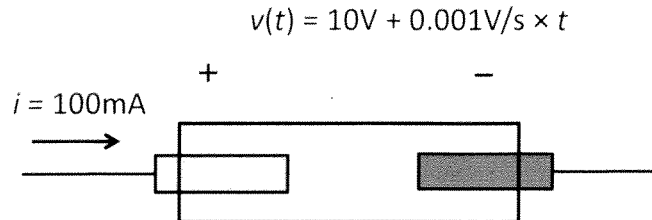


NAME \_\_\_\_\_ McGill ID# \_\_\_\_\_

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram below, representing a battery. Answer the questions.



- At what time  $t$  is the battery voltage  $v(t)$  equal to +12V ? [1pt]
- What is the total charge  $q$  delivered into the "+" terminal of the battery over the time interval  $0\text{s} \leq t \leq 4000\text{s}$ ? [1pt]
- Plot the instantaneous power  $p(t)$  absorbed by the battery versus the time  $t$ , for the time interval  $0\text{s} \leq t \leq 4000\text{s}$ . Clearly label the axes of your plot with SI units. [3pts]
- At what time  $t$  is the battery absorbing 1.3W of power? [2pts]
- What is the energy absorbed by the battery over the time interval  $0\text{s} \leq t \leq 4000\text{s}$ ? [3pts]

$$a) \quad 12\text{V} = 10\text{V} + \frac{0.001\text{V}}{\text{s}} \cdot t$$

$$t = \frac{12\text{V} - 10\text{V}}{0.001\text{V/s}} = 2000\text{s} \quad [+1]$$

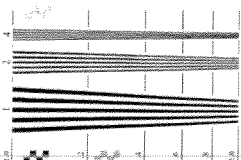
$$b) \quad q = i \cdot \Delta t$$

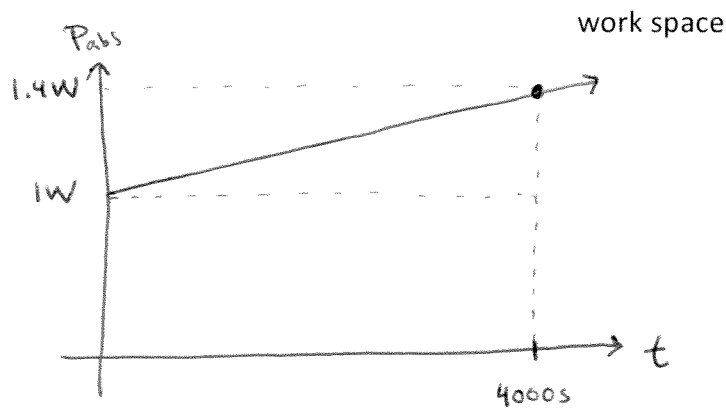
$$= 100\text{mA} \cdot 4000\text{s}$$

$$= 400\text{C} \quad [+1]$$

$$c) \quad P_{\text{abs}}(t) = v(t) \cdot i = \left(10\text{V} + \frac{0.001\text{V}}{\text{s}} \cdot t\right) \cdot 100\text{mA}$$

$$= 1\text{W} + \frac{100\mu\text{W}}{\text{s}} \cdot t$$





[+1] for shape

[+1] for  $P_{abs}(0) = 1W$

[+1] for  $P_{abs}(4000) = 1.4W$

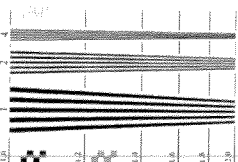
$$d) \quad 1.3W = 1W + \frac{100\mu W}{s} \times t \quad [+1]$$

$$t = \frac{1.3W - 1W}{100\mu W/s} = 3000s \quad [+1]$$

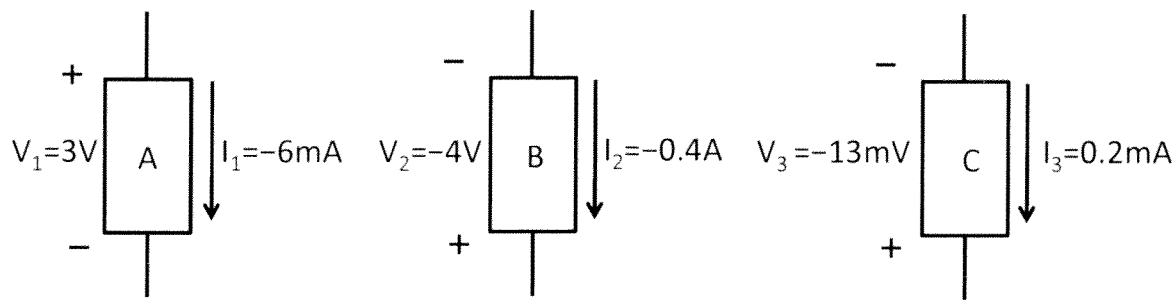
$$e) \quad U_{abs} = \int_0^{4000} P_{abs}(t') dt' \quad [+1]$$

$$P_{abs}(t') = 1W + \frac{100\mu W}{s} \times t' \quad [+1]$$

$$\begin{aligned} \therefore U_{abs} &= \int_0^{4000} 1W + \frac{100\mu W}{s} \times t' dt' \\ &= 1W \cdot 4000s + \frac{1}{2} \cdot \frac{100\mu W}{s} \times (4000s)^2 \\ &= 4.8 kJ \quad [+1] \end{aligned}$$



2. Consider the circuit diagrams below. Answer the questions, clearly indicating power delivery or absorption.



- Do the variables  $V_1$  and  $I_1$  satisfy passive sign convention? [1pt]
- How much power is the element A delivering or absorbing? [1pt]
- Do the variables  $V_2$  and  $I_2$  satisfy passive sign convention? [1pt]
- How much power is the element B delivering or absorbing? [1pt]
- Do the variables  $V_3$  and  $I_3$  satisfy passive sign convention? [1pt]
- How much power is the element C delivering or absorbing? [1pt]

a) yes [1]

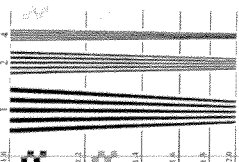
b)  $P_{abs} = V_1 \cdot I_1 = (3V)(-6mA) = -18mW$  absorbing [1]  
or  $+18mW$  is delivered by A

c) no [1]

d)  $P_{del} = V_2 \cdot I_2 = (-4V)(-0.4A) = +1.6W$  is delivered by B [1]

e) no [1]

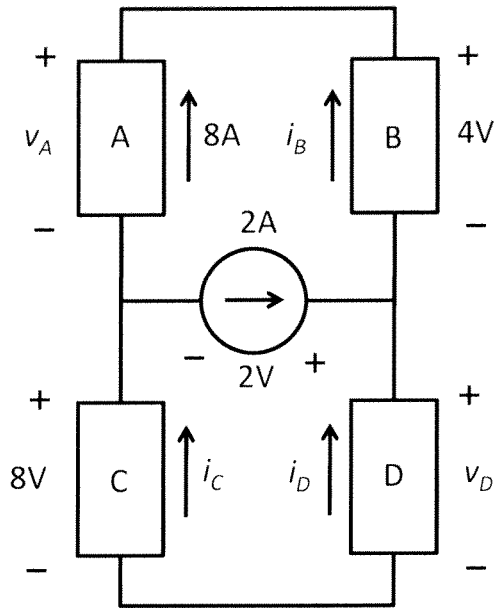
f)  $P_{del} = V_3 \cdot I_3 = (-13mV)(0.2mA) = -2.6\mu W$  delivered [1]  
or  $+2.6\mu W$  is absorbed by C



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READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram below. Answer the questions.



a) Use KCL to find the value of  $i_B$ . [1pt]

b) Use KCL to find the value of  $i_C$ . [1pt]

c) Use KCL to find the value of  $i_D$ . [1pt]

d) Use KVL to find the value of  $v_A$ . [1pt]

e) Use KVL to find the value of  $v_D$ . [1pt]

f) Element B is a resistor. What is the resistance of element B? [1pt]

g) Element D is a resistor. What is the resistance of element D? [1pt]

h) Is it possible that element A is a resistor? (yes or no) [1pt]

$$a) \quad 0 = -8A - i_B \quad i_B = -8A \quad [+1]$$

$$b) \quad 0 = -i_C + 2A + 8A \quad i_C = +10A \quad [+1]$$

$$c) \quad 0 = 10A + i_D \quad i_D = -10A \quad [+1]$$

$$d) \quad 0 = -v_A + 4V + 2V \quad v_A = +6V \quad [+1]$$

$$e) \quad 0 = -8V - 2V + v_D \quad v_D = +10V \quad [+1]$$

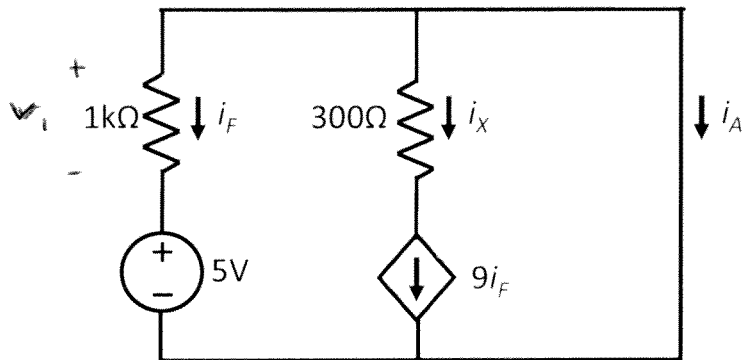
$$f) \quad R_B = -\frac{4V}{-8A} = 0.5\Omega \quad [+1]$$

$$g) \quad R_D = -\frac{10V}{-10A} = 1\Omega \quad [+1]$$

$$h) \quad \text{no} \quad [+1]$$

element A is delivering power

2. Consider the circuit diagram below. Answer the questions.



- What is the value of  $i_F$ ? [2pts]
- What is the power absorbed by the  $1k\Omega$  resistor? [2pts]
- What is the power delivered by the independent voltage source? [2pts]
- What is the value of  $i_X$ ? [2pts]
- What is the value of  $i_A$ ? [2pts]

a) KVL:  $0 = -5V - v_i + 0$  [1]

$$v_i = -5V$$

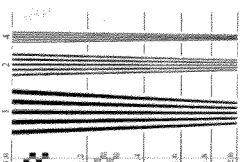
Ohm:  $i_F = \frac{-5V}{1k\Omega} = -5mA$  [1]

b)  $P_{abs} = i_F^2 \cdot 1k\Omega$  [1]  
 $= 25mW$  [1]

d)  $i_X = 9i_F$  [1]  
 $= -45mA$  [1]

c)  $P_{del} = (-i_F) \cdot 5V$  [1]  
 $= 25mW$  [1]

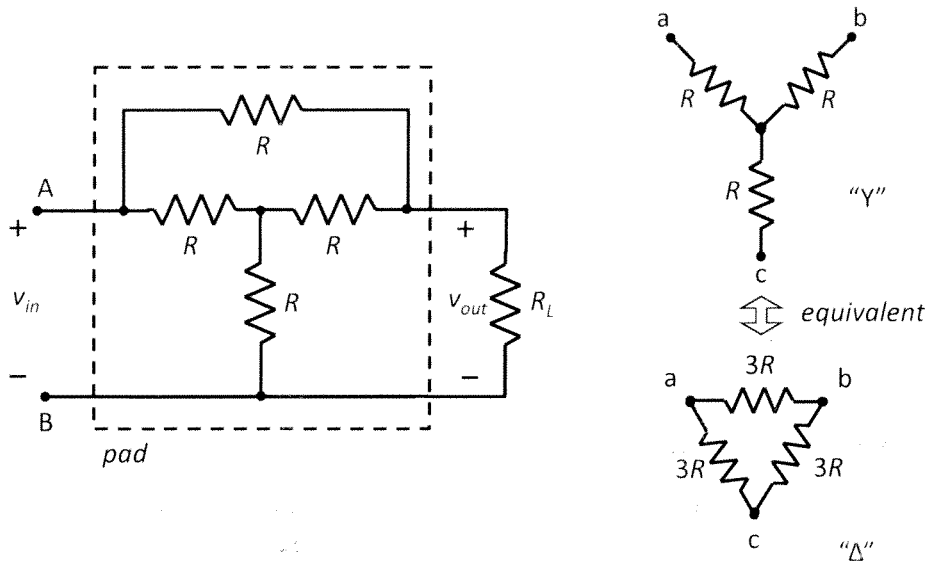
e) KCL:  
 $0 = i_F + i_X + i_A$  [1]  
 $i_A = -i_F - i_X$   
 $= 50mA$  [1]



NAME \_\_\_\_\_ McGill ID# \_\_\_\_\_

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram below, including the Y-to- $\Delta$  equivalent resistance transformation. The boxed area is known as a "fixed attenuator pad", and is used in volume control circuits. Answer the questions.



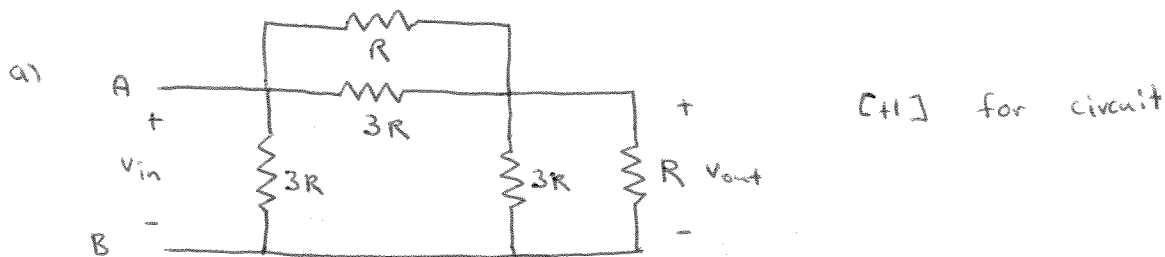
a) If  $R_L = R$ , what is the equivalent resistance between the terminals A and B? [2pts]

**HINT:** You may find it useful to use a Y-to- $\Delta$  equivalent resistance transformation.

b) If  $R_L = R$ , what is the ratio  $v_{out} / v_{in}$ ? Your answer should be a number. [2pts]

c) If  $R_L = R = 600\Omega$ , and  $v_{in} = 2V$ , how much power is absorbed by the resistor  $R_L$ ? [2pts]

d) If  $R_L = R = 600\Omega$ , and  $v_{in} = 2V$ , how much power is absorbed by the entire circuit of resistors? [2pts]



$$R_{AB} = 3R // (3R // R + 3R // R)$$

$$3R // R = \frac{3R \cdot R}{3R + R} = \frac{3}{4} R$$

$$3R // R + 3R // R = \frac{3}{2} R$$

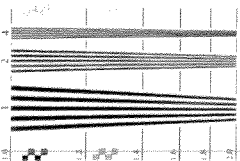
work space

$$R_{AB} = 3R // \frac{3}{2}R = \frac{3R \cdot \frac{3}{2}R}{3R + \frac{3}{2}R} = R \quad [+1]$$

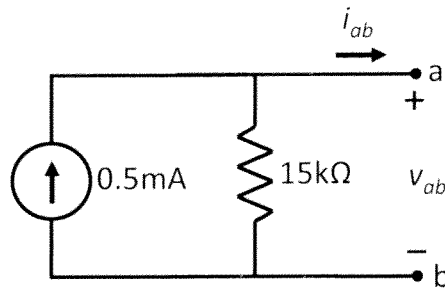
$$\begin{aligned} \text{b)} \quad \frac{v_{out}}{v_{in}} &= \frac{3R // R}{3R // R + 3R // R} \quad \text{voltage divider} \\ &= \frac{1}{2} \quad [+2] \end{aligned}$$

$$\text{c)} \quad P_{abs} = \frac{v_{out}^2}{R_L} = \frac{\left(\frac{1}{2}v_{in}\right)^2}{R_L} = \frac{(1V)^2}{600\Omega} = 1.667 \text{ mW} \quad [+2]$$

$$\text{d)} \quad P_{abs} = \frac{v_{in}^2}{R_{AB}} = \frac{v_{in}^2}{R} = \frac{(2V)^2}{600\Omega} = 6.667 \text{ mW} \quad [+2]$$



2. Consider the circuit diagram below. Answer the questions.



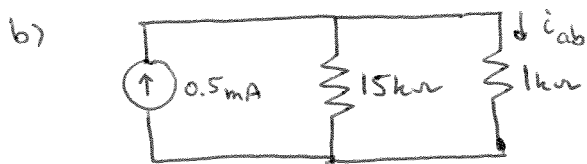
a) An ideal ammeter is connected to terminals a and b. What is the measured value of  $i_{ab}$ ? [1pt]

b) An ammeter with  $1k\Omega$  equivalent resistance is connected to terminals a and b. What is the measured value of  $i_{ab}$ ? [1pt]

c) An ideal voltmeter is connected to terminals a and b. What is the measured value of  $v_{ab}$ ? [1pt]

d) You have used a voltmeter connected to terminals a and b, and measure  $v_{ab} = 6V$ . What is the equivalent resistance of your voltmeter? [1pts]

a)  $i_{ab} = 0.5mA$  [1]

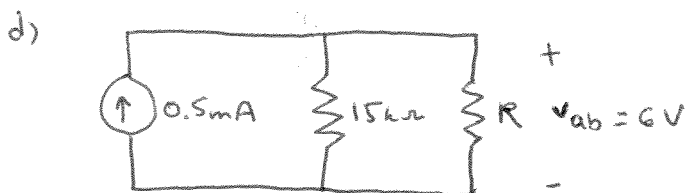


$$i_{ab} = 0.5mA \cdot \frac{15k\Omega}{1k\Omega + 15k\Omega}$$

$$= 0.4688mA$$
 [1]

c)  $v_{ab} = 0.5mA \cdot 15k\Omega$

$$= 7.5V$$
 [1]

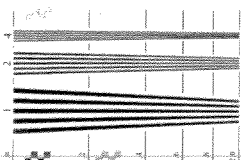


$$R \parallel 15k\Omega = \frac{6V}{0.5mA} = 12k\Omega$$

$$\therefore \frac{1}{R} + \frac{1}{15k\Omega} = \frac{1}{12k\Omega}$$

$$R = \left( \frac{1}{12k\Omega} - \frac{1}{15k\Omega} \right)^{-1}$$

$$= 60k\Omega$$
 [1]

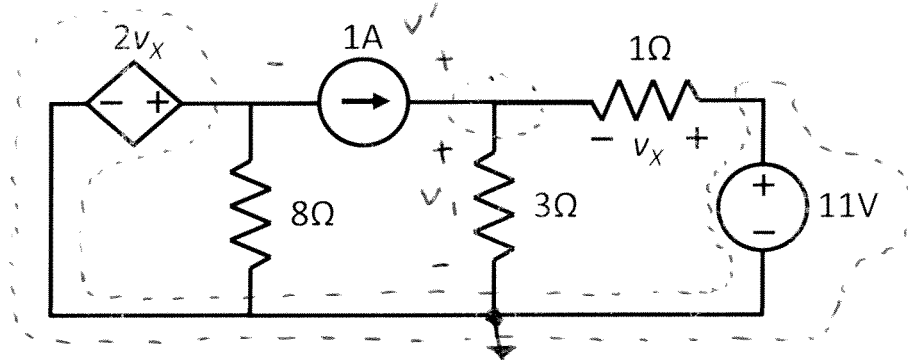




NAME \_\_\_\_\_ McGill ID# \_\_\_\_\_

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram below. Answer the questions.



a) How many supernodes are there in the circuit? How many nodes (which are not part of a supernode) are there in the circuit? [2pts]

b) What is the voltage across the  $3\Omega$  resistor? Be sure to clearly define your voltage variable on the diagram. [3pts]

c) How much power does the 1A current source deliver or absorb? [2pts]

a) 1 super node (+1) 1 node (+1)

b)

$$0 = -1A + \frac{v_1}{3\Omega} + \frac{v_1 - 11V}{1\Omega} \quad (+2)$$

$$v_1 = \frac{1A + 11V/1\Omega}{1/3\Omega + 1/1\Omega} = 9V \quad (+1)$$

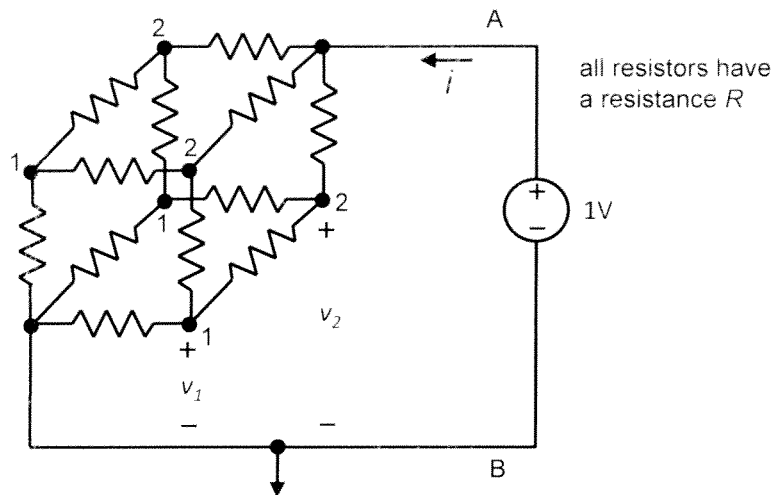
c) KVL.  $v_x = 11V - v_1 = 11V - 9V = 2V$

KVL.  $0 = -2v_x - v' + v_1$

$$v' = v_1 - 2v_x = 9V - 4V = 5V \quad (+1)$$

$$P_{del} = v' \cdot 1A = +5W \text{ delivered} \quad (+1)$$

2. In this problem, you will find the equivalent resistance for a cube of resistors. Each resistor has a resistance  $R$ . Answer the questions.



By symmetry, it follows that each node labeled 1 is equivalent, having the same potential  $v_1$  with respect to reference. It also follows that each node labeled 2 is equivalent, having the same potential  $v_2$  with respect to reference.

- What is the node equation for  $v_1$ ? [2pts] **HINT:** All nodes labeled 1 have the same node equation.
- What is the node equation for  $v_2$ ? [2pts] **HINT:** All nodes labeled 2 have the same node equation.
- What are the values of  $v_1$  and  $v_2$ ? [2pts]
- What is the current  $i$ ? Your answer will depend on  $R$ . [2pts]
- What is the equivalent resistance  $R_{eq}$  of the cube of resistors with respect to terminals A and B? [2pts]

$$a) \quad 0 = \frac{v_1}{R} + \frac{v_1 - v_2}{R} + \frac{v_1 - v_A}{R} \quad [+2]$$

$$b) \quad 0 = \frac{v_2 - v_1}{R} + \frac{v_2 - v_1}{R} + \frac{v_2 - 1V}{R} \quad [+2]$$

$$c) \quad \begin{cases} 0 = 3v_1 - 2v_2 \\ 1V = 3v_2 - 2v_1 \end{cases} \quad \left. \begin{aligned} v_2 &= \frac{3}{2}v_1 \\ 1V &= 3\left(\frac{3}{2}v_1\right) - 2v_1 = \frac{5}{2}v_1 \end{aligned} \right\}$$

$$\therefore v_1 = \frac{2}{5}V \quad v_2 = \frac{3}{5}V$$

work space

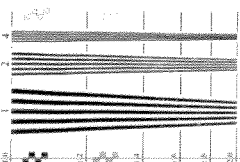
$$d) \quad i = 3 \cdot \frac{V_1}{R} = \frac{\frac{6}{5} V}{R} = \frac{1.2 V}{R} \quad [+2]$$

$$\text{or } i = 3 \cdot \frac{(1V - V_2)}{R}$$

$$e) \quad R_{eq} = \frac{1V}{i} = \frac{1V}{(1.2V/R)} = \frac{5}{6} R \quad [+1]$$

[+1]

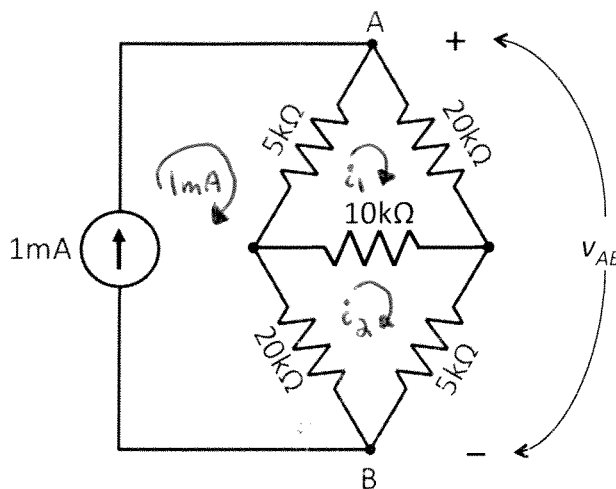
work space



NAME \_\_\_\_\_ McGill ID# \_\_\_\_\_

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

1. Consider the circuit diagram below. This circuit is a *resistance bridge*. Answer the questions.



- Which technique, nodal or mesh analysis, gives fewer equations to solve for the circuit? [1pt]
- Use the technique that you identified in a) to solve for the circuit variables (ie. mesh currents or node voltages). Clearly identify your variables in the diagram. [2pts]
- What is the value of the voltage  $v_{AB}$ ? [1pt]
- What is the equivalent resistance of the resistor network between terminals A and B? [2pts]
- What is the equivalent resistance of the resistor network between terminals A and B if an open circuit replaces the  $10k\Omega$  resistor? [2pts]
- What is the equivalent resistance of the resistor network between terminals A and B if a short circuit replaces the  $10k\Omega$  resistor? [2pts]

a) mesh analysis (2 equations) (+1)

$$b) \quad 0 = 5k\Omega (i_1 - 1mA) + 20k\Omega \cdot i_1 + 10k\Omega (i_1 - i_2)$$

$$0 = 20k\Omega (i_2 - 1mA) + 10k\Omega (i_2 - i_1) + 5k\Omega \cdot i_2$$

$$5 = 35i_1 - 10i_2$$

$$20 = -10i_1 + 35i_2$$

$$i_1 = \frac{\begin{vmatrix} 5 & -10 \\ 20 & 35 \end{vmatrix}}{\begin{vmatrix} 35 & -10 \\ -10 & 35 \end{vmatrix}} = 0.333 \text{ mA} \quad (+1)$$

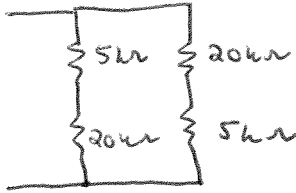
$$i_2 = \frac{\begin{vmatrix} 35 & 5 \\ -10 & 20 \end{vmatrix}}{\begin{vmatrix} 35 & -10 \\ -10 & 35 \end{vmatrix}} = 0.666 \text{ mA} \quad (+1)$$

work space

$$\begin{aligned} c) \quad V_{AB} &= 5k\Omega (1mA - i_1) + 20k\Omega (1mA - i_2) \\ &= 10.00 \text{ V } [+1] \end{aligned}$$

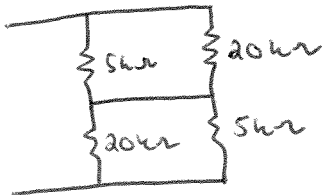
$$\begin{aligned} d) \quad R_{AB} &= \frac{V_{AB}}{1mA} \quad [+1] \\ &= \frac{10V}{1mA} = 10k\Omega \quad [+1] \end{aligned}$$

e)

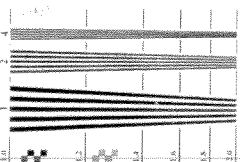


$$\begin{aligned} R_{AB} &= (5k\Omega + 20k\Omega) \parallel (5k\Omega + 20k\Omega) \quad [+1] \\ &= 12.5k\Omega \quad [+1] \end{aligned}$$

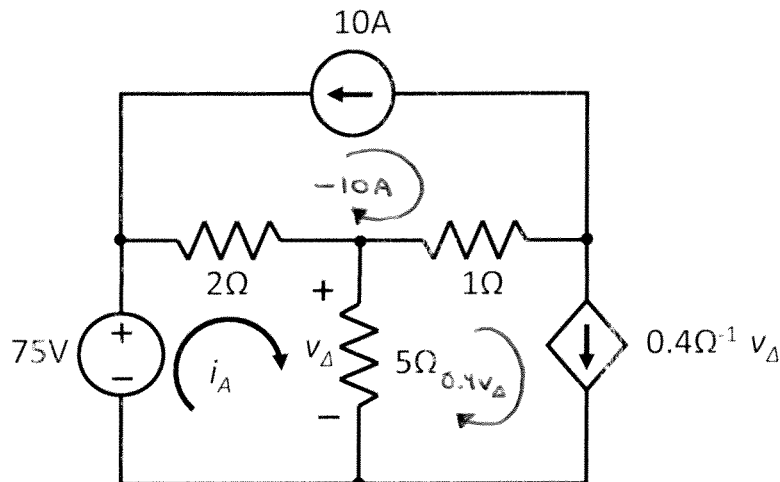
f)



$$\begin{aligned} R_{AB} &= (5k\Omega \parallel 20k\Omega) + (5k\Omega \parallel 20k\Omega) \quad [+1] \\ &= 8k\Omega \quad [+1] \end{aligned}$$



2. Consider the circuit below. Answer the questions.



- What is the KVL equation for the mesh current  $i_A$ ? [3pts]
- What is the value of the mesh current  $i_A$ ? [1pt]
- What is the value of the control variable  $v_\Delta$ ? [1pt]
- How much power does the  $5\Omega$  resistor absorb? [1pt]

$$a) \quad 0 = -75V + 2\Omega \cdot (i_A + 10A) + 5\Omega (i_A - 0.4v_\Delta) \quad \begin{matrix} [+1] & [+1] & [+1] \end{matrix}$$

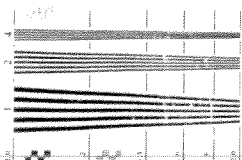
$$b) \quad v_\Delta = 5\Omega \cdot (i_A - 0.4v_\Delta) \quad \therefore \quad v_\Delta = \frac{5}{3}\Omega \cdot i_A$$

$$0 = -75V + 2\Omega (i_A + 10A) + 5\Omega (i_A - 0.4 \cdot \frac{5}{3} \cdot i_A)$$

$$i_A = \frac{75 - 20}{2 + 5 - 10/3} A = 15 A \quad [+1]$$

$$c) \quad v_\Delta = \frac{5}{3}\Omega \cdot 15A = 25V \quad [+1]$$

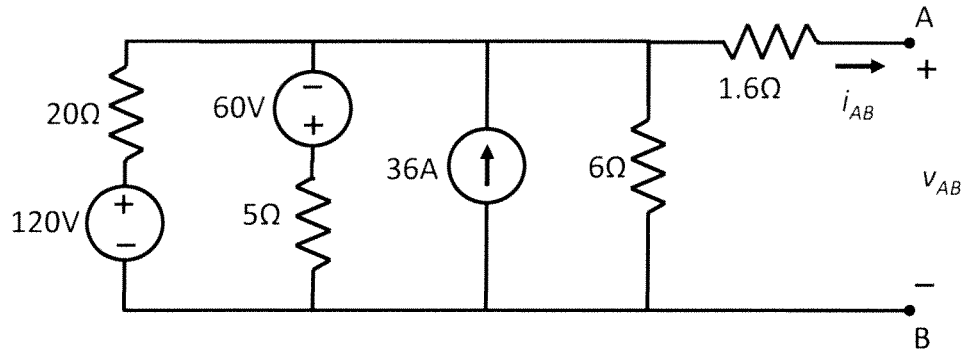
$$d) \quad P_{abs} = \frac{v_\Delta^2}{5\Omega} = \frac{(25V)^2}{5\Omega} = 125W \quad [+1]$$



NAME \_\_\_\_\_ McGill ID# \_\_\_\_\_

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).

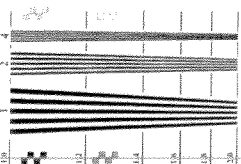
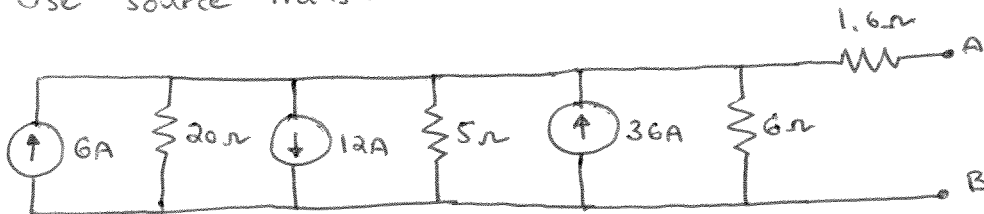
1. Consider the circuit diagram below. Answer the questions.



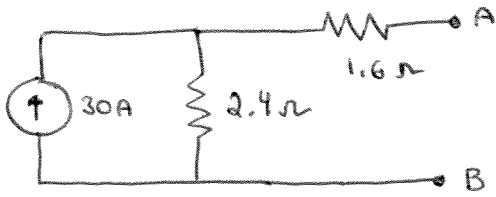
- What property must a circuit have in order for the principle of superposition to apply? [1pt]
- What is Thévenin's theorem? (one sentence is sufficient) [1pt]
- Find the Thévenin equivalent circuit with respect to the terminals A and B. Clearly label the terminals A and B on your equivalent circuit. [3pts]  
**NOTE:** There are numerous ways to solve this problem.
- Draw the  $i_{AB}$ - $V_{AB}$  diagram for the circuit above, clearly identifying the short-circuit current and open-circuit voltage on your diagram. [3pts]
- Use a source transformation on the Thévenin equivalent circuit to find the Norton equivalent circuit with respect to the terminals A and B. [2pts]

- It must be linear. [1]
- Any two-terminal circuit composed of independent sources, dependent sources and ideal resistors is equivalent to a Thévenin equivalent circuit. [1]

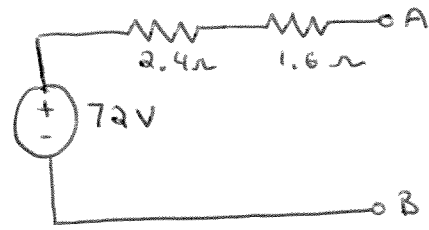
c) Use source transformations:



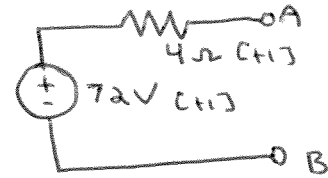
work space



$\Rightarrow$

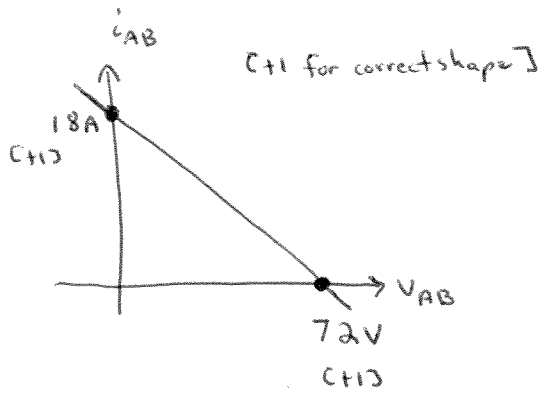


$\Downarrow$

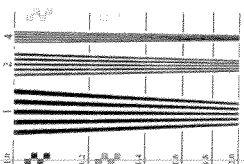
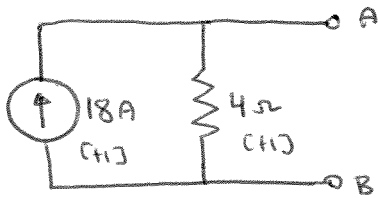


[+1 for Thévenin circuit form]

d)

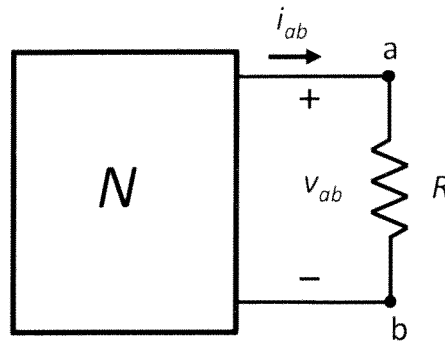


e)





2. Consider the circuit below. Answer the questions.



The circuit  $N$  is known to be composed of independent sources, dependent sources and ideal resistors.

When the resistance  $R = 25\Omega$ , it is found that 250mW of power is absorbed by the resistor  $R$ .

When the resistance  $R = 125\Omega$ , it is found that 312.5mW of power is absorbed by the resistor  $R$ .

a) What is the Thévenin resistance of  $N$ ? [6pts]

b) What is the open-circuit voltage of  $N$ ? [2pts]

$$R = 25\Omega \quad P = 250\text{mW}$$

$$\frac{V_1^2}{25\Omega} = 250\text{mW}$$

$$V_1 = 2.5\text{V} \quad (+1)$$

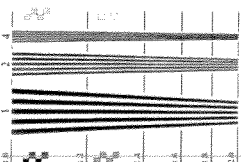
$$I_1 = \frac{2.5\text{V}}{25\Omega} = 100\text{mA} \quad (+1)$$

$$R = 125\Omega \quad P = 312.5\text{mW}$$

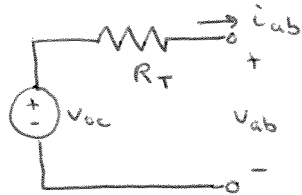
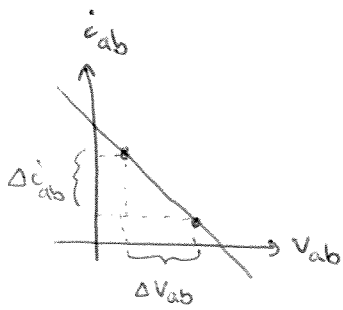
$$\frac{V_a^2}{125\Omega} = 312.5\text{mW}$$

$$V_a = 6.25\text{V} \quad (+1)$$

$$I_a = \frac{6.25\text{V}}{125\Omega} = 50\text{mA} \quad (+1)$$



work space



$$V_{ab} = V_{oc} - i_{ab} R_T$$

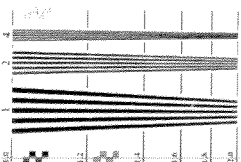
$$\frac{dV_{ab}}{di_{ab}} = -R_T \quad [1]$$

$$\therefore -R_T = \frac{\Delta V_{ab}}{\Delta i_{ab}} = \frac{V_1 - V_2}{i_1 - i_2} = \frac{2.5V - 6.25V}{100mA - 50mA} = -75\Omega$$

$$R_T = 75\Omega \quad [1]$$

b)  $V_{ab} = V_{oc} - i_{ab} \cdot R_T \quad [1]$

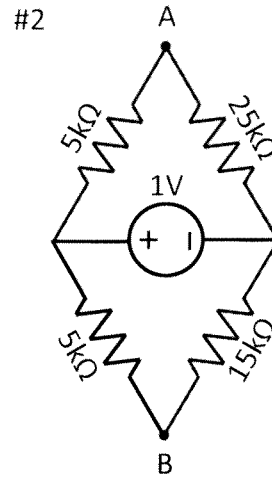
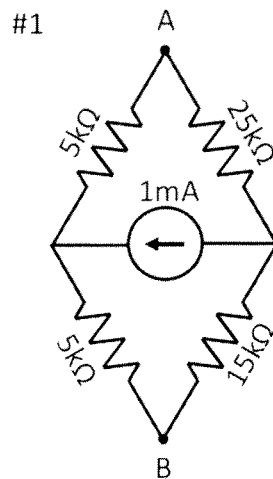
$$\begin{aligned} V_{oc} &= V_{ab} + i_{ab} R_T \\ &= 6.25V + 50mA \cdot 75\Omega \\ &= 10V \quad [1] \end{aligned}$$



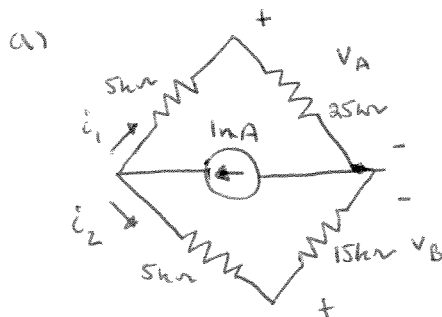
NAME \_\_\_\_\_ McGill ID# \_\_\_\_\_

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate). THINK ABOUT YOUR TECHNIQUE BEFORE YOU SOLVE!

1. Consider the circuit diagrams below. Answer the questions.



- What is the Thévenin equivalent of circuit #1 with respect to the terminals A and B? [3pts]
- What is the maximum power that circuit #1 can deliver to an optimally chosen load resistor connected across terminals A and B? [2pts]
- What is the Thévenin equivalent of circuit #2 with respect to the terminals A and B? [3pts]
- What is the maximum power that circuit #2 can deliver to an optimally chosen load resistor connected across terminals A and B? [2pts]



Find  $V_{oc}$  and  $R_{TH}$ . (11)

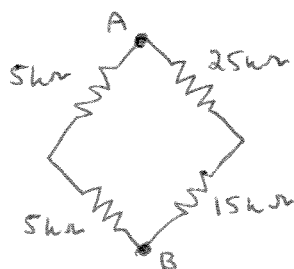
$$i_1 = 1\text{mA} \cdot \frac{(5+15)}{(5+15) + (5+25)} = 0.400\text{mA}$$

$$i_2 = 1\text{mA} \cdot \frac{(5+25)}{(5+25) + (5+15)} = 0.600\text{mA}$$

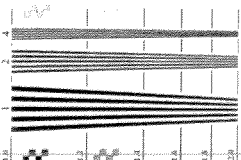
$$V_A = 25\text{k}\Omega \cdot 0.400\text{mA} = 10\text{V}$$

$$V_B = 15\text{k}\Omega \cdot 0.600\text{mA} = 9\text{V}$$

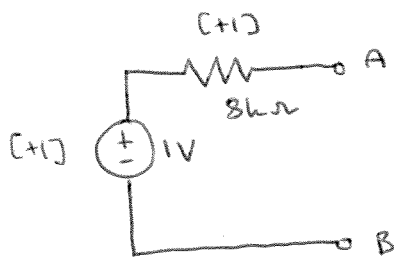
$$V_{oc} = V_A - V_B = 1\text{V}$$



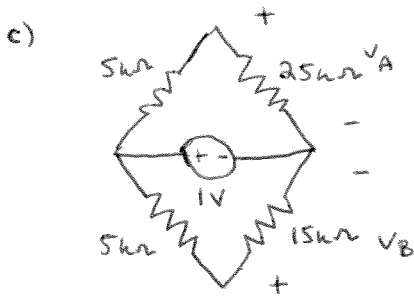
$$R_{TH} = R_{AB} = (5\text{k}\Omega + 5\text{k}\Omega) \parallel (15\text{k}\Omega + 25\text{k}\Omega) = 8\text{k}\Omega$$



work space



$$\begin{aligned}
 b) \quad P_{max} &= \frac{V_{oc}}{2} \cdot \frac{i_{sc}}{2} \quad [+1] \\
 &= \frac{V_{oc}^2}{4R_{TH}} \\
 &= 31.25 \mu W \quad [+1]
 \end{aligned}$$

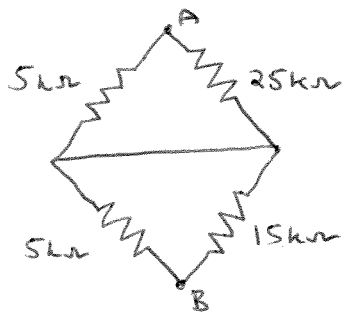


$$V_A = 1V \cdot \frac{25}{5+25} = 0.833V$$

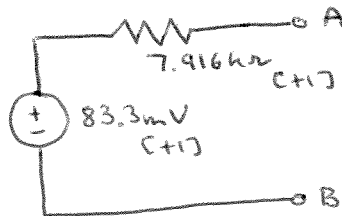
Find  $V_{oc}$  and  $R_{TH}$ .  
[+1]

$$V_B = 1V \cdot \frac{15}{5+15} = 0.750V$$

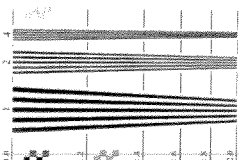
$$V_{oc} = V_A - V_B = 83.3 mV$$



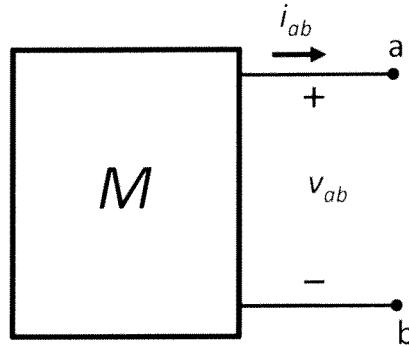
$$\begin{aligned}
 R_{TH} &= (5k\Omega // 25k\Omega) + (5k\Omega // 15k\Omega) \\
 &= 7.916 k\Omega
 \end{aligned}$$



$$\begin{aligned}
 d) \quad P_{max} &= \frac{V_{oc}}{2} \cdot \frac{i_{sc}}{2} \quad [+1] \\
 &= \frac{V_{oc}^2}{4R_{TH}} \\
 &= 219 nW \quad [+1]
 \end{aligned}$$



2. Consider the circuit below. Answer the questions.



The circuit  $M$  is known to be composed of independent sources, dependent sources and ideal resistors.

When an ammeter with  $15\text{k}\Omega$  internal resistance is connected across terminals  $a$  and  $b$ , a current  $i_{ab}=3\text{mA}$  is measured.

When an ammeter with  $5\text{k}\Omega$  internal resistance is connected across terminals  $a$  and  $b$ , a current  $i_{ab}=5\text{mA}$  is measured.

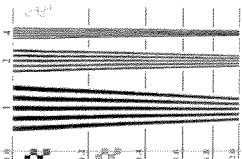
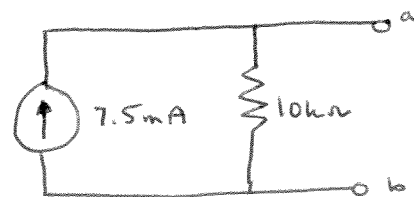
- What is the Norton equivalent circuit of  $M$ ? [5pts]
- What is the maximum power that the circuit  $M$  can deliver to an optimally chosen load resistor connected to terminals  $a$  and  $b$ ? [2pts]
- What is the maximum power that the circuit  $M$  can deliver to an optimally chosen load resistor connected to terminals  $a$  and  $b$  if it is also required that  $v_{ab} \geq 60\text{V}$ ? [2pts]

a) #1  $v_{ab} = 3\text{mA} \cdot 15\text{k}\Omega = 45\text{V}$   $i_{ab} = 3\text{mA}$  (+1)

#2  $v_{ab} = 5\text{mA} \cdot 5\text{k}\Omega = 25\text{V}$   $i_{ab} = 5\text{mA}$  (+1)

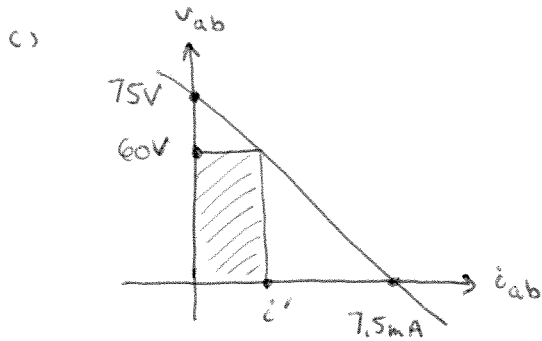
[+1 for either] 
$$\begin{cases} v_{ab} = v_{oc} - i_{ab} R_T \\ i_{ab} = i_{sc} - \frac{v_{ab}}{R_T} \end{cases} \quad \therefore R_T = -\frac{\Delta v_{ab}}{\Delta i_{ab}} = -\frac{(45\text{V} - 25\text{V})}{(3\text{mA} - 5\text{mA})} = 10\text{k}\Omega \text{ (+1)}$$

$$i_{sc} = i_{ab} + \frac{v_{ab}}{R_T} = 3\text{mA} + \frac{45\text{V}}{10\text{k}\Omega} = 7.5\text{mA} \text{ (+1)}$$



work space

$$b) P_{max} = \frac{V_{oc}}{2} \cdot \frac{i_{sc}}{2} = \frac{i_{sc}^2 \cdot R_T}{4} = 140.6 \text{ mW} \quad [+1]$$



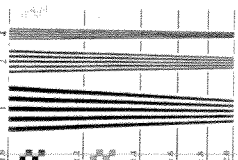
$$V_{oc} = i_{sc} \cdot R_T = 75 \text{ V}$$

$$\frac{i'}{7.5 \text{ mA}} = \frac{75 \text{ V} - 60 \text{ V}}{75 \text{ V}}$$

$$i' = 1.5 \text{ mA}$$

[+1] for diagram showing  
max power area

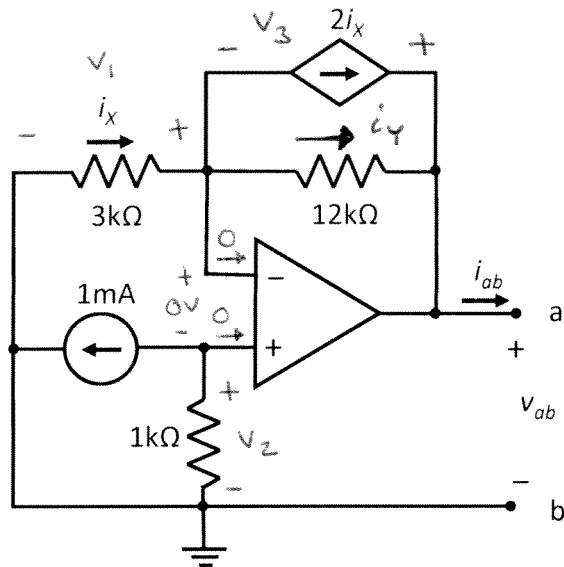
$$P_{max} = 60 \text{ V} \cdot 1.5 \text{ mA} \\ = 90 \text{ mW} \quad [+1]$$



NAME \_\_\_\_\_ McGill ID# \_\_\_\_\_

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate). THINK ABOUT YOUR TECHNIQUE BEFORE YOU SOLVE!

1. Consider the circuit diagram below. Assume ideal op-amp behaviour. Answer the questions.



- What is the value of the current  $i_x$ ? [3pts]
- What is the value of  $v_{ab}$  if a  $3k\Omega$  resistor is connected across the terminals a and b? [3pts]
- What is the value of  $i_{ab}$  if a  $3k\Omega$  resistor is connected across the terminals a and b? [2pts]
- What is the Thévenin equivalent of the circuit above with respect to the terminals a and b? [1pt]

$$a) \quad v_a = -1mA \cdot 1k\Omega = -1V \quad (+1)$$

$$v_1 = v_a = -1V \quad (+1)$$

$$i_x = -\frac{v_1}{3k\Omega} = -\frac{(-1V)}{3k\Omega} = 0.333mA \quad (+1)$$

$$b) \quad KCL: \quad 0 = -i_x + 2i_x + i_y \quad \left. \begin{array}{l} (+2) \\ \text{either} \end{array} \right\}$$

$$\text{or:} \quad 0 = \frac{v_1}{3k\Omega} + 2i_x + \frac{v_1 - v_{AB}}{12k\Omega}$$

$$i_y = -i_x = -0.333mA$$

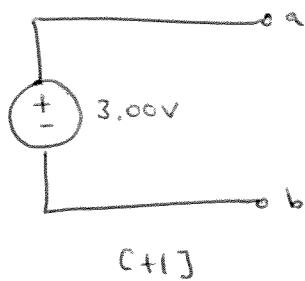
work space

$$V_{ab} = -i_x \cdot 3k\Omega - i_y \cdot 12k\Omega = 3.00V \quad [t+1]$$

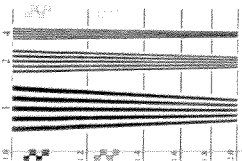
$$c) \quad i_{ab} = \frac{V_{ab}}{3k\Omega} \quad [t+1]$$

$$= \frac{3.00V}{3k\Omega} = 1mA \quad [t+1]$$

d)

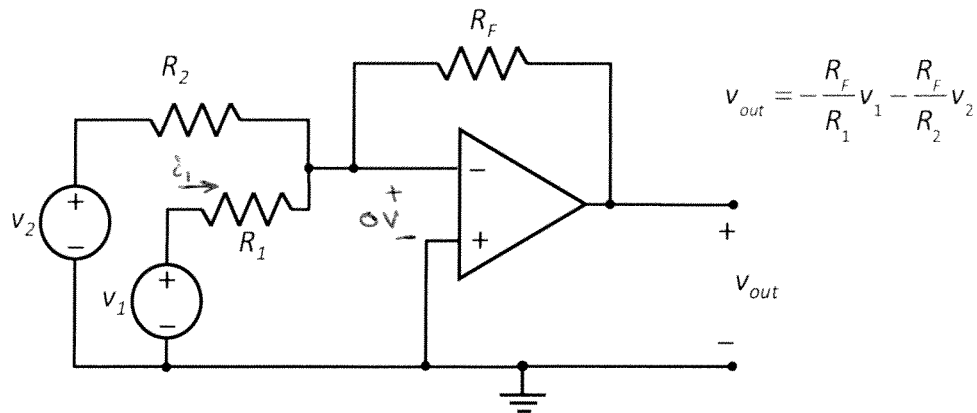


$R_T = 0\Omega$  since  $V_{ab}$  is independent of  $i_{ab}$ .





2. Consider the circuit below. Assume ideal op-amp behaviour. Answer the questions.



You are to design the above circuit to implement the function  $v_{out} = -10 v_1 - 20 v_2$ . It is also desired that the source  $v_1$  delivers  $5\mu\text{W}$  when  $v_1 = 100\text{mV}$ .

a) What is the value of  $R_1$ ,  $R_2$  and  $R_F$ ? [6pts]

b) Assume that the power supplies to the op-amp are  $+10\text{V}$  and  $-10\text{V}$ . If  $v_1 = 0\text{V}$ , over what range of input voltage  $v_2$  will the output voltage be given by  $v_{out} = -20 v_2$ ? (Equivalently, over what range of input voltage  $v_2$  will the op-amp behaviour be ideal?) [2pts]

c) Give one reason why negative feedback is used in op-amp circuits. [1pt]

a)  $i_1 = v_1 / R_1$

$$P_{del} = v_1 \cdot i_1 = v_1^2 / R_1 \quad \therefore R_1 = \frac{v_1^2}{P_{del}} \quad (+1)$$

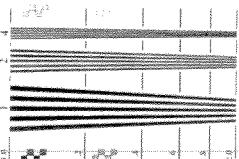
$$= \frac{(100\text{mV})^2}{5\mu\text{W}} = 2\text{k}\Omega \quad (+1)$$

Since  $v_{out} = -\frac{R_F}{R_1} v_1 - \frac{R_F}{R_2} v_2$

$$= -10 v_1 - 20 v_2$$

we have:  $\frac{R_F}{R_1} = 10 \quad (+1) \quad R_F = 10 R_1 = 20\text{k}\Omega \quad (+1)$

also:  $\frac{R_F}{R_2} = 20 \quad (+1) \quad R_2 = \frac{R_F}{20} = 1\text{k}\Omega \quad (+1)$



work space

b)  $-10V < -20V_a < +10V$  [ +1 ]

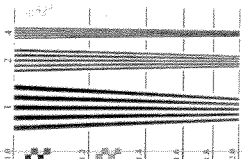
$$0.5V > V_a > -0.5V \quad [ +1 ]$$

c) stability

programmable output

output independent / weakly dependent on open-loop gain

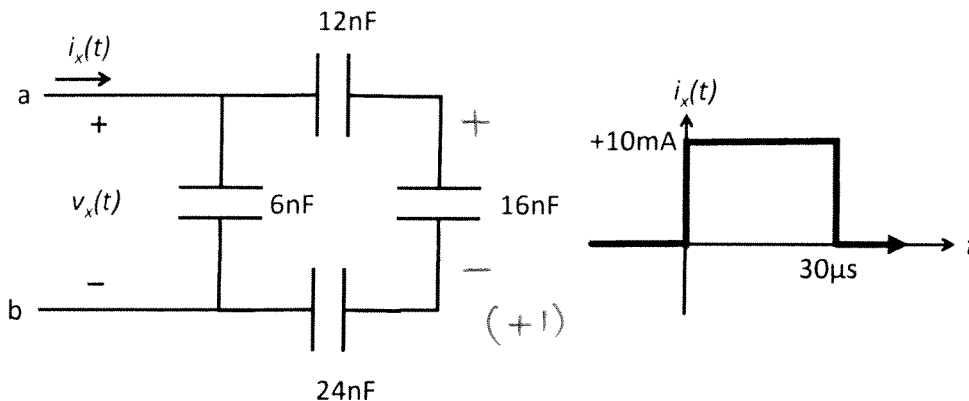
[ +1 ] for any



NAME \_\_\_\_\_ McGill ID# \_\_\_\_\_

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate). THINK ABOUT YOUR TECHNIQUE BEFORE YOU SOLVE!

1. Consider the circuit and plot below. The capacitors are storing zero energy at  $t=0$ . Answer the questions.



- Give one physical reason why voltage is continuous on a capacitor. [1pt]
- What is the equivalent capacitance between terminals a and b? [2pts]
- What is the value of  $v_x(t)$  at  $t = 30\mu\text{s}$ ? [2pts]
- What is the charge separation on the 6nF capacitor at  $t = 30\mu\text{s}$ ? Indicate the polarity of the charge separation on the circuit diagram. [2pts]
- What is the charge separation on the 16nF capacitor at  $t = 30\mu\text{s}$ ? Indicate the polarity of the charge separation on the circuit diagram. [3pts]

a) Conservation of Energy or charge (+)

b)

$$\frac{1}{C'} = \frac{1}{12\text{nF}} + \frac{1}{16\text{nF}} + \frac{1}{24\text{nF}}$$

$$C' = 5.33\text{nF} (+)$$

$$C_{eq} = 5.33\text{nF} + 6\text{nF} = 11.33\text{nF} (+)$$

work space

$$c) i = C \frac{dv}{dt} \rightarrow v(30\mu s) = \frac{1}{C} \int_0^{30\mu s} i(t) dt \quad (+1)$$

$$= \frac{1}{11.33nF} \int_0^{30\mu s} 10mA dt = \frac{1}{11.33nF} [10mA \times t]_0^{30\mu s}$$

$$= 26.47V \quad (+1)$$

$$d) Q = CV \quad (+1)$$

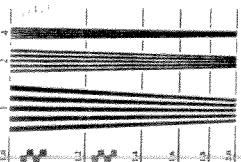
$$Q = 6nF \times 26.47V = 158.82nC \quad (+1)$$

$$e) \text{ Total Charge} = C_{eq}V = 11.3nF \times 26.47V = 3 \times 10^{-7}C = 0.3\mu C \quad (+1)$$

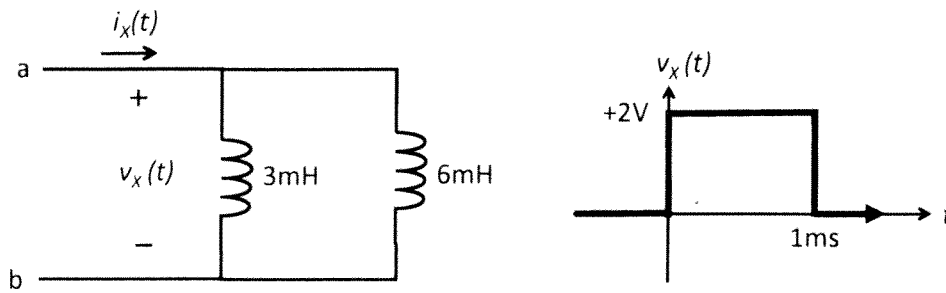
$$q_{16nF} = Q - q_{6nF}$$

$$= 0.3\mu C - 0.15882\mu C = 0.14118\mu C \quad (+1)$$

$$= 141.18nC$$



2. Consider the circuit and plot below. The inductors are storing zero energy at  $t=0$ s. There is no mutual inductance between the inductors. Answer the questions.



- Give one physical reason why current is continuous in an inductor. [1pt]
- What is the equivalent inductance between terminals a and b? [2pts]
- What is the value of  $i_x(t)$  at  $t = 1\text{ms}$ ? [2pts]
- What is the energy stored in the 3mH inductor at  $t=0.5\text{ms}$ ? [2pts]
- What is the energy stored in the 3mH inductor at  $t=1\text{ms}$ ? [2pts]

a) Conservation of Energy (+1)

$$b) \frac{1}{L_{eq}} = \frac{1}{3\text{mH}} + \frac{1}{6\text{mH}} \quad (+1)$$

$$L_{eq} = 2\text{mH} \quad (+1)$$

$$c) V = L \frac{di}{dt} \rightarrow i(t) = \frac{1}{L} \int_0^{1\text{ms}} V(t) dt \quad (+1)$$

$$i(t) = \frac{1}{2\text{mH}} \int_0^{1\text{ms}} 2 dt = \frac{1}{2\text{mH}} [2t]_0^{1\text{ms}} = 1\text{A} \quad (+1)$$

$$d) U = \frac{1}{2} L i^2 \quad (+1)$$

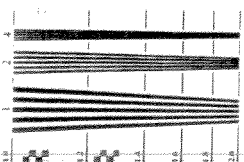
$$i(0.5\text{ms}) = \frac{1}{3\text{mH}} [2t]_0^{0.5\text{ms}} = 0.333\text{A}$$

$$U = \frac{1}{2} \times 3\text{mH} \times 0.333^2 = 0.1667\text{mJ} \quad (+1)$$

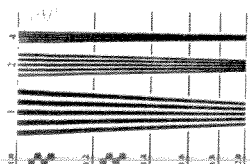
$$e) U = \frac{1}{2} L i^2 \quad (+1)$$

$$i(1\text{ms}) = \frac{1}{3\text{mH}} [2t]_0^{1\text{ms}} = 0.666\text{A}$$

$$U = \frac{1}{2} \times 3\text{mH} \times 0.666^2 = 0.666\text{mJ} \quad (+1)$$



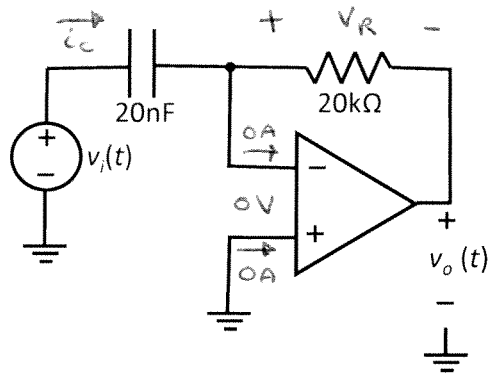
work space



NAME ANSWER KEY McGill ID# \_\_\_\_\_

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate). THINK ABOUT YOUR TECHNIQUE BEFORE YOU SOLVE!

1. Consider the circuit below. Assume ideal op-amp behaviour. Answer the questions.



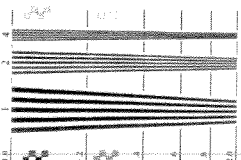
- Express the output voltage  $v_o(t)$  in terms of the input voltage  $v_i(t)$ . [3pts]
- If the input voltage  $v_i(t) = 1V \cos(2\pi ft)$  with  $f = 1\text{kHz} = 1000\text{s}^{-1}$ , what is the output voltage  $v_o(t)$ ? [2pts]
- If the input voltage  $v_i(t) = 1V \cos(2\pi ft)$  with  $f = 1\text{kHz} = 1000\text{s}^{-1}$  (as above), what is the **maximum** instantaneous power absorbed by the resistor? [2pts]
- The power supply voltages to the op-amp are +10V and -10V. If the input is  $v_i(t) = 1V \cos(2\pi ft)$ , what is the maximum frequency  $f$  for which the op-amp will still operate as an ideal op-amp? [1pt]

$$\begin{aligned} \text{a) } i_c &= C \frac{dv_i}{dt} \quad \text{KVL} \Rightarrow 0 = V_i(t) - v_o(t) - R \left( \frac{dv_o(t)}{dt} - v_i(t) \right) \\ V_R &= R C \frac{dv_i}{dt} \\ \Rightarrow v_o(t) &= -R C \frac{dv_i(t)}{dt} \quad \text{C11} \\ &= -4 \times 10^{-4} \frac{dv_i(t)}{dt} \quad \text{C11} \end{aligned}$$

OR using KCL

$$\begin{aligned} \Rightarrow i_c &= C \frac{dv_i}{dt} = C \frac{d(0 - v_o)}{dt} = -C \frac{dv_o(t)}{dt} \\ i_R &= \frac{0 - v_o(t)}{R} = -\frac{v_o(t)}{R} \\ \Rightarrow -C \frac{dv_o(t)}{dt} &= -\frac{v_o(t)}{R} \end{aligned}$$

$$\Rightarrow v_o(t) = -R C \frac{dv_i(t)}{dt} = -4 \times 10^{-4} \frac{dv_i(t)}{dt}$$



$$b) V_i(t) = 1 \cos(2\pi ft)$$

$$V_o(t) = -RC \frac{dV_i(t)}{dt}$$

$$\frac{dV_i(t)}{dt} = -2\pi f \sin(2\pi ft) \quad [+1]$$

$$\Rightarrow V_o(t) = -4 \times 10^{-4} [-2\pi f \sin(2\pi ft)]$$

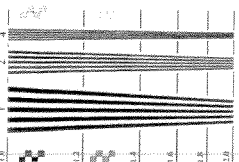
$$= 8 \times 10^{-4} \pi (1000) \sin(2\pi ft) = 0.8\pi \sin(2000\pi t) \text{ V} \quad [+1]$$

$$c) P_{\text{avg}} = \frac{V_{\text{max}}^2}{R} \quad [+1]$$

$$= \frac{(0.8\pi)^2}{20000} = 3.158 \times 10^{-4} \text{ W} \quad [+1]$$

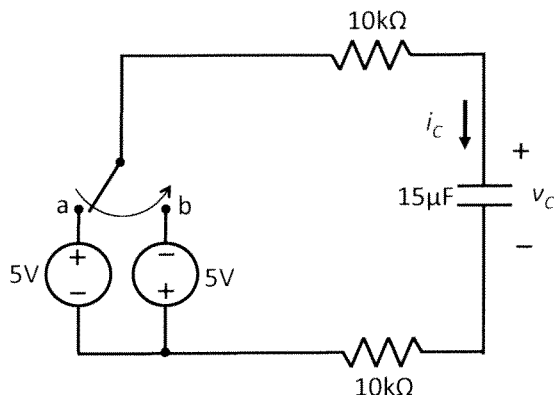
$$d) \max V_o(t) = 10 \text{ V} = (0.4 \text{ m}) (2\pi f)$$

$$\Rightarrow f = \frac{10}{(0.4 \text{ m}) (2\pi)} = 3978.8 \text{ Hz} \quad [+1]$$

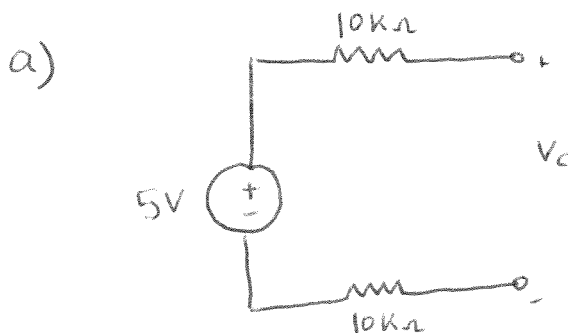




2. Consider the circuit below. The capacitor is in dc steady state for  $t < 0$ , with the switch in position a. At  $t=0$ s, the switch moves instantaneously from position a to position b. Answer the questions.



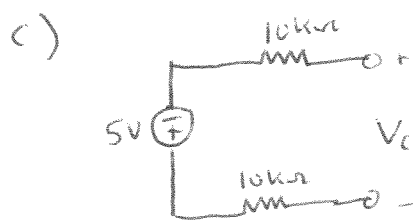
- What is the dc steady state value of  $v_c(t)$  for  $t < 0$ ? [1pt]
- What is  $v_c(0+)$ ? [2pts]
- What is the dc steady state value of  $v_c(t)$  as  $t \rightarrow \infty$ ? [1pt]
- What is the time constant  $\tau$  of this RC circuit? [2pts]
- It can be shown that  $v_c(t) = c_1 + c_2 \exp(-t/\tau)$  for  $t > 0$ . What are the constants  $c_1$  and  $c_2$ ? [2pts]
- At what time  $t$  does  $v_c(t) = 0V$ ? [1pt]
- What is the current  $i_c(t)$  for  $t > 0$ ? [2pts]



$V_c(t)$  for  $t < 0$

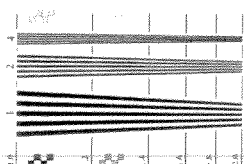
$$V_c(t) = 5V \quad [+1]$$

- b)  $V_c(0-) = V_c(0+)$  Continuity of capacitor voltage [+1]  
 $V_c(0+) = 5V$  [+1]



$V_c(t)$  for  $t \rightarrow \infty$

$$V_c = -5V \quad [+1]$$

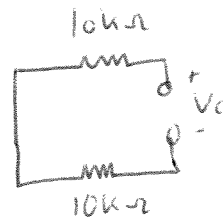


$$d) \tau = R_T C$$

$$R_T = 10k\Omega + 10k\Omega = 20k\Omega \quad [+1]$$

$$C = 15\mu F$$

$$\Rightarrow \tau = (20k\Omega)(15\mu F) \\ = 0.3s \quad [+1]$$



$$e) C_1 = V(\infty) = -5V \quad [+1]$$

$$C_2 = V(0+) - V(\infty) = 5 - (-5) = 10V \quad [+1]$$

$$f) V_c(t) = C_1 + C_2 \exp(-t/\tau) \\ = -5 + 10 \exp(-t/0.3)$$

$$V_c(t) = 0 \Rightarrow -5 + 10 \exp(-t/0.3) = 0$$

$$\exp(-t/0.3) = 1/2$$

$$-t/0.3 = \ln(0.5)$$

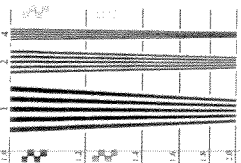
$$\Rightarrow t = 0.2079s \quad [+1]$$

$$g) i_c = C \frac{dV_c(t)}{dt} \quad [+1]$$

$$= 15\mu F \frac{d[-5 + 10 \exp(-t/0.3)]}{dt}$$

$$= 15\mu F \left[ \frac{10}{-0.3} \exp(-t/0.3) \right]$$

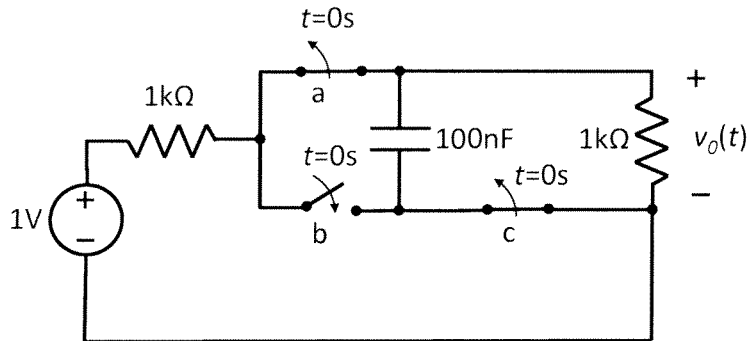
$$= -5 \times 10^{-4} \exp(-t/0.3) A \quad [+1]$$



NAME \_\_\_\_\_ McGill ID# \_\_\_\_\_

READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate). THINK ABOUT YOUR TECHNIQUE BEFORE YOU SOLVE!

1. Consider the circuit below. The circuit is in dc steady state for  $t < 0$ , with switch a closed, switch b opened, and switch c closed. At  $t = 0$ , switch a opens, switch b closes, and switch c opens. Answer the questions.



a) How much energy is stored in the capacitor at  $t = 0$ ? [2pts]

**HINT:** You may find it useful to draw the circuit for  $t < 0$ .

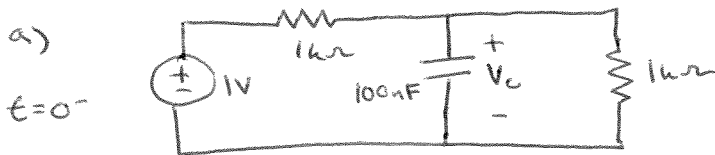
b) What is  $v_o(0+)$ ? [1pt]

**HINT:** You may find it useful to draw the circuit for  $t = 0+$ .

c) What is  $v_o(\infty)$ ? [1pt]

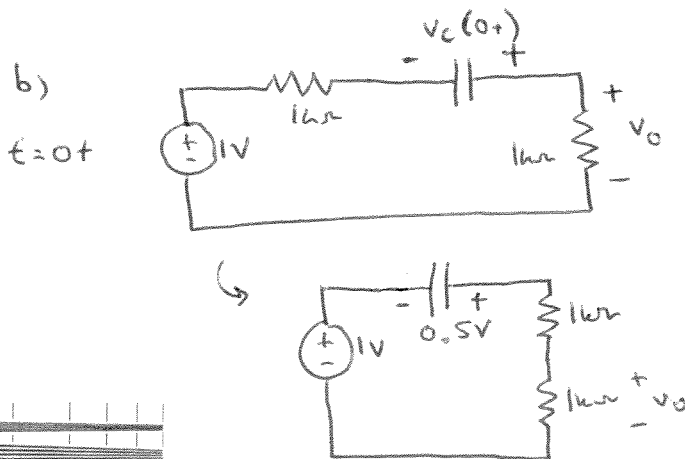
d) What is the time constant  $\tau$  for the evolution of  $v_o(t)$  for  $t > 0$ ? [2pts]

e) What is  $v_o(t)$  for  $t > 0$ ? [3pts]



$$v_c = 1V \cdot \frac{1k\Omega}{1k\Omega + 1k\Omega} = 0.5V \quad (+1)$$

$$U = \frac{1}{2} C v_c^2 = 12.5 \text{ nJ} \quad (+1)$$



$$v_c(0+) = v_c(0-) = 0.5V$$

$$v_o = \frac{1k\Omega}{1k\Omega + 1k\Omega} \cdot (1V + 0.5V)$$

$$= 0.75V \quad (+1)$$

work space

$$c) \quad v_o(\infty) = 0V \quad (+1)$$

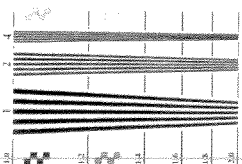
$$d) \quad R_{TH} = 1k\Omega + 1k\Omega = 2k\Omega \quad (+1)$$

$$\tau = R_{TH} \cdot C = 2k\Omega \cdot 100nF = 0.2ms \quad (+1)$$

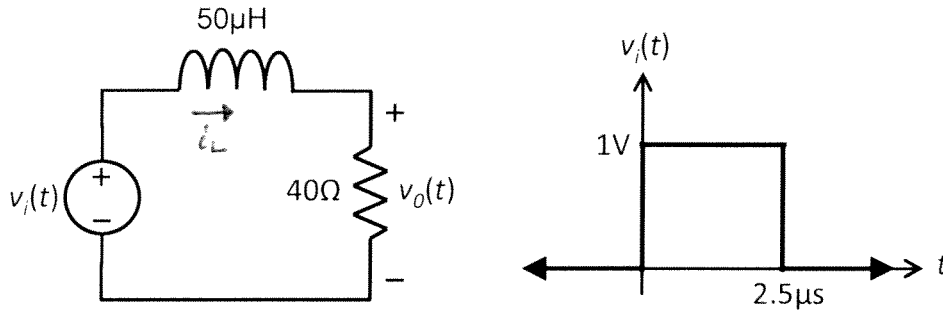
$$e) \quad v_o(0+) = 0.75V \quad v_o(\infty) = 0V \quad \tau = 0.2ms$$

$$v_o(t) = v_o(\infty) + (v_o(0+) - v_o(\infty)) \exp(-t/\tau) \quad (+1)$$

$$= 0.75V \exp(-t/0.2ms) \quad (+2)$$



2. Consider the circuit and plot below. The inductor is in dc steady state for  $t < 0$ . Answer the questions.



- What is  $v_o(0+)$ ? [1pt]
- What is  $v_o(2.5\mu s-)$ ? [1pt]
- What is  $v_o(2.5\mu s+)$ ? [1pt]
- What is  $v_o(\infty)$ ? [1pt]
- What is the time constant  $\tau$  of this circuit? [1pt]
- What is  $v_o(t)$  for  $0 < t < 2.5\mu s$ ? [3pts]
- What is  $v_o(t)$  for  $2.5\mu s < t$ ? [3pts]

$$v_o(0+) = i_L(0+) \cdot 40\Omega = i_L(0-) \cdot 40\Omega = 0V \quad [+1]$$

$$v_o(\infty) \text{ assuming no further transients} = 1V$$

$$\tau = \frac{L}{R} = \frac{50\mu H}{40\Omega} = 1.25\mu s \quad [+1]$$

$$\text{For } 0 < t < 2.5\mu s: v_o(0+) = 0V \quad v_o(\infty) = 1V \quad \tau = 1.25\mu s \quad [+1]$$

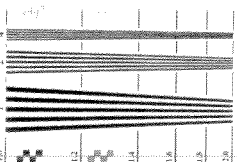
(no further transients)

$$v_o(t) = v_o(\infty) + (v_o(0+) - v_o(\infty))\exp(-t/\tau) \quad [+1]$$

$$= 1V - 1V \exp(-t/1.25\mu s) \quad [+1]$$

$$v_o(2.5\mu s-) = 1V - 1V \exp(-2.5\mu s/1.25\mu s) = 0.865V \quad [+1]$$

$$v_o(2.5\mu s+) = i_L(2.5\mu s+) \cdot 40\Omega = i_L(2.5\mu s-) \cdot 40\Omega = v_o(2.5\mu s-) = 0.865V \quad [+1]$$



work space

$$v_o(\phi) = 0V$$

$$\text{For } t < 2.5\mu s: \quad v_o(2.5\mu s+) = 0.865V \quad v_o(\phi) = 0V \quad \tau = 1.25\mu s \quad [11]$$

$$v_o(t) = v_o(\phi) + (v_o(2.5\mu s+) - v_o(\phi)) \exp(-(t - 2.5\mu s)/\tau) \quad [11]$$

$$= 0.865V \exp(-(t - 2.5\mu s)/1.25\mu s) \quad [11]$$

