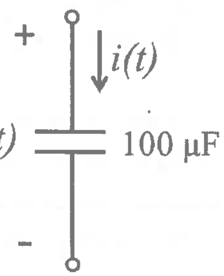
ECSE-200 Electric Circuits 1 - Quiz #8 (Mar. 15, 2019)

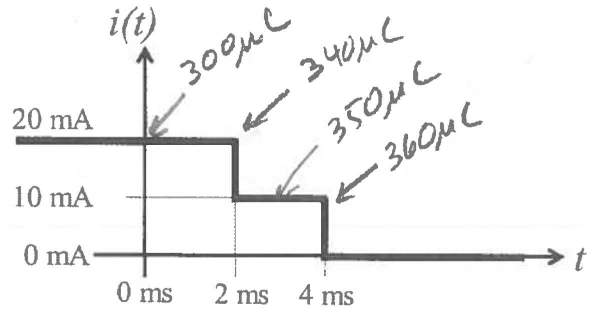
LAST NAME ____________________ **MCGILL ID#** ____________

FIRST NAME ____________________ **SIGNATURE** ____________________

- *Only Faculty standard calculator accepted*
- *No cellphone allowed*
- *Show all your work*
- *Clearly indicate your final answer with the SI unit and multiplier*
- *You have 45 minutes to complete this quiz*

Question 1: Consider the 100 µF capacitor (C = 100 µF) shown below along with the diagram illustrating the current $i(t)$ flowing through the capacitor. A constant current of 20 mA flows through the capacitor up to time t = 2 ms. At that time t = 2 ms, the current drops to 10 mA until time t = 4 ms. For t > 4 ms, the current is 0 A. The capacitor is initially charged at t = 0 ms and has a voltage of 3 V across it. Answer the following questions.





a) At what time t will the capacitor have a charge separation of 350 µC (q = 350 µC)? [2 pt]
b) What is the instantaneous power at t = 1 ms? [2 pt]
c) What is the energy stored as electric potential energy $U(t)$ in the capacitor at time t = 5 ms? [2 pt]

a) $v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(\tau)\,d\tau$ and $q = Cv$ and $i = \frac{dq}{dt}$

at $t = 0ms$ $q(0) = 100\mu F \cdot 3V = 300\mu C$

at $t = 2ms$ $v(2ms) = 3V + \frac{1}{100\mu F}\int_0^{2ms} 20mA\,dt = 3V + \frac{20mA}{100\mu F}\cdot 2ms = 3.4V$

$q(2ms) = 100\mu F \cdot 3.4V = 340\mu C$

at $t = 4ms$ $v(4ms) = 3.4V + \frac{1}{100\mu F}\int_{2ms}^{4ms} 10mA\,dt = 3.4V + \frac{10mA}{100\mu F}\cdot 2ms = 3.6V$

$q(4ms) = 360\mu F$

$t - 2ms = \frac{q - Cv(2ms)}{i} = \frac{350\mu C - 340\mu C}{10mA} = 1ms$ $\boxed{t = 3ms}$

b) $p(t) = i(t)\cdot v(t)$ $v(1ms) = 3V + \frac{1}{100\mu F}\int_0^{1ms} 20mA\,d\tau = 3V + \frac{20mA \cdot 1ms}{100\mu F} = 3.2V$

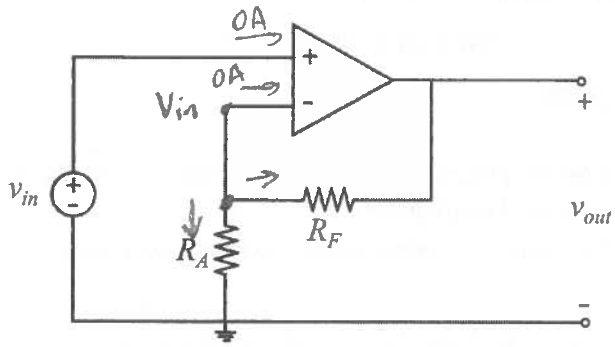
$p(1ms) = 20mA * 3.2V = \boxed{64mW}$

c) for $t > 4ms$, $i(t) = 0$ $v(5ms) = 3.6V + 0 \rightarrow$ $U(5ms) = \frac{1}{2}100\mu F\,(3.6V)^2$

$U(5ms) = \frac{1}{2}Cv^2$

$\boxed{U = 648\mu J}$

Question 2: The circuit shown below is a common operational amplifier (op-amp) circuit. Answer the following questions.



a) $\frac{Vin}{RA} + \frac{Vin - Vout}{RF} = 0$ (KCL at inv node)

$Vout = Vin\left(1 + \frac{RF}{RA}\right)$

$\boxed{\frac{Vout}{Vin} = 1 + \frac{RF}{RA}}$

a) Using the ideal op-amp model, derive the expression for the voltage gain v_{out}/v_{in} as a function of the two resistors R_A and R_F. [1 pt]

b) The op-amp circuit above is configured as a non-inverting amplifier. Using the ideal op-amp model, the voltage gain is $v_{out}/v_{in} = 1 + R_F/R_A$ as you found in part a). Choose resistance values for resistors R_A and R_F to design the op-amp circuit such that the voltage gain is +8 V/V and the total power absorbed by the two resistors is 500 μW when v_{out} = 2 V. [2 pt]

c) Using the practical op-amp circuit model simplified by setting the input resistance to infinity ($R_i \to \infty$) and the output resistance to zero ($R_o = 0\ \Omega$), what is the output voltage value v_{out} if $R_A \to \infty$, $R_F \to 0\ \Omega$, v_{in} = 5 V, and the open-loop gain A is 15 V/V? [2 pt]

d) In the circuit described in part c), what is the output voltage value v_{out} using the ideal op-amp model? [1 pt]

b) $\frac{Vout}{Vin} = 1 + \frac{RF}{RA} = +8\ V/V \rightarrow \frac{RF}{RA} = 7 \therefore RF = 7RA$ & $Vin = \frac{1}{8} Vout = \frac{1}{4} V = 0.25V$

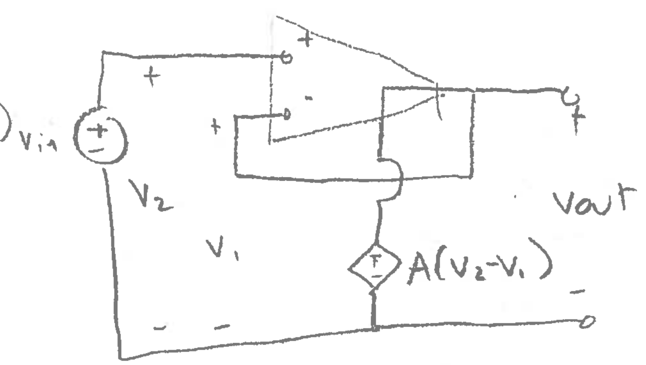
$P_{RA} = Vin \cdot \frac{Vin}{RA} = \frac{Vin^2}{RA}$ $\quad P_{RF} = (Vin - Vout) \cdot \frac{(Vin - Vout)}{RF} = \frac{(Vin - Vout)^2}{RF}$

$P_{tot} = P_{RA} + P_{RF} = \frac{(0.25V)^2}{RA} + \frac{(0.25 - 2V)^2}{7\,RA} = 500 \times 10^{-6}$

$\frac{0.4375 + 3.0625}{7RA} = 500 \times 10^{-6}$

$\boxed{\begin{array}{l} RA = 1K\Omega \\ RF = 7K\Omega \end{array}}$

c)



$Vout = A(V_2 - V_1)$

$V_2 = Vin \quad V_1 = Vout$

$Vout = A(Vin - Vout) \rightarrow Vout = \frac{A}{A+1} Vin$

$Vout = \frac{15 V/V}{15 V/V + 1} \cdot 5V = \boxed{4.6875V}$

d) $Vout = Vin$ $\quad \boxed{Vout = 5V}$