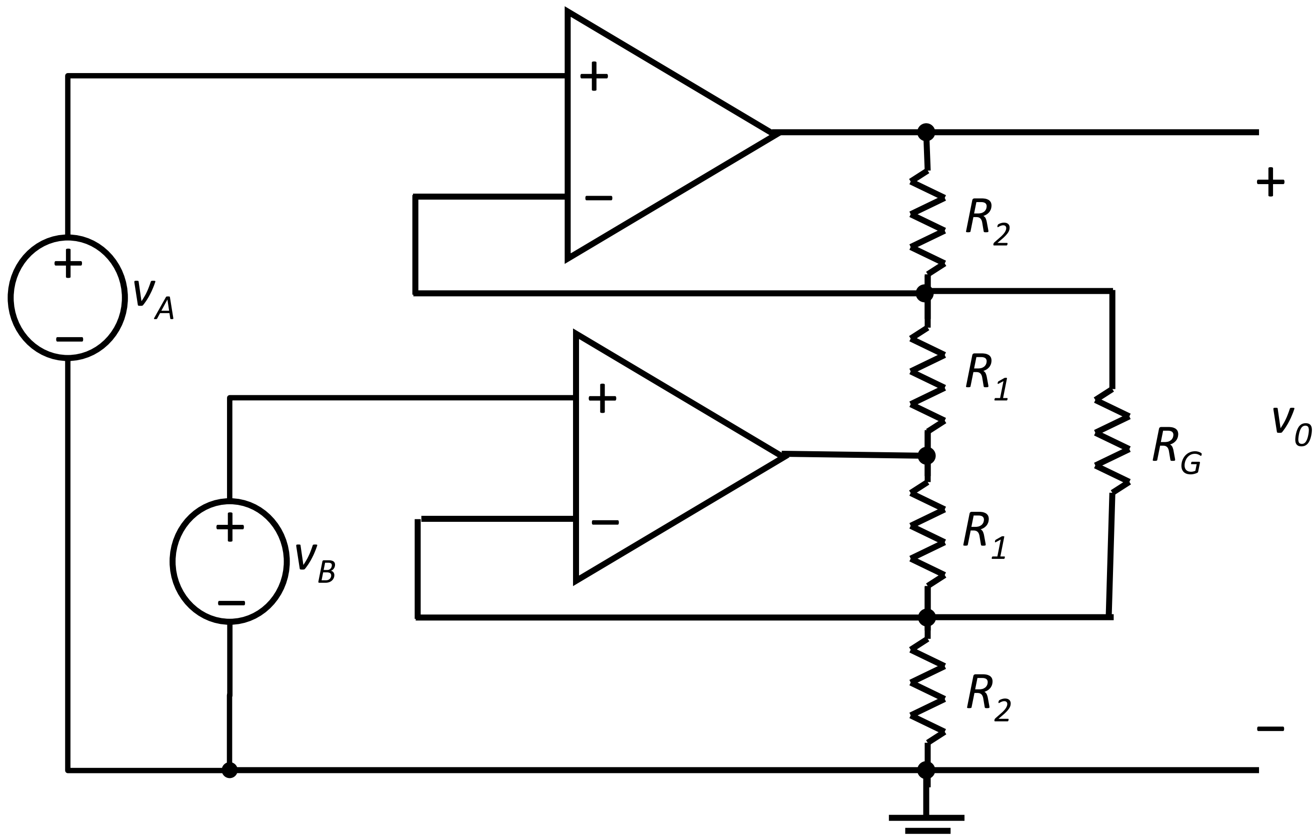
# Today's Outline

## 5. Operational Amplifiers

- Op-Amp Circuits
- Practical Op-Amp Model
- Negative Feedback (revisited)

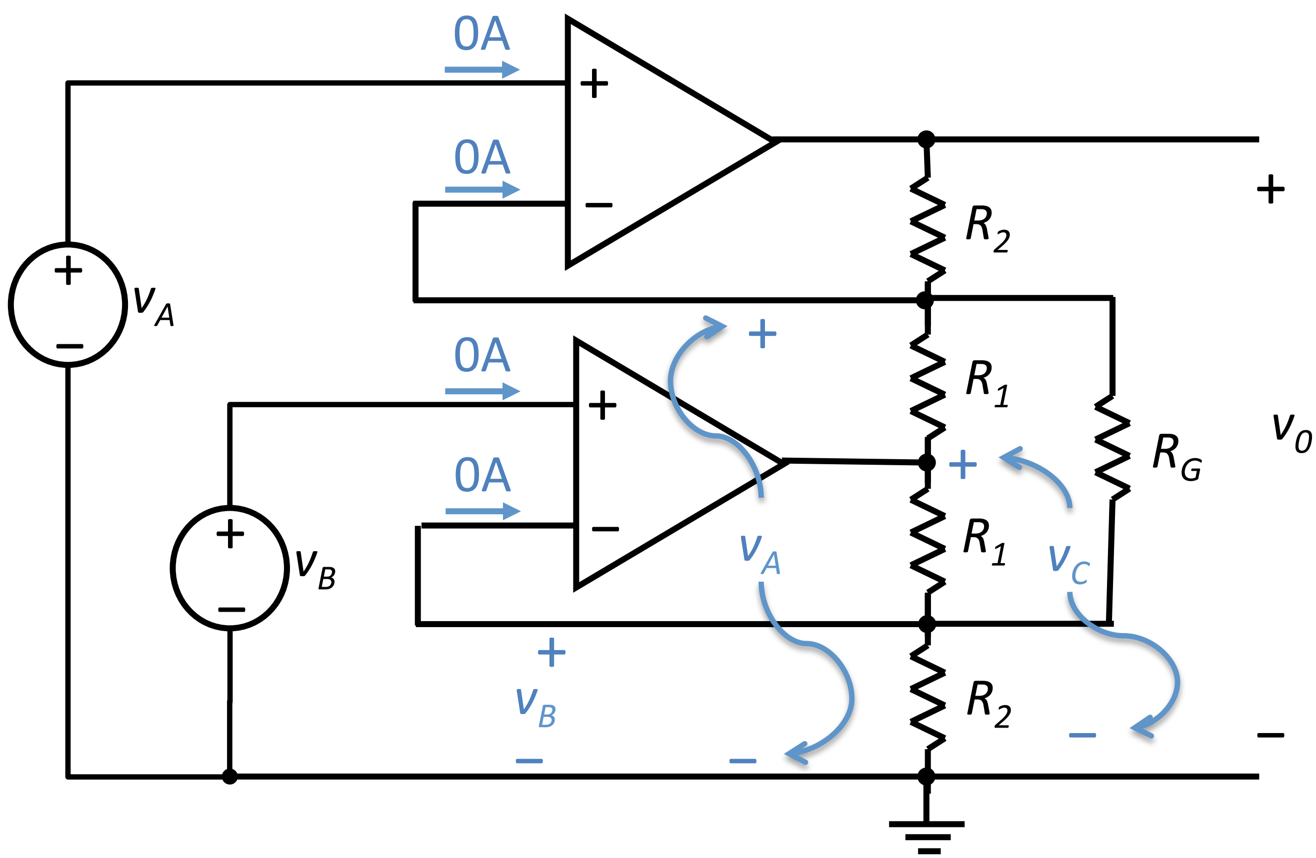
# Example 8

Assuming ideal op-amp behaviour, what is the relationship between the output $v_0$ and the inputs $v_A$ and $v_B$?

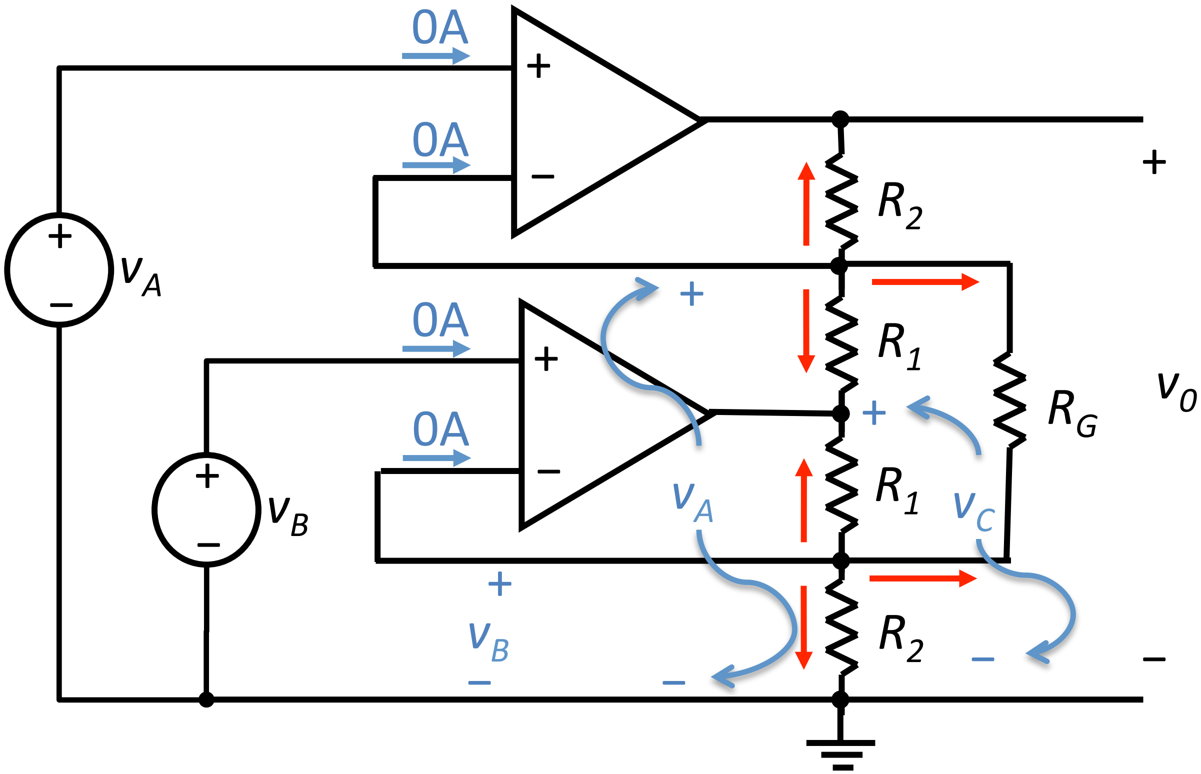


# Example 8

Apply ideal op-amp equations.



# Example 8

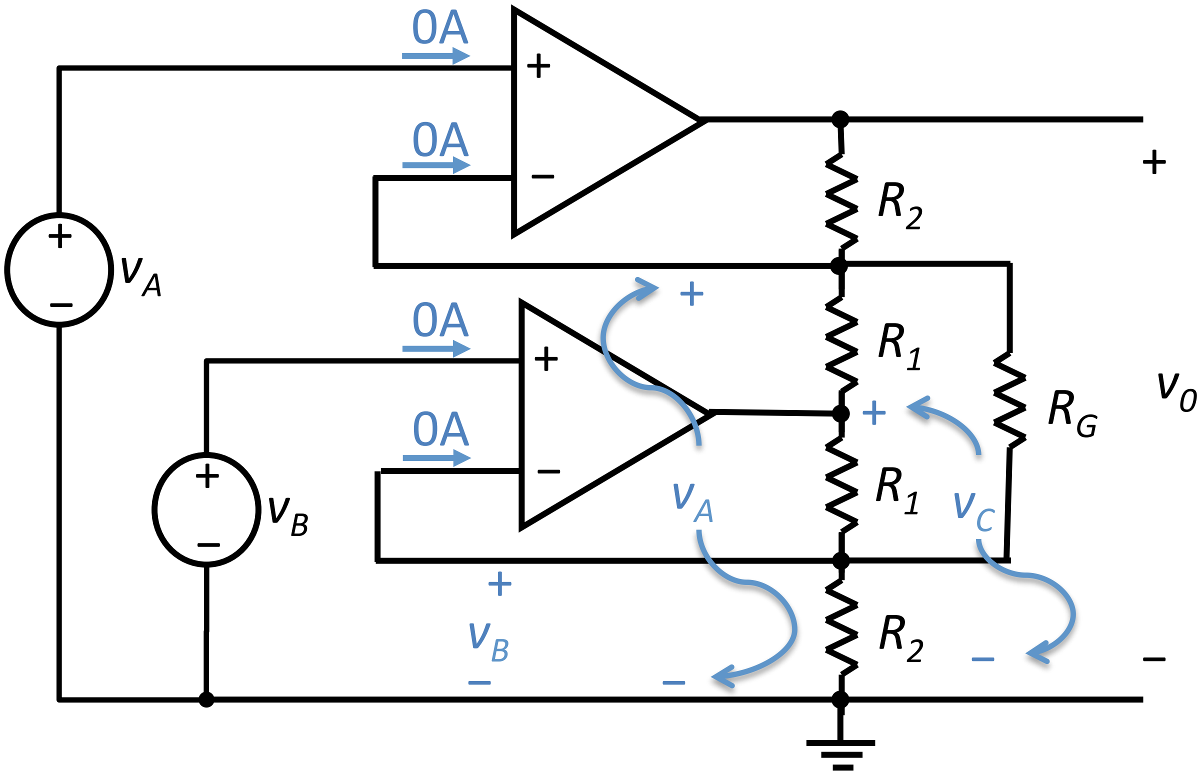



Write out node-voltage equations:

$$B: \quad 0 = \frac{v_B}{R_2} + \frac{v_B - v_A}{R_G} + \frac{v_B - v_C}{R_1} \quad \rightarrow \quad v_C = R_1\left(\frac{v_B}{R_2} + \frac{v_B - v_A}{R_G} + \frac{v_B}{R_1}\right)$$

$$A: \quad 0 = \frac{v_A - v_0}{R_2} + \frac{v_A - v_B}{R_G} + \frac{v_A - v_C}{R_1} \quad \rightarrow \quad v_0 = R_2\left(\frac{v_A}{R_2} + \frac{v_A - v_B}{R_G} + \frac{v_A - v_C}{R_1}\right)$$

# Example 8

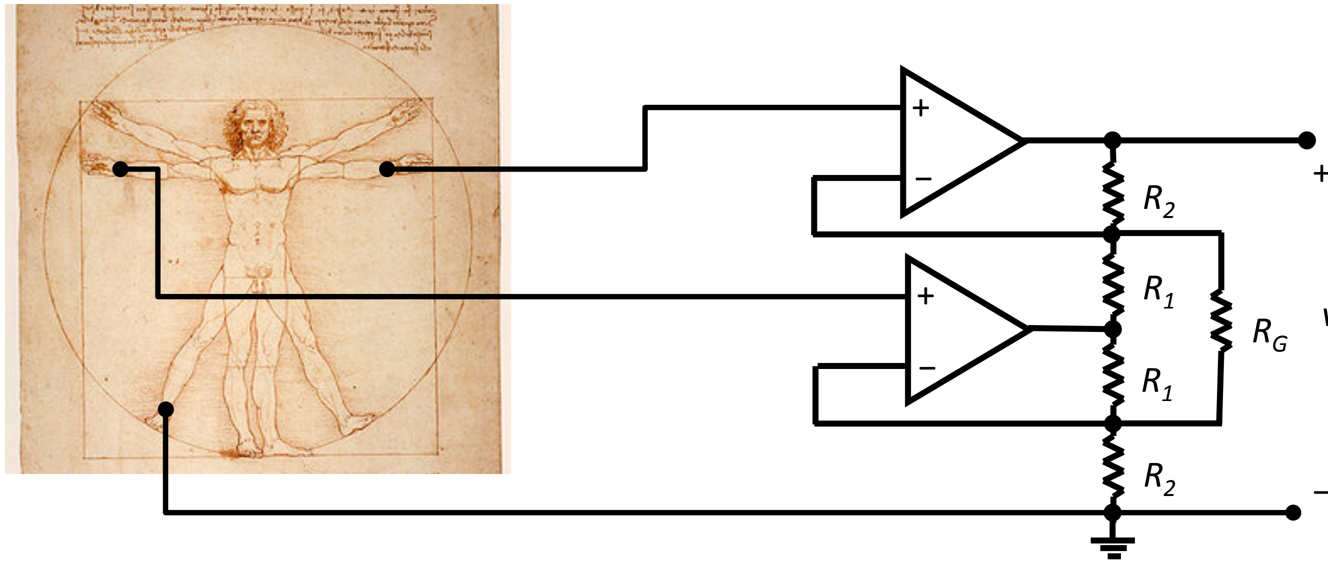


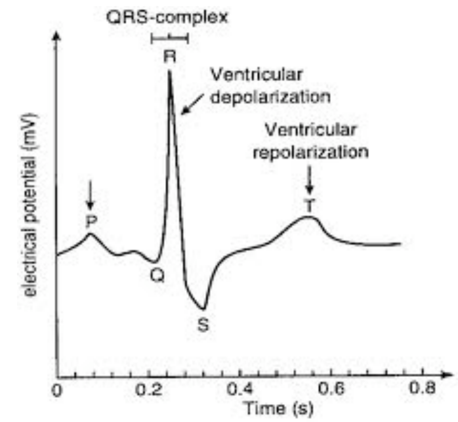

Substitution gives:

$$v_0 = \left(v_A - v_B\right)\left(1 + \frac{R_2}{R_1} + 2\frac{R_2}{R_G}\right)$$

This circuit is a ***precision differential voltage amplifier***, with the output being proportional to the difference in input voltages.

# McGill Application: Electrocardiography



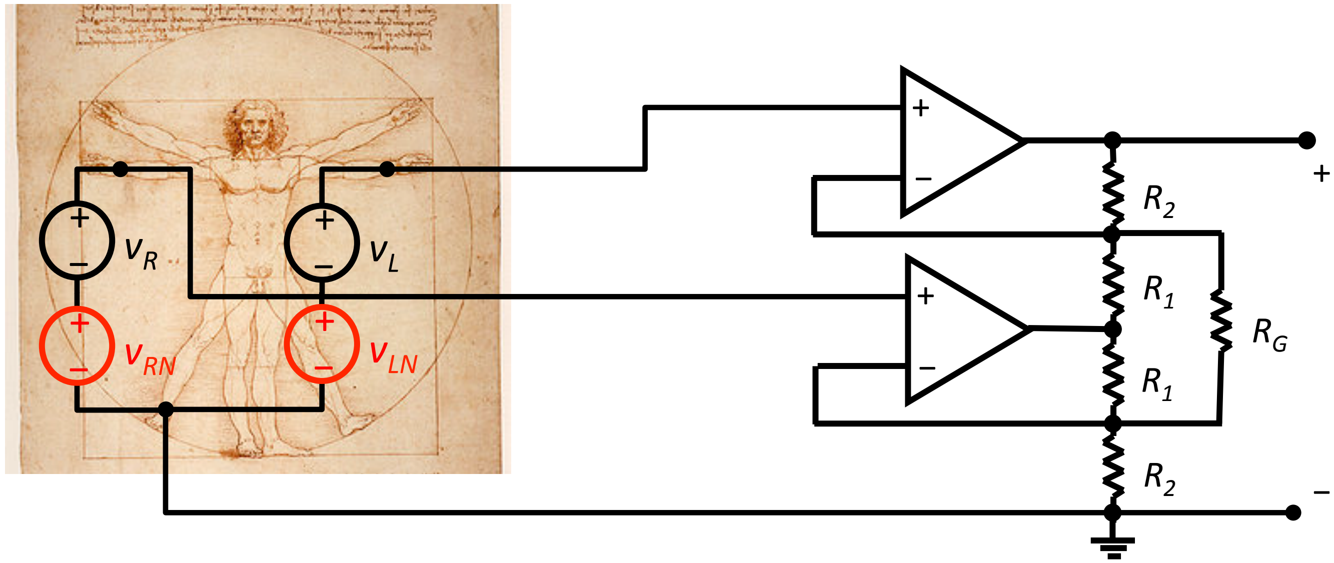




A simplified electrocardiogram (ECG or EKG) setup.

The heart generates an electric potential (voltage) across the arms, which can be measured and used to diagnose disorders. There are many other electrode configurations possible.

Willem Einthoven
1860-1927

# Application: Electrocardiography




$v_0 = (\, v_L + \color{red}{v_{LN}}\,) - (\, v_R + \color{red}{v_{RN}}\,)$

$= (\, v_L - v_R\,) + \color{red}{(\, v_{LN} - v_{RN}\,)}$

$v_L$ = left arm signal

$v_{LN}$ = left arm noise

$v_R$ = right arm signal

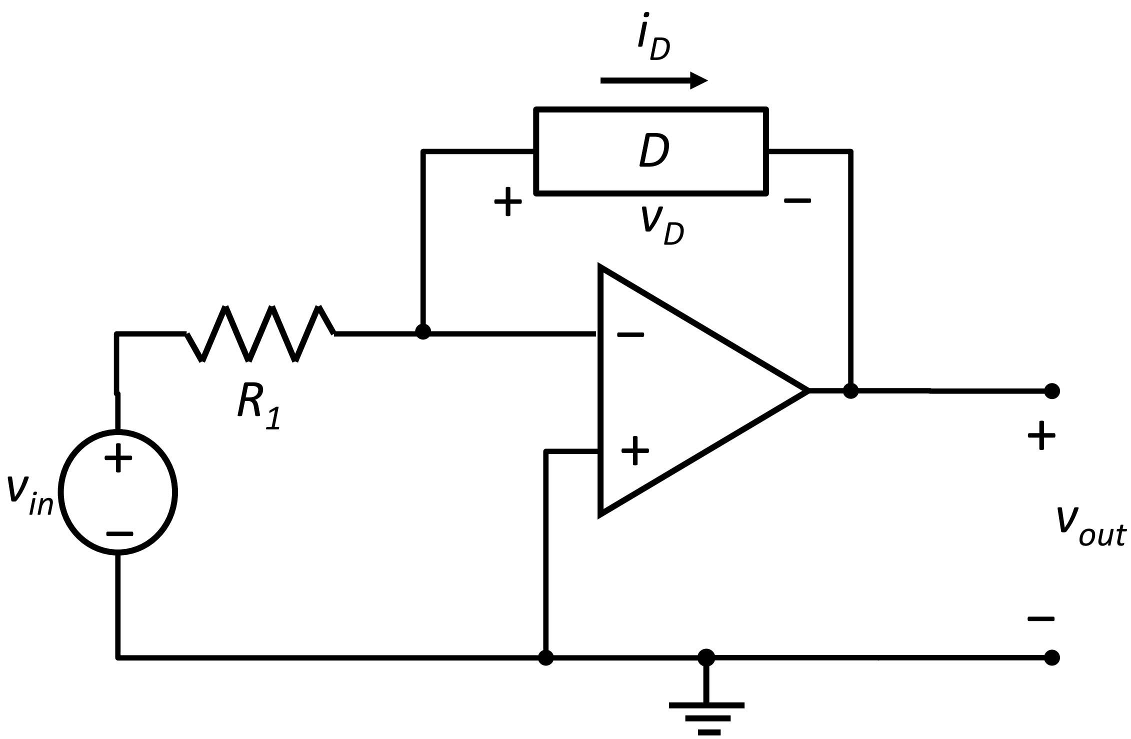
$v_{RN}$ = right arm noise

The output $v_0$ includes a differential signal $v_L - v_R$ .

The output $v_0$ also includes the ***differential noise*** $v_{LN} - v_{RN}$ . The average noise that is common to both electrodes $\frac{1}{2}(v_{LN}+v_{RN})$, called ***common mode noise***, is rejected by the differential amplifier.

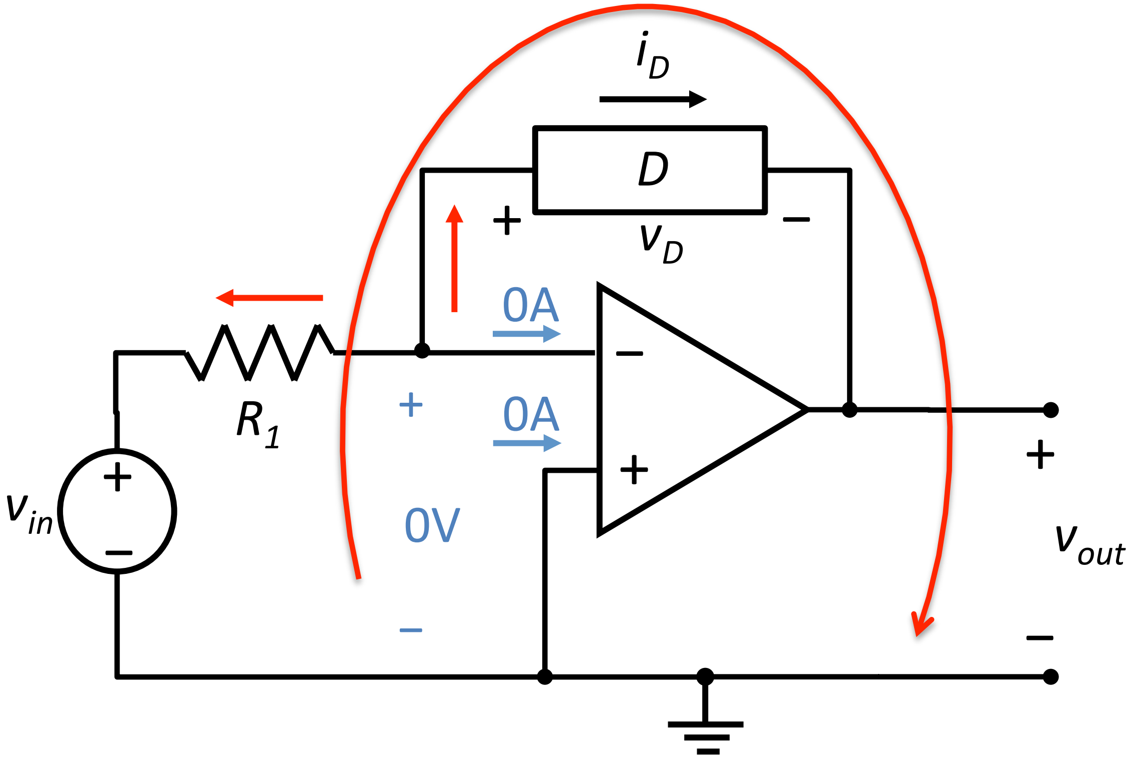
# Example 9

Assume the circuit element D terminal variables follow the relationship $v_D$=F($i_D$). How is $v_{out}$ related to $v_{in}$?



# Example 9

Apply ideal op-amp equations, write out node equation and KVL equation.




$$node: \quad 0 = \frac{0 - v_{in}}{R_1} + i_D$$

$$i_D = \frac{v_{in}}{R_1}$$

$$v_D = F(i_D) = F\left(\frac{v_{in}}{R_1}\right)$$

$$KVL: \quad 0 = v_D + v_{out}$$

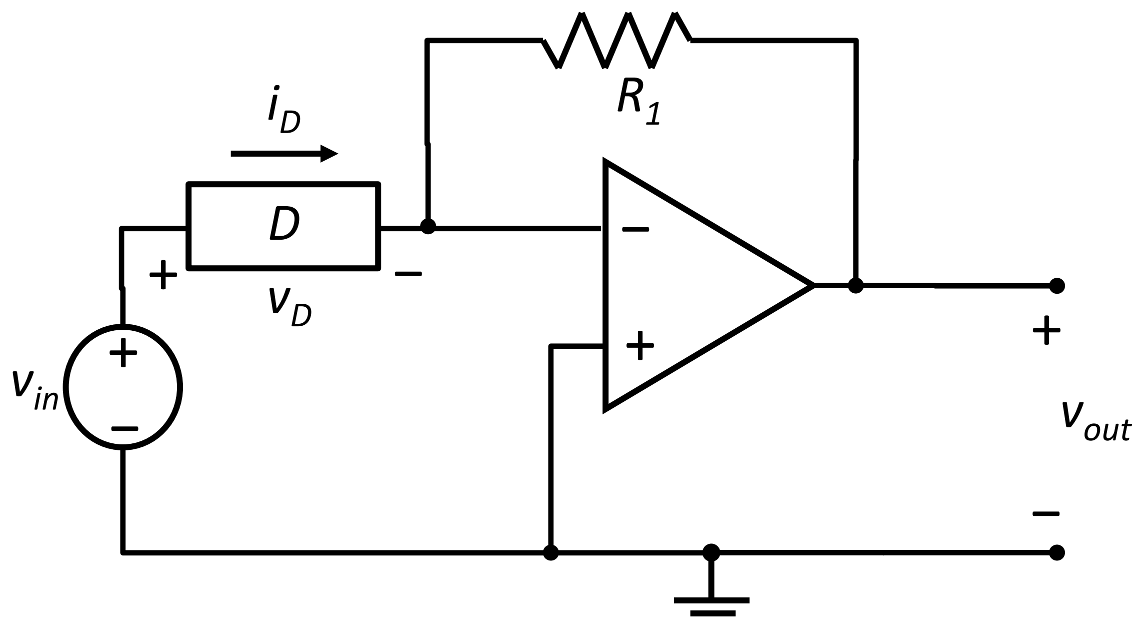
$$\boxed{v_{out} = -F\left(\frac{v_{in}}{R_1}\right)}$$

This configuration can be very useful. If *D* is a ***diode***, then the circuit can take logarithms:

$$i_D \approx 1\text{nA} \cdot \exp\left(v_D / 26\text{mV}\right) \quad \rightarrow \quad v_D \approx 26\text{mV} \cdot \ln\left[\frac{i_D}{1\text{nA}}\right] \quad \rightarrow \quad v_{out} = -26\text{mV} \cdot \ln\left[\frac{v_{in}}{1\text{nA} \cdot R_1}\right]$$

# Example 10

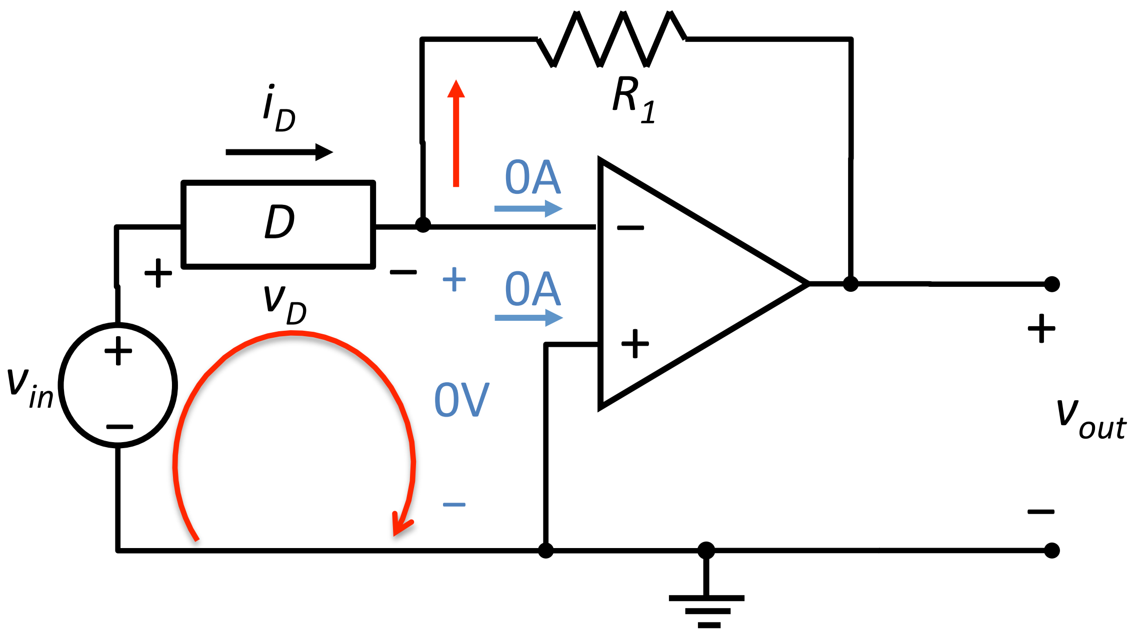
Assume the circuit element D terminal variables follow the relationship $i_D$=G($v_D$). How is $v_{out}$ related to $v_{in}$?



McGill

# Example 10

Apply ideal op-amp equations, write out KVL equation and node equation.




$$KVL: \quad 0 = -v_{in} + v_D$$

$$v_D = v_{in}$$

$$node: \quad 0 = -i_D + \frac{0 - v_{out}}{R_1}$$

$$v_{out} = -R_1 i_D$$

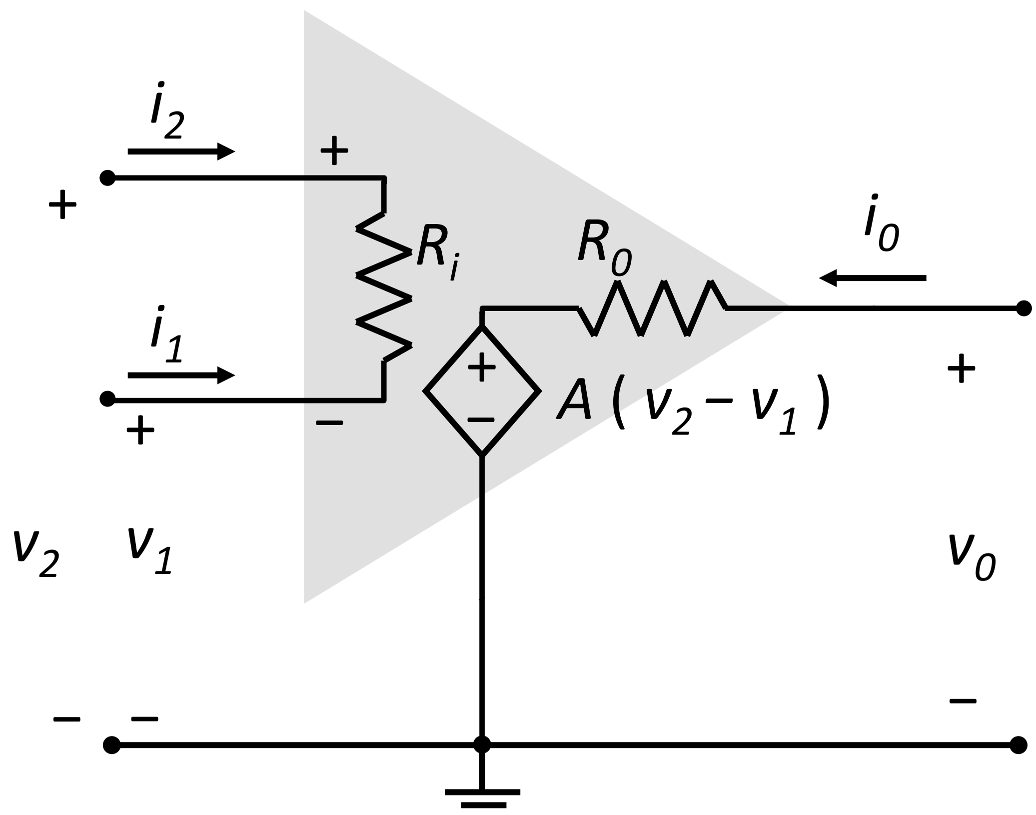
$$\boxed{v_{out} = -R_1 G(v_{in})}$$

This configuration can be very useful. If *D* is a ***diode***, then the circuit can take exponentials:

$$i_D \approx 1\text{nA} \cdot \exp(v_D / 26\text{mV}) \quad \rightarrow \quad v_{out} = -R_1 \cdot 1\text{nA} \cdot \exp(v_D / 26\text{mV})$$

# "Practical" Op-Amp Model

**Practical Op-Amp Circuit Model:** A circuit model for the op-amp that consists of an **input resistance** $R_i$, an **output resistance** $R_0$, and an **open-loop gain** $A$.




**Note:**

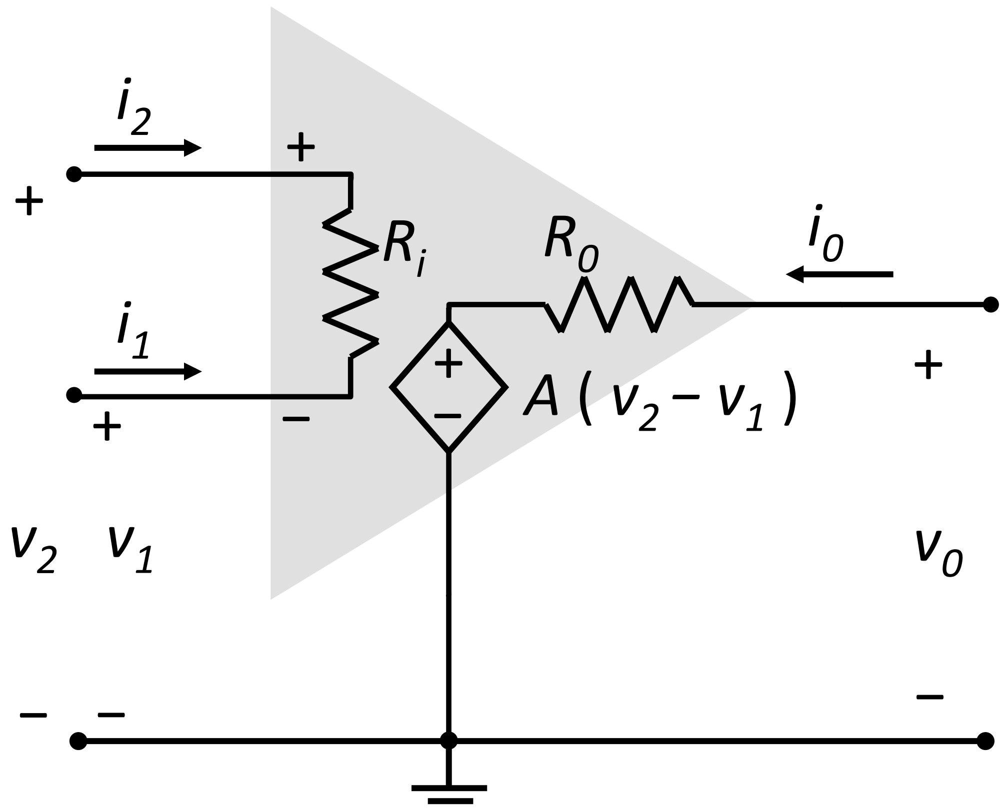
- the voltage across the source is proportional to the difference $v_2 - v_1$
- the connection to the reference terminal is explicit
- the theorems of section 4 can be applied to op-amp circuits

# "Practical" Op-Amp Model

Op-amp model values are often in the range:

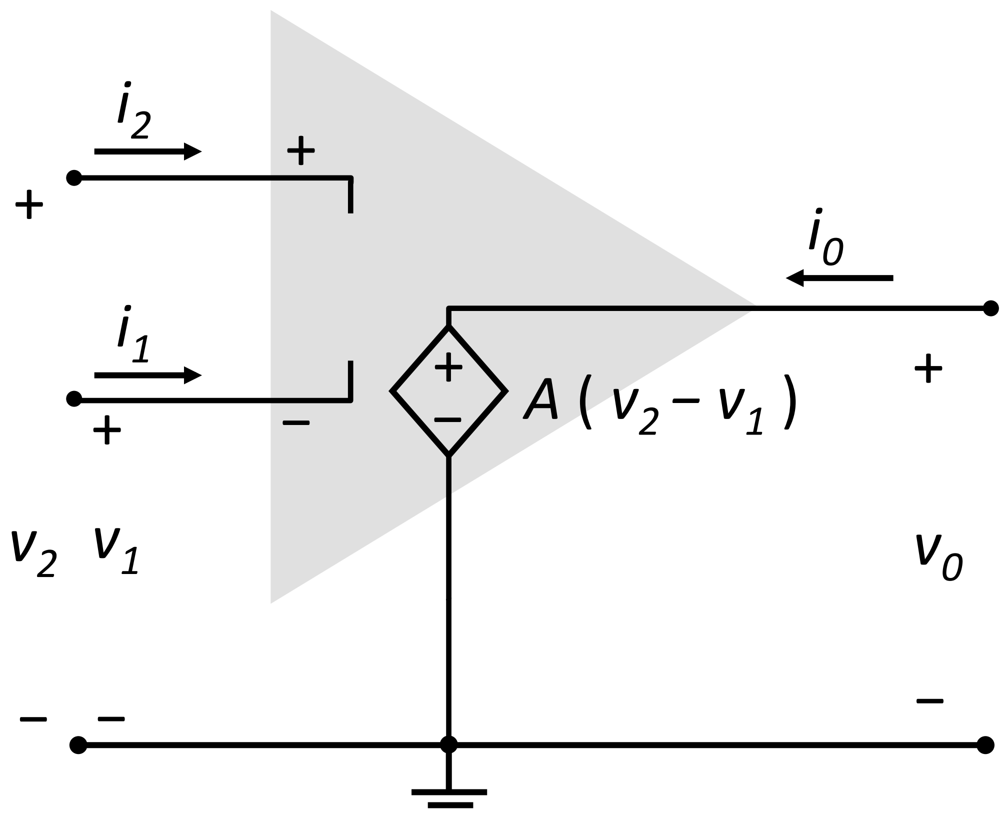
- input resistance $R_i$ = 100kΩ to 1TΩ ($10^{12}$Ω)
- output resistance $R_0$ = 1Ω to 50Ω
- open loop gain $A$ = $10^5$V/V to $10^7$V/V

*this suggests a simpler model…*



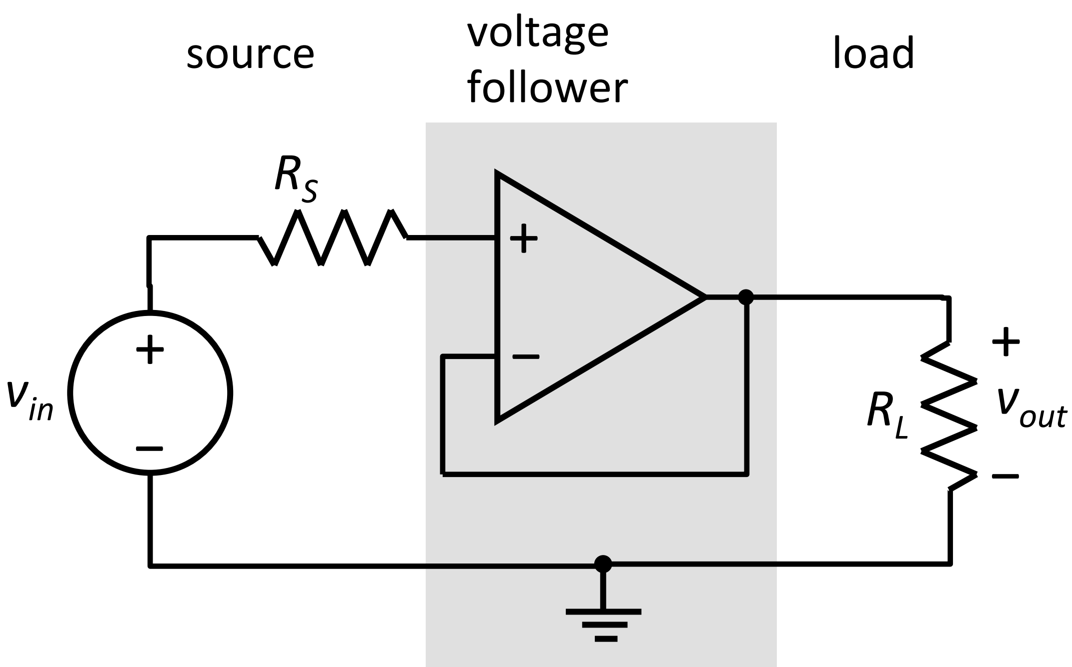
# Finite Gain Op-Amp Model

A simpler circuit model arises in the limit:
$$R_i \rightarrow \infty$$
$$R_0 \rightarrow 0$$



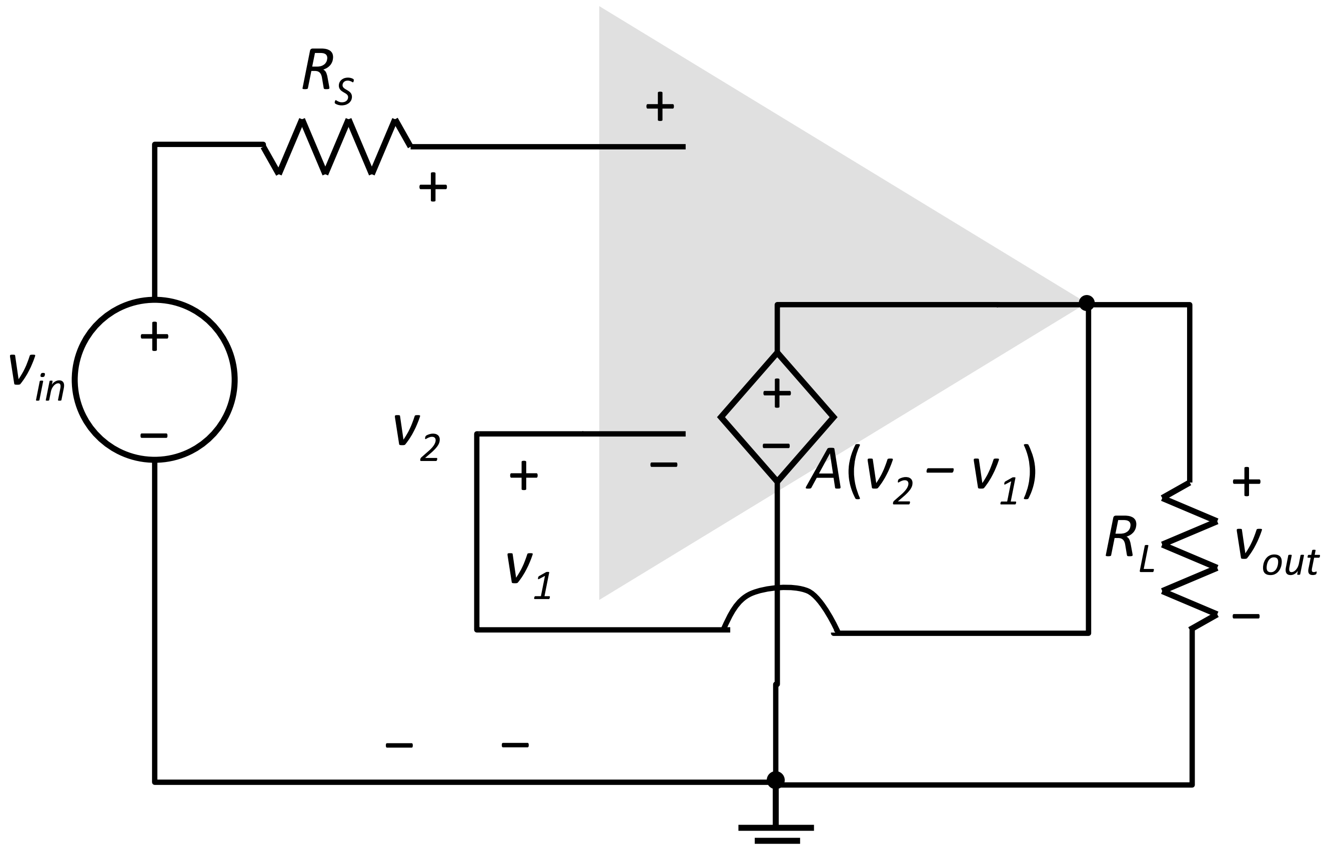
# Example

Find the ratio $v_{out}$ / $v_{in}$, assuming a finite gain op-amp model.



# Example

Replace the op-amp with the finite gain model.



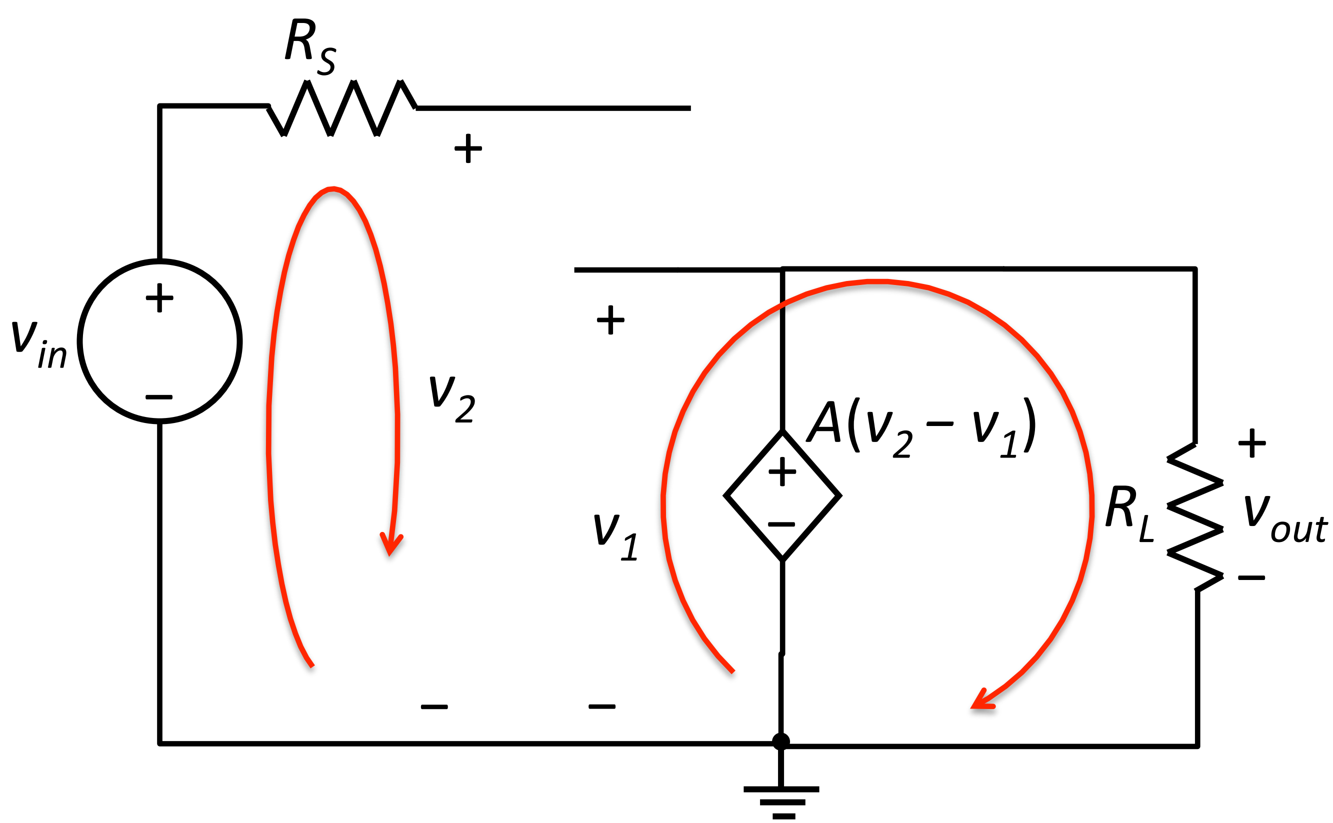
# Example

Redraw the circuit. KVL gives: $0 = -v_{in} + 0 \times R_S + v_2$ thus: $v_2 = v_{in}$

$v_{out} = v_1 = A\,(v_2 - v_1)$ thus: $v_1(A+1) = Av_2$

$$v_1 = \frac{A}{A+1} v_2$$

$$v_{out} = \frac{A}{A+1} v_{in}$$

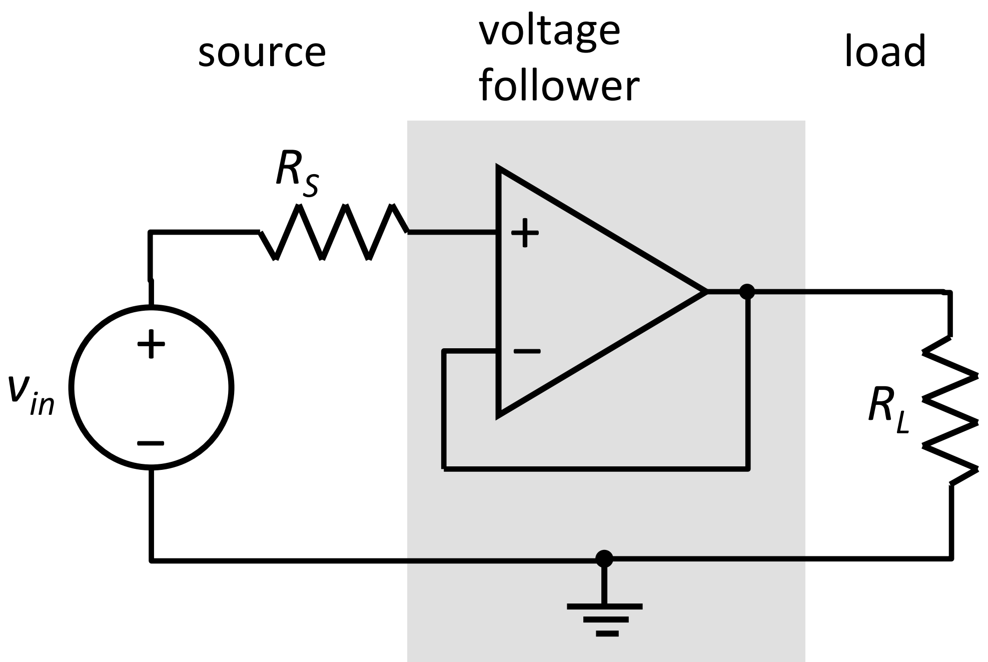


# Example

For "small" open-loop gain *A*, the circuit output is explicitly dependent upon the value of *A*.

If A >> 1, the circuit output becomes independent of the open-loop gain A, and behaves as predicted by the ideal op-amp model.

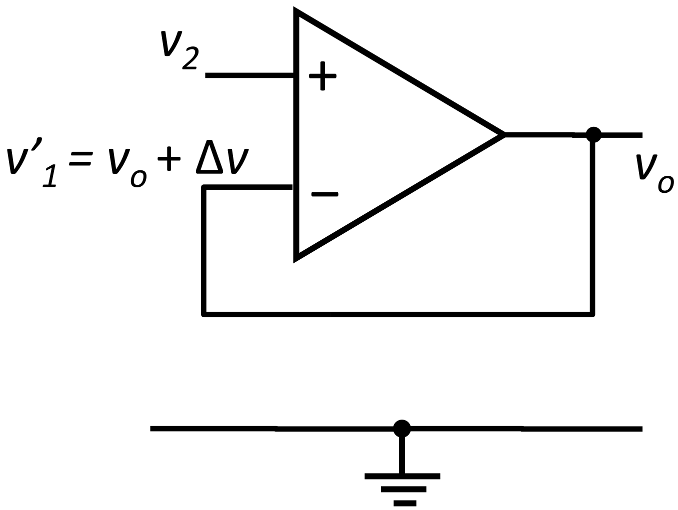
***Note:*** *So long as A is large enough, circuit behaviour is relatively insensitive to variation in A (due to temperature, age, manufacturer, unit to unit variability). This is one reason why we use op-amp circuits with a negative feedback loop.*

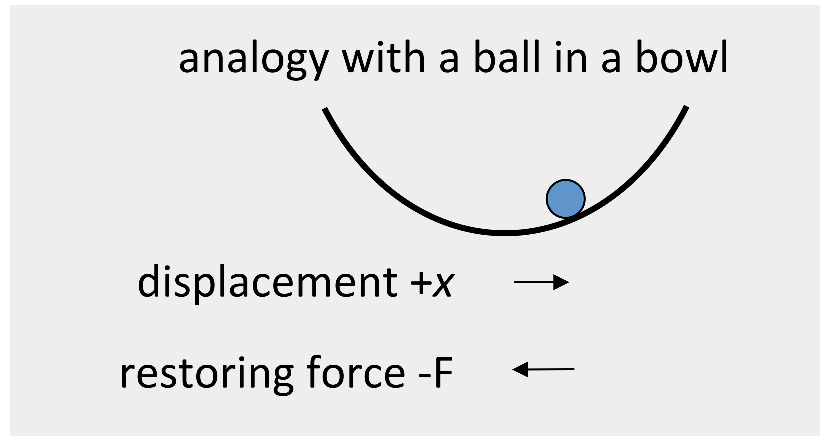



$$v_{out} = \frac{Av_{in}}{A+1} \rightarrow v_{in} \quad \text{as} \quad A \rightarrow \infty$$

# Negative Feedback

The negative feedback loop is **stable** against output fluctuations.
Consider the voltage follower as an example:




“displacement” $v_1 \rightarrow v_1' = v_0 + \Delta v$

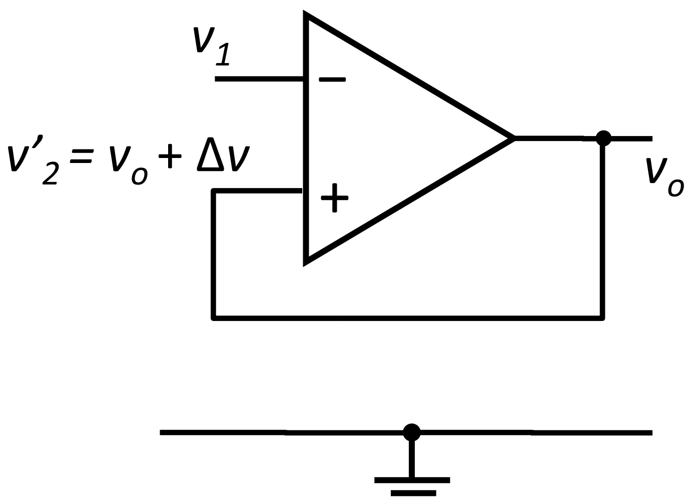
“restoring force” $v_0 \rightarrow v_0' = A\left[v_2 - v_1'\right] = A\left[v_2 - (v_1 + \Delta v)\right] = v_0 - A\Delta v$

$$v_1' \rightarrow v_1'' = v_1' - A\Delta v$$

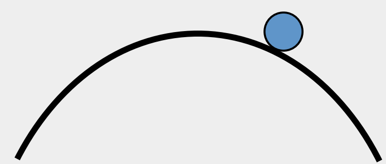
A rigorous analysis is required for quantitative understanding of stability, but it is clear negative feedback suppresses fluctuations.

# Positive Feedback

In contrast, a **positive feedback loop** is **unstable** against output fluctuations (just like audio feedback into a microphone). The following circuit is unstable and difficult to control (*take careful note of input terminal labels!*)




analogy with a ball on a bowl



displacement $+x$ $\longrightarrow$

reinforcing force +F $\longrightarrow$

"displacement" $v_2 \rightarrow v'_2 = v_0 + \Delta v$

"reinforcing force" $v_0 \rightarrow v'_0 = A\left[v'_2 - v_1\right] = A\left[(v_2 + \Delta v) - v_1\right] = v_0 + A\Delta v$

$$v'_2 \rightarrow v''_2 = v'_2 + A\Delta v$$

# Section 5 Summary

Ideal Op-Amp Model: The open input and virtual short approximations give a simple pair of op-amp equations.

Negative Feedback: The connection of output to inverting input through a circuit element, which gives rise to stability and programmable function in an op-amp circuit.

Op-Amp Circuits: Programmed by external elements, op-amp circuits can perform a wide range of useful signal operations. Easily analyzed with node voltage analysis.

Practical Op-Amp Model: An op-amp model that incorporates finite open-loop gain, input resistance and output resistance.