McGill

Today's Outline

3. Analysis Methods

- Nodal Analysis with Voltage Sources

Recall the Node Voltage Method

Step #1: Define a reference node.

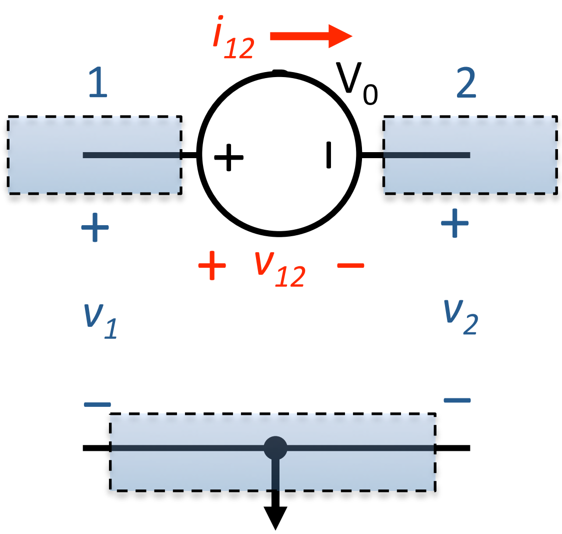
Step #2: Label remaining nodes, and define node voltage variables with respect to reference node.

Step #3: Write KCL equations in terms of node voltage variables, intrinsically using KVL and terminal laws (such as Ohm's law).

Step #4: Solve the linear system of equations, and use the node voltages to calculate the desired quantity.

Voltage Sources and Node Voltage Method

Consider what happens when a voltage source is located between two nodes:



v_{12} and i_{12} = temporary variables

KVL: $-v_1 + v_{12} + v_2 = 0$

$$v_{12} = v_1 - v_2$$

terminal law: $v_{12} = V_0$

$$i_{12} = \text{anything}$$

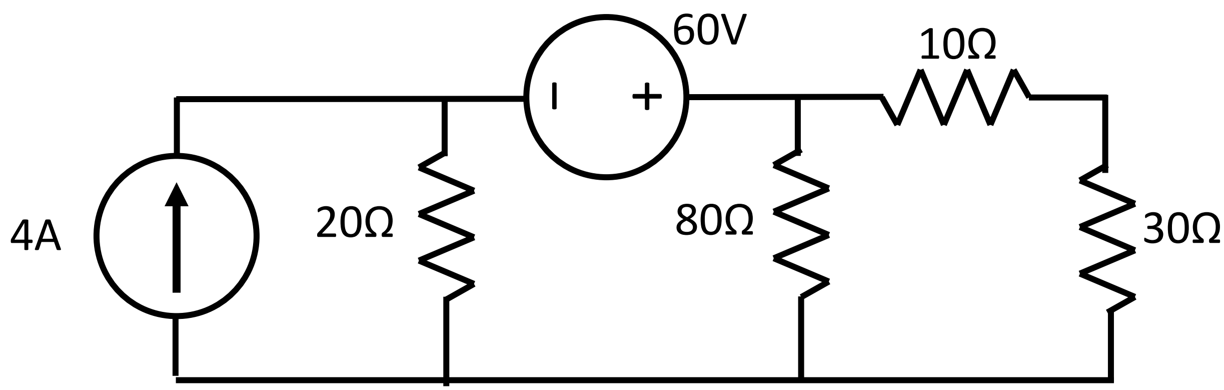
There are two consequences:

We have a very simple relationship between node voltages, $v_1 - v_2 = V_0$ that is independent of i_{12}.

It is *impossible* to express the current i_{12} between node 1 and 2 in terms of v_1 and v_2.

Voltage Sources and Node Voltage Method

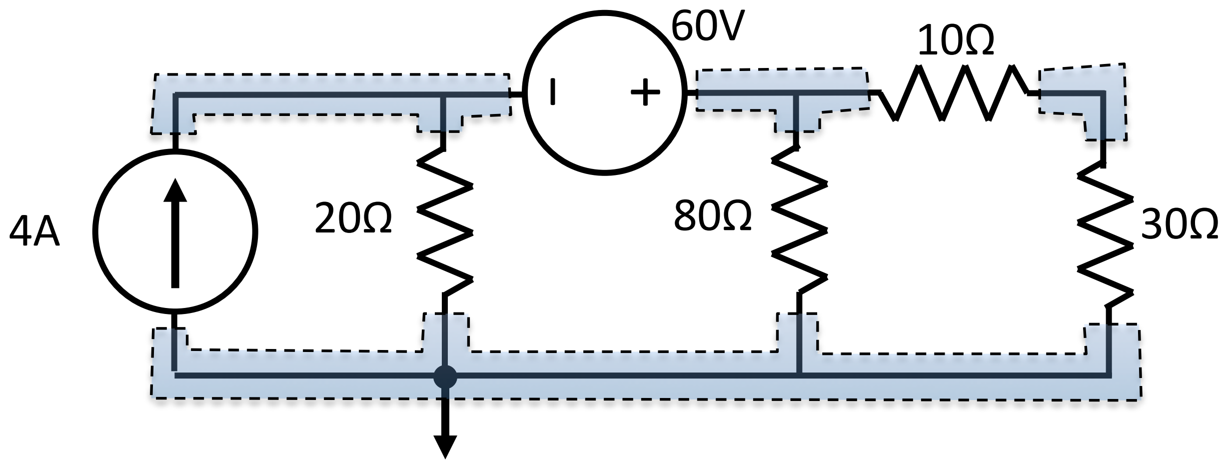
The node voltage method can be generalized to incorporate voltage sources. We illustrate the method with the example below.



Node Voltage Method

Step #1:

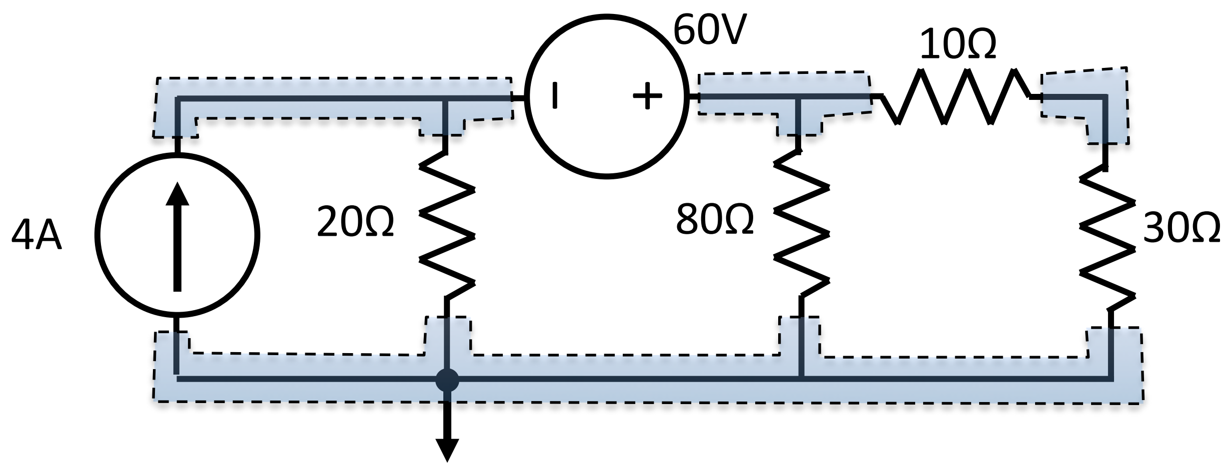
Choose a reference node.



Node Voltage Method

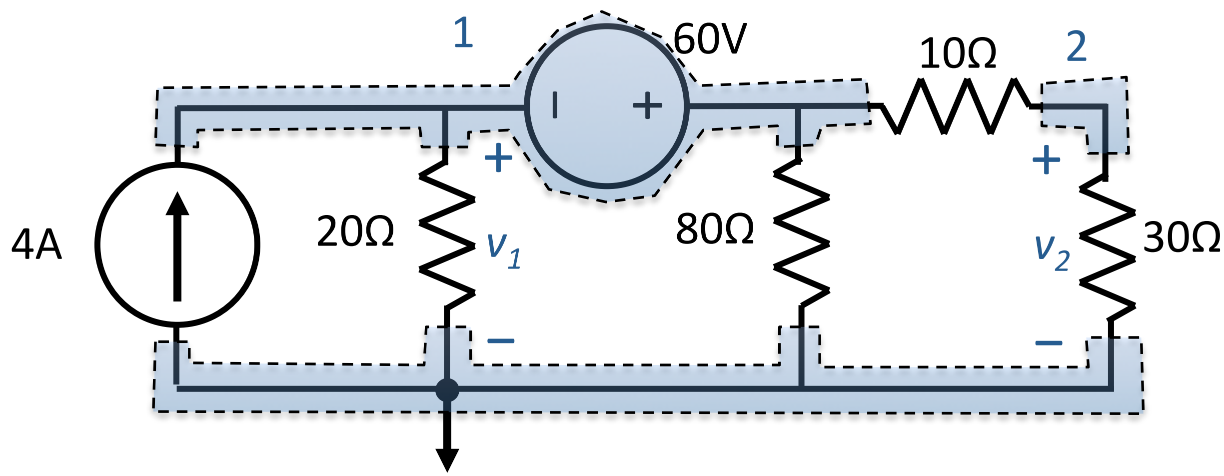
Step #2:

Label all remaining nodes. Combine nodes that are connected by a voltage source into a “super-node”. Identify node voltages, including only one node voltage for a “super-node”.



Node Voltage Method

In this example, there are two nodes combined into a super-node (labeled 1), and one node (labeled 2). We define a single voltage variable, v_1, to one of the nodes in the super-node.



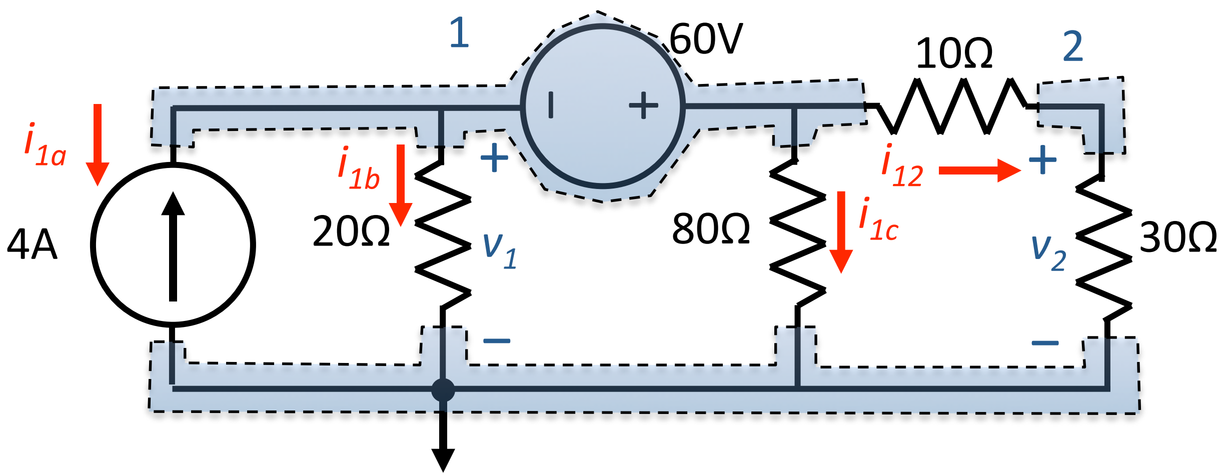
Note that we could define the voltage variable v_1 as any node voltage within the super-node 1.

Node Voltage Method

Step #3: Write KCL equations for each labeled node and super-node. Use only the defined voltage variables to express each current by implicitly using KVL and terminal laws of the elements.

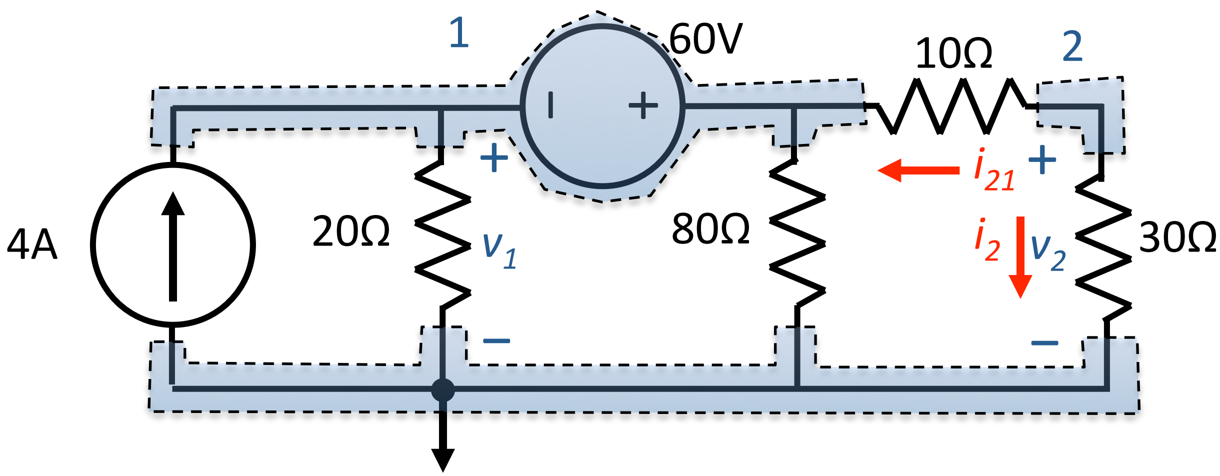
Node Voltage Method

KCL at super-node 1: $$0 = \underbrace{-4\text{A}}_{i_{1a}} + \underbrace{\frac{v_1}{20\Omega}}_{i_{1b}} + \underbrace{\frac{(v_1 + 60\text{V})}{80\Omega}}_{i_{1c}} + \underbrace{\frac{(v_1 + 60\text{V}) - v_2}{10\Omega}}_{i_{12}}$$



Node Voltage Method

KCL at node 2: $$0 = \underbrace{\frac{v_2 - (v_1 + 60\text{V})}{10\Omega}}_{i_{21}} + \underbrace{\frac{v_2}{30\Omega}}_{i_2}$$



Node Voltage Method

Step #4:

Solve for the node voltages, and calculate any quantity of interest.

node 1: $$0 = -4\text{A} + \frac{v_1}{20\Omega} + \frac{(v_1 + 60\text{V})}{80\Omega} + \frac{(v_1 + 60\text{V}) - v_2}{10\Omega}$$

node 2: $$0 = \frac{v_2 - (v_1 + 60\text{V})}{10\Omega} + \frac{v_2}{30\Omega}$$

Note that in this case, the entire circuit problem is organized into a system of 2 equations with 2 unknowns.

Node Voltage Method

Use substitution to find the value v_1 and v_2.

node 2: $$0 = \frac{v_2 - (v_1 + 60\text{V})}{10\Omega} + \frac{v_2}{30\Omega}$$

$$0 = 3v_2 - 3v_1 - 180\text{V} + v_2$$

$$v_1 = \frac{4}{3}v_2 - 60\text{V}$$

super-node 1: $$0 = -4\text{A} + \frac{v_1}{20\Omega} + \frac{(v_1 + 60\text{V})}{80\Omega} + \frac{(v_1 + 60\text{V}) - v_2}{10\Omega}$$

$$0 = -4\text{A} + \frac{(\frac{4}{3}v_2 - 60\text{V})}{20\Omega} + \frac{((\frac{4}{3}v_2 - 60\text{V}) + 60\text{V})}{80\Omega} + \frac{((\frac{4}{3}v_2 - 60\text{V}) + 60\text{V}) - v_2}{10\Omega}$$ substitution

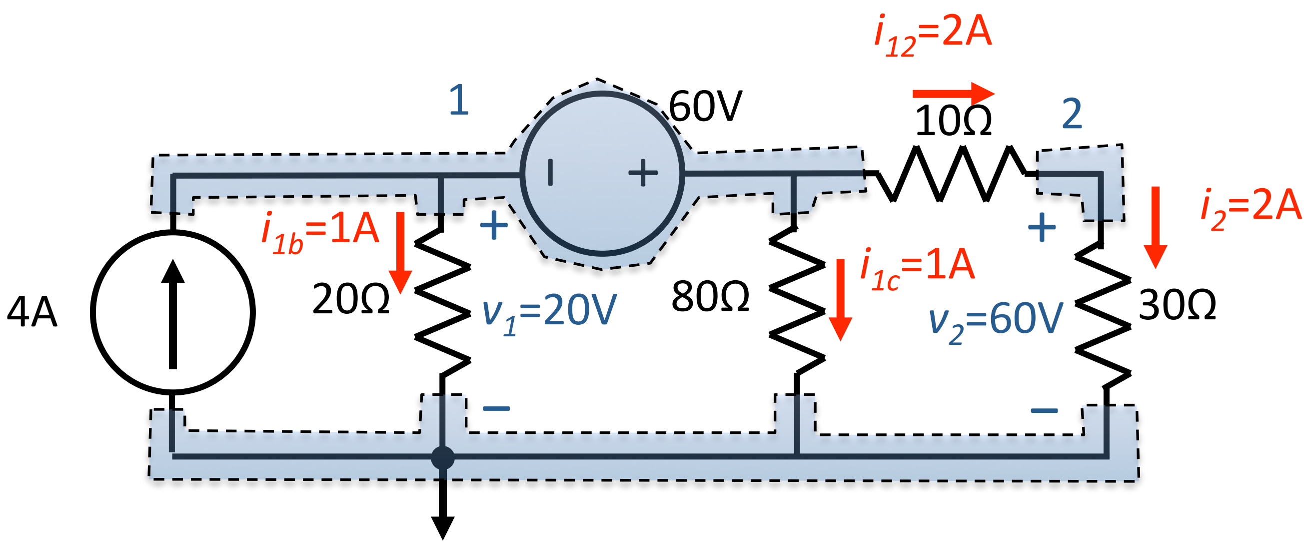
$$0 = -320\text{V} + (\tfrac{16}{3}v_2 - 240\text{V}) + \tfrac{4}{3}v_2 + \tfrac{8}{3}v_2$$ multiply by 80Ω

$$v_2 = \frac{560\text{V}}{28/3} = 60\text{V}$$

node 2: $$v_1 = \frac{4}{3}v_2 - 60\text{V} = \frac{4}{3}(60\text{V}) - 60\text{V} = 20\text{V}$$

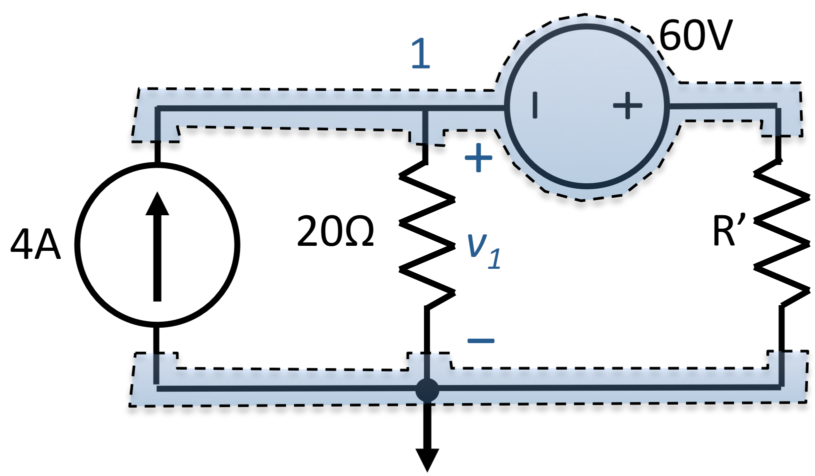
Node Voltage Method

Any quantity (for example, the branch currents) can now be easily calculated from v_1 and v_2.



Node Voltage Method

Note that we could use equivalent resistance for the right hand side to simplify the problem further.



equivalent resistance:

$$R' = 80\Omega \,//\, (10\Omega + 30\Omega)$$
$$= \frac{80 \cdot 40}{80 + 40}\Omega$$
$$= 26\tfrac{2}{3}\Omega$$

single super-node voltage equation:

$$0 = -4\text{A} + \frac{v_1}{20\Omega} + \frac{(v_1 + 60\text{V})}{26\tfrac{2}{3}\Omega}$$

$$v_1 = 20\text{V}$$

Summary of Node Voltage Method

Step #1: Define a reference node.

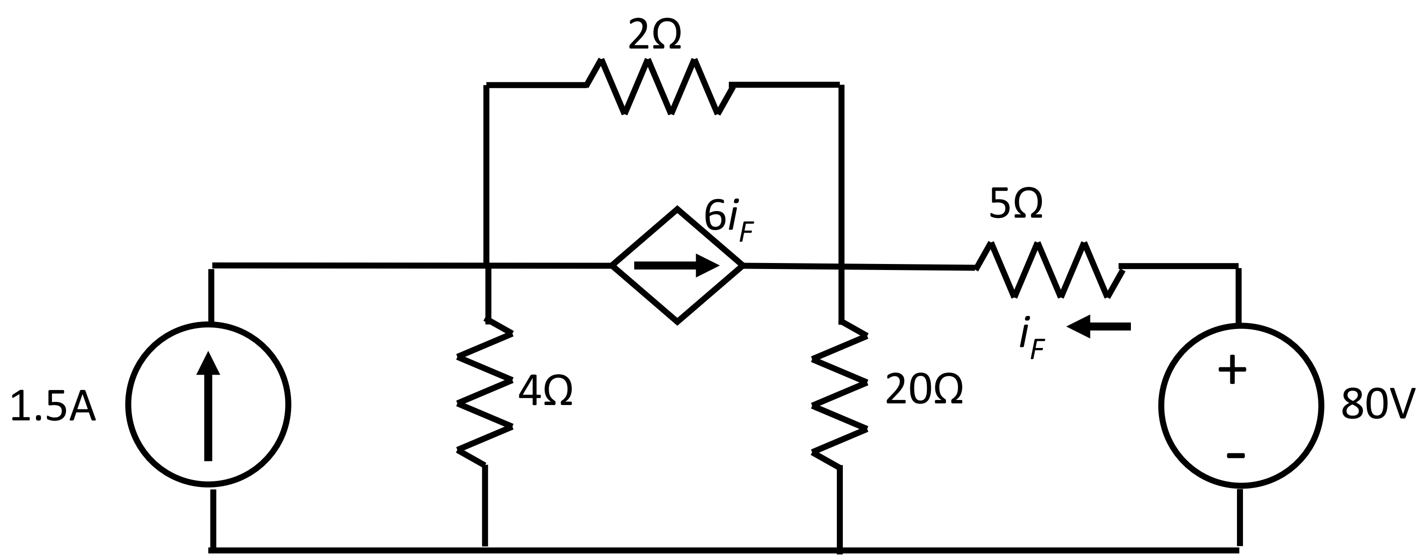
Step #2: Label remaining nodes, *grouping together nodes separated by voltage sources into super-nodes*. Identify node voltage variables (*one node voltage per super-node*).

Step #3: Write KCL equations in terms of node voltage variables, intrinsically using KVL and terminal laws (such as Ohm's law).

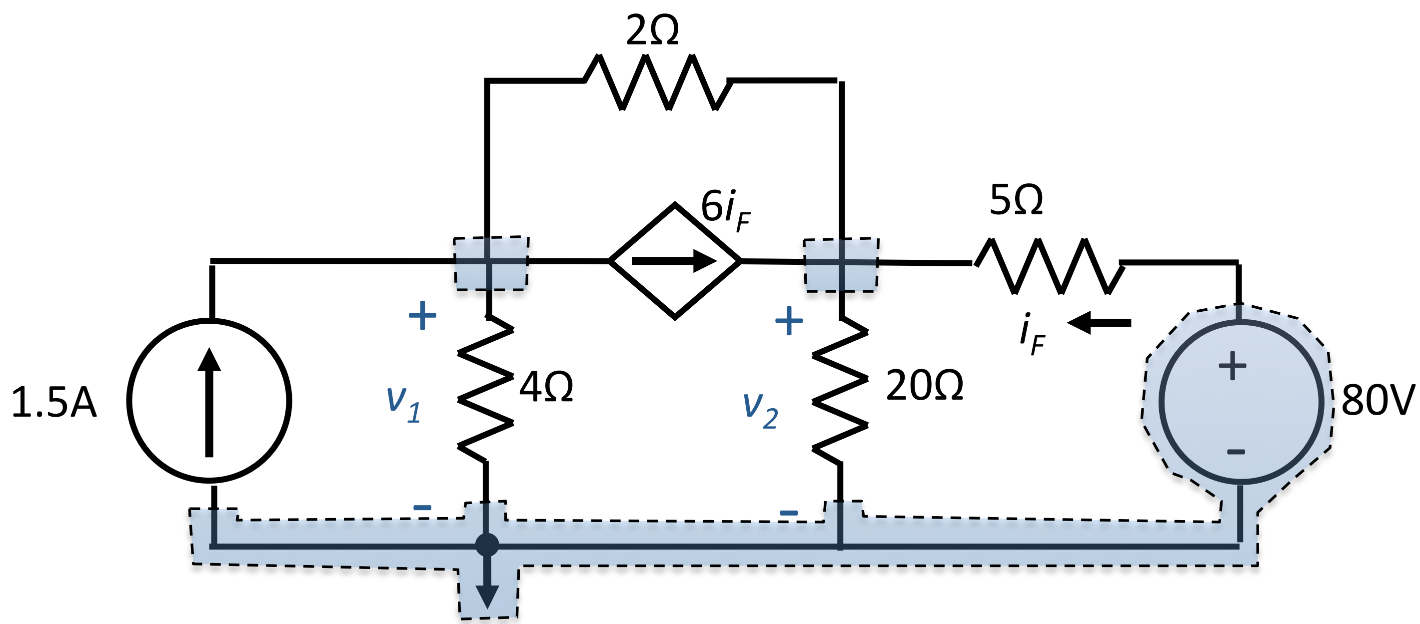
Step #4: Solve the linear system of equations, and use the node voltages to calculate the desired quantity.

Dependent Sources and Node Voltage

In the presence of dependent voltage sources or dependent current sources, express each controlling circuit variable in terms of the node voltages. For example, write a set of node voltage equations for the following circuit:



Dependent Sources and Node Voltage



node 1: $0 = -1.5\text{A} + \dfrac{v_1}{4\Omega} + \dfrac{v_1 - v_2}{2\Omega} + 6i_F$

node 2: $0 = \dfrac{v_2 - v_1}{2\Omega} - 6i_F + \dfrac{v_2}{20\Omega} + \dfrac{v_2 - 80\text{V}}{5\Omega}$

control variable: $i_F = \dfrac{80\text{V-}v_2}{5\Omega}$

three linear equations for three unknowns