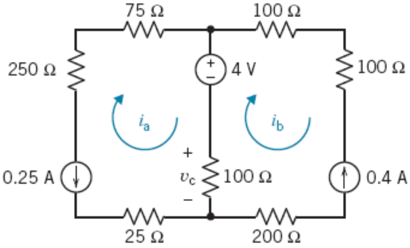
ECSE 200 - Electric Circuits 1
Problem set 5

ECE Dept., McGill University

Part 1: Mesh Current Analysis with Voltage and Current Sources

Problem P 4.6-2

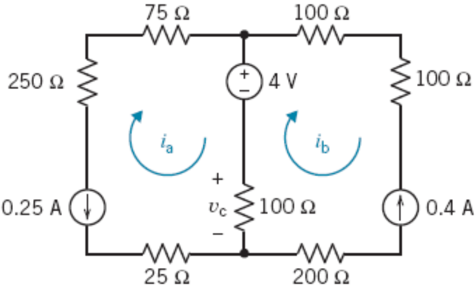
Find v_c for the circuit shown in the figure below.



Problem P 4.6-2

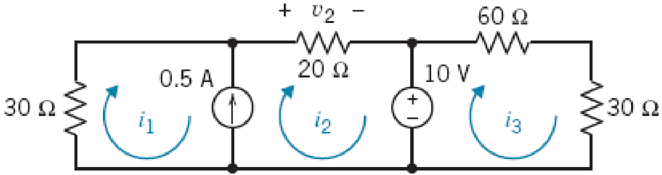
Solution

- Mesh a: $i_a = -0.25$ (A).
- Mesh b: $i_b = -0.4$ (A).
- Apply Ohm's Law: $v_c = 100(i_a - i_b) = 100 \times 0.15 = 15$ (V).



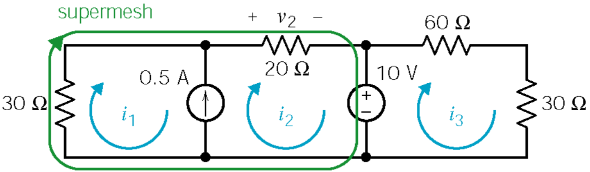
Problem P 4.6-3

Find v_2 for the circuit shown in the figure below.



Problem P 4.6-3

Solution



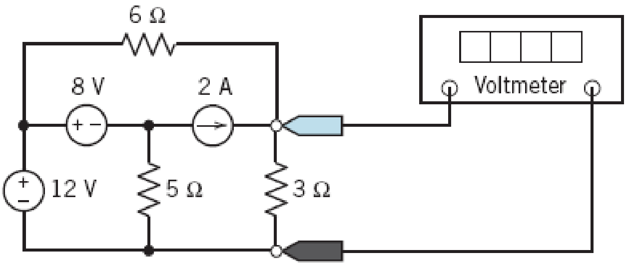
- From mesh 1 and 2 we have $i_2 = i_1 + 0.5$ (A).
- Apply KVL to the supermesh:

$$30i_1 + 20i_2 + 10 = 0$$
$$30i_1 + 20(i_1 + 0.5) + 10 = 0$$
$$i_1 = -0.4 \text{ (A)}$$

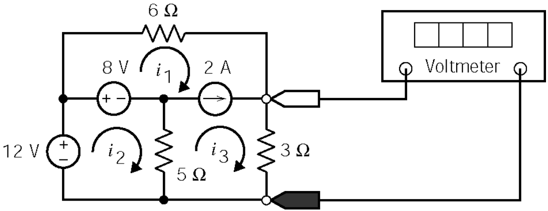
- $v_2 = 20i_2 = 20(i_1 + 0.5) = 20 \times 0.1 = 2$ (V)

Problem P 4.6-5

Determine the value of the voltage measured by the voltmeter shown in the figure below. *Answer:* 8 (V).



Problem P 4.6-5



(Long) solution

- Express the current source current in terms of the mesh currents:

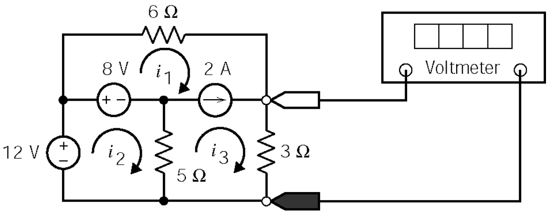
$$i_3 = i_1 + 2 \ (1)$$

- Supermesh (formed by mesh 1 and mesh 3):

$$6i_1 + 3i_3 - 5(i_2 - i_3) - 8 = 0 \Rightarrow 6i_1 - 5i_2 + 8i_3 = 8 \ (2)$$

- Mesh 2: $12 - 8 - 5(i_2 - i_3) = 0 \Rightarrow i_2 - i_3 = 0.2$ (3)
- Solving (1), (2) and (3) results in: $i_3 = \frac{8}{3}$ (A) $\Rightarrow$ the voltage measured by the voltmeter is: $3i_3 = 8$ (V)

Problem P 4.6-5



(Short) solution

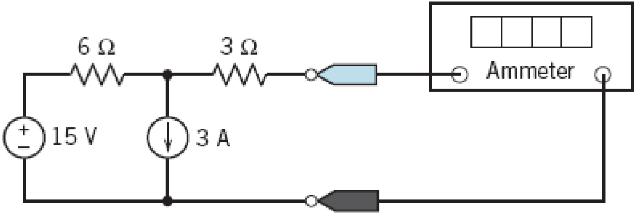
- Consider only the outer supermesh:

$$12 - 6i_1 - 3i_3 = 0$$
$$12 - 6(i_3 - 2) - 3i_3 = 0$$
$$i_3 = \frac{8}{3} \text{ (A)}$$

- The voltage measured by the voltmeter is: $3i_3 = 8$ (V).

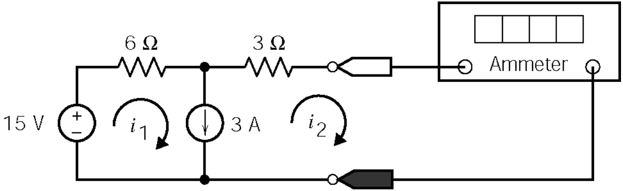
Problem P 4.6-12

Determine the value of the current measured by the ammeter shown in the figure below.



Problem P 4.6-12

Solution



- Express the current source current in terms of the mesh currents:

$$i_2 + 3 = i_1 \ (1)$$

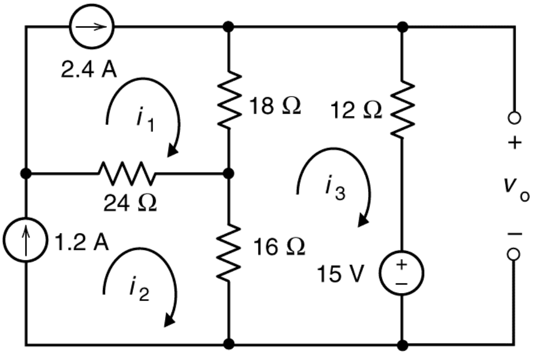
- Apply KVL for the supermesh (formed by mesh 1 and mesh 2):

$$15 - 6i_1 - 3i_2 = 0 \Rightarrow 5 = 2i_1 + i_2 \ (2)$$

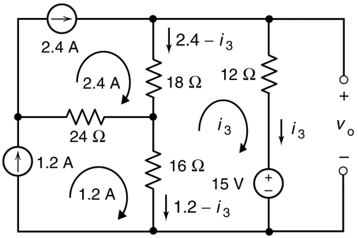
- Solving (1) and (2) results in $i_2 = -\frac{1}{3}$ (A), which is the current measured by the ammeter.

Problem P 4.6-13

Determine the values of the mesh currents i_1, i_2 and i_3 and the output voltage v_o in the circuit shown in the figure below.



Problem P 4.6-13



- Express the current source currents as the mesh currents:

$$i_1 = 2.4 \text{ (A) (1)}$$
$$i_2 = 1.2 \text{ (A) (2)}$$

- Apply KVL to mesh 3:

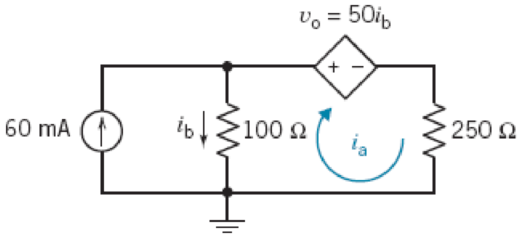
$$12i_3 + 15 - 16(i_2 - i_3) - 18(i_1 - i_3) = 0 \Rightarrow 18i_1 + 16i_2 - 46i_3 = 15 \text{ (3)}$$

- Solving (1),(2), and (3) results in $i_1 = 1.369$ (A), $i_2 = 0.169$ (A), and $i_3 = 1.0304$ (A).
- Therefore, $v_o = 12i_3 + 15 = 27.3648$ (V).

Part 2: Mesh Current Analysis with Dependent Sources

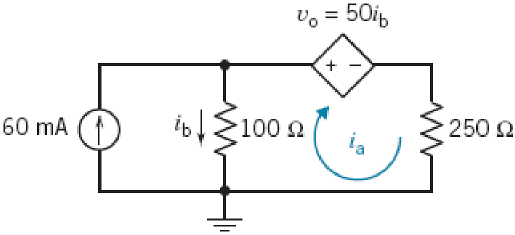
Problem P 4.7-3

Find v_0 for the circuit shown in the figure below.



Problem P 4.7-3

Solution



- Express the control current i_b as a function of the mesh current i_a:

$$i_b = 0.06 - i_a \ (1)$$

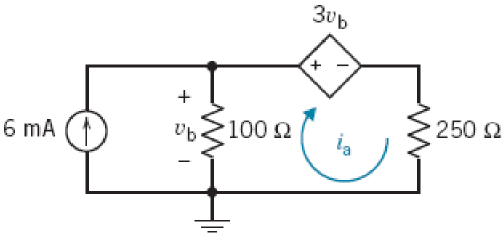
- Apply KVL to the right mesh:

$$50i_b + 250i_a - 100i_b = 0 \ (2)$$

- Solving (1) and (2) results in $i_a = 0.01$ (A) and $i_b = 0.05$ (A).
- Therefore, $v_o = 50i_b = 50 \times 0.05 = 2.5$ (V).

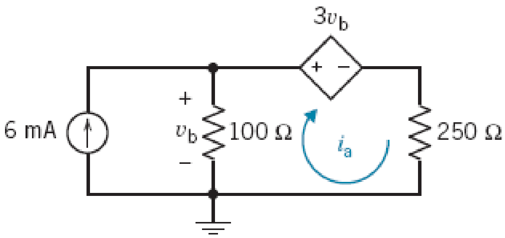
Problem P 4.7-4

Determine the mesh current i_a for the circuit shown in the figure below.



Problem P 4.7-4

Solution



- Express the control voltage v_b as a function of the mesh current i_a:

$$\frac{v_b}{100} + i_a = 0.006 \Rightarrow v_b = 0.6 - 100 i_a \ (1)$$

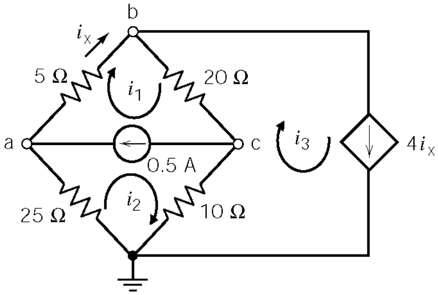
- Apply KVL to the right mesh:

$$v_b - 3v_b - 250 i_a = 0 \Rightarrow -2v_b = 250 i_a \ (2)$$

- Solving (1) and (2) results in $i_a = -0.024$ (A).

Problem P 4.7-11

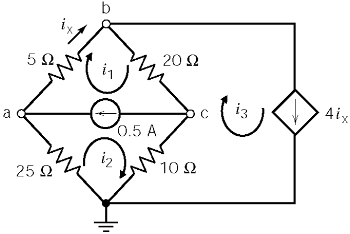
Determine the values of the mesh currents of the circuit shown in the figure below.



Problem P 4.7-11

Solution

- Determine the mesh current variables:
  - Mesh 1: $i_1 = i_x$
  - Mesh 2 (apply KCL to node a): $i_2 = i_x - 0.5$
  - Mesh 3: $i_3 = 4i_x$
- Apply KVL to the supermesh formed by mesh 1 and mesh 2:



$$0 = v_{ab} + v_{bc} + v_{cd} + v_{da}$$

$$0 = 5i_1 + 20(i_1 - i_3) + 10(i_2 - i_3) + 25i_2$$

$$0 = 5i_x + 20(i_x - 4i_x) + 10(i_x - 0.5 - 4i_x) + 25(i_x - 0.5)$$

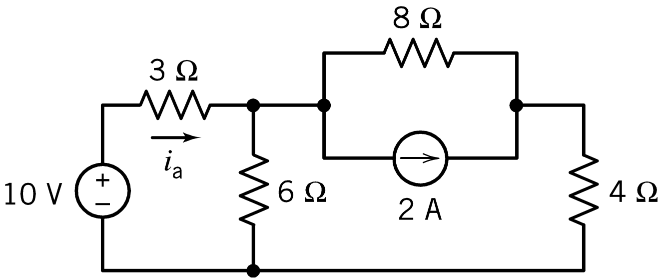
$$i_x = -0.29167 \text{ (A)}$$

Therefore, $i_1 = -0.29167$ (A), $i_2 = -0.79167$ (A), and $i_3 = -1.16668$ (A).

Part 3: Source Transformations

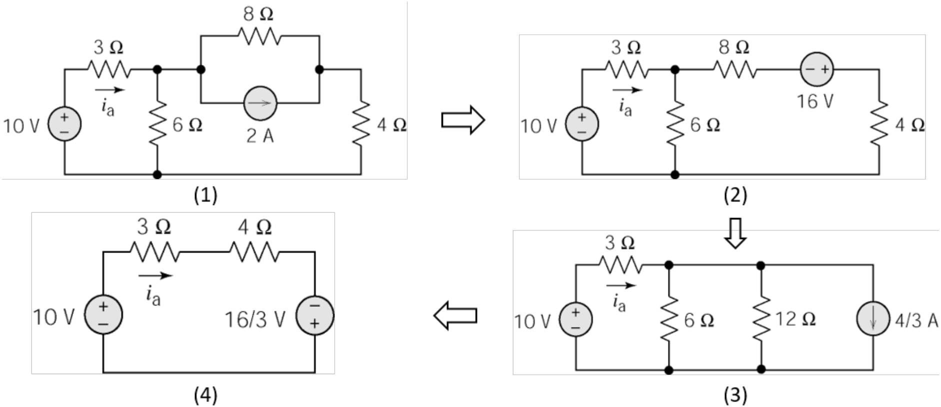
Problem P 5.2-2

Consider the circuit shown in the figure below. Find i_a by simplifying the circuit (using source transformations) to a single-loop circuit so that you need to write only one KVL equation to find i_a.



Problem P 5.2-2

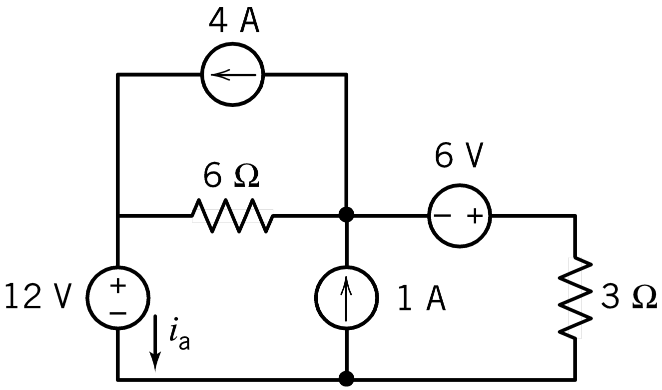
Solution



Apply KVL to the circuit shown in (4): $7i_a - \frac{16}{3} - 10 = 0 \Rightarrow i_a = 2.19$ (A).

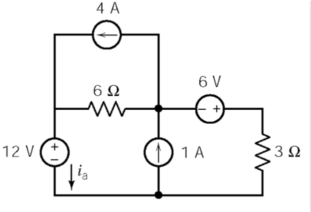
Problem P 5.2-5

Use source transformations to find the current i_a in the following circuit.

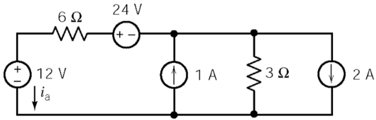


Problem P 5.2-5

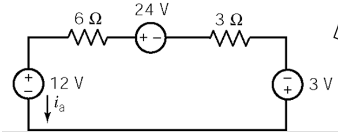
Solution



(1)



(2)



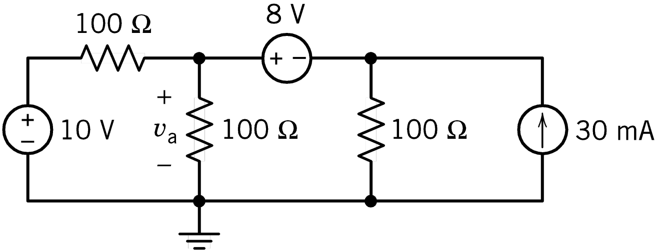
(3)

Apply KVL to the final transformed circuit in (3):

$12 + 3 + 3i_a - 24 + 6i_a = 0 \Rightarrow i_a = 1$ (A).

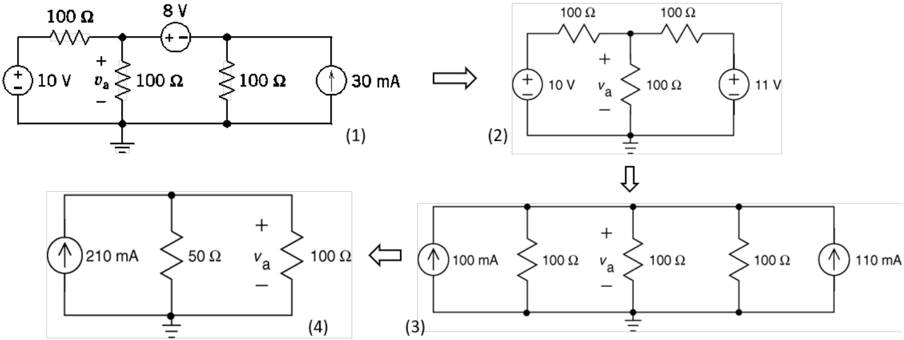
Problem P 5.2-6

Use source transformations to find v_a in the following circuit.



Problem P 5.2-6

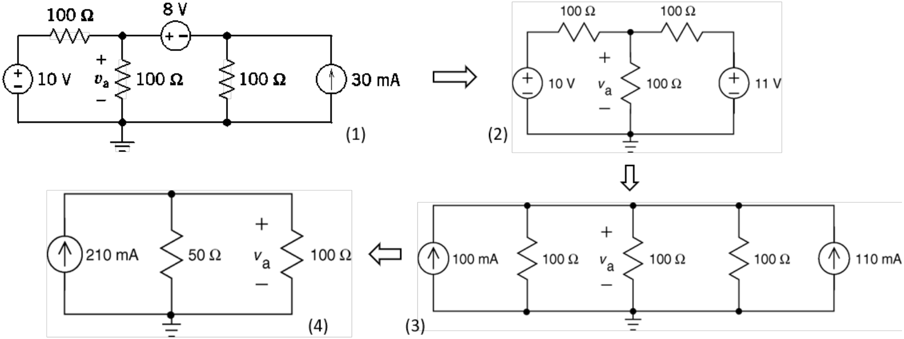
Solution



- (1)⇒(2):A source transformation on the right side of the circuit, followed by replacing serial voltage sources with an equivalent voltage source.

Problem P 5.2-6

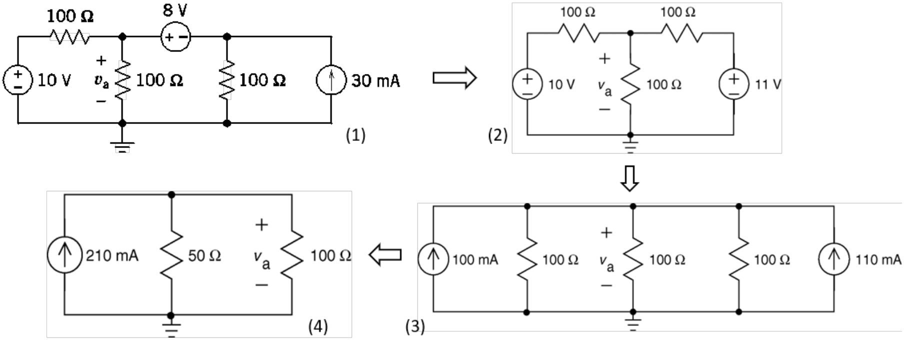
Solution (cnt.)



- (2)⇒(3):Apply source transformations to the voltage sources on both side of the circuit in (2).

Problem P 5.2-6

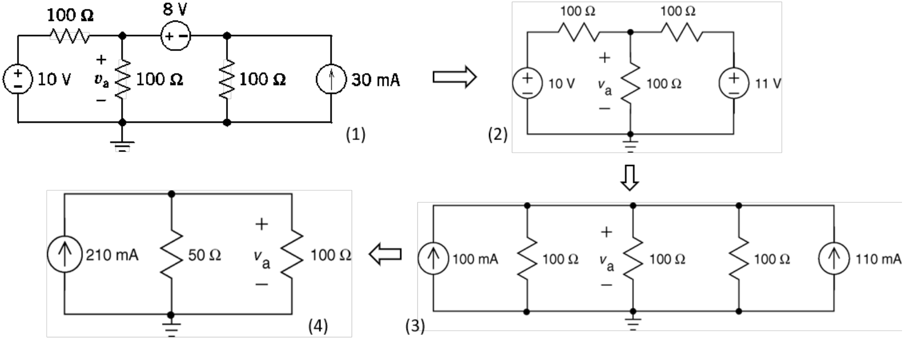
Solution (cnt.)



- (3)⇒(4):Replacing parallel resistors and parallel current sources with an equivalent resistor and an equivalent current source, respectively.

Problem P 5.2-6

Solution (cnt.)



- From the final transformed circuit in (4): $v_a = 0.21 \times \frac{50 \times 100}{50+100} = 7$ (V).

Thank you !