Today's Outline

1. **Fundamentals**

- Energy
- Voltage
- Power

Electric Potential Energy

Electric Potential Energy = the **work** required to move a particle from a point A to a point B in the presence of **electric forces** *without any change in kinetic energy*

– SI unit is the Joule (abbreviated J)

$1\ \text{J} = 1\ \text{N m} = 1\ \text{kg m}^2/\text{s}^2$

– potential energy variables are usually given the symbol *U*

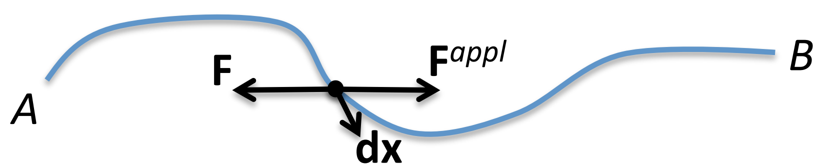
– work variables are usually given the symbol *W*

$$U_{AB} = W_{AB} = \int_A^B \mathbf{F}^{appl} \cdot \mathbf{dx} = -\int_A^B \mathbf{F} \cdot \mathbf{dx}$$

James Prescott Joule (1818 –1889)

Potential Energy Example

In the example below, we need to apply a force $\mathbf{F}^{appl}$ to push the particle from A to B against the force **F**. As we move the particle from A to B, we perform a work W_{AB} which is equal to the increase in the potential energy U_{AB} of the particle.



$$U_{AB} = W_{AB} = \int_A^B \mathbf{F}^{applied} \cdot \mathbf{dx} = -\int_A^B \mathbf{F} \cdot \mathbf{dx} > 0$$

What would happen if the direction of the force **F** was reversed?

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Electric Potential (Voltage)

Electric Potential = work *per unit charge* to move a particle in an electric field from a point A to a point B, *without any change in kinetic energy*

– also called "voltage"

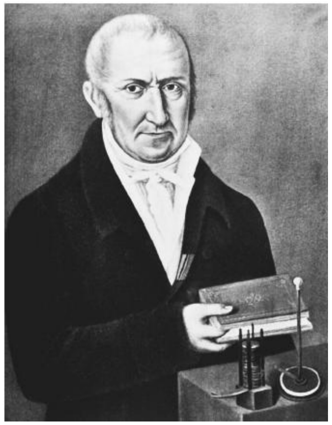
– voltage variables are usually given the symbol *V*

– SI unit is the Volt (abbreviated V)

1 V = 1 J / C

– the definition can be written:

$$V_{AB} = \frac{W_{AB}}{Q} \quad \rightarrow \quad W_{AB} = QV_{AB}$$



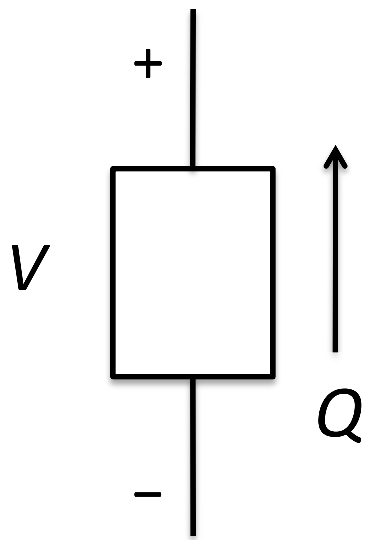
Alessandro Volta (1745-1827)

Voltage

In circuits, we often define voltage variables across circuit elements:

"−" terminal identifies the initial point A

"+" terminal identifies the final point B



Passing a charge Q from "−" to "+" requires work:

$$W = QV$$

The increase in potential energy of the charge Q as it is passed from "−" to "+" is:

$$U = W = QV$$

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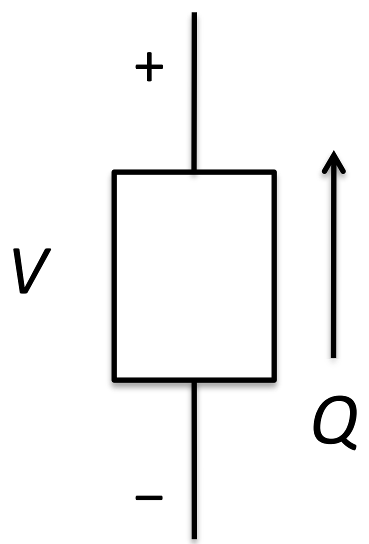
Voltage

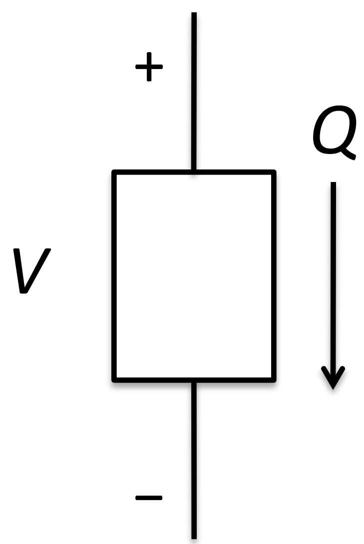
The voltage variable *V* can be positive or negative; the meaning of the sign is easily understood in terms of potential energy.



V	Q	$U=W=QV$
+	+	+ potential energy of the charge is increased, circuit element ***delivers*** energy to the charge
−	+	− potential energy of the charge is decreased, circuit element ***absorbs*** energy from the charge

Voltage

When the direction of charge movement is reversed, from “+” to “−”, we reverse the flow of energy (similar to rolling downhill versus rolling uphill).



V	Q	$U=W=-QV$
+	+	− potential energy of the charge is decreased, circuit element ***absorbs*** energy from the charge
−	+	+ potential energy of the charge is increased, circuit element ***delivers*** energy to the charge

Important: Both the ***definition*** and ***value*** of variables V and Q are required to determine the physical situation.

Power

Power = rate of change in energy, *per unit time*

– power is usually given the symbol P

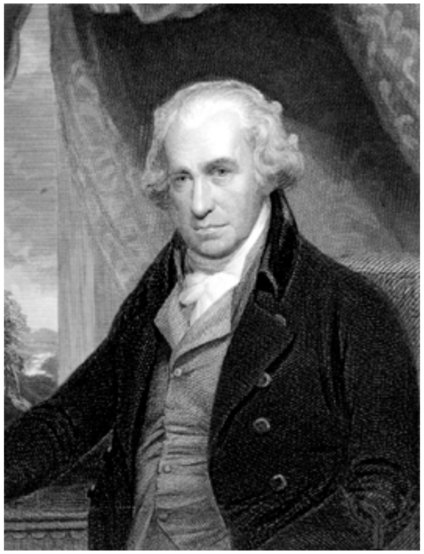
– SI unit is the Watt (abbreviated W)

1 W = 1 J / s

– the definition can be written:

$$P = \pm \frac{dU}{dt} = \pm \frac{dQ}{dt}\frac{dU}{dQ} = \pm IV$$

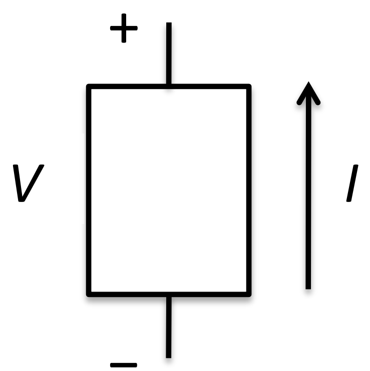
where the sign depends on whether we are calculating the rate at which energy is *delivered* or *absorbed* by a circuit element, and on the relative *orientation of current and voltage variable definitions*



James Watt
(1736–1819)

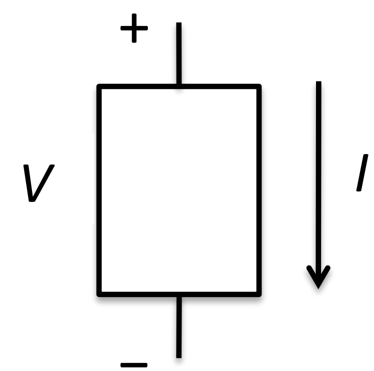
Power

To understand the role of V and I variable definitions, consider the following two situations (see slides 6 and 7).



$P = IV$

= power delivered by circuit element to charges



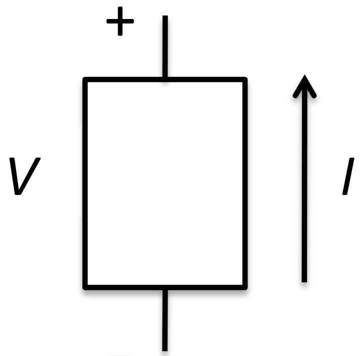
$P = IV$

= power absorbed by circuit element from charges

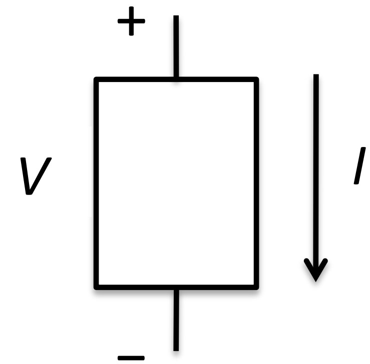
In each case, I, V, and thus P can be either positive of negative. The power absorbed or delivered by the element is completely determined by I and V.

Power

An analogy to power in hydraulics can be made.

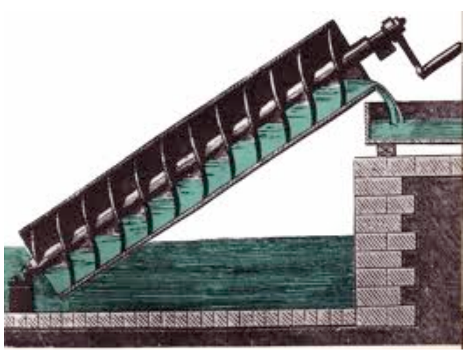


$P = IV$
= power delivered by element to charges



$P = IV$
= power absorbed by element from charges

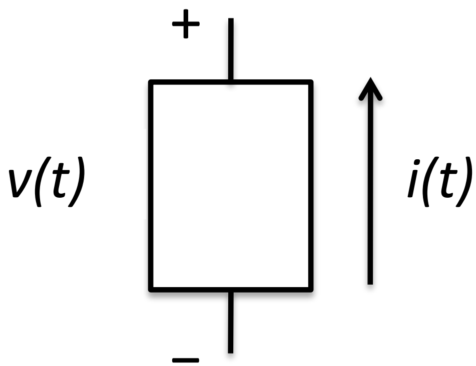
power delivered by screw to water



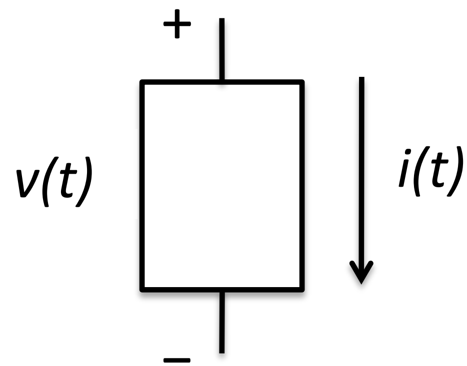
power absorbed by wheel from water

Instantaneous Power

If the voltage *v(t)* and current *i*(*t*) at the terminals of a circuit element are functions of time, then the **instantaneous power** delivered or absorbed by the element is $p(t) = i(t)v(t)$.



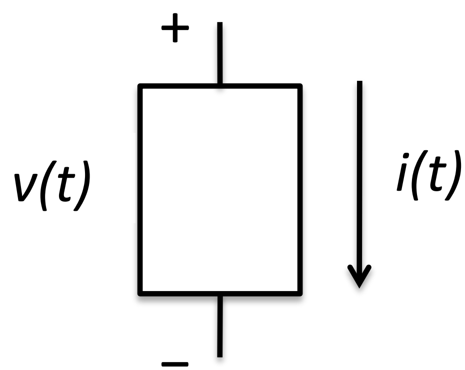
$p(t) = i(t)v(t)$
= power delivered by circuit element at time t



$p(t) = i(t)v(t)$
= power absorbed by circuit element at time t

Passive Sign Convention

Passive sign convention: convention for defining the voltage variable $v(t)$ and the current variable $i(t)$ at the terminals of a circuit element such that the $i(t)$ reference direction flows from "+" to "−".



With the passive sign convention:

$p(t) = i(t)v(t) > 0$ corresponds to power absorption by the element

$p(t) = i(t)v(t) < 0$ corresponds to power delivery by the element

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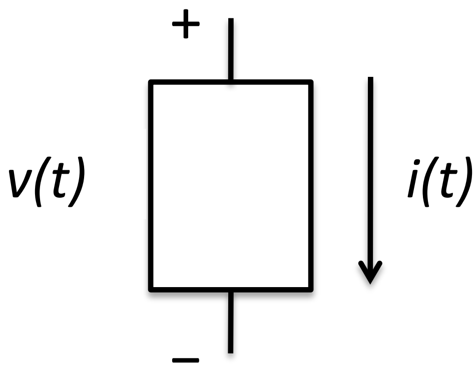
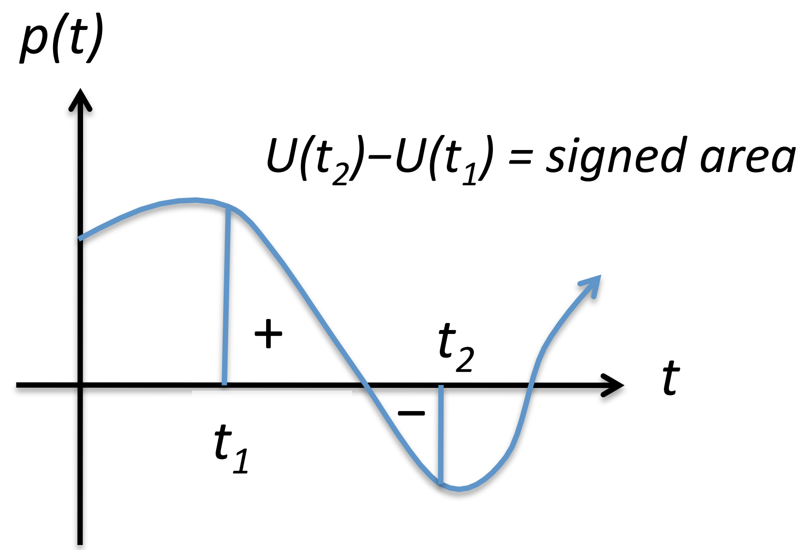
Energy from Instantaneous Power

The energy delivered or absorbed over a time interval from t_1 to t_2 is:

$$U(t_2) - U(t_1) = \int_{t_1}^{t_2} p(t')\,dt' = \int_{t_1}^{t_2} i(t')v(t')\,dt'$$

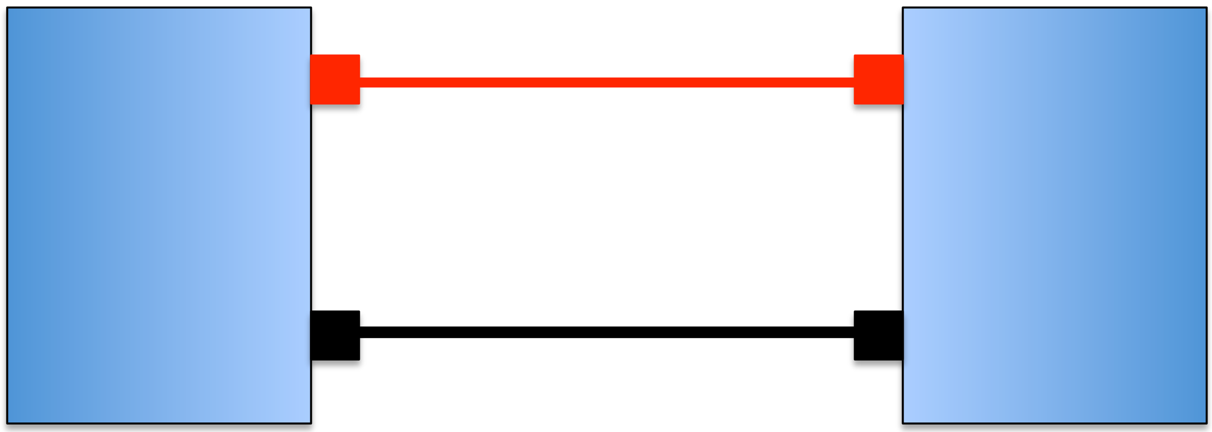


$p(t) = i(t)v(t)$
= power absorbed by circuit element at time t



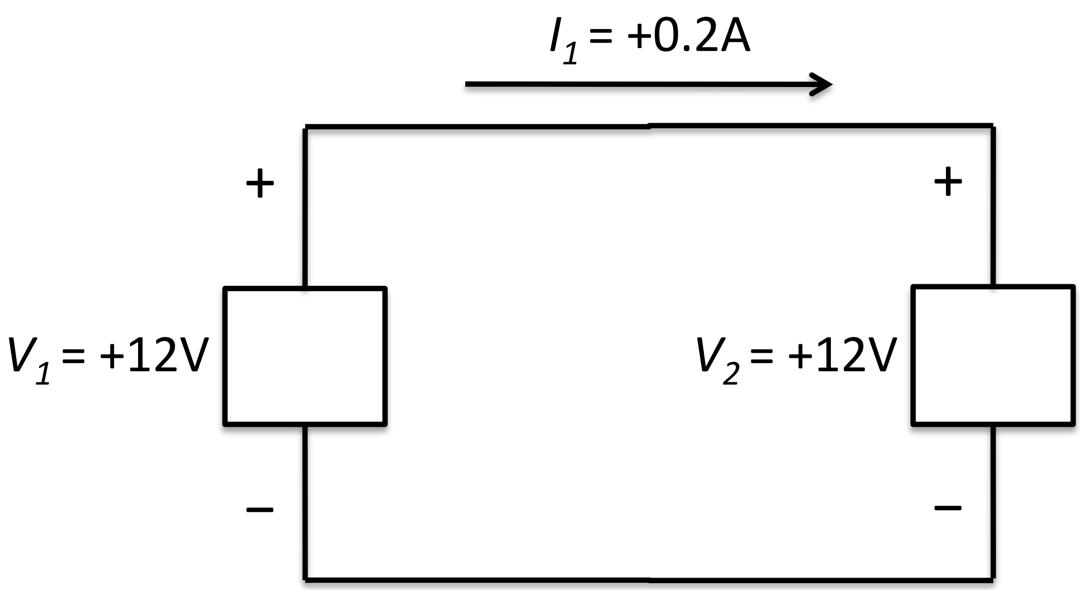
Example 1

Two circuits are connected, as below. A 0.2A current flows in a clockwise fashion. The red wire is at a potential 12V greater than the black wire. Which circuit is delivering electrical power, and how much?



Example 1

We redraw a circuit diagram with variables labeled.



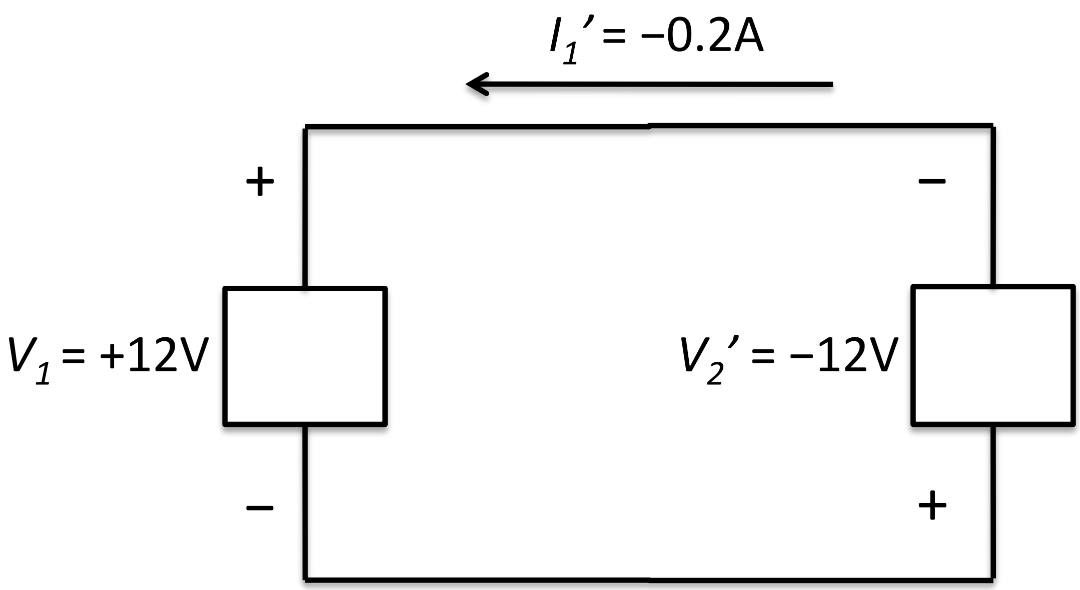
$P_1 = I_1 V_1 = (+0.2A)(+12V)$ delivered
$= +2.4W$ delivered

$P_2 = I_1 V_2 = (+0.2A)(+12V)$ absorbed
$= +2.4W$ absorbed

Therefore, the left cicruit delivers 2.4W of power to the right circuit.

Example 1

We can chose different *I*,*V* variables ($I_1' = -I_1$ and $V_2' = -V_2$) to describe the *exact same physical situation*.



$P_1' = I_1' V_1 = (-0.2A)(+12V)$ absorbed
= −2.4W absorbed
equivalent to 2.4W delivered

$P_2' = I_1' V_2' = (-0.2A)(-12V)$ absorbed
= +2.4W absorbed

As before, the left circuit delivers 2.4W of power to the right circuit.

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Example 3

Consider a record breaking Tour de France cyclist, producing 0.65 horsepower* over 40 minutes to climb a mountain (Alpe d'Huez).

1) How much current would a 12 V battery need to pass in order to provide the same quantity of power?

2) How much charge would a 12 V battery need to pass in order to provide the same quantity of energy?

* 1 hp = 746 W

Example 3

Consider a record breaking Tour de France cyclist, producing 0.65 horsepower* over 40 minutes to climb a mountain (Alpe d'Huez).

1) How much current would a 12 V battery need to pass in order to provide the same quantity of power?

P = 0.65hp x (746 W / hp) = 485 W

$P = IV$, therefore

$I = P / V$ = 485W / 12V = 40.4A

* 1 hp = 746 W

Example 3

Consider a record breaking Tour de France cyclist, producing 0.65 horsepower* over 40 minutes to climb a mountain (Alpe d'Huez).

2) How much charge would a 12 V battery need to pass in order to provide the same quantity of energy?

$W = P\,\Delta t$ = 485 W x 40 min x (60s/min)

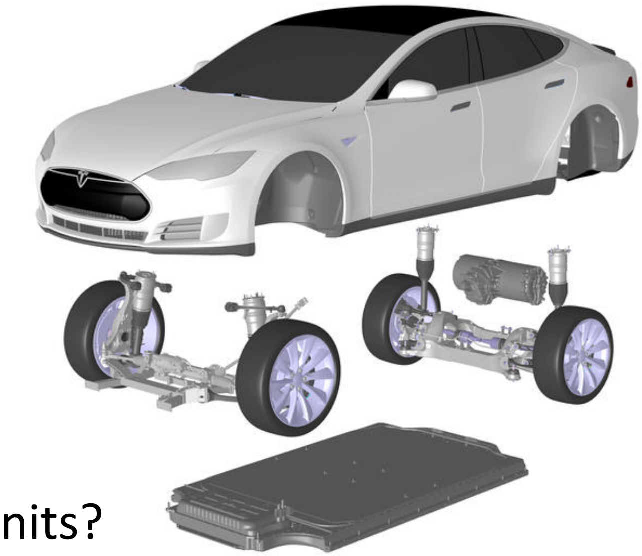
= 1.16 MJ

1 MJ ~ energy content of a candy bar

$W = QV$, therefore

$Q = W / V$ = 1.16MJ / 12V = 97kC

Example 2



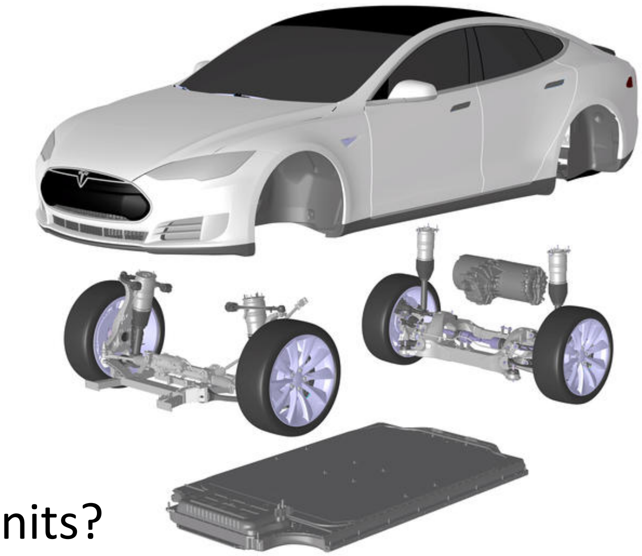
An automobile battery is rated at 85 kW-hrs and a nominal voltage of 402 V.

1) How much energy can the battery deliver, in SI units?

2) How much charge can the battery deliver, in SI units?

3) The maximum current that can be drawn through the battery is 925 A. What is the maximum power that the battery can deliver?

Example 2



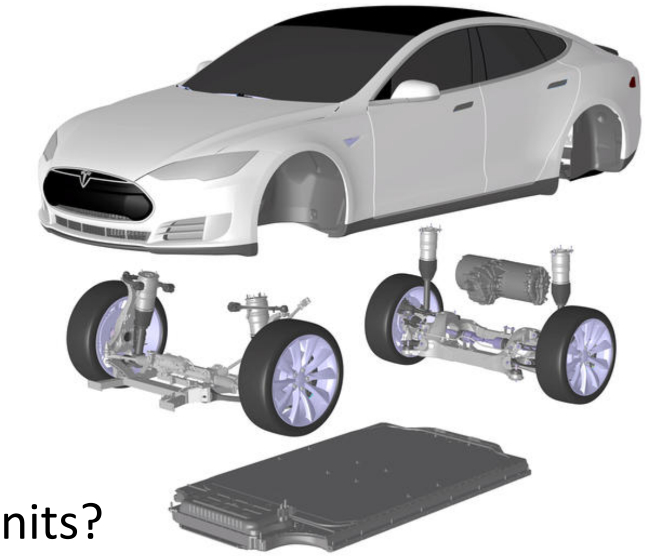
An automobile battery is rated at 85 kW-hrs and a nominal voltage of 402 V.

1) How much energy can the battery deliver, in SI units?

W = 85 kW-hrs x (60 min / hr) x (60 s / min)

= 306 MJ

Example 2



An automobile battery is rated at 85 kW-hrs and a nominal voltage of 402 V.

2) How much charge can the battery deliver, in SI units?

$W = Q\,V$

Therefore, if we assume V is constant (in reality, this is a poor approximation for a battery):

$Q = W / V$

$= 306\text{ MJ} / 402\text{V} = 761\text{ kC}$

Example 2

An automobile battery is rated at 85 kW-hrs and a nominal voltage of 402 V.

3) The maximum current that can be drawn through the battery is 925 A. What is the maximum power that the battery can deliver?

$P = I\ V$ = 925 A x 402 V = 372 kW

This power is equivalent to 499 hP. In reality, battery voltage (and thus power) drops for several reasons related to the internal electrochemistry of the battery itself.