McGill

December 2010
Final Examination

Electric Circuits 1

ECSE-200 Section: 1, 2

8 Dec 2010, 9:00AM

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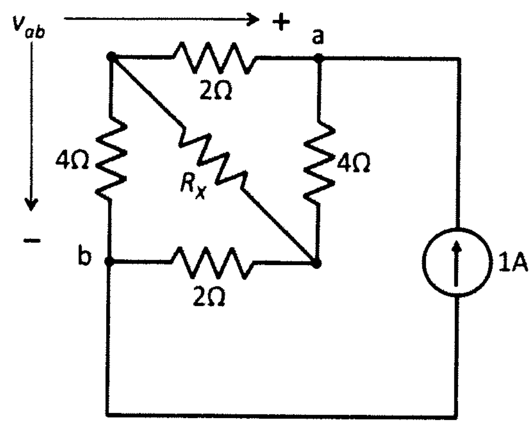
Student Name:		McGill ID:									

INSTRUCTIONS:

- This is a **CLOSED BOOK** examination.
- **NO CRIB SHEETS** are permitted.
- **SPACE IS PROVIDED** on the examination to answer all questions.
- **STANDARD CALCULATOR** permitted ONLY.
- This examination consists of 4 questions, with a total of 18 pages, including the cover page.
- This examination is **PRINTED ON BOTH SIDES** of the paper
- This examination paper **MUST BE RETURNED**

1. Consider the circuit below. Answer the questions. [12 pts]



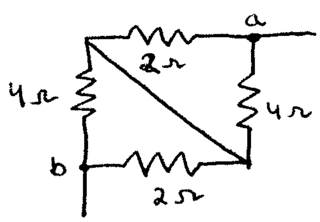
a) What is the voltage v_{ab} if $R_X = 0\Omega$? [2pts]

b) What is the voltage v_{ab} if $1/R_X = 0\Omega^{-1}$? [2pts]

c) What is the voltage v_{ab} if $R_X = 6\Omega$? [5pts]

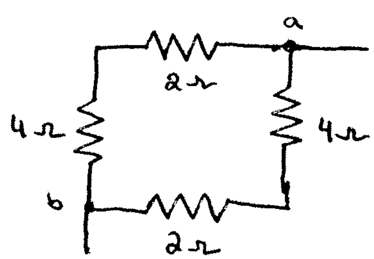
d) What should the value of R_X be in order to absorb the maximum power by R_X possible? [3pts]

a)



$$R_{ab} = (2\Omega // 4\Omega) + (2\Omega // 4\Omega) \quad [+1]$$
$$= \frac{8}{3}\,\Omega$$
$$V_{ab} = 1A \cdot \frac{8}{3}\,\Omega = \frac{8}{3}\,V \quad [+1]$$

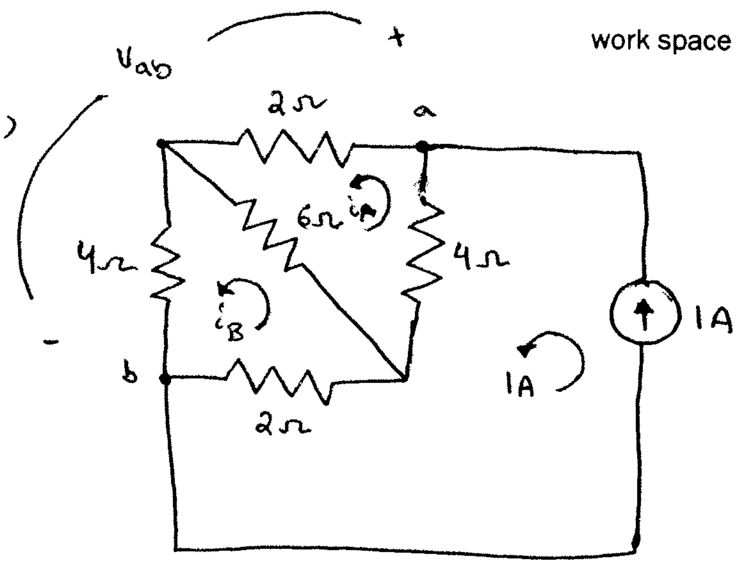
b)



$$R_{ab} = (2\Omega + 4\Omega) // (2\Omega + 4\Omega) \quad [+1]$$
$$= 3\,\Omega$$
$$V_{ab} = 1A \cdot 3\Omega = 3V \quad [+1]$$

work space

c)



$$0 = 4\Omega\,(i_A - 1A) + 2\Omega\, i_A + 6\Omega\,(i_A - i_B) \qquad [+1]$$

$$0 = 2\Omega\,(i_B - 1A) + 6\Omega\,(i_B - i_A) + 4\Omega\, i_B \qquad [+1]$$

$$\begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 12 & -6 \\ -6 & 12 \end{pmatrix} \begin{pmatrix} i_A \\ i_B \end{pmatrix}$$

$$i_A = \frac{\begin{vmatrix} 4 & -6 \\ 2 & 12 \end{vmatrix}}{\begin{vmatrix} 12 & -6 \\ -6 & 12 \end{vmatrix}} = \frac{5}{9}\,A \qquad [+1/2]$$

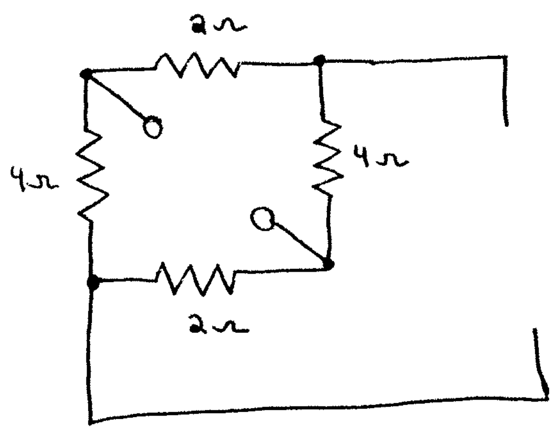
$$i_B = \frac{\begin{vmatrix} 12 & 4 \\ -6 & 2 \end{vmatrix}}{\begin{vmatrix} 12 & -6 \\ -6 & 12 \end{vmatrix}} = \frac{4}{9}\,A \qquad [+1/2]$$

$$V_{ab} = 2\Omega\, i_A + 4\Omega\, i_B \qquad [+1]$$

$$= 2\Omega \cdot \frac{5}{9}A + 4\Omega\, \frac{4}{9}A$$

$$= 2\,\frac{8}{9}\,V \qquad [+1]$$

d) Turn off sources, look at equivalent resistance:



[+1]

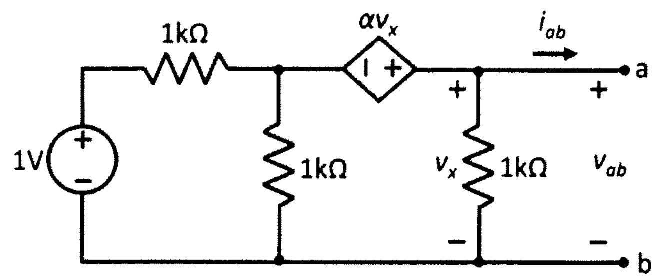
$R_{TH} = R_{eq} = (2\Omega + 4\Omega) \,//\, (2\Omega + 4\Omega)$ [+1]

$= 3\Omega$

Maximum power transfer theorem: $R_L = R_{TH} = 3\Omega$ [+1]

work space

2. Consider the circuit below. Answer the questions. [12 pts]



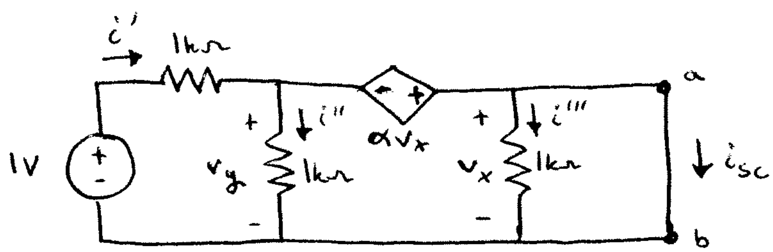
a) What is the short circuit current i_{sc} at the terminals a and b? [3pts]

b) If the open circuit voltage v_{oc}=0.5V at the terminals a and b, what is the value of α? [3pts]

c) For the value of α satisfying part b), what is the power delivered or absorbed by the dependent source under open circuit conditions? [3pts]

d) Assume again that v_{oc}=0.5V. What is the maximum power that can be delivered to a load resistance attached to terminals a and b, under the restriction that $v_{ab}\geq 0.4$V? [3pts]

a)



KVL : $v_x = 0V$ [+1]

$\therefore \ \alpha v_x = 0V \quad , \quad i''' = 0mA$

KVL: $0 = -v_y - \alpha v_x + v_x$

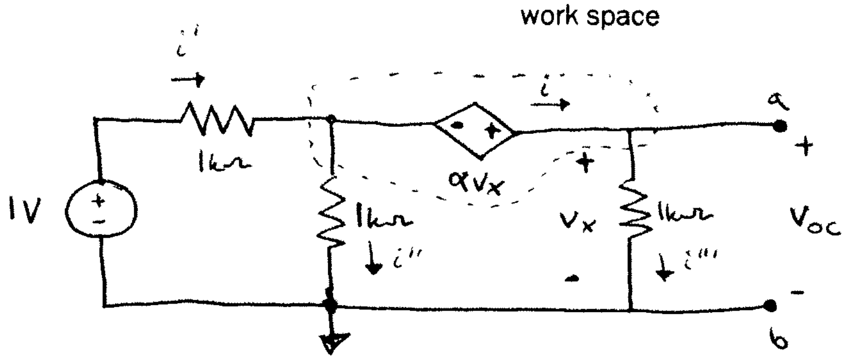
$v_y = 0V \quad , \quad i'' = 0mA$

KVL + Ohm : $i' = 1V/1k\Omega = 1mA$ [+1]

KCL: $0 = -i' + i'' + i''' + i_{sc}$

$i_{sc} = i' = 1mA$ [+1]

b)



$V_{oc} = 0.5V$

node equation:

$$\frac{V_x}{1k\Omega} + \frac{V_x - \alpha V_x}{1k\Omega} + \frac{(V_x - \alpha V_x) - 1V}{1k\Omega} = 0 \qquad [+2]$$

$$(3 - 2\alpha)\frac{V_x}{1k\Omega} - \frac{1V}{1k\Omega} = 0$$

$$\alpha = \frac{3 - 1V/V_x}{2}$$

$$\alpha = \frac{3 - 1V/0.5V}{2}$$

$$\alpha = \frac{1}{2} \quad [+1]$$

c) Ohm: $i''' = \dfrac{0.5V}{1k\Omega} = 0.5mA$

KCL: $0 = -i + i'''$

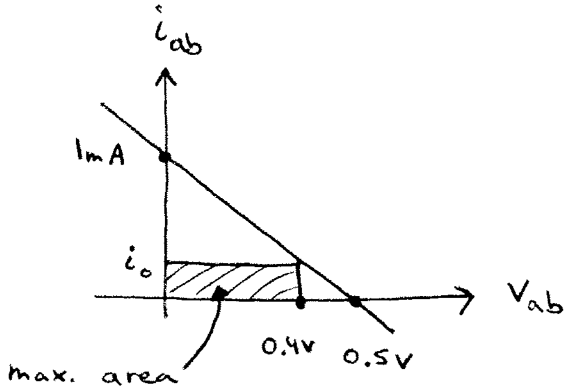
$i = i''' = 0.5mA \quad [+1]$

Power delivered by source $= (\alpha V_x) \cdot i \quad [+1]$

$= (0.5 \cdot 0.5V) \cdot 0.5mA$

$= 125\ \mu W \quad [+1]$

d)



[+1] for diagram

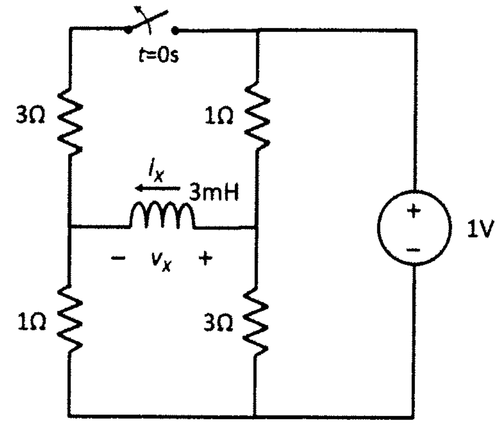
By triangles: $i_o = 1\text{mA} \cdot \dfrac{0.5\text{V} - 0.4\text{V}}{0.5\text{V}} = 0.2\text{mA}$ [+1]

$$P_{delivered} = 0.4\text{V} \cdot 0.2\text{mA}$$

$$= 80\,\mu\text{W}$$ [+1]

work space

3. Consider the circuit below. The circuit is in dc steady state for $t<0$ with the switch closed. At $t=0s$, the switch opens instantaneously and remains open. Answer the questions. [12 pts]



a) What is the inductor current $i_x(0+)$? [3pts]

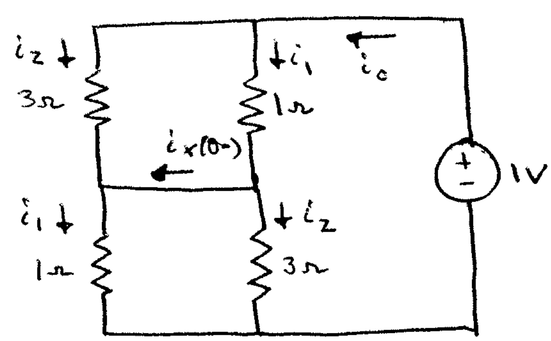
b) What is the inductor current $i_x(\infty)$? [3pts]

c) What is the time constant for the first order response for $t>0$? [2pts]

d) What is the inductor current $i_x(t)$ for $t>0$? [2pts]

e) What is the inductor voltage $v_x(t)$ for $t>0$? [2pts]

a)



$$i_o = \frac{1V}{(1\Omega // 3\Omega) + (1\Omega // 3\Omega)} = \frac{2}{3} A$$

$$i_1 = \frac{3\Omega}{1\Omega + 3\Omega} \cdot i_o = \frac{1}{2} A \quad [+1]$$

$$i_2 = \frac{1\Omega}{1\Omega + 3\Omega} \cdot i_o = \frac{1}{6} A \quad [+1]$$

dc steady state L → short

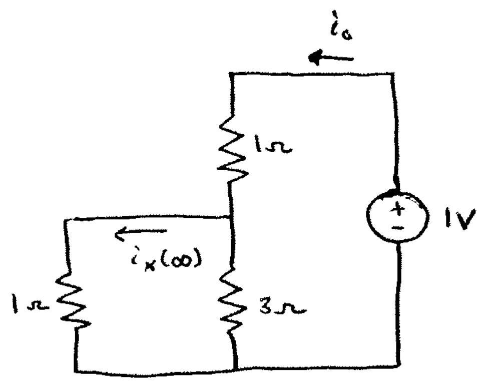
KCL: $0 = -i_1 + i_x + i_2$

$$i_x = i_1 - i_2 = \frac{1}{3} A$$

$$i_x(0+) = i_x(0-) = \frac{1}{3} A \quad [+1]$$

work space

b)



$$i_o = \frac{1V}{1\Omega + 1\Omega // 3\Omega} \quad [+1]$$

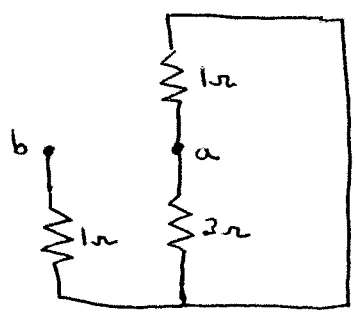
$$= \frac{4}{7} A$$

$$i_x(\infty) = \frac{3\Omega}{1\Omega + 3\Omega} \cdot i_o \quad [+1]$$

$$= \frac{3}{7} A \quad [+1]$$

dc steady state $L \rightarrow$ short

c) Find R_{TH} from inductor terminals.



$$R_{TH} = 1\Omega + 1\Omega // 3\Omega \quad [+\tfrac{1}{2}]$$

$$= \frac{7}{4} \Omega$$

$$\tau = \frac{L}{R_{TH}} \quad [+1/2]$$

$$= \frac{3mH}{7/4 \, \Omega}$$

$$= \frac{12}{7} ms \quad [+1]$$

d) $i_x(t) = i_x(\infty) + [i_x(0+) - i_x(\infty)] \exp(-t/\tau)$ [+1]

$= \frac{3}{7} A + \left(\frac{1}{3} - \frac{3}{7}\right) A \exp\left(-\frac{t}{12/7 \text{ ms}}\right)$

$= \frac{3}{7} A - \frac{2}{21} A \exp\left(\frac{-t}{12/7 \text{ ms}}\right)$ [+1] $t > 0$

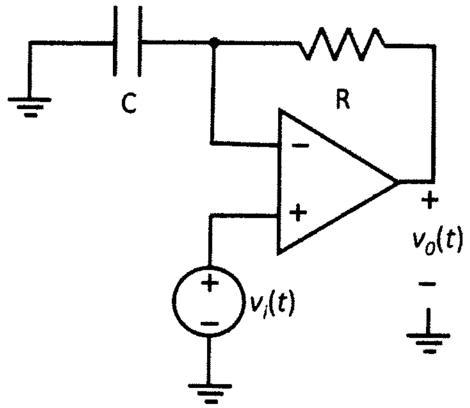
e) $v_x(t) = L \frac{di_x}{dt}$ [+1]

$= 3\text{mH} \cdot \left(-\frac{2}{21} A \cdot \frac{-1}{12/7 \text{ms}}\right) \exp\left(\frac{-t}{12/7 \text{ ms}}\right)$

$= \frac{1}{6} V \exp\left(\frac{-t}{12/7 \text{ ms}}\right)$ [+1]

work space

4. Consider the circuit below. Assume ideal op-amp behavior. Answer the questions. [12 pts]



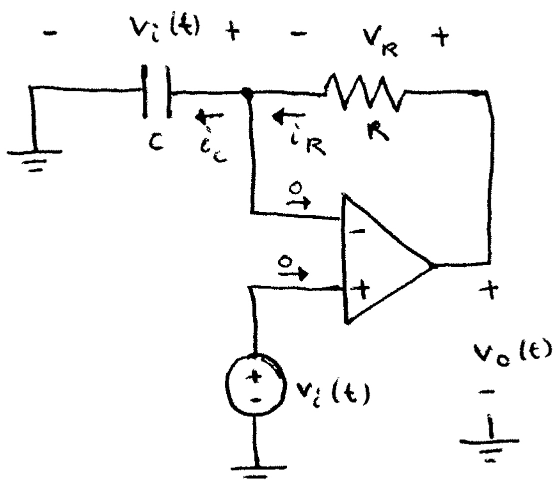
a) What is the output voltage $v_O(t)$ as a function of the input voltage $v_I(t)$? [5pts]

For the remaining questions, consider the case of R=10kΩ and C=10nF.

b) If $v_I(t)$=5V, what is the energy stored in the capacitor? [2pts]

c) If at an instantaneous time t_O it is known that $v_I(t_O)$=2V and $dv_I/dt(t_O)$=100mV/μs, what is the instantaneous power delivered or absorbed by the op-amp at the time t_O? [5pts]

a)



By ideal op-amp conditions, voltage across capacitor is $v_i(t)$ (virtual short). [+1]

$$i_c = C \frac{dv_i}{dt} \quad [+1]$$

KCL: $0 = +i_c - i_R \quad \therefore \quad i_R = i_c \quad [+1]$

Ohm: $v_R = i_R \cdot R = RC \dfrac{dv_i}{dt} \quad [+1]$

KVL: $0 = -v_i - v_R + v_o$

$v_o = v_i + v_R$

$= v_i(t) + RC\frac{dv_i}{dt}$ [+1]

b) $U = \frac{1}{2} C v_i^2$ [+1]

$= \frac{1}{2} \cdot 10nF \cdot (5V)^2$

$= 125nJ$ [+1]

c) Power delivered by op-amp $= P_{OA} = i_R \cdot v_o$ [+2]

$i_R = C\frac{dv_i}{dt} = 10nF \cdot \frac{100mV}{1\mu s}$

$= 1mA$ [+1]

$v_o = v_i + RC\frac{dv_i}{dt}$

$= 2V + 10k\Omega \cdot 10nF \cdot \frac{100mV}{1\mu s}$

$= 12V$ [+1]

$P_{OA} = 12V \cdot 1mA = 12mW$ [+1]

work space

work space

work space