Today's Outline

6. Energy Storage Elements

- dc steady state, continuity and switched circuits

definitions

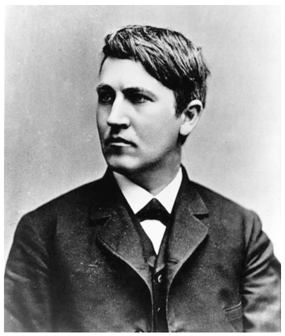
steady state: Regime of circuit operation where circuit variables are unchanging in time. All ***transients*** have subsided.

dc steady state: Regime of circuit operation where voltages and currents are constant in time, ie. $dv/dt = 0$ and $di/dt = 0$. "dc" refers to "direct current."

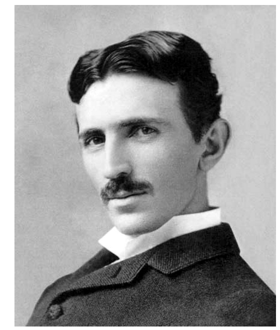
ac steady state: regime of circuit operation where voltages and currents are sinusoids, e.g. $x(\mathrm{t}) = A \sin (\omega t + \phi)$, with amplitude A and phase ϕ constant in time, ie. $dA/dt = 0$ and $d\phi/dt = 0$. "ac" refers to "alternating current."

war of currents: AC/DC

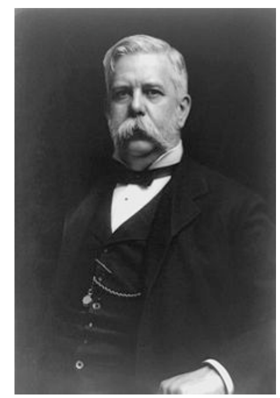
In the 1880's, an industrial war was waged for electricity distribution via dc or ac. ac generation and distribution was more efficient due to the abilitiy change voltages with transformers. Efficient dc distribution required the advent of high voltage electronics.



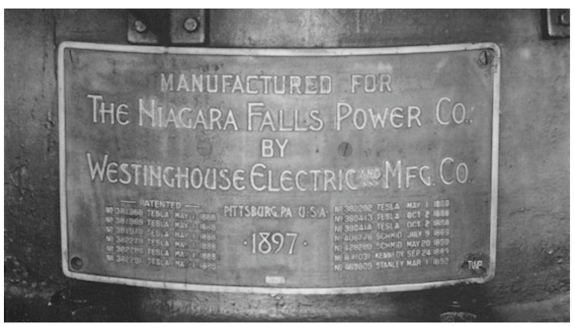
Thomas Edison developed and invested in dc distribution (1847-1931)



Nikola Tesla developed ac distribution (1856-1943)



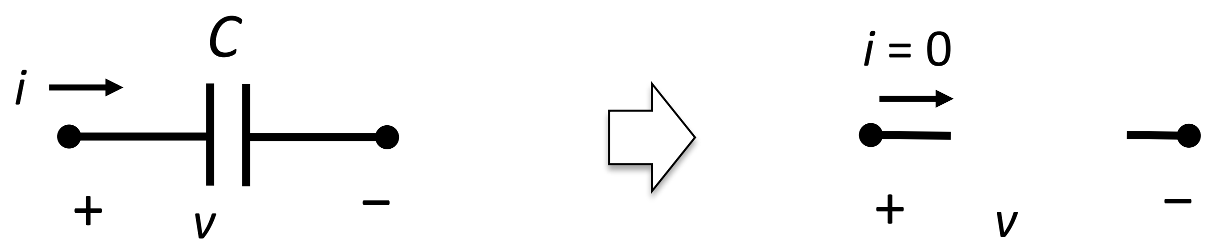
George Westinghouse invested in ac distribution (1846-1914)



Westinghouse won a contract for hydro-electric generation at Niagara Falls in 1893. Lord Kelvin served on the commission deciding the contract, financed by J. P. Morgan, Lord Rothschild and others.

dc steady state capacitor

At dc steady state, the capacitor is equivalent to an open circuit, meaning *i*=0 independent of the voltage *v* across the capacitor terminals.



dc steady state:

$$i = C\frac{dv}{dt} = 0$$

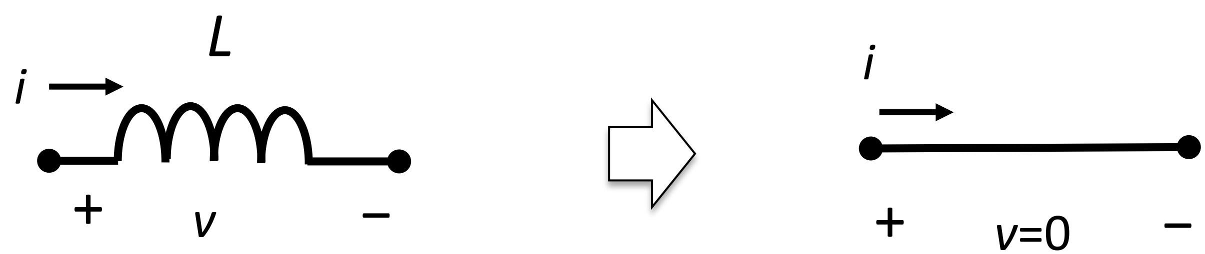
v = any constant

Note that unlike an open circuit, the energy stored by a capacitor at dc steady state is given by:

$$U = \frac{1}{2}Cv^2$$

dc steady state inductor

At dc steady state, the inductor is equivalent to a short circuit, meaning v=0 independent of the current i through the inductor terminals.



dc steady state:

$$v = L\frac{di}{dt} = 0$$

i = any constant

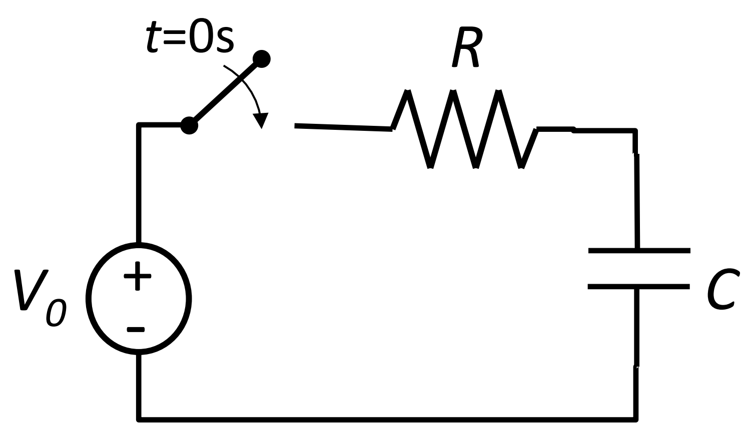
Note that unlike a short circuit, the energy stored by an inductor at dc steady state is given by:

$$U = \frac{1}{2}Li^2$$

switched circuits

A common application of capacitors and inductors is in ***switched circuits*** (we will see why in sections 7 and 8). The dc steady state behaviour that the circuit exhibits when the switch is in a fixed position for a ***"long time"*** is easily found.

The continuity of capacitor voltage and the continuity of inductor current permit us to understand how some circuit variables behave immediately after a switching event.

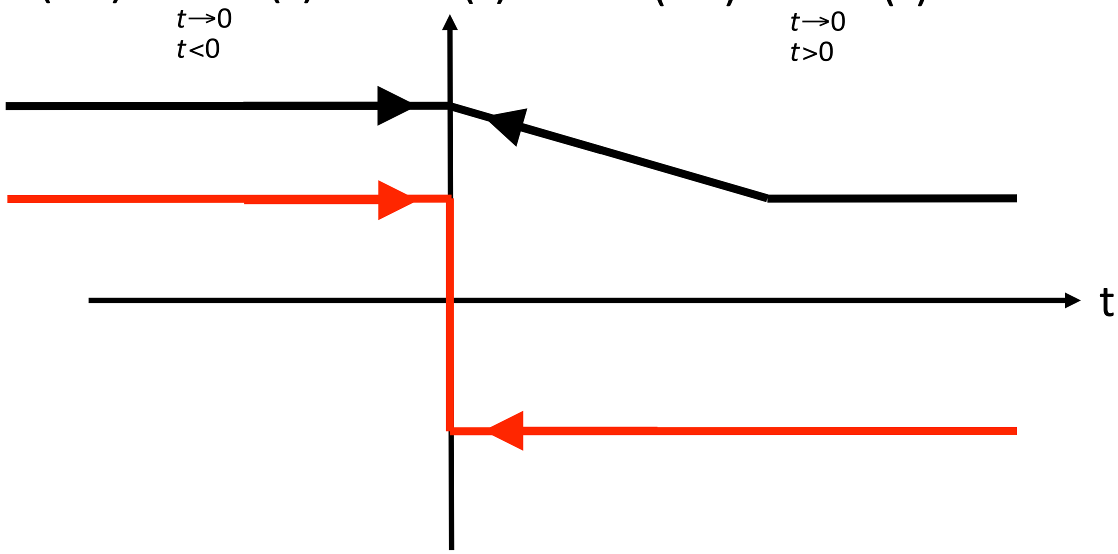


We will make precise the meaning of a "long time" in section 7.

a reminder about notation

The notation t = 0+ and t = 0− are to be understood as follows. The value of a circuit variable $x(t)$ as t approaches 0 from the past (left) or from the future (right) are identified separately:

$$x(0-) = \lim_{\substack{t \to 0 \\ t<0}} x(t) \qquad x(0+) = \lim_{\substack{t \to 0 \\ t>0}} x(t)$$

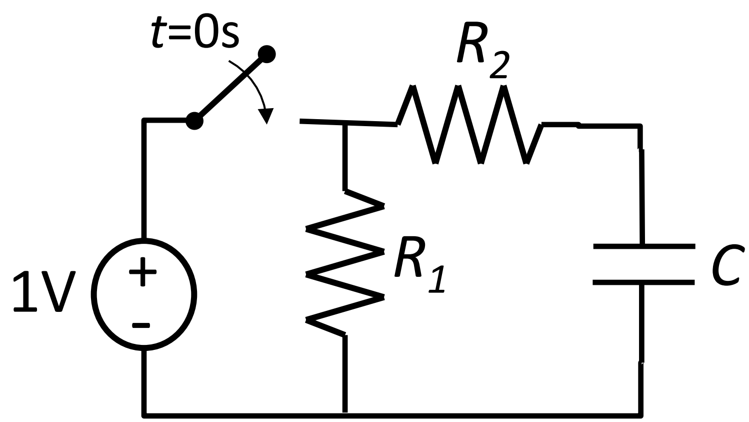


continuous at t=0
$x(0-)=x(0+)$

discontinuous at t=0
$x(0-) \neq x(0+)$

example 1

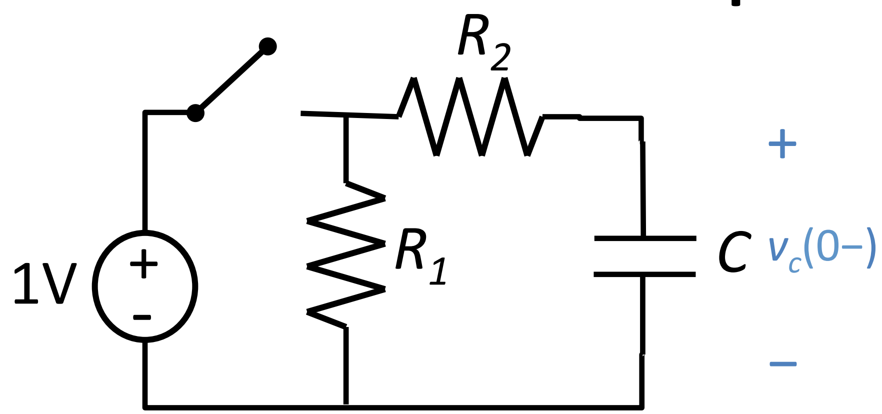
In the circuit below, the switch closes instantaneously at t=0s. What is the voltage across R_2 immediately after the switch closes (t = 0+) ? Assume the circuit is in steady state for t<0. What is the rate of change in capacitor voltage versus time at t=0+?



McGill

example 1

t=0−



KCL:

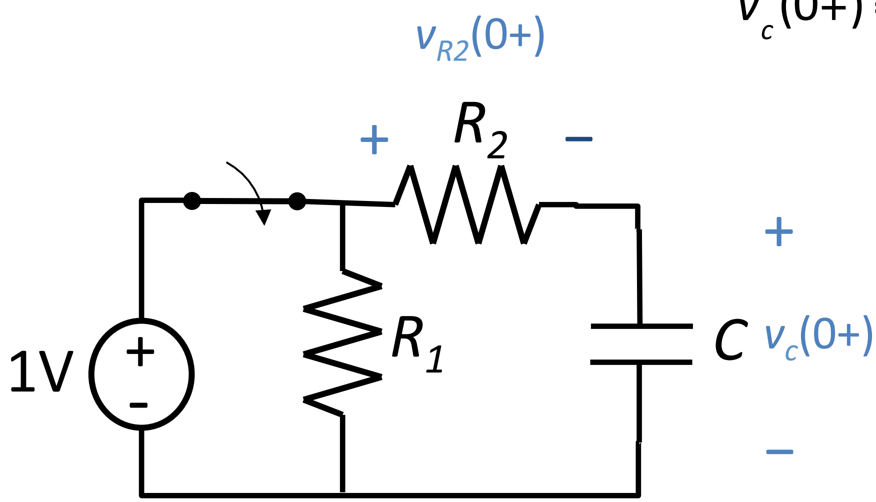
$$0 = \frac{v_c(0-)}{R_1 + R_2} + C\left.\frac{dv_c}{dt}\right|_{t=0-}$$

$$0 = \frac{v_c(0-)}{R_1 + R_2} + 0 \quad \text{steady-state}$$

$$v_c(0-) = 0\text{V}$$

$$v_c(0+) = v_c(0-) = 0\text{V}$$ continuity of capacitor voltage

t=0+



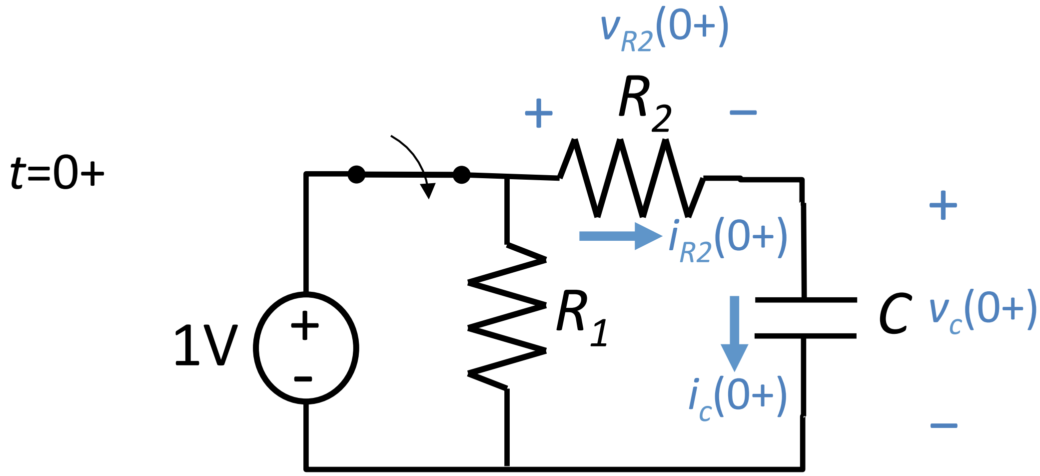
KVL:

$$0 = -1\text{V} + v_{R2}(0+) + v_C(0+)$$

$$v_{R2}(0+) = 1\text{V} - v_C(0+)$$

$$v_{R2}(0+) = 1\text{V}$$

example 1



Ohm: $i_{R2}(0+) = \dfrac{v_{R2}(0+)}{R_2} = \dfrac{1\text{V}}{R_2}$

KCL: $i_C(0+) = i_{R2}(0+) = \dfrac{1\text{V}}{R_2}$

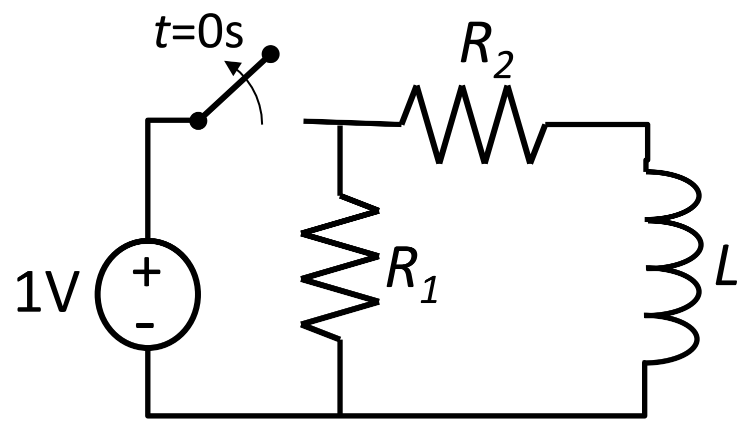
capacitor equation: $i_C(0+) = C\left.\dfrac{dv_C}{dt}\right|_{t=0+}$

$$\left.\frac{dv_C}{dt}\right|_{t=0+} = \frac{i_C(0+)}{C}$$

$$= \frac{1\text{V}}{R_2 C}$$

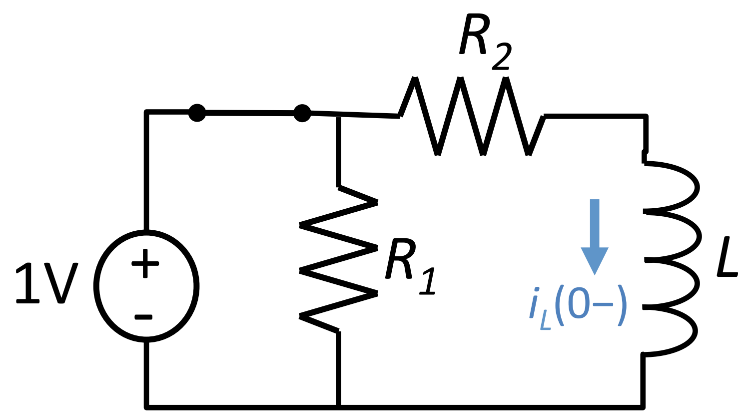
example 2

In the circuit below, the switch opens instantaneously at t=0s. What is the voltage across L at t = 0+? Assume the circuit is in steady state for t<0.



example 2

t=0−

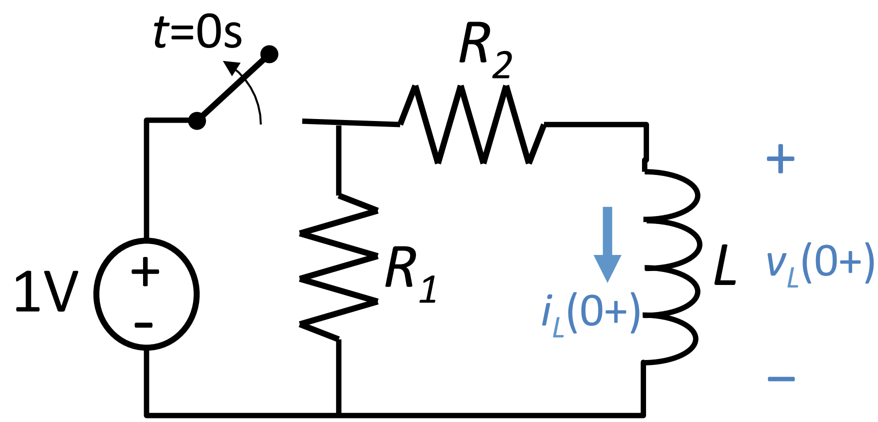


KVL: $$0 = -1\text{V} + R_2 i_L(0-) + L\left.\frac{di_L}{dt}\right|_{t=0-}$$

$$i_L(0-) = \frac{1\text{V}}{R_2}$$ steady-state

$$i_L(0+) = i_L(0-) = \frac{1\text{V}}{R_2}$$ continuity of inductor current

t=0+



KVL:

$$0 = R_1 i_L(0+) + R_2 i_L(0+) + v_L(0+)$$

$$v_L(0+) = -R_1 i_L(0+) - R_2 i_L(0+)$$

$$= -1\text{V}\left(\frac{R_1}{R_2} + 1\right)$$