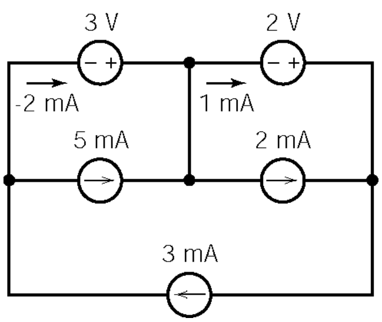


Figure P 3.2-6

Solution:



$$P_{2\,\text{V}} = +\left[2\times\left(1\times10^{-3}\right)\right] = 2\times10^{-3} = 2\text{ mW}$$
$$P_{3\,\text{V}} = +\left[3\times\left(-2\times10^{-3}\right)\right] = -6\times10^{-3} = -6\text{ mW}$$

(checked using LNAP 8/16/02)

P 3.2-7 What is the value of the resistance R in Figure P 3.2-7?
Hint: Assume an ideal ammeter. An ideal ammeter is equivalent to a short circuit.
Answer: $R = 4\ \Omega$

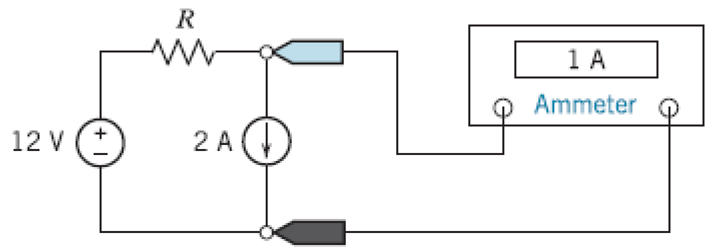
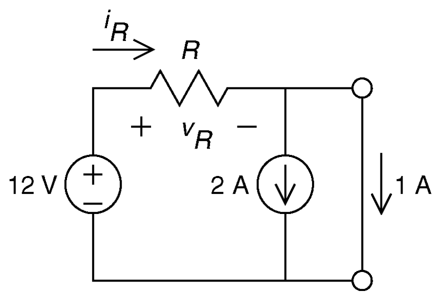


Figure P 3.2-7

Solution:



KCL: $i_R = 2+1 \quad \Rightarrow \quad i_R = 3\ \text{A}$

KVL: $v_R + 0 - 12 = 0 \quad \Rightarrow \quad v_R = 12\ \text{V}$

$$\therefore \quad R = \frac{v_R}{i_R} = \frac{12}{3} = 4\ \Omega$$

(checked using LNAP 8/16/02)

P 3.2-8 The voltmeter in Figure P 3.2-8 measures the value of the voltage across the current source to be 56 V. What is the value of the resistance *R*?

Hint: Assume an ideal voltmeter. An ideal voltmeter is equivalent to an open circuit.

Answer: $R = 10\ \Omega$

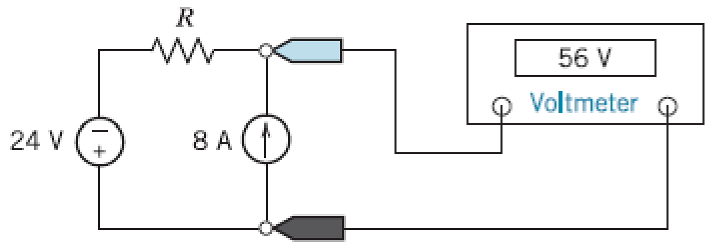
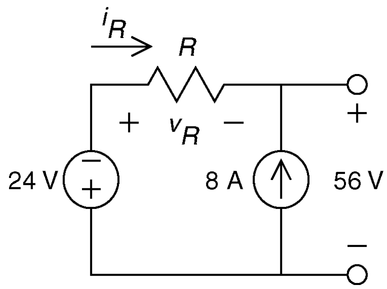


Figure P 3.2-8

Solution:



KVL: $v_R + 56 + 24 = 0 \quad \Rightarrow \quad v_R = -80\text{ V}$

KCL: $i_R + 8 = 0 \quad \Rightarrow \quad i_R = -8\text{ A}$

$\therefore \quad R = \dfrac{v_R}{i_R} = \dfrac{-80}{-8} = 10\ \Omega$

(checked using LNAP 8/16/02)

P 3.2-9 Determine the values of the resistances R_1 and R_2 in Figure P 3.2-9.

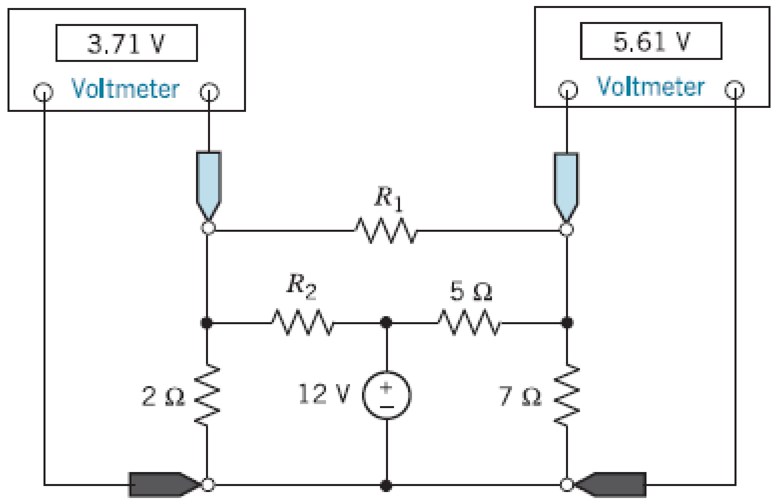
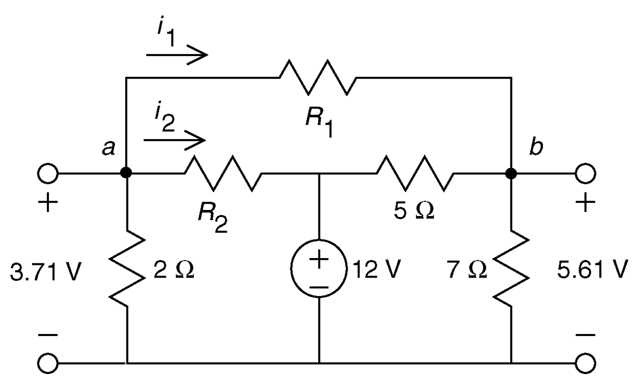


Figure P 3.2-9

Solution:



KCL at node *b*:

$$\frac{5.61}{7}=\frac{3.71-5.61}{R_1}+\frac{12-5.61}{5} \quad\Rightarrow\quad 0.801=\frac{-1.9}{R_1}+1.278$$

$$\Rightarrow\quad R_1=\frac{1.9}{1.278-0.801}=3.983\approx 4\,\Omega$$

KCL at node *a*:

$$\frac{3.71}{2}+\frac{3.71-5.61}{4}+\frac{3.71-12}{R_2}=0 \quad\Rightarrow\quad 1.855+\left(-0.475\right)+\frac{-8.29}{R_2}=0$$

$$\Rightarrow\quad R_2=\frac{8.29}{1.855-0.475}=6.007\approx 6\,\Omega$$

(checked using LNAP 8/16/02)

P 3.2-10 The circuit shown in Figure P 3.2-10 consists of five voltage sources and four current sources. Express the power supplied by each source in terms of the voltage source voltages and the current source currents.

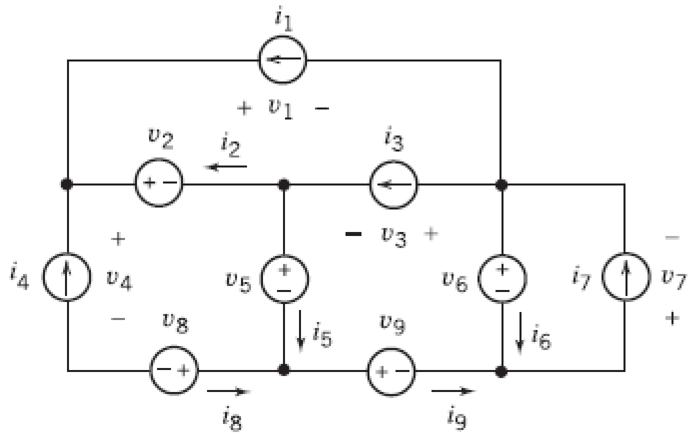


Figure P 3.2-10

Solution:

The subscripts suggest a numbering of the sources. Apply KVL to get

$$v_1 = v_2 + v_5 + v_9 - v_6$$

i_1 and v_1 do not adhere to the passive convention, so

$$p_1 = i_1\, v_1 = i_1\left(v_2 + v_5 + v_9 - v_6\right)$$

is the power supplied by source 1. Next, apply KCL to get

$$i_2 = -\left(i_1 + i_4\right)$$

i_2 and v_2 do not adhere to the passive convention, so

$$p_2 = i_2\, v_2 = -\left(i_1 + i_4\right)v_2$$

is the power supplied by source 2. Next, apply KVL to get

$$v_3 = v_6 - \left(v_5 + v_9\right)$$

i_3 and v_3 adhere to the passive convention, so

$$p_3 = -i_3\, v_3 = -i_3\left(v_6 - \left(v_5 + v_9\right)\right)$$

is the power supplied by source 3. Next, apply KVL to get

$$v_4 = v_2 + v_5 + v_8$$

i_4 and v_4 do not adhere to the passive convention, so

$$p_4 = i_4\, v_4 = i_4\left(v_2 + v_5 + v_8\right)$$

is the power supplied by source 4. Next, apply KCL to get

$$i_5 = i_3 - i_2 = i_3 - \left(-\left(i_1 + i_4\right)\right) = i_1 + i_3 + i_4$$

i_5 and v_5 adhere to the passive convention, so

$$p_5 = -i_5 v_5 = -\left(i_1 + i_3 + i_4\right) v_5$$

is the power supplied by source 5. Next, apply KCL to get

$$i_6 = i_7 - \left(i_1 + i_3\right)$$

i_6 and v_6 adhere to the passive convention, so

$$p_6 = -i_6 v_6 = -\left(i_7 - \left(i_1 + i_3\right)\right) v_6$$

is the power supplied by source 6. Next, apply KVL to get

$$v_7 = -v_6$$

i_7 and v_7 adhere to the passive convention, so

$$p_7 = -i_7 v_7 = -i_7\left(-v_6\right) = i_7 v_6$$

is the power supplied by source 7. Next, apply KCL to get

$$i_8 = -i_4$$

i_8 and v_8 do not adhere to the passive convention, so

$$p_8 = i_8 v_8 = \left(-i_4\right) v_8 = -i_4 v_8$$

is the power supplied by source 8. Finally, apply KCL to get

$$i_9 = i_1 + i_3$$

i_9 and v_9 adhere to the passive convention, so

$$p_9 = -i_9 v_9 = -\left(i_1 + i_3\right) v_9$$

is the power supplied by source 9.

(Check: $\sum_{n=1}^{9} p_n = 0$.)

P 3.2-11 Determine the power received by each of the resistors in the circuit shown in Figure P 3.2-11.

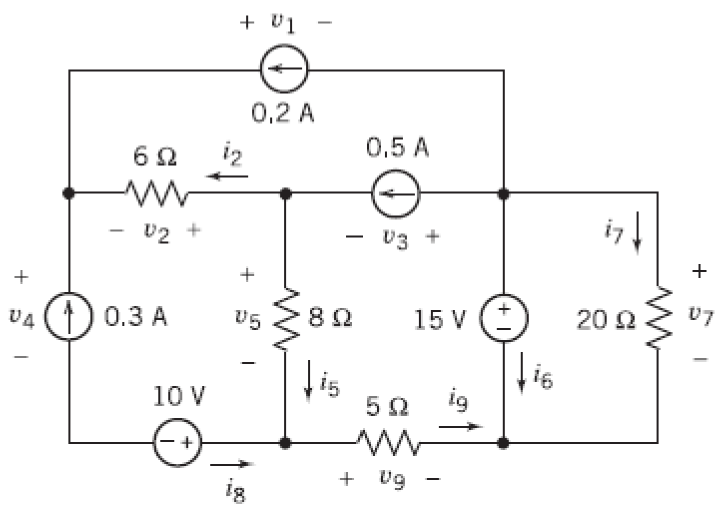


Figure P 3.2-11

Solution

The subscripts suggest a numbering of the circuit elements. Apply KCL to get

$$i_2 + 0.2 + 0.3 = 0 \quad \Rightarrow \quad i_2 = -0.5 \text{ A}$$

The power received by the 6 Ω resistor is

$$p_2 = 6 i_2^{\,2} = 6(-0.5)^2 = 1.5 \text{ W}$$

Next, apply KCL to get

$$i_5 = 0.2 + 0.3 + 0.5 = 1.0 \text{ A}$$

The power received by the 8 Ω resistor is

$$p_5 = 8 i_5^{\,2} = 8(1)^2 = 8 \text{ W}$$

Next, apply KVL to get

$$v_7 = 15 \text{ V}$$

The power received by the 20 Ω resistor is

$$p_7 = \frac{v_7^{\,2}}{20} = \frac{15^2}{20} = 11.25 \text{ W}$$

is the power supplied by source 7. Finally, apply KCL to get

$$i_9 = 0.2 + 0.5 = 0.7 \text{ A}$$

The power received by the 5 Ω resistor is

$$p_9 = 5 i_9^{\,2} = 5(0.7)^2 = 2.45 \text{ W}$$

P 3.2-12 Determine the voltage and current of each of the circuit elements in the circuit shown in Figure P 3.2-12.

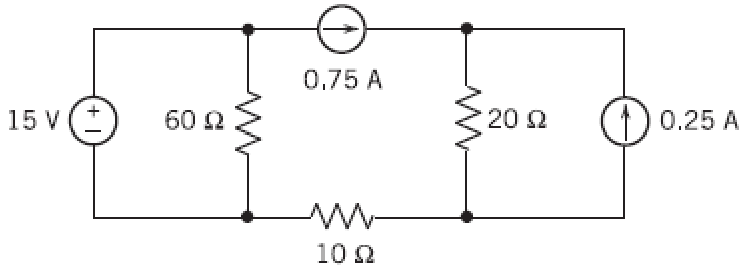
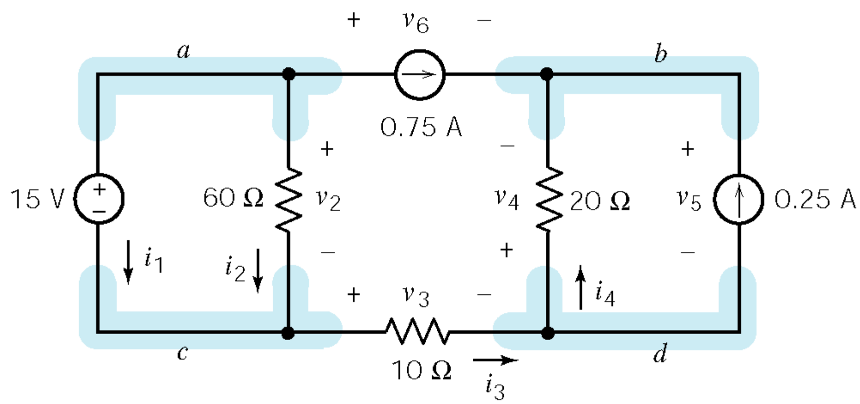


Figure P 3.2-12

Solution: We can label the circuit as follows:



The subscripts suggest a numbering of the circuit elements. Apply KCL at node *b* to get

$$i_4 + 0.25 + 0.75 = 0 \quad \Rightarrow \quad i_4 = -1.0 \text{ A}$$

Next, apply KCL at node *d* to get

$$i_3 = i_4 + 0.25 = -1.0 + 0.25 = -0.75 \text{ A}$$

Next, apply KVL to the loop consisting of the voltage source and the 60 Ω resistor to get

$$v_2 - 15 = 0 \quad \Rightarrow \quad v_2 = 15 \text{ V}$$

Apply Ohm's law to each of the resistors to get

$$i_2 = \frac{v_2}{60} = \frac{15}{60} = 0.25 \text{ A}, \qquad v_3 = 10\, i_3 = 10(-0.75) = -7.5 \text{ V}$$

and

$$v_4 = 20\, i_4 = 20(-1) = -20 \text{ V}$$

Next, apply KCL at node *c* to get

$$i_1 + i_2 = i_3 \quad \Rightarrow \quad i_1 = i_3 - i_2 = -0.75 - 0.25 = -1.0 \text{ A}$$

Next, apply KVL to the loop consisting of the 0.75 A current source and three resistors to get

$$v_6 - v_4 - v_3 - v_2 = 0 \quad \Rightarrow \quad v_6 = v_4 + v_3 + v_2 = -20 + (-7.5) + 15 = -12.5 \text{ V}$$

Finally, apply KVL to the loop consisting of the 0.25 A current source and the 20 Ω resistor to get

$$v_5 + v_4 = 0 \quad \Rightarrow \quad v_5 = -v_4 = -(-20) = 20 \text{ V}$$

(Checked: LNAPDC 8/28/04)

P 3.2-13 Determine the value of the current that is measured by the meter in Figure P 3.2-13.

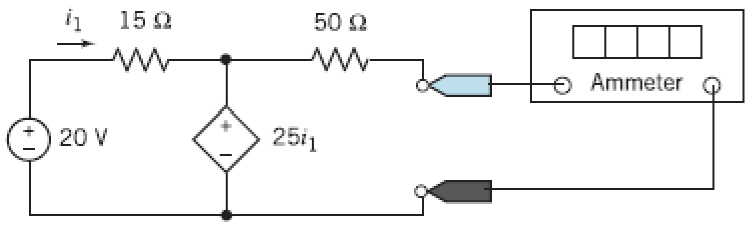
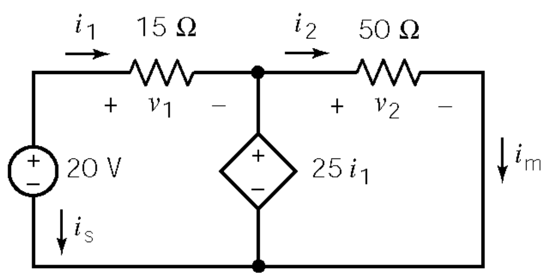


Figure P 3.2-13

Solution:

We can label the circuit as shown.



The subscripts suggest a numbering of the circuit elements. Apply KVL to node the left mesh to get

$$15i_1 + 25i_1 - 20 = 0 \quad \Rightarrow \quad i_1 = \frac{20}{40} = 0.5 \text{ A}$$

Apply KVL to node the left mesh to get

$$v_2 - 25i_1 = 0 \quad \Rightarrow \quad v_2 = 25i_1 = 25(0.5) = 12.5 \text{ V}$$

Apply KCL to get $i_m = i_2$. Finally, apply Ohm's law to the 50 Ω resistor to get

$$i_m = i_2 = \frac{v_2}{50} = \frac{12.5}{50} = 0.25 \text{ A}$$

(Checked: LNAPDC 9/1/04)

P 3.2-14 Determine the value of the voltage that is measured by the meter in Figure P 3.2-14.

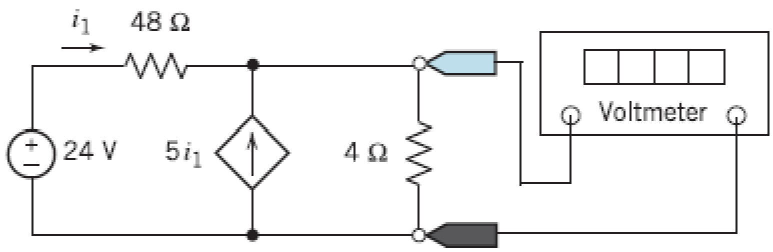
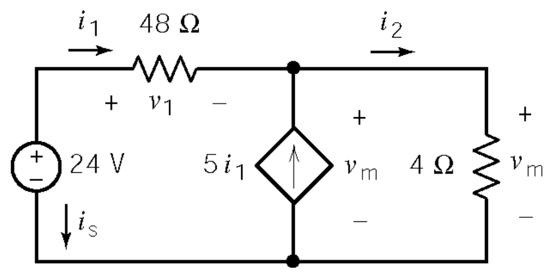


Figure P 3.2-14

Solution:

We can label the circuit as shown.



The subscripts suggest a numbering of the circuit elements. Ohm's law to the 48 Ω resistor to get

$$v_1 = 48 i_1$$

Apply KCL at the top node of the CCCS to get

$$i_1 + 5 i_1 = i_2 \quad \Rightarrow \quad i_2 = 6 i_1$$

Ohm's law to the 4 Ω resistor to get

$$v_m = 4 i_2 = 4(6 i_1) = 24 i_1$$

Apply KVL to the outside loop to get

$$v_1 + v_m - 24 = 0 \quad \Rightarrow \quad 48 i_1 + 24 i_1 = 24 \quad \Rightarrow \quad i_1 = \frac{24}{72} = \frac{1}{3} \text{ A}$$

Finally,

$$v_m = 24 i_1 = 24\left(\frac{1}{3}\right) = 8 \text{ V}$$

(Checked: LNAPDC 9/1/04)

P 3.2-15 Determine the value of the voltage that is measured by the meter in Figure P 3.2-15.

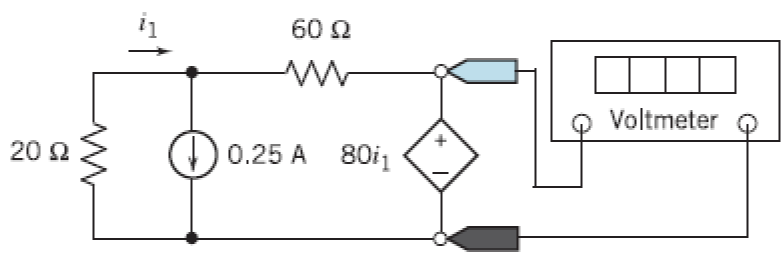
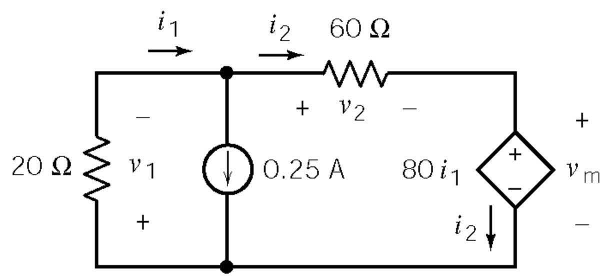


Figure P 3.2-15

Solution:

We can label the circuit as shown.



The subscripts suggest a numbering of the circuit elements. Apply KCL at the top node of the current source to get

$$i_1 = i_2 + 0.25$$

Apply Ohm's law to the resistors to get

$$v_1 = 20i_1 \quad \text{and} \quad v_2 = 60i_2 = 60(i_1 - 0.25) = 60i_1 - 15$$

Apply KVL to the outside to get

$$v_2 + 80i_1 + v_1 = 0 \quad \Rightarrow \quad (60i_1 - 15) + 80i_1 + 20i_1 = 0 \quad \Rightarrow \quad i_1 = \frac{15}{160} = 0.09375 \text{ A}$$

Finally, $$v_m = 80i_1 = 80(0.09375) = 7.5 \text{ V}$$

(Checked: LNAPDC 9/1/04)

P 3.2-16 The voltage source in Figure P 3.2-16 supplies 4.8 W of power. The current source supplies 3.6 W. Determine the values of the resistances, R_1 and R_2.

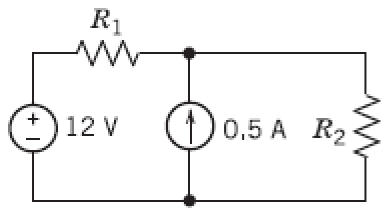
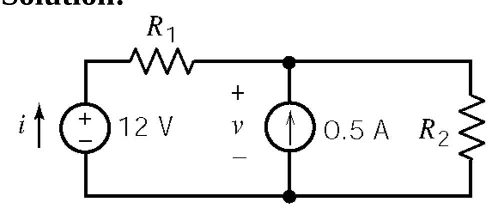


Figure P 3.2-16

Solution:



$$i = \frac{3.6}{12} = 0.3\text{ A} \quad \text{and} \quad v = \frac{4.8}{0.5} = 9.6\text{ V}$$

$$R_1 = \frac{12-9.6}{0.3} = 8\ \Omega \quad \text{and} \quad R_2 = \frac{9.6}{0.3+0.5} = 12\ \Omega$$

(Checked: LNAPDC 7/27/08)

P 3.2-17 Determine the current *i* in Figure P 3.3-17.
Answer: $i = 4$ A

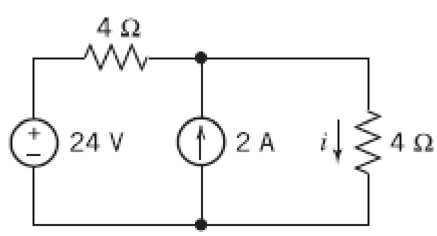
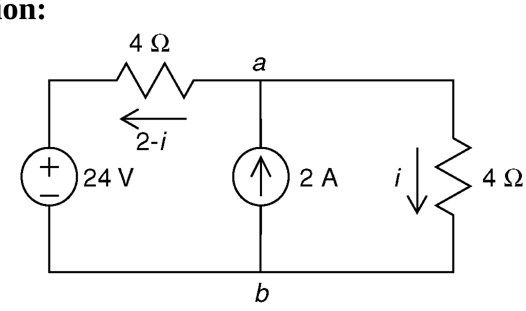


Figure P 3.3-17

Solution:



Apply KCL at node a to determine the current in the horizontal resistor as shown.

Apply KVL to the loop consisting of the voltages source and the two resistors to get

$-4(2-i) + 4(i) - 24 = 0 \Rightarrow i = 4$ A

P 3.2-18 Determine the value of the current i_m in Figure P 3.2-18*a*.

Hint: Apply KVL to the closed path a-b-d-c-a in Figure P 3.2-18*b* to determine v_a. Then apply KCL at node b to find i_m.

Answer: $i_m = 9$ A.

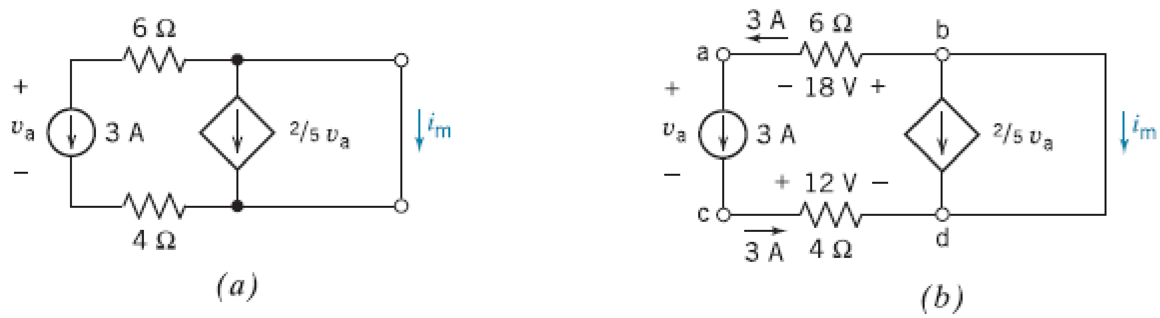


Figure P 3.2-18

Solution:

$$-18+0-12-v_a=0 \quad\Rightarrow\quad v_a=-30\text{ V} \quad\text{and}\quad i_m=\frac{2}{5}v_a+3 \quad\Rightarrow\quad i_m=9\ \text{A}$$

P3.2-19 Determine the value of the voltage v_6 for the circuit shown in Figure P3.2-19.

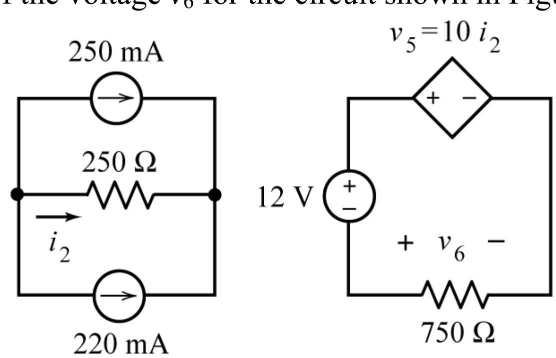
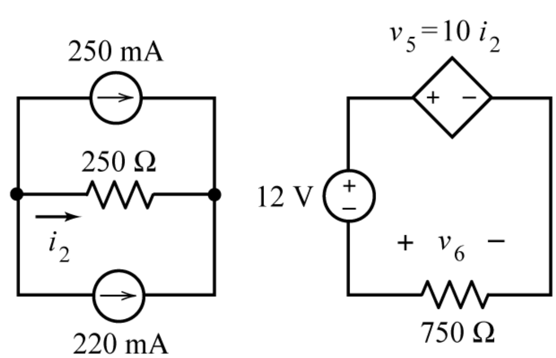


Figure P3.2-23

Solution:

Apply KCL at the left node:

$$0.25+i_2+0.22=0 \quad\Rightarrow\quad i_2=-0.47\ \text{A}$$



Use the element equation of the dependent source:

$$v_5=10\,i_2=10(-0.47)=-4.7\text{ V}$$

Apply KVL to the right mesh

$$v_5-v_6-12=0 \quad\Rightarrow\quad v_6=v_5-12=-4.7-12=-16.7\text{ V}$$

P3.2-20 Determine the value of the voltage v_6 for the circuit shown in Figure P3.2-20.

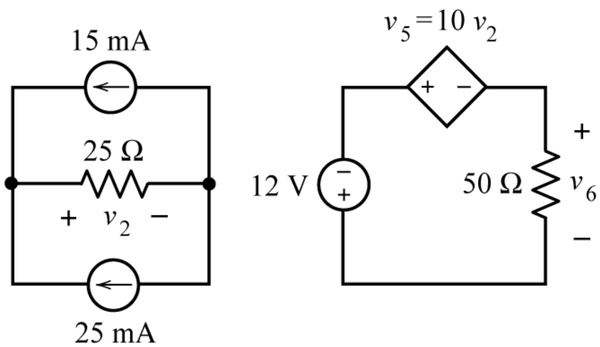
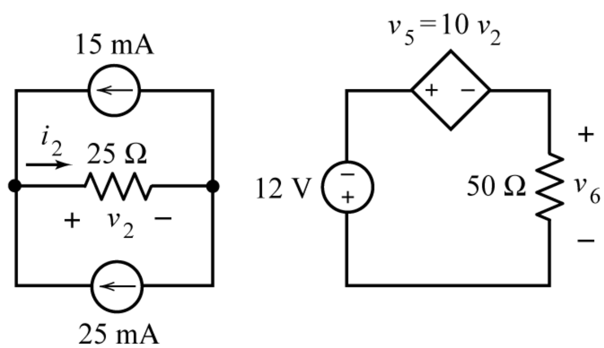


Figure P3.2-20

Solution:

Apply KCL at the left node:

$$0.015 + 0.025 = i_2 \quad \Rightarrow \quad i_2 = 0.040 \text{ A}$$



From Ohm's law

$$v_2 = 25 i_2 = 25(0.04) = 1 \text{ V}$$

Use the element equation of the dependent source:

$$v_5 = 10 v_2 = 10(1) = 10 \text{ V}$$

Apply KVL to the right mesh

$$v_5 + v_6 + 12 = 0 \quad \Rightarrow \quad v_6 = -v_5 - 12 = -10 - 12 = -22 \text{ V}$$

P3.2-21 Determine the value of the voltage v_5 for the circuit shown in Figure P3.2-21.

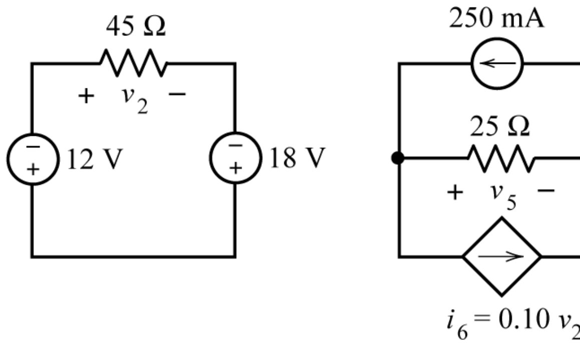
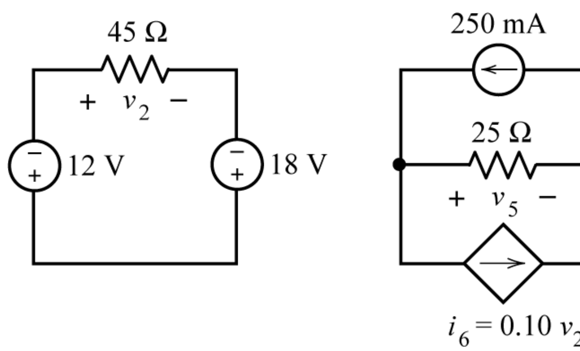


Figure P3.2-21

Solution:

Apply KVL to the left mesh:



$$v_2 - 18 + 12 = 0 \quad \Rightarrow \quad v_2 = 6 \text{ V}$$

Use the element equation of the dependent source:

$$i_6 = 0.10 v_2 = 0.10(6) = 0.6 \text{ A}$$

Apply KCL at the right node

$$\frac{v_5}{25} + i_6 = 0.25 \quad \Rightarrow \quad v_5 = 25(0.25 - i_6) = 25(0.25 - 0.6) = -8.75 \text{ V}$$

P3.2-22 Determine the value of the voltage v_5 for the circuit shown in Figure P3.2-22.

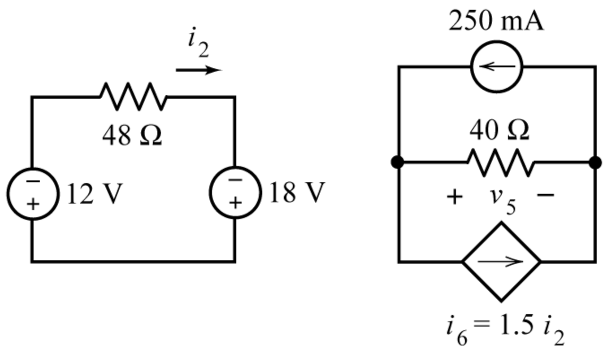
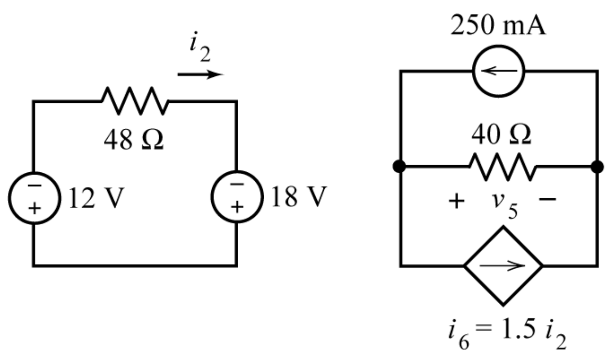


Figure P3.2-22

Solution:

Apply KVL to the left mesh:

$$48\,i_2 - 18 + 12 = 0 \quad \Rightarrow \quad i_2 = 0.125\ \text{A}$$



Use the element equation of the dependent source:

$$i_6 = 1.5\,i_2 = 1.5(0.125) = 0.1875\ \text{A}$$

Apply KCL at the right node

$$\frac{v_5}{40} + i_6 = 0.25 \quad \Rightarrow \quad v_5 = 40\left(0.25 - i_6\right) = 40\left(0.25 - 0.1875\right) = 2.5\ \text{V}$$

P3.2-23 Determine the value of the voltage v_6 for the circuit shown in Figure P3.2-23.

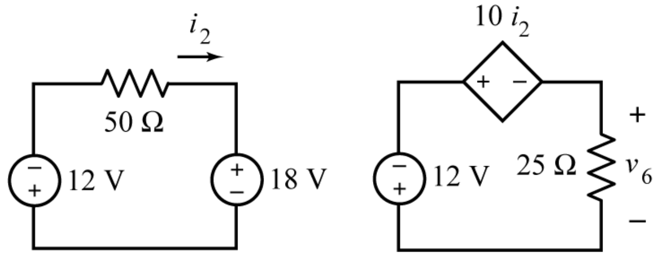
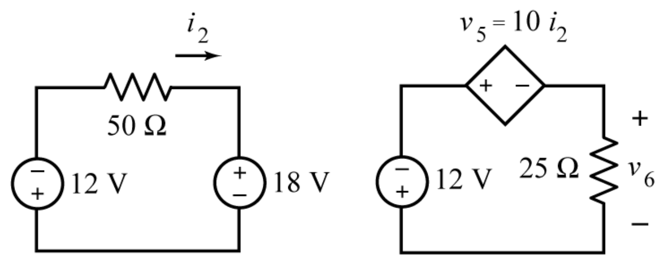


Figure P3.2-23

Solution:

Apply KVL to the left mesh:

$$50i_2 + 18 + 12 = 0 \quad \Rightarrow \quad i_2 = \frac{-30}{50} = -0.6 \text{ A}$$



Use the element equation of the dependent source:

$$v_5 = 10i_2 = 10(-0.6) = -6 \text{ V}$$

Apply KVL to the right mesh

$$v_5 + v_6 + 12 = 0 \quad \Rightarrow \quad v_6 = -v_5 - 12 = -(-6) - 12 = -6 \text{ V}$$

P3.2-24 Determine the value of the voltage v_5 for the circuit shown in Figure P3.2-24.

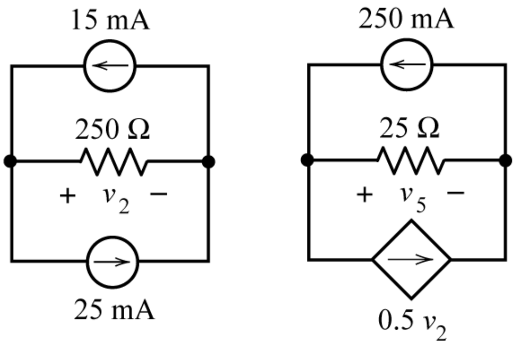
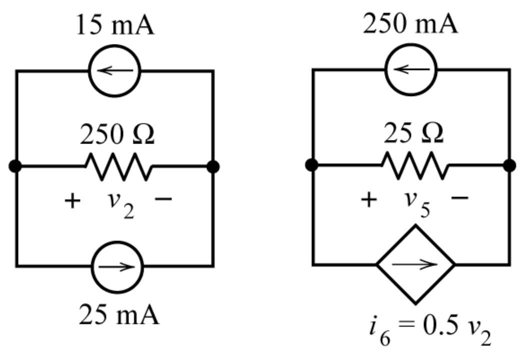


Figure P3.2-24

Solution:

Apply KCL at the left node:



$$0.015 = 0.025 + \frac{v_2}{250} \quad \Rightarrow \quad v_2 = 250(-0.01) = -2.5 \text{ V}$$

Use the element equation of the dependent source:

$$i_6 = 0.5 v_2 = 0.5(-2.5) = -1.25 \text{ A}$$

Apply KCL at the right node

$$\frac{v_5}{25} + i_6 = 0.25 \quad \Rightarrow \quad v_5 = 25(0.25 - i_6) = 25(0.25 - (-1.25)) = 37.5 \text{ V}$$

P3.2-25 The voltage source in the circuit shown in Figure P3.2-25 supplies 2 W of power. The value of the voltage across the 25 Ω is $v_5 = 4$ V. Determine the values of the resistance R_1 and of the gain, G, of the CCVS.

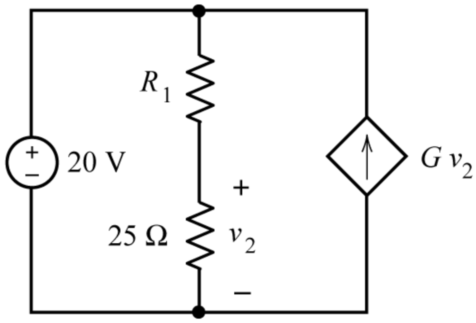
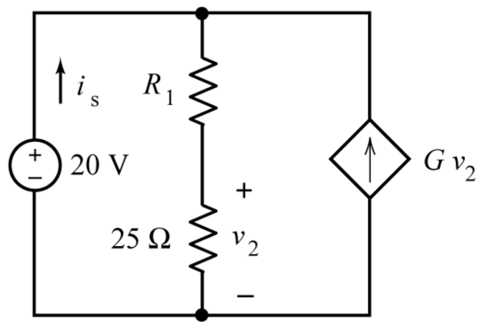


Figure P3.2-25

Solution:

The voltage source current is calculated from the values of the source voltage and power:

$$i_s = \frac{2}{20} = 0.1 \text{ A}$$



Apply KCL at the bottom node to get

$$i_s + G v_2 = \frac{v_2}{25} \quad \Rightarrow \quad G v_2 = \frac{4}{25} - 0.1 = 0.06 \text{ A}$$

Then

$$G = \frac{G v_2}{v_2} = \frac{0.06}{4} = 0.015 \text{ A/V} = 15 \text{ mA/V}$$

Next, use Ohm's law to determine the value of the resistance R_1:

$$R_1 = \frac{20 - v_2}{\frac{v_2}{25}} = \frac{20 - 4}{\frac{4}{25}} = 100 \text{ }\Omega$$

P3.2-26 Consider the circuit shown in Figure P3.2-26. Determine the values of

(a) The current i_a in the 20-Ω resistor.

(b) The voltage v_b across the 10-Ω resistor.

(c) The current i_c in the independent voltage source.

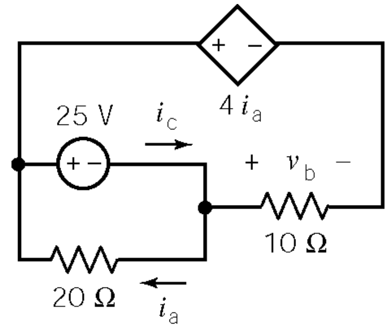
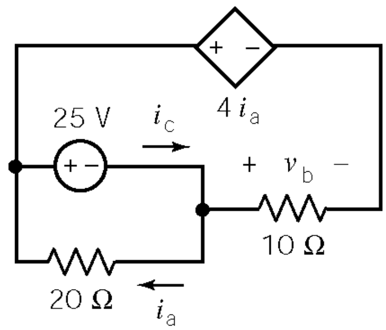


Figure P3.2-26

Solution:



(a) From Ohm's law $i_a = -\frac{25}{20} = -1.25$ A .

(b) From KVL $4i_a - v_b + 20i_a = 0 \quad \Rightarrow \quad v_b = 24i_a = 24(-1.25) = -30$ V

(c) From KCL $i_c = i_a + \frac{v_b}{10} = -1.25 + \frac{-30}{10} = -4.25$ A

P3.2-27 Consider the circuit shown in Figure 3.2-27.

a.) Determine the values of the resistances.

b.) Determine the values of the power supplied by each current source.

c.) Determine the values of the power received by each resistor.

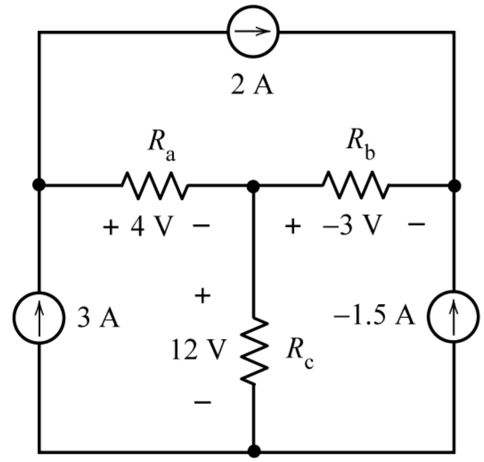
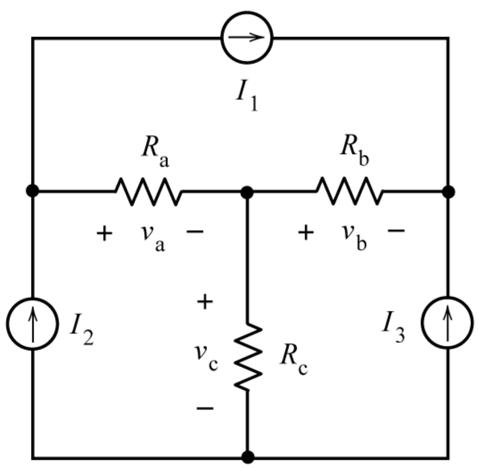


Figure 3.2-27

Solution:



Using KCL and Ohm's law:

$$v_a = R_a(I_2 - I_1),\ \ v_b = -R_b(I_1 + I_3)$$

and

$$v_c = R_c(I_2 + I_3)$$

Using KVL, the power supplied the current sources are:

$I_2(v_a + v_c)$, $-I_1(v_a + v_b)$ and $I_3(-v_b + v_c)$

The power received the resistors are:

$$v_a(I_2 - I_1),\ \ -v_b(I_1 + I_3)\ \text{ and }\ v_c(I_2 + I_3)$$

a.) $R_a = \dfrac{4}{3-2} = 4\ \Omega$, $R_b = -\dfrac{-3}{2+(-1.5)} = 6\ \Omega$

and $R_c = \dfrac{12}{3+(-1.5)} = 8\ \Omega$

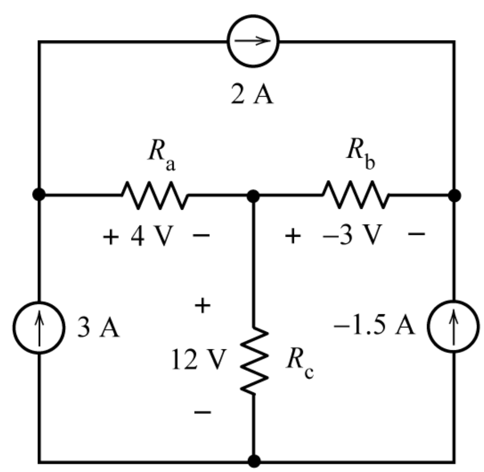
b.) $3(4+12) = 48\ \text{W}$, $-2(4+(-3)) = -2\ \text{W}$

and

$$-1.5(-(-3)+12) = -22.5\ \text{W}$$

c.) $4(3-2) = 4\ \text{W}$, $-(-3)(2+(-1.5)) = 1.5\ \text{W}$

and $12(3+(-1.5)) = 18\ \text{W}$



P3.2-28 Consider the circuit shown in Figure 3.2-28.

a. Determine the value of the power supplied by each independent source.

b. Determine the value of the power received by each resistor.

c. Is power conserved?

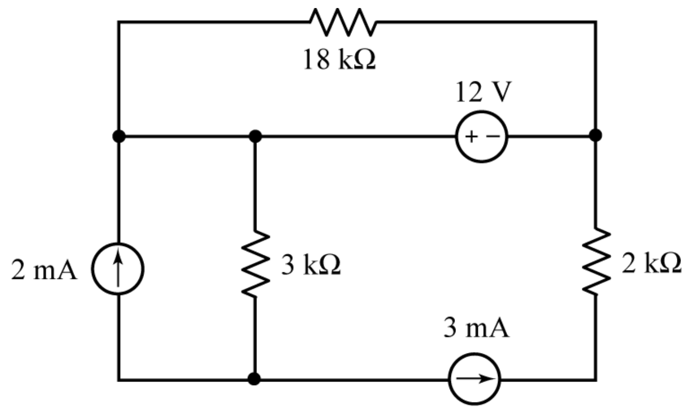
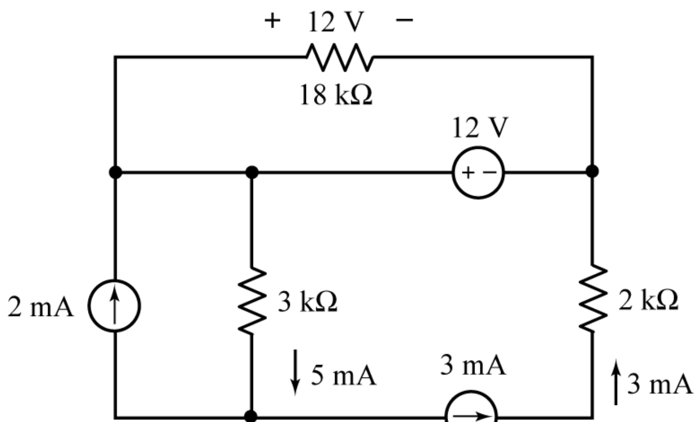
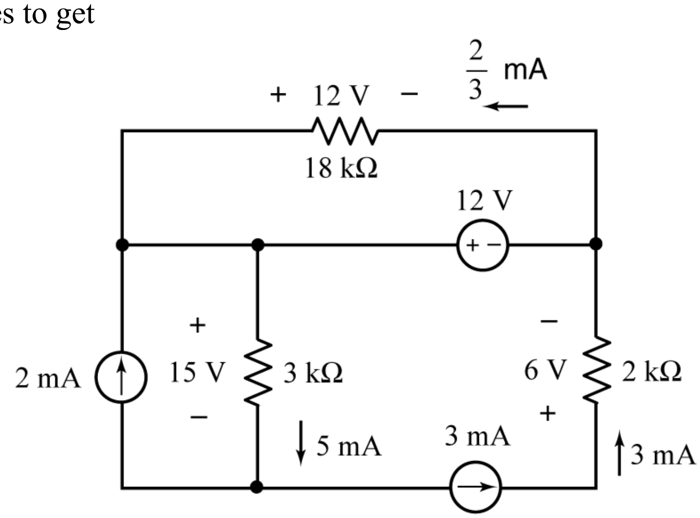


Figure 3.2-28

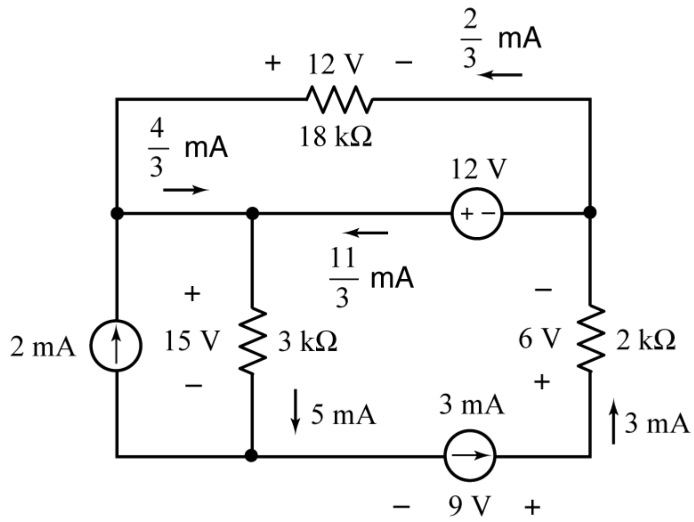
Solution:
Apply KCL twice and KVL to get



Apply Ohm's law 3 times to get



Apply KCL twice and KVL to get



a. Notice that the voltage and current reference directions for each independent source do not adhere to passive convention. Consequently the power supplied by the 2 mA current source is

$$15\ (2) = 30 \text{ mW}$$

The power supplied by the 3 mA current source is

$$9\ (3) = 27 \text{ mW}$$

The power supplied by the 12 V voltage source is

$$12\ (11/3) = 44 \text{ mW}$$

b. Notice that the voltage and current reference directions for each resistor do adhere to passive convention. Consequently the power received by the 18 kΩ resistor is

$$12\ (2/3) = 8 \text{ mW}$$

The power received by the 3 kΩ resistor is

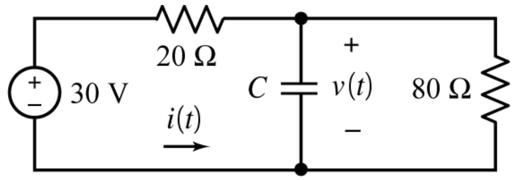
$$15\ (5) = 75 \text{ mW}$$

The power received by the 2 kΩ resistor is

$$6\ (3) = 18 \text{ mW}$$

c. Power is conserved; the sum of the powers supplied by the independent sources is equal to the sum of the powers received by the resistors.

$$30 + 27 + 44 = 101 \text{ mW} = 8 + 75 + 18$$

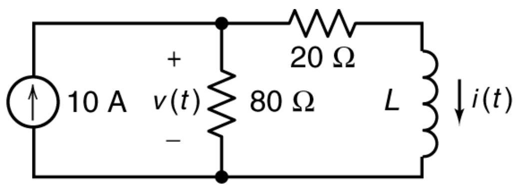


P3.2-29

P3.2-29 The voltage across the capacitor in Figure P3.2-29 is $v(t) = 24 - 10e^{-25t}$ V for $t \geq 0$. Determine the voltage source current $i(t)$ for $t > 0$.

Solution: Notice that $i(t)$ is the current in the 20 Ω resistor. Apply KVL to the left mesh to get

$$-20i(t) + \left[24 - 10e^{-25t}\right] - 30 = 0 \quad \Rightarrow \quad i(t) = \frac{\left[24 - 10e^{-25t}\right] - 30}{20} = -0.3 - 0.5e^{-25t} \quad \text{for } t > 0.$$



P3.2-30

P3.2-30 The current the inductor in Figure P3.2-30 is given by $i(t) = 8 - 6e^{-25t}$ A for $t \geq 0$. Determine the voltage $v(t)$ across the 80 Ω resistor for $t > 0$.

Solution: Notice that $i(t)$ is the current in the 20 Ω resistor. Apply KCL at the top node of the 80 Ω resistor to get

$$\frac{v(t)}{80} = 10 - \left(8 - 6e^{-25t}\right) \quad \Rightarrow \quad v(t) = 80\left[10 - \left(8 - 6e^{-25t}\right)\right] = 160 + 480e^{-25t} \quad \text{for } t \geq 0$$

Section 3-3 Series Resistors and Voltage Division

P 3.3-1 Use voltage division to determine the voltages v_1, v_2, v_3, and v_4 in the circuit shown in Figure P 3.3-1.

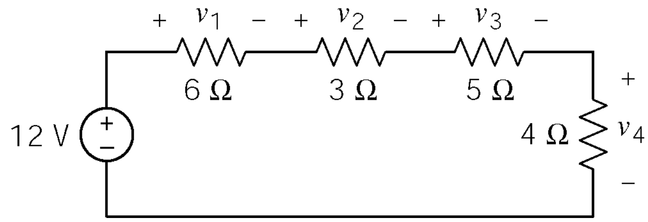
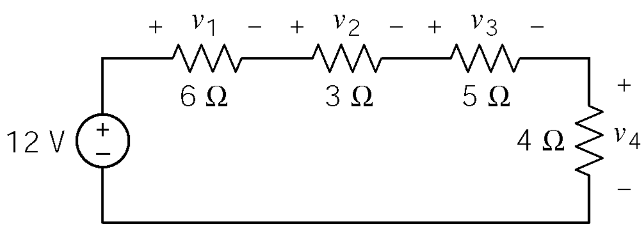


Figure P 3.3-1.

Solution:



$$v_1 = \frac{6}{6+3+5+4}\,12 = \frac{6}{18}\,12 = \underline{4\text{ V}}$$

$$v_2 = \frac{3}{18}\,12 = \underline{2\text{ V}}\ ;\ v_3 = \frac{5}{18}\,12 = \underline{\frac{10}{3}\text{ V}}$$

$$v_4 = \frac{4}{18}12 = \underline{\frac{8}{3}\text{ V}}$$

(checked using LNAP 8/16/02)

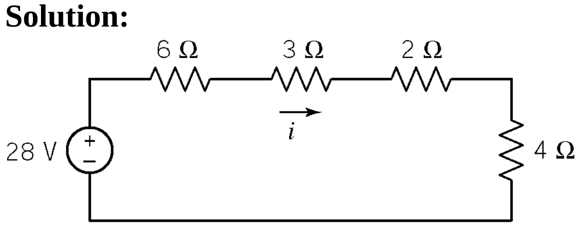
P 3.3-2 Consider the circuits shown in Figure P 3.3-2.

(a) Determine the value of the resistance R in Figure P 3.3-2*b* that makes the circuit in Figure P 3.3-2*b* equivalent to the circuit in Figure P 3.3-2*a*.

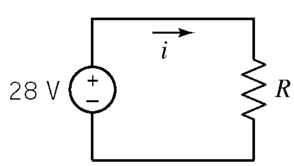
(b) Determine the current i in Figure P 3.3-2*b*. Because the circuits are equivalent, the current i in Figure P 3.3-2*a* is equal to the current i in Figure P 3.3-2*b*.

(c) Determine the power supplied by the voltage source.

Solution:



(*a*)



(*b*)

$(a)\ R = 6+3+2+4 = \underline{15\ \Omega}$

$(b)\ i = \frac{28}{R} = \frac{28}{15} = \underline{1.867\text{ A}}$

$(c)\ p = 28\cdot i = 28(1.867) = \underline{52.27\text{ W}}$

(28 V and i do not adhere to the passive convention.)

(checked using LNAP 8/16/02)

P 3.3-3 The ideal voltmeter in the circuit shown in Figure P 3.3-3 measures the voltage v.

(a) Suppose $R_2 = 100\ \Omega$. Determine the value of R_1.

(b) Suppose, instead, $R_1 = 100\ \Omega$. Determine the value of R_2.

(c) Suppose, instead, that the voltage source supplies 1.2 W of power. Determine the values of both R_1 and R_2.

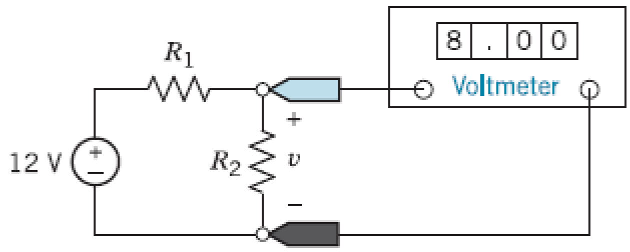
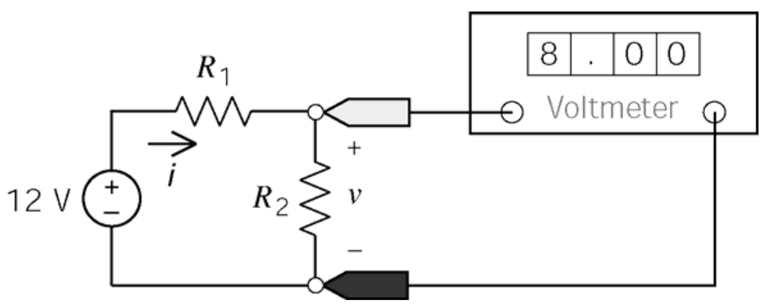


Figure P 3.3-3

Solution:



$$i\,R_2 = v = 8\text{ V}$$

$$12 = i\,R_1 + v = i\,R_1 + 8$$

$$\Rightarrow 4 = i\,R_1$$

$$(a)\ i = \frac{8}{R_2} = \frac{8}{50}\ ;\ R_1 = \frac{4}{i} = \frac{4\cdot 50}{8} = \underline{25\ \Omega}$$

$$(b)\ i = \frac{4}{R_1} = \frac{4}{50}\ ;\ R_2 = \frac{8}{i} = \frac{8\cdot 50}{4} = \underline{100\ \Omega}$$

$$(c)\ 1.2 = 12\,i \Rightarrow i = 0.1\text{ A}\ ;\ R_1 = \frac{4}{i} = \underline{40\ \Omega};\ R_2 = \frac{8}{i} = \underline{80\ \Omega}$$

(Checked using LNAP 8/16/02)

P 3.3-4 Determine the voltage v in the circuit shown in Figure P 3.3-4.

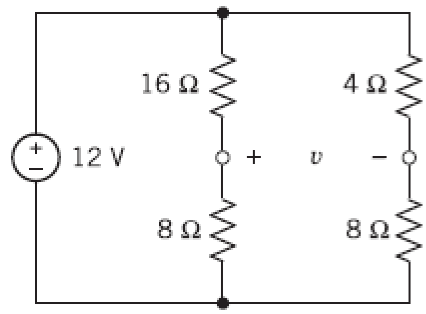
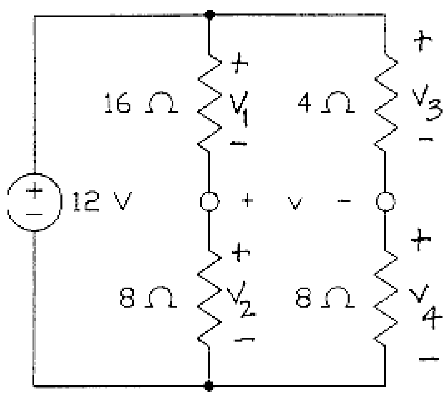


Figure P 3.3-4

Solution:



Voltage division

$$v_1 = \frac{16}{16+8} 12 = 8 \text{ V}$$

$$v_3 = \frac{4}{4+8} 12 = 4 \text{ V}$$

KVL: $v_3 - v - v_1 = 0$

$$\underline{v = -4 \text{ V}}$$

(checked using LNAP 8/16/02)

P 3.3-5 The model of a cable and load resistor connected to a source is shown in Figure P 3.3-5. Determine the appropriate cable resistance, R, so that the output voltage, v_o, remains between 9 V and 13 V when the source voltage, v_s, varies between 20 V and 28 V. The cable resistance can only assume integer values in the range $20 < R < 100\ \Omega$.

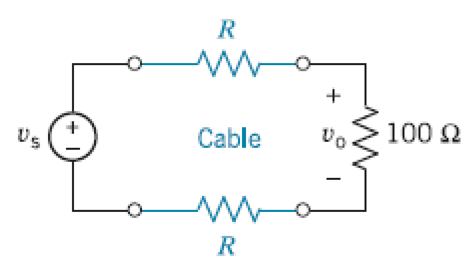


Figure P 3.3-5

Solution:

using voltage divider: $v_0 = \left(\dfrac{100}{100+2R}\right) v_s \quad \Rightarrow \quad R = 50\left(\dfrac{v_s}{v_o} - 1\right)$

$$\left.\begin{array}{l} \text{with } v_s = 20 \text{ V and } v_0 > 9 \text{ V}, \quad R < 61.1\,\Omega \\ \text{with } v_s = 28 \text{ V and } v_0 < 13 \text{ V}, \quad R > 57.7\,\Omega \end{array}\right\} \underline{R = 60\,\Omega}$$

P 3.3-6 The input to the circuit shown in Figure P 3.3-6 is the voltage of the voltage source, v_a. The output of this circuit is the voltage measured by the voltmeter, v_b. This circuit produces an output that is proportional to the input, that is

$$v_b = k\, v_a$$

where k is the constant of proportionality.

(a) Determine the value of the output, v_b, when $R = 240\ \Omega$ and $v_a = 18$ V.

(b) Determine the value of the power supplied by the voltage source when $R = 240\ \Omega$ and $v_a = 18$ V.

(c) Determine the value of the resistance, R, required to cause the output to be $v_b = 2$ V when the input is $v_a = 18$ V.

(d) Determine the value of the resistance, R, required to cause $v_b = 0.2 v_a$ (that is, the value of the constant of proportionality is $k = \frac{2}{10}$).

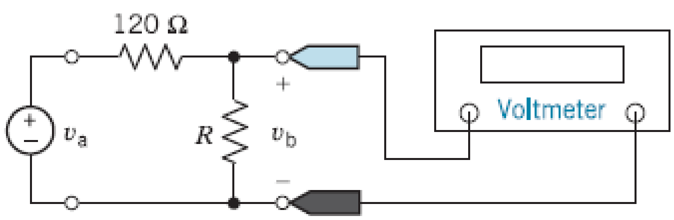
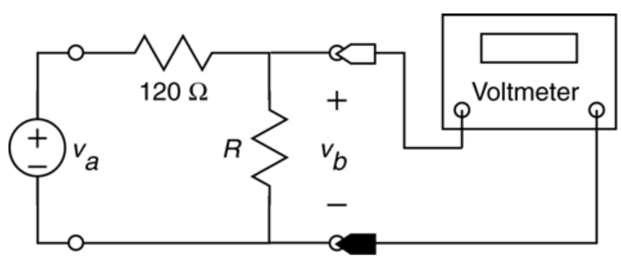


Figure P 3.3-6

Solution:



a.) $\left(\dfrac{180}{120+180}\right)18 = 10.8\ \text{V}$

b.) $18\left(\dfrac{18}{120+180}\right) = 1.08\ \text{W}$

c.) $\left(\dfrac{R}{R+120}\right)18 = 2 \quad\Rightarrow\quad 18\,R = 2\,R + 2\,(120) \quad\Rightarrow\quad R = 15\ \Omega$

d.) $0.2 = \dfrac{R}{R+120} \quad\Rightarrow\quad (0.2)(120) = 0.8\,R \quad\Rightarrow\quad R = 30\ \Omega$

(Checked using LNAP 8/16/02)

P 3.3-7 Determine the value of voltage v in the circuit shown in Figure P 3.3-7.

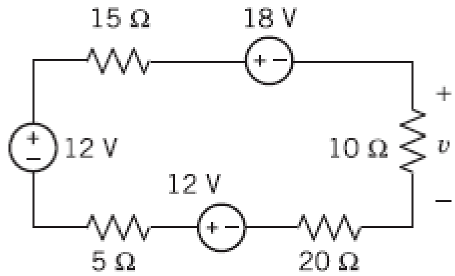


Figure P 3.3-7

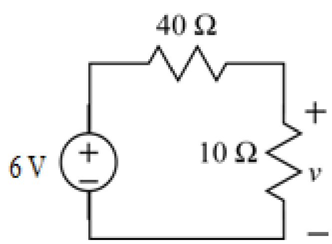
Solution:
All of the elements are connected in series.

Replace the series voltage sources with a single equivalent voltage having voltage

$$12 + 12 - 18 = 6 \text{ V}.$$

Replace the series 15 Ω, 5 Ω and 20 Ω resistors by a single equivalent resistance of

$$15 + 5 + 20 = 40 \ \Omega.$$



By voltage division

$$v = \left(\frac{10}{10+40}\right)6 = \frac{6}{5} = 1.2 \text{ V}$$

(Checked: LNAP 2/6/07)

P 3.3-8 Determine the power supplied by the dependent source in the circuit shown in Figure P 3.3-8.

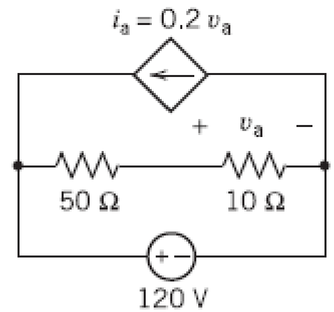


Figure P 3.3-8

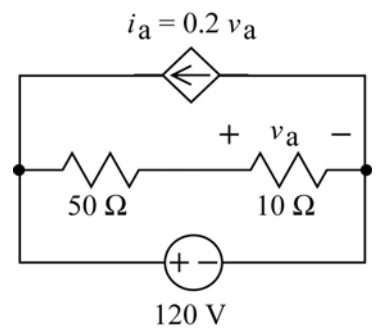
Solution:
Use voltage division to get

$$v_a = \left(\frac{10}{10+50}\right)(120) = 20 \text{ V}$$

Then $$i_a = 0.2(20) = 4 \text{ A}$$

The power supplied by the dependent source is given by

$$p = (120)i_a = 480 \text{ W}$$



(Checked: LNAP 6/21/04)

P 3.3-9 A potentiometer can be used as a transducer to convert the rotational position of a dial to an electrical quantity. Figure P 3.3-9 illustrates this situation. Figure P 3.3-9*a* shows a potentiometer having resistance R_p connected to a voltage source. The potentiometer has three terminals, one at each end and one connected to a sliding contact called a wiper. A voltmeter measures the voltage between the wiper and one end of the potentiometer.

Figure P 3.3-9*b* shows the circuit after the potentiometer is replaced by a model of the potentiometer that consists of two resistors. The parameter *a* depends on the angle, θ, of the dial. Here $a = \frac{\theta}{360°}$, and θ is given in degrees. Also, in Figure P 3.3-9*b*, the voltmeter has been replaced by an open circuit and the voltage measured by the voltmeter, v_m, has been labeled. The input to the circuit is the angle θ, and the output is the voltage measured by the meter, v_m.

(a) Show that the output is proportional to the input.

(b) Let R_p = 1 kΩ and v_s = 24 V. Express the output as a function of the input. What is the value of the output when θ = 45°? What is the angle when v_m = 10 V?

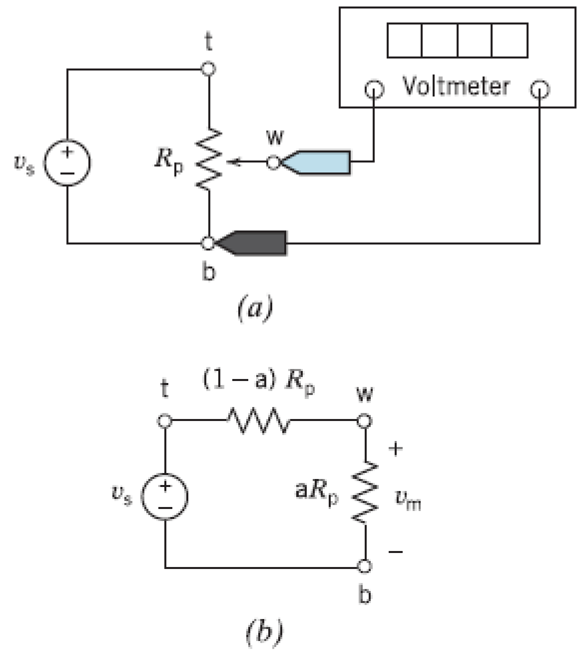
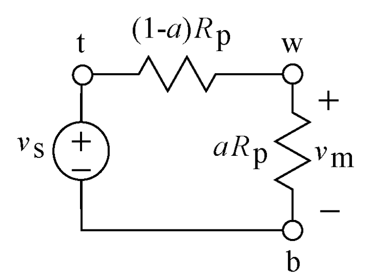


Figure P 3.3-9

Solution:

(a) Use voltage division to get

$$v_m = \frac{aR_p}{(1-a)R_p + R_p} v_s = a v_s$$



Therefore $$v_m = \left(\frac{v_s}{360}\right)\theta$$

So the input is proportional to the input.

(b) When v_s = 24 V then $v_m = \left(\frac{1}{15}\right)\theta$. When $\theta = 45°$ then v_m = 3 V. When v_m = 10 V then $\theta = 150°$.

(Checked: LNAP 6/12/04)

P 3.3-10 Determine the value of the voltage measured by the meter in Figure P 3.3-10.

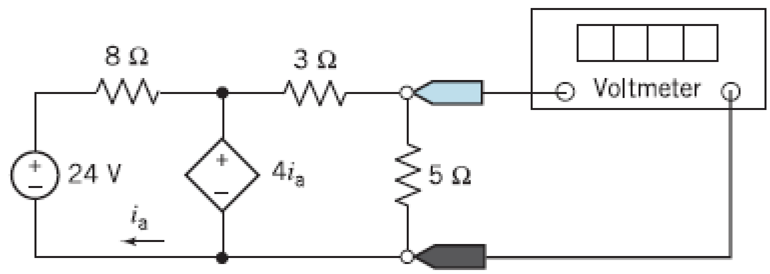
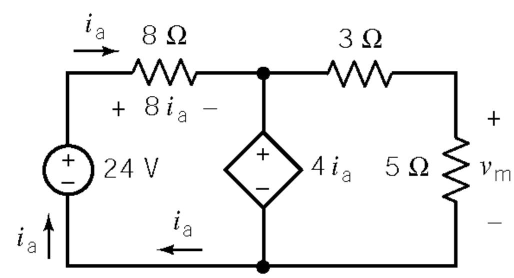


Figure P 3.3-10

Solution:

Replace the (ideal) voltmeter with the equivalent open circuit. Label the voltage measured by the meter. Label some other element voltages and currents.



Apply KVL the left mesh to get

$$8i_a + 4i_a - 24 = 0 \quad \Rightarrow \quad i_a = 2 \text{ A}$$

Use voltage division to get

$$v_m = \frac{5}{5+3} 4i_a = \frac{5}{5+3} 4(2) = 5 \text{ V}$$

(Checked using LNAP 9/11/04)

P 3.3-11 For the circuit of Figure P 3.3-11, find the voltage v_3 and the current i and show that the power delivered to the three resistors is equal to that supplied by the source.

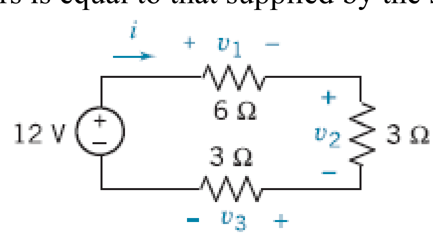
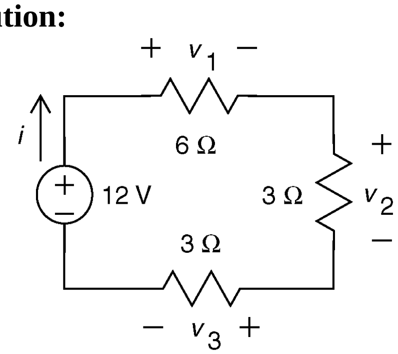


Figure P 3.3-11

Solution:



From voltage division $v_3 = 12\left(\frac{3}{3+9}\right) = \underline{3 \text{ V}}$

then $i = \frac{v_3}{3} = \underline{1 \text{ A}}$

The power absorbed by the resistors is: $(1^2)(6)+(1^2)(3)+(1^2)(3)=12$ W

The power supplied by the source is (12)(1) = 12 W.

P 3.3-12 Consider the voltage divider shown in Figure P 3.3-12 when $R_1 = 6\ \Omega$. It is desired that the output power absorbed by $R_1 = 6\ \Omega$ be 6 W. Find the voltage v_o and the required source v_s.
Answer: $v_s = 14$ V, $v_o = 6$ V

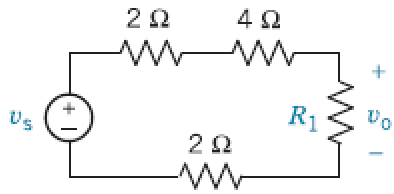
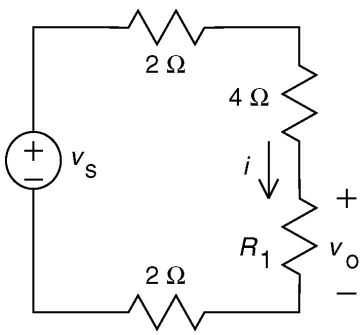


Figure P 3.3-12

Solution:



$P = 4.5$ W and $R_1 = 8\ \Omega$

$$i^2 = \frac{P}{R_1} = \frac{4.5}{8} = 0.5625 \text{ or } i = 0.75 \text{ A}$$

$$v_0 = i\, R_1 = (0.75)\,(8) = \underline{6 \text{ V}}$$

from KVL: $-v_s + i\,(2+4+8+2) = 0$

$$\Rightarrow v_s = 16\,i = 16(0.75) = 12 \text{ V}$$

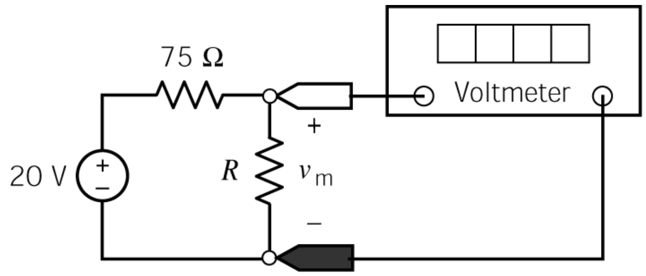


Figure P3.3-13

P3.3-13

Consider the voltage divider circuit shown in Figure P3.3-13. The resistor *R* represents a temperature sensor. The resistance *R*, in Ω, is related to the temperature *T*, in °C, by the equation

$$R = 50 + \frac{1}{2}T$$

a) Determine the meter voltage, v_m, corresponding to temperatures 0 °C, 75 °C and 100 °C.

b) Determine the temperature, *T*, corresponding to the meter voltages 8 V, 10 V and 15 V.

P3.3-13

Using voltage division $$v_m = \left(\frac{R}{75+R}\right)20$$

Solving for *R* yields $$R = \frac{75\,v_m}{20 - v_m}$$

The temperature can be calculated from the resistance using

$$T = 2(R-50) = 2\left(\frac{75\,v_m}{20-v_m} - 50\right) = \frac{150\,v_m}{20-v_m} - 100$$

a) At 0 °C the resistance is *R* = 50 Ω so $v_m = \left(\frac{50}{75+50}\right)20 = 8\text{ V}$. At 75 °C the resistance is

R = 87.5 Ω so $v_m = \left(\frac{87.5}{75+87.5}\right)20 = 10.77\text{ V}$. At 100 °C the resistance is *R* = 100 Ω so

$$v_m = \left(\frac{100}{75+100}\right)20 = 11.43\text{ V}.$$

b) When $v_m = 8$ V, the temperature is $T = \frac{150(8)}{20-8} - 100 = 0\ °\text{C}$. When $v_m = 10$ V, the temperature is

$T = \frac{150(10)}{20-10} - 100 = 50\ °\text{C}$. When $v_m = 15$ V, the temperature is $T = \frac{150(15)}{20-15} - 100 = 350\ °\text{C}$.

P3.3-14 Consider the circuit shown in Figure P3.3-14.

(a) Determine the value of the resistance R required to cause $v_o = 17.07$ V.

(b) Determine the value of the voltage v_o when $R = 14\ \Omega$.

(c) Determine the power supplied by the voltage source when $v_o = 14.22$ V.

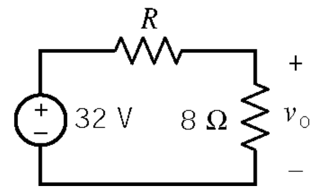


Figure P3.3-14

Solution:

Voltage division indicates that $v_o = \left(\dfrac{8}{8+R}\right)32$.

(a) When $v_o = 17.07$ V, then $17.07 = \left(\dfrac{8}{8+R}\right)32 \quad \Rightarrow \quad R = \dfrac{(8)32}{17.07} - 8 = 6.997 \simeq 7\ \Omega$.

(b) When $R = 14\ \Omega$ then $v_o = \left(\dfrac{8}{8+14}\right)32 = 11.6363$ V.

(c) The power supplied by the voltage source is given by $32\dfrac{v_o}{8} = 4v_o$ since $\dfrac{v_o}{8}$ is the current in each of the series elements. When $v_o = 14.22$ V the voltage source supplies 56.88 W.

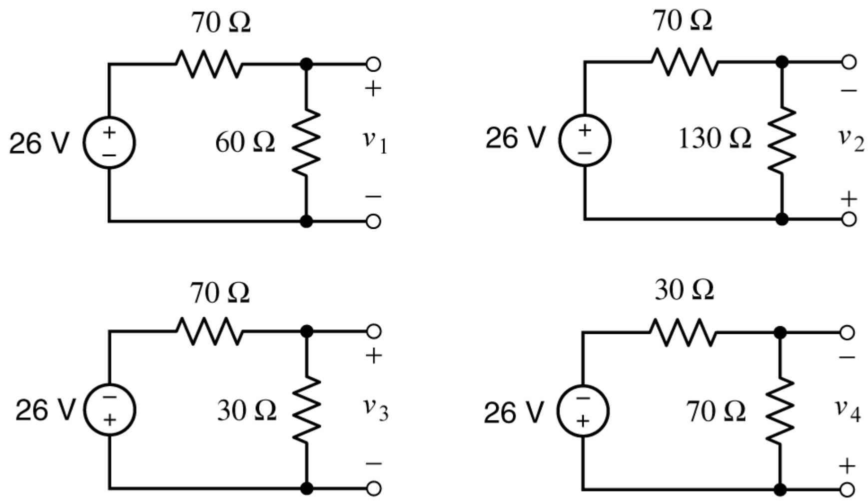


Figure P3.3-15

P3.3-15. Figure P3.3-15 shows four similar but slightly different circuits. Determine the values of the voltages v_1, v_2, v_3 and v_4.

Solution: Using voltage division:

$$v_1 = \left(\frac{60}{60+70}\right)26 = 12 \text{ V}, \quad v_2 = -\left(\frac{130}{70+130}\right)26 = -16.9 \text{ V},$$

$$v_3 = -\left(\frac{30}{70+30}\right)26 = -7.8 \text{ V} \quad \text{and} \quad v_4 = -\left(\frac{70}{30+70}\right)26 = 18.2 \text{ V}$$

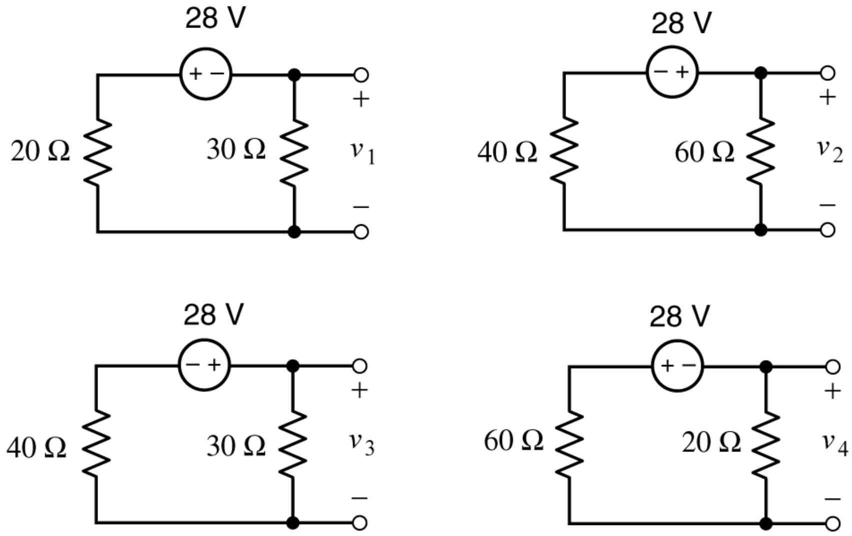


Figure P3.3-16

P3.3-16. Figure P3.3-16 shows four similar but slightly different circuits. Determine the values of the voltages v_1, v_2, v_3 and v_4.

Solution: Using voltage division:

$$v_1 = -\left(\frac{30}{20+30}\right)28 = -16.8 \text{ V}, \quad v_2 = -\left(\frac{60}{40+60}\right)28 = 16.8 \text{ V},$$

$$v_3 = \left(\frac{30}{40+30}\right)28 = 12 \text{ V} \quad \text{and} \quad v_4 = -\left(\frac{20}{60+20}\right)28 = -7 \text{ V}$$

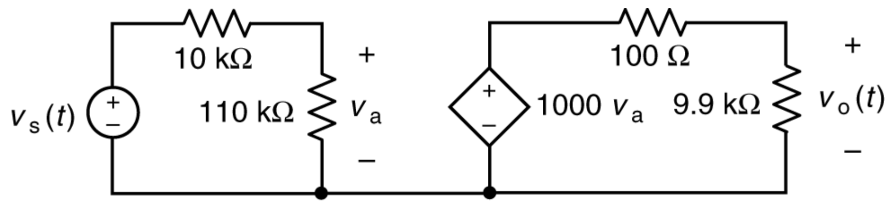


Figure P3.3-17

P3.3-17. The input to the circuit shown in Figure P3.3-19 is the voltage source voltage

$$v_s(t) = 12\cos(377\,t) \text{ mV}$$

The output is the voltage $v_o(t)$. Determine $v_o(t)$.

P3.3-17

Using voltage division: $v_a(t) = \frac{110}{10+110} 12\cos(377\,t) = 11\cos(377\,t)$ mV

Using voltage division again: $v_o(t) = \frac{9900}{100+9900} 1000\, v_a(t)$

Therefore:

$$v_o(t) = \frac{9900}{100+9900}(1000)11\cos(377\,t) = 10890\cos(377\,t) \text{ mV}$$

$$= 10.89\cos(377\,t) \text{ V}$$

Section 3-4 Parallel Resistors and Current Division

P 3.4-1 Use current division to determine the currents i_1, i_2, i_3, and i_4 in the circuit shown in Figure P 3.4-1.

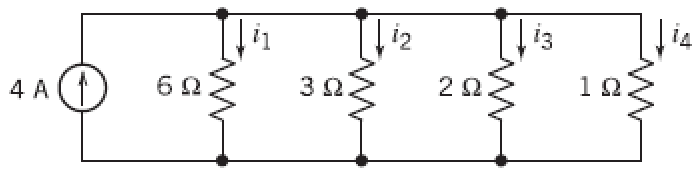
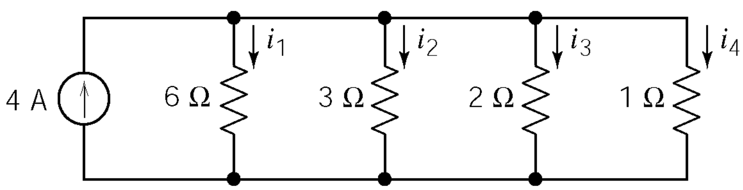


Figure P 3.4-1.

Solution:



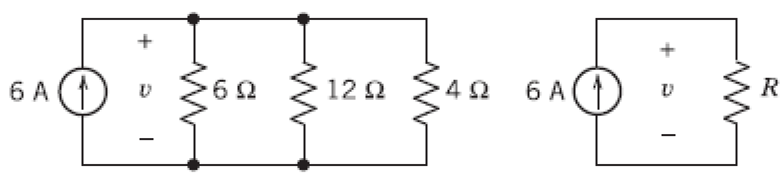
$$i_1 = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\frac{1}{1}}4 = \frac{1}{1+2+3+6}\,4 = \underline{\frac{1}{3}\text{ A}}$$

$$i_2 = \frac{\frac{1}{3}}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\frac{1}{1}}\,4 = \underline{\frac{2}{3}\text{ A}};$$

$$i_3 = \frac{\frac{1}{2}}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+\frac{1}{1}}\,4 = \underline{1\text{ A}}$$

$$i_4 = \frac{1}{\frac{1}{6}+\frac{1}{3}+\frac{1}{2}+1}\,4 = \underline{2\text{ A}}$$

P 3.4-2 Consider the circuits shown in Figure P 3.4-2.



(a) *(b)*

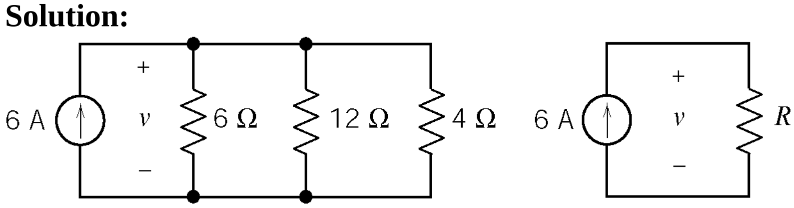
Figure P 3.4-2

(a) Determine the value of the resistance R in Figure P 3.4-2*b* that makes the circuit in Figure P 3.4-2*b* equivalent to the circuit in Figure P 3.4-2*a*.

(b) Determine the voltage v in Figure P 3.4-2*b*. Because the circuits are equivalent, the voltage v in Figure P 3.4-2*a* is equal to the voltage v in Figure P 3.4-2*b*.

(c) Determine the power supplied by the current source.

Solution:



(a) *(b)*

$$(a)\quad \frac{1}{R} = \frac{1}{6}+\frac{1}{12}+\frac{1}{4} = \frac{1}{2} \Rightarrow \underline{R\ =\ 2\,\Omega}$$

$$(b)\quad v = 6\cdot 2 = \underline{12\text{ V}}$$

$$(c)\quad p = 6\cdot 12 = \underline{72\text{ W}}$$

P 3.4-3 The ideal voltmeter in the circuit shown in Figure P 3.4-3 measures the voltage v.

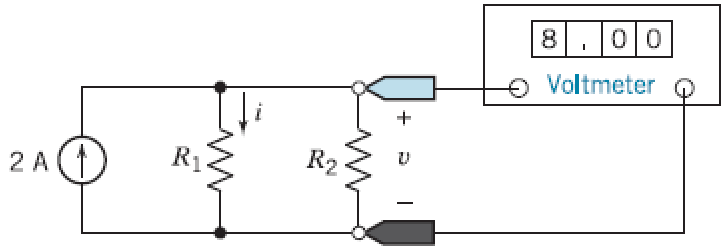
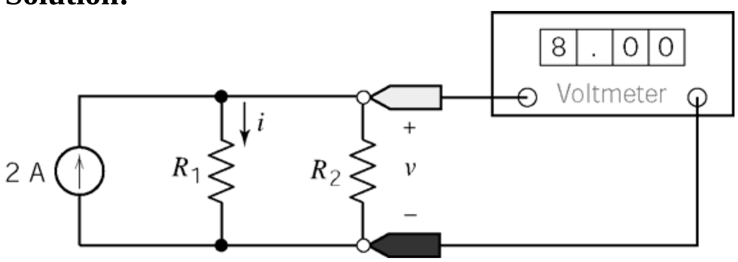


Figure P 3.4-3

(a) Suppose $R_2 = 12\ \Omega$. Determine the value of R_1 and of the current i.
(b) Suppose, instead, $R_1 = 12\ \Omega$. Determine the value of R_2 and of the current i.
(c) Instead, choose R_1 and R_2 to minimize the power absorbed by any one resistor.

Solution:



$$i = \frac{8}{R_1} \text{ or } R_1 = \frac{8}{i}$$

$$8 = R_2(2-i) \Rightarrow i = 2 - \frac{8}{R_2} \text{ or } R_2 = \frac{8}{2-i}$$

$$(a)\ i = 2 - \frac{8}{6} = \underline{\frac{2}{3}\text{ A}}\ ;\ R_1 = \frac{8}{\frac{2}{3}} = \underline{12\,\Omega}$$

$$(b)\ i = \frac{8}{6} = \underline{\frac{4}{3}\text{ A}}\ ;\ R_2 = \frac{8}{2-\frac{4}{3}} = \underline{12\,\Omega}$$

(c) $R_1 = R_2$ will cause $i = \frac{1}{2} 2 = 1$ A. The current in both R_1 and R_2 will be 1 A.

$$2 \cdot \frac{R_1\, R_2}{R_1 + R_2} = 8\ ;\quad R_1 = R_2 \quad \Rightarrow \quad 2 \cdot \frac{1}{2} R_1 = 8 \Rightarrow R_1 = 8 \ \therefore \underline{R_1 = R_2 = 8\,\Omega}$$

P 3.4-4 Determine the current i in the circuit shown in Figure P 3.4-4.

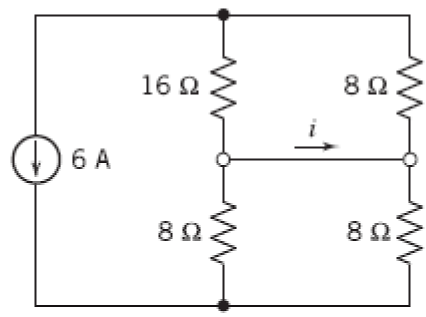
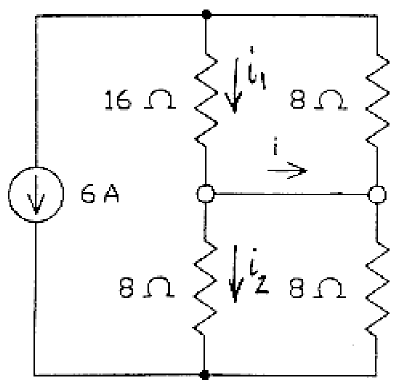


Figure P 3.4-4

Solution:



Current Division:

$$i_1 = \frac{8}{16+8}(-6) = -2 \text{ A}$$

$$i_2 = \frac{8}{8+8}(-6) = -3 \text{ A}$$

KCL:

$$i = i_1 - i_2 = -2 - (-3) = 1 \text{ A}$$

P 3.4-5 Consider the circuit shown in Figure P 3.4-5 when 4 Ω $\leq R_1 \leq$ 6 Ω and R_2 = 10 Ω. Select the source i_s so that v_o remains between 9 V and 13 V

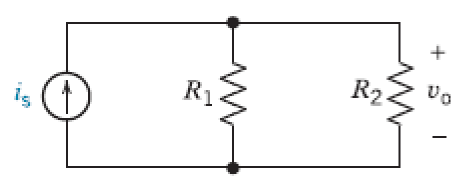
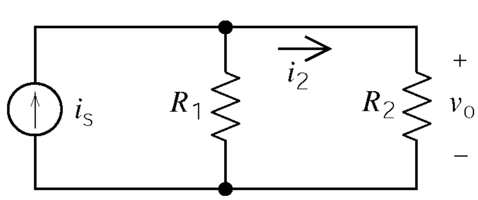


Figure P 3.4-5

Solution:



current division: $i_2 = \left(\dfrac{R_1}{R_1 + R_2}\right) i_s$ and

Ohm's Law: $v_o = i_2 R_2$ yields

$$i_s = \left(\frac{v_o}{R_2}\right)\left(\frac{R_1 + R_2}{R_1}\right)$$

plugging in $R_1 = 4\Omega$, $v_o > 9$ V gives $i_s > 3.15$ A

and $R_1 = 6\Omega$, $v_o < 13$ V gives $i_s < 3.47$ A

So any $\underline{3.15 \text{ A} < i_s < 3.47 \text{ A}}$ keeps $9 \text{ V} < v_o < 13$ V.

P 3.4-6 Figure P 3.4-6 shows a transistor amplifier. The values of R_1 and R_2 are to be selected. Resistances R_1 and R_2 are used to bias the transistor, that is, to create useful operating conditions. In this problem, we want to select R_1 and R_2 so that $v_b = 5$ V. We expect the value of i_b to be approximately 10 μA. When $i_1 \le 10 i_b$, it is customary to treat i_b as negligible, that is, to assume $i_b = 0$. In that case R_1 and R_2 comprise a voltage divider.

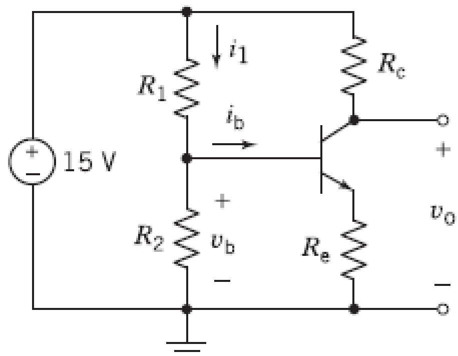


Figure P 3.4-6

(a) Select values for R_1 and R_2 so that $v_b = 5$ V and the total power absorbed by R_1 and R_2 is no more than 5 mW.

(b) An inferior transistor could cause i_b to be larger than expected. Using the values of R_1 and R_2 from part (a), determine the value of v_b that would result from $i_b = 15$ μA.

Solution:

(a) To insure that i_b is negligible we require

$$i_1 = \frac{15}{R_1 + R_2} \ge 10\left(10\times10^{-6}\right) = 10^{-3}$$

so

$$R_1 + R_2 \le 150 \text{ k}\Omega$$

To insure that the total power absorbed by R_1 and R_2 is no more than 5 mW we require

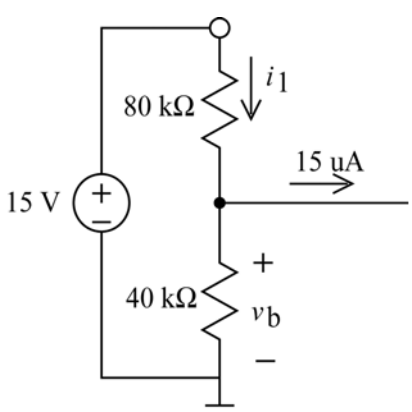
$$\frac{15^2}{R_1 + R_2} \le 5\times10^{-3} \quad \Rightarrow \quad R_1 + R_2 \ge 45 \text{ k}\Omega$$

Next to cause $v_b = 5$ V we require

$$5 = v_b = \frac{R_2}{R_1 + R_2} 15 \quad \Rightarrow \quad R_1 = 2R_2$$

For example, $R_1 = 40 \text{ k}\Omega,\ R_2 = 80 \text{ k}\Omega,$ satisfy all three requirements.

(b)



KVL gives $\left(80\times10^3\right) i_1 + v_b - 15 = 0$

KCL gives $i_1 = \dfrac{v_b}{40\times10^3} + 15\times10^{-6}$

Therefore $\left(80\times10^3\right)\left(\dfrac{v_b}{40\times10^3} + 15\times10^{-6}\right) + v_b = 15$

Finally $3v_b + 1.2 = 15 \quad \Rightarrow \quad v_b = \dfrac{13.8}{3} = 4.6 \text{ V}$

P 3.4-7 Determine the value of the current i in the circuit shown in Figure P 3.4-7.

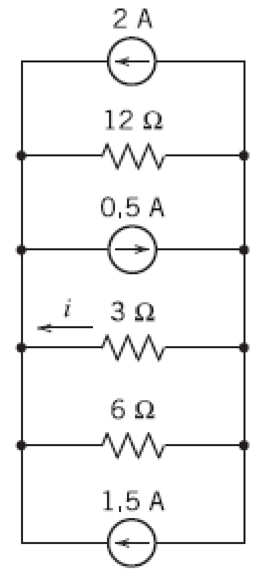
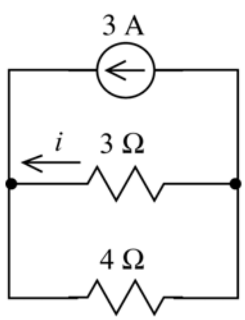


Figure P 3.4-7

Solution:

All of the elements of this circuit are connected in parallel. Replace the parallel current sources by a single equivalent 2 – 0.5 + 1.5 = 3 A current source. Replace the parallel 12 Ω and 6 Ω resistors by a single $\frac{12\times 6}{12+6}=4\ \Omega$ resistor.



By current division

$$i=-\left(\frac{4}{3+4}\right)3=-\frac{12}{7}=-1.714\ \text{A}$$

(checked: LNAP 62/6/07)

P 3.4-8 Determine the value of the voltage v in Figure P 3.4-8.

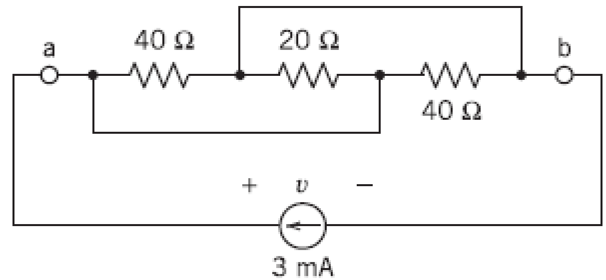
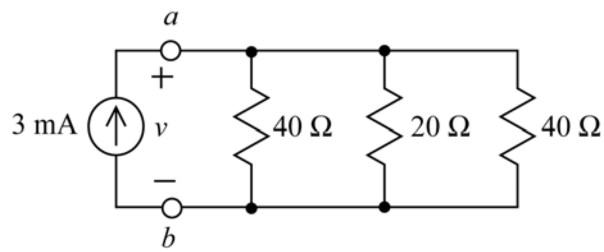


Figure P 3.4-8

Solution:
Each of the resistors is connected between nodes a and b. The resistors are connected in parallel and the circuit can be redrawn like this:



Then $$40 \,||\, 20 \,||\, 40 = 10 \ \Omega$$

so $$v = 10(0.003) = 0.03 = 30 \text{ mV}$$

(checked: LNAP 6/21/04)

P 3.4-9 Determine the power supplied by the dependent source in Figure P 3.4-9.

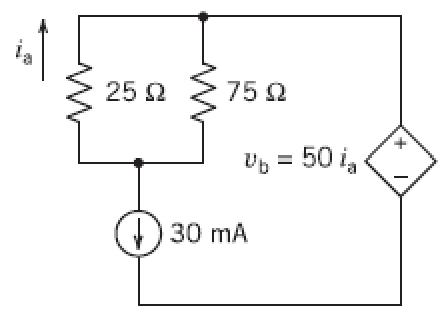
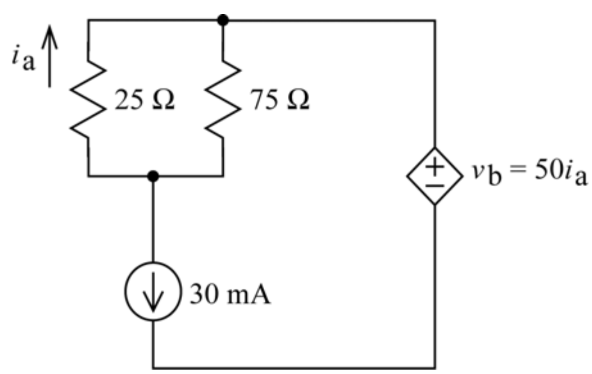


Figure P 3.4-9

Solution:
Use current division to get

$$i_a = -\frac{75}{25+75}\left(30\times10^{-3}\right) = -22.5 \text{ mA}$$

so $$v_b = 50\left(-22.5\times10^{-3}\right) = -1.125 \text{ V}$$



The power supplied by the dependent source is given by

$$p = -\left(30\times10^{-3}\right)\left(-1.125\right) = 33.75 \text{ mW}$$

(checked: LNAP 6/12/04)

.

P 3.4-10 Determine the values of the resistances R_1 and R_2 for the circuit shown in Figure P 3.4-10.

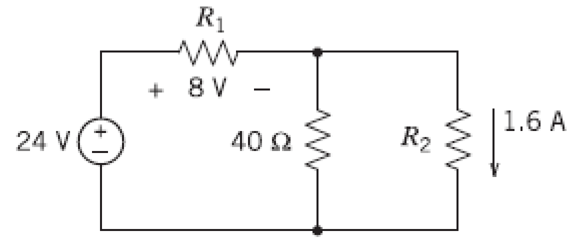


Figure P 3.4-10

Solution:
Using voltage division

$$8=\frac{R_1}{R_1+\dfrac{40R_2}{R_2+40}}\times 24 \quad\Rightarrow\quad \frac{1}{3}=\frac{R_1\left(R_2+40\right)}{R_1R_2+40\left(R_1+R_2\right)}$$

$$\Rightarrow\quad R_1R_2+40\left(R_1+R_2\right)=3R_1R_2+120R_1 \quad\Rightarrow\quad R_1=\frac{40R_2}{2R_2+80}$$

Using KVL

$$24=8+R_2\left(1.6\right) \quad\Rightarrow\quad R_2=10\ \Omega$$

Then

$$R_1=\frac{40\left(10\right)}{2\left(10\right)+80}=4\ \Omega$$

P 3.4-11 Determine the values of the resistances R_1 and R_2 for the circuit shown in Figure P 3.4-11.

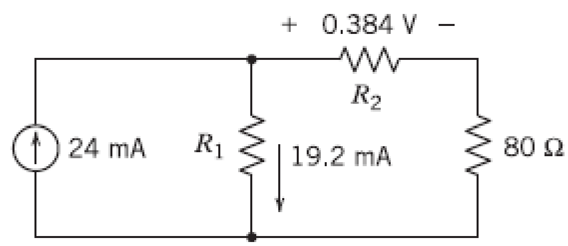


Figure P 3.4-11

Solution:
Using KCL

$$.024=0.0192+\frac{0.384}{R_2} \quad\Rightarrow\quad R_2=\frac{0.384}{0.0048}=80\ \Omega$$

Using current division

$$\frac{0.384}{R_2}=\frac{R_1}{R_1+\left(R_2+80\right)}\times 0.024 \quad\Rightarrow\quad 16=\frac{R_1R_2}{R_1+R_2+80}=\frac{80R_1}{R_1+160} \quad\Rightarrow\quad R_1=40\ \Omega$$

P 3.4-12 Determine the value of the current measured by the meter in Figure P 3.4-12.

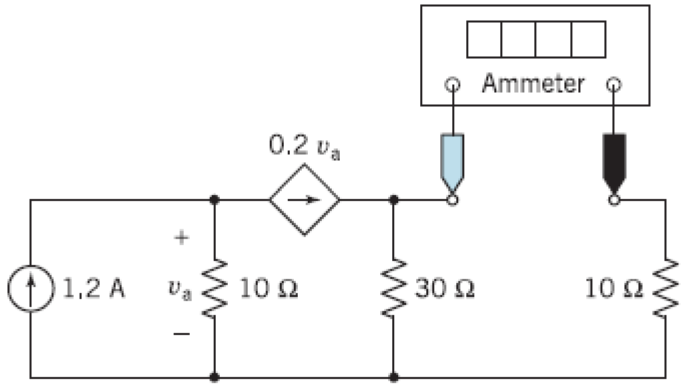
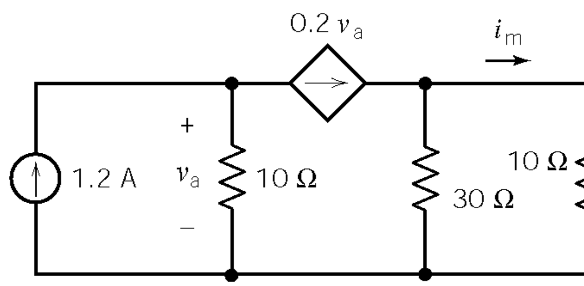


Figure P 3.4-12

Solution:

Replace the (ideal) ammeter with the equivalent short circuit. Label the current measured by the meter.



Apply KCL at the left node of the VCCS to get

$$1.2 = \frac{v_a}{10} + 0.2\,v_a = 0.3\,v_a \quad \Rightarrow \quad v_a = \frac{1.2}{0.3} = 4 \text{ V}$$

Use current division to get

$$i_m = \frac{30}{30+10}\,0.2\,v_a = \frac{30}{30+10}\,0.2(4) = 0.6 \text{ A}$$

(checked using LNAP 9/11/04)

P3.4-13 Consider the combination of resistors shown in Figure P3.4-13. Let R_p denote the equivalent resistance.

(a) Suppose $20\ \Omega \le R \le 320\ \Omega$. Determine the corresponding range of values of R_p.

(b) Suppose instead $R = 0$ (a short circuit). Determine the value of R_p.

(c) Suppose instead $R = \infty$ (an open circuit). Determine the value of R_p.

(d) Suppose instead the equivalent resistance is $R_p = 40\ \Omega$. Determine the value of R.

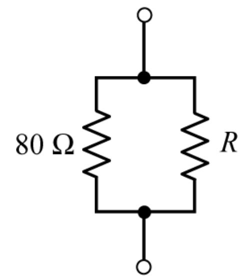


Figure P3.4-13

Solution:

(a) First, when $R = 20\ \Omega$ then $R_p = 80 \,||\, 20 = \dfrac{80(20)}{80+20} = 16\ \Omega$. Next, when $R = 320\ \Omega$ then

$R_p = 80 \,||\, 320 = \dfrac{80(320)}{80+320} = 64\ \Omega$. Consequently

$$16\ \Omega \le R_p \le 64\ \Omega.$$

(b) When $R = 0$ then $R_p = 80 \,||\, 0 = \dfrac{80(0)}{80+0} = 0\ \Omega$.

(c) When $R = \infty$ then $R_p = \dfrac{80(\infty)}{80+\infty} = \dfrac{80}{\dfrac{80}{\infty}+1} = 80\ \Omega$.

(d) When $R_p = 40\ \Omega$ then $40 = 80 \,||\, R = \dfrac{80R}{80+R} \quad \Rightarrow \quad 80 + R = 2R \quad \Rightarrow \quad R = 80\ \Omega$

P3.4-14 Consider the combination of resistors shown in Figure P3.4-14. Let R_p denote the equivalent resistance.

(a) Suppose $40\ \Omega \le R \le 400\ \Omega$. Determine the corresponding range of values of R_p.

(b) Suppose instead $R = 0$ (a short circuit). Determine the value of R_p.

(c) Suppose instead $R = \infty$ (an open circuit). Determine the value of R_p.

(d) Suppose instead the equivalent resistance is $R_p = 80\ \Omega$. Determine the value of R.

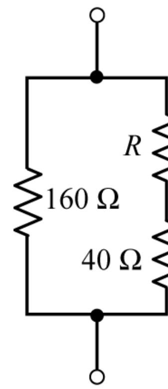


Figure P3.4-14

Solution:

(a) First, when $R = 40\ \Omega$ then $R_p = 160 \,||\, (40+40) = 160 \,||\, 80 = \dfrac{160(80)}{160+80} = \dfrac{160}{3} = 53.33\ \Omega$. Next, when $R = 400\ \Omega$ then $R_p = 160 \,||\, (40+400) = 160 \,||\, 440 = \dfrac{160(440)}{160+440} = \dfrac{16(44)}{6} = 117.33\ \Omega$.

Consequently $53.33\ \Omega \le R_p \le 117.33\ \Omega$.

(b) When $R = 0$ then $R_p = 160 \,||\, (0+40) = 160 \,||\, 40 = \dfrac{160(40)}{160+40} = \dfrac{16(4)}{2} = 32\ \Omega$.

(c) When $R = \infty$ then $R_p = 160 \,||\, (\infty+40) = 160 \,||\, \infty = \dfrac{1}{\frac{1}{160}+\frac{1}{\infty}} = 160\ \Omega$.

(d) When $R_p = 80\ \Omega$ then

$$80 = 160 \,||\, (R+40) = \frac{160(R+40)}{160+R+40} \quad \Rightarrow \quad R+200 = \frac{160}{80}(R+40) = 2R+80 \quad \Rightarrow \quad R = 120\ \Omega.$$

P3.4-15 Consider the combination of resistors shown in Figure P3.4-15. Let R_p denote the equivalent resistance.

(a) Suppose $50\ \Omega \le R \le 800\ \Omega$. Determine the corresponding range of values of R_p.

(b) Suppose instead $R = 0$ (a short circuit). Determine the value of R_p.

(c) Suppose instead $R = \infty$ (an open circuit). Determine the value of R_p.

(d) Suppose instead the equivalent resistance is $R_p = 150\ \Omega$. Determine the value of R.

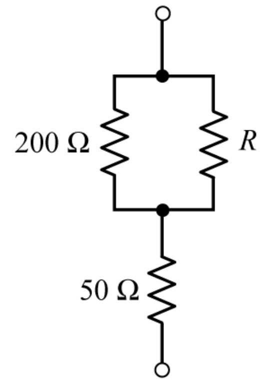


Figure P3.4-15

Solution:

(a) First, when $R = 50\ \Omega$ then $R_p = 50 + (200 \| 50) = 50 + \dfrac{200(50)}{200+50} = 50 + \dfrac{200}{5} = 90\ \Omega$. Next, when $R = 800\ \Omega$ then $R_p = 50 + (200 \| 800) = 50 + \dfrac{200(800)}{200+800} = 50 + \dfrac{800}{5} = 210\ \Omega$.

Consequently $\quad 90\ \Omega \le R_p \le 210\ \Omega$.

(b) When $R = 0$ then $R_p = 50 + (200 \| 0) = 50 + \dfrac{200(0)}{200+0} = 50\ \Omega$.

(c) When $R = \infty$ then $R_p = 50 + (200 \| \infty) = 50 + \dfrac{200(\infty)}{200+\infty} = 50 + \dfrac{200}{\dfrac{200}{\infty}+1} = 250\ \Omega$.

(d) When $R_p = 150\ \Omega$ then

$$150 = 50 + (200 \| R) = 50 + \frac{200R}{200+R} \quad \Rightarrow \quad 100(200+R) = 200R \quad \Rightarrow \quad R = 200\ \Omega.$$

P3.4-16 The input to the circuit shown in Figure P3.4-16 is the source current, i_s. The output is the current measured by the meter, i_o. A current divider connects the source to the meter. Given the following observations:

A. The input $i_s = 5$ A causes the output to be $i_o = 2$ A.

B. When $i_s = 2$ A the source supplies 48 W.

Determine the values of the resistances R_1 and R_2.

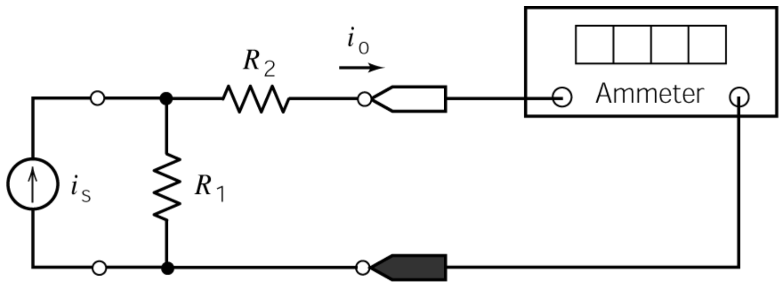


Figure P3.4-16

Solution:

From current division, $i_o = \left(\frac{R_1}{R_1 + R_2}\right) i_s$. When $i_s = 5$ A and $i_o = 2$ A then $\frac{2}{5} = \frac{R_1}{R_1 + R_2}$ so $2\left(R_1 + R_2\right) = 5\,R_1$ or $2\,R_2 = 3\,R_1$.

The power supplied by the source is given by $i_s\left[\left(\frac{R_1\,R_2}{R_1 + R_2}\right) i_s\right]$. When $i_s = 2$ A the source supplies 48 W, so $48 = 2\left[\left(\frac{R_1\,R_2}{R_1 + R_2}\right) 2\right] \quad \Rightarrow \quad 12 = \frac{R_1\,R_2}{R_1 + R_2}$.

Combining these results gives $12 = \frac{R_1\left(\frac{3}{2} R_1\right)}{R_1 + \left(\frac{3}{2} R_1\right)} = \frac{\frac{3}{2} R_1}{\frac{5}{2}} = \frac{3}{5} R_1 \quad \Rightarrow \quad R_1 = \frac{5}{3}(12) = 20\ \Omega$ and

$\frac{3\,R_1}{2} = 30\ \Omega$.

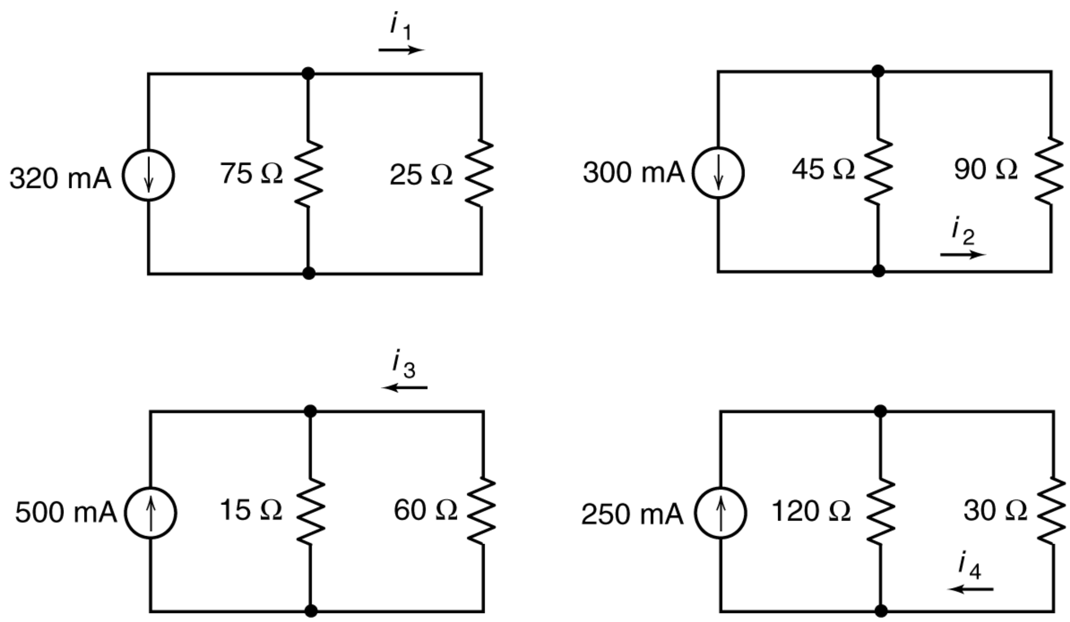


Figure P3.4-17

P3.4-17. Figure P3.4-17 shows four similar but slightly different circuits. Determine the values of the currents i_1, i_2, i_3 and i_4.

Solution:

Using current division:

$$i_1 = -\left(\frac{75}{75+25}\right)320 = -240 \text{ mA}, \quad i_2 = \left(\frac{45}{45+90}\right)300 = 100 \text{ mA},$$

$$i_3 = -\left(\frac{15}{15+60}\right)500 = -100 \text{ mA} \quad \text{and} \quad i_4 = \left(\frac{120}{120+30}\right)250 = 200 \text{ mA}$$

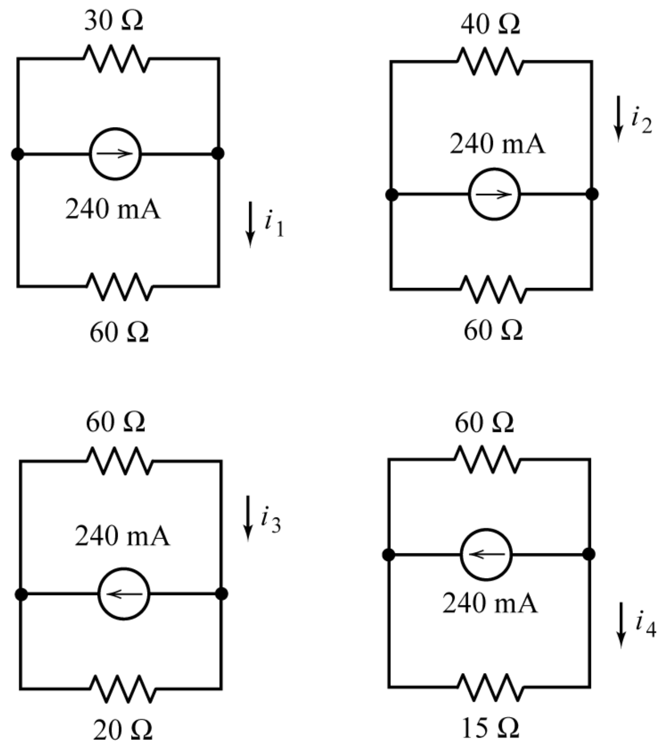


Figure P3.4-18

P3.4-18. Figure P3.4-18 shows four similar but slightly different circuits. Determine the values of the currents i_1, i_2, i_3 and i_4.

Solution:
Using current division:

$$i_1 = \left(\frac{30}{30+60}\right)240 = 80 \text{ mA}, \quad i_2 = -\left(\frac{60}{60+40}\right)240 = -144 \text{ mA},$$

$$i_3 = \left(\frac{20}{60+20}\right)240 = 60 \text{ mA} \quad \text{and} \quad i_4 = -\left(\frac{60}{60+15}\right)240 = -192 \text{ mA}$$

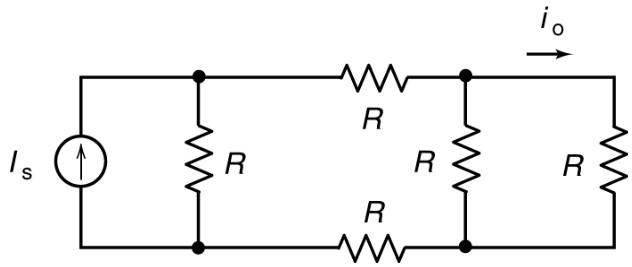


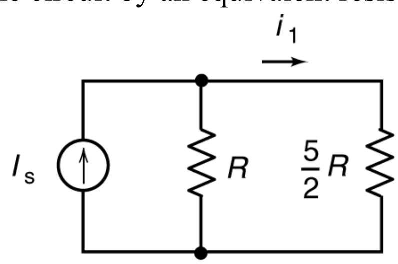
Figure P3.4-19

P3.4-19. The input to the circuit shown in Figure P3.4-19 is the current source current I_s. The output is the current i_o. The output of this circuit is proportion to the input, that is

$$i_o = k\, I_s$$

Determine the value of the constant of proportionality, *k*.

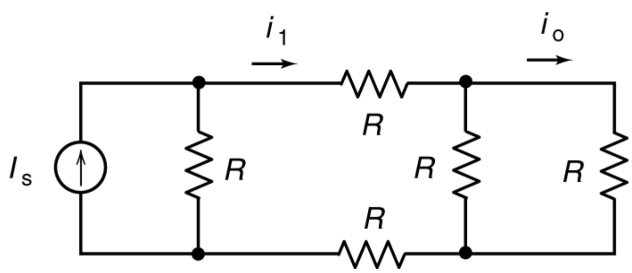
Solution:
Replace six resistors at the right of the circuit by an equivalent resistance to get



Using current division:

$$i_1 = \left(\frac{\frac{5}{2}R}{R+\frac{5}{2}R}\right) I_s = \frac{2}{7} I_s$$

Return to the original circuit



Using current division:

$$i_o = \frac{R}{R+R} i_1 = \frac{1}{2} i_1 = \frac{1}{2}\left(\frac{2}{7} I_s\right) = \frac{1}{7} I_s$$

Therefore $$k = \frac{1}{7}$$

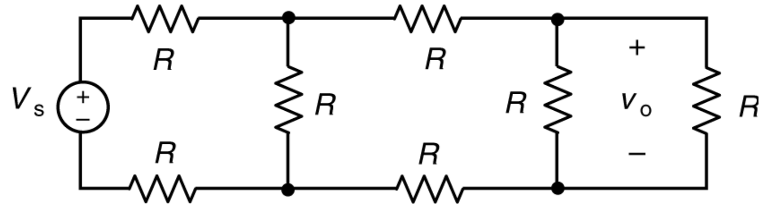


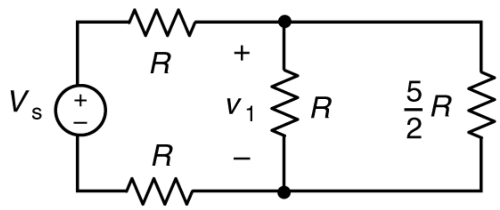
Figure P3.4-20

P3.4-20. The input to the circuit shown in Figure P3.4-20 is the voltage source voltage V_s. The output is the voltage v_o. The output of this circuit is proportion to the input, that is

$$v_o = k V_s$$

Determine the value of the constant of proportionality, k.

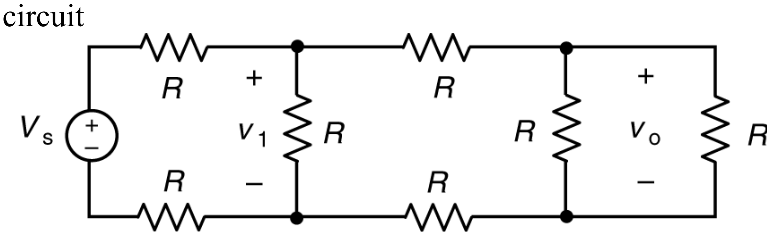
Solution:
Replace six resistors at the right of the circuit by an equivalent resistance to get



Using voltage division:

$$v_1 = \left(\frac{R \| \frac{5}{2} R}{2R + R \| \frac{5}{2} R} \right) V_s \left(\frac{\frac{5}{7} R}{2R + \frac{5}{7} R} \right) V_s = \frac{5}{19} V_s$$

Return to the original circuit



Using current division:

$$v_o = \frac{\frac{R}{2}}{2R + \frac{R}{2}} v_1 = \frac{1}{5} v_1 = \frac{1}{5} \left(\frac{5}{19} I_s \right) = \frac{1}{19} V_s$$

Therefore $$k = \frac{1}{19}$$

Section 3-5 Series Voltage Sources and Parallel Current Sources

P 3.5-1 Determine the power supplied by each source in the circuit shown in Figure P 3.5-1.

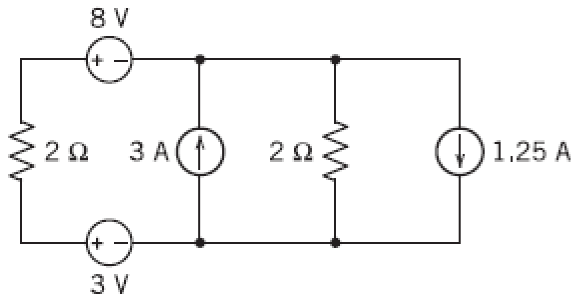
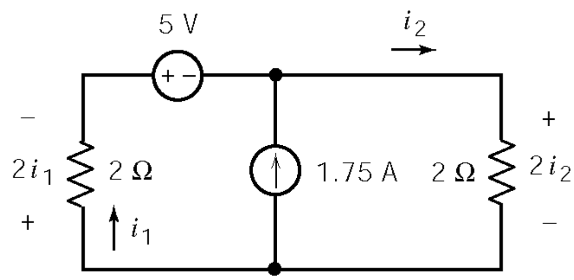


Figure P 3.5-1

Solution:

The voltage sources are connected in series and can be replaced by a single equivalent voltage source. Similarly, the parallel current sources can be replaced by an equivalent current source.

After doing so, and labeling the resistor currents, we have the circuit shown.



Apply KCL at the top node of the current source to get

$$i_1 + 1.75 = i_2$$

Apply KVL to the outside loop to get

$$5 + 2i_2 + 2i_1 = 0$$

so $$5 + 2(i_1 + 1.75) + 2i_1 = 0 \quad \Rightarrow \quad i_1 = -\frac{8.5}{4} = -2.125 \text{ A}$$

and $$i_2 = -2.125 + 1.75 = -0.375 \text{ A}$$

The power supplied by each sources is:

Source	Power delivered
8-V voltage source	$-8i_1 = 17$ W
3-V voltage source	$3i_1 = -6.375$ W
3-A current source	$3 \times 2i_2 = -2.25$ W
1.25-A current source	$-1.25 \times 2i_2 = 0.9375$ W

(Checked using LNAP, 9/14/04)

P 3.5-2 Determine the power supplied by each source in the circuit shown in Figure P 3.5-2.

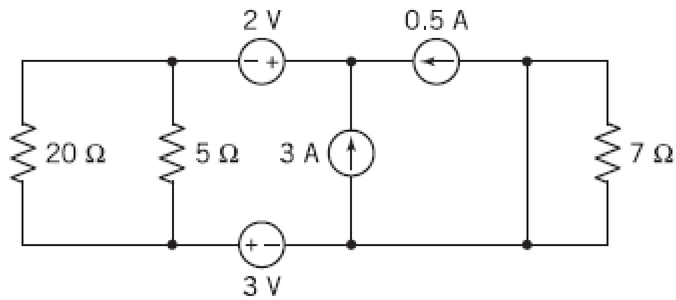


Figure P 3.5-2

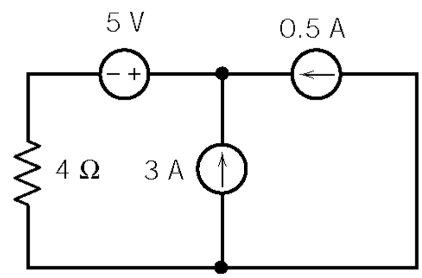
Solution:

The 20-Ω and 5-Ω resistors are connected in parallel. The equivalent resistance is $\frac{20\times5}{20+5}=4\ \Omega$.

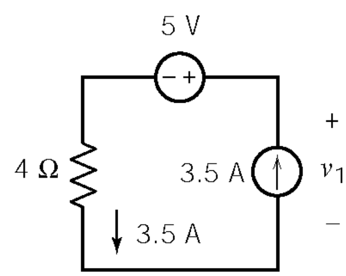
The 7-Ω resistor is connected in parallel with a short circuit, a 0-Ω resistor. The equivalent resistance is $\frac{0\times7}{0+7}=0\ \Omega$, a short circuit.

The voltage sources are connected in series and can be replaced by a single equivalent voltage source.

After doing so, and labeling the resistor currents, we have the circuit shown.



The parallel current sources can be replaced by an equivalent current source.



Apply KVL to get

$$-5+v_1-4(3.5)=0 \quad\Rightarrow\quad v_1=19\text{ V}$$

The power supplied by each sources is:

Source	Power delivered
8-V voltage source	$-2(3.5)=-7$ W
3-V voltage source	$-3(3.5)=-10.5$ W
3-A current source	$3\times19=57$ W
0.5-A current source	$0.5\times19=9.5$ W

(Checked using LNAP, 9/15/04)

P 3.5-3 Determine the power received by each resistor in the circuit shown in Figure P 3.5-3.

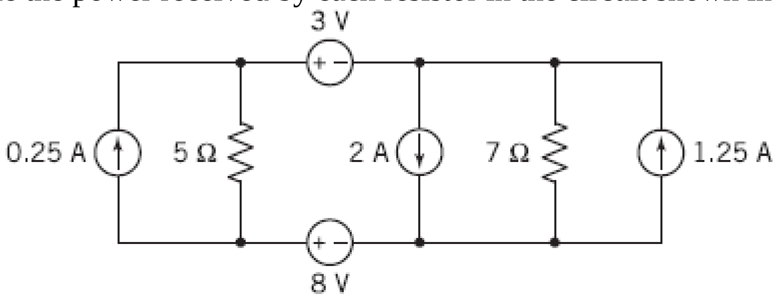
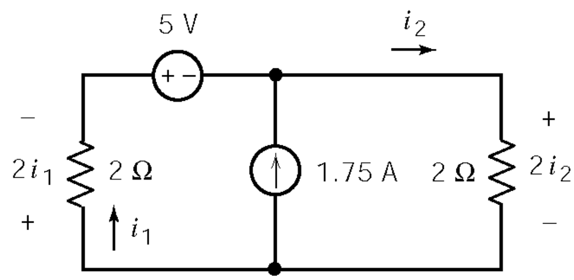


Figure P 3.5-3

Solution:

The voltage sources are connected in series and can be replaced by a single equivalent voltage source. Similarly, the parallel current sources can be replaced by an equivalent current source.

After doing so, and labeling the resistor currents, we have the circuit shown.



Apply KCL at the top node of the current source to get

$$i_1 + 1.75 = i_2$$

Apply KVL to the outside loop to get

$$5 + 2i_2 + i_1 = 0$$

so

$$5 + 2(i_1 + 1.75) + 2i_1 = 0 \quad \Rightarrow \quad i_1 = -\frac{8.5}{4} = -2.125 \text{ A}$$

and

$$i_2 = -2.125 + 1.75 = -0.375 \text{ A}$$

The power supplied by each sources is:

Source	Power delivered
8-V voltage source	$-8i_1 = 17$ W
3-V voltage source	$3i_1 = -6.375$ W
3-A current source	$3 \times 2i_2 = -2.25$ W
1.25-A current source	$-1.25 \times 2i_2 = 0.9375$ W

(Checked using LNAP, 9/14/04)

Section 3-6 Circuit Analysis

P 3.6-1 The circuit shown in Figure P 3.6-1*a* has been divided into two parts. In Figure P 3.6-1*b*, the right-hand part has been replaced with an equivalent circuit. The left-hand part of the circuit has not been changed.

(a) Determine the value of the resistance R in Figure P 3.6-1*b* that makes the circuit in Figure P 3.6-1*b* equivalent to the circuit in Figure P 3.6-1*a*.

(b) Find the current i and the voltage v shown in Figure P 3.6-1*b*. Because of the equivalence, the current i and the voltage v shown in Figure P 3.6-1*a* are equal to the current i and the voltage v shown in Figure P 3.6-1*b*.

(c) Find the current i_2 shown in Figure P 3.6-1*a* using current division.

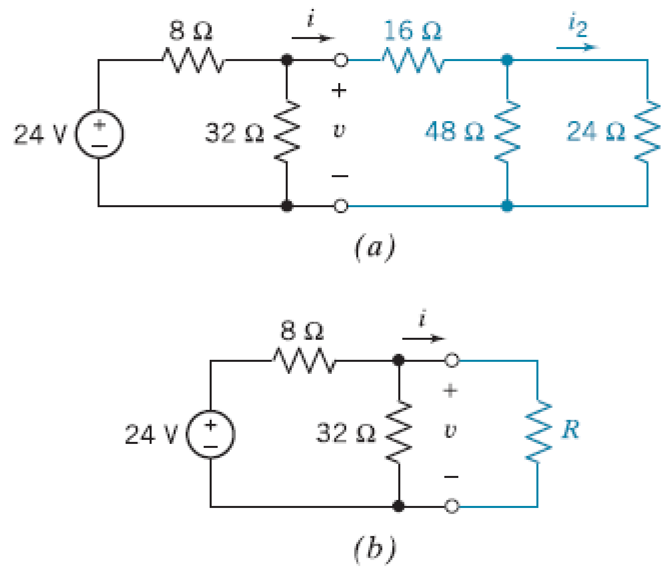
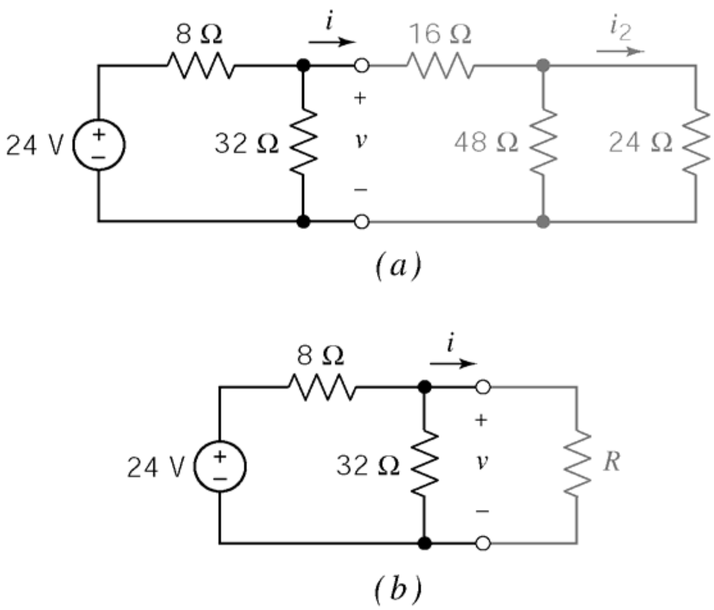


Figure P 3.6-1

Solution:



$$(a)\quad R = 16+\frac{48\cdot 24}{48+24} = \underline{32\ \Omega}$$

$$(b)\quad v = \frac{\dfrac{32\cdot 32}{32+32}}{8+\dfrac{32\cdot 32}{32+32}}24 = \underline{16\ \text{V}}\ ;$$

$$i=\frac{16}{32}=\underline{\frac{1}{2}\ \text{A}}$$

$$(c)\quad i_2 = \frac{48}{48+24}\cdot\frac{1}{2} = \underline{\frac{1}{3}\ \text{A}}$$

P 3.6-2 The circuit shown in Figure P 3.6-2*a* has been divided into three parts. In Figure P 3.6-2*b,* the rightmost part has been replaced with an equivalent circuit. The rest of the circuit has not been changed. The circuit is simplified further in Figure 3.6-2*c*. Now the middle and rightmost parts have been replaced by a single equivalent resistance. The leftmost part of the circuit is still unchanged.

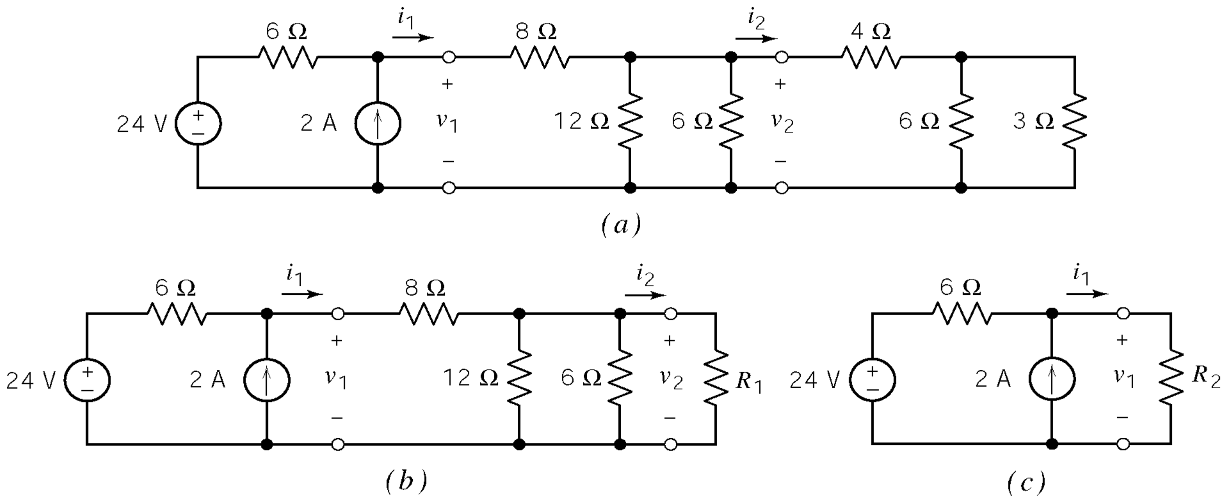


Figure P 3.6-2

(a) Determine the value of the resistance R_1 in Figure P 3.6-2*b* that makes the circuit in Figure P 3.6-2*b* equivalent to the circuit in Figure P 3.6-2*a*.

(b) Determine the value of the resistance R_2 in Figure P 3.6-2*c* that makes the circuit in Figure P 3.6-2*c* equivalent to the circuit in Figure P 3.6-2*b*.

(c) Find the current i_1 and the voltage v_1 shown in Figure P 3.6-2*c*. Because of the equivalence, the current i_1 and the voltage v_1 shown in Figure P 3.6-2*b* are equal to the current i_1 and the voltage v_1 shown in Figure P 3.6-2*c*.
Hint: $24 = 6(i_1–2) + i_1R_2$

(d) Find the current i_2 and the voltage v_2 shown in Figure P 3.6-2*b*. Because of the equivalence, the current i_2 and the voltage v_2 shown in Figure P 3.6-2*a* are equal to the current i_2 and the voltage v_2 shown in Figure P 3.6-2*b*.
Hint: Use current division to calculate i_2 from i_1.

(e) Determine the power absorbed by the 3-Ω resistance shown at the right of Figure P 3.6-2*a*.

Solution:

$$(a)\ R_1 = 4 + \frac{3 \cdot 6}{3+6} = \underline{6\,\Omega}$$

$$(b)\ \frac{1}{R_p} = \frac{1}{12} + \frac{1}{6} + \frac{1}{6} \Rightarrow R_p = 2.4\,\Omega \quad \text{then} \quad R_2 = 8 + R_p = \underline{10.4\,\Omega}$$

$$(c)\ \text{KCL: } i_2 + 2 = i_1 \quad \text{and} \quad -24 + 6\,i_2 + R_2 i_1 = 0$$

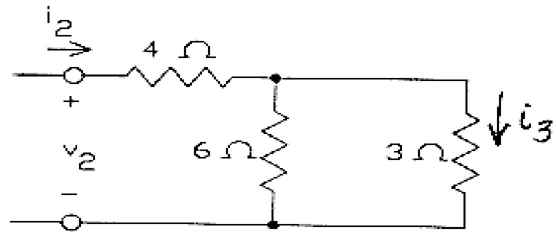
$$\Rightarrow -24 + 6\,(i_1 - 2) + 10.4 i_1 = 0$$

$$\Rightarrow i_1 = \frac{36}{16.4} = \underline{2.195\text{ A}} \Rightarrow v_1 = i_1\,R_2 = 2.2\,(10.4) = \underline{22.83\text{ V}}$$

$$(d)\ \ i_2 = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{6}+\frac{1}{12}}(2.195) = \underline{0.878\text{ A}},$$

$$v_2 = (0.878)(6) = \underline{5.3\text{ V}}$$

$$(e)\ \ i_3 = \frac{6}{3+6} i_2 = 0.585\text{ A} \quad \Rightarrow \quad P = 3\, i_3^{\,2} = \underline{1.03\text{ W}}$$



P 3.6-3 Find i using appropriate circuit reductions and the current divider principle for the circuit of Figure P 3.6-3.

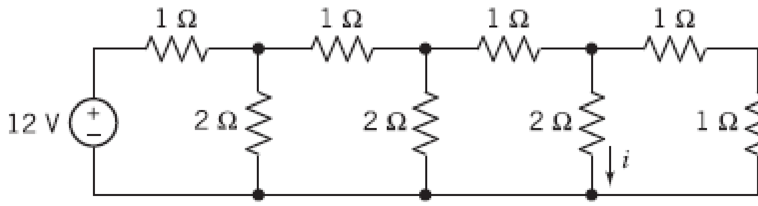
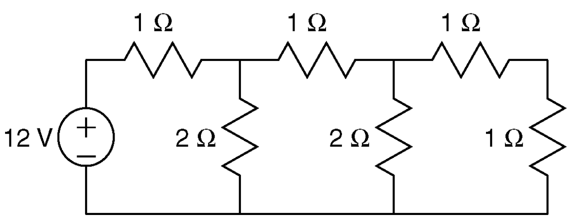
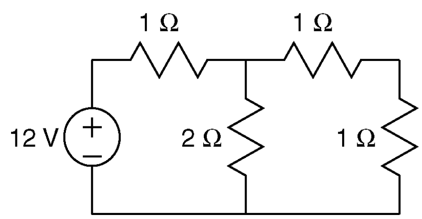


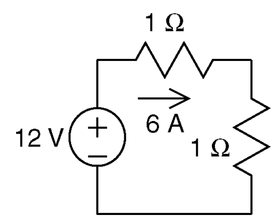
Figure P 3.6-3

Solution:

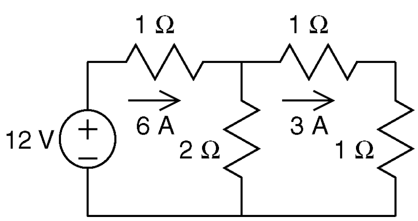
Reduce the circuit from the right side by repeatedly replacing series 1 Ω resistors in parallel with a 2 Ω resistor by the equivalent 1 Ω resistor

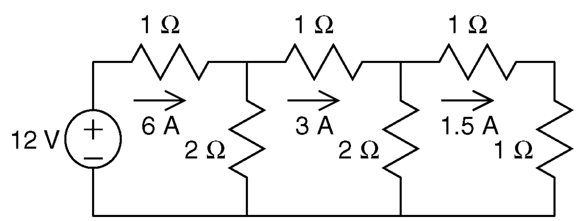


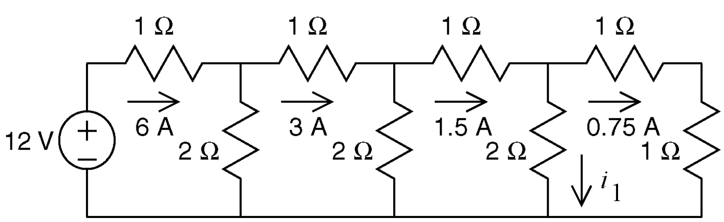




This circuit has become small enough to be easily analyzed. The vertical 1 Ω resistor is equivalent to a 2 Ω resistor connected in parallel with series 1 Ω resistors:







$$i_1 = \frac{1+1}{2+(1+1)}(1.5) = 0.75 \text{ A}$$

P 3.6-4

(a) Determine values of R_1 and R_2 in Figure P 3.6-4*b* that make the circuit in Figure P 3.6-4*b* equivalent to the circuit in Figure P 3.6-4*a*.

(b) Analyze the circuit in Figure P 3.6-4*b* to determine the values of the currents i_a and i_b

(c) Because the circuits are equivalent, the currents i_a and i_b shown in Figure P 3.6-4*b* are equal to the currents i_a and i_b shown in Figure P 3.6-4*a*. Use this fact to determine values of the voltage v_1 and current i_2 shown in Figure P 3.6-4*a*.

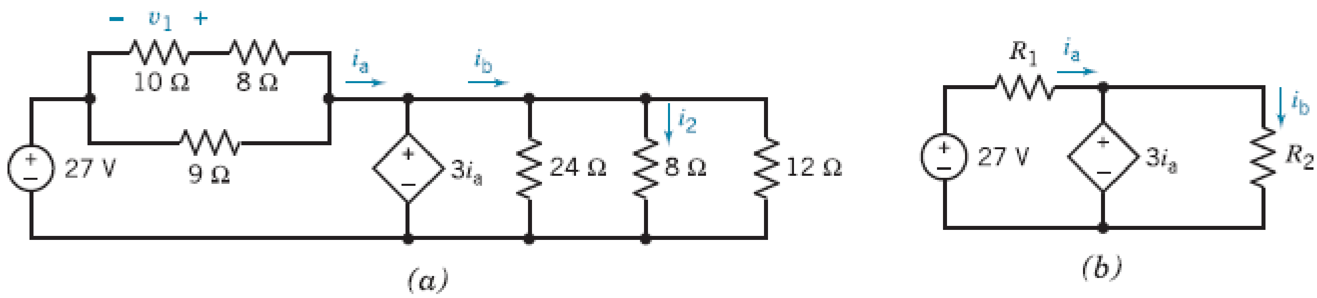
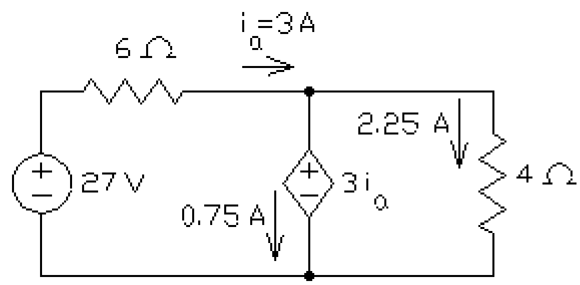


Figure P 3.6-4

Solution:

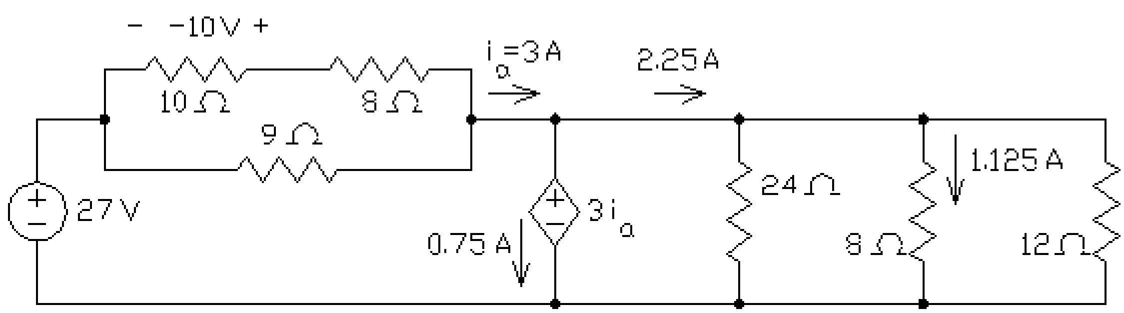
(a) $$\frac{1}{R_2}=\frac{1}{24}+\frac{1}{12}+\frac{1}{8}\quad\Rightarrow\quad R_2=4\,\Omega \qquad \text{and} \qquad R_1=\frac{(10+8)\cdot 9}{(10+8)+9}=6\,\Omega$$

(b)



First, apply KVL to the left mesh to get $-27+6i_a+3i_a=0 \quad\Rightarrow\quad i_a=3\text{ A}$. Next, apply KVL to the left mesh to get $4i_b-3i_a=0 \quad\Rightarrow\quad i_b=2.25\text{ A}$.

(c)



$$i_2=\frac{\frac{1}{8}}{\frac{1}{24}+\frac{1}{8}+\frac{1}{12}}2.25=1.125\text{ A} \qquad \text{and} \qquad v_1=-(10)\left[\frac{9}{(10+8)+9}3\right]=-10\text{ V}$$

P 3.6-5 The voltmeter in the circuit shown in Figure P 3.6-5 shows that the voltage across the 30-Ω resistor is 6 volts. Determine the value of the resistance R_1.
Hint: Use the voltage division twice.
Answer: $R_1 = 40\ \Omega$

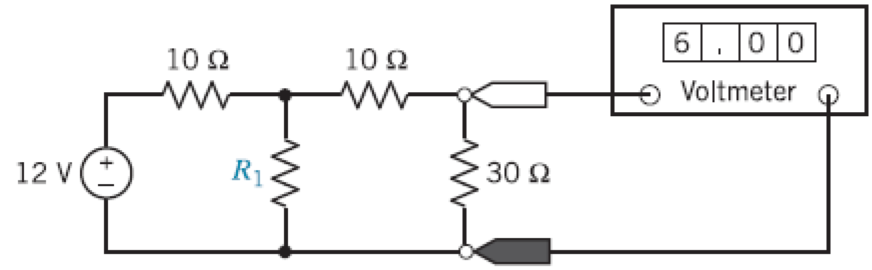
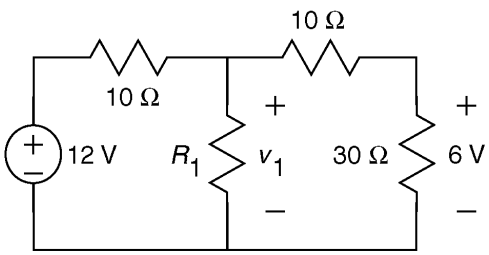
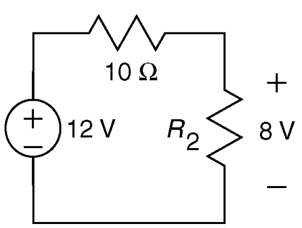


Figure P 3.6-5

P3.6-5



$$\frac{30}{10+30} v_1 = 6 \quad \Rightarrow \quad v_1 = 8\ \text{V}$$



$$\frac{R_2}{R_2+10} 12 = 8 \quad \Rightarrow \quad R_2 = 20\ \Omega$$

$$20 = \frac{R_1(10+30)}{R_1+(10+30)} \quad \Rightarrow \quad R_1 = 40\ \Omega$$

Alternate values that can be used to change the numbers in this problem:

meter reading, V	Right-most resistor, Ω	R_1, Ω
6	30	40
4	30	10
4	20	15
4.8	20	30

P 3.6-6 Determine the voltages v_a and v_c and the currents i_b and i_d for the circuit shown in Figure P 3.6-6.

Answer: $v_a = -2$ V, $v_c = 6$ V, $i_b = -16$ mA, and $i_d = 2$ mA

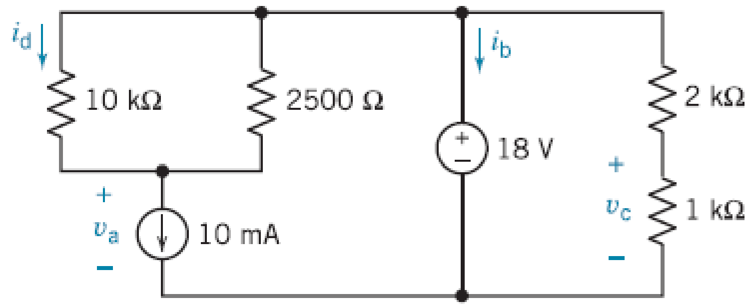
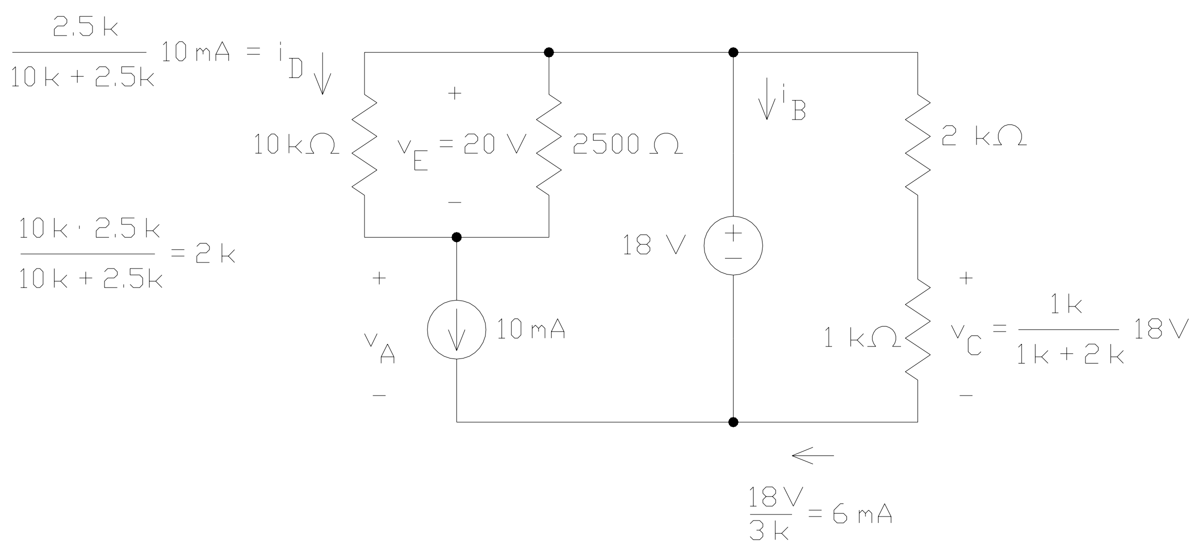


Figure P 3.6-6

Solution:



P 3.6-7 Determine the value of the resistance R in Figure P 3.6-7.
Answer: $R = 28\ \text{k}\Omega$

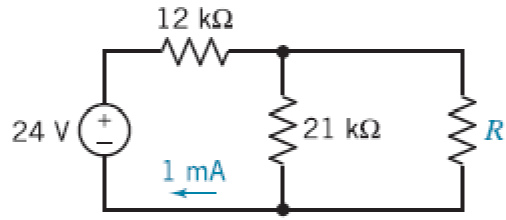
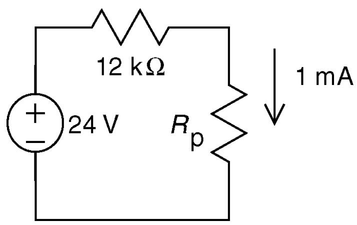


Figure P 3.6-7

Solution:



$$1\times10^{-3} = \frac{24}{12\times10^{3} + R_p} \Rightarrow R_p = 12\times10^{3} = 12\ \text{k}\Omega$$

$$12\times10^{3} = R_p = \frac{\left(21\times10^{3}\right)R}{\left(21\times10^{3}\right)+R} \Rightarrow R = 28\ \text{k}\Omega$$

P 3.6-8 Most of us are familiar with the effects of a mild electric shock. The effects of a severe shock can be devastating and often fatal. Shock results when current is passed through the body. A person can be modeled as a network of resistances. Consider the model circuit shown in Figure P 3.6-8. Determine the voltage developed across the heart and the current flowing through the heart of the person when he or she firmly grasps one end of a voltage source whose other end is connected to the floor.

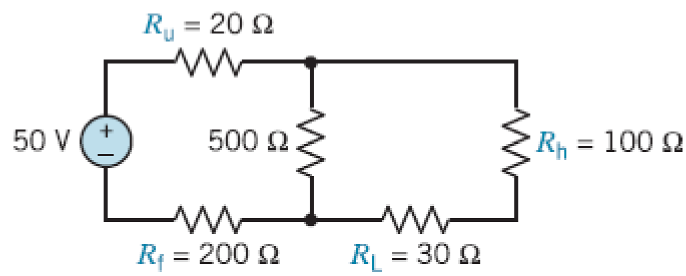
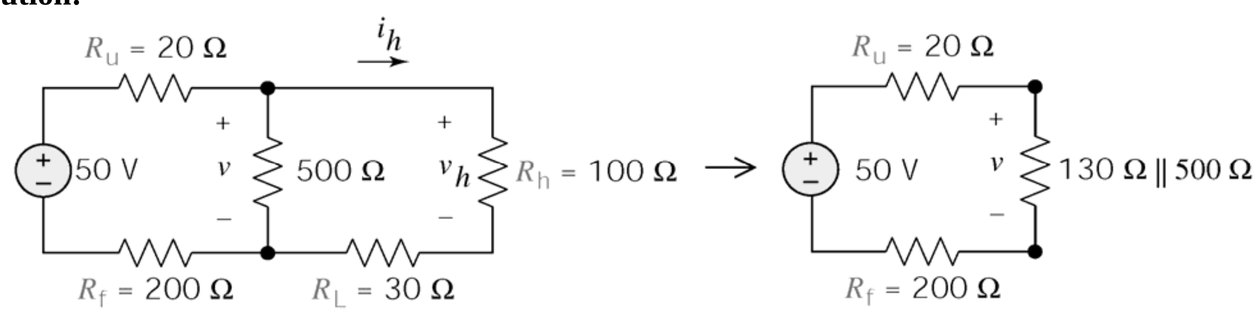


Figure P 3.6-8

The heart is represented by R_h. The floor has resistance to current flow equal to R_f, and the person is standing barefoot on the floor. This type of accident might occur at a swimming pool or boat dock. The upper-body resistance R_u and lower-body resistance R_L vary from person to person.

Solution:



$$\text{Voltage division } \Rightarrow v = 50\left(\frac{130\|500}{130\|500+200+20}\right) = 15.963\ V$$

$$\therefore v_h = v\left(\frac{100}{100+30}\right) = (15.963)\left(\frac{10}{13}\right) = \underline{12.279\text{ V}}$$

$$\therefore\ i_h = \frac{v_h}{100} = \underline{.12279\text{ A}}$$

P 3.6-9 Determine the value of the current i in Figure 3.6-9.

Answer: $i = 0.5$ mA

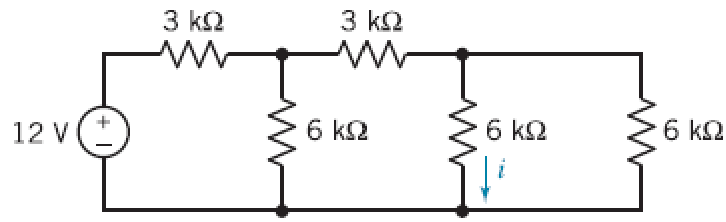
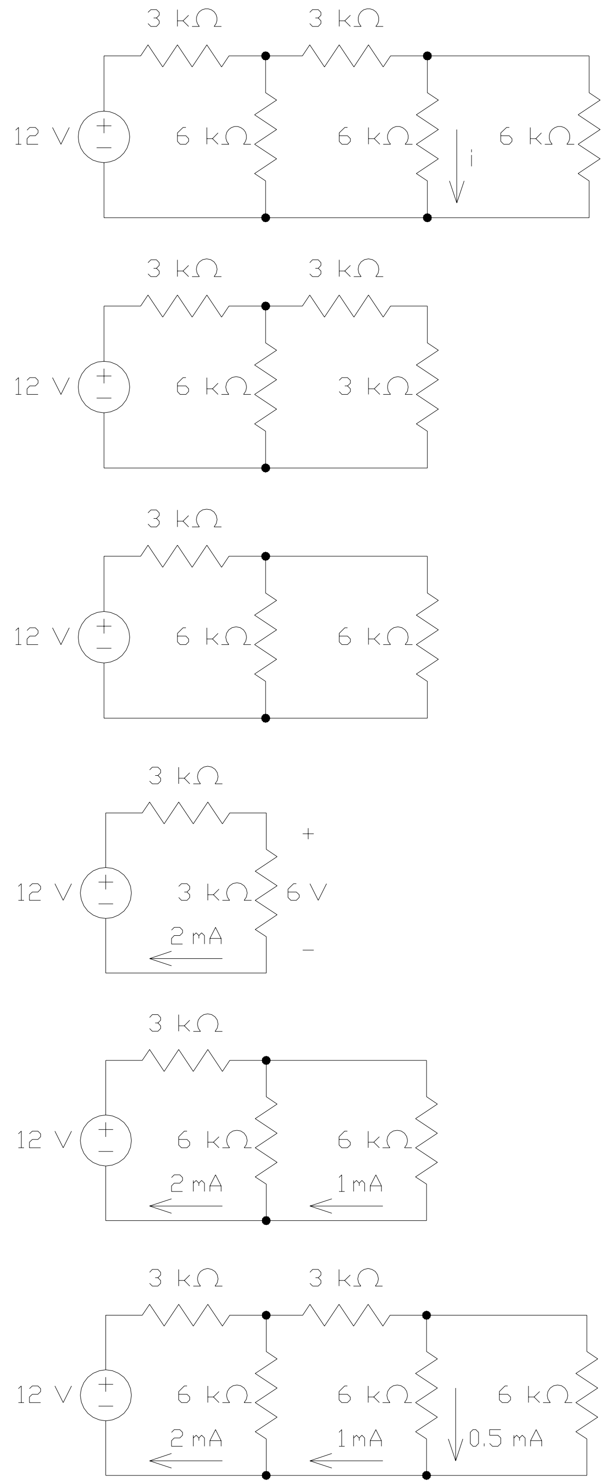


Figure 3.6-9

Solution:



P 3.6-10 Determine the values of i_a, i_b, and v_c in Figure P 3.6-10.

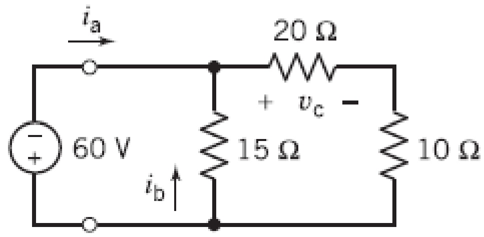
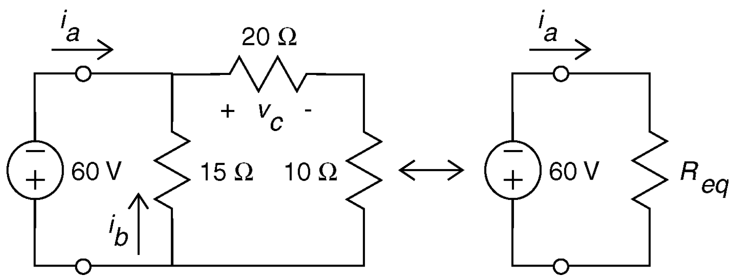


Figure P 3.6-10

Solution:



$$R_{eq} = \frac{15\left(20+10\right)}{15+\left(20+10\right)} = 10\ \Omega$$

$$i_a = -\frac{60}{R_{eq}} = -6\text{ A},\quad i_b = \left(\frac{30}{30+15}\right)\left(\frac{60}{R_{eq}}\right) = 4\text{ A},\quad v_c = \left(\frac{20}{20+10}\right)\left(-60\right) = -40\text{ V}$$

P 3.6-11 Find i and $R_{\text{eq a–b}}$ if v_{ab} = 40 V in the circuit of Figure P 3.6-11.
Answer: $R_{\text{eq a–b}} = 8\ \Omega$, $i = 5/6$ A

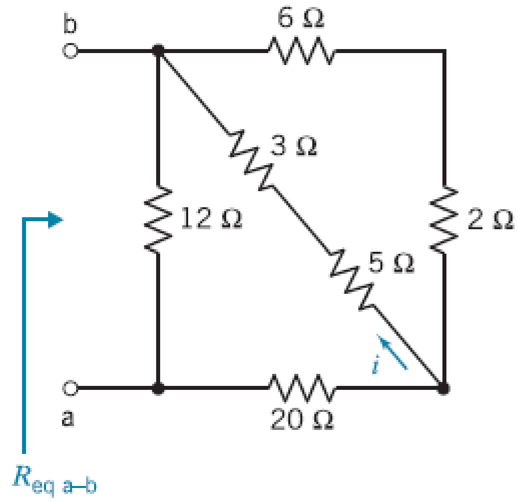
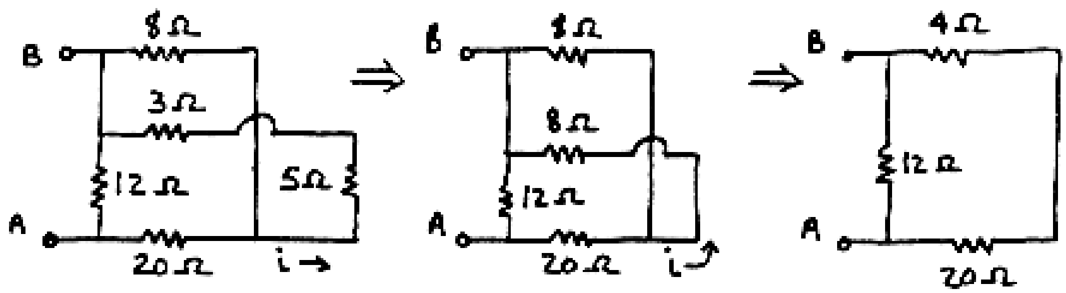


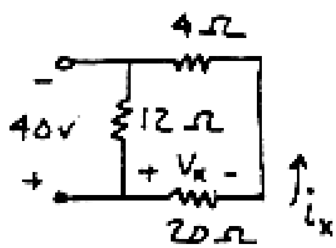
Figure P 3.6-11

Solution:
a)



$$R_{eq} = 24 \| 12 = \frac{(24)(12)}{24 + 12} = \underline{8\ \Omega}$$

b)



from voltage division:

$$v_x = 40\left(\frac{20}{20+4}\right) = \frac{100}{3}\,\text{V} \therefore i_x = \frac{\frac{100}{3}}{20} = \frac{5}{3}\,\text{A}$$

from current division: $i = i_x \left(\frac{8}{8+8}\right) = \underline{\frac{5}{6}\ \text{A}}$

P 3.6-12 The ohmmeter in Figure P 3.6-12 measures the equivalent resistance, R_{eq}, of the resistor circuit. The value of the equivalent resistance, R_{eq}, depends on the value of the resistance R.

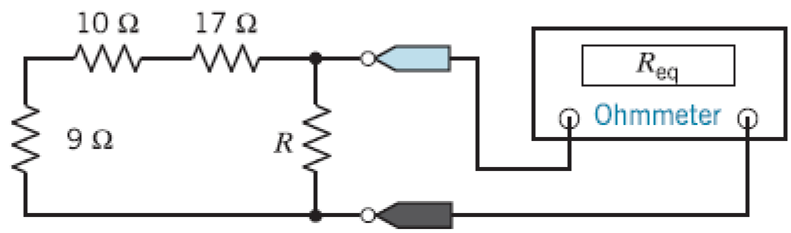
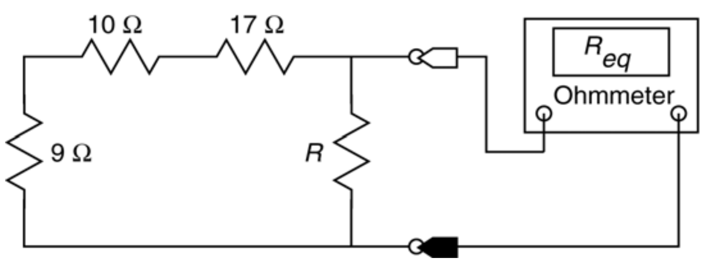


Figure P 3.6-12

(a) Determine the value of the equivalent resistance, R_{eq}, when $R = 18\ \Omega$.
(b) Determine the value of the resistance R required to cause the equivalent resistance to be $R_{eq} = 18\ \Omega$.

Solution:



$$9+10+17=36\ \Omega$$

a.) $$\frac{36(9)}{36+9}=7.2\ \Omega$$

b.) $$\frac{36\ R}{36+R}=12 \Rightarrow 24\ R=(12)(36) \Rightarrow R=18\ \Omega$$

P 3.6-13 Find the R_{eq} at terminals a–b in Figure P 3.6-13. Also determine i, i_1, and i_2.

Answer: $R_{eq} = 8\ \Omega$, $i = 5$ A, $i_1 = 5/3$ A, $i_2 = 5/2$ A

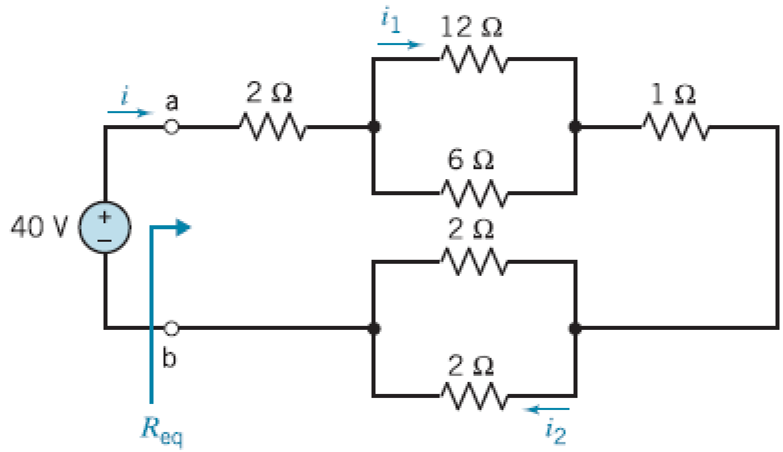


Figure P 3.6-13

Solution:

$$R_{eq} = 2+1+\left(6\|12\right)+\left(2\|2\right) = 3+4+1 = \underline{8\ \Omega} \quad \text{so} \quad i = \frac{40}{R_{eq}} = \frac{40}{8} = \underline{5\ \text{A}}$$

Using current division

$$i_1 = i\left(\frac{6}{6+12}\right) = (5)\left(\tfrac{1}{3}\right) = \underline{\tfrac{5}{3}\ \text{A}} \quad \text{and} \quad i_2 = i\left(\frac{2}{2+2}\right) = (5)\left(\tfrac{1}{2}\right) = \underline{\tfrac{5}{2}\ \text{A}}$$

P 3.6-14 All of the resistances in the circuit shown in Figure P 3.6-14 are multiples of R. Determine the value of R.

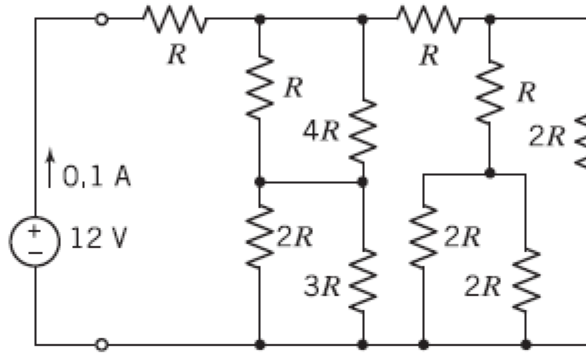


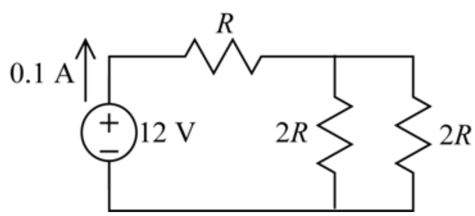
Figure P 3.6-14

Solution:

$$(R \| 4R)+(2R \| 3R)=\frac{4}{5}R+\frac{6}{5}R=2R$$

$$R+\left(2R \| \left(R+\left(2R \| 2R\right)\right)\right)=R+\left(2R \| 2R\right)=2R$$

So the circuit is equivalent to



Then

$$12=0.1\left(R+\left(2R \| 2R\right)\right)=0.1\left(2R\right) \quad \Rightarrow \quad R=60\ \Omega$$

(checked: ELAB 5/31/04)

P 3.6-15 The circuit shown in Figure P 3.6-15 contains seven resistors, each having resistance R. The input to this circuit is the voltage source voltage, v_s. The circuit has two outputs, v_a and v_b. Express each output as a function of the input.

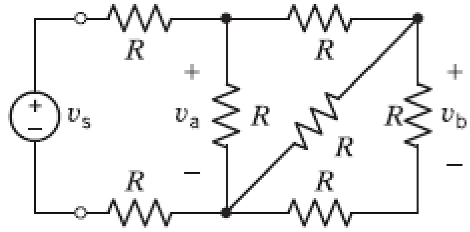
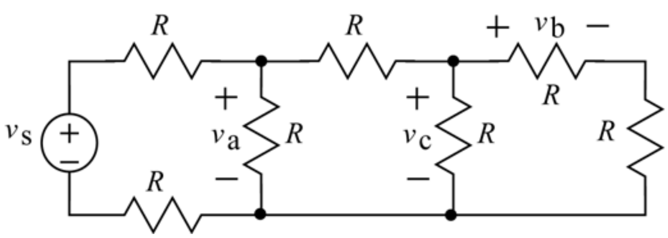


Figure P 3.6-15

Solution:

The circuit can be redrawn as



$$v_a = \frac{R \| \left(R + \left(R \| 2R\right)\right)}{2R + R \| \left(R + \left(R \| 2R\right)\right)} v_s = \frac{5}{21} v_s$$

$$v_c = \frac{R \| 2R}{R + \left(R \| 2R\right)} v_s = \frac{2}{5} v_a = \frac{2}{21} v_s$$

$$v_b = \frac{R}{R + R} v_c = \frac{1}{2} v_c = \frac{1}{21} v_s$$

(Checked using LNAP 5/23/04)

P 3.6-16 The circuit shown in Figure P 3.6-16 contains three 10-Ω, 1/4-W resistors. (Quarter-watt resistors can dissipate 1/4 W safely.) Determine the range of voltage source voltages, v_s, such that none of the resistors absorbs more than 1/4 W of power.

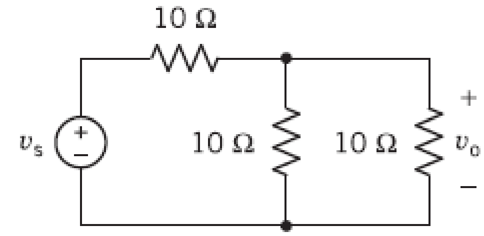
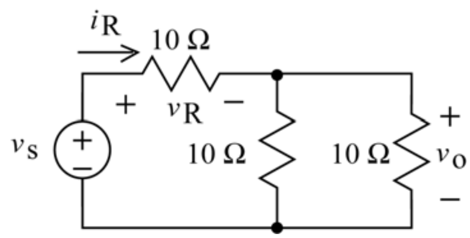


Figure P 3.6-16

Solution:



$$v_o = \frac{(10 \| 10)}{10 + (10 \| 10)} v_s = \frac{5}{15} v_s = \frac{v_s}{3}$$

$$v_R + v_o - v_s = 0 \quad \Rightarrow \quad v_R = \frac{2}{3} v_s$$

$$i_R = \frac{v_R}{10} = \frac{2}{30} v_s$$

$$P = \left(\frac{2}{30} v_s \right)^2 (10) = \frac{4}{90} v_s^{\,2} \le \frac{1}{4} \quad \Rightarrow \quad |v_s| \le \sqrt{\frac{90}{16}} = \frac{3\sqrt{10}}{4} = 2.37 \text{ V}$$

(checked: LNAP 5/31/04)

P 3.6-17 The four resistors shown in Figure P 3.6-17 represent strain gauges. Strain gauges are transducers that measure the strain that results when a resistor is stretched or compressed. Strain gauges are used to measure force, displacement, or pressure. The four strain gauges in Figure P 3.6-17 each have a nominal (unstrained) resistance of 120 **Ω** and can each absorb 0.2 mW safely. Determine the range of voltage source voltages, v_s, such that no strain gauge absorbs more than 0.2 mW of power.

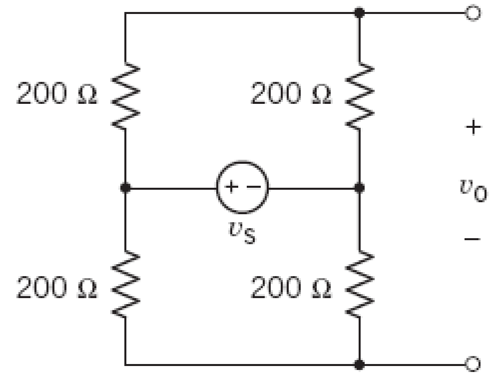


Figure P 3.6-17

Solution:

The voltage across each strain gauge is $\frac{v_s}{2}$ so the current in each strain gauge is $\frac{v_s}{400}$. The power dissipated by each resistor is given by $\frac{v_s}{2}\left(\frac{v_s}{400}\right)=\frac{{v_s}^2}{800}$ so we require $0.5\times10^{-3}\le\frac{{v_s}^2}{800}$ or

$\left|v_s\right|\le\sqrt{0.4}=0.6325\text{ V}$.

(checked: LNAP 6/9/04)

P 3.6-18 The circuit shown in Figure P 3.6-18*b* has been obtained from the circuit shown in Figure P 3.6-18*a* by replacing series and parallel combinations of resistances by equivalent resistances.

(a) Determine the values of the resistances R_1, R_2, and R_3 in Figure P 3.6-18*b* so that the circuit shown in Figure P 3.6-18*b* is equivalent to the circuit shown in Figure P 3.6-18*a*.

(b) Determine the values of v_1, v_2, and i in Figure P 3.6-18*b*.

(c) Because the circuits are equivalent, the values of v_1, v_2, and i in Figure P 3.6-18*a* are equal to the values of v_1, v_2, and i in Figure P 3.6-18*b*. Determine the values of v_4, i_5, i_6, and v_7 in Figure P 3.6-18*a*.

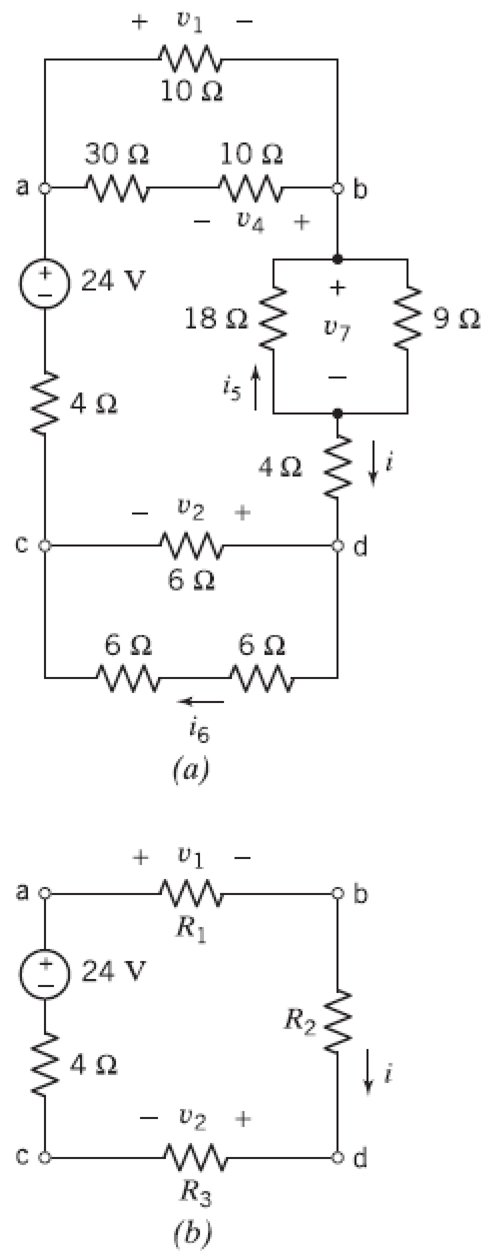


Figure P 3.6-18

Solution:

(a)

$$R_1 = 10 \,||\, (30+10) = 8\ \Omega, \qquad R_2 = 4 + (18 \,||\, 9) = 10\ \Omega \quad \text{and}$$

$$R_3 = 6 \,||\, (6+6) = 4\ \Omega$$

(b)

$$i = 1\text{ A}, \quad v_1 = 8\text{ V} \quad \text{and} \quad v_2 = 4\text{ V}$$

(c)

$$v_4 = -\frac{10}{10+30} 8 = -2\text{ V}, \quad i_5 = -\frac{9}{9+18} 1 = -\frac{1}{3}\text{ A},$$

$$v_7 = -18\left(-\frac{1}{3}\right) = +6\text{ V} \quad \text{and} \quad i_6 = \frac{4}{12} = \frac{1}{3}\text{ A}$$

(checked: LNAP 6/6/04)

P 3.6-19 Determine the values of v_1, v_2, i_3, v_4, v_5, and i_6 in Figure P 3.6-19.

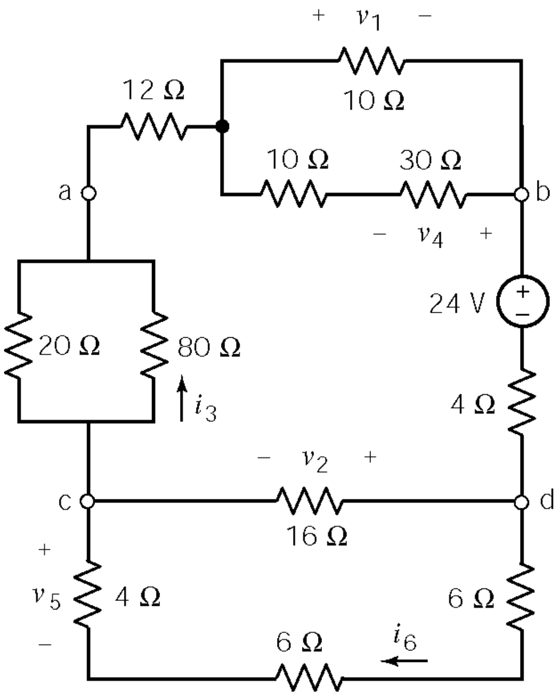
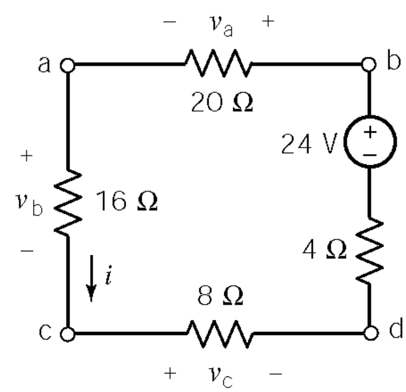


Figure P 3.6-19

Solution:

Replace series and parallel combinations of resistances by equivalent resistances. Then KVL gives



$$(20+4+8+16)i = 48 \quad \Rightarrow \quad i = 0.5 \text{ A}$$

$$v_a = 20i = 10 \text{ V},\ v_b = 16i = 8 \text{ V and } v_c = 8i = 4 \text{ V}$$

Compare the original circuit to the equivalent circuit to get

$$v_1 = -\left(\frac{10\,\|\,(10+30)}{12+10\,\|\,(10+30)}\right)v_a = -\left(\frac{8}{12+8}\right)10 = -4 \text{ V}$$

$$v_2 = -v_c = -4 \text{ V}$$

$$i_3 = -\left(\frac{20}{20+80}\right)i = -\left(\frac{1}{5}\right)(0.5) = -0.1 \text{ A}$$

$$v_4 = -\left(\frac{30}{10+30}\right)v_1 = -\left(\frac{1}{4}\right)(-4) = 1 \text{ V}$$

$$v_5 = \left(\frac{4}{5+6+6}\right)v_c = \left(\frac{1}{4}\right)(4) = 1 \text{ V}$$

$$i_6 = -\left(\frac{16}{16+(4+6+6)}\right)i = -\left(\frac{1}{2}\right)(0.5) = -0.25 \text{ A}$$

(checked: LNAP 6/10/04)

P 3.6-20 Determine the values of i, v, and R_{eq} by the circuit model shown in Figure P 3.6-20, given that v_{ab} = 18 V.

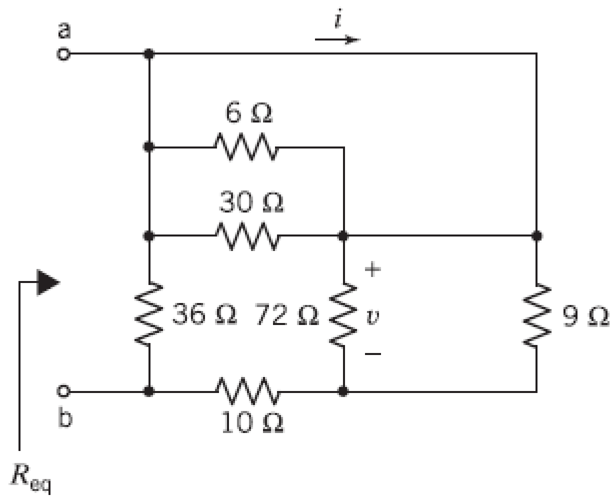
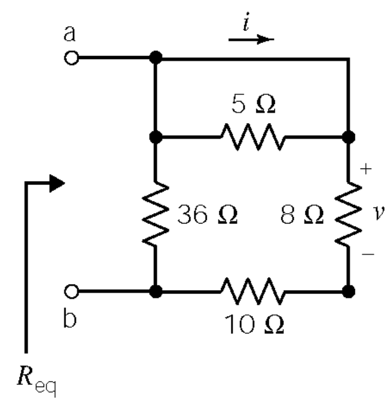


Figure P 3.6-20

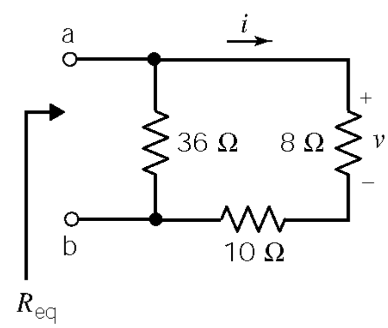
Solution:

Replace parallel resistors by equivalent resistors:

$$6 \parallel 30 = 5\ \Omega \text{ and } 72 \parallel 9 = 8\ \Omega$$



A short circuit in parallel with a resistor is equivalent to a short circuit.



$$R_{eq} = 36 \parallel (8+10) = 12\ \Omega$$

Using voltage division when v_{ab} = 18 V:

$$v = \frac{8}{8+10} v_{ab} = \frac{4}{9}(18) = 8\ \text{V}$$

$$i = \frac{v}{8} = 1\ \text{A}$$

(checked: LNAP 6/21/04)

P 3.6-22 Determine the value of the resistance R in the circuit shown in Figure P 3.6-22, given that $R_{eq} = 9\ \Omega$.
Answer: $R = 15\ \Omega$

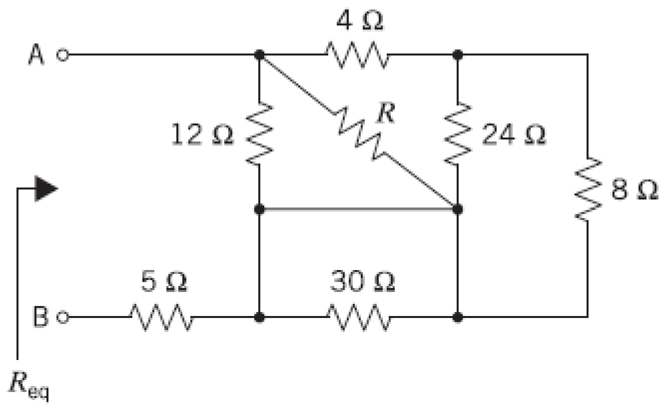
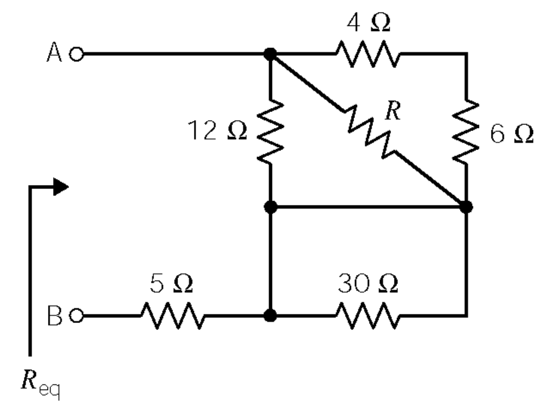


Figure P 3.6-22

Solution:

Replace parallel resistors by an equivalent resistor:

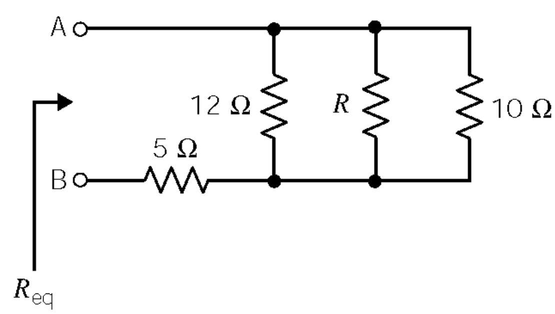
$$8 \,||\, 24 = 6\ \Omega$$



A short circuit in parallel with a resistor is equivalent to a short circuit.

Replace series resistors by an equivalent resistor:

$$4+6 = 10\ \Omega$$



Now

$$9 = R_{eq} = 5 + (12 \,||\, R \,||\, 10)$$

so

$$4 = \frac{R \times \frac{60}{11}}{R + \frac{60}{11}} \quad \Rightarrow \quad R = 15\ \Omega$$

(checked: LNAP 6/21/04)

P 3.6-22 Determine the value of the resistance R in the circuit shown in Figure P 3.6-22, given that $R_{eq} = 50\ \Omega$.

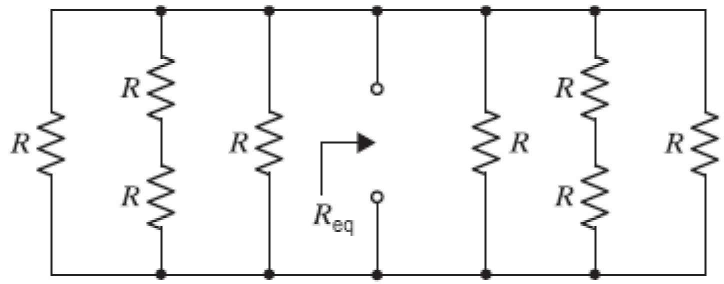


Figure P 3.6-22

Solution:

$$R_{eq} = \left(R \parallel (R+R) \parallel R\right) \parallel \left(R \parallel (R+R) \parallel R\right)$$

$$R \parallel (R+R) \parallel R = 2R \parallel \frac{R}{2} = \frac{2}{5} R$$

$$R_{eq} = \frac{2}{5} R \parallel \frac{2}{5} R = \frac{R}{5} \quad \Rightarrow \quad R = 5\, R_{eq} = 200\ \Omega$$

(checked: LNAP 6/21/04)

P 3.6-23 Determine the values of r, the gain of the CCVS, and g, the gain of the VCCS, for the circuit shown in Figure P 3.6-23.

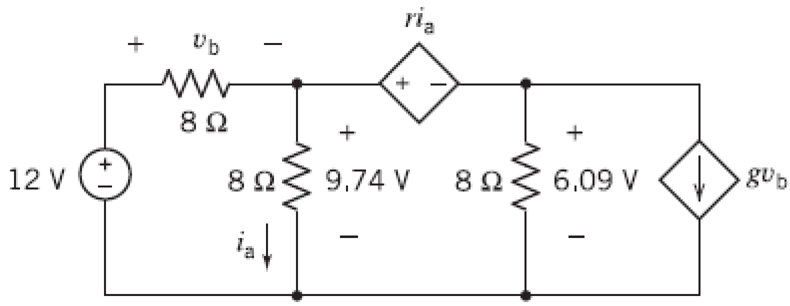


Figure P 3.6-23

Solution:

$$i_a = \frac{9.74}{8} = 1.2175 \text{ A}$$

$$9.74 - 6.09 = r\,i_a = r\left(\frac{9.74}{8}\right) \quad \Rightarrow \quad r = \left(\frac{9.74\text{-}6.09}{9.74}\right)8 = 3\ \frac{\text{V}}{\text{A}}$$

$$v_b = 12 - 9.74 = 2.26 \text{ V}$$

$$gv_b + \frac{6.09}{8} + \frac{9.74}{8} - \frac{2.26}{8} = 0 \quad \Rightarrow \quad gv_b = -1.696 \text{ A}$$

$$g = \frac{gv_b}{v_b} = \frac{-6.696}{2.26} = -0.75$$

(checked: LNAP 6/21/04)

P 3.6-24 The input to the circuit in Figure P 3.6-24 is the voltage of the voltage source, v_s. The output is the voltage measured by the meter, v_o. Show that the output of this circuit is proportional to the input. Determine the value of the constant of proportionality.

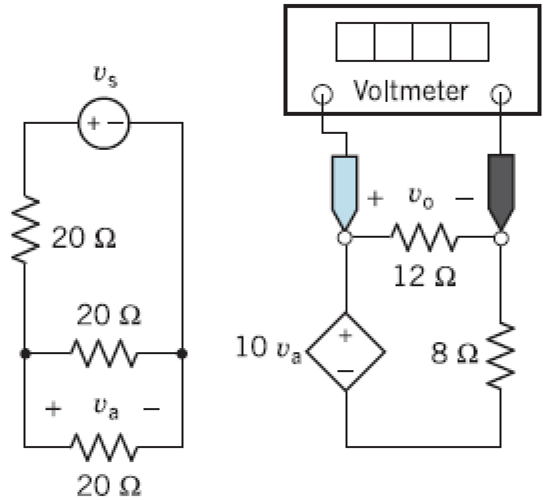


Figure P 3.6-24

Solution:

$$v_a = \frac{20 \| 20}{20 + (20 \| 20)} v_s = \frac{1}{3} v_s$$

$$v_o = \left(\frac{12}{12+8}\right)(10 v_a) = \frac{3}{5} \times 10 \times \frac{1}{3} v_s = 2 v_s$$

So v_o is proportional to v_s and the constant of proportionality is $2 \ \frac{\text{V}}{\text{V}}$.

P 3.6-25 The input to the circuit in Figure P 3.6-25 is the voltage of the voltage source, v_s. The output is the current measured by the meter, i_o. Show that the output of this circuit is proportional to the input. Determine the value of the constant of proportionality.

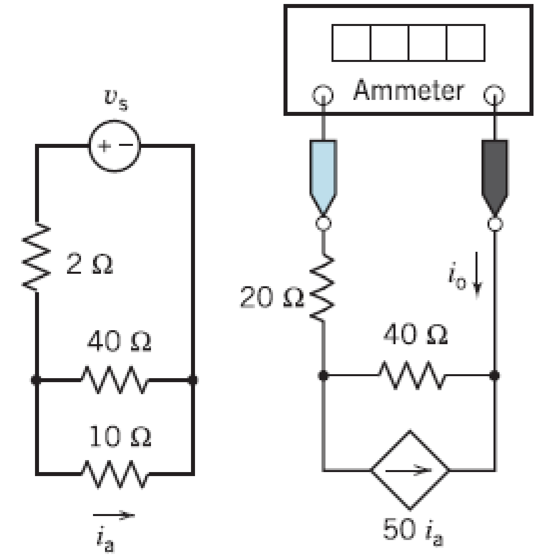


Figure P 3.6-25

Solution:

$$i_a = \left(\frac{40}{40+10}\right)\frac{v_s}{2+(40\,||\,10)} = \left(\frac{4}{5}\right)\left(\frac{v_s}{10}\right) = \frac{4}{50}v_s$$

$$i_o = -\left(\frac{40}{20+40}\right)(50 i_a) = -\frac{100}{3}\left(\frac{4}{50}\right)v_s = -\frac{8}{3}v_s$$

The output is proportional to the input and the constant of proportionality is $-\frac{8}{3}\ \frac{\text{A}}{\text{V}}$.

P 3.6-26 Determine the voltage measured by the voltmeter in the circuit shown in Figure P 3.6-26.

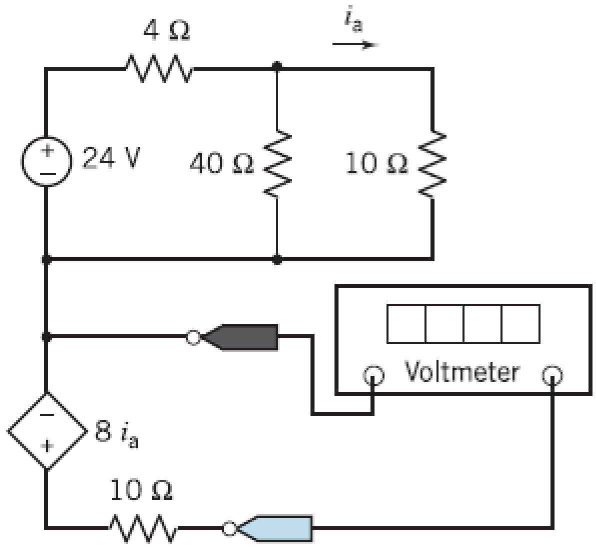
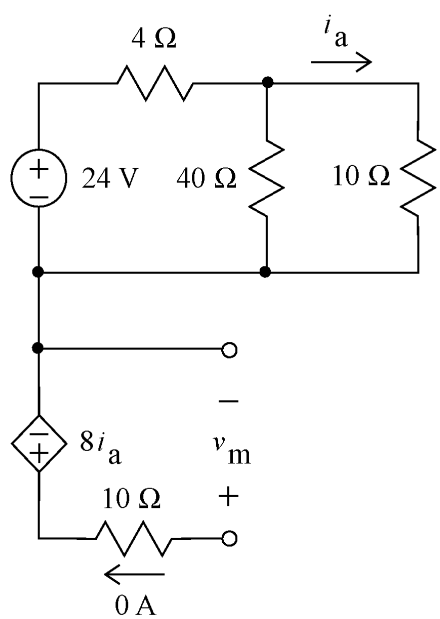


Figure P 3.6-26

Solution:



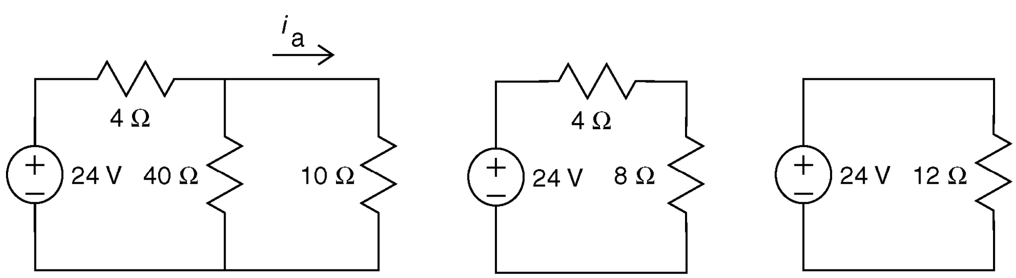
Replace the voltmeter by the equivalent open circuit and label the voltage measured by the meter as v_m.

The 10-Ω resistor at the right of the circuit is in series with the open circuit that replaced the voltmeter so it's current is zero as shown. Ohm's law indicates that the voltage across that 10-Ω resistor is also zero. Applying KVL to the mesh consisting of the dependent voltage source, 10-Ω resistor and open circuit shows that

$$v_m = 8\,i_a$$

The 10-Ω resistor and 40-Ω resistor are connected in parallel. The parallel combination of these resistors is equivalent to a single resistor with a resistance equal to

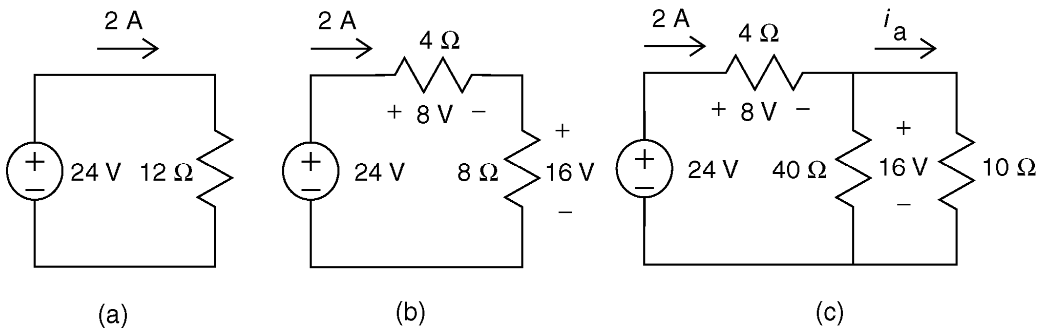
$$\frac{40 \times 10}{40+10} = 8\ \Omega$$



(a) (b) (c)

Figure a shows part of the circuit. In Figure b, an equivalent resistor has replaced the parallel resistors. Now the 4-Ω resistor and 8-Ω resistor are connected in series. The series combination of these resistors is equivalent to a single resistor with a resistance equal to $4+8=12\ \Omega$. In Figure c, an equivalent resistor has replaced the series resistors.

Here the same three circuits with the order reversed. The earlier sequence of figures illustrates the process of simplifying the circuit by repeatedly replacing series or parallel resistors by an equivalent resistor. This sequence of figures illustrates an analysis that starts with the simplified circuit and works toward the original circuit.



Consider Figure a. Using Ohm's law, we see that the current in the 12-Ω resistor is 2 A. The current in the voltage source is also 2 A. Replacing series resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the current in the voltage source must also be 2 A in Figure b. The currents in resistors in Figure b are equal to the current in the voltage source. Next, Ohm's law is used to calculate the resistor voltages as shown in Figure b.

Replacing parallel resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the current in the 4-Ω resistor in Figure c must be equal to the current in the 4-Ω resistor in Figure b. Using current division in Figure c are yields

$$i_a = \left(\frac{40}{40+10}\right) 2 = 1.6 \text{ A}$$

Finally,

$$v_m = 8\, i_a = 8 \times 1.6 = 12.8 \text{ V}$$

P 3.6-27 Determine the current measured by the ammeter in the circuit shown in Figure P 3.6-27.

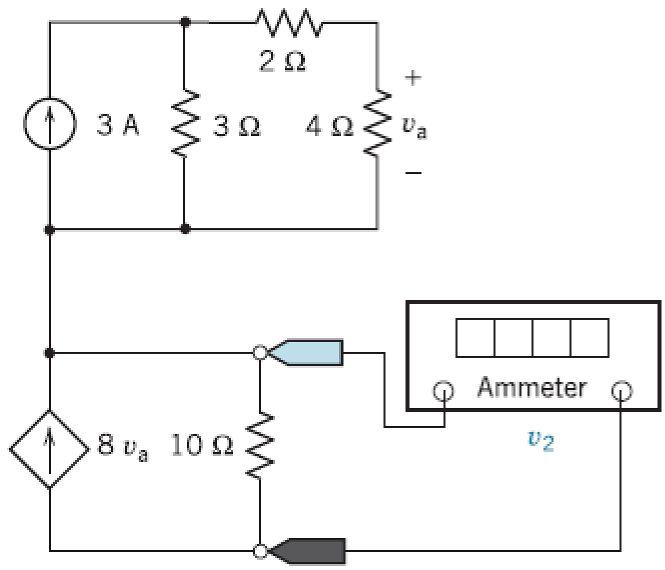


Figure P 3.6-27

Solution:

Replace the ammeter by the equivalent short circuit and label the current measured by the meter as i_m.

The 10-Ω resistor at the right of the circuit is in parallel with the short circuit that replaced the ammeter so it's voltage is zero as shown. Ohm's law indicates that the current in that 10-Ω resistor is also zero. Applying KCL at the top node of that 10-Ω resistor shows that

$$i_m = 0.8\, v_a$$

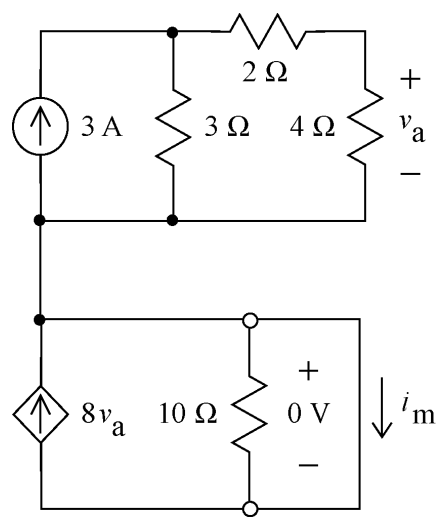
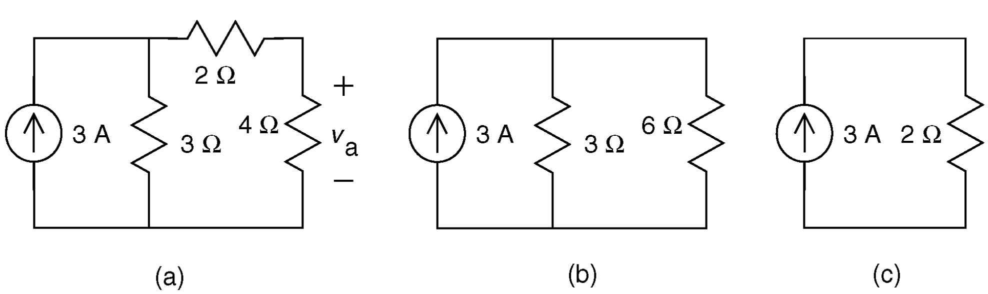


Figure a shows part of the circuit. The 2-Ω resistor and 4-Ω resistor are connected in series. The series combination of these resistors is equivalent to a single 6-Ω resistor.

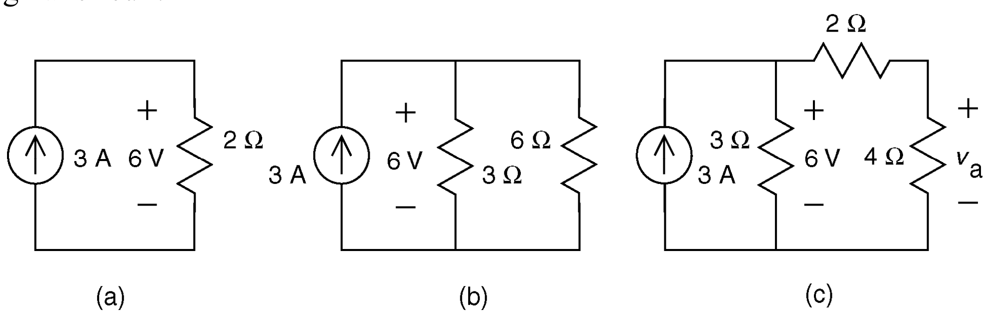


In Figure b, an equivalent resistor has replaced the series resistors. Now the 3-Ω resistor and 6-Ω resistor are connected in parallel. The parallel combination of these resistors is equivalent to a single resistor with a resistance equal to

$$\frac{3 \times 6}{3+6} = 2\ \Omega$$

In Figure c, an equivalent resistor has replaced the parallel resistors.

Here the same three circuits with the order reversed. The earlier sequence of figures illustrates the process of simplifying the circuit by repeatedly replacing series or parallel resistors by an equivalent resistor. This sequence of figures illustrates an analysis that starts with the simplified circuit and works toward the original circuit.



Consider Figure a. Using Ohm's law, we see that the voltage across the 2-Ω resistor is 6 V. The voltage across the current source is also 6 V. Replacing parallel resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the voltage across the current source must also be 6 V in Figure b. The voltage across each resistor in Figure b is equal to the voltage across the current source.

Replacing series resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit, so the voltage across the 3-Ω resistor in Figure c must be equal to the voltage across the 3-Ω resistor in Figure b. Using voltage division in Figure c yields

$$v_a = \left(\frac{4}{2+4}\right) 6 = 4\ \text{V}$$

Finally,

$$i_m = 0.8\, v_a = 0.8 \times 4 = 3.2\ \text{V}$$

P 3.6-28 Determine the value of the resistance R that causes the voltage measured by the voltmeter in the circuit shown in Figure P 3.6-28 to be 6 V.

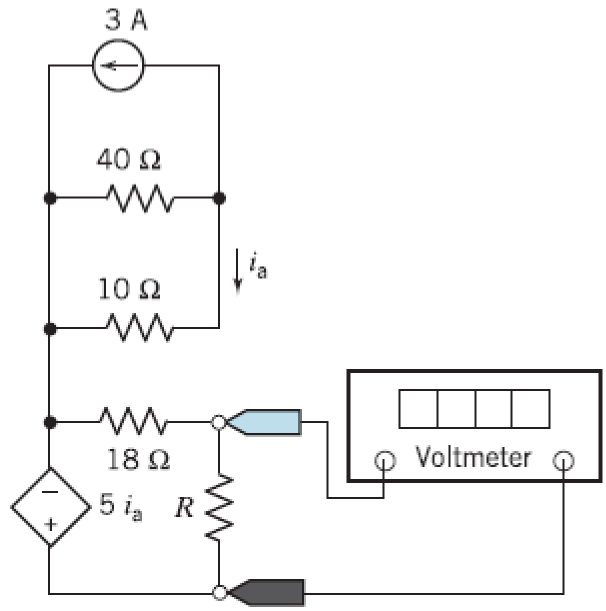


Figure P 3.6-28

Solution:

Use current division in the top part of the circuit to get

$$i_a = \left(\frac{40}{40+10}\right)(-3) = -2.4 \text{ A}$$

Next, denote the voltage measured by the voltmeter as v_m and use voltage division in the bottom part of the circuit to get

$$v_m = \left(\frac{R}{18+R}\right)(-5\, i_a) = \left(\frac{-5\, R}{18+R}\right) i_a$$

Combining these equations gives:

$$v_m = \left(\frac{-5\, R}{18+R}\right)(-2.4) = \frac{12\, R}{18+R}$$

When $v_m = 6$ V,

$$6 = \frac{12\, R}{18+R} \quad \Rightarrow \quad R = \frac{6\times 18}{12-6} = 18 \ \Omega$$

P 3.6-29 The input to the circuit shown in Figure P 3.6-29 is the voltage of the voltage source, v_s. The output is the current measured by the meter, i_m.

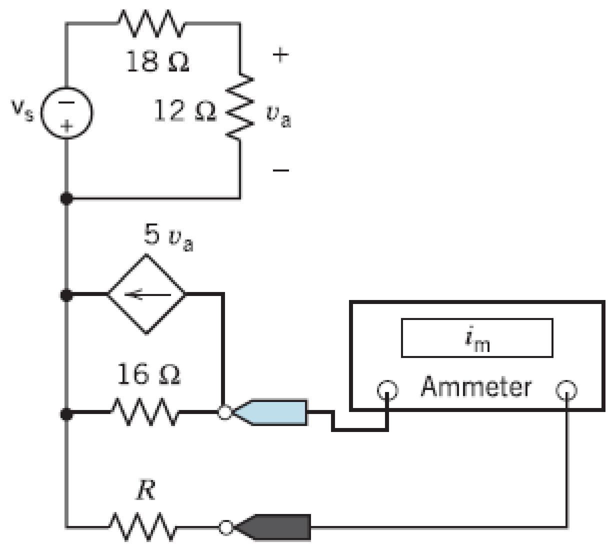


Figure P 3.6-29

(a) Suppose v_s = 15 V. Determine the value of the resistance R that causes the value of the current measured by the meter to be i_m = 5 A.

(b) Suppose v_s = 15 V and R = 24 Ω. Determine the current measured by the ammeter.

(c) Suppose R = 24 Ω. Determine the value of the input voltage, v_s, that causes the value of the current measured by the meter to be i_m = 3 A.

Soluton:

Use voltage division in the top part of the circuit to get

$$v_a = \left(\frac{12}{12+18}\right)(-v_s) = -\frac{2}{5}v_s$$

Next, use current division in the bottom part of the circuit to get

$$i_m = -\left(\frac{16}{16+R}\right)(5\,v_a) = \left(-\frac{80}{16+R}\right)v_a$$

Combining these equations gives:

$$i_m = \left(-\frac{80}{16+R}\right)\left(-\frac{2}{5}v_s\right) = \left(\frac{32}{16+R}\right)v_s$$

a. When v_s = 15 V and i_m = 12 A

$$12 = \left(\frac{32}{16+R}\right)15 \quad\Rightarrow\quad 192 + 12\,R = 480 \quad\Rightarrow\quad R = \frac{288}{12} = 24\ \Omega$$

b. When v_s = 15 V and R = 80 Ω

$$i_m = \left(\frac{32}{16+80}\right)15 = 5\ \text{A}$$

c. When i_m = 3 A and R = 24 Ω

$$3 = \left(\frac{32}{16+24}\right)v_s = \frac{4}{5}v_s \quad\Rightarrow\quad v_s = \frac{15}{4} = 3.75\ \text{V}$$

P 3.6-30 The ohmmeter in Figure P 3.6-30 measures the equivalent resistance of the resistor circuit connected to the meter probes.

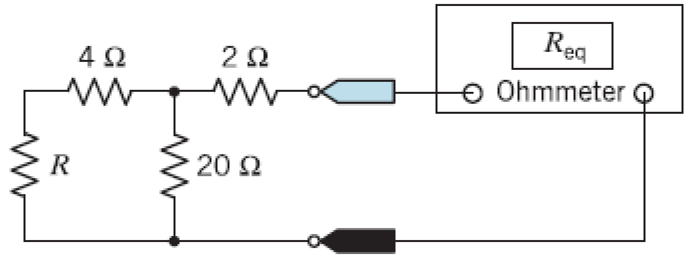


Figure P 3.6-31

(a) Determine the value of the resistance R required to cause the equivalent resistance to be R_{eq} = 12 Ω.
(b) Determine the value of the equivalent resistance when R = 14 Ω.

Solution:

$$R_{eq} = \left((R+4)\,||\,20\right)+2 = \frac{(R+4)\times 20}{(R+4)+20}+2 = \frac{20\,R+80}{R+24}+2$$

(a) $$12 = \frac{20\,R+80}{R+24}+2 \quad\Rightarrow\quad 10 = \frac{20\,R+80}{R+24} \quad\Rightarrow\quad R+24 = 2\,R+8 \quad\Rightarrow\quad R = 16\ \Omega$$

(b) $$R_{eq} = \frac{20(14)+80}{14+24}+2 = 11.5\ \Omega$$

(Checked: LNAPDC 9/28/04)

P 3.6-31 The voltmeter in Figure P 3.6-31 measures the voltage across the current source.

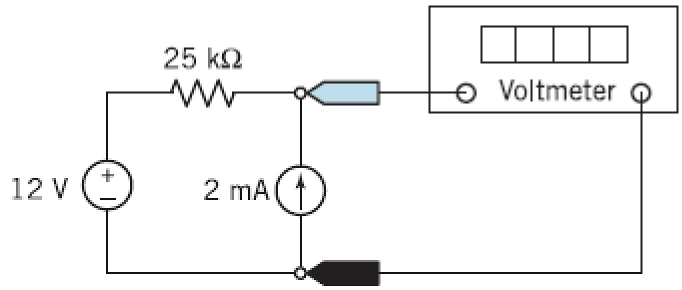
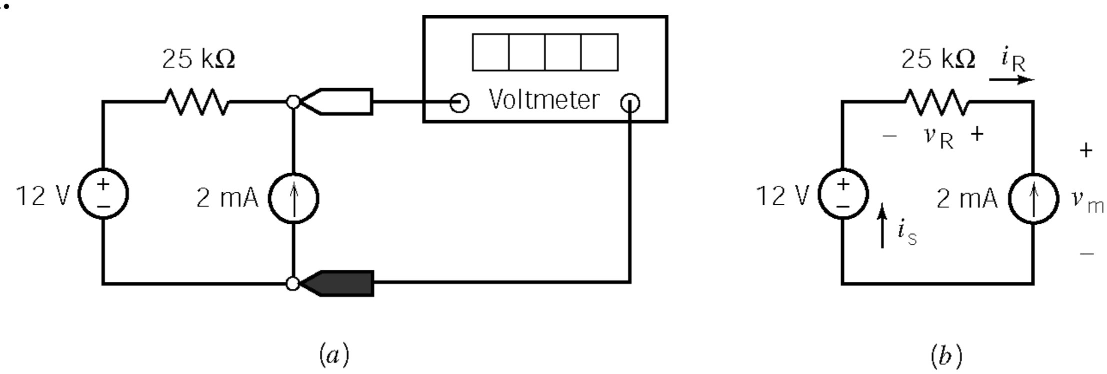


Figure P 3.6-31

(a) Determine the value of the voltage measured by the meter.
(b) Determine the power supplied by each circuit element.

Solution:



Replace the ideal voltmeter with the equivalent open circuit and label the voltage measured by the meter. Label the element voltages and currents as shown in (***b***).

Using units of V, A, Ω and W:	Using units of V, mA, kΩ and mW:
a.) Determine the value of the voltage measured by the meter. Kirchhoff's laws give $12+v_R=v_m$ and $-i_R=-i_s=2\times10^{-3}$ A Ohm's law gives $v_R=-\left(25\times10^3\right)i_R$ Then $v_R=-\left(25\times10^3\right)i_R=-\left(25\times10^3\right)\left(-2\times10^{-3}\right)=50$ V $v_m=12+v_R=12+50=62$ V	a.) Determine the value of the voltage measured by the meter. Kirchhoff's laws give $12+v_R=v_m$ and $-i_R=-i_s=2$ mA Ohm's law gives $v_R=-25\,i_R$ Then $v_R=-25\,i_R=-25(-2)=50$ V $v_m=12+v_R=12+50=62$ V

b.) Determine the power supplied by each element.

voltage source	$12(i_s) = -12(-2\times10^{-3})$ $= -24\times10^{-3}$ W
current source	$62(2\times10^{-3}) = 124\times10^{-3}$ W
resistor	$v_R\, i_R = 50(-2\times10^{-3})$ $= -100\times10^{-3}$ W
total	0

b.) Determine the power supplied by each element.

voltage source	$12(i_s) = -12(-2)$ $= -24$ mW
current source	$62(2) = 124$ mW
resistor	$v_R\, i_R = 50(-2)$ $= -100$ mW
total	0

P 3.6-32 Determine the resistance measured by the ohmmeter in Figure P 3.6-32.

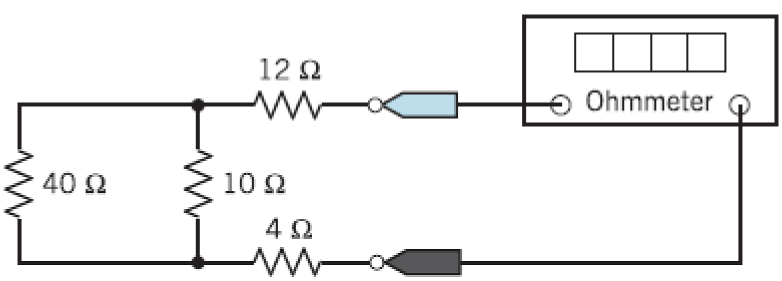


Figure P 3.6-32

Solution:

$$12 + \frac{40\times10}{40+10} + 4 = 12\ \Omega$$

P 3.6-33 Determine the resistance measured by the ohmmeter in Figure P 3.6-33.

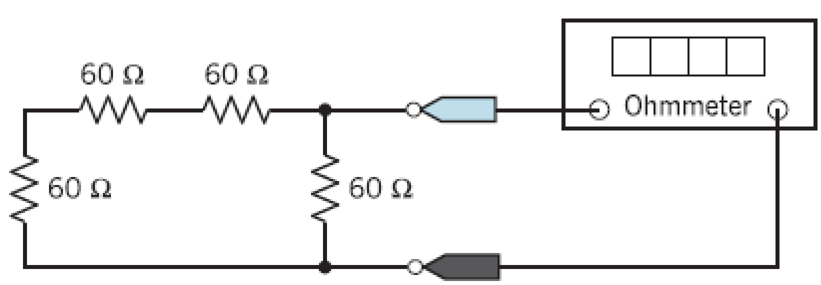


Figure P 3.6-33

Solution:

$$\frac{(60+60+60)\times60}{(60+60+60)+60} = 45\ \Omega$$

P3.6-34

Consider the circuit shown in Figure P3.6-34. Given the values of the following currents and voltages:

$$i_1 = 0.625 \text{ A},\ v_2 = -25 \text{ V},\ i_3 = -1.25 \text{ A} \quad \text{and} \quad v_4 = -18.75 \text{ V}$$

Determine the values of R_1, R_2, R_3 and R_4.

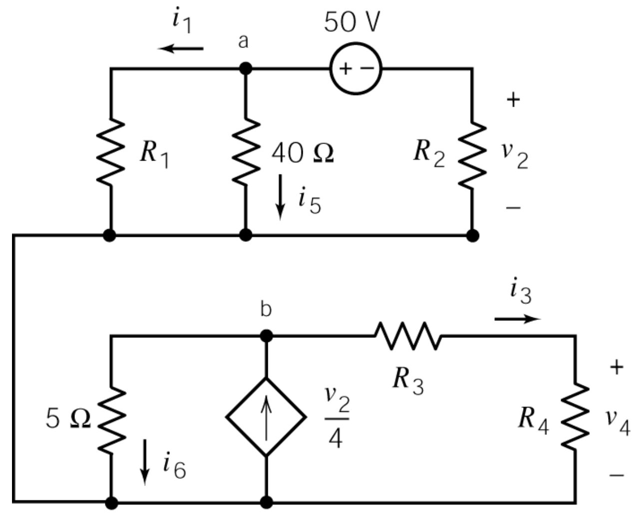
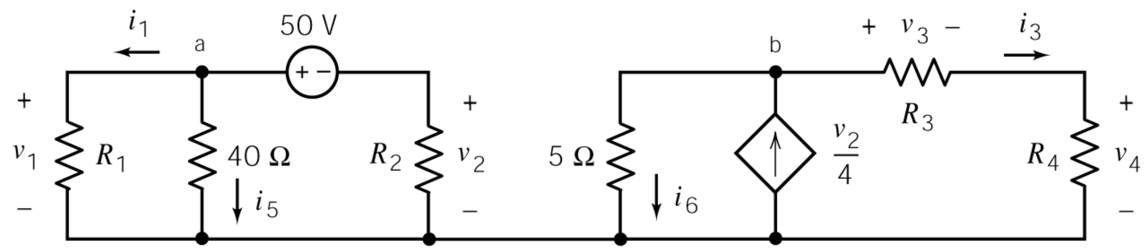


Figure P3.6-34

Solution:



From KVL $\quad 50 + v_2 - v_1 = 0 \;\Rightarrow\; v_1 = 50 + (-25) = 25 \text{ V}$

From Ohm's law $\quad R_1 = \dfrac{v_1}{i_1} = \dfrac{25}{0.625} = 40\ \Omega$

From KCL

$$i_1 + i_5 + i_2 = 0 \;\Rightarrow\; i_2 = -(i_1 + i_5) = -\left(0.625 + \frac{v_1}{40}\right) = -\left(0.625 + \frac{25}{40}\right) = -1.25 \text{ A}$$

From Ohm's law $\quad R_2 = \dfrac{v_2}{i_2} = \dfrac{-25}{-1.25} = 20\ \Omega$

From KCL $\quad \dfrac{v_2}{4} = i_6 + i_3 \;\Rightarrow\; i_6 = -i_3 + \dfrac{v_2}{4} = -(-1.25) + \dfrac{-25}{4} = -5 \text{ A}$

From KVL $\quad v_3 + v_4 - 5i_6 = 0 \;\Rightarrow\; v_3 = -v_4 + 5i_6 = -(-18.75) + 5(-5) = -6.25 \text{ V}$

From Ohm's law $\quad R_3 = \dfrac{v_3}{i_3} = \dfrac{-6.25}{-1.25} = 5\ \Omega \quad \text{and} \quad R_4 = \dfrac{v_4}{i_3} = \dfrac{-18.75}{-1.25} = 15\ \Omega$

P3.6-35

Consider the circuits shown in Figure P3.6-35. The equivalent circuit on the right is obtained from the original circuit on the left by replacing series and parallel combinations of resistors by equivalent resistors. The value of the current in the equivalent circuit is $i_s = 0.8$ A. Determine the values of R_1, R_2, R_5, v_2 and i_3.

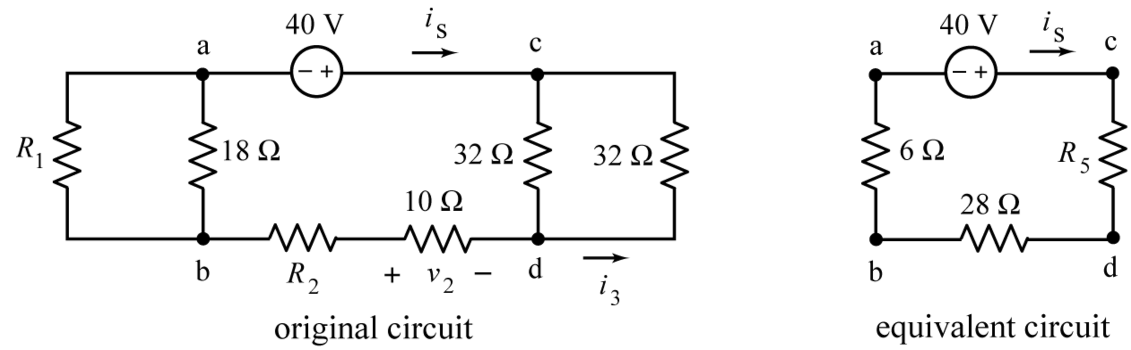


Figure P3.6-35

Solution:

$$R_1 \| 18 = 6 \quad \Rightarrow \quad \frac{18R_1}{18+R_1} = 6 \quad \Rightarrow \quad 3R_1 = 18 + R_1 \quad \Rightarrow \quad R_1 = 9\ \Omega$$

$$R_2 + 10 = 28 \quad \Rightarrow \quad R_2 = 18\ \Omega$$

$$40 = (6 + 28 + R_5)i_s \quad \Rightarrow \quad \frac{40}{0.8} = 34 + R_5 \quad \Rightarrow \quad R_5 = 50 - 34 = 16\ \Omega$$

$$v_2 = -10i_s = -10(0.8) = -8\text{ V} \text{ and } i_3 = -\frac{32}{32+32}i_s = -\frac{0.8}{2} = -0.4\text{ A}$$

P3.6-36 Consider the circuit shown in Figure P3.6-36. Given

$$v_2 = \frac{2}{3} v_s, \; i_3 = \frac{1}{5} i_1 \text{ and } v_4 = \frac{3}{8} v_2 .$$

Determine the values of R_1, R_2 and R_4.

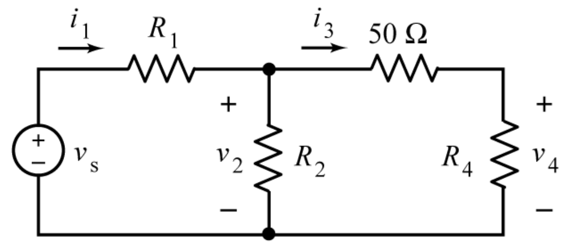


Figure P3.6-36

Hint: Interpret $v_2 = \frac{2}{3} v_s$, $i_3 = \frac{1}{5} i_1$ and $v_4 = \frac{3}{8} v_2$ as current and voltage division.

Solution:

From voltage division $v_4 = \dfrac{R_4}{50 + R_4} v_2$

so $$\frac{R_4}{50 + R_4} = \frac{3}{8} \Rightarrow 8 R_4 = 3\left(50 + R_4\right) \Rightarrow R_4 = \frac{150}{8-3} = 30\ \Omega .$$

From current division $i_3 = \dfrac{R_2}{R_2 + \left(50 + R_4\right)} i_1 = \dfrac{R_2}{R_2 + 80} i_1$

so $$\frac{R_2}{R_2 + 80} = \frac{1}{5} \Rightarrow 5 R_2 = R_2 + 80 \Rightarrow R_2 = 20\ \Omega .$$

Notice that $R_2 \,||\left(50 + R_4\right) = 20 \,||\left(50 + 30\right) = 20 \,||\, 80 = 16\ \Omega$. From voltage division

$$v_1 = \frac{R_2 \,||\left(50 + R_4\right)}{R_1 + \left(R_2 \,||\left(50 + R_4\right)\right)} v_s = \frac{16}{R_1 + 16} v_s$$

so $$\frac{16}{R_1 + 16} = \frac{2}{3} \Rightarrow 48 = 2\left(R_1 + 16\right) \Rightarrow R_1 = \frac{48 - 32}{2} = 8\ \Omega .$$

P3.6-37 Consider the circuit shown in Figure P3.6-37. Given

$$i_2 = \frac{2}{5} i_s, \ v_3 = \frac{2}{3} v_1 \text{ and } i_4 = \frac{4}{5} i_2 .$$

Determine the values of R_1, R_2 and R_4.

Hint: Interpret $i_2 = \frac{2}{5} i_s$, $v_3 = \frac{2}{3} v_1$ and $i_4 = \frac{4}{5} i_2$ as current and voltage division.

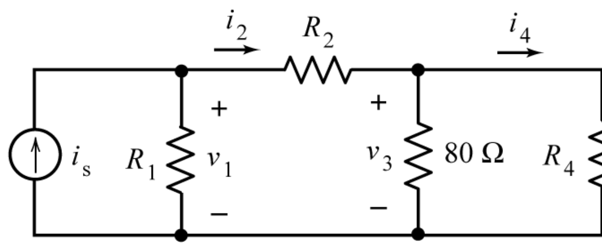


Figure P3.6-37

Solution:

From current division $i_4 = \dfrac{80}{80+R_4} i_2$

so $$\frac{80}{80+R_4} = \frac{4}{5} \quad \Rightarrow \quad 400 = 4\left(80+R_4\right) \quad \Rightarrow \quad R_4 = \frac{400-320}{4} = 20\ \Omega .$$

From voltage division $v_3 = \dfrac{80 \,||\, R_4}{R_2 + \left(80 \,||\, R_4\right)} v_1 = \dfrac{80 \,||\, 20}{R_2 + \left(80 \,||\, 20\right)} v_1 = \dfrac{16}{R_2+16} v_1$

so $$\frac{16}{R_2+16} = \frac{2}{3} \quad \Rightarrow \quad 48 = 2\left(R_2+16\right) \quad \Rightarrow \quad R_2 = \frac{48-32}{2} = 8\ \Omega .$$

Notice that $R_2 + \left(80 \,||\, R_4\right) = 8 + \left(80 \,||\, 20\right) = 8 + 16 = 24\ \Omega$. From current division

$$i_1 = \frac{R_1}{R_1 + \left(R_2 + \left(80 \,||\, R_4\right)\right)} i_s = \frac{R_1}{R_1+24} i_s$$

so $$\frac{R_1}{R_1+24} = \frac{2}{5} \quad \Rightarrow \quad 5R_1 = 2\left(R_1+24\right) \quad \Rightarrow \quad R_1 = \frac{48}{3} = 16\ \Omega$$

P3.6-38 Consider the circuit shown in Figure P3.6-38.

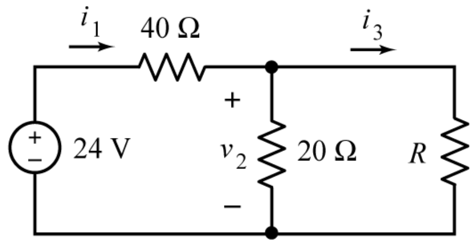


Figure P3.6-38

(a) Suppose $i_3 = \frac{1}{3} i_1$. What is the value of the resistance R?

(b) Suppose instead $v_2 = 4.8$ V. What is the value of the equivalent resistance of the parallel resistors?

(c) Suppose instead $R = 20\ \Omega$. What is the value of the current in the 40 Ω resistor?

Hint: Interpret $i_3 = \frac{1}{3} i_1$ as current division.

Solution:

(a) From current division $i_3 = \frac{20}{20+R} i_1$ so $\frac{20}{20+R} = \frac{1}{3} \Rightarrow 60 = 20 + R \Rightarrow R = 40\ \Omega$.

(b) From voltage division $v_2 = \frac{R_p}{40+R_p} 24$

so $$4.8 = \frac{R_p}{40+R_p} 24 \Rightarrow \frac{4.8}{24}\left[40 + R_p\right] = R_p \Rightarrow R_p = \frac{(0.2)40}{1-0.2} = 10\ \Omega.$$

(c) $$i_1 = \frac{24}{40 + (20 \parallel 20)} = \frac{24}{40+10} = \frac{24}{50} = 0.48\ \text{A}$$

P3.6-39 Consider the circuit shown in Figure P3.6-39.

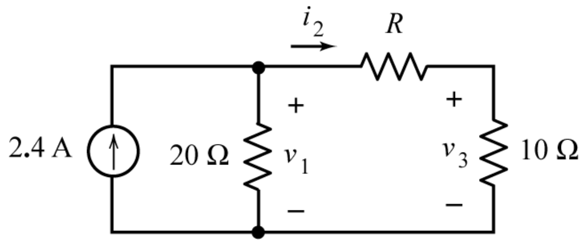


Figure P3.6-39

(a) Suppose $v_3 = \frac{1}{4} v_1$. What is the value of the resistance *R*?

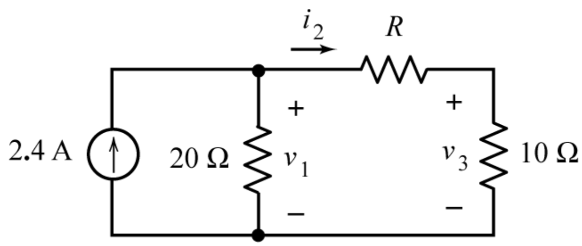
(b) Suppose $i_2 = 1.2\text{ A}$. What is the value of the resistance *R*?

(c) Suppose $R = 70\ \Omega$. What is the voltage across the 20 Ω resistor?

(d) Suppose $R = 30\ \Omega$. What is the value of the current in this 30 Ω resistor?

Hint: Interpret $v_3 = \frac{1}{4} v_1$ as voltage division.

Solution:



(a) From voltage division $v_3 = \dfrac{10}{10+R} v_1$ so $\dfrac{10}{10+R} = \dfrac{1}{4} \quad \Rightarrow \quad 40 = 10 + R \quad \Rightarrow \quad R = 30\ \Omega$.

(b)
$$1.2 = \frac{20}{20+(R+10)} 2.4 = \frac{20}{R+30} 2.4 \quad \Rightarrow \quad R+30 = \frac{20(2.4)}{1.2} = 40 \quad \Rightarrow \quad R = 10\ \Omega$$

(c)
$$20 \,||\, (70+10) = \frac{20(80)}{20+80} = 16\ \Omega \quad \text{so} \quad v_1 = (16)2.4 = 38.4\text{ V}$$

(d)
$$i_2 = \frac{20}{20+(R+10)} 2.4 = \frac{20}{20+(30+10)} 2.4 = \frac{20}{60} 2.4 = 0.8\text{ A}$$

P3.6-40 Consider the circuit shown in Figure P3.6-40. Given that the voltage of the dependent voltage source is $v_a = 8$ V, determine the values of R_1 and v_o.

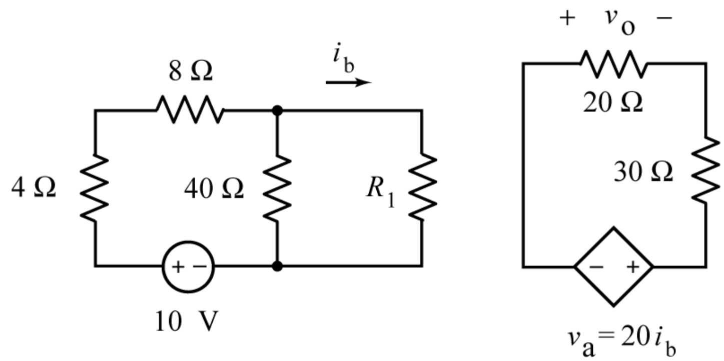


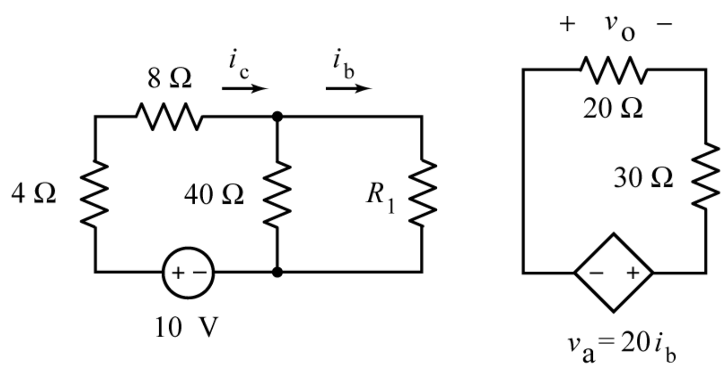
Figure P3.6-40

Solution:

First,

$$v_o = -\frac{20}{20+30} 8 = -3.2 \text{ V}$$

Next,



$$\frac{8}{20} = i_b = \frac{40}{40+R_1} i_c = \frac{40}{40+R_1}\left(\frac{10}{12+40 \,\|\, R_1}\right) = \frac{40}{40+R_1}\left(\frac{10}{12+\dfrac{40\,R_1}{40+R_1}}\right) = \frac{400}{12\left(40+R_1\right)+40\,R_1} = \frac{400}{480+52\,R_1}$$

then

$$\frac{8}{20} = \frac{400}{480+52\,R_1} \quad \Rightarrow \quad 480+52\,R_1 = \frac{400(20)}{8} = 1000 \quad \Rightarrow \quad \frac{1000-480}{52} = 10\ \Omega$$

P3.6-41 Consider the circuit shown in Figure P3.6-41. Given that the current of the dependent current source is $i_a = 2$ A, determine the values of R_1 and i_o.

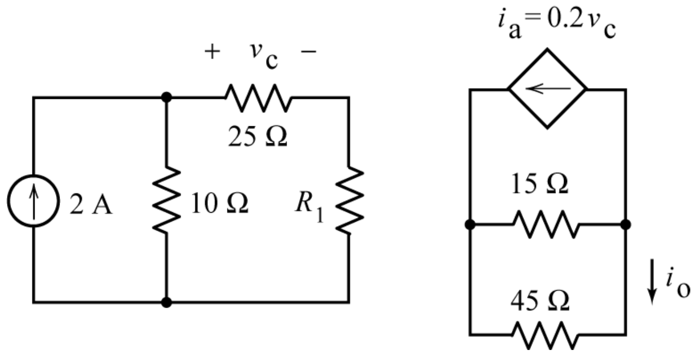
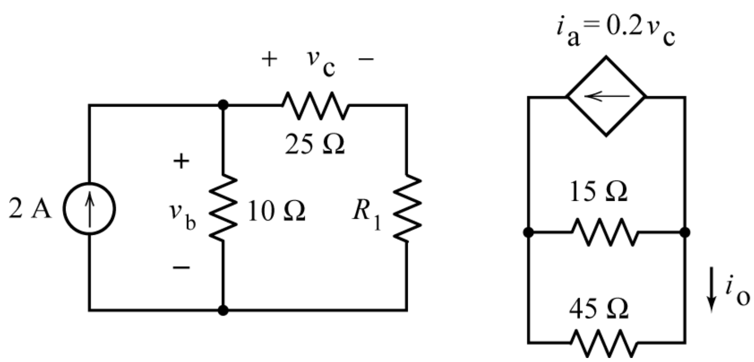


Figure P3.6-41

Solution:
First,

$$i_o = -\frac{15}{15+45} 2 = -0.5 \text{ A}$$

Next,



$$\frac{2}{0.2} = v_c = \frac{25}{25+R_1} v_b = \frac{25}{25+R_1}\left(2\left(10 \,\|\, 25+R_1\right)\right) = \frac{50}{25+R_1}\left(\frac{10\left(25+R_1\right)}{10+\left(25+R_1\right)}\right) = \frac{500}{35+R_1}$$

then

$$\frac{2}{0.2} = \frac{500}{35+R_1} \quad \Rightarrow \quad 35+R_1 = 50 \quad \Rightarrow \quad R_1 = 15 \ \Omega$$

P3.6-42 Determine the values of i_a, i_b, i_2, and v_1 in the circuit shown in Figure P3.6-42.

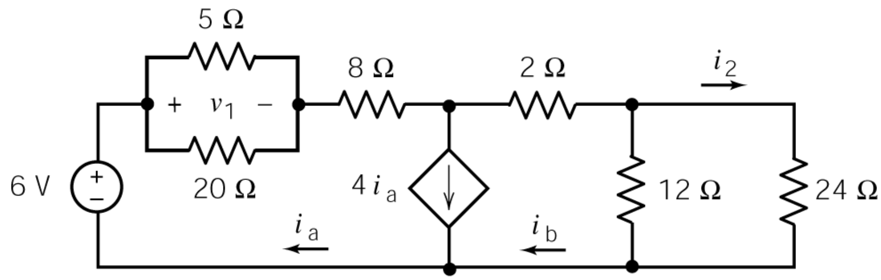
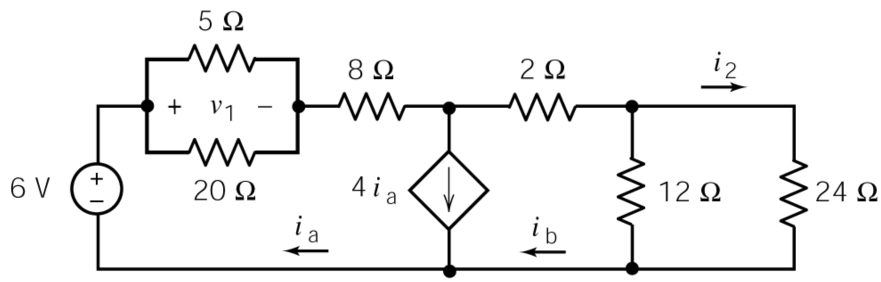
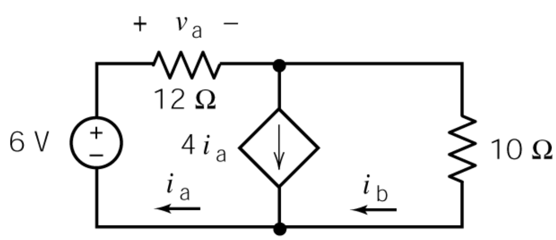


Figure P3.6-42

Solution:



Use equivalent resistances to reduce the circuit to



From KCL $i_b = 4i_a + i_a \quad \Rightarrow \quad i_b = -3i_a$.

From KVL

$$12i_a + 10i_b - 6 = 0 \quad \Rightarrow \quad 12i_a + 10(-3i_a) = 6$$

So $i_a = -\dfrac{1}{3}$ A, $v_a = -4$ A and $i_b = 1$ A .

Returning our attention to the original circuit, notice that i_a and i_b were not changed when the circuit was reduced. Now $v_1 = (5 \,\|\, 20)\, i_a = (4)(-0.333) = -1.333$ V and $i_2 = \dfrac{12}{12+24} i_b = 0.333$ A .

P3.6-43 The Ohmmeter in Figure P3.6-43 measures R_{eq}, the equivalent resistance of the part of the circuit to the left of the terminals.

(a) Suppose R_{eq} = 12 Ω. Determine the value of the resistance *R*.
(b) Suppose instead that *R* = 14 Ω. Determine the value of the equivalent resistance R_{eq}.

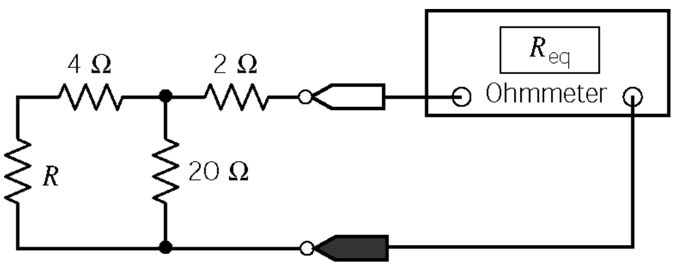


Figure P3.6-43

Solution:

$$R_{eq} = 2 + \left(20 \,\|\, (4+R)\right).$$

(a) When $R_{eq} = 12\ \Omega$ then

$$12 = 2 + \left(20 \,\|\, (4+R)\right) \;\Rightarrow\; 10 = \frac{20(4+R)}{20+(4+R)} \;\Rightarrow\; 24 + R = 2(4+R) \;\Rightarrow\; R = 16\ \Omega$$

(b) When $R = 14\ \Omega$ then

$$R_{eq} = 2 + \left(20 \,\|\, (4+14)\right) = 2 + (20 \,\|\, 18) = 2 + \frac{20(18)}{20+18} = 11.4736\ \Omega$$

P3.6-43. Determine the values of the resistance R and current i_a in the circuit shown in Figure P3.6-43.

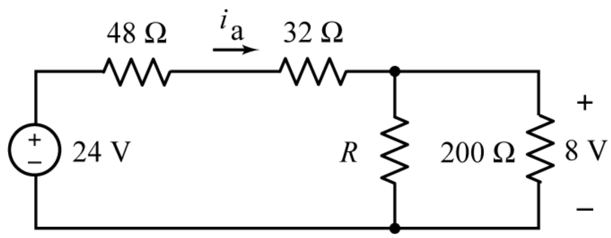
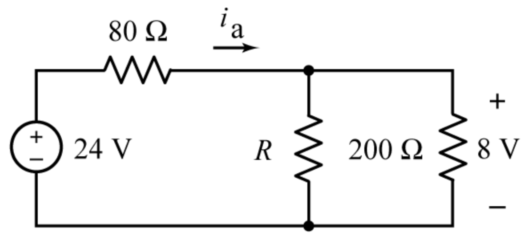


Figure P3.6-43

Solution. Replace the series resistors by an equivalent resistor. Then use KVL to write



$$80\,i_a + 8 - 24 = 0 \quad \Rightarrow \quad i_a = \frac{24-8}{80} = 0.2 \text{ A}$$

Use KCL to write

$$\frac{24-8}{80} = \frac{8}{R} + \frac{8}{200} \quad \Rightarrow \quad \frac{8}{R} = \frac{16}{80} - \frac{8}{200} = 0.16 \quad \Rightarrow \quad R = \frac{8}{0.16} = 50\ \Omega$$

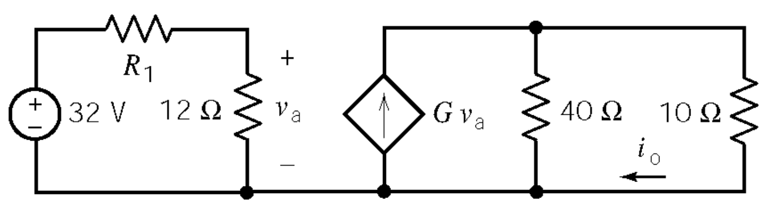


Figure P3.6-44

P3.6-44 The input to the circuit shown in Figure P3.6-44 is the voltage of the voltage source, 32 V. The output is the current in the 10 Ω resistor, i_o. Determine the values of the resistance, R_1, and of the gain of the dependent source, G, that cause both the value of voltage across the 12 Ω to be v_a = 10.38 V and the value of the output current to be i_o = 0.4151 A.

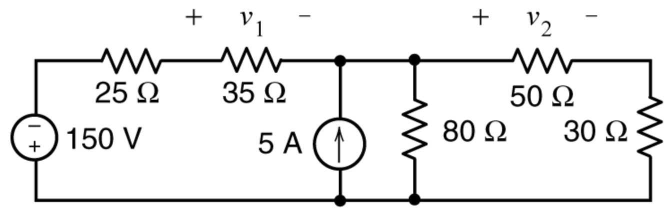
Solution:
Using voltage division

$$10.38 = v_a = \frac{12}{R_1+12}(32) \quad \Rightarrow \quad R_1 + 12 = \frac{12(32)}{10.38} = 36.9942 \approx 37\ \Omega \quad \Rightarrow \quad R_1 = 25\ \Omega$$

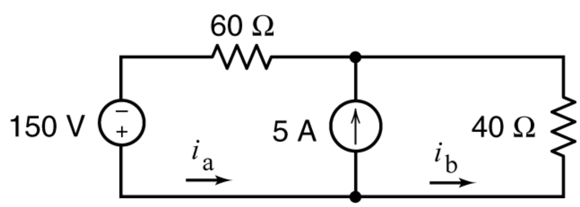
Using current division

$$0.4151 = i_o = \frac{40}{40+10} G v_a = (0.8)G(10.38) \quad \Rightarrow \quad G = \frac{0.4151}{(0.8)10.38} = 0.05\ \frac{\text{A}}{\text{V}}$$

P3.6-45 The equivalent circuit in Figure 3.6-45 is obtained from the original circuit by replacing series and parallel combinations of resistors by equivalent resistors. The values of the currents in the equivalent circuit are $i_a = 3.5$ A and $i_b = -1.5$ A. Determine the values of the voltages v_1 and v_2 in the original circuit.



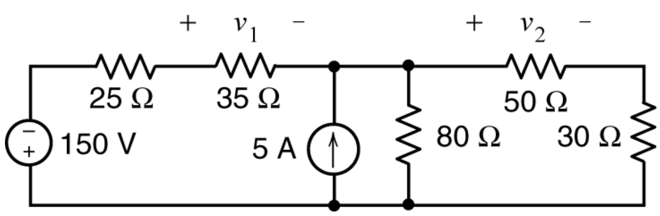
original circuit

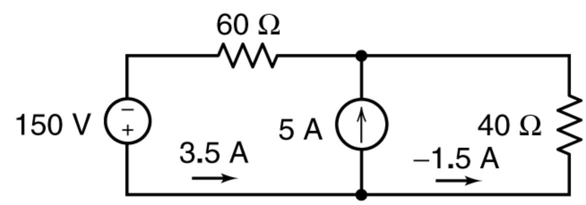


equivalent circuit

Figure P3.6-45

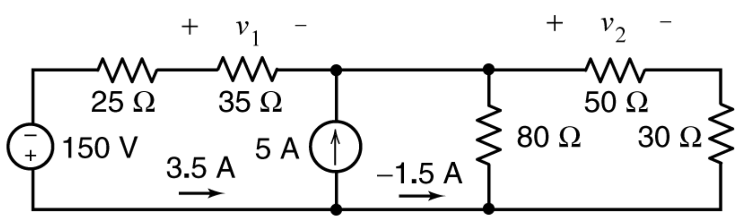
Solution:





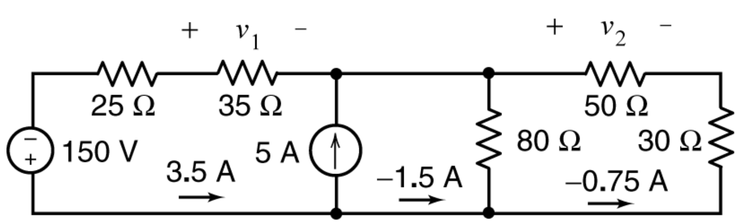
original circuit　　　　equivalent circuit

Label the currents in the equivalent circuit that correspond to the give currents in the equivalent circuit:



original circuit

Use current division:



original circuit

Using Ohm's law:

$$v_1 = -35 i_a = -35(3.5) = -122.5 \text{ V} \quad \text{and} \quad v_2 = -50(-0.75) = 37.5 \text{ V}$$

P3.6-46 Figure 3.6-46 shows three separate, similar circuits. In each a 12 V source is connected to a subcircuit consisting of three resistors. Determine the values of the voltage source currents i_1, i_2 and i_3. Conclude that while the voltage source voltage is 12 V in each circuit, the voltage source current depends on the subcircuit connected to the voltage source.

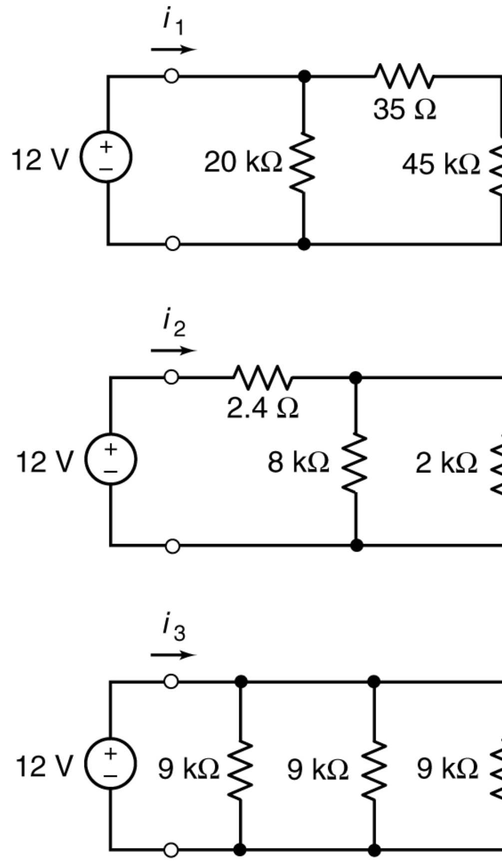
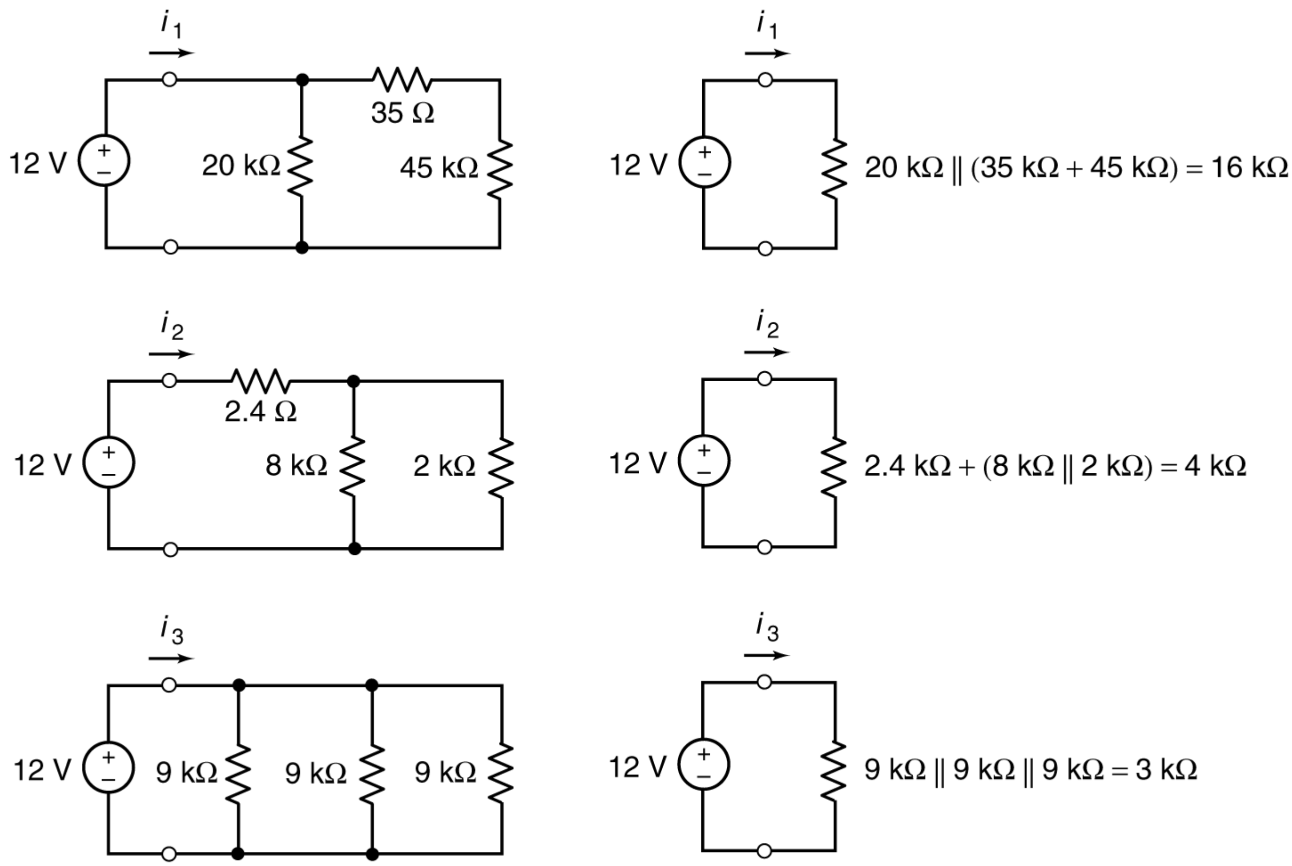


Figure P3.6-46

Solution:
Replace the resistor subcircuit by an equivalent resistor in each circuit:



Using Ohm's law:

$$i_1 = \frac{12}{16\ \text{k}\Omega} = 0.75\ \text{mA}\ ,\ i_2 = \frac{12}{4\ \text{k}\Omega} = 3\ \text{mA} \quad \text{and} \quad i_3 = \frac{12}{3\ \text{k}\Omega} = 4\ \text{mA}$$

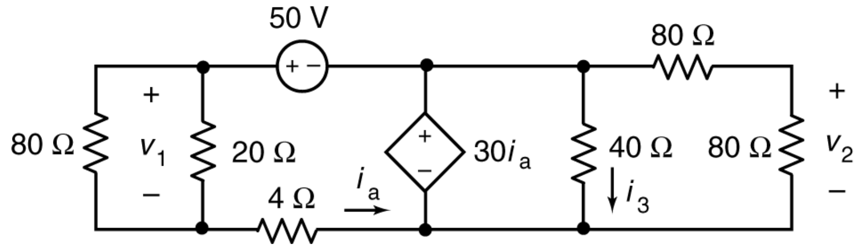
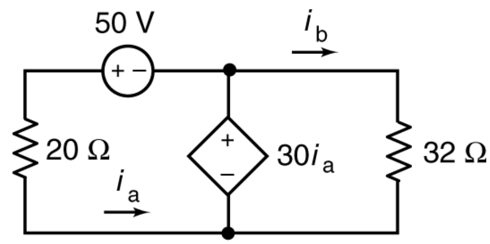


Figure P3.6-47

P3.6-47 Determine the values of the voltages, v_1 and v_2, and of the current, i_3, in the circuit shown in Figure P3.6-47.

Solution:
Replace series and parallel combinations of resistors by equivalent resistors to get

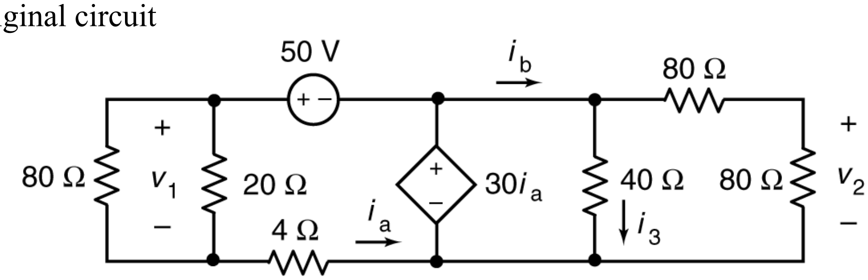


($4+(80\,||\,20)=4+16=20\ \Omega$ and $40||(80+80)=40\,||\,160=32\ \Omega$.) Next, apply KVL to the left mesh to get

$$50+30i_a-20i_a=0 \quad\Rightarrow\quad i_a=\frac{50}{20-10}=-5\ \text{A} \quad \text{and} \quad 30i_a=-150\ \text{V}$$

Ohm's law gives $$i_b=\frac{30i_a}{32}=\frac{-150}{32}=-4.6875\ \text{A}$$

Label i_b on the original circuit



Finally $$v_1=(80\,||\,20)i_a=16(-5)=-80\ \text{V}\,,\ v_2=\frac{1}{2}(30i_a)=-75\ \text{V}$$

and $$i_3=\frac{80+80}{40+(80+80)}i_b=\frac{4}{5}(-4.6875)=-3.75\ \text{A}$$

Section 3-7 Analyzing Resistive Circuits using MATLAB

P3.7-1 Determine the power supplied by each of the sources, independent and dependent, in the circuit shown in Figure P3.7-1.
Hint: Use the guidelines given in Section 3.7 to label the circuit diagram. Use MATLAB to solve the equations representing the circuit.

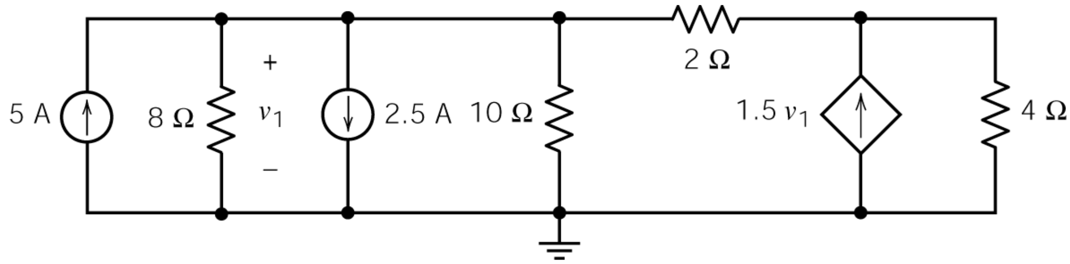
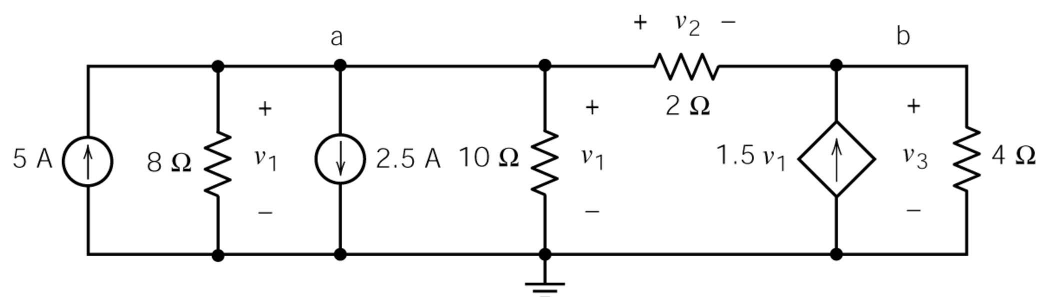


Figure 3.7-1

Solution:
We'll begin by choosing the bottom node to be the reference node. Next we'll label the other nodes and some element voltages:



Notice that the 8 Ω resistor, the 10 Ω resistor and the two independent current sources are all connected in parallel. Consequently, the element voltages of theses elements can be labeled so that they are equal. Similarly, the 4 Ω resistor and the dependent current source are connected in parallel so their voltages can be labeled so as to be equal.

Using Ohm's Law we see that the current directed downward in the 8 Ω resistor is $\frac{v_1}{8}$, current directed downward in the 10 Ω resistor is $\frac{v_1}{10}$, and the current directed from left to right in the 2 Ω resistor is $\frac{v_2}{2}$.
Applying Kirchhoff's Current Law (KCL) at node a gives

$$5 = \frac{v_1}{8} + 2.5 + \frac{v_1}{10} + \frac{v_2}{2} \quad \Rightarrow \quad 0.225\,v_1 + 0.5\,v_2 = 2.5 \qquad (1)$$

Using Ohm's Law we see that the current directed downward in the 4 Ω resistor is $\frac{v_3}{4}$. Applying Kirchhoff's Current Law (KCL) at node a gives

$$\frac{v_2}{2} + 1.5 v_1 = \frac{v_3}{4} \quad \Rightarrow \quad 1.5 v_1 + 0.5 v_2 - 0.25 v_3 = 0 \tag{2}$$

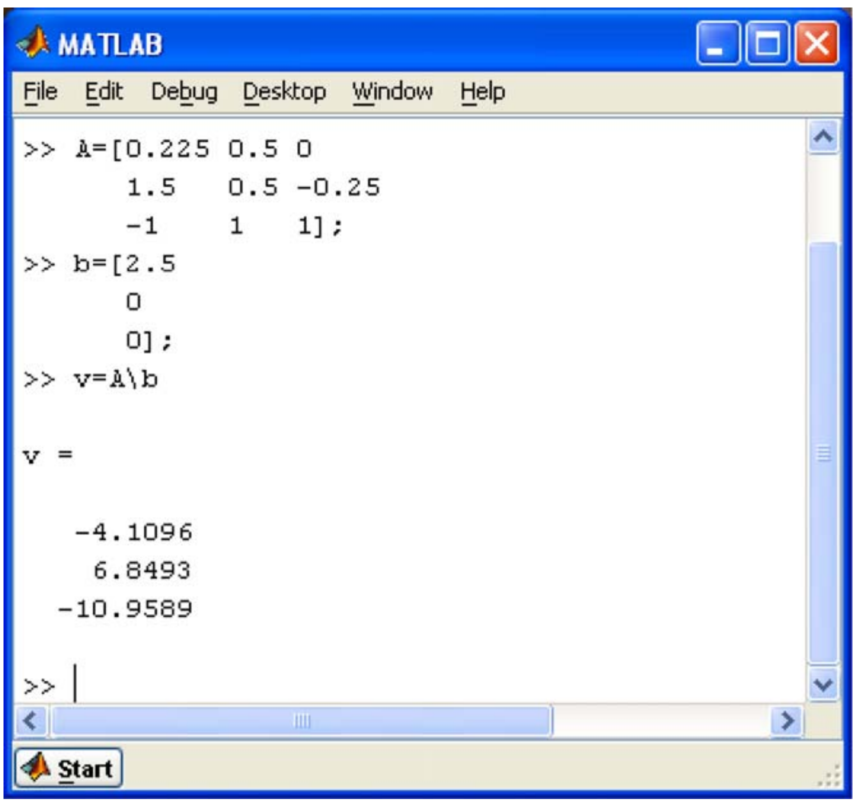
Applying Kirchhoff's Voltage Law (KVL) to the mesh consisting of the 10 Ω resistor, the 2 Ω resistor and the dependent source to get

$$v_2 + v_3 - v_1 = 0 \tag{3}$$

Equations 1, 2 and 3 comprise a set of three simultaneous equations in the three unknown voltages v_1, v_2 and v_3. We can write these equations in matrix form as

$$\begin{bmatrix} 0.225 & 0.5 & 0 \\ 1.5 & 0.5 & -0.25 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \\ 0 \end{bmatrix}$$

We can solve this matrix equation using MATLAB:



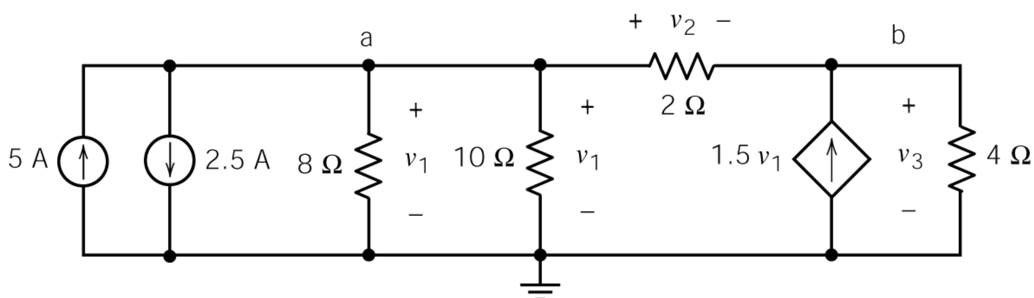
Hence

$$v_1 = -4.1096 \text{ V}, \quad v_2 = 6.8493 \text{ V} \quad \text{and} \quad v_3 = -10.9589 \text{ V}$$

The power supplied by the 5 A current source is $5v_1 = 5(-4.1096) = -20.548$ W . The power supplied by the 2.5 A current source is $-2.5v_1 = -2.5(-4.1096) = 10.247$ W . The power supplied by the dependent current source is $(1.5v_1)v_3 = 1.5(-4.1096)(-10.9589) = 67.555$ W .

Observation: Changing the order of the 8 Ω resistor, the 10 Ω resistor and the two independent current sources only changes the order of the terms in the KCL equation at node a. We know that addition is commutative, so change the order of the terms will not affect the values of the voltages v_1, v_2 and v_3.

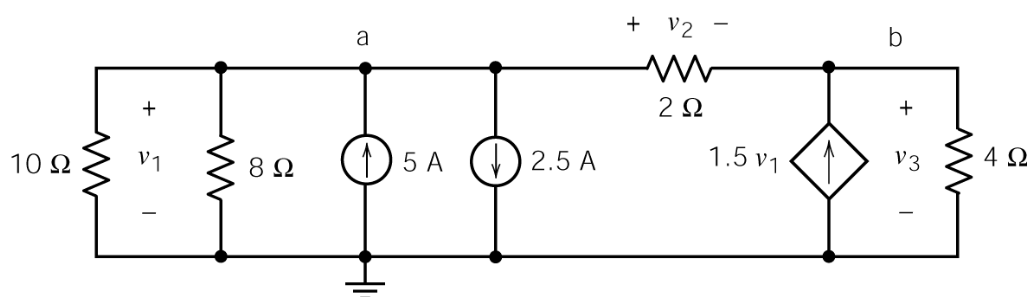
For example, if the positions of the 2.5 A current source and 8 Ω resistor are swithched:



The KCL equation at node a is

$$5 = 2.5 + \frac{v_1}{8} + \frac{v_1}{10} + \frac{v_2}{2} \quad \Rightarrow \quad 0.225v_1 + 0.5v_2 = 2.5$$

Similarly, when the circuit is drawn as



The KCL equation at node a is

$$5 = \frac{v_1}{10} + \frac{v_1}{8} + 2.5 + \frac{v_2}{2} \quad \Rightarrow \quad 0.225v_1 + 0.5v_2 = 2.5$$

The changes do not affect the values of the voltages v_1, v_2 and v_3.

P3.7-2 Determine the power supplied by each of the sources, independent and dependent, in the circuit shown in Figure P3.7-2.
Hint: Use the guidelines given in Section 3.7 to label the circuit diagram. Use MATLAB to solve the equations representing the circuit.

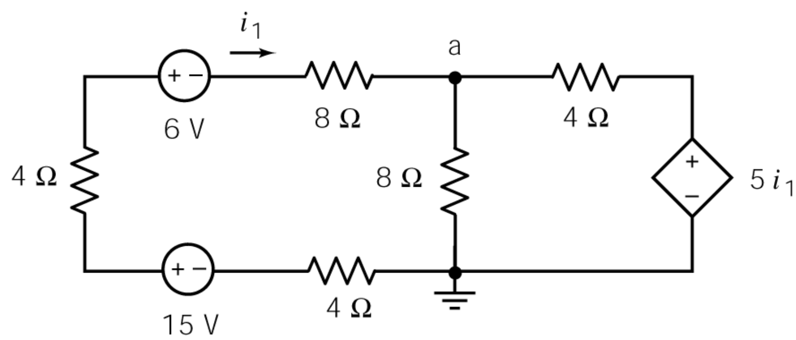
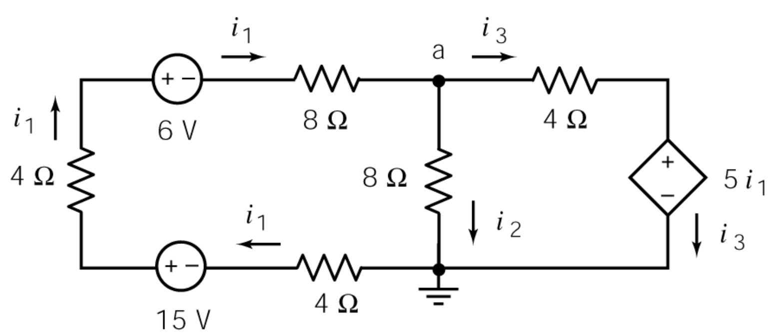


Figure 3.7-2

Solution:
We'll begin by choosing the bottom node to be the reference node. Next we'll label the other nodes and some element currents:



Notice that two 4 Ω resistors, an 8 Ω resistor and the two independent voltage sources are all connected in series. Consequently, the element currents of theses elements can be labeled so that they are equal. Similarly, a 4 Ω resistor and the dependent voltage source are connected in series so their currents can be labeled so as to be equal.

The current in each resistor has been labeled so we can use Ohm's Law to calculate resistor voltages from the resistor currents and the resistances. Apply Kirchhoff's Voltage Law (KVL) to the left mesh to get

$$6+8i_1+8i_2+4i_1-15+4i_1=0 \quad \Rightarrow \quad 16i_1+8i_2=9 \tag{1}$$

Apply Kirchhoff's Voltage Law (KVL) to the right mesh to get

$$4i_3+5i_1-8i_2=0 \tag{2}$$

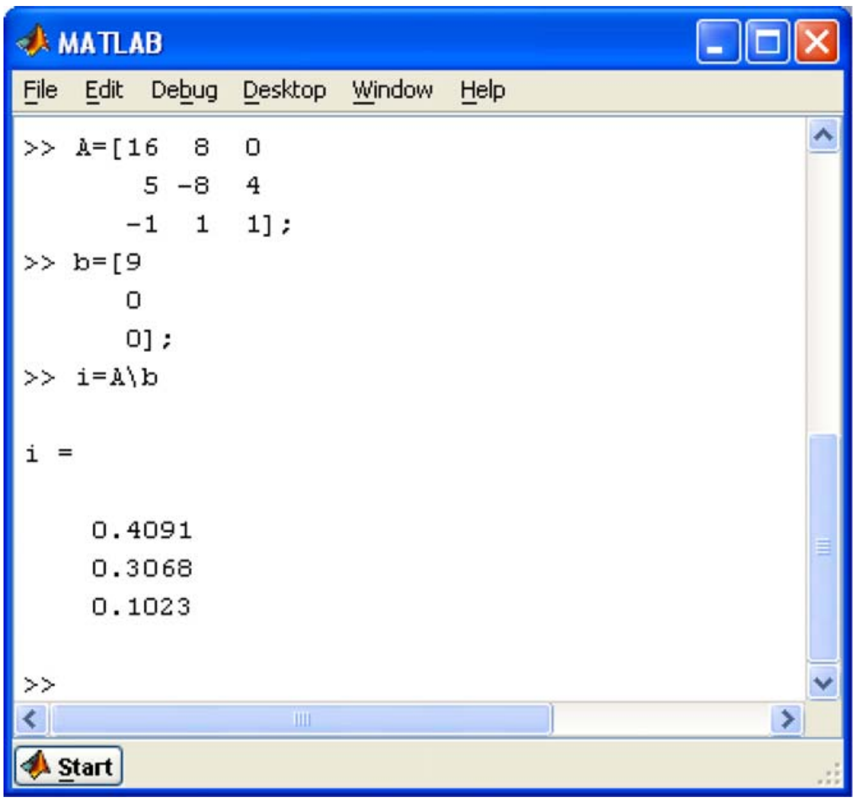
Applying Kirchhoff's Current Law (KCL) at node a to get

$$i_1=i_2+i_3 \quad \Rightarrow \quad -i_1+i_2+i_3=0 \tag{3}$$

Equations 1, 2 and 3 comprise a set of three simultaneous equations in the three unknown voltages v_1, v_2 and v_3. We can write these equations in matrix form as

$$\begin{bmatrix} 16 & 8 & 0 \\ 5 & -8 & 4 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 0 \end{bmatrix}$$

We can solve this matrix equation using MATLAB:

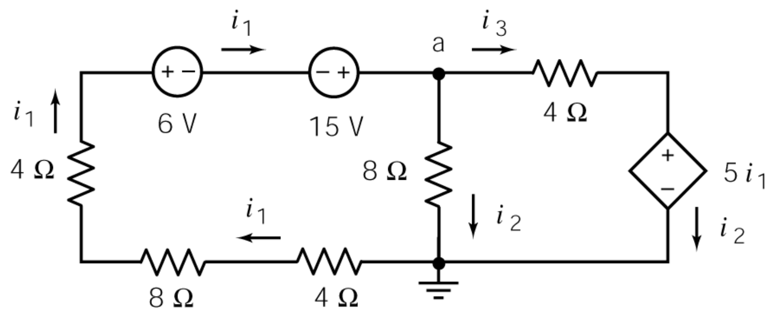


Hence

$$i_1 = 0.4091 \text{ A}, \quad i_2 = 0.3068 \text{ A} \quad \text{and} \quad i_3 = 0.1023 \text{ A}$$

The power supplied by the 15 V voltage source is $15 i_1 = 15(0.4091) = 6.1365$ W. The power supplied by the 6 V voltage source is $-6 i_1 = -6(0.4091) = -2.4546$ W. The power supplied by the dependent voltage source is $-(5 i_2) i_3 = -5(0.3068)(0.1023) = 0.1569$ W.

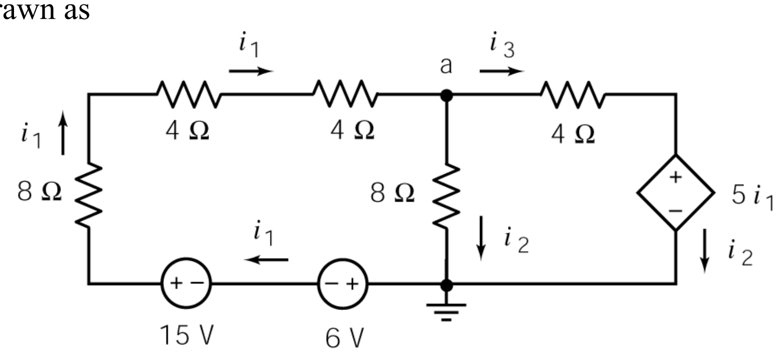
Observation: Changing the order of the two 4 Ω resistors, an 8 Ω resistor and the two independent voltage sources in the left mesh changes the order of the terms in the KVL equation for that mesh. We know that addition is commutative, so change the order of the terms will not affect the values of the currents i_1, i_2 and i_3. For example, when the circuit is drawn as



The KVL equation for the left mesh is

$$6-15+8i_2+4i_1+8i_1+4i_1=0 \quad\Rightarrow\quad 16i_1+8i_2=9$$

When the circuit is drawn as



The KVL equation for the left mesh is

$$4i_1+4i_1+8i_2+6-15+8i_1=0 \quad\Rightarrow\quad 16i_1+8i_2=9$$

These changes do not affect the values of the currents i_1, i_2 and i_3.

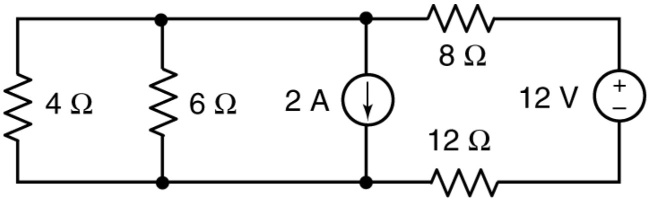
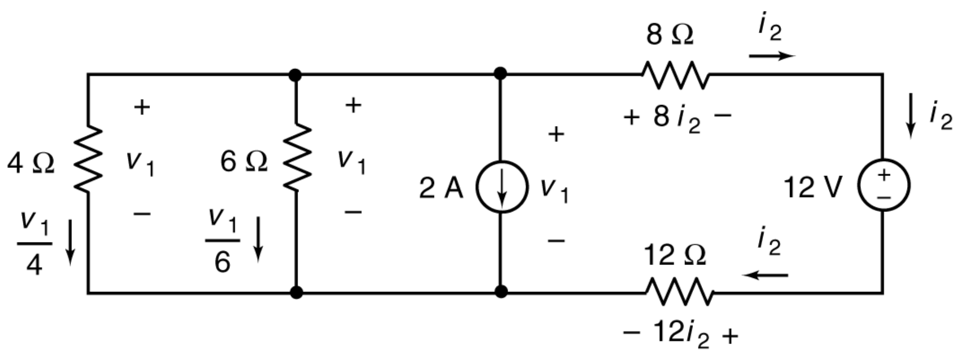


Figure P3.7-3

P3.7-3. Determine the power supplied by each of the independent sources in the circuit shown in Figure P3.7-3.

P3.7-3

Label the element currents and voltages as suggested in Table 3.7-1 Guidelines for Labeling Circuit Variables:



Apply KCL at the top left node: $\frac{v_1}{4}+\frac{v_1}{4}+2+i_2=0$

Apply KVL to the left mesh: $8\,i_2-12+12\,i_2-v_1=0$

In matrix form: $\begin{bmatrix} \frac{1}{4}+\frac{1}{6} & 1 \\ -1 & 8+12 \end{bmatrix}\begin{bmatrix} v_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} -2 \\ 12 \end{bmatrix}$

Solving using MATLAB: $\begin{bmatrix} v_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} -5.5714 \\ 0.3214 \end{bmatrix}$

The current source **supplies** $-2\,v_1=-2\left(-5.5714\right)=11.1428 \text{ W}$

The voltage source supplies $12\,i_2=12\left(0.3214\right)=3.8568 \text{ W}$

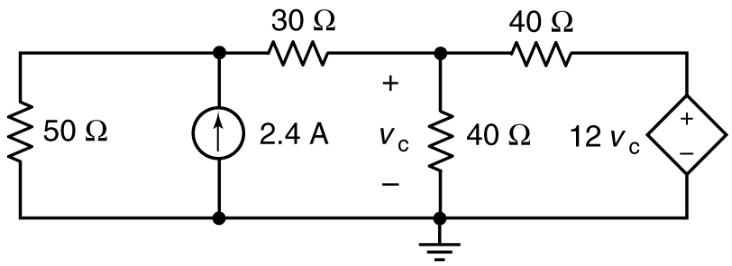
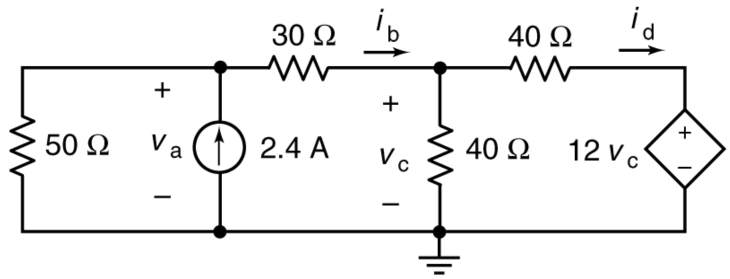


Figure P3.7-4

P3.7-4. Determine the power supplied by each of the sources in the circuit shown in Figure P3.7-4.

P3.7-4
Label the element currents and voltages as suggested in Table 3.7-1 Guidelines for Labeling Circuit Variables:



Apply KVL to the loop consisting of 30 Ω, 40 Ω and 50 Ω resistors: $30\,i_b + v_c - v_a = 0$

Apply KVL to the right mesh: $v_c = 40\,i_d + 12\,v_c$

Apply KCL at the top node of the current source: $\dfrac{v_a}{50} + i_b = 2.4$

Apply KCL at the top node of a 40 Ω resistor: $i_b = \dfrac{v_c}{40} + i_d$

In matrix form:
$$\begin{bmatrix} -1 & 30 & 1 & 0 \\ 0 & 0 & 11 & 40 \\ 0.02 & 1 & 0 & 0 \\ 0 & -1 & 0.025 & 1 \end{bmatrix}\begin{bmatrix} v_a \\ i_b \\ v_c \\ i_d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2.4 \\ 0 \end{bmatrix}$$

Solving using MATLAB:
$$\begin{bmatrix} v_a \\ i_b \\ v_c \\ i_d \end{bmatrix} = \begin{bmatrix} 41.0526 \\ 1.5789 \\ -6.3158 \\ 1.7368 \end{bmatrix}$$

The current source **supplies** $2.4\,v_a = 2.4(41.0526) = 98.5262$ W

The VCVS supplies $-(12\,v_c)\,i_d = -12(-6.3158)(1.7368) = 131.6314$ W

Section 3-8 How Can We Check …

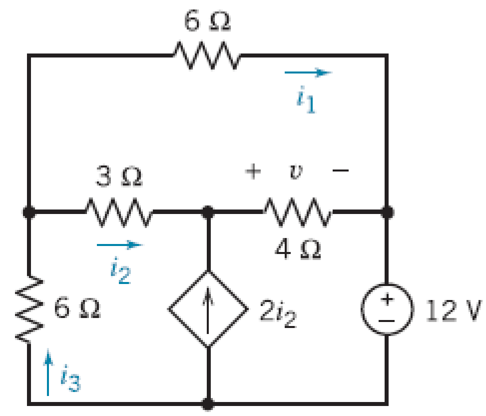


Figure P 3.8-1

P 3.8-1 A computer analysis program, used for the circuit of Figure P 3.8-1, provides the following branch currents and voltages:

$$i_1 = -0.833 \text{ A},\ i_2 = -0.333 \text{ A},\ i_3 = -1.167 \text{ A},$$

and

$$v = -2.0 \text{ V}.$$

Are these answers correct?

Hint: Verify that KCL is satisfied at the center node and that KVL is satisfied around the outside loop consisting of the two 6-Ω resistors and the voltage source.

Solution:

KCL at node a: $i_3 = i_1 + i_2$

$$-1.167 = -0.833 + (-0.333)$$

$$-1.167 = -1.166 \text{ OK}$$

KVL loop consisting of the vertical 6 Ω resistor, the 3 Ω and4Ω resistors, and the voltage source:

$$6i_3 + 3i_2 + v + 12 = 0$$

yields $\underline{v = -4.0 \text{ V}}$ not $v = -2.0$ V

The answers are not correct.

P 3.8-2 The circuit of Figure P 3.8-2 was assigned as a homework problem. The answer in the back of the textbook says the current, *i,* is 1.25 A. Verify this answer using current division.

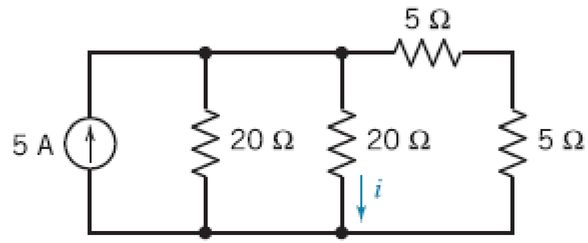


Figure P 3.8-2

Solution:

Apply current division to get: $i=\left(\dfrac{\dfrac{1}{20}}{\dfrac{1}{20}+\dfrac{1}{20}+\dfrac{1}{5+5}}\right)5=\left(\dfrac{1}{4}\right)5=1.25\text{ A}\quad\surd$

The answer is correct.

P 3.8-3 The circuit of Figure P 3.8-3 was built in the lab and v_o was measured to be 6.25 V. Verify this measurement using the voltage divider principle.

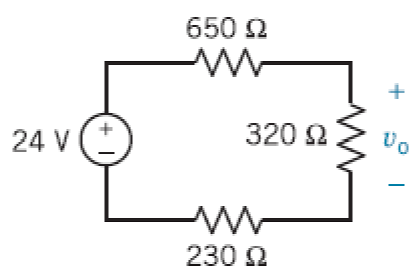


Figure P 3.8-3

Solution:

Apply voltage division to get: $v=\left(\dfrac{320}{650+320+230}\right)24=\left(\dfrac{320}{1200}\right)24=6.4\text{ V}\neq 6.25\text{ V}$

The measurement is not correct.

P 3.8-4 The circuit of Figure P 3.8-4 represents an auto's electrical system. A report states that

$$i_H = 9\text{ A},\quad i_B = -9\text{ A},\quad \text{and}\quad i_A = 19.1\text{ A}.$$

Verify that this result is correct.

Hint: Verify that KCL is satisfied at each node and that KVL is satisfied around each loop.

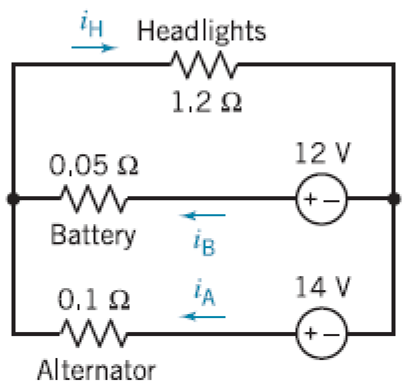


Figure P 3.8-4

Solution:

KVL bottom loop: $-14+0.1i_A+1.2i_H=0$

KVL right loop: $-12+0.05i_B+1.2i_H = 0$

KCL at left node: $i_A+i_B=i_H$

Solving the three above equations yields:

$\underline{i_A=16.8\text{ A}}$ $\underline{i_H=10.3\text{ A}}$ and $\underline{i_B=-6.49\text{ A}}$

These are not the given values. Consequently***, the report is incorrect***.

P 3.8-5 Computer analysis of the circuit in Figure P 3.8-5 shows that

$$i_a = -0.5\text{ mA and } i_b = -2\text{ mA}.$$

Was the computer analysis done correctly?

Hint: Verify that the KVL equations for all three meshes are satisfied when $i_a = -0.5$ mA and $i_b = -2$ mA.

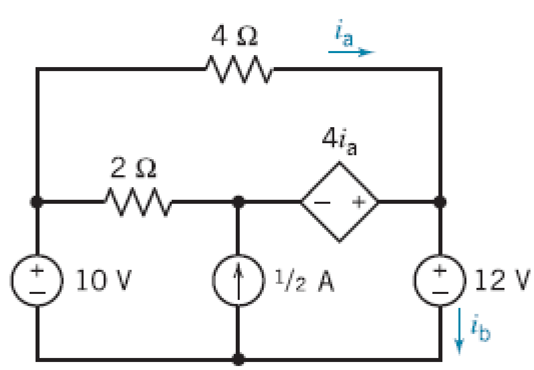


Figure P 3.8-5

Solution:

Top mesh: $0 = 4\,i_a + 4\,i_a + 2\left(i_a + \frac{1}{2} - i_b\right) = 10(-0.5) + 1 - 2(-2)$

Lower left mesh: $v_s = 10 + 2\left(i_a + 0.5 - i_b\right) = 10 + 2(2) = 14\text{ V}$

Lower right mesh: $v_s + 4\,i_a = 12 \quad \Rightarrow \quad v_s = 12 - 4(-0.5) = 14\text{ V}$

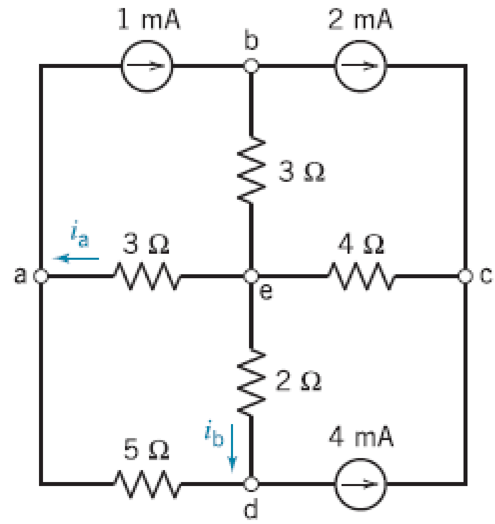
The KVL equations are satisfied so ***the analysis is correct***.

P 3.8-6 Computer analysis of the circuit in Figure P 3.8-6. shows that

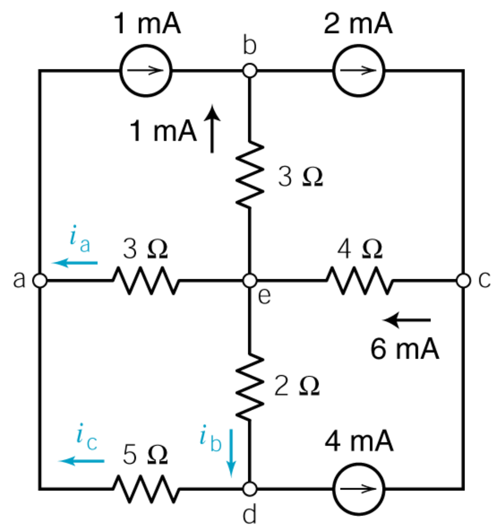
$$i_a = 0.5 \text{ mA and } i_b = 4.5 \text{ mA.}$$

Was the computer analysis done correctly?

Hint: First, verify that the KCL equations for all five nodes are satisfied when $i_a = 0.5$ mA and $i_b = 4.5$ mA. Next, verify that the KVL equation for the lower left mesh (a-e-d-a) is satisfied. (The KVL equations for the other meshes aren't useful because each involves an unknown voltage.)



Solution: Apply KCL at nodes b and c to label the circuit as



Apply KCL to get

$$i_c + 4 = 4.5 \quad \Rightarrow \quad i_c = 0.5 \text{ A} \qquad \text{(node d)}$$

$$i_a + i_c = 1 \quad \Rightarrow \quad 0.5 + 0.5 = 1 \surd \qquad \text{(node a)}$$

$$6 = 1 + i_a + i_b = 1 + 0.5 + 4.5 \surd \qquad \text{(node e)}$$

Apply KVL to get to mesh (a-e-d-a)

$$-3(0.5) + 2(4.5) - 5(0.5) = -1.5 + 9 - 2.5 \neq 0$$

The given currents ***do not satisfy*** these five Kirchhoff's laws equations and therefore ***are not correct***.

P 3.8-7 Verify that the element currents and voltages shown in Figure P 3.8-7 satisfy Kirchhoff's laws:

(a) Verify that the given currents satisfy the KCL equations corresponding to nodes a, b, and c.

(b) Verify that the given voltages satisfy the KVL equations corresponding to loops a-b-d-c-a and a-b-c-d-a.

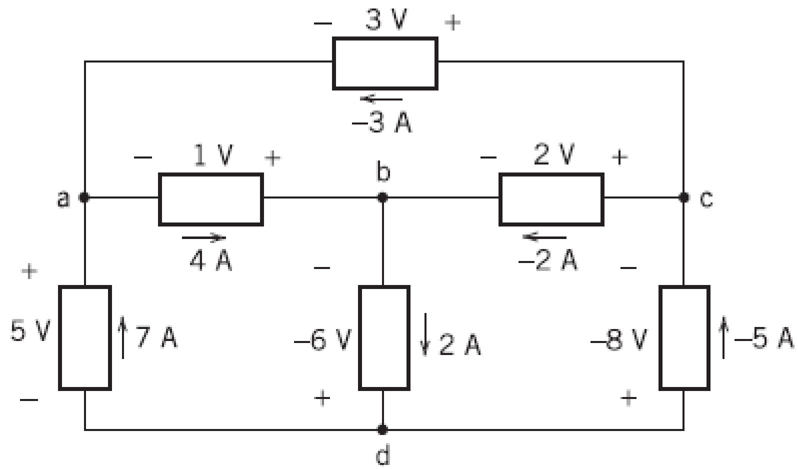


Figure P 3.8-7

Solution:

(a)

$$
\begin{aligned}
7+(-3)&=4 \qquad (\text{node } a)\\
4+(-2)&=2 \qquad (\text{node } b)\\
-5&=-2+(-3) \qquad (\text{node } c)
\end{aligned}
$$

(b)

$$
\begin{aligned}
-1-(-6)+(-8)+3&=0 \qquad (\text{loop } a\text{-}b\text{-}d\text{-}c\text{-}a)\\
-1-2-(-8)-5&=0 \qquad (\text{loop } a\text{-}b\text{-}c\text{-}d\text{-}a)
\end{aligned}
$$

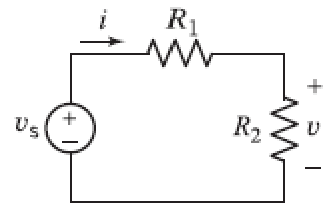
The given currents and voltages satisfy these five Kirchhoff's laws equations

P 3.8-8 Figure P 3.8-8 shows a circuit and some corresponding data. The tabulated data provides values of the current, i, and voltage, v, corresponding to several values of the resistance R_2.

(a) Use the data in rows 1 and 2 of the table to find the values of v_s and R_1.

(b) Use the results of part (a) to verify that the tabulated data are consistent.

(c) Fill in the missing entries in the table.



(a)

R_2, Ω	i, A	v, V
0	2.4	0
10	1.2	12
20	0.8	16
30	?	18
40	0.48	?

(b)

Figure P 3.8-8

Solution:

(a)

$$i = \frac{v_s}{R_1 + R_2}$$

from row 1

$$2.4 = \frac{v_s}{R_1}$$

from row 2

$$1.2 = \frac{v_s}{R_1 + 10}$$

so

$$2.4 R_1 = v_s = 1.2\left(R_1 + 10\right) \quad \Rightarrow \quad R_1 = 10\ \Omega$$

then

$$v_s = 2.4\left(10\right) = 24\ \text{V}$$

(b)

$$i = \frac{24}{10 + R_2} \quad \text{and} \quad v = \frac{24 R_2}{10 + R_2}$$

When $R_2 = 20\ \Omega$ then $i = \frac{24}{30} = 0.8$ A and $v = \frac{480}{30} = 16$ V.

When $R_2 = 30\ \Omega$ then $v = \frac{720}{40} = 18$ V.

When $R_2 = 40\ \Omega$ the $i = \frac{24}{50} = 0.48$ A.

(c) When $R_2 = 30\ \Omega$ then $i = \frac{24}{40} = 0.6$ A.

When $R_2 = 40$ W then $v = \frac{960}{50} = 19.2$ V.

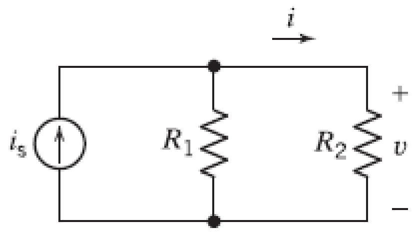
(checked: LNAP 6/21/04)

P 3.8-9 Figure P 3.8-9 shows a circuit and some corresponding data. The tabulated data provide values of the current, i, and voltage, v, corresponding to several values of the resistance R_2.

(a) Use the data in rows 1 and 2 of the table to find the values of i_s and R_1.

(b) Use the results of part (a) to verify that the tabulated data are consistent.

(c) Fill in the missing entries in the table.



(a)

R_2, Ω	i, A	v, V
10	4/3	40/3
20	6/7	120/7
40	1/2	20
80	?	?

(b)

Figure P 3.8-9

Solution:

(a)
$$i=\frac{R_1}{R_1+R_2}i_s$$

From row 1
$$\frac{4}{3}=\frac{R_1}{R_1+10}i_s \quad\Rightarrow\quad 4R_1+40=3R_1 i_s$$

From row 2
$$\frac{6}{7}=\frac{R_1}{R_1+20}i_s \quad\Rightarrow\quad 6R_1+120=7R_1 i_s$$

So
$$\frac{4R_1+40}{3R_1}=i_s=\frac{6R_1+120}{7R_1} \quad\Rightarrow\quad 28R_1+280=18R_1+360 \quad\Rightarrow\quad R_1=8\ \Omega$$

Then
$$\frac{4}{3}=\frac{8}{8+10}i_s \quad\Rightarrow\quad i_s=3\text{ A}$$

(b)
$$i=\frac{8}{8+R_2}(3)=\frac{24}{8+R_2} \quad\text{and}\quad v=R_2\, i=\frac{24R_2}{8+R_2}$$

When $R_2 = 40\ \Omega$ then $i=\dfrac{24}{48}=0.5\text{ A}$ and $v=\dfrac{960}{48}=20\text{ V}$. These are the values in the table so tabulated data is consistent.

(c) When $R_2 = 80\ \Omega$ then $i=\dfrac{24}{88}=\dfrac{3}{11}\text{ A}$ and $v=\dfrac{24(80)}{88}=\dfrac{240}{11}\text{ V}$.

(checked: LNAP 6/21/04)