McGill

December 2011
Final Examination

Electric Circuits 1

ECSE-200 Section: 1, 2

13 Dec 2011, 9:00AM

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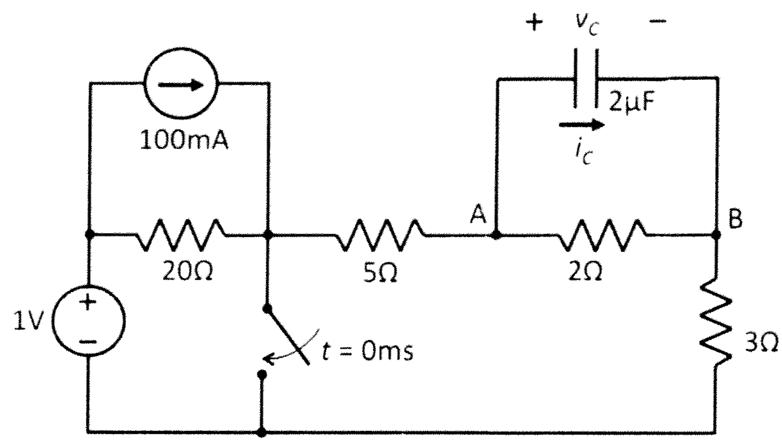
Assoc Examiner: Zetian Mi

INSTRUCTIONS:

- This is a **CLOSED BOOK** examination.
- **NO CRIB SHEETS** are permitted.
- Provide your answers in an **EXAM BOOKLET**.
- **STANDARD CALCULATOR** permitted ONLY.
- This examination consists of 4 questions, with a total of 6 pages, including the cover page.
- This examination is **PRINTED ON BOTH SIDES** of the paper

1. Consider the circuit below. The circuit is in dc steady state for $t < 0$. The switch closes instantaneously at $t = 0$. Answer the questions. [12 pts]



a) Give one physical reason why capacitor voltage must be continuous. [1pt]

b) Draw an equivalent circuit for $t < 0$. Your circuit should be a Thévenin equivalent circuit connected to a capacitor. [2pts]

c) Draw an equivalent circuit for $t > 0$. Your circuit should be a Thévenin equivalent circuit connected to a capacitor. [1pt]

d) What is $v_C(t)$ for $t > 0$? [4pts]

e) What is $i_C(t)$ for $t > 0$? [2pts]

f) Consider the equivalent circuit in part c). What is the total energy absorbed by the Thévenin resistance over the time interval $0 < t < \infty$? [2pts]

a) conservation of energy

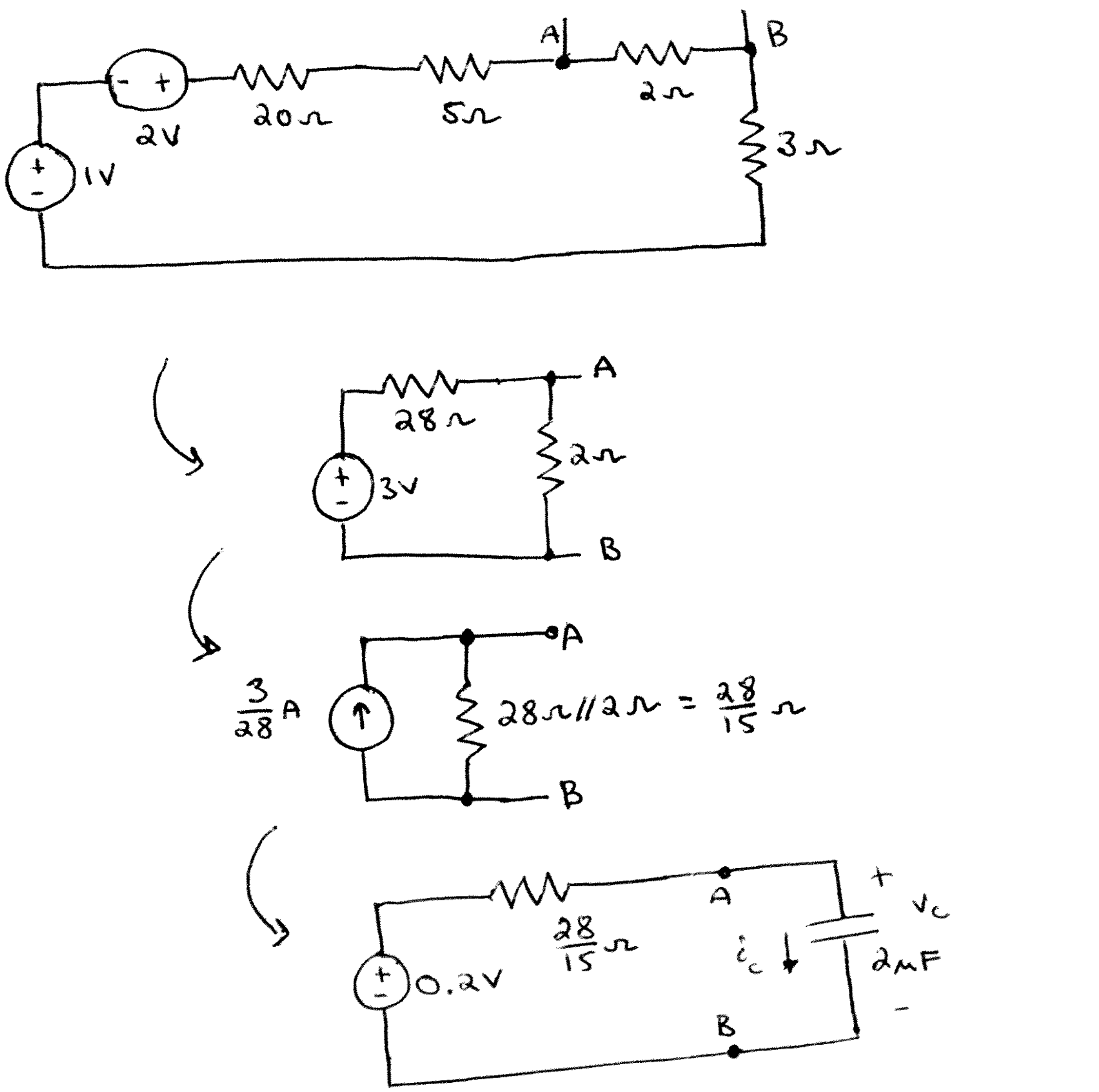
conservation of charge

finite current

finite power

$\Big\}$ [+1]

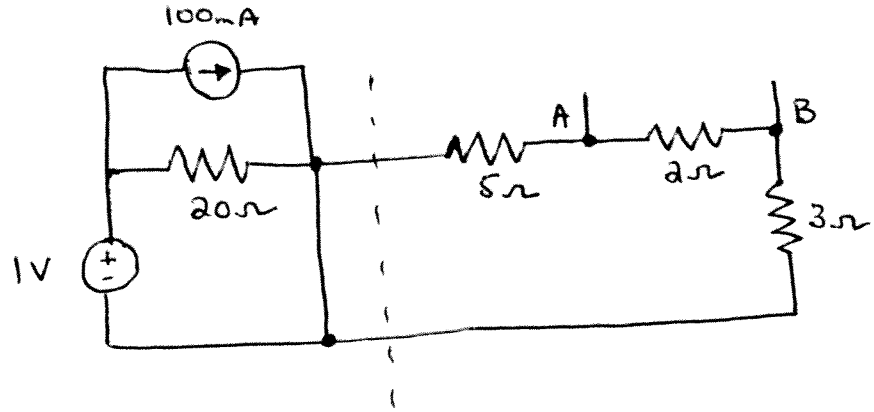
b) Use source transformations and equivalent resistance:



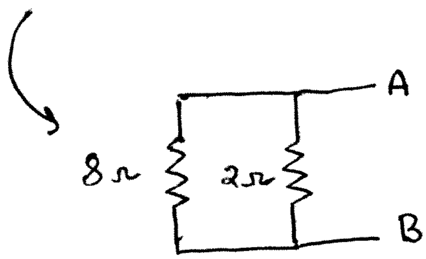
[+1] for V_{oc}

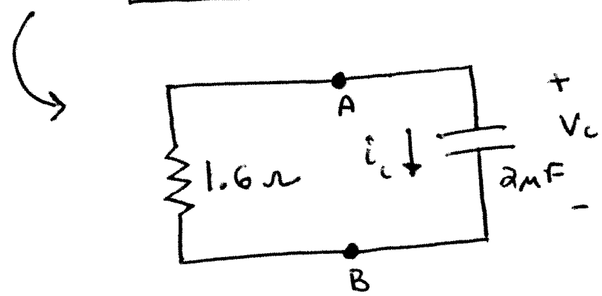
[+1] for R_T

c) Use equivalent resistance and Thévenin's theorem:



← equivalent to a short circuit





[+1] for R_T

d) $V_c(0+) = V_c(0-) = 0.2\,V$ (from b))

[+1]

$V_c(\infty) = -i_c(\infty)\cdot 1.6\Omega = 0A\cdot 1.6\Omega = 0V$ (from c))

[+1]

$\tau = R_{TH}\cdot C = 1.6\Omega \cdot 2\mu F$

$= 3.2\,\mu s$ [+1]

$$v_c(t) = v_c(\infty) + [v_c(0+) - v_c(\infty)]e^{-t/\tau} \quad [+1/2]$$

$$= 0.2V\, e^{-t/3.2\mu s} \quad t > 0 \quad [+1]$$

e) $$i_c(t) = C\frac{dv_c}{dt} \quad [+1]$$

$$= 2\mu F \cdot \frac{d}{dt}\left(0.2V\, e^{-t/3.2\mu s}\right)$$

$$= 2\mu F \cdot 0.2V \cdot -\frac{1}{3.2\mu s} \cdot e^{-t/3.2\mu s}$$

$$= -125mA\, e^{-t/3.2\mu s} \quad [+1]$$

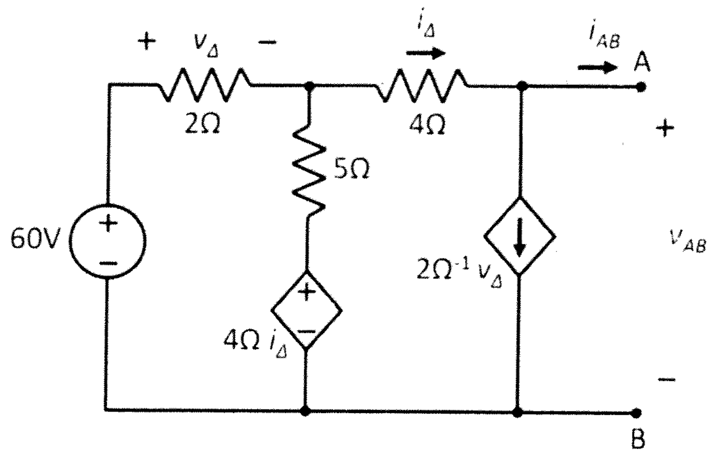
f) By conservation of energy:

$$U_{abs} = \frac{1}{2}C(v_c(0+))^2 \quad [+1]$$

$$= \frac{1}{2} \cdot 2\mu F \cdot (0.2V)^2$$

$$= 40nJ \quad [+1]$$

2. Consider the circuit below. Answer the questions. [12 pts]



a) What is the physical principle behind Kirchoff's Voltage Law? [1pt]

b) What is the physical principle behind Kirchoff's Current Law? [1pt]

c) Name one circuit theorem or principle that follows from the linearity of a circuit. [1pt]

d) What is the Thévenin equivalent circuit with respect to the terminals A and B? Clearly label A and B on your circuit diagram. [4pts]

e) What is the Norton equivalent circuit with respect to the terminals A and B? Clearly label A and B on your circuit diagram. [1pt]

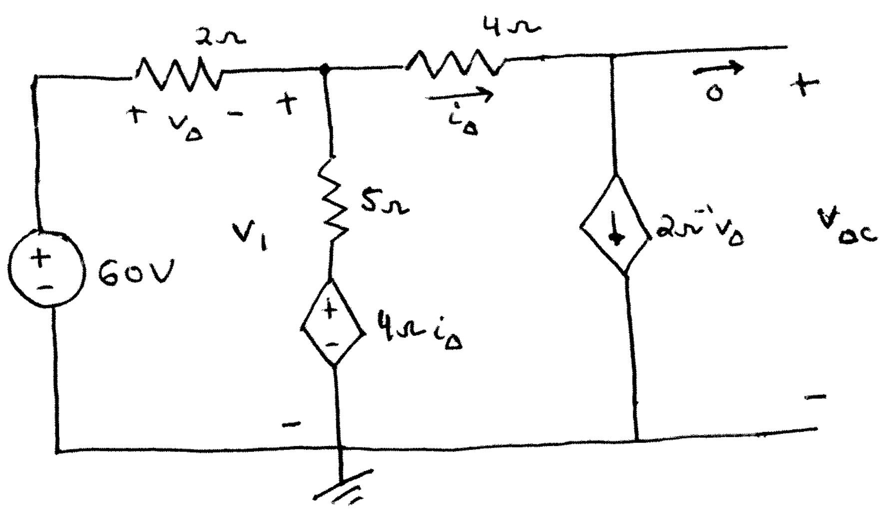
f) What is the maximum power that the circuit can deliver to an optimally chosen load resistor attached to the terminals A and B? What is the value of the optimally chosen load resistor. [4pts]

a) conservation of energy [+1]

b) conservation of charge [+1]

c) principle of linear superposition
Thévenin's theorem
Norton's theorem
source transformations
maximum power transfer theorem [+1]

d) Find V_{oc}. [+1/2]



node:

$$\frac{v_1 - 60V}{2\Omega} + \frac{v_1 - 4\Omega\, i_\Delta}{5\Omega} + 2\Omega^{-1} v_\Delta = 0$$

control: $i_\Delta = 2\Omega^{-1} v_\Delta$

$$v_\Delta = 60V - v_1$$

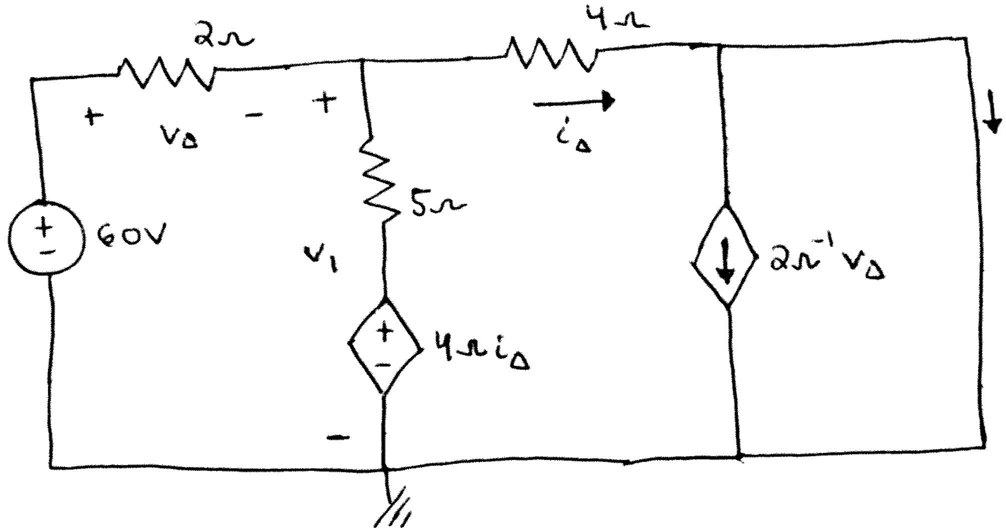
$$\frac{v_1 - 60V}{2\Omega} + \frac{v_1 - 4\Omega(2\Omega^{-1}(60V - v_1))}{5\Omega} + 2\Omega^{-1}(60V - v_1) = 0$$

$$v_1\left(\frac{1}{2} + \frac{1}{5} + \frac{2\cdot 4}{5} - 2\right) + \left(-\frac{60}{2} - \frac{4\cdot 2\cdot 60}{5} + 2\cdot 60\right) = 0$$

$$v_1 = \frac{+6}{0.3} = +20V$$

KVL: $V_{oc} = v_1 - i_\Delta \cdot 4\Omega = v_1 - 4\Omega \cdot 2\Omega^{-1}(60V - v_1)$

$= -300V$ [+1]



Find isc.
[+1/2]

node: $\dfrac{v_1 - 60V}{2\Omega} + \dfrac{v_1 - 4\Omega i_\Delta}{5\Omega} + \dfrac{v_1}{4\Omega} = 0$

control: $i_\Delta = v_1 / 4\Omega$

$v_\Delta = 60V - v_1$

$$\frac{v_1 - 60V}{2\Omega} + \frac{v_1 - v_1}{5\Omega} + \frac{v_1}{4\Omega} = 0$$

$$v_1 = \frac{(60/2)}{(1/2 + 1/4)} = 40V$$

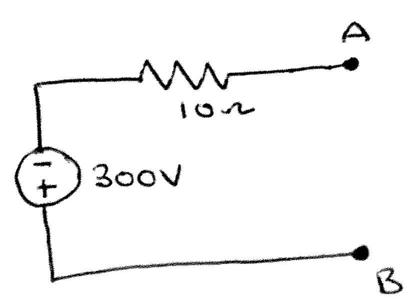
KCL: $i_{sc} = i_\Delta - 2\Omega^{-1} v_\Delta$

$= \dfrac{v_1}{4\Omega} - 2\Omega^{-1}(60V - v_1)$

$= -30A$ [+1]

$R_T = v_{oc} / i_{sc}$ [+1]

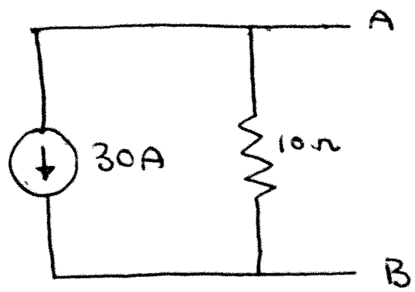
$= 10\Omega$



[max points = 4]

[+1] for circuit

e)



[+1]

f)

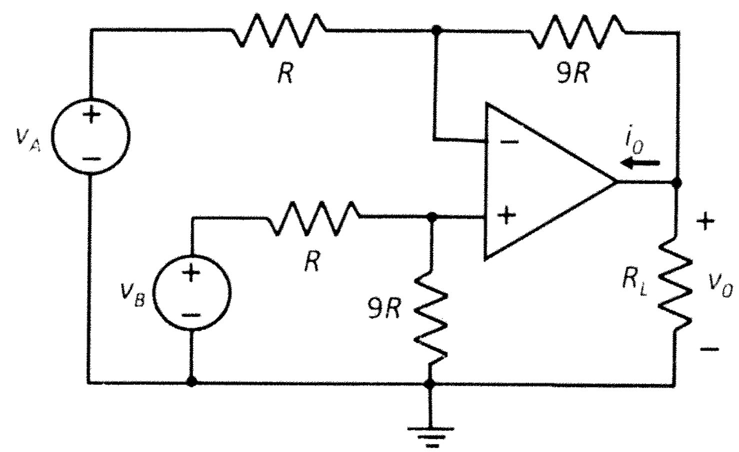
$$P_{max} = \frac{v_{oc}}{2} \cdot \frac{i_{sc}}{2} \quad [+1]$$

$$= 2.25\,kW \quad [+1]$$

$$R_L = R_T \quad [+1]$$

$$= 10\Omega \quad [+1]$$

3. Consider the circuit below. Assume ideal op-amp behaviour. Answer the questions. [12 pts]



a) Express v_O as a function of v_A and v_B. [2pts]

Assume v_A = 1V, v_B = 2V, R = 1kΩ and R_L = 1kΩ for the remainder of this question.

b) What is the total power absorbed by the resistors in this circuit? [2pts]

c) What is the total power delivered or absorbed by the independent voltage sources? [2pts]

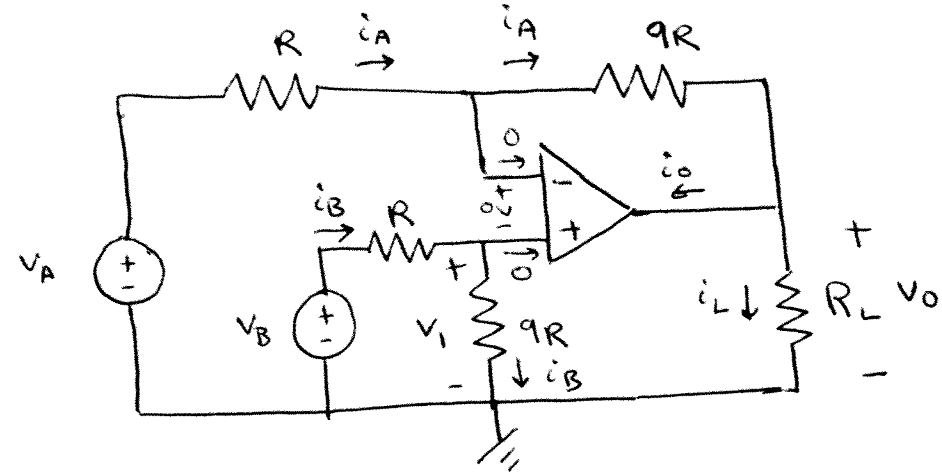
d) What is the current i_O? [2pts]

e) A voltmeter with 18kΩ internal resistance is used to measure the voltage of the non-inverting input terminal of the op-amp relative to the reference terminal. What is the measured voltage? [2pts]

f) Does the circuit have a negative feedback loop? [1pt]

g) Give one benefit of using negative feedback in op-amp circuits. [1pt]

a)



$$V_1 = V_B \cdot \frac{9R}{R+9R} = 9V_B/10 \quad [+1]$$

KCL at inverting node:

$$0 = \frac{V_1 - V_A}{R} + \frac{V_1 - V_o}{9R}$$

$$0 = \frac{9V_B}{10} - V_A + \frac{1}{9}\left(\frac{9V_B}{10} - V_o\right)$$

$$V_o = 9\left(\frac{9V_B}{10} - V_A\right) + \frac{9V_B}{10}$$

$$V_o = 9(V_B - V_A) \quad [+1]$$

b)

$$i_B = \frac{V_B}{R+9R} = \frac{2V}{10k\Omega} = 0.2mA$$

$$i_A = \frac{V_A - V_1}{R} = \frac{1V - \frac{9}{10}\cdot 2V}{1k\Omega} = -0.8\,mA$$

$$i_L = \frac{V_o}{R_L} = \frac{9(2V - 1V)}{1k\Omega} = 9\,mA$$

$P_{abs} = i^2R = v^2/R$ [+1], sum for each resistor:

$$P_{abs,\,total} = i_B^2 \cdot R + i_B^2 \cdot 9R + i_A^2 \cdot R + i_A^2 \cdot 9R + i_L^2 \cdot R_L$$

$$= 40\,\mu W + 360\,\mu W + 640\,\mu W + 5760\,\mu W + 81000\,\mu W$$

$$= 87.8\,mW \quad [+1]$$

c) $P_{del} = i_A \cdot v_A + i_B \cdot v_B \quad [+1]$

$$= -0.8mA \cdot 1V + 0.2mA \cdot 2V$$

$$= -400\,\mu W \quad [+1]$$

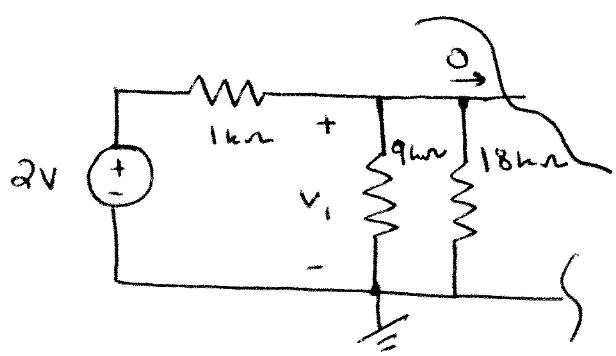
(or $+400\,\mu W$ absorbed)

d) KCL: $i_o = i_A - i_L \quad [+1]$

$$= -0.8mA - 9mA$$

$$= -9.8\,mA \quad [+1]$$

e)



[+1] for circuit

$$v_1 = 2V \cdot \frac{9k\Omega \,//\, 18k\Omega}{1k\Omega + 9k\Omega \,//\, 18k\Omega}$$

$$= 1.714\,V \quad [+1]$$

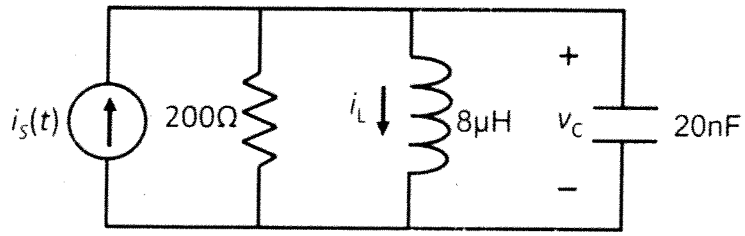
f) yes [+1]

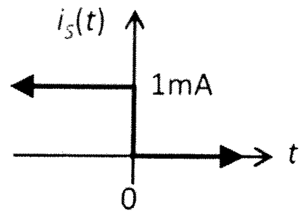
g) stability

programmability

independence from open-loop op-amp parameters [+1]

4. Consider the circuit and the plot below. Assume dc steady state behaviour for $t<0$. Answer the questions. [12 pts]





$$i_s(t) = \begin{cases} 1\text{mA} & t < 0 \\ 0\text{mA} & t > 0 \end{cases}$$

a) What is $v_C(0+)$? [1pt]

b) What is $i_L(0+)$? [1pt]

c) What is the total energy stored in this circuit at $t = 0+$? [2pts]

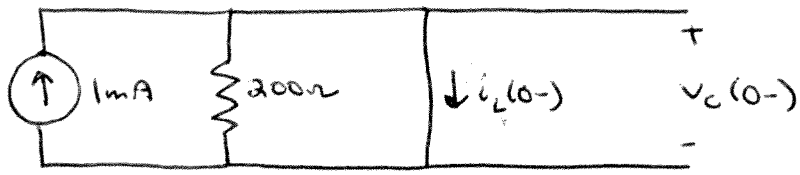
d) What is the differential equation for $v_C(t)$ for the time interval $t > 0$? [3pts]

e) Find $v_C(t)$ for $t > 0$. Note that it can be shown that $dv_C/dt(0+) = -50\text{kV/s}$. [4pts]

f) Is this RLC circuit's natural response underdamped, critically damped, or overdamped? [1pt]

a) b)

t<0



$i_L(0+) = i_L(0-) = 1mA$ [+1]

$v_c(0+) = v_c(0-) = 0V$ [+1]

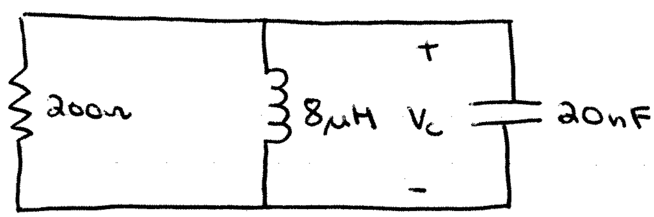
c) $U = \frac{1}{2}C\left(v_c(0+)\right)^2 + \frac{1}{2}L\left(i_L(0+)\right)^2$ [+1]

$= \frac{1}{2}\cdot 20nF\,(0V)^2 + \frac{1}{2}\cdot 8\mu H\,(1mA)^2$

$= 4\,pJ$ [+1]

d)

t>0



$$0 = \frac{V_c}{200\Omega} + \frac{V_c}{s\cdot 8\mu H} + \frac{V_c}{1/s\cdot 20nF}$$

multiply by s/20nF

$$0 = s^2\cdot V_c + \frac{1}{200\Omega\cdot 20nF}\cdot s\cdot V_c + \frac{1}{8\mu H\cdot 20nF}\cdot V_c$$

$$0 = \left(s^2 + 2.5\times10^5 s + 6.25\times10^{12}\right)V_c$$

$$0 = \left[\frac{d^2}{dt^2} + 2.5\times10^5\frac{d}{dt} + 6.25\times10^{12}\right]V_c$$ [+1] for each term

e)

$$s = \frac{-2.5\times10^5 \pm \sqrt{(2.5\times10^5)^2 - 4\cdot1\cdot6.25\times10^{12}}}{2\cdot1}$$

$$= -1.25\times10^5 \pm j\,2.497\times10^6 \quad (s^{-1}) \qquad [+1]$$

$$v_c(t) = e^{-1.25\times10^5 s^{-1} t}\left[A\cos(2.497\times10^6 s^{-1} t) + B\sin(2.497\times10^6 s^{-1} t)\right]$$

[+1]

initial conditions: $v_c(0+) = 0V$

$$\therefore \quad v_c(0) = 1\cdot[A\cdot1 + B\cdot0]$$

$$= A$$

$$\therefore \quad A = 0 \qquad [+1]$$

$$\frac{dv_c}{dt}(0+) = -\frac{50kV}{s}$$

$$\therefore \quad \frac{dv_c}{dt} = \frac{d}{dt}\left[e^{-1.25\times10^5 s^{-1} t}\,B\sin(2.497\times10^6 s^{-1} t)\right]$$

$$= -1.25\times10^5 s^{-1} t\cdot e^{-1.25\times10^5 s^{-1} t}\cdot B\sin(2.497\times10^6 s^{-1} t)$$

$$+ 2.497\times10^6 s^{-1}\cdot e^{-1.25\times10^5 s^{-1} t}\,B\cos(2.497\times10^6 s^{-1} t)$$

$$\frac{dv_c}{dt}(0) = 2.497\times10^6 s^{-1}\cdot1\cdot B\cdot1$$

$$B = \frac{-50kV/s}{2.497\times10^6 s^{-1}} = -20.0\,mV \qquad [+1]$$

$$\therefore v_c(t) = e^{-1.25\times10^5 s^{-1} t}\left[-20.0mV\sin(2.497\times10^6 s^{-1}\cdot t)\right]$$

f) underdamped [+1]

end