McGill University

Faculty of Engineering

June 2015

Final Examination

ELECTRIC CIRCUITS 1

ECSE-200S

June 12th 2015, 9:35am-12:35pm

Examiner: Professor O. Liboiron-Ladouceur

Student Name:	SOLUTIONS	**McGill ID:**									

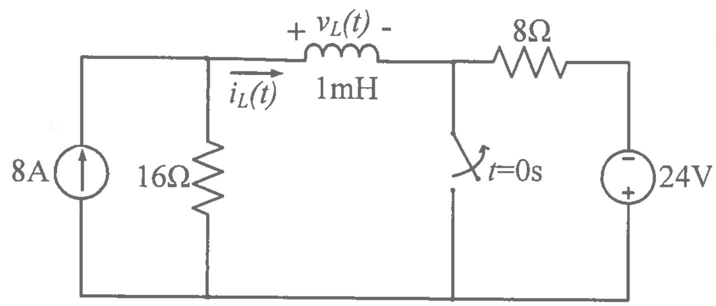
Signature: ______________________________

INSTRUCTIONS:

- Print your name, fill in your student ID number and sign on the line above.
- You have three hours to complete this examination.
- This is a **CLOSED BOOK** examination. **NO CRIB SHEET** allowed.
- FACULTY **STANDARD CALCULATOR** permitted **ONLY**.
- Answer the problems in this examination. Show your work and clearly indicate your answer.
- Read through all questions and ensure that you have a complete examination.
- The examination consists of a total of 10 pages, including this cover page.
- The examination consists of 5 short questions (2 pts each) and 3 problems (10 pts each).
- The examination is worth a total of 40 points.

PART 2 – Problems

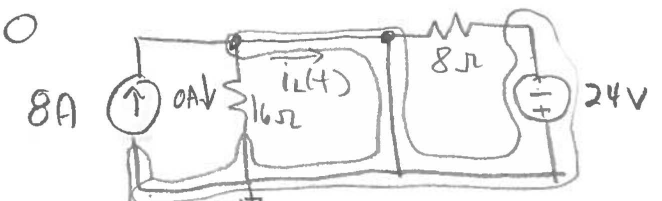
2.1. Consider the circuit shown below. The switch is closed for $t<0$s. Assume dc steady state behaviour for $t<0$. The switch opens at $t=0$s. Answer the following questions. [10pts]



(a) Find the current $i_L(t)$ and plot it versus time t.
(b) Find the voltage $v_L(t)$ for $t > 0$.
(c) If the inductor was replaced by a capacitor, what would be the voltage across the capacitor at $t = 0^+$?

a) find initial condition from current continuity in inductor

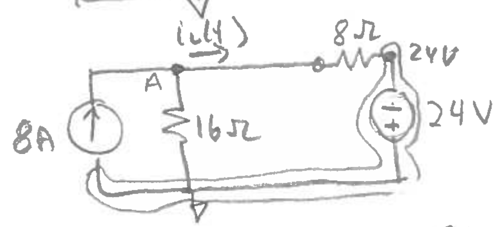
$t<0$



$i_L(t) = 8A$ for $t=0^-$ & $t=0^+$

$i_L(0^+) = 8A$

$t \to \infty$



KCL at A: $-8A + \frac{V_A}{16\Omega} + \frac{V_A + 24V}{8\Omega} = 0$

$-128V + V_A + 2V_A + 48V = 0$

$3V_A = 80$

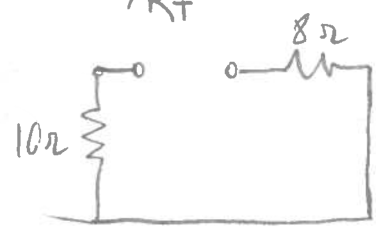
$V_A = 26\,\tfrac{2}{3}\,V$

$i_L(\infty) = \frac{V_A + 24V}{8\Omega} = \frac{26\,2/3 + 24}{8}$ $\quad i_L(\infty) = 6\tfrac{1}{3}\,A$

time constant L/R_T

$t>0$

turn of supplies



$\tau = \frac{1mH}{(8+16)\Omega} = \frac{1}{24{,}000}\,s = 41\tfrac{2}{3}\,\mu sec$

$$\boxed{i_L(t) = 6\tfrac{1}{3}A + 1\tfrac{2}{3}e^{-24K \cdot t}\,A,\ t>0}$$

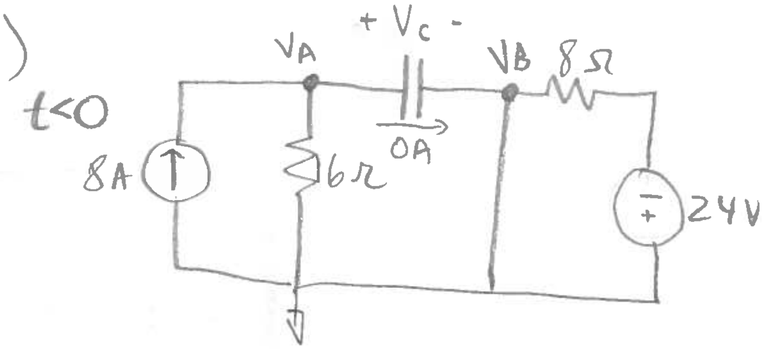
b) $V_L(t) = L\dfrac{di_L(t)}{dt}$

$$= 1\times10^{-3}\frac{d}{dt}\left[6\tfrac{1}{3}A + 2\tfrac{2}{3}e^{-24k\cdot t}\right]$$

$$= \cancel{10^{-3}}\left[2\tfrac{2}{3}\cdot(-24{,}\cancel{000})e^{-24k\cdot t}\right]$$

$$\boxed{V_L(t) = 64e^{-24{,}000t}\ V,\quad t>0}$$

c) $t<0$

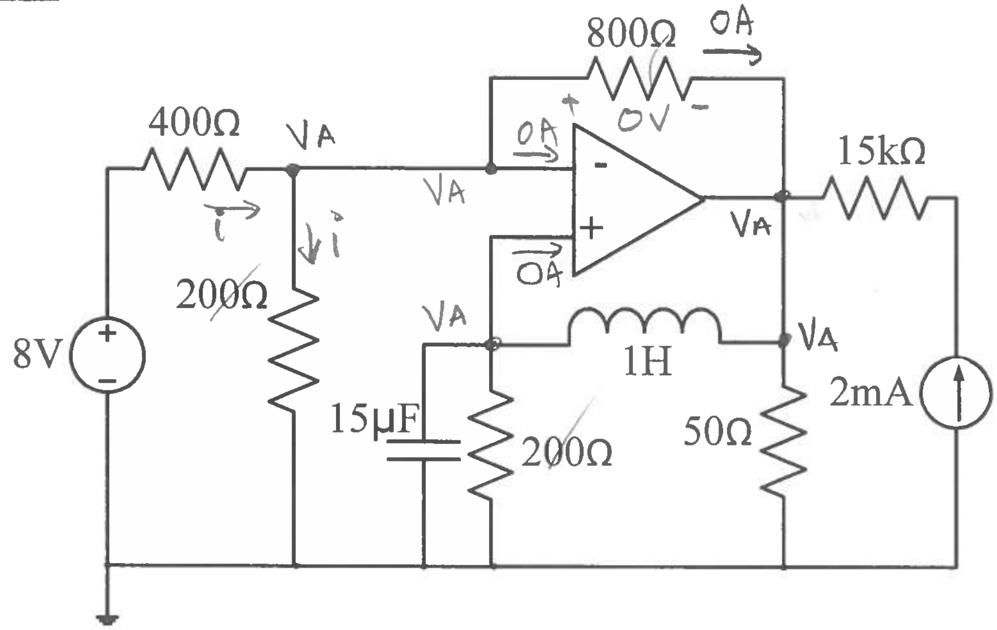


$V_A = 8A\cdot16\Omega = 128\,V$

$V_B = 0V$

$$\boxed{V_C(0^+) = 128V}$$

2.2. Consider the circuit shown below. Assume that the op-amp is ideal and that the circuit is in dc steady state. Answer the following questions. [10pts]



(a) Find the power supplied by the voltage source and the current source?
(b) Which resistor dissipates least amount of energy?
(c) What is the energy stored in the capacitor and in the inductor?

a) 8V supply: $i = \frac{8V}{600\Omega} = \frac{4V}{300} = \frac{1}{75} A$ Pwr $\frac{1}{75} A \cdot 8V = \boxed{106.\frac{2}{3} mW}$

2mA supply $V_A = \frac{200\Omega}{75} \cdot A = 2\frac{2}{3} V$ Pwr: $2mA \cdot 2\frac{2}{3} V = \boxed{5\frac{1}{3} mW}$

b) $P = \frac{V^2}{R} = I^2 \cdot R$. $\frac{V_A^2}{50\Omega} = \frac{\left(\frac{200}{75}\right)^2}{50} = 142.\bar{2} mW$

$(2mA)^2 \cdot 15k\Omega = 60 mW$ $\boxed{400\Omega}$

$\left(\frac{1}{75}\right)^2 \cdot 400\Omega = 71.\bar{1} mW$

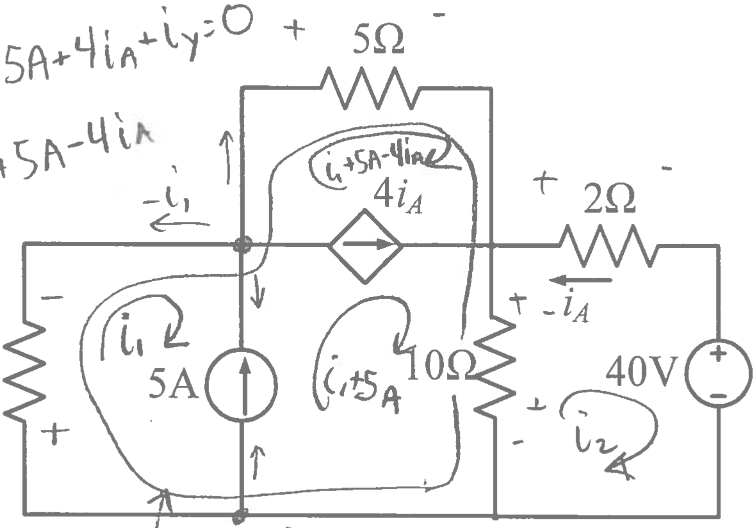
c) $U = \frac{1}{2} CV^2 = \frac{1}{2} 15\times10^{-3} \left(\frac{200}{75}\right)^2 = 53\frac{1}{3} mJ$ ← capacitor

$U = \frac{1}{2} L i^2 = \frac{1}{2} 1H \left(\frac{200}{75} \cdot \frac{1}{200}\right)^2 = 88.\bar{8} \mu J$ ← inductor

2.3. Consider the circuit shown belowAnswer the following questions. [10pts]

$-i_1-5A+4i_A+i_y=0$

$i_y=i_1+5A-4i_A$



$i_2=-i_A$

$i_1+5i_A+i_x=0 \quad i_x=-(i_1+5i_A)$

(a) Identify the super-mesh in the circuit above.
(b) Find the current i_A using mesh analysis.
(c) What is the power delivered by the current-dependent current source?

b) Super-mesh KVL: $6i_1+5(i_1+5+4i_2)+10(i_1+5-i_2)=0$

$6i_1+5i_1+25+24i_2+10i_1+50-10i_2=0$

$11i_1+14i_2+75=0 \quad -(1)$

othermesh KVL:

$10(i_2-i_1-5)+2i_2=0$

$10i_2-10i_1-50+2i_2=0$

$-10i_1+12i_2-50=0.$

$i_1=\frac{12}{10}i_2-5 \quad -(2)$

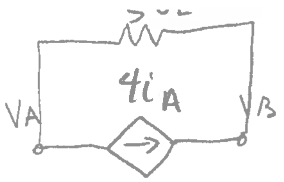
(2) in (1) $\frac{132i_2}{10}-55+14i_2+75=0$

$132i_2+200+140i_2=0$

$i_2=-\frac{200}{272}A$

$\boxed{i_A=735.3\,mA}$

c)



$I = 4 i_A = 2.94\ A$

$\boxed{\text{Pwr delivered } 56.2\ W}$

$V_A - V_B = 5\Omega(i_1 + 5 - 4 i_A) = -19.12\ V.$

$\frac{12}{10}\left(\frac{-200}{272}\right) - 5 = -5.88\ A$