Today's Outline

2. Resistive Circuits

- Ammeters
- Voltmeters
- Wheatstone Bridge

Measurement

– Precise measurement of current, voltage or resistance requires the use of a properly chosen/designed instrument

– Ideal ammeters, voltmeters, ohmmeters are elements (ideal models) permitting the perfect measurement of current, voltage or resistance

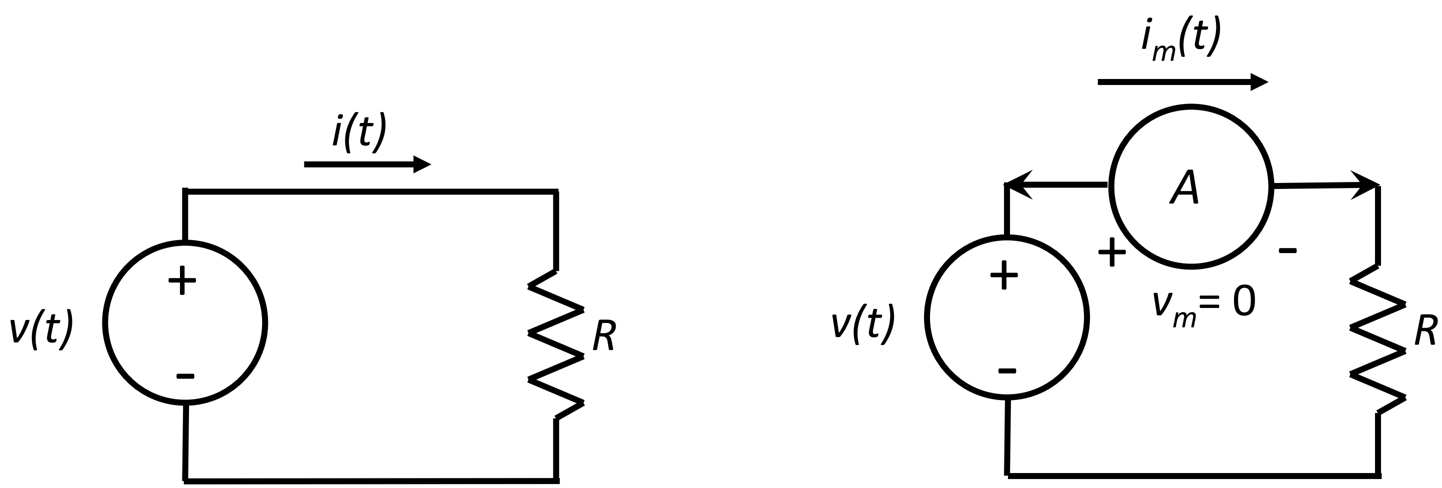
– Real, physical instruments introduce errors; choosing the correct instrument in the laboratory requires an understanding of physical measurement instruments

Ideal Ammeter

Ideal ammeter: an element that measures the current flowing through its terminals with zero power absorbed, meaning the ammeter voltage drop $v_m = 0$, equivalent to a short-circuit

– ammeter is placed in series with branch current, *i(t)* to be measured

– the measured current $i_m(t) = i(t)$ because the ideal ammeter is equivalent to a short circuit in series, leaving circuit voltages unchanged



circuit

circuit with ideal ammeter

Constructing an Ammeter

How can a current be "reported"?

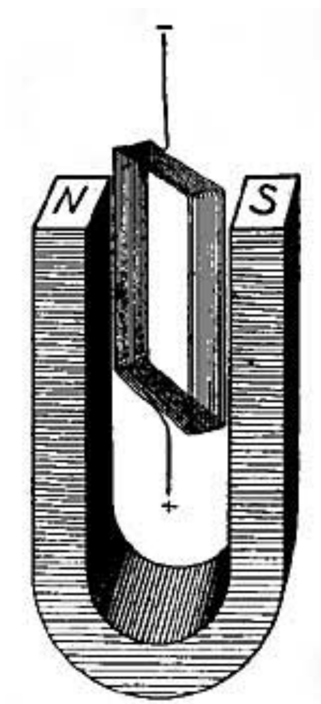
Magnetic field generated by current in a wire can be "seen" by it's effect on ferromagnets ("permanent" magnets).

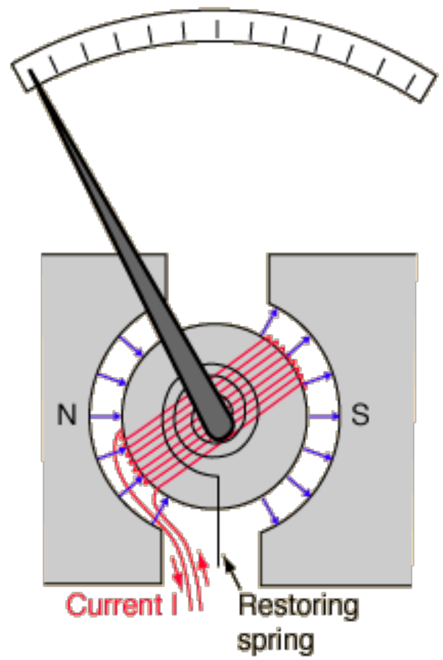
D'Arsonval Galvanometer

A coil carrying a current in the presence of a magnetic field can be arranged to produce an observable movement.

Hans Christian Ørsted (1777-1851)

Jacques-Arsène d'Arsonval (1851-1940)





torque = $n\ I\ A\ B$

n = number of loops

I = loop current

A = loop area

B = magnetic field

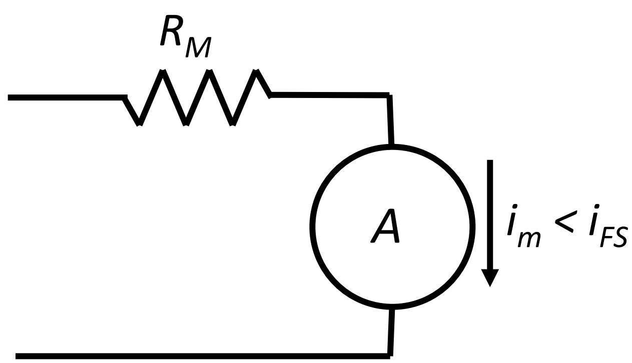
"Hyperphysics", C.R. Nave, Georgia State University

McGill

Physical Ammeter

A physical ammeter is characterized by:

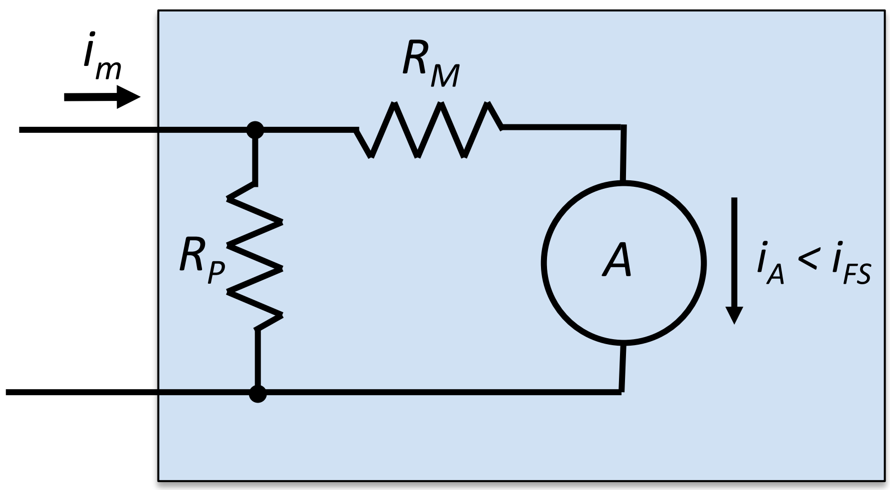
- an **internal resistance** R_M (due to wire loop resistance for a galvanometer)
- a **full-scale current** limit i_{FS} (due to the limit of mechanical deflection for a galvanometer)
- modern ammeters based on microelectronic circuits have internal resistance and a full-scale current limit arising from other physical reasons



physical ammeter modeled with
a resistor and ideal ammeter

Practical Ammeter

To measure a current i_m greater than the full-scale current i_{FS} of a galvanometer, a *current divider* is used to divert a known fraction of the current from the galvanometer



practical ammeter incorporating a current divider and galvanometer

internal resistance:

$R_{int} = R_P || R_M$

ratio of i_m to i_A:

$$i_A = i_m \frac{R_P}{R_P + R_M}$$

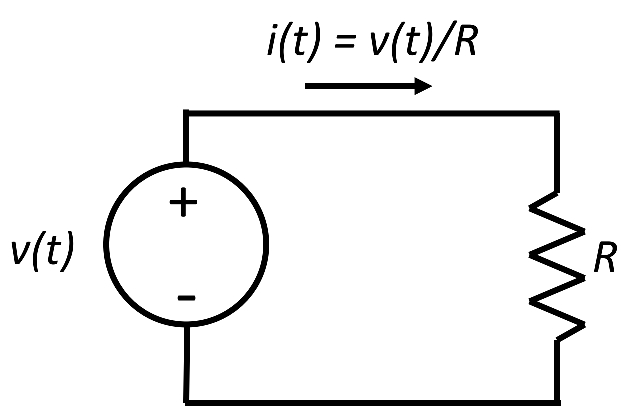
$$i_m = i_A \frac{R_P + R_M}{R_P}$$

$$= i_A \left(1 + R_M / R_P\right)$$

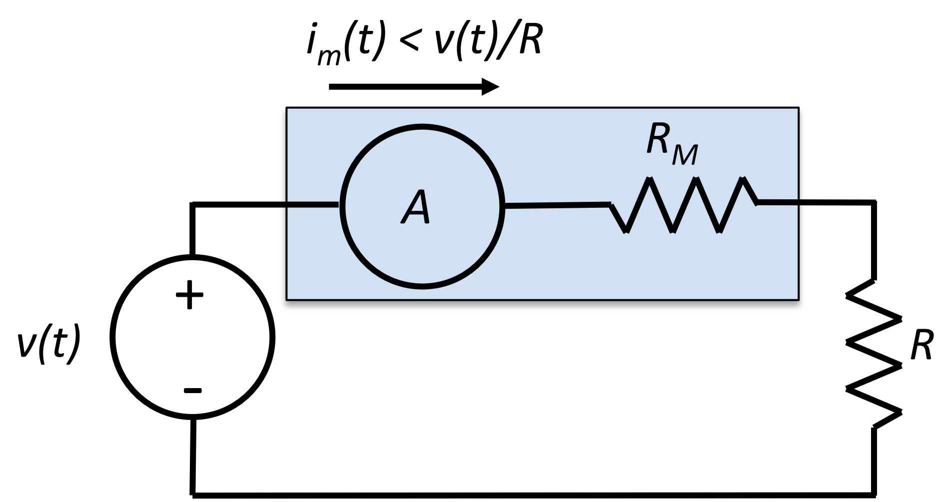
Measurement Error

Practical meters perturb the circuit being measured, causing a change in the flow of current and distribution of voltage, leading to **measurement error**.

In the case of the circuit below, the finite resistance of the ammeter clearly modifies the current flow in the circuit.



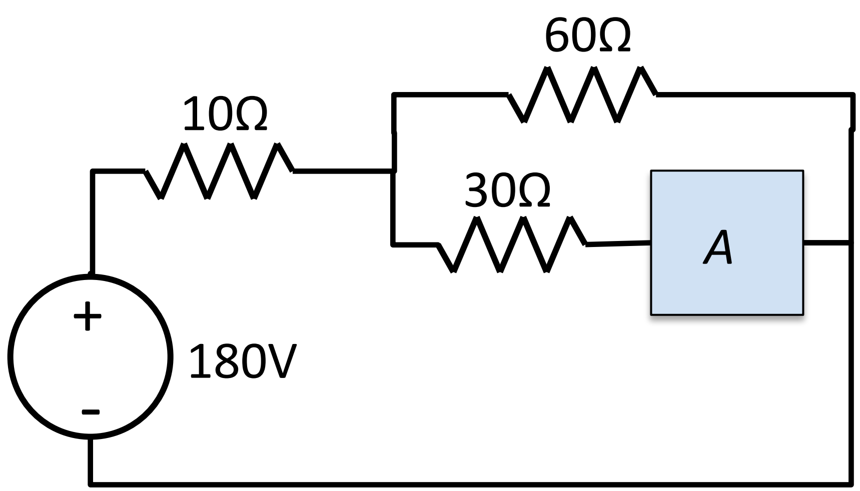
circuit



circuit with practical ammeter

Example

The ammeter below has a total internal resistance of 0.5Ω. Find the percentage error in the ammeter measurement.

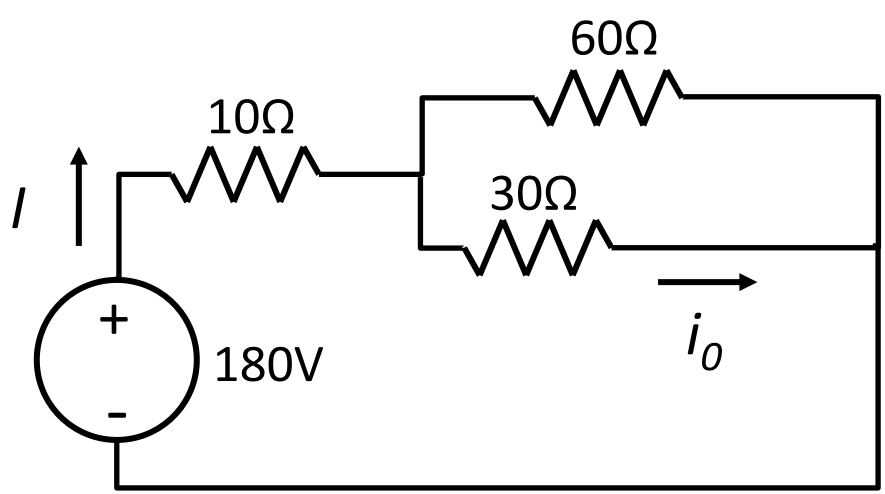


Strategy:

−find the ideal current and the actual measured current

Example

Find the ideal current.



The equivalent resistance "seen" by the source:

$$R_{eq} = 10\Omega + 30\Omega \,||\, 60\Omega$$
$$= 10\Omega + 20\Omega$$
$$= 30\Omega$$

Find the total current drawn:

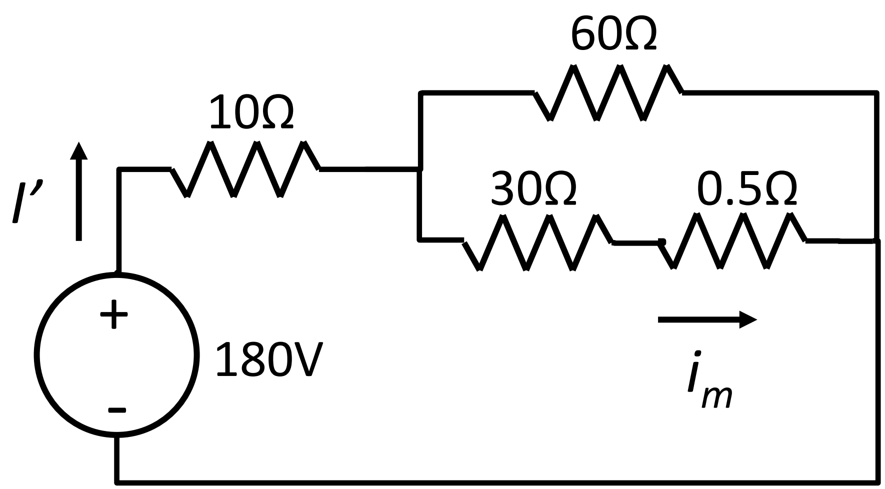
$$I = 180\text{V} / 30\Omega = 6\text{A}$$

Use the current divider:

$$i_0 = 6\text{A}\frac{60\Omega}{30\Omega + 60\Omega} = 4\text{A}$$

Example

Find the measured current.



The equivalent resistance "seen" by the source:

$$R_{eq} = 10\Omega + (30\Omega + 0.5\Omega)\,||\,60\Omega$$
$$= 10\Omega + 20.22\Omega$$
$$= 30.22\Omega$$

Find the total current drawn:

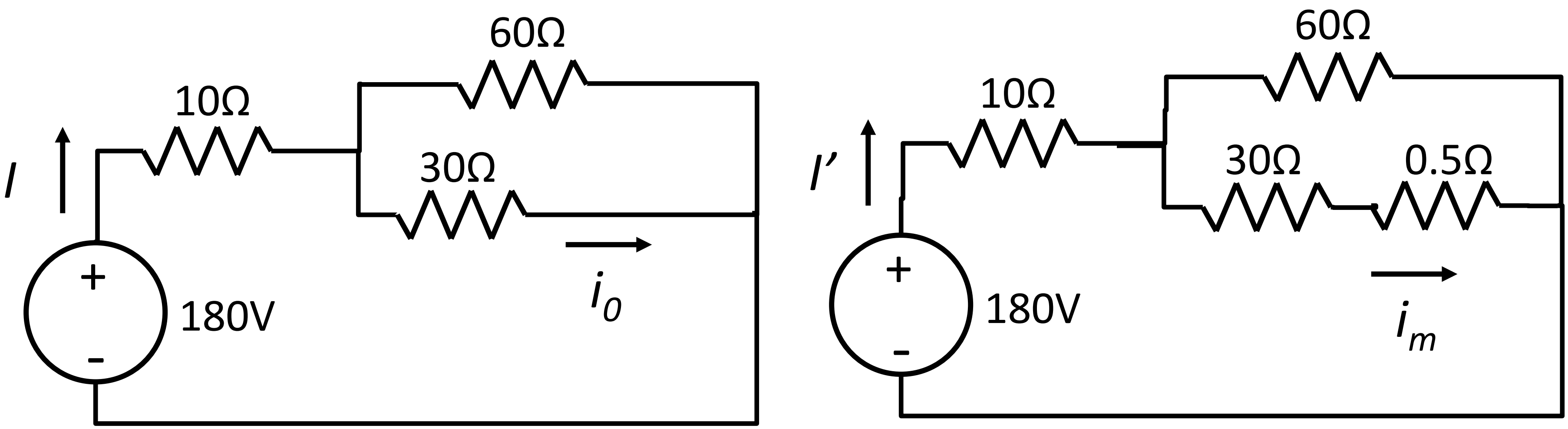
$$I' = 180\text{V} / 30.22\Omega = 5.956\text{A}$$

Use the current divider:

$$i_m = 5.956\text{A}\frac{60\Omega}{30.5\Omega + 60\Omega} = 3.949\text{A}$$

Example

Compare the ideal current and the actual measured current.



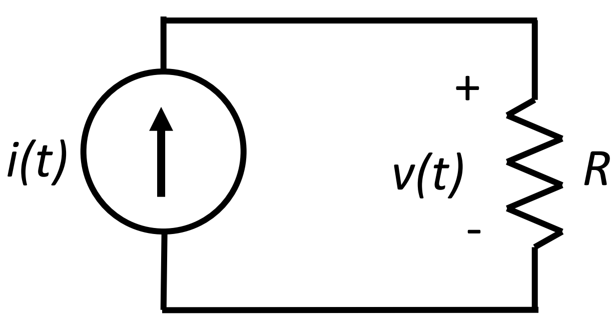
$$\text{percentage error} = \frac{3.949\text{A} - 4\text{A}}{4\text{A}} \times 100\% = -1.30\%$$

Ideal Voltmeter

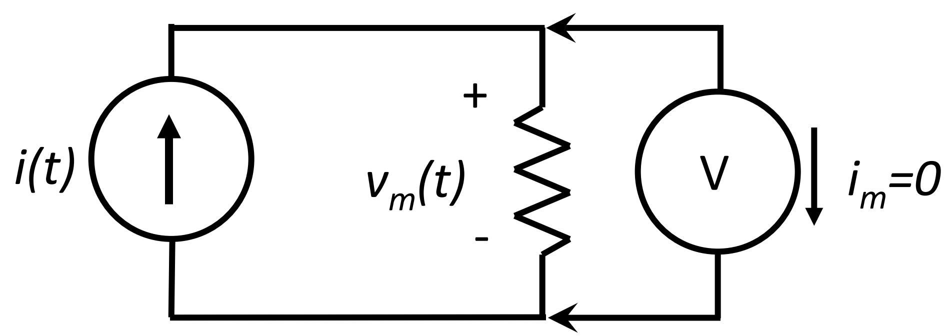
Ideal voltmeter: an element that measures the voltage across its terminals with zero power absorbed, meaning the voltmeter current drawn $i_m = 0$, equivalent to an open-circuit

– voltmeter is placed in parallel with branch voltage, *v(t)* to be measured

– the measured voltage $v_m(t) = v(t)$ because the ideal voltmeter is equivalent to an open circuit in parallel, leaving circuit currents unchanged



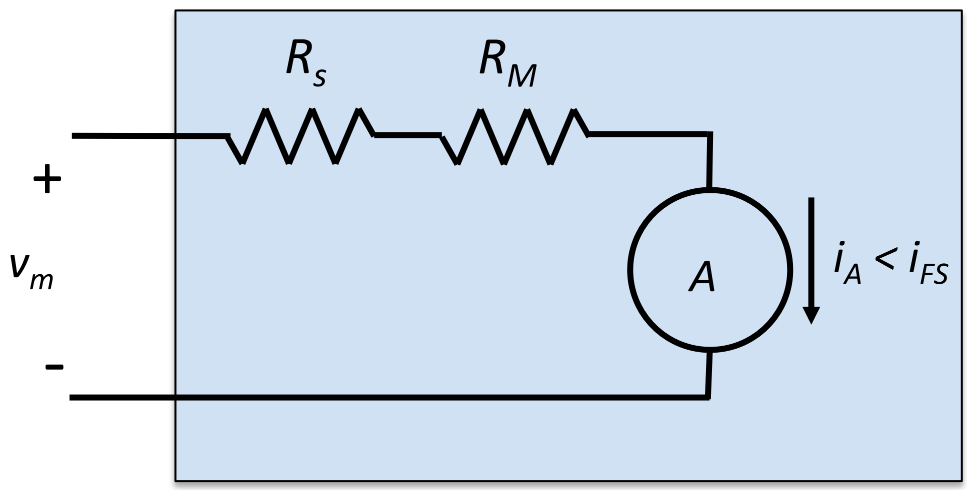
circuit



circuit with ideal voltmeter

Practical Voltmeter

– To measure a voltage with a galvanometer, a *potential divider* is used to reduce the potential, and thus current flow, through the galvanometer
– A practical voltmeter thus has a finite internal resistance



practical voltmeter incorporating a voltage divider and galvanometer

internal resistance:

$R_{int} = R_S + R_M$

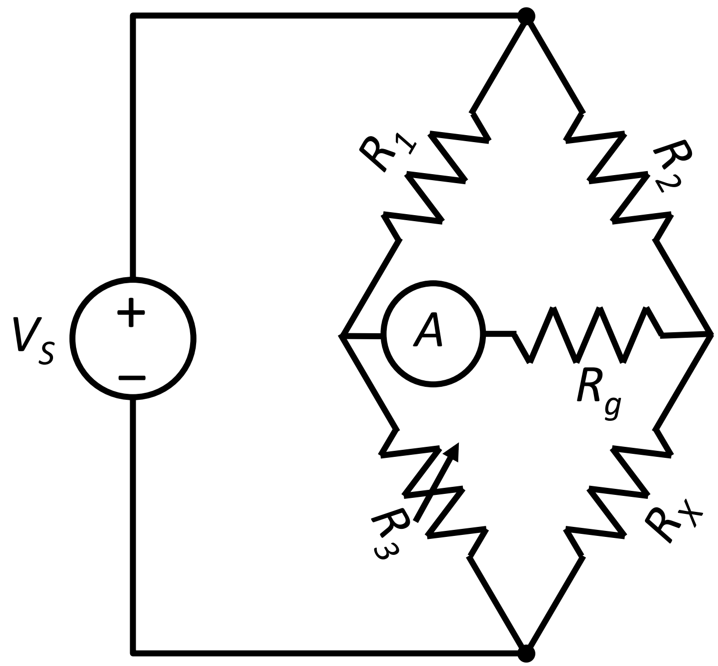
ratio of v_m to i_A:

$$i_A = \frac{v_m}{R_S + R_M}$$

$$v_m = i_A\left(R_S + R_M\right)$$

Wheatstone Bridge

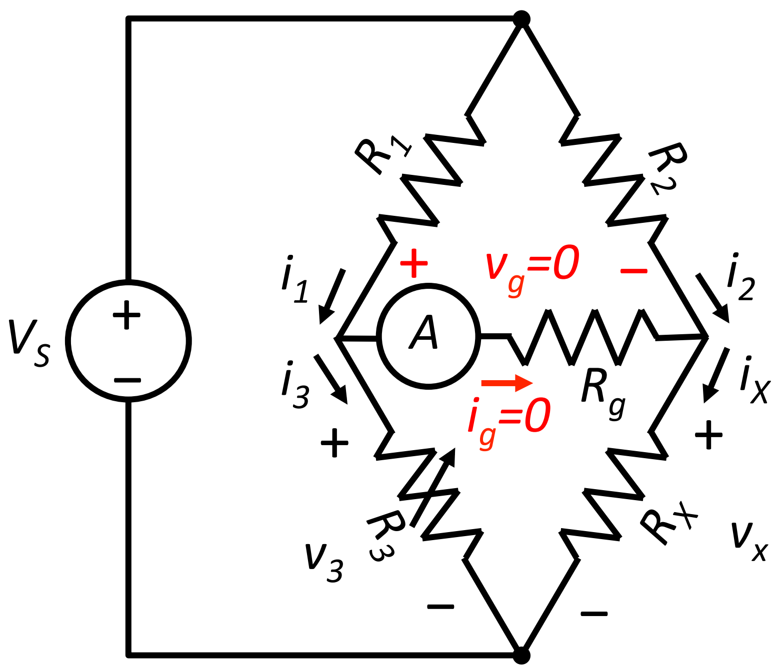
Wheatstone bridge: the circuit below that can be used to determine an unknown resistance (R_X) by adjusting resistances to produce zero current flow (zero voltage difference) at the terminals of a galvanometer.



Charles Wheatstone
(1802-1875)

Wheatstone Bridge

Consider the condition when there is no current flow through the galvanometer and the bridge is "balanced" : $i_g = 0 \;\rightarrow\; v_g = i_g R_g = 0$



KVL: $0 = -v_3 + v_g + v_X$

$v_3 = v_X$

KCL: $i_1 = i_3 \qquad i_2 = i_X$

voltage divider:

$$\frac{v_3}{V_S} = \frac{R_3}{R_3 + R_1} \qquad \frac{v_X}{V_S} = \frac{R_X}{R_X + R_2}$$

Wheatstone Bridge

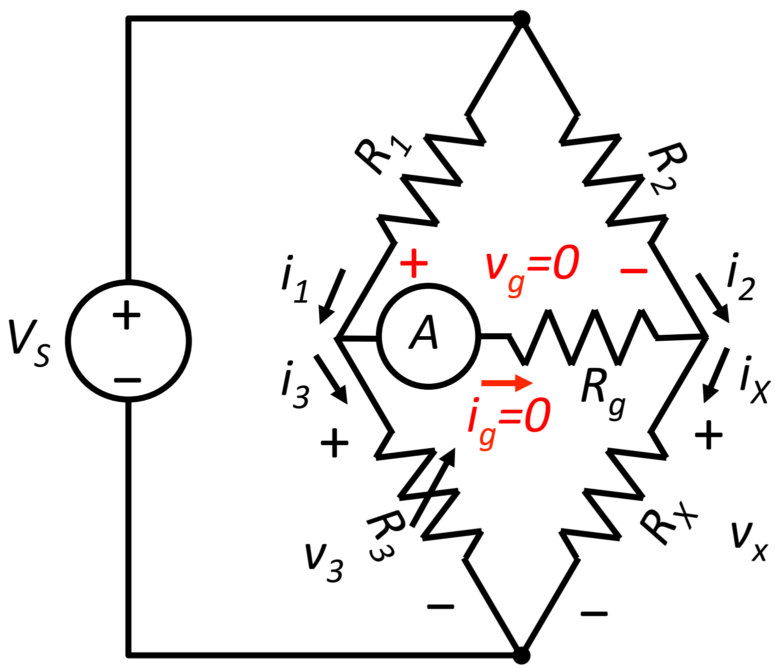
With the bridge in a balanced condition:

$$v_3 = v_X$$

$$\frac{R_3}{R_3 + R_1} = \frac{R_X}{R_X + R_2}$$

$$R_3 R_x + R_3 R_2 = R_x R_3 + R_x R_1$$

$$R_x = R_3 \cdot \frac{R_2}{R_1}$$



Typically, R_1 and R_2 are calibrated resistors, and R_3 is a calibrated rheostat that is adjusted to achieve balance ($i_g = 0$). R_x can then be calculated.

Section 2 Summary

Kirchoff's Current Law and Kirchoff's Voltage Law: Arising from physical conservation of charge and energy, these laws allow us to write equations relating algebraic current and voltage variables. These laws are the basis of all circuit analysis.

Equivalent Resistance: Any two-terminal circuit composed of ideal resistors has identical terminal characteristics to that of a single equivalent resistor, a fact that can be used to simplify circuit analysis problems. The equivalent resistance for series and parallel combinations of resistors are particularly useful.

Voltage Division and Current Division: Series resistors divide voltage as the ratio of resistance. Parallel resistors divide current as the ratio of conductance.

Ammeters, Voltmeters: Ideal meters do not transfer any power to/from the circuit under test, but practical meters do. The finite resistance of ammeters and voltmeters perturbs the circuit under test.