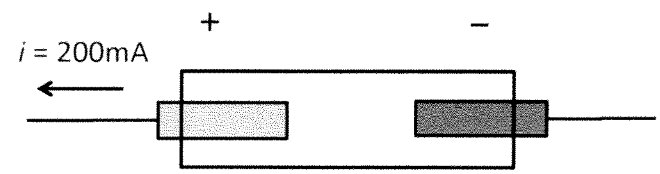
ECSE-200 Quiz #1 (9 Sept 2011)

NAME______________________________ McGill ID#______________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram below, representing a battery. Answer the questions.

$v(t) = 11.5\text{V} - 0.001\text{V/s} \times t$



a) Plot the instantaneous power $p(t)$ delivered by the battery versus the time t, for the time interval $0\text{s} \le t \le 1000\text{s}$. Clearly label the axes of your plot with SI units. [3pts]

b) At what time t is the battery delivering 2.2W of power? [2pts]

c) What is the energy delivered by the battery over the time interval $0\text{s} \le t \le 1000\text{s}$? [3pts]

d) How long will it take for 96kC of charge to exit the battery from the "+" terminal? [2pts]

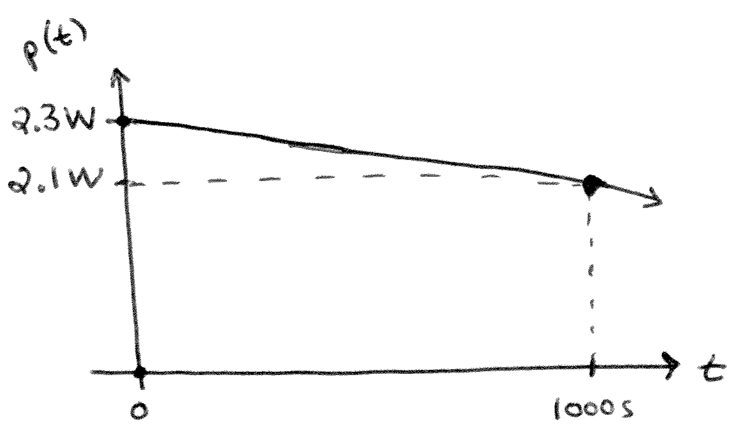
a) $p(t) = i \cdot v(t)$ [+1]

$= 200\text{mA} \cdot \left[11.5\text{V} - 0.001\frac{\text{V}}{\text{s}} \cdot t \right]$

$= 2.3\text{W} - 0.2\frac{\text{mW}}{\text{s}} \cdot t$

[+1] for values

[+1] for shape



b) $p(t) = 2.3\,W - 0.2\,\frac{mW}{s} \cdot t = 2.2\,W$ [+1]

$$t = \frac{0.1\,W}{0.2\,mW/s}$$

$$= 500\,s \quad [+1]$$

c) $U = \int_0^{1000s} p(t)\,dt$ [+1]

$$= \int_0^{1000s} \left(2.3W - 0.2\,\frac{mW}{s}\,t\right) dt$$

$$= \left(2.3W\,t - \frac{1}{2} \cdot 0.2\,\frac{mW}{s}\,t^2\right)\Bigg|_{t=0}^{t=1000s} \quad [+1]$$

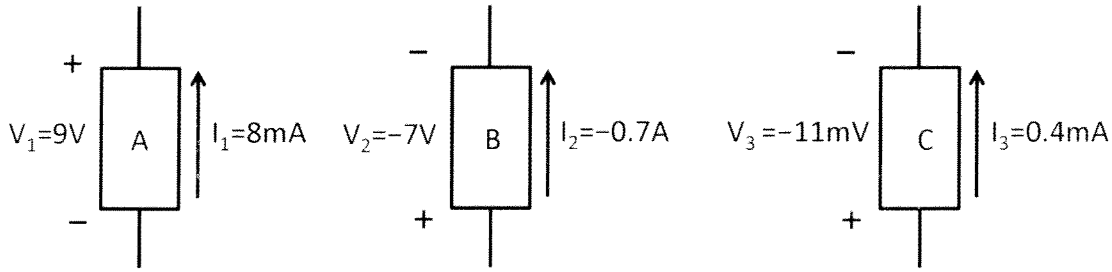
$$= 2.2\,kJ \quad [+1]$$

d) $q = i \cdot \Delta t$ [+1]

$$\Delta t = \frac{96\,kC}{200\,mA}$$

$$= 480\,000\ s \quad [+1]$$

2. Consider the circuit diagrams below. Answer the questions.



a) Do the variables V_1 and I_1 satisfy passive sign convention? [1pt]

b) How much power is the element A delivering or absorbing? [2pts]

c) Do the variables V_2 and I_2 satisfy passive sign convention? [1pt]

d) How much power is the element B delivering or absorbing? [2pts]

e) Do the variables V_3 and I_3 satisfy passive sign convention? [1pt]

f) How much power is the element C delivering or absorbing? [2pts]

a) no [+1]

b) $P_{del} = V_1 \cdot I_1 = 9V \cdot 8mA = 72\ mW$ [+1]

delivered by A [+1]

c) yes [+1]

d) $P_{abs} = V_2 \cdot I_2 = (-7V)\cdot(-0.7A) = 4.9\ W$ [+1]

absorbed by B [+1]

e) yes [+1]

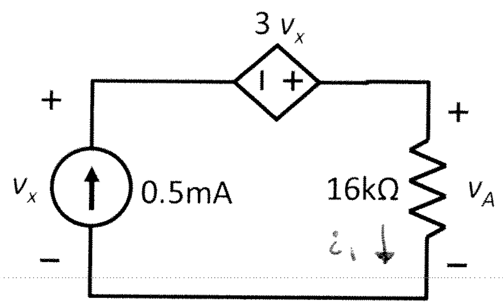
f) $P_{abs} = V_3 \cdot I_3 = (-11\ mV)\cdot(0.4mA) = -4.4\ \mu W$ [+1]

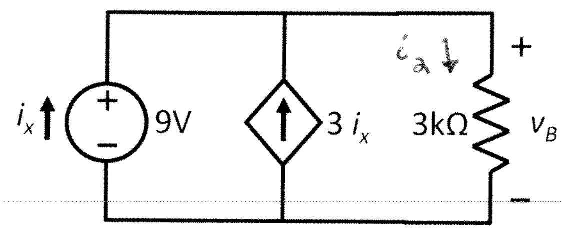
absorbed by C [+1]

NAME______________________________ McGill ID#__________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagrams below. Answer the questions.





a) What is the value of v_A? [3pts]

b) What is the value of v_X? [2pts]

c) What is the value of v_B? [2pts]

d) What is the value of i_x? [3pts]

a) KCL: $0 = 0.5mA - i_1$ [+1]

Ohm: $v_A = i_1 \cdot 16k\Omega$ [+1]

$= 0.5mA \cdot 16k\Omega = 8V$ [+1]

b) KVL: $0 = -v_x - 3v_x + v_A$ [+1]

$v_x = \frac{v_A}{4} = 2V$ [+1]

c) KVL: $0 = -9V + v_B$ [+1]

$v_B = 9V$ [+1]

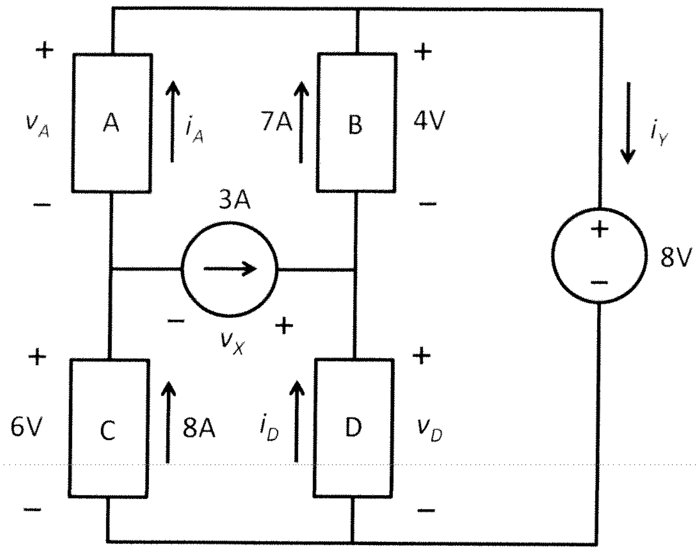
d) Ohm: $v_B = i_2 \cdot 3k\Omega$ [+1]

$i_2 = 9V / 3k\Omega = 3mA$

KCL: $0 = -i_x - 3i_x + i_2$ [+1]

$i_x = i_2/4 = 0.75mA$ [+1]

2. Consider the circuit diagram below. Answer the questions. **HINT:** You may find it useful to apply KCL and KVL at appropriately chosen nodes and loops in the circuit.



a) What is the value of i_A? [2pts]

b) What is the value of i_D? [2pts]

c) What is the value of v_x? [2pts]

d) What is the value of v_D? [2pts]

e) Identify the elements that are absorbing power. What is the total power absorbed by these elements? [2pts]

a) KCL: $0 = -8A + 3A + i_A$ [+1]

$i_A = 5A$ [+1]

b) KCL: $0 = -3A + 7A - i_D$ [+1]

$i_D = 4A$ [+1]

c) KVL: $0 = -6V - v_x - 4V + 8V$ [+1]

$v_x = -2V$ [+1]

d) KVL: $0 = -v_D - 4V + 8V$ [+1]

$v_D = 4V$ [+1]

e) By KVL, $v_A = 2V$. Therefore elements A, B, C, D are all delivering power ($i_A \cdot v_A > 0$ and i_A, v_A do not satisfy passive sign convention, same for B, C, D).

By KCL: $0 = i_Y - i_A - 7A = i_Y - 12A$

$i_Y = 12A$

Power <u>absorbed</u> <u>by</u> independent voltage source:

$$P_{8V} = 8V \cdot i_Y = 8V \cdot 12A = 96\ W$$

Power <u>delivered</u> <u>by</u> independent current source:

$$P_{3A} = V_x \cdot 3A = (-2V) \cdot 3A = -6\ W$$

∴ The independent current source and voltage source are both absorbing power. [+1]

The total power absorbed by these elements is:

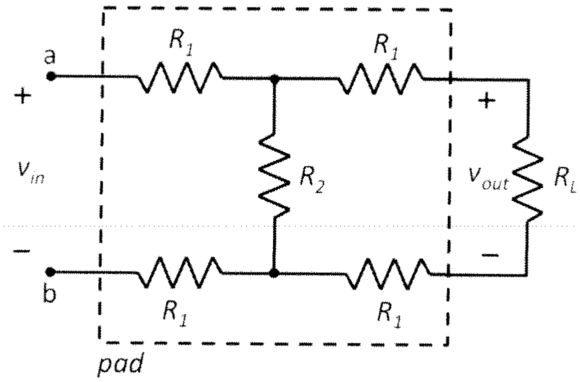
$$P_{abs} = 96W + 6\ W = 102\ W \quad [+1]$$

ECSE-200 Quiz #3 (23 Sept 2011)

NAME____ McGill ID#__________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

Consider the circuit below. The boxed area is known as a "pad", and is used in volume control circuits. Answer the questions.



a) What is the equivalent resistance R_{eq} between the terminals *a* and *b*, expressed in terms of R_L, R_1 and R_2 ? [1pt]

b) What is the ratio v_{out}/v_{in}, expressed in terms of R_L, R_1 and R_2 ? [1pt]

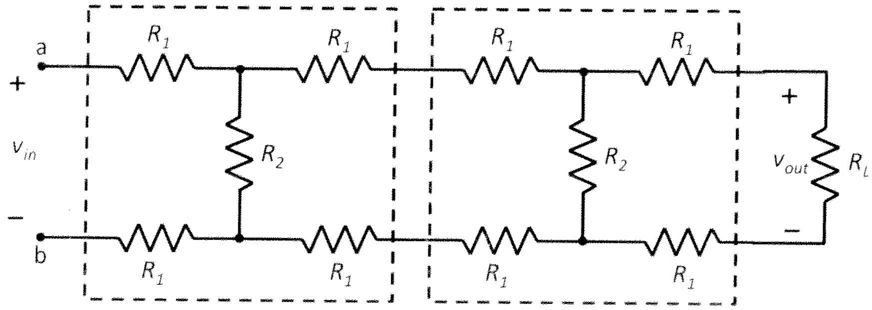
For the remainder of the questions, assume that $R_L = 600\Omega$, and that the values of R_1 and R_2 have been designed such that $R_{eq} = R_L = 600\Omega$ and $v_{out}/v_{in} = 0.6$.

c) What are the values of R_1 and R_2 ? [4pts]

d) How much power would be absorbed by the entire network of resistors if a 24V independent voltage source was connected across the terminals *a* and *b*? [2pts]

e) How much power would be absorbed by the resistor R_L if a 24V independent voltage source was connected across the terminals *a* and *b*? [2pts]

f) A second identical pad circuit is inserted between the load resistor and the first pad, as below. For this new circuit, what is the equivalent resistance R_{eq} between the terminals *a* and *b* and what is the ratio v_{out}/v_{in}. Give numerical values. [2pts]



work space

a) $R_{eq} = 2R_1 + R_2 \parallel (2R_1 + R_L)$ [+1]

b) $$\frac{V_{out}}{V_{in}} = \frac{R_L}{R_L + 2R_1} \cdot \frac{R_2 \parallel (2R_1 + R_L)}{R_2 \parallel (2R_1 + R_L) + 2R_1}$$ [+1]

c) $R_{eq} = R_L = 2R_1 + R_2 \parallel (2R_1 + R_L)$

$\therefore R_2 \parallel (2R_1 + R_L) = R_L - 2R_1$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{R_L}{R_L + 2R_1} \cdot \frac{R_L - 2R_1}{R_L - 2R_1 + 2R_1} = \frac{R_L - 2R_1}{R_L + 2R_1}$$

$\frac{V_{out}}{V_{in}} \cdot (R_L + 2R_1) = R_L - 2R_1$

$$2R_1 = \frac{R_L\left(1 - \frac{V_{out}}{V_{in}}\right)}{\left(1 + \frac{V_{out}}{V_{in}}\right)}$$ [+1]

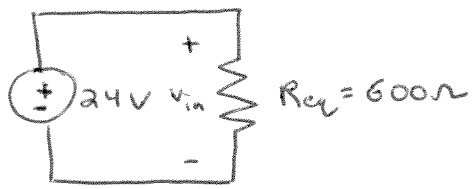
$$R_1 = \frac{600\Omega}{2} \cdot \frac{(1 - 0.6)}{(1 + 0.6)} = 75\Omega$$ [+1]

Returning above: $$\frac{1}{R_2} + \frac{1}{2R_1 + R_L} = \frac{1}{R_L - 2R_1}$$ [+1]

$$R_2 = \left[\frac{1}{600\Omega - 150\Omega} - \frac{1}{600\Omega + 150\Omega}\right]^{-1}$$

$= 1125\Omega$ [+1]

d)

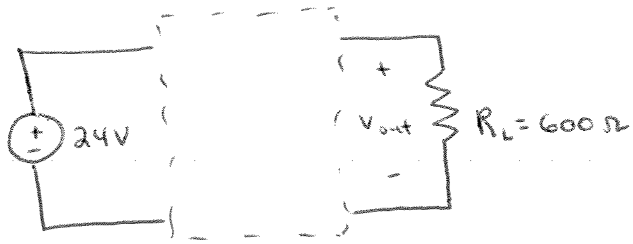


$$P_{abs} = \frac{v_{in}^2}{R_{eq}} \quad [+1]$$

$$= \frac{(24V)^2}{600\Omega}$$

$$= 0.96W \quad [+1]$$

e)



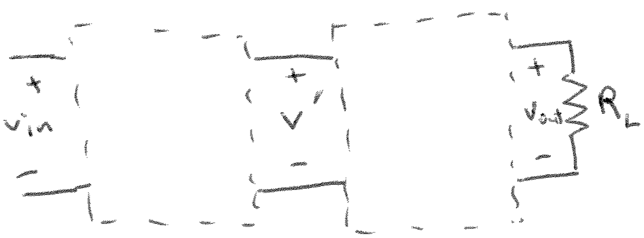
$$P_{abs} = \frac{V_{out}^2}{R_L}$$

$$V_{out} = 0.6 \cdot V_{in} \quad [+1]$$

$$P_{abs} = \frac{(0.6 \cdot 24V)^2}{600\Omega}$$

$$= 0.3456\ W \quad [+1]$$

f)



$$\frac{V_{out}}{V_{in}} = \frac{V_{out}}{V'} \cdot \frac{V'}{V_{in}}$$

$$= 0.6 \times 0.6$$

$$= 0.36 \quad [+1]$$

$R_{eq} = 600\Omega$ [+1]

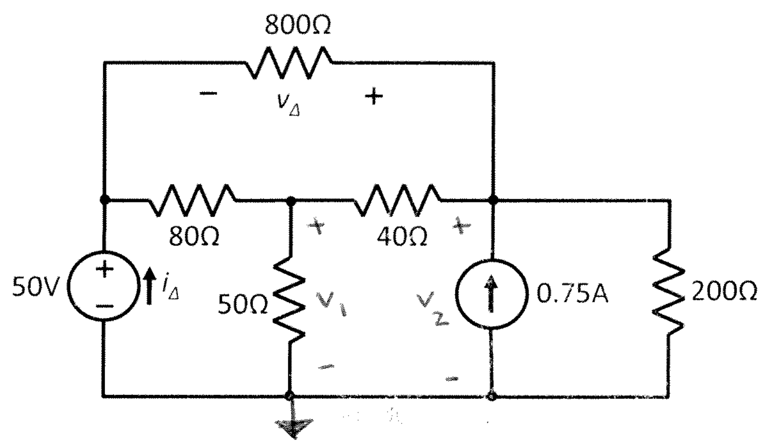
since the equivalent resistance to the first pad is 600Ω, and therefore also for the second pad.

ECSE-200 Quiz #4 (30 Sept 2011)

NAME______________________________ McGill ID#______________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit below. Answer the questions.



a) Use the node-voltage technique to determine the value of node-voltage variables. Clearly define your node voltage variables on the diagram above. [4pts]

b) What is the value of the voltage v_Δ? [2pts]

c) What is the value of the current i_Δ? [2pts]

a) $$\frac{v_1}{50\Omega} + \frac{v_1 - 50V}{80\Omega} + \frac{v_1 - v_2}{40\Omega} = 0 \qquad [+1]$$

$$-0.75A + \frac{v_2 - v_1}{40\Omega} + \frac{v_2 - 50V}{800\Omega} + \frac{v_2}{200\Omega} = 0 \qquad [+1]$$

$$0.0575 v_1 - 0.025 v_2 = 0.625 V$$
$$-0.025 v_1 + 0.03125 v_2 = 0.8125 V$$

$$v_1 = \frac{\begin{vmatrix} 0.625 & -0.025 \\ 0.8125 & 0.03125 \end{vmatrix}}{\begin{vmatrix} 0.0575 & -0.025 \\ -0.025 & 0.03125 \end{vmatrix}} = 34V \qquad [+1]$$

work space

$$v_2 = \frac{\begin{vmatrix} 0.0575 & 0.625 \\ -0.025 & 0.8125 \end{vmatrix}}{\begin{vmatrix} 0.0575 & -0.025 \\ -0.025 & 0.03125 \end{vmatrix}} = 53.2V \quad [+1]$$

b) KVL: $0 = -50V - v_\Delta + v_2 \quad [+1]$

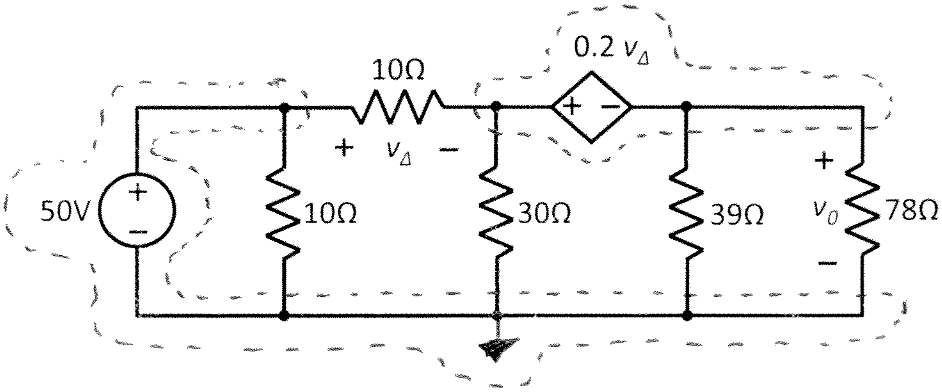
$$v_\Delta = v_2 - 50V$$

$$= 3.2V \quad [+1]$$

c) KCL + Ohm: $i_\Delta = \frac{50V - v_1}{80\Omega} + \frac{50V - v_2}{800\Omega} \quad [+1]$

$$i_\Delta = 0.196A \quad [+1]$$

2. Consider the circuit below. Answer the questions.



[+1] for node identification.

a) What is the minimal number of node voltage variables required to solve this circuit by the node-voltage technique? Clearly identify your nodes and super-nodes in the diagram above. [2 pts]

b) Write down the node-voltage and control variable equations required to solve this circuit. [2pts]

c) What is the value of the voltage v_O? [1pt]

d) What is the value of the voltage v_Δ? [1pt]

e) How much power does the dependent voltage source deliver or absorb? [2pts]

f) How much power does the independent voltage source deliver or absorb? [2pts]

a) Only 1 node voltage variable is required. [+1]

There is also 1 control variable (v_Δ).

b)

$$\frac{v_o}{78\Omega} + \frac{v_o}{39\Omega} + \frac{v_o + 0.2v_\Delta}{30\Omega} + \frac{v_o + 0.2v_\Delta - 50V}{10\Omega} = 0$$

[+1]

$$v_\Delta = 50V - (0.2v_\Delta + v_o) \quad [+1]$$

c) Use control variable equation:

$$v_\Delta = \frac{50V - v_o}{1.2}$$ => substitute into node voltage equation.

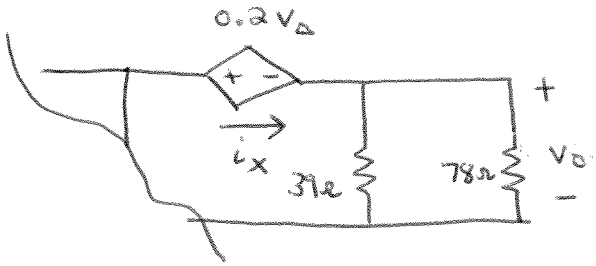
$$\frac{v_o}{78\Omega} + \frac{v_o}{39\Omega} + \frac{v_o + \frac{0.2}{1.2}(50V - v_o)}{30\Omega} + \frac{v_o + \frac{0.2}{1.2}(50V - v_o) - 50V}{10\Omega} = 0$$

$$v_o \cdot 0.14957 + (-3.889\,V) = 0$$

$$v_o = 26V \quad [+1]$$

d) $$v_\Delta = \frac{50V - v_o}{1.2} = 20\,V \quad [+1]$$

e)



$$i_x = \frac{v_o}{39\Omega} + \frac{v_o}{78\Omega} \quad [+1]$$

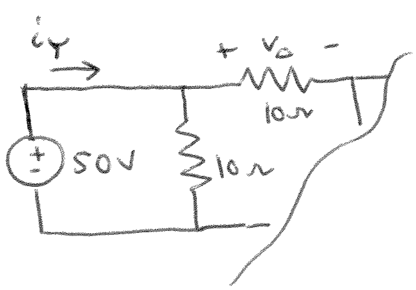
$$= 1A$$

P_{abs} = power absorbed by dependent source

$$= 0.2v_\Delta \cdot i_x$$

$$= 0.2 \cdot 20V \cdot 1A = 4W \quad [+1]$$

f)



$$i_Y = \frac{50V}{10\Omega} + \frac{v_\Delta}{10\Omega} \quad [+1]$$

$$= 7A$$

P_{del} = power delivered by independent source

$$= 50V \cdot i_Y$$

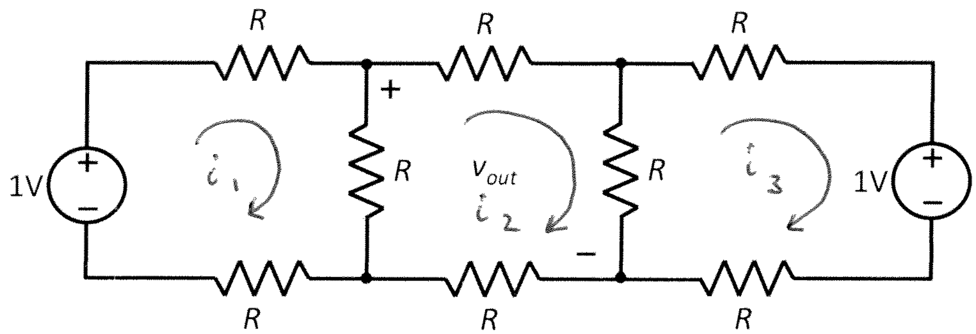
$$= 350\,W \quad [+1]$$

ECSE-200 Quiz #5 (7 Oct 2011)

NAME______________________________ McGill ID#______________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit below. Answer the questions.



a) What minimum number of node-voltage equations is required to solve the circuit above? [1pt]

b) What minimum number of mesh-current equations is required to solve the circuit above? [1pt]

For the remainder of this problem, assume that $R = 10\Omega$.

c) Use the technique with the fewest number of equations to solve the circuit. [6pts]

d) What is the value of v_{out} ? [2pts]

a) 5 node equations [+1]

b) 3 mesh equations [+1]

c)

$$0 = -1V + 10\Omega i_1 + 10\Omega (i_1 - i_2) + 10\Omega i_1 \quad [+1]$$

$$0 = 10\Omega (i_2 - i_1) + 10\Omega i_2 + 10\Omega (i_2 - i_3) + 10\Omega i_2 \quad [+1]$$

$$0 = 10\Omega (i_3 - i_2) + 10\Omega i_3 + 1V + 10\Omega i_3 \quad [+1]$$

$$+0.1 = 3 \cdot i_1 - i_2 + 0 \cdot i_3$$
$$0 = -i_1 + 4 \cdot i_2 - i_3$$
$$-0.1 = 0 \cdot i_1 - i_2 + 3 \cdot i_3$$

work space

$$i_1 = \frac{\begin{vmatrix} 0.1 & -1 & 0 \\ 0 & 4 & -1 \\ -0.1 & -1 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 3 \end{vmatrix}} = \frac{0.1 \cdot \begin{vmatrix} 4 & -1 \\ -1 & 3 \end{vmatrix} - (-1) \begin{vmatrix} 0 & -1 \\ -0.1 & 3 \end{vmatrix}}{3 \cdot \begin{vmatrix} 4 & -1 \\ -1 & 3 \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ 0 & 3 \end{vmatrix}} = 33.33\ mA$$

[+1]

$$i_2 = \frac{\begin{vmatrix} 3 & 0.1 & 0 \\ -1 & 0 & -1 \\ 0 & -0.1 & 3 \end{vmatrix}}{\begin{vmatrix} 3 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 3 \end{vmatrix}} = \frac{3 \cdot \begin{vmatrix} 0 & -1 \\ -0.1 & 3 \end{vmatrix} - (0.1) \begin{vmatrix} -1 & -1 \\ 0 & 3 \end{vmatrix}}{3 \cdot \begin{vmatrix} 4 & -1 \\ -1 & 3 \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ 0 & 3 \end{vmatrix}} = 0\ mA$$

[+1]

$$i_3 = \frac{\begin{vmatrix} 3 & -1 & 0.1 \\ -1 & 4 & 0 \\ 0 & -1 & -0.1 \end{vmatrix}}{\begin{vmatrix} 3 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 3 \end{vmatrix}} = \frac{3 \cdot \begin{vmatrix} 4 & 0 \\ -1 & -0.1 \end{vmatrix} - (-1) \begin{vmatrix} -1 & 0.1 \\ -1 & -0.1 \end{vmatrix}}{3 \cdot \begin{vmatrix} 4 & -1 \\ -1 & 3 \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ 0 & 3 \end{vmatrix}} = -33.33\ mA$$

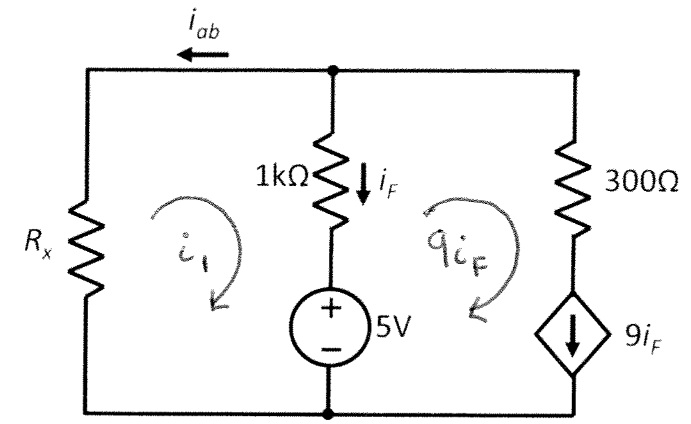
[+1]

There are many other ways to solve this system of equations.

d) $V_{out} = i_2 \cdot 10\Omega + (i_2 - i_3) \cdot 10\Omega$ [+1]

$= 0.333\ V$ [+1]

2. Consider the circuit below. Answer the questions.



a) What minimum number of node-voltage equations is required to solve the circuit above? [1pt]

b) What minimum number of mesh-current equations is required to solve the circuit above? [1pt]

c) Write down the mesh-current and control variable equations that are required to solve the circuit. Be sure to clearly identify any mesh current variables on the diagram above. [2pts]

d) It is known that i_{AB} = 10mA. What is the value of R_x? [3pts]

NOTE: Although you are encouraged to use mesh analysis, you may use any technique that you see fit. Be sure to show the steps of your analysis.

a) 2 node equations [+1]

b) 1 mesh equation [+1]

c)

$$i_1 \cdot R_x + (i_1 - 9i_F) \cdot 1k\Omega + 5V = 0 \quad [+1]$$

$$i_F = i_1 - 9i_F \quad [+1]$$

d)

$$i_1 = -10\,mA \quad [+1]$$

From control equation: $i_F = \dfrac{i_1}{10}$ [+1]

Substitute into mesh equation:

(next page)

work space

$$-10mA \cdot R_X + (-10mA - (-9mA)) \cdot 1k\Omega + 5V = 0$$

$$R_X = \frac{5V + (-10mA - (-9mA)) \cdot 1k\Omega}{10mA}$$

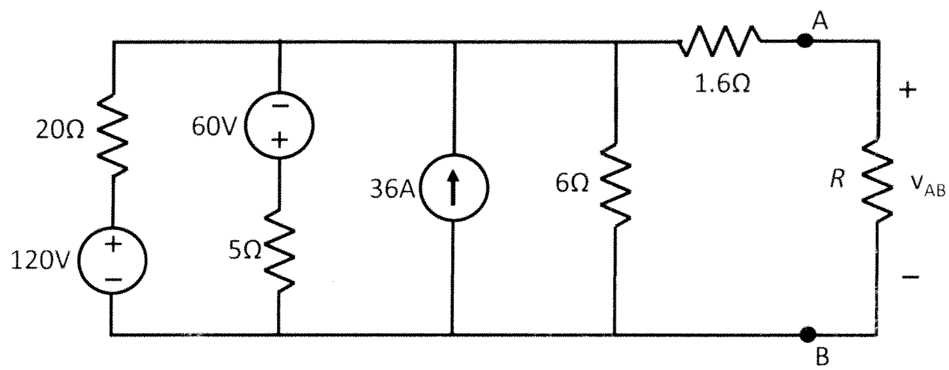
$$= 400\Omega \quad [+1]$$

ECSE-200 Quiz # $-24/(1+i)^4 = 6$ (14 Oct 2011)

NAME____ McGill ID#____________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit below. Answer the questions.

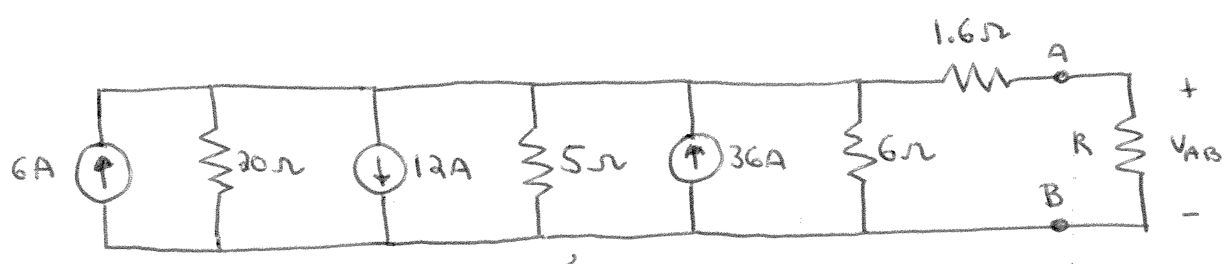


a) If $R = 8\Omega$, what is the voltage v_{AB}? [7pts]

b) If $R = 16\Omega$, what is the voltage v_{AB}? [1pt]

c) If $R = 4\Omega$, what is the voltage v_{AB}? [1pt]

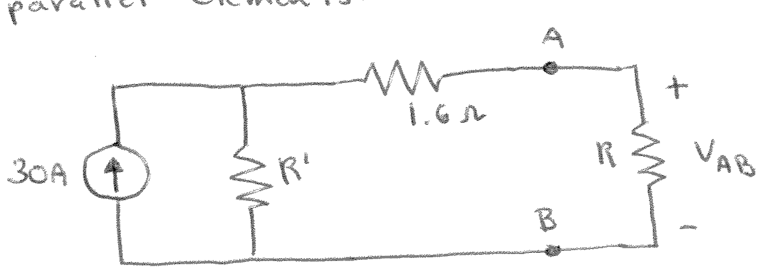
Use source transformations:



[+1] [+1]

$$\frac{120V}{20\Omega} = 6A \qquad \frac{60V}{5\Omega} = 12A$$

Combine parallel elements:



[+1]

$R' = 20\Omega \parallel 5\Omega \parallel 6\Omega$

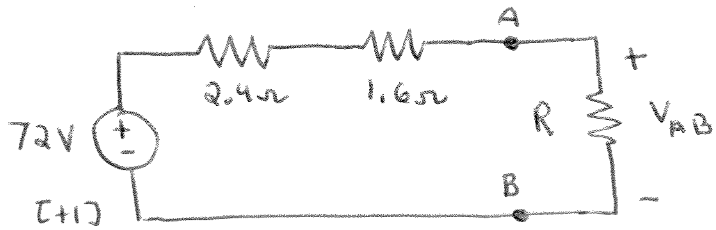
$$\frac{1}{R'} = \frac{1}{20} + \frac{1}{5} + \frac{1}{6}$$

$R' = 2.4\Omega$

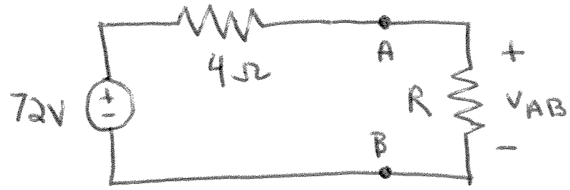
[+1]

Convert to Thévenin equivalent:

$30A \cdot 2.4\Omega = 72V$



Combine series resistors:



Apply voltage divider:

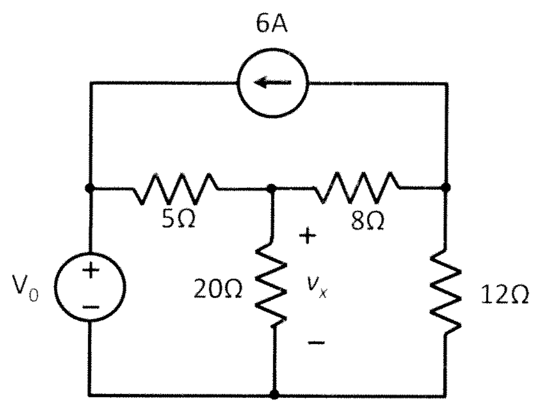
a) $V_{AB} = 72V \cdot \dfrac{R}{R + 4\Omega}$ [+1]

$= 72V \cdot \dfrac{8\Omega}{8\Omega + 4\Omega} = 48V$ [+1]

b) $V_{AB} = 72V \cdot \dfrac{16\Omega}{16\Omega + 4\Omega} = 57.6V$ [+1]

c) $V_{AB} = 72V \cdot \dfrac{4\Omega}{4\Omega + 4\Omega} = 36V$ [+1]

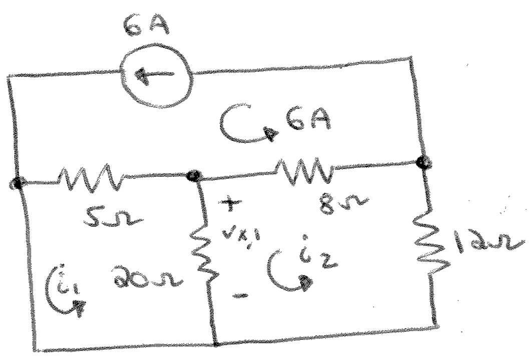
2. Consider the circuit below. Answer the question.



What should the value of V_0 be in order that v_x = +38V? [6pts]

HINT: You may find it useful to apply the principle of superposition. Be sure to show the steps of your analysis.

Turn current source on, voltage source off.



[+1 for diagram]

$$0 = (i_1 - 6A)\cdot 5\Omega + (i_1 - i_2)\cdot 20\Omega$$

$$0 = i_2 \cdot 12\Omega + (i_2 - 6A)\cdot 8\Omega + (i_2 - i_1)\cdot 20\Omega$$

$$30 = 25 i_1 - 20 i_2$$

$$48 = -20 i_1 + 40 i_2$$

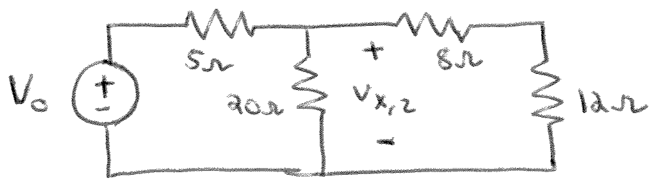
$$i_1 = \frac{\begin{vmatrix} 30 & -20 \\ 48 & 40 \end{vmatrix}}{\begin{vmatrix} 25 & -20 \\ -20 & 40 \end{vmatrix}} = 3.6A$$

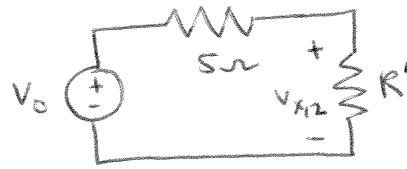
$$V_{x,1} = 20\Omega \cdot (i_2 - i_1)$$

$$= -12V \quad [+1]$$

$$i_2 = \frac{\begin{vmatrix} 25 & 30 \\ -20 & 48 \end{vmatrix}}{\begin{vmatrix} 25 & -20 \\ -20 & 40 \end{vmatrix}} = 3A$$

Turn voltage source on, current source off.





[+1 for diagram]

$$v_{x,2} = V_o \cdot \frac{R'}{R' + 5\Omega} \qquad R' = 20\Omega \,//\, (8+12)\Omega$$
$$= 10\Omega$$

$$v_{x,2} = \frac{10}{15} \cdot V_o = \frac{2}{3} V_o \quad [+1]$$

We require V_o such that $v_x = 38\,V$:

$$38V = v_{x,1} + v_{x,2} \quad [+1]$$

$$38V = -12V + \frac{2}{3} V_o$$

$$V_o = \frac{3}{2} \cdot (38V + 12V)$$

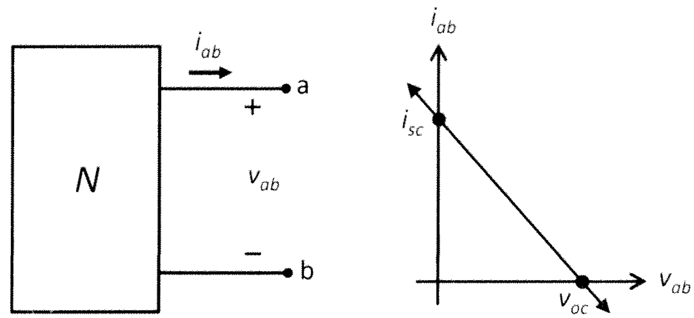
$$= 75\,V \quad [+1]$$

ECSE-200 Quiz # 7 (21 Oct 2011)

NAME______________________________ McGill ID#________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit below. The circuit "N" is composed of resistors, independent sources and dependent sources. Answer the questions.



In a first experiment, an ammeter with a total internal resistance of 5kΩ is used to measure the current i_{ab}, giving a measurement of +0.7mA.

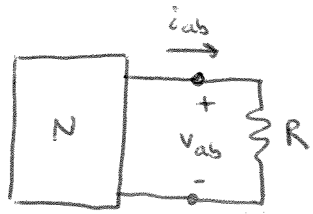
In a second experiment, a voltmeter with a total internal resistance of 125kΩ is used to measure the voltage v_{ab}, giving a measurement of +17.5V.

a) What is the Thévenin resistance of "N"? [4pts]

b) What is the short circuit current i_{sc} of "N"? [2pts]

c) What is the open circuit voltage v_{oc} of "N"? [2pts]

d) What is the maximum power that "N" can deliver to an ideally chosen load resistance connected across a and b? [2pts]

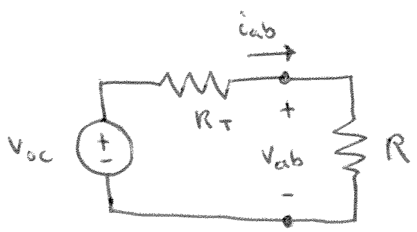


First data: $i_{ab} = 0.7mA$

$V_{ab} = 5k\Omega \cdot 0.7mA = 3.5V$ [+1]

Second data: $V_{ab} = 17.5V$

$i_{ab} = \frac{17.5V}{125k\Omega} = 0.14mA$ [+1]



$V_{ab} = V_{oc} - i_{ab} \cdot R_T$

$\therefore \frac{\Delta V_{ab}}{\Delta i_{ab}} = -R_T$ or $\frac{\Delta i_{ab}}{\Delta V_{ab}} = -\frac{1}{R_T}$

(slope) [+1 for either]

work space

$$R_T = -\frac{\Delta V_{ab}}{\Delta i_{ab}} = -\frac{(3.5V - 17.5V)}{(0.7mA - 0.14mA)} = 25\,k\Omega \; [+1]$$

b) $V_{ab} = V_{oc} - i_{ab} \cdot R_T$

$V_{ab} = i_{sc} \cdot R_T - i_{ab} R_T$

$$i_{sc} = \frac{V_{ab}}{R_T} + i_{ab} \quad [+1]$$

$$i_{sc} = \frac{3.5V}{25k\Omega} + 0.7mA = 0.84\,mA \quad [+1]$$

(substitution)

c) $V_{oc} = i_{sc} \cdot R_T = 0.84mA \cdot 25k\Omega = 21V$

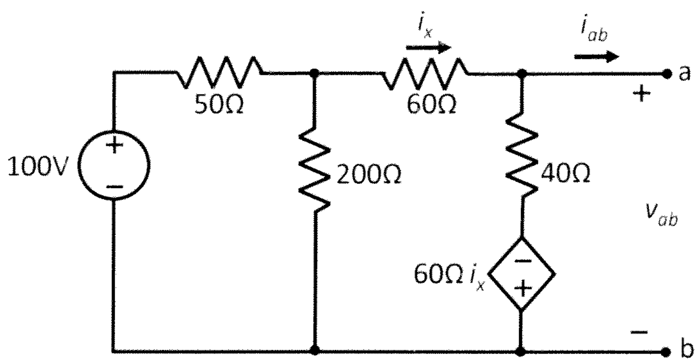
[+1] [+1]

d) Maximum power that can be delivered is:

$$P_{max} = \frac{V_{oc}}{2} \cdot \frac{i_{sc}}{2} \quad [+1]$$

$$= 4.41\,mW \quad [+1]$$

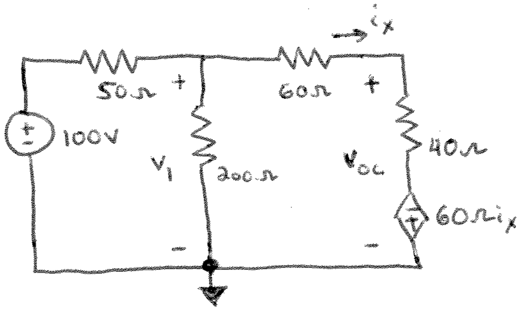
2. Consider the circuit below. Answer the questions.



a) What is the Thévenin equivalent circuit with respect to the terminals a and b? [7pts]

b) A load resistance R is connected across the terminals a and b, and 1.5W of power is delivered to the load resistance. What are the two possible values of R? [3pts]

a) Find the open circuit voltage: [+1]



$$0 = \frac{V_1 - 100V}{50\Omega} + \frac{V_1}{200\Omega} + \frac{V_1 + 60\Omega i_x}{60\Omega + 40\Omega}$$

$$i_x = \frac{V_1 + 60\Omega \cdot i_x}{60\Omega + 40\Omega}$$

$$\rightarrow V_1 = 40\Omega\, i_x$$

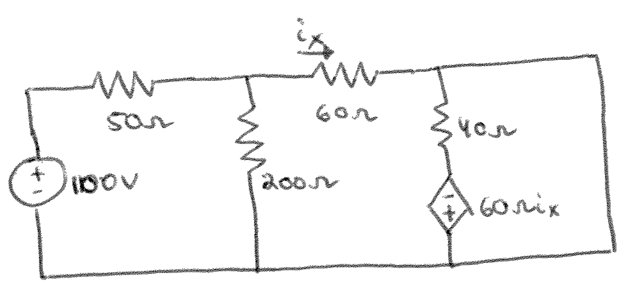
$$\rightarrow 0 = \frac{4}{5} i_x - 2A + \frac{1}{5} i_x + i_x$$

$$i_x = 1A$$

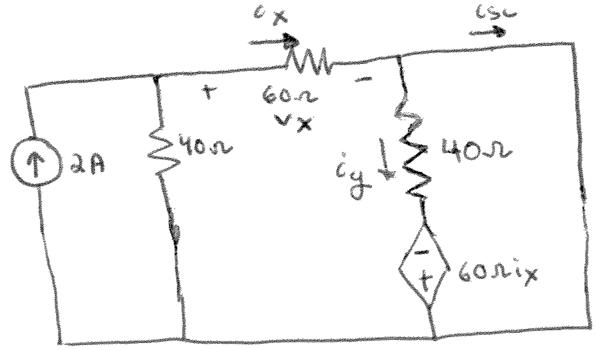
$$V_{oc} = 40\Omega \cdot i_x - 60\Omega\, i_x = -20V$$

[+1] [+1]

Find the short circuit current: [+1]



i_{sc} ⇒



current divider:

work space

$$i_x = 2A \cdot \frac{40\Omega}{40\Omega + 60\Omega} = 0.8A$$

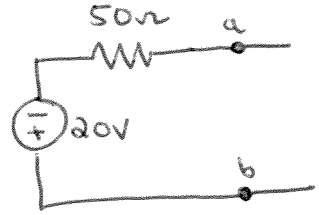
KVL: $0 = 40\Omega \cdot i_y - 60\Omega \cdot i_x \qquad \therefore\ i_y = \frac{6}{4} i_x = 1.2A$

KCL: $i_{sc} = i_x - i_y = -0.4A$

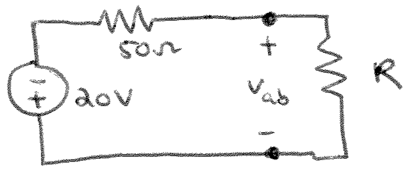
[+1] [+1]

$$R_T = \frac{V_{oc}}{i_{sc}} = \frac{-20V}{-0.4A} = 50\Omega$$

[+1]



b)



$$v_{ab} = -20V \cdot \frac{R}{(R + 50\Omega)}$$

$$P_{abs} = \frac{v_{ab}^2}{R} = (-20V)^2 \frac{R}{(R+50\Omega)^2} \quad [+1]$$

$$\frac{1.5W}{(20V)^2} = \frac{R}{(R+50\Omega)^2}$$

$$R = 3.75\times10^{-3}\,(R + 50\Omega)^2$$

$$0 = 3.75\times10^{-3} R^2 - 0.625R + 9.375$$

$$R = \frac{+0.625 \pm \sqrt{(0.625)^2 - 4 \cdot 3.75\times10^{-3} \cdot 9.375}}{2 \cdot 3.75\times10^{-3}}$$

$$= 150\Omega,\ 16.67\Omega$$

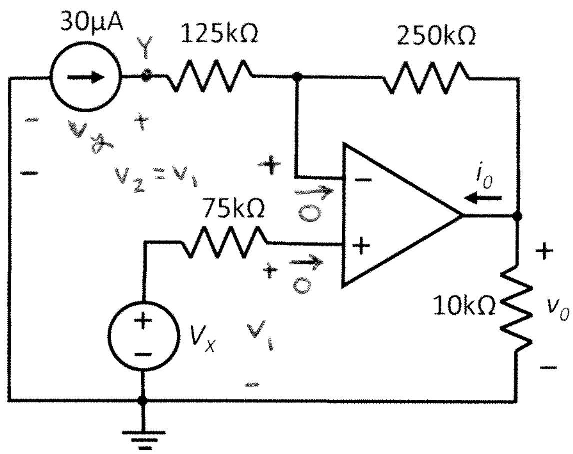
[+1] [+1]

ECSE-200 Quiz # 8 (28 Oct 2011)

NAME____ McGill ID#____________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit below. Assume ideal op-amp behaviour. Answer the questions.



a) Is there a negative feedback loop in this circuit? [1pt]

Assume V_X = 0V for part b.

b) What is the value of the voltage v_O? [4pts]

Assume V_X = 3.75V for the remainder of this question.

c) What is the value of the voltage v_O? [3pts]

d) How much power does the 30μA current source deliver or absorb? [2pts]

a) yes [1pt]

b) $i_1 = i_2 = 0$ [+1] $v_1 = v_2$ [+1]

KVL: $v_1 = V_X - i_1 \cdot 75k\Omega = V_X = 0V$ and $v_2 = 0V$ [+1]

KCL: $0 = -30\mu A + \dfrac{v_2 - v_o}{250k\Omega}$

$\therefore v_o = -30\mu A \cdot 250k\Omega + v_2$

work space

$$v_o = -7.5\,V + 0\,V = -7.5\,V \quad [+1]$$

c) KVL: $v_1 = v_x - i_1 \cdot 75\,k\Omega = v_x = +3.75\,V$ and $v_2 = 3.75\,V$ [+1]

KCL: $0 = -30\,\mu A + \dfrac{v_2 - v_o}{250\,k\Omega}$ [+1]

$$v_o = -30\,\mu A \cdot 250\,k\Omega + v_2$$

$$= -7.5\,V + 3.75\,V$$

$$= -3.75\,V \quad [+1]$$

d) KCL at node Y: $0 = -30\,\mu A + \dfrac{v_y - v_2}{125\,k\Omega}$ (+1)

$$0 = -30\,\mu A + \frac{v_y - (3.75\,V)}{125\,k\Omega}$$

$$v_y = 30\,\mu A \cdot 125\,k\Omega + 3.75\,V$$

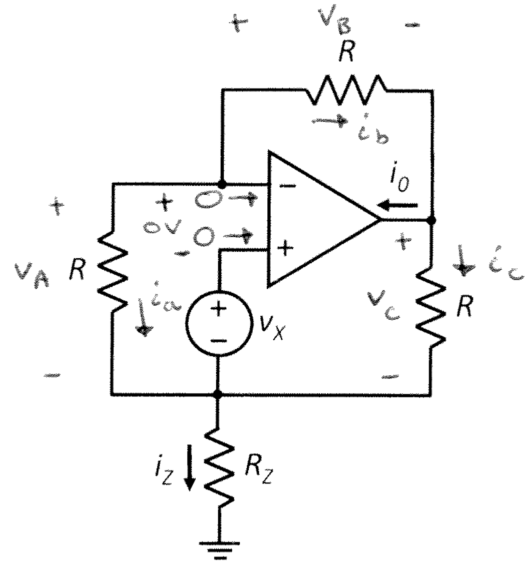
$$= 7.5\,V$$

$$P_{del} = 30\,\mu A \cdot v_y$$

$$= 30\,\mu A \cdot 7.5\,V$$

$= 225\,\mu W$ delivered by $30\,\mu A$ current source [+1]

2. Consider the circuit below. Assume ideal op-amp behaviour. Answer the questions.



a) Is there a negative feedback loop in this op-amp circuit? [1pt]

b) The current i_Z is defined to be positive entering the reference node. If $i_Z \neq 0$, is KCL violated? [1pt]

c) What is the current i_Z, expressed in terms of v_X, R, and R_Z? [5pts]

HINT: You may find it useful to apply the ideal op-amp equations, KCL, KVL, and Ohm's law.

d) What is the current i_O, expressed in terms of i_Z? [1pt]

a) yes [+1] b) no [+1]

c) KVL: $0 = -V_x + 0V + V_A$

$\therefore V_A = V_x$

Ohm: $i_a = \frac{V_A}{R} = \frac{V_x}{R}$ [+1]

KCL: $0 = i_a + 0A + i_b$

$\therefore i_b = -i_a = -V_x/R$

Ohm: $V_B = i_b \cdot R = \frac{-V_x}{R} \cdot R = -V_x$

KVL: $0 = -V_A + V_B + V_c$

$\therefore V_c = V_A - V_B$

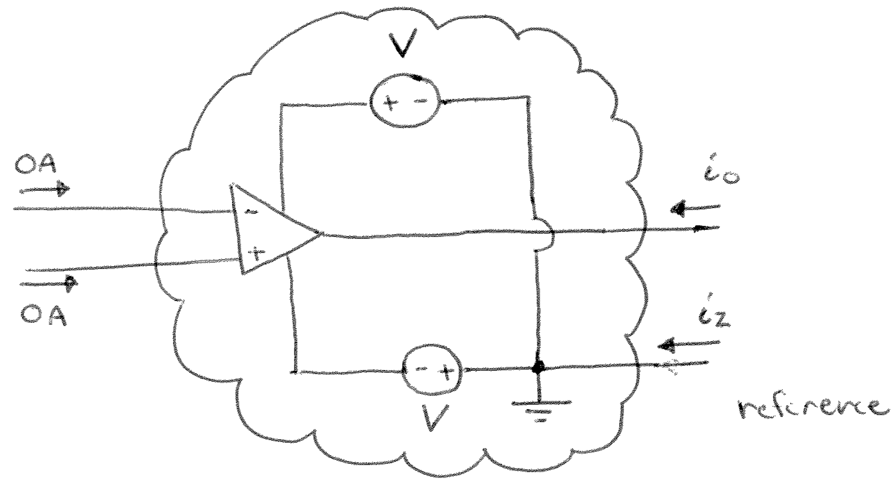
$$v_c = v_x - (-v_x) = 2v_x \quad [+1]$$

Ohm: $i_c = \frac{v_c}{R} = \frac{2v_x}{R}$ [+1]

KCL: $0 = -i_a - i_c + i_z$ [+1]

$\therefore\ i_z = i_a + i_c = \frac{v_x}{R} + \frac{2v_x}{R} = \frac{3v_x}{R}$ [+1]

d)



KCL (charge conservation) requires: $i_o = -i_z$ [+1]

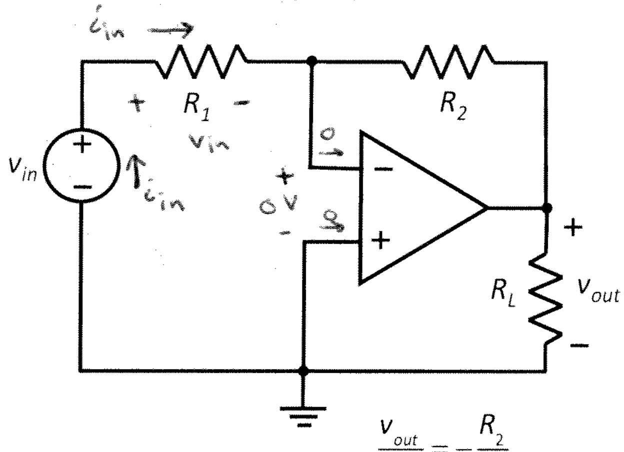
ECSE-200 Quiz # $10/\sum_{n=0}^{\infty} 0.1^n = 9$ (4 Nov 2011)

NAME____ McGill ID#____________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

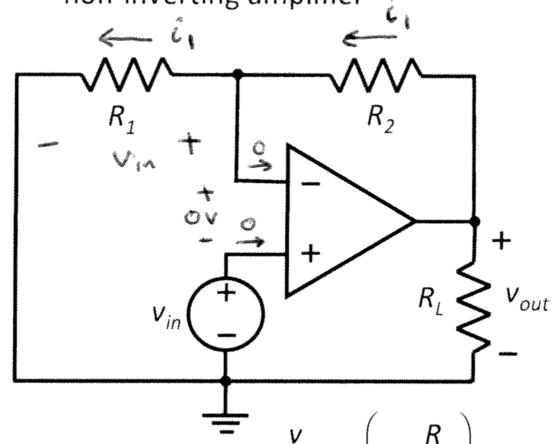
1. Consider the circuits below. Assume ideal op-amp behaviour. Answer the questions.

inverting amplifier



$$\frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1}$$

non-inverting amplifier



$$\frac{v_{out}}{v_{in}} = \left(1+\frac{R_2}{R_1}\right)$$

You are to design an inverting amplifier with the following specifications: a voltage gain of −80 V/V, and 50nW of power delivered by the input voltage source when v_{in} = 10mV.

a) What are appropriate values of R_1 and R_2? [4pts]

You are to now design a non-inverting amplifier with the following specifications: a voltage gain of +30 V/V, and 200nW of power absorbed by R_1 when v_{in} = 10mV.

b) What are appropriate values of R_1 and R_2? [4pts]

c) If the load resistor R_L=3kΩ is connected to your non-inverting amplifier, what is the ratio of the power absorbed in R_1 and R_2 to the power absorbed in R_L? Assume $v_{in} \neq 0$. [2pts]

a) $-\frac{R_2}{R_1} = -80$ [+1]

$i_{in} = \frac{v_{in}}{R_1}$ $\quad \therefore P_{del} = v_{in} \cdot i_{in} = \frac{v_{in}^2}{R_1}$ [+1]

$R_1 = \frac{(10\,mV)^2}{50\,nW} = 2\,k\Omega$ [+1]

$R_2 = 80 R_1 = 160\,k\Omega$ [+1]

b) $$1 + \frac{R_2}{R_1} = 30$$

$$i_1 = \frac{v_{in}}{R_1} \quad \therefore \quad P_{abs} = v_{in} \cdot i_1 = \frac{v_{in}^2}{R_1} \quad [+1]$$

$$R_1 = \frac{(10mV)^2}{200nW} = 500\,\Omega \quad [+1]$$

$$R_2 = (30-1) \cdot R_1 = 14500\,\Omega \quad [+1]$$

c) Power absorbed by R_1 and R_2:

$$P_{abs} = i_1^2 \cdot R_1 + i_1^2 \cdot R_2$$
$$= i_1^2 (R_1 + R_2)$$
$$= \frac{v_{out}^2}{(R_1 + R_2)}$$

since $v_{out} = i_1 \cdot R_1 + i_1 \cdot R_2 = i_1 (R_1 + R_2)$

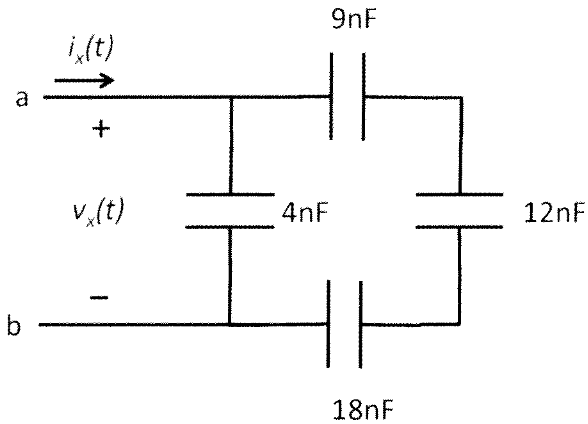
Power absorbed by R_L:

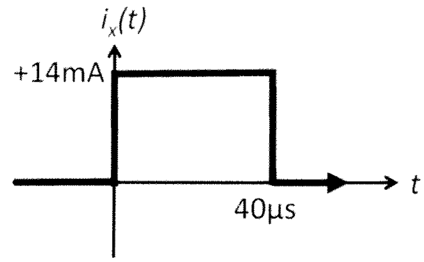
$$P_{abs}' = \frac{v_{out}^2}{R_L}$$

Power ratio:

$$\frac{P_{abs}}{P_{abs}'} = \underbrace{\frac{1/(R_1+R_2)}{1/R_L}}_{[+1]} = \frac{3k\Omega}{0.5k\Omega + 14.5k\Omega} = 0.20 \quad [+1]$$

2. Consider the circuit and diagram below. The capacitors are initially uncharged. Answer the questions.





a) Plot the voltage $v_x(t)$ versus t, showing clearly the behavior for $t < 0$, $0 < t < 40\mu s$, and $40\mu s < t$. Be sure to label your axes. [4pts]

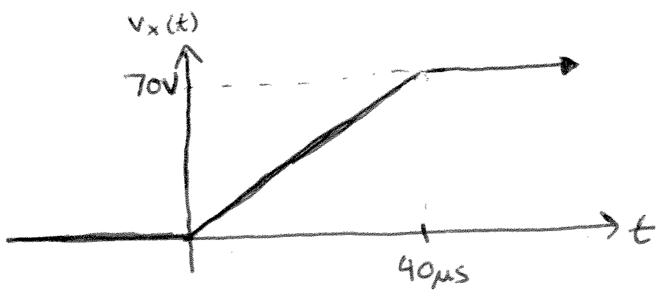
b) What is the maximum power that is absorbed by the capacitors? [2pts]

c) What is the energy stored in the capacitors (ie. the sum energy over all capacitors) after the current pulse? [2pts]

d) What is the charge stored on the 12nF capacitor after the current pulse? [2pts]

a) $$C_{eq} = 4nF + \left(\frac{1}{9nF} + \frac{1}{12nF} + \frac{1}{18nF}\right)^{-1} = 8nF \quad [+1]$$

$$V_x(t) = \frac{q(t)}{C_{eq}} = \frac{\int_0^t i_x(\tau)\,d\tau}{C_{eq}} \quad [+1]$$



$$V_x(40) = \frac{14mA \cdot 40\mu s}{8nF} = 70\,V \quad [+1]$$

[+1] for shape

b) Maximum power is absorbed at 40μs. [+1]

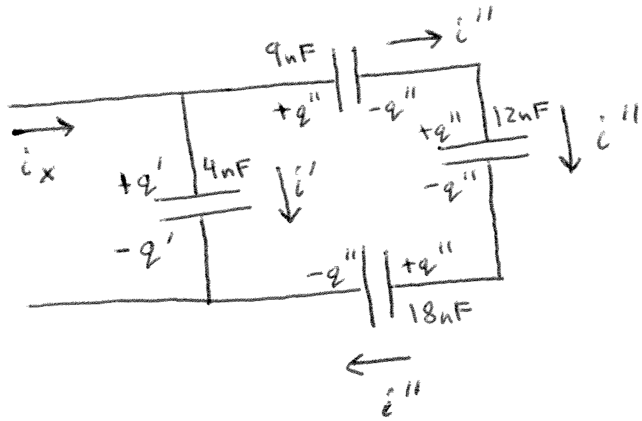
$$P_{abs} = i_x(40\mu s) \cdot v_x(40\mu s) = 980\ mW \quad [+1]$$

c) $U = \frac{1}{2} C_{eq} V_x^2$ [+1]

$= \frac{1}{2} \cdot 8nF \cdot (70V)^2$

$= 19.6\,\mu J$ [+1]

d)



total charge delivered: $14mA \cdot 40\mu s = 560nC = Q$

charge on 4nF : $4nF \cdot 70V = 280nC = q'$

KCL: $i_x = i' + i''$ (+1)

$\int i_x dt = \int i' dt + \int i'' dt$ since capacitors are initially uncharged

$Q = q' + q''$

$q'' = Q - q' = 560nC - 280nC$

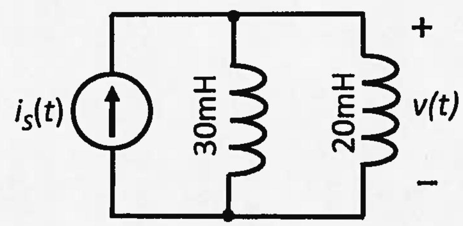
$= 280nC$ [+1]

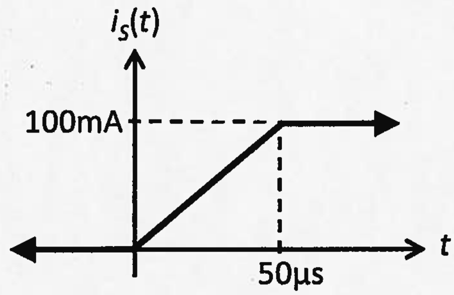
ECSE-200 Quiz # 10 (11 Nov 2011)

NAME____ McGill ID#____________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit and diagram below. The initial flux linkage in the inductors is zero. The mutual inductance between the inductors is zero. Answer the questions.





a) What is the total energy stored in the inductors for 50µs < t ? [3pts]

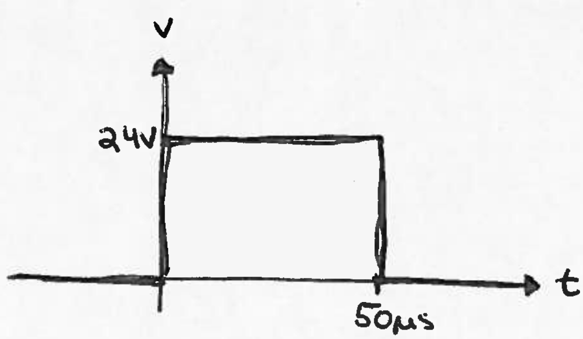
b) Plot the voltage $v(t)$ versus t, showing clearly the behavior for $t < 0$, $0 < t < 50$µs, and 50µs < t. Be sure to label your axes. [3pts]

a) $\frac{1}{L'} = \frac{1}{30\text{mH}} + \frac{1}{20\text{mH}}$ $L' = 12\text{ mH}$ [+1]

$U = \frac{1}{2} L' i_s^2$ [+1]

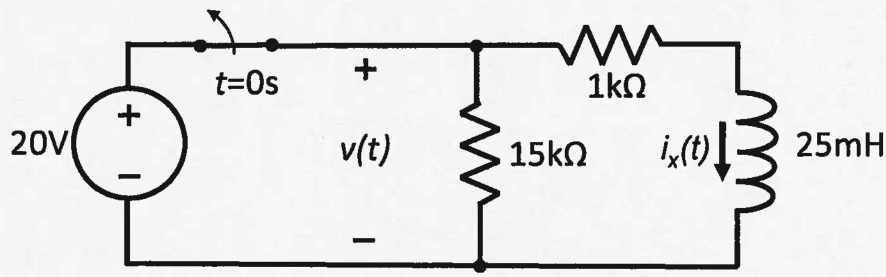
$= 60\,\mu J$ [+1]

b) $v = L' \frac{di_s}{dt}$ [+1] $v_{max} = 12\text{mH} \cdot \frac{100\text{mA}}{50\mu s} = 24\text{V}$ [+1]



[+1] for shape

2. Consider the circuit below. ~~The initial flux linkage in the inductor is zero.~~ The circuit is initially in dc steady state. The switch is closed for $t < 0$, and opens instantaneously at $t = 0s$. Answer the questions.

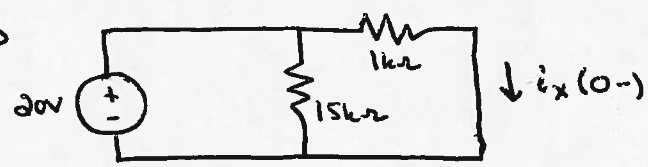


a) What is the inductor current $i_X(t)$ at $t = 0-$? [2pts]

b) What is the inductor current $i_X(t)$ at $t = 0+$? [1pt]

c) What is the voltage $v(t)$ at $t = 0+$? [3pts]

a)

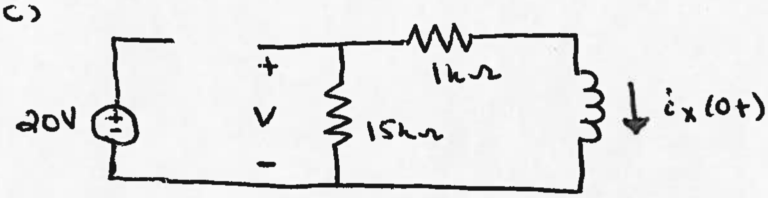


$$i_x(0-) = \frac{20V}{1k\Omega} = 20mA$$

[+1] for circuit

b) $i_x(0+) = i_x(0-) = 20mA$ [+1]

c)



KCL + Ohm's Law:

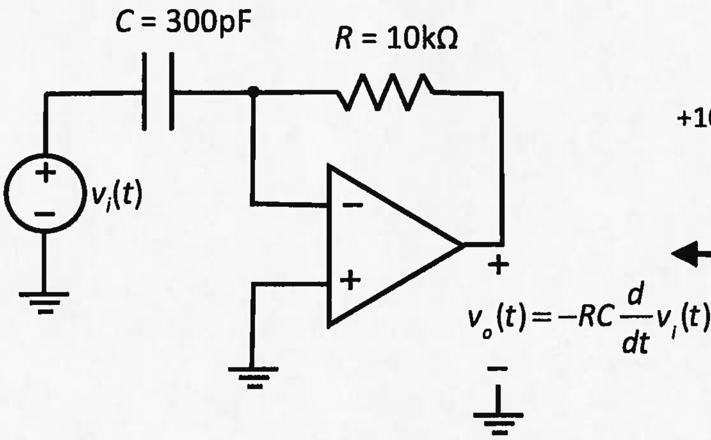
$V = -i_x \cdot 15k\Omega$ [+1]

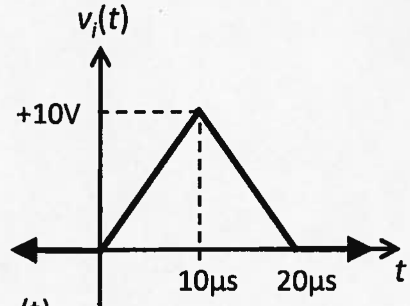
$= -20mA \cdot 15k\Omega$

$= -300V$ [+1]

[+1] for circuit

3. Consider the circuit and diagram below. The capacitor is initially uncharged. Assume ideal op-amp behaviour. Answer the questions.





a) Plot the voltage $v_o(t)$ versus t, showing clearly the behavior for $t < 0$, $0 < t < 20\mu s$, and $20\mu s < t$. Be sure to label your axes. [4pts]

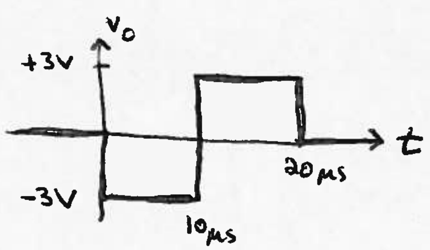
b) What is the energy stored on the capacitor at $t = 5\mu s$? [2pts]

c) What is the power dissipated in the resistor at $t = 5\mu s$? [2pts]

a) $V_o = -10k\Omega \cdot 300pF \cdot \frac{dv_i}{dt} = -3\mu s\,\frac{dv_i}{dt}$ [+1]

$\frac{dv_i}{dt} = \pm\frac{10V}{10\mu s}$

$V_o = \mp 3\mu s \cdot \frac{10V}{10\mu s} = \mp 3V$ [+1]



[+2] for shape

b) $U = \frac{1}{2}CV^2 = \frac{1}{2}C\,v_i^2(5\mu s)$ [+1] $\qquad v_i(5\mu s) = 5V$

$= 3.75\,nJ$ [+1]

c) $P = i^2R = \frac{V^2}{R} = \frac{V_o^2(5\mu s)}{R}$ [+1] $\qquad V_o(5\mu s) = -3V$

$= 900\,\mu W$ [+1]

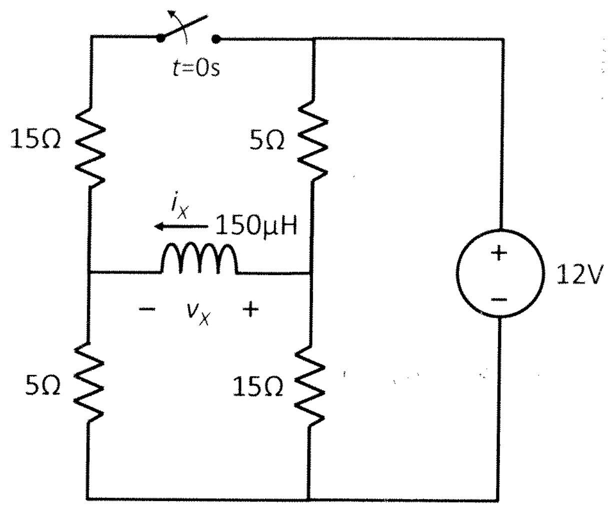
work space

ECSE-200 Quiz # 11 (18 Nov 2011)

NAME______________________________ McGill ID#__________________

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit below. The circuit is in dc steady state for $t < 0$, and the switch opens instantaneously at $t = 0$s. Answer the questions.



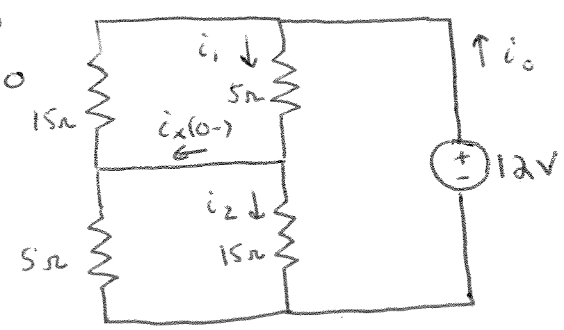
a) What is the current $i_X(t)$ for $t > 0$? [4pts]

b) What is the voltage $v_X(t)$ for $t > 0$? [3pts]

c) What is the energy stored in the inductor at $t = 0$? [2pts]

a)

$t<0$



$$i_0 = \frac{12V}{5\Omega \| 15\Omega + 5\Omega \| 15\Omega} = \frac{12V}{7.5\Omega}$$

$$= 1.6A$$

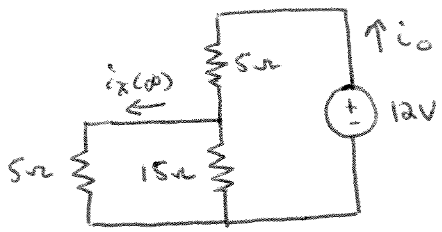
$$i_1 = i_0 \cdot \frac{15\Omega}{5\Omega + 15\Omega} = 1.2A$$

$$i_2 = i_0 \cdot \frac{5\Omega}{5\Omega + 15\Omega} = 0.4A$$

$$i_x(0-) = i_1 - i_2 \quad [+1]$$

$$\therefore i_x(0+) = i_x(0-) = 0.8\,A$$

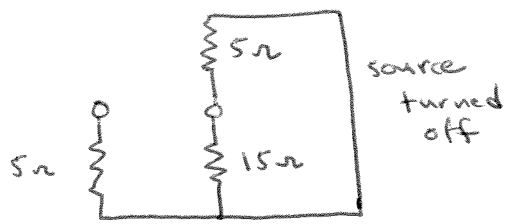
$t \to \infty$



$$i_o = \frac{12V}{5\Omega + 5\Omega \| 15\Omega} = 1.371\ A$$

$$i_x(\infty) = i_o \cdot \frac{15\Omega}{5\Omega + 15\Omega} = 1.029\ A$$

[+1]



$$R_{TH} = 5\Omega + 15\Omega \| 5\Omega = 8.75\Omega$$

$$\tau = \frac{L}{R_{TH}} = \frac{150\mu H}{8.75\Omega} = 17.14\mu s$$

[+1]

$t > 0$ $\quad i_x(t) = i_x(\infty) + [i_x(0+) - i_x(\infty)] \exp(-t/\tau)$

$$= 1.029A - 0.229A \exp(-t/17.14\mu s) \quad [+1]$$

b) $\quad V_x(t) = L \dfrac{di_x(t)}{dt} \quad [+2]$

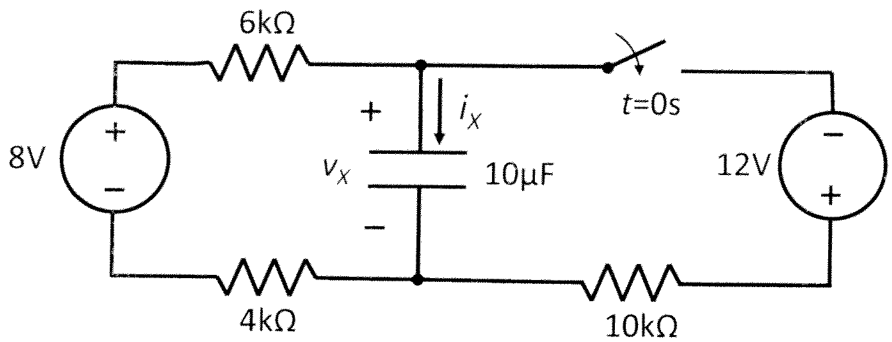
$$= 150\mu H \cdot \frac{d}{dt}\left[1.029A - 0.229A \exp(-t/17.14\mu s)\right]$$

$$= 2.00V \exp(-t/17.14\mu s) \quad [+1]$$

c) $\quad U = \frac{1}{2} L\, i_x^2(0) \quad [+1]$

$$= 48\ \mu J \quad [+1]$$

2. Consider the circuit below. The circuit is in dc steady state for $t < 0$, and the switch closes instantaneously at t = 0s. Answer the questions.



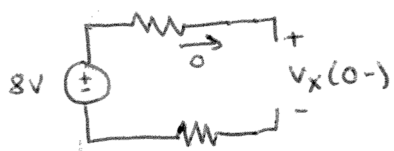
a) What is the voltage $v_X(t)$ for $t > 0$? [4pts]

b) What is the current $i_X(t)$ for $t > 0$? [3pts]

c) What is the energy stored on the capacitor at $t = 0$? [2pts]

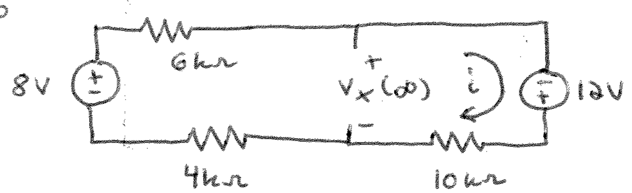
d) What is the total energy absorbed by the capacitor over the time interval $0 < t < \infty$? [2pts]

a) $t < 0$



$V_x(0+) = V_x(0-) = 8V$ [+1]

$t \to \infty$

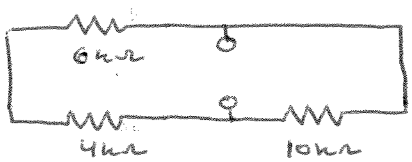


$$i = \frac{12V + 8V}{10k\Omega + 4k\Omega + 6k\Omega} \quad [+1]$$

$$= 1mA$$

$$0 = -V_x(\infty) - 12V + 1mA \cdot 10k\Omega$$

$$V_x(\infty) = -2V$$



$$\tau = R_{TH} \cdot C \quad [+1]$$

$$= (10k\Omega \,||\, (4k\Omega + 6k\Omega)) \cdot 10\mu F$$

$$= 50\,ms$$

$$V_x(t) = V_x(\infty) + [V_x(0+) - V_x(\infty)] \exp(-t/\tau)$$

$$= -2V + 10V \exp(-t/50ms) \quad [+1]$$

b) $i_x = C \dfrac{dv_x(t)}{dt}$ [+2]

$= 10\mu F \cdot \dfrac{d}{dt}\left[-2V + 10V \exp(-t/50ms) \right]$

$= -2mA \exp(-t/50ms)$ [+1]

c) $U = \frac{1}{2} C v_x^2(0)$ [+1]

$= 320 \mu J$ [+1]

d) net energy absorbed:

$\Delta U = U(\infty) - U(0)$

$= \frac{1}{2} C v_x^2(\infty) - \frac{1}{2} C v_x^2(0)$ [+1]

$= -300 \mu J$ [+1]

in other words the capacitor <u>delivers</u> $300\mu J$ to the circuit.

ECSE-200 Quiz # 12 (25 Nov 2011)

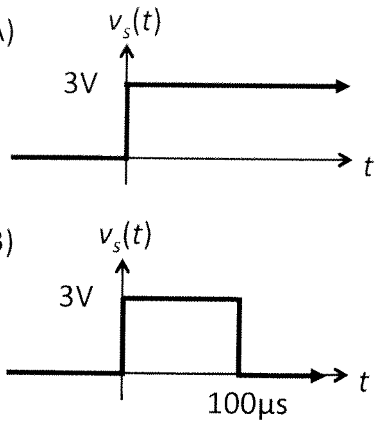
NAME____ McGill ID#____________________

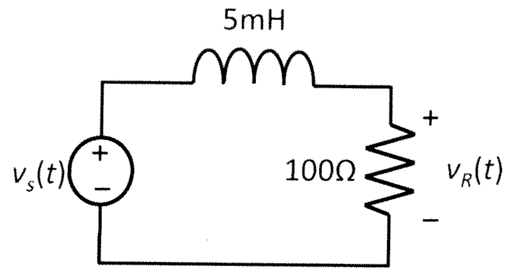
READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

Consider the circuit and diagrams below. The circuit is in dc steady state for $t < 0$ with the inductor storing zero energy. Answer the questions.

(A)

(B)





a) Express the $v_s(t)$ shown in (A) in terms of the unit step function $u(t)$. [1pt]

b) Express the $v_s(t)$ shown in (B) in terms of the unit step function $u(t)$. [1pt]

For the remainder of this question, assume $v_s(t)$ is equal to that shown in (B). Please note that you can solve the remainder of this question without the answers to parts a) and b).

c) Solve for the voltage $v_R(t)$ for $0 < t < 100\mu s$. [4pts]

d) Solve for the voltage $v_R(t)$ for $100\mu s < t$. [4pts]

e) Plot the voltage $v_R(t)$ versus time t. [2pts]

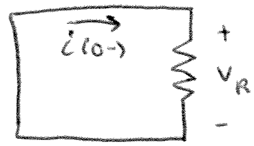
f) What is the maximum energy stored by the inductor during the circuit response to the voltage pulse? [2pts]

a) $V_s(t) = 3V \cdot u(t)$ (+1)

b) $V_s(t) = 3V \cdot u(t) - 3V \cdot u(t - 100\mu s)$ (+1)

c) $t = 0-$

inductor is a short



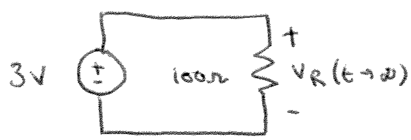
$i(0-) = 0A$

$i(0+) = i(0-) = 0A$

$V_R(0+) = 100\Omega \cdot i(0+) = 0V$ [+1]

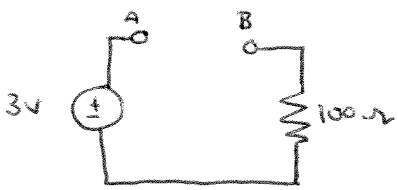
$t \to \infty$

inductor is a short



$V_R(t \to \infty) = 3V$ [+1]

time constant



$R_{TH} = 100\Omega$

$$\tau = \frac{L}{R_{TH}} = \frac{5mH}{100\Omega} = 50\mu s \quad [+1]$$

$$V_R(t) = V_R(\infty) + [V_R(0+) - V_R(\infty)]\exp(-t/\tau)$$

$$= 3V - 3V\exp(-t/50\mu s) \qquad 0 < t < 100\mu s \qquad [+1]$$

d) $t = 100\mu s -$

$$i(100\mu s-) = \frac{V_R(100\mu s-)}{100\Omega} = \frac{3V - 3V\exp(-100\mu s/50\mu s)}{100\Omega}$$

$$= 2.594V / 100\Omega$$

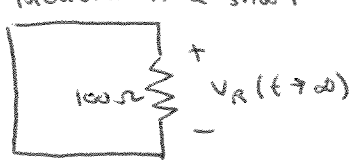
$$i(100\mu s+) = i(100\mu s-) = 2.594V/100\Omega$$

$$V_R(100\mu s+) = 100\Omega \cdot i(100\mu s+)$$

$$= 2.594V \quad [+1]$$

$t \to \infty$

inductor is a short



$V_R(t \to \infty) = 0V$ [+1]

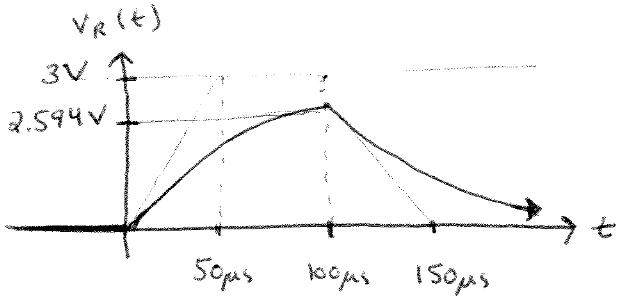
work space

$$\tau = \frac{L}{R_{TH}} = \frac{5mH}{100\Omega} = 50\mu s \quad [+1]$$

$$v_R(t) = v_R(\infty) + [v_R(100\mu s+) - v_R(\infty)] \exp(-(t-100\mu s)/\tau)$$

$$= 2.594\,V \exp(-(t-100\mu s)/50\mu s) \quad 100\mu s < t \quad [+1]$$

e)



[+1] for shape

[+1] for 2.594V @ 100μs

f) $i(t) = \frac{v_R(t)}{100\Omega}$ $\therefore$ maximum $i(t)$ occurs at maximum $v_R(t)$.

$$i_{max} = \frac{v_{max}}{100\Omega} = 25.94\ mA \quad [+1]$$

$$U_{max} = \frac{1}{2} L\, i_{max}^2$$

$$= \frac{1}{2} \cdot 5mH \cdot (25.94mA)^2$$

$$= 1.68\ \mu J$$