McGill University
Faculty of Engineering

COURSE: ECSE200 – Fundamentals of Electrical Engineering

FINAL EXAMINATION

Examiner: Thomas Szkopek ______________________

Date: 20 June 2008 Time: 9:35-12:35

INSTRUCTIONS

A) *Read* all four exam questions *very carefully*.

B) Answer *all four* (×4) questions in this exam paper in the space provided. Use the reverse side of exam sheets if necessary.

C) Show all solution steps and indicate your final answer clearly. Partial marks can only be awarded for correct methodology that is presented clearly.

D) Individual question values are indicated. The total mark value of this exam is 36 points.

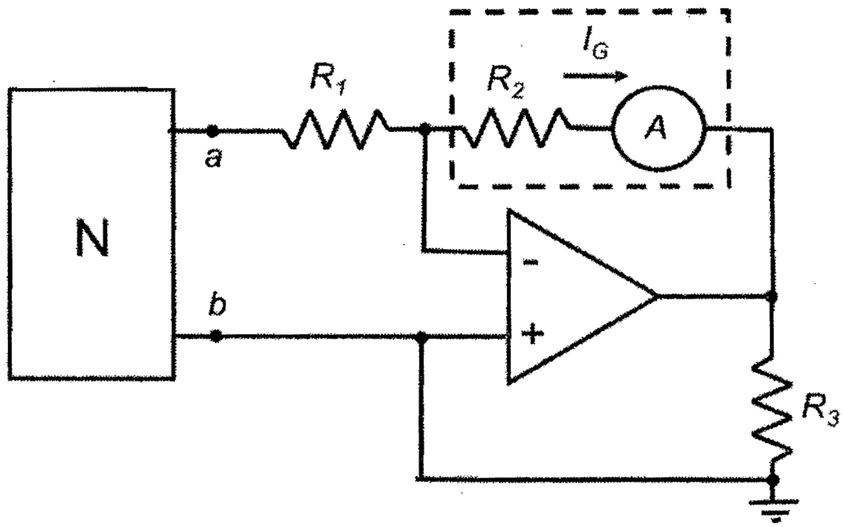
E) Closed book examination.

IDENTIFICATION

NAME: ______________________________________

ID NUMBER: _________________________________

(9/36) 1. For the circuit below, assume ideal op-amp behaviour and that the network N is ***linear***.



(3/9) (a) What is the current through the galvanometer, I_G? Express your answer in terms of the following quantities as necessary: known resistances R_1, R_2, R_3 and Thévenin equivalent network parameters of N.

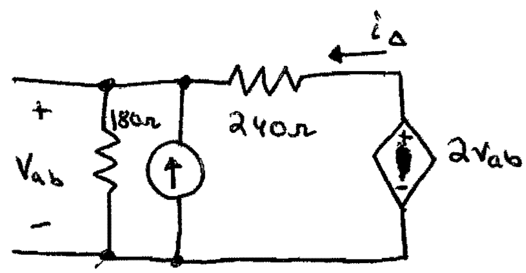
(3/9) (b) The internal galvanometer resistance is R_2=24Ω, and it is also known that R_3=1.37kΩ. The galvanometer measures I_G = 5mA with R_1=490Ω, and it is also known that the galvanometer measures I_G = 3mA with R_1 =1490Ω. What is the Thévenin resistance of network N?

(3/9) (c) Using the data of part (b), what is the open circuit voltage of the linear network N? Indicate clearly the polarity with respect to the nodes a and b.

work space

b) The current passing through the dependent source is:

$$i_\Delta = \frac{2V_{ab} - V_{ab}}{240\Omega} = \frac{V_{ab}}{240\Omega} = \frac{0.72V}{240\Omega} = 3mA$$



$$q = \overbrace{\int_0^{1hr}}^{(+1)} i_\Delta\, dt = i_\Delta \cdot 1hr = 3mA \times 1hr \times \frac{3600s}{hr}$$

$$= \underline{10.8\,C} \qquad +3$$

c) The network N is at the threshold of conduction when $I_{ab} = 0A$, $V_{ab} = 0.72V$.

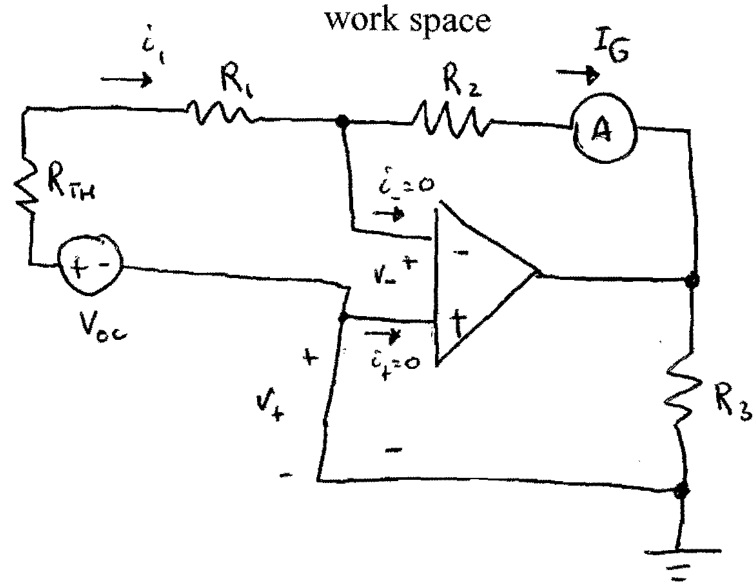
$$I_{ab} + V_{ab}\left(\frac{1}{180\Omega} - \frac{1}{240\Omega}\right) - I_o = 0 \qquad \text{(from part a), KCL).}$$

$$I_o = I_{ab} + V_{ab}\left(\frac{1}{180\Omega} - \frac{1}{240\Omega}\right)$$

$$= 0A + 0.72V\left(\frac{1}{180\Omega} - \frac{1}{240\Omega}\right) = \underline{1mA} \qquad +3$$

work space

1. a)



ideal op-amp:

$v_+ = v_- \ (=0)$

$i_+ = i_- = 0$

KVL: $-V_{oc} + i_1 R_{TH} + i_1 R_1 + v_- = 0$

$$i_1 = V_{oc} / (R_1 + R_{TH})$$

KCL: $-i_1 + i_- + I_G = 0$

$$\therefore \ \underline{I_G = i_1 = \frac{V_{oc}}{R_1 + R_{TH}}} \quad +3$$

(+2 for correct but incomplete expression)

b) From part a), we have:

$$I_G R_1 + I_G R_{TH} = V_{oc} = \text{constant}$$

$$\therefore \ I_G R_1 + I_G R_{TH} = I_G' R_1' + I_G' R_{TH}$$

(+2 for correct technique)

$$5mA \cdot 490\Omega + 5mA \, R_{TH} = 3mA \cdot 1490\,\Omega + 3mA \cdot R_{TH}$$

$$R_{TH} = \frac{3mA \cdot 1490\,\Omega - 5mA \cdot 490\,\Omega}{5mA - 3mA}$$

$$\underline{= 1.01\,k\Omega} \quad +3$$

work space

c) From part b) we have:

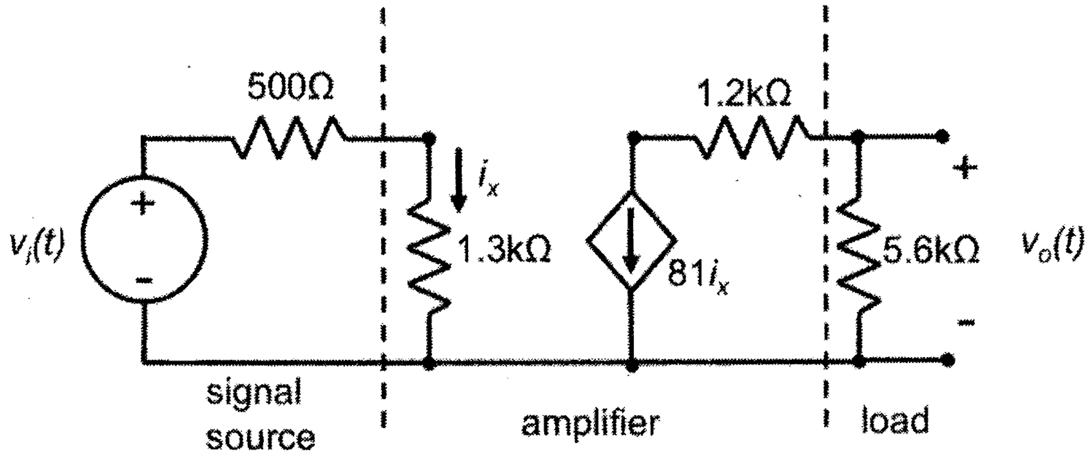
$$V_{oc} = I_G R_1 + I_G R_{TH} = 5mA \times 490\Omega + 5mA \times 1010\Omega$$

$$= \underline{7.5V}$$ (could also use 3mA, 1490Ω data).

+3

work space

(9/36) 2. Consider the circuit model below for a signal source, amplifier, and load.



(3/9) (a) What is the voltage gain $v_0(t) / v_i(t)$?

(3/9) (b) ~~What~~ The power dissipated in the 5.6kΩ load resistor is what percentage of the total power supplied to the circuit by both sources?

(3/9) (c) A 4nF capacitor is placed in parallel with the load resistance. What is the time constant for the output voltage transient $v_0(t)$, if the input voltage switches from a constant 0V to a constant 100mV?

work space

a) $$i_x = \frac{v_i}{500\Omega + 1.3k\Omega} = \frac{v_i}{1.8k\Omega}$$ (KCL, KVL, Ohm's Law)

$$v_o = -81\, i_x \cdot 5.6k\Omega$$ (KCL, Ohm's Law)

$$= -81\, v_i \frac{5.6k\Omega}{1.8k\Omega}$$

$$= -252\, v_i$$

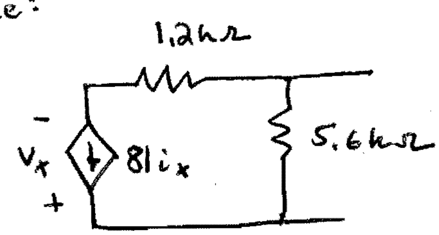
$$\therefore \underline{\frac{v_o}{v_i} = -252}$$ +3 (+2 with sign error)

b) Power supplied by v_i source:

$$P_{v_i} = v_i \cdot i_x$$ (active sign convention)

$$= \frac{v_i^2}{1.8k\Omega}$$

Power supplied by $81\, i_x$ source:



$$P_{81 i_x} = v_x \cdot 81\, i_x$$

$$= [81\, i_x \cdot (1.2k\Omega + 5.6k\Omega)] \cdot 81\, i_x$$

$v_x = 81\, i_x \cdot (1.2k\Omega + 5.6k\Omega)$ by KVL and Ohm.

$$= 6561 \cdot 6.8k\Omega \cdot i_x^2$$

$$= 6561 \cdot 6.8k\Omega \cdot \left(v_i / 1.8k\Omega \right)^2 = 24786 \times \frac{v_i^2}{1.8k\Omega}$$

(+1 for correct P_{v_i} + $P_{81 i_x}$)

work space

Power absorbed by 5.6kΩ resistor:

$$P_{5.6k\Omega} = (81\ i_x)^2 \cdot 5.6k\Omega$$

$$= \left(81\ \frac{v_i}{1.8k\Omega}\right)^2 \cdot 5.6k\Omega$$

$$= 20412 \times \frac{v_i^2}{1.8k\Omega} \quad (+1)$$

% absorbed in 5.6 kΩ versus power supplied:

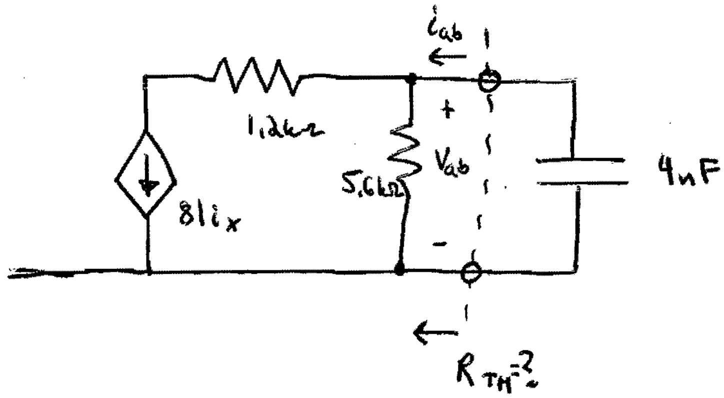
$$100\% \times \frac{P_{5.6k\Omega}}{P_{v_i} + P_{81 i_x}} = 100\% \times \frac{20412\,(v_i^2/1.8k\Omega)}{[1 + 24786]\,(v_i^2/1.8k\Omega)}$$

$$= \underline{82.35\ \%} \quad +3$$

Powers can be computed in terms of other variables as well, such as i_x, which will cancel to give the same result.

work space

c)



KCL gives: $81 i_x + \frac{V_{ab}}{5.6k\Omega} - i_{ab} = 0$

$$i_{ab} = 81 i_x + V_{ab}/5.6k\Omega$$

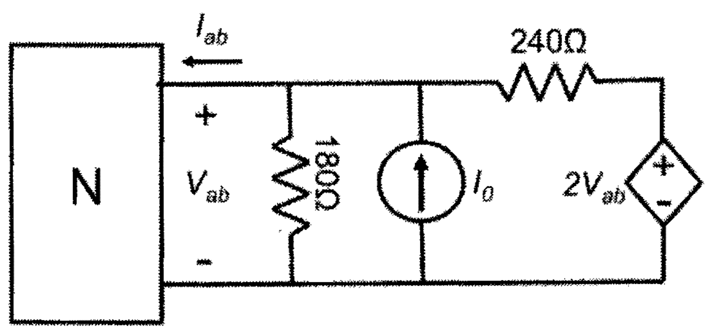
But the Norton equivalent has the i_{ab}, v_{ab} relationship:

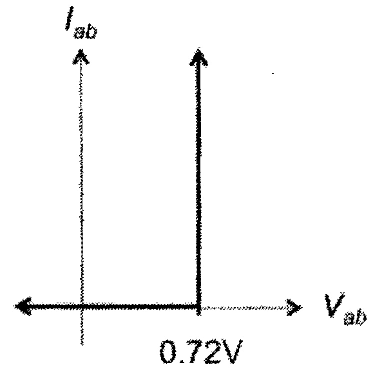
$$i_{ab} = i_{sc} + V_{ab}/R_{TH}$$

$$\therefore R_{TH} = 5.6 k\Omega$$

$$\tau = \underbrace{R_{TH} \cdot C}_{(+1)} = 5.6k\Omega \cdot 4nF = \underline{22.4 \mu s} \quad +3$$

(9/36) 3. Consider the circuit below, with the current-voltage characteristic of the network N specified below.





(3/9) (a) Assume $I_0 = 28\text{mA}$ for this part. How much power is absorbed by the network N?

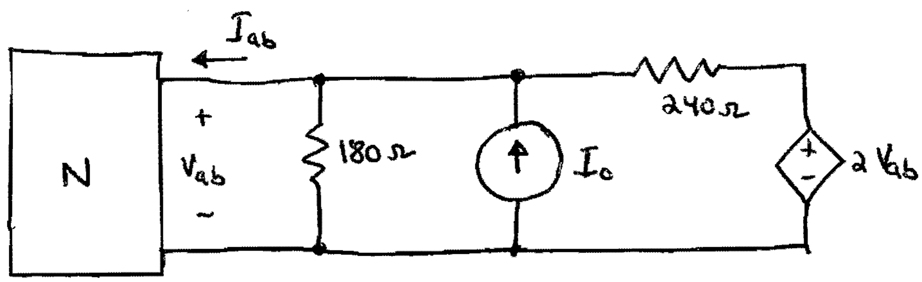
(3/9) (b) Assuming still that $I_0 = 28\text{mA}$, how much charge passes through the dependent voltage source over the duration of 1 hour?

(3/9) (c) At what value of I_0 is the network N at the threshold of conducting current? In other words, any increment in I_0 above this value will cause a non-zero current to flow through N.

work space

a) N is non-linear, with two linear piece-wise portions of the I_{ab}, V_{ab} curve.

Assume $V_{ab} = 0.72\,V$ and I_{ab} is undetermined.



KCL gives: $$I_{ab} + \frac{V_{ab}}{180\Omega} - I_o + \frac{V_{ab} - 2V_{ab}}{240\Omega} = 0$$

$$I_{ab} + V_{ab}\left(\frac{1}{180\Omega} - \frac{1}{240\Omega}\right) - 28mA = 0$$

$$I_{ab} = 28\,mA - 0.72V\left(\frac{1}{180\Omega} - \frac{1}{240\Omega}\right)$$

$$= 27\,mA$$

(+2 for correct expression for I_{ab})

Since $I_{ab} > 0\,mA$, we have a solution consistent with the I_{ab}, V_{ab} relationship for N.

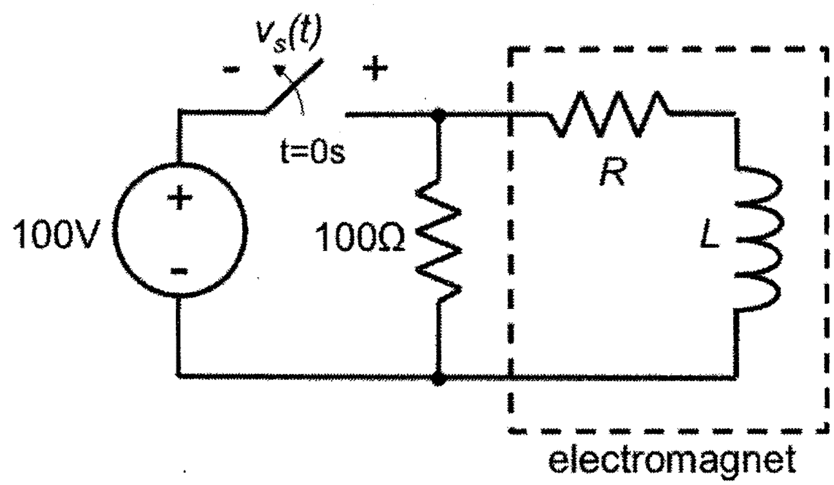
Power absorbed $= I_{ab} \cdot V_{ab}$ (passive sign convention)

$= 27mA \cdot 0.72V$

$= \underline{19.44\,mW}$ +3

work space

(9/36) 4. You have purchased a used electromagnet at a discount price, and consequently you do not know its specifications. Consider the circuit below for testing your magnet, in which the magnet is modeled as an ideal solenoid of inductance L in series with a wire resistance R.



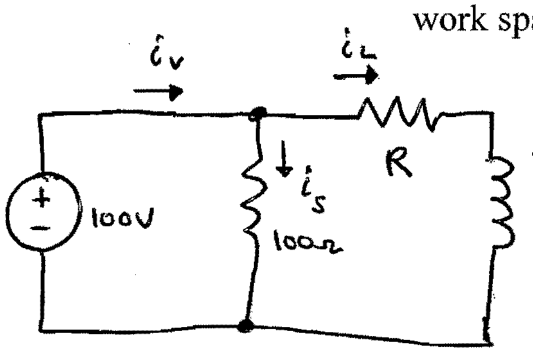
(3/9) (a) Assume steady state operation with the switch closed in this part. You find that the power supplied by the ideal voltage source is 2.1kW. You also find that the energy dissipated by the magnet over a time of 30ms is equal to the energy stored in the magnet. What are the magnet parameters L and R?

(3/9) (b) After reaching steady state with the switch closed, the switch is opened instantaneously (to become an open circuit) at $t = 0\text{s}$. What is the voltage across the switch, as defined by the variable $v_s(t)$ for time $t > 0\text{s}$? Note carefully the polarity of $v_s(t)$.

(3/9) (c) The 100Ω resistor is rated by the manufacturer to operate with a maximum power dissipation of 18kW. After the switch is opened at the instant $t = 0\text{s}$, will the 100Ω resistor be operating within the manufacturer's specifications? Justify your answer.

work space

a)



$V_L = L\dfrac{di_L}{dt} = 0$ (steady state)

KCL: $i_V = i_L + i_s$

Power supplied = $100V \times (i_L + i_s)$

$i_s = \dfrac{100V}{100\Omega} = 1A$ (Ohm's Law)

$i_L = \dfrac{100V}{R}$ (Ohm's Law)

$\therefore\ 2.1kW = 100V\left(\dfrac{100V}{R} + 1A\right)$

$21A = \dfrac{100V}{R} + 1A$

$R^{-1} = \dfrac{21A - 1A}{100V} = 0.2 A/V \rightarrow R = \dfrac{1}{0.2A/V} = \underline{5\Omega}$ +1.5

Energy dissipated in magnet over 30ms at steady state:

$$E_{dis} = \int_0^{30ms} P_{dis}\,dt = \frac{(100V)^2}{R} \times 30ms$$

$$= \frac{(100V)^2}{5\Omega} \times 30ms = 60J$$

work space

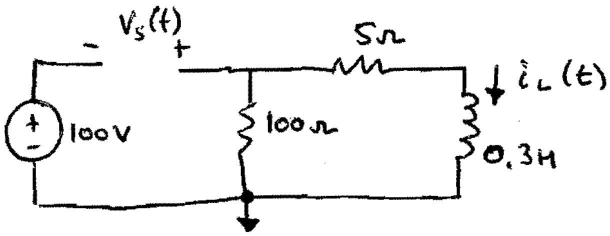
Energy stored in magnet:

$$E_{stor} = \frac{1}{2} L i_L^2 = \frac{1}{2} \cdot L \cdot (100V/5\Omega)^2 = L \times 200\,A^2$$

$$E_{dis} = E_{stor} \rightarrow L = \frac{60J}{200A^2} = \underline{0.3H} \quad +1.5$$

b) At $t=0^-$, $i_L(0^-) = 100V/5\Omega = 20A$.

Continuity of inductor current gives $i_L(0^+) = i_L(0^-) = 20A$.



KVL gives: $-100V - V_S - i_L \times 100\Omega = 0$

$$V_S = -100V - i_L \times 100\Omega$$

At $t=0^+$, $V_S(0^+) = -100V - 20A \times 100\Omega = -2.1kV$

At $t \to \infty$, $V_S(\infty) = -100V - i_L(\infty) \times 100\Omega = -100V - 0A \times 100\Omega = -100V$

$$\underbrace{\tau = L/R_{TH}}_{(+1)} = 0.3H / (100\Omega + 5\Omega) = 2.857\,ms.$$

$V_s(t) = c_1 + c_2 \exp(-t/\tau) \quad t > 0$

$V_s(\infty) = c_1 = -100V$; $V_s(0^+) = c_1 + c_2 = -2.1kV \rightarrow c_2 = -2.0kV$

$\therefore \quad \underline{V_s(t) = -100V - 2.0kV \exp(-t/2.857ms) \quad t > 0} \quad +3$

(+2 for expression with sign error)

work space

c) At $t=0^+$, $i_L = 20\,A$ and the power dissipated in the 100Ω resistor is:

$$P_{100\Omega} = i_L^2 \cdot 100\Omega = (20A)^2 \cdot 100\Omega = 40\,kW$$ (+2 if wrong $P_{100\Omega}$ value due to sign error)

The power dissipated in the 100Ω resistor at $t=0_s^+$ is $40\,kW > 18\,kW$, out of the manufacturer's specifications.

+3