McGill

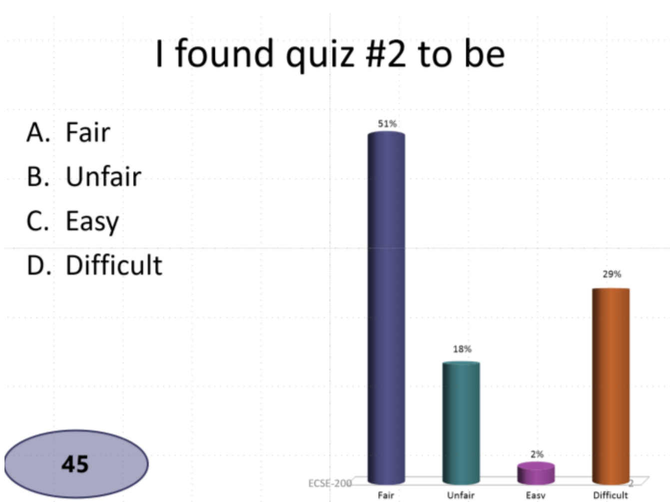
ECSE-200
Electric Circuits 1

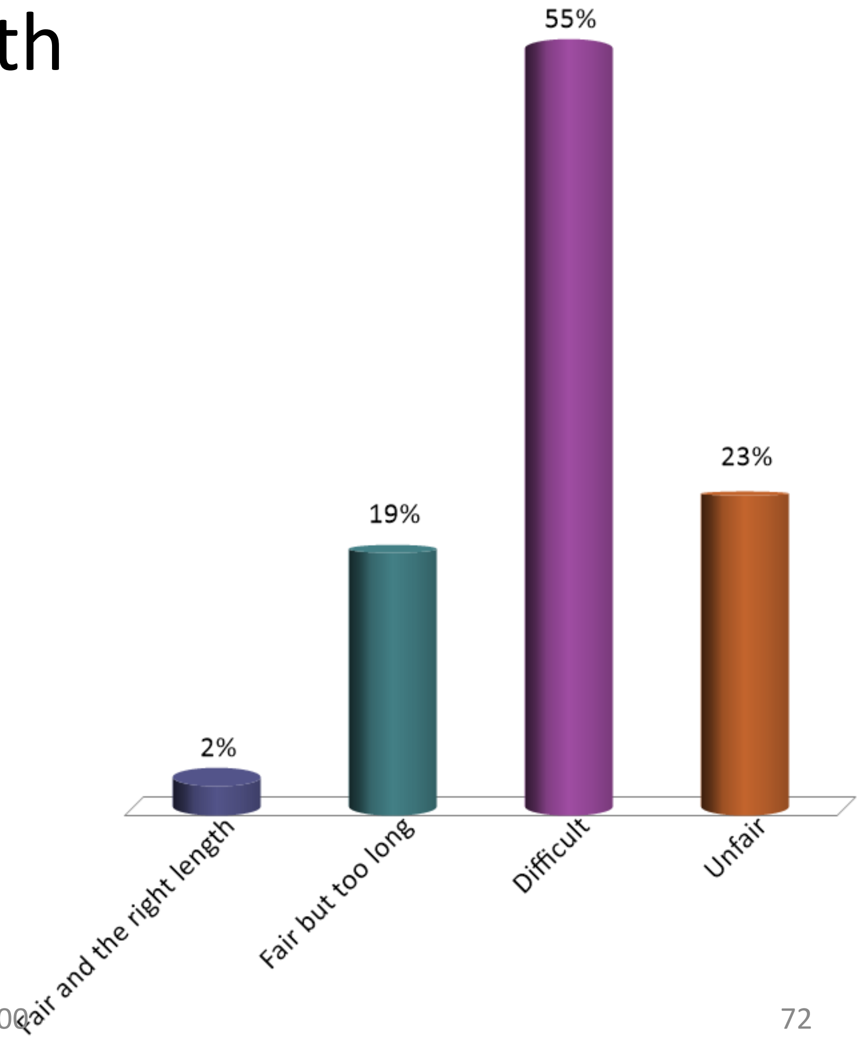
Monday, February 4 2019

Lecture 13

I found quiz #3

A. Fair and the right length
B. Fair but too long
C. Difficult
D. Unfair





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3. Basic Circuit Analysis

- Node Voltage Method
- Nodal Analysis
  - resistors and independent current sources
  - resistors and independent current and voltage sources
  - resistors and mix of independent and dependent sources
- Mesh Current Method

- Mesh's Analysis
  - resistors and independent voltage sources
  - resistors and independent current and voltage sources
  - resistors and mix of independent and dependent sources
- Node Voltage versus Mesh Current

Today's lecture
Homework:
Read 4.5-4.8

Mesh Current Method

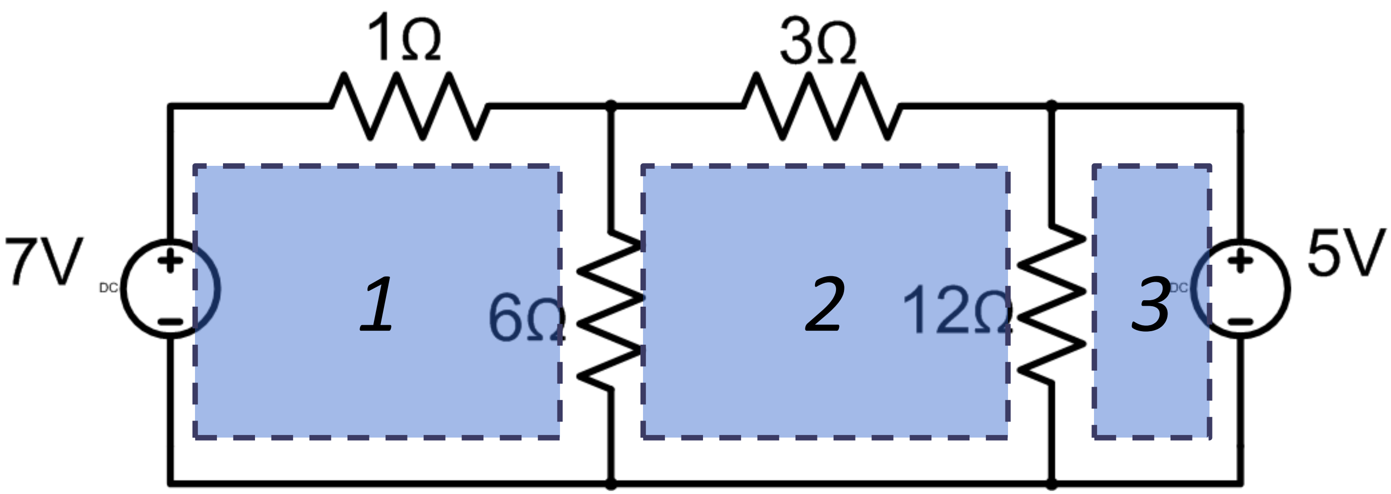
Step #1: Identify and label meshes. Define mesh current variables circulating in each mesh.

Step #2: Write KVL equations for each mesh using mesh current variables only, by intrinsically using KCL and terminal laws (such as Ohm's law).

Step #3: Solve the linear system of equations, and use the mesh currents to calculate the desired quantity.

Mesh Current Method

Meshes are identified below, and can also be thought of as the smallest loops that can be used to tile the circuit.

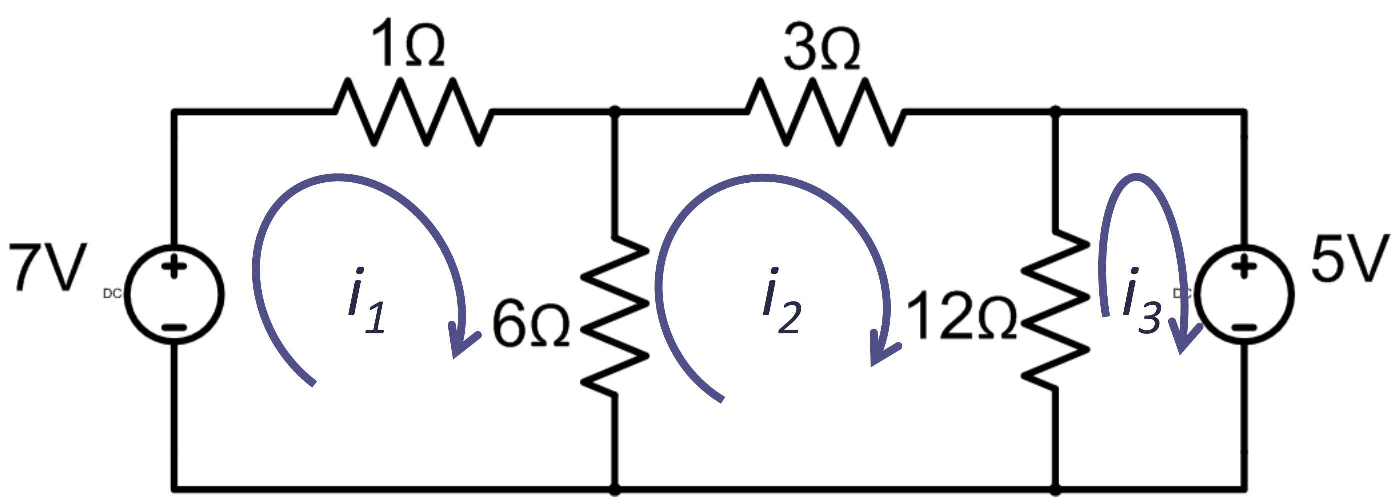


Mesh Current Method

KVL on mesh 1: $0 = -7V + i_1 1\Omega + (i_1 - i_2)6\Omega$

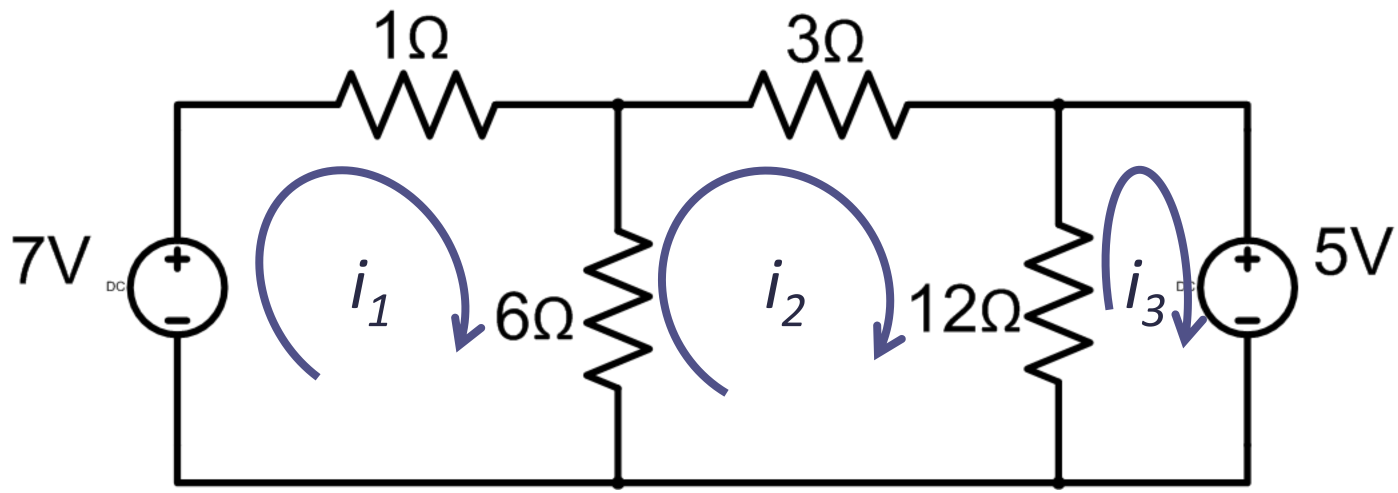
KVL on mesh 2: $0 = (i_2 - i_1)6\Omega + i_2 3\Omega + (i_2 - i_3)12\Omega$

KVL on mesh 3: $0 = (i_3 - i_2)12\Omega + 5V$



Mesh Current Method

We can now calculate the power delivered by the 7V voltage source (the original question about this circuit).



i_1=1.222A i_2=0.2592A i_3=-0.1574A

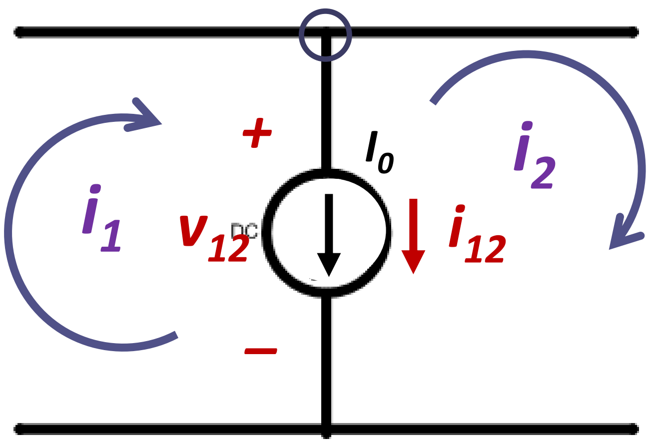
P_{7V} = power delivered by 7V voltage source

= (7V)(i_1)

= 7Vx1.222A = +8.554W

Current Sources and Mesh Current Method

Consider what happens when a current source is located between two meshes:



v_{12} and i_{12} = temporary variables

KCL @ node: $-i_1+i_{12}+i_2=0$

$i_{12}=i_1-i_2$

terminal law: $i_{12}= I_0$

v_{12}= anything

There are two consequences:

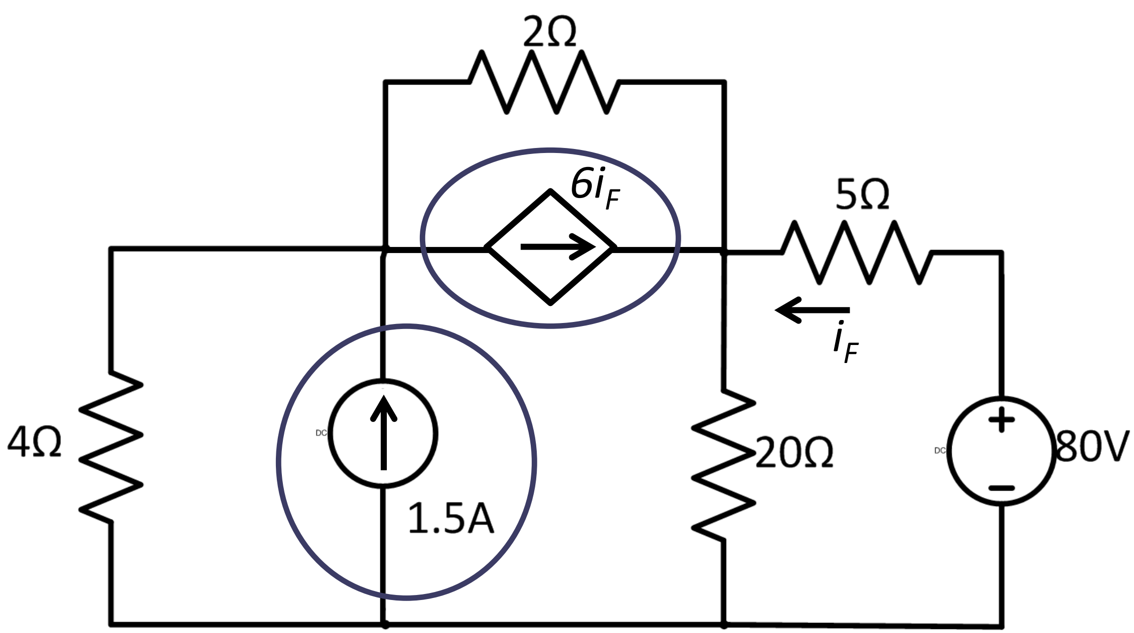
We have a *very simple* relationship between mesh currents, $i_1-i_2=I_0$ that is independent of v_{12}.

It is *impossible* to express the voltage v_{12} shared by mesh 1 and 2 in terms of i_1 and i_2.

Current Sources and Mesh Current Method

The mesh current method can be generalized to incorporate current sources.

We illustrate the method with the example below.



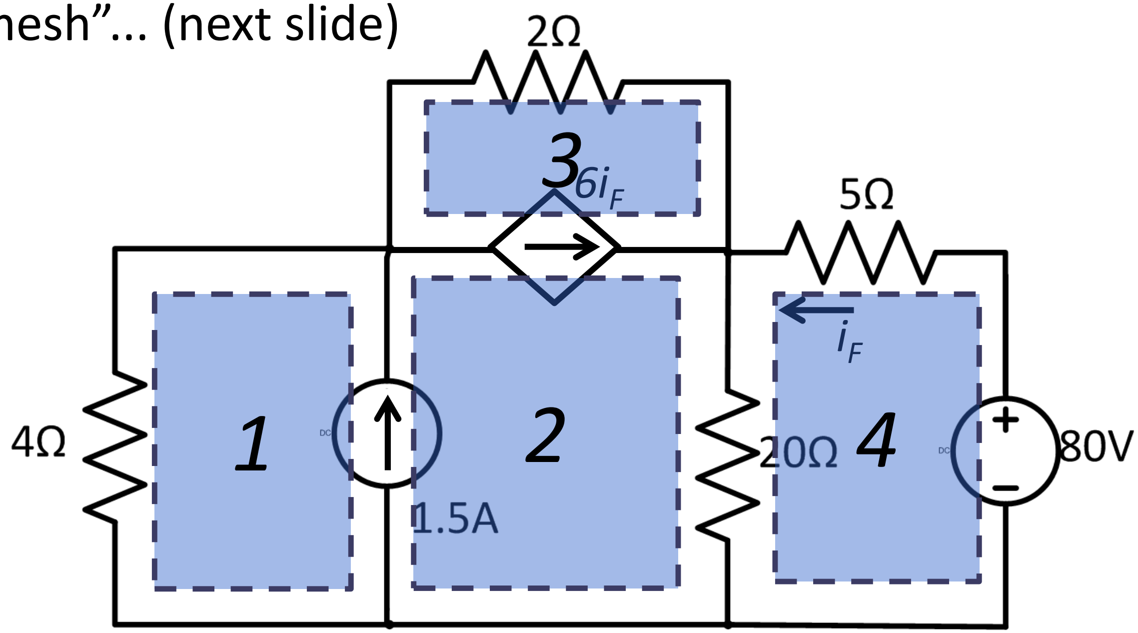
Mesh Current Method

Step #1:

Identify the meshes of the circuit.

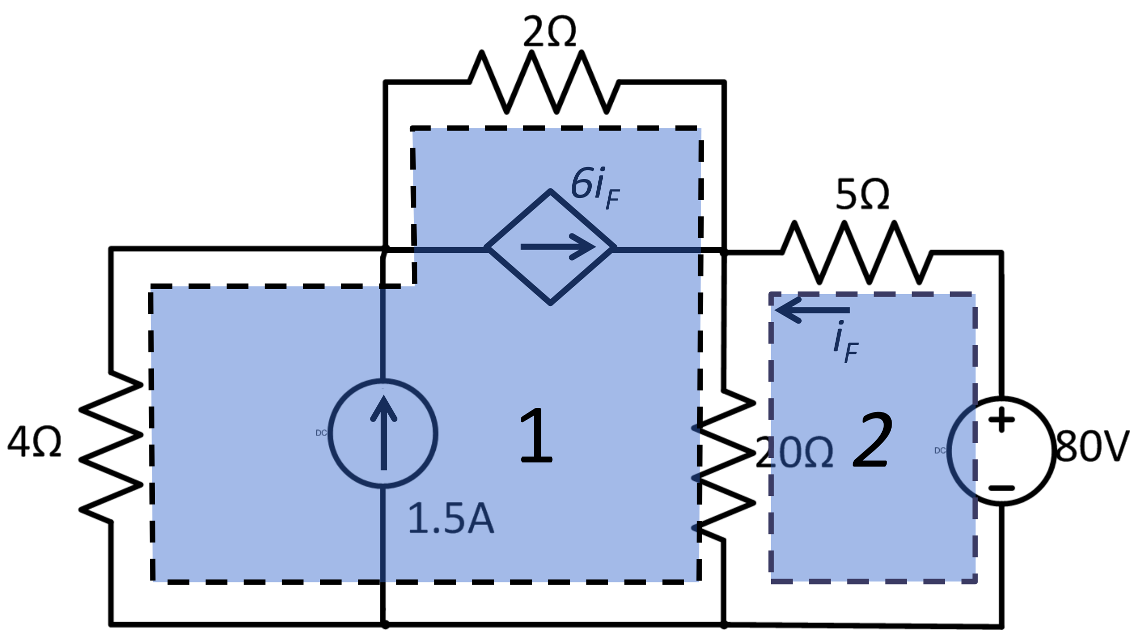
Combine meshes that have current sources with neighboring meshes into "super-meshes".

Identify mesh currents, including only one mesh current variable for a "super-mesh"... (next slide)



Mesh Current Method

In this example, there are three meshes combined into a super-mesh (labeled 1), and one remaining mesh (labeled).

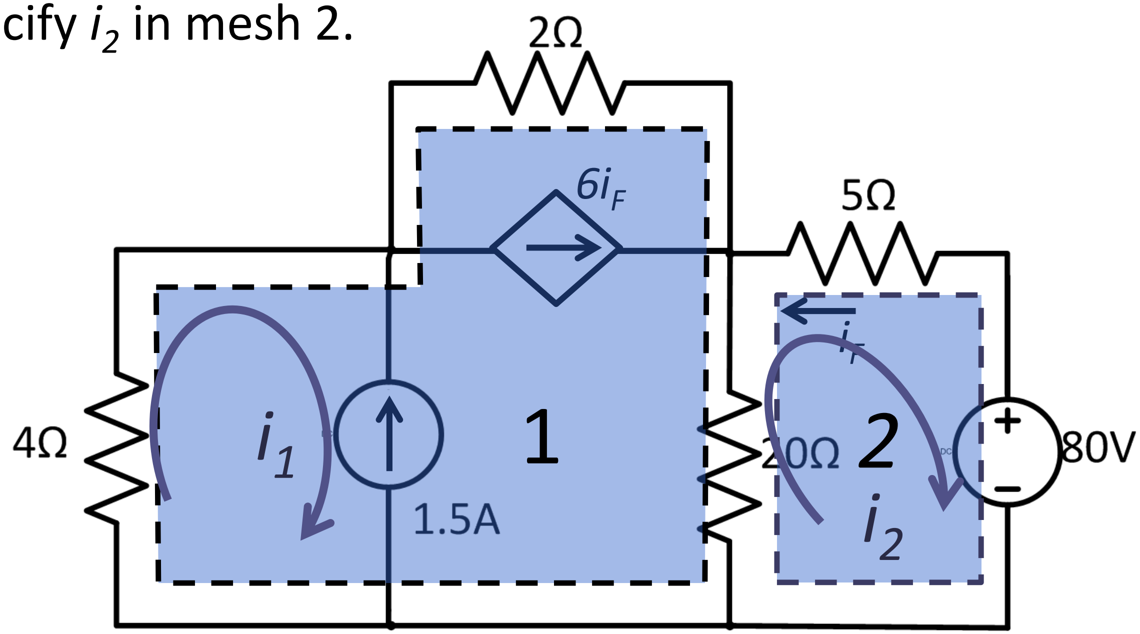


Mesh Current Method

We specify one mesh current variable i_1 for the super-mesh.

We choose the left-most mesh to carry i_1 (this is one of several possible choices).

We also specify i_2 in mesh 2.

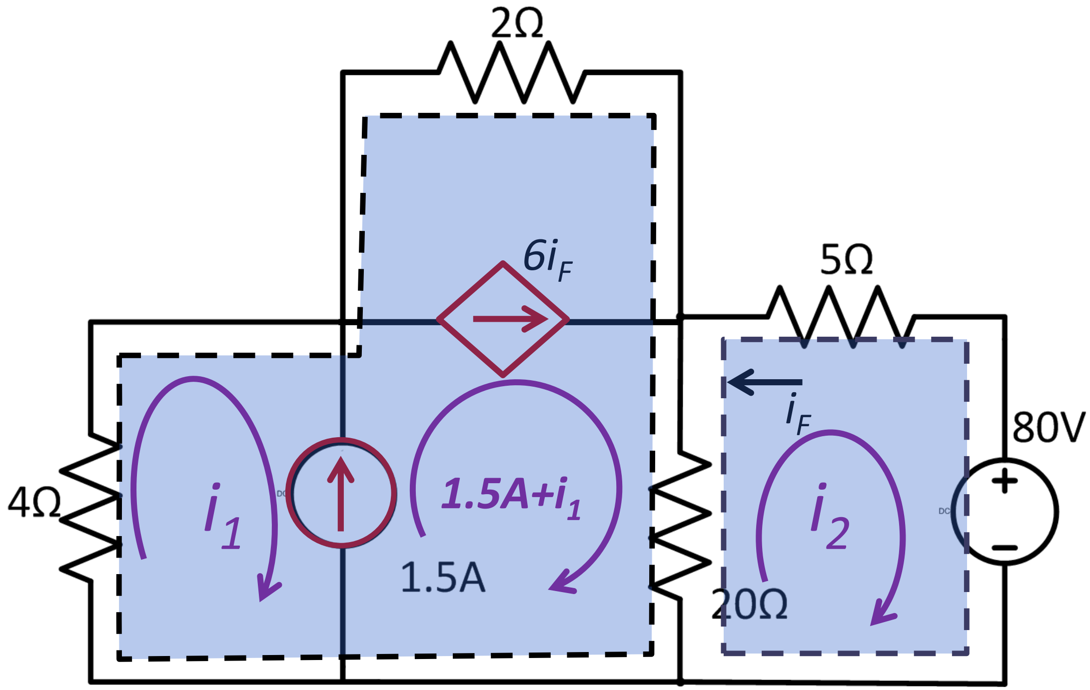


Note that we treat the dependent current source like an independent current source at this stage of the calculation.

Mesh Current Method

The remaining mesh currents in the super-mesh are determined by the current sources.

For example, the "middle" mesh current must be 1.5A+i_1 in order for the current through the independent source to be 1.5A.



Note that we treat the dependent current source like an independent current source at this stage of the calculation.

What is the “top” mesh current?

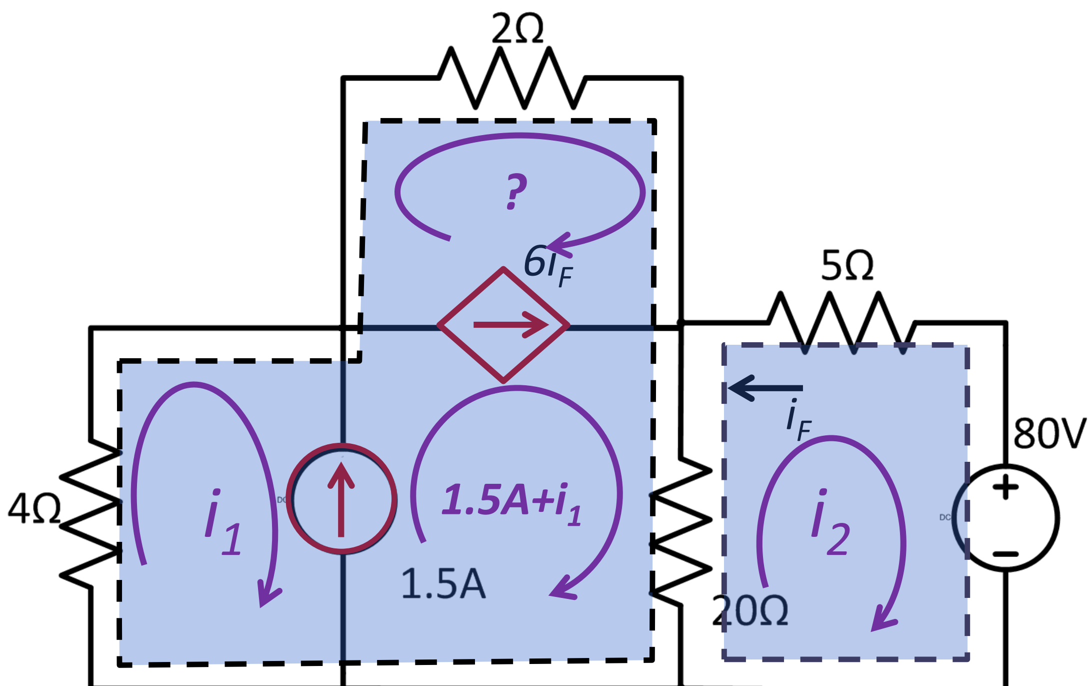
A. $1.5A+i_1+6i_F$

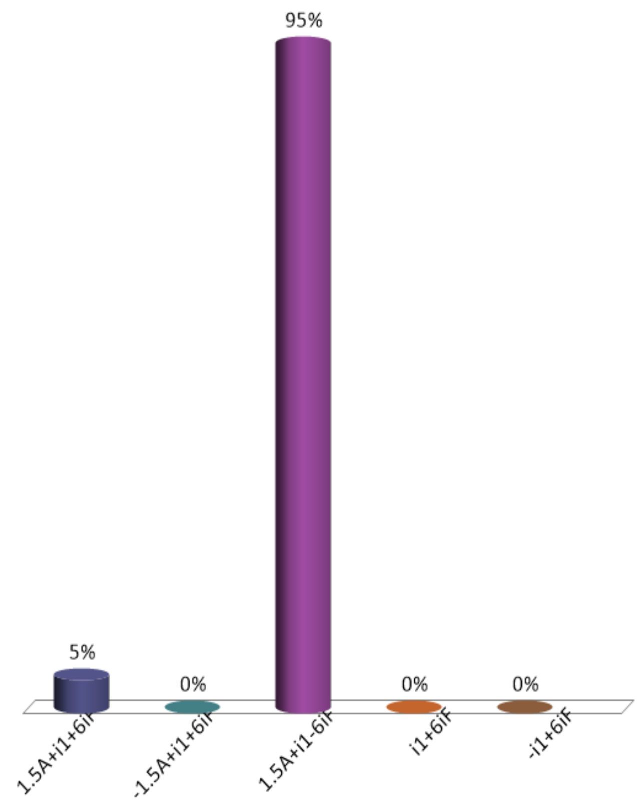
B. $-1.5A+i_1+6i_F$

C. $\boxed{1.5A+i_1-6i_F}$

D. i_1+6i_F

E. $-i_1+6i_F$





Mesh Current Method

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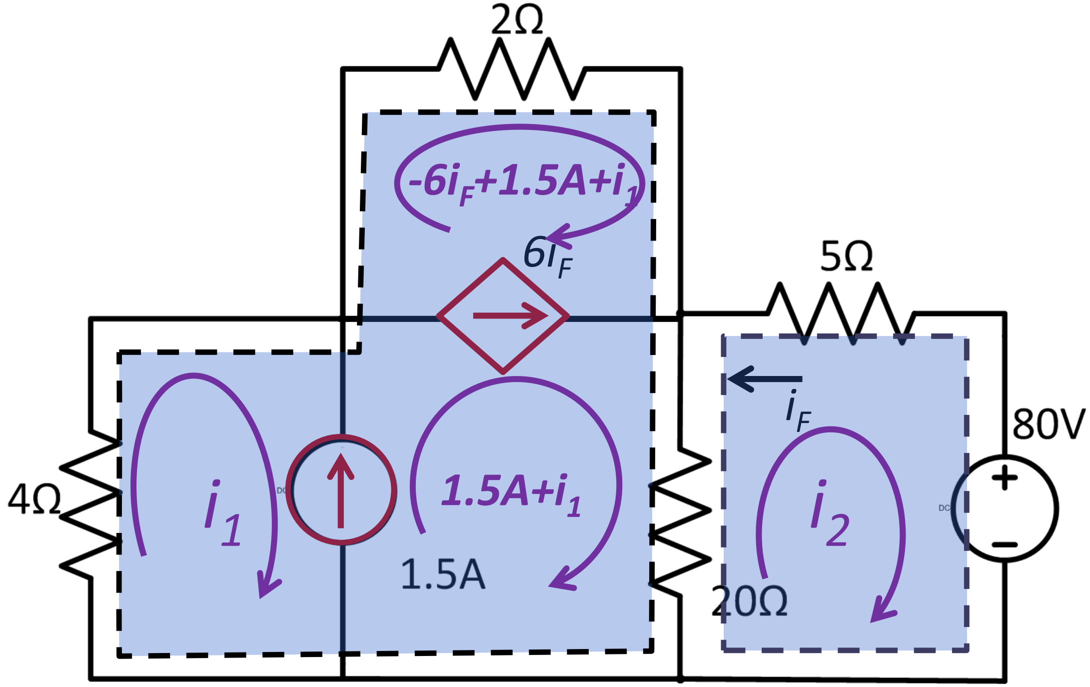


Note that we treat the dependent current source like an independent current source at this stage of the calculation.

Mesh Current Method

Step #2:

Write KVL equations for each mesh and each super-mesh.

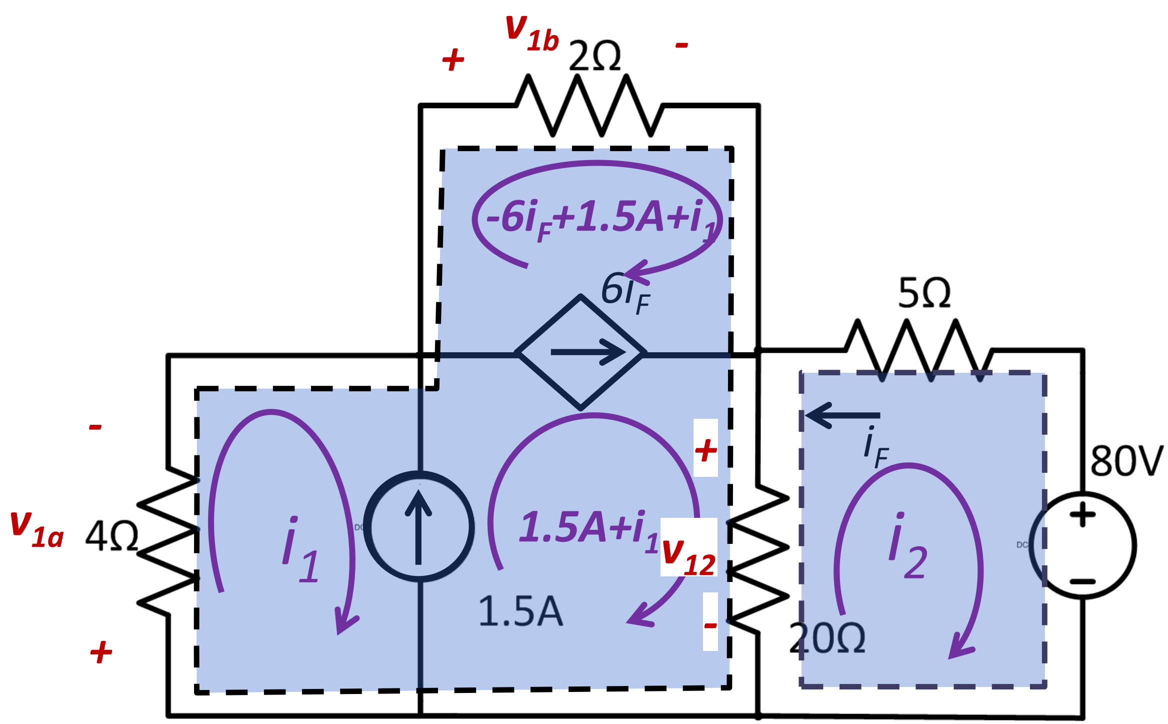
For the super-mesh, traverse the loop that does not pass through any current source.

<u>Use only the defined current variables</u> to express each voltage by implicitly using KCL and the terminal laws of the elements.

In the presence of dependent sources, express any source variables in terms of mesh currents.

Mesh Current Method

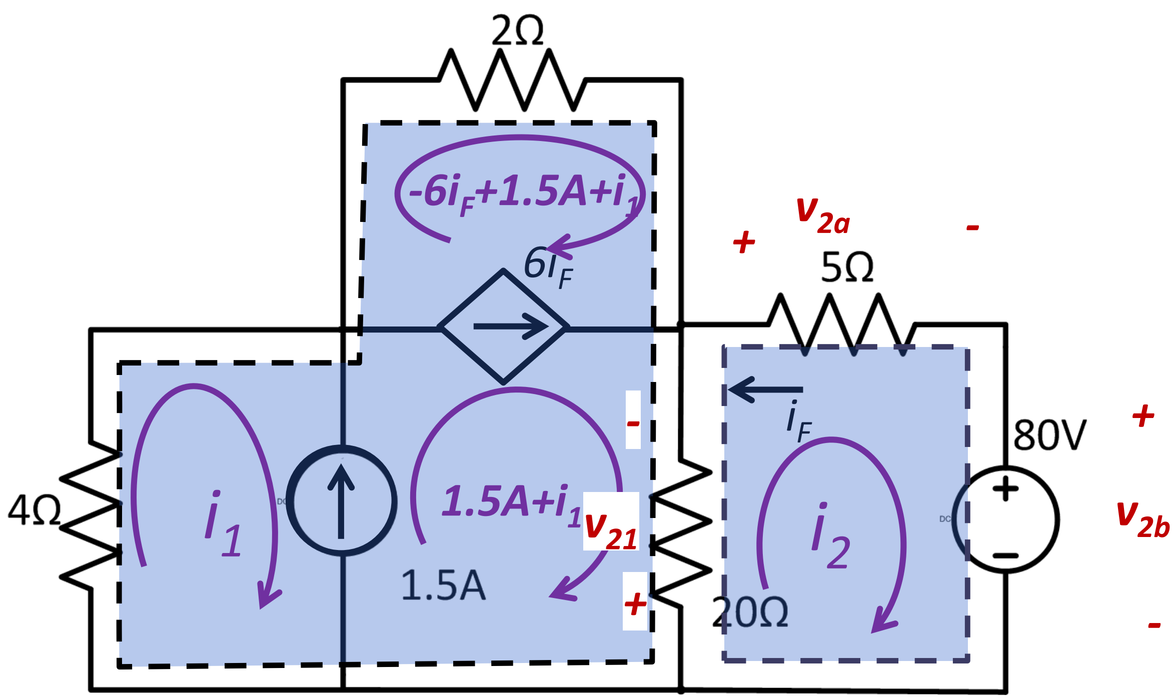
KVL on super-mesh 1: $$0 = \underbrace{i_1 4\Omega}_{v_{1a}} + \underbrace{\left(-6i_F + 1.5A + i_1\right)2\Omega}_{v_{1b}} + \underbrace{\left(1.5A + i_1 - i_2\right)20\Omega}_{v_{12}}$$



Mesh Current Method

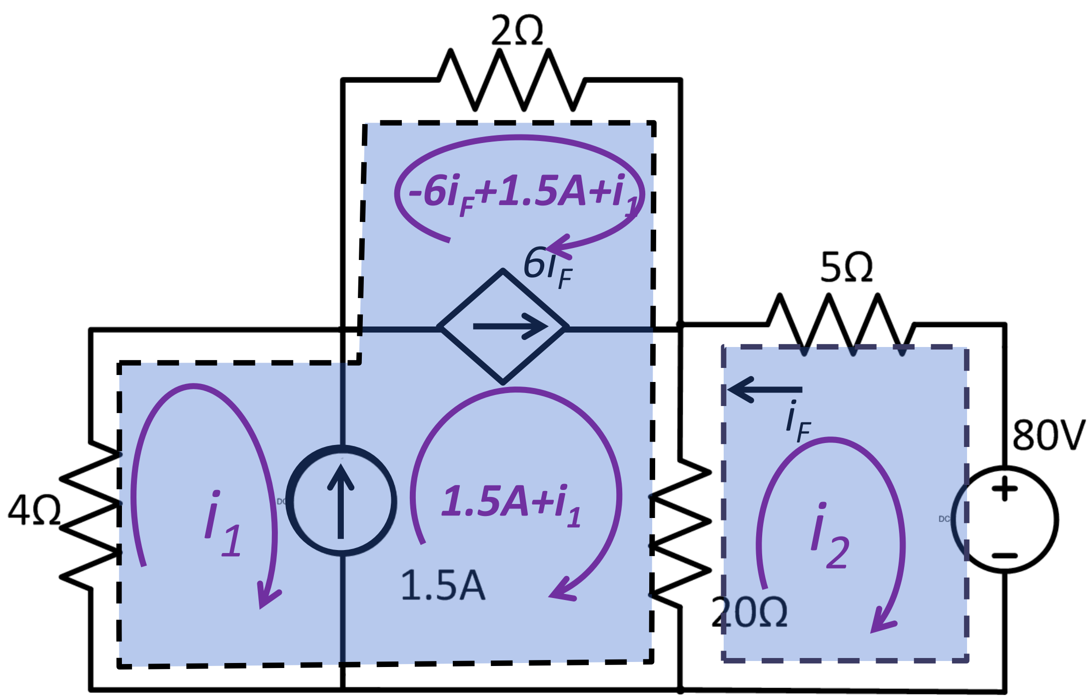
KVL on mesh 2:

$$0=\underbrace{\left(i_2-\left(1.5A+i_1\right)\right)20\Omega}_{v_{21}}+\underbrace{i_2 5\Omega}_{v_{2a}}+\underbrace{80V}_{v_{2b}}$$



Mesh Current Method

Source variable definition: $i_F = -i_2$



Mesh Current Method

Step #3: Solve for the mesh current variables.

Super-mesh 1: $0 = i_1 4\Omega + \left(-6i_F + 1.5A + i_1\right)2\Omega + \left(1.5A + i_1 - i_2\right)20\Omega$

Mesh 2: $0 = \left(i_2 - \left(1.5A + i_1\right)\right)20\Omega + i_2 5\Omega + 80V$

Source equation: $i_F = -i_2$

The entire circuit problem is organized into a system of 3 equations with unknowns.

This system trivially reduced to a system of 2 equations with 2 unknowns (i_1, i_2).

Mesh Current Method

We can use substitution to find the values of i_1 and i_2.

mesh 2: $$0 = (i_2 - (1.5A + i_1))20\Omega + i_2 5\Omega + 80V$$
$$= -i_1 20\Omega + i_2 25\Omega + 50V$$
$$i_2 25\Omega = i_1 20\Omega - 50V$$
$$i_2 = \tfrac{4}{5} i_1 - 2A$$
i_2 as a function of i_1

super-mesh 1: $$0 = i_1 4\Omega + (-6i_F + 1.5A + i_1)2\Omega + (1.5A + i_1 - i_2)20\Omega$$
$$= i_1 26\Omega - i_2 20\Omega - i_F 12\Omega + 33V$$
$$= i_1 26\Omega - i_2 20\Omega - (-i_2)12\Omega + 33V$$
substitute i_F with $-i_2$
$$= i_1 26\Omega - i_2 8\Omega + 33V$$
$$= i_1 26\Omega - (\tfrac{4}{5} i_1 - 2A)8\Omega + 33V$$
substitute i_2
$$= i_1 19.6\Omega + 49V$$

$$\boxed{i_1 = -\tfrac{49}{19.6} A = -2.50A}$$
$$\boxed{i_2 = \tfrac{4}{5}(-2.50A) - 2A = -4.00A}$$

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Mesh Current Method

We can also use **Cramer's rule**:

Restructure equation in form of $b=Ax_1+Bx_2$

$$\begin{cases} 0 = i_1 4\Omega + \left(-6\left(-i_2\right)+1.5A+i_1\right)2\Omega + \left(1.5A+i_1-i_2\right)20\Omega \\ -33A = 26i_1 - 8i_2 \qquad \left[b_1 = A_{11}x_1 + A_{12}x_2\right] \\ 0 = \left(i_2 - \left(1.5A+i_1\right)\right)20\Omega + i_2 5\Omega + 80V \\ -50A = -20i_1 + 25i_2 \qquad \left[b_2 = A_{21}x_1 + A_{22}x_2\right] \end{cases}$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} -33 \\ -50 \end{bmatrix} = \begin{bmatrix} 26 & -8 \\ -20 & 25 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$x_1 = \frac{\begin{vmatrix} b_1 & A_{12} \\ b_2 & A_{22} \end{vmatrix}}{\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix}} = \frac{b_1 A_{22} - A_{12} b_2}{A_{11}A_{22} - A_{12}A_{21}}$$

$$i_1 = \frac{\begin{vmatrix} -33 & -8 \\ -50 & 25 \end{vmatrix}}{\begin{vmatrix} 26 & -8 \\ -20 & 25 \end{vmatrix}} = \frac{(-33)\cdot 25 - (-8)\cdot(-50)}{26\cdot 25 - (-8)\cdot(-20)} = \frac{-1225}{490} = -2.5A$$

$$i_2 = \frac{\begin{vmatrix} 26 & -33 \\ -20 & 50 \end{vmatrix}}{\begin{vmatrix} 26 & -8 \\ -20 & 25 \end{vmatrix}} = \frac{26\cdot(-50) - (-33)\cdot(-20)}{26\cdot 25 - (-8)\cdot(-20)} = \frac{-1960}{490} = -4A$$

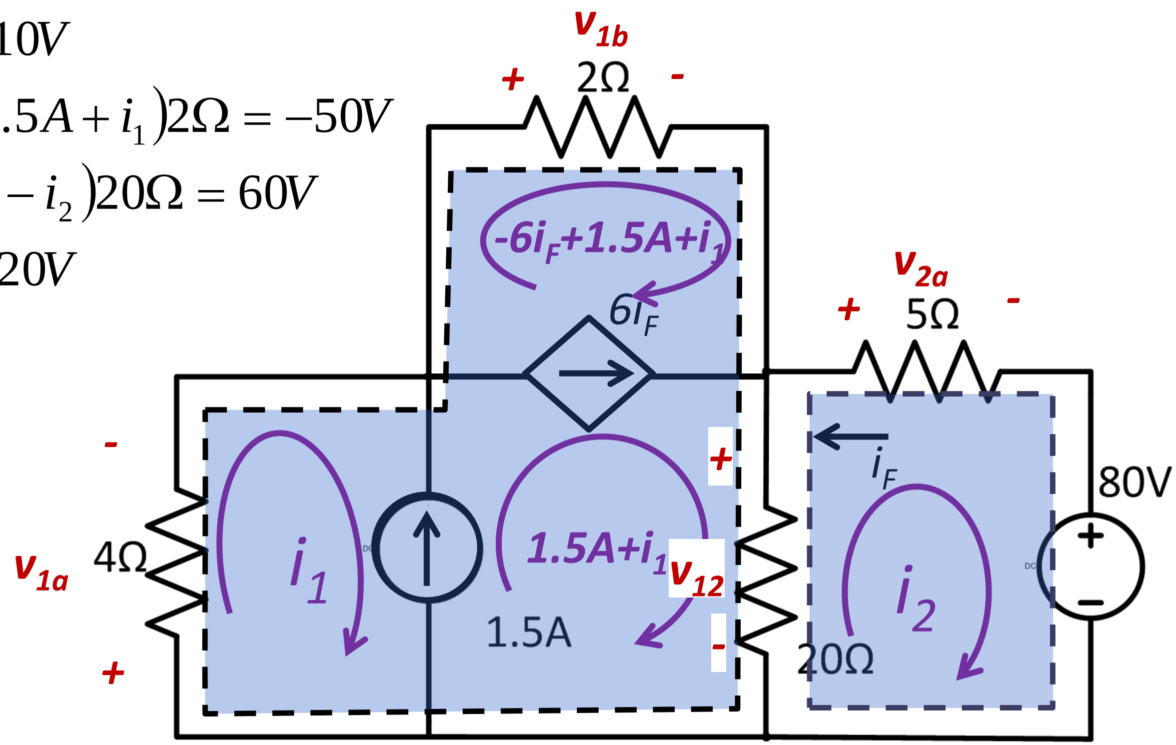
$$x_2 = \frac{\begin{vmatrix} A_{11} & b_1 \\ A_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix}} = \frac{A_{11} b_2 - b_1 A_{21}}{A_{11}A_{22} - A_{12}A_{21}}$$

Mesh Current Method

Any quantity (for example, the branch voltages) can now be easily calculated from i_1 and i_2.

For the circuit below, we can verify that KVL is satisfied for any loop.

$$v_{1a} = i_1 4\Omega = -10V$$
$$v_{1b} = \left(-6i_F + 1.5A + i_1\right)2\Omega = -50V$$
$$v_{12} = \left(1.5A + i_1 - i_2\right)20\Omega = 60V$$
$$v_{2a} = i_2 5\Omega = -20V$$



$$i_1 = -2.50A$$

$$i_2 = -4.00A$$
$$i_F = 4.00A$$

Summary of Mesh Current Method

Step #1:

i) Label meshes, grouping any meshes sharing a current source into *super-meshes*.

ii) Define mesh current variables circulating in each mesh (and express all currents in a *super-mesh* using a single mesh current).

Step #2: Write KVL equations for each mesh using mesh current variables only, by intrinsically using KCL and terminal laws (such as Ohm's law). Traverse the single loop in the *super-mesh* that does not involve a current source.

Step #3: Solve the linear system of equations, and use the mesh currents to calculate the desired quantity.

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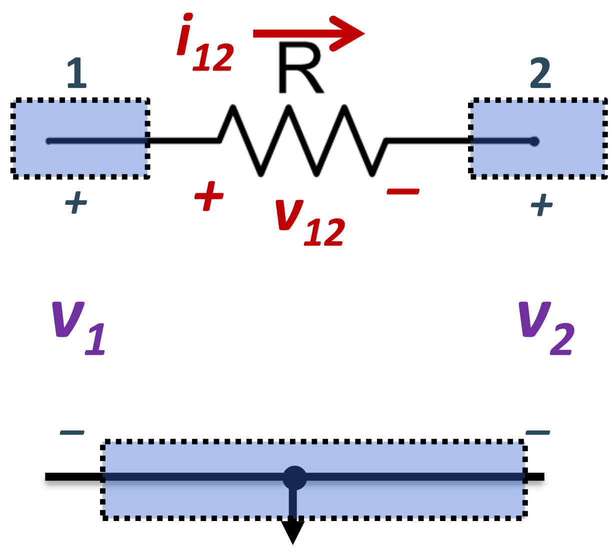
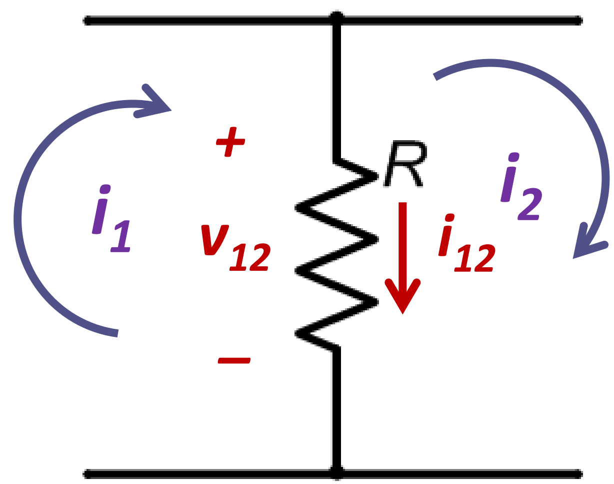
Node Voltage versus Mesh Current Method

	Node Voltage	Mesh Current
explicit variables:	node voltage v_x	mesh current i_x
number of variables:	number of nodes minus reference	number of meshes
explicit equations:	KCL	KVL
implicit equations	KVL + element laws (e.g. Ohm's Law) to express element currents in terms of v_x	KCL + element laws (e.g. Ohm's Law) to express element voltages in terms of i_x

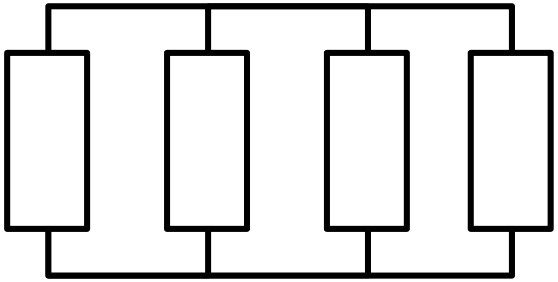




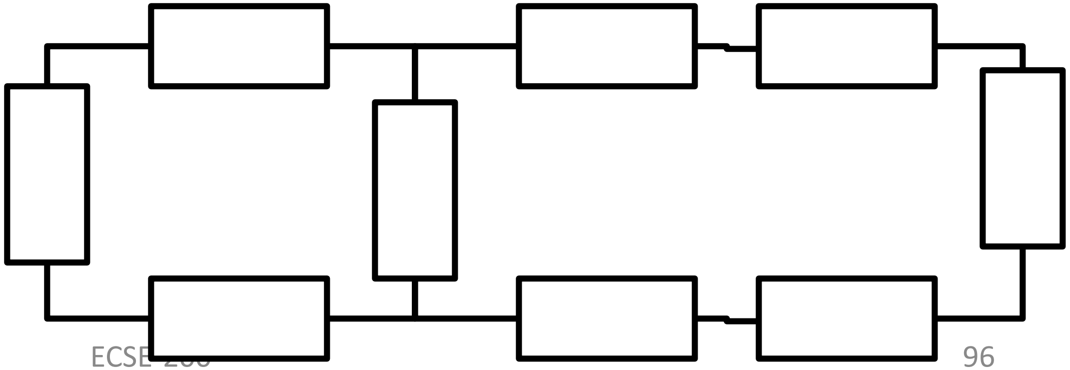
Node Voltage versus Mesh Current Method

Circuit topology should be considered when choosing methods:

1) Circuits with many elements in parallel and a few nodes can be solved with very few node voltage variables.



2) Circuits with many elements in series arranged in a few meshes can be solved with very few mesh current variables.



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Section 3 Summary

Node Voltage Method: A systematic way to express KCL, KVL, Ohm's Law and other element laws. Only node voltage variables are used throughout the analysis process.

Mesh Current Method: Another systematic way to express KCL, KVL, Ohm's Law and other element laws. Only mesh current variables are used throughput the analysis process.