ECSE 200 - Electric Circuits 1
Turotial 1 - Problem set 1

ECE Dept., McGill University

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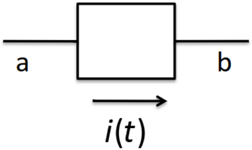
Outline

Outline for section 1

Electric charge and current

- Recall



$q(t)$ = charge passed from "a" to "b"

  - $q(t)$: charge that has passed through the element
  - $i(t)$: current that has passed through the element

$$i(t) = \frac{\partial q(t)}{\partial t}$$

$$\Rightarrow \Delta q(t) = q(t_2) - q(t_1) = \int_{t_1}^{t_2} i(t) \partial t$$

Electric charge and current

- **Problem 1.2-2**
  The current, in amperes, of a circuit element is represented as follow:

$$i(t) = \begin{cases} 0 & t < 0 \\ 4(1 - e^{-5t}) & t \geq 0. \end{cases}$$

  Determine the total charge that has entered this circuit element for $t \geq 0$.

Electric charge vs. current

- **Problem 1.2-2**
  Solution:

$$
\begin{aligned}
\Delta q|_0^t = \Delta q &= \int_0^t i(t')\partial t' + q(0) = \int_0^t i(t')\partial t' + \int_{-\infty}^0 i(t')\partial t' \\
&= \int_0^t 4(1 - e^{-5t'})\partial t' + \int_{-\infty}^0 0\partial t' \\
&= \int_0^t 4\partial t' - \int_0^t 4e^{-5t'}\partial t' \\
&= 4t + 0.8(e^{-5t} - 1) \\
&= 4t + 0.8e^{-5t} - 0.8
\end{aligned}
$$

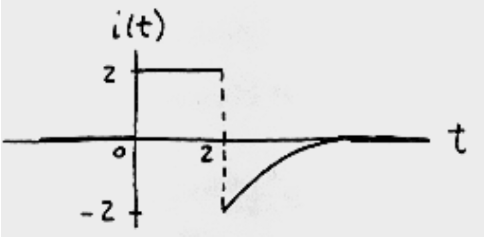
So $\Delta q = 4t + 0.8e^{-5t} - 0.8$ (C).

Electric charge and current

- **Problem 1.2-5**
  The total charge $q(t)$, in coulombs, that enters the terminal of an element is
  $$q(t) = \begin{cases} 0 & t < 0 \\ 2t & 0 \le t \le 2 \\ 3 + e^{-2(t-2)} & t > 2. \end{cases}$$
  Find the current $i(t)$ and sketch its waveform for $t \ge 0$.

Electric charge vs. current

- **Problem 1.2-5**
  Solution:
  $$i(t) = \frac{\partial q(t)}{\partial t} = \begin{cases} 0 & t < 0 \\ 2 & 0 \leq t \leq 2 \\ -2e^{-2(t-2)} & t > 2. \end{cases}$$
  The waveform of $i(t)$ for $t \geq 0$:



Electric charge and current

- **Problem 1.2-6**
  Given an electro-plating bath that receives a constant current of 450 A for 20 minutes, and each coulomb transports 1.118 mg of silver. What is the weight of silver deposited in grams ?

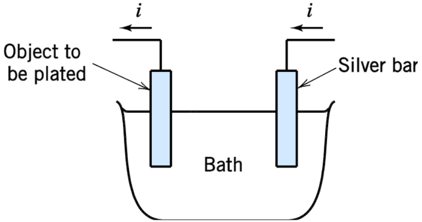


Figure P 1.2-6

Electric charge and current

- **Problem 1.2-6**
  Given an electro-plating bath that receives a constant current of 450 A for 20 minutes, and each coulomb transports 1.118 mg of silver. What is the weight of silver deposited in grams ?
  <u>Solution:</u>
  - The depositing time is:
    $t_d = 20 \text{ (minutes)} = 1200 \text{ (s)}$
  - The total number of charges is:
    $\Delta q = \int_0^{t_d} 450 \partial t + 0 = 450 \times 1200 = 5.4 \times 10^5 \text{ (C)}$
  - The total weight of deposited silver is:
    $m_{silver} = \Delta q \times 1.118 = 603.72 \text{ (g)}$

Electric charge and current

- **Problem 1.3-1**
  A constant current of $3.2\mu A$ flows through an element. What is the charge that has passed through the element in the first millisecond?

Electric charge and current

- **Problem 1.3-1**
  A constant current of $3.2\mu A$ flows through an element. What is the charge that has passed through the element in the first millisecond?
  Solution:

$$\begin{aligned}\Delta q &= \int_0^{10^{-3}} i(t)\partial t \\ &= \int_0^{10^{-3}} 3.2 \times 10^{-6}\partial t \\ &= 3.2 \times 10^{-6} \times (10^{-3} - 0) \\ &= 3.2 \times 10^{-9}(C)\end{aligned}$$

Outline for section 2

Recall

- Electric potential: $\mathbf{W_{AB} = QV_{AB}}$ J
- Power: $\mathbf{P} = \frac{\partial \mathbf{W}}{\partial \mathbf{t}} = \frac{\partial \mathbf{Q}}{\partial \mathbf{t}} \frac{\partial \mathbf{W}}{\partial \mathbf{Q}} = IV$ W.
- Instantaneous power: $\mathbf{p(t) = i(t)v(t)}$ W.
- Energy: $\mathbf{E} = \mathbf{U(t_2)} - \mathbf{U(t_1)} = \int_{\mathbf{t_1}}^{\mathbf{t_2}} \mathbf{p(t)} \partial \mathbf{t}$ J.

Voltage, energy and power

- **Problem 1.5-1**
  Figure P1.5-1 shows four circuit elements identified by the letters A, B, C, and D.
  (a) Which of the devices supply 30 mW?
  (b) Which of the devices absorb 0.03 W?
  (c) What is the value of the power received by device B?
  (d) What is the value of the power delivered by device B?
  (e) What is the value of the power delivered by device C?

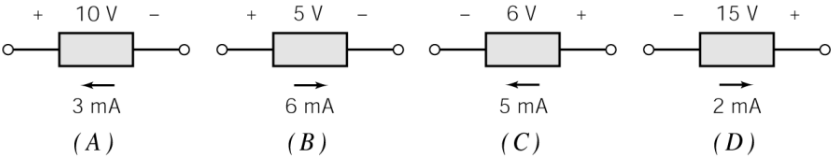


Figure P1.5-1

Voltage, energy and power

- **Problem 1.5-1**
  Figure P1.5-1 shows four circuit elements identified by the letters A, B, C, and D.
  (a) Which of the devices supply 30 mW? **A, and D**
  (b) Which of the devices absorb 0.03 W? **B, and C**
  (c) What is the value of the power received by device B? **30 mW**
  (d) What is the value of the power delivered by device B? **-30 mW**
  (e) What is the value of the power delivered by device C? **-30 mW**

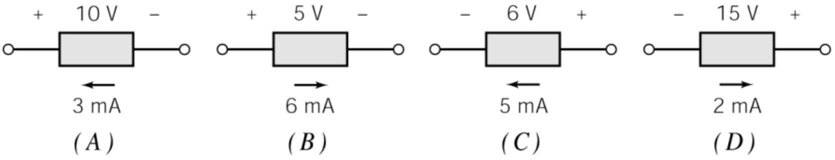


Figure P1.5-1

Voltage, energy and power

- **Problem 1.5-4**
  The current through and voltage across an element vary with time as shown in Figure P1.5-4. Sketch the power delivered to the element for $t > 0$. What is the total energy delivered to the element between $t = 0$ and $t = 25$ s? The element voltage and current adhere to the passive convention.

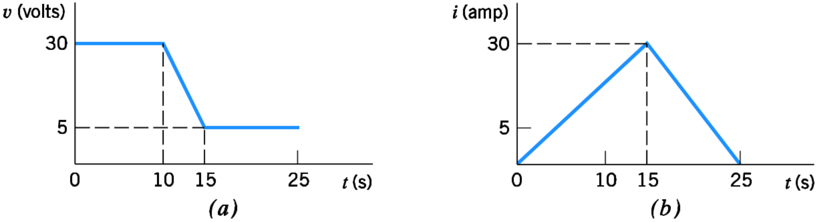


Figure P 1.5-4

Voltage, energy and power

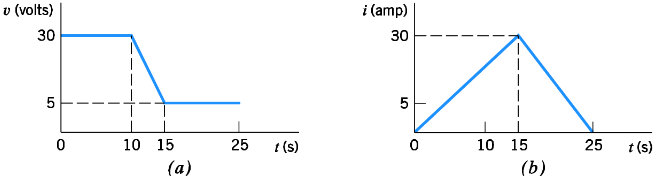


Figure P 1.5-4

- **Problem 1.5-4**

  <u>Solution</u>

  From Fig. P 1.5-4, the the element voltage $v(t)$ and current $i(t)$ can be represented as follow:

$$v(t) = \begin{cases} 30 & 0 \leq t \leq 10 \\ -5t + 80 & 10 < t \leq 15 \\ 5 & 15 < t \leq 25 \end{cases} \text{ and } i(t) = \begin{cases} 2t & 0 \leq t \leq 15 \\ -3t + 75 & 15 < t \leq 25 \end{cases}$$

Voltage, energy and power

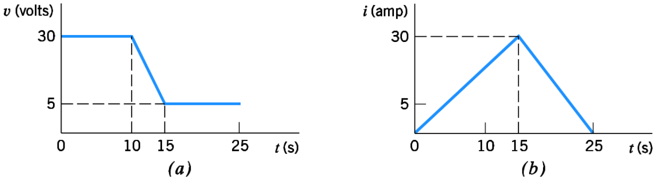


Figure P 1.5-4

- **Problem 1.5-4**

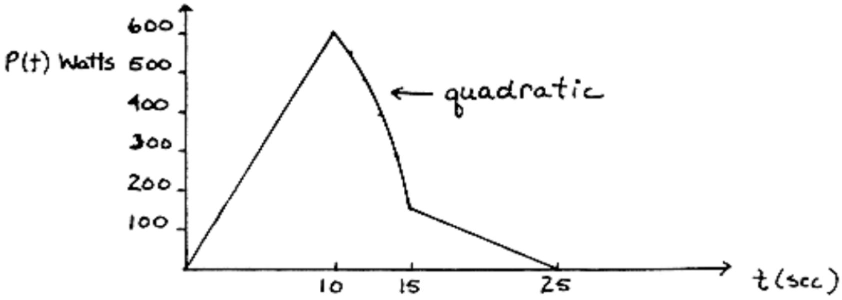
  Solution (cnt.)

  Therefore,

$$P(t) = \begin{cases} 60t & 0 \leq t \leq 10 \\ -10t^2 + 160t & 10 < t \leq 15 \\ -15t + 375 & 15 < t \leq 25 \end{cases}$$

Voltage, energy and power

- **Problem 1.5-4**
  Solution (cnt.)



$$\text{Energy} = \int P\,dt = \int_0^{10} 60t\,dt + \int_{10}^{15}\left(160t - 10t^2\right)dt + \int_{15}^{25}\left(375 - 15t\right)dt$$

$$= 30t^2\Big|_0^{10} + 80t^2 - \frac{10}{3}t^3\Big|_{10}^{15} + 375t - \frac{15}{2}t^2\Big|_{15}^{25} = \underline{5833.\bar{3}\ \text{J}}$$

Voltage, energy and power

- **Problem 1.5-6(modified)**
  Given an element that has $v(t) = 4\sin(3t)$ V and $i(t) = \frac{1}{12}\cos(3t)$ A, evaluate the power $p(t)$ of this element at $t = 5$ s and $t = 35$ s. Show that the power supplied by this element has a positive value at some times and a negative value at other times.

Voltage, energy and power

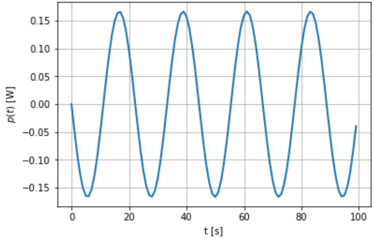
- **Problem 1.5-6(modified)**

  Solution:

  We have

$$p(t) = v(t)i(t) = 4\sin(3t)\frac{1}{12}\cos(3t) = \frac{1}{3}\sin(3t)\cos(3t)$$
$$= \frac{1}{6}\sin(6t) \text{ (W)}$$

  - At $t = 5$: $p(5) = \frac{1}{6}\sin(30) = 0.083$ W
  - At $t = 35$: $p(35) = \frac{1}{6}\sin(210) = -0.083$ W



Voltage, energy and power

- **Problem 1.5-10**
  Given that $u(t) = 500(V)$ and $i(t) = 2 + 30e^{-at}(mA)$ where $a = 0.85\ \frac{1}{hr}$. Determine the energy E supplied by the voltage when the procedure lasts 3 hours.

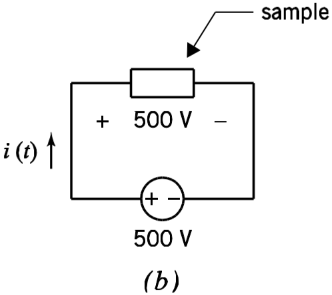


Figure 1.5-10 (*a*) An image of a gel and (*b*) the electric circuit used to preparation a gel.

Voltage, energy and power

- **Problem 1.5-10**
  Given that $v(t) = 500(V)$ and $i(t) = 2 + 30e^{-at}(mA)$ where $a = 0.85\ \frac{1}{hr}$. Determine the energy E supplied by the voltage when the procedure lasts 3 hours.
  <u>Solution</u>

$$
\begin{aligned}
E &= \int_0^3 P(t)\partial t = \int_0^3 v(t)i(t)\partial t \\
&= \int_0^3 500(2 + 30e^{-at}) \times 10^{-3} \partial t = t|_0^3 + \frac{-15e^{-at}}{a}|_0^3 \\
&= 3 + \frac{-15}{0.85}(e^{-0.85\times 3} - 1) = 19.27 \text{ Wh}
\end{aligned}
$$

Thank you !