Fundamentals

P1.3-6 The current in a circuit element is plotted in Figure P1.3-6. Determine the total charge that flows through the circuit element between 300 and 1200 μs.

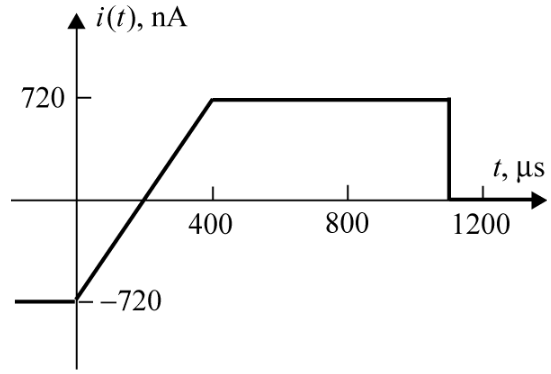
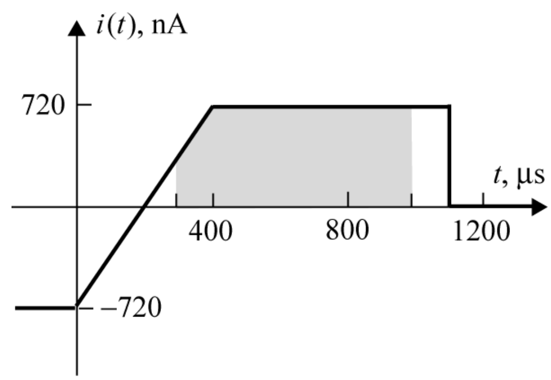


Figure P1.3-6

Solution:

$$q(t) = \int_{300\ \mu s}^{1000\ \mu s} i(\tau)\, d\tau = \text{"area under the curve between 300 } \mu\text{s and 1000 } \mu\text{s"}$$



$$q(t) = \left(\frac{360+720}{2}\times 10^{-9}\right)\left(100\times 10^{-6}\right) + \left(720\times 10^{-9}\right)\left(600\times 10^{-6}\right) = (54+432)\times 10^{-12} = 486 \text{ pC}$$

P 2.5-1 A current source and a voltage source are connected in parallel with a resistor as shown in Figure P 2.5-1. All of the elements connected in parallel have the same voltage, v_s in this circuit. Suppose that v_s = 15 V, i_s = 3 A, and R = 5 Ω. (a) Calculate the current i in the resistor and the power absorbed by the resistor. (b) Change the current source current to i_s = 5 A and recalculate the current, i, in the resistor and the power absorbed by the resistor.

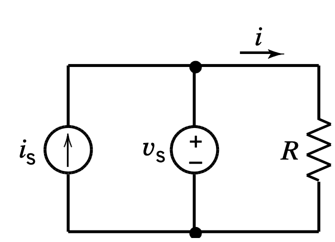


Figure P 2.5-1

Answer: i = 3 A and the resistor absorbs 45 W both when i_s = 3 A and when i_s = 5 A.

Solution:

(a) $i = \dfrac{v_s}{R} = \dfrac{15}{5} = \underline{3\text{ A}}$ and $P = R\,i^2 = 5(3)^2 = \underline{45\text{ W}}$

(b) i and P do not depend on i_s.

The values of i and P are 3 A and 45 W, both when $i_s = 3$ A and when $i_s = 5$ A.

P 2.6-5 The voltmeter in Figure P 2.6-5*a* measures the voltage across the current source. Figure P 2.6-5*b* shows the circuit after removing the voltmeter and labeling the voltage measured by the voltmeter as v_m. Also, the other element voltages and currents are labeled in Figure P 2.6-5*b*.

Given that

$$12 = v_R + v_m \text{ and } -i_R = i_s = 2\text{ A}$$

and

$$v_R = 25 i_R$$

(a) Determine the value of the voltage measured by the meter.

(b) Determine the power supplied by each element.

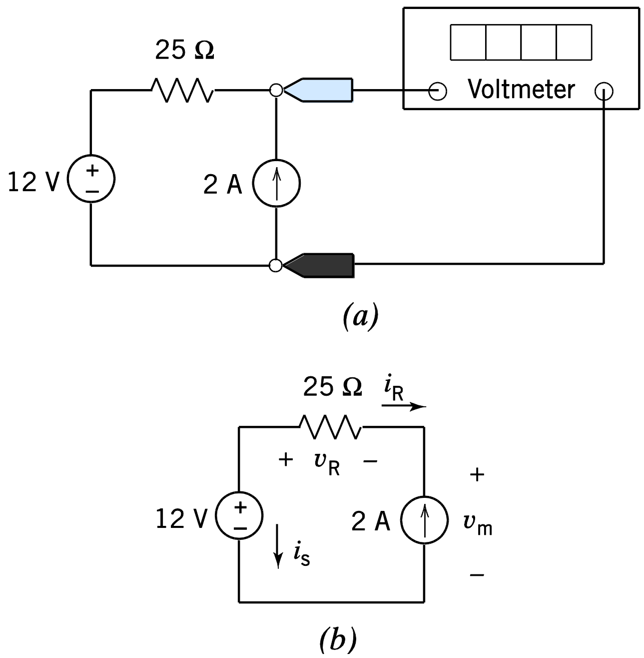


Figure P 2.6-5

Solution:

a.)

$$v_R = 25\,i_R = 25(-2) = -50\text{ V}$$

$$v_m = 12 - v_R = 12 - (-50) = 62\text{ V}$$

b.)

Element	Power supplied

voltage source	$-12(i_s) = -12(2) = -24$ W
current source	$62(2) = 124$ W
resistor	$-v_R \times i_R = -(-50)(-2) = -100$ W
total	0

P2.7-8 The circuit shown in Figure P2.7-8 contains a dependent source. Determine the value of the gain *k* of that dependent source.

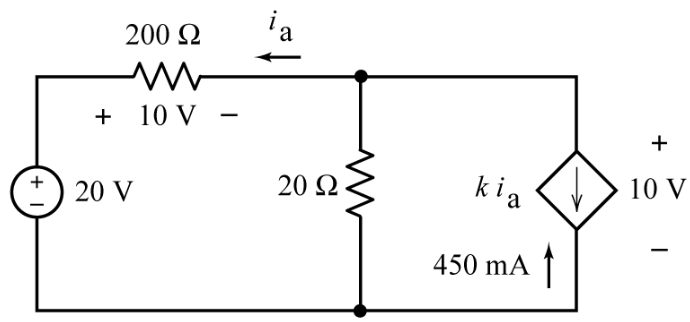


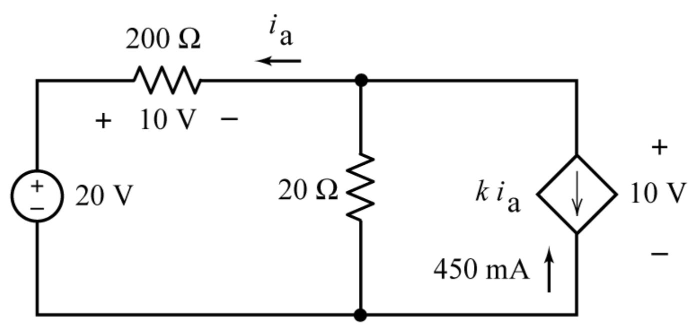
Figure P2.7-8

Solution:

$$i_a = -\frac{10}{200} = -0.05 \text{ A} = -50 \text{ mA}$$

$$k\,i_a = -450 \text{ mA}$$

$$k = \frac{k\,i_a}{i_a} = \frac{-450}{-50} = 9 \ \frac{\text{A}}{\text{A}}$$



Resistive Circuits

P 3.2-6 Determine the power supplied by each voltage source in the circuit of Figure P 3.2-6.

Answer: The 2-V voltage source supplies 2 mW and the 3-V voltage source supplies –6 mW.

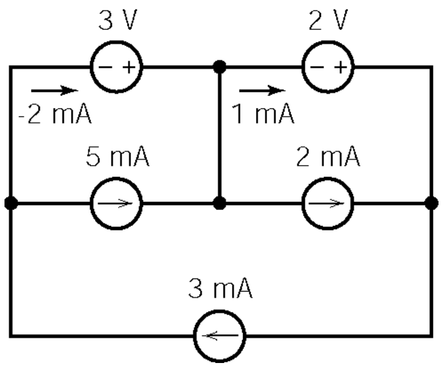
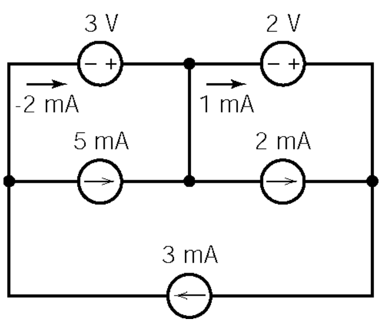


Figure P 3.2-6

Solution:



$$P_{2\,\mathrm{V}} = +\left[2\times\left(1\times10^{-3}\right)\right] = 2\times10^{-3} = 2\text{ mW}$$

$$P_{3\,\mathrm{V}} = +\left[3\times\left(-2\times10^{-3}\right)\right] = -6\times10^{-3} = -6\text{ mW}$$

(checked using LNAP 8/16/02)

P 3.2-9 Determine the values of the resistances R_1 and R_2 in Figure P 3.2-9.

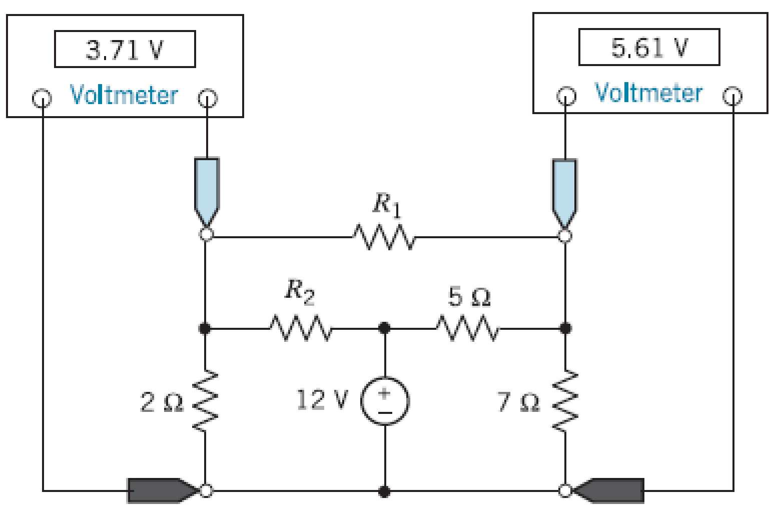
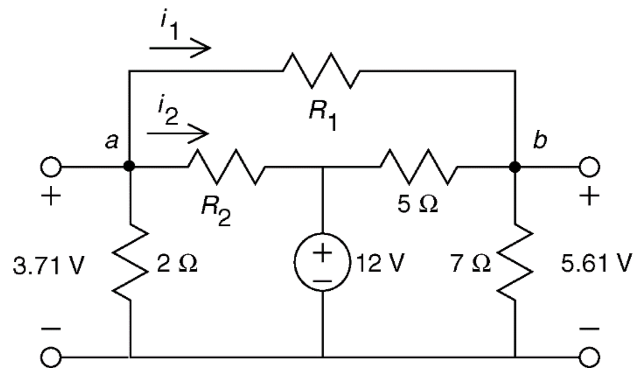


Figure P 3.2-9

Solution:



$$\frac{5.61}{7}=\frac{3.71-5.61}{R_1}+\frac{12-5.61}{5} \quad \Rightarrow \quad 0.801=\frac{-1.9}{R_1}+1.278$$

$$\Rightarrow \quad R_1=\frac{1.9}{1.278-0.801}=3.983\approx 4\,\Omega$$

KCL at node *b*:

$$\frac{3.71}{2}+\frac{3.71-5.61}{4}+\frac{3.71-12}{R_2}=0 \quad \Rightarrow \quad 1.855+(-0.475)+\frac{-8.29}{R_2}=0$$

$$\Rightarrow \quad R_2=\frac{8.29}{1.855-0.475}=6.007\approx 6\,\Omega$$

KCL at node *a*:

(checked using LNAP 8/16/02)

P 3.2-15 Determine the value of the voltage that is measured by the meter in Figure P 3.2-15.

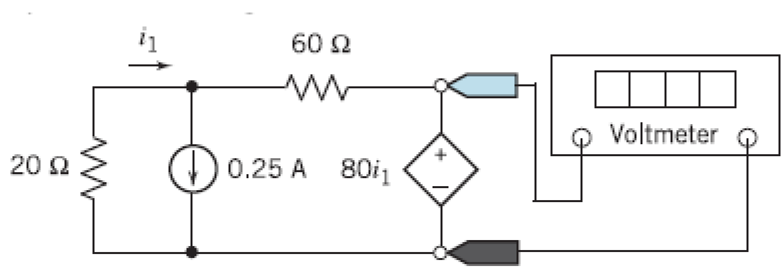
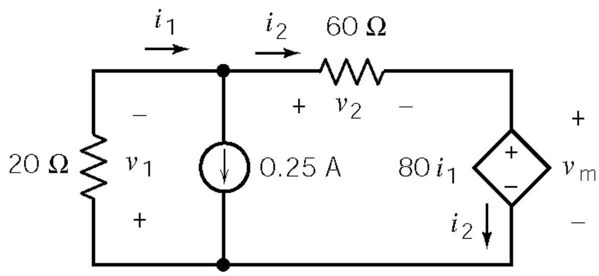


Figure P 3.2-15

Solution:

We can label the circuit as shown.



The subscripts suggest a numbering of the circuit elements. Apply KCL at the top node of the current source to get

$$i_1 = i_2 + 0.25$$

Apply Ohm's law to the resistors to get

$$v_1 = 20\,i_1 \quad \text{and} \quad v_2 = 60\,i_2 = 60\left(i_1 - 0.25\right) = 60\,i_1 - 15$$

Apply KVL to the outside to get

$$v_2 + 80\,i_1 + v_1 = 0 \quad \Rightarrow \quad \left(60\,i_1 - 15\right) + 80\,i_1 + 20\,i_1 = 0 \quad \Rightarrow \quad i_1 = \frac{15}{160} = 0.09375 \text{ A}$$

Finally, $$v_\text{m} = 80\,i_1 = 80\left(0.09375\right) = 7.5 \text{ V}$$

(Checked: LNAPDC 9/1/04)

P 3.3-8 Determine the power supplied by the dependent source in the circuit shown in Figure P 3.3-8.

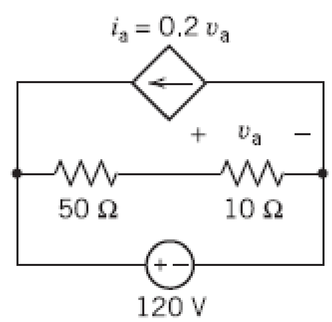
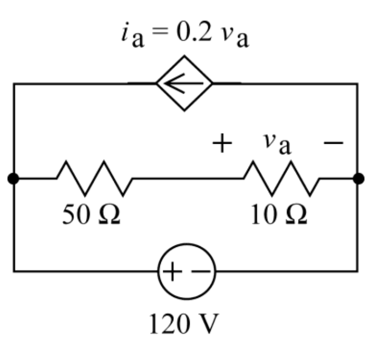


Figure P 3.3-8

Solution:

Use voltage division to get



$$v_\text{a} = \left(\frac{10}{10+50}\right)\left(120\right) = 20 \text{ V}$$

Then $$i_\text{a} = 0.2\left(20\right) = 4 \text{ A}$$

The power supplied by the dependent source is given by

$$p = (120)i_a = 480 \text{ W}$$

(Checked: LNAP 6/21/04)

P 3.4-8 Determine the value of the voltage v in Figure P 3.4-8.

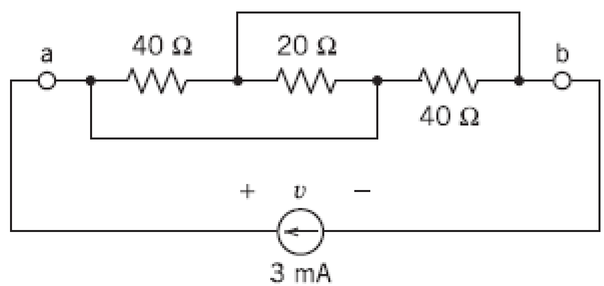
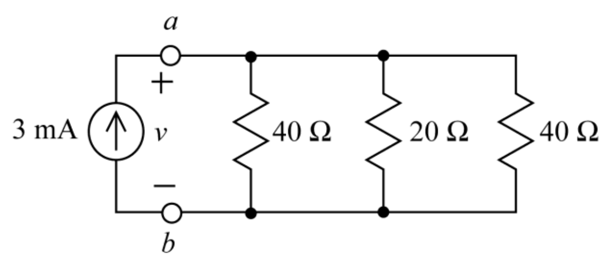


Figure P 3.4-8

Solution:

Each of the resistors is connected between nodes a and b. The resistors are connected in parallel and the circuit can be redrawn like this:



Then $$40 \parallel 20 \parallel 40 = 10\ \Omega$$

so $$v = 10(0.003) = 0.03 = 30 \text{ mV}$$

(checked: LNAP 6/21/04)

P 3.6-13 Find the R_{eq} at terminals a–b in Figure P 3.6-13. Also determine i, i_1, and i_2.
Answer: $R_{eq} = 8\ \Omega$, $i = 5$ A, $i_1 = 5/3$ A, $i_2 = 5/2$ A

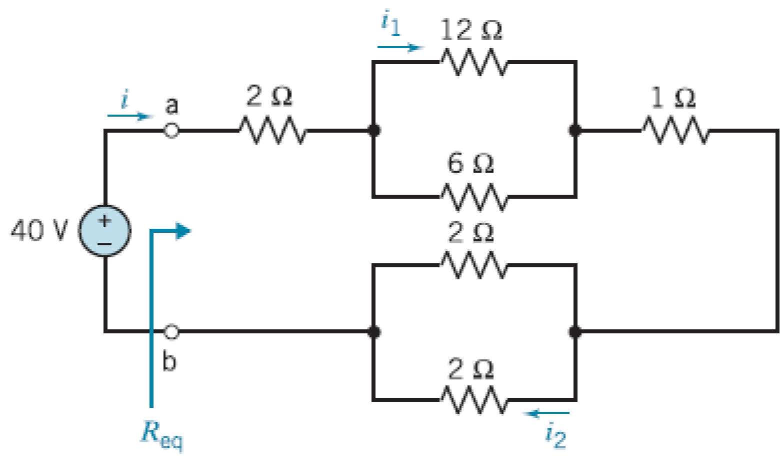


Figure P 3.6-13

Solution:

$$R_{eq} = 2+1+\left(6\|12\right)+\left(2\|2\right)=3+4+1=\underline{8\ \Omega} \quad \text{so} \quad i=\frac{40}{R_{eq}}=\frac{40}{8}=\underline{5\ \text{A}}$$

Using current division

$$i_1 = i\left(\frac{6}{6+12}\right) = (5)\left(\tfrac{1}{3}\right) = \underline{\tfrac{5}{3}\ \text{A}} \quad \text{and} \quad i_2 = i\left(\frac{2}{2+2}\right)=(5)\left(\tfrac{1}{2}\right)=\underline{\tfrac{5}{2}\ \text{A}}$$

P 3.6-20 Determine the values of i, v, and R_{eq} by the circuit model shown in Figure P 3.6-20, given that v_{ab} = 18 V.

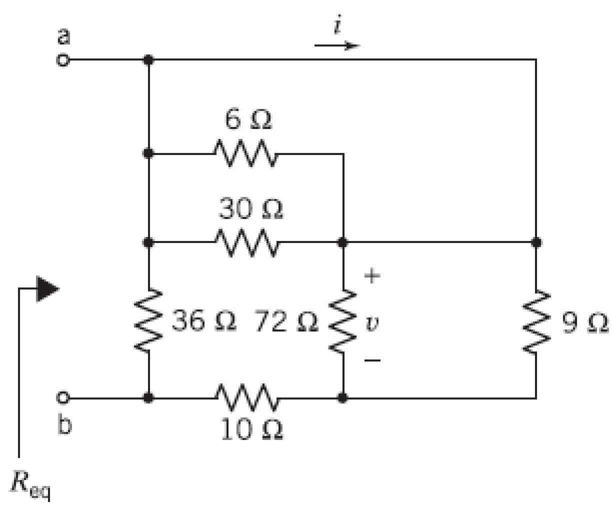
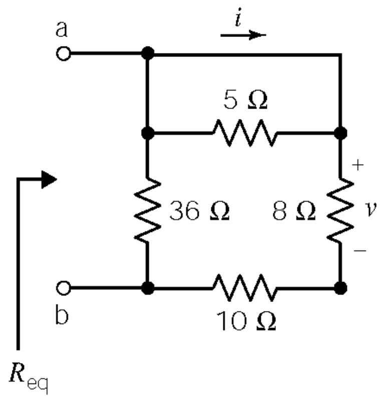


Figure P 3.6-20

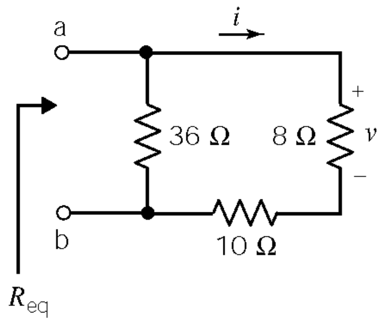
Solution:

Replace parallel resistors by equivalent resistors:

$$6 \parallel 30 = 5\ \Omega \quad \text{and} \quad 72 \parallel 9 = 8\ \Omega$$



A short circuit in parallel with a resistor is equivalent to a short circuit.



$$R_{\text{eq}} = 36 \parallel (8+10) = 12\ \Omega$$

Using voltage division when v_{ab} = 18 V:

$$v = \frac{8}{8+10} v_{ab} = \frac{4}{9}(18) = 8\text{ V}$$

$$i = \frac{v}{8} = 1\text{ A}$$

(checked: LNAP 6/21/04)

Analysis Methods

P 4.2-7 The node voltages in the circuit shown in Figure P 4.2-7 are $v_a = 7$ V and $v_b = 10$ V. Determine values of the current source current, i_s, and the resistance, R.

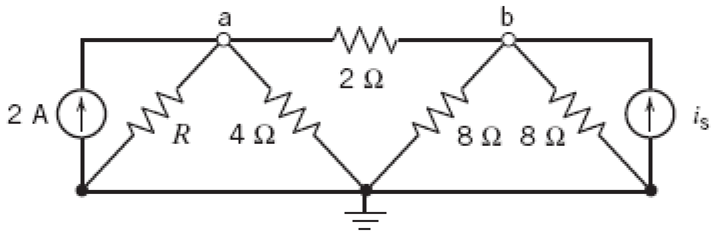


Figure P 4.2-7

Solution

Apply KCL at node a to get

$$2 = \frac{v_a}{R} + \frac{v_a}{4} + \frac{v_a - v_b}{2} = \frac{7}{R} + \frac{7}{4} + \frac{7-10}{2} = \frac{7}{R} + \frac{1}{4} \quad \Rightarrow \quad R = 4\ \Omega$$

Apply KCL at node b to get

$$i_s + \frac{v_a - v_b}{2} = \frac{v_b}{8} + \frac{v_b}{8} = i_s + \frac{7-10}{2} = \frac{10}{8} + \frac{10}{8} \quad \Rightarrow \quad i_s = 4\ \text{A}$$

(checked: LNAP 6/21/04)

P 4.4-4 The circled numbers in Figure P 4.4-4 are node numbers. The node voltages of this circuit are $v_1 = 10$ V, $v_2 = 14$ V, and $v_3 = 12$ V.

(a) Determine the value of the current i_b.

(b) Determine the value of r, the gain of the CCVS.

Answers: (a) –2 A (b) 4 V/A

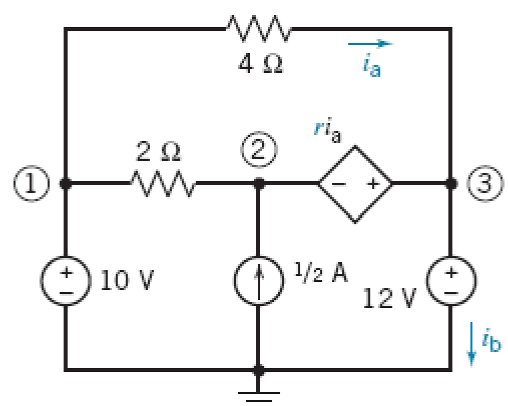


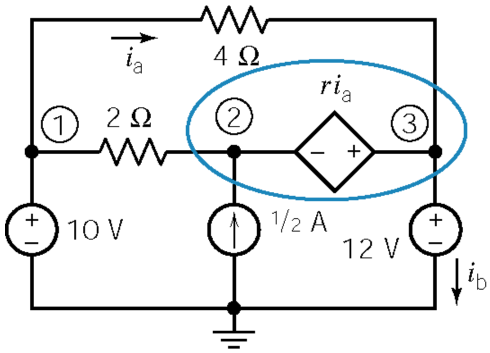
Figure P 4.4-4

Solution:

Apply KCL to the supernode of the CCVS to get

$$\frac{12-10}{4} + \frac{14-10}{2} - \frac{1}{2} + i_b = 0 \quad \Rightarrow \quad i_b = -2\ \text{A}$$

Next



$$\left.\begin{array}{l} i_a = \dfrac{10-12}{4} = -\dfrac{1}{2} \\ r\,i_a = 12-14 \end{array}\right\} \Rightarrow \quad r = \dfrac{-2}{-\dfrac{1}{2}} = 4\ \dfrac{\text{V}}{\text{A}}$$

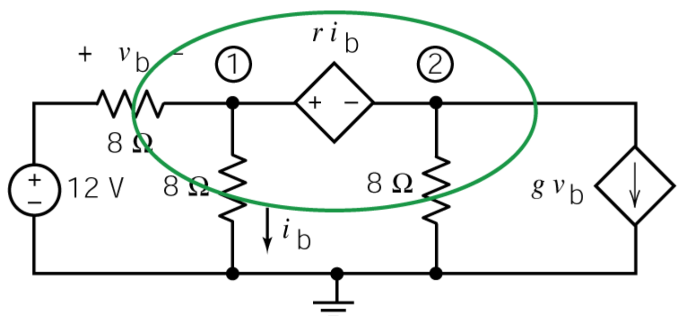
(checked using LNAP 8/14/02)

P4.4-7 The encircled numbers in the circuit shown Figure 4.4-27 are node numbers. The corresponding node voltages are:

v_1 = 9.74 V and v_2 = 6.09 V

Determine the values of the gains of the dependent sources, r and g.

Solution:



Using Ohm's law, $i_\text{b} = \dfrac{v_1}{8} = \dfrac{9.74}{8} = 1.2175\ \text{A}$. Using KVL, the voltage across the CCVS is

$$r\,i_\text{b} = v_1 - v_2 = 9.74 - 6.09 = 3.65\ \text{V}$$

Then
$$r = \frac{r\,i_\text{b}}{i_\text{b}} = \frac{3.65}{1.2175} = 2.9979\ \text{V/A}$$

Using KVL, $v_b = 12 - v_1 = 12 - 9.74 = 2.26$ V . Apply KCL to the supernode corresponding to the CCVS to get

$$\frac{12 - v_1}{8} = \frac{v_1}{8} + \frac{v_2}{8} + g\,v_b \quad \Rightarrow \quad \frac{12 - 9.74}{8} = \frac{9.74}{8} + \frac{6.09}{8} + g\,v_b \quad \Rightarrow \quad g\,v_b = -1.6963 \text{ A}$$

Then
$$g = \frac{g\,v_b}{v_b} = \frac{-1.6963}{2.26} = -0.7506 \text{ A/V}$$

P 4.7-7 The currents i_1, i_2 and i_3 are the mesh currents of the circuit shown in Figure P 4.7-7. Determine the values of i_1, i_2, and i_3.

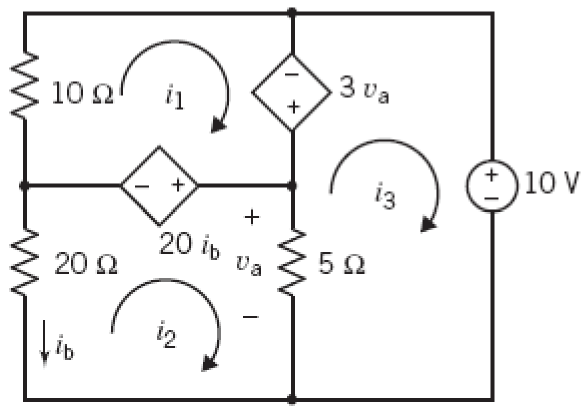


Figure P 4.7-7

Solution:

Express v_a and i_b, the controlling voltage and current of the dependent sources, in terms of the mesh currents

$$v_a = 5(i_2 - i_3) \;\text{ and }\; i_b = -i_2$$

Next express 20 i_b and 3 v_a, the controlled voltages of the dependent sources, in terms of the mesh currents

$$20\;i_b = -20\;i_2 \;\text{ and }\; 3\;v_a = 15(i_2 - i_3)$$

Apply KVL to the meshes

$$-15(i_2 - i_3) + (-20\;i_2) + 10\;i_1 = 0$$
$$-(-20\;i_2) + 5(i_2 - i_3) + 20\;i_2 = 0$$
$$10 - 5(i_2 - i_3) + 15\;(i_2 - i_3) = 0$$

These equations can be written in matrix form

$$\begin{bmatrix} 10 & -35 & 15 \\ 0 & 45 & -5 \\ 0 & 10 & -10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix}$$

Solving, e.g. using MATLAB, gives

$$i_1 = -1.25 \text{ A},\ i_2 = +0.125 \text{ A, and } i_3 = +1.125 \text{ A}$$

(checked: MATLAB & LNAP 5/19/04)

P 4.7-12 The currents i_1, i_2, and i_3 are the mesh currents corresponding to meshes 1, 2, and 3 in Figure P 4.7-12. Determine the values of these mesh currents.

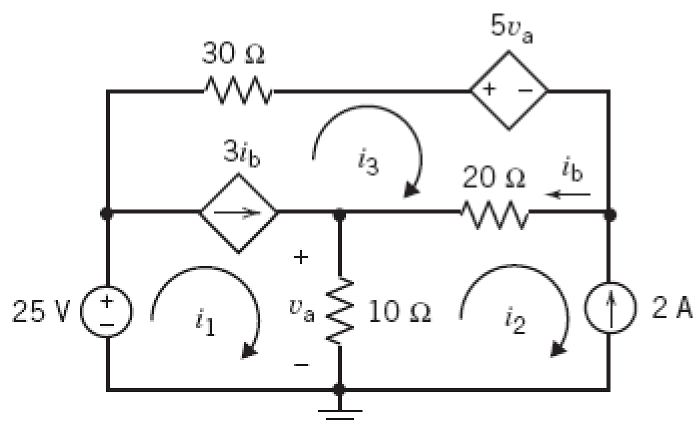
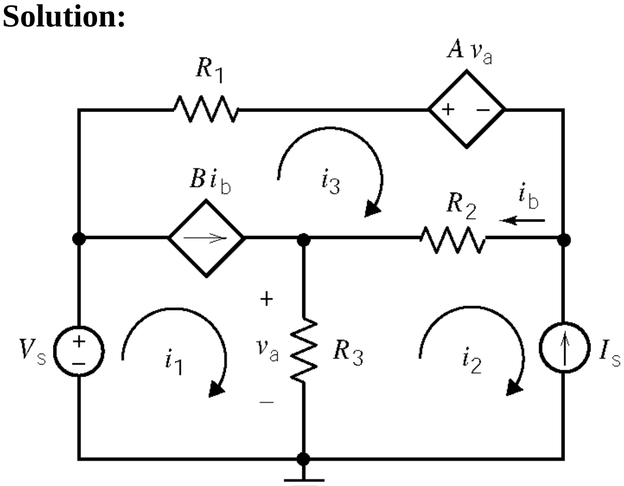


Figure P 4.7-12

Solution:



Express the controlling voltage and current of the dependent sources in terms of the mesh currents:

$$v_a = R_3\left(i_1 - i_2\right) \quad \text{and} \quad i_b = i_3 - i_2$$

Express the current source currents in terms of the mesh currents:

$$i_2 = -I_s \quad \text{and} \quad i_1 - i_3 = B\,i_b = B\left(i_3 - i_2\right)$$

Consequently

$$i_1 - \left(B+1\right)i_3 = B\,I_s$$

Apply KVL to the supermesh corresponding to the dependent current source

$$R_1\, i_3 + A R_3 \left(i_1 - i_2\right) + R_2 \left(i_3 - i_2\right) + R_3 \left(i_1 - i_2\right) - V_s = 0$$

or

$$\left(A+1\right) R_3\, i_1 - \left(R_2 + \left(A+1\right) R_3\right) i_2 + \left(R_1 + R_2\right) i_3 = V_s$$

Organizing these equations into matrix form:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -\left(B+1\right) \\ \left(A+1\right) R_3 & -\left(R_2 + \left(A+1\right) R_3\right) & R_1 + R_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -I_s \\ B\, I_s \\ V_s \end{bmatrix}$$

With the given values:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -4 \\ 60 & -80 & 50 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ 25 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -0.8276 \\ -2 \\ -1.7069 \end{bmatrix} \quad \text{A}$$

(Checked using LNAP 9/29/04)

Circuit Theorems

P 5.2-5 Use source transformations to find the current i_a in the circuit shown in Figure P 5.2-5.

Answer: $i_a = 1$ A

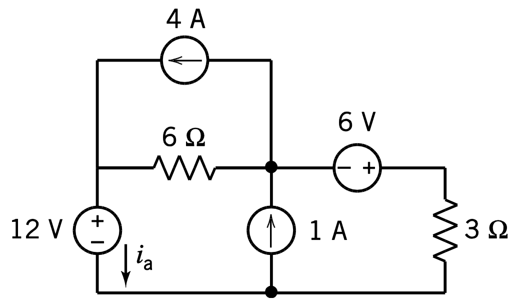
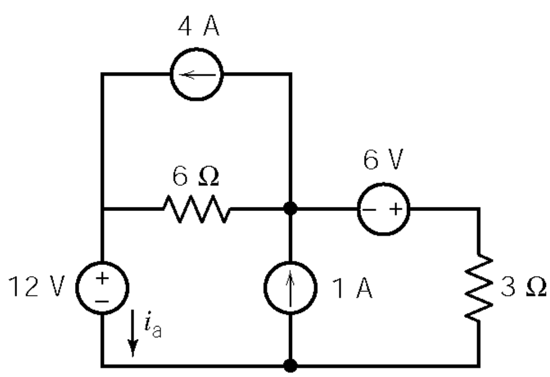
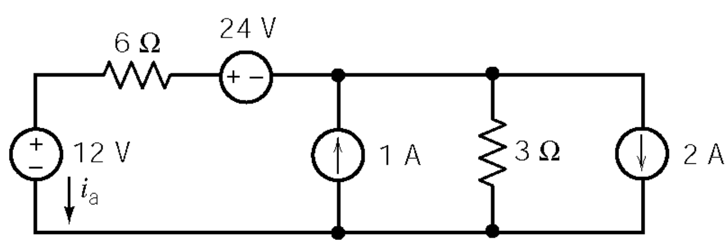
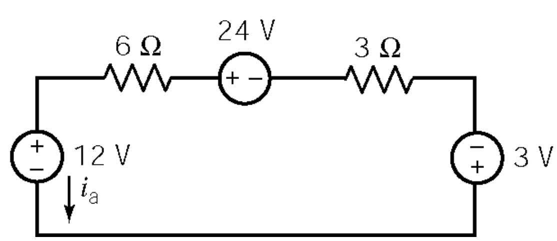


Figure P 5.2-5.

Solution:







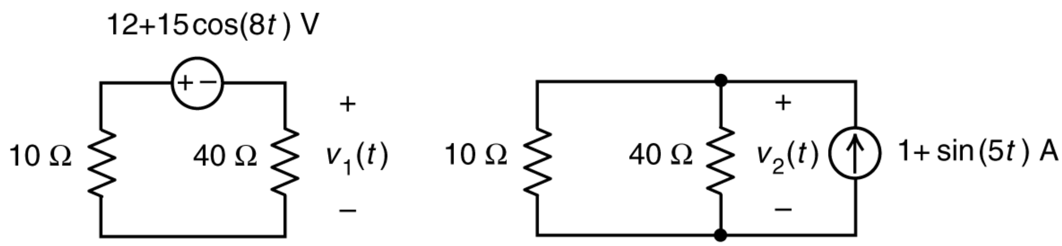
$$-12 - 6\,i_a + 24 - 3\,i_a - 3 = 0 \Rightarrow i_a = 1\text{ A}$$

(checked using LNAP 8/15/02)

P5.3-7 Determine $v(t)$, the voltage across the 40 Ω resistor in the circuit in Figure P5.3-7.

Solution:

We'll use superposition. Let $v_1(t)$ the be the part of $v(t)$ due to the voltage source acting alone. Similarly, let $v_2(t)$ the be the part of $v(t)$ due to the voltage source acting alone. We can use these circuits to calculate $v_1(t)$ and $v_2(t)$.



Using voltage division we calculate

$$v_1(t) = -\frac{40}{10+40}\left(12+15\cos(8t)\right) = -9.6-12\cos(8t)$$

Using equivalent resistance we first determine 10||40 = 8 Ω and then calculate

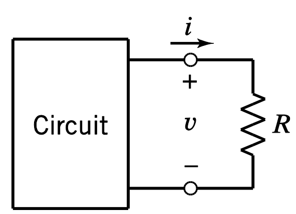
$$v_2(t) = 8\left(1+\sin(5t)\right) = 8+8\sin(5t)$$

Using superposition $$v(t) = v_1(t)+v_2(t) = -1.6+8\sin(5t)-12\cos(8t)\ \text{V}$$

P 5.4-9 A resistor, *R*, was connected to a circuit box as shown in Figure P 5.4-9. The current, *i*, was measured. The resistance was changed, and the current was measured again. The results are shown in the table.

(a) Specify the value of *R* required to cause *i* = 2 mA.

(b) Given that *R* > 0, determine the maximum possible value of the current *i*.



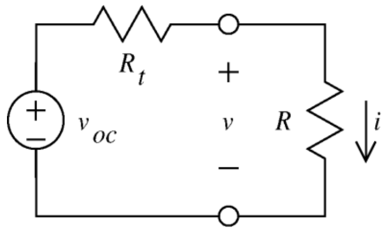
R	*i*
2 kΩ	4 mA
4 kΩ	3 mA

Figure P 5.4-9

Hint: Use the data in the table to represent the circuit by a Thévenin equivalent.

Solution:

From the given data:



$$0.004 = \frac{v_{oc}}{R_t + 2000}, \quad 0.003 = \frac{v_{oc}}{R_t + 4000} \;\Rightarrow\; \begin{cases} v_{oc} = 24 \text{ V} \\ R_t = 4000\ \Omega \end{cases}$$

$$i = \frac{v_{oc}}{R_t + R}$$

(a) When i = 0.002 A:

$$0.002 = \frac{24}{4000 + R} \quad \Rightarrow \quad R = 8000\ \Omega$$

(b) Maximum i occurs when R = 0:

$$\frac{24}{4000} = 0.006 = 6 \text{ mA} \quad \Rightarrow \quad i \le 6 \text{ mA}$$

P 5.4-13 The circuit shown in Figure P 5.4-13 consists of two parts, the source (to the left of the terminals) and the load. The load consists of a single adjustable resistor having resistance $0 \le R_L \le 20\ \Omega$. The resistance R is fixed, but unspecified. When $R_L = 4\ \Omega$, the load current is measured to be i_o = 0.375 A. When $R_L = 8\ \Omega$, the value of the load current is
i_o = 0.300 A.

(a) Determine the value of the load current when $R_L = 10\ \Omega$.

(b) Determine the value of R.

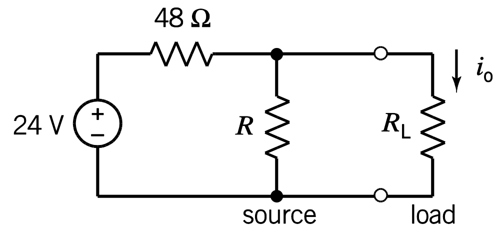
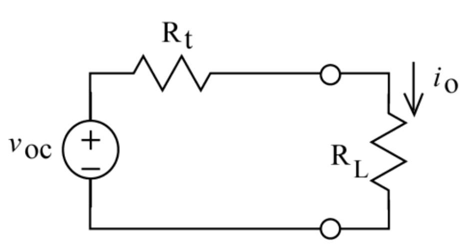


Figure P 5.4-13

Solution:
Replace the source by it's Thevenin equivalent circuit to get



$$i_o = \frac{v_{oc}}{R_t + R_L}$$

Using the given formation

$$\left.\begin{aligned} 0.375 &= \frac{v_{oc}}{R_t + 4} \\ 0.300 &= \frac{v_{oc}}{R_t + 8} \end{aligned}\right\} \quad \Rightarrow \quad 0.375\left(R_t + 4\right) = 0.300\left(R_t + 8\right)$$

So

$$R_t = \frac{(0.300)8 - (0.375)4}{0.075} = 12\ \Omega \text{ and } v_{oc} = 0.3(12+8) = 6\text{ V}$$

(a) When $R_L = 10\ \Omega$, $i_o = \dfrac{6}{12+10} = 0.27\overline{27}$ A.

(b) $12\ \Omega = R_t = 48\ 11R \quad \Rightarrow \quad R = 16\ \Omega$.

(checked: LNAP 5/24/04)

P 5.4-15 Consider the circuit shown in Figure P 5.4-15. Replace the part of the circuit to the left of terminals a–b by its Thévenin equivalent circuit. Determine the value of the current i_o.

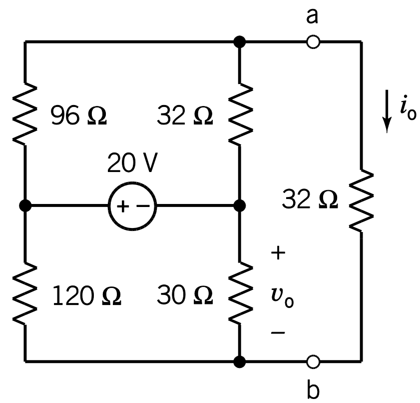
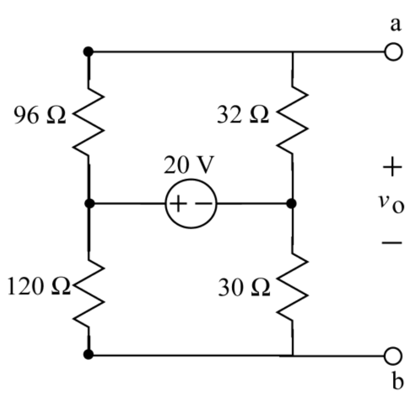


Figure P 5.4-15

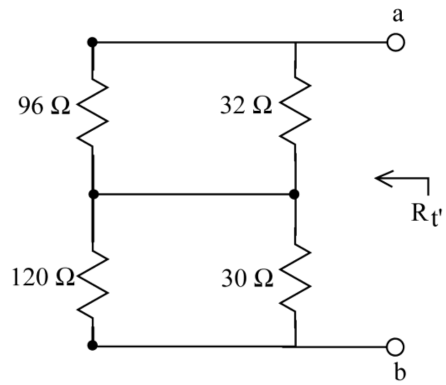
Solution:

Find the Thevenin equivalent circuit for the part of the circuit to the left of the terminals a-b.



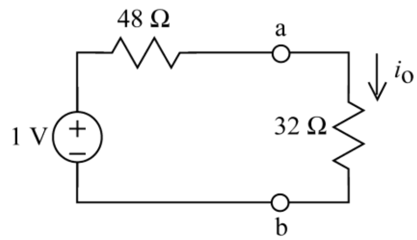
Using voltage division twice

$$v_{oc} = \frac{32}{32+96}20 - \frac{30}{120+30}20 = 5 - 4 = 1\text{ V}$$



$$R_t = (96 \| 32) + (120 \| 30) = 24 + 24 = 48\ \Omega$$

Replacing the part of the circuit to the left of terminals a-b by its Thevenin equivalent circuit gives



$$i_o = \frac{1}{48+32} = 0.0125\text{ A } = 12.5\text{ mA}$$

(checked: LNAP 5/24/04)

P 5.5-6 The circuit shown in Figure P 5.5-6*b* is the Norton equivalent circuit of the circuit shown in Figure P 5.5-6*a*. Find the value of the short-circuit current, i_{sc}, and Thévenin resistance, R_t.

Answer: $i_{sc} = -24$ A and $R_t = -3$ Ω

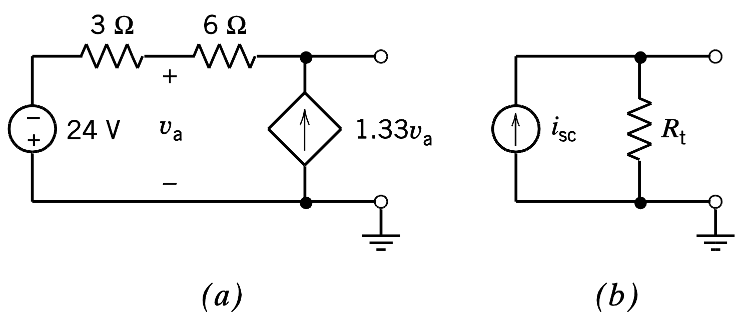


Figure P 5.5-6

Solution:

To determine the value of the short circuit current, I_{sc}, we connect a short circuit across the terminals of the circuit and then calculate the value of the current in that short circuit. Figure (a) shows the circuit from Figure 4.6-5a after adding the short circuit and labeling the short circuit current. Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure (a), node voltage v_1 is equal to the negative of the voltage source voltage. Consequently, $v_1 = -24$ V. The voltage at node 3 is equal to the voltage across a short, $v_3 = 0$. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the voltage across a short, i.e. $v_3 = 0$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad -48 = 3\,v_a \quad \Rightarrow \quad v_a = -16 \text{ V}$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3} v_2 = i_{sc} \quad \Rightarrow \quad \frac{9}{6} v_a = i_{sc} \quad \Rightarrow \quad i_{sc} = \frac{9}{6}(-16) = -24 \text{ A}$$

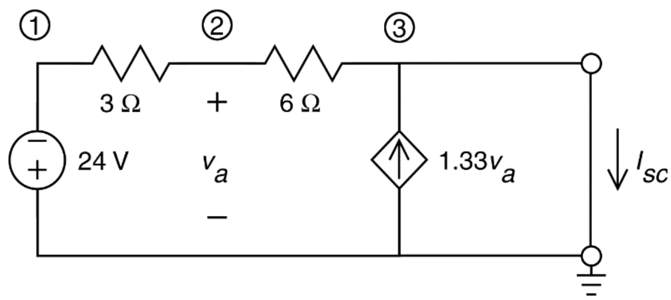


Figure (a) Calculating the short circuit current, I_{sc}, using mesh equations.

To determine the value of the Thevenin resistance, R_{th}, first replace the 24 V voltage source by a 0 V voltage source, i.e. a short circuit. Next, connect a current source circuit across the terminals of the circuit and then label the voltage across that current source as shown in Figure (b). The Thevenin resistance will be calculated from the current and voltage of the current source as

$$R_{th} = \frac{v_T}{i_T}$$

Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure (b), node voltage v_1 is equal to the across a short circuit, i.e. $v_1 = 0$. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the voltage across the current source, i.e. $v_3 = v_T$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\,v_1 + v_3 = 3\,v_2 \quad \Rightarrow \quad v_T = 3\,v_a$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3} v_2 + i_T = 0 \quad \Rightarrow \quad 9\, v_2 - v_3 + 6\, i_T = 0$$

$$\Rightarrow \quad 9\, v_a - v_T + 6\, i_T = 0$$

$$\Rightarrow \quad 3\, v_T - v_T + 6\, i_T = 0 \quad \Rightarrow \quad 2\, v_T = -6\, i_T$$

Finally,

$$R_t = \frac{v_T}{i_T} = -3\,\Omega$$

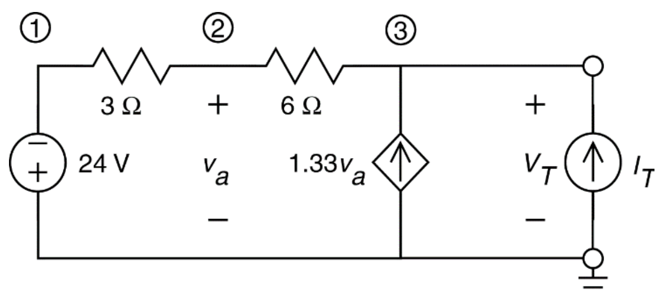


Figure (b) Calculating the Thevenin resistance, $R_{th} = \dfrac{v_T}{i_T}$, using mesh equations.

To determine the value of the open circuit voltage, v_{oc}, we connect an open circuit across the terminals of the circuit and then calculate the value of the voltage across that open circuit. Figure (c) shows the circuit from Figure P 4.6-5a after adding the open circuit and labeling the open circuit voltage. Also, the nodes have been identified and labeled in anticipation of writing node equations. Let v_1, v_2 and v_3 denote the node voltages at nodes 1, 2 and 3, respectively.

In Figure (c), node voltage v_1 is equal to the negative of the voltage source voltage. Consequently, $v_1 = -24$ V. The controlling voltage of the VCCS, v_a, is equal to the node voltage at node 2, i.e. $v_a = v_2$. The voltage at node 3 is equal to the open circuit voltage, i.e. $v_3 = v_{oc}$.

Apply KCL at node 2 to get

$$\frac{v_1 - v_2}{3} = \frac{v_2 - v_3}{6} \quad \Rightarrow \quad 2\, v_1 + v_3 = 3\, v_2 \quad \Rightarrow \quad -48 + v_{oc} = 3\, v_a$$

Apply KCL at node 3 to get

$$\frac{v_2 - v_3}{6} + \frac{4}{3} v_2 = 0 \quad \Rightarrow \quad 9\,v_2 - v_3 = 0 \quad \Rightarrow \quad 9\,v_a = v_{oc}$$

Combining these equations gives

$$3\left(-48 + v_{oc}\right) = 9\,v_a = v_{oc} \quad \Rightarrow \quad v_{oc} = 72 \text{ V}$$

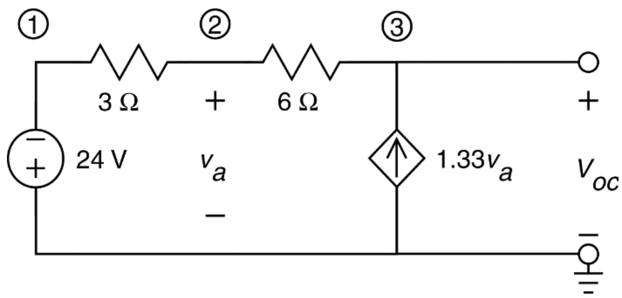


Figure (c) Calculating the open circuit voltage, v_{oc}, using node equations.

As a check, notice that

$$R_{th}\, I_{sc} = (-3)(-24) = 72 = V_{oc}$$

(checked using LNAP 8/16/02)

P 5.6-5 Determine the maximum power that can be absorbed by a resistor, *R*, connected to terminals a–b of the circuit shown in Figure P 5.6-5. Specify the required value of *R*.

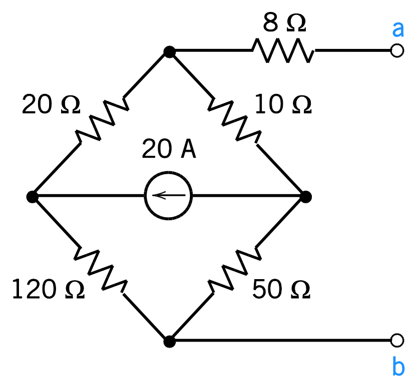
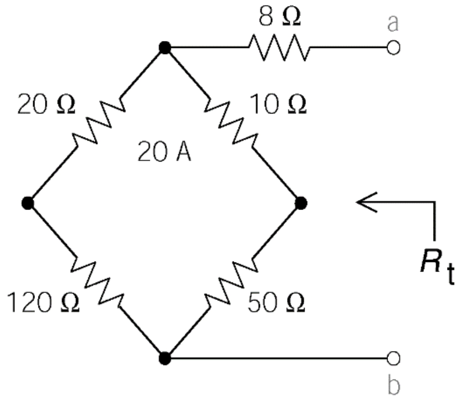
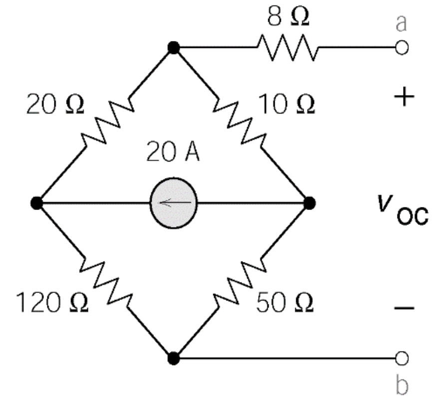


Figure P 5.6-5

Solution:





The required value of R is

$$R = R_t = 8 + \frac{(20+120)(10+50)}{(20+120)+(10+50)} = 50\ \Omega$$

$$v_{oc} = \left[\frac{170}{170+30}(20)\right]10 - \left[\frac{30}{170+30}(20)\right]50$$
$$= \frac{170(20)(10) - 30(20)(50)}{200} = \frac{4000}{200} = 20\text{ V}$$

The maximum power is given by

$$p_{\max} = \frac{v_{oc}^{\ 2}}{4\,R_t} = \frac{20^2}{4(50)} = 2\text{ W}$$

P 5.6-8 For the circuit of Figure P 5.6-8, find the power delivered to the load when R_L is fixed and R_t may be varied between 1 Ω and 5 Ω. Select R_t so that maximum power is delivered to R_L.

Answer: 13.9 W

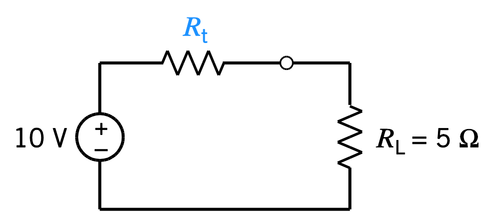


Figure P 5.6-8

Solution:

$$p = i\,v = \left(\frac{10}{R_t + R_L}\right)\left[\frac{R_L}{R_t + R_L}(10)\right] = \frac{100\,R_L}{(R_t + R_L)^2}$$

The power increases as R_t decreases so choose R_t = 1 Ω. Then

$$p_{\max} = i\,v = \frac{100\,(5)}{(1+5)^2} = 13.9 \text{ W}$$

Op-amps

P 6.3-4 Find v and i for the circuit of Figure P 6.3-4.

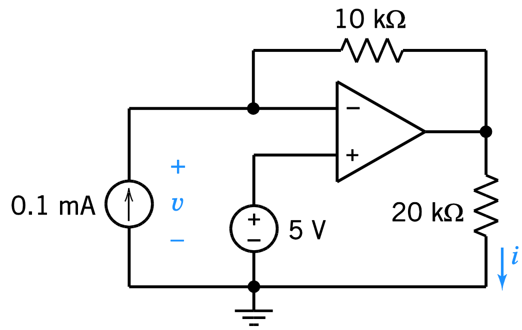
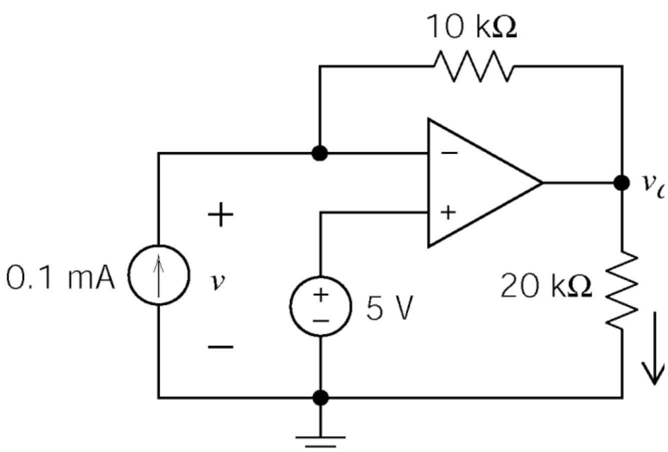


Figure P 6.3-4

Solution:



The voltages at the input nodes of an ideal op amp are equal so $v = 5$ V.

Apply KCL at the inverting input node of the op amp:

$$-\left(\frac{v_a - 5}{10000}\right) - 0.1\times10^{-3} - 0 = 0 \quad \Rightarrow \quad v_a = 4 \text{ V}$$

Apply Ohm's law to the 20 kΩ resistor

$$i = \frac{v_a}{20000} = \frac{1}{5} \text{mA}$$

(checked using LNAP 8/16/02)

P 6.3-13 The circuit shown in Figure P 6.3-13 has one input, v_s, and one output, v_o. Show that the output is proportional to the input. Design the circuit so that $v_o = 5\ v_s$.

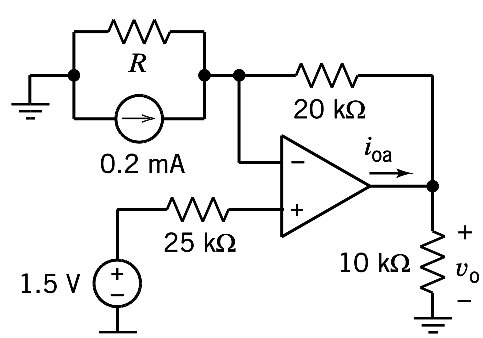
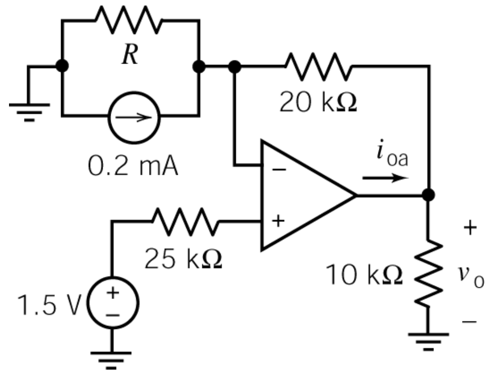


Figure P 6.3-13

Solution:

The output of this circuit is $v_o = 3.5$ V.



(a) The current in the 25 kΩ resistor is 0 A because this current is also the input current of an ideal op amp. Consequently, the voltage at the input nodes of the op amp is 1.5 V. Apply KCL at the inverting input of the op amp to get

$$\frac{1.5}{R} = 0.2 + \frac{3.5-1.5}{20} = 0.3$$

so

$$R = \frac{1.5}{0.3} = 5 \text{ k}\Omega$$

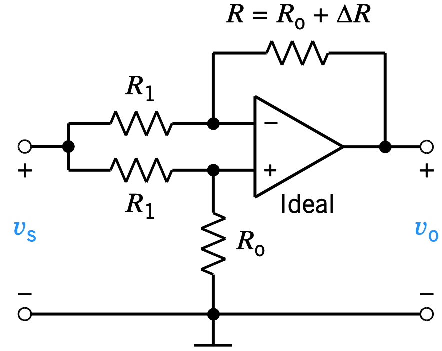
(b) The voltage source current is 0A so the voltage source supplies 0 W of power. The voltage across the current source is equal to the node voltage at the inverting input of the op amp, 1.5 V. Notice that this voltage and the given current source current do not adhere to the passive convention so (0.2)(1.5) = 0.3 mW is the power **supplied** by the current source.

(c) Apply KCL at the output node of the op amp to get

$$i_{oa} = \frac{3.5}{10} + \frac{3.5-1.5}{20} = 0.45 \text{ A}$$

The op amp supplies $p_{oa} = i_{oa} \times v_o = (0.45)(3.5) = 1.575$ W

P 6.4-10 The circuit shown in Figure P 6.4-10 includes a simple strain gauge. The resistor *R* changes its value by ΔR when it is twisted or bent. Derive a relation for the voltage gain v_o/v_s and show that it is proportional to the fractional change in *R*, namely $\Delta R/R_o$.

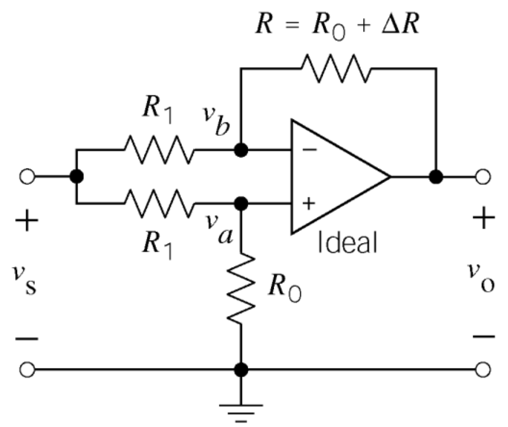


Answer: $v_o = \frac{R_o}{R_o + R_1} \frac{\Delta R}{R_o}$

Figure P 6.4-10

Solution:

By voltage division (or by applying KCL at node *a*)



$$v_a = \frac{R_0}{R_1+R_0} v_s$$

Applying KCL at node *b*:

$$\frac{v_b - v_s}{R_1} + \frac{v_b - v_0}{R_0+\Delta R} = 0$$

$$\Rightarrow \quad \frac{R_0+\Delta R}{R_1}(v_b - v_s) + v_b = v_0$$

The node voltages at the input nodes of an ideal op amp are equal so $v_b = v_a$.

$$v_0 = \left[\left(\frac{R_0+\Delta R}{R_1}+1\right)\frac{R_0}{R_1+R_0} - \frac{R_0+\Delta R}{R_1}\right] v_s = -\frac{\Delta R}{R_1+R_0} v_s = \left(-v_s \frac{R_0}{R_1+R_0}\right)\frac{\Delta R}{R_0}$$

P 6.4-14 The circuit shown in Figure P 6.4-14 has one input, v_s, and one output, v_o. Show that the output is proportional to the input. Design the circuit so that $v_o = 20 v_s$.

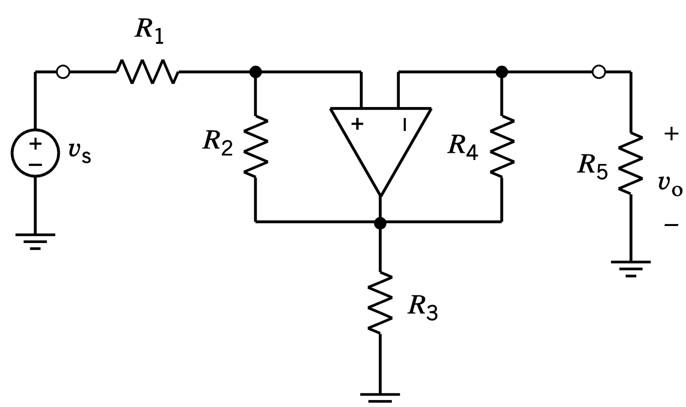
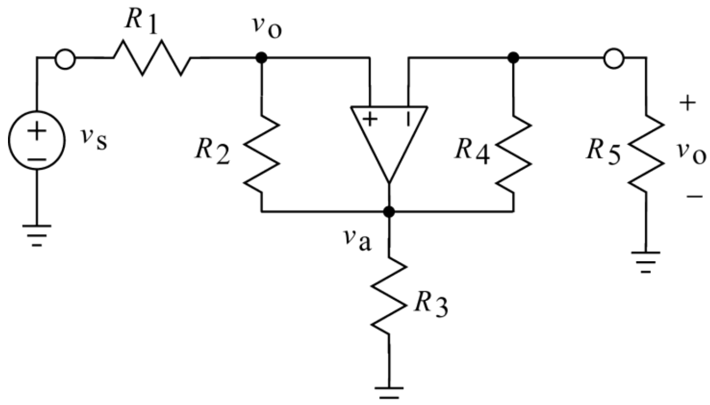


Figure P 6.4-14

Solution:



Represent this circuit by node equations.

$$\frac{v_o - v_a}{R_2} + \frac{v_o - v_s}{R_1} = 0 \quad \Rightarrow \quad R_2 v_s = \left(R_1 + R_2\right) v_o - R_1 v_a$$

$$\frac{v_o - v_a}{R_4} + \frac{v_o}{R_5} = 0 \quad \Rightarrow \quad v_a = \left(1 + \frac{R_4}{R_5}\right) v_o$$

So $\quad v_s = \left(1 + \frac{R_1}{R_2}\right) v_o - \left(\frac{R_1}{R_2}\right)\left(1 + \frac{R_4}{R_5}\right) v_o = \frac{\left(R_1 + R_2\right) R_5 - R_1\left(R_4 R_5\right) v_o}{R_2 R_5} = \frac{R_2 R_5}{R_2 R_5 - R_1 R_4} v_s$

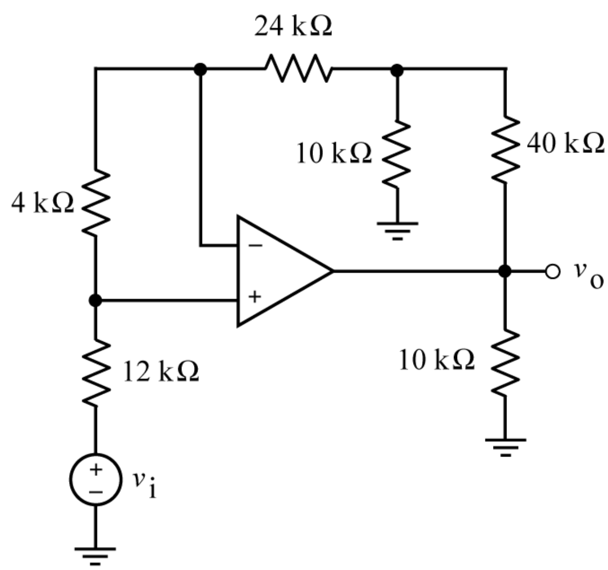
Then $\quad 20 = \frac{R_2 R_5}{R_2 R_5 - R_1 R_4} \quad \Rightarrow \quad \frac{19}{20} = \frac{R_1 R_4}{R_2 R_5}$

For example $\quad R_1 = 19$ kΩ, $R_4 = 10$ kΩ, $R_2 = 20$ kΩ, $R_5 = 10$ kΩ, $R_3 = 10$ kΩ.

(checked: LNAP 5/24/04)

P6.4-27

The input to the circuit shown in Figure P6.4-27 is the voltage source voltage, v_i . The output is the node voltage, v_o .The output is related to the input by the equation $v_o = k\, v_i$ where $k = \dfrac{v_o}{v_i}$ is called the gain of the circuit. Determine the value of the gain *k*.



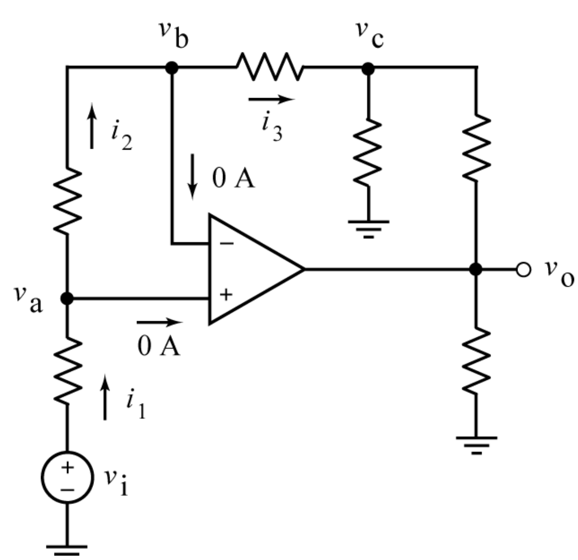
FigureP6.4-27

Solution:

Label the node voltages as shown. The node voltages at the input nodes of an ideal op amp are equal so

$$v_a = v_b$$

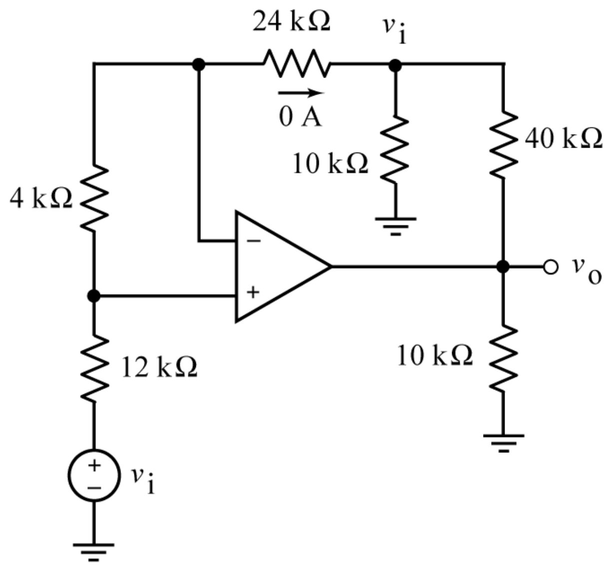
Consequently $\quad i_2 = \dfrac{v_a - v_b}{R} = 0$



The input currents of an ideal op amp are 0 A, so applying KCL at nodes a and b shows that $i_1 = 0$ and $i_3 = 0$. Consequently, the voltages across the corresponding resistor are 0 V.

Finally

$$v_a = v_b = v_c = v_i$$



Write an node equation at the right node of the 24 kΩ resistor:

$$0 = \frac{v_i}{10} + \frac{v_i - v_o}{40} \quad \Rightarrow \quad v_o = 5 v_i$$

Finally

$$k = \frac{v_o}{v_i} = 5$$

P 6.5-10 For the op amp circuit shown in Figure P 6.5-10, find and list all the possible voltage gains that can be achieved by connecting the resistor terminals to either the input or the output voltage terminals.

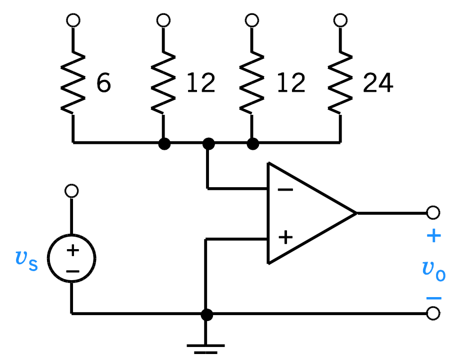


Figure P 6.5-10

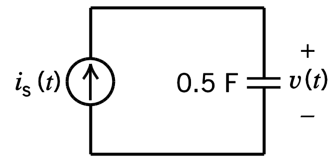
Solution:

R_1	6	12	24	6‖12	6‖24
R_2	12‖12‖24	6‖12‖24	6‖12‖12	12‖24	12‖12
$-v_o/v_s$	0.8	0.286	0.125	2	1.25

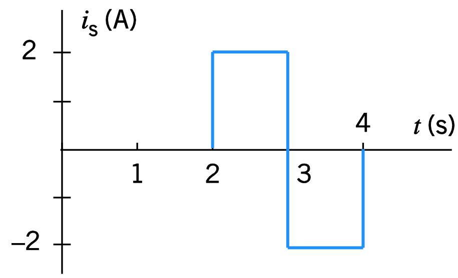
R_1	12‖12	12‖24	6‖12‖12	6‖12‖24	12‖12‖24
R_2	6‖24	6‖12	24	12	6
$-v_o/v_s$	0.8	0.5	8	3.5	1.25

Energy Storage Elements

P 7.2-9 Determine $v(t)$ for $t \geq 0$ for the circuit of Figure P 7.2-9*a* when $i_s(t)$ is the current shown in Figure P 7.2-9*b* and $v(0) = 1$ V.



(a)



(b)

Figure P 7.2-9

Solution:

$$v(t) = 2\int_0^t i(t)\, dt + 1$$

$$= 1 \qquad \text{for} \qquad 0 \le t \le 2$$

$$= 2\int_2^t 2\, dt + 1 = 4(t-2) + 1 = 4t - 7 \qquad \text{for} \qquad 2 \le t \le 3$$

$$= 2\int_2^3 2\, dt + 2\int_3^t -2\, dt + 1 = 4 - 4(t-3) + 1 = -4t + 17 \qquad \text{for} \qquad 3 \le t \le 4$$

$$= 2\int_2^3 2\, dt + 2\int_3^4 -2\, dt + 1 = 1 \qquad \text{for} \qquad t \ge 4$$

In summary

$$v(t) = \begin{cases} 1 & 0 \le t \le 2 \\ 4t - 7 & 2 \le t \le 3 \\ -4t + 17 & 3 \le t \le 4 \\ 1 & 4 \le t \end{cases}$$

P7.2-20 The input to the circuit shown in Figure P7.2-20 is the voltage:

$$v(t)=3+4e^{-2t} \text{ A} \quad \text{for } t>0$$

The output is the current: $i(t)=0.3-1.6e^{-2t}$ V for $t>0$

Determine the values of the resistance and capacitance

Answer: $R =$ 10 Ω and $C =$ 0.25 F.

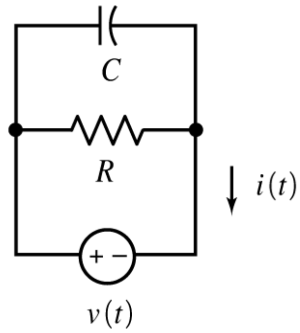


Figure P7.2-20

Solution: Apply KCL at either node to get

$$0.3-1.6e^{-2t}=\frac{3+4e^{-2t}}{R}+C\frac{d}{dt}\left(3+4e^{-2t}\right)$$
$$=\frac{3+4e^{-2t}}{R}+(-2)4Ce^{-2t}=\frac{3}{R}+\left(\frac{4}{R}-8C\right)e^{-2t}$$

Equating coefficients:

$$0.3=\frac{3}{R} \quad\Rightarrow\quad R=10\ \Omega \quad \text{and} \quad -1.6=\frac{4}{10}-8C \quad\Rightarrow\quad C=0.25\text{ F}$$

P 7.5-10 Determine $i(t)$ for $t \geq 0$ for the current of Figure P 7.5-10*a* when $i(0)$ = 1 A and $v_s(t)$ is the voltage shown in Figure P 7.5-10*b*.

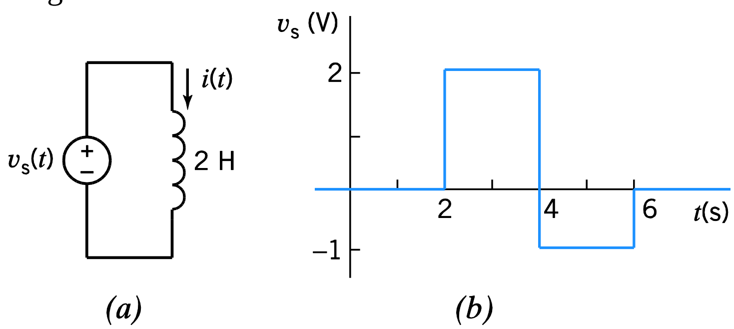


Figure P 7.5-10

Solution:

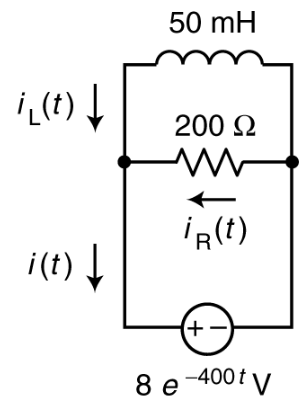
$$i(t)=\frac{1}{2}\int_0^t v(t)\,dt+1=\begin{cases} 1 & t\leq 2 \\ \int_2^t d\tau+1=(t-2)+1=t-1 & 2\leq t\leq 4 \\ \frac{1}{2}\int_4^t d\tau+3=-\frac{1}{2}t+5 & 4\leq t\leq 6 \\ 2 & 6\leq t \end{cases}$$

P7.5-18 . The source voltage the circuit shown in Figure P7.5-18 is $v(t) = 8\, e^{-400t}$ V after time $t = 0$. The initial inductor current is $i_L(0) = 210$ mA. Determine the source current $i(t)$ for $t > 0$.

Answer: $i(t) = 360 e^{-400t} - 190$ mA for $t > 0$

Solution:

Label the resistor current as shown. The resistor, inductor and voltage source are connected in parallel so the voltage across each is $v(t) = 2.5\, e^{-400t}$ V. Notice that the labeled voltage and current of both the resistor and inductor do not adhere to the passive convention.



The resistor current is $i_R(t) = -\dfrac{8\, e^{-400t}}{200} = -40\, e^{-400t}$ mA

The inductor current is $i_L(t) = 0.21 + \dfrac{1}{0.05}\displaystyle\int_0^t -8\, e^{-400\tau}\, d\tau$ A

$$i_L(t) = 0.21 + \frac{-8}{0.05(-400)}\int_0^t e^{-400\tau}\, d\tau \text{ A}$$
$$= 0.21 + 0.4\left(e^{-400t} - 1\right) \text{ A}$$
$$= 400 e^{-400t} - 190 \text{ mA}$$

Using KCL $\quad i(t) = 360 e^{-400t} - 190$ mA for $t > 0$

P 7.8-2 The switch in Figure P 7.8-2 has been open for a long time before closing at time $t = 0$. Find $v_c(0^+)$ and $i_L(0^+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_c(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. Find $v_c(\infty)$ and $i_L(\infty)$.

Answer: $v_c(0^+) = 6$ V, $i_L(0^+) = 1$ mA, $v_c(\infty) = 3$ V, and $i_L(\infty) = 1.5$ mA

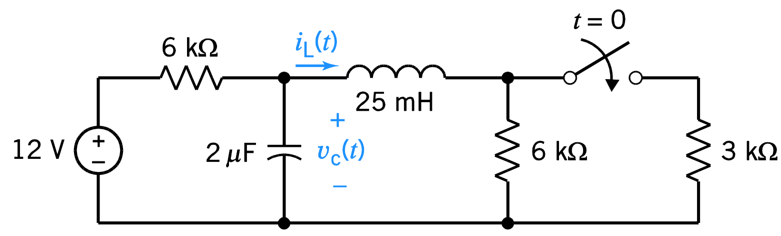
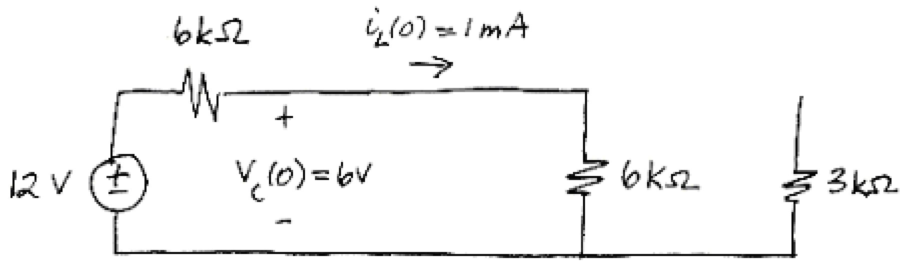


Figure P7.8-2

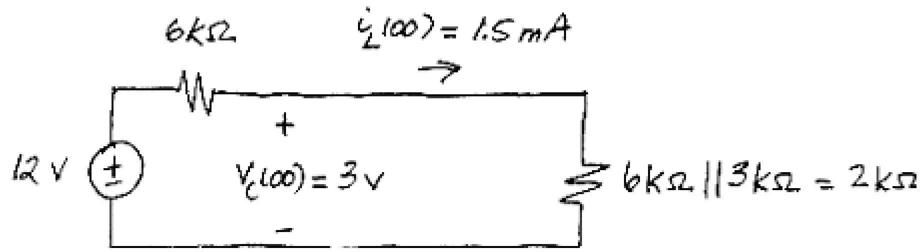
Solution:



Then

$$i_L\left(0^+\right) = i_L\left(0^-\right) = 1\text{ mA} \quad \text{and} \quad v_C\left(0^+\right) = v_C\left(0^-\right) = 6\text{ V}$$

Next



P7.8-5 . The switch in the circuit shown in Figure P7.8-5 has been open for a long time before it closes at time t = 0. Determine the values of i_R(0-) and i_C(0-), the current in one of the 20 Ω resistors and in the capacitor immediately before the switch closes and the values of i_R(0+) and i_C(0+), the current in that 20 Ω resistor and in the capacitor immediately after the switch closes.

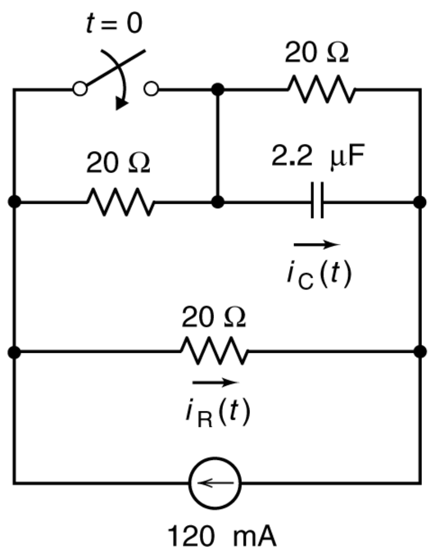
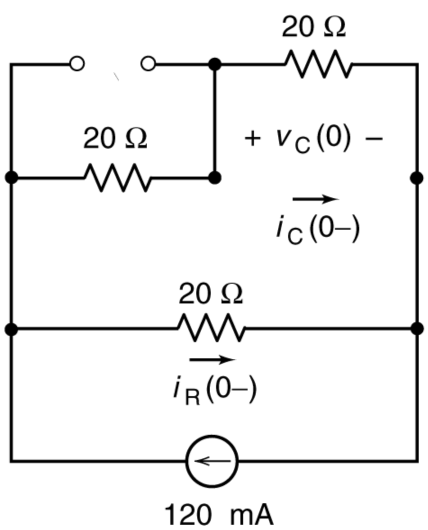


Figure P7.8-5

Solution:

The circuit is at steady state immediately before the switch closes. We have



The capacitor acts like an open circuit so $i_C(0-) = 0$.

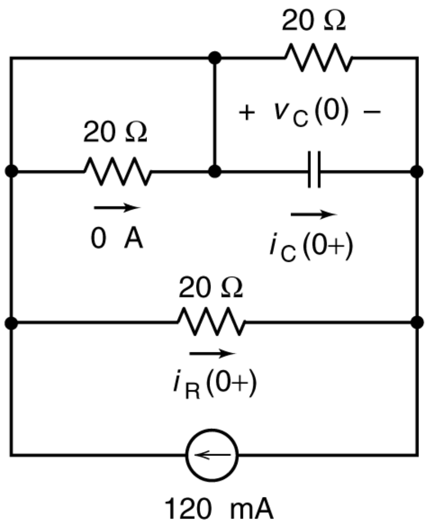
Noticing that two 20 Ω are connected in series and using current division:

$$i_R(0-) = \frac{(20+20)}{20+(20+20)}(120) = \frac{2}{3}(120) = 80 \text{ mA}$$

Using current division and Ohm's law:

$$v_C(0) = \left[\frac{20}{20+(20+20)}(120)\right](20) = 0.8 \text{ V}$$

The capacitor does not change instantaneously so $v_C(0+) = v_C(0-) @ v_C(0)$. Immediately after the switch closes we have:



Applying KVL to the loop consisting of the closed switch, the capacitor and a 20 Ω resistor gives

$$0+v_C(0)-20\,i_R(0+)=0$$

$$0+0.8=20\,i_R(0+)$$

$$i_R(0+)=40\text{ mA}$$

Applying KCL at the node at the right side of the circuit gives:

$$\frac{v_C(0+)}{20}+i_C(0+)+i_R(0+)=0.120$$

$$\frac{0.8}{20}+i_C(0+)+0.04=0.120$$

$$i_C(0+)=0.04=40\text{ mA}$$

P7.8-11

The circuit shown in Figure 7.8-11 has reached steady state before the switch opens at time t = 0. Determine the values of $i_L(t)$, $v_C(t)$ and $v_R(t)$ immediately before the switch opens and the value of $v_R(t)$ immediately after the switch opens.

Answers: $i_L(0-)$= 1.25 A, $v_C(0-)$ = 20 V, $v_R(0-)$ = –5 V and $v_R(0+)$ = –4 V

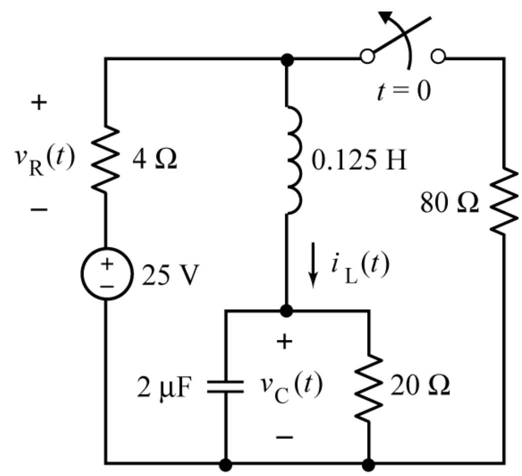
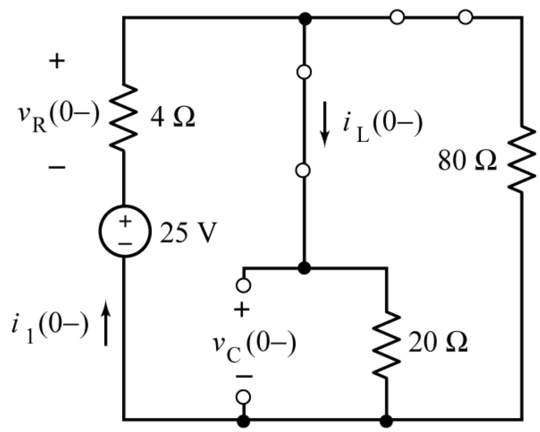


Figure 7.8-11

Solution: Because

- This **circuit has reached steady state** before the switch opens at time $t = 0$.
- The only source is a **constant** voltage **source**.



At t=0–, t**he capacitor acts like an open circuit** and **the inductor acts like a short circuit**. From the circuit

$$i_1(0-) = \frac{25}{4+(20\|80)} = \frac{25}{4+16} = 1.25\text{ A},$$

$$i_L(0-) = \left(\frac{80}{20+80}\right) i_1(0-) = 1\text{ A},$$

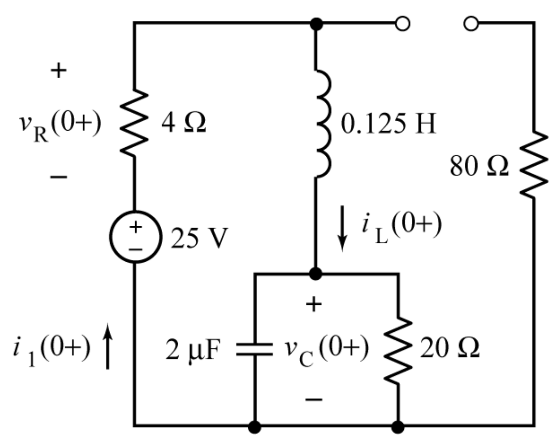
$$v_C(0-) = 20\, i_L(0-) = 20\text{ V}$$

and

$$v_R(0-) = -4\, i_1(0-) = -5\text{ V}$$

The **capacitor voltage and inductor current don't change instantaneously** so

$$v_C(0+) = v_C(0-) = 20\text{ V} \text{ and}$$
$$i_L(0+) = i_L(0-) = 1\text{ A}$$



Apply KCL at the top node to see that

$$i_1(0+) = i_L(0+) = 1\text{ A}$$

From Ohm's law

$$v_R(0+) = -4\, i_1(0+) = -4\text{ V}$$

(Notice that the resistor voltage did change instantaneously.)

First Order Circuits

P 8.3-9 The circuit shown in Figure P 8.3-9 is at steady state before the switch closes at time
$t = 0$. The input to the circuit is the voltage of the voltage source, 24 V. The output of this circuit, the voltage across the 3-Ω resistor, is given by

$$v_o(t) = 6 - 3e^{-0.35t} \text{ V} \quad \text{when} \quad t > 0$$

Determine the value of the inductance, L, and of the resistances, R_1 and R_2.

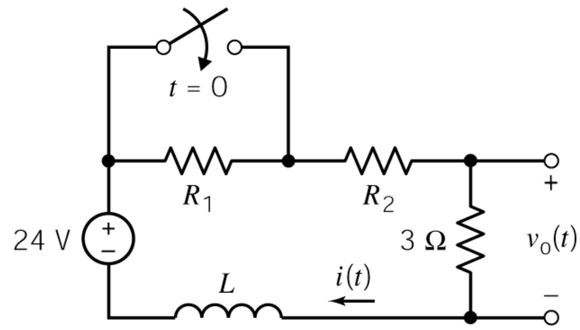


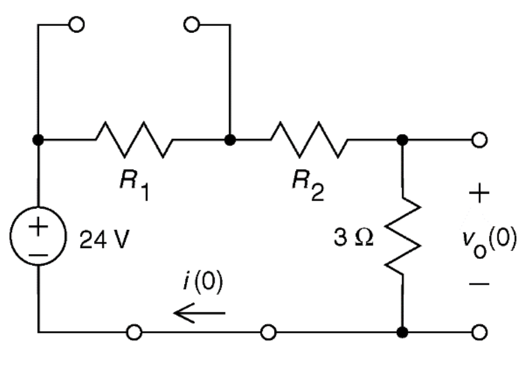
Figure P 8.3-9

Solution: Before the switch closes, the circuit will be at steady state. Because the only input to this circuit is the constant voltage of the voltage source, all of the element currents and voltages, including the inductor current, will have constant values. Closing the switch disturbs the circuit by shorting out the resistor R_1. Eventually the disturbance dies out and the circuit is again at steady state. All the element currents and voltages will again have constant values, but probably different constant values than they had before the switch closed.

The inductor current is equal to the current in the 3 Ω resistor. Consequently,

$$i(t) = \frac{v_o(t)}{3} = \frac{6 - 3e^{-0.35t}}{3} = 2 - e^{-0.35t} \text{ A when } t > 0$$

In the absence of unbounded voltages, the current in any inductor is continuous. Consequently, the value of the inductor current immediately before $t = 0$ is equal to the value immediately after $t = 0$.



Here is the circuit before $t = 0$, when the switch is open and the circuit is at steady state. The open switch is modeled as an open circuit. An inductor in a steady-state dc circuit acts like a short circuit, so a short circuit replaces the inductor. The current in that short circuit is the steady state inductor current, $i(0)$. Apply KVL to the loop to get

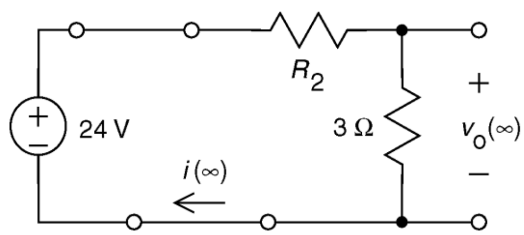
$$R_1\, i(0) + R_2\, i(0) + 3\, i(0) - 24 = 0$$

$$\Rightarrow \quad i(0) = \frac{24}{R_1 + R_2 + 3}$$

The value of $i(0)$ can also be obtained by setting $t = 0$ in the equation for $i(t)$. Do so gives

$$i(0) = 2 - e^{0} = 1 \text{ A}$$

Consequently, $$1 = \frac{24}{R_1 + R_2 + 3} \quad \Rightarrow \quad R_1 + R_2 = 21$$



Next, consider the circuit after the switch closes. Here is the circuit at $t = \infty$, when the switch is closed and the circuit is at steady state. The closed switch is modeled as a short circuit. The combination of resistor and a short circuit connected is equivalent to a short circuit. Consequently, a short circuit replaces the switch and the resistor R_1.

An inductor in a steady-state dc circuit acts like a short circuit, so a short circuit replaces the inductor. The current in that short circuit is the steady state inductor current, $i(\infty)$. Apply KVL to the loop to get

$$R_2\, i(\infty) + 3\, i(\infty) - 24 = 0 \quad \Rightarrow \quad i(\infty) = \frac{24}{R_2 + 3}$$

The value of $i(\infty)$ can also be obtained by setting $t = \infty$ in the equation for $i(t)$. Doing so gives

$$i(\infty) = 2 - e^{-\infty} = 2 \text{ A}$$

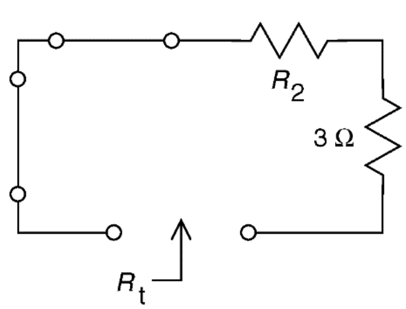
Consequently

$$2 = \frac{24}{R_2 + 3} \quad \Rightarrow \quad R_2 = 9 \ \ \Omega$$

Then

$$R_1 = 12 \ \ \Omega$$

Finally, the exponential part of $i(t)$ is known to be of the form $e^{-t/\tau}$ where $\tau = \dfrac{L}{R_t}$ and R_t is the Thevenin resistance of the part of the circuit that is connected to the inductor.



Here is shows the circuit that is used to determine R_t. A short circuit has replaced combination of resistor R_1 and the closed switch. Independent sources are set to zero when calculating R_t, so the voltage source has been replaced by an short circuit.

$$R_t = R_2 + 3 = 9 + 3 = 12 \ \ \Omega$$

so

$$\tau = \frac{L}{R_t} = \frac{L}{12}$$

From the equation for $i(t)$

$$-0.35\,t = -\frac{t}{\tau} \quad \Rightarrow \quad \tau = 2.857 \text{ s}$$

Consequently,

$$2.857 = \frac{L}{12} \quad \Rightarrow \quad L = 34.28 \text{ H}$$

P8.3-28 After time $t = 0$, a given circuit is represented by the circuit diagram shown in FigureP8.3-28.

a.) Suppose that the inductor current is

$$i(t) = 21.6 + 28.4e^{-4t} \text{ mA} \quad \text{for } t \geq 0$$

Determine the values of R_1 and R_3.

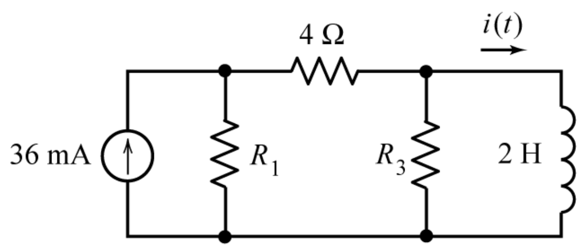


Figure P8.3-28

b.) Suppose instead that $R_1 = 16\ \Omega$, $R_3 = 20\ \Omega$ and the initial condition is $i(0) = 10$ mA. Determine the inductor current for $t \geq 0$.

Solution: The inductor current is given by $i(t) = i_{sc} + \left(i(0) - i_{sc}\right)e^{-at}$ for $t \geq 0$ where $a = \frac{1}{\tau} = \frac{R_t}{L}$.

a. Comparing this to the given equation gives $21.6 = i_{sc} = \frac{R_1}{R_1 + 4}(36) \quad \Rightarrow \quad R_1 = 6\ \Omega$ and

$4 = \frac{R_t}{2} \quad \Rightarrow \quad R_t = 8\ \Omega$. Next $8 = R_t = \left(R_1 + 4\right) \| R_3 = 10 \| R_3 \quad \Rightarrow \quad R_3 = 40\ \Omega$.

b. $R_t = \left(16 + 4\right) \| 20 = 10\ \Omega$ so $a = \frac{1}{\tau} = \frac{10}{2} = 5$ s. also $i_{sc} = \frac{16}{16+4}(36) = 28.8$ mA. Then

$i(t) = i_{sc} + \left(i(0) - i_{sc}\right)e^{-at} = 28.8 + \left(10 - 28.8\right)e^{-5t} = 28.2 - 18.8e^{-5t}$.

P8.3-29 Consider the circuit shown in Figure P8.3-29.

a.) Determine the time constant, τ, and the steady state capacitor voltage, $v(\infty)$, when the switch is **open**.

b.) Determine the time constant, τ, and the steady state capacitor voltage, $v(\infty)$, when the switch is **closed**.

Answers: a.) τ = 3 s and $v(\infty)$ = 24 V; b.) τ = 2.25 s and $v(\infty)$ = 2 V;

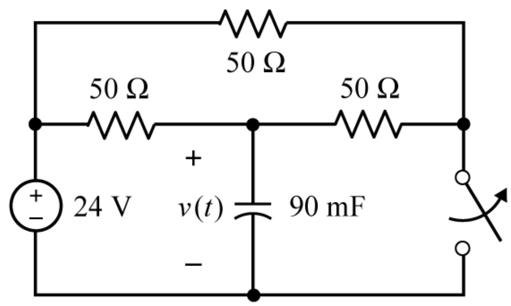
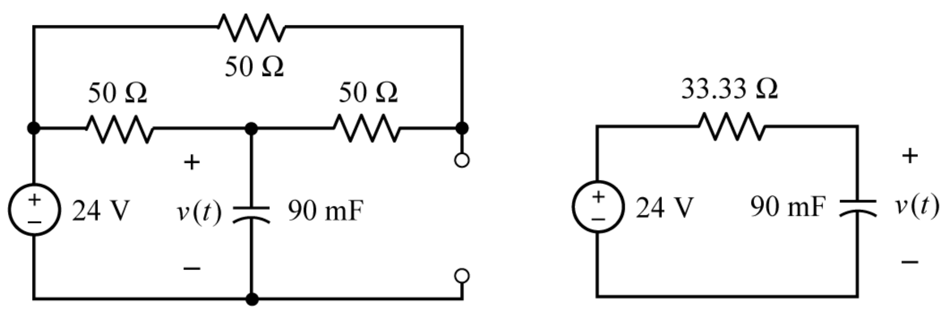


Figure P8.3-29

Solution: a.) When the switch is open we have

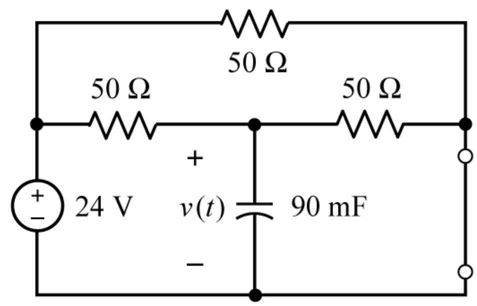


After replacing series and parallel resistors by equivalent resistors, the part of the circuit connected to the capacitor is a Thevenin equivalent circuit with $R_t = 33.33\ \Omega$. The time constant is

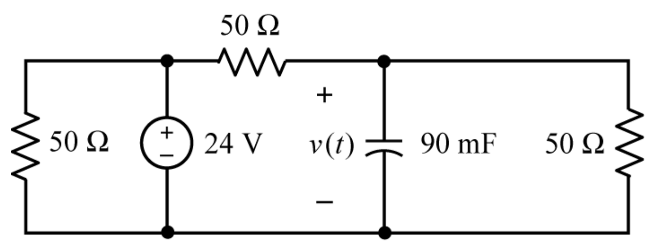
$$\tau = R_t C = 33.33(0.090) = 3\text{ s}.$$

Since the input is constant, the capacitor acts like an open circuit when the circuit is at steady state. Consequently, there is zero current in the 33.33 Ω resistor and KVL gives $v(\infty)$ = 24 V.

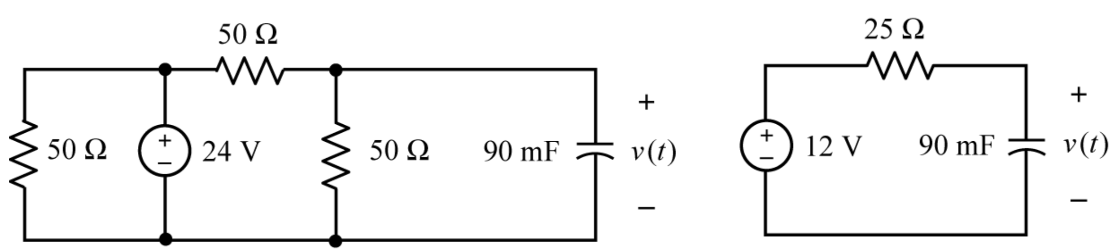
b.) When the switch is closed we have



This circuit can be redrawn as



Now we find the Thevenin equivalent of the part of the circuit connected to the capacitor:



So $R_t = 25\ \Omega$ and

$$\tau = R_t\, C = 25(0.090) = 2.25 \text{ s}$$

Since the input is constant, the capacitor acts like an open circuit when the circuit is at steady state. Consequently, there is zero current in the 25 Ω resistor and KVL gives $v(\infty) = 12$ V.

P 8.4-5 The circuit shown in Figure P 8.4-5 is at steady state before the switch opens at $t = 0$. The switch remains open for 0.5 second and then closes. Determine $v(t)$ for $t \geq 0$.

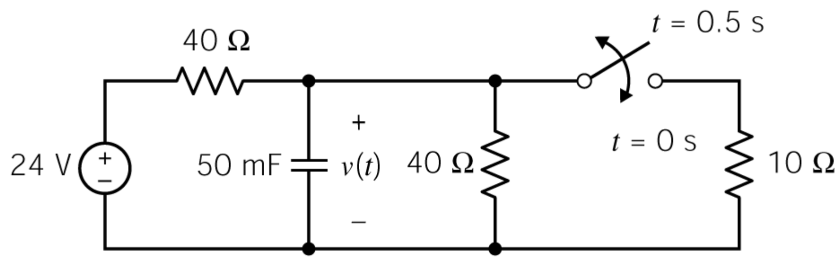
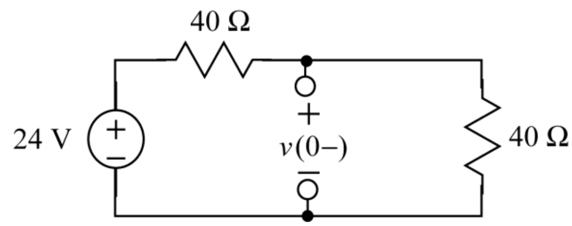


Figure P 8.4-5

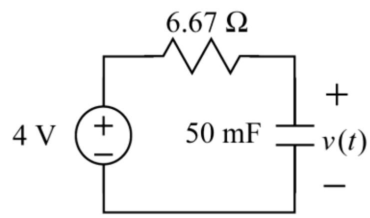
Solution:

The circuit is at steady state before the switch closes. The capacitor acts like an open circuit. The initial condition is

$$v(0+) = v(0-) = \left(\frac{40}{40+40}\right) 24 = 12\ \text{V}$$



After the switch closes, replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.



Recognize that $R_t = 6.67\ \Omega$ and $v_{oc} = 4\ \text{V}$

The time constant is

$$\tau = R_t C = (6.67)(0.05) = 0.335\ \text{s} \quad \Rightarrow \quad \frac{1}{\tau} = 2.988\ ;\ 3\ \frac{1}{\text{s}}$$

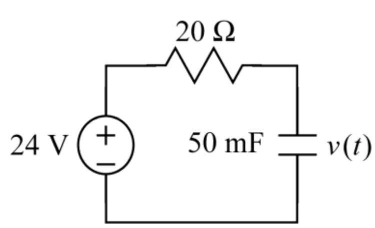
The capacitor voltage is

$$v(t) = \left(v(0+) - v_{oc}\right)e^{-t/\tau} + v_{oc} = (12-4)e^{-3t} + 4 = 4 + 8e^{-3t}\ \text{V} \quad \text{for}\ \ 0 \ge t \ge 0.5\ \text{s}$$

When the switch opens again at time t = 0.5 the capacitor voltage is

$$v(0.5+) = v(0.5-) = 4 + 8e^{-3(0.5)} = 5.785\ \text{V}$$

After time t = 0.5 s, replace the part of the circuit connected to the capacitor by its Thevenin equivalent circuit.



Recognize that $R_t = 20\ \Omega$ and $v_{oc} = 12\text{ V}$

The time constant is

$$\tau = R_t C = 20(0.05) = 1 \quad \Rightarrow \quad \frac{1}{\tau} = 1\ \frac{1}{\text{s}}$$

The capacitor voltage is

$$v(t) = \left(v(0.5+) - v_{oc}\right)e^{-(t-0.5)/\tau} + v_{oc} = (5.785 - 12)e^{-10(t-0.5)} + 12$$
$$= 12 - 6.215e^{-10(t-0.5)}\text{ V} \quad \text{for } t \geq 0.5\text{ s}$$

so

$$v(t) = \begin{cases} 12\text{ V} & \text{for } t \geq 0 \\ 4 + 8e^{-3t}\text{ V} & \text{for } 0 \leq t \leq 0.5\text{ s} \\ 12 - 6.215e^{-(t-0.5)}\text{ V} & \text{for } t \geq 0.5\text{ s} \end{cases}$$

P 8.6-2 The input to the circuit shown in Figure P 8.6-2 is the voltage of the voltage source, $v_s(t)$. The output is the voltage across the capacitor, $v_o(t)$. Determine the output of this circuit when the input is $v_s(t) = 3 + 3\,u(t)$ V.

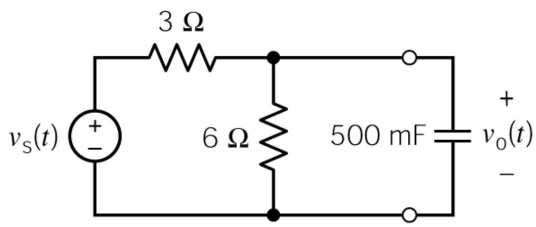


Figure P 8.6-2

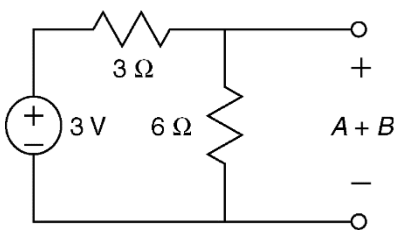
Solution:

The value of the input is one constant, 3 V, before time $t = 0$ and a different constant, 6 V, after time $t = 0$. The response of the first order circuit to the change in the value of the input will be

$$v_o(t) = A + B\,e^{-a\,t} \quad \text{for } t > 0$$

where the values of the three constants A, B and a are to be determined.

The values of A and B are determined from the steady state responses of this circuit before and after the input changes value.



Capacitors act like open circuits when the input is constant and the circuit is at steady state. Consequently, the capacitor is replaced by an open circuit.

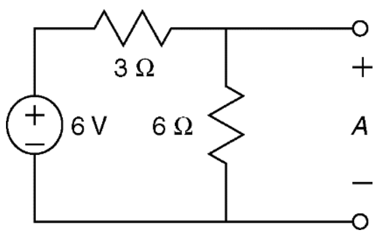
The value of the capacitor voltage at time t = 0, will be equal to the steady state capacitor voltage before the input changes. At time $t = 0$ the output voltage is

$$v_o(0) = A + B\,e^{-a(0)} = A + B$$

The steady-state circuit for t < 0.

Consequently, the capacitor voltage is labeled as *A* + *B*. Analysis of the circuit gives

$$A + B = \frac{6}{3+6}(3) = 2 \text{ V}$$



Capacitors act like open circuits when the input is constant and the circuit is at steady state. Consequently, the capacitor is replaced by an open circuit.

The value of the capacitor voltage at time t = ∞, will be equal to the steady state capacitor voltage after the input changes. At time $t = \infty$ the output voltage is

$$v_o(\infty) = A + B\,e^{-a(\infty)} = A$$

The steady-state circuit for t > 0.

Consequently, the capacitor voltage is labeled as *A*.

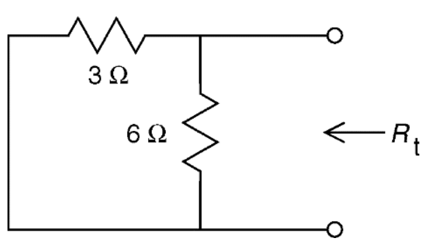
Analysis of the circuit gives

$$A = \frac{6}{3+6}(6) = 4 \text{ V}$$

Therefore $B = -2$ V

The value of the constant *a* is determined from the time constant, τ, which is in turn calculated from the values of the capacitance *C* and of the Thevenin resistance, R_t, of the circuit connected to the capacitor.

$$\frac{1}{a} = \tau = R_t\,C$$



Here is the circuit used to calculate R_t.

$$R_t = \frac{(3)(6)}{3+6} = 2 \;\Omega$$

Therefore

$$a = \frac{1}{(2)(.5)} = 1 \;\frac{1}{\text{s}}$$

(The time constant is $\tau = (2)(0.5) = 1$ s .)

Putting it all together:

$$v_o(t) = \begin{cases} 2 \text{ V} & \text{for } t \le 0 \\ 4 - 2\, e^{-t} \text{ V} & \text{for } t \ge 0 \end{cases}$$

P 8.6-5 The initial voltage of the capacitor of the circuit shown in Figure P 8.6-5 is zero. Determine the voltage $v(t)$ when the source is a pulse, described by

$$v_s = \begin{cases} 0 & t < 1 \text{ s} \\ 4\text{ V} & 1 < t < 2 \text{ s} \\ 0 & t > 2 \text{ s} \end{cases}$$

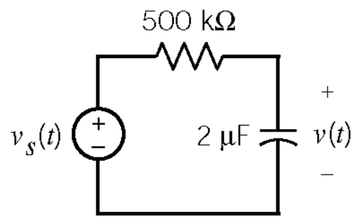


Figure P 8.6-5

Solution

$$\tau = R\,C = (5\times10^{5})(2\times10^{-6}) = 1 \text{ s}$$

Assume that the circuit is at steady state at t = 1⁻. Then

$$v(t) = 4 - 4\,e^{-(t-1)} \text{ V} \quad \text{for } 1 \le t \le 2$$

so $$v(2) = 4 - 4\,e^{-(2-1)} = 2.53 \text{ V}$$

and $$v(t) = 2.53\,e^{-(t-2)} \text{ V} \quad \text{for } t \ge 2$$

Finally $$v(t) = \begin{cases} 0 & t \le 1 \\ 4 - 4e^{-(t-1)} & 1 \le t \le 2 \\ 2.53e^{-(t-2)} & t \ge 2 \end{cases}$$

P 8.6-10 Determine the voltage $v(t)$ for $t \geq 0$ for the circuit shown in Figure P 8.6-10.

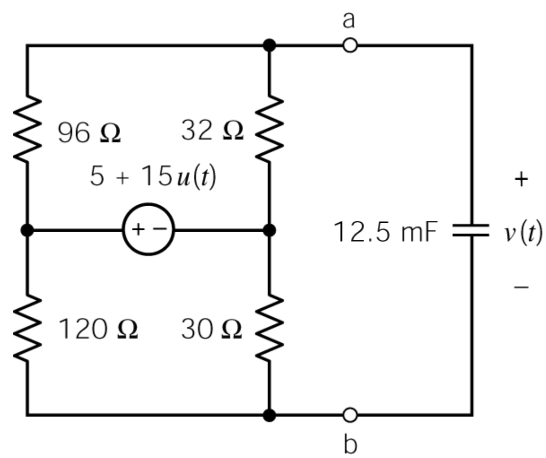
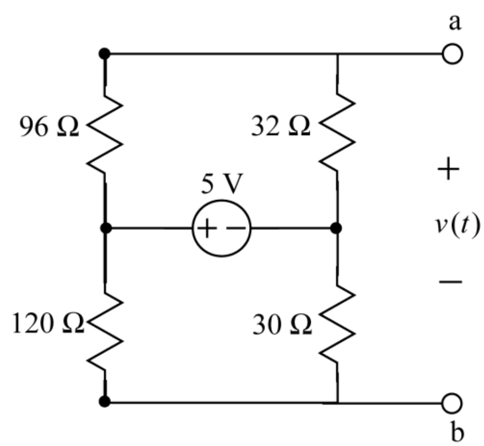


Figure P 8.6-10

Solution:

For $t < 0$



Using voltage division twice

$$v(t) = \frac{32}{32+96}5 - \frac{30}{120+30}5 = 0.25 \text{ V}$$

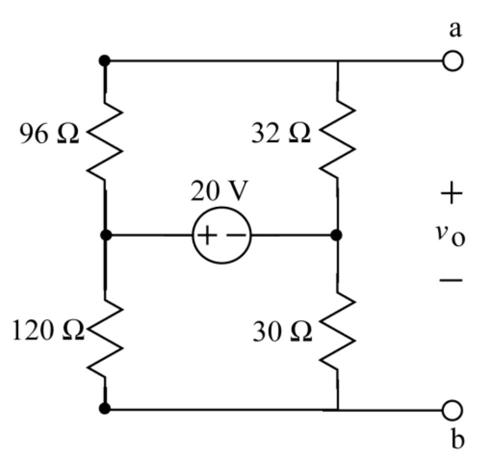
so

$$v(0-) = 0.25 \text{ V}$$

and

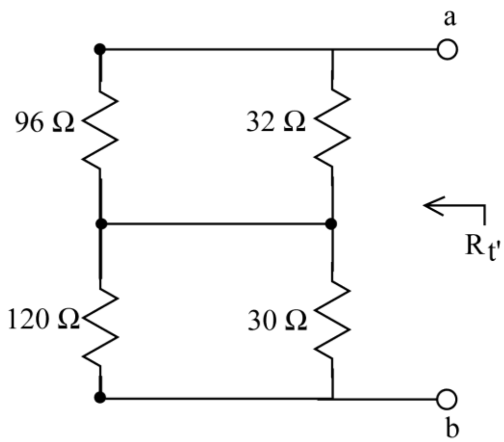
$$v(0+) = v(0-) = 0.25 \text{ V}$$

For $t > 0$, find the Thevenin equivalent circuit for the part of the circuit connected to the capacitor.



Using voltage division twice

$$v_{oc} = \frac{32}{32+96}20 - \frac{30}{120+30}20 = 5 - 4 = 1 \text{ V}$$



$$R_t = (96 \| 32) + (120 \| 30) = 24 + 24 = 48\ \Omega$$

then

$$\tau = 48 \times 0.0125 = 0.6\ \text{s}$$

so

$$\frac{1}{\tau} = 1.67\ \frac{1}{\text{s}}$$

Now

$$v(t) = [0.25 - 1]e^{-1.67t} + 1 = 1 - 0.75e^{-1.67t}\ \text{V} \quad \text{for } t \geq 0$$

(checked: LNAP 7/1/04)

Second Order Circuits

P 9.2-4 The input to the circuit shown in Figure P 9.2-4 is the voltage of the voltage source, V_s. The output is the inductor current $i(t)$. Represent the circuit by a second-order differential equation that shows how the output of this circuit is related to the input, for $t > 0$.

Hint: Use the direct method.

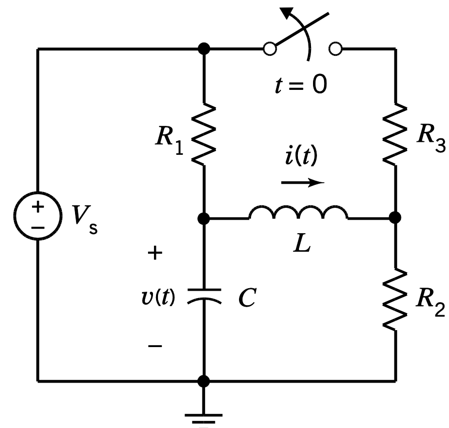


Figure P 9.2-4

Solution:

After the switch opens, apply KCL and KVL to get

$$R_1\left(i(t)+C\frac{d}{dt}v(t)\right)+v(t)=V_s$$

Apply KVL to get

$$v(t)=L\frac{d}{dt}i(t)+R_2\,i(t)$$

Substituting $v(t)$ into the first equation gives

$$R_1\left(i(t)+C\frac{d}{dt}\left(L\frac{d}{dt}i(t)+R_2\,i(t)\right)\right)+L\frac{d}{dt}i(t)+R_2\,i(t)=V_s$$

then

$$R_1\,C\,L\frac{d^2}{dt^2}i(t)+\left(R_1\,C\,R_2+L\right)\frac{d}{dt}i(t)+\left(R_1+R_2\right)i(t)=V_s$$

Dividing by $R_1\,C\,L$:

$$\frac{d^2}{dt^2}i(t)+\left(\frac{R_1\,C\,R_2+L}{R_1\,C\,L}\right)\frac{d}{dt}i(t)+\left(\frac{R_1+R_2}{R_1\,C\,L}\right)i(t)=\frac{V_s}{R_1\,C\,L}$$

P 9.2-10 The input to the circuit shown in Figure P 9.2-10 is the voltage of the voltage source, v_s. The output is the capacitor voltage $v(t)$. Represent the circuit by a second-order differential equation that shows how the output of this circuit is related to the input, for $t > 0$.
Hint: Find a Thévenin equivalent circuit.

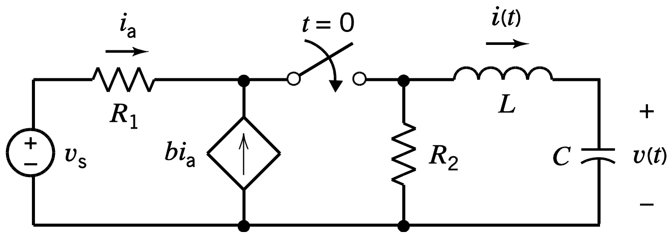
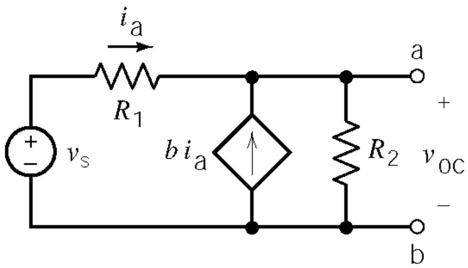
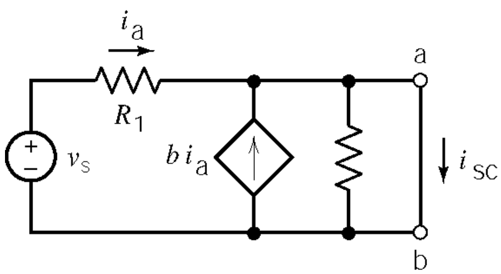


Figure P 9.2-10

Solution:
Find the Thevenin equivalent circuit for the part of the circuit to the left of the inductor.

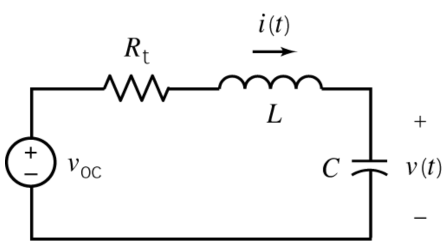


$$\left.\begin{aligned} i_a &= \frac{v_s - v_{oc}}{R_1} \\ i_a + b\,i_a &= \frac{v_{oc}}{R_2} \end{aligned}\right\} \Rightarrow \quad v_{oc} = \frac{v_s\,R_2\left(1+b\right)}{R_1 + R_2\left(1+b\right)}$$



$$i_{sc} = i_a\left(1+b\right) = \frac{v_s}{R_1}\left(1+b\right)$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{\dfrac{v_s\,R_2\left(1+b\right)}{R_1 + R_2\left(1+b\right)}}{\dfrac{v_s}{R_1}\left(1+b\right)} = \frac{R_1\,R_2}{R_1 + R_2\left(1+b\right)}$$



$$R_t\,i\left(t\right) + L\,\frac{d\,i\left(t\right)}{d\,t} + v\left(t\right) - v_{oc} = 0$$

$$i\left(t\right) = C\,\frac{d\,v\left(t\right)}{d\,t}$$

$$R_t\,C\,\frac{d\,v\left(t\right)}{d\,t} + L\,C\,\frac{d^2\,v\left(t\right)}{d\,t^2} + v\left(t\right) = v_{oc} \quad \Rightarrow \quad \frac{d^2\,v\left(t\right)}{d\,t^2} + \frac{R_t}{L}\,\frac{d\,v\left(t\right)}{d\,t} + \frac{1}{L\,C}\,v\left(t\right) = \frac{v\left(t\right)}{L\,C}$$

Finally,

$$\frac{d^2 v(t)}{d t^2}+\frac{R_1 R_2}{L\left(R_1+R_2(1+b)\right)} \frac{d v(t)}{d t}+\frac{1}{L C} v(t)=\frac{v(t)}{L C}$$

P 9.2-11 The input to the circuit shown in Figure P 9.2-11 is the voltage of the voltage source, $v_s(t)$. The output is the voltage $v_2(t)$. Derive the second-order differential equation that shows how the output of this circuit is related to the input.

Hint: Use the direct method.

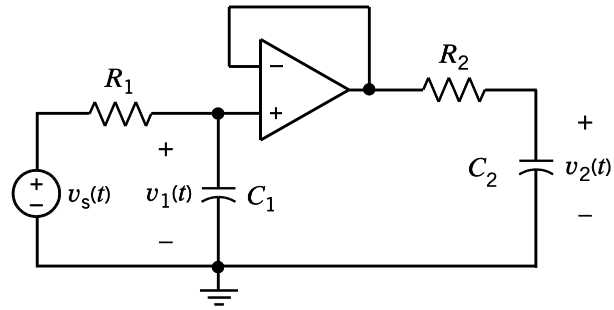


Figure P 9.2-11

Solution:
KCL gives

$$\frac{v_s(t)-v_1(t)}{R_1}=C_1 \frac{d}{dt} v_1(t) \quad \Rightarrow \quad v_s(t)=R_1 C_1 \frac{d}{dt} v_1(t)+v_1(t)$$

and

$$\frac{v_1(t)-v_2(t)}{R_2}=C_2 \frac{d}{dt} v_2(t) \quad \Rightarrow \quad v_1(t)=R_2 C_2 \frac{d}{dt} v_2(t)+v_2(t)$$

Substituting gives

$$v_s(t)=R_1 C_1 \frac{d}{dt}\left[R_2 C_2 \frac{d}{dt} v_2(t)+v_2(t)\right]+R_2 C_2 \frac{d}{dt} v_2(t)+v_2(t)$$

so

$$\frac{1}{R_1 R_2 C_1 C_2} v_s(t)=\frac{d^2}{dt^2} v_2(t)+\left(\frac{1}{R_1 C_1}+\frac{1}{R_2 C_2}\right) v_2(t)+\frac{1}{R_1 R_2 C_1 C_2} v_2(t)$$

P 9.3-1 Find the characteristic equation and its roots for the circuit of Figure P 9.2-2.

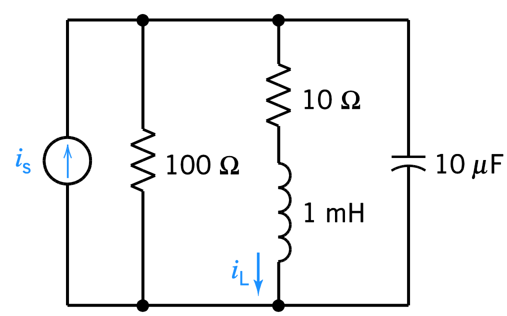


Figure P 9.2-2

Solution:
From Problem P 9.2-2 the characteristic equation is

$$\underline{1.1\times10^8 + 11000s + s^2 \;=\; 0} \quad \Rightarrow \quad s_1,\, s_2 = \frac{-11000 \pm \sqrt{(11000)^2 - 4(1.1\times10^8)}}{2} \;=\; \underline{-5500 \pm j8930}$$

P 9.3-2 Find the characteristic equation and its roots for the circuit of Figure P 9.3-2.

Answer: $s^2 + 400s + 3 \times 10^4 = 0$
roots: $s = -300, -100$

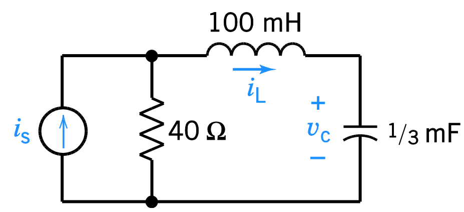
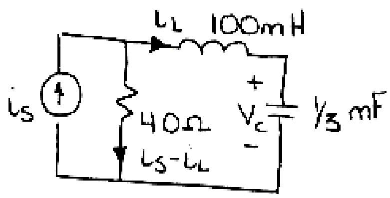


Figure P 9.3-2

Solution:



$$\text{KVL: } 40\,(i_s - i_L) = 100\times10^{-3}\frac{di_L}{dt} + v_c$$

$$i_L \;=\; i_c \;=\; \left(\frac{1}{3}\times10^{-3}\right)\frac{dv_c}{dt}$$

$$i_L \;=\; \frac{40}{3}\times10^{-3}\frac{di_s}{dt} - \frac{40}{3}\times10^{-3}\frac{di_L}{dt} - \frac{100}{3}\times10^{-6}\frac{d^2 i_L}{dt^2}$$

$$\frac{d^2 i_L}{dt^2} + 400\frac{di_L}{dt} + 30000 i_L \;=\; 400\frac{di_s}{dt}$$

$$\underline{s^2 + 400s + 30000 \;=\; 0} \quad \Rightarrow \quad (s+100)(s+300) \;=\; 0 \quad \Rightarrow \quad \underline{s_1 = -100, \quad s_2 = -300}$$

P 9.3-3 Find the characteristic equation and its roots for the circuit shown in Figure P 9.3-3.

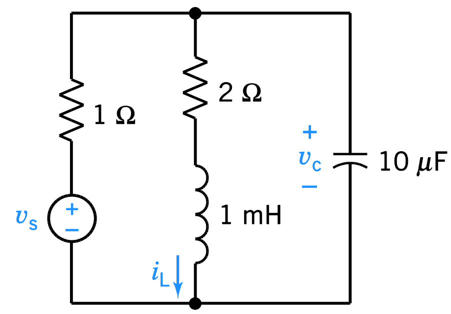
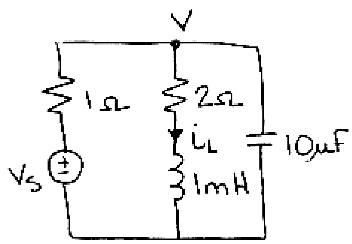


Figure P 9.3-3

Solution:



$$\text{KCL:} \quad \frac{v - v_s}{1} + i_L + 10\times10^{-6}\frac{dv}{dt} = 0$$

$$\text{KVL:} \quad v = 2i_L + 10^{-3}\frac{di_L}{dt}$$

$$0 = 2i_L + 10^{-3}\frac{di_L}{dt} - v_s + i_L + 10\times10^{-6}\cdot 2\frac{di_L}{dt} + 10\times10^{-6}\times10^{-3}\frac{d^2 i_L}{dt}$$

$$v_s = 3i_L + .00102\frac{di_L}{dt} + 1\times10^{-8}\frac{d^2 i_L}{dt^2}$$

$$\frac{d^2 i_L}{dt} + 102000\frac{di_L}{dt} + 3\times10^{-8} i_L = 1\times10^{8}\, v_s$$

$$\underline{s^2 + 102000s + 3\times10^8 = 0}, \quad \therefore \underline{s_1 = 3031, \; s_2 = -98969}$$

P 9.4-5 The circuit shown in Figure P 9.4-5 is used to detect smokers in airplanes who surreptitiously light up before they can take a single puff. The sensor activates the switch, and the change in the voltage $v(t)$ activates a light at the flight attendant's station. Determine the natural response $v(t)$.

Answer: $v(t) = -1.16e^{-2.7t} + 1.16e^{-37.3t}$ V

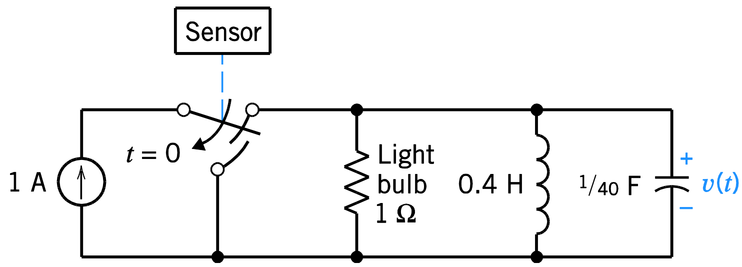
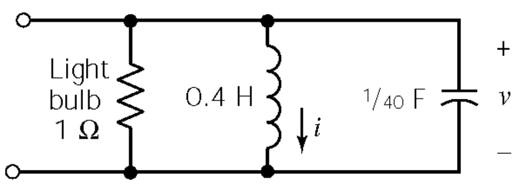


Figure P 9.4-5

Solution:



$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s^2 + 40s + 100 = 0$$

$$s = -2.7\ ,\ \ -37.3$$

The initial conditions are $v(0) = 0$, $i(0)\ 1$ A .

$$v_n = A_1e^{-2.7t} + A_2e^{-37.3t}\ ,\ \ v(0) = 0 = A_1 + A_2 \qquad (1)$$

$$\text{KCL at } t = 0^+ \text{ yields : } \frac{v(0^+)}{1} + i(0^+) + \frac{1}{40}\frac{dv(0^+)}{dt} = 0$$

$$\therefore \frac{dv(0^+)}{dt} = -40v(0^+) - 40i(0^+) = -40(1) = -2.7A_1 - 37.3A_2 \qquad (2)$$

from (1) and (2) $\Rightarrow A_1 = -1.16\ ,\ A_2 = 1.16$

So $v(t) = v_n(t) = -1.16e^{-2.7t} + 1.16e^{-37.3t}$

P 9.5-3 Police often use stun guns to incapacitate potentially dangerous felons. The hand-held device provides a series of high-voltage, low-current pulses. The power of the pulses is far below lethal levels, but it is enough to cause muscles to contract and put the person out of action. The device

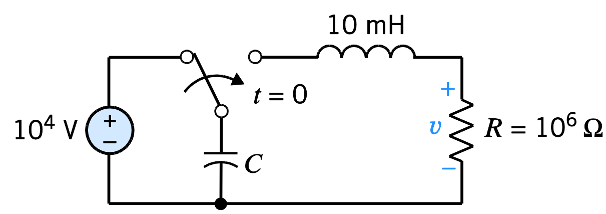


Figure P 9.5-3

provides a pulse of up to 50,000 V, and a
current of 1 mA
flows through an arc. A model of the circuit for one period is shown in Figure P 9.5-3. Find $v(t)$ for $0 < t < 1$ ms. The resistor R represents the spark gap. Select C so that the response is critically damped.

Solution:

Assume steady − state at $t = 0^-$ $\therefore v_c(0^-) = 10^4$ V & $i_L(0^-) = 0$

$\underline{t > 0}$

$$\text{KVL a: } -v_c + .01\frac{di_L}{dt} + 10^6 i_L = 0 \qquad (1)$$

$$\text{Also } : i_L = -C\, {dv_c}/{dt} = -C\left[.01\frac{d^2 i_L}{dt^2} + 10^6\frac{di_L}{dt}\right] \qquad (2)$$

$$\therefore \quad 0.01C\frac{d^2 i_L}{dt^2} + 10^6 C\frac{di_L}{dt} + i_L = 0$$

$$\text{Characteristic eq. } \Rightarrow 0.01C\, s^2 + 10^6 s + 1 = 0 \Rightarrow s = \frac{-10^6 C \pm \sqrt{\left(10^6 C\right)^2 - 4(.01C)}}{2(.01C)}$$

for critically damped: $10^{12}C^2 - .04C = 0$

$$\Rightarrow C = 0.04 \text{ pF} \quad \therefore s = -5\times10^7, -5\times10^7$$

So $i_L(t) = A_1 e^{-5\times10^7 t} + A_2 t e^{-5\times10^7 t}$

Now from (1) $\Rightarrow \frac{di_L}{dt}(0^+) = 100\left[v_c(0^+) - 10^6 i_L(0^+)\right] = 10^6\, A/s$

So $i_L(0) = 0 = A_1$ and $\frac{di_L(0)}{dt} = 10^6 = A_2 \quad \therefore i_L(t) = 10^6 t e^{-5\times10^7 t}$ A

Now $\underline{v(t) = 10^6 i_L(t) = 10^{12} t e^{-5\times10^7 t} \text{ V}}$

P 9.6-4 The natural response of a parallel *RLC* circuit is measured and plotted as shown in Figure P 9.6-4. Using this chart, determine an expression for $v(t)$.

Hint: Notice that $v(t)$ = 260 mV at t = 5 ms and that $v(t)$ = – 200 mV at t = 7.5 ms. Also, notice that the time between the first and third zero-crossings is 5 ms.

Answer:

$v(t) = 544e^{-276t} \sin 1257t$ V

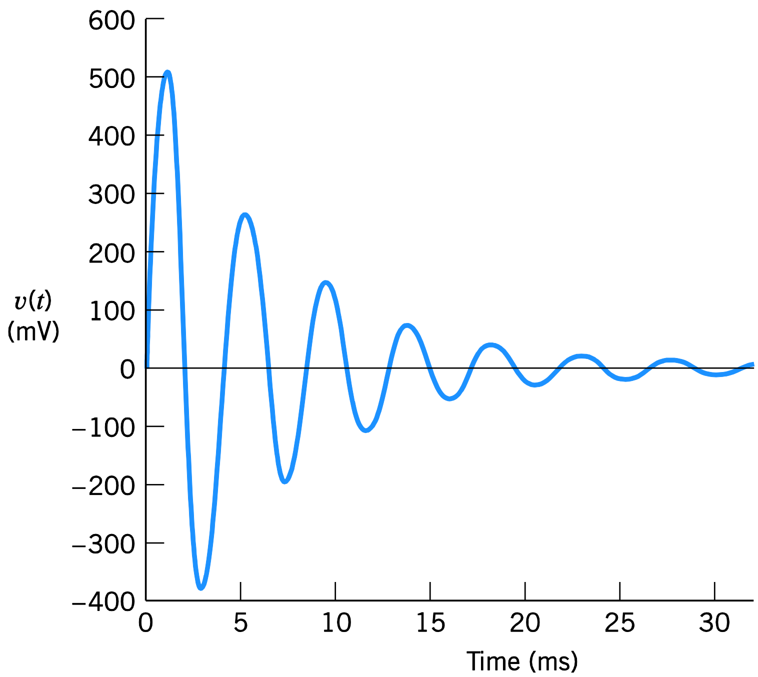


Figure P 9.6-4

Solution:

The response is underdamped so

$$\therefore\; v(t) = e^{-\alpha t}\left[k_1 \cos\,\omega t + k_2 \sin \omega t\;\right] + k_3$$

$$v(\infty) = 0 \;\Rightarrow\; k_3 = 0,\;\; v(0) = 0 \;\Rightarrow\; k_1 = 0$$

$$\therefore\; v(t) = k_2 e^{-\alpha t} \sin \omega t$$

From Fig. P 9.6-4

$$t \approx 5\text{ms} \;\leftrightarrow\; v \approx 260\text{mV} \quad (\text{max})$$
$$t \approx 7.5\text{ms} \leftrightarrow v \approx -200 \text{ mV (min)}$$

$\therefore$ distance between adjacent maxima is $\approx \omega = \dfrac{2\pi}{T} = 1257 \;\text{rad}/\text{s}$

so

$$0.26 = k_2 e^{-\alpha\,(.005)} \sin\left(1257\,(.005)\right) \qquad (1)$$

$$-0.2 = k_2 e^{-\alpha\,(.0075)} \sin\left(1257\,(.0075)\right) \qquad (2)$$

Dividing (1) by (2) gives

$$-1.3 = e^{\alpha\,(0.0025)}\left(\frac{\sin\,(6.29\;rad)}{\sin\,(9.43\;rad)}\right) \;\Rightarrow\; e^{0.0025\,\alpha} = 1.95 \;\Rightarrow\; \alpha = 267$$

From (1) $k_2 = 544$ so

$$\underline{v(t) = 544e^{-267t}\sin 1257t} \qquad (\text{approx. answer})$$