4. Circuit Theorems

- Source Transformations
- Linearity and the Principle of Superposition
- Thévenin's Theorem
- Norton's Theorem
- Maximum Power Transfer Theorem

Motivation

- Circuit theorems can greatly simplify circuit analysis and provide insight into the operation of circuits
- For example: What are the consequences of a circuit being composed of linear circuit elements?

Today's Outline

4. Circuit Theorems

- Practical Sources
- Source Transformations

Practical Sources

It is commonly observed in practical sources that:

– power is dissipated when a load is attached to the source (for example, a battery warms up when discharged)

– the voltage from a practical voltage source decreases as current is drawn from the source (as from an audio amplifier)

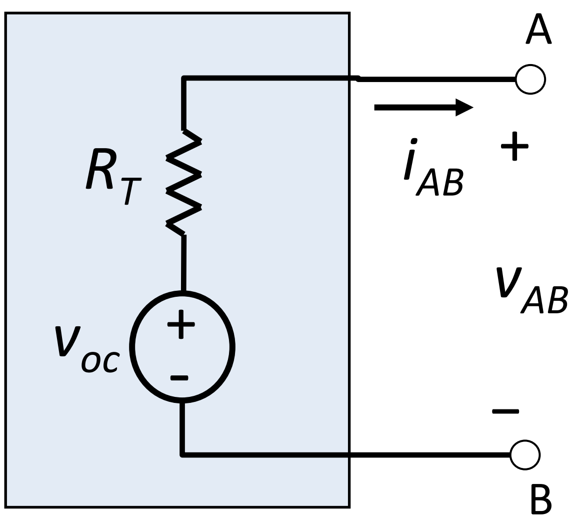
– the current from a practical current source decreases as voltage develops across the terminals of the source (as from a solar cell)

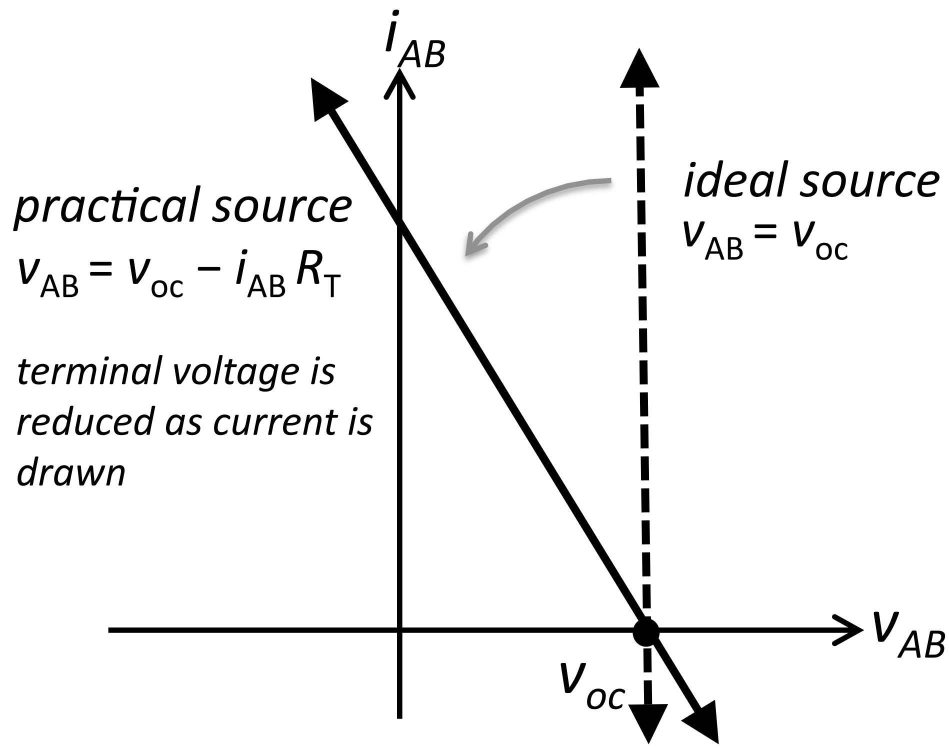
While the origin and nature of these effects can be very complex, the Thévenin and Norton equivalent circuits are very useful models for practical sources.

McGill

Practical Voltage Source

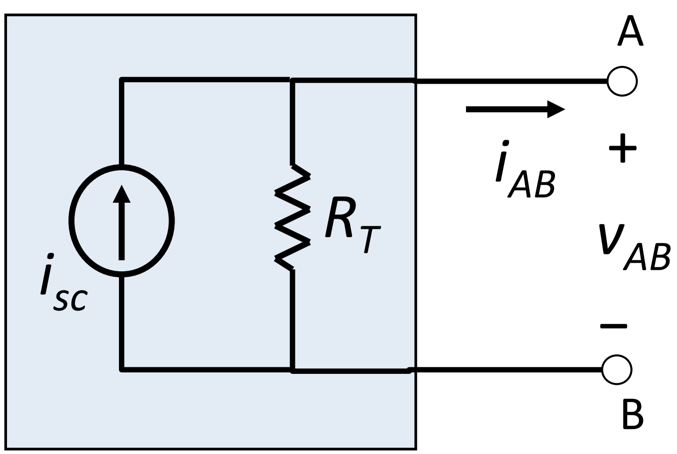
A practical voltage source can often be modeled as a **Thévenin circuit**: a voltage source v_{oc} called the "**open circuit voltage**" in series with a resistance R_T, called the "**Thévenin resistance**." An ideal voltage source is recovered as R_T -> 0Ω (short).

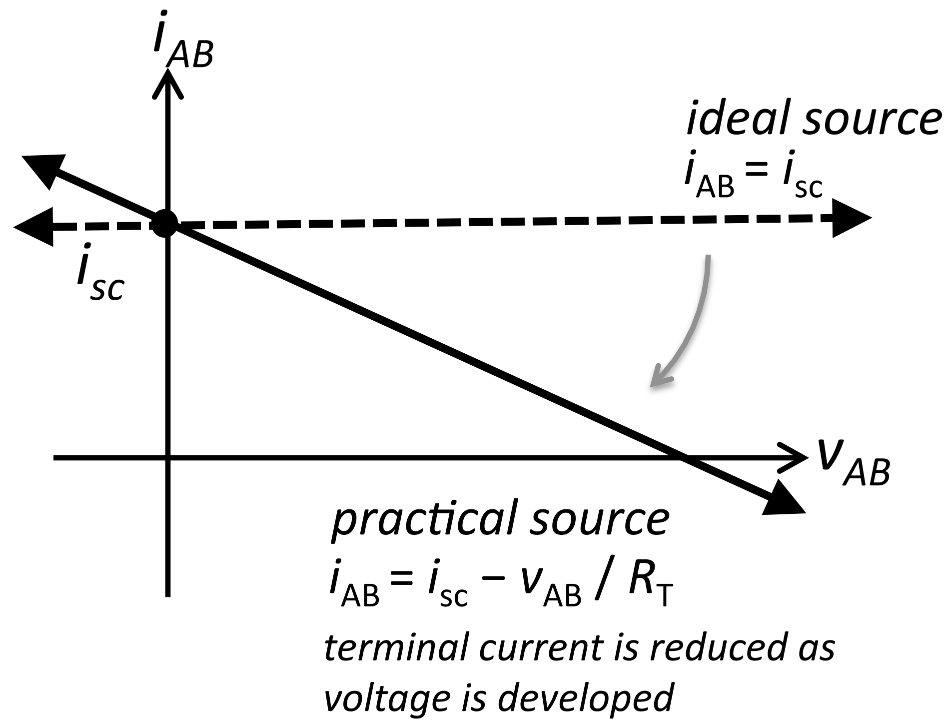




Practical Current Source

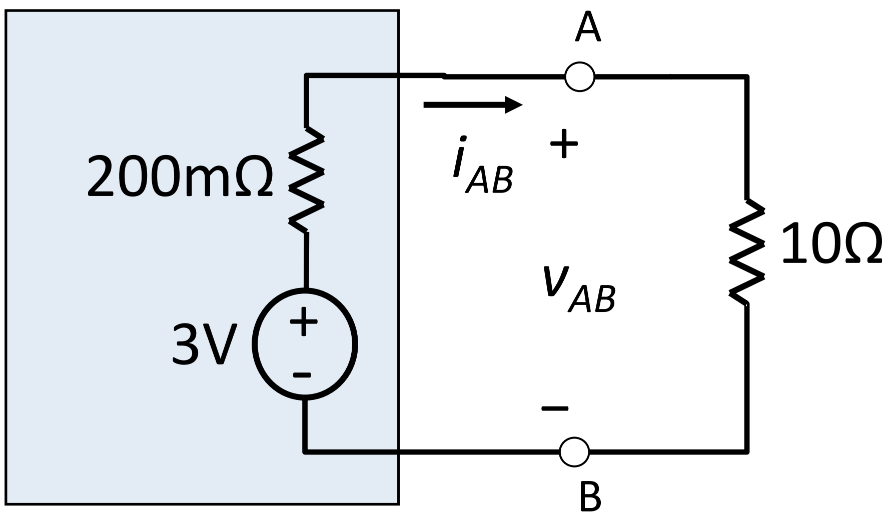
A practical current source can often be modeled as a **Norton circuit:** an independent current source i_{sc} called the "**short circuit current**" in parallel with a resistance R_T also called the "**Thévenin resistance**." An ideal current source is recovered as $R_T \rightarrow \infty$ (open).





Example 1

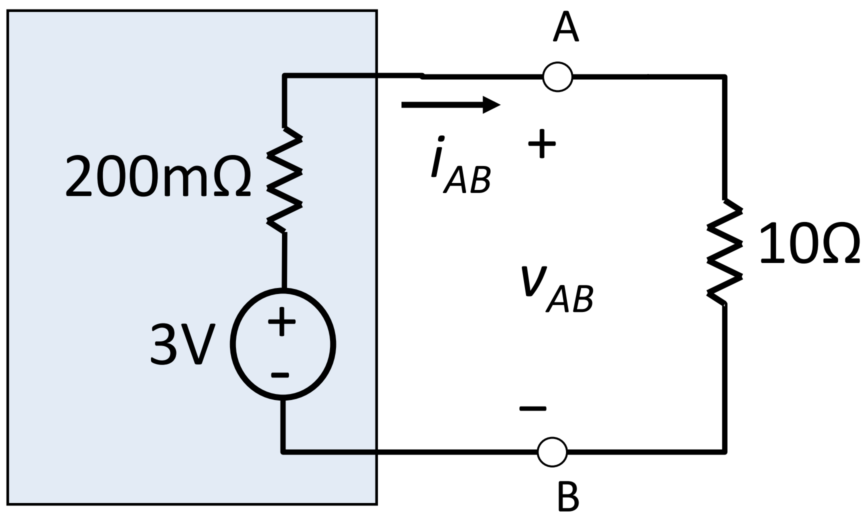
A portable battery is characterized by an open circuit voltage of 3V. The internal (Thévenin) resistance is known to be 200mΩ. If a resistive load with 10Ω equivalent resistance is attached to the battery, what voltage is applied to the load?



Strategy:

– apply the voltage divider equation

Example 1



Voltage divider:

$$\frac{v_{AB}}{3V} = \frac{10\Omega}{10\Omega + 0.2\Omega}$$

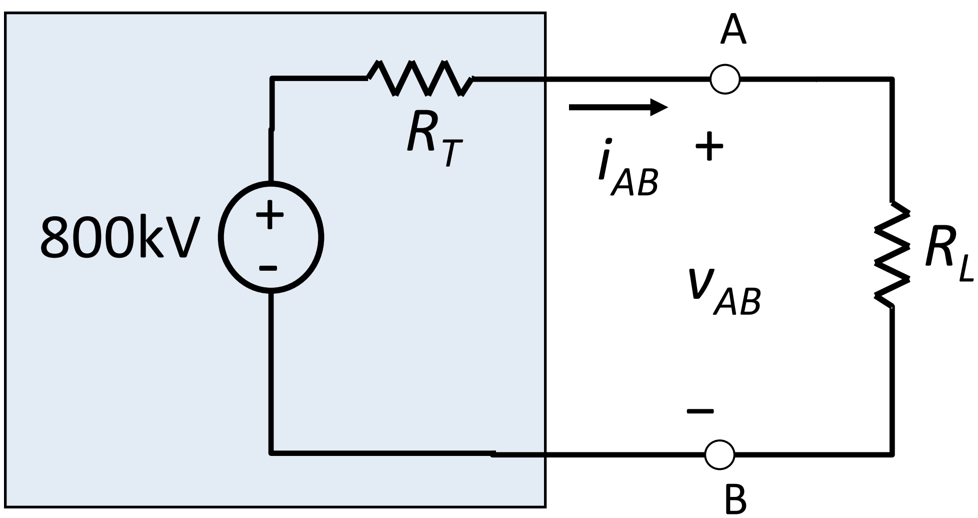
$$v_{AB} = 0.980 \cdot 3V$$

$$= 2.94V$$

this is a 2% drop in voltage

Example 2

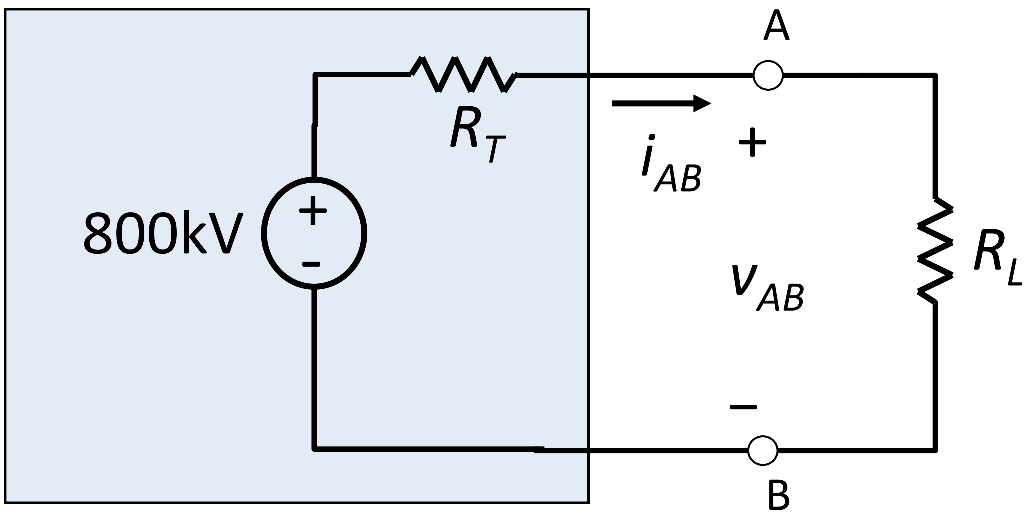
An HVDC (high-voltage direct-current) power supply line is driven by an 800kV source with 2Ω internal resistance. Of the total 5GW produced by the HVDC source, 10% of the power is lost in internal resistance, including a 3000km transmission line to the load. What is the resistance of the load, the resistance of the transmission lines, and the voltage at the load?



Strategy:

– use power and voltage to find resistance

Example 2



$$P_{del} = i_{AB} \cdot 800\text{kV}$$

$$i_{AB} = \frac{5\text{GW}}{800\text{kV}} = 6.25\text{kA}$$

$$P_{load} = i_{AB}^2 \cdot R_L$$

$$R_L = \frac{0.9 \times 5\text{GW}}{(6.25\text{kA})^2}$$

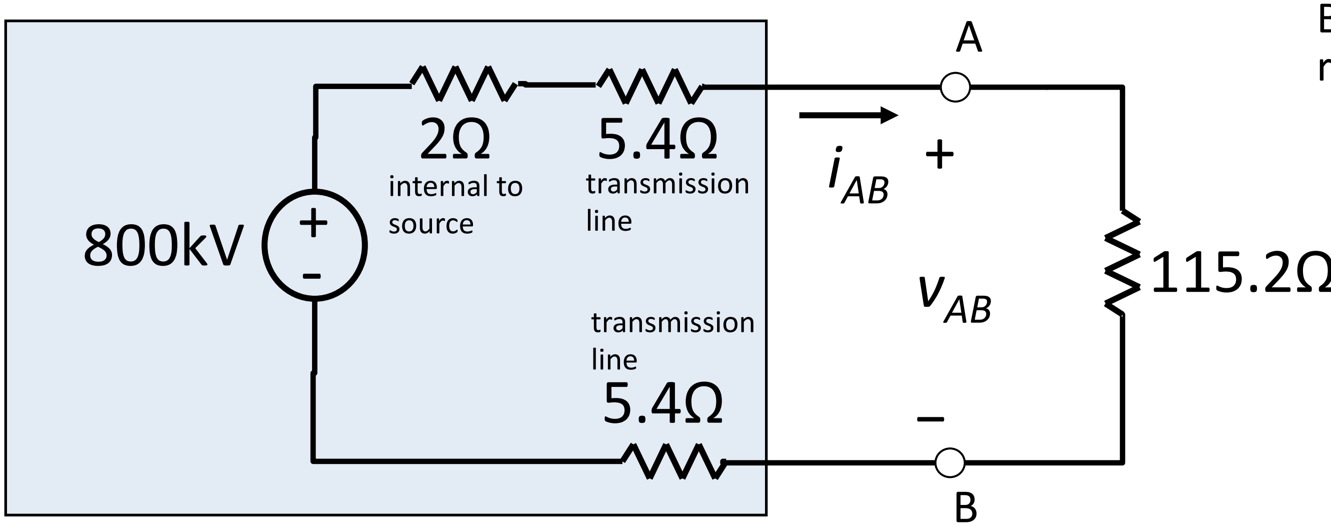
$$= 115.2\Omega$$

$$P_{\text{internal}} = i_{AB}^2 \cdot R_T$$

$$R_T = \frac{0.1 \times 5\text{GW}}{(6.25\text{kA})^2}$$

$$= 12.8\Omega$$

Example 2



Breakdown of internal resistance contributions:

$$R_T = R_{source} + 2R_{line}$$
$$= 2\Omega + 2R_{line}$$
$$R_{line} = \frac{12.8 - 2\Omega}{2}$$
$$= 5.4\Omega$$

Voltage divider:

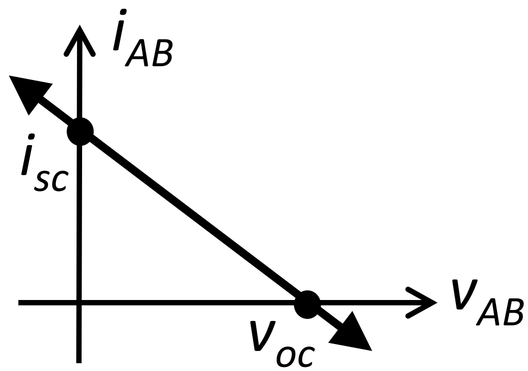
$$\frac{v_{AB}}{800\text{kV}} = \frac{115.2\Omega}{12.8\Omega + 115.2\Omega}$$
$$v_{AB} = 0.900 \cdot 800\text{kV}$$
$$= 720\text{kV}$$

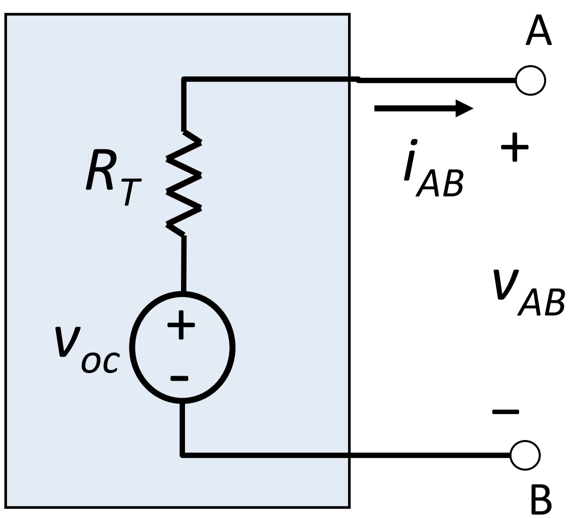
this is a 10% drop in voltage

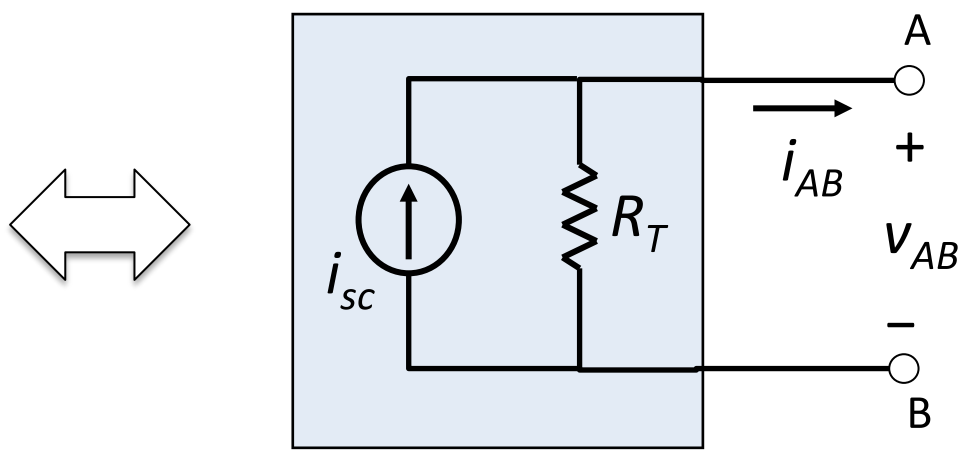
Source Transformation

Source Transformation: A Thévenin circuit and a Norton circuit are equivalent when their i_{AB}-v_{AB} diagrams are identical:

$$\boxed{v_{oc} = i_{sc} R_T}$$



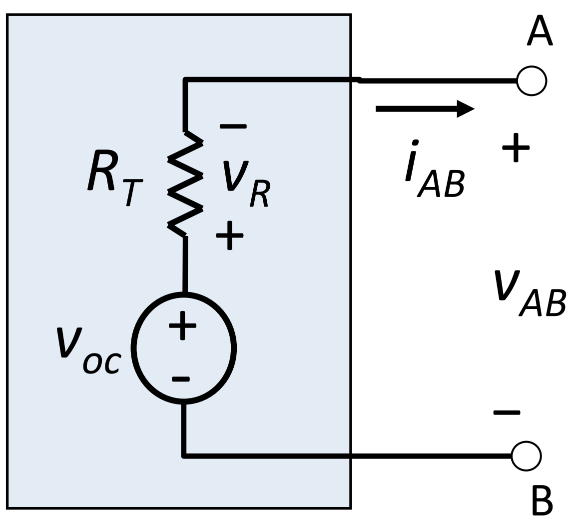




Source Transformation

Proof: Show that the terminal equations relating v_{AB} and i_{AB} are identical for appropriately chosen component values. In other words, the i_{AB}-v_{AB} diagrams are identical.

First analyze the Thévenin circuit:



KVL: $0 = -v_{oc} + v_R + v_{AB}$

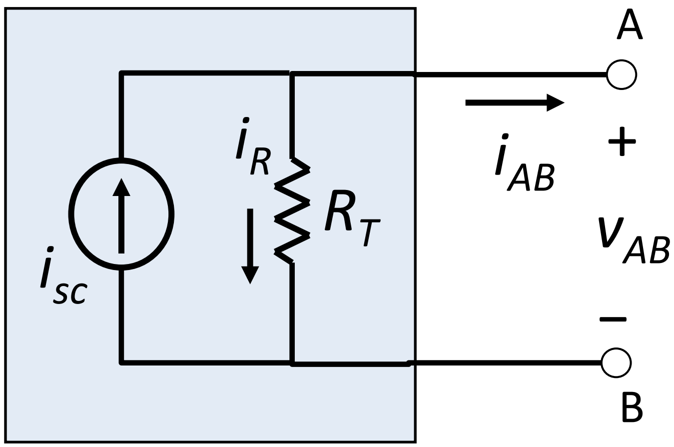
Ohm: $v_R = i_{AB} R_T$

Combining the above:

$v_{AB} = v_{oc} - i_{AB} R_T$

Source Transformation

Next analyze the Norton circuit:



KCL: $0 = -i_{sc} + i_R + i_{AB}$

Ohm: $i_R = v_{AB} / R_T$

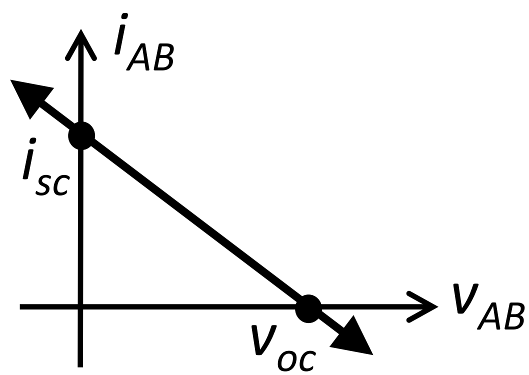
Combining the above:

$i_{AB} = i_{sc} - v_{AB} / R_T$

Comparing the two circuit terminal laws:

$v_{AB} = v_{oc} - i_{AB} R_T$

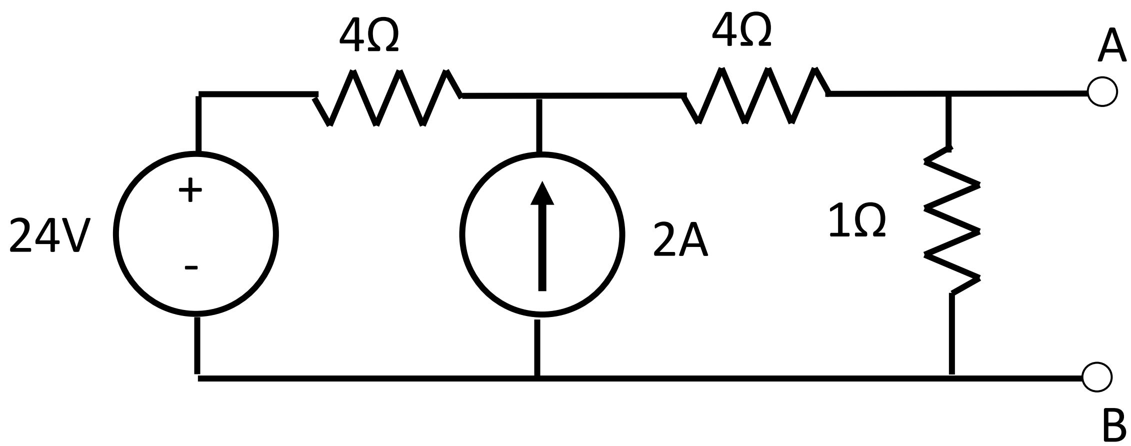
$i_{AB} = i_{sc} - v_{AB} / R_T$ -> $v_{AB} = i_{sc} R_T - i_{AB} R_T$



The two circuits are thus equivalent when $v_{oc} = i_{sc} R_T$.

Example

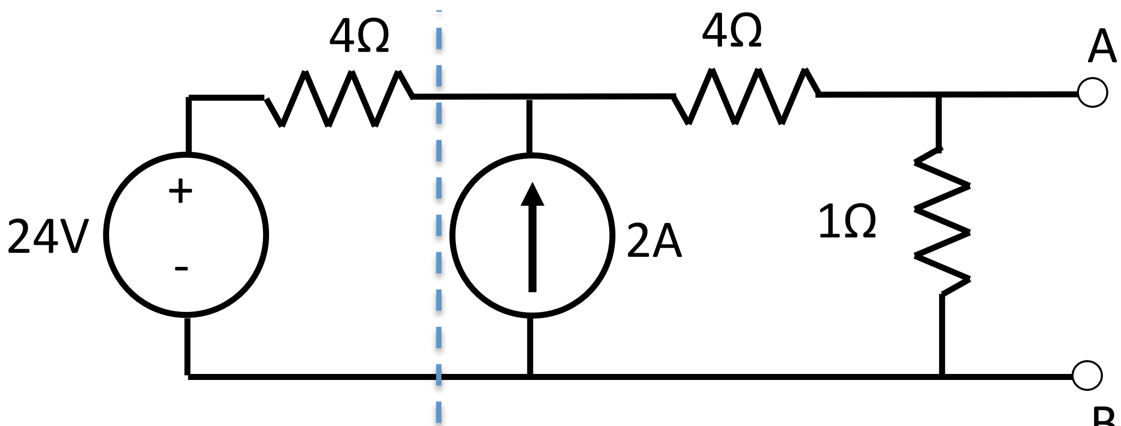
Reduce the following circuit to a single Norton equivalent circuit with respect to the terminals A and B.

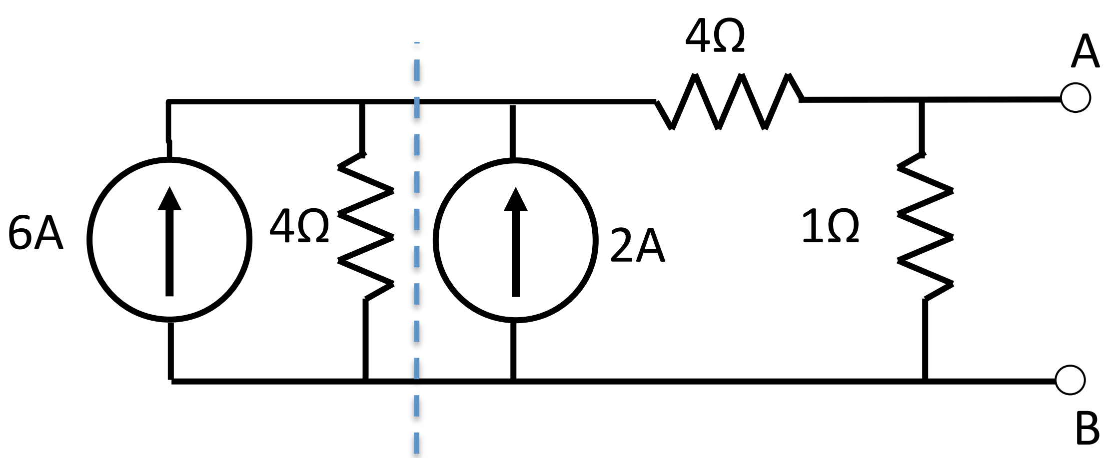


Strategy:

– use transformations between Thévenin and Norton equivalent circuits, working from left to right

Example





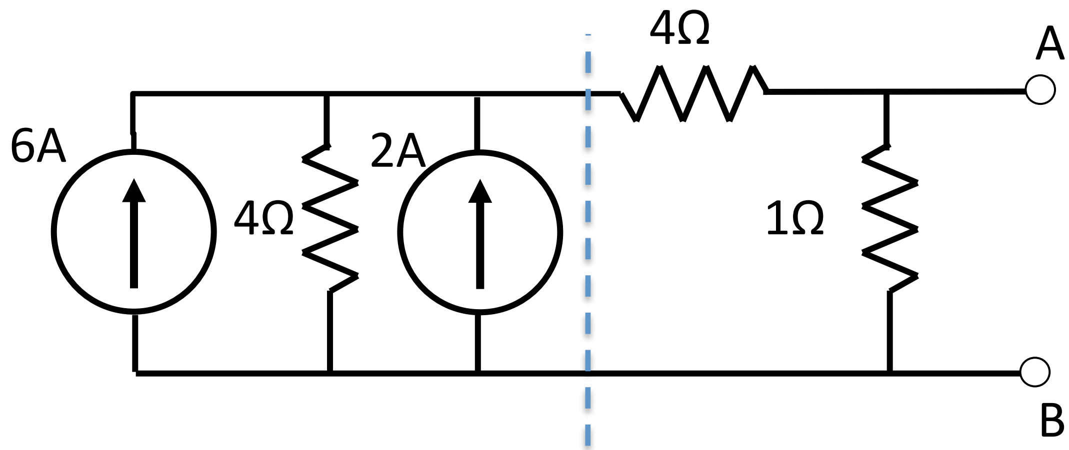
Transform Thévenin circuit on the left into a Norton circuit.

$i_{sc} = v_{oc} / R_T$

$= 24V/4\Omega$

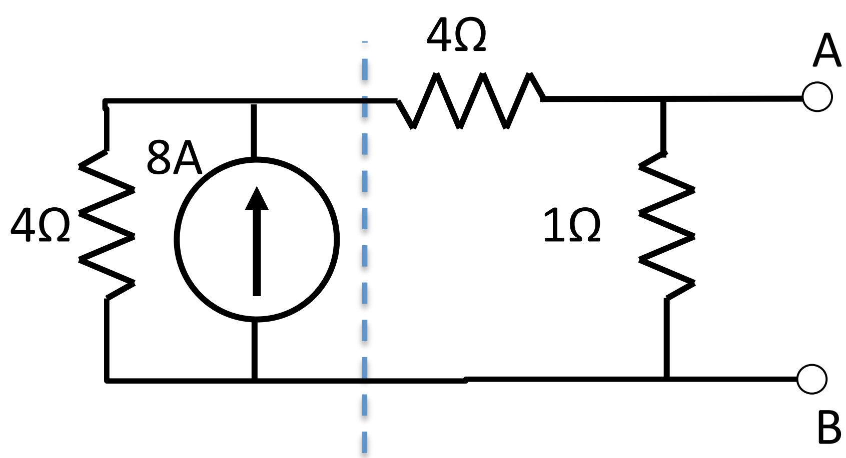
$= 6A$

Example

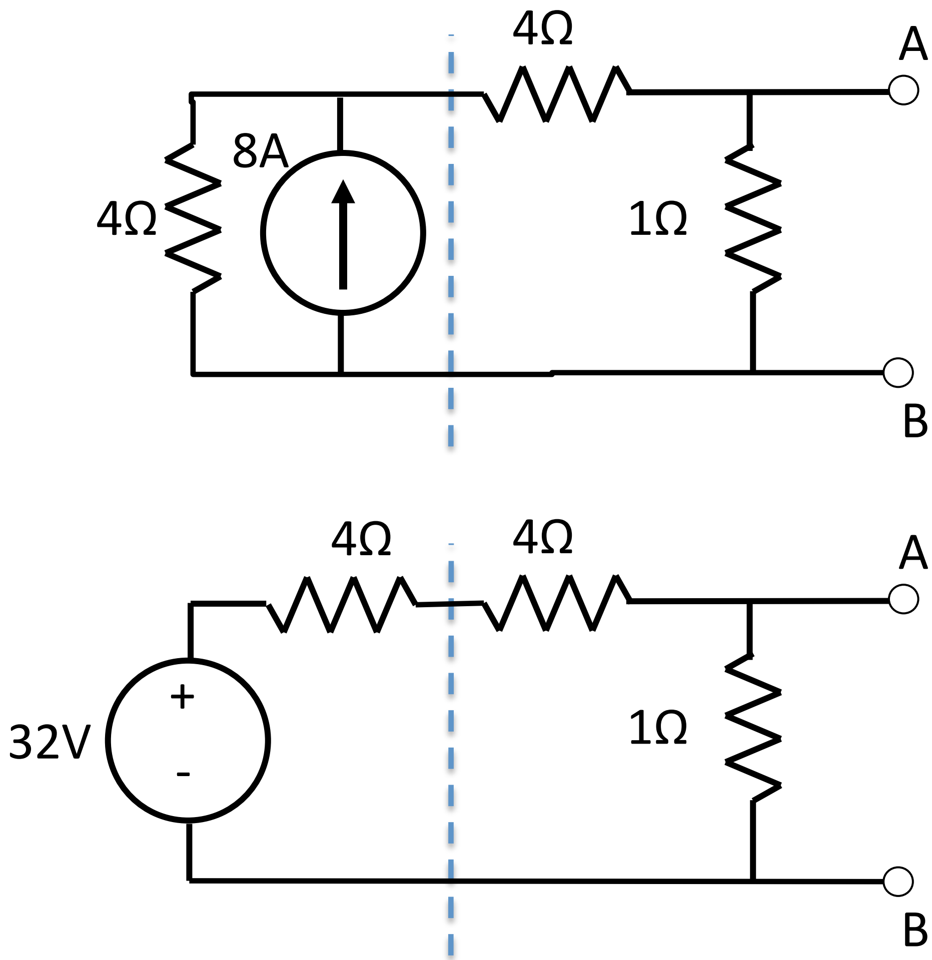


Transform circuit on the left into a Norton circuit.

i_{sc}= 2A + 6A = 8A



Example



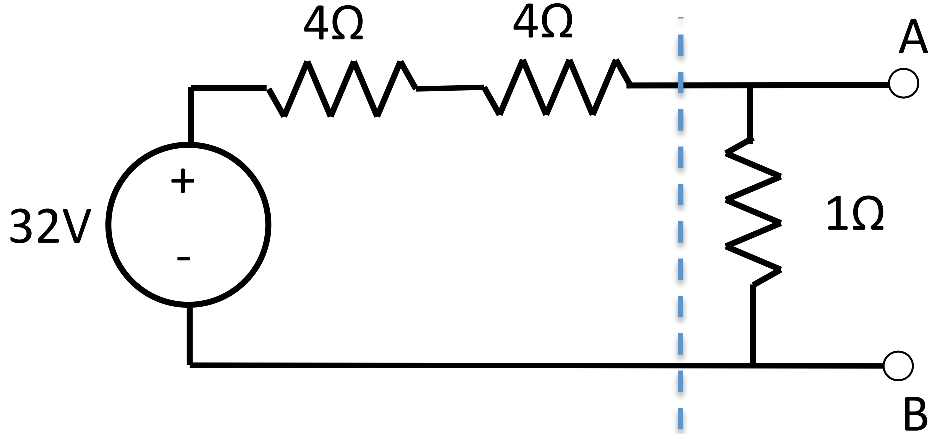
Transform circuit on the left into a Thévenin circuit.

$v_{oc} = i_{sc} R_T$

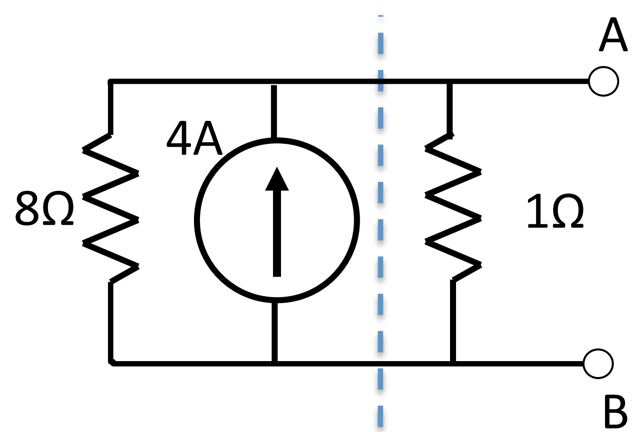
$= 8A\ 4\Omega$

$= 32\ V$

Example



Use series equivalent resistance and transform circuit on the left into a Norton circuit.



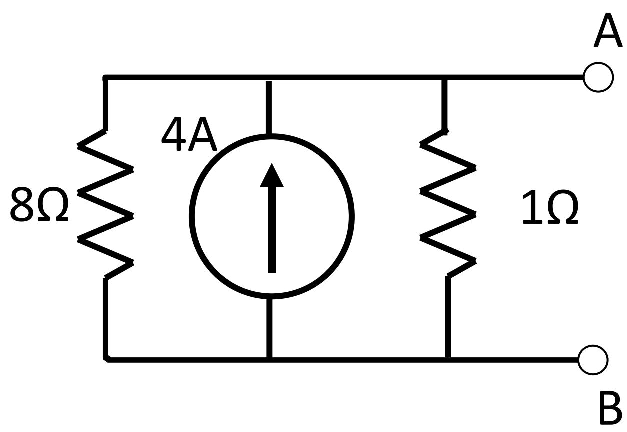
$R_T = 4\Omega + 4\Omega = 8\Omega$

$i_{sc} = v_{oc} / R_T$

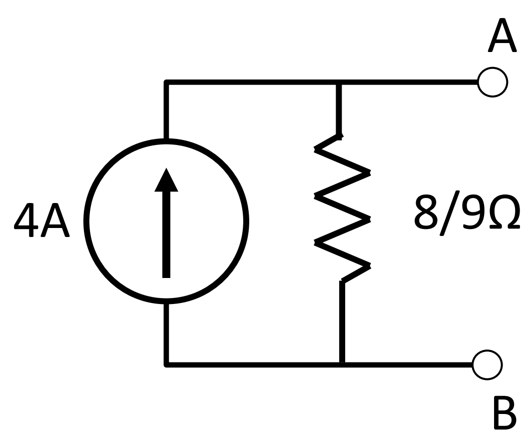
$= 32V/8\Omega$

$= 4A$

Example



Use parallel equivalent resistance to create a Norton circuit.



$$R_T = 8\Omega \,||\, 1\Omega$$

$$= \frac{8\Omega \cdot 1\Omega}{8\Omega + 1\Omega} = \frac{8}{9}\Omega$$