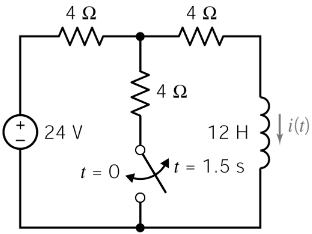
ECSE 200 - Electric Circuits 1
Tutorial 11

ECE Dept., McGill University

Problem P 8.4-2

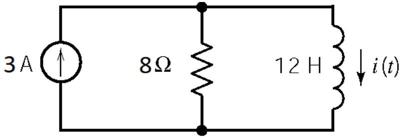
The circuit shown in Figure P 8.4-2 is at steady state before the switch closes at time t = 0. The switch remains closed for 1.5 s and then opens. Determine the inductor current, i(t), for t > 0.

Answer: $i(t) = \begin{cases} 2 + e^{-0.5t} \ A \ for \ 0 < t < 1.5 \ s \\ 3 - 0.53e^{-0.667(t-1.5)} \ A \ for \ 1.5 \ s \ < t \end{cases}$

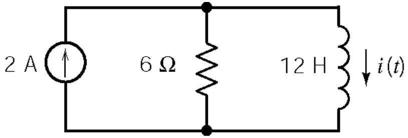


Problem P 8.4-2 Solution

Replace the part of the circuit connected to the inductor by its Norton equivalent circuit to get:



$t < 0$ and $t > 1.5$ s



$0 < t < 1.5$ s

Before the switch closes at t = 0 the circuit is at steady state so i(0) $= 3\ A$. For $0 < t < 1.5\ s$, i_{sc} =2 A and $R_t = 6\ \Omega$so $\tau = \frac{12}{6} = 2$ s .
Therefore

i(t) = $i_{sc} + (i(0) - i_{sc})e^{-\frac{t}{\tau}} = 2 + e^{-0.5t}\ A\ for\ 0 < t < 1.5\ s.$
At t =1.5 s, i(1.5) $= 2 + e^{-0.5(1.5)}$ =2.47 A. For 1.5 s < t, $i_{sc} = 3\ A$ and ${}_R t = 8\ \Omega$so $\tau = \frac{12}{8} = 1.5$ s. Therefore,

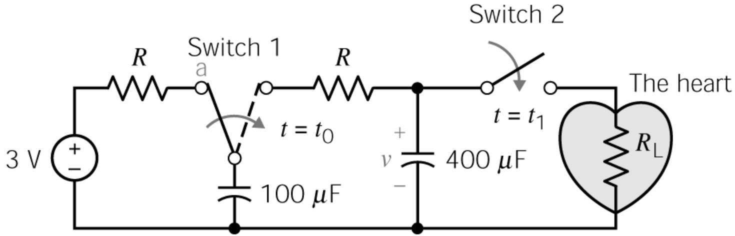
i(t) = $i_{sc} + (i(1.5) - i_{sc})e^{\frac{-(t-1.5)}{\tau}} =$
$3 - 0.53e^{-0.667(t-1.5)}\ V\ for\ 1.5\ s < t$
Finally,

$$\text{i(t)=}\begin{cases} 2 + e^{-0.5t}\ A\ for\ 0 < t < 1.5\ s \\ 3 - 0.53e^{-0.667(t-1.5)}\ A\ for\ 1.5\ s < t \end{cases}$$

Problem P 8.4-3

Cardiac pacemakers are used by people to maintain regular heart rhythm when they have a damaged heart. The circuit of a pacemaker can be represented as shown in Figure P 8.4-3. The resistance of the wires, R, can be neglected since R $<$ 1 mΩ. The hearts load resistance, R_L, is 1 kΩ. The first switch is activated at t $=$ t_0, and the second switch is activated at $t_1 = t_0 + 10$ ms. This cycle is repeated every second. Find v(t) for $t_0 \leq t \leq 1$. Note that it is easiest to consider $t_0 = 0$ for this calculation. The cycle repeats by switch 1 returning to position a and switch 2 returning to its open position.

Hint: Use q $=$ Cv to determine v(0) for the 100 μF capacitor.

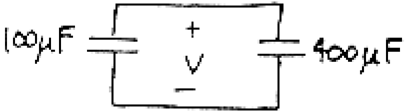


Problem P 8.4-3 Solution

At t = 0-: Assume that the circuit has reached steady state so that the voltage across the 100 μF capacitor is 3 V. The charge stored by the capacitor is

$q(0^-) \ = \ (100 \times 10^{-6})(3) \ = 300 \times 10^{-6} \ C$

$0 < t < 10 \ ms$: With R negligibly small, the circuit reaches steady state almost immediately (i.e. at t = 0^+). The voltage across the parallel capacitors is determined by considering charge conservation:



$$q(0^-) \ = \ (100\mu F)v(0^+) \ + \ (400)\mu F v(0^+)$$

$$v(0^+) \ = \ \frac{q(0^+)}{(100 \times 10^{-6}) + (400 \times 10^{-6})} \ = \ \frac{q(0^-)}{(500 \times 10^{-6})} \ = \ \frac{300 \times 10^{-6}}{500 \times 10^{-6}}$$

$v(0^+) \ = \ 0.6 \ V$

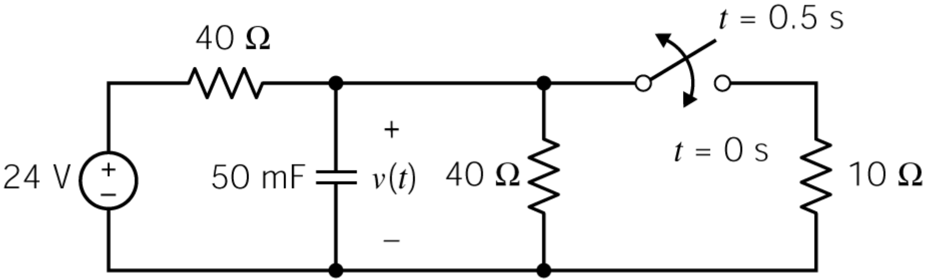
Problem P 8.4-3 Solution

10 ms $< t < 1$ s: Combine 100 μF & 400 μF in parallel to obtain

$$v(t) = v(0^+)e^{\frac{-(t-0.1)}{RC}} = 0.6e^{\frac{-(t-0.1)}{(1000)(5\times10^{-4})}} = 0.6e^{-2(t-.01)}\ V$$

Problem P 8.4-5

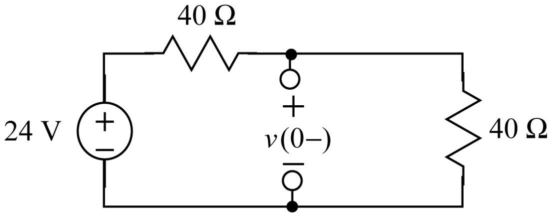
The circuit shown in Figure P 8.4-5 is at steady state before the switch closes at $t = 0$. The switch remains closed for 0.5 second and then opens. Determine $v(t)$ for $t \geq 0$.

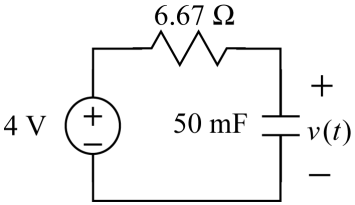


Problem P 8.4-5 Solution

The circuit is at steady state before the switch closes. The capacitor acts like an open circuit. The initial condition is

$$v(0^+) \;=\; v(0^-) \;= (\frac{40}{40+40})24 \;=\; 12\ V$$



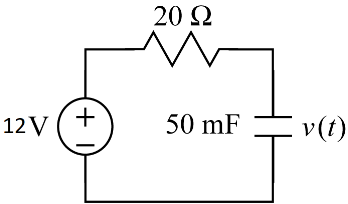


Problem P 8.4-5 Solution

After the switch closes, replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.
Recognize that $R_t = 6.67\ \Omega$ and $v_{oc} = 4\ V$
The time constant is $\tau = R_t C = (6.67)(0.05) = 0.335$ s
The capacitor voltage is

$$v(t) = (v(0^+) - v_{oc})e^{\frac{-t}{\tau}} + v_{oc} = (12-4)e^{-3t} + 4 =$$

$4 + 8e^{-3t}\ V$ *for* $0 \geq t \geq 0.5$ s. When the switch opens again at time t $=$ 0.5 the capacitor voltage is
$v(0.5+) = v(0.5-) = 4 + 8e^{-3(0.5)} = 5.785$ V



Problem P 8.4-5 Solution

After time t = 0.5 s, replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.
Recognize that $R_t = 20\ \Omega$ and $v_{oc} = 12\ V$

The time constant is $\tau = R_t C = (20)(0.05) = 1 \Longrightarrow \dfrac{1}{\tau} = 1$

The capacitor voltage is

$$v(t) = (v(0.5+) - v_{oc})e^{\frac{-(t-0.5)}{\tau}} + v_{oc} =$$

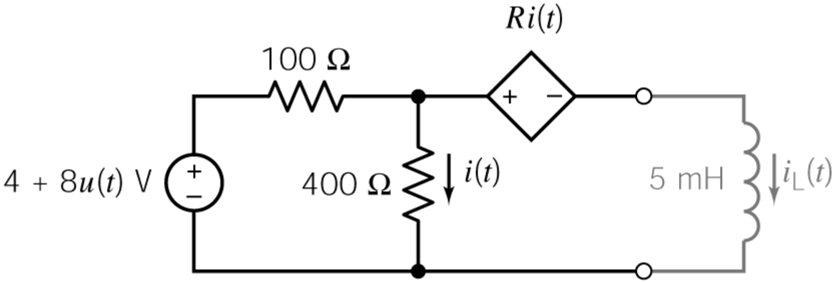
$(5.785 - 12)e^{-10(t-0.5)} + 12 = 12 - 6.215e^{-10(t-0.5)}\ V\ for\ t \geq 0.5$ s.

$$\text{Hence, v(t)} = \begin{cases} 12\ V\ for\ t \geq 0 \\ 4 + 8e^{-3t}\ V\ for\ 0 \leq t \leq 0.5\ s \\ 12 - 6.215e^{-(t-0.5)}\ V\ for\ t \geq 0.5\ s \end{cases}$$

Problem P 8.5-1

The circuit in Figure P 8.5-1 contains a current-controlled voltage source. What restriction must be placed on the gain, R, of this dependent source in order to guarantee stability.

Answer: R $< 400\ \Omega$

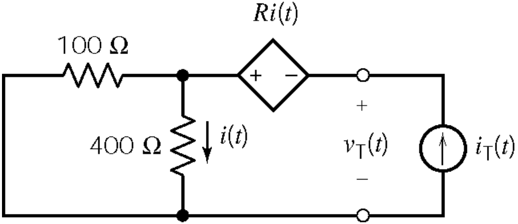


Problem P 8.5-1 Solution

This circuit will be stable if the Thvenin equivalent resistance of the circuit connected to the inductor is positive. The Thvenin equivalent resistance of the circuit connected to the inductor is calculated as

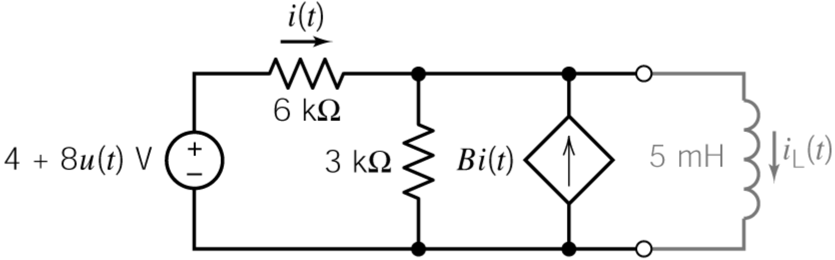
$$R_t = \frac{v_T}{i_T} = \frac{400i(t) - Ri(t)}{\frac{100}{100+400} i_T} = \frac{(400-R)100}{100+400}$$

The circuit is stable when R < 400 Ω



Problem P 8.5-2

The circuit in Figure P 8.5-2 contains a current-controlled current source. What restriction must be placed on the gain, B, of this dependent source in order to guarantee stability



Problem P 8.5-2 Solution

The Thvenin equivalent resistance of the circuit connected to the inductor is calculated as

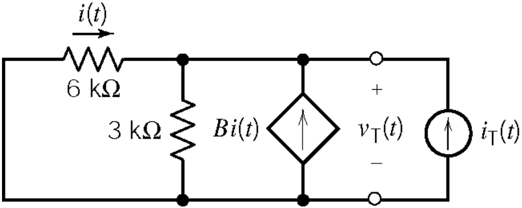
Ohm's law: $i(t) \;=\; -\dfrac{v_T(t)}{6000}$

KCL: $i(t) + Bi(t) + i_T(t) \;=\; \dfrac{v_T(t)}{3000}$

$\therefore\;\; i_T(t) = -(B+1)\left(\dfrac{v_T(t)}{6000}\right) + \dfrac{v_T(t)}{3000} \;=\; \dfrac{(B+3)v_T(t)}{6000}$

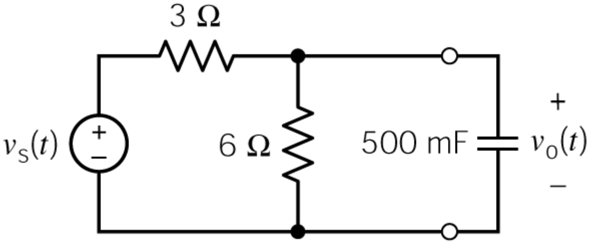
$R_t \;=\; \dfrac{v_T(t)}{i_T(t)} \;=\; \dfrac{6000}{B+3}$

The circuit is stable when B $>$ -3 A/A.



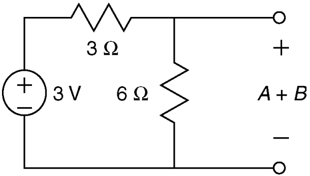
Problem P 8.6-2

The input to the circuit shown in Figure P 8.6-2 is the voltage of the voltage source, $v_s(t)$. The output is the voltage across the capacitor, $v_o(t)$. Determine the output of this circuit when the input is $v_s(t) \ = 3 + 3$ u(t) V.



Problem P 8.6-2 Solution

The value of the input is one constant, 3 V, before time t = 0 and a different constant, 6 V, after time t = 0. The response of the first order circuit to the change in the value of the input will be
$v_o(t) \ = \ A + Be^{-\alpha t}$ *for* $t > 0$
where the values of the three constants A, B and α are to be determined. The values of A and B are determined from the steady state responses of this circuit before and after the input changes value.



Capacitors act like open circuits when the input is constant and the circuit is at steady state. Consequently, the capacitor is replaced by an open circuit.

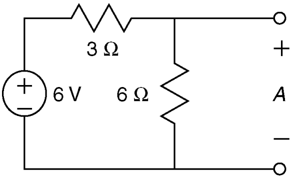
Problem P 8.6-2 Solution

The value of the capacitor voltage at time t = 0, will be equal to the steady state capacitor voltage before the input changes. At time t = 0 the output voltage is

$$v_o(t) \ = \ A + Be^{-\alpha(0)} \ = \ A + B$$

Consequently, the capacitor voltage is labeled as A + B. Analysis of the circuit gives

$$A + B \ = \ \frac{6}{3+6}(3) \ = 2 \text{ V}$$



Problem P 8.6-2 Solution

Capacitors act like open circuits when the input is constant and the circuit is at steady state. Consequently, the capacitor is replaced by an open circuit.

The value of the capacitor voltage at time t $= \infty$, will be equal to the steady state capacitor voltage after the input changes. At time t $= \infty$ the output voltage is

$v_o(\infty) \ = \ A + Be^{-\alpha(\infty)} \ = \ A$

Consequently, the capacitor voltage is labeled as A. Analysis of the circuit gives

$A \ = \ \dfrac{6}{3+6}(6) \ = 4$ V

Therefore, B = -2 V

The value of the constant α is determined from the time constant, τ, which is in turn calculated from the values of the capacitance C and of the Thevenin resistance, R_t, of the circuit connected to the capacitor.

$\dfrac{1}{\alpha} \ = \ \tau \ = \ R_t C$

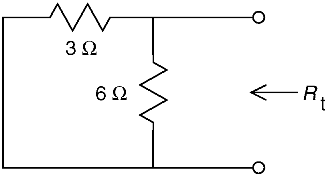
Problem P 8.6-2 Solution

Here is the circuit used to calculate R_t.

$R_t \ = \ \dfrac{(3)(6)}{3+6} \ = 2\ \Omega$

Therefore, $\alpha \ = \ \dfrac{1}{(2)(0.5)} \ = 1;$

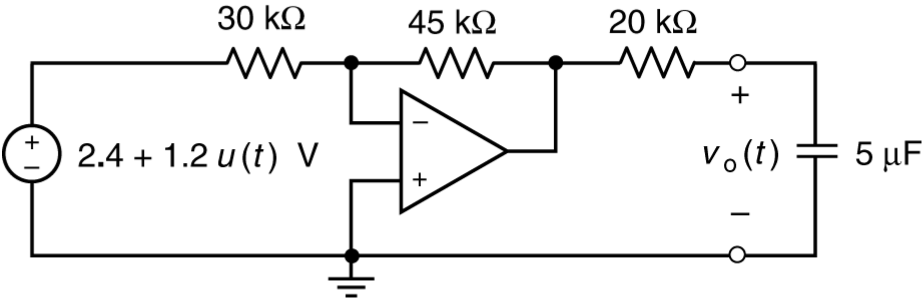
(The time constant is $\tau \ = \ (2)(0.5) \ = \ 1\ s$.)



Putting it all together: $v_0(t) \ = \ \begin{cases} 2\ V\ \mathit{for}\ t \ \leq \ 0 \\ 4 - 2e^{-t}\ V\ \mathit{for}\ \ t \ \geq \ 0 \end{cases}$

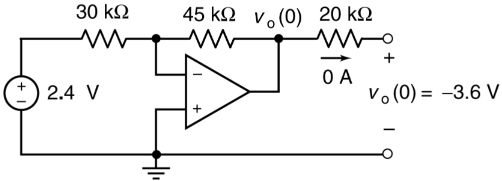
Problem P 8.6-4

Determine $v_o(t)$ for t $>$ 0 for the circuit shown in Figure P8.6-4.



Problem P 8.6-4 Solution

Determine the initial condition, $v_o(0)$, by considering the circuit when t < 0 and the circuit is at steady state. Since a capacitor in a dc circuit acts like a open circuit , we have

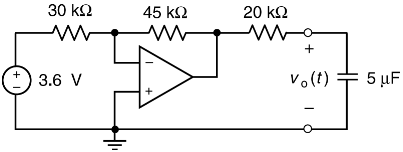


Recognizing the inverting amplifier, we have

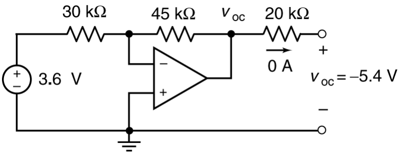
$$v_o(0) \;=\; \frac{-45}{30} 2.4 \;= \text{-}3.6 \text{ V}$$

Problem P 8.6-4 Solution

Next, consider the circuit when t $>$ 0 and the circuit is not at steady state:

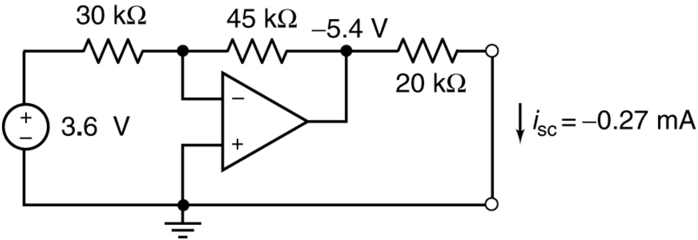


To find the Thevenin equivalent of the part of the circuit connected to the capacitor we determine both the open circuit voltage and the short circuit current:



and

Problem P 8.6-4 Solution



Now we calculate the Thevenin resistance:

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{-5.4}{-0.27 \times 10^{-3}} = 20 \text{ k}\Omega$$

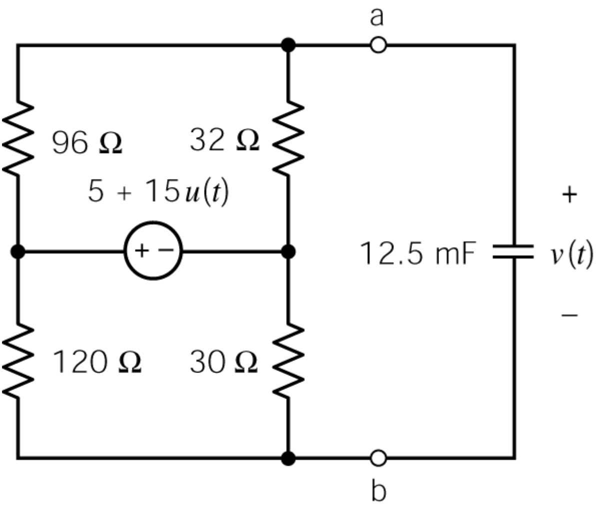
and the time constant: $\tau = R_t C = (20 \times 10^3)(5 \times 10^{-6}) = 0.1$ s

The capacitor voltage is given by

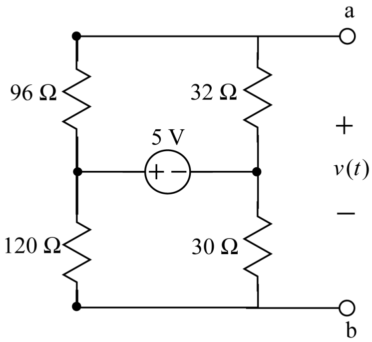
$$v_0(t) = (v_0(0) - v_{oc})e^{\frac{-t}{\tau}} + v_{oc} = (-3.6 - (-5.4))e^{-10t} - 5.4 = 1.8e^{-10t} - 5.4 \; V \; for \; t \geq 0.$$

Problem P 8.6-10

Determine the voltage v(t) for $t \geq 0$ for the circuit shown in Figure P 8.6-10.



Problem P 8.6-10 Solution



For t $<$ 0: Using voltage division twice

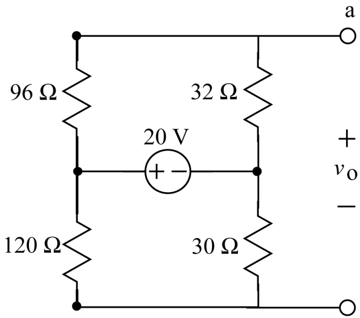
$$v(t) = \frac{32}{32+96}5 - \frac{30}{120+30}5 = 0.25 \text{ V}$$

so v(0-) $= 0.25$ V

and v(0+) $=$ $v(0-)$ $= 0.25$ V

Problem P 8.6-10 Solution

For t > 0, find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor.

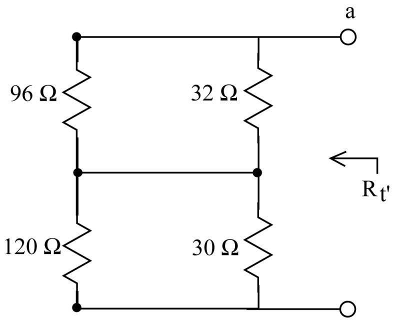


For t > 0: Using voltage division twice

$$v_{oc} = \frac{32}{32+96}20 - \frac{30}{120+30}20 = 1 \text{ V}$$

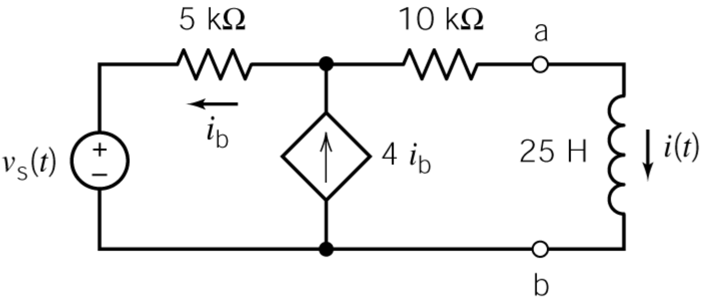
Problem P 8.6-10 Solution

$$R_t = (96 \parallel 32) + (120 \parallel 30) = 24 + 24 = 48\ \Omega$$
Then, $\tau = 48 \times 0.0125 = 0.6$ s
so, $\dfrac{1}{\tau} = 1.67$
Now $v(t) = (0.25 - 1)e^{-1.67t} + 1 = 1 - 0.75e^{-1.67t}\ V$ *for* $t \geq 0$.



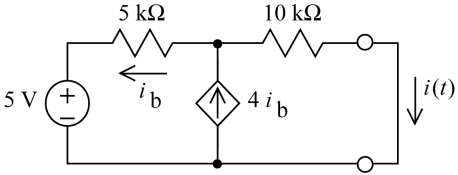
Problem P 8.6-11

The voltage source voltage in the circuit shown in Figure P 8.6-11 is
$v_s(t) = 5 + 20\ \mathrm{u(t)}$
Determine i(t) for t $\geq$ 0.



Problem P 8.6-11 Solution

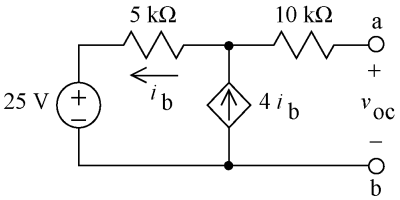
For t $<$ 0 the circuit is at steady state so the inductor acts like a short circuit:



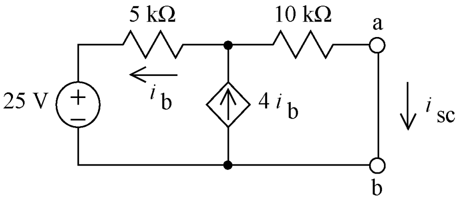
Apply KVL to the supermesh corresponding to the dependent source to get
$-5000i_b + 10000(3i_b) - 5 \;=\; 0 \;\implies\; i_b \;= 0.2$ mA
Apply KVL to get i(t) $=\; 3i_b \;= 0.6$ mA
so, i(0-)$=$ 0.6 mA and i(0$+$)$=$ $i(0-) \;= 0.6$ mA

Problem P 8.6-11 Solution

For t > 0 find the Norton equivalent circuit for the part of the circuit that is connected to the inductor.



Apply KCL at the top node of the dependent source to see that $i_b = 0$ A .
Then $v_{oc} = 25 - 5000(i_b) = 25$ V

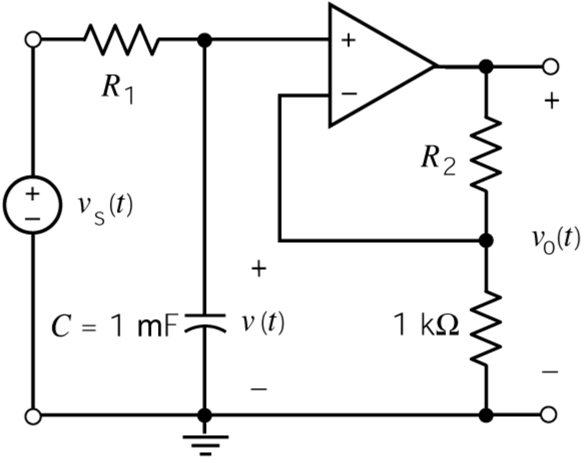


Problem P 8.6-11 Solution

Apply KVL to the supermesh corresponding to the dependent source to get
$-5000 i_b + 1000(3 i_b) - 25 \ = \ 0 \ \Longrightarrow \ i_b \ = 1 \text{ mA}$
Apply KCL to get $i_{sc} \ = \ 3 i_b \ = 3 \text{ mA}$
Then $R_t \ = \ \dfrac{v_{oc}}{i_{sc}} \ = 8.33 \text{ k}\Omega$
Then $\tau \ = \ \dfrac{25}{8333} \ = 3 \text{ ms}$
So $\dfrac{1}{\tau} \ = \ 333 \ Hz$
Now $i(t) \ = \ (0.6 - 3)e^{-333t} + 3 \ = \ (3 - 2.4)e^{-333t} \ mA \ for \ t \ \geq 0.$

Problem P 8.6-27

When the input to the circuit shown in Figure P 8.6-27 is the voltage source voltage $v_s(t) = $ 3-u(t) V, the output is the voltage
$v_o(t) = 10 + 5e^{-50t}\ V\ for\ t \geq 0$
Determine the values of R_1 and R_2.



Problem P 8.6-27 Solution

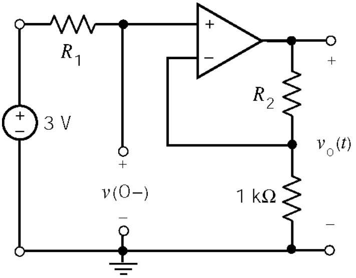
Apply KCL at the inverting input of the op amp to get

$$\frac{v_o(t) - v(t)}{R_2} = \frac{v(t)}{1000} \implies v_o(t) = (1 + \frac{R_2}{1000})v(t)$$

We will determine the capacitor voltage first and then use it to determine the output voltage.

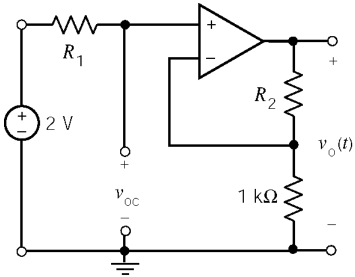
When t $<$ 0 and the circuit is at steady state, the capacitor acts like an open circuit. Apply KCL at the noninverting input of the op amp to get

$$\frac{3 - v(0-)}{R_1} = 0 \implies v(0-) = 3 \text{ V}$$



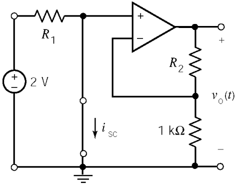
Problem P 8.6-27 Solution

The initial condition is v(0+) = $v(0-)$ $= 3$ V
For t≥0 , we find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor.



$$\frac{2 - v_{oc}}{R_1} = 0 \implies v_{oc} = 2 \text{ V}$$

Problem P 8.6-27 Solution



$$\frac{2}{R_1} = i_{sc} \implies R_t = \frac{v_{oc}}{i_{sc}} = R_1$$

The time constant is $\tau = R_t C = R_t(10^{-3})$. From the given equation for $v_0(t)$, $\dfrac{1}{\tau} = 50\ Hz$, so

$$R_t(10^{-3}) = \frac{1}{50} \implies R_1 = R_t = \frac{10^3}{50} = 20\ \Omega$$

The capacitor voltage is given by

$$v(t) = (v(0) - v_{oc})e^{\frac{-t}{\tau}} + v_{oc} = (3-2)e^{-50t} + 2 =$$

$2 + e^{-50t}\ V\ for\ t\ \geq 0.$

So, $v_o(t) = 5v(t) \implies 5 = 1 + \dfrac{R_2}{1000} \implies R_2 = 4\ \text{k}\Omega$

Thank you !