Electric Circuits 1

ECSE-200 Section: 1

11 December 2012, 6:00PM

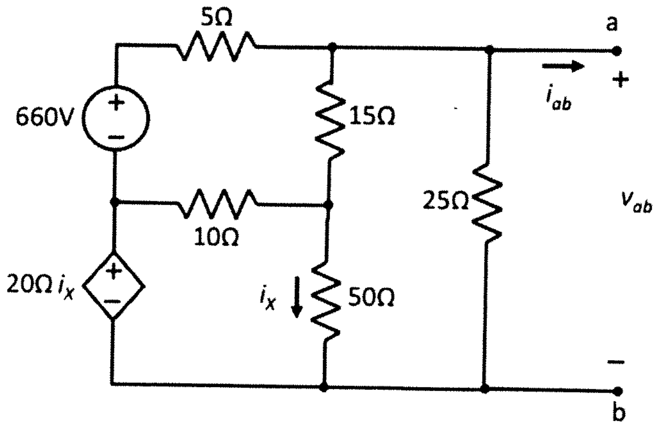
Examiner: Thomas Szkopek

Assoc Examiner: Zetian Mi

INSTRUCTIONS:

- This is a **CLOSED BOOK** examination.
- **NO CRIB SHEETS** are permitted.
- Provide your answers in an **EXAM BOOKLET**.
- **STANDARD CALCULATOR** permitted ONLY.
- This examination consists of 4 questions, with a total of 6 pages, including the cover page.
- This examination is **PRINTED ON BOTH SIDES** of the paper

1. Consider the circuit below. Answer the questions. [12 pts]



a) What is the physical basis of Kirchoff's Voltage Law? [1pt]

b) What is the physical basis of Kirchoff's Current Law? [1pt]

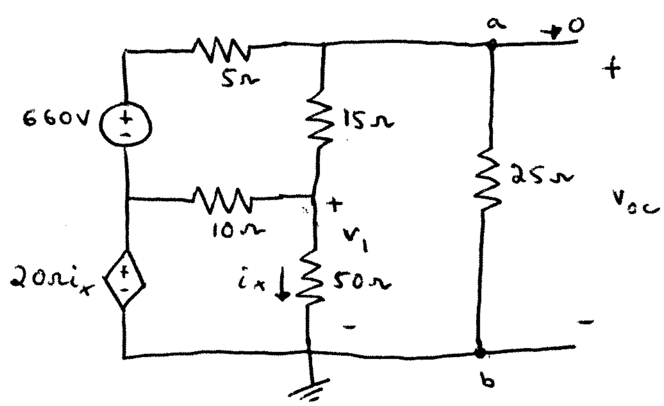
c) What is the Thévenin equivalent circuit with respect to the terminals a and b? Clearly label a and b on your circuit diagram. [6pts]

d) What is the maximum power that the circuit can deliver to an optimally chosen load resistor attached to the terminals a and b? What is the value of the optimally chosen load resistor? [4pts]

a) conservation of energy [+1]

b) conservation of charge [+1]

c) Find open circuit voltage.



node equations:

$$0 = \frac{v_1}{50\Omega} + \frac{v_1 - 20\Omega i_x}{10\Omega} + \frac{v_1 - V_{oc}}{15\Omega}$$

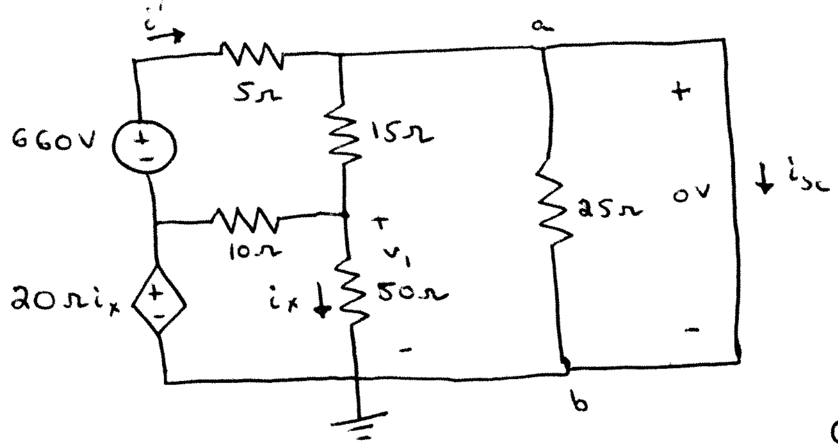
$$0 = \frac{V_{oc}}{25\Omega} + \frac{V_{oc} - v_1}{15\Omega} + \frac{V_{oc} - (20\Omega i_x + 660V)}{5\Omega}$$

control equation: $i_x = v_1 / 50\Omega$

$$0 = \left(\frac{1}{50} + \frac{1}{10}\left(1 - \frac{2}{5}\right) + \frac{1}{15}\right) v_1 - \frac{1}{15} \cdot V_{oc}$$

$$\frac{660}{5} = \left(-\frac{1}{15} - \frac{1}{5} \cdot \frac{2}{5}\right) v_1 + \left(\frac{1}{25} + \frac{1}{15} + \frac{1}{5}\right) \cdot V_{oc}$$

V_{oc} = **550 V** [+1]



node equation:

$$0 = \frac{v_1}{50\Omega} + \frac{v_1 - 20\Omega i_x}{10\Omega} + \frac{v_1}{15\Omega}$$

control equation: $i_x = v_1/50$

$$0 = \left(\frac{1}{50} + \frac{1}{10}\left(1 - \frac{2}{5}\right) + \frac{1}{15}\right) v_1$$

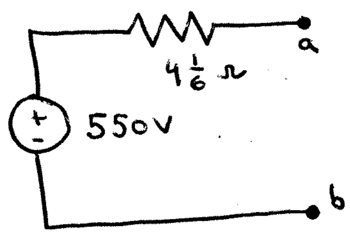
Find short circuit current.

$v_1 = 0\text{V}$ $\therefore$ $i_x = 0\text{ A}$

$$i_{sc} = i' + \frac{v_1}{15\Omega} = \frac{660\text{V}}{5\Omega} + 0\text{A} = 132\text{ A} \quad [+1]$$

$$R_T = v_{oc}/i_{sc} = \frac{550\text{V}}{132\text{A}} = 4\tfrac{1}{6}\,\Omega = 4.1667\Omega \quad [+1]$$

[+1]



[+2]

d) Maximum power deliverable by circuit:

$$P_{max} = \frac{v_{oc}}{2} \cdot \frac{i_{sc}}{2} \quad (+1)$$

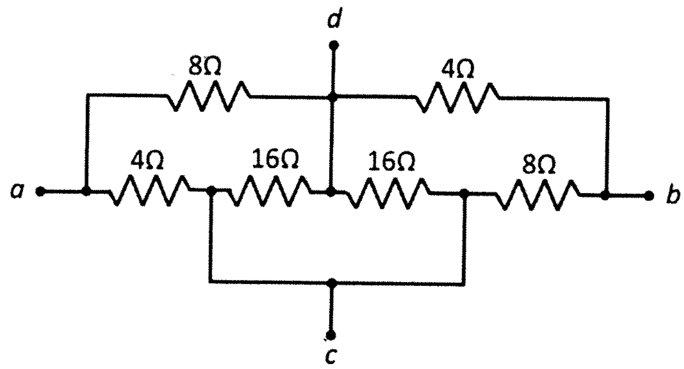
$$= \frac{550\text{V} \cdot 132\text{A}}{4}$$

$$= 18.15\text{ kW} \quad [+1]$$

Load resistor should be: $R_L = R_T = 4\frac{1}{6}\,\Omega$

[+1] [+1]

2. Consider the circuit below. Answer the questions. [12 pts]



a) What is the definition of a passive element? [1pt]

b) What is the definition of a linear element? [1pt]

c) What is the equivalent resistance between terminals c and d? [2pts]

d) What is the equivalent resistance between terminals a and c? [2pts]

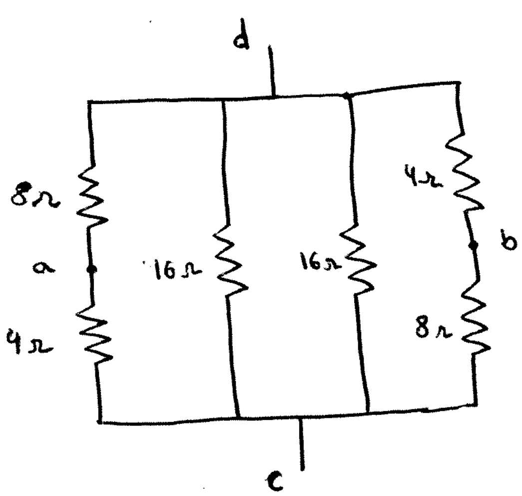
e) What is the equivalent resistance between terminals b and c? [2pts]

f) What is the equivalent resistance between terminals a and b? [4pts]

a) An element that cannot deliver more energy than it receives from a circuit.

b) An element for which terminal voltage and terminal current are related by a linear equation or linear operator.

c)



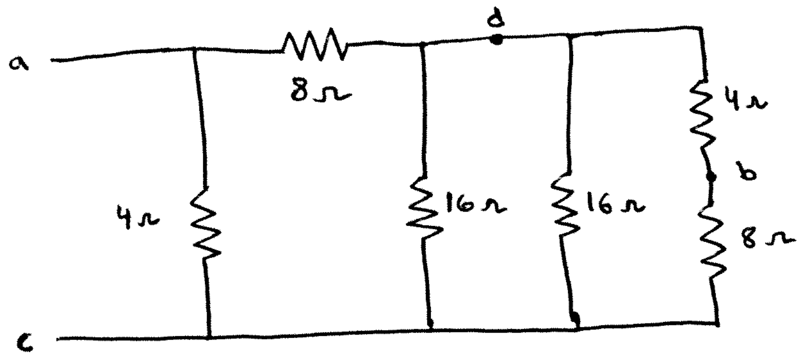
$$R_{dc} = (4\Omega + 8\Omega) // 16\Omega // 16\Omega // (4\Omega + 8\Omega)$$ [+1]

$$= 6\Omega // 8\Omega$$

$$= 3\tfrac{3}{7}\Omega$$ [+1]

$$= 3.429\Omega$$

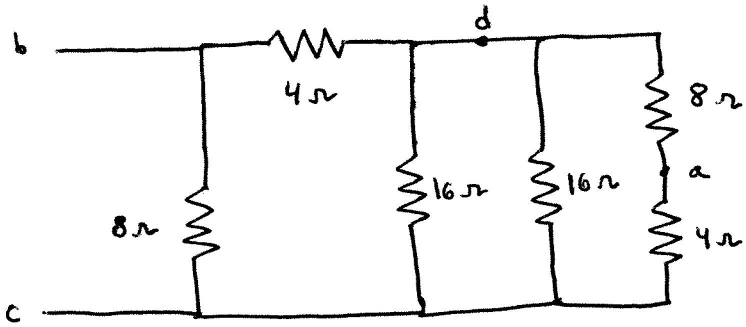
d)



$R_{ac} = 4\Omega \,//\, [\, 8\Omega + 16\Omega // 16\Omega // (4+8)\Omega \,]$ [+1]

$= 3.048\,\Omega$ [+1]

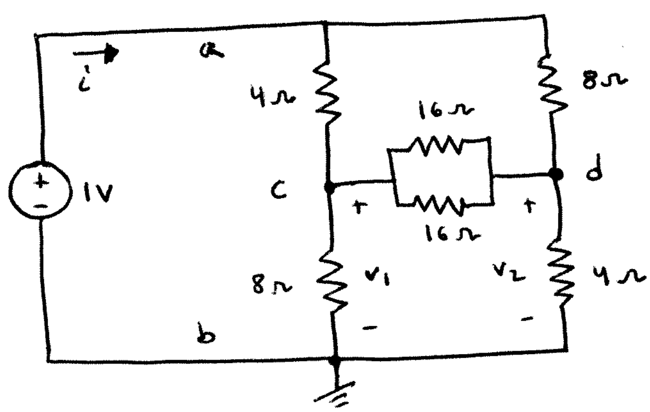
e)



$R_{bc} = 8\Omega \,//\, [\, 4\Omega + 16\Omega // 16\Omega // (4+8)\Omega \,]$ [+1]

$= 4.190\,\Omega$ [+1]

f)



[+1 for diagram]

Find response to applied source.

$$0 = \frac{v_1}{8\Omega} + \frac{v_1 - v_2}{8\Omega} + \frac{v_1 - 1V}{4\Omega}$$

$$0 = \frac{v_2}{4\Omega} + \frac{v_2 - v_1}{8\Omega} + \frac{v_2 - 1V}{8\Omega}$$

$$0.25\,V = 0.5\,v_1 - 0.125\,v_2$$

$$0.125\,V = -0.125\,v_1 + 0.5\,v_2$$

$v_1 = 0.6V \qquad v_2 = 0.4\,V$

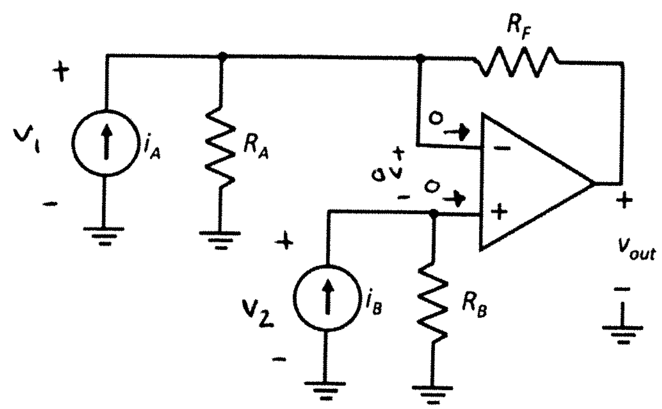
KCL + Ohm:

$$i = \frac{v_1}{8\Omega} + \frac{v_2}{4\Omega} = 175\,mA$$

$$R_{eq} = \frac{1V}{i} = 5.714\,\Omega$$ [+2]

[+1]

3. Consider the circuit below. Assume ideal op-amp behaviour. Answer the questions. [12 pts]



a) Give one reason why positive feedback is not used in op-amp circuits. [1pt]

b) Find an expression for v_{out} in terms of i_A, i_B and resistor values. [5pts]

c) It is desired that v_{out} = 4kΩ ($i_B - i_A$) with R_A = 1kΩ. What are the required values of R_B and R_F? [2pts]

Assume the resistor values satisfy the design requirements of part c) for the remaining parts.

d) If i_A = 200μA and i_B = 500μA, how much power does the op-amp deliver to the circuit? [3pts]

e) The power supply voltages to the op-amp are +10V and −10V. At what values of ($i_B - i_A$) does the op-amp output saturate? [1pt]

a) Positive feedback gives unstable behaviour. [+1]

b) Apply ideal op-amp conditions: $i_+ = i_- = 0\,A$ [+1]

$v_2 = v_1$ [+1]

KCL + KVL + Ohm: $v_2 = i_B \cdot R_B$ [+1]

node equation at inverting input node:

$$0 = -i_A + \frac{v_2}{R_A} + \frac{v_2 - v_{out}}{R_F} \quad [+1]$$

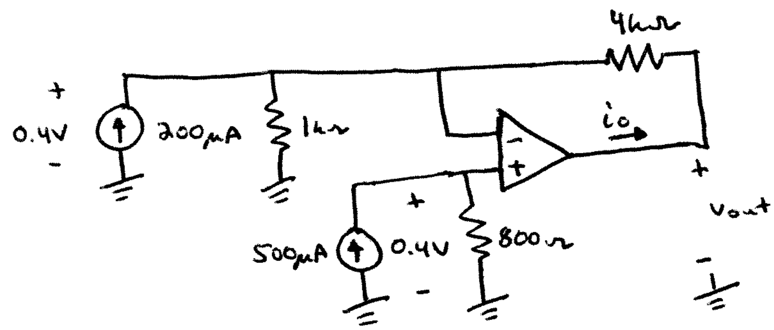
$$\therefore \quad v_{out} = -R_F i_A + \left(\frac{R_F}{R_A} + 1\right) v_2 = \left(1 + \frac{R_F}{R_A}\right) \cdot R_B i_B - R_F i_A \quad [+1]$$

c) $V_{out} = 4k\Omega\,(i_B - i_A) = \left(1 + \frac{R_F}{1k\Omega}\right) \cdot R_B \cdot i_B - R_F \cdot i_A$

$\therefore\ R_F = 4k\Omega$ [+1]

$$R_B = \frac{4k\Omega}{1 + R_F/1k\Omega} = \frac{4k\Omega}{1 + 4k\Omega/1k\Omega} = 800\Omega \quad [+1]$$

d) All power transfer occurs at the output of the op-amp.



$V_1 = V_2 = 500\mu A \cdot 800\Omega = 0.4V$

$V_{out} = 4k\Omega\,(500\mu A - 200\mu A)$

$= 1.2V$ [+1/2]

$$i_o = \frac{V_{out} - V_1}{R_F} = \frac{1.2V - 0.4V}{4k\Omega}$$

$= 200\,\mu A$ [+1/2]

Power delivered $= V_{out} \cdot i_o$ [+1]

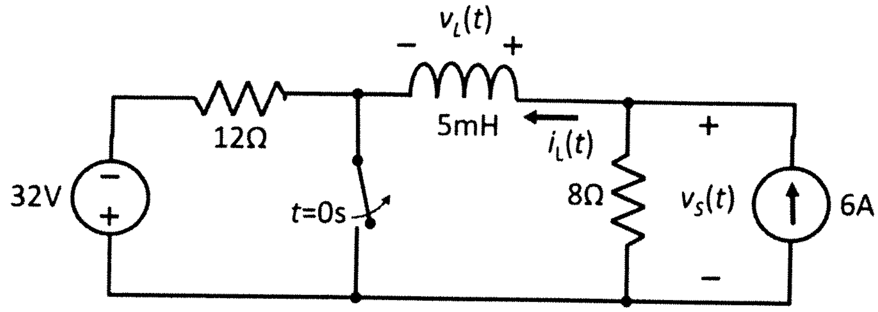
$= 1.2V \times 200\mu A = 240\,\mu W$ [+1]

e) $\pm 10V = 4k\Omega \cdot (i_B - i_A)$ at saturation

$\therefore\ |i_B - i_A| \geq 2.5\,mA$ results in saturation of output.

[+1]

4. Consider the circuit below. The switch is closed for $t<0s$, and opens instantaneously at $t=0s$. Assume dc steady state behaviour for $t<0$. Answer the questions. [12 pts]



a) What is the current $i_L(t)$ for $t > 0$? [4pts]

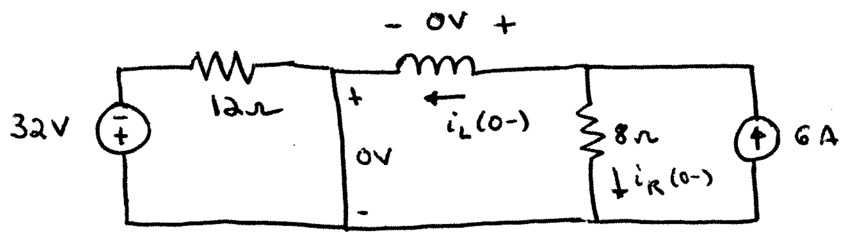
b) Plot $i_L(t)$ versus t. Be sure to label your axes. [2pts]

c) What is the voltage $v_L(t)$ for $t > 0$? [2pts]

d) What is the voltage $v_S(t)$ for $t > 0$? [2pts]

e) If the inductor was replaced by a capacitor, what would be the value of $v_s(0+)$? [2pts]

a) $t = 0-$

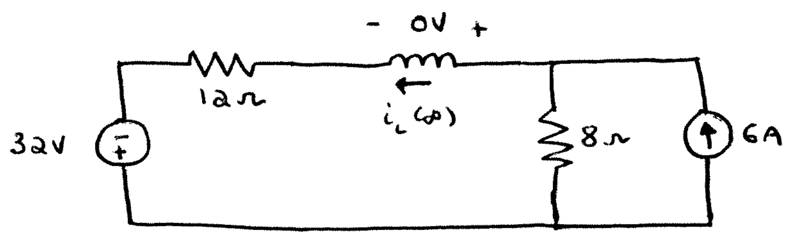


$i_R(0-) = \frac{0V}{8\Omega} = 0A$

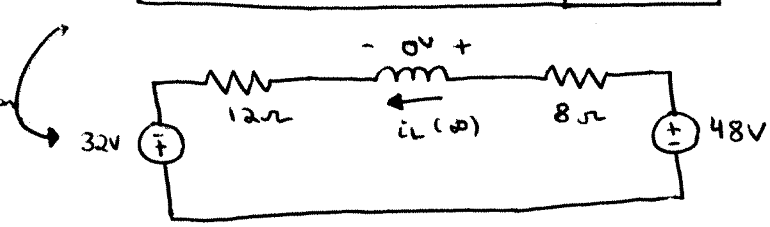
$i_L(0-) = 6A$

$i_L(0+) = i_L(0-) = 6A$ [+1]

$t \to \infty$



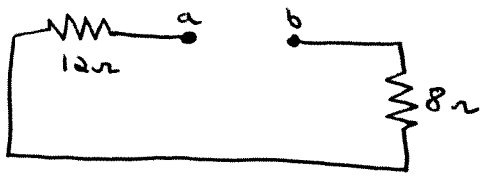
Source transformation



$i_L(\infty) = \frac{48V + 32V}{8\Omega + 12\Omega}$

$= 4A$ [+1]

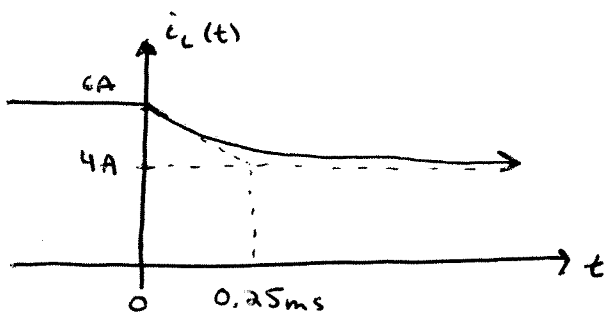
$$\tau = \frac{L}{R_{TH}} = \frac{5mH}{12\Omega + 8\Omega} = 0.25ms \quad [+1]$$



For $t > 0$: $$i_L(t) = i_L(\infty) + [i_L(0+) - i_L(\infty)] \exp(-t/\tau)$$

$$= 4A + 2A \exp(-t/0.25ms) \quad [+1]$$

b)



[+1 for shape, +1 for axis labels]

c) $$v_L(t) = L \cdot \frac{di_L}{dt} \quad [+1]$$

$$= 5mH \cdot \frac{d}{dt}\left[4A + 2A \exp(-t/0.25ms)\right]$$

$$= -40V \exp(-t/0.25ms) \quad [+1]$$

$0 < t$

d) By KVL: $$0 = -V_S + V_L + 12\Omega \cdot i_L - 32V \quad [+1]$$

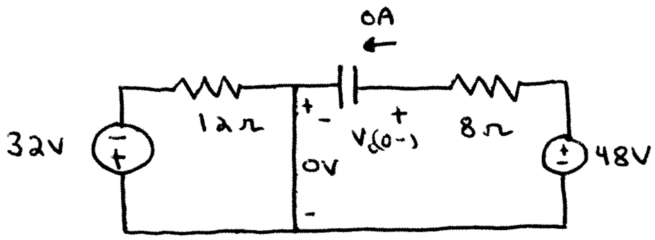
$$V_S = V_L + 12\Omega \cdot i_L - 32V$$

$$= -40V \exp(-t/0.25ms) + 48V + 24V \exp(-t/0.25ms) - 32V$$

$$= 16V - 16V \exp(-t/0.25ms) \quad [+1]$$

$0 < t$

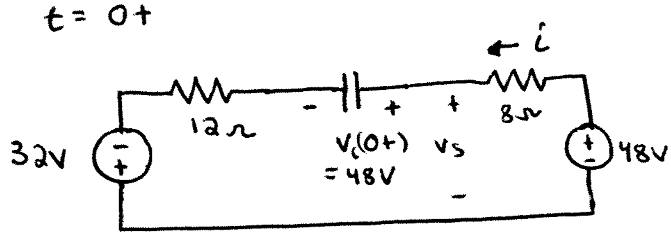
e) $t = 0-$



$V_c(0-) = 48V$

$V_c(0+) = V_c(0-) = 48V$ [+1]

$t = 0+$



$$0 = -48V + i \cdot 8\Omega + 48V + i \cdot 12\Omega - 32V$$

$$i = \frac{32V}{20\Omega} = 1.6A$$

$$V_s(0+) = 48V - i \cdot 8\Omega$$

$$= 35.2V \quad [+1]$$

end