ECSE-200
Electric Circuits 1

February 6, 2019

Lecture 14

McGill

4. Circuit Theorems

- Source Transformations (5.2) Today's lecture

- Linearity and the Principle of Superposition (5.3)
- Thévenin's Theorem (5.4)
- Norton's Theorem (5.5)
- Maximum Power Transfer Theorem (5.6)

(subsections in Svoboda & Dorf reference textbook)

Motivation

- Circuit theorems can **greatly simplify** circuit analysis and provide insight into the operation of circuits

Practical Sources

It is commonly observed in practical sources (as opposed to ideal sources) that:

- power is dissipated when a load is attached to the source (for example, a battery warms up when discharged)
- the voltage from a practical voltage source decreases as current is drawn from the source
- the current from a practical current source decreases as voltage develops across the terminals of the source

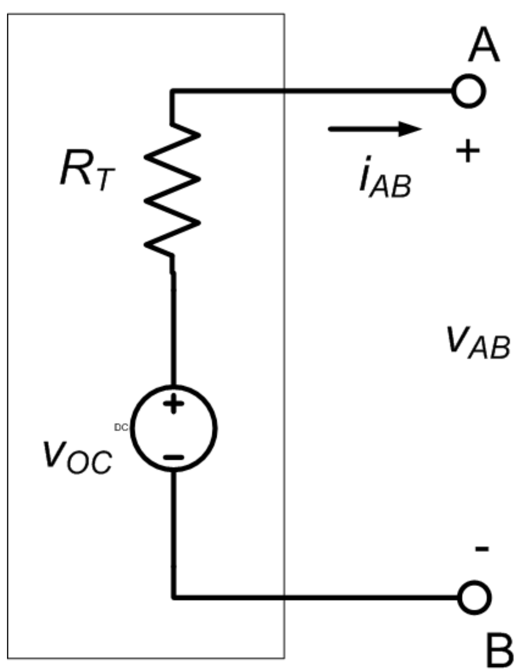
While the origin and nature of these effects can be very complex, the **Thévenin and Norton equivalent circuits** are **very useful models** for practical sources.

Practical Voltage Source

A practical voltage source can often be modeled as a **Thévenin circuit**:

→ a voltage source v_{OC} called the "**open circuit voltage**" in series with a resistance R_T, called the "**Thévenin resistance**".

An ideal voltage source is recovered as $R_T \rightarrow 0\Omega$ (corresponding to a short).

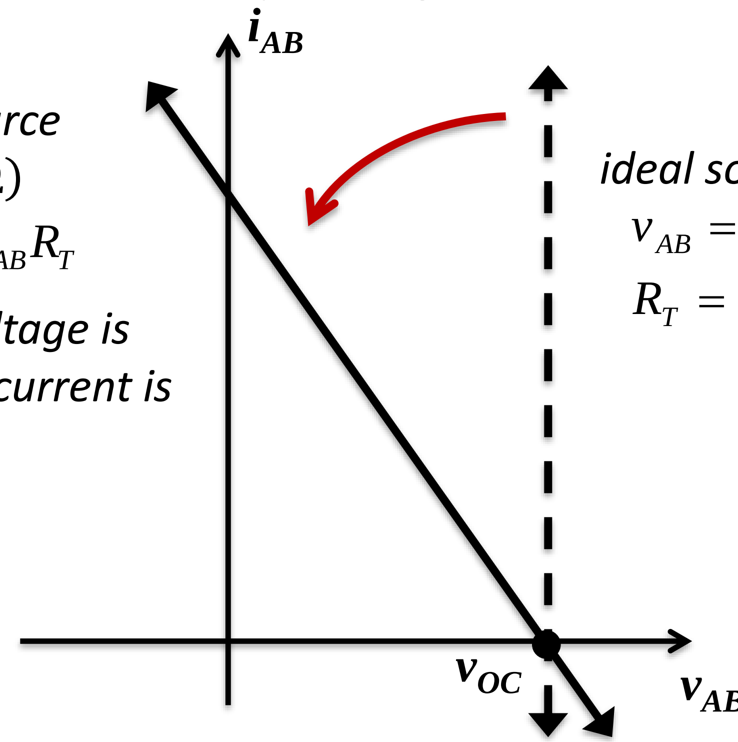


practical source

$(R_T \neq 0\Omega)$

$v_{AB} = v_{OC} - i_{AB} R_T$

terminal voltage is reduced as current is drawn



ideal source

$v_{AB} = v_{OC}$

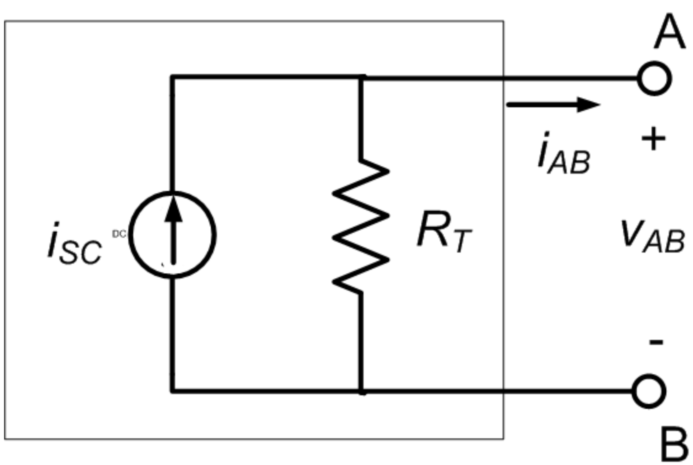
$R_T = 0\Omega$

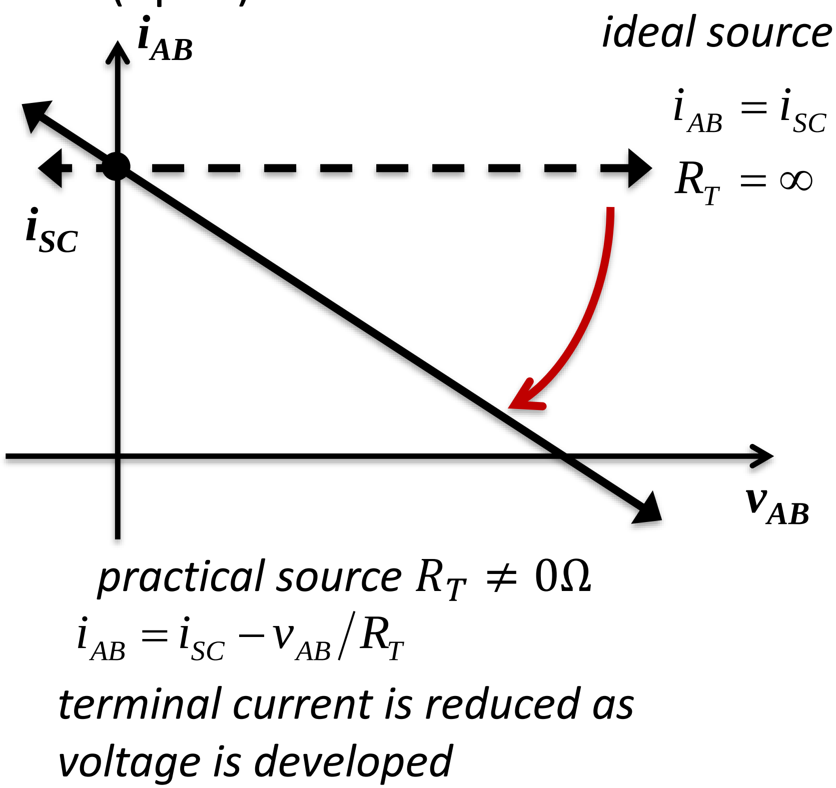
Practical Current Source

A practical current source can often be modeled as a **Norton circuit**:

→an independent current source i_{SC} called the "**short circuit current**" in parallel with a resistance R_T, called the "**Thévenin resistance.**"

An ideal current source is recovered as $R_T \to \infty$ (open).



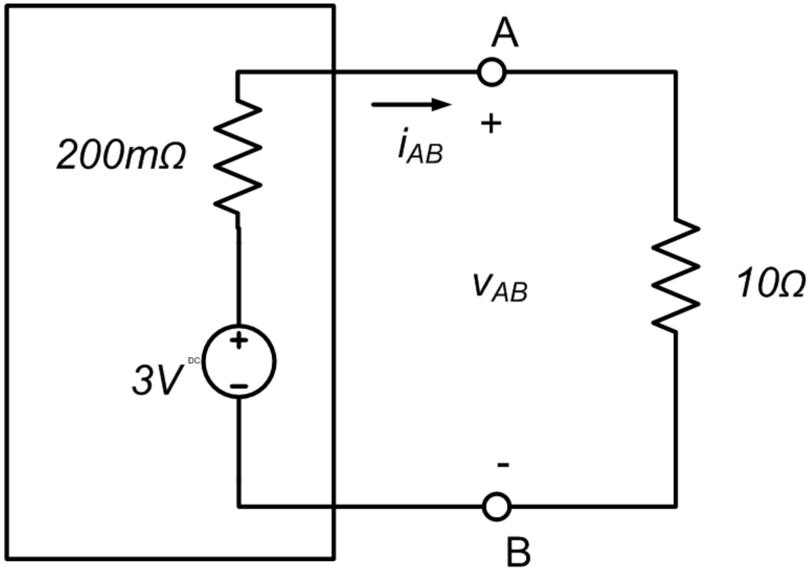


Example 1

A portable battery is characterized by an open circuit voltage of 3 V.

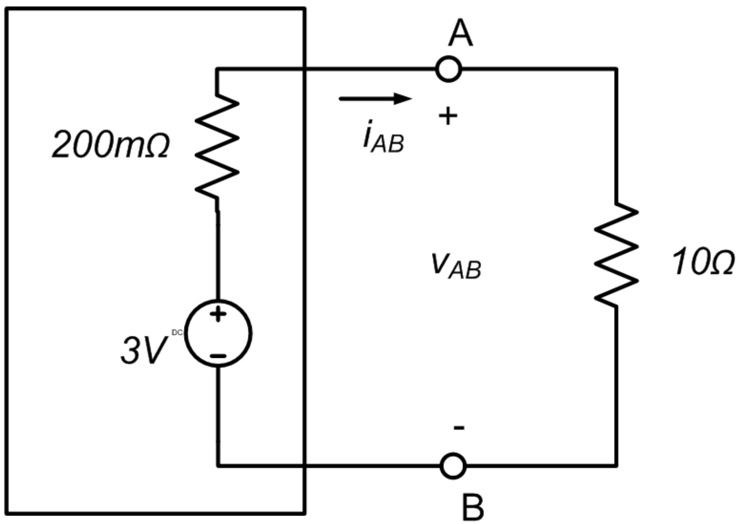
The internal (Thévenin) resistance is known to be 200 mΩ.

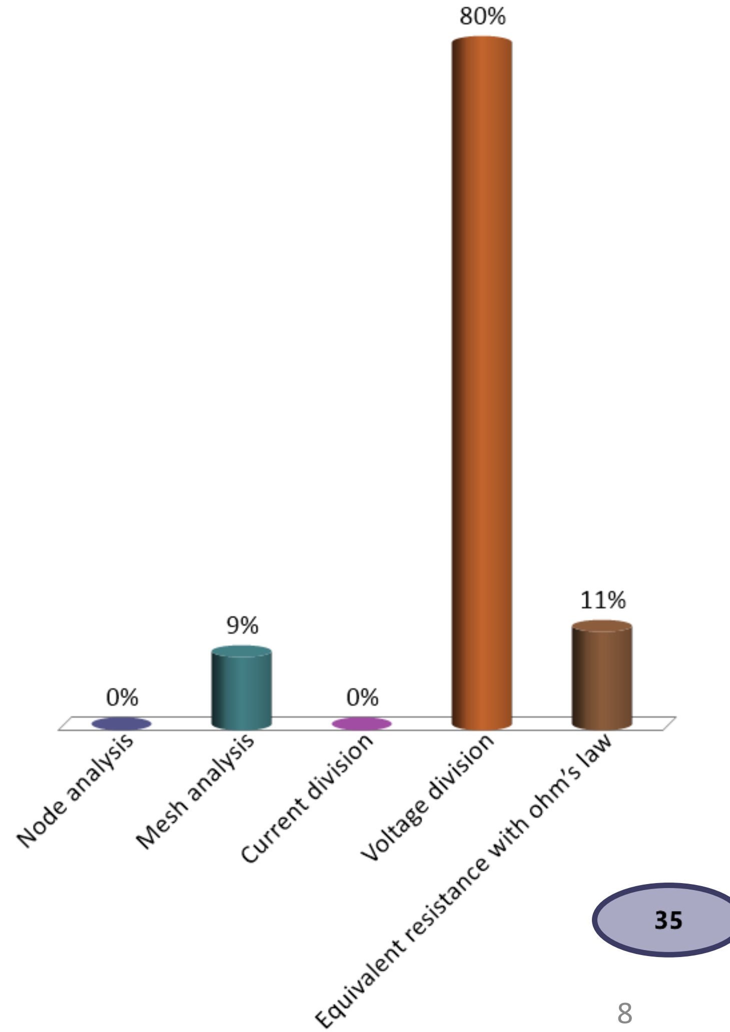
If a resistive load with 10 Ω equivalent resistance is attached to the battery, what voltage is applied to the load?



Which circuit analysis would you use to find the voltage to the load (i.e. v_{AB})?

A. Node analysis
B. Mesh analysis
C. Current division
D. Voltage division
E. Equivalent resistance with ohm's law





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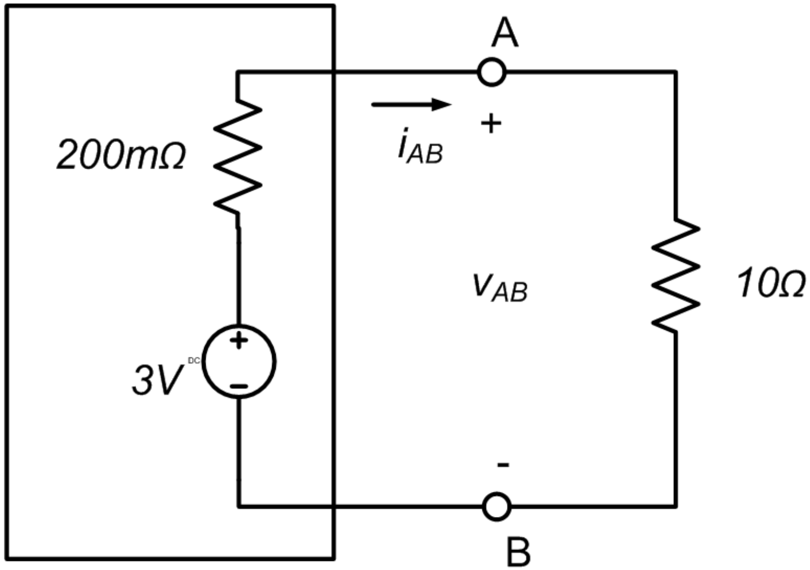
Example 1



Strategy:

- apply the voltage divider equation

Voltage divider:
$$\frac{v_{AB}}{3V} = \frac{10\Omega}{10\Omega + 0.2\Omega}$$
$$v_{AB} = 0.980 \cdot 3V$$
$$\boxed{v_{AB} = 2.94V}$$

this is a 2% drop in voltage

Example 2

An HVDC (high-voltage direct-current) power supply line is driven by a 800kV source with 2Ω internal resistance.

Of the total 5GW produced by the HVDC source, 10% of the power is lost in internal resistance, including a 3000km transmission line to the load.

What is the resistance of the load, the resistance of the transmission lines, and the voltage at the load?

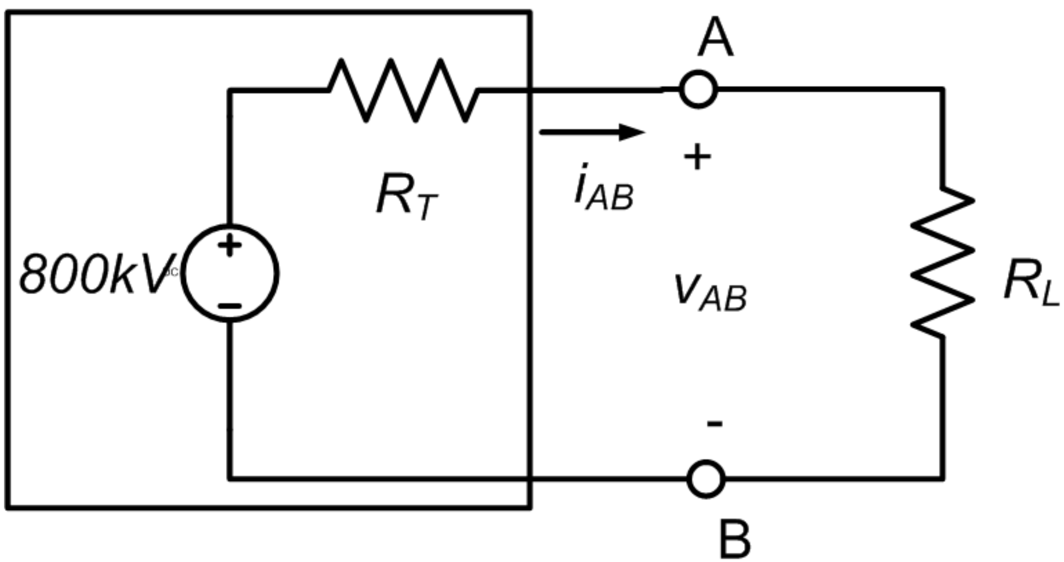
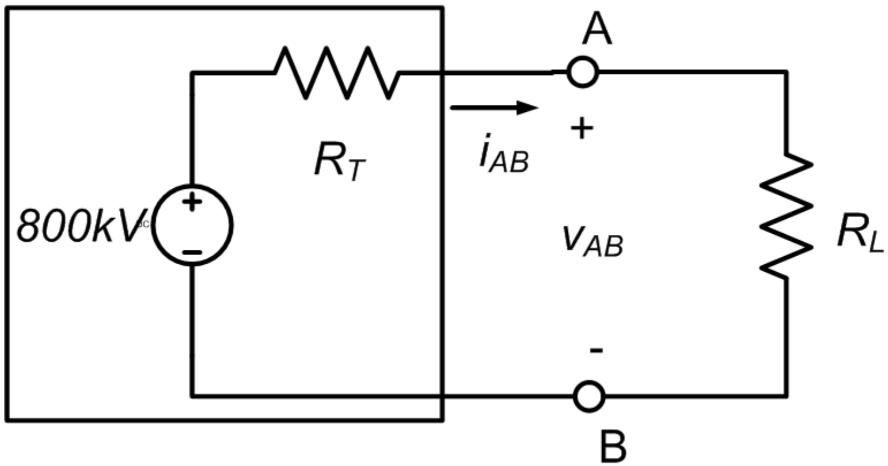


photo: www.alstom.com

Strategy:

- use power and voltage to find resistance

Example 2



From info given, find current, load and all internal resistances.

$$P_{del} = i_{AB} \cdot 800kV$$

$$i_{AB} = \frac{5GW}{800kV} = 6.25kA$$

$$P_{load} = i_{AB}^2 \cdot R_L$$

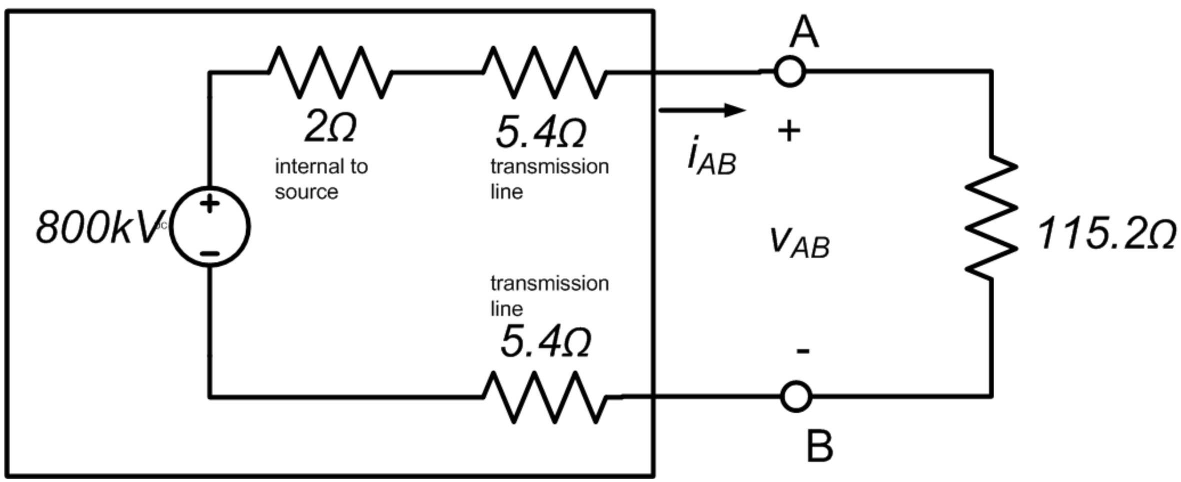
$$R_L = \frac{0.9 \times 5GW}{(6.25kA)^2} = 115.2\Omega$$

$$P_{internal} = i_{AB}^2 \cdot R_T$$

$$R_T = \frac{0.1 \times 5GW}{(6.25kA)^2} = 12.8\Omega$$

Example 2

Model of physical system adjusted based on value found:



Breakdown of internal resistance contributions to find resistance of transmission lines:

$$R_T = R_{source} + 2R_{line} = 12.8\Omega$$

$$R_T = 2\Omega + 2R_{line} == 12.8\Omega$$

$$R_{line} = \frac{12.8 - 2\Omega}{2} = 5.4\Omega$$

voltage divider to find voltage at the load:

$$\frac{v_{AB}}{800kV} = \frac{115.2\Omega}{12.8\Omega + 115.2\Omega}$$

$$\boxed{v_{AB} = 0.900 \cdot 800kV = 720kV}$$

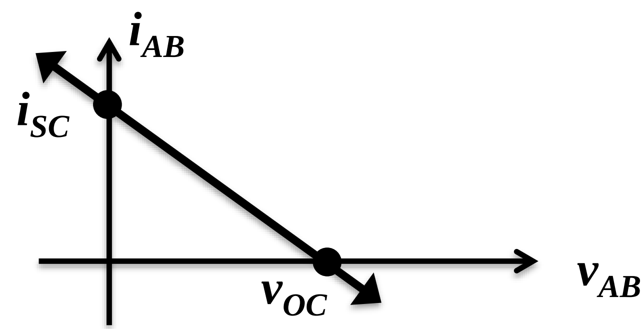
this is a 10% drop in voltage

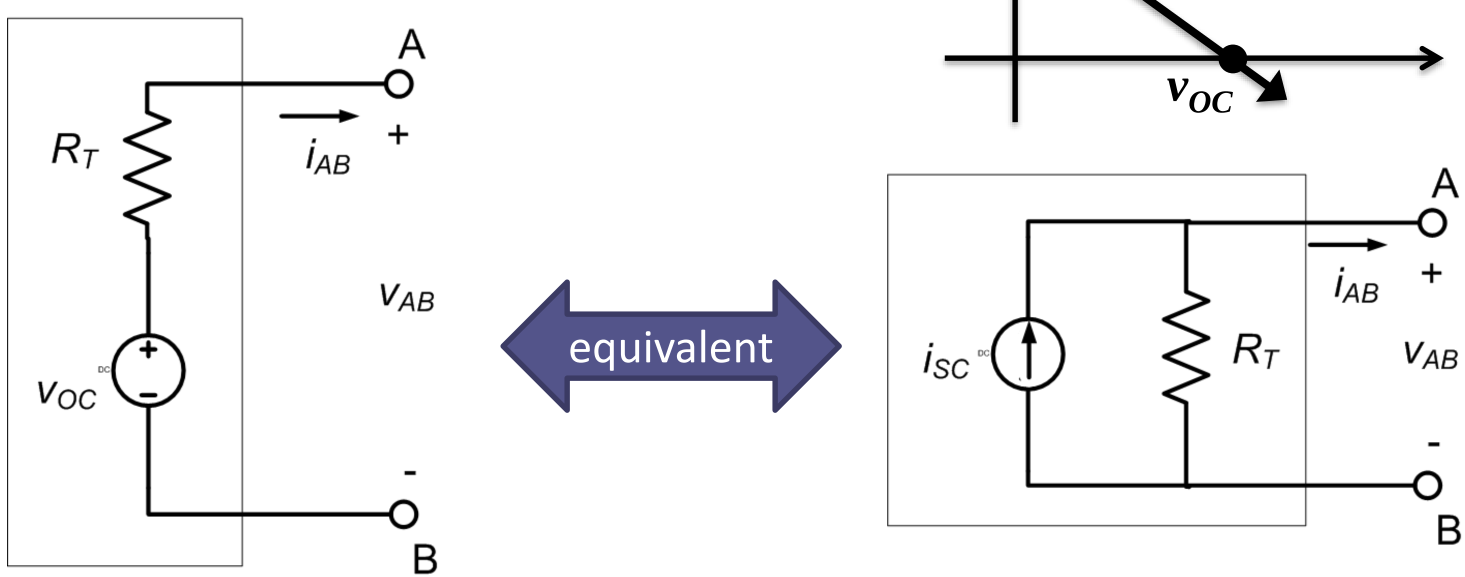
Source Transformation

Source Transformation:

A Thévenin circuit and a Norton circuit are actually equivalent when their i_{AB}-v_{AB} diagrams are identical!

$$\boxed{v_{OC} = i_{SC} R_T}$$

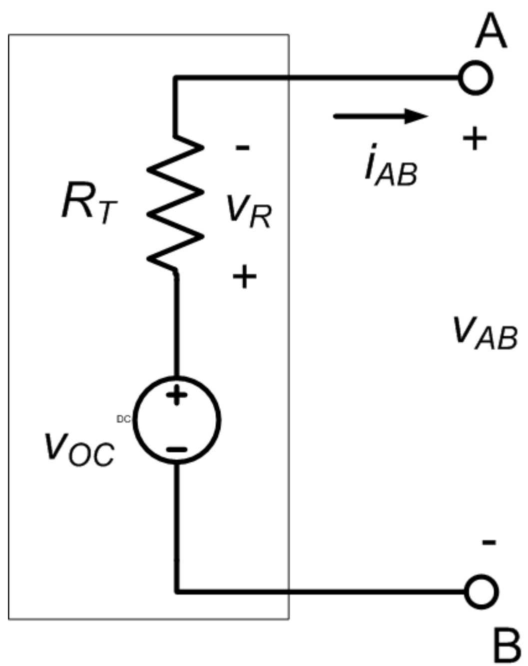




Source Transformation

Proof: Show that the terminal equations relating v_{AB} and i_{AB} are identical for appropriately chosen component values.

In other words, show that the i_{AB}-v_{AB} diagrams are identical:



KVL: $0 = -v_{OC} + v_R + v_{AB}$

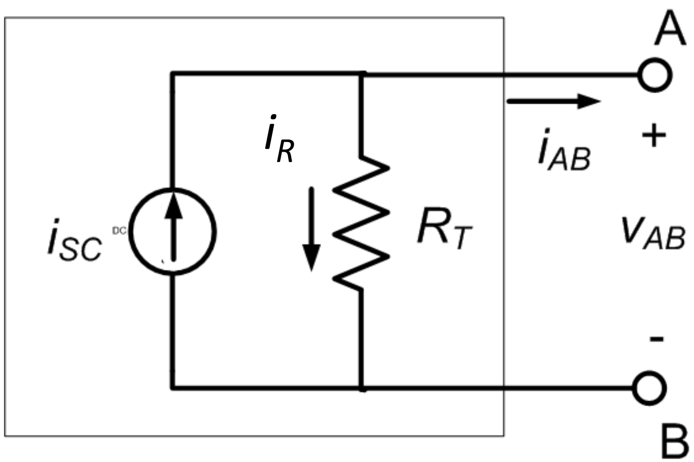
Ohm: $v_R = i_{AB} R_T$

Combining the above:

$$v_{AB} = v_{OC} - i_{AB} R_T$$

Source Transformation

Next, analyze the Norton circuit:



KCL: $0 = -i_{SC} + i_R + i_{AB}$

Ohm: $i_R = v_{AB}/R_T$

Combining the above:

$$i_{AB} = i_{SC} - v_{AB}/R_T$$

Comparing the two circuit terminal laws:

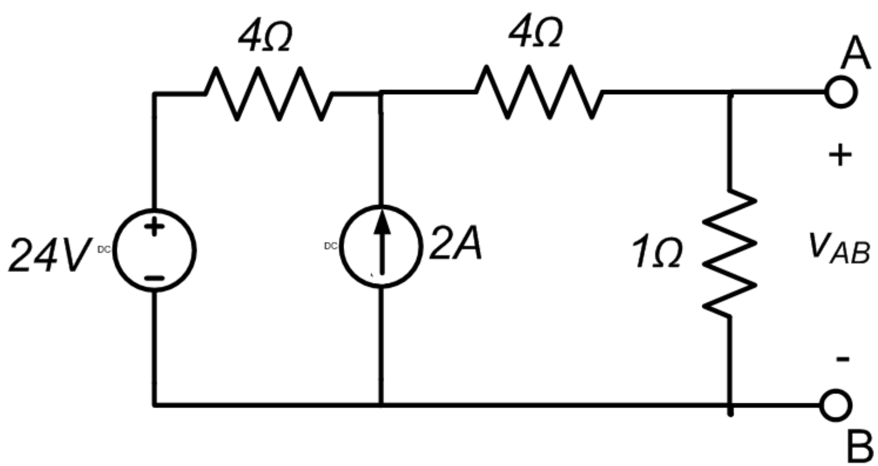
$$v_{AB} = \boxed{v_{OC}} - i_{AB}R_T$$

$$i_{AB} = i_{SC} - v_{AB}/R_T \rightarrow v_{AB} = \boxed{i_{SC}R_T} - i_{AB}R_T$$

The two circuits are thus equivalent when: $\boxed{v_{OC} = i_{SC}R_T}$

Example

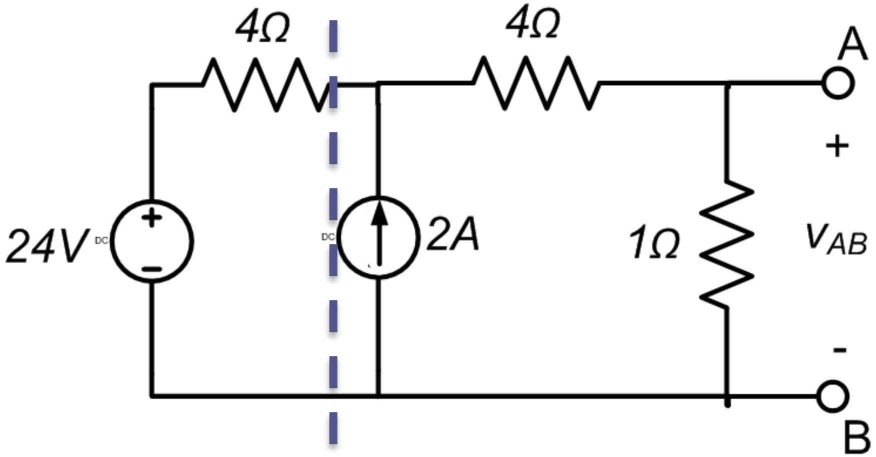
Reduce the following circuit to a single **Norton equivalent circuit** with respect to the terminals A and B.

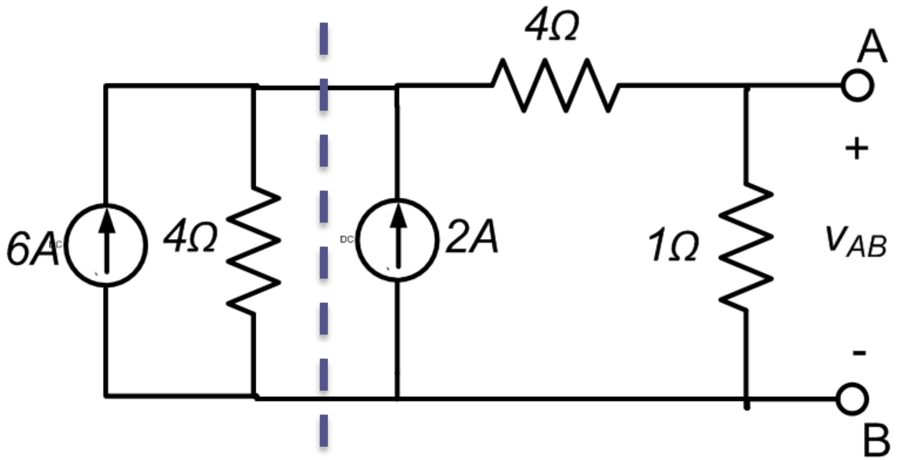


Strategy:

- use transformations between Thévenin and Norton equivalent circuits, working from left to right

Example

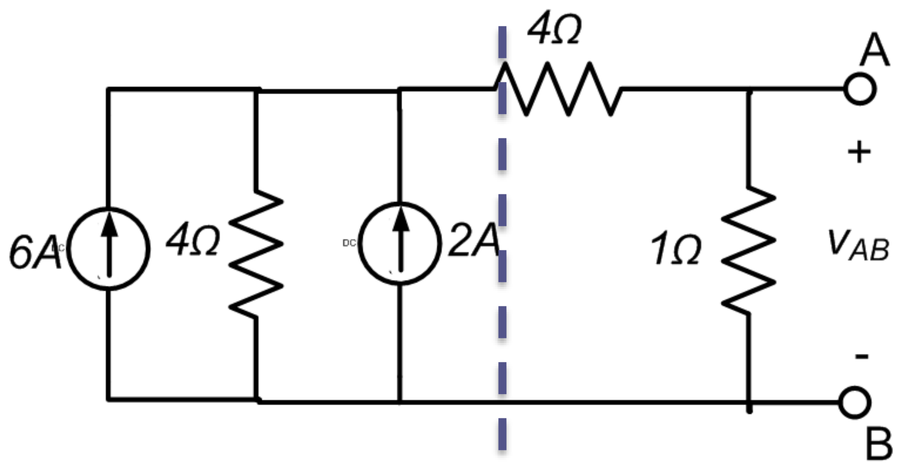




Transform Thévenin circuit on the left into a Norton circuit.

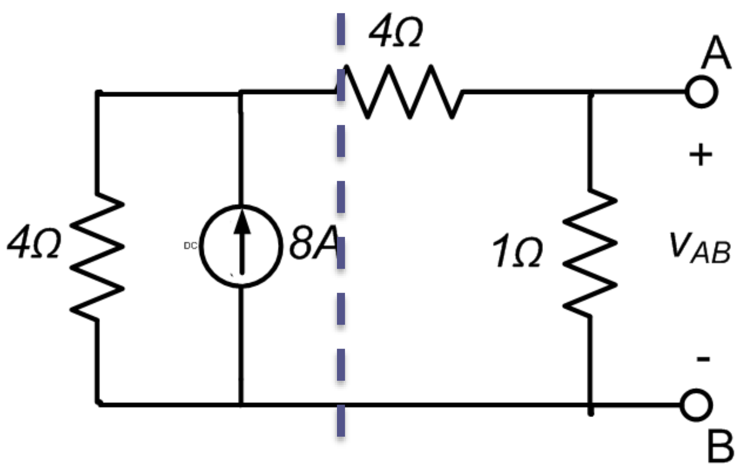
$$
\begin{aligned}
i_{SC} &= v_{OC}/R_T \\
&= 24V/4\Omega \\
&= 6A
\end{aligned}
$$

Example

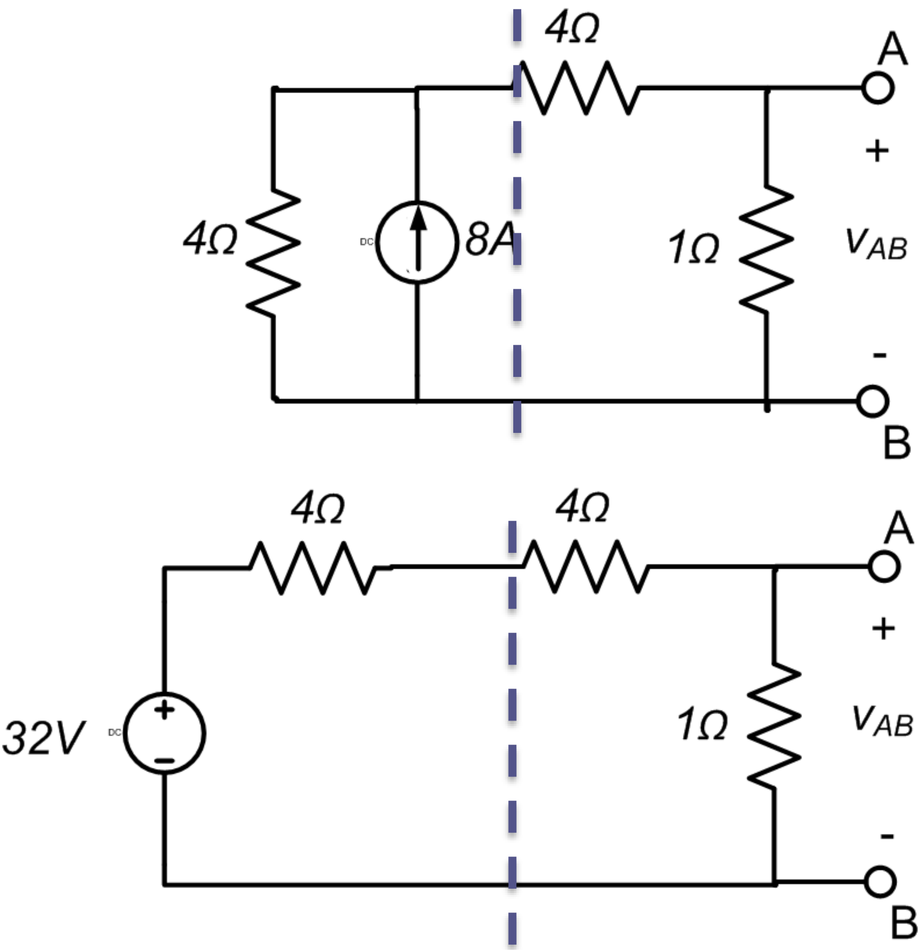


Transform circuit
on the left into
a Norton circuit.

$$i_{SC} = 2A + 6A = 8A$$



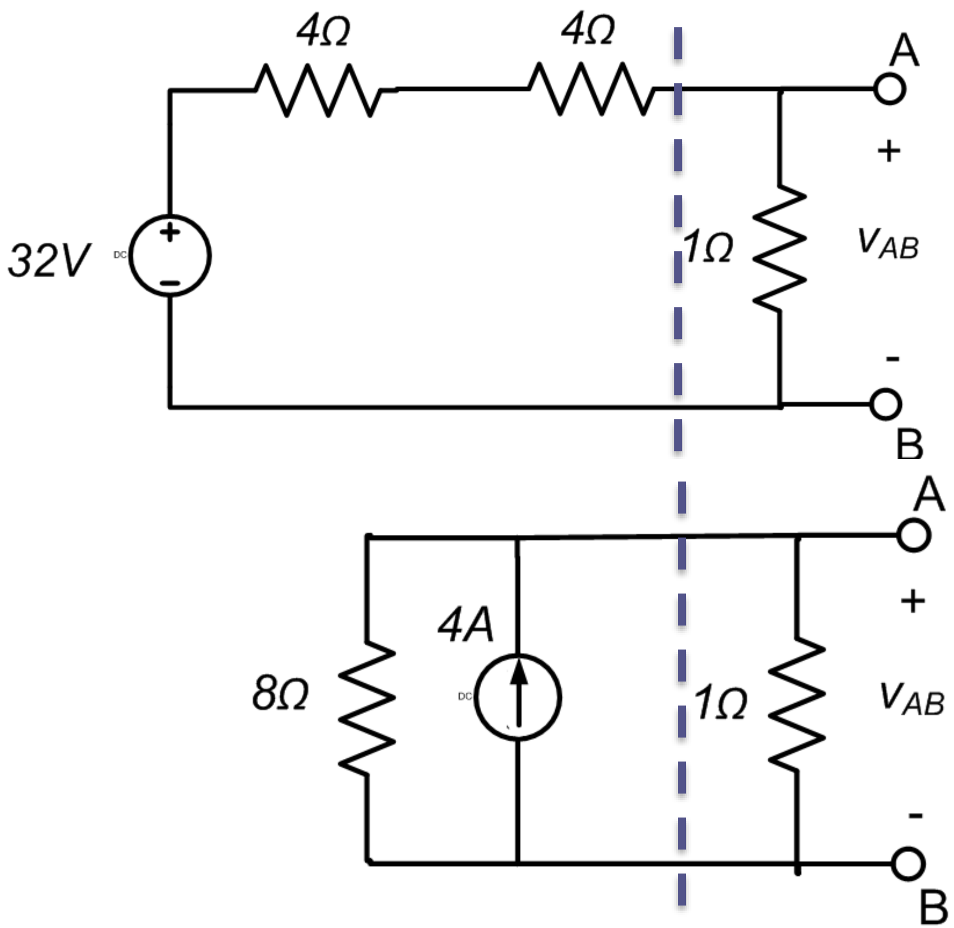
Example



Transform circuit on the left into a Thévenin circuit.

$$
\begin{aligned}
v_{OC} &= i_{SC} R_T \\
&= 8A \cdot 4\Omega \\
&= 32V
\end{aligned}
$$

Example



Use series equivalent resistance and transform circuit on the left into a Norton circuit

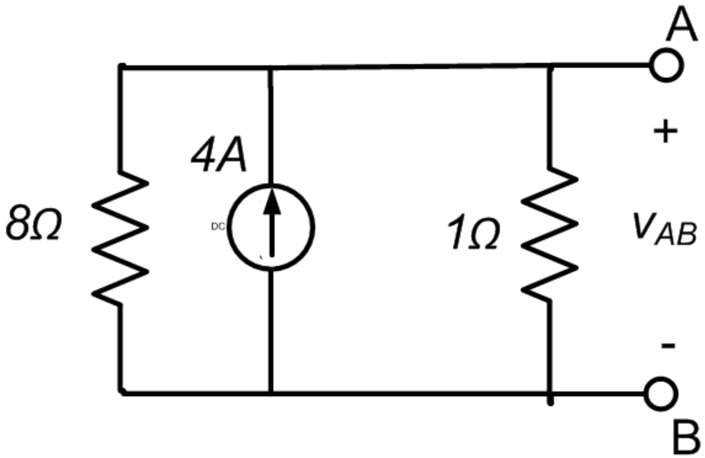
$$R_T = 4\Omega + 4\Omega = 8\Omega$$

$$i_{SC} = v_{OC}/R_T$$

$$= 32V/8\Omega$$

$$= 4A$$

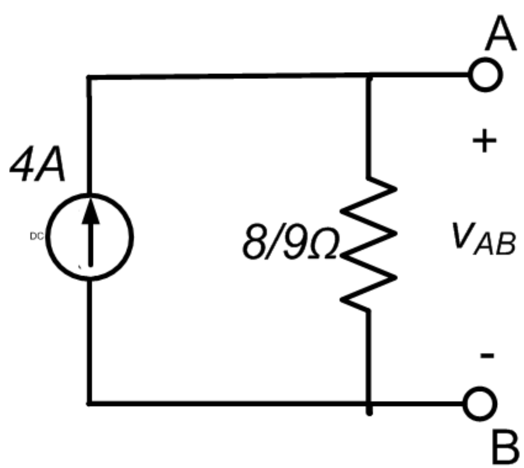
Example

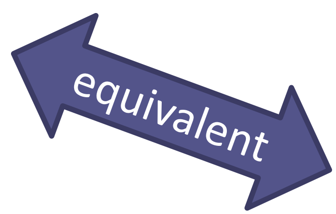


Use parallel equivalent resistance to create a Norton circuit.

$$R_T = 8\Omega \,\|\, 1\Omega$$

$$R_T = \frac{8\Omega \cdot 1\Omega}{8\Omega + 1\Omega} = \frac{8}{9}\Omega$$





Circuit at the beginning: