McGill University

Faculty of Engineering

December 2013
Final Examination

Version #1

ELECTRIC CIRCUIT 1

ECSE-200A

16 December 2013, 18:00 – 21:00

Examiner: Professor O. Liboiron-Ladouceur

Co-Examiner: Professor G. Roberts

Student Name:		**McGill ID:**									

Signature: ____

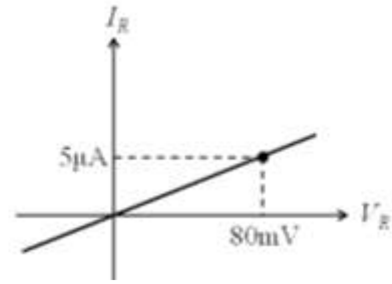
INSTRUCTIONS:

- Print your name, fill in your student ID number and sign on the line.
- **Initialize each page** of the exam at the top right corner and be sure to have a complete exam.
- This is a **CLOSED BOOK** examination. **NO CRIB SHEET** allowed.
- FACULTY **STANDARD CALCULATOR** permitted **ONLY**.
- This is, in part, a **MULTIPLE** CHOICE examination. As such, the following warning applies:

  The Examination Security Monitor Program detects pairs of students with unusually similar answer patterns on multiple-choice exams. Data generated by this program can be used as admissible evidence, either to initiate or corroborate an investigation or a charge of cheating under Section 16 of the Code of Student Conduct and Disciplinary Procedures.
- **Mark your answer to the multiple choice questions on the computer sheet PENCIL ONLY.** Each question is worth 1 mark. No answer or incorrect answer to a multiple choice question receives 0 mark.
- Answer the problems in the exam booklet provided. Show your work and clearly indicate your answer.
- Read through all of the questions and ensure that you have a complete examination (see page number at the bottom). The examination consists of a total of 7 pages including this cover page.
- The examination consists of 2 parts; *Part 1* consists of 15 multiple choice questions (15 marks), part 2 consists of 3 problems (20 marks). This examination consists of a total of 35 marks.
- This examination paper **MUST BE RETURNED**

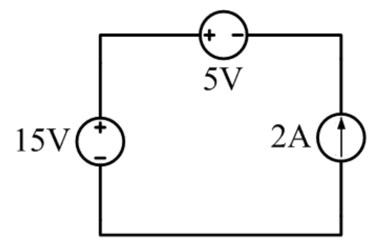
PART 1 –Indicate your answer to each question on the computer sheets provided. There is only one possible answer.

1.1. What is the resistance value of a resistor exhibiting the I-V diagram shown to the right (assume passive sign convention)?



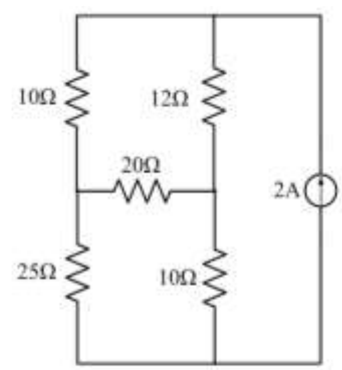
a) 16mΩ
b) 62.5Ω
c) 16kΩ
d) 62.5kΩ

1.2. What is the power delivered by the current source in the circuit shown on the right?



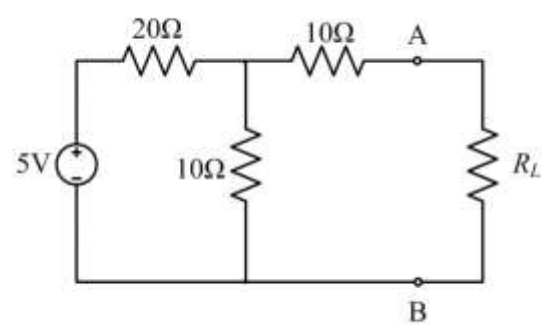
a) 10W
b) 20W
c) 30W
d) 40W

1.3. What circuit analysis method between mesh analysis and node analysis should be used to get the fewer equations to solve when analyzing the circuit on the right?



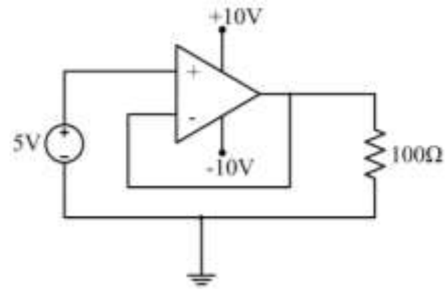
a) Both circuit analysis (mesh and node analysis) will give the same number of equations
b) Mesh analysis
c) Node analysis
d) Neither, principle of superposition should be used

1.4. What is the current source value for the Norton circuit equivalence at terminals A and B in the circuit shown to the right?



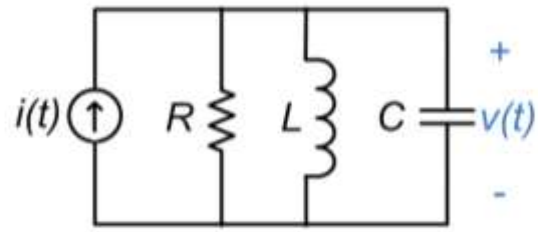
a) 5mA
b) 10mA
c) 62.5mA
d) 100mA

1.5. What is the power delivered by the 5V independent source in the circuit shown to the right?



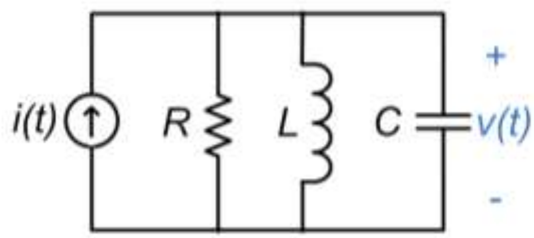
a) 0W
b) 0.25W
c) 1W
d) 4W

1.6. What is the characteristic equation of the circuit shown to the right?



a) $\boldsymbol{s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0}$
b) $s^2 + \frac{1}{L/R}s + \frac{1}{LC} = 0$
c) $s^2 + \frac{1}{2RC}s + \frac{1}{\sqrt{LC}} = 0$
d) $s^2 + \frac{1}{\sqrt{L/R}}s + \frac{1}{RC} = 0$

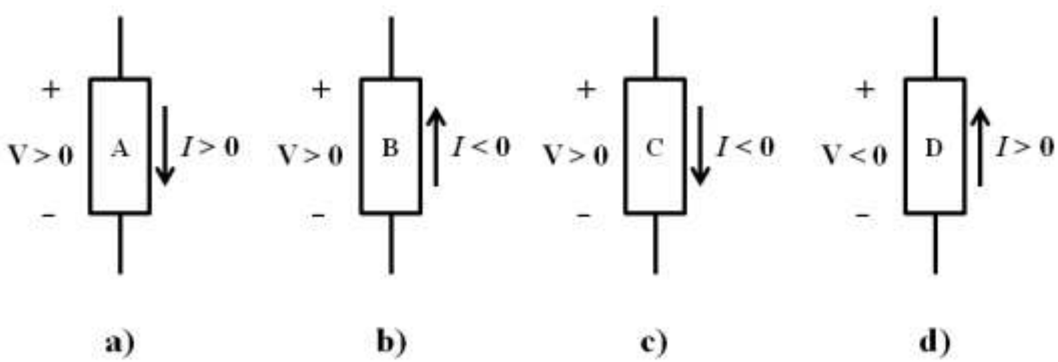
1.7. What happens to the behavior of the circuit shown to the right if the resistor R becomes really large ($R \rightarrow \infty$)?



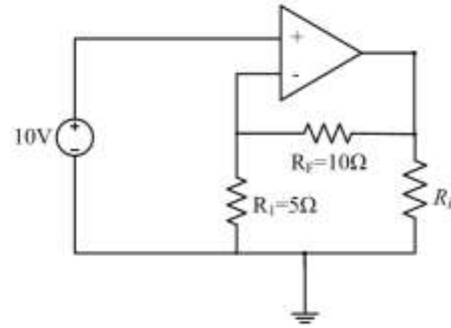
a) The natural response is critically damped
b) The natural response is underdamped
c) The natural response is overdamped
d) The natural response is undamped

1.8. What does 1 Volt represent?
a) The current required in moving a Coulomb of charge from point A to point B
b) The power absorbed associated in moving a Coulomb of charge from point A to point B
c) The power delivered in moving a Coulomb of charge from point A to point B
d) The work associated in moving a Coulomb of charge from point A to point B

1.9. Which one of the elements below (A, B, **C**, or D) must be an active element?

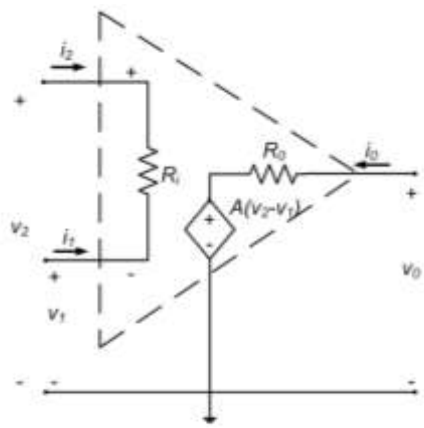


1.10. How will the voltage gain be affected if the feedback resistance (R_F) is changed to 20Ω and the other resistance (R_1) changes to 10Ω in the circuit shown to the right? Assume that the op-amp is ideal.



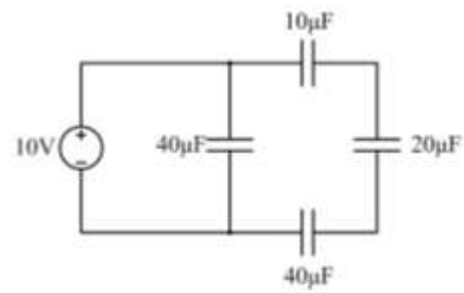
a) The magnitude of the voltage gain decreases
b) The magnitude of the voltage gain increases
c) The magnitude of the voltage gain remains unchanged
d) It depends on the load resistor (R_L)

1.11. In the model of a practical op-amp as shown to the right, what statement below is correct?



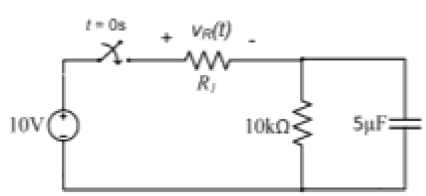
a) The input resistance R_i is small while the output resistance R_o is large
b) For a small open loop gain, the op-amp behaves as predicted by the ideal op-amp model
c) The gain A depends on the load resistance
d) None of the above

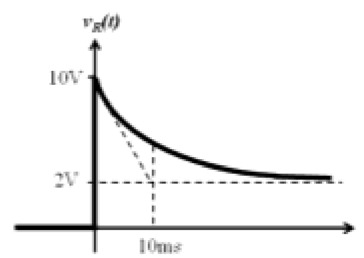
1.12. What is the total energy stored in the capacitances of the circuit shown to the right? (Assume circuit is in dc steady state)



a) 1.2725mJ
b) 2.286mJ
c) 2.545mJ
d) 4.571mJ

1.13. What is the value of the resistor R_1 in the circuit shown along with the voltage $v_R(t)$ versus time t (time constant τ is 10ms)?





a) 2.5kΩ
b) 6.7kΩ
c) 10kΩ
d) R_1 can be any value

1.14. What is capacitance?

a) The charge separation between the two metallic plates of a capacitor
b) The constant of proportionality between charge and voltage
c) The constant of proportionality between charge and current
d) The constant of proportionality between magnetic flux and voltage

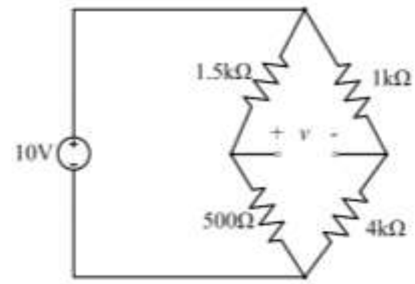
1.15. What is the voltage *v* in the circuit shown to the right?

a) -5.5V

b) -10V

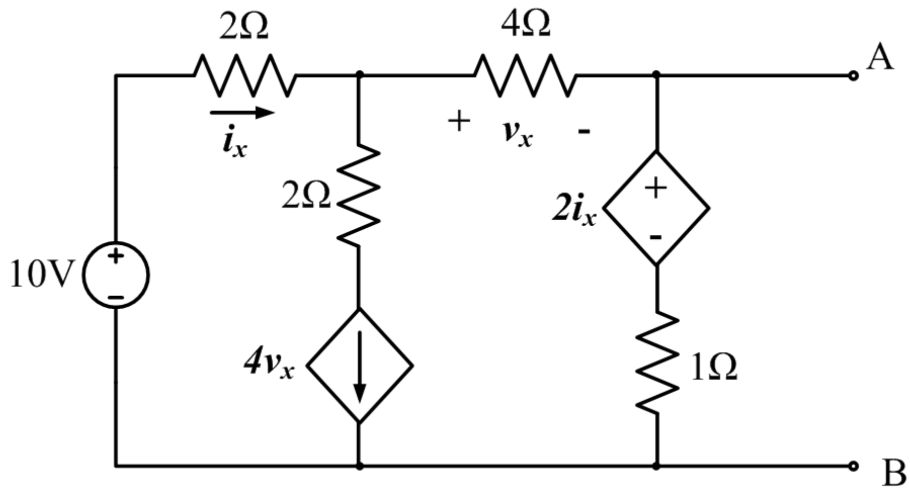
c) 2V

d) 5.5V



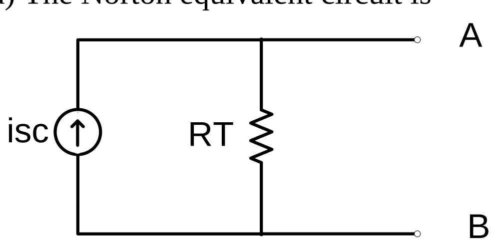
PART 2 – This part consists of 3 questions. Write your answer in the exam booklet. Show your work and clearly indicate your answer.

2.1. Consider the circuit shown below and answer the questions [8pts]

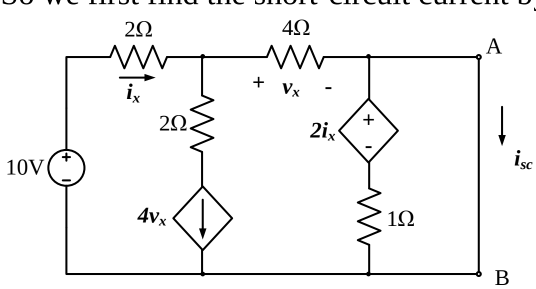


(a) What is the Norton equivalent circuit with respect to the terminals A and B?
(b) What is the maximum power that the circuit can deliver to an optimally chosen load resistor connected at terminals A and B?

a) The Norton equivalent circuit is



So we first find the short-circuit current by shorting the two terminals



Because of the short, the node voltage is vx. Using KCL, we find:

$$0 = -i_x + 4v_x + \frac{v_x}{4\Omega}$$

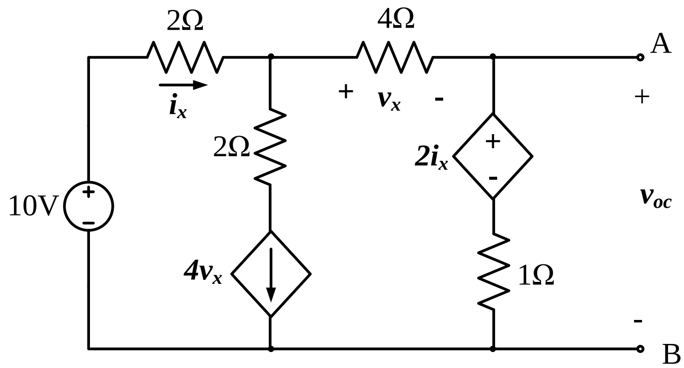
From Ohm's law: $i_x = \frac{10V - v_x}{2\Omega}$

So the KCL equation becomes:

$0 = \frac{v_x - 10V}{2\Omega} + 4v_x + \frac{v_x}{4\Omega} \rightarrow 0 = 2v_x - 20 + 16v_x + v_x = 19v_x - 20 \rightarrow v_x = \frac{20}{19}V = 1.053V$

From Ohm's law: $i_{sc} = \frac{v_x}{4\Omega} = \frac{20}{19\cdot 4}V = 0.2632A$

To find the Thevenin resistance, we also need to find the open-circuit voltage voc:



The open-circuit voltage is the same voltage across the dependent voltage source and the 1ohm resistor. We first to a KCL to get the first equation:

$0 = -i_x + 4v_x + \frac{v_x}{4\Omega} = -4i_x + 17v_x \rightarrow v_x = \frac{4}{17}i_x$

Then, a KVL is done to get the 2nd equation:

$$0 = -10V + 2\Omega \cdot i_x + v_x + 2i_x + \frac{v_x}{4\Omega} \cdot 1\Omega$$

We replace one variable from the 1st equation:

$$0 = -10V + 2\Omega \cdot i_x + \frac{4}{17}i_x + 2i_x + \frac{4}{17} \cdot \frac{1}{4\Omega} \cdot 1\Omega \cdot i_x$$

$$0 = -10V + 2 \cdot i_x + \frac{4}{17}i_x + 2 \cdot i_x + \frac{1}{17} \cdot i_x = -170 + 34 \cdot i_x + 4 \cdot i_x + 34 \cdot i_x + i_x$$

$$i_x = \frac{170}{73}A = 2.329A$$

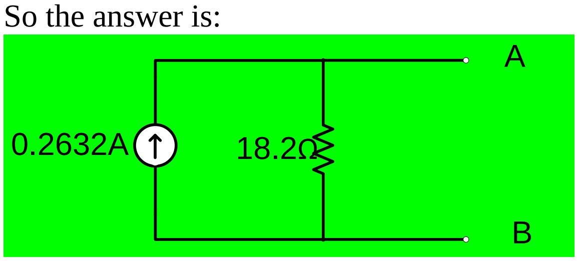
So:

$$v_{OC} = 2i_x + \frac{v_x}{4\Omega} \cdot 1\Omega = 4.658\text{V} + \frac{1}{17}2.329A = 4.795V$$

So the Thevenin resistance is:

$$R_T = \frac{v_{OC}}{i_{sc}} = \frac{4.795V}{0.2632A} = 18.2\Omega$$

So the answer is:

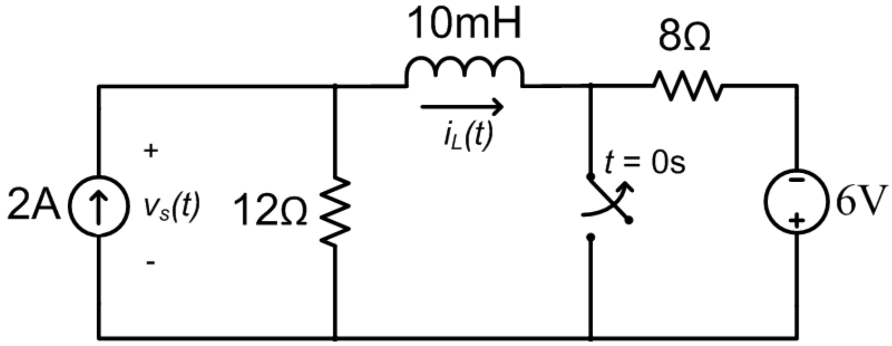


b) The maximum power that the circuit can deliver to an optimally chosen load resistor connected at terminals A and B is when the load resistance is 18.2ohm:

$$P = \frac{i_{sc}}{2} \cdot \frac{v_{oc}}{2} = \frac{0.2632A}{2} \cdot \frac{4.795V}{2} = 0.3159W$$

$$P = 315.9mW$$

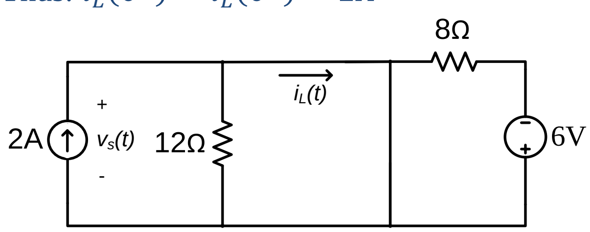
2.2. Consider the circuit shown below. The circuit is in dc steady state for $t < 0$ with the switch closes. At $t = 0$s, the switch opens and remains open for $t > 0$s. Answer the questions. [7pts]



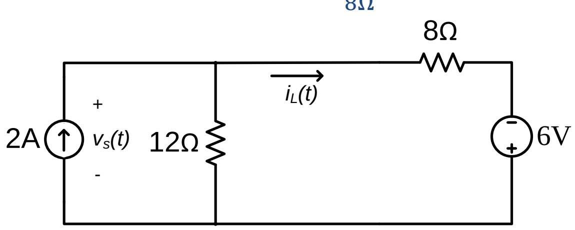
(a) Find the inductor current $i_L(t)$ and voltage $v_s(t)$ across the current source?
(b) Find the energy stored in the inductor at $t = 1$ms?

a) First, find the inductor current $i_L(t)$ using the general procedure building the solution:
$i_L(t) = i_L(\infty) + [i_L(0+) - i_L(\infty)]exp\left(-\frac{L}{R_T}t\right)$

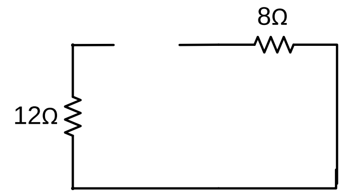
For t<0: There is a short to the ground reference with no current going through the 12ohm resistor. Thus: $i_L(0^-) = i_L(0^+) = 2A$



For t→∞: the inductor current can be found using KCL at the node or through source transformation to simplify the circuit with the two resistances in parallel.
KCL: $0 = -2A + \frac{V_{12\Omega}}{12\Omega} + \frac{V_{12\Omega}-(-6V)}{8\Omega}$
solve for the node voltage: $V_{12\Omega} = 6V$
The current is: $i_L(\infty) = \frac{6V-(-6V)}{8\Omega} = 1.5A$



Now, the Thevenin resistance seen by the inductor can be found by turning off the independent source



and finding the 'seen' resistance by the inductor: 18 Ω

Building the solution: $i_L(t) = i_L(\infty) + [i_L(0+) - i_L(\infty)]exp\left(-\frac{t}{\frac{L}{R_T}}\right)$

$$i_L(t) = 1.5A + [2A - 1.5A]exp\left(-\frac{t}{\frac{10mH}{18\ \Omega}}\right)$$

$$i_L(t) = 1.5A + 0.5Aexp(-2000t)$$

To find the voltage at the current supply $v_s(t)$, we find its relationship to the inductor current using KCL:

$$0 = -2A + \frac{v_s(t)}{12\Omega} + i_L(t)$$

So the voltage is

$$v_s(t) = 24V - 12i_L(t) = 24 - 12[1.5 + 0.5exp(-2000t)]$$

$$v_s(t) = 6 - 6exp(-2000t)$$

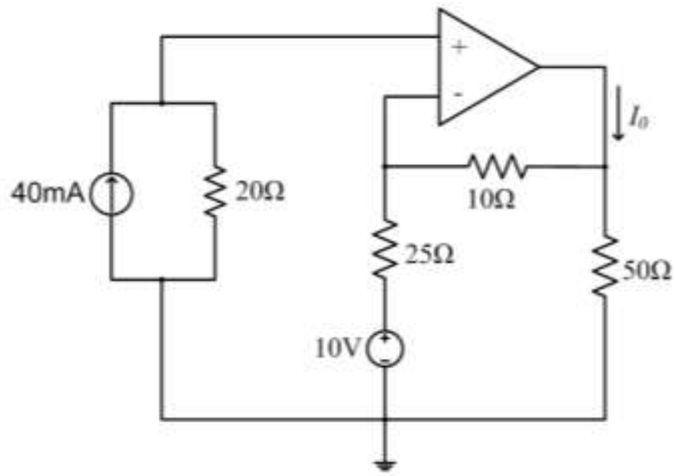
c) The energy stored in the inductor at t = 1ms is: $\frac{1}{2}Li_L^2$

$$i_L(1ms) = 1.5A + 0.5Aexp(-2000 \times 10^{-3}) = 1.57\ A$$

$$\frac{1}{2}Li_L^2 = \frac{1}{2} \cdot 10 \times 10^{-3} \cdot 1.57^{2}$$

$$U = 12.29\ mJ$$

2.3. Consider the circuit shown below. Assume that the op-amp is ideal. Answer the questions. [5pts]



(a) Find the power absorbed by the 50Ω load resistor.
(b) Find the op-amp output current labelled I_0?

a) Find the power by finding the voltage at the load resistor node (V_0).

i) voltage at the non-inverting node using KCL (ideal opamp):

$$i_{20\Omega} = 40mA;\ V_+ = 20\Omega \cdot i_{20\Omega} = 800mV = 0.8V$$

ii) Current through the 10ohm resistor using KCL (ideal opamp) at the inverting node:

$$0 = i_{25\Omega} + i_{10\Omega} = \frac{0.8V - 10V}{25\Omega} + \frac{0.8V - V_0}{10\Omega} = \frac{-9.2V}{25\Omega} + \frac{0.8V}{10\Omega} - \frac{V_0}{10\Omega}$$

$$V_0 = \frac{-9.2V}{25\Omega} \cdot 10\Omega + 0.8V = -3.68V + 0.8V = -2.88V$$

iii) Power absorbed: $P_{abs} = V_0 \cdot i_{50\Omega} = \frac{V_0^2}{50\Omega} = \frac{(-2.88V)^2}{50\Omega} = \frac{8.2944}{50} 0.1659W = 165.9mW$

$$P_{abs} = 165.9mW$$

b) Find the current I_0 using a KCL at the opamp output node:

$$0 = -I_0 + \frac{V_0 - V^-}{10\Omega} + \frac{V_0}{50\Omega}$$

$$I_0 = \frac{-2.88V - 0.8V}{10\Omega} + \frac{-2.88V}{50\Omega} = \frac{-3.68V}{10\Omega} + \frac{-2.88V}{50\Omega} = -0.368A - 0.0576A = -0.4256A$$

$$I_0 = -425.6mA$$