

ECSE 200 - Electronic Circuits 1

Tutorial 1 - Problem set 1

ECE Dept., McGill University

Sept 10, 2018

Outline

1 Electric charge and current

2 Voltage, energy and power

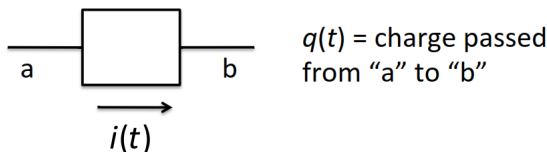
Outline for section 1

1 Electric charge and current

2 Voltage, energy and power

Electric charge and current

- Recap



- ▶ $q(t)$: charge that has passed through the element
- ▶ $i(t)$: current that has passed through the element

$$i(t) = \frac{\partial q(t)}{\partial t}$$

$$\Rightarrow \Delta q(t) = q(t_2) - q(t_1) = \int_{t_1}^{t_2} i(t) \partial t$$

Electric charge and current

- **Problem 1.2-2**

The current, in amperes, of a circuit element is represented as follow:

$$i(t) = \begin{cases} 0 & t < 0 \\ 4(1 - e^{-5t}) & t \geq 0. \end{cases}$$

Determine the total charge that has entered this circuit element for $t \geq 0$.

Electric charge vs. current

• Problem 1.2-2

Solution:

$$\begin{aligned}\Delta q|_0^t &= \Delta q = \int_0^t i(t) dt + q(0) = \int_0^t i(t) dt + \int_{-\infty}^0 i(t) dt \\&= \int_0^t 4(1 - e^{-5t}) dt + \int_{-\infty}^0 0 dt \\&= \int_0^t 4 dt - \int_0^t 4e^{-5t} dt \\&= 4t - (-0.8e^{-5t} + 0.8) \\&= 4t + 0.8e^{-5t} - 0.8\end{aligned}$$

So $\Delta q = 4t + 0.8e^{-5t} - 0.8$ (C).

Electric charge and current

- **Problem 1.2-5**

The total charge $q(t)$, in coulombs, that enters the terminal of an element is

$$q(t) = \begin{cases} 0 & t < 0 \\ 2t & 0 \leq t \leq 2 \\ 3 + e^{-2(t-2)} & t > 2. \end{cases}$$

Find the current $i(t)$ and sketch its waveform for $t \geq 0$.

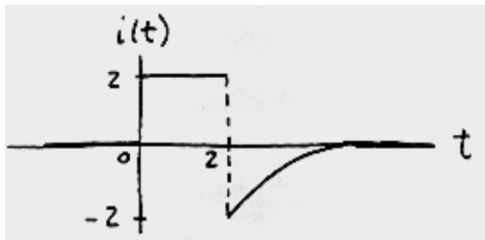
Electric charge vs. current

● Problem 1.2-5

Solution:

$$i(t) = \frac{\partial q(t)}{\partial t} = \begin{cases} 0 & t < 0 \\ 2 & 0 \leq t \leq 2 \\ -2e^{-2(t-2)} & t > 2. \end{cases}$$

The waveform of $i(t)$ for $t \geq 0$:



Electric charge and current

● Problem 1.2-6

Given an electro-plating bath that receives a constant current of 450 A for 20 minutes, and each coulomb transports 1.118 mg of silver. What is the weight of silver deposited in grams ?

Solution:

- ▶ The depositing time is:

$$t_d = 20 \text{ (minutes)} = 1200 \text{ (s)}$$

- ▶ The total number of charges is:

$$\Delta q = \int_0^{t_d} 450 \partial t + 0 = 450 \times 1200 = 5.4 \times 10^5 \text{ (C)}$$

- ▶ The total weight of deposited silver is:

$$m_{\text{silver}} = \Delta q \times 1.118 = 603.72 \text{ (g)}$$

Electric charge and current

- **Problem 1.3-1**

A constant current of $3.2\mu A$ flows through an element. What is the charge that has passed through the element in the first millisecond?

Electric charge and current

- **Problem 1.3-1**

A constant current of $3.2\mu A$ flows through an element. What is the charge that has passed through the element in the first millisecond?

Solution:

$$\begin{aligned}\Delta q &= \int_0^t i(t) dt \\ &= \int_0^t 3.2 \times 10^{-6} dt \\ &= 3.2 \times 10^{-6} \times (10^{-3} - 0) \\ &= 3.2 \times 10^{-9} (C)\end{aligned}$$

Outline for section 2

1 Electric charge and current

2 Voltage, energy and power

Voltage, energy and power

● Problem 1.5-1

Figure P1.5-1 shows four circuit elements identified by the letters A, B, C, and D.

- (a) Which of the devices supply 30 mW?
- (b) Which of the devices absorb 0.03 W?
- (c) What is the value of the power received by device B?
- (d) What is the value of the power delivered by device B?
- (e) What is the value of the power delivered by device C?

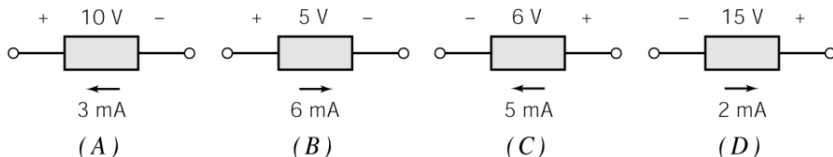


Figure P1.5-1

Voltage, energy and power

• Problem 1.5-1

Figure P1.5-1 shows four circuit elements identified by the letters A, B, C, and D.

- (a) Which of the devices supply 30 mW? **A, and D**
- (b) Which of the devices absorb 0.03 W? **B, and C**
- (c) What is the value of the power received by device B? **30 mW**
- (d) What is the value of the power delivered by device B? **-30 mW**
- (e) What is the value of the power delivered by device C? **-30 mW**

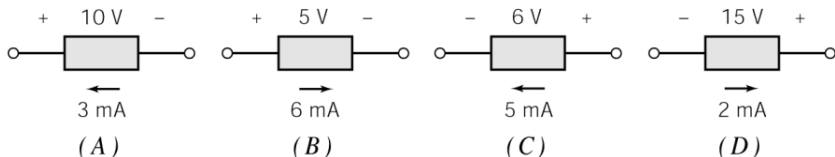


Figure P1.5-1

Voltage, energy and power

● Problem 1.5-4

The current through and voltage across an element vary with time as shown in Figure P1.5-4. Sketch the power delivered to the element for $t > 0$. What is the total energy delivered to the element between $t = 0$ and $t = 25$ s? The element voltage and current adhere to the passive convention.

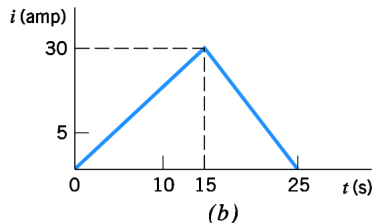
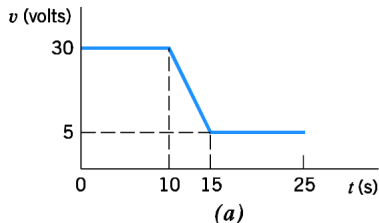


Figure P 1.5-4

Voltage, energy and power

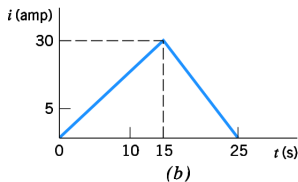
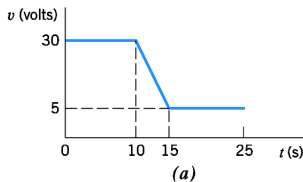


Figure P 1.5-4

● Problem 1.5-4

Solution

From Fig. P 1.5-4, the the element voltage $v(t)$ and current $i(t)$ can be represented as follow:

$$v(t) = \begin{cases} 30 & 0 \leq t \leq 10 \\ -5t + 80 & 10 < t \leq 15 \\ 5 & 15 < t \leq 25 \end{cases} \text{ and } i(t) = \begin{cases} 2t & 0 \leq t \leq 15 \\ -3t + 75 & 15 < t \leq 25 \end{cases}$$

Voltage, energy and power

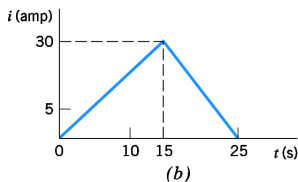
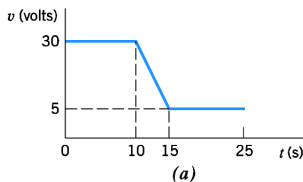


Figure P 1.5-4

• Problem 1.5-4

Solution (cnt.)

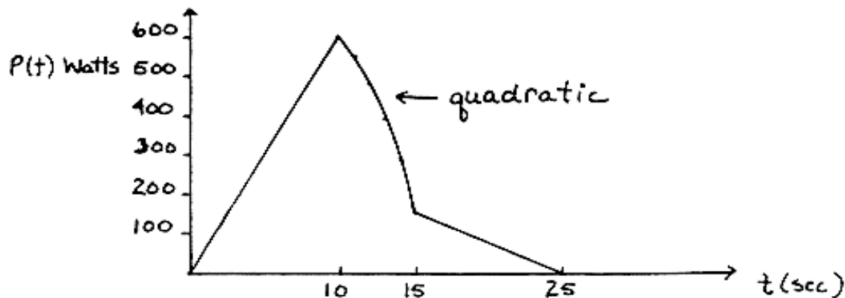
Therefore,

$$P(t) = \begin{cases} 60t & 0 \leq t \leq 10 \\ -10t^2 + 160t & 10 < t \leq 15 \\ -15t + 375 & 15 < t \leq 25 \end{cases}$$

Voltage, energy and power

• Problem 1.5-4

Solution (cnt.)



$$\begin{aligned}\text{Energy} &= \int P dt = \int_0^{10} 60t dt + \int_{10}^{15} (160t - 10t^2) dt + \int_{15}^{25} (375 - 15t) dt \\ &= 30t^2 \Big|_0^{10} + 80t^2 - \frac{10}{3}t^3 \Big|_{10}^{15} + 375t - \frac{15}{2}t^2 \Big|_{15}^{25} = \underline{5833.3 \text{ J}}\end{aligned}$$

Voltage, energy and power

● Problem 1.5-10

Given that $u(t) = 500(V)$ and $i(t) = 2 + 30e^{-at}(mA)$ where $a = 0.85 \frac{1}{hr}$. Determine the energy supplied E by the voltage when the procedure lasts 3 hours.

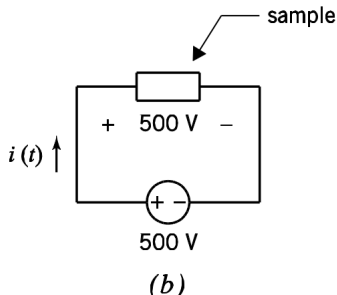


Figure 1.5-10 (a) An image of a gel and (b) the electric circuit used to preparation a gel.

Voltage, energy and power

• Problem 1.5-10

Given that $u(t) = 500(V)$ and $i(t) = 2 + 30e^{-at}(mA)$ where $a = 0.85 \frac{1}{hr}$. Determine the energy supplied E by the voltage when the procedure lasts 3 hours.

Solution

$$\begin{aligned} E &= \int_0^t P(t) dt = \int_0^t u(t)i(t) dt \\ &= \int_0^t 500(2 + 30e^{-at}) \times 10^{-3} dt = t \Big|_0^3 + \frac{-15e^{-at}}{a} \Big|_0^3 \\ &= 3 + \frac{-15}{0.85}(e^{-0.85 \times 3} - 1) = 19.27 \text{ Wh} \end{aligned}$$

Thank you !