ECSE-200 Electric Circuits 1 - Quiz #11 (April 5, 2019)

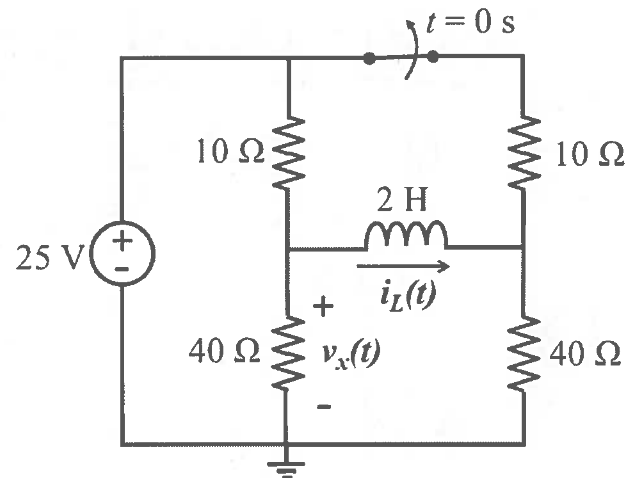
LAST NAME ______________________________ **MCGILL ID#** ______________

FIRST NAME ______________________ **SIGNATURE** ______________________

- ***Only Faculty standard calculator accepted***
- ***No cellphone allowed***
- ***Show all your work***
- ***Clearly indicate your final answer with the SI unit and multiplier***
- ***You have 45 minutes to complete this quiz***

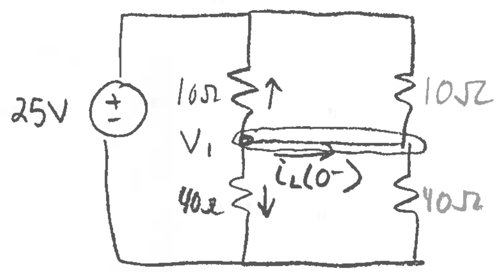
Question 1: Consider the circuit shown. The switch opens the connection at $t = 0\ s$. The circuit reaches steady state before the switch changes its connection state. Answer the following questions.

a) Find the solution of the current $i_L(t)$ for $t > 0\ s$. [3 pt]
b) Plot the current $i_L(t)$ with respect to time t indicating the time constant, the initial and final current values. [2 pt]
c) Find the voltage $v_x(t)$ for $t > 0\ s$. [2 pt]



a) $t<0s$

FIND INITIAL CONDITION



KCL: $\frac{V_1 - 25V}{10\Omega} + \frac{V_1}{40} + i_L(0^-) = 0$ $\quad V_1 = 25V \cdot \frac{40\Omega // 40\Omega}{40\Omega//40\Omega + 10\Omega // 10\Omega} = 25V \cdot \frac{20\Omega}{25\Omega} = 20V$

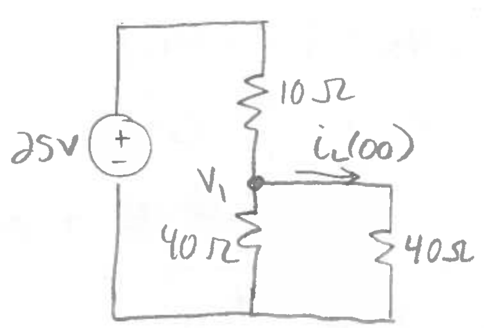
$\frac{20-25}{10} + \frac{20}{40} = -i_L(0^-)$

$-\frac{5}{10} + \frac{1}{2} = 0$

$i_L(0^-) = i_L(0^+) = 0A$ from continuity of current in voltage

$t \to \infty$

FIND FORCED RESPONSE



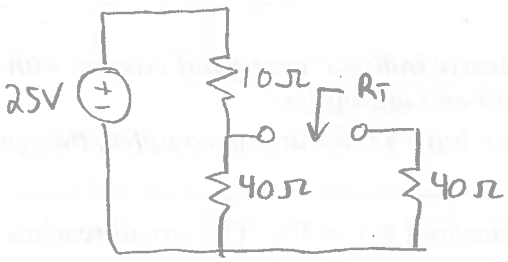
$i_L(\infty) = \frac{V_1}{40\Omega}$ $\quad V_1 = 25V \cdot \frac{40\Omega // 40\Omega}{40\Omega//40\Omega + 10\Omega}$

$V_1 = 25V \cdot \frac{20\Omega}{30\Omega} = \frac{50}{3} V$

$i_L(\infty) = \frac{5}{12} V = 0.417 A$

time constant $\tau = L/R_T$

R_T



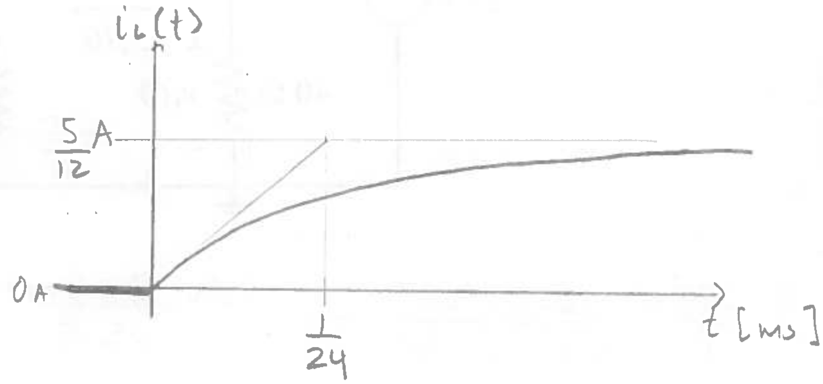
$R_T: 10\Omega // 40\Omega + 40\Omega = 48\Omega$

$R_T = 48\Omega \quad \tau = \frac{2H}{48\Omega} = \frac{1}{24}s = 41.67ms$

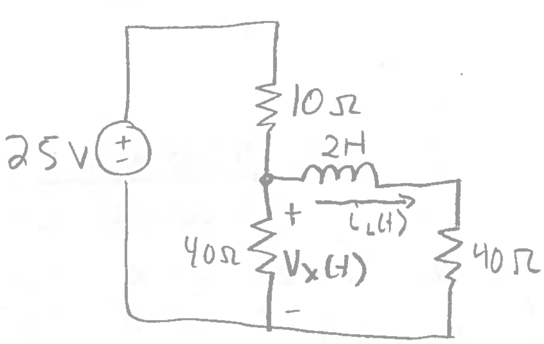
$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}\ A$$

$$\boxed{i_L(t) = \frac{5}{12}A - \frac{5}{12}e^{-24t}\ A}$$

b)



c) $t > 0$



KCL: $\frac{V_x(t)}{40\Omega} + \frac{V_x(t) - 25V}{10\Omega} + i_L(t) = 0$

$V_x + 4V_x - 100 + 40i_L = 0$

$5V_x = 100 - 40i_L$

$V_x = 20 - 8i_L$

$= 20 - \cancel{8}^{2} \cdot \frac{5}{\cancel{12}_{3}} + \cancel{8}^{2} \cdot \frac{5}{\cancel{12}_{3}} e^{-24t}$

$$\boxed{V_x(t) = \frac{50}{3}V + \frac{10}{3}e^{-24t}\ V}$$

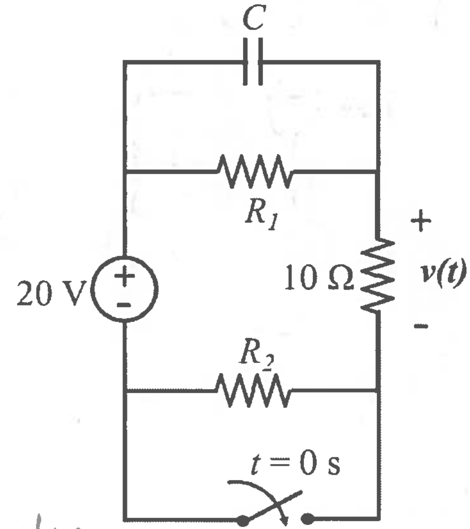
Question 2: Consider the circuit shown. The switch closes and makes the connection at time $t = 0$ s. The circuit reaches steady state before the switch changes its connection.

The voltage across the 10 Ω resistor as shown in the circuit is

$$v(t) = 6 + 3e^{-2t}\ V \quad \text{for } t > 0\ s$$

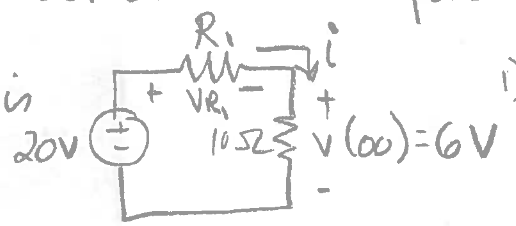
Answer the following questions.

a) Determine the time constant of this circuit. [1 pt]
b) Find R_1, R_2, and C. [3 pt]
c) Determine the energy stored in the capacitor at $t = 0$ s. [2 pt]



a) $\tau = \frac{1}{2} s = 0.5\ s$

b) i) $t \to \infty$ to find R_1 since steady-state blocks current thru capacitor and $V_{R2} = 0V$ so no current
↓
equiv circuit is

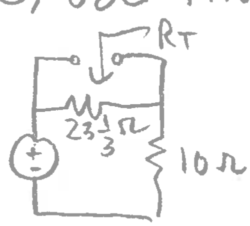


1) current through R_1
$i = \frac{6V}{10\Omega} = 0.6V$

2) Voltage across R_1 is (KVL)
$-20V + V_{R_1} + 6V = 0 \quad V_{R_1} = 14V$

3) $R_1 = \frac{V_{R_1}}{i} = \frac{14V}{0.6V}$ $\boxed{R_1 = 23\frac{1}{3}\Omega}$

ii) To find C, use time constant and R_T
t>0



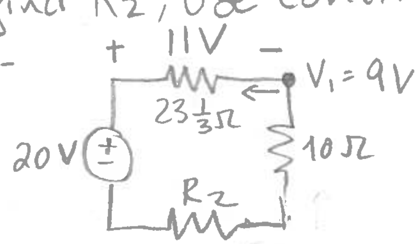
$R_T = 23\frac{1}{3} \,//\, 10\Omega = 7\Omega$ $\quad \frac{1}{2} s = RC = 7\Omega \cdot C$

$\boxed{C = \frac{1}{14} F = 71.429uF}$

iii) To find R_2, use continuity of voltage in a capacitor

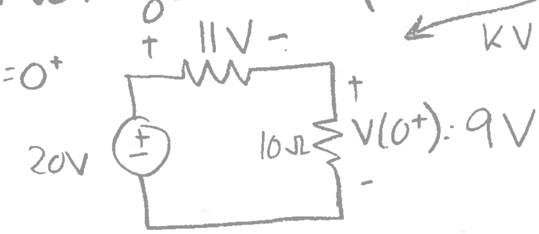
KVL → $V_{R_1} = +11V$
↓
$V_C(0^+) = 11V$
since in parallel

t=0-



←

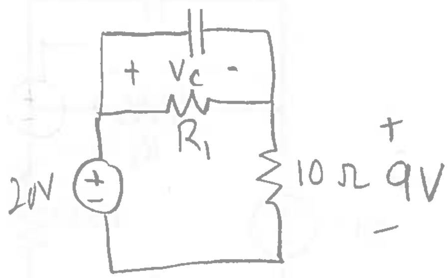
t=0+



KCL: $\frac{-11V}{23\frac{1}{3}} + \frac{9V}{10\Omega + R_2} = 0 \rightarrow 9V = 0.471(10\Omega + R_2)$

$\boxed{R_2 = 9.\frac{1}{11}\Omega}$

c) at $t = 0^+ s$



$V_c(0^+) = 11V$

$\frac{1}{2} C V^2 = \frac{1}{2} \cdot \frac{1}{14} \cdot (11)^2$

$\boxed{U = \frac{121}{28} J = 4.321 J}$