ELECTRIC CIRCUITS 1

ECSE 200

24 APRIL 2018, 14h00

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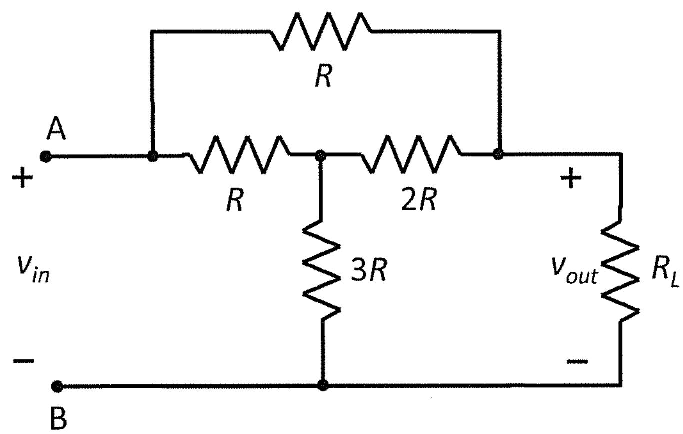
STUDENT NAME:		McGILL ID:									

INSTRUCTIONS:

EXAM:	CLOSED BOOK
	PRINTED ON BOTH SIDES OF THE PAGE
	ANSWER IN BOOKLET , EXTRA BOOKLETS PERMITTED
	THE EXAM SHOULD BE RETURNED
CRIB SHEETS:	NOT PERMITTED
DICTIONARIES:	NONE
CALCULATORS:	PERMITTED (Non-Programmable)
ANY SPECIAL INSTRUCTIONS:	NA

1. Consider the circuit below. Answer the questions. [12 pts]



a) What is the physical law at the origin of Kirchoff's current law? [1pt]

b) What is the physical law at the origin of Kirchoff's voltage law? [1pt]

c) What is the equivalent resistance between nodes A and B if R_L is an open circuit? [2pts]

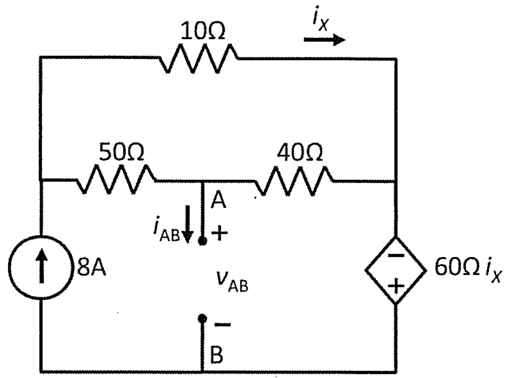
d) Assume v_{in} = 1V and R_L is an open circuit. What is the total power absorbed by the resistors in the circuit? [2pts]

e) What is the equivalent resistance between nodes A and B if R_L is a short circuit? [2pts]

f) What is the equivalent resistance between nodes A and B if $R_L = R$? [3pts]

g) What is the ratio v_{out}/v_{in} if $R_L = R$? [1pt]

2. Consider the circuit below. Answer the questions. [12 pts]



a) What is Thévenin's theorem? [1pt]

b) Write the equation that relates the quantities v_{oc}, i_{sc} and R_T. [1pt]

c) What is the open circuit voltage v_{oc} at the terminals A and B? [2pts]

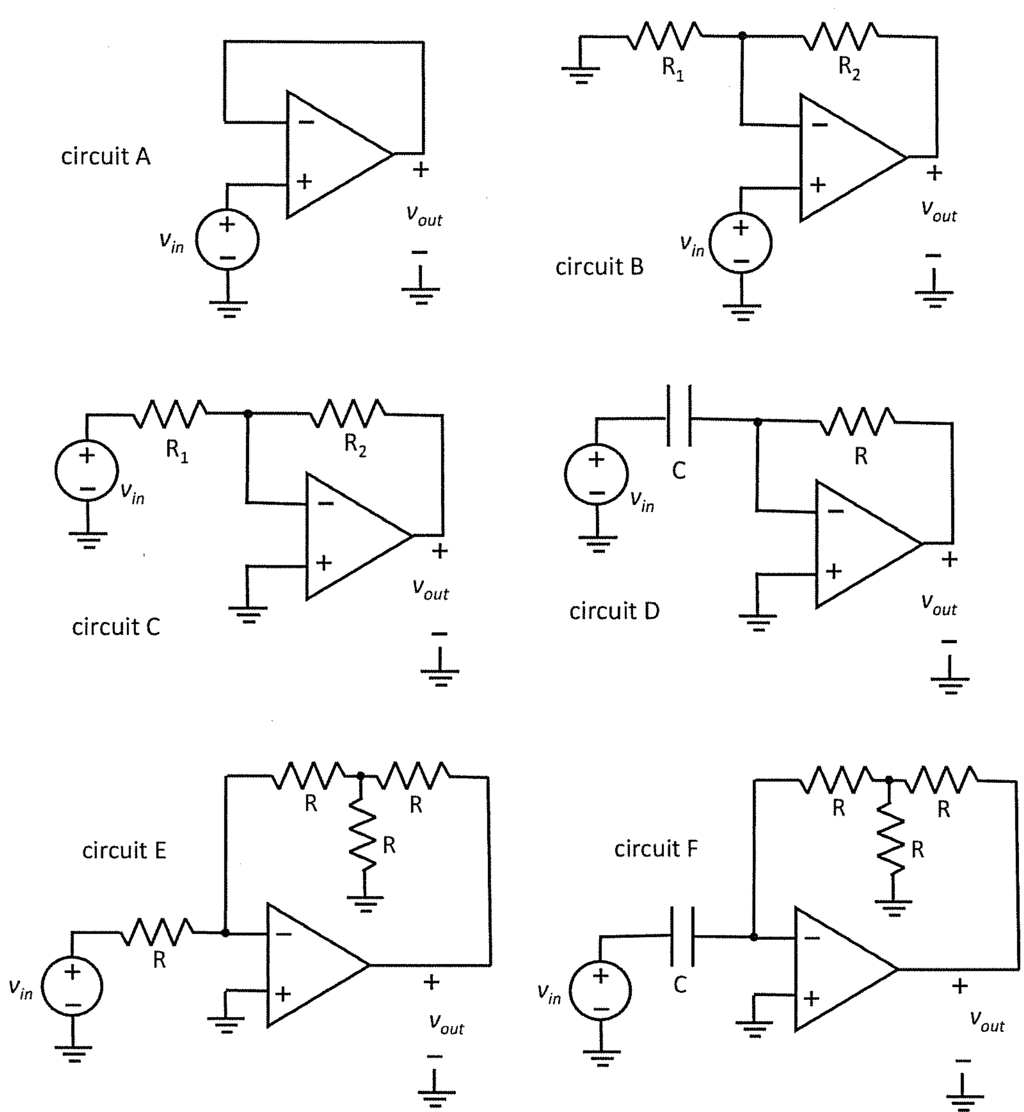
d) What is the short circuit current i_{sc} at the terminals A and B? [2pts]

e) Draw the i_{AB} versus v_{AB} diagram for the circuit. Label your axes. [2pts]

f) Draw the Thévenin equivalent circuit with respect to the terminals A and B. [2pts]

g) What is the maximum power that the circuit can deliver to an optimally chosen load resistor attached to the nodes A and B? [2pts]

3. Consider the circuits below. Assume ideal op-amp behaviour. Answer the questions. [12 pts]



a) Express v_{out} in terms of v_{in} for circuit A. [2pts]

b) Express v_{out} in terms of v_{in} for circuit B. [2pts]

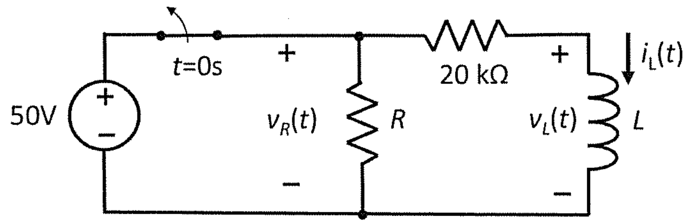
c) Express v_{out} in terms of v_{in} for circuit C. [2pts]

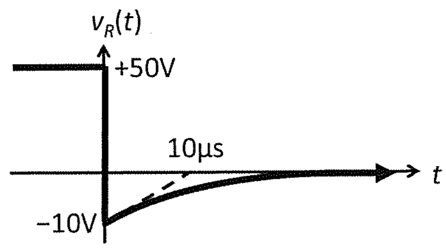
d) Express v_{out} in terms of v_{in} for circuit D. [2pts]

e) Express v_{out} in terms of v_{in} for circuit E. [2pts]

f) Express v_{out} in terms of v_{in} for circuit F. [2pts]

4. Consider the circuit below. The circuit is in dc steady state for $t < 0$ with the switch closed. The switch opens instantaneously at $t = 0$s. The circuit reaches a new dc steady state as $t \to \infty$. The voltage $v_R(t)$ versus time t is plotted below. Answer the questions. [12 pts]





a) Give one reason why inductor current must be continuous versus time. [1pt]

b) What is $i_L(t)$ at $t = 0-$? [1pt]

c) What is $i_L(t)$ at $t = 0+$? [1pt]

d) What is $i_L(t)$ as $t \to \infty$? [1pt]

e) What is the value of R ? [2pts]

f) What is the value of L ? [2pts]

g) What is $i_L(t)$ for $t > 0$? [2pts]

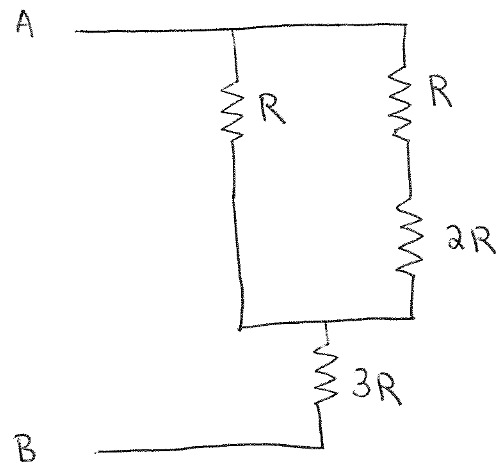
h) What is $v_L(t)$ for $t > 0$? [2pts]

end

1. a) conservation of charge [+1]

b) conservation of energy [+1]

c)

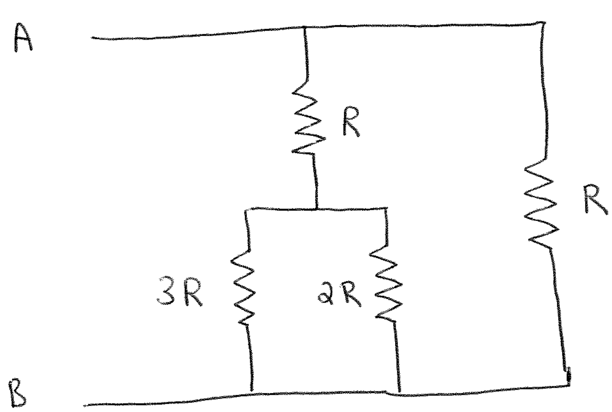


$$R_{AB} = 3R + R // (R + 2R) \quad [+1]$$
$$= 3R + \frac{3}{4}R$$
$$= 3\frac{3}{4}R \quad [+1]$$

d) $P_{abs} = (1V)^2 / R_{AB}$ [+1]

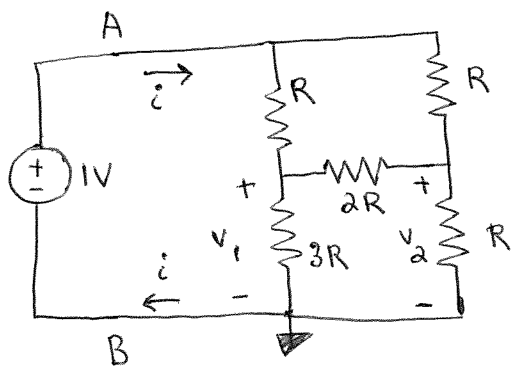
$= \frac{0.267}{R}$ W absorbed [+1]

e)



$$R_{AB} = R // (R + 3R // 2R) \quad [+1]$$
$$= R // \frac{11}{5}R$$
$$= \frac{11}{16}R \quad [+1]$$

f)



Apply test source. [+1]

$$0 = \frac{v_1}{3R} + \frac{v_1 - v_2}{2R} + \frac{v_1 - 1V}{R}$$

$$0 = \frac{v_2}{R} + \frac{v_2 - v_1}{2R} + \frac{v_2 - 1V}{R}$$

$$1 = \frac{11}{6} v_1 - \frac{1}{2} v_2$$

$$1 = -\frac{1}{2} v_1 + \frac{5}{2} v_2$$

$$\therefore \quad v_1 = \frac{\begin{vmatrix} 1 & -1/2 \\ 1 & 5/2 \end{vmatrix}}{\begin{vmatrix} 11/6 & -1/2 \\ -1/2 & 5/2 \end{vmatrix}} = 0.6923V$$

$$v_2 = \frac{\begin{vmatrix} 11/6 & 1 \\ -1/2 & 1 \end{vmatrix}}{\begin{vmatrix} 11/6 & -1/2 \\ -1/2 & 5/2 \end{vmatrix}} = 0.5385V$$

$$i = \frac{v_1}{3R} + \frac{v_2}{R}$$

$$= \frac{0.7693\,V}{R}$$

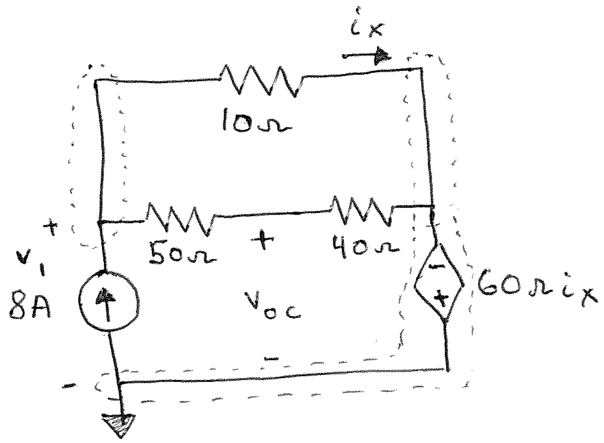
$$R_{AB} = \frac{1V}{i} \quad [+1]$$

$$= 1.300\,R \quad [+1]$$

g) $\frac{V_{out}}{V_{in}} = 0.5385$ [+1]

2. a) Any two-terminal circuit composed of resistors, independent sources, and dependent sources is equivalent to a voltage source in series with a resistor. [+1]

b) $V_{oc} = i_{sc} \cdot R_T$ [+1]

c) 

Apply an open circuit. [+1]

$$\frac{v_1 + 60\Omega \cdot i_x}{10\Omega} + \frac{v_1 + 60\Omega i_x}{90\Omega} - 8A = 0$$

$$i_x = \frac{v_1 + 60\Omega \cdot i_x}{10\Omega}$$

$$8A = \frac{1}{9\Omega} \cdot v_1 + 6\tfrac{2}{3} \cdot i_x$$

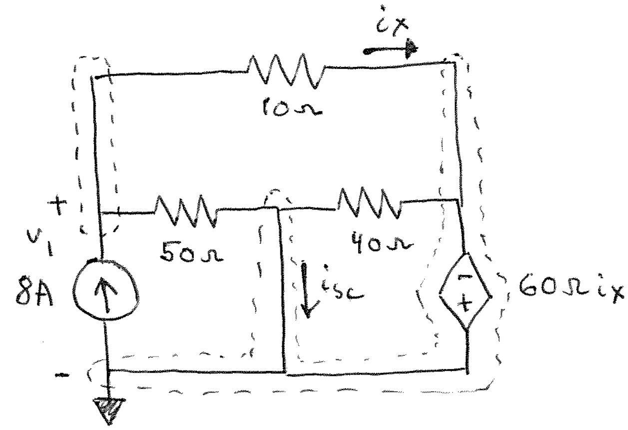
$$0 = \frac{1}{10\Omega} \cdot v_1 + 5 \cdot i_x$$

$\therefore$

$$v_1 = \frac{\begin{vmatrix} 8 & 6\frac{2}{3} \\ 0 & 5 \end{vmatrix}}{\begin{vmatrix} 1/9 & 6\frac{2}{3} \\ 1/10 & 5 \end{vmatrix}} = -360V \qquad i_x = \frac{\begin{vmatrix} 1/9 & 8 \\ 1/10 & 0 \end{vmatrix}}{\begin{vmatrix} 1/9 & 6\frac{2}{3} \\ 1/10 & 5 \end{vmatrix}} = 7.200A$$

$$V_{oc} = v_1 - 50\Omega \cdot \left(\frac{v_1 + 60\Omega \cdot i_x}{90\Omega}\right) = -400V \quad [+1]$$

d)



Apply a short circuit. [+1]

$$\frac{v_1 + 60\Omega \cdot i_x}{10\Omega} + \frac{v_1}{50\Omega} - 8A = 0$$

$$i_x = \frac{v_1 + 60\Omega \cdot i_x}{10\Omega}$$

$$8A = \frac{3}{25\Omega} \cdot v_1 + 6 \cdot i_x$$

$$0 = \frac{1}{10\Omega} \cdot v_1 + 5 \cdot i_x$$

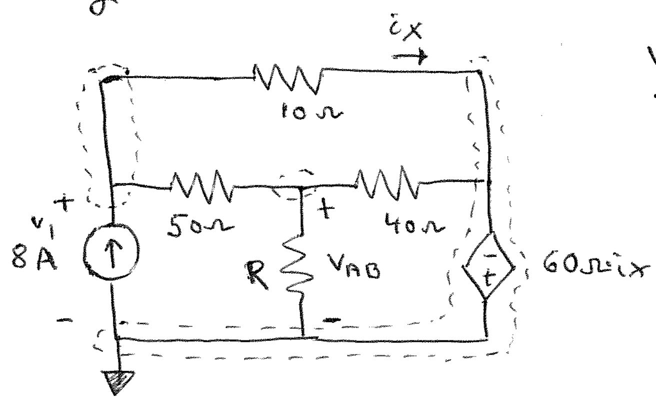
$\therefore$

$$v_1 = \frac{\begin{vmatrix} 8 & 6 \\ 0 & 5 \end{vmatrix}}{\begin{vmatrix} 3/25 & 6 \\ 1/10 & 5 \end{vmatrix}} = \text{undefined} \qquad i_x = \frac{\begin{vmatrix} 3/25 & 8 \\ 1/10 & 0 \end{vmatrix}}{\begin{vmatrix} 3/25 & 6 \\ 1/10 & 5 \end{vmatrix}} = \text{undefined}$$

If short-circuit conditions lead to no solution, the circuit must be behaving as a voltage source.

i_{sc} is undefined. [+1]

Easily confirmed:



$$\frac{v_1 + 60\Omega \cdot i_x}{10\Omega} + \frac{v_1 - V_{AB}}{50\Omega} - 8A = 0$$

$$i_x = v_1 + 60\Omega \cdot i_x$$

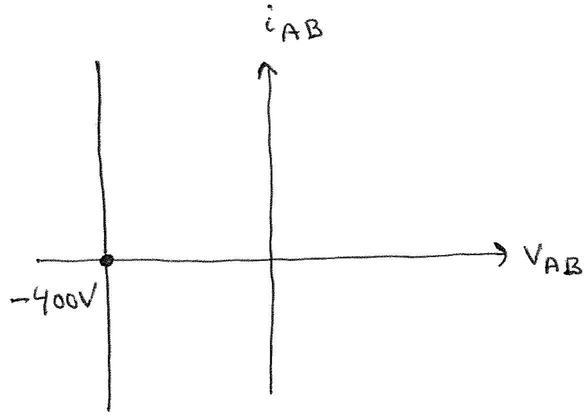
$$\therefore \quad i_x = -v_1/50\Omega$$

$$\frac{v_1 + 60\Omega \cdot (-v_1/50\Omega)}{10\Omega} + \frac{v_1 - V_{AB}}{50\Omega} - 8A = 0$$

$$-\frac{V_{AB}}{50\Omega} = 8A$$

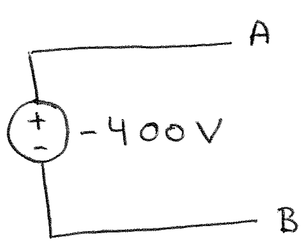
$$V_{AB} = -400\text{ V}$$

e)



[+1] for an i-v diagram
[+1] for correct diagram

f)



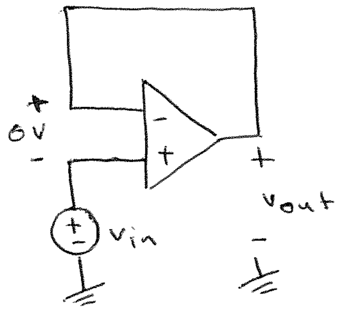
[+1] for a Thévenin circuit
[+1] for correct circuit

g) $P_{max} = \frac{V_{oc}}{2} \cdot \frac{i_{sc}}{2}$ [+1]

$\Rightarrow$ no limit because the Thévenin equivalent circuit is a voltage source. [+1]

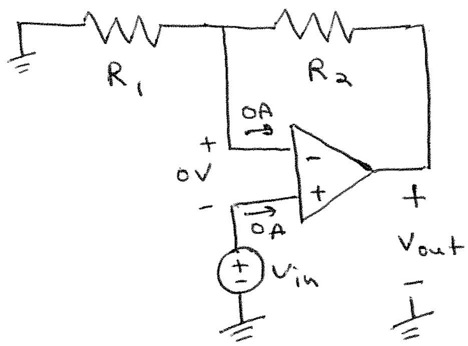
3.

a)



$$V_{out} = V_{in} \qquad [+2]$$

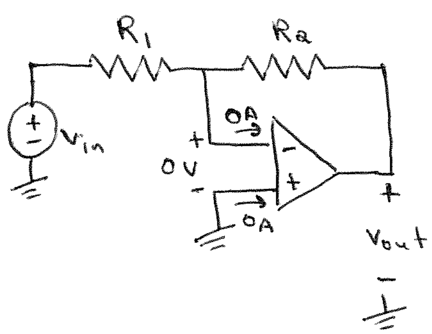
b)



$$\frac{V_{in}}{R_1} + \frac{V_{in} - V_{out}}{R_2} = 0$$

$$V_{out} = V_{in} \cdot \left(1 + \frac{R_2}{R_1}\right) \qquad [+2]$$

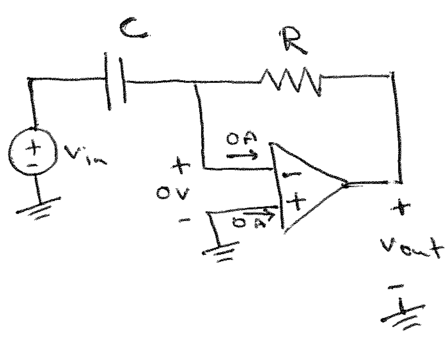
c)



$$\frac{0 - V_{in}}{R_1} + \frac{0 - V_{out}}{R_2} = 0$$

$$V_{out} = V_{in} \cdot \left(-\frac{R_2}{R_1}\right) \qquad [+2]$$

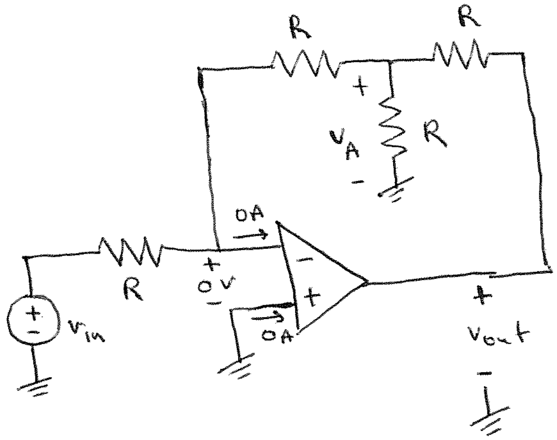
d)



$$C \frac{d}{dt}(0 - V_{in}) + \frac{0 - V_{out}}{R} = 0$$

$$V_{out} = -RC \cdot \frac{dV_{in}}{dt} \qquad [+2]$$

e)



$$\frac{0-v_{in}}{R} + \frac{0-v_A}{R} = 0$$

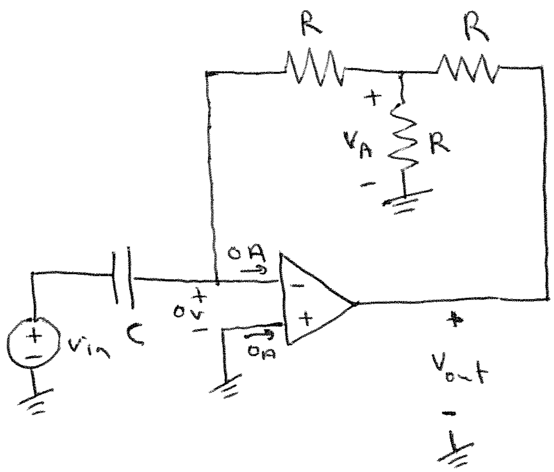
$$V_A = -V_{in}$$

$$\frac{V_A}{R} + \frac{V_A}{R} + \frac{V_A - V_{out}}{R} = 0$$

$$V_{out} = 3V_A$$

$$\therefore \quad V_{out} = -3\, v_{in} \qquad [+2]$$

f)



$$C\frac{d}{dt}(0-v_{in}) + \frac{0-v_A}{R} = 0$$

$$V_A = -RC\frac{dv_{in}}{dt}$$

$$\frac{V_A}{R} + \frac{V_A}{R} + \frac{V_A - V_{out}}{R} = 0$$

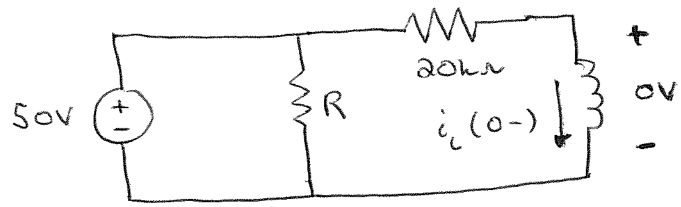
$$V_{out} = 3V_A$$

$$\therefore \quad V_{out} = -3RC\frac{dv_{in}}{dt} \qquad [+2]$$

4. a) finite voltage
finite power
conservation of energy

[+1]

b) $t < 0$

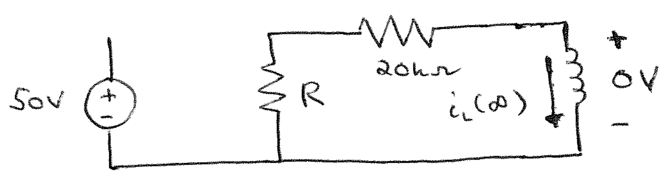


$i_L(0-) = \frac{50V}{20k\Omega}$

$= 2.5mA$ [+1]

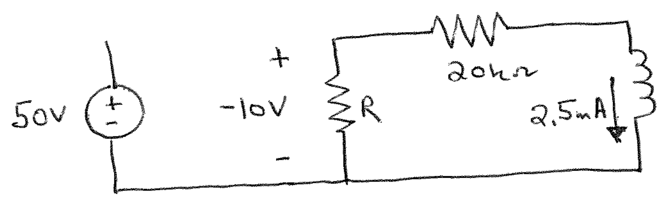
c) $i_L(0+) = i_L(0-) = 2.5mA$ [+1]

d) $t \to \infty$



$i_L(\infty) = 0A$ [+1]

e) $t = 0+$ [+1]



$(-10V) = -2.5mA \cdot R$

$R = 4k\Omega$ [+1]

f) $t > 0$

$R_T = 24k\Omega$

$\tau = \frac{L}{R_T}$ [+1]

$L = 10\mu s \cdot 24k\Omega = 240mH$ [+1]

g) $i_L(t) = i_L(\infty) + [i_L(0+) - i_L(\infty)] \exp(-t/\tau)$ [+1]

$= 2.5\text{mA} \exp(-t/10\,\mu\text{s})$ [+1]

h) $v_L(t) = L \frac{d}{dt} i_L(t)$ [+1]

$= 240\text{mH} \cdot \left(\frac{-1}{10\,\mu\text{s}}\right) \cdot 2.5\text{mA} \exp(-t/10\,\mu\text{s})$

$= -60\text{V} \exp(-t/10\,\mu\text{s})$ [+1]