Today’s Outline

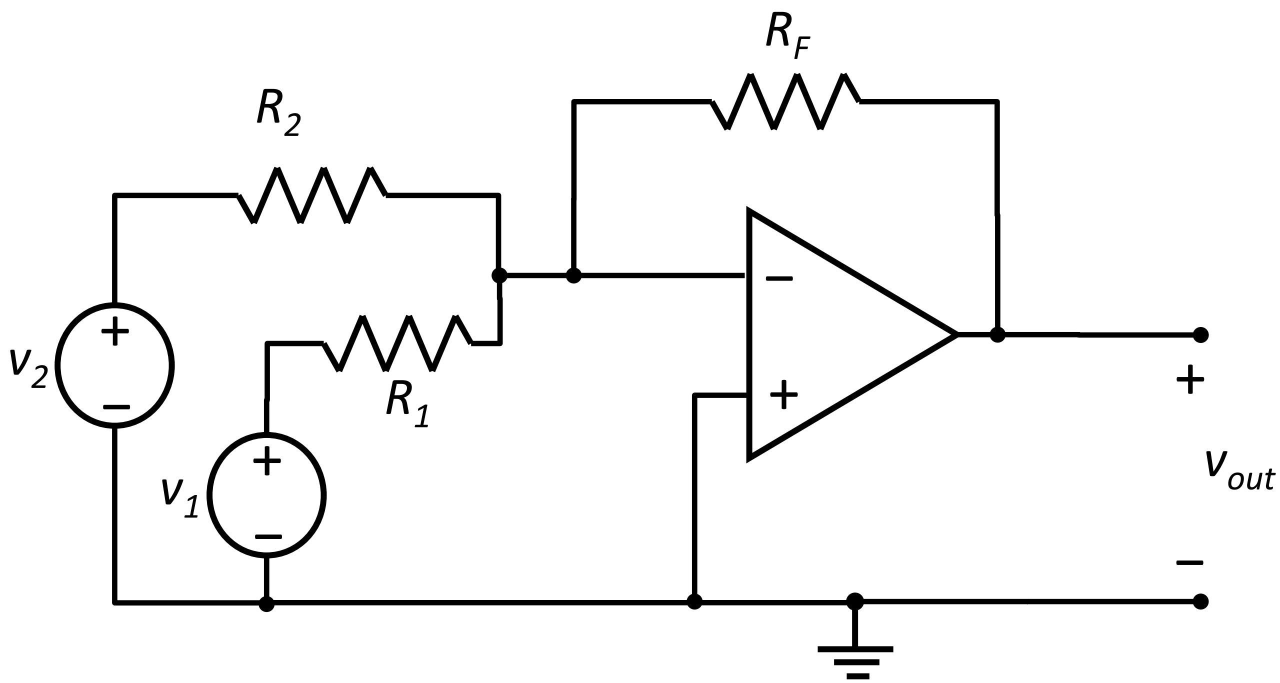
5. Operational Amplifiers

- Op-Amp Circuits

McGill

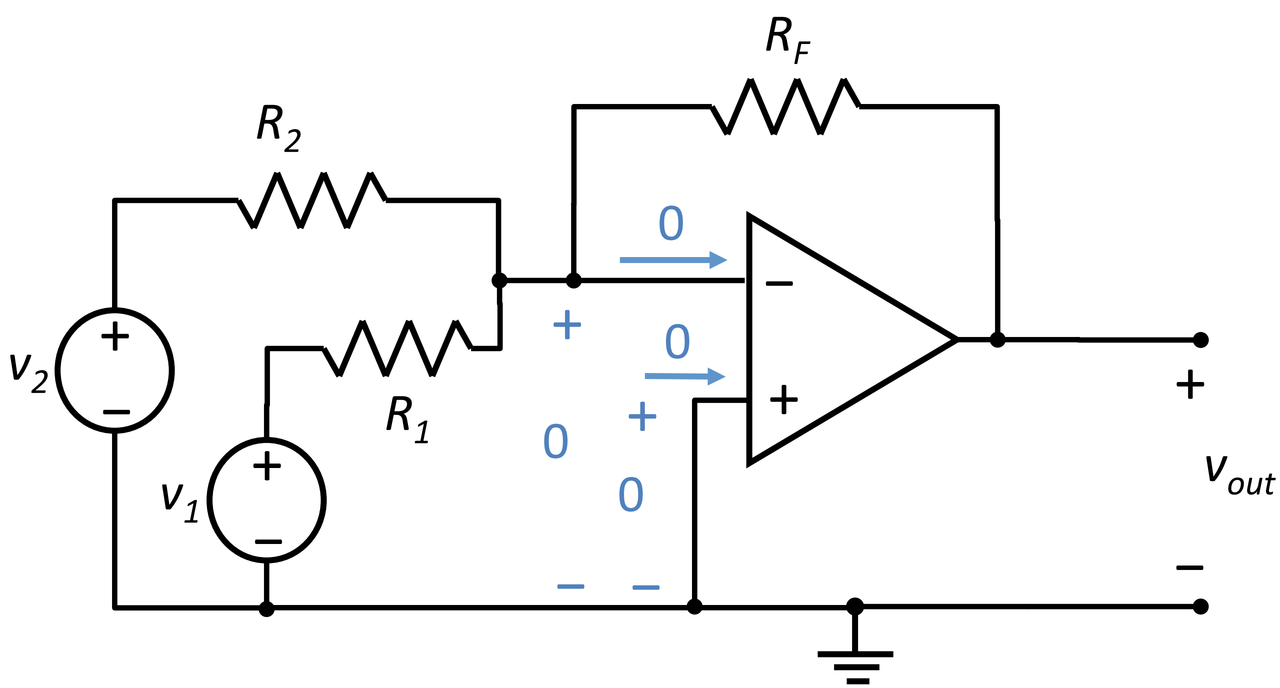
Example 4

Assuming ideal op-amp behaviour, find v_{out} as a function of v_1 and v_2.

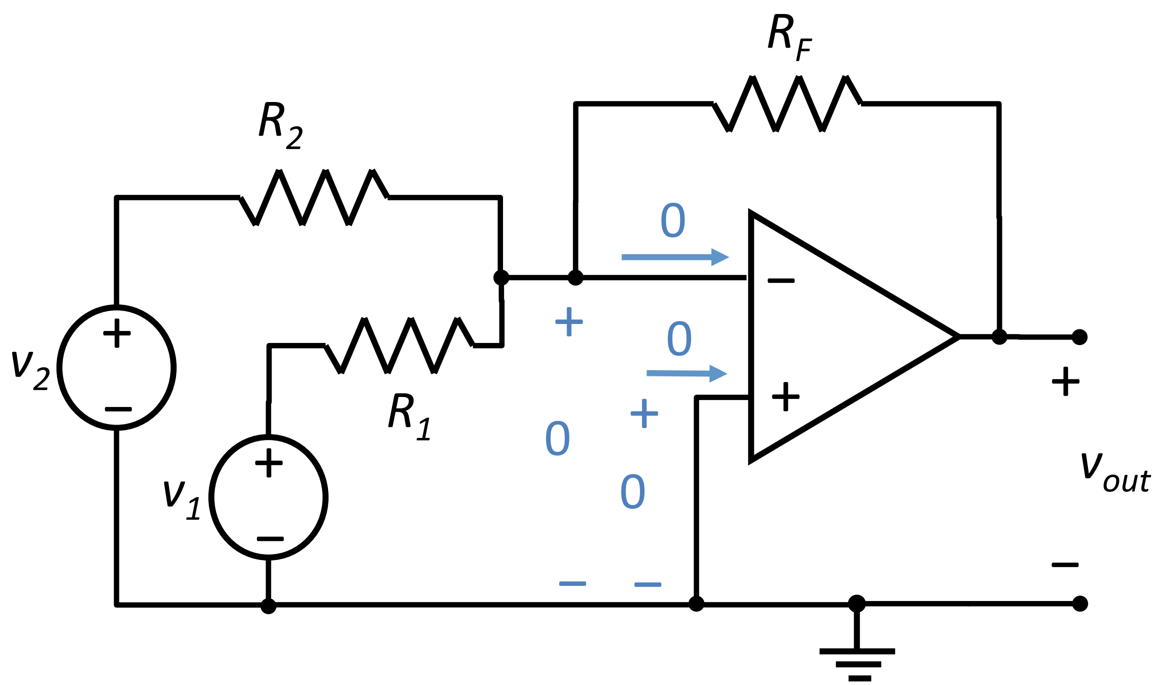


Example 4

Apply ideal op-amp conditions.



Example 4



Node voltage equation at the inverting input:

$$0 = \frac{0 - v_1}{R_1} + \frac{0 - v_2}{R_2} + \frac{0 - v_{out}}{R_F}$$

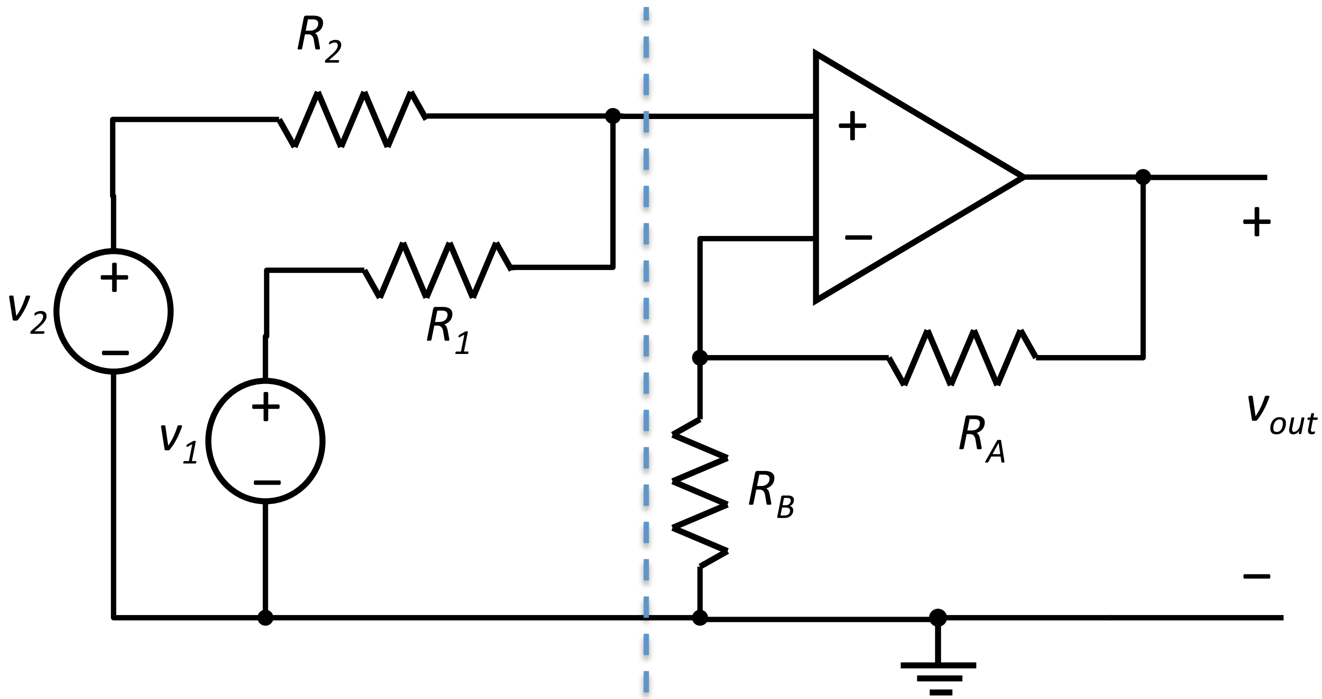
$$\boxed{v_{out} = -\frac{R_F}{R_1} v_1 - \frac{R_F}{R_2} v_2}$$

This op-amp circuit is configured as a ***summing inverting amplifier***. The ratio of R_F to R_1 and to R_2 programs the contribution to v_{out} from v_1 and v_2.

The principle of superposition could also be easily applied here.

Example 5

Assuming ideal op-amp behaviour, find v_{out} as a function of v_1 and v_2.

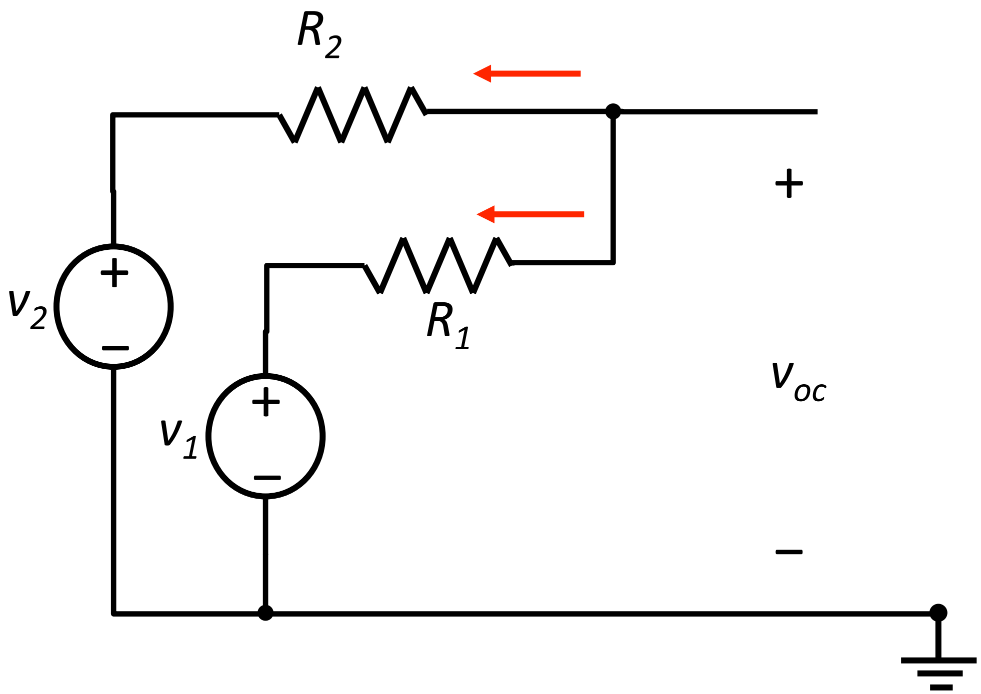


Strategy:

- *find the Thévenin equivalent for the input network (left of dotted line)*
- *apply the analysis for the simplified circuit*

Example 5

Find open circuit voltage of the input network.



A single node-voltage equation gives:

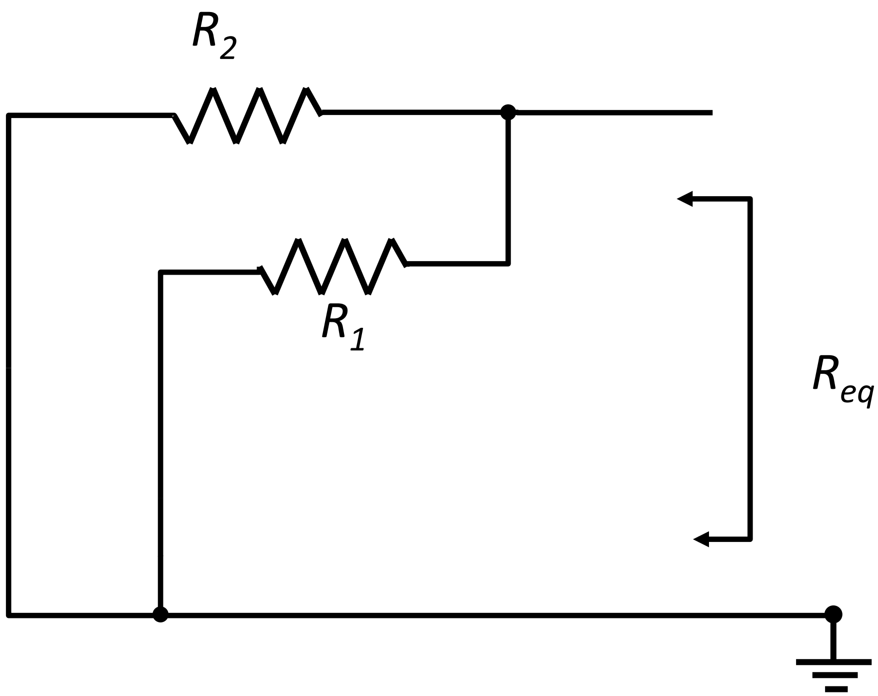
$$0 = \frac{v_{oc} - v_1}{R_1} + \frac{v_{oc} - v_2}{R_2}$$

$$0 = v_{oc}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{v_1}{R_1} - \frac{v_2}{R_2}$$

$$v_{oc} = \frac{R_1 v_2 + R_2 v_1}{R_1 + R_2}$$

Example 5

Find the Thévenin resistance of the input network.

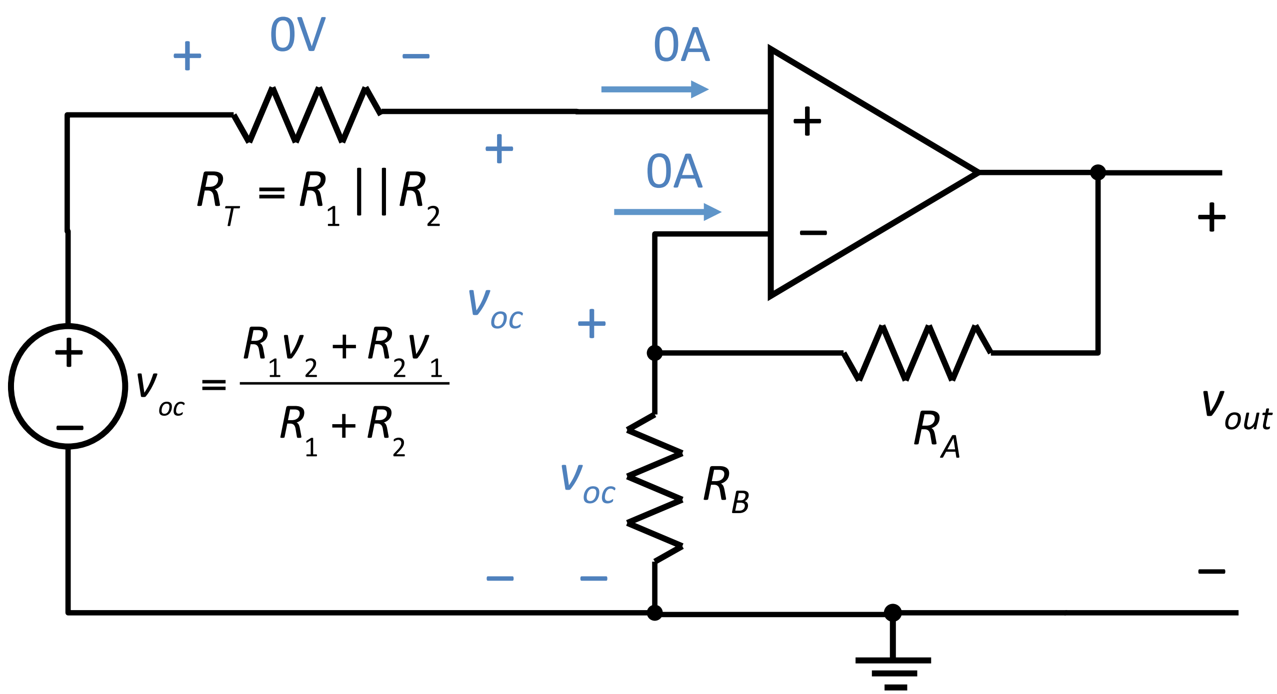


Turn-off voltage sources, and find equivalent resistance.

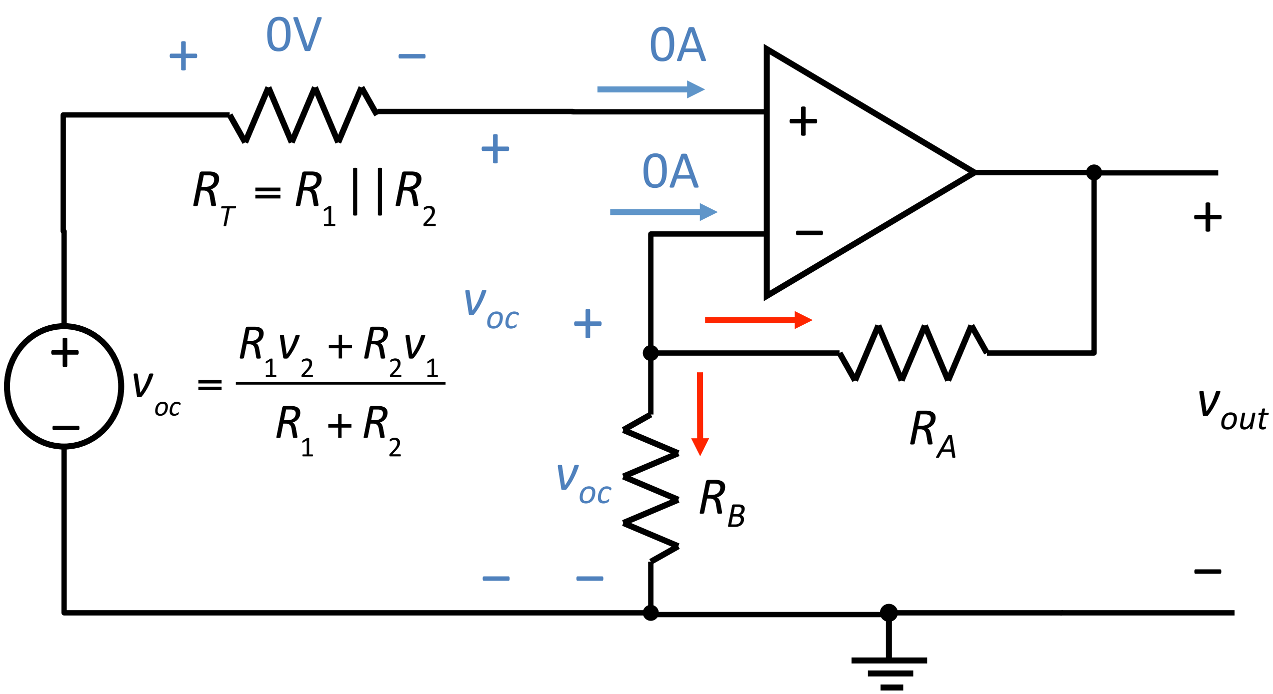
$$R_T = R_{eq}$$
$$= R_1 \,||\, R_2$$
$$= \frac{R_1 R_2}{R_1 + R_2}$$

Example 5

The circuit is thus simplified. We then apply ideal op-amp conditions.



Example 5



Node voltage equation at the inverting input:

$$0 = \frac{v_{oc}}{R_B} + \frac{v_{oc} - v_{out}}{R_A}$$

$$v_{out} = \left(1 + \frac{R_A}{R_B}\right) v_{oc}$$

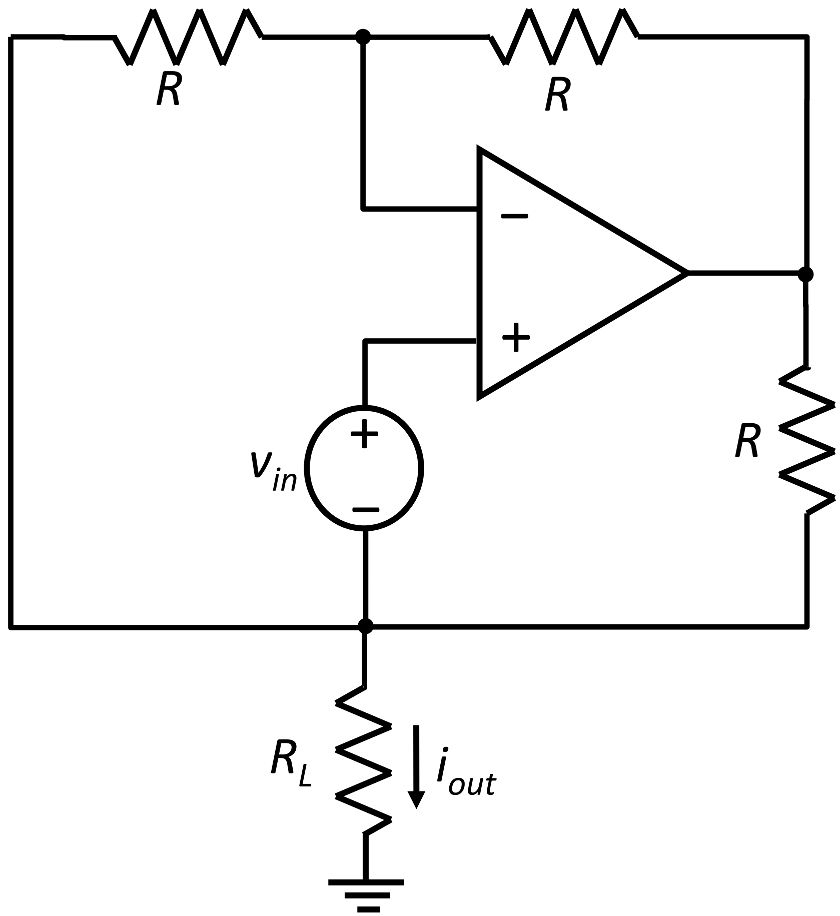
This circuit is configured as a ***summing non-inverting amplifier***:

$$v_{out} = \left(1 + \frac{R_A}{R_B}\right)\left(\frac{R_2}{R_1 + R_2} v_1 + \frac{R_1}{R_1 + R_2} v_2\right)$$

The principle of superposition could also be applied.

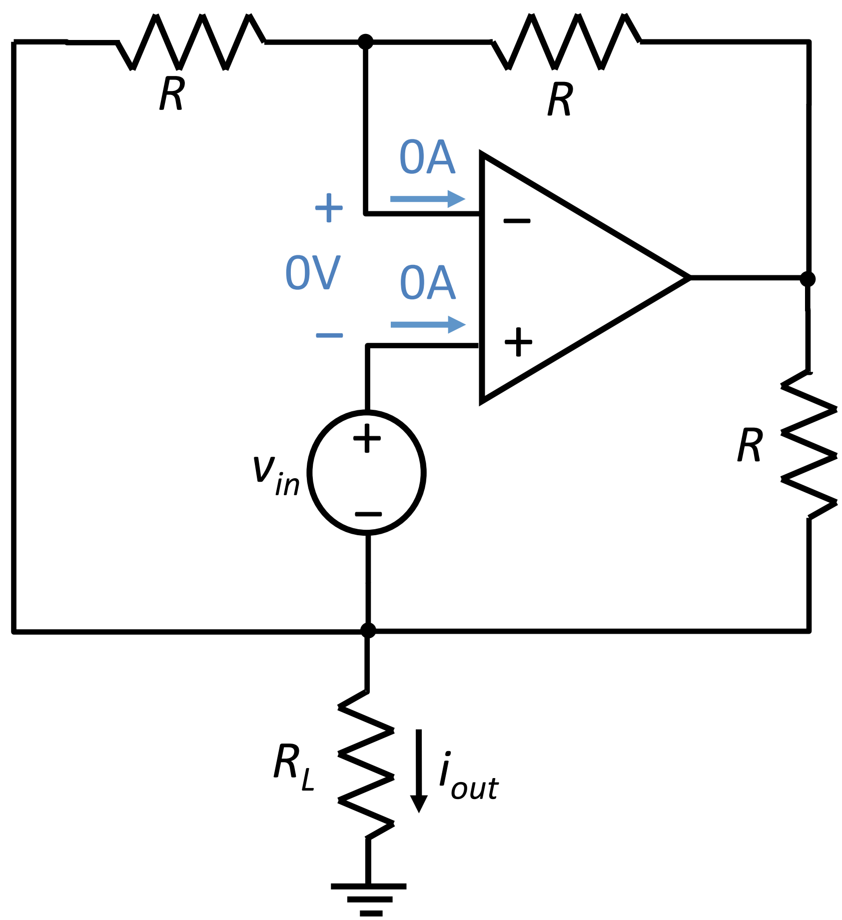
Example 6

Assuming ideal op-amp behaviour, find i_{out} as a function of v_{in}.

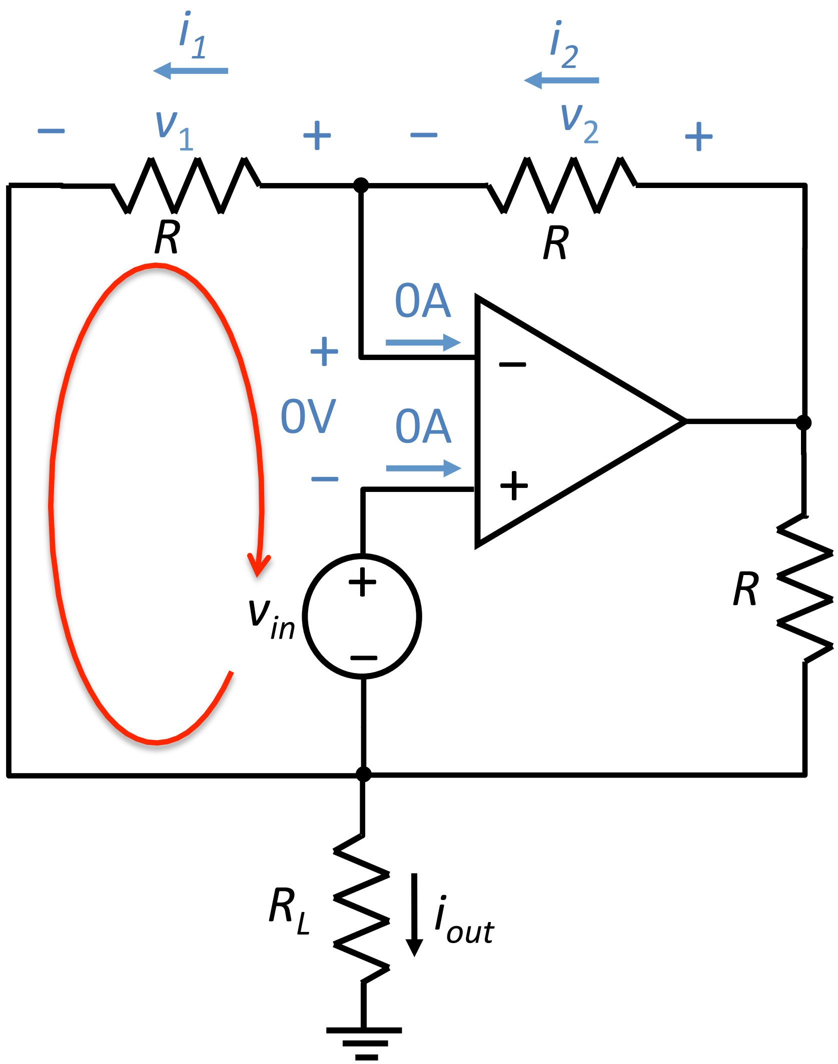


Example 6

Apply ideal op-amp equations.



Example 6



KVL:

$$0 = -v_1 + v_{in}$$

$$v_1 = v_{in}$$

Ohm:

$$i_1 = \frac{v_1}{R} = \frac{v_{in}}{R}$$

KCL:

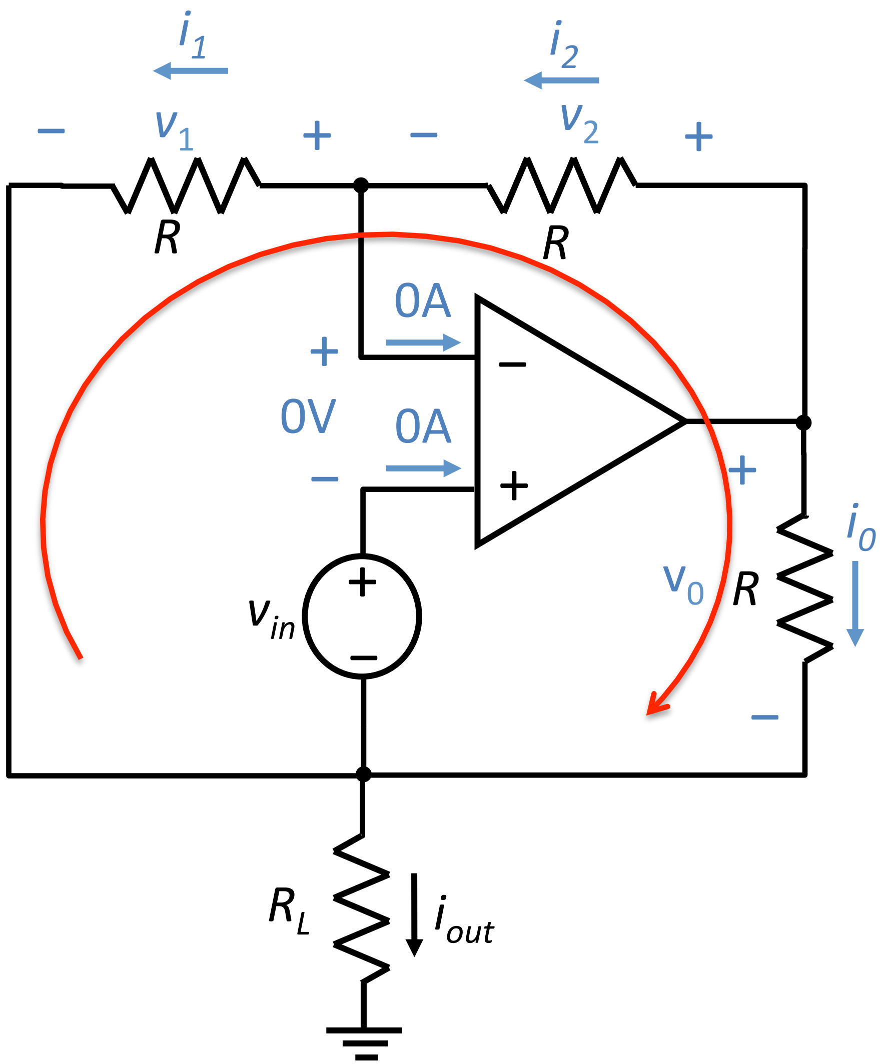
$$0 = i_1 - i_2$$

$$i_2 = i_1$$

Ohm:

$$v_2 = i_2 R = i_1 R = v_{in}$$

Example 6



KVL:

$$0 = -v_1 - v_2 + v_0$$

$$v_0 = v_1 + v_2 = 2v_{in}$$

Ohm:

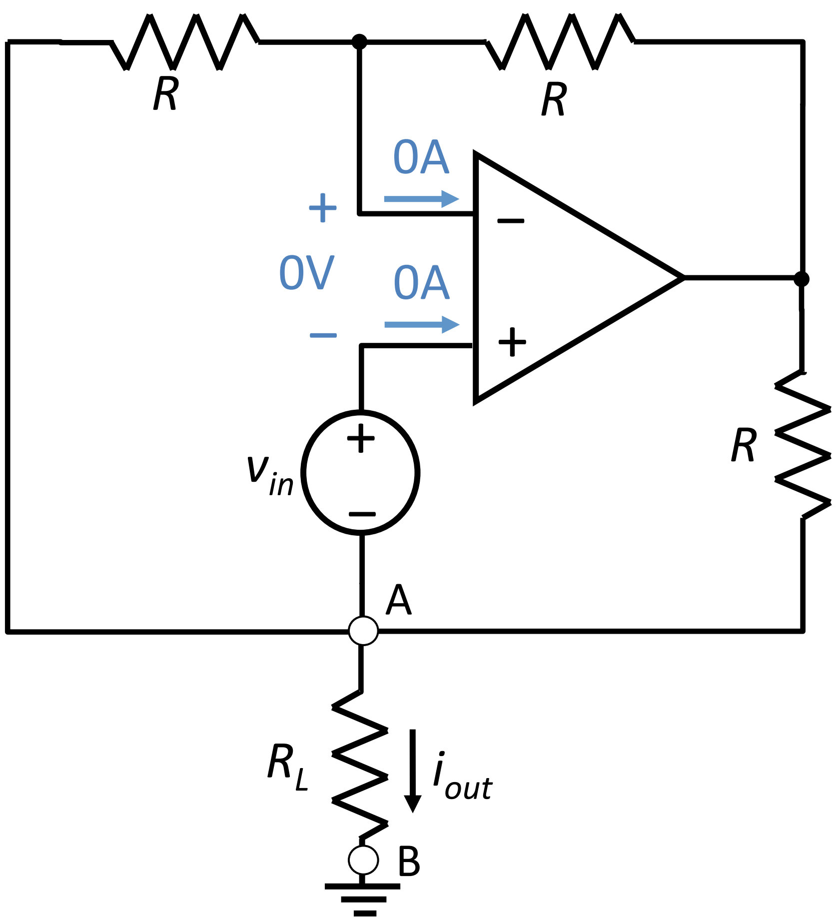
$$i_0 = \frac{v_0}{R} = \frac{2v_{in}}{R}$$

KCL:

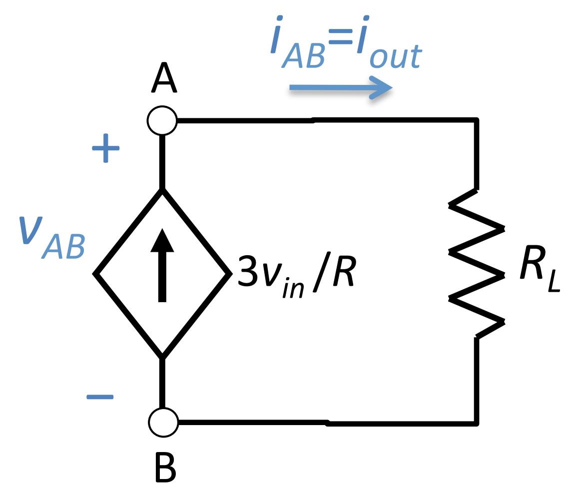
$$0 = i_{out} - i_1 - i_0$$

$$i_{out} = i_1 + i_0 = 3\frac{v_{in}}{R}$$

Example 6

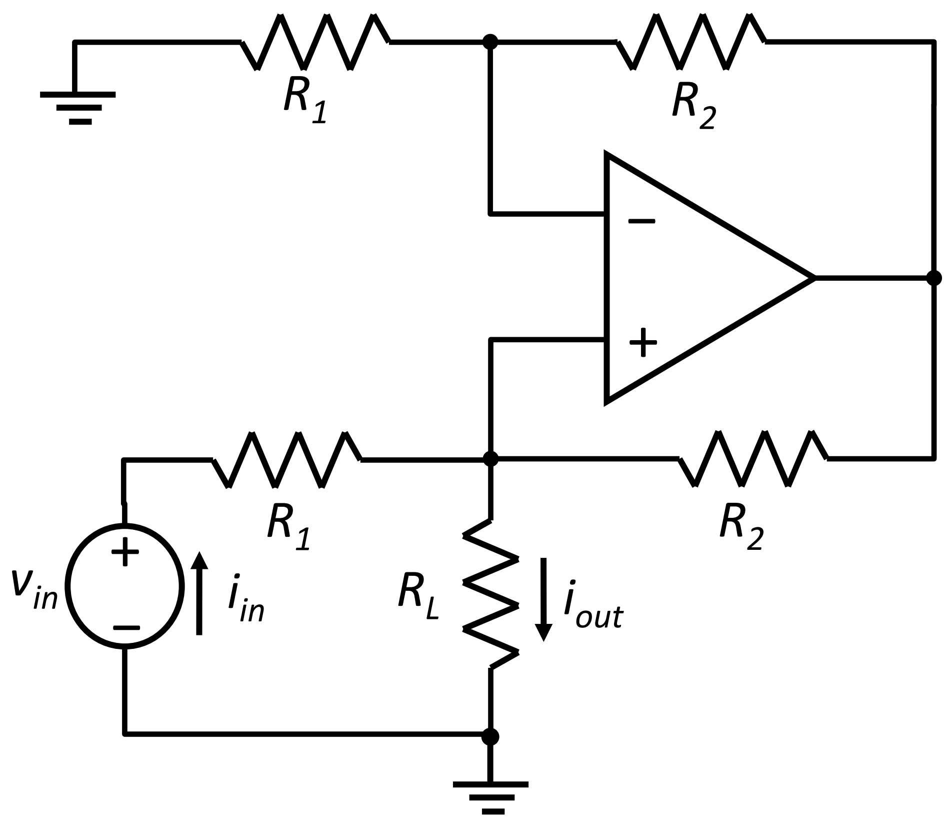


This circuit behaves like a voltage dependent ***current source*** at the terminals A and B. An equivalent circuit is:



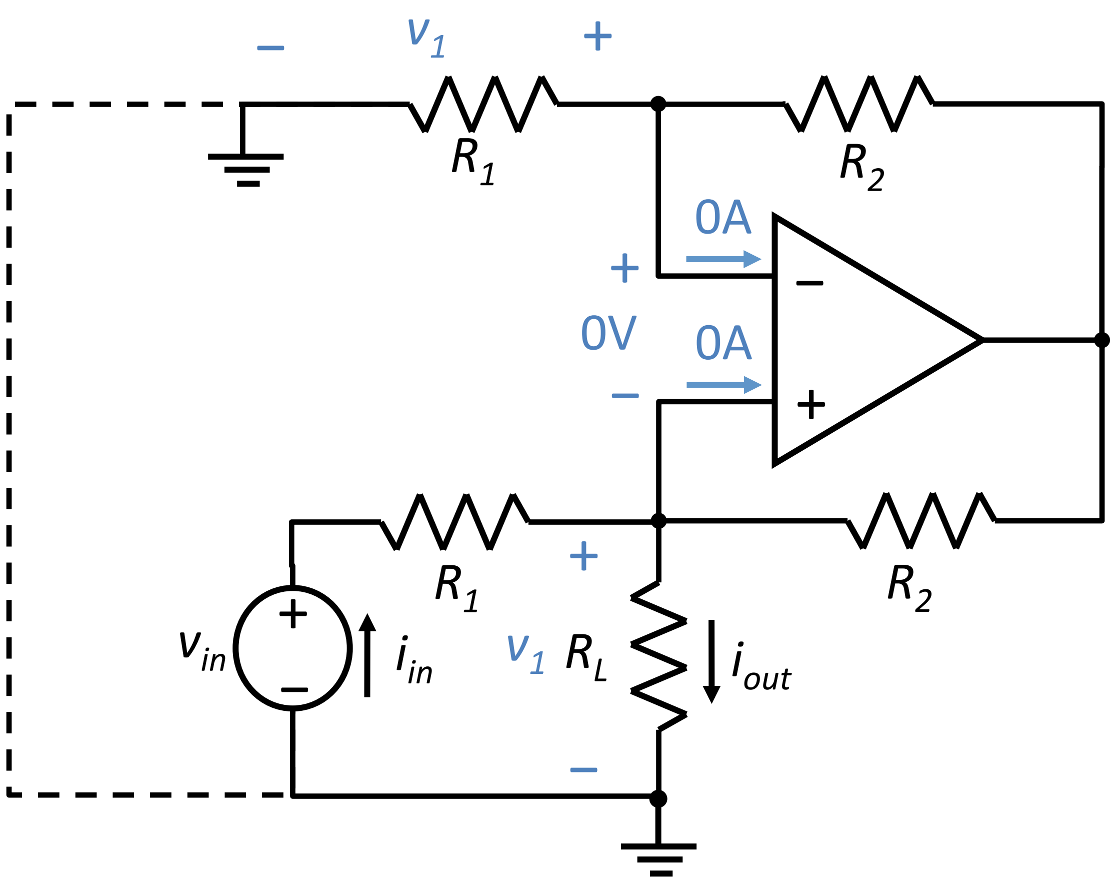
Example 7

Assuming ideal op-amp behaviour, find i_{out} and i_{in} as a function of v_{in}.



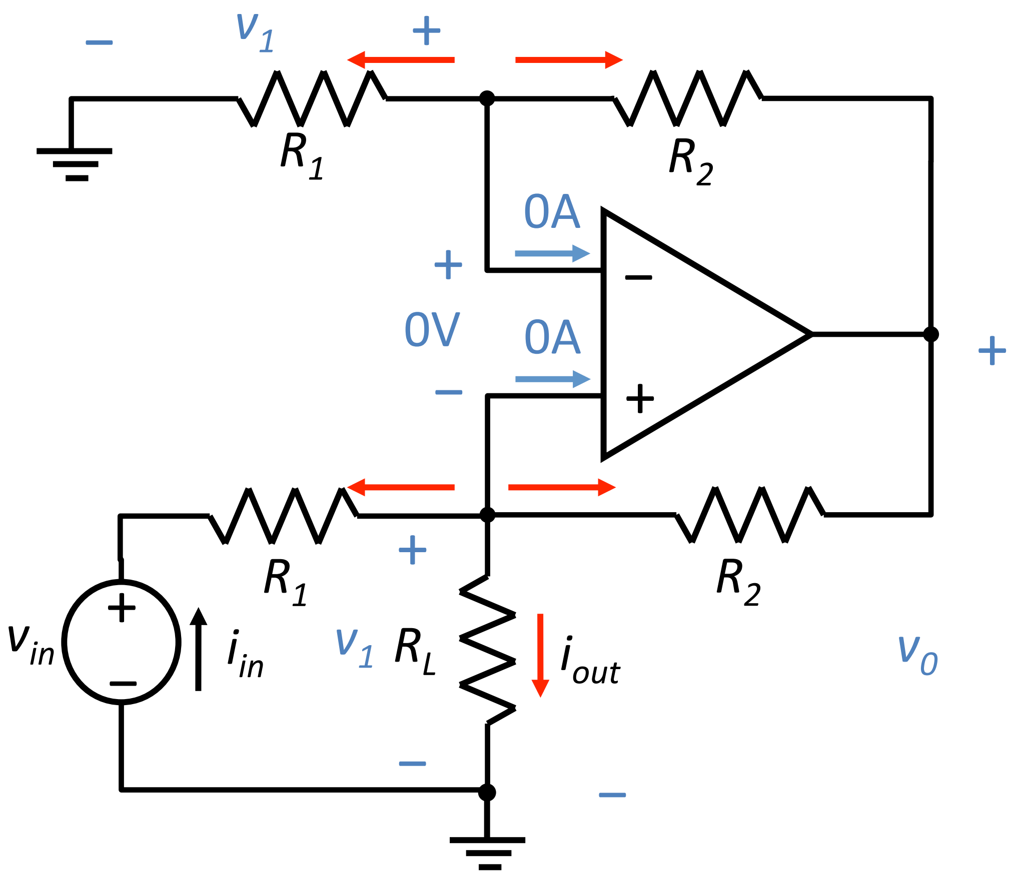
Example 7

Apply ideal op-amp conditions.



Note that all instances of the reference terminal are at the same potential.

Example 7



Inverting input node equation:

$$0 = \frac{v_1}{R_1} + \frac{v_1 - v_0}{R_2}$$

Non-inverting input node equation:

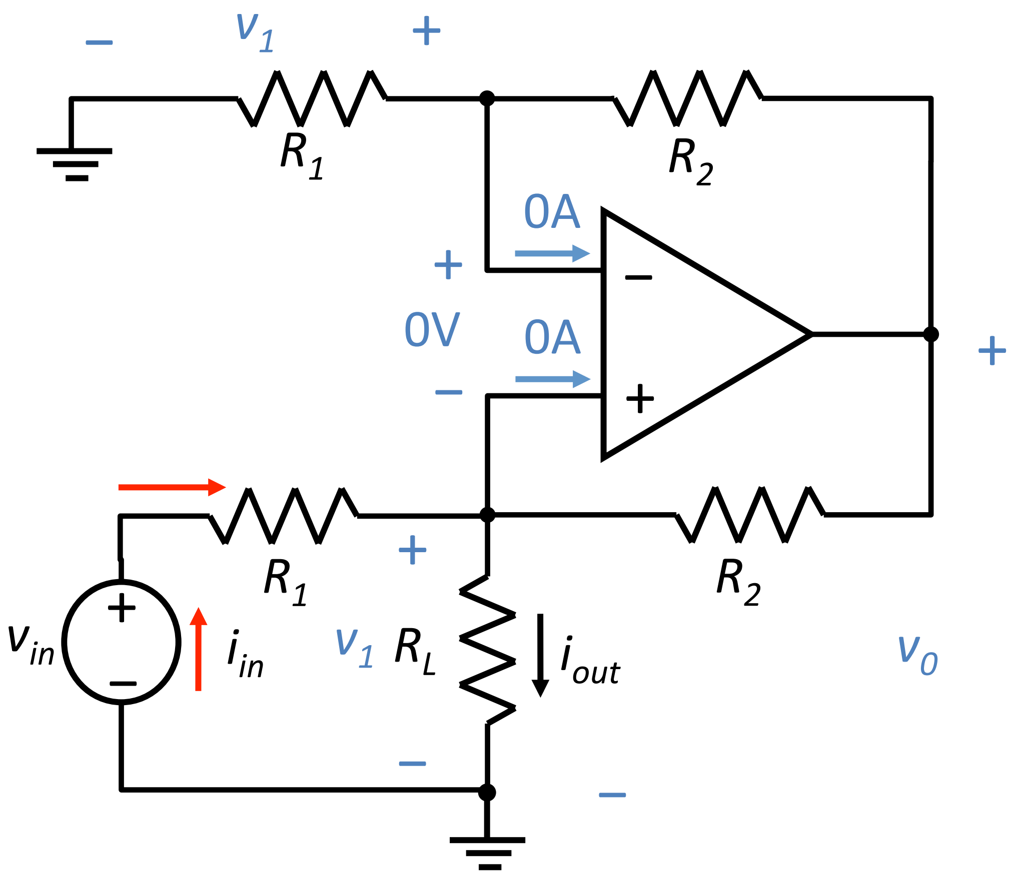
$$0 = \frac{v_1 - v_{in}}{R_1} + \frac{v_1 - v_0}{R_2} + i_{out}$$

$$0 = \frac{v_1}{R_1} + \frac{-v_{in}}{R_1} + \frac{v_1 - v_0}{R_2} + i_{out}$$

$$0 = \underbrace{\frac{v_1}{R_1} + \frac{v_1 - v_0}{R_2}}_{0} + \frac{-v_{in}}{R_1} + i_{out}$$

$$i_{out} = \frac{v_{in}}{R_1}$$

Example 7



Input and output currents:

$$i_{in} = \frac{v_{in} - v_1}{R_1} \qquad i_{out} = \frac{v_{in}}{R_1} = \frac{v_1}{R_L}$$

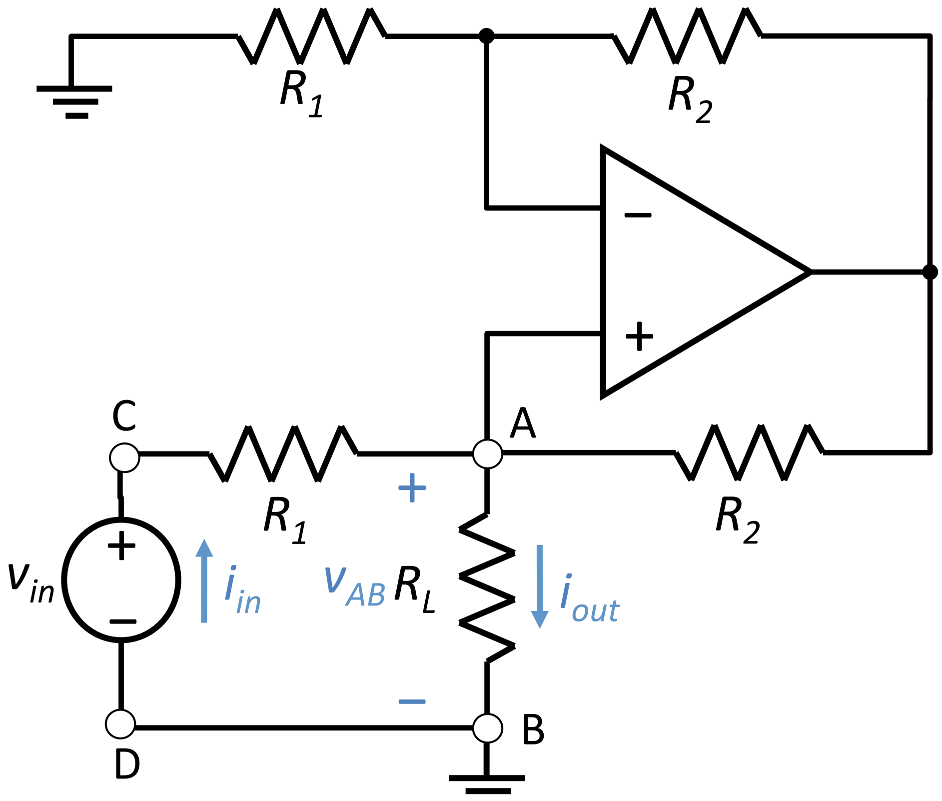
$$= \frac{v_{in}}{R_1} - \frac{v_1}{R_1} \qquad v_1 = \frac{R_L}{R_1} v_{in}$$

Input current:

$$i_{in} = \frac{v_{in}}{R_1}\left(1 - \frac{R_L}{R_1}\right)$$

$$= \frac{v_{in}}{\left(\dfrac{R_1^2}{R_1 - R_L}\right)}$$

Example 7



This op-amp circuit is configured as a ***voltage dependent current source***.

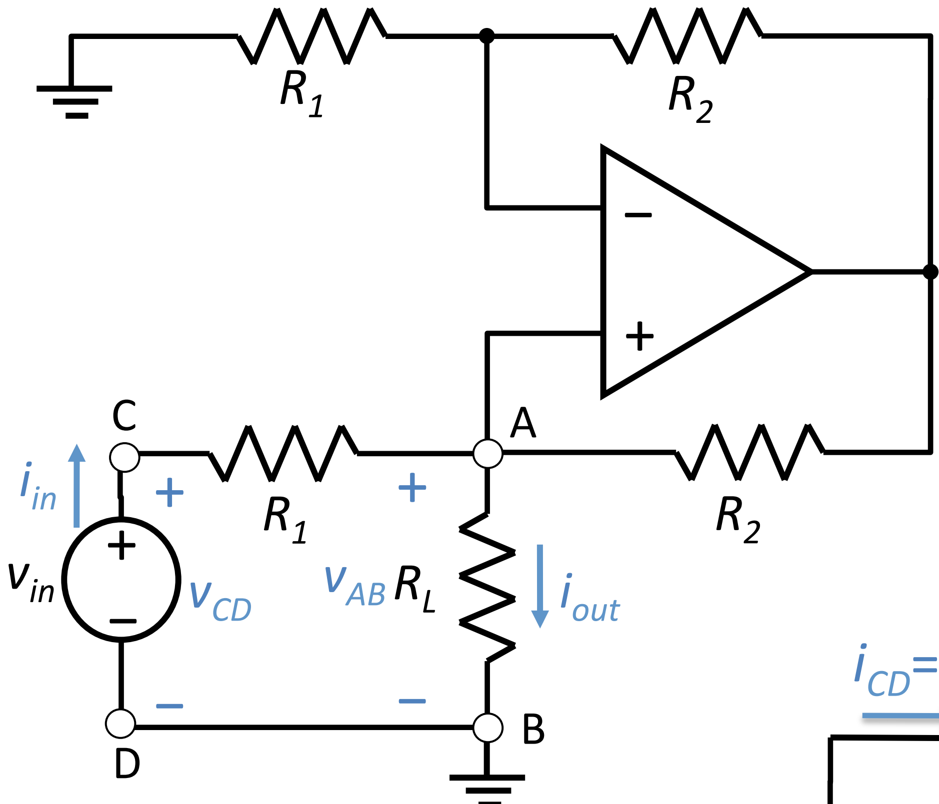
$$i_{out} = \frac{v_{in}}{R_1}$$

The current i_{out} is independent of the voltage v_{AB} across the load resistor R_L (under ideal conditions).

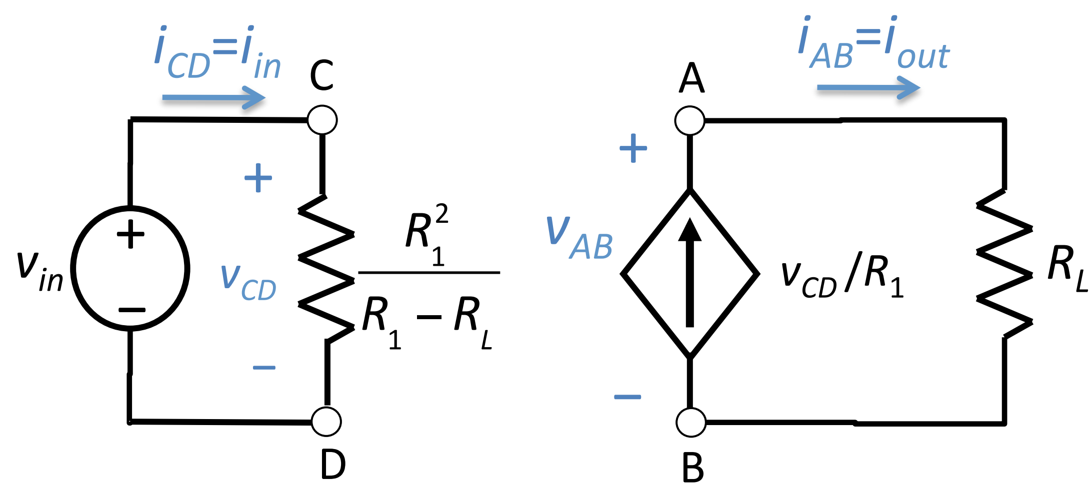
The i_{in}-v_{in} relationship is that of a resistance:

$$i_{in} = \frac{v_{in}}{R_T} \qquad R_T = \frac{R_1^2}{R_1 - R_L}$$

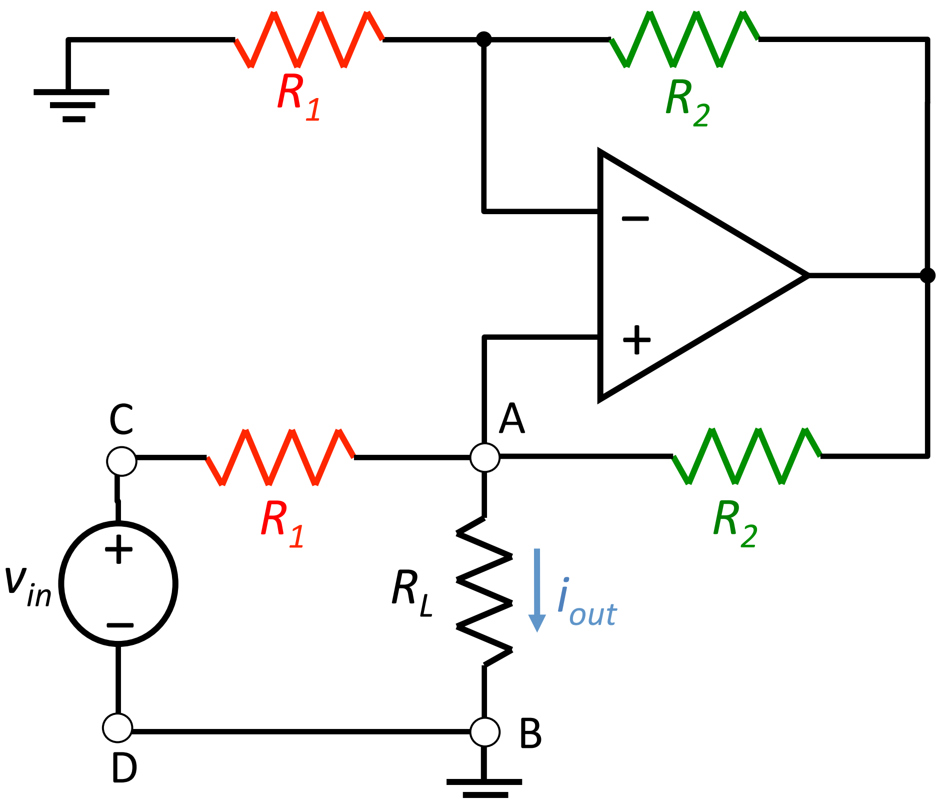
Example 7



This circuit is known as a ***Howland current source***. An equivalent circuit is:



Example 7



Question: What happens to i_{out} if the resistors R_1 are not ***matched*** (equal)?

In practice, this circuit can be difficult to implement because the resistors R_1 must be carefully matched with high precision, as must the resistors R_2.

Question

- Is there another way to design a voltage controlled current source with an op-amp?