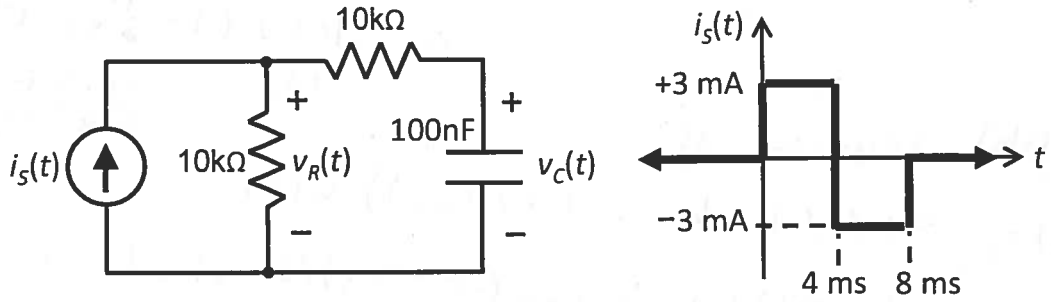
ECSE-200 Quiz #11 ([minimum number of randomly chosen people such that the probability of two people having the same birthday is greater than ½, ignoring leap years] Nov 2018)

NAME_____ McGill ID#_____

READ each question carefully. Do your work independently. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

Consider the circuit diagram and the plot of current $i_S(t)$ versus time t. The circuit is in dc steady-state for $t < 0$. Answer the questions.



1) Express the input $i_S(t)$ shown in the figure above in terms of the unit step function $u(t)$. [3pts]

2) What is the voltage $v_C(t)$ in response to the input $i_S(t) = 1A\ u(t)$? [2pts]

3) What is the voltage $v_C(t)$ in response to the input $i_S(t)$ as shown in the figure above? [3pts]

4) What is the voltage $v_R(t)$ in response to the input $i_S(t)$ as shown in the figure above? [2pts]

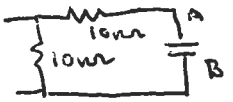
1) $i_S(t) = 3mA \cdot u(t) - 6mA\, u(t-4ms) + 3mA\, u(t-8ms)$
[+1] [+1] [+1]

2) initial condition: $v_C(0+) = v_C(0-) = 0V$

final condition:

$v_C(\infty) = 1A \cdot 10k\Omega = 10kV$

time constant:

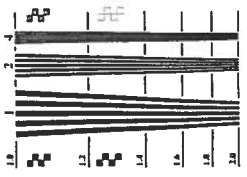


$\tau = R_T C = 20k\Omega \cdot 100nF = 2ms$ [+1]

$v_C(t) = 10kV\,(1 - \exp(-t/2ms))\,u(t)$ [+1]

3) $v_C(t) = 30V\,(1 - \exp(-t/2ms))\,u(t)$
$+ (-60V)(1 - \exp(-(t-4ms)/2ms))\,u(t-4ms)$
$+ 30V\,(1 - \exp(-(t-8ms)/2ms))\,u(t-8ms)$

[+1 for correct amplitudes]
[+1 for correct delays]
[+1 for correct functional form]

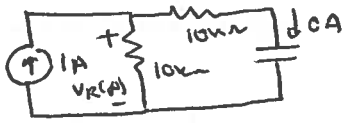


4) Find unit step response. $i_s = 1A\,u(t)$

$t = 0+$

$V_R(0+) = 1A \cdot (10k\Omega \,//\, 10k\Omega) = 5kV$

$t = \infty$



$V_R(\infty) = 10kV$

$\tau = 2ms$ $\quad \therefore$ unit response $\quad V_R(t) = (10kV - 5kV \exp(-t/2ms))\,u(t)$

$= 10kV\,(1 - \frac{1}{2}\exp(-t/2ms))\,u(t)$

[+1 for correct form of step response]

The total response is:

$$V_R(t) = 30V\,(1 - \tfrac{1}{2}\exp(-t/2ms))\,u(t)$$
$$+ (-60V)(1 - \tfrac{1}{2}\exp(-(t-4ms)/2ms))\,u(t-4ms)$$
$$+ 30V\,(1 - \tfrac{1}{2}\exp(-(t-8ms)/2ms))\,u(t-8ms)$$

[+1]