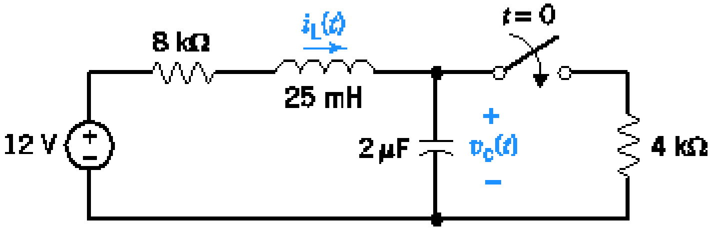
ECSE 200 - Electric Circuits 1
Tutorial 10

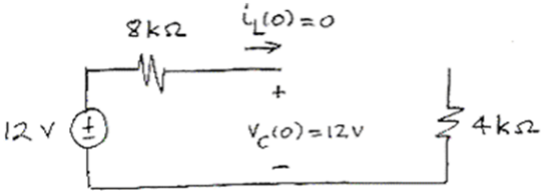
ECE Dept., McGill University

Problem P 7.8-1

The switch in the figure has been open for a long time before closing at time t = 0. Find $v_c(0^+)$ and $i_L(0^+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_c(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. $v_c(\infty)$ and $i_L(\infty)$.



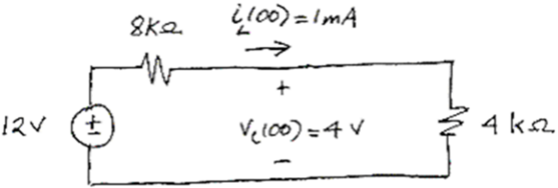
Problem P 7.8-1 Solution



Then

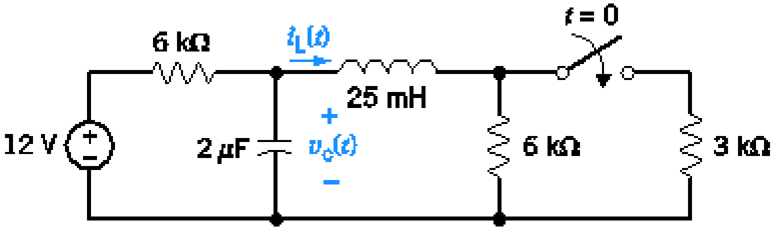
$$i_L(0^+)=i_L(0^-)=0 \quad \text{and} \quad v_C(0^+)=v_C(0^-)=12\text{ V}$$

Next

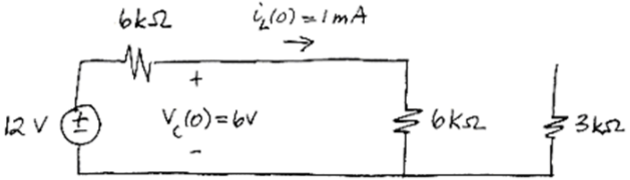


Problem P 7.8-2

The switch in the figure has been open for a long time before closing at time t = 0. Find $v_c(0^+)$ and $i_L(0^+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_c(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. $v_c(\infty)$ and $i_L(\infty)$.



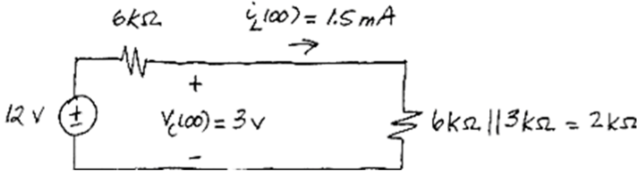
Problem P 7.8-2 Solution



Then

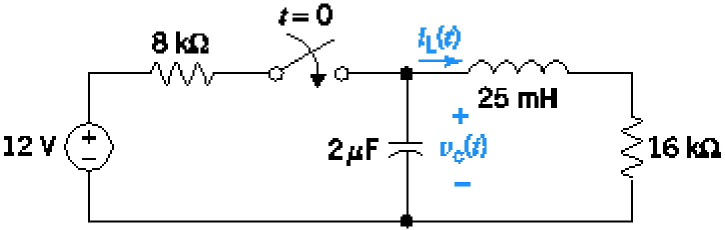
$$i_L(0^+) = i_L(0^-) = 1 \text{ mA} \quad \text{and} \quad v_C(0^+) = v_C(0^-) = 6 \text{ V}$$

Next

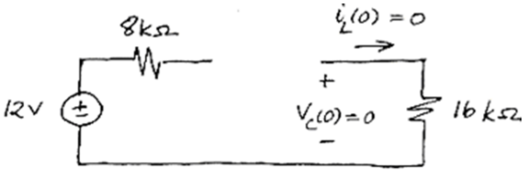


Problem P 7.8-3

The switch in the figure has been open for a long time before closing at time t = 0. Find $v_c(0^+)$ and $i_L(0^+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_c(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. $v_c(\infty)$ and $i_L(\infty)$.



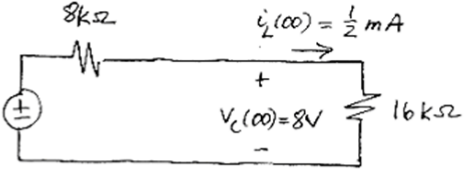
Problem P 7.8-3 Solution



Then

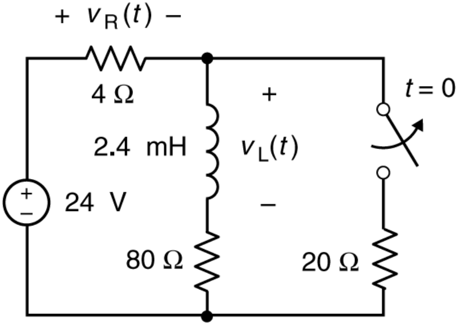
$$i_L(0^+) = i_L(0^-) = 0 \quad \text{and} \quad v_C(0^+) = v_C(0^-) = 0 \text{ V}$$

Next



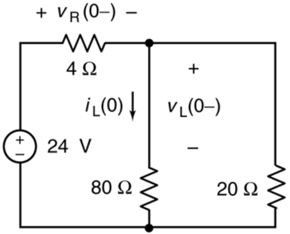
Problem P 7.8-4

The switch in the circuit shown in the figure has been closed for a long time before it opens at time $t = 0$. Determine the values of $v_R(0^-)$ and $v_L(0^-)$, the voltage across the 4Ω resistor and the inductor immediately before the switch opens and the values of $v_R(0^+)$ and $v_L(0^+)$, the voltage across the 4Ω resistor and the inductor immediately after the switch opens.



Problem P 7.8-4 Solution

The circuit is at steady state immediately before the switch opens. We have



The inductor acts like a short circuit so $v_L(0-) = 0$.

Noticing that the 80 Ω and 20 Ω are connected in parallel and using voltage division:

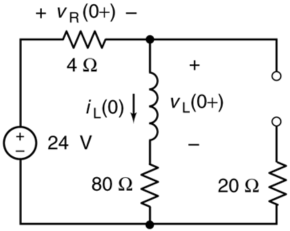
$$v_R(0-) = \frac{4}{4+(80 \parallel 20)}(24) = \frac{4}{4+16}(24) = 4.8 \text{ V}$$

Using current division:

$$i_L(0) = \left(\frac{20}{80+20}\right)\frac{24}{4+(80 \parallel 20)} = \frac{1}{5}\left(\frac{24}{4+16}\right) = 0.24 \text{ A}$$

The inductor current does not change instantaneously so $i_L(0+) = i_L(0-) \triangleq i_L(0)$. Immediately after the switch opens we have:

Problem P 7.8-4 Solution



$$v_R(0+) = 4\,i_L(0) = 4(0.24) = 0.96 \text{ V}$$

Using KVL:

$$v_R(0+) + v_L(0+) + 80\,i_L(0) - 24 = 0$$

$$0.96 + v_L(0+) + 80(0.24) - 24 = 0$$

$$v_L(0+) = 3.84 \text{ V}$$

$$v(t) = 75 - 82\,e^{-7t} = R\left(5 + 2\,e^{-7t}\right) + L\frac{d}{dt}\left(5 + 2\,e^{-7t}\right)$$

$$= R\left(5 + 2\,e^{-7t}\right) + L\left((-7)\,2\,e^{-7t}\right) = 5\,R + \left(2\,R - 14\,L\right)e^{-7t}$$

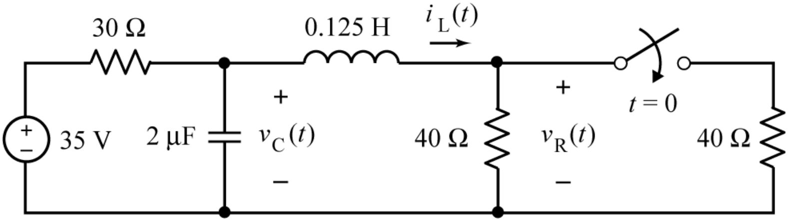
Equation coefficients gives $75 = 5\,R \;\Rightarrow\; R = 15\ \Omega$ and

and $-82 = 2\,R - 14\,L = 30 - 14\,L \;\Rightarrow\; L = \dfrac{82 + 30}{14} = 8 \text{ H}$

Problem P 7.8-12

The circuit shown in the figure has reached steady state before the switch closes at time $t = 0$.

- Determine the values of $i_L(t)$, $v_c(t)$ and $v_R(t)$ immediately before the switch closes.
- Determine the value of $v_R(t)$ immediately after the switch closes.

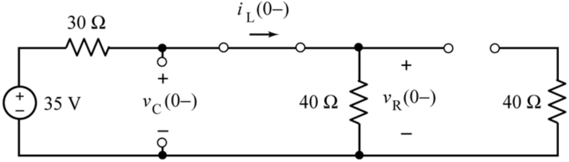


Problem P 7.8-12 Solution

Solution: Because

- This **circuit has reached steady state** before the switch closes at time $t = 0$.
- The only source is a **constant** voltage **source**.

At t=0–, **the capacitor acts like an open circuit** and **the inductor acts like a short circuit**.



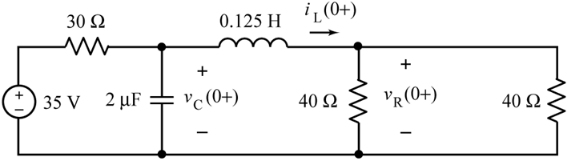
From the circuit $$i_L(0-) = \frac{35}{30+40} = 0.5\ \text{A}\ ,\ v_R(0-) = 40\, i_L(0-) = 20\ \text{V}\ ,$$

And $$v_C(0-) = v_R(0-) = 20\ \text{V}$$

Problem P 7.8-12 Solution

The **capacitor voltage and inductor current don't change instantaneously** so

$$v_C(0+) = v_C(0-) = 20\text{ V} \text{ and } i_L(0+) = i_L(0-) = 0.5\text{ A}$$



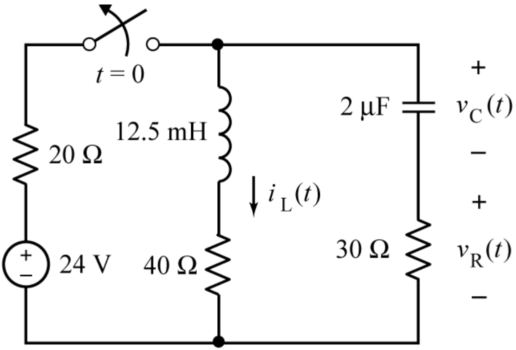
$$v_R(0+) = 40\left(\frac{40}{40+40}\right)i_L(0+) = 10\text{ V}$$

(Notice that the resistor voltage did change instantaneously.)

Problem P 7.8-13

The circuit shown in the figure has reached steady state before the switch opens at time $t = 0$.

- Determine the values of $i_L(t)$, $v_c(t)$ and $v_R(t)$ immediately before the switch closes.
- Determine the value of $v_R(t)$ immediately after the switch closes.



Problem P 7.8-13 Solution

Solution: Because

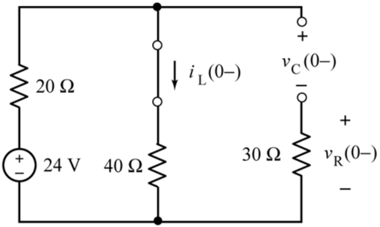
- This **circuit has reached steady state** before the switch opens at time $t = 0$.
- The only source is a **constant** voltage **source**.

At t=0–, **the capacitor acts like an open circuit** and **the inductor acts like a short circuit**.

The current in the 30 Ω resistor is zero so $v_R(0-) = 0$ V . Next

$$i_1(0-) = \frac{24}{20+40} = 0.4 \text{ A} \quad \text{and}$$

$$v_C(0-) = 40\, i_L(0-) = 16 \text{ V}$$



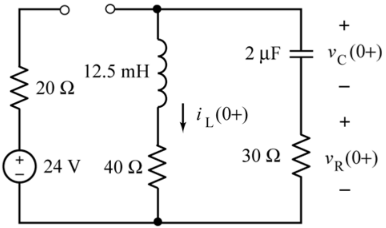
Problem P 7.8-13 Solution

The **capacitor voltage and inductor current don't change instantaneously** so

$$v_C(0+) = v_C(0-) = 16 \text{ V} \text{ and}$$
$$i_L(0+) = i_L(0-) = 0.4 \text{ A}$$

Apply KCL at the bottom node and then Ohm's law to get

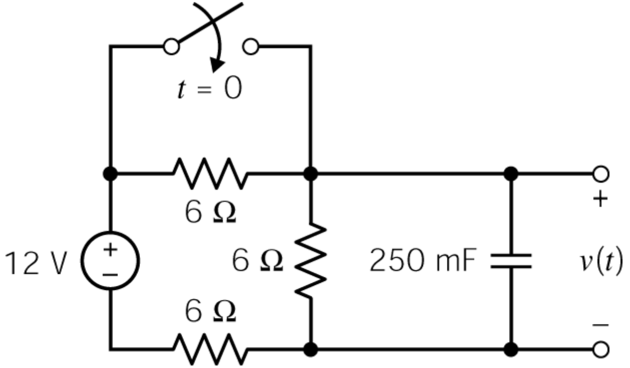
$$v_R(0+) = -30\, i_L(0+) = -12 \text{ V}$$



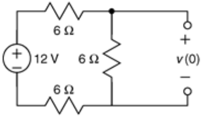
(Notice that the resistor voltage did change instantaneously.)

Problem P 8.3-1

The circuit shown in the figure is at steady state before the switch closes at time $t = 0$. The input to the circuit is the voltage of the voltage source, 12 V. The output of this circuit is the voltage across the capacitor, $v(t)$. Determine $v(t)$ for $t > 0$.



Problem P 8.3-1 Solution

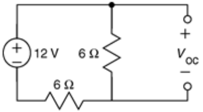


Here is the circuit before $t = 0$, when the switch is open and the circuit is at steady state. The open switch is modeled as an open circuit.

A capacitor in a steady-state dc circuit acts like an open circuit, so an open circuit replaces the capacitor. The voltage across that open circuit is the initial capacitor voltage, $v\,(0)$.

By voltage division

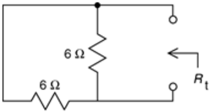
$$v(0) = \frac{6}{6+6+6}(12) = 4 \text{ V}$$



Next, consider the circuit after the switch closes. The closed switch is modeled as a short circuit.

We need to find the Thevenin equivalent of the part of the circuit connected to the capacitor. Here's the circuit used to calculate the open circuit voltage, V_{oc}.

$$V_{oc} = \frac{6}{6+6}(12) = 6 \text{ V}$$



Here is the circuit that is used to determine R_t. A short circuit has replaced the closed switch. Independent sources are set to zero when calculating R_t, so the voltage source has been replaced by a short circuit.

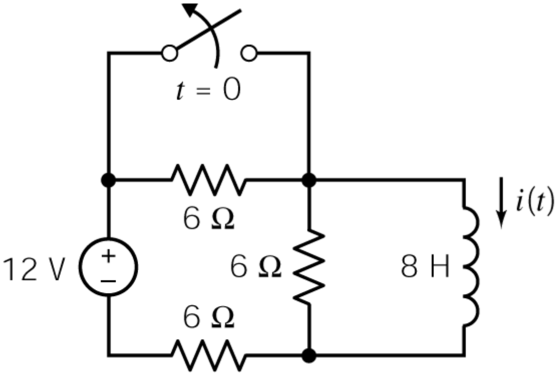
$$R_t = \frac{(6)(6)}{6+6} = 3 \ \Omega$$

Then $\tau = R_t\, C = 3\,(0.25) = 0.75$ s

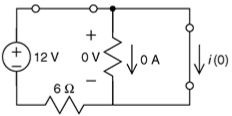
Finally, $v(t) = V_{oc} + \left(v(0) - V_{oc}\right) e^{-t/\tau} = 6 - 2\, e^{-1.33\,t}$ V for $t > 0$

Problem P 8.3-2

The circuit shown in the figure is at steady state before the switch opens at time $t = 0$. The input to the circuit is the voltage of the voltage source, 12 V. The output of this circuit is the current in the inductor, $i(t)$. Determine $i(t)$ for $t > 0$.



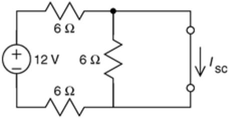
Problem P 8.3-2 Solution



Here is the circuit before $t = 0$, when the switch is closed and the circuit is at steady state. The closed switch is modeled as a short circuit.

An inductor in a steady-state dc circuit acts like an short circuit, so a short circuit replaces the inductor. The current in that short circuit is the initial inductor current, $i(0)$.

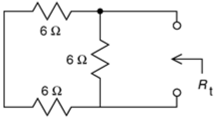
$$i(0) = \frac{12}{6} = 2 \text{ A}$$



Next, consider the circuit after the switch opens. The open switch is modeled as an open circuit.

We need to find the Norton equivalent of the part of the circuit connected to the inductor. Here's the circuit used to calculate the short circuit current, I_{sc}.

$$I_{sc} = \frac{12}{6+6} = 1 \text{ A}$$



Here is the circuit that is used to determine R_t. An open circuit has replaced the open switch. Independent sources are set to zero when calculating R_t, so the voltage source has been replaced by an short circuit.

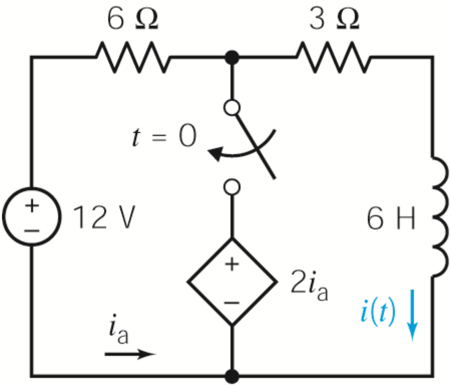
$$R_t = 6 \,||\, (6+6) = \frac{(6+6)(6)}{(6+6)+6} = 4 \ \Omega$$

Then $$\tau = \frac{L}{R_t} = \frac{8}{4} = 2 \text{ s}$$

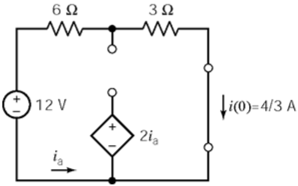
Finally, $$i(t) = I_{sc} + \left(i(0) - I_{sc}\right) e^{-t/\tau} = 1 + e^{-0.5\,t} \text{ A for } t > 0$$

Problem P 8.3-4

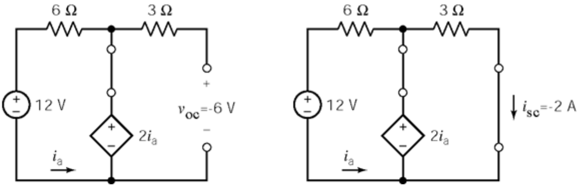
The circuit shown in the figure is at steady state before the switch closes at time $t = 0$. Determine the inductor current, $i(t)$, for $t > 0$.



Problem P 8.3-4 Solution



After the switch closes:

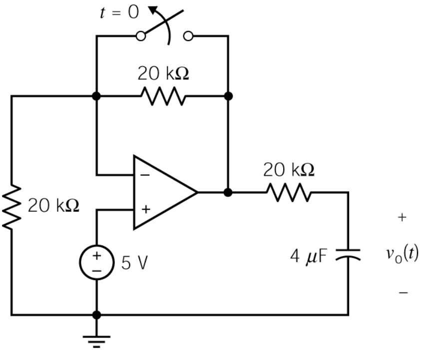


Therefore $R_t = \dfrac{-6}{-2} = 3\ \Omega$ so $\tau = \dfrac{6}{3} = 2$ s.

Finally, $i(t) = i_{sc} + (i(0) - i_{sc})\, e^{-\frac{t}{\tau}} = -2 + \dfrac{10}{3} e^{-0.5t}$ A for $t > 0$

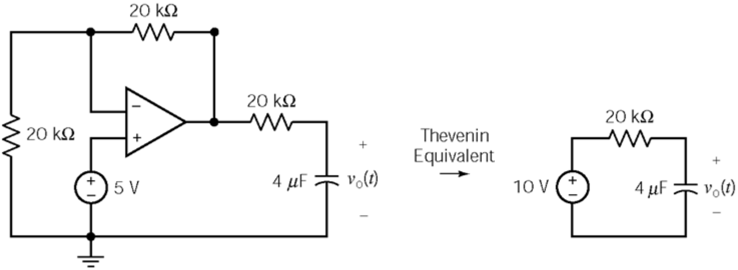
Problem P 8.3-5

The circuit shown in the figure is at steady state before the switch opens at time $t = 0$. Determine the voltage, $v_o(t)$, for $t > 0$.



Problem P 8.3-5 Solution

Solution: Before the switch opens, $v_o(t) = 5\text{ V} \Rightarrow v_o(0) = 5\text{ V}$. After the switch opens the part of the circuit connected to the capacitor can be replaced by it's Thevenin equivalent circuit to get:



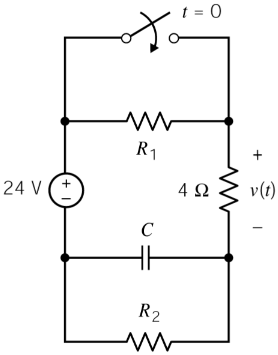
Therefore $\tau = (20\times10^3)(4\times10^{-6}) = 0.08\text{ s}$.

Next, $v_C(t) = v_{oc} + (v(0) - v_{oc})\, e^{-\frac{t}{\tau}} = 10 - 5\, e^{-12.5t}\text{ V} \quad \text{for } t > 0$

Finally, $v_0(t) = v_C(t) = 10 - 5\, e^{-12.5t}\text{ V} \quad \text{for } t > 0$

Problem P 8.3-11

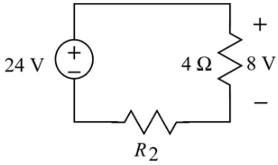
The voltage v(t) in the circuit shown in the figure is given by $v(t) = 8 + 4\exp(2t)$ Vfor $t > 0$. Determine the values of R1, R2, and C.



Problem P 8.3-11 Solution

Solution: As $t \to \infty$ the circuit reaches steady state and the capacitor acts like an open circuit. Also, from the given equation, $v(t) \to 8$ V , as labeled on the drawing to the right, then

$$8 = \frac{4}{R_2 + 4} 24 \quad \Rightarrow \quad R_2 = 8\ \Omega$$



After $t = 0$

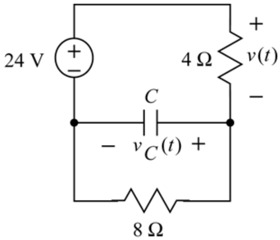
$$v_C(t) = 24 - v(t) = 16 - 4e^{-2t}$$

Immediately after $t = 0$

$$v_C(0+) = 16 - 4 = 12 \text{ V}$$

The capacitor voltage cannot change instantaneously so

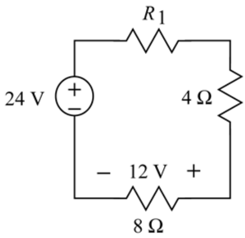
$$v(0-) = 12 \text{ V}$$



Problem P 8.3-11 Solution

The circuit is at steady state just before the switch closes so the capacitor acts like an open circuit. Then

$$12 = \frac{8}{R_1 + 4 + 8} 24 \quad \Rightarrow \quad R_1 = 4\ \Omega$$



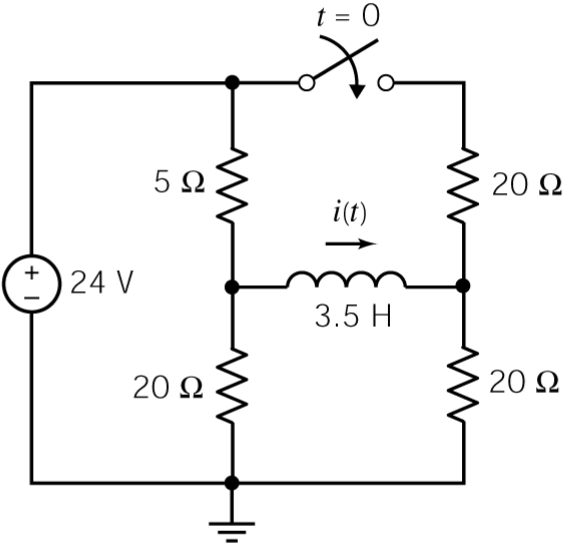
After $t = 0$ the Thevenin resistance seen by the capacitor is

$$R_t = 8 \,||\, 4 = \frac{8}{3}\ \Omega$$

so
$$2 = \frac{1}{\frac{8}{3} C} \quad \Rightarrow \quad C = \frac{3}{16}\ \text{F}$$

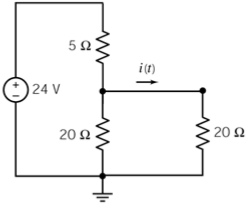
Problem P 8.3-20

The circuit shown in the figure is at steady state before the switch closes. Determine $i(t)$ for $t \geq 0$.



Problem P 8.3-20 Solution

Solution: Before the switch closes the circuit is at steady state so the inductor acts like a short circuit. We have

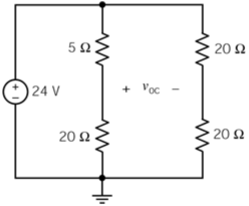


$$i(t) = \frac{1}{2}\left(\frac{24}{5+(20 \parallel 20)}\right) = 0.8\ \text{A}$$

so

$$i(0+) = i(0-) = 0.8\ \text{A}$$

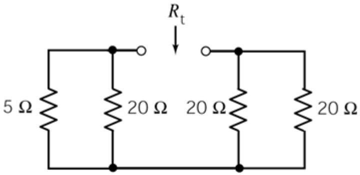
After the switch closes, find the Thevenin equivalent circuit for the part of the circuit connected to the inductor.



Using voltage division twice

$$v_{oc} = \left(\frac{20}{25} - \frac{1}{2}\right)24 = 7.2\ \text{V}$$

Problem P 8.3-20 Solution



$$R_t = (5 \parallel 20) + (20 \parallel 20) = 14\ \Omega$$

$$i_{sc} = \frac{v_{oc}}{R_t} = \frac{7.2}{14} = 0.514\ \text{A}$$

Then

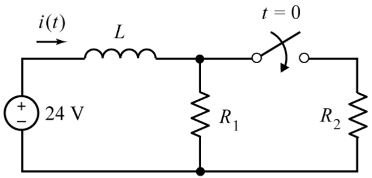
$$\tau = \frac{L}{R_t} = \frac{3.5}{14} = \frac{1}{4}\ \text{s} \quad \Rightarrow \quad \frac{1}{\tau} = 4\ \frac{1}{\text{s}}$$

and

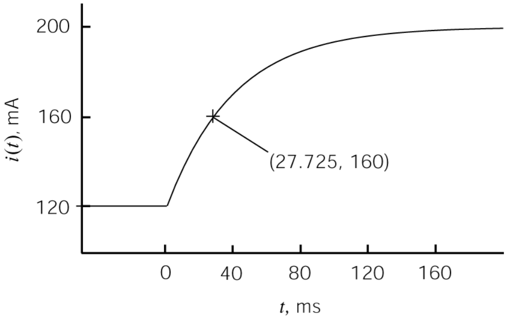
$$i(t) = \left(i(0+) - i_{sc}\right)e^{-t/\tau} + i_{sc} = (0.8 - 0.514)e^{-4t} + 0.514 = 0.286e^{-4t} + 0.514\ \text{A} \quad \text{for } t \geq 0$$

Problem P 8.3-24

Consider the circuit shown in the Figure (a) and corresponding plot of the inductor current shown in the Figure (b). Determine the values of L, R1 and R2.



(a)



(b)

Problem P 8.3-24 Solution

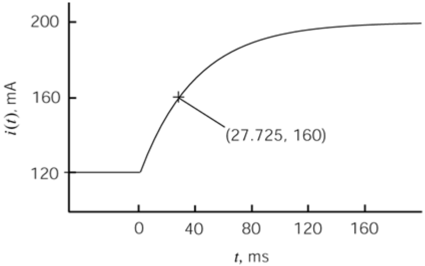
Solution: From the plot

$$D = i(t) \text{ for } t < 0 = 120 \text{ mA} = 0.12 \text{ A},$$

$$E + F = i(0+) = 120 \text{ mA} = 0.12 \text{ A}$$

and

$$E = \lim_{t\to\infty} i(t) = 200 \text{ mA} = 0.2 \text{ A} .$$

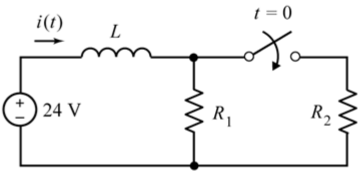


The point labeled on the plot indicates that $i(t) = 160$ mA when $t = 27.725$ ms $= 0.027725$ s. Consequently

$$160 = 200 - 80\,e^{-a(0.027725)} \quad \Rightarrow \quad a = \frac{\ln\left(\dfrac{160-200}{80}\right)}{-0.027725} = 25 \ \frac{1}{\text{s}}$$

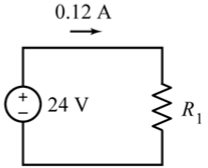
Then

$$i(t) = \begin{cases} 120 \text{ mA} & \text{for } t \le 0 \\ 200 - 80\,e^{-25t} \text{ mA} & \text{for } t \ge 0 \end{cases}$$



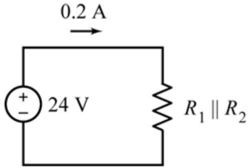
Problem P 8.3-24 Solution

When $t < 0$, the circuit is at steady state so the inductor acts like a short circuit.



$$R_1 = \frac{24}{0.12} = 200\ \Omega$$

As $t \to \infty$, the circuit is again at steady state so the inductor acts like a short circuit.



$$R_1 \| R_2 = \frac{24}{0.2} = 120\ \Omega$$

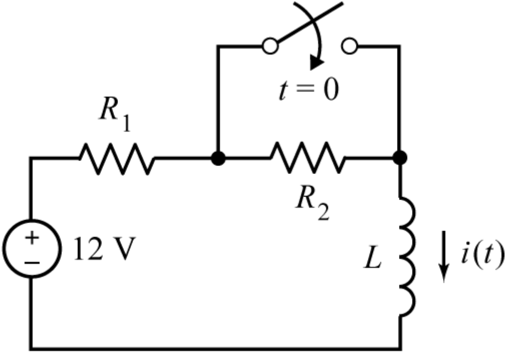
$$120 = 200 \| R_2 \quad \Rightarrow \quad R_2 = 300\ \Omega$$

Next, the inductance can be determined using the time constant:

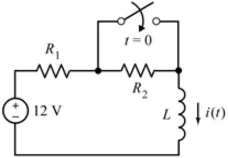
$$25 = a = \frac{1}{\tau} = \frac{R_1 \| R_2}{L} = \frac{120}{L} \quad \Rightarrow \quad L = \frac{120}{25} = 4.8\ \text{H}$$

Problem P 8.3-27

The circuit shown in the figure is at steady state before the switch closes at time $t = 0$. After the switch closes, the inductor current is given by $v(t) = 0.6 - 0.2\exp(5t)$ A. Determine the values of R1, R2 and L.



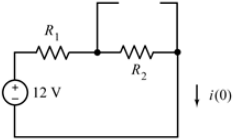
Problem P 8.3-27 Solution



The steady state current before the switch closes is equal to $i(0)=0.6-0.2e^{-5(0)}=0.4$ A .

The inductor will act like a short circuit when this circuit is at steady state so

$$0.4=i(0)=\frac{12}{R_1+R_2} \quad \Rightarrow \quad R_1+R_2=30\ \Omega$$

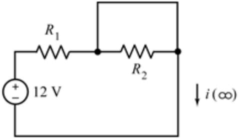


After the switch has been open for a long time, the circuit will again be at steady state. The steady state inductor current will be $i(\infty)=0.6-0.2e^{-5(\infty)}=0.6$ A

The inductor will act like a short circuit when this circuit is at steady state so

$$0.6=i(\infty)=\frac{12}{R_1} \quad \Rightarrow \quad R_1=20\ \Omega$$

Then $R_2=10\ \Omega$.



After the switch is closed, the Thevenin resistance of the part of the circuit connected to the inductor is $R_t=R_1$. Then

$$5=\frac{1}{\tau}=\frac{R_t}{L}=\frac{R_1}{L}=\frac{20}{L} \quad \Rightarrow \quad L=4\ \mathrm{H}$$

Thank you !