Today's Outline

2. Resistive Circuits

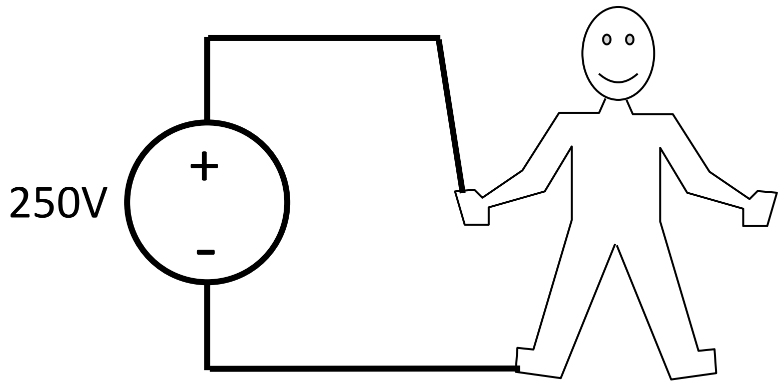
- Equivalent Resistance in Circuit Analysis

Example 1

Should there be any concern about the following situation?

Physiological Reaction	Current
Barely Perceptible	3-5mA
Extreme Pain	35-50mA
Muscle Paralysis	50-70mA
Heart Stoppage	500mA

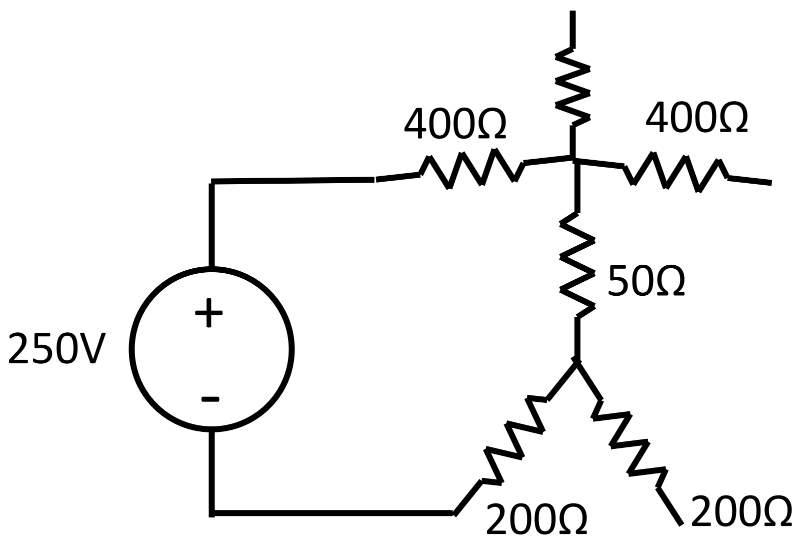
W.F. Cooper, *Electrical Safety Engineering*, 2nd ed., Butterworth, 1986.



To answer this question, we need an electrical model…

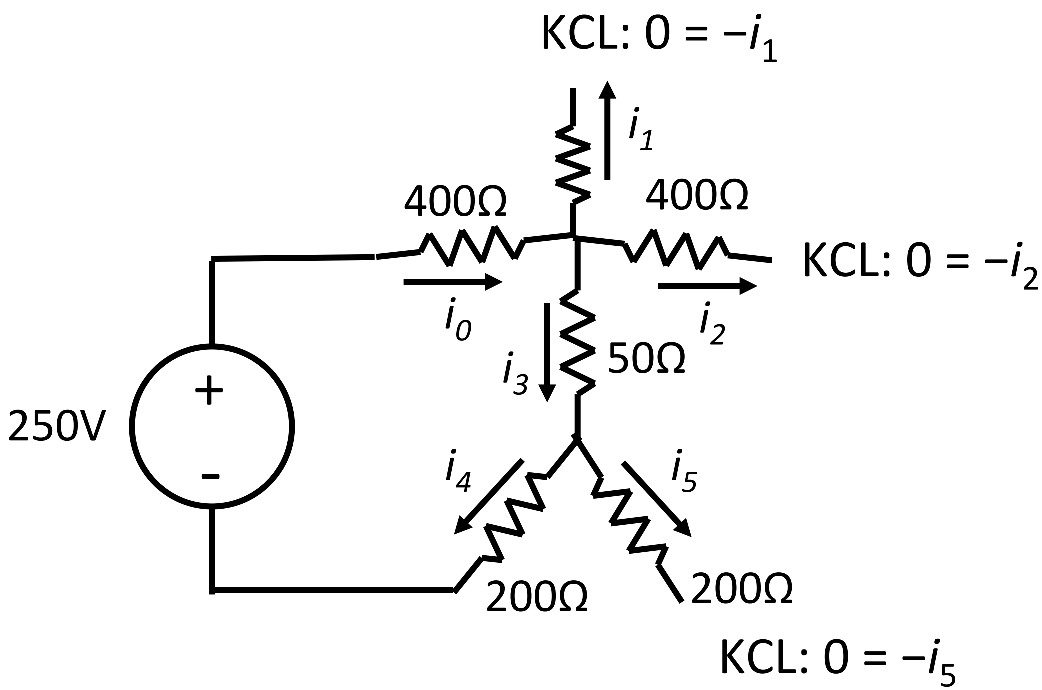
Example 1

An approximate electrical model is given by a resistor network from experimental data on a “typical” human, and the current can be easily found.



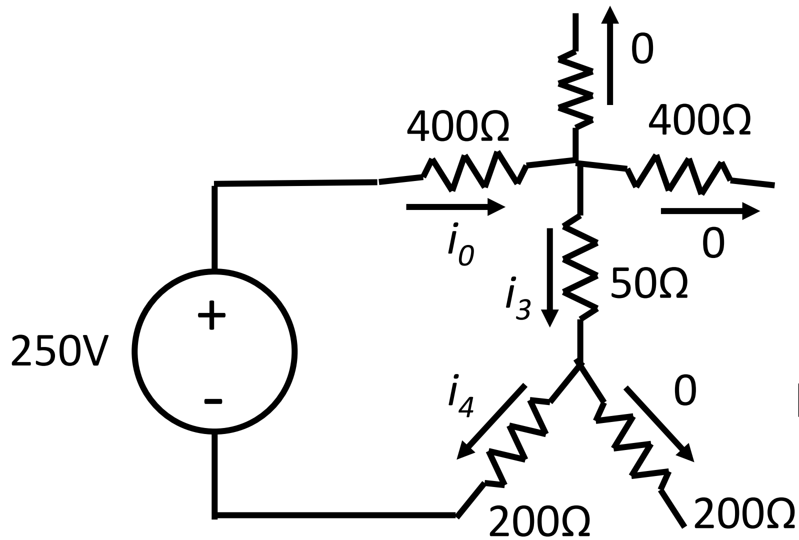
Note that we are neglecting the resistance of contact with the skin here. Note also that human tissue is very crudely modeled by ideal resistors.

Example 1



KCL: $0 = -i_1$
i_1
400Ω
400Ω
KCL: $0 = -i_2$
i_0
i_2
i_3
50Ω
250V
+
-
i_4
i_5
200Ω
200Ω
KCL: $0 = -i_5$

Example 1



KCL: $0 = -i_0 + 0 + 0 + i_3$

$i_0 = i_3$

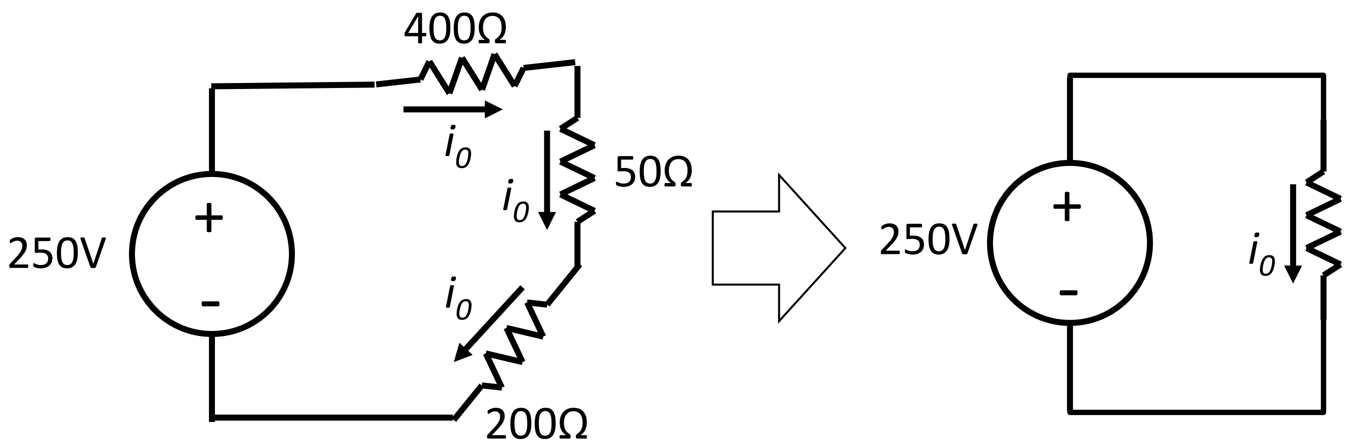
KCL: $0 = -i_3 + 0 + i_4$

$i_3 = i_4$

From KCL, the "dangling" resistors do not conduct current. We therefore have a current $i_0 = i_3 = i_4$ flowing through the series combination of 400Ω, 50Ω and 200Ω resistors.

Recall that resistors in series carry the same current.

Example 1



R_{eq} = 400Ω+50Ω+200Ω

= 650Ω

Ohm's Law: i_0 = 250V / 650Ω = 385mA

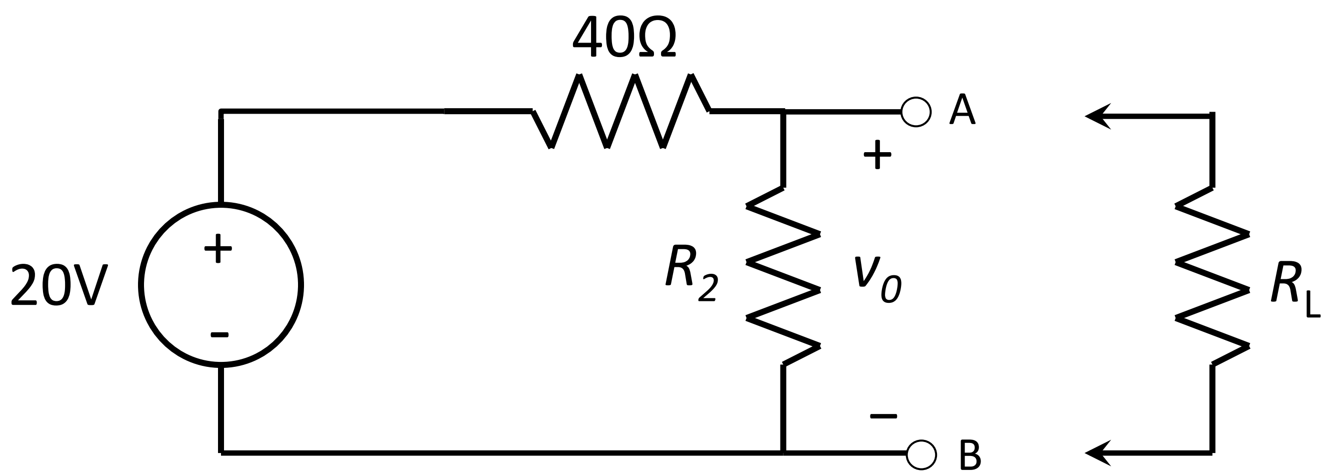
The current flow is dangerous

Physiological Reaction	**Current**
Barely Perceptible	3-5mA
Extreme Pain	35-50mA
Muscle Paralysis	50-70mA
Heart Stoppage	500mA

W.F. Cooper, *Electrical Safety Engineering*, 2nd ed., Butterworth, 1986.

Example 2

In the absence of the load resistor R_L, the voltage divider produces v_0 = 4V. When the load resistor is attached to the terminals A B, the voltage v_0 = 3V. What is the value of R_L?



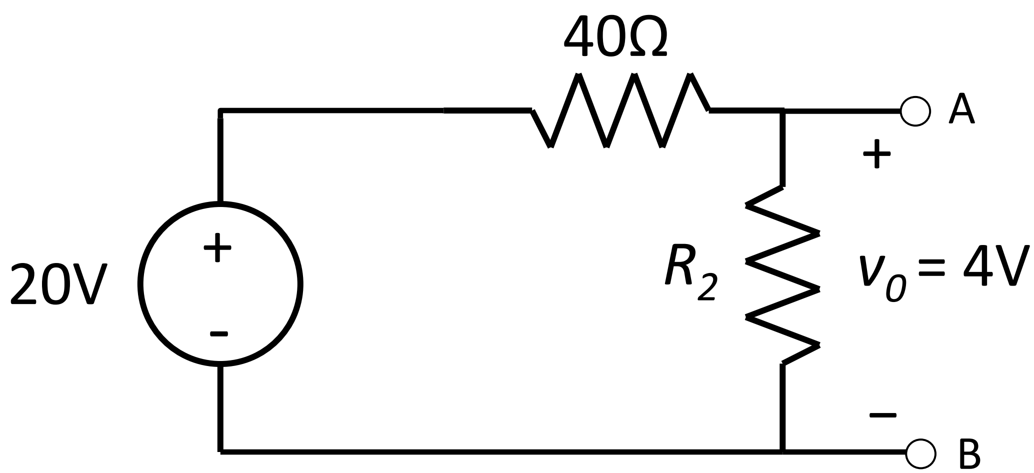
Strategy:

– find R_2 using the voltage divider equation

–find $R_2 || R_L$ using the voltage divider equation, and thus find R_L

Example 2

Find R_2 using the voltage divider equation.



$$\frac{4V}{20V} = \frac{R_2}{40\Omega + R_2}$$

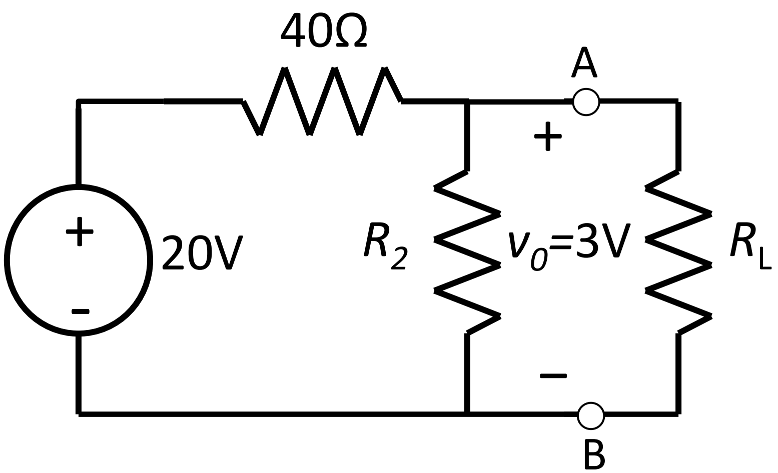
$$\frac{1}{5}\left(40\Omega + R_2\right) = R_2$$

$$8\Omega = R_2\left(1 - \frac{1}{5}\right)$$

$$R_2 = \frac{5}{4}\cdot 8\Omega = 10\Omega$$

Example 2

Find $R_2||R_L$ using the voltage divider equation.



$$\frac{3V}{20V} = \frac{R_2||R_L}{40\Omega + R_2||R_L}$$

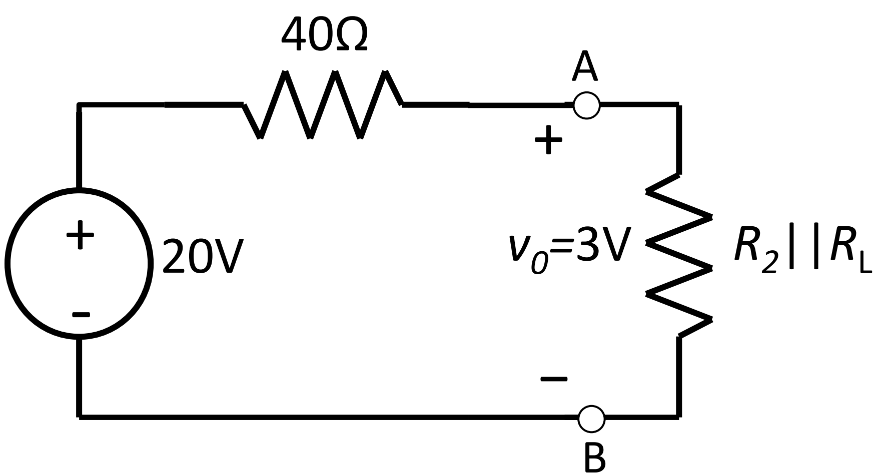
$$\frac{3}{20}\left(40\Omega + R_2||R_L\right) = R_2||R_L$$

$$6\Omega = R_2||R_L\left(1 - \frac{3}{20}\right)$$

$$R_2||R_L = \frac{20}{17}\cdot 6\Omega$$

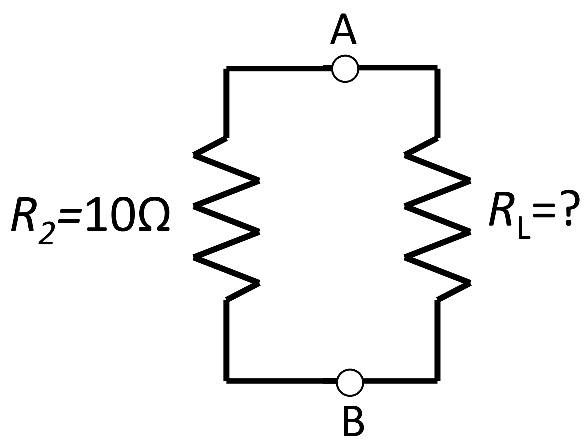
$$= \frac{120}{17}\Omega$$

$$= 7.059\Omega$$



Example 2

Find R_L from the value of R_2 and $R_2||R_L$.



$R_2||R_L = (120/17)\Omega$

$$\frac{1}{R_2||R_L} = \frac{1}{R_2} + \frac{1}{R_L}$$

For parallel combinations, we add conductances.

$$\frac{17}{120\Omega} = \frac{1}{10\Omega} + \frac{1}{R_L}$$

$$\frac{1}{R_L} = \frac{17}{120\Omega} - \frac{1}{10\Omega} = \frac{17}{120\Omega} - \frac{12}{120\Omega}$$

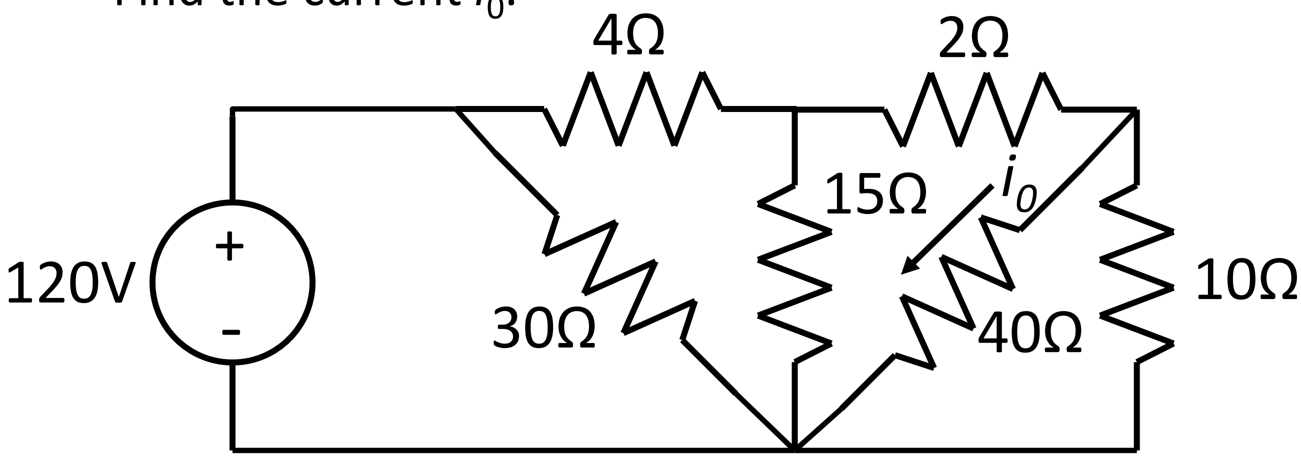
$$= \frac{5}{120\Omega}$$

$$R_L = 24\Omega$$

We could of course use decimals or fractions for this calculation. If you elect to use decimals, use 4 significant digits (*A*.*BCD*).

Example 3

Find the current i_0.



Strategy:

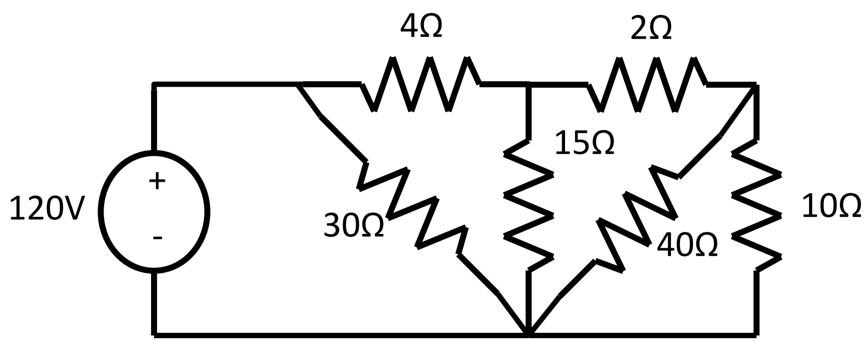
– reduce the circuit to a source and single equivalent resistor

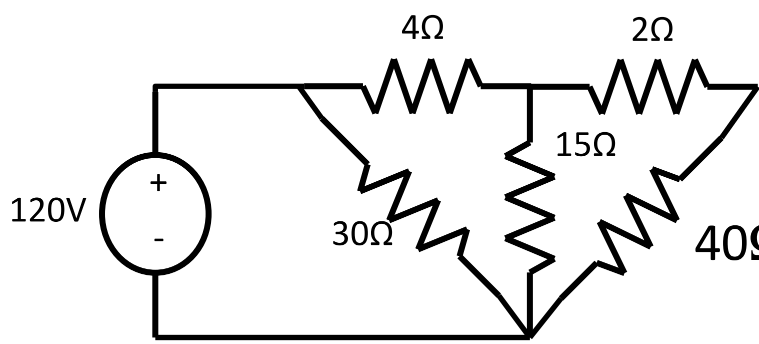
– work through the equivalent circuits to find i_0

We could also use KVL and KCL directly, but this example shows how creative one can be in using resistor equivalence and voltage/ current dividers to solve a problem.

Example 3

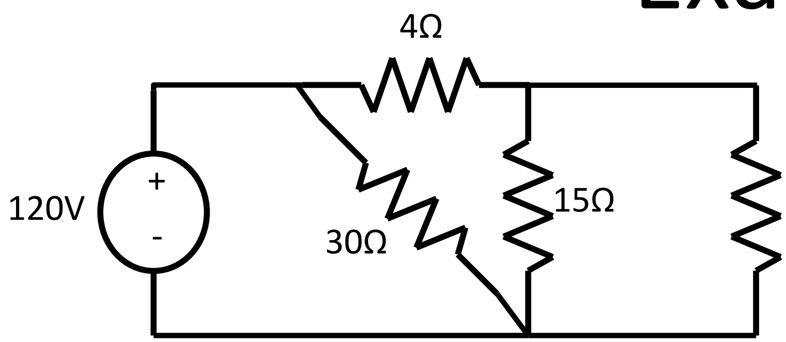
Reduce the circuit to a single equivalent resistor.



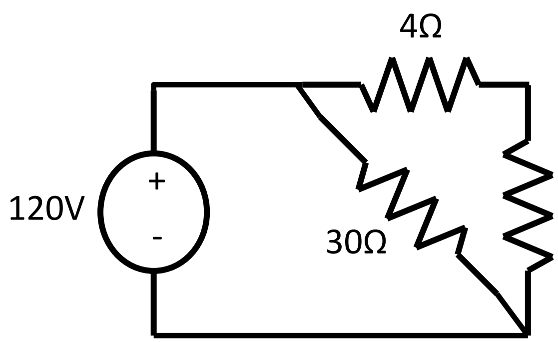


$$40\Omega \,||\, 10\Omega = \frac{10\Omega \cdot 40\Omega}{10\Omega + 40\Omega} = 8\Omega$$

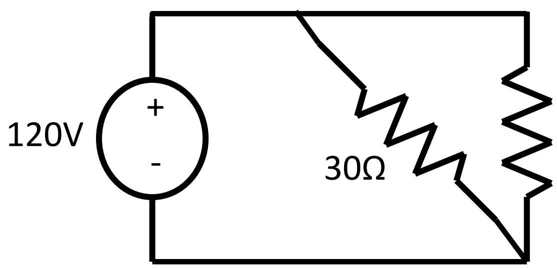
Example 3



$$8\Omega + 2\Omega = 10\Omega$$



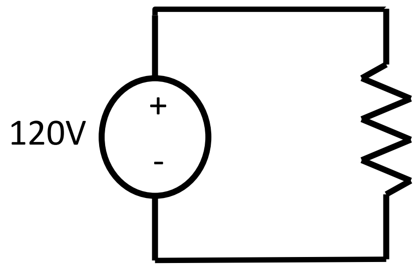
$$10\Omega \,||\, 15\Omega = \frac{10\Omega \cdot 15\Omega}{10\Omega + 15\Omega} = 6\Omega$$



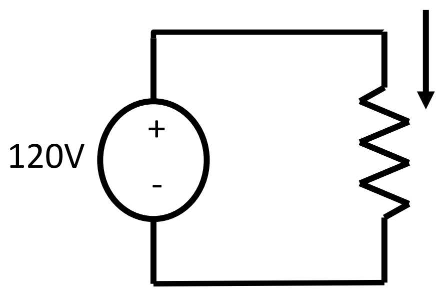
$$6\Omega + 4\Omega = 10\Omega$$

McGill

Example 3



$$10\Omega\,||\,30\Omega = \frac{10\Omega \cdot 30\Omega}{10\Omega + 30\Omega} = 7.5\Omega$$



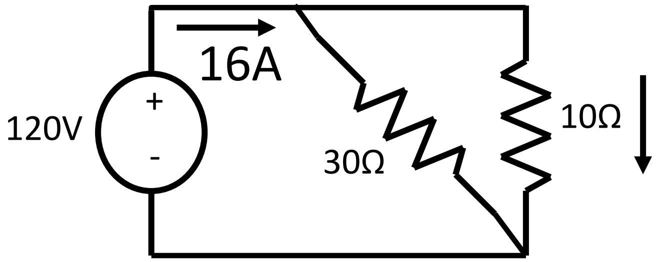
$i_A = 120V / 7.5\Omega = 16A$

7.5Ω

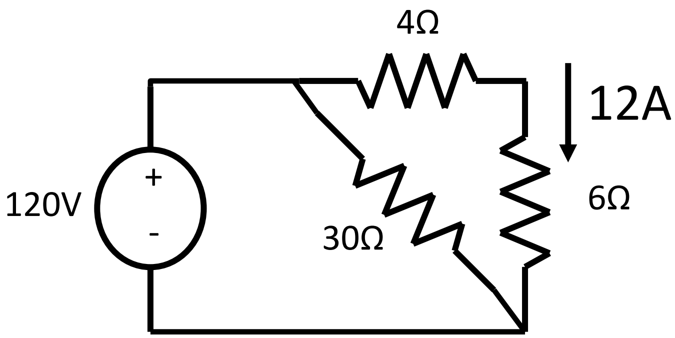
Note that the entire network of *resistors* is equivalent to a single 7.5Ω resistor across the terminals of the 120V source.

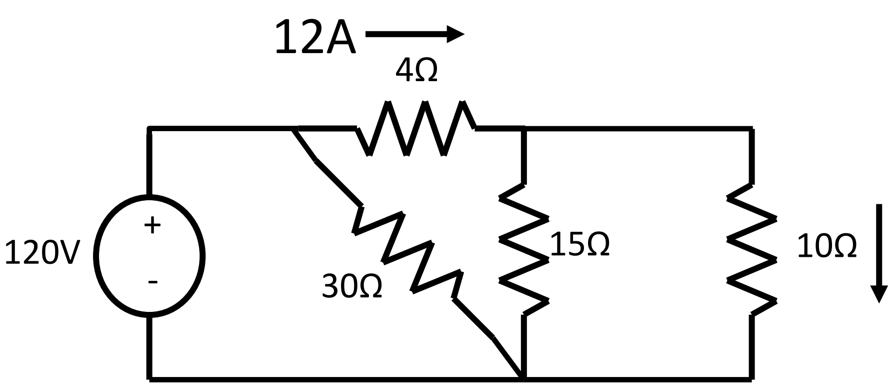
We now work forward through the equivalent circuits, solving for currents along the way.

Example 3



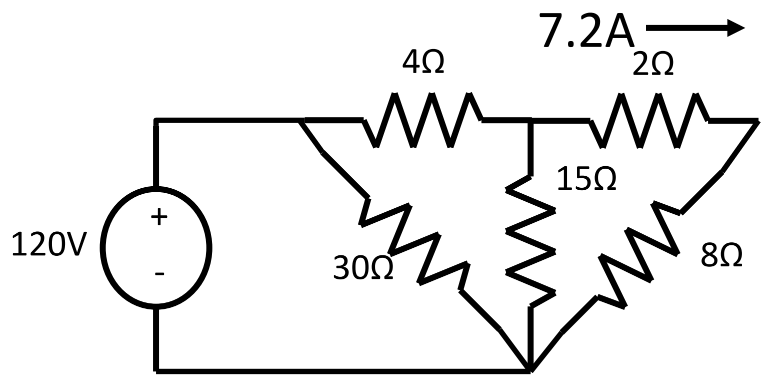
$$i_B = 16\text{A}\,\frac{30\Omega}{10\Omega + 30\Omega} = 12\text{A}$$

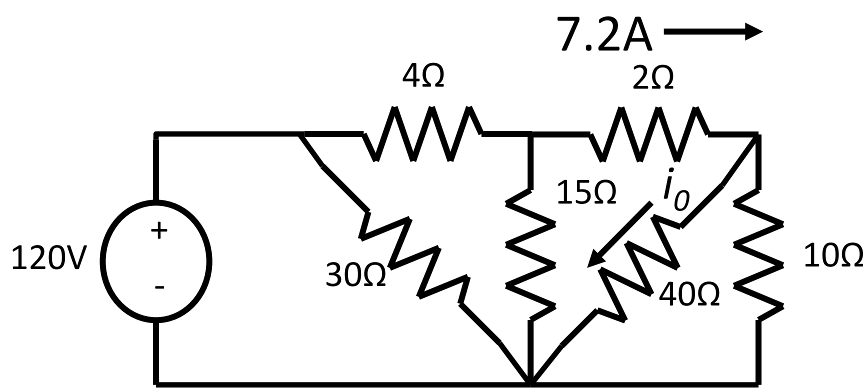




$$i_C = 12\text{A}\,\frac{15\Omega}{10\Omega + 15\Omega} = 7.2\text{A}$$

Example 3



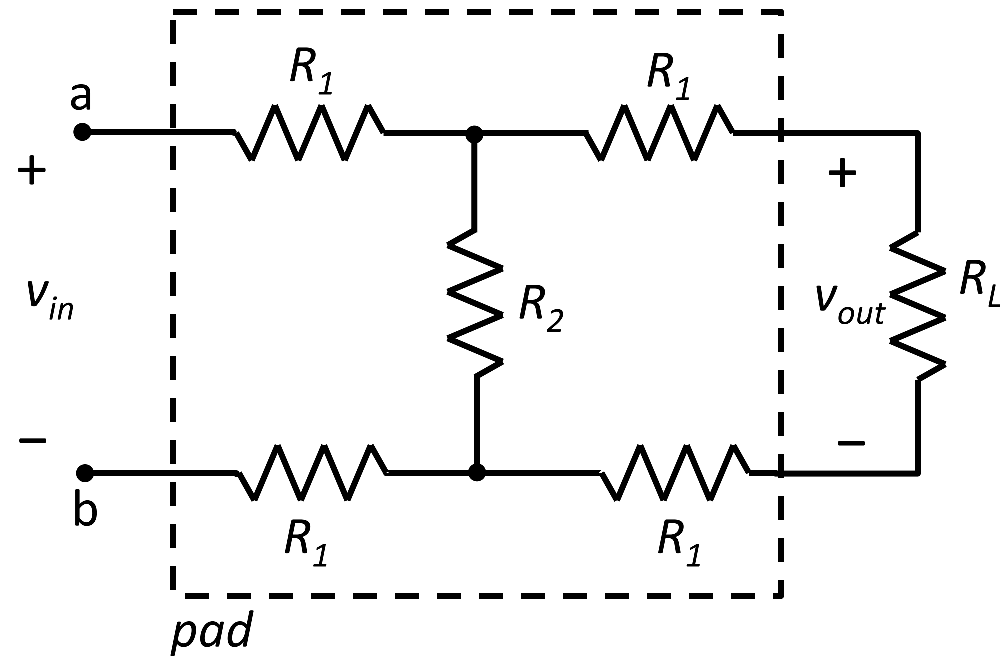


$$i_0 = 7.2\text{A}\frac{10\Omega}{10\Omega + 40\Omega} = 1.44\text{A}$$

Note that we have reduced the “large” circuit problem to a series of small, easily solvable problems.

Example 4

The circuit below is an ***attenuator pad***, often used in volume control circuits. What is the equivalent resistance at terminals a and b? What is the ratio of v_{out} to v_{in}?

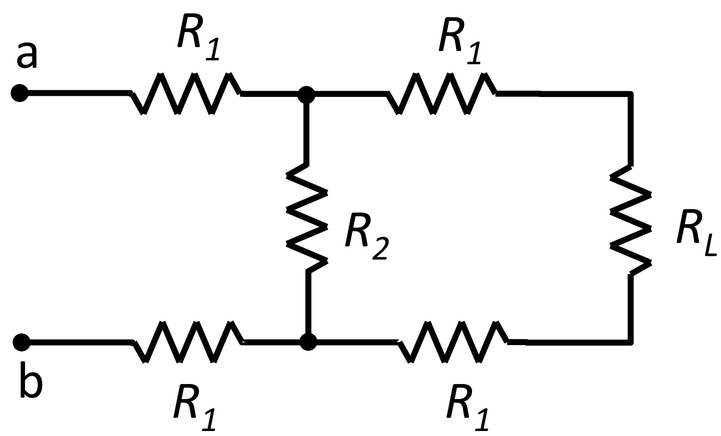


Strategy:

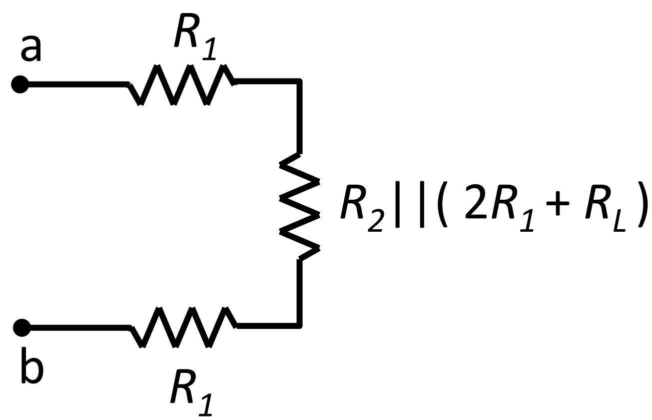
– apply equivalent resistance rules for series and parallel (if that doesn't work, revert to definition of equivalent resistance)

Example 4

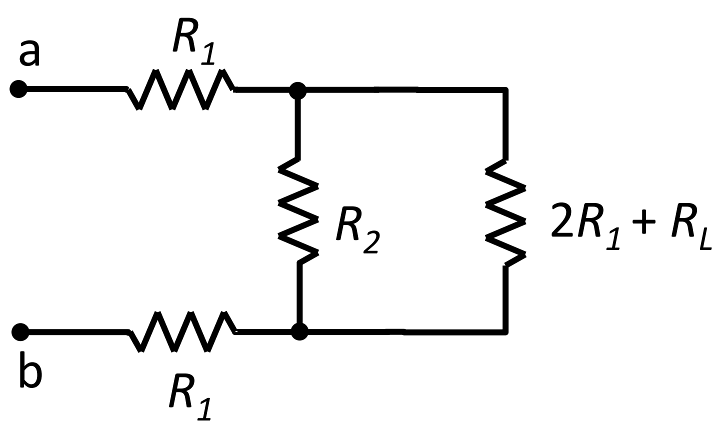
step #1



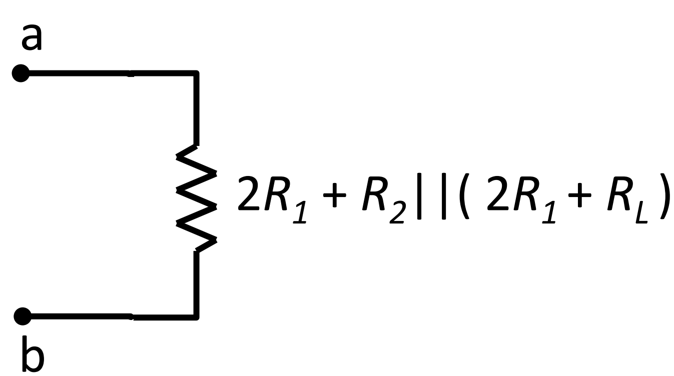
step #3



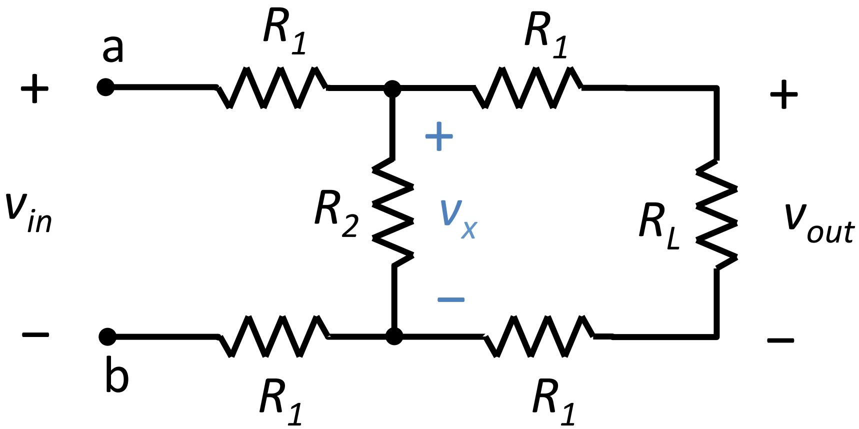
step #2



step #4

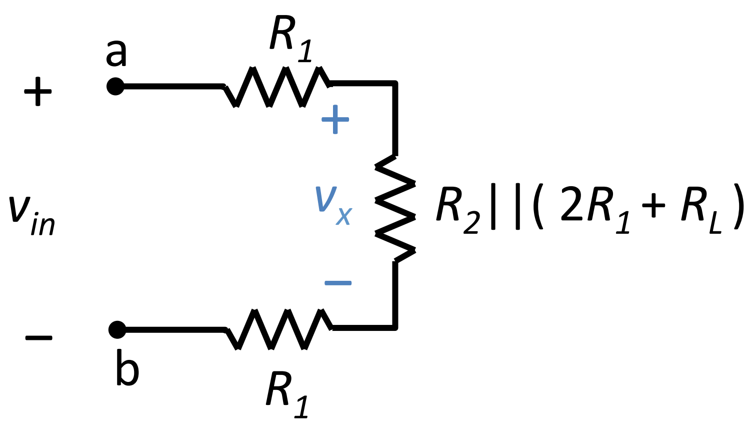


Example 4



Applying a voltage divider equation:

$$v_{out} = v_x \frac{R_L}{2R_1 + R_L}$$



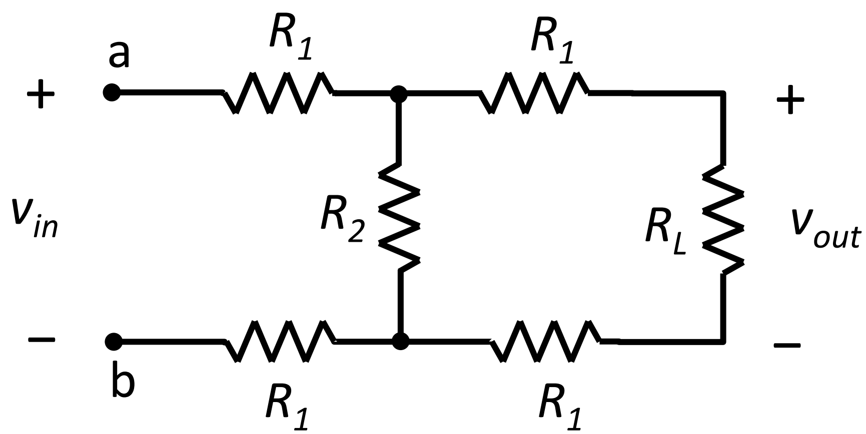
Applying another voltage divider equation:

$$v_x = v_{in} \frac{R_2 \,||\, (2R_1 + R_L)}{2R_1 + R_2 \,||\, (2R_1 + R_L)}$$

Combining results:

$$\frac{v_{out}}{v_{in}} = \frac{v_{out}}{v_x}\frac{v_x}{v_{in}} = \frac{R_L}{2R_1 + R_L} \cdot \frac{R_2 \,||\, (2R_1 + R_L)}{2R_1 + R_2 \,||\, (2R_1 + R_L)}$$

Example 4



The equivalent resistance at terminals a and b, and the voltage ratio v_{out}/v_{in} are given by:

$$R_{ab} = 2R_1 + R_2 || (2R_1 + R_L)$$

$$\frac{v_{out}}{v_{in}} = \frac{R_L}{2R_1 + R_L} \cdot \frac{R_2 || (2R_1 + R_L)}{2R_1 + R_2 || (2R_1 + R_L)}$$

In practice, one specifies the load resistance R_L, the equivalent resistance R_{ab} and the voltage ratio v_{out}/v_{in} to determine the required R_1 and R_2 of the pad.

Note that there are many other attenuator pad geometries.