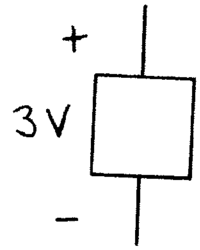
ECSE-200 Quiz #1 (9 January 2009)

NAME______________________________ McGill ID#________________

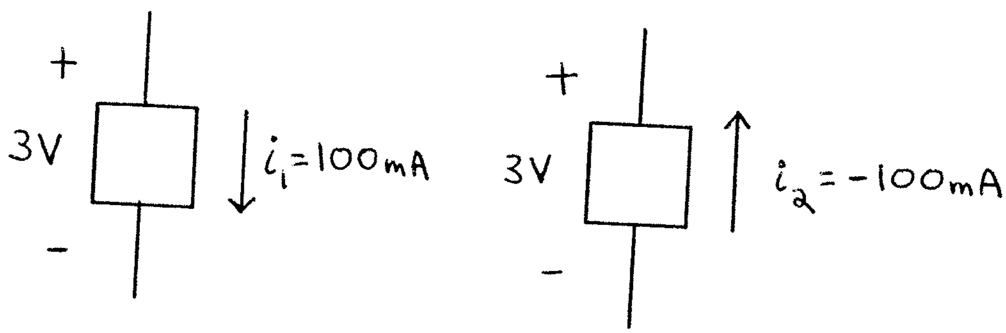
1. Consider the 3V battery below, which is rated with a capacity of 2000 mA-hours, and answer all parts of the question.



a) Assume for this part alone that a current of 100mA is flowing into the positive terminal and out of the negative terminal of the battery. Make two diagrams showing two different but equivalent descriptions of the current flow in terms of algebraic current variables (the variable names are of your choosing).

b) What is the maximum energy that can be delivered by this battery? Give your answer in SI units.

a)



(+1) (+1)

b) $Q = 2000\,\text{mA-hrs} \times \frac{3600\,\text{s}}{\text{hr}}$

$= 2\,\text{A} \times 3.6\,\text{ks}$

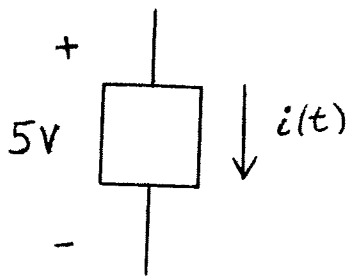
$= 7.2\,\text{kC}$ (+1)

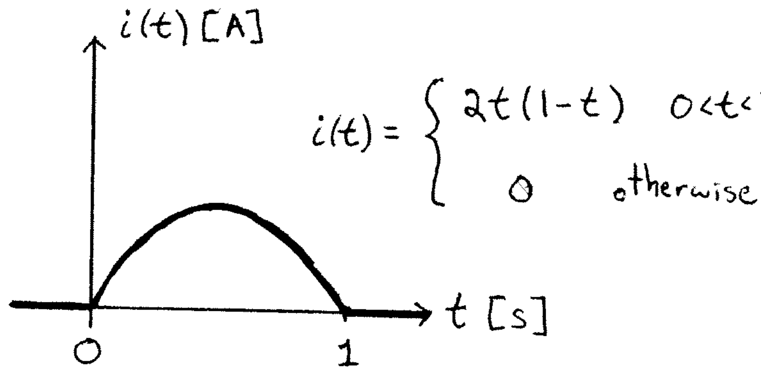
$U = QV$ = energy delivered by battery

$= 7.2\,\text{kC} \cdot 3\,\text{V}$

$= \underline{21.6\,\text{kJ}}$ (+1)

2. Consider the circuit element and plot below, answer all parts of the question.





a) What is the total charge passed through the circuit element over the time interval 0s < t < 1s ?

b) Over the time interval 0s < t < 1s, what is the expression for the power delivered by the circuit element?

c) What is the total energy delivered by the circuit element over the time interval 0s < t < 1s? Consider carefully the sign of your answer.

a)

$$Q = \int_0^1 i(t)\,dt \qquad (+1)$$

$$= \int_0^1 2t(1-t)\,dt$$

$$= \int_0^1 2t - 2t^2\,dt$$

$$= \left[2\,\tfrac{1}{2}t^2 - 2\,\tfrac{1}{3}t^3 \right]_0^1$$

$$= \left[t^2 - \tfrac{2}{3}t^3 \right]_0^1$$

$$= \left(1 - \tfrac{2}{3}\right) - (0 - 0)$$

$$= \underline{\tfrac{1}{3}\ [C]} \qquad (+1)$$

b) $P =$ power delivered by element

$= (+5)(-i(t)) \qquad (+1)$

$= (+5)(-2t(1-t))$

$= \underline{-10t(1-t)\ [W]} \qquad (+1)$

c) $U =$ energy delivered by element

$= \int_0^1 P(t)\,dt$

$= \int_0^1 (+5)(-i(t))\,dt$

$= (+5)\left(-\int_0^1 i(t)\,dt\right)$

$= (+5)\ (-Q)$

$= (+5)\left(-\frac{1}{3}\right)$

$= -\frac{5}{3}\ [J]$

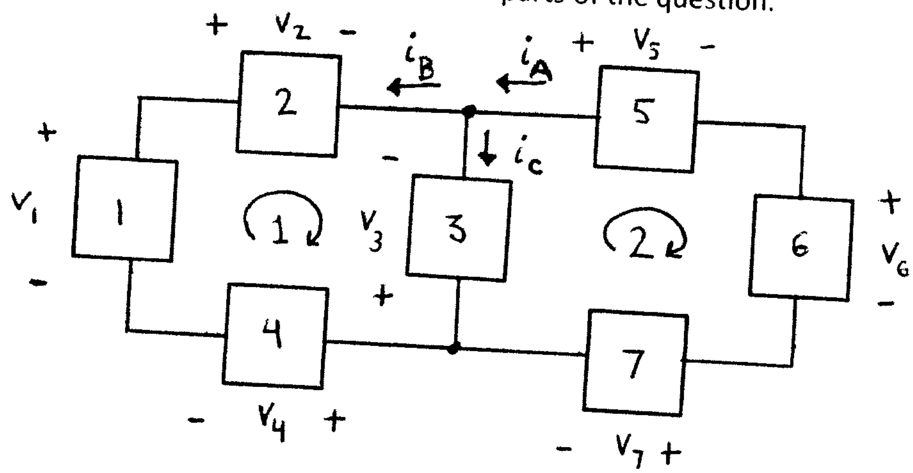
$= \underline{-1.667[J]} \qquad (+1)$

ECSE-200 Quiz #2 (13 January 2009)

NAME________________________________ McGill ID#______________

READ each question carefully. Give units on your answers (where appropriate).

1. Consider the circuit below, and answer all parts of the question.



a) Write down the KVL equations for loops 1 and 2. Use any form of KVL that you prefer. [2pts]

b) If i_B = 5A, i_C = −2A and v_6 = 10V, what is the power absorbed by element 6? [3pts]

a) KVL on loop 1: $-V_1 + V_2 - V_3 + V_4 = 0$ [1pt]

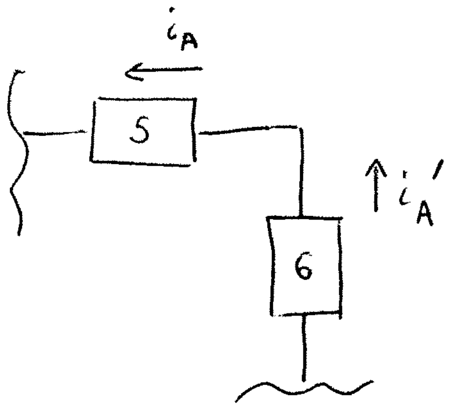
KVL on loop 2: $V_3 + V_5 + V_6 + V_7 = 0$ [1pt]

b) KCL at top: $-i_A + i_B + i_C = 0$

$i_A = i_B + i_C$ [1pt]

$= 5A + (-2A)$

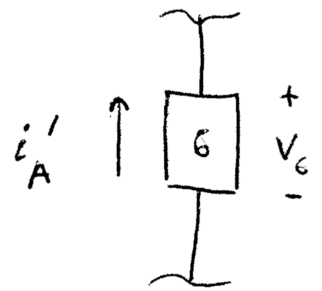
$= 3A$



KCL: $i_A - i_A' = 0$

$i_A' = i_A = 3A$

[1pt]



$P_{6,abs}$ = power absorbed by element 6

$= - i_A' \cdot V_6$

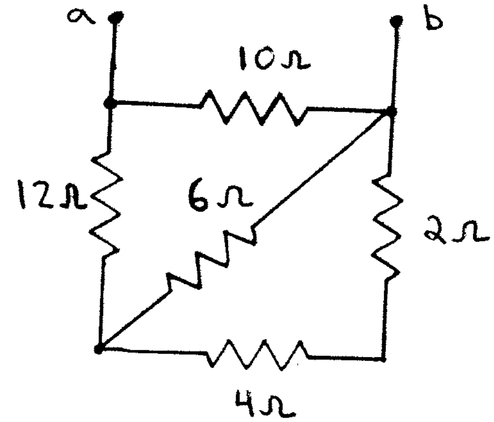
(negative sign because i_A' and V_6 are <u>not</u> oriented with passive sign convention)

$= -(3A) \cdot (10V)$

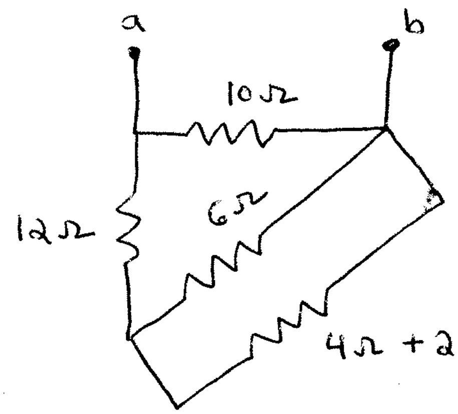
$= \underline{-30\text{ W} \quad \text{absorbed}}$ [1 pt]

(equivalent to +30W of delivered power)

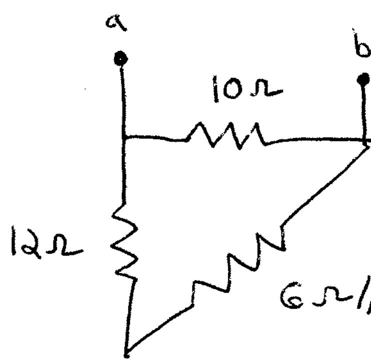
2. Consider the circuit below, and answer the question.



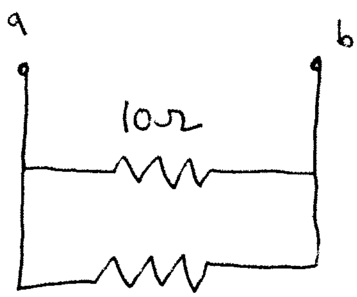
What is the equivalent resistance between the terminals a and b? Show each step in your work. [2pts]



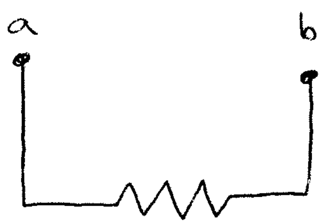
$4\Omega + 2\Omega = 6\Omega$



$$6\Omega // 6\Omega = \frac{6\Omega \cdot 6\Omega}{6\Omega + 6\Omega} = \frac{36}{12}\Omega = 3\Omega$$



$12\Omega + 3\Omega = 15\Omega$

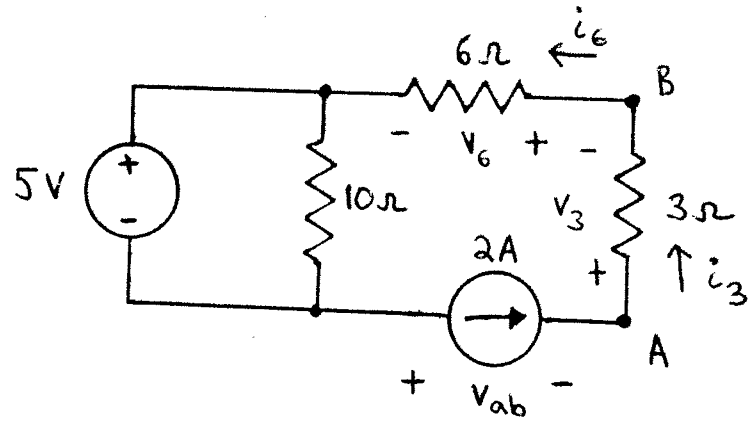


$$R_{eq} = 10\Omega \,//\, 15\Omega = \frac{10\Omega \cdot 15\Omega}{10\Omega + 15\Omega} = \frac{150}{25}\,\Omega$$

$$\underline{R_{eq} = 6\Omega}$$

[1 pt correct answer
+ 1 pt showing work]

3. Consider the circuit below, and answer the question below.



[+1 pt]

What is the voltage v_{ab}? Show all your work and consider carefully the sign of your answer. Indicate in the circuit diagram above any variables that you introduce in order to arrive at your solution. Indicate clearly which equations in your work, if any, correspond to KVL, KCL, Ohm's law or other principles. [4pts]

KCL at node A: $0 = i_3 - 2A$

$i_3 = 2A$

[+1 pt]

KCL at node B: $0 = -i_3 + i_6$

$i_6 = i_3 = 2A$

Ohm's Law: $V_3 = 3\Omega \cdot i_3 = 3\Omega \cdot 2A = 6V$

$V_6 = 6\Omega \cdot i_6 = 6\Omega \cdot 2A = 12V$

[+1 pt]

KVL around outer loop:

$0 = -5V - V_6 - V_3 - V_{ab}$

$V_{ab} = -5V - V_6 - V_3 = -5V - 12V - 6V$

$= \underline{-23V}$

[+1pt]

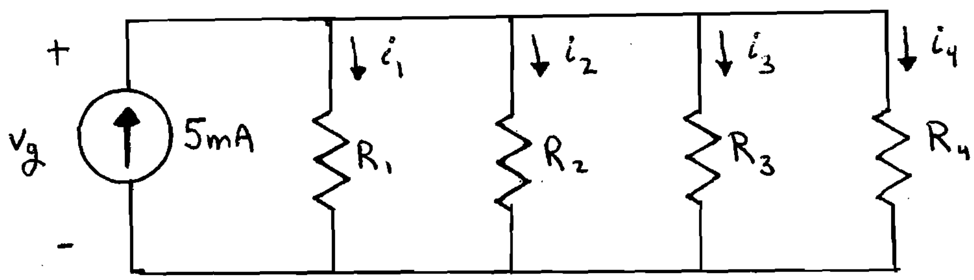
work space

ECSE-200 Quiz #3 (23 January 2009)

NAME______________________________ McGill ID#________________

READ each question carefully. Give units on your answers (where appropriate).

1. Consider the circuit below and answer the question.



What should the values of R_1, R_2, R_3 and R_4 be, if it is desired that i_1=4i_2, i_2=8i_3, i_3=5i_4, and v_g=+1V. Show all steps of your work. [4pts]

$i_3 = 5 i_4$

$i_2 = 8 i_3 = 40 i_4$

$i_1 = 4 i_2 = 160 i_4$

KCL: $0 = -5mA + i_1 + i_2 + i_3 + i_4$ [+1]

$5mA = i_1 + i_2 + i_3 + i_4$

$= 160 i_4 + 40 i_4 + 5 i_4 + i_4$ [+1]

$= 206 i_4$

$i_4 = \frac{5mA}{206} = 24.27 \mu A$

$i_3 = 5 i_4 = 121.36 \mu A$

$i_2 = 8 i_3 = 0.9709 mA$

$i_1 = 4 i_2 = 3.883 mA$

[+1]

Ohm's Law:

$$R_1 = \frac{v_g}{i_1} = \frac{1V}{3.883\,mA}$$

$$= 257.5\,\Omega$$

$$R_2 = \frac{v_g}{i_2} = \frac{1V}{0.9709\,mA}$$

$$= 1.030\,k\Omega$$

$$R_3 = \frac{v_g}{i_3} = \frac{1V}{121.36\,\mu A}$$

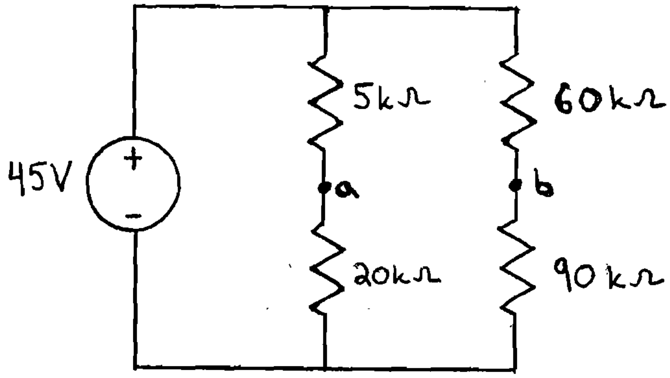
$$= 8.240\,k\Omega$$

$$R_4 = \frac{v_g}{i_4} = \frac{1V}{24.27\,\mu A}$$

$$= 41.20\,k\Omega$$

[+1]

2. Consider the circuit below, and answer the questions below.

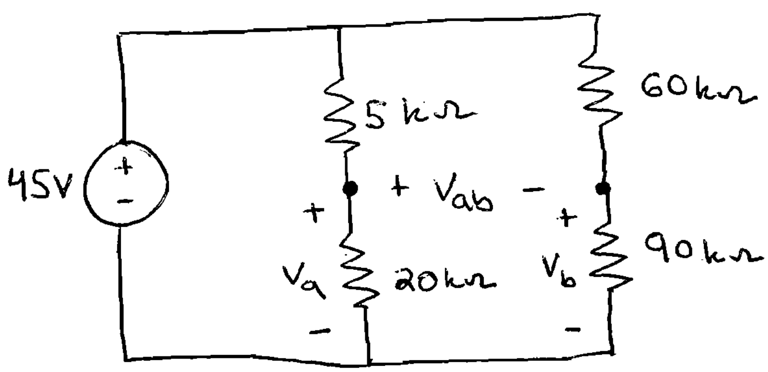


a) What is the open circuit voltage across the terminals a and b? Indicate clearly the polarity of your answer (variable definition and value). [3pts]

b) What is the Thévenin equivalent resistance between nodes a and b? [2pts]

c) What is the Thévenin equivalent circuit with respect to the nodes a and b? Indicate clearly the polarity of any sources. [1pt]

a) Two voltage dividers :



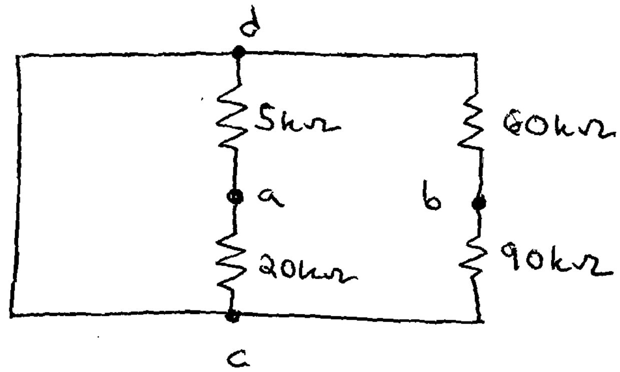
$$V_a = 45V \cdot \frac{20k\Omega}{20k\Omega + 5k\Omega} = 36\,V \qquad [+1]$$

$$V_b = 45V \cdot \frac{90k\Omega}{90k\Omega + 60k\Omega} = 27V \qquad [+1]$$

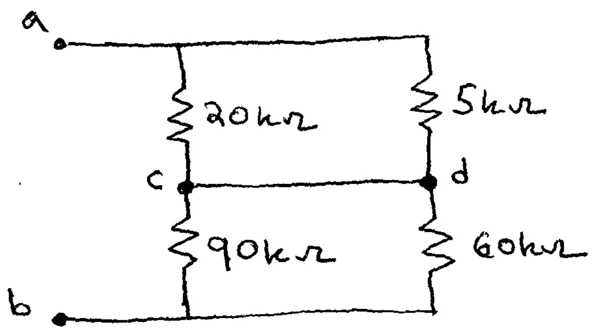
KVL : $0 = -V_a + V_{ab} + V_b \longrightarrow V_{ab} = V_a - V_b$ [+1]

$$= +9V = V_{oc}$$

b) Turn off source:



Redraw circuit:

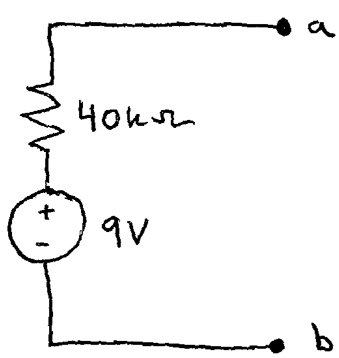


$$R_{eq} = R_T = (20k\Omega \,//\, 5k\Omega) + (90k\Omega \,//\, 60k\Omega) \quad [+1]$$

$$= \frac{20 \cdot 5}{20+5}\,k\Omega + \frac{90 \cdot 60}{90+60}\,k\Omega$$

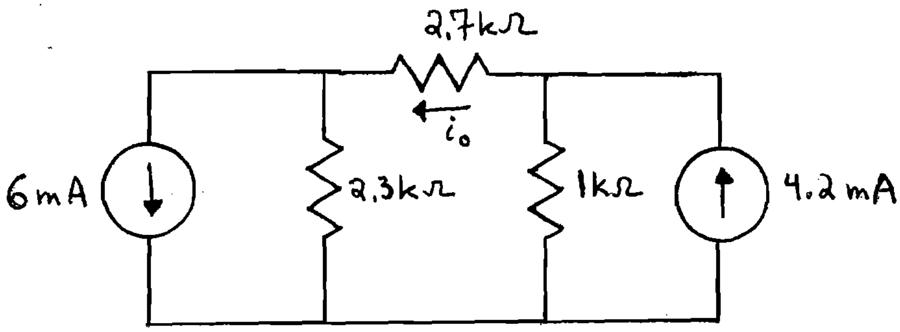
$$= 40k\Omega \quad [+1]$$

c)

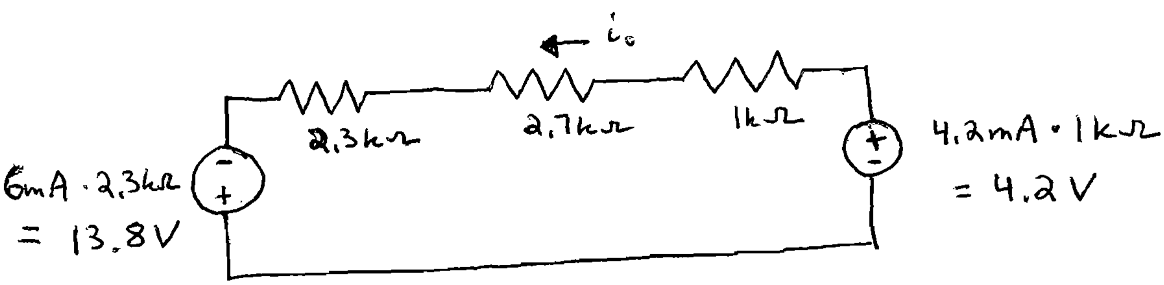


[+1]

3. Consider the circuit below, and answer the question below.

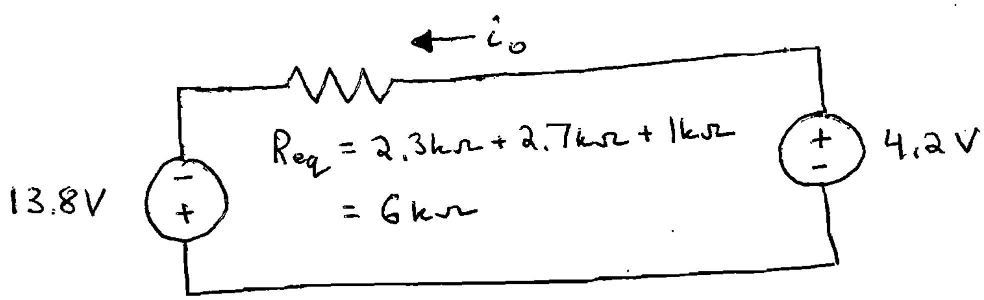


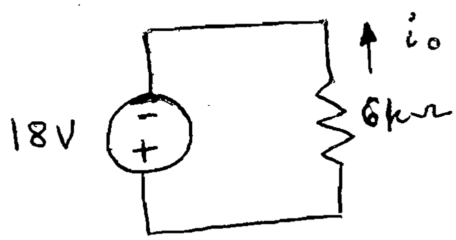
Use a series of source transformations (Thévenin <-> Norton) to find the current i_0. Show all source transformations with diagrams. [5pts]



[+1 for values
+1 for sign/polarity]

[+1 for values
+1 for sign/polarity]





$$i_o = \frac{18V}{6k\Omega} = +3mA$$

[+1]

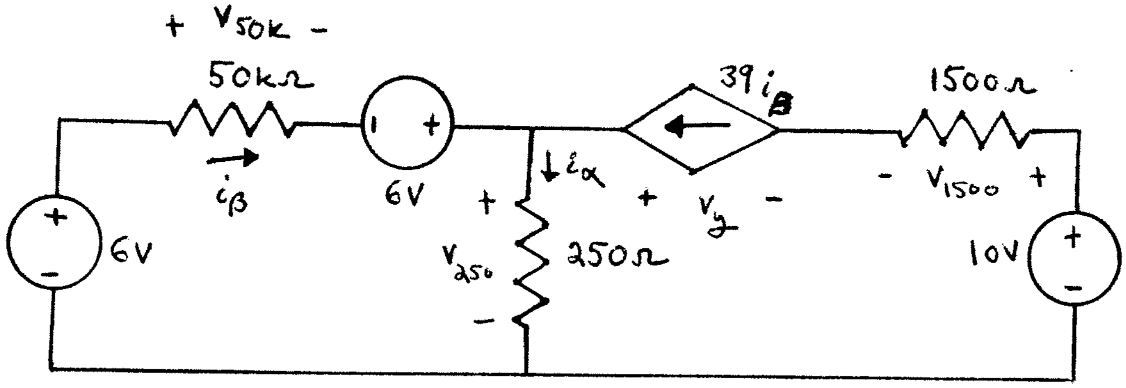
work space

ECSE-200 Quiz #4 (30 January 2009)

NAME____ McGill ID#________________

READ each question carefully. Give units on your answers (where appropriate).

1. Consider the circuit below and answer the question.



a) What is the current i_β? [5pts]

b) What is the voltage v_y? [2pts]

Show all your work, including any instance of KCL, KVL and element terminal laws.

KCL: $-i_\beta - 39\, i_\beta + i_\alpha = 0$ [+1]

KVL: $-6V + V_{50k} - 6V + V_{250} = 0$ [+1]

$-V_{250} + V_y - V_{1500} + 10V = 0$ [+1]

Ohm's Law:

$$\left.\begin{aligned} V_{50k} &= 50k\Omega\, i_\beta \\ V_{250} &= 250\Omega\, i_\alpha \end{aligned}\right\} \text{[+1]}$$

$V_{1500} = 1500\Omega\,(39\, i_\beta)$ [+1]

From KCL, $i_\alpha = 40\, i_\beta$

KVL + Ohm's Law + KCL, expressing in terms of i_β:

$$-6V + 50k\Omega\, i_\beta - 6V + 250\Omega\,(40\, i_\beta) = 0$$

$$-12V + 60k\Omega\, i_\beta = 0$$

$$i_\beta = \frac{12V}{60k\Omega}$$

$$= \underline{0.2mA \quad [+1]}$$

KVL + Ohm's Law + KCL:

$$v_y = V_{250} + V_{1500} - 10V$$

$$= 250\Omega\, i_\alpha + 1500\Omega\,(39\, i_\beta) - 10V$$

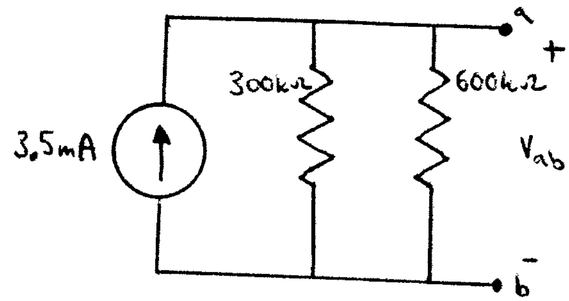
$$= 250\Omega\,(40\, i_\beta) + 1500\Omega\,(39\, i_\beta) - 10V$$

$$= 10k\Omega\, i_\beta + 58.5k\Omega\, i_\beta - 10V$$

$$= 68.5k\Omega \cdot 0.2mA - 10V$$

$$= \underline{3.7\,V} \quad [+1]$$

2. Consider the circuit below, and answer the questions below.

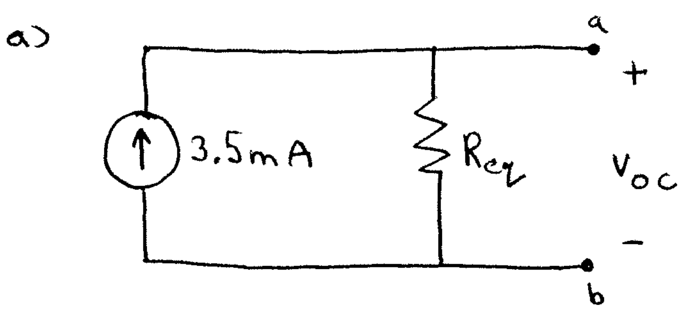


a) What is the open circuit voltage v_{ab} across the terminals a and b? Show all your work. [2pts]

b) When a voltmeter with 800kΩ internal resistance is used to measure the voltage v_{ab}, what is the measured value? Draw a circuit diagram to represent the circuit above and voltmeter (connected properly), and show all your work. [3pts]

c) What is the percentage error in the measurement? [1pt]

a)



$$R_{eq} = 300k\Omega \,//\, 600k\Omega$$

$$= \frac{300 \cdot 600}{300 + 600} k\Omega$$

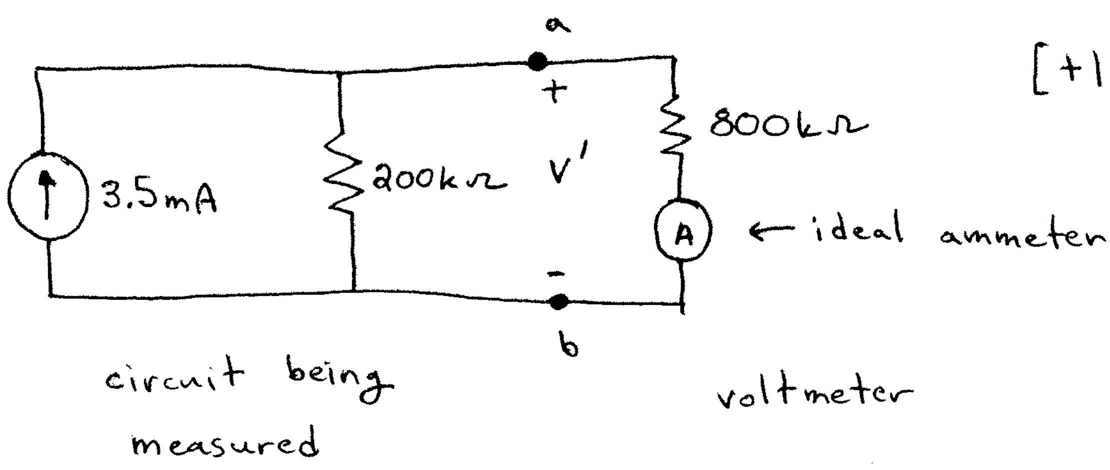
$$= 200\ k\Omega \quad [+1]$$

$$V_{oc} = 3.5mA \cdot R_{eq} = 3.5mA \cdot 200k\Omega$$

$$= \underline{700\ V} \quad [+1]$$

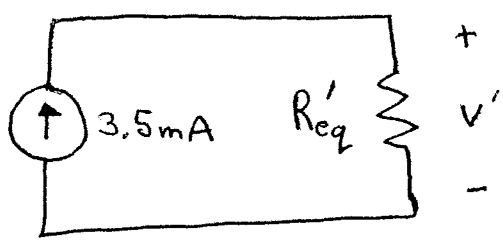
b)

work space



[+1]

We simplify the circuit with equivalent resistance.



$$R_{eq}' = 200\,k\Omega \,//\, 800\,k\Omega$$

$$= \frac{200 \cdot 800}{200 + 800}\,k\Omega$$

$$= 160\,k\Omega \qquad [+1]$$

$$V' = 3.5mA \cdot R_{eq}' = 3.5mA \cdot 160k\Omega$$

$$= \underline{560V} \qquad [+1]$$

This is the <u>measured</u> voltage.

c) Measurement error (as a percentage):

$$\frac{V' - V_{oc}}{V_{oc}} \times 100\% = \frac{560\,V - 700\,V}{700\,V} \times 100\%$$

$$= -0.2 \times 100\%$$

$$= -20\% \quad [+1]$$

↑

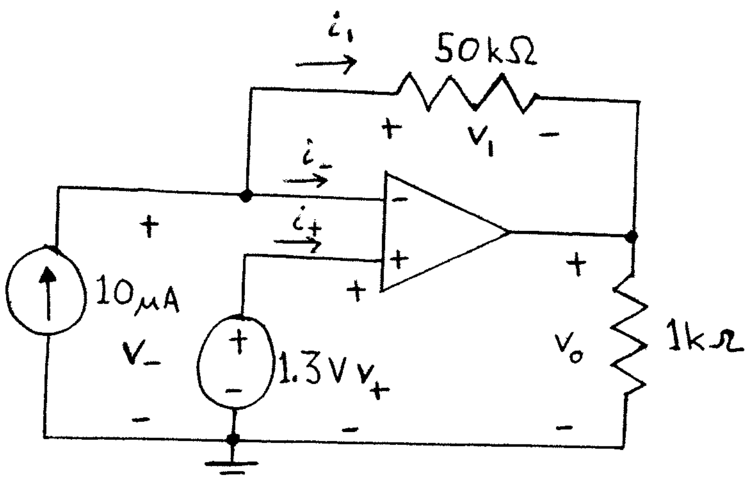
the measured value is less than the actual value of the open circuit voltage

ECSE-200 Quiz #5 (6 February 2009)

NAME________________________________ McGill ID#____________

READ each question carefully. Give units on your answers (where appropriate).

1. Consider the circuit below and answer the question.



What is the voltage v_O? Assume ideal op-amp behaviour. Show all your work. [4pts]

ideal op-amp: $v_- = v_+ = 1.3V$ [+1]

KCL: $0 = -10\mu A + i_- + i_1$

ideal op-amp: $i_+ = i_- = 0 A$

$\therefore\ i_1 = 10\ \mu A$ [+1]

Ohm's law: $v_1 = i_1 \times 50k\Omega$

$= 10\mu A \times 50k\Omega$

$= 500\ mV$ [+1]

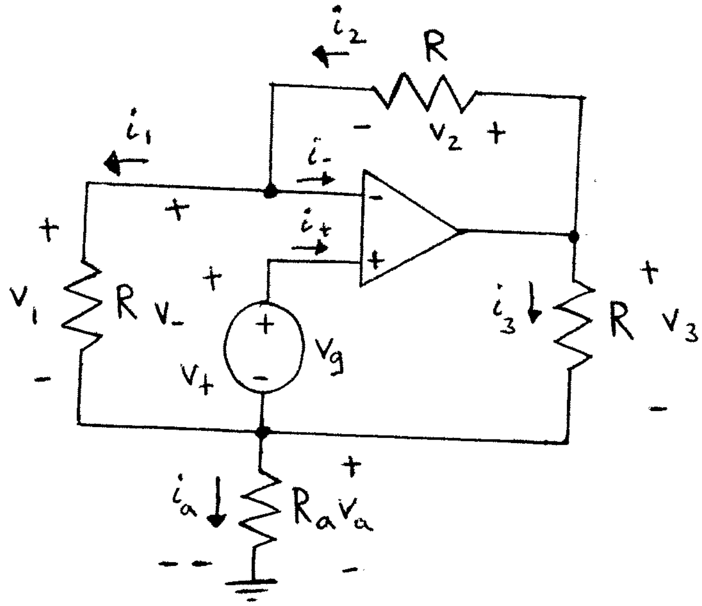
KVL: $0 = -v_- + v_1 + v_o$

$v_o = V_- - v_1$

$= 1.3V - 0.5V$

$= 0.8\ V$ [+1]

2. Consider the circuit below and answer the question below.



Find the current i_a in terms of the voltage v_g and resistor values. Assume ideal op-amp behaviour. Show all your work. [10pts]

Ideal op-amp: $\left.\begin{array}{l} V_+ = V_- \\ i_+ = i_- = 0 \end{array}\right\}$ [+1]

Ohm's Law: $\left.\begin{array}{ll} V_1 = i_1 R & V_3 = i_3 R \\ V_2 = i_2 R & V_a = i_a R_a \end{array}\right\}$ [+1]

KCL: $0 = i_1 + i_- - i_2$

$i_1 = i_2$ [+1]

KVL: $0 = -V_a - V_g + V_+$

$V_+ = V_a + V_g$ [+1]

KVL: $0 = -v_a - v_1 + v_-$

$v_- = v_a + v_1$ [+1]

Combining the previous KVL equations; and ideal op-amp gives $v_+ = v_- = v_a + v_g = v_a + v_1 \rightarrow v_1 = v_g$ [+1]

Combining KCL and Ohm's Law: $i_1 = i_2$

$$\frac{v_1}{R} = \frac{v_2}{R}$$

$$v_g = v_1 = v_2 \quad [+1]$$

KVL: $0 = -v_1 - v_2 + v_3$

$v_3 = v_1 + v_2 = 2v_g$ [+1]

KCL: $0 = -i_1 + i_+ - i_3 + i_a$

$i_a = i_1 + i_3 - i_+$ [+1]

$= \frac{v_g}{R} + \frac{2v_g}{R} - 0$ (from earlier eqns.)

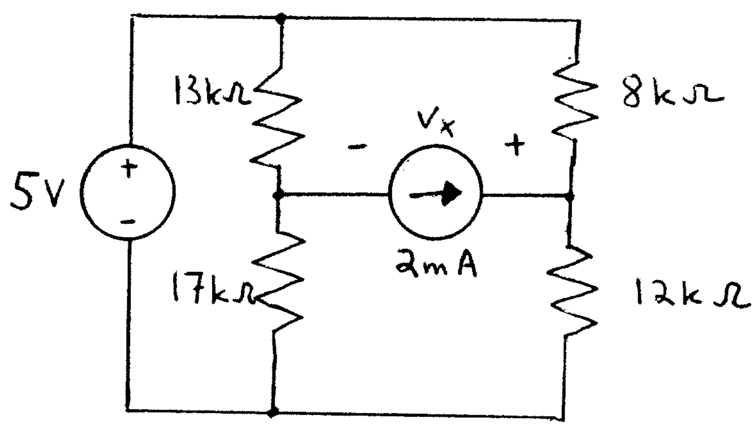
$= \frac{3v_g}{R}$ [+1]

ECSE-200 Quiz #6 (13 February 2009)

NAME________________________________ McGill ID#________________

READ each question carefully. Give units on your answers (where appropriate).

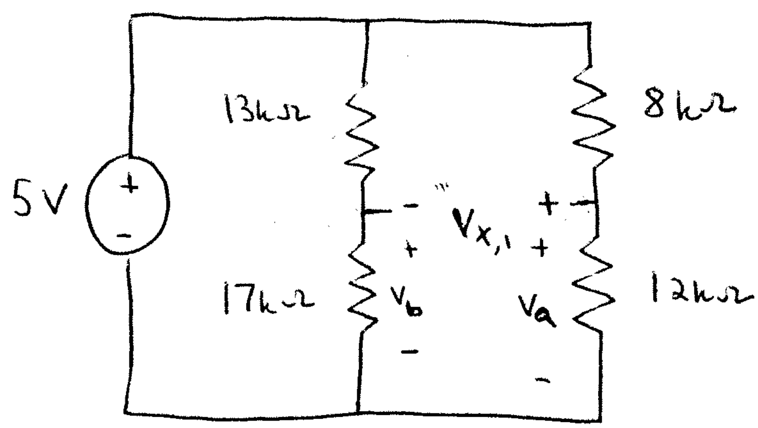
1. Consider the circuit below and answer the question.



What is the voltage v_x? Show all your work. [8pts]

Hint: *Consider applying the principle of superposition.*

2mA → 0



$V_{x,1} = V_a - V_b$ (KVL)

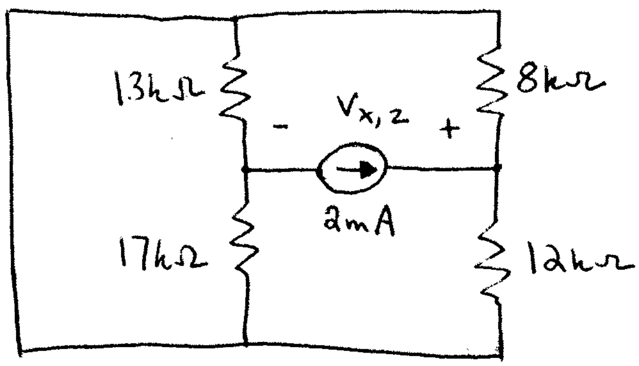
$= 5V \dfrac{12k\Omega}{8k\Omega + 12k\Omega}$ [+1]

$- 5V \dfrac{17k\Omega}{13k\Omega + 17k\Omega}$ [+1]

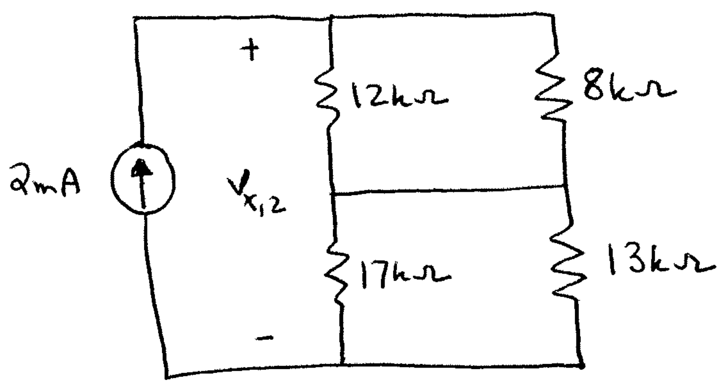
$= 0.0333 \times 5V$

$= 0.1667\ V$ [+1]

$5V \rightarrow 0$



$\Downarrow$



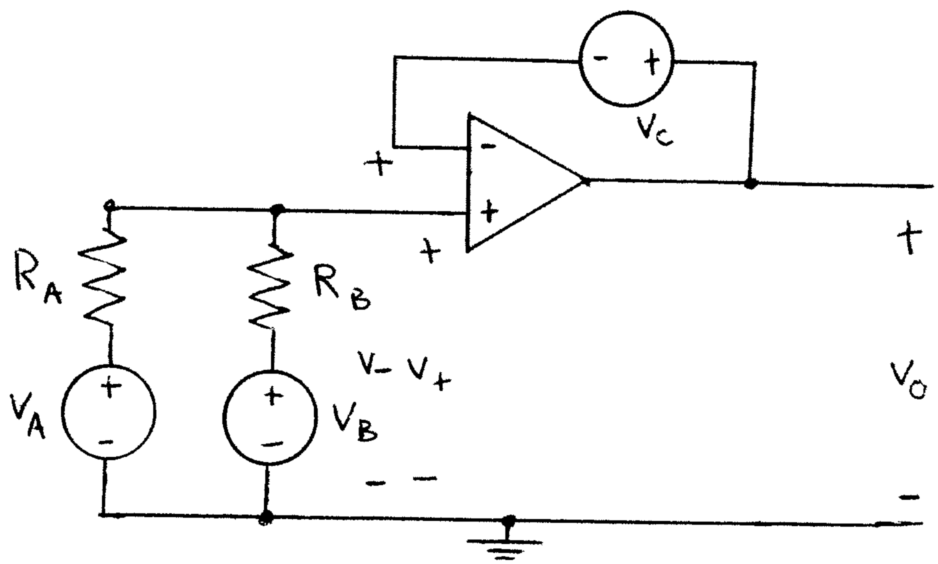
$$R_{eq} = (8k\Omega \,//\, 12k\Omega) + (17k\Omega \,//\, 13k\Omega)$$

$$= 12.1667\,k\Omega \quad [+2]$$

$$V_{x,2} = 2mA \cdot R_{eq} = 24.333\,V \quad [+1]$$

$$V_x = \underbrace{V_{x,1} + V_{x,2}}_{[+1]} = \underbrace{24.5\ V}_{[+1]}$$

2. Consider the circuit below and answer the question below.



It is known that $v_O = 0.75$V if $v_A = 2$V, $v_B = 0$V and $v_C = 0.25$V. It is also known that $v_O = 7.75$V if $v_A = 0$V, $v_B = 10$V and $v_C = 0.25$V.

Find the value of v_O (give a numerical answer) for the case where $v_A = 1$V, $v_B = 5$V and $v_C = 0.25$V. Assume ideal op-amp behaviour. Show all your work. [5pts]

Hint: *Consider the principle of superposition very carefully with respect to the* ***three*** *independent sources in this problem.*

Ideal op-amp: $V_+ = V_-$

KVL: $0 = -V_- - V_c + V_o$

$V_- = V_o - V_c$ [+1]

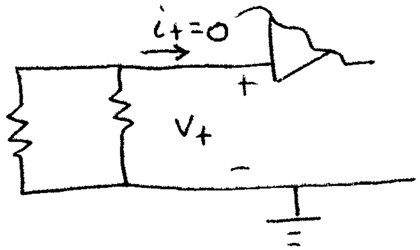
[V] for all values

V_A	V_B	V_c	V_-	V_o
2	0	0.25	0.50	0.75
0	10	0.25	7.50	0.75
2	0	0	0.50	0.50
0	10	0	7.50	7.50

[+1] (points awarded
[+1] for any multiple of this)

We also know $v_+ = 0$ if $v_A = 0$ and $v_B = 0$

since we have:



By ideal op-amp, $v_- = 0$.

Thus, $v_o = v_c$ if $v_A = 0, v_B = 0$. [+1]

We have the following table of responses:

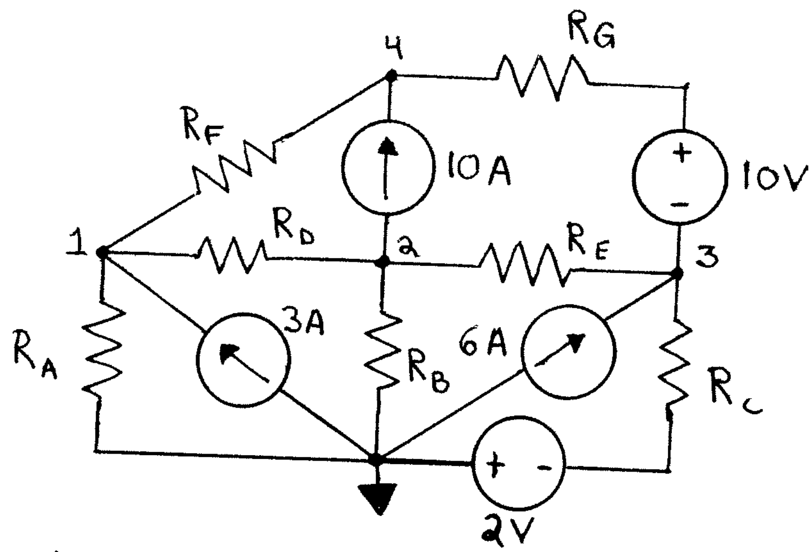
V_A	V_B	V_C	V_o
1	0	0	$0.50/2 = 0.25$
0	1	0	$7.50/10 = 0.75$
0	0	1	1.00
1	5	0.25	by principle of superposition: $1 \times 0.25\,V + 5 \times 0.75\,V + 0.25 \times 1.00\,V = 4.25V$ [+1]

ECSE-200 Quiz #7 (20 February 2009)

NAME________________________________ McGill ID#______________

READ each question carefully. Give units on your answers (where appropriate).

1. Consider the circuit below and answer the question.



Write the node voltage equations for the circuit above. The nodes have already been identified. *Do not solve for the voltage variables.* [4pts]

1: $$\frac{V_1}{R_A} - 3A + \frac{(V_1 - V_2)}{R_D} + \frac{(V_1 - V_4)}{R_F} = 0$$ [+1]

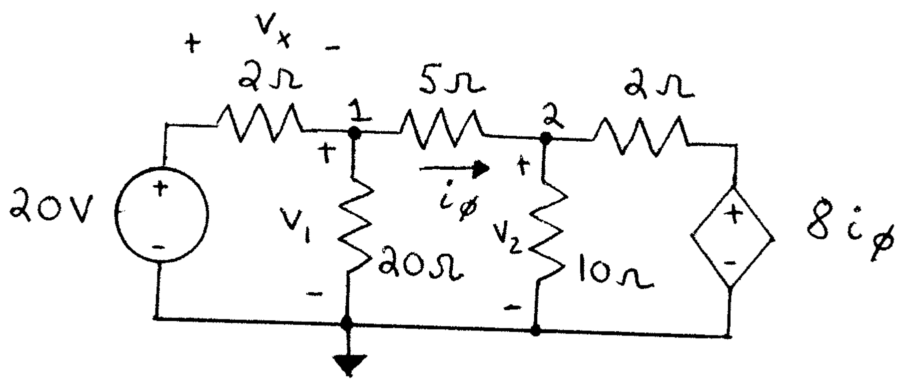
2: $$\frac{V_2}{R_B} + \frac{(V_2 - V_3)}{R_E} + 10A + \frac{(V_2 - V_1)}{R_D} = 0$$ [+1]

3: $$\frac{(V_3 + 2V)}{R_C} - 6A + \frac{(V_3 - V_2)}{R_E} + \frac{(V_3 + 10V - V_4)}{R_G} = 0$$ [+1]

4: $$\frac{(V_4 - V_1)}{R_F} - 10A + \frac{(V_4 - (V_3 + 10V))}{R_G} = 0$$ [+1]

work space

2. Consider the circuit below and answer the question below.



Find the power absorbed by the 2Ω resistor on the left (ie. the 2Ω resistor that is connected to the 20V independent source). Show all your work. [7pts]

Use node voltage method:

KCL at node 1:

$$\frac{V_1 - 20V}{2\Omega} + \frac{V_1}{20\Omega} + \frac{V_1 - V_2}{5\Omega} = 0 \qquad [+1]$$

$$V_1\left(\frac{1}{2\Omega} + \frac{1}{20\Omega} + \frac{1}{5\Omega}\right) - \frac{V_2}{5\Omega} = 10A$$

$$V_1 \frac{3}{4\Omega} - V_2 \frac{1}{5\Omega} = 10A$$

KCL at node 2:

$$\frac{V_2 - V_1}{5\Omega} + \frac{V_2}{10\Omega} + \frac{V_2 - 8\Omega\, i_\phi}{2\Omega} = 0 \qquad [+1]$$

control variable equation:

$$i_\phi = \frac{V_1 - V_2}{5\Omega} \qquad [+1]$$

$$\therefore \quad \frac{V_2 - V_1}{5\Omega} + \frac{V_2}{10\Omega} + \frac{V_2 - \frac{8}{5}(V_1 - V_2)}{2\Omega} = 0$$

$$v_2\left(\frac{1}{5\Omega}+\frac{1}{10\Omega}+\frac{1}{2\Omega}+\frac{8}{10\Omega}\right)-v_1\left(\frac{1}{5\Omega}+\frac{8}{10\Omega}\right)=0$$

$$v_2=\frac{\left(\frac{2}{10}+\frac{8}{10}\right)v_1}{\left(\frac{2}{10}+\frac{1}{10}+\frac{5}{10}+\frac{8}{10}\right)} = \frac{5}{8}\,v_1$$

Substitute back into node 1 equation:

$$10V = v_1\,\frac{3}{4} - v_2\,\frac{1}{5}$$

$$= v_1\,\frac{3}{4} - \left(\frac{5}{8}v_1\right)\frac{1}{5}$$

$$= v_1\left(\frac{3}{4}-\frac{1}{8}\right)$$

$$\therefore\ v_1=\frac{10V}{5/8}=\underbrace{16V}_{[+1]} \qquad v_2=\frac{5}{8}v_1 = 10V$$

KVL: $0=-20V+v_x+v_1$

$$v_x = 20V - v_1 = 4V \qquad [+1]$$

Power absorbed by 2Ω resistor $= \dfrac{v_x^2}{2\Omega}$

$$= \frac{(4V)^2}{2\Omega}$$

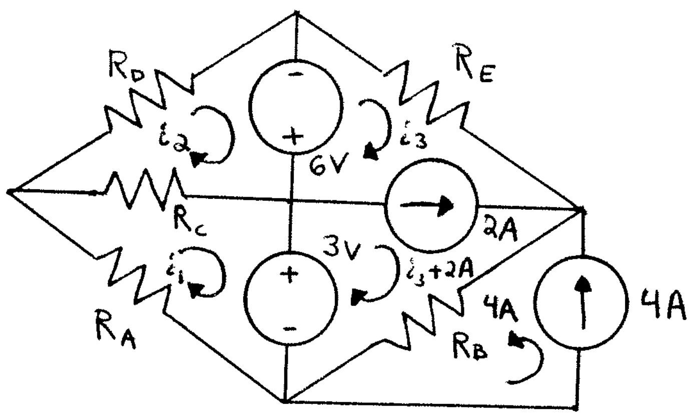
$$= \underline{+8W} \qquad [+2]$$

ECSE-200 Quiz #8 (6 March 2009)

NAME______________________________ McGill ID#__________________

READ each question carefully. Give units on your answers (where appropriate). Give only one answer to every question (multiple answers will not be accepted).

1. Consider the circuit below and answer the question.



[+1]

Write the mesh current equations for the circuit above. Indicate clearly in the diagram the definitions of your mesh currents.

Hint: *Note carefully the number of meshes and super-meshes.*

Do not solve for the mesh variables. [4pts]

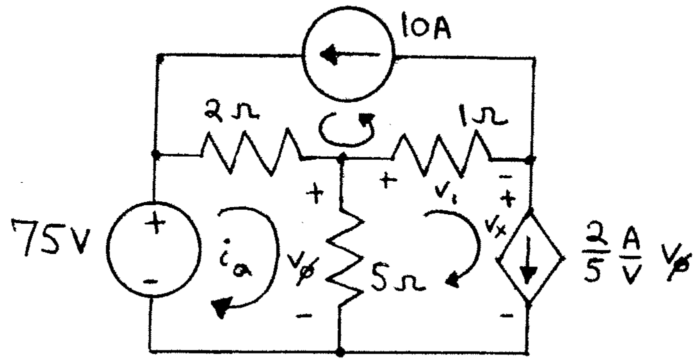
$$i_1 \cdot R_A + (i_1 - i_2) R_C + 3V = 0 \qquad [+1]$$

$$i_2 \cdot R_D + (-6V) + (i_2 - i_1) R_C = 0 \qquad [+1]$$

$$i_3 \cdot R_E + ((i_3 + 2A) + 4A) R_B + (-3V) + 6V = 0 \qquad [+1]$$

work space

2. Consider the circuit below and answer the questions.



a) Find the mesh current i_a. Show all your work. [4pts]

b) How much power is the dependent current source delivering to the circuit or absorbing from the circuit? State your answer clearly. [4 pts]

a) 1) $-75V + (i_a + 10A)2\Omega + (i_a - \frac{2}{5}\frac{A}{V}V_\phi)5\Omega = 0$ [+1]

2) $V_\phi = \left(i_a - \frac{2}{5}\frac{A}{V}V_\phi\right)5\Omega$ [+1]

$V_\phi = i_a \cdot 5\Omega - 2V_\phi \quad \therefore \quad V_\phi = \frac{5}{3}\Omega\, i_a$ [+1]

Substitution back into 1) gives:

$-75V + 2\Omega i_a + 20V + 5\Omega i_a - 2V_\phi = 0$

$-55V + 7\Omega i_a - 2 \cdot \frac{5}{3}\Omega i_a = 0$

$i_a = \frac{55V}{(7 - {}^{10}/_{3})\Omega} = 15A$ [+1]

b) $V_\phi = \frac{5}{3}\Omega \cdot i_a = \frac{5}{3}\Omega \cdot 15A = 25V$ [+1]

$$V_1 = \left(\frac{2}{5}\frac{A}{V} \cdot V_\phi + 10A\right) \cdot 1\Omega$$
$$= \left(\frac{2}{5}\frac{A}{V} \cdot 25V + 10A\right) \cdot 1\Omega$$
$$= 20V$$

KVL: $-V_\phi + V_1 + V_x = 0$

$$V_x = V_\phi - V_1$$
$$= 25V - 20V = 5V$$ [+1]

Power absorbed by the dependent source is: $P_{abs} = V_x \cdot \left(\frac{2}{5}\frac{A}{V} \cdot V_\phi\right)$ (note passive sign convention)

$$= 5V \cdot \left(\frac{2}{5}\frac{A}{V} \cdot 25V\right)$$
$$= \underline{+50W}$$

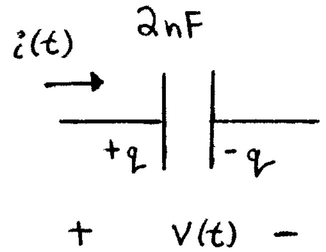
[+1 for value
+1 for sign/explanation]

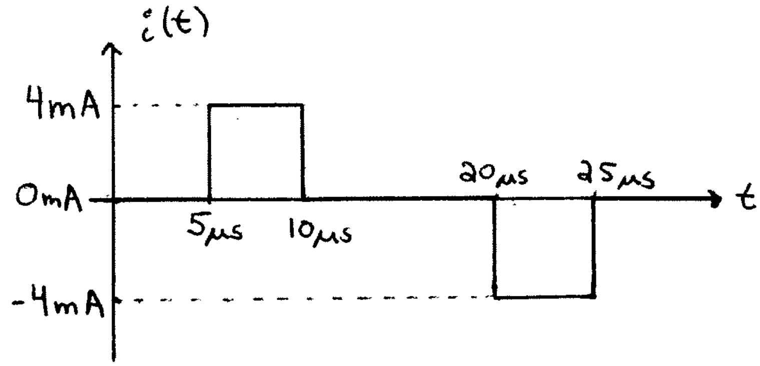
ECSE-200 Quiz #9 (13 March 2009)

NAME________________________________ McGill ID#________________

READ each question carefully. Give units on your answers (where appropriate). Give only one answer to every question (multiple answers will not be accepted).

1. Consider the ideal capacitor and the plot below, and answer the question.





The capacitor current is given above, note that it is a piece-wise linear function of time. Assume the capacitor stores 0 J of energy at time t=0 s.

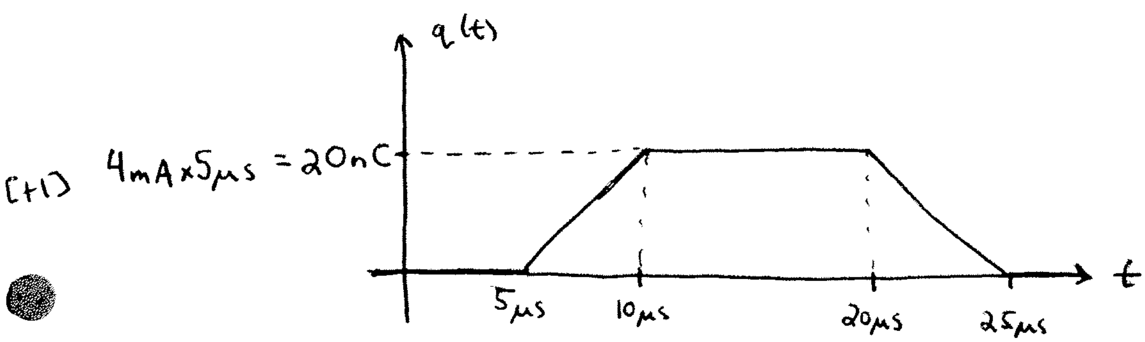
a) Plot the charge separation $q(t)$ versus time t, indicating clearly in SI units points of interest on the time axis and on the charge axis. Give the equation that you are using to arrive at your plot. [4pts]

b) Plot the voltage $v(t)$ versus time t, indicating clearly in SI units points of interest on the time axis and on the voltage axis. Give the equation that you are using to arrive at your plot. [4pts]

a) $q(t) - q(0) = \int_0^t i(t')\,dt'$ [+1]

$q(0) = 0$ since $U(0) = \frac{1}{2}\frac{q^2(0)}{C} = 0$ [+1]

$\therefore$ $q(t)$ = integral of $i(t)$ versus t curve.

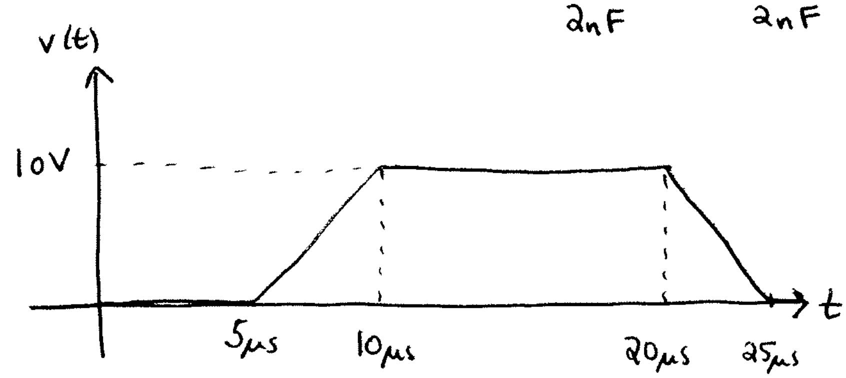


[+1] for shape and values

b) $q = Cv \quad \therefore \quad v = \dfrac{q}{C}$ [+1]

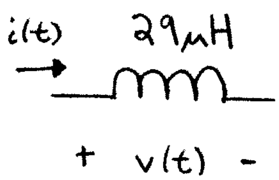
$v(t)$ has the same shape as $q(t)$.

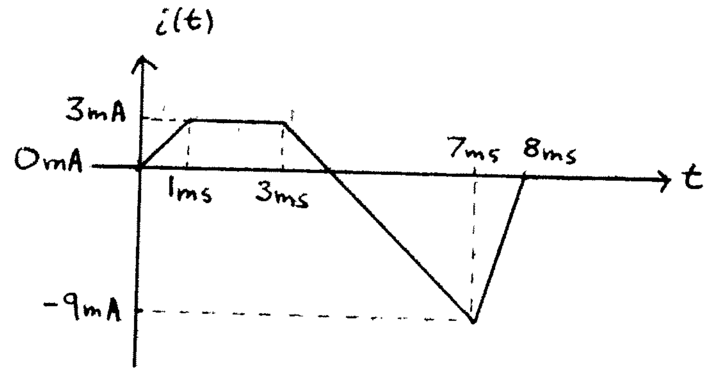
$$v(10\mu s) = \frac{q(10\mu s)}{2nF} = \frac{20nC}{2nF} = 10V \quad [+1]$$



[+2] for shape and values

2. Consider the ideal inductor and the plot below, and answer the question.





a) At what point in time does the inductor store the most energy? What is the value of this maximum in stored energy? [3pts]

b) What is the maximum (most positive) value of the voltage *v*(*t*)? Show your work. Over what time interval is *v*(*t*) equal to its maximum value? [3 pts]

c) What is the minimum (most negative) value of the voltage *v*(*t*)? Over what time interval is *v*(*t*) equal to its minimum value? [2 pts]

d) How much power is being absorbed or delivered by the inductor at t=2ms? Justify your answer. [2pts]

a) $U(t) = \frac{1}{2} L i^2(t)$ [+1] ∴ we want the point of maximum $i^2(t)$

$t = 7ms$ [+1]

$U_{max} = \frac{1}{2} \cdot 29mH \cdot (-9mA)^2$

$= 1.175\ nJ$ [+1]

b) $v(t) = L \cdot \frac{di}{dt}$ ∴ we want the interval with the most positive slope of i(t) versus t.

$7ms < t < 8ms$ [+1]

$V_{max} = 29\mu H \cdot \frac{9mA}{1ms} = 261\ \mu V$ [+1]

c) we want the interval of most negative slope of $i(t)$ versus t.

$3ms < t < 7ms$ [+1]

$$V_{min} = 29\mu H \cdot \frac{-9\,mA - (+3mA)}{7ms - 3ms}$$

$$= 29\mu H \cdot \frac{-12mA}{4ms}$$

$$= \underline{-87\mu V}$$ [+1]

d) Power absorbed at $t = 2ms$ = $i(t) \cdot v(t) \Big|_{t=2ms}$

[+1] $\left.\begin{aligned} i(t) &= 3mA \\ v(t) &= L\frac{di}{dt} = 0V \end{aligned}\right\}$ at $t = 2ms$ from graph

$P(2ms) = i(2ms)\,v(2ms) = 0W$ [+1]

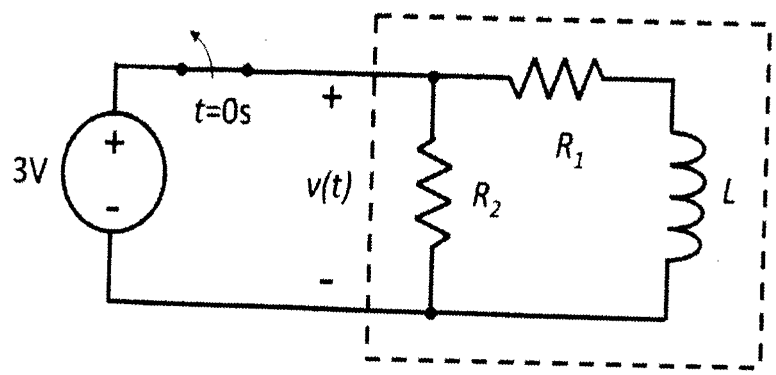
$\therefore$ There is 0W absorbed (delivered) by the inductor.

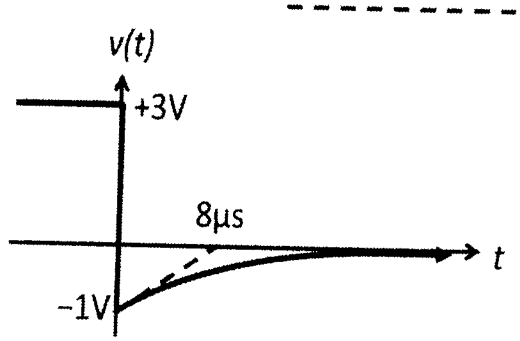
ECSE-200 Quiz #10 (20 March 2009)

NAME______________________________ McGill ID#__________________

READ each question carefully. Give units on your answers (where appropriate). Give only one answer to every question (multiple answers will not be accepted).

Consider the circuit and plot below.



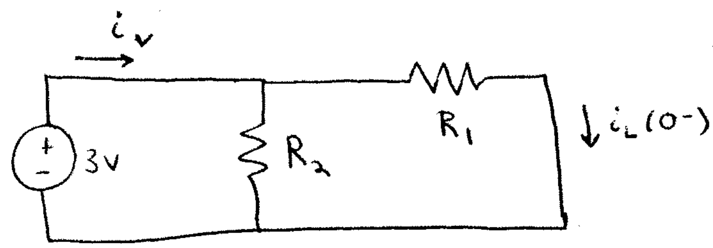


The circuit is in dc steady state for $t < 0$. The power delivered by the voltage source for $t<0$ is 120mW. Use the information supplied to answer the following questions.

a) What are the values of R_1 and R_2? Show ***all*** of your work. [8pts]

b) What is the value of L? Show ***all*** of your work. [2pts]

a) For $t < 0$:



Power delivered by source

$$P = 3V \cdot i_v$$

$$120mW = 3V \cdot i_v$$

$$i_v = 120mW / 3V = 40mA \quad [+1]$$

We therefore know the equivalent resistance:

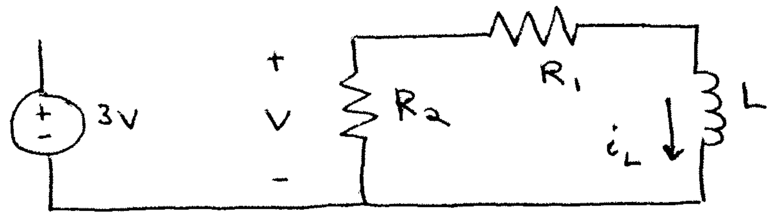
$$\frac{3V}{i_v} = R_1 // R_2$$

$$R_1 // R_2 = \frac{3V}{40mA} = 75\Omega \quad [+1]$$

We also know the inductor current at $t = 0^-$:

$$i_L(0^-) = \frac{3V}{R_1} \quad [+1]$$

For $t > 0$:



Continuity of inductor current gives $i_L(0+) = i_L(0-) = \frac{3V}{R_1}$

↑ [+1]

Ohm's Law gives: $v(t) = -i_L(t)\, R_2$ $\quad (t>0)$

$$\therefore\ v(0+) = -i_L(0+)\, R_2$$

$$v(0+) = -\frac{3V}{R_1} \cdot R_2 \qquad [+1]$$

From the graph $v(0+) = -1V$, therefore:

$$\frac{R_2}{R_1} = \frac{-1V}{-3V} = \frac{1}{3} \qquad [+1]$$

We have two equations for R_1 and R_2:

$$R_1 \,//\, R_2 = \frac{R_1 R_2}{R_1 + R_2} = 75\Omega$$

$$\frac{R_2}{R_1} = \frac{1}{3}$$

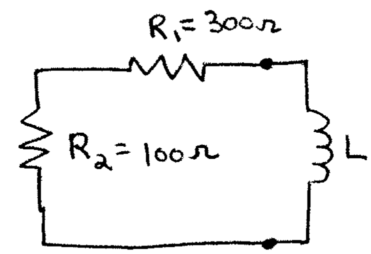
Combine: $$\frac{\frac{1}{R_1} \cdot R_1 R_2}{\frac{1}{R_1}(R_1 + R_2)} = \frac{R_2}{1 + R_2/R_1} = 75\Omega$$

$$\frac{R_2}{1+\frac{1}{3}} = \frac{R_2}{\frac{4}{3}} = 75\,\Omega \qquad R_2 = \frac{4}{3}\cdot 75\,\Omega = 100\,\Omega \quad [+1]$$

$$R_1 = 3R_2 = 300\,\Omega \quad [+1]$$

b) From the graph, we know $\tau = 8\,\mu s$.

We also know $\tau = \dfrac{L}{R_{eq}}$ [+½]



$$R_{eq} = R_1 + R_2 \quad [+½]$$
$$= 400\,\Omega$$

$$\therefore\ 8\,\mu s = \frac{L}{400\,\Omega}$$

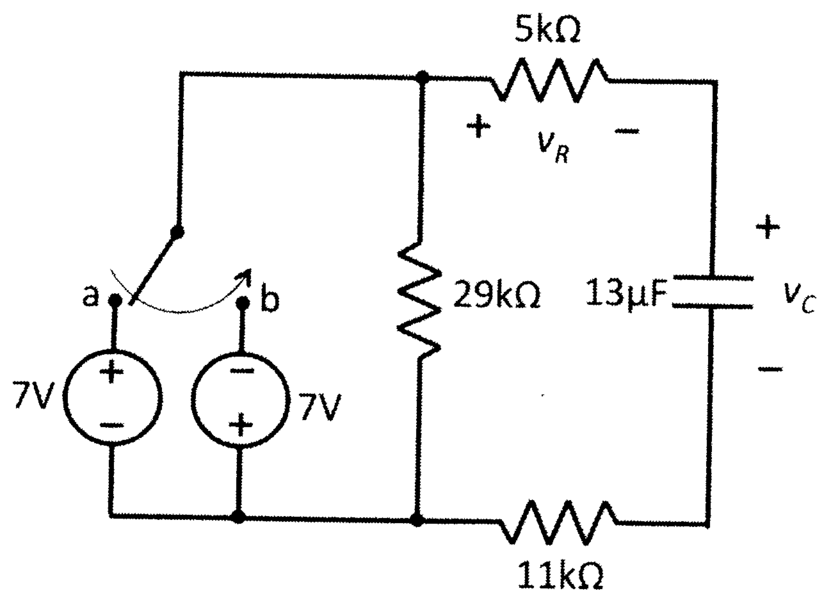
$$L = 3.2\ mH \quad [+1]$$

ECSE-200 Quiz #11 (27 March 2009)

NAME________________________________ McGill ID#____________________

READ each question carefully. Give units on your answers (where appropriate). Give only one answer to every question (multiple answers will not be accepted).

1. Consider the circuit below.



a) With the switch fixed in the position "a", what is the dc steady state value of the indicated capacitor voltage v_C? [2pts]

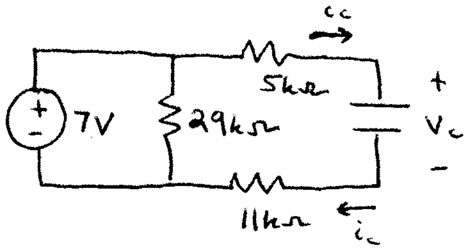
b) With the switch fixed in the position "b", what is the dc steady state value of the indicated capacitor voltage v_C? [2pts]

c) With the switch in position "b", what is the time constant for this RC circuit? **Hint:** Consider very carefully the value of Thévenin resistance. [3pts]

d) Assume dc steady state conditions for t<0 with the switch in position "a". The switch moves instantaneously from position "a" to "b" at t=0. What is $v_C(t)$ for t < 0 and for t > 0? [4 pts]

e) Making the same assumptions as in part d), and using your results of part d) if necessary, what is $v_R(t)$ for t<0 and for t>0? [3 pts]

a)

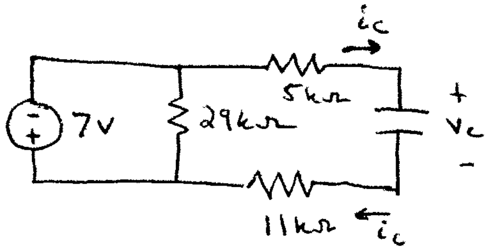


$V_c = 7V - i_c(5k\Omega + 11k\Omega)$

$i_c = 0$ because capacitor is open in dc steady state [+1]

$\therefore V_c = 7V$ [+1]

b)

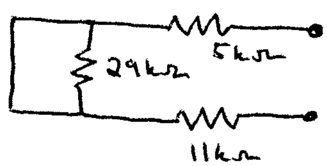


$V_c = -7V - i_c(5k\Omega + 11k\Omega)$

$i_c = 0$ because capacitor is open in dc steady state [+1]

$\therefore V_c = -7V$ [+1]

c) Turn off source:



$R_{eff} = 5k\Omega + 0k\Omega // 29k\Omega + 11k\Omega$ [+1]

$= 16k\Omega$ [+1]

$\tau = R_{eff} \cdot C = 16k\Omega \cdot 13\mu F$
$= 208\,ms$ [+1]

d) $v_c(0-) = v_c(0+) = 7V$ $\quad v_c(\infty) = -7V$ [+1]

$t < 0$ $\quad v_c(t) = 7V$ [+1]

$t > 0$ $\quad v_c(t) = v_c(\infty) + [v_c(0) - v_c(\infty)]\exp(-t/\tau)$ [+1]

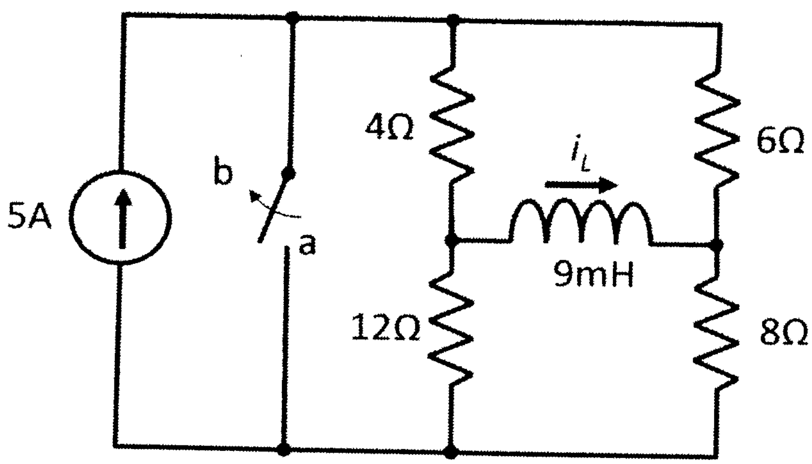
$= -7V + 14V\exp(-t/208\,ms)$ [+1]

e) $v_R(t) = i_c(t) \times 5k\Omega$ [+1]

[+1] $\left\{ i_c(t) = C\dfrac{dv_c}{dt} = \begin{cases} 13\mu F \times \dfrac{d}{dt}\left(-7V + 14V\exp(-t/208\,ms)\right) = -875\mu A e^{\frac{-t}{208ms}} \\ 0 \quad t<0 \text{ because of steady state} \end{cases} \right.$

[+1] $t<0$ $\quad v_R(t) = i_c(t) \times 5k\Omega = 0V$ $\qquad t>0$ $\quad v_R(t) = i_c(t)\cdot 5k\Omega = -4.375V\exp\left(\frac{-t}{208ms}\right)$

2. Consider the circuit below.



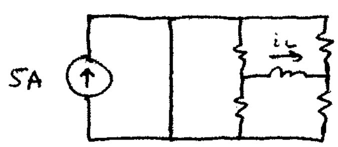
a) With the switch fixed in the closed position "a", what is the dc steady state value of the indicated inductor current i_L? [1pt]

b) With the switch fixed in the open position "b", what is the dc steady state value of the indicated inductor current i_L? [3pts]

c) With the switch in the open position "b", what is the time constant for this RL circuit? **Hint:** Consider very carefully the value of Thévenin resistance. [3pts]

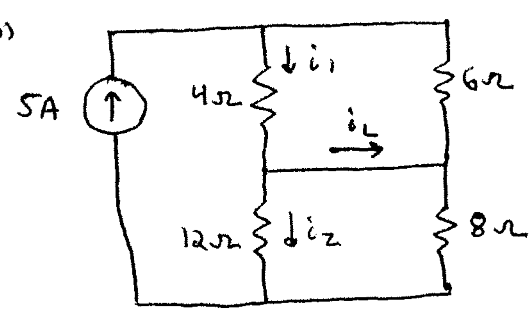
d) Assume dc steady state conditions for t<0 with the switch in the closed position "a". The switch moves instantaneously from the closed position "a" to the open position "b" at t=0. What is $i_L(t)$ for t < 0 and for t > 0? [4 pts]

a)



voltage across all resistors is zero (inductor is a short at dc steady state), hence no currents flow and $i_L = 0\,A$. [+1]

b)



(inductor is shown as short for emphasis)

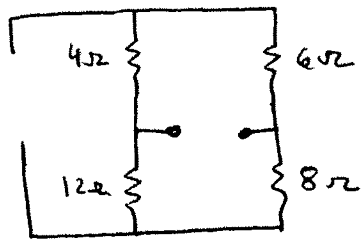
$$i_1 = 5A \times \frac{6\Omega}{4\Omega + 6\Omega} = 3A \quad [+1]$$

$$i_2 = 5A \times \frac{8\Omega}{12\Omega + 8\Omega} = 2A \quad [+1]$$

KCL : $-i_1 + i_L + i_2 = 0$

$$i_L = i_1 - i_2 = 1A \quad [+1]$$

c) Turn source off:



$$R_{eq} = (4\Omega + 6\Omega) // (8\Omega + 12\Omega) \quad [+1]$$
$$= 10\Omega // 20\Omega$$
$$= 6.667\ \Omega \quad [+1]$$

$$\tau = \frac{L}{R_{eq}} = \frac{9\,mH}{6.667\Omega} = 1.35\ ms \quad [+1]$$

d) $i_L(0^-) = i_L(0^+) = 0\ A$ by continuity of inductor current [+1]

$i_L(\infty) = 1\ A$

$t < 0 \quad i_L(t) = 0\ A$ [+1]

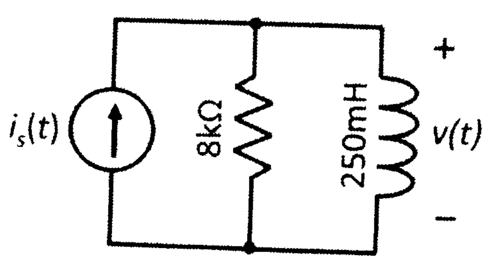
$t > 0 \quad i_L(t) = i_L(\infty) + (i_L(0) - i_L(\infty))\exp(-t/\tau)$ [+1]

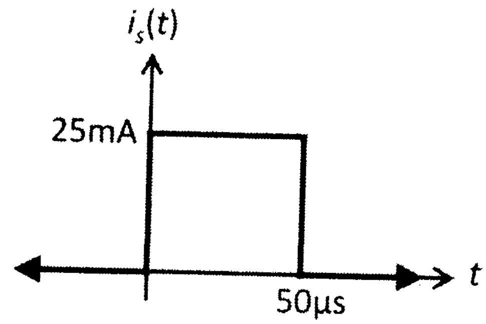
$= 1A - 1A\exp(-t/1.35\,ms)$ [+1]

NAME________________________________ McGill ID#__________________

READ each question carefully. Give units on your answers (where appropriate). Give only one answer to every question (multiple answers will not be accepted).

1. Consider the circuit and current plot below. The circuit is in dc steady state for $t < 0$.





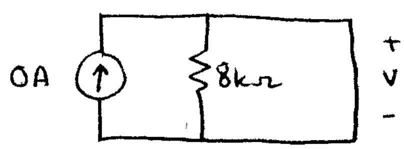
a) What is the voltage $v(t)$ for $t < 0$? [2pts]

b) What is the voltage $v(t)$ for $0 < t < 50\mu s$? [5pts]

c) What is the voltage $v(t)$ for $50\mu s < t$? [5pts]

d) Plot $v(t)$ versus t. [3pts]

a) $t < 0$



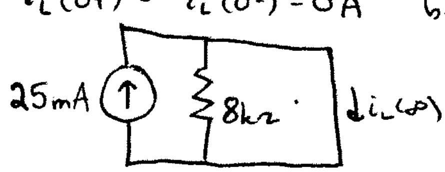
$\underline{v(t) = 0}$ [+1]

in dc steady state, inductor acts as a short [+1]

b) $0 < t < 50\mu s$

initial conditions: $i_L(0-) = 0\text{ A}$ $\quad (t<0,\ i_s = 0\text{A})$

$i_L(0+) = i_L(0-) = 0\text{A}$ by continuity of inductor current [+1]



final condition: $i_L(\infty) = 25\text{mA}$ (current divider with a short) [+1]

$\tau = \frac{L}{R_T}$ $\quad R_T = 8k\Omega \quad$ (Norton equivalent circuit drives inductor)

$\tau = 250mH / 8k\Omega = 31.25\mu s \quad [+1]$

$i_L(t) = i_L(\infty) + (i_L(0+) - i_L(\infty)) \exp(-t/\tau)$

$= 25mA - 25mA \exp(-t/31.25\mu s) \quad [+1]$

$v(t) = 250mH \cdot \frac{di_L}{dt} = 250mH \left(-25mA \cdot -\frac{1}{31.25\mu s} \exp(-t/31.25\mu s) \right)$

$= 200V \exp(-t/31.25\mu s) \quad [+1]$

c) $50\mu s < t$

initial conditions: $i_L(50\mu s+) = i_L(50\mu s-) = 25mA - 25mA \exp\left(-\frac{50\mu s}{31.25\mu s}\right)$

$= 19.95mA \quad [+1]$

final conditions: same as for $t < 0$, $i_L(\infty) = 0A \quad [+1]$

$\tau = 31.25\mu s \quad [+1]$

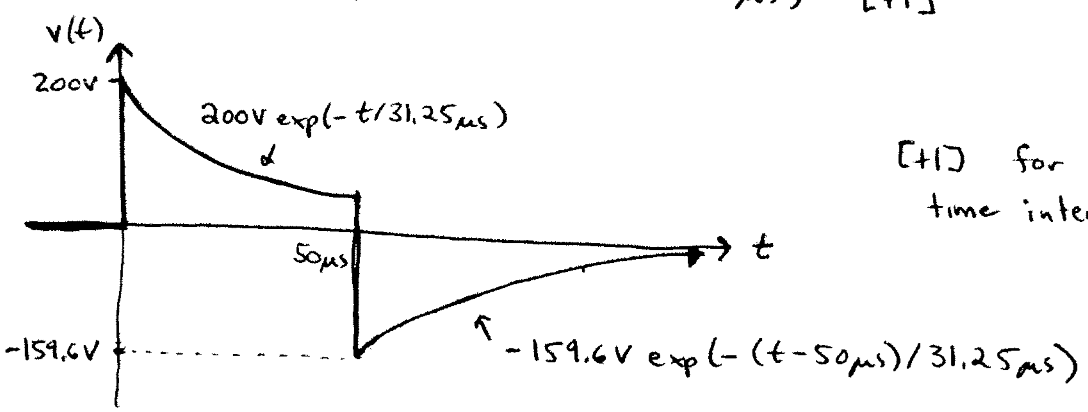
$i_L(t) = i_L(\infty) + (i_L(50\mu s+) - i_L(\infty)) \exp(-(t - 50\mu s)/\tau)$

$= 19.95mA \exp(-(t - 50\mu s)/31.25\mu s) \quad [+1]$

$v(t) = 250mH \cdot \frac{di_L}{dt} = 250mH \left(19.95mA \cdot \frac{-1}{31.25\mu s} \exp(-(t-50\mu s)/31.25\mu s) \right)$

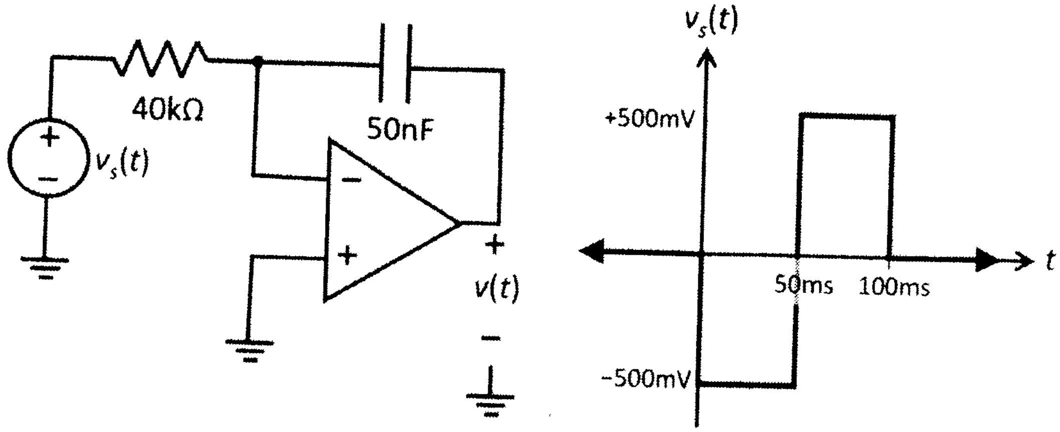
$= -159.6V \exp(-(t - 50\mu s)/31.25\mu s) \quad [+1]$

d)



[+1] for each time interval

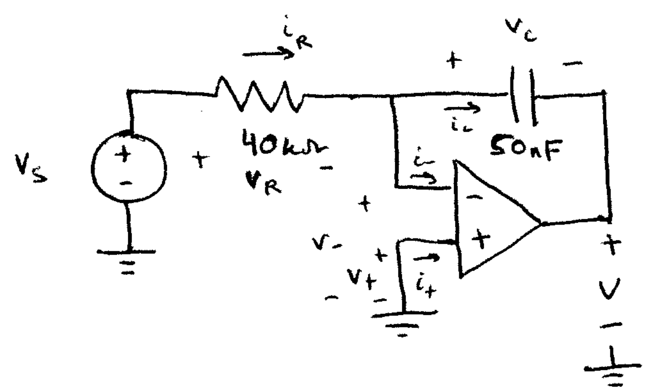
2. Consider the circuit below. Assume ideal op-amp behaviour.



a) Express the voltage $v(t)$ for $t>0$ explicitly in terms of an arbitrary $v_s(t)$. Assume that the capacitor stores zero energy at $t = 0$s. [5pts]

b) For the $v_s(t)$ indicated in the plot above, give a plot of the resulting $v(t)$ versus t. Assume again that the capacitor stores zero energy at $t = 0$s. Indicate clearly all points of interest (for example: maxima, minima, zero-crossings) on the voltage axis and on the time axis. [3pts]

a)



ideal op-amp:

$i_+ = i_- = 0A$

$V_+ = V_-$

$V_+ = 0V \therefore V_- = V_+ = 0V$

KVL: $-V_s + V_R + V_- = 0 \rightarrow V_R = +V_s$ [+1]

Ohm: $i_R = \dfrac{V_R}{40k\Omega}$

KCL: $i_c = i_R - i_- = i_R = V_R/40k\Omega = V_s/40k\Omega$ [+1]

capacitor: $i_c = 50nF \dfrac{dv_c}{dt}$ or $v_c(t) - v_c(0) = \dfrac{1}{50nF}\int_0^t i_c(t')\,dt'$ [+1]

KVL: $-V_- + V_c + V = 0 \rightarrow V = -V_c$ [+1]

$$\therefore \quad v(t) - v(0) = -(v_c(t) - v_c(0))$$

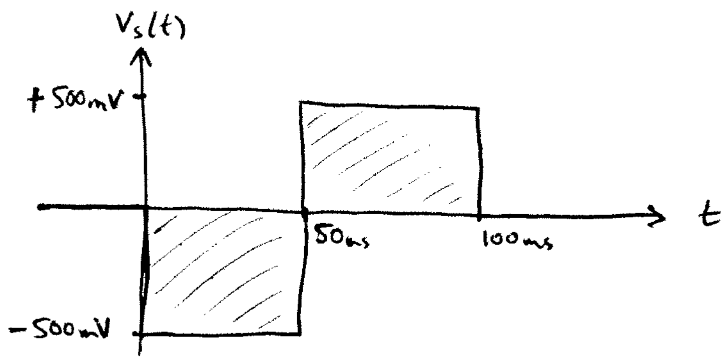
$$v(t) = \frac{-1}{50nF} \int_0^t i_c(t')\,dt'$$ where we use $v_c(\ ) = -v($

$$= \frac{-1}{50nF \cdot 40k\Omega} \int_0^t v_s(t')\,dt'$$

$$= \frac{-1}{2ms} \int_0^t v_s(t')\,dt'$$ [+1]

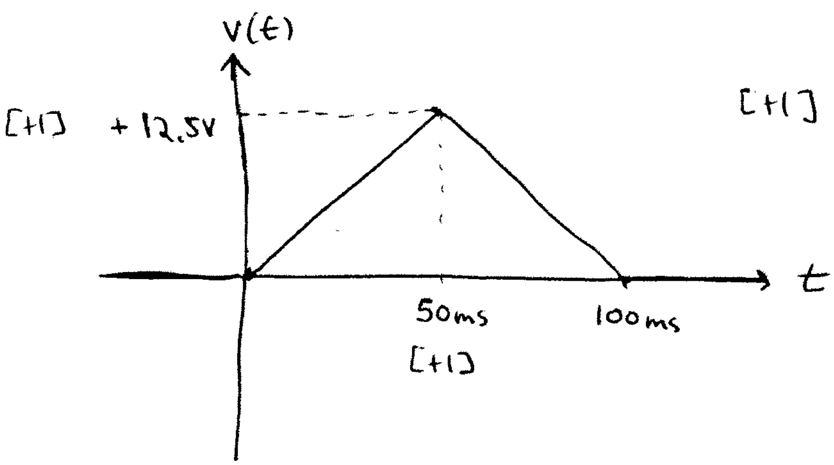
this circuit is an integrator.

b)



$$v(50_{ms}) = \frac{-1}{2ms} \int_0^{50ms} -500mV\,dt' = \frac{-1}{2ms} \times -500mV \cdot 50ms$$

$$= 12.5V$$



[+1] for shape