ECSE 200 - Electric Circuits 1
Tutorial 4

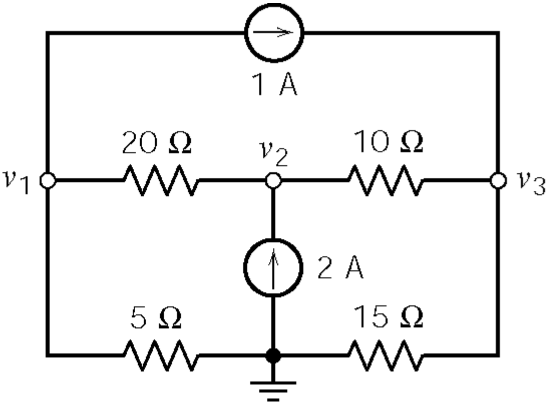
ECE Dept., McGill University

Recall

- **Nodal Analysis**
  - Define a reference node.
  - Label remaining nodes, and define node voltage variables with respect to reference node (Grouping together nodes separated by voltage sources into supernodes and identify node voltage variables (one node voltage per supernode)).
  - Write KCL equations for each node using node voltage variables only, by intrinsically using KVL and terminal laws (such as Ohms law).
  - Solve the linear system of equations, and use the node voltages to calculate the desired quantity.
- **Mesh Analysis**
  - Label meshes, and grouping any meshes sharing a current source into supermeshes. Define mesh current variables circulating in each mesh (and express all currents in a supermesh using a single mesh current.
  - Write KVL equation for each mesh using mesh current variables only, by intrinsically using KCL and terminal laws (such as Ohms law). Traverse the single loop in the supermesh that does not involve a current source.
  - Solve the linear system of equation, and use the mesh currents to calculate the desired quantity.

Problem P 4.2-2

Determine the node voltages for the circuit of the figure.



Problem P 4.2-2 Solution

KCL at node 1: $$\frac{v_1 - v_2}{20} + \frac{v_1}{5} + 1 = 0 \quad \Rightarrow \quad 5\,v_1 - v_2 = -20$$

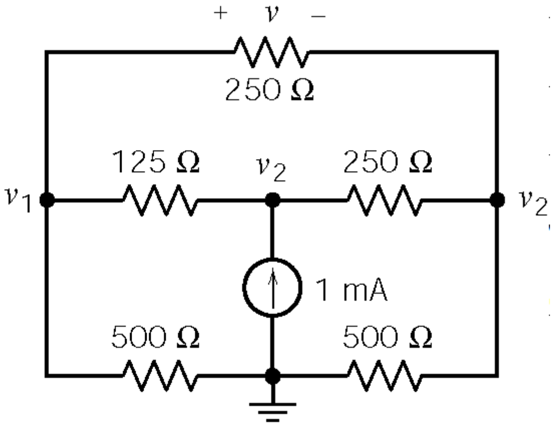
KCL at node 2: $$\frac{v_1 - v_2}{20} + 2 = \frac{v_2 - v_3}{10} \quad \Rightarrow \quad -v_1 + 3\,v_2 - 2\,v_3 = 40$$

KCL at node 3: $$\frac{v_2 - v_3}{10} + 1 = \frac{v_3}{15} \quad \Rightarrow \quad -3\,v_2 + 5\,v_3 = 30$$

Solving gives $v_1 = 2$ V, $v_2 = 30$ V and $v_3 = 24$ V.

Problem P 4.2-5

Find the voltage v for the circuit shown in the figure.



Problem P 4.2-5 Solution

Write node equations:

$$\frac{v_1}{500}+\frac{v_1-v_2}{125}+\frac{v_1-v_3}{250}=0$$

$$-\frac{v_1-v_2}{125}-.001+\frac{v_2-v_3}{250}=0$$

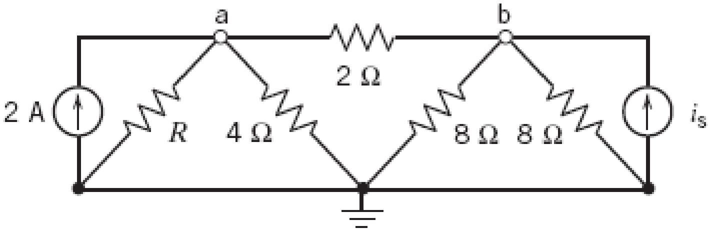
$$-\frac{v_2-v_3}{250}-\frac{v_1-v_3}{250}+\frac{v_3}{500}=0$$

Solving gives: $v_1=0.261\text{ V},\quad v_2=0.337\text{ V},\quad v_3=0.239\text{ V}$

Finally: $v=v_1-v_3=\underline{0.022\text{ V}}$

Problem P 4.2-7

The node voltages in the circuit shown in the figure are $v_a = 7V$ and $v_b = 10V$. Determine values of the current source current, i_s, and the resistance, R.



Problem P 4.2-7 Solution

Apply KCL at node a to get

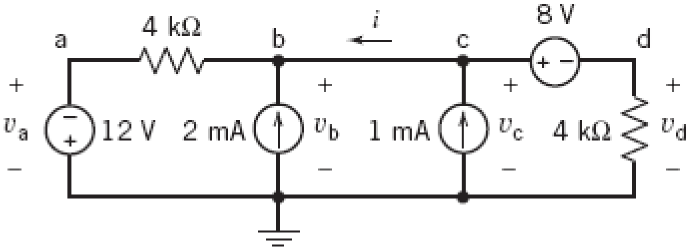
$$2 = \frac{v_a}{R} + \frac{v_a}{4} + \frac{v_a - v_b}{2} = \frac{7}{R} + \frac{7}{4} + \frac{7-10}{2} = \frac{7}{R} + \frac{1}{4} \quad \Rightarrow \quad R = 4\ \Omega$$

Apply KCL at node b to get

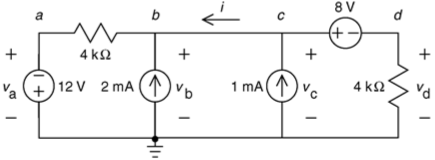
$$i_s + \frac{v_a - v_b}{2} = \frac{v_b}{8} + \frac{v_b}{8} = i_s + \frac{7-10}{2} = \frac{10}{8} + \frac{10}{8} \quad \Rightarrow \quad i_s = 4\ \text{A}$$

Problem P 4.3-2

The voltages v_a, v_b, v_c, and v_d in the figure are the node voltages corresponding to nodes a, b, c, and d. The current i is the current in a short circuit connected between nodes b and c. Determine the values of v_a, v_b, v_c, and v_d and of i.



Problem P 4.3-2 Solution



Express the branch voltage of each voltage source in terms of its node voltages to get:

$$v_a = -12 \text{ V}, \;\; v_b = v_c = v_d + 8$$

KCL at node b:

$$\frac{v_b - v_a}{4000} = 0.002 + i \;\;\Rightarrow\;\; \frac{v_b - (-12)}{4000} = 0.002 + i \;\;\Rightarrow\;\; v_b + 12 = 8 + 4000\, i$$

KCL at the supernode corresponding to the 8 V source:

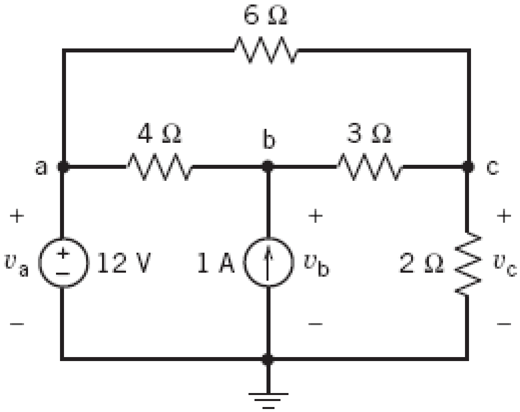
$$0.001 = \frac{v_d}{4000} + i \;\;\Rightarrow\;\; 4 = v_d + 4000\, i$$

so $$v_b + 4 = 4 - v_d \;\;\Rightarrow\;\; (v_d + 8) + 4 = 4 - v_d \;\;\Rightarrow\;\; v_d = -4 \text{ V}$$

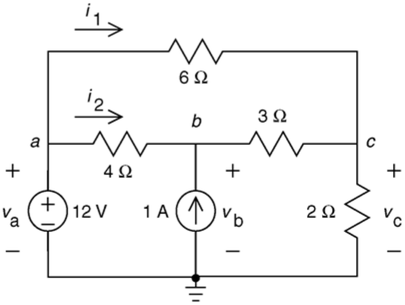
Consequently $v_b = v_c = v_d + 8 = 4$ V and $i = \dfrac{4 - v_d}{4000} = 2$ mA

Problem P 4.3-5

The voltages v_a, v_b, and v_c in the figure are the node voltages corresponding to nodes a, b, and c. The values of these voltages are: $v_a = 12V$, $v_b = 9.882V$,and$v_c = 5.294V$. Determine the power supplied by the voltage source.



Problem P 4.3-5 Solution



The power supplied by the voltage source is

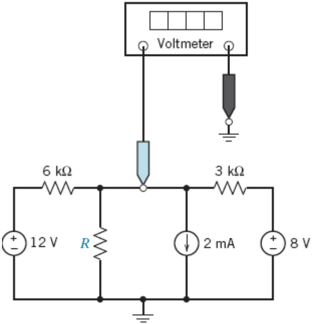
$$v_a\left(i_1+i_2\right)=v_a\left(\frac{v_a-v_b}{4}+\frac{v_a-v_c}{6}\right)=12\left(\frac{12-9.882}{4}+\frac{12-5.294}{6}\right)$$

$$=12(0.5295+1.118)=12(1.648)=19.76 \text{ W}$$

Problem P 4.3-6

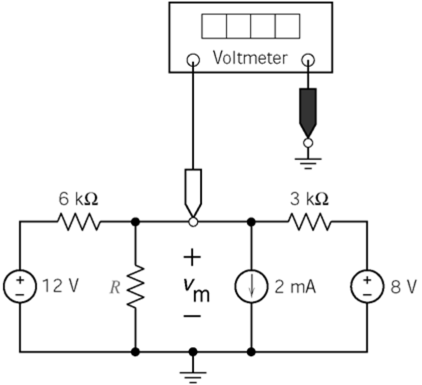
The voltmeter in the circuit of the figure measures a node voltage. The value of that node voltage depends on the value of the resistance R.

- Determine the value of the resistance R that will cause the voltage measured by the voltmeter to be 4 V.
- Determine the voltage measured by the voltmeter when $R = 1.2k\Omega$.



Problem P 4.3-6 Solution

Label the voltage measured by the meter. Notice that this is a node voltage.



Write a node equation at the node at which the node voltage is measured.

$$-\left(\frac{12-v_m}{6000}\right)+\frac{v_m}{R}+0.002+\frac{v_m-8}{3000}=0$$

That is

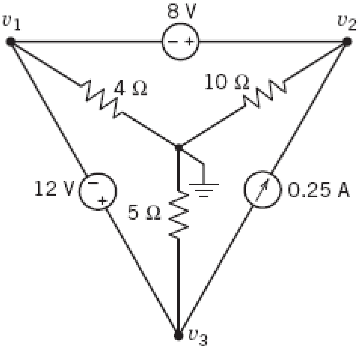
$$\left(3+\frac{6000}{R}\right)v_m=16 \Rightarrow R=\frac{6000}{\frac{16}{v_m}-3}$$

(a) The voltage measured by the meter will be 4 volts when $R = 6$ kΩ.

(b) The voltage measured by the meter will be 2 volts when $R = 1.2$ kΩ.

Problem P 4.3-12

Determine the values of the node voltages of the circuit shown in the figure.



Problem P 4.3-12 Solution

Express the voltage source voltages in terms of the node voltages:

$$v_2 - v_1 = 8 \text{ and } v_3 - v_1 = 12$$

Apply KVL to the supernode to get

$$\frac{v_2}{10} + \frac{v_1}{4} + \frac{v_3}{5} = 0 \quad \Rightarrow \quad 2v_2 + 5v_1 + 4v_3 = 0$$

so

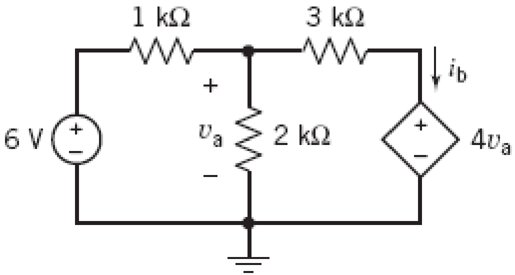
$$2(8 + v_1) + 5v_1 + 4(12 + v_1) = 0 \quad \Rightarrow \quad v_1 = -\frac{64}{11} \text{ V}$$

The node voltages are

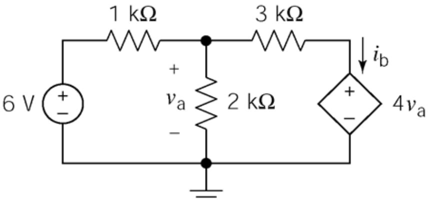
$$v_1 = -5.818 \text{ V}$$
$$v_2 = 2.182 \text{ V}$$
$$v_3 = 6.182 \text{ V}$$

Problem P 4.4-2

Find i_b for the circuit shown in the figure.



Problem P 4.4-2 Solution



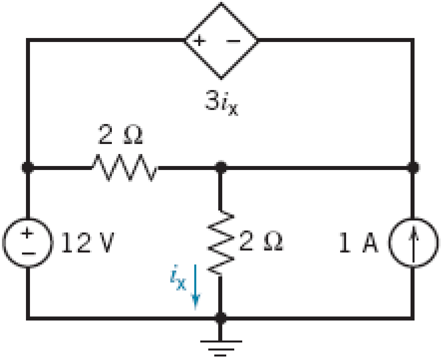
Write and solve a node equation:

$$\frac{v_a-6}{1000}+\frac{v_a}{2000}+\frac{v_a-4v_a}{3000}=0 \quad\Rightarrow\quad v_a=12\text{ V}$$

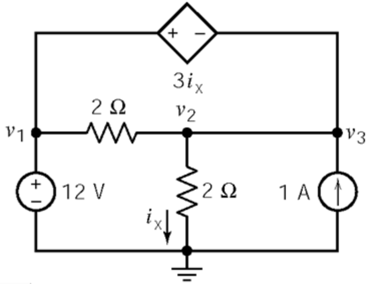
$$i_b=\frac{v_a-4v_a}{3000}=\underline{-12\text{ mA}}$$

Problem P 4.4-5

Determine the value of the current i_x in the circuit of the figure.



Problem P 4.4-5 Solution



First, express the controlling current of the CCVS in terms of the node voltages: $i_x = \dfrac{v_2}{2}$

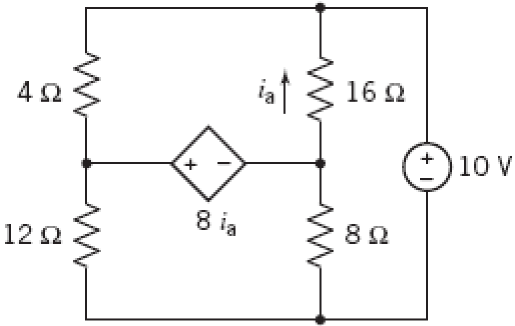
Next, express the controlled voltage in terms of the node voltages:

$$12 - v_2 = 3\,i_x = 3\,\frac{v_2}{2} \quad \Rightarrow v_2 = \frac{24}{5}\ \text{V}$$

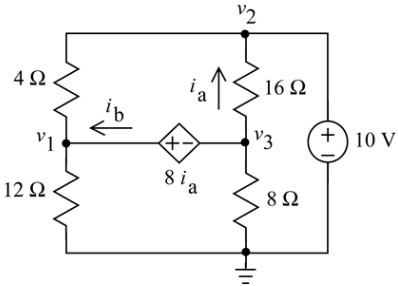
so $i_x = 12/5\ \text{A} = 2.4\ \text{A}$.

Problem P 4.4-8

Determine the value of the power supplied by the dependent source in the figure.



Problem P 4.4-8 Solution



Label the node voltages.

First, $v_2 = 10$ V due to the independent voltage source. Next, express the controlling current of the dependent source in terms of the node voltages:

$$i_a = \frac{v_3 - v_2}{16} = \frac{v_3 - 10}{16}$$

Now the controlled voltage of the dependent source can be expressed as

$$v_1 - v_3 = 8\, i_a = 8\left(\frac{v_3 - 10}{16}\right) \quad \Rightarrow \quad v_1 = \frac{3}{2} v_3 - 5$$

Apply KCL to the supernode corresponding to the dependent source to get

$$\frac{v_1 - v_2}{4} + \frac{v_1}{12} + \frac{v_3 - v_2}{16} + \frac{v_3}{8} = 0$$

Problem P 4.4-8 Solution

Multiplying by 48 and using $v_2 = 10$ V gives

$$16v_1 + 9v_3 = 150$$

Substituting the earlier expression for v_1

$$16\left(\frac{3}{2}v_3 - 5\right) + 9v_3 = 150 \quad \Rightarrow \quad v_3 = 6.970 \text{ V}$$

Then $v_1 = 5.455$ V and $i_a = -0.1894$ A. Applying KCL at node 2 gives

$$\frac{v_1}{12} = i_b + \frac{10 - v_1}{4} \quad \Rightarrow \quad 12\, i_b = -3 + 4\, v_1 = -30 + 4(5.455)$$

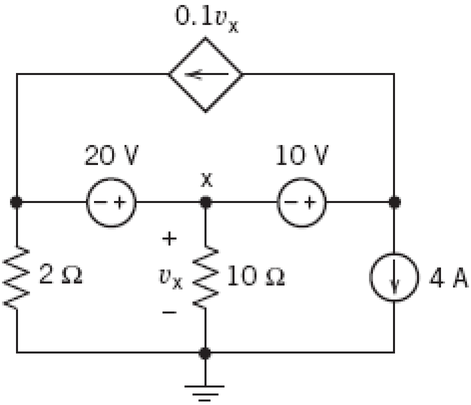
So $$i_b = -0.6817 \text{ A.}$$

Finally, the power supplied by the dependent source is

$$p = (8\, i_a)\, i_b = 8(-0.1894)(-0.6817) = 1.033 \text{ W}$$

Problem P 4.4-11

Determine the power supplied by the dependent source in the circuit shown in the figure.



Problem P 4.4-11 Solution

This circuit contains two ungrounded voltage sources, both incident to node x. In such a circuit it is necessary to merge the super-nodes corresponding to the two ungrounded voltage sources into a single supernode. That single supernode separates the two voltage sources and their nodes from the rest of the circuit. It consists of the two resistors and the current source. Apply KCL to this supernode to get

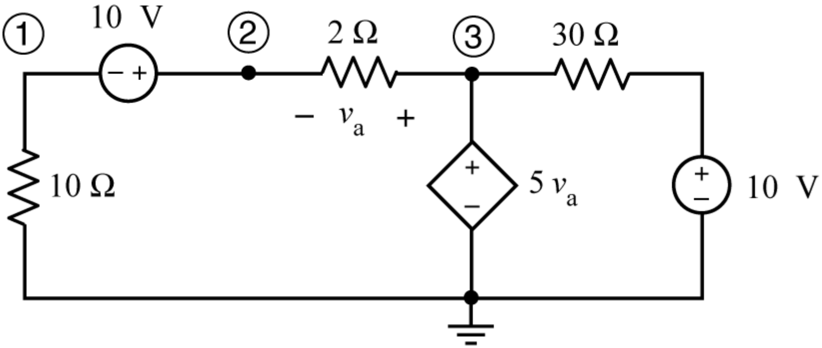
$$\frac{v_x - 20}{2} + \frac{v_x}{10} + 4 = 0 \quad \Rightarrow \quad v_x = 10 \text{ V}.$$

The power supplied by the dependent source is

$$(0.1\, v_x)(-30) = -30 \text{ W}.$$

Problem P 4.4-20

The encircled numbers in the figure are node numbers. Determine the values of v_1, v_2 and v_3, the node voltages corresponding to nodes 1, 2 and 3.



Problem P 4.4-20 Solution

Apply KCL to the supernode corresponding to the horizontal voltage source to get

$$\frac{v_1}{10} = \frac{v_a}{2} = \frac{v_3 - v_2}{2} = \frac{v_3 - (v_1 + 10)}{2} \quad \Rightarrow \quad v_1 = 5\left(v_3 - \left(v_1 + 10\right)\right) \quad \Rightarrow \quad 50 = -6\,v_1 + 5\,v_3$$

Looking at the dependent source we notice that

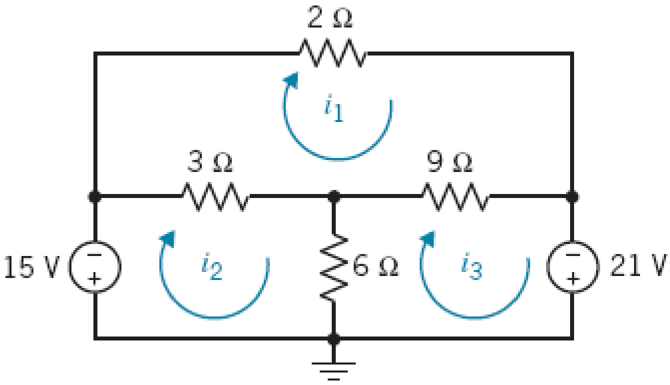
$$v_3 = 5\,v_a = 5\left(v_3 - v_2\right) = 5\left(v_3 - \left(v_1 + 10\right)\right) \quad \Rightarrow \quad 50 = -5\,v_1 + 4\,v_3$$

Consequently $\qquad v_1 = -50 \text{ V} \;$ and $v_3 = -50 \text{ V}$

Then $\qquad v_2 = v_1 + 10 = -40 \text{ V}$

Problem P 4.5-1

Determine the mesh currents, i_1, i_2, and i_3, for the circuit shown in the figure.



Problem P 4.5-1 Solution

The mesh equations are

$$2\,i_1 + 9\,(i_1 - i_3) + 3\,(i_1 - i_2) = 0$$
$$15 - 3\,(i_1 - i_2) + 6\,(i_2 - i_3) = 0$$
$$-6\,(i_2 - i_3) - 9\,(i_1 - i_3) - 21 = 0$$

or

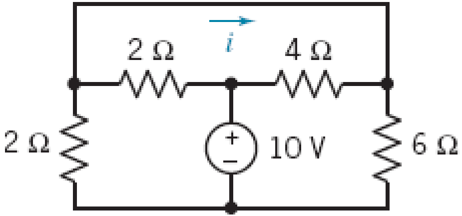
$$14\,i_1 - 3\,i_2 - 9\,i_3 = 0$$
$$-3\,i_1 + 9\,i_2 - 6\,i_3 = -15$$
$$-9\,i_1 - 6\,i_2 + 15\,i_3 = 21$$

so

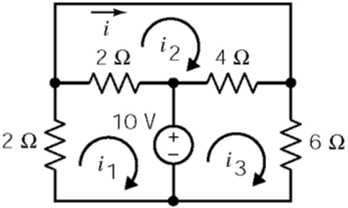
$$i_1 = 3 \text{ A}, \; i_2 = 2 \text{ A and } i_3 = 4 \text{ A}.$$

Problem P 4.5-5

Find the current i for the circuit of the figure. *Hint:A short circuit can be treated as a 0-V voltage source.*



Problem P 4.5-5 Solution



Mesh Equations:

$$\text{mesh 1}: \ 2i_1 + 2(i_1 - i_2) + 10 = 0$$

$$\text{mesh 2}: \ 2(i_2 - i_1) + 4(i_2 - i_3) = 0$$

$$\text{mesh 3}: \ -10 + 4(i_3 - i_2) + 6i_3 = 0$$

Solving:

$$i = i_2 \ \Rightarrow \ i = -\frac{5}{17} = -0.294 \text{ A}$$

Thank you !