Today’s Outline

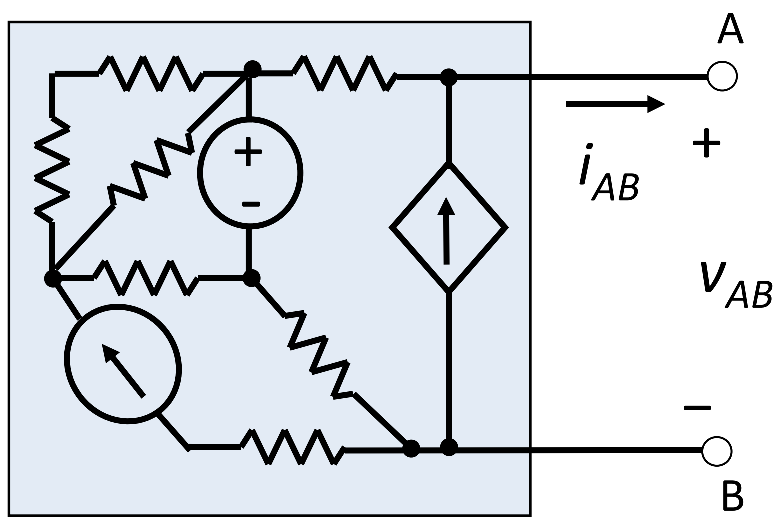
4. Circuit Theorems

- Thévenin’s Theorem

Equivalent Circuits

So far, we have considered equivalent circuits for circuits that are composed only of ideal resistors.

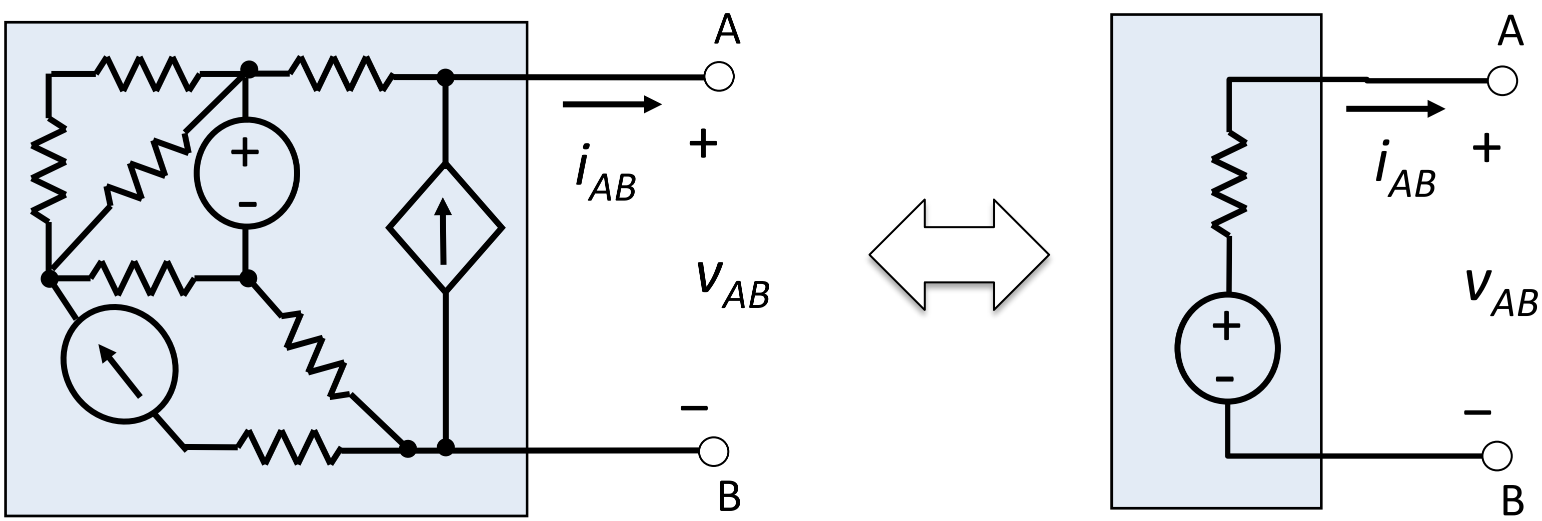
What about circuits that are composed of independent sources, dependent sources, and resistors?



Thévenin's Theorem

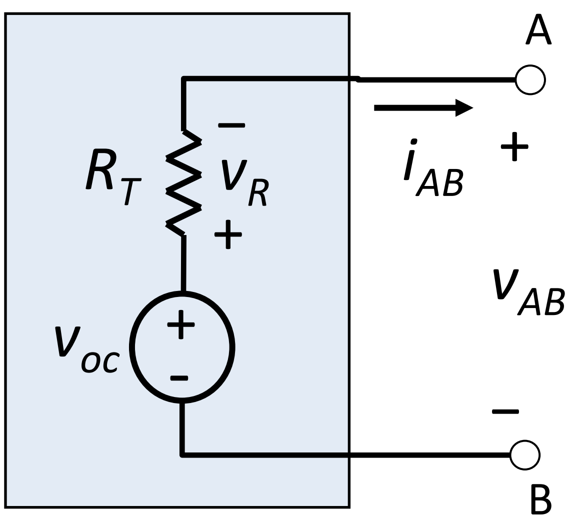
Thévenin's Theorem: any two terminal circuit composed of sources (dependent and independent) and ideal resistors is equivalent to a series combination of an independent voltage source and an ideal resistor, known as a **Thévenin equivalent circuit**

Léon Charles Thévenin (1857-1926)



Thévenin Equivalent Circuit

To understand the meaning of the voltage source, which we denote v_{oc}, and the resistance, which we denote R_T, we analyze the terminal law for the Thévenin equivalent circuit.



KVL: $0 = -v_{oc} + v_R + v_{AB}$

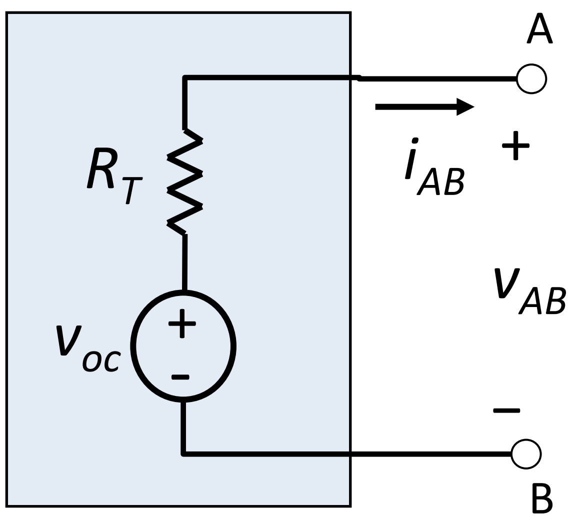
Ohm: $v_R = i_{AB} R_T$

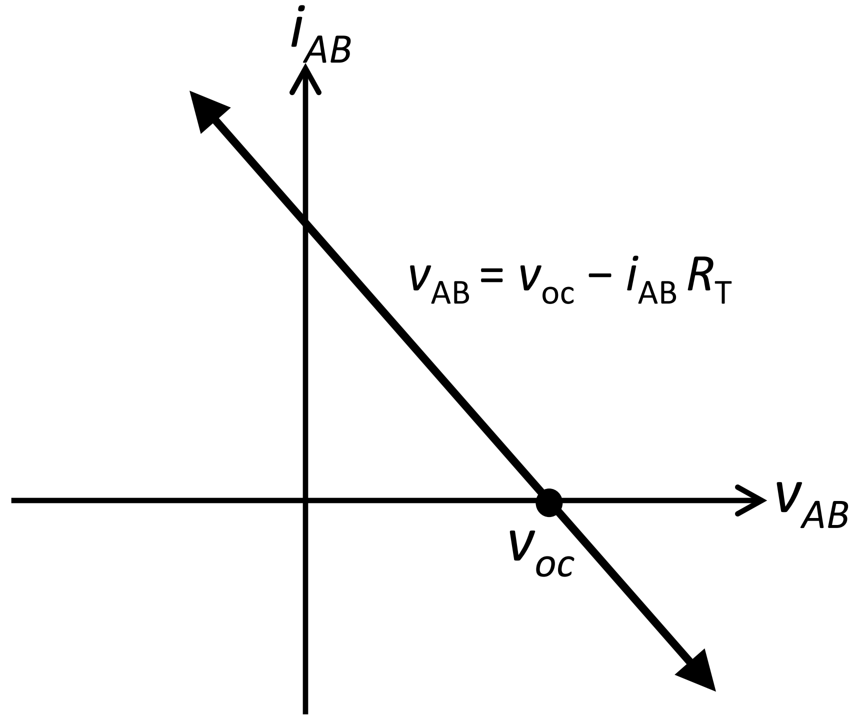
Combining the above:

$v_{AB} = v_{oc} - i_{AB} R_T$

McGill

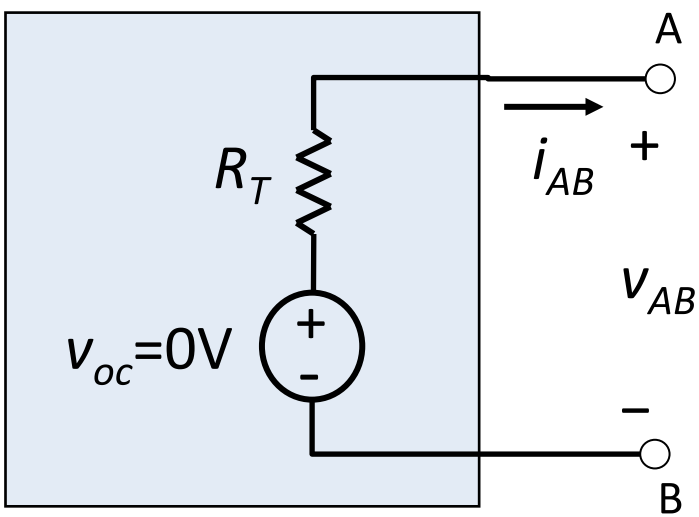
Open Circuit Voltage

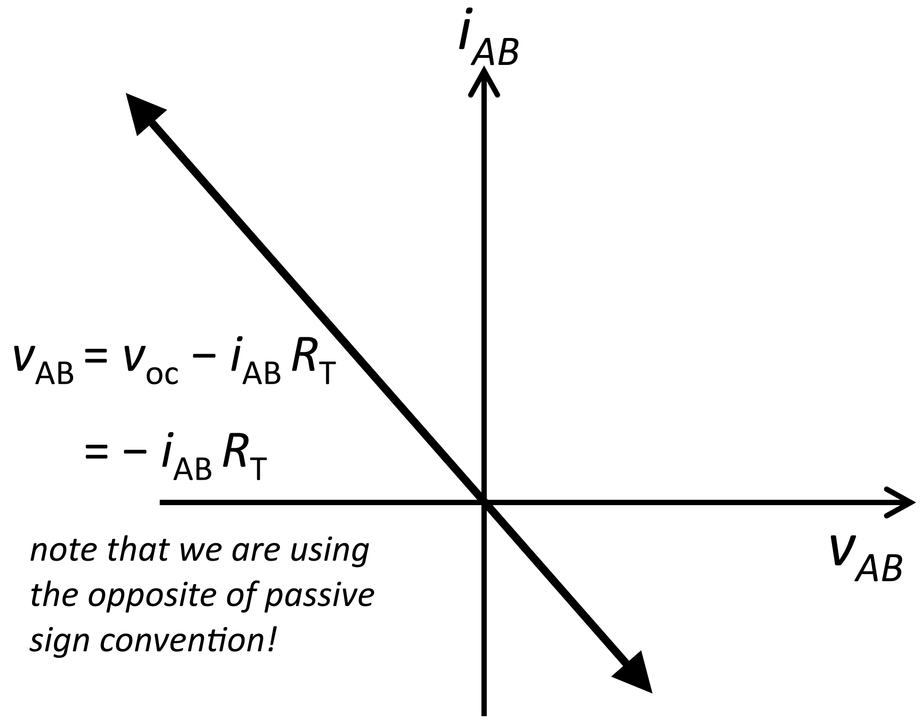




The voltage v_{oc} is the **open circuit voltage**, the voltage that appears across the terminals AB when there is no current i_{AB} drawn from the circuit (the terminals AB are said to be *open*).

Thévenin Resistance





If the voltage source is *turned off*, v_{oc}=0V, then the circuit behaves like an ideal resistor. We call this the **Thévenin resistance**, R_T.

Finding a Thévenin Equivalent Circuit

Does the circuit contain **dependent** sources?

no

1. Find v_{oc} .
2. Find R_T as the equivalent resistance with all sources turned off.

or

1. Use source transformations.

yes

1. Find v_{oc} . Is v_{oc} = 0?

no

2. Find i_{sc}.
3. Then, $R_T = v_{oc}/i_{sc}$.

yes

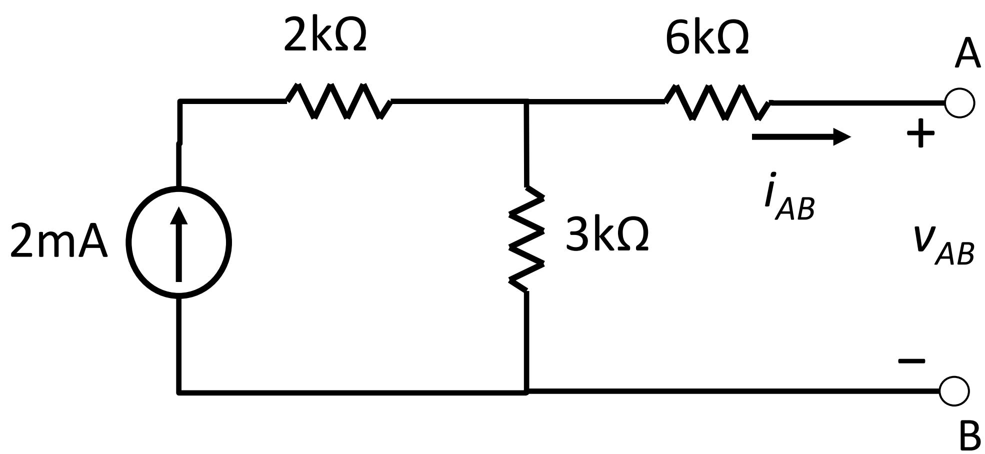
2. Apply a known voltage (eg. v_{ab} = 1V), and find i_{ab}.
3. Find R_T using:

$$v_{ab} = v_{oc} - i_{ab} R_T$$

These are not the only ways to find a Thévenin equivalent circuit. A combination of techniques and ingenuity can be used. For example, you can solve for the v_{AB}-i_{AB} equation directly.

Example 1

Find the Thévenin equivalent for the following circuit with respect to the terminals AB.



Strategy:

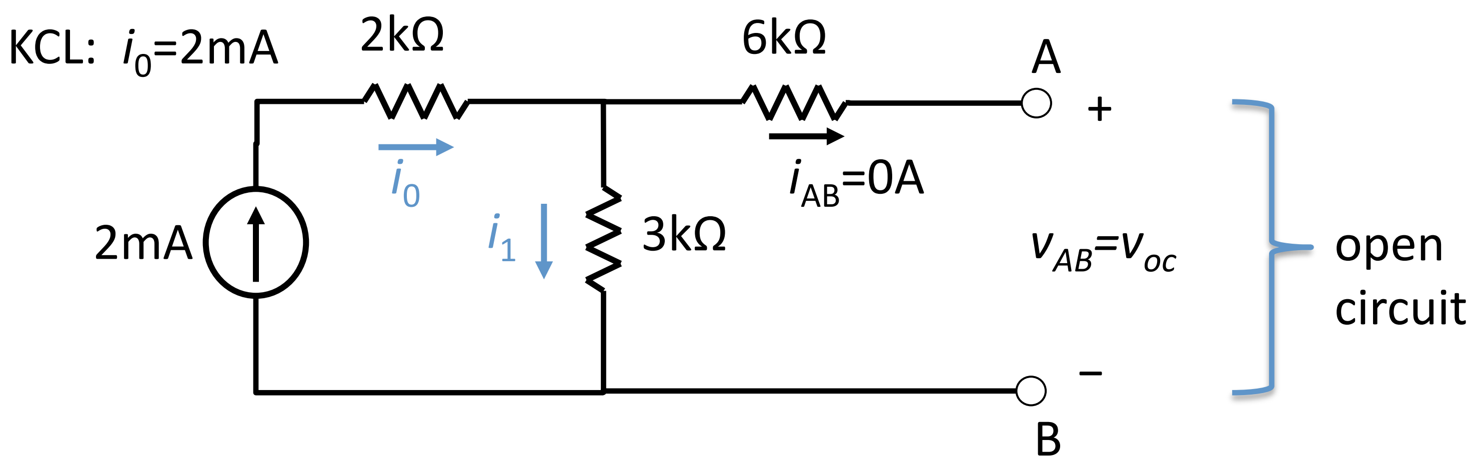
– *find the open circuit voltage*

– *find the Thévenin resistance*

Note that you need to know what to do with a current source in series with a resistor in order to apply source transformations.

Example 1

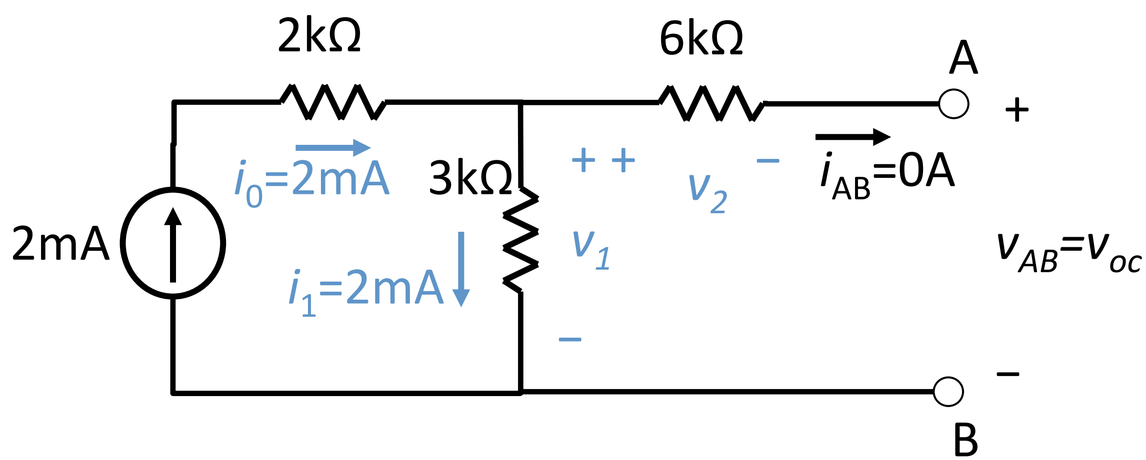
An open circuit is presented to the terminals A and B, and we solve for the open circuit voltage v_{oc}.



KCL: $0 = -i_0 + i_1 + i_{AB}$

$$i_1 = i_0 - i_{AB} = 2\text{mA} - 0\text{A} = 2\text{mA}$$

Example 1



Ohm: $v_1 = i_1\, 3k\Omega = 2mA\, 3k\Omega = 6V$

$v_2 = i_{AB}\, 6k\Omega = 0A\, 6k\Omega = 0V$

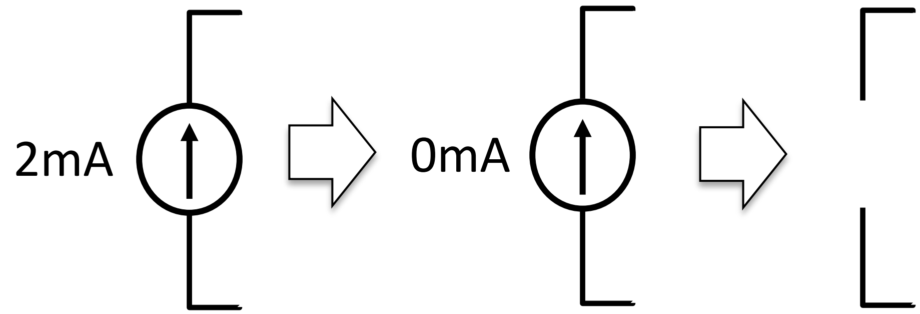
KVL: $0 = -v_1 + v_2 + v_{oc}$

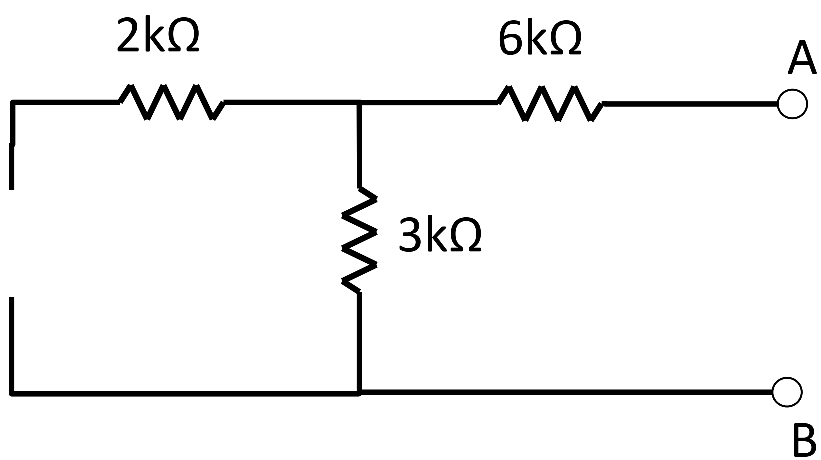
$v_{oc} = v_1 - v_2$

thus: $v_{oc} = v_1 - v_2 = 6V - 0V = 6V$

Example 1

We find the Thévenin resistance, turning off the current source.





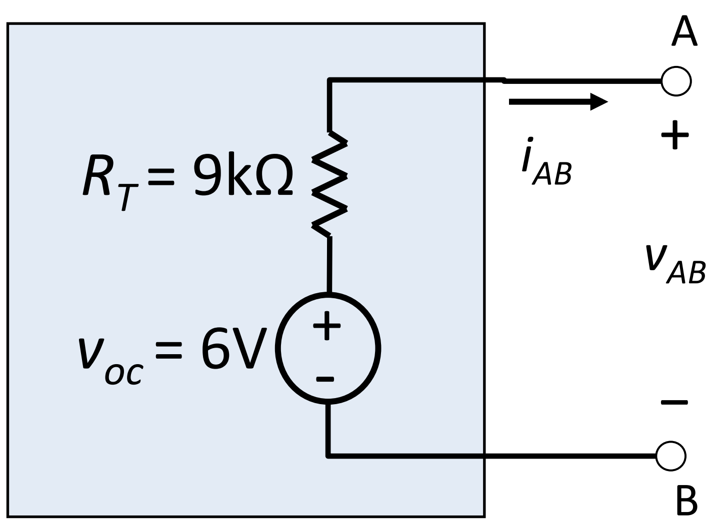
Equivalent resistance seen at AB:

$R_T = 6k\Omega + 3k\Omega = 9k\Omega$

note that the 2kΩ resistor is ignored because no current can flow through it (the 2kΩ is in series with an open = ∞Ω)

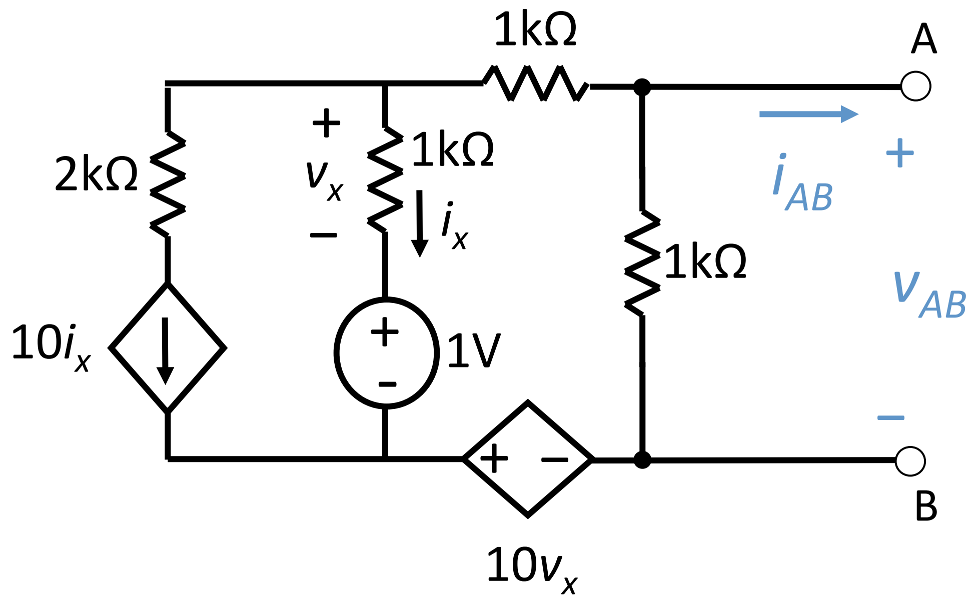
Example 1

The Thévenin equivalent circuit is thus:



Example 2

Find the Thévenin equivalent for the following circuit with respect to the terminals AB.



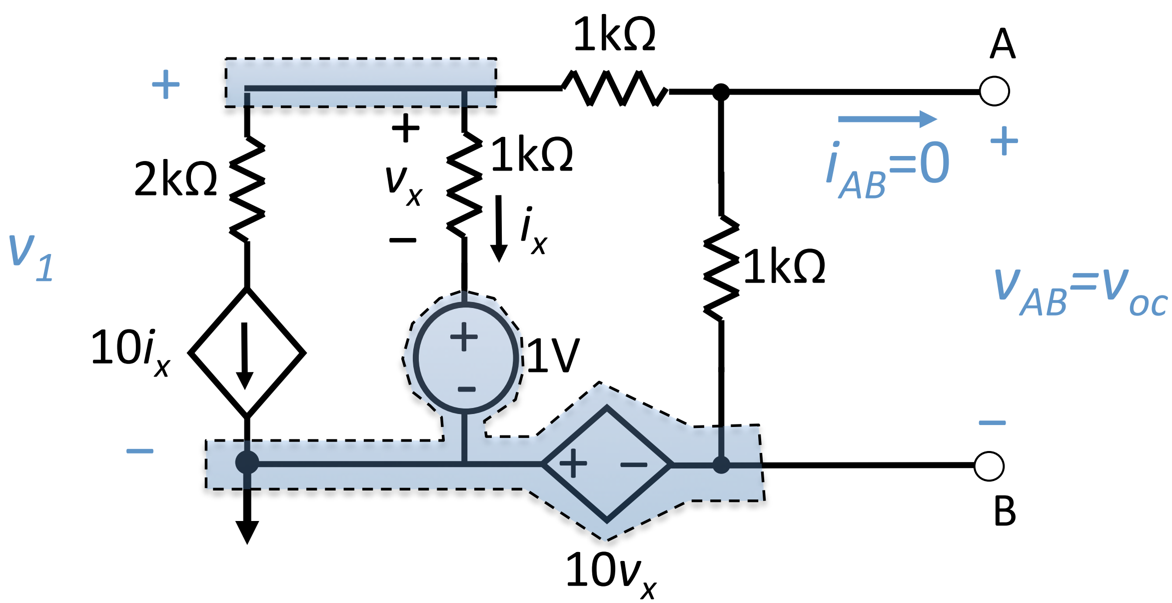
Strategy:

Find the open circuit voltage v_{oc}, the short circuit current i_{sc}, and then $R_T = v_{oc} / i_{sc}$.

If this fails, solve for i_{AB}-v_{AB} equation, or apply a test source to AB.

Example 2

First, apply an open circuit to AB and find the open circuit voltage.



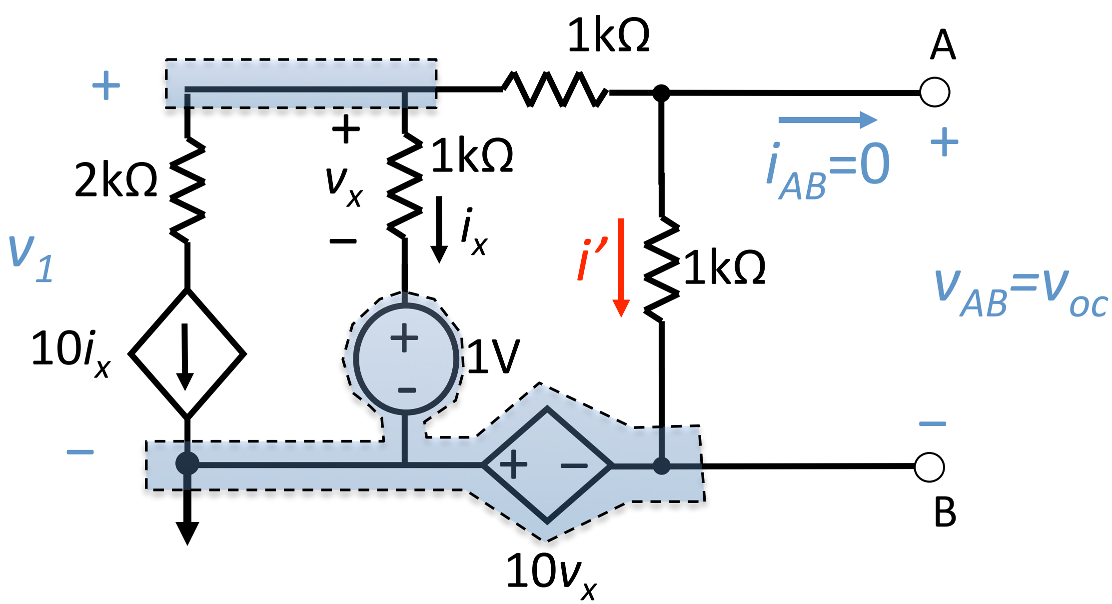
A single node voltage equation can be used, with two control variable equations:

$$0 = 10i_x + \frac{v_1 - 1\text{V}}{1\text{k}\Omega} + \frac{v_1 + 10v_x}{2\text{k}\Omega}$$

$$v_x = v_1 - 1\text{V}$$

$$i_x = (v_1 - 1\text{V}) / 1\text{k}\Omega$$

Example 2



$$0 = 10i_x + \frac{v_1 - 1\text{V}}{1\text{k}\Omega} + \frac{v_1 + 10v_x}{2\text{k}\Omega}$$

$$v_x = v_1 - 1\text{V}$$

$$i_x = (v_1 - 1\text{V}) / 1\text{k}\Omega = v_x / 1\text{k}\Omega$$

Substitution:

$$0 = 10\frac{v_x}{1\text{k}\Omega} + \frac{v_x}{1\text{k}\Omega} + \frac{1\text{V} + 11v_x}{2\text{k}\Omega}$$

$$0 = (10 + 1 + 5.5)v_x + 0.5\text{V}$$

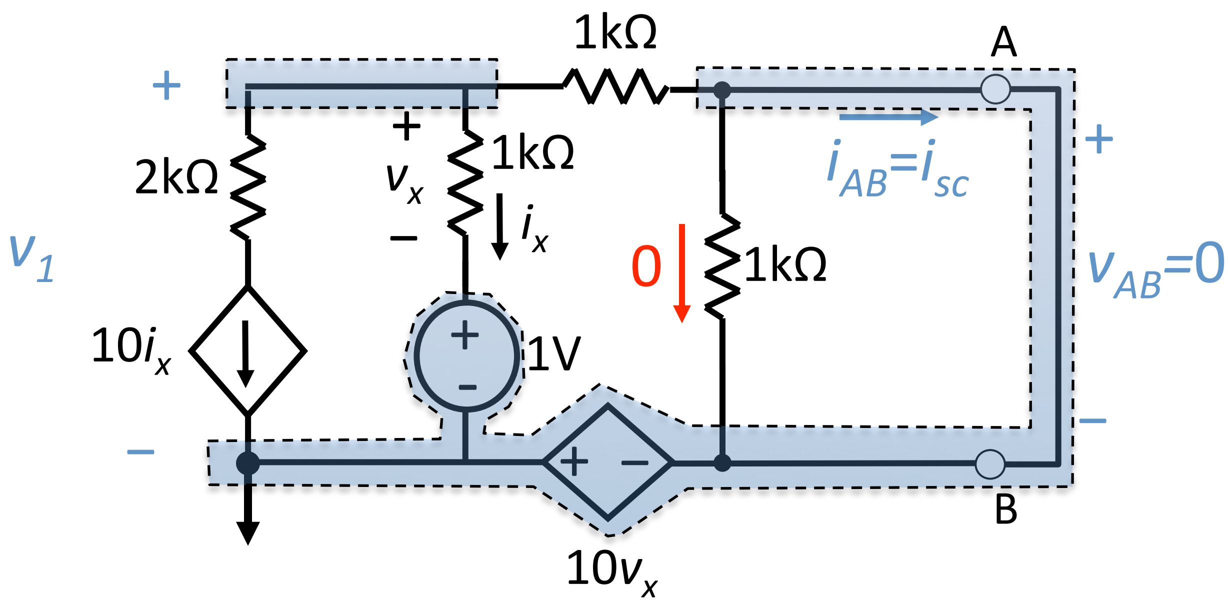
$$v_x = -\frac{1}{33}\text{V}$$

Substitution: $v_1 = v_x + 1\text{V} = \frac{32}{33}\text{V}$

Ohm's law and current i': $v_{oc} = 1\text{k}\Omega\left(\frac{v_1 + 10v_x}{2\text{k}\Omega}\right) = \frac{1}{3}\text{V}$

Example 2

Second, apply a short circuit to AB and find the short circuit current. Note that no current flows through the 1kΩ resistor in parallel with the short (why?).



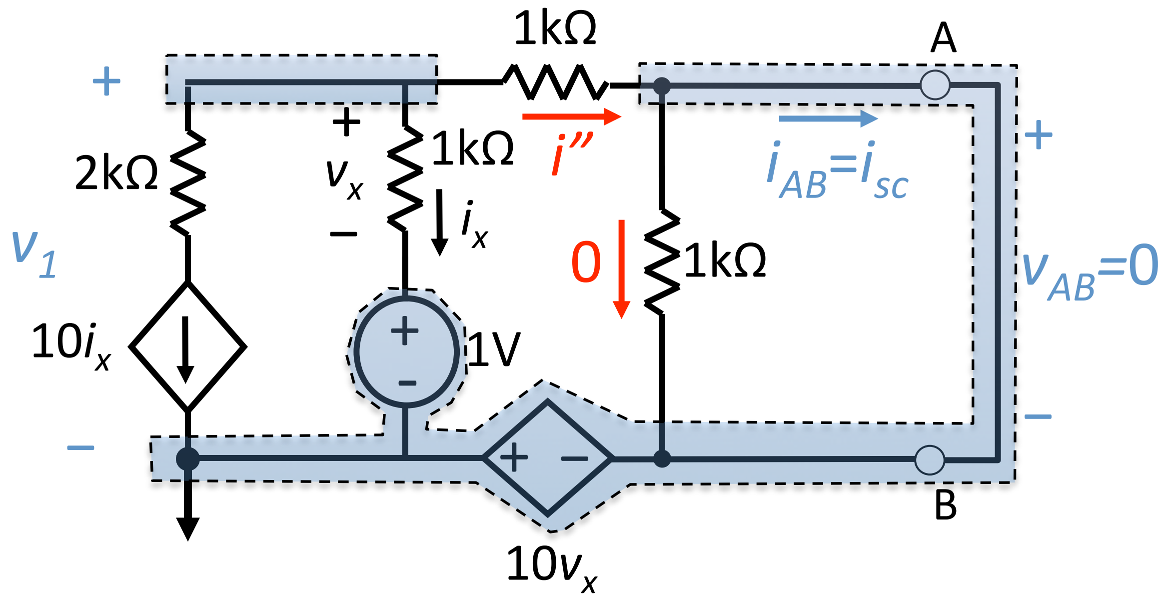
A single node voltage equation can again be used, with two control variable equations:

$$0 = 10i_x + \frac{v_1 - 1\text{V}}{1\text{k}\Omega} + \frac{v_1 + 10v_x}{1\text{k}\Omega}$$

$$v_x = v_1 - 1\text{V}$$

$$i_x = (v_1 - 1\text{V}) / 1\text{k}\Omega$$

Example 2



$$0 = 10i_x + \frac{v_1 - 1\text{V}}{1\text{k}\Omega} + \frac{v_1 + 10v_x}{1\text{k}\Omega}$$

$$v_x = v_1 - 1\text{V}$$

$$i_x = (v_1 - 1\text{V}) / 1\text{k}\Omega = v_x / 1\text{k}\Omega$$

Substitution:

$$0 = 10\frac{v_x}{1\text{k}\Omega} + \frac{v_x}{1\text{k}\Omega} + \frac{1\text{V} + 11v_x}{1\text{k}\Omega}$$

$$0 = (10 + 1 + 11)v_x + 1\text{V}$$

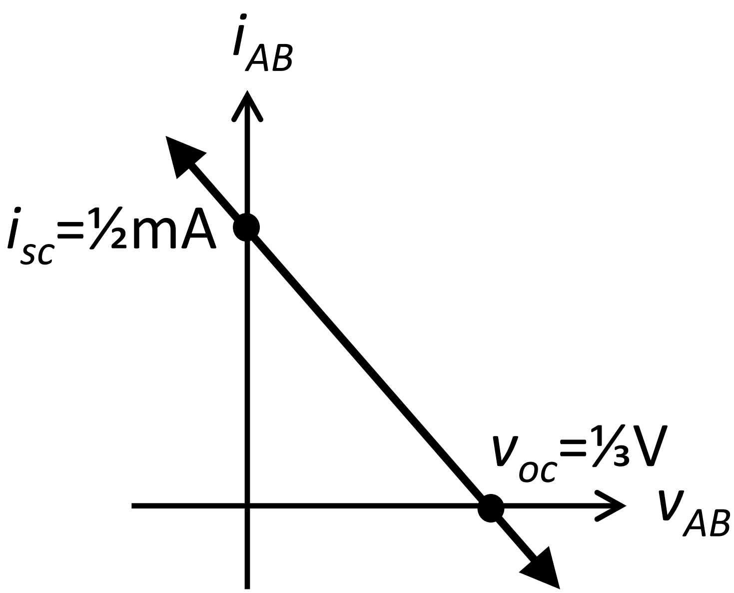
$$v_x = -\frac{1}{22}\text{V}$$

Substitution: $v_1 = v_x + 1\text{V} = \frac{21}{22}\text{V}$

KCL with current i'': $i_{sc} = \frac{v_1 + 10v_x}{1\text{k}\Omega} = \frac{1}{2}\text{mA}$

Example 2

i_{AB}-v_{AB} diagram:

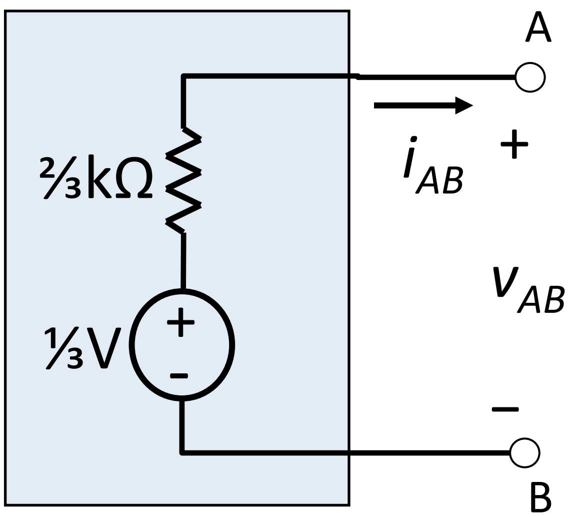


We can thus find the Thévenin resistance:

$$R_T = \frac{v_{oc}}{i_{sc}} = \frac{\frac{1}{3}\text{V}}{\frac{1}{2}\text{mA}} = \frac{2}{3}\text{k}\Omega$$

Example 2

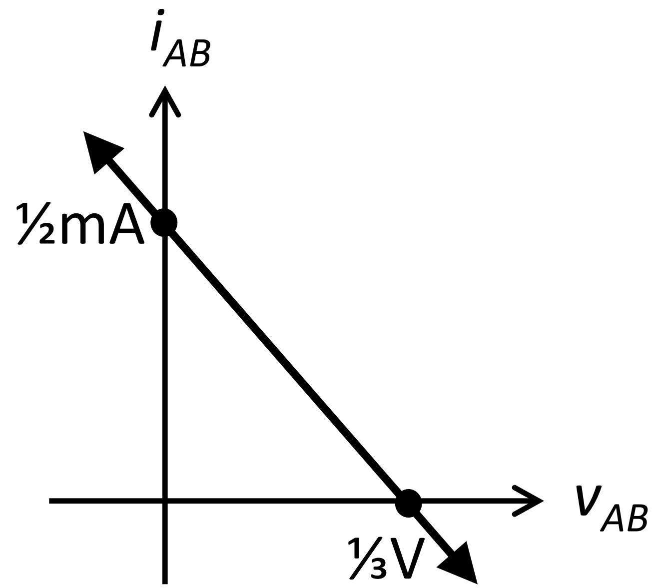
Thévenin equivalent:



i_{AB}-v_{AB} terminal equation:

$$v_{AB} = \frac{1}{3}\text{V} - \frac{2}{3}\text{k}\Omega i_{AB}$$

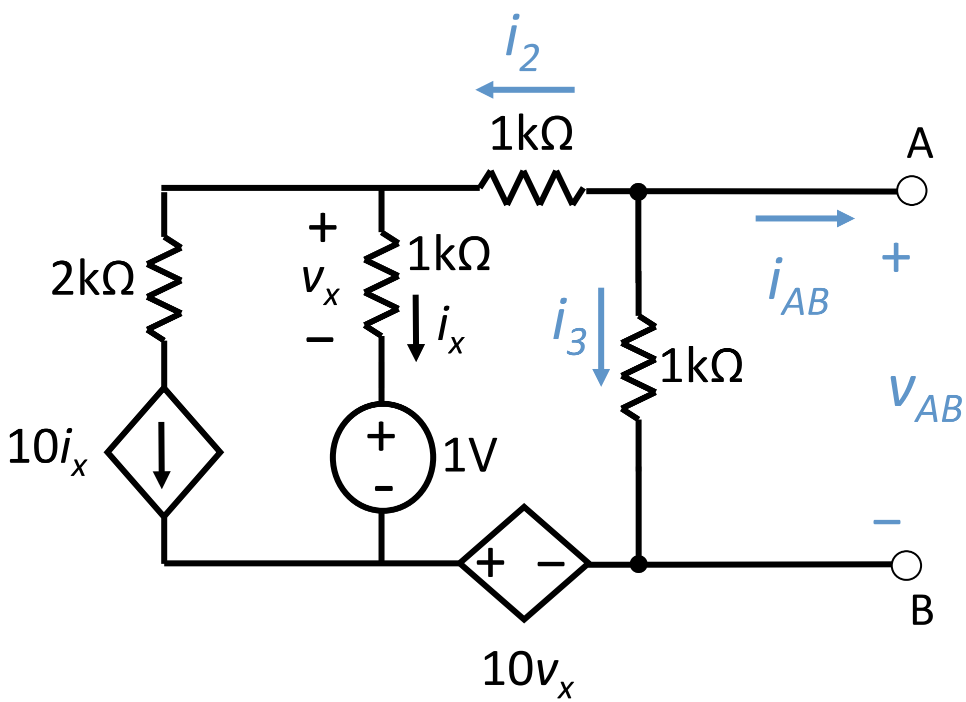
i_{AB}-v_{AB} diagram:



IMPORTANT: We see that the Thévenin equivalent circuit is just a compact way to represent a linear two-terminal circuit (ie. a circuit with a linear i_{AB}-v_{AB} diagram), no matter the complexity.

Example 2 (alternative solution)

Alternatively, we could solve for the i_{AB}-v_{AB} diagram directly, making no assumptions about i_{AB} and v_{AB} in the process.



KCL:

$$0 = -i_2 + i_x + 10i_x$$

$$i_2 = 11i_x$$

KCL:

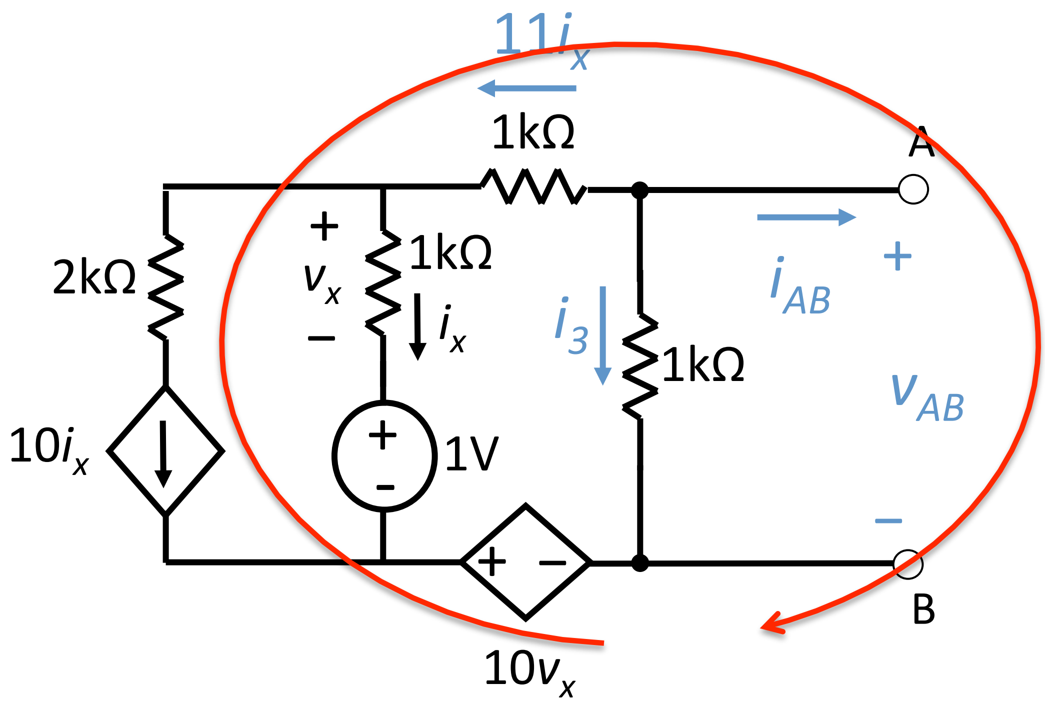
$$0 = i_2 + i_3 + i_{AB}$$

Ohm: $i_3 = v_{AB} / 1\text{k}\Omega$

thus: $0 = 11i_x + v_{AB} / 1\text{k}\Omega + i_{AB}$

$$i_x = \frac{-i_{AB}}{11} - \frac{v_{AB}}{11\text{k}\Omega}$$

Example 2 (alternative solution)



KVL + Ohm:

$$0 = -10v_x - 1\text{V} - v_x - 11i_x \times 1\text{k}\Omega + v_{AB}$$

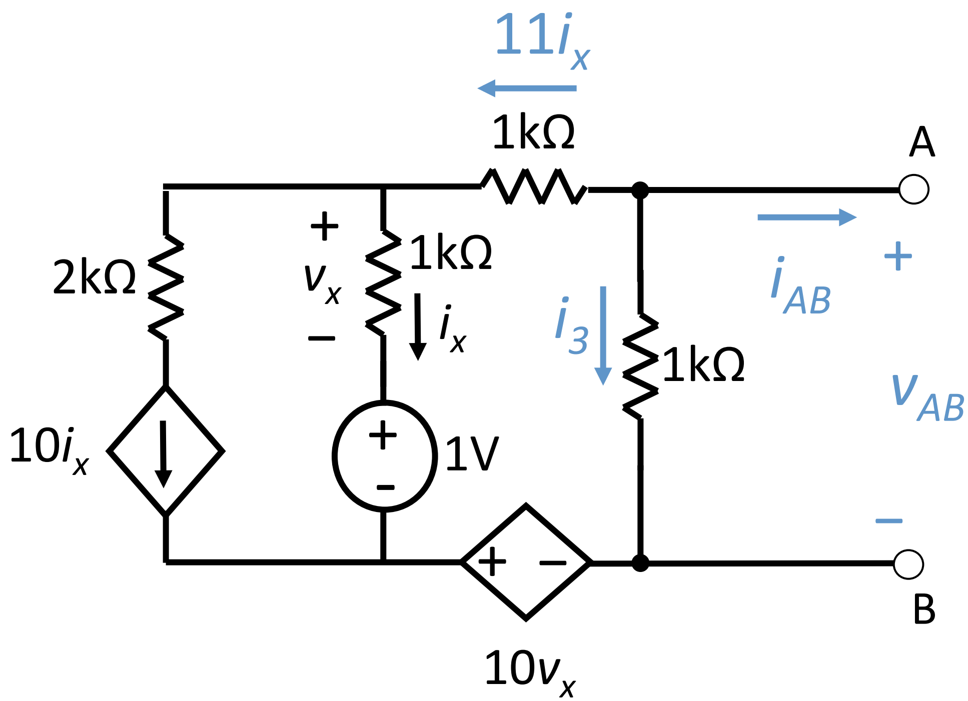
Ohm: $v_x = i_x \times 1\text{k}\Omega$

thus:

$$0 = -10\text{k}\Omega\, i_x - 1\text{V} - 1\text{k}\Omega\, i_x - 11\text{k}\Omega i_x + v_{AB}$$

$$i_x = \frac{v_{AB} - 1\text{V}}{22\text{k}\Omega}$$

Example 2 (alternative solution)



We thus have the two expressions for i_x:

$$i_x = \frac{-i_{AB}}{11} - \frac{v_{AB}}{11\text{k}\Omega}$$

$$i_x = \frac{v_{AB} - 1\text{V}}{22\text{k}\Omega}$$

Combining these:

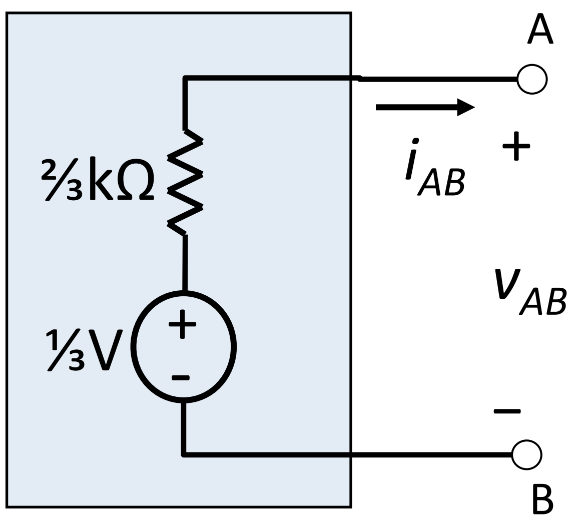
$$\frac{v_{AB} - 1\text{V}}{22\text{k}\Omega} = \frac{-i_{AB}}{11} - \frac{v_{AB}}{11\text{k}\Omega}$$

$$v_{AB} - 1\text{V} = -2\text{k}\Omega i_{AB} - 2v_{AB}$$

$$v_{AB} = \frac{1}{3}\text{V} - \frac{2}{3}\text{k}\Omega i_{AB}$$

Example 2 (alternative solution)

Thévenin equivalent:



i_{AB}-v_{AB} terminal equation:

$$v_{AB} = \frac{1}{3}\text{V} - \frac{2}{3}\text{k}\Omega\, i_{AB}$$

i_{AB}-v_{AB} diagram: