Electric Circuits 1

ECSE-200 Section: 1, 2

20 Apr 2011, 2:00PM

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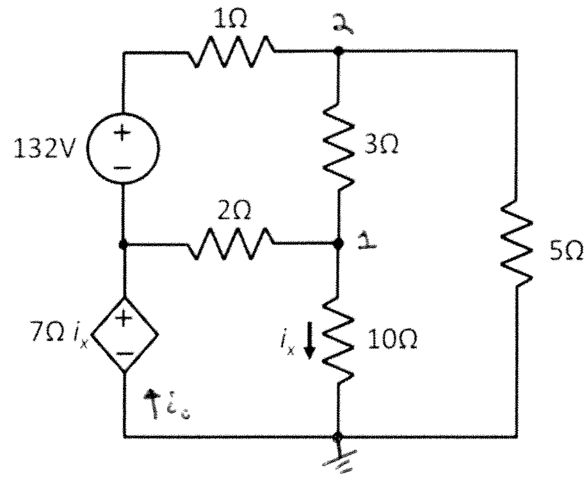
Student Name:		**McGill ID:**									

INSTRUCTIONS:

- This is a **CLOSED BOOK** examination.
- **NO CRIB SHEETS** are permitted.
- **SPACE IS PROVIDED** on the examination to answer all questions.
- **STANDARD CALCULATOR** permitted ONLY.
- This examination consists of 4 questions, with a total of 18 pages, including the cover page.
- This examination is **PRINTED ON BOTH SIDES** of the paper
- This examination paper **MUST BE RETURNED**

1. Consider the circuit below. Answer the questions. [12 pts]



a) What is the physical principle behind Kirchoff's Voltage Law? [1pt]

b) What is the physical principle behind Kirchoff's Current Law? [1pt]

c) How many equations are required in order to solve the circuit: by mesh current analysis, and by node voltage analysis? [2pts]

d) What is the value of the current i_x? [5pts]

e) What is the power delivered or absorbed by the dependent source? [3pts]

a) Conservation of energy [+1]

b) Conservation of charge [+1]

c) 3 mesh equations + 1 control equation [+1]

2 node equations + 1 control equation [+1]

d) $$0 = \frac{V_1}{10\Omega} + \frac{V_1 - 7\Omega i_x}{2\Omega} + \frac{V_1 - V_2}{3\Omega} \quad [+1]$$

$$0 = \frac{V_2 - 132V - 7\Omega i_x}{1\Omega} + \frac{V_2 - V_1}{3\Omega} + \frac{V_2}{5\Omega} \quad [+1]$$

$$i_x = \frac{V_1}{10\Omega} \quad [+1]$$

$$0 = 35 v_1 - 20 v_2$$

$$3960 = -31 v_1 + 46 v_2$$

$$v_1 = \frac{\begin{vmatrix} 0 & -20 \\ +3960 & +46 \end{vmatrix}}{\begin{vmatrix} +35 & -20 \\ -31 & +46 \end{vmatrix}} = 80V \quad [+1]$$

$$i_x = \frac{v_1}{10\Omega} = 8A \quad [+1]$$

e) $$v_2 = \frac{35}{20} v_1 = 140V$$

$$i_o = i_x + \frac{v_2}{5\Omega} \quad [+1]$$

$$= 8A + \frac{140V}{5\Omega}$$

$$= 36A$$

$$P_{del} = 7\Omega i_x \cdot i_o \quad [+1]$$

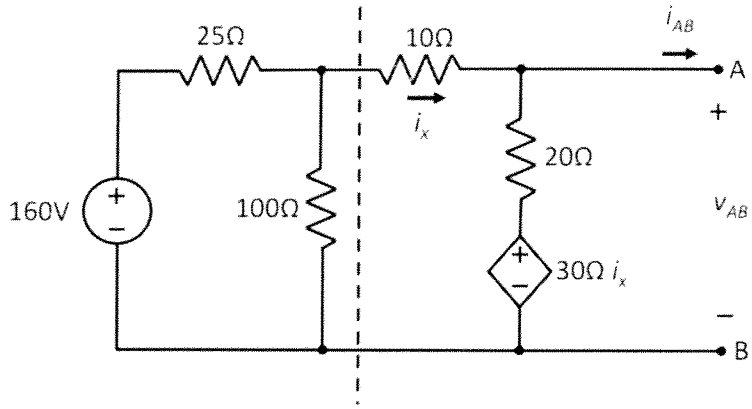
$$= 2.016 \text{ kW} \quad [+1]$$

delivered by
dependent source

work space

work space

2. Consider the circuit below. Answer the questions. [12 pts]



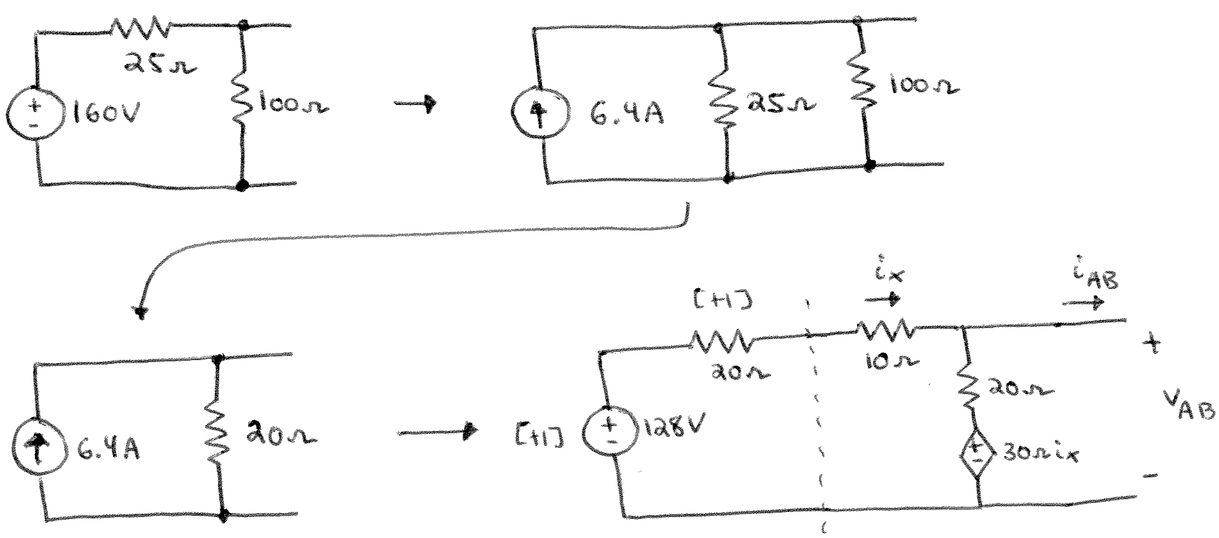
a) Redraw the circuit, with the sub-circuit to the left of the dashed line replaced by a Thévenin equivalent circuit. [2pts]

b) What is the Thévenin equivalent circuit with respect to the terminals A and B? [5pts]

c) What is the maximum power that this circuit can deliver to a load resistor R_L attached to terminals A and B? [2pts]

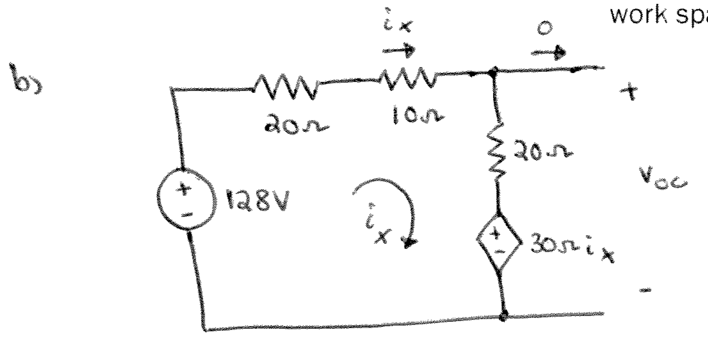
d) It is desired that 250W is dissipated in a load resistor R_L connected to the terminals A and B. What two values of R_L satisfy this condition? [3pts]

a)



work space

b)



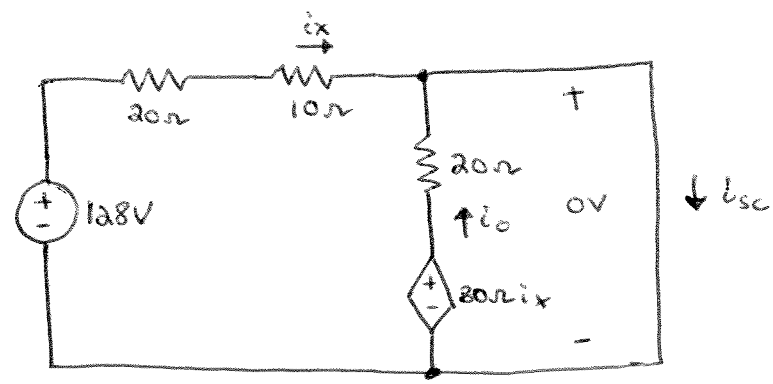
Find V_{oc} and i_{sc}. [+1]

KVL:

$$0 = -128V + 20\Omega \cdot i_x + 10\Omega \cdot i_x + 20\Omega\, i_x + 30\Omega \cdot i_x$$

$$i_x = \frac{128V}{80\Omega} = 1.6A$$

$$V_{oc} = 20\Omega \cdot i_x + 30\Omega \cdot i_x = 80V \quad [+1]$$



KVL:

$$0 = -128V + 20\Omega \cdot i_x + 10\Omega \cdot i_x + 0V$$

$$i_x = \frac{128V}{30\Omega} = 4.267\ A$$

KVL:

$$0 = -30\Omega \cdot i_x + 20\Omega \cdot i_o$$

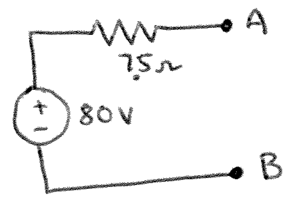
$$i_o = \frac{3}{2} \cdot i_x = 6.4\ A$$

KCL:

$$i_{sc} = i_o + i_x = 10.667\ A$$

[+1] [+1]

$$R_T = \frac{v_{oc}}{i_{sc}} = 7.5\,\Omega \quad [+1]$$



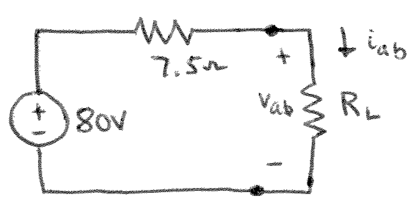
c) $$P_{max} = \frac{V_{oc}}{2} \cdot \frac{i_{sc}}{2} \quad [+1]$$

$$= \frac{80V}{2} \cdot \frac{10.667A}{2}$$

$$= 213.3\,W \quad [+1]$$

d) 250 W is greater than $P_{max} = 213.3\,W$; it is therefore not possible to choose a load resistor R_L that absorbs 250 W from the circuit. [+3]

Mathematically:



$$P_L = V_{ab} \cdot i_{ab}$$

$$= V_{oc} \cdot \frac{R_L}{R_L + 7.5\Omega} \times i_{sc} \frac{7.5\Omega}{R_L + 7.5\Omega}$$

$$= \frac{V_{oc} \cdot i_{sc}}{4} \times \frac{4 R_L \cdot 7.5\Omega}{(R_L + 7.5\Omega)^2}$$

$$\frac{250W}{213.3W}(R_L + 7.5\Omega)^2 = 4 R_L \cdot 7.5\Omega$$

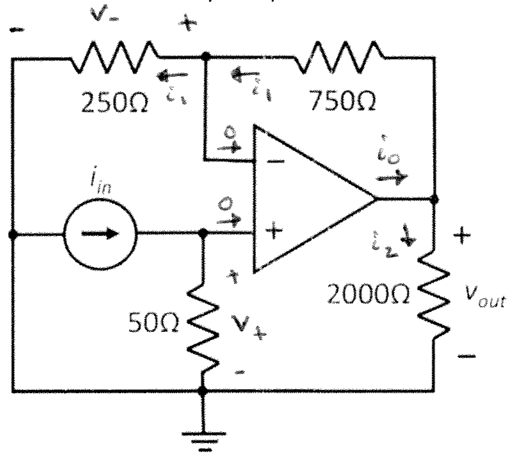
$$1.17 R_L^2 - 12.4 R_L + 65.9 = 0$$

$$R_L = \frac{12.4 \pm \sqrt{12.4^2 - 4 \cdot 1.17 \cdot 65.9}}{2 \cdot 1.17} \ \Omega$$

$$= 5.3 \pm 5.3i \ \Omega$$

$\rightarrow$ no real solution

3. Consider the circuit below. Assume ideal op-amp behaviour. Answer the questions. [12 pts]



a) Give one reason why negative feedback is used in op-amp circuits. [1pt]

b) What is the ratio v_{out} / i_{in}? Be sure to give correct units on your answer. [4pts]

For the remainder of this question, assume i_{in} = 10mA.

c) What is the power delivered by the independent current source? [2pts]

d) What is the power delivered by the op-amp? [4pts]

e) A voltmeter with 4kΩ internal resistance is used to measure v_{out}. What is the voltage value that is measured by the voltmeter? [1pt]

a) Operations can be programmed.
Circuit behaves independent of open-loop gain. [+1]
Circuit is stable.

b) $v_+ = 50\Omega \cdot i_{in}$ [+1]

$v_- = v_+ = 50\Omega \cdot i_{in}$ [+1]

$0 = \frac{v_-}{250\Omega} + \frac{v_- - v_{out}}{750\Omega}$ [+1]

$v_{out} = v_- \left(1 + \frac{750\Omega}{250\Omega}\right) = v_- \cdot 4$

$= 200\Omega \cdot i_{in}$ [+1]

c) $P_{\substack{\text{current}\\ \text{del}}} = i_{in} \cdot v_+$ [+1]

$= i_{in}^2 \cdot 50\Omega$

$= (10mA)^2 \cdot 50\Omega$

$= 5mW$ [+1]

d) $i_o = i_1 + i_2$ [+1]

$= \frac{V_-}{250\Omega} + \frac{V_{out}}{2000\Omega}$ [+1]

$= \frac{50\Omega \cdot 10mA}{250\Omega} + \frac{4 \cdot 50\Omega \cdot 10mA}{2000\Omega}$

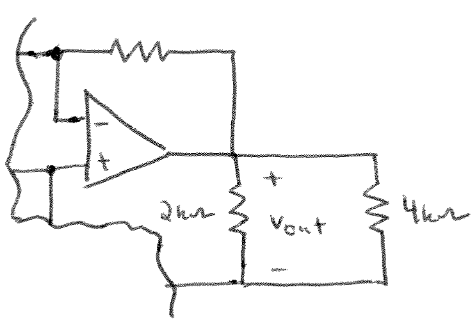
$= 3mA$

$P_{\substack{\text{op-amp}\\ \text{del}}} = i_o \cdot V_{out}$ [+1]

$= 3mA \cdot 4 \cdot 50\Omega \cdot 10mA$

$= 6mW$ [+1]

e)



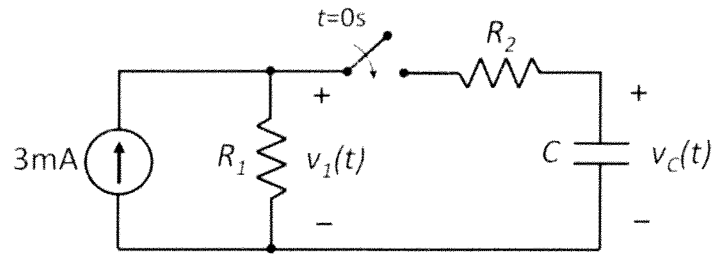
$V_{out} = 200\Omega \cdot i_{in}$ [+1/2]

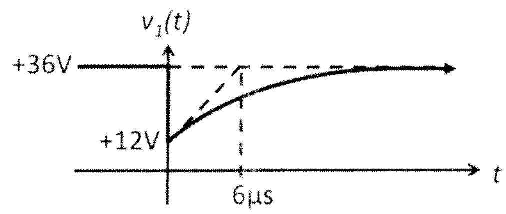
$= 2V$ [+1/2]

work space

work space

4. Consider the circuit and the plot below. The switch closes instantaneously at t=0s. Assume dc steady state behaviour for $t<0$. Assume the capacitor stores zero energy for $t<0$. Answer the questions. [12 pts]





a) Plot $v_C(t)$. Be sure to include the time intervals before and after the switching event. [2pts]
HINT: Consider the initial and final states of the capacitor.

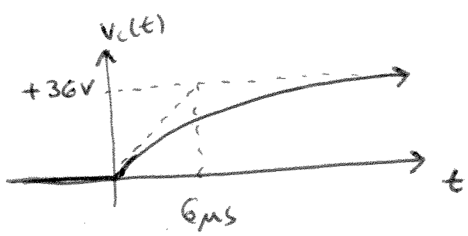
b) What are the values of R_1, R_2 and C ? [6pts]

c) What is the energy stored by the capacitor as $t \to \infty$? [2pts]

d) What is the total energy dissipated by the resistor R_2 on the time interval $0 < t$? [2pts]

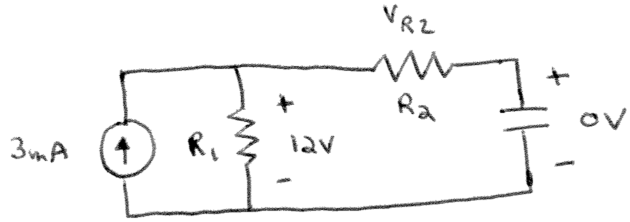
a) $V_C(0^-) = 0V$ (zero energy for $t<0$)

$V_C(\infty) = V_1(\infty) = +36V$ (C is an open circuit as $t \to \infty$)



[+1, initial condition]
[+1, final condition]

$t = 0^+$

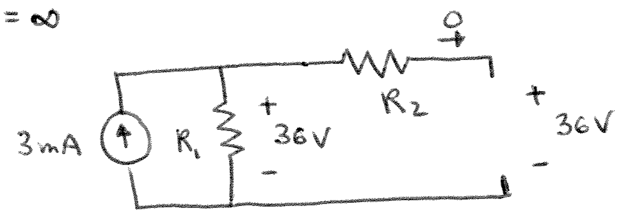


$$V_{R2} = 12V$$

$$3mA = \frac{12V}{R_1} + \frac{12V}{R_2} \quad [+1]$$

$$\frac{1}{4k\Omega} = \frac{1}{R_1} + \frac{1}{R_2}$$

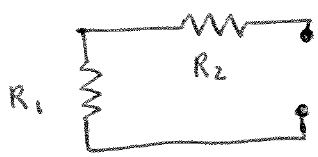
$t = \infty$



$$3mA = \frac{36V}{R_1} \quad [+1]$$

$$R_1 = 12k\Omega \quad [+1]$$

$$\therefore \quad R_2 = \left(\frac{1}{4k\Omega} - \frac{1}{R_1}\right)^{-1} = 6k\Omega \quad [+1]$$



$$R_{TH} = R_1 + R_2$$

$$\tau = 6\mu s = (R_1 + R_2) \cdot C \quad [+1]$$

$$C = \frac{6\mu s}{12k\Omega + 6k\Omega} = 333.3\ pF$$

c) $U = \frac{1}{2} C V_c^2(\infty)$ [+1]

$= \frac{1}{2} \cdot 333.3\,pF \cdot (36V)^2$

$= 216\ nJ$ [+1]

d) $V_c(t) = 36V - 36V \exp(-t/6\mu s)$

$i_c(t) = C \frac{dv_c}{dt} = 333.3\,pF \cdot -36V \cdot \frac{-1}{6\mu s} \exp(-t/6\mu s)$

$p_{abs} = i_c^2(t) \cdot R_2 = \left[\frac{333.3\,pF \cdot 36V}{6\mu s} \exp(-t/6\mu s) \right]^2 \cdot 6k\Omega$

$= 24mW \exp(-t/3\mu s)$

[+1 for technique]

$U_{abs} = \int_0^\infty p_{abs}(t)\, dt$

$= \int_0^\infty 24mW \exp(-t/3\mu s)\, dt$

$= 24mW \cdot (-3\mu s) \cdot [0 - 1]$

$= 72\ nJ$ [+1]

work space

work space