McGill

June 2009
Final Examination

Fundamentals of Electrical Engineering
ECSE 200
Section 1

12 June 2009, 9:35

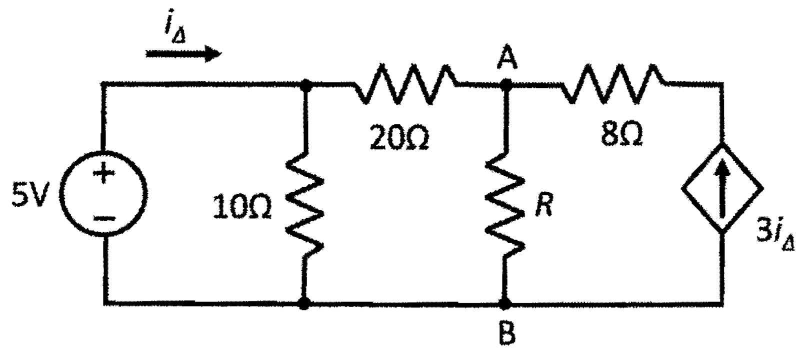
Examiner: Thomas Szkopek

Student Name:		McGill ID:									

INSTRUCTIONS:

- This is a **CLOSED BOOK** examination.
- **SPACE IS PROVIDED** on the examination to answer all questions.
- **STANDARD FACULTY CALCULATOR** permitted ONLY.
- This examination consists of 4 questions, each with multiple parts, for a total of 17 numbered pages, including the cover page.
- Show ALL your work and indicate your final answer CLEARLY.
- This examination is **PRINTED ON BOTH SIDES** of the paper.
- This examination paper **MUST BE RETURNED**.

1. Consider the circuit below. [12pts]

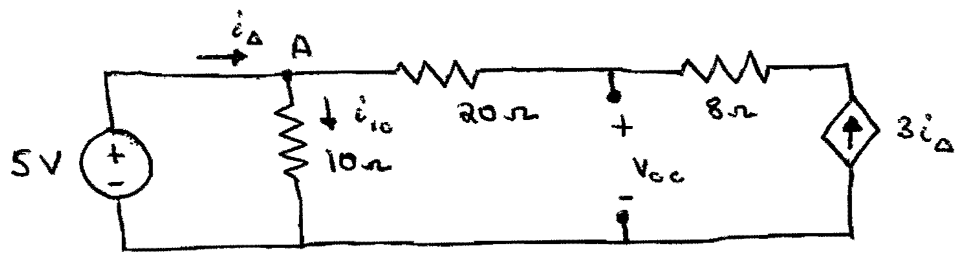


a) What is the maximum power that can be delivered to the resistor *R*, if the value of *R* is chosen optimally? [4pts]

b) If the resistor *R* is removed and replaced with an ammeter with 0.5Ω internal resistance, what is the measured current? Indicate both the magnitude and direction of the measured current. [4pts]

c) If now the ammeter is removed and replaced with a 2A independent current source (with the current directed upwards from node B to node A), what is the power that is delivered or absorbed by the dependent current source? [4pts]

a)

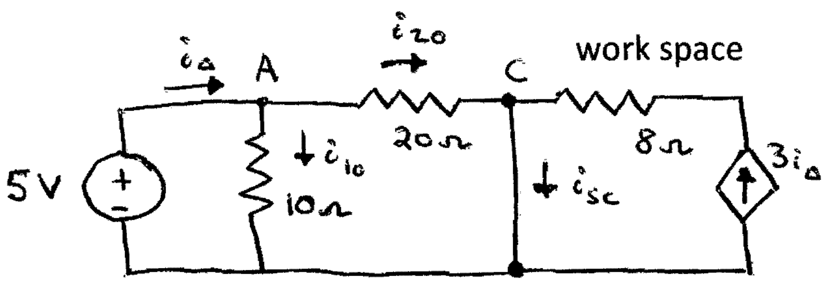


$$i_{10} = 5V/10\Omega = 0.5A$$

KCL: at A $\quad 0 = -i_\Delta - 3i_\Delta + 0.5A$

$$i_\Delta = \frac{0.5A}{4} = 0.125A$$

KVL : $\quad V_{oc} = 5V + 3i_\Delta \cdot 20\Omega = 12.5V$ [+1]



$$i_{10} = \frac{5V}{10\Omega} = 0.5A$$

$$i_{20} = \frac{5V}{20\Omega} = 0.25A$$

KCL at A : $i_\Delta = i_{10} + i_{20} = 0.75\,A$

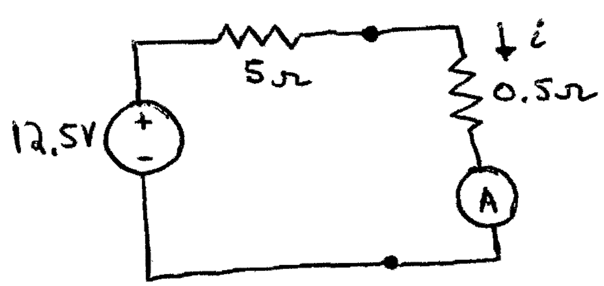
$3i_\Delta = 2.25A$

KCL at C : $i_{sc} = i_{20} + 3i_\Delta = 2.5A$ [+1]

$$\therefore\ P_{max} = \frac{V_{oc}}{2} \cdot \frac{i_{sc}}{2} = \frac{12.5V \cdot 2.5A}{4} = 7.813\,W \quad [+1]$$

[+1]

b) $R_{TH} = \dfrac{V_{oc}}{i_{sc}} = \dfrac{12.5V}{2.5A} = 5\Omega$ [+1]



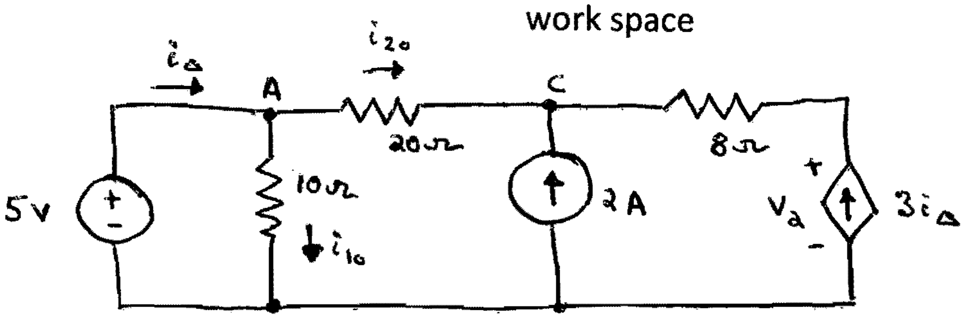
$$i = \frac{V_{oc}}{R_{TH} + 0.5\Omega} = 2.273\,A \quad [+1]$$

[+1]

[+1 for equiv. circuit]

work space

c)



$$i_{10} = \frac{5V}{10\Omega} = 0.5A$$

KCL at A: $i_{20} = i_\Delta - i_{10} = i_\Delta - 0.5A$

KCL at C: $(i_\Delta - 0.5A) + 2A + 3i_\Delta = 0$ [+1]

$$i_\Delta = \frac{-1.5A}{4} = -0.375A$$

KVL: $-5V + 20\Omega(i_\Delta - 0.5A) - 3i_\Delta \cdot 8\Omega + v_a = 0$ [+1]

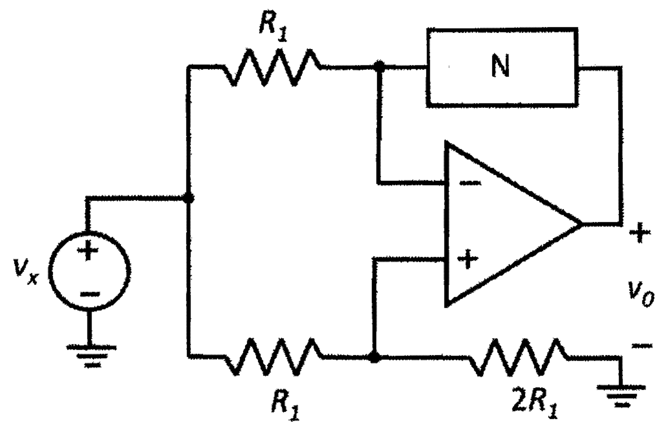
$$v_a = 13.5V$$

Power delivered by dependent source:

$$P_{del} = 3i_\Delta \cdot v_a = -15.188W \text{ [+1]}$$

[+1]

2. Consider the circuit below. Assume ideal op-amp behaviour. The network N is composed of resistors, independent sources and dependent sources. [12pts]



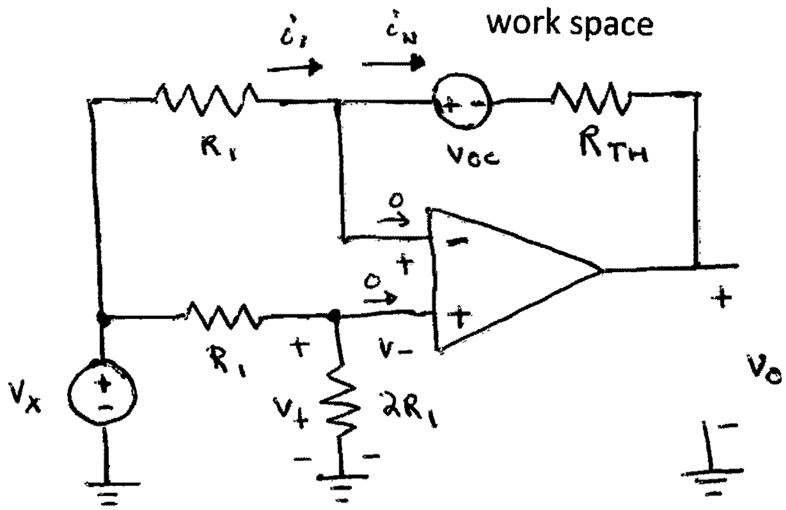
a) Find v_O as a function of v_x, R_1 and the Thévenin equivalent circuit parameters of the network N. Indicate clearly with a diagram the Thévenin equivalent circuit for N that you are using for your calculations. [4pts]

b) The network N is replaced with an inductance L. What is v_O in terms of v_x, R_1 and L? Note that v_x may be time dependent. [4pts]

c) Now the inductance L is replaced with a capacitance C. What is v_O in terms of v_x, R_1 and C? Note again that v_x may be time dependent. [4pts]

a)

work space



ideal op-amp

$i_+ = i_- = 0$

$V_+ = V_-$

$$V_- = V_+ = \frac{2R_1}{R_1 + 2R_1} V_x = \frac{2}{3} V_x \quad [+1]$$

$$i_1 = \frac{V_x - V_-}{R_1} = \frac{1}{3} \frac{V_x}{R_1}$$

$$i_N = \frac{(V_- - V_{oc}) - V_o}{R_{TH}} = \frac{\frac{2}{3} V_x - V_{oc} - V_o}{R_{TH}}$$

$$i_1 = i_N \quad [+1]$$

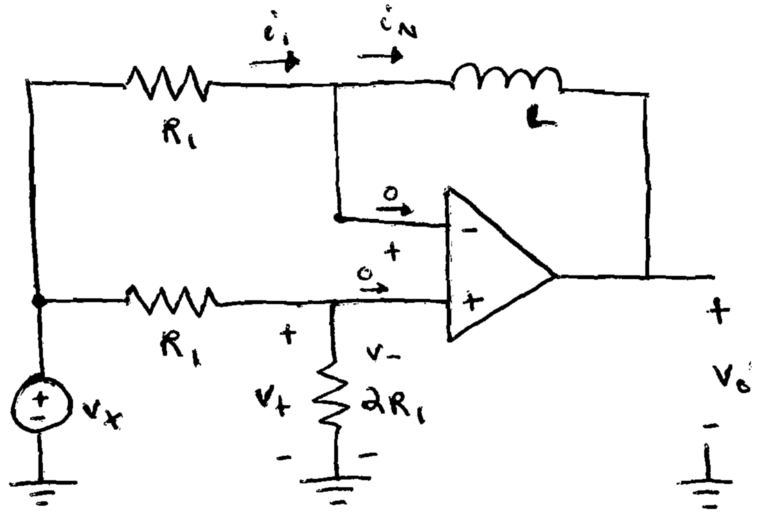
$$\frac{\frac{1}{3} V_x}{R_1} = \frac{\frac{2}{3} V_x - V_{oc} - V_o}{R_{TH}}$$

$$\therefore \quad V_o = \left(\frac{2}{3} - \frac{1}{3} \frac{R_{TH}}{R_1} \right) V_x - V_{oc}$$

[+1] [+1]

work space

b)



ideal op-amp

$i_+ = i_- = 0$

$v_+ = v_-$

$$v_- = v_+ = \frac{2R_1}{R_1 + 2R_1} v_x = \frac{2}{3} v_x \quad [+1]$$

$$i_1 = \frac{v_x - v_-}{R_1} = \frac{1}{3} \frac{v_x}{R_1}$$

$$v_- - v_o = L \frac{di_N}{dt} = L \frac{di_1}{dt} = \frac{1}{3} \frac{L}{R_1} \frac{dv_x}{dt}$$

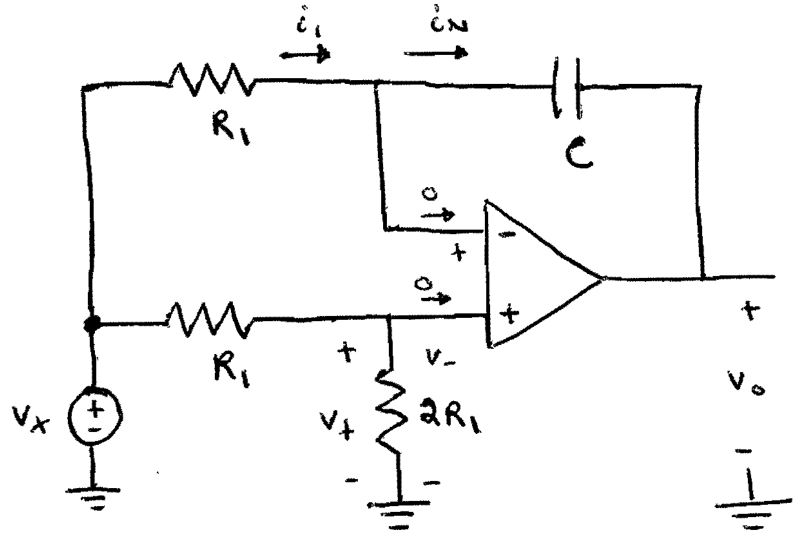
[+1]

$$\therefore \quad v_o = \frac{2}{3} v_x - \frac{1}{3} \frac{L}{R_1} \frac{dv_x}{dt}$$

[+1] [+1]

work space

c)



ideal op-amp

$i_+ = i_- = 0$

$v_+ = v_-$

$$v_- = v_+ = \frac{2R_1}{R_1 + 2R_1} v_x = \frac{2}{3} v_x \quad [+1]$$

$$i_1 = \frac{v_x - v_-}{R_1} = \frac{1}{3}\frac{v_x}{R_1}$$

$$i_N = C \frac{d}{dt}(v_- - v_o) \quad [+1]$$

$$i_1 = i_N$$

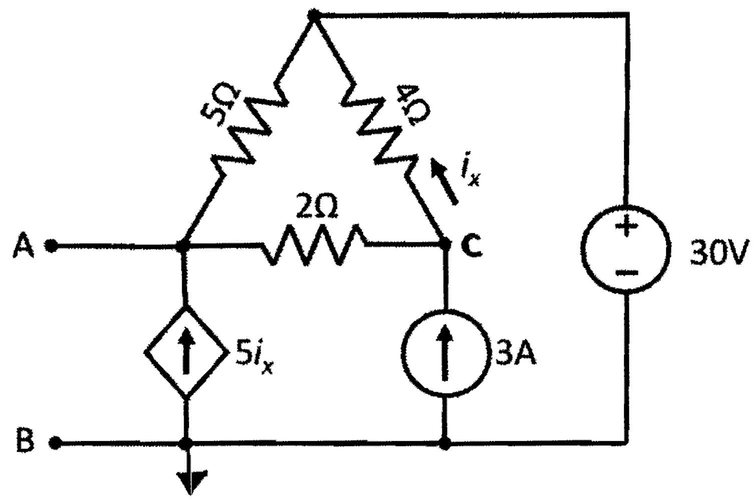
$$\frac{1}{3}\frac{v_x}{R_1} = C\,\frac{2}{3}\frac{dv_x}{dt} - C\frac{dv_o}{dt}$$

$$\frac{dv_o}{dt} = \frac{2}{3}\frac{dv_x}{dt} - \frac{1}{3R_1C} v_x$$

$$v_o(t) - v_o(t_o) = \frac{2}{3}\left(v_x(t) - v_x(t_o)\right) - \frac{1}{3R_1C}\int_{t_o}^{t} v_x(t')\,dt'$$

[+1] [+1]

3. Consider the circuit below. [12pts]



a) What is the current i_x? [4pts]

b) What is the open circuit voltage with respect to the terminals A,B? [3pts]

c) What is the Thévenin resistance with respect to the terminals A,B? [4pts]

d) How would the Thévenin resistance change if: the $5i_x$ source remains unchanged, the 30V source is replaced with a 60V source, and the 3A source is replaced with a 6A source? ***Justify*** your answer. [1pt]

a) b)

$$-5i_x + \frac{V_A - 30V}{5\Omega} + \frac{V_A - V_C}{2\Omega} = 0 \qquad [+2]$$

$$\frac{V_C - V_A}{2\Omega} + \frac{V_C - 30V}{4\Omega} - 3A = 0 \qquad [+2]$$

$$i_x = \frac{V_C - 30V}{4\Omega} \qquad [+1]$$

work space

Substitution for i_x and simplifying gives:

$$\frac{7}{10} V_A - \frac{7}{4} V_c = -\frac{63}{2} V$$

$$-\frac{1}{2} V_A + \frac{3}{4} V_c = \frac{21}{2} V$$

Solving:

$$V_A = 15V$$

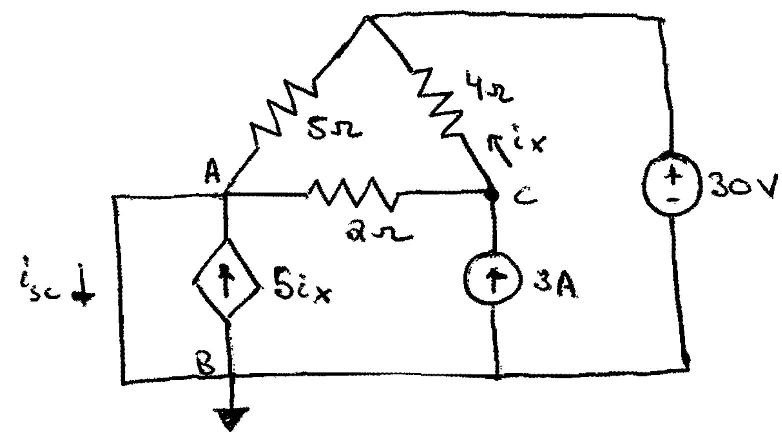
$$V_c = 24V$$

$$i_x = \frac{V_c - 30V}{4\Omega} = -1.5A \quad [+1]$$

$$V_{oc} = V_{AB} = V_A = 15V \quad [+1]$$

c)

work space



$$\frac{V_c - 30V}{4\Omega} + \frac{V_c}{2\Omega} - 3A = 0 \quad [+1]$$

$$V_c = \frac{3A + \frac{30V}{4\Omega}}{\frac{1}{4\Omega} + \frac{1}{2\Omega}} = 14V$$

$$i_x = \frac{14V - 30V}{4\Omega} = -4A$$

KCL at A:

$$0 = i_{sc} - 5i_x - \frac{V_c}{2\Omega} - \frac{30V}{5\Omega} \quad [+1]$$

$$i_{sc} = 5(-4A) + \frac{14V}{2\Omega} + \frac{30V}{5\Omega} = -7A$$

$$\therefore \quad R_{TH} = \frac{V_{oc}}{i_{sc}} = \frac{15V}{-7A} = -2.143\Omega \quad [+1]$$

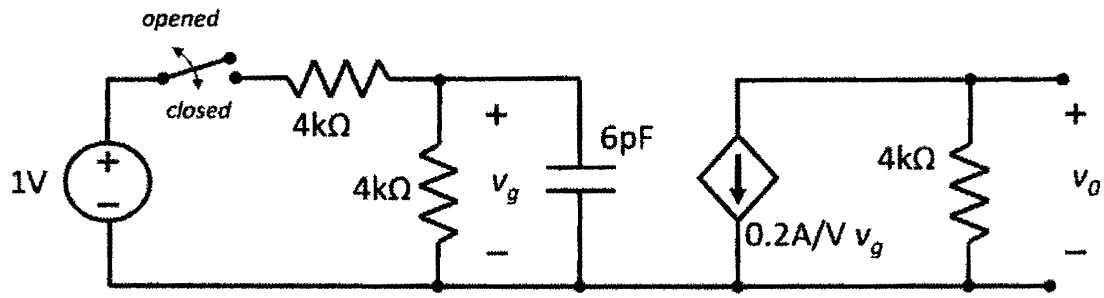
[+1]

d) By linearity, $V_{oc}' = 2V_{oc}$ and $i_{sc}' = 2i_{sc}$, therefore

$$R_{TH}' = \frac{V_{oc}'}{i_{sc}'} = \frac{V_{oc}}{i_{sc}} = R_{TH}$$ ie. no change.

[+1]

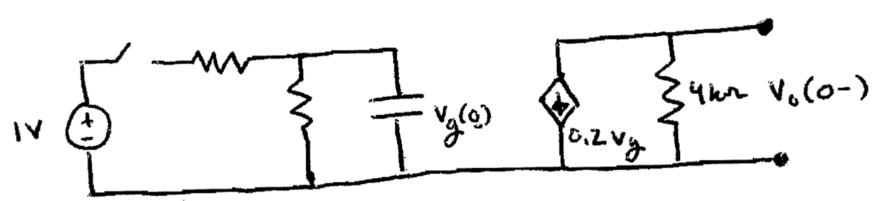
4. Consider the circuit below. The capacitor is in dc steady state for $t < 0$ with the switch opened. The switch closes at t=0s, and then opens again at t=10ns. [12pts]



a) What is the voltage $v_O(t)$ for all t? Plot your answer versus time, indicating important times, voltages, and tangents on your plot. [9pts]

b) If the switch was left closed sufficiently long to establish dc steady state conditions, and then the switch was instantaneously opened, how long from the time of the switch opening would it take for the capacitor to lose 90% of its initially stored energy? [3pts]

a) $t < 0$



$v_g(0) = 0V$ [+1]

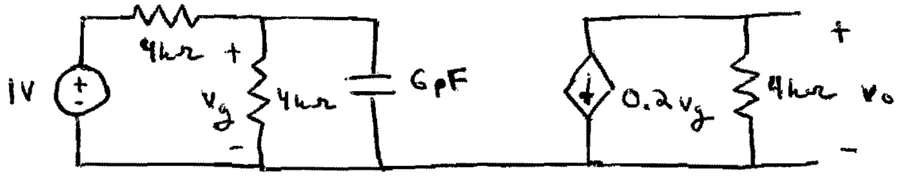
$v_o = -0.2 \frac{A}{V} v_g \cdot 4k\Omega = -800 v_g$ [+1]

(this relation is valid for all time)

$v_o(0-) = 0V$

work space

$0 < t < 10ns$



$v_g(0_+) = v_g(0_-) = 0V$ [+1/2]

$v_g(\infty) = \frac{4k\Omega}{4k\Omega + 4k\Omega} \cdot 1V = 0.5V$ [+1/2]

$\tau = (4k\Omega // 4k\Omega) \cdot 6pF = 12ns$ [+1/2]

$v_g(t) = 0.5V(1 - \exp(-t/12ns))$

$V_o(t) = -800\, v_g(t) = -400V\,(1 - \exp(-t/12ns))$

$10ns < t$

$v_g(10ns+) = v_g(10ns-) = 0.5\,V\,(1 - \exp(-10/12)) = 0.283V$ [+1/2]

$v_g(\infty) = 0V$ [+1/2]

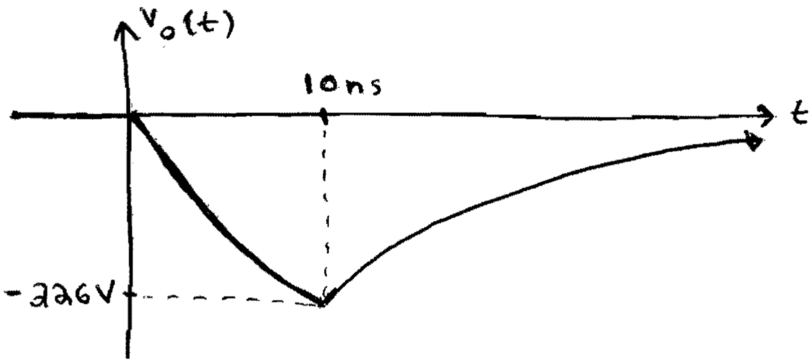
$\tau = 4k\Omega \cdot 6pF = 24ns$ [+1/2]

$v_g(t) = 0.283V \exp(-(t - 10ns)/24ns)$

$v_o(t) = -800\, v_g(t) = -226V \exp(-(t-10ns)/24ns)$

work space

$$v_o(t) = \begin{cases} 0\,\text{V} & t < 0 & [+1] \\ -400\,\text{V}\,(1 - \exp(-t/12\text{ns})) & 0 < t < 10\text{ns} & [+1] \\ -226\,\text{V}\,\exp(-(t-10\text{ns})/24\text{ns}) & 10\text{ns} < t & [+1] \end{cases}$$



[-1 for sign errors]

[+1] for values and shape

b) $U(0) = \frac{1}{2} C v_g^2(0)$ [+1/2]

$$U(t) = \frac{1}{2} C v_g^2(t) = \frac{1}{2} C \left[v_g(0) \exp(-t/\tau) \right]^2$$

$$= \frac{1}{2} C v_g^2(0) \exp(-2t/\tau) \quad [+1/2]$$

90% energy lost implies 10% remains.

$$U(t) = \frac{1}{10} U(0) \quad [+1]$$

$$\frac{1}{10} = \exp(-2t/\tau)$$

$$t = \frac{\tau}{2} \ln 10 \qquad \tau = 4\text{k}\Omega \cdot 6\text{pF} = 24\text{ns}$$

$$t = \frac{24\text{ns}}{2} \cdot \ln 10 = 27.6\text{ns} \quad [+1]$$