McGill

April 2017
Final Examination

ELECTRIC CIRCUIT 1

ECSE-200-001

April 21, 2017; 9am to 12pm

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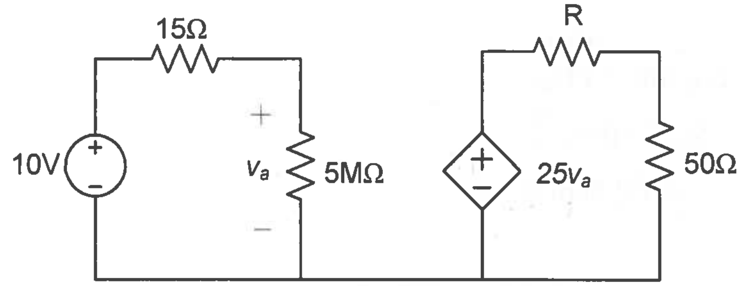
STUDENT NAME:		**McGILL ID:**	

INSTRUCTIONS:

EXAM:	CLOSED BOOK ☒ OPEN BOOK ☐
	SINGLE-SIDED ☒ PRINTED ON BOTH SIDES OF THE PAGE ☐
	MULTIPLE CHOICE ☐ NOTE: The Examination Security Monitor Program detects pairs of students with unusually similar answer patterns on multiple-choice exams. Data generated by this program can be used as admissible evidence, either to initiate or corroborate an investigation or a charge of cheating under Section 16 of the Code of Student Conduct and Disciplinary Procedures. ANSWER IN BOOKLET ☐ EXTRA BOOKLETS PERMITTED: YES ☐ NO ☒ ANSWER ON EXAM ☒
	SHOULD THE EXAM BE: RETURNED ☒ KEPT BY STUDENT ☐
CRIB SHEETS:	NOT PERMITTED ☒ PERMITTED ☐ e.g. one 8 1/2X11 handwritten double-sided sheet Specifications:
DICTIONARIES:	TRANSLATION ONLY ☒ REGULAR ☐ NONE ☐
CALCULATORS:	NOT PERMITTED ☐ PERMITTED (Non-Programmable) ☒
ANY SPECIAL INSTRUCTIONS: e.g. molecular models	

PART 1 – Short questions (clearly show your final answer).

1.1. In the circuit shown below, assume that the power absorbed by the load resistor of 50 Ω is 5W. Find the resistance value of the resistor R? [2pts]



$V_a = 10V \cdot \frac{5M\Omega}{15\Omega + 5M\Omega} \approx 10V$ [voltage division]

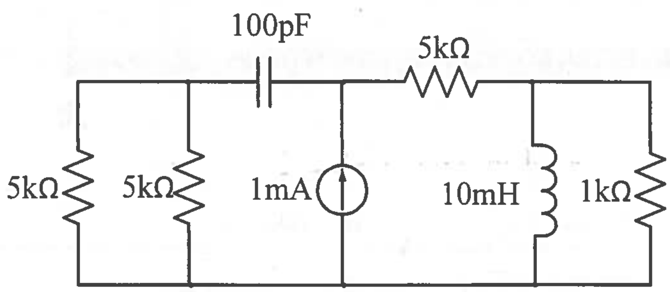
$V_{50\Omega} \rightarrow \frac{(V_{50\Omega})^2}{50\Omega} = 5W \rightarrow V_{50\Omega} = \sqrt{250}\,V$ $[P = V \cdot I = V(V/R) = V^2/R]$

$\sqrt{250}\,V = 25V_a \cdot \frac{50\Omega}{R+50} \Rightarrow 250 \cdot \frac{50}{R+50} = \sqrt{250} \Rightarrow \sqrt{250} \cdot 50 - 50 = R$

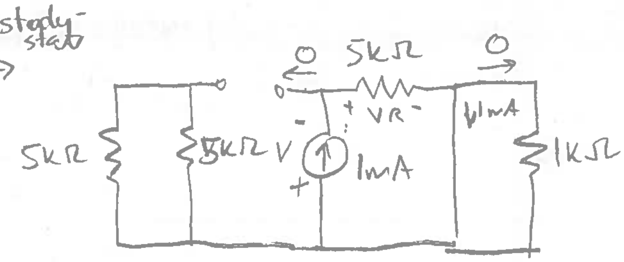
[voltage div.]

$\boxed{R = 740.57\,\Omega}$

1.2. Assume that the circuit below has reached dc steady state. What is the power delivered by the independent current source in the circuit? [2pts]



in DC steady-state →

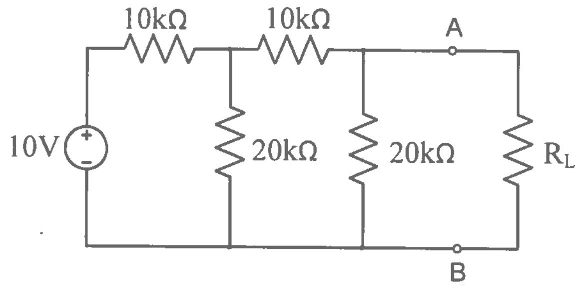


[KVL] $V + V_R = 0 \rightarrow V = -(1mA \times 5k\Omega) = -5V$

[P=VI] $P = (-5V)(1mA) = -5mW$

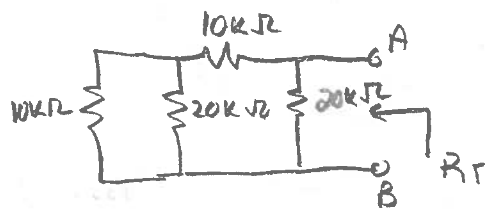
$\boxed{\text{Power delivered: } 5mW}$

1.3. A load resistor R_L is connected to the circuit at the terminals A and B of a resistive circuit with a voltage supply, as shown below. What should the load resistor R_L be for maximum power transfer from the circuit? [2pts]



$R_L = R_T$

Turn off supply

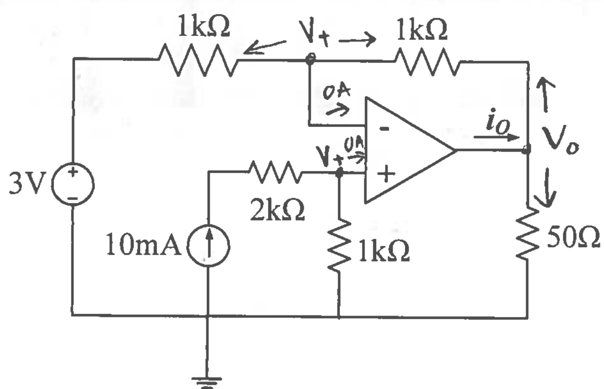


$R_T = (20k\Omega) // (10k\Omega + (10k\Omega // 20k\Omega))$

$R_T = (20k\Omega) // (10k\Omega + 6.67k\Omega)$

$\boxed{R_T : 9.09\ k\Omega}$

1.4. Assume an ideal op-amp in the circuit shown below. What is the output current i_O? [2pts]



$V_+ = 1k\Omega \times 10mA = 10V$

KCL @ inv. node:

$$\frac{V_+ - 3V}{1k\Omega} + \frac{V_+ - V_o}{1k\Omega} = 0$$

$$2V_+ - 3V = V_o$$

$$17\,V = V_o$$

KCL @ output node:

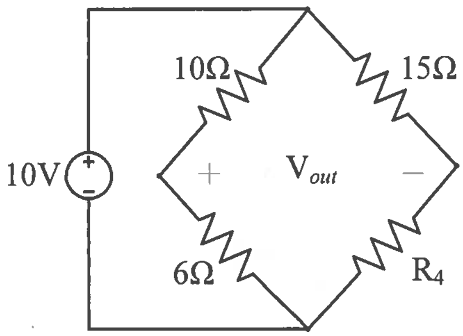
$$-i_o + \frac{V_o - V_+}{1k\Omega} + \frac{V_o}{50\Omega} = 0$$

$$i_o = \frac{7V}{1k\Omega} + \frac{17V}{50\Omega} = 347mA$$

$\boxed{i_o = 347mA}$

1.5. The circuit below is called the Wheatstone Bridge which was explored in your lab. The circuit is said to be balanced when $V_{out} = 0V$. What is the resistance value R4 for the Wheatstone Bridge to be balanced? [2pts]

R_4



$$V_{out}^{+} = 10V \cdot \frac{6\Omega}{6\Omega + 10\Omega}$$

$$V_{out}^{-} = 10V \cdot \frac{R_4}{15\Omega + R_4}$$

$$V_{out} = V_{out}^{+} - V_{out}^{-}$$

$$V_{out} = 10V \left[\frac{6\Omega}{16\Omega} - \frac{R_4}{15\Omega + R_4} \right]$$

Balanced → $V_{out} = 0V$

$$\frac{6\Omega}{16\Omega} = \frac{R_4}{15\Omega + R_4}$$

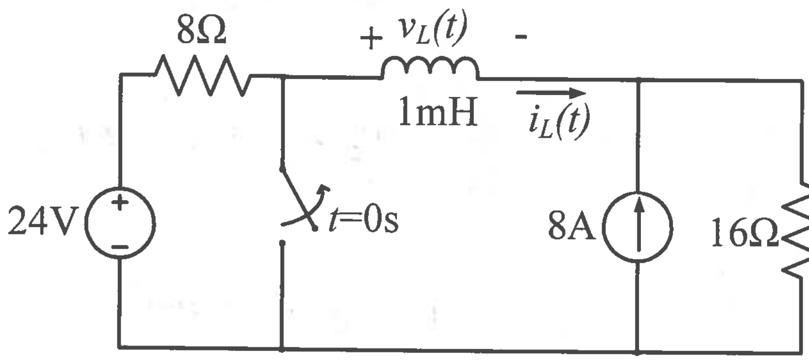
$$90 + 6R_4 = 16R_4$$

$$90R_4 = 10R_4$$

$$\boxed{R_4 = 9\Omega}$$

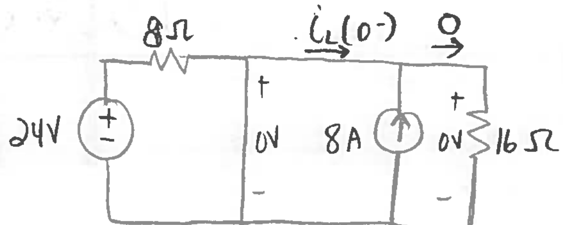
PART 2 – Problems

2.1. Consider the circuit shown below. The switch is closed for $t < 0$s. Assume dc steady state behaviour for $t < 0$. The switch opens at $t = 0$s. Answer the following questions. [10pts]



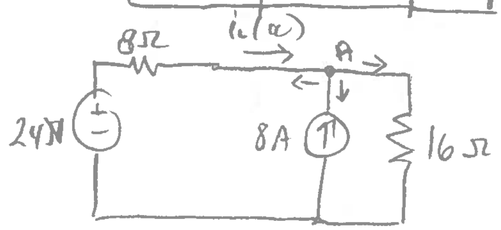
(a) Find the solution for the current $i_L(t)$ for $t > 0$.
(b) Plot your solution in (a) versus time t. Indicate the value of the current at $t = 0^+$, $t \rightarrow \infty$, and indicate the time constant τ.
(c) Find the voltage $v_L(t)$ for $t > 0$.
(d) If the inductor was replaced by a capacitor, what would be the voltage across the capacitor at $t = 0^+$?

a) $t > 0$



$i_L(0^-) = -8A = i_L(0^+)$

$t \to \infty$



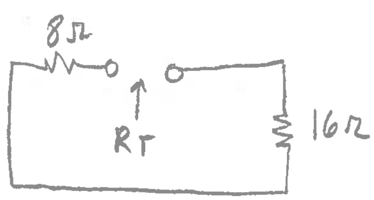
using node analysis

KCL @ A: $\frac{V_A - 24V}{8\Omega} - 8A + \frac{V_A}{16\Omega} = 0$

$2V_A - 48V - 128V + V_A = 0$

$3V_A = 176V \quad V_A = \frac{176}{3} V$

$\tau = L/R_T$



$i_L(\infty) = \frac{24V - 176/3\,V}{8\Omega} = -4\frac{1}{3} A$

R_T: 8 + 16 = 24 ohm

$\tau = \frac{1mH}{24 \text{ ohm}} =$ 41.67 us

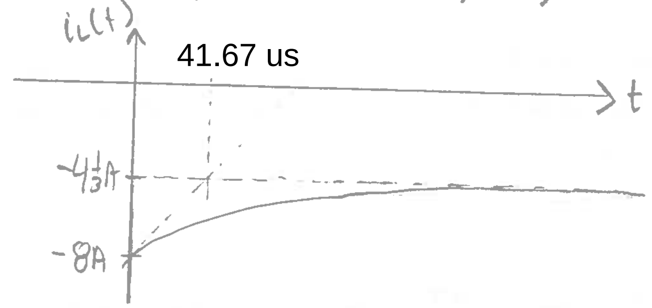
Solution: $i_L(t) = i_L(\infty) + [i_L(0^+) - i_L(\infty)]\, e^{-t/\tau}$

$i_L(t) = -4\frac{1}{3} A + [-8A + 4\frac{1}{3} A]\, e^{-t/41.67\,us}$

$i_L(t) = -4\frac{1}{3} - 3\frac{2}{3} e^{-t/41.67\,us}$ A ; $t > 0s$

$i_L(t) = -4\frac{1}{3} - 3\frac{2}{3}\, e^{-t/41.67\ \mu s}\ A, \quad t>0$

plot:

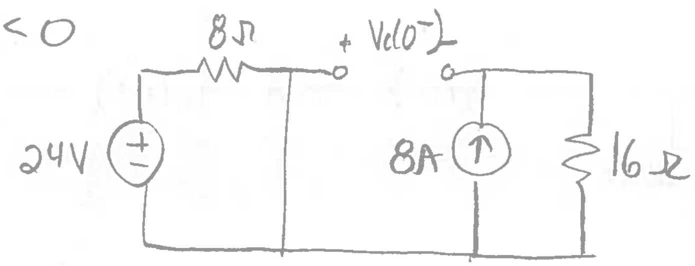


b) $v_L(t) = L\dfrac{d\, i_L(t)}{dt} = 1\text{mH}\,\dfrac{d}{dt}\left[-4\frac{1}{3} - 3\frac{2}{3}\, e^{-t/41.67\ \mu s}\right] V$

$= 1\text{mH} \cdot \left[-3\frac{2}{3} \cdot \dfrac{-1}{187.5\mu s}\, e^{-t/41.67\ \mu s}\right] V$

$$\boxed{v_L(t) = 19.56\,V e^{-t/41.67\ \mu s}\ V, \quad t>0}$$

c) $t<0$



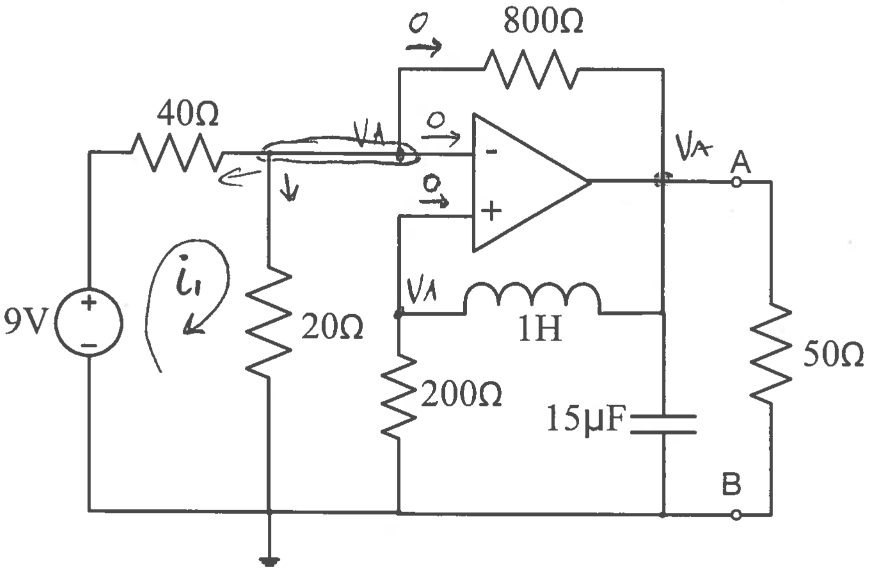
$V_c(0^-) = V_c(0^+)$

$V_c^+(0^-) = 0V \qquad V_c^-(0^-): 8A \cdot 16\Omega = 128V$

$V_c(0^-) = V_c(0^+) = 0 - 128V = -128V$

$$\boxed{V_c(0^+) = -128V}$$

2.2. Consider the circuit shown below. Assume that the op-amp is ideal and that the circuit is in dc steady state. Answer the following questions. [10pts]



(a) Find the power dissipated by the independent voltage source of ~~8~~ 9V?
(b) Which resistor dissipates the largest amount of energy?
(c) What is the energy stored in the capacitor and in the inductor?

a) dc steady state → inductor is a short and capacitor are open.
output voltage of opamp sets voltage at non-inverting node, which sets voltage at inverting node. Thus, there is no voltage drop across feedback resistor 800 Ω

KCL @ inv node: $\frac{V_A - 9V}{40\Omega} + \frac{V_A}{20\Omega} = 0 \rightarrow V_A - 9V + 2V_A = 0 \Rightarrow V_A = 3V$

KVL @ 9V mesh: $-9V + 40 i_1 + 20 i_1 = 0 \rightarrow i_1 = \frac{9}{60} A \Rightarrow i_1 = 150 mA$

$P = VI = 9V \cdot 150mA = 1.35W$

$\boxed{P_{9V} = 1.35W}$

b) 40Ω: $6V \cdot 150mA = 0.9W$
20Ω: $3V \cdot 150mA = 0.45W$
800Ω: $0V \cdot 0A = 0W$
200Ω: $\frac{V_A^2}{R} = \frac{9V^2}{200\Omega} = 45mW$
50Ω: $\frac{V_A^2}{R} = \frac{9V^2}{50\Omega} = 0.18W$

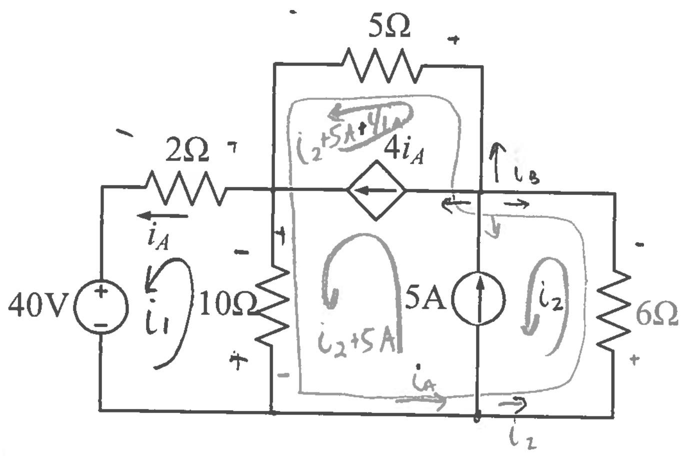
$\boxed{\text{40Ω resistor}}$

c) capacitor: $\frac{1}{2}CV^2 = \frac{1}{2} \cdot 15\times10^{-6} \cdot 3^2$

inductor
$\frac{1}{2}Li^2 = \frac{1}{2} \cdot 1H \cdot \left(\frac{3}{200}\right)^2$
$= 112.5\mu J$

$\boxed{\begin{array}{ll} \text{cap}: & 67.5\mu J \\ \text{ind}: & 112.5\mu J \end{array}}$

2.3. Consider the circuit shown below. Answer the following questions. [10pts]



$i_B - i_2 - 5A + 4i_A$

$i_B = i_2 + 5A - 4i_A$

$-i_A + i_2 + 5A$

(a) Find the current i_A using mesh analysis.

(b) What is the power delivered by the current-dependent current source?

a) using super-mesh, KVL:

$$6\Omega \cdot i_2 + 5\Omega[i_2 + 5A + 4i_A] + 10\Omega[i_2 + 5A - i_1] = 0$$

$$6i_2 + 5i_2 + 20 + 20i_A + 10i_2 + 50 - 10i_1 = 0$$

control variable: $i_1 = i_A$

$$11i_2 + 20 + 20i_1 + 10i_2 + 50 - 10i_1 = 0$$

$$10i_1 + 21i_2 + 70 = 0$$

other mesh: $40V + 10\Omega \cdot i_1 + 2\Omega i_1 = 0$ ← sufficient to answer (a)

$$i_1 = \frac{40V}{12\Omega} = 3\tfrac{1}{3}A$$

$\boxed{i_A = 3\tfrac{1}{3}A}$

b) Voltage accross 5Ω will be the same as the $4i_A$ source

cont mesh analysis in (a) to solve for i_2

$$10(3\tfrac{1}{3}) + 21i_2 + 70 \rightarrow i_2 = \frac{-1}{21}[33\tfrac{1}{3} + 70] =$$

$i_2 = -4.92A$

$$V_{5\Omega} = 5\Omega[i_2 + 5A + 4i_A] = 50[-4.92 + 5A + 4(3\tfrac{1}{3})]$$

$$= 670.67V$$

$$P = 4i_A \times 670.67V = 8.94W$$

$\boxed{P = 8.94W}$