# Electric Circuits 1

## ECSE-200 Section: 1

**15 December 2014, 9:00AM**

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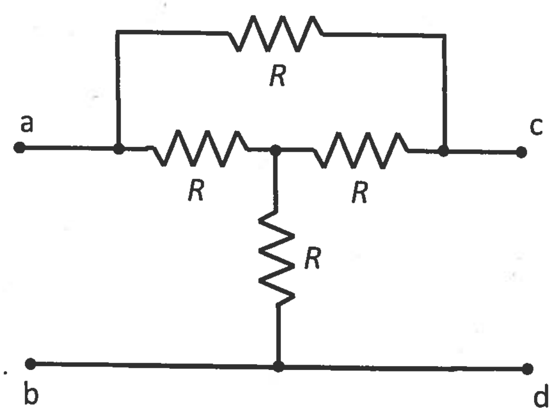
INSTRUCTIONS:

- This is a **CLOSED BOOK** examination.
- **NO CRIB SHEETS** are permitted.
- Provide your answers in an **EXAM BOOKLET.**
- **STANDARD CALCULATOR** permitted ONLY.
- This examination consists of 4 questions, with a total of 6 pages, including the cover page.
- This examination is **PRINTED ON BOTH SIDES** of the paper

1. Consider the circuit below. Answer the questions. [12 pts]




a) What is the definition of a passive element? [1pt]

b) What is the definition of a linear element? [1pt]

c) What is the physical law at the origin of Kirchoff's current law? [1pt]

d) What is the physical law at the origin of Kirchoff's voltage law? [1pt]

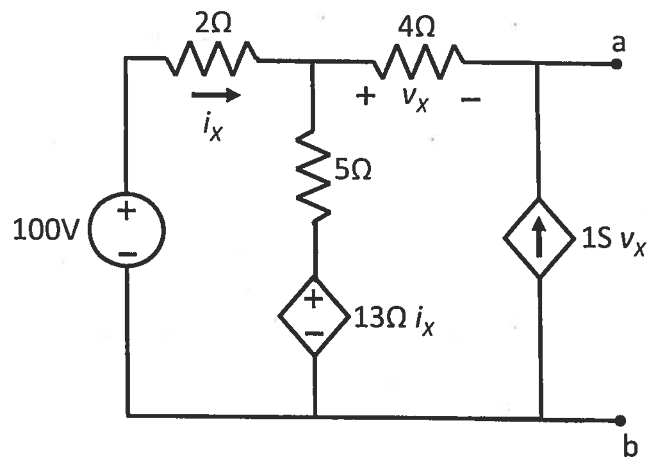
e) What is the equivalent resistance between terminals a and b, if there is an open circuit between the terminals c and d ? [2pts]

f) What is the equivalent resistance between terminals a and b, if there is a short circuit attached between the terminals c and d ? [2pts]

g) What is the equivalent resistance between terminals a and b, if a resistance $R$ is attached between the terminals c and d ? [2pts]

h) What is the equivalent resistance between terminals a and b, if a resistance $2R$ is attached between the terminals c and d ? [2pts]

2. Consider the circuit below. Answer the questions. [12 pts]



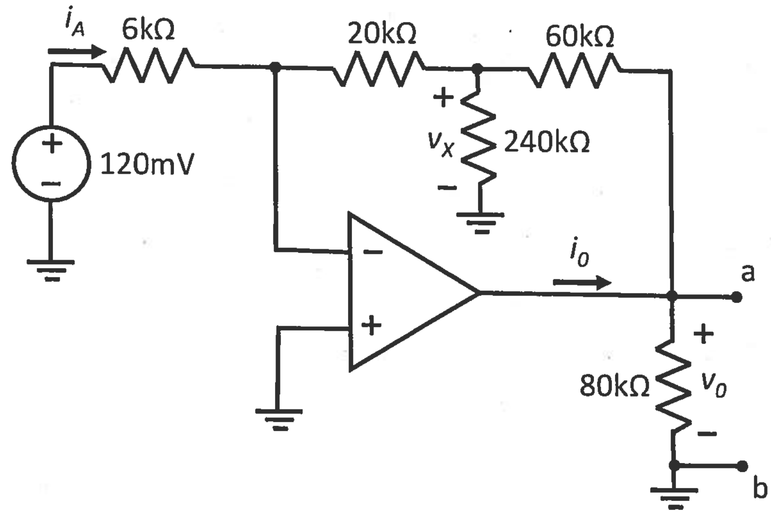

a) What is Thévenin's theorem? [1pt]

b) Draw the Thévenin equivalent circuit with respect to terminals a and b. Be sure to label the terminals a and b in your diagram. [5pts]

c) What is the maximum power that can be delivered to an optimally chosen load resistor attached to the terminals a and b? [2pts]

d) A load resistor $R$ is attached to the terminals a and b. What are the two values of $R$ that will cause a power of 1kW to be absorbed by $R$? [4pts]

3. Consider the circuit below. Assume ideal op-amp behaviour. Answer the questions. [12 pts]




a) Give one reason why negative feedback is used in op-amp circuits. [2pts]

b) What is the current $i_A$ ? [2pts]

c) What is the voltage $v_X$ ? [2pts]

d) What is the voltage $v_O$ ? [1pt]

e) What is the current $i_O$ ? [1pt]

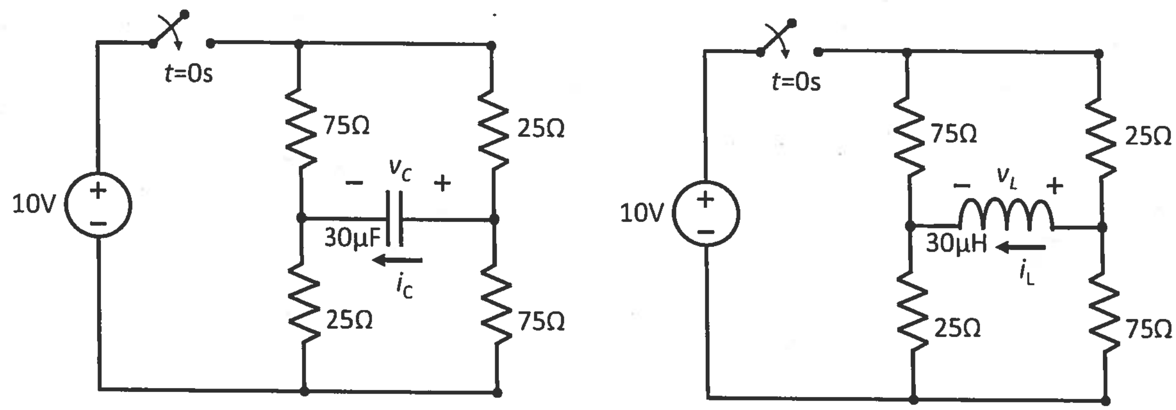
f) How much power does the op-amp deliver? [2pts]

For parts g) and h), a voltmeter with 80kΩ internal resistance is connected to the terminals a and b.

g) What is the voltage measured by the voltmeter? [1pt]

h) How much power does the op-amp deliver? [1pt]

4. Consider the circuits below. The switches are open for $t < 0s$, and close instantaneously at $t = 0s$. Assume dc steady state behaviour for $t < 0$. Answer the questions. [12 pts]




a) What is the voltage $v_C(t)$ for $t > 0$? Plot your solution for $v_C(t)$ versus $t$. Label your axes. [5pts]

b) What is the current $i_L(t)$ for $t > 0$? Plot your solution for $i_L(t)$ versus $t$. Label your axes. [5pts]

c) What is the maximum power absorbed by the capacitor? [1pt]

d) What is the maximum power absorbed by the inductor? [1pt]

end

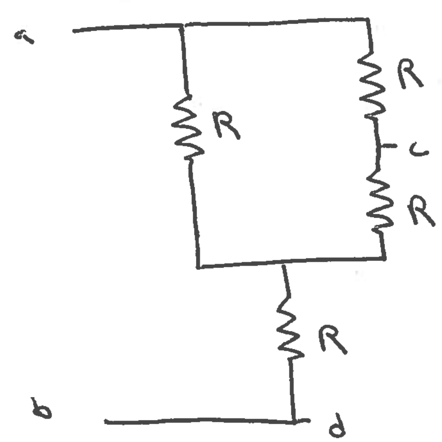
1 a) An element that never delivers more energy to a circuit than it has absorbed from a circuit. [+1]

b) An element where terminal current and voltage are related by a linear function or linear operator. [+1]

c) Conservation of charge. [+1]

d) Conservation of energy. [+1]

e)

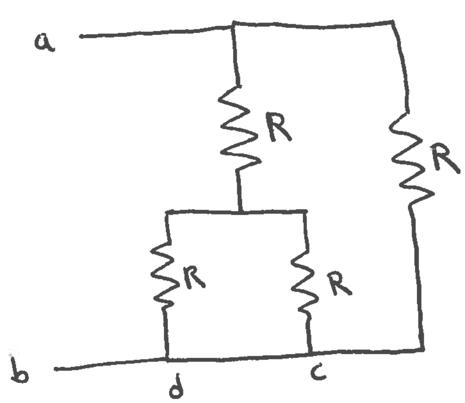



$$R_{ab} = R // (R+R) + R \quad [+1]$$

$$= \frac{R \cdot 2R}{R + 2R} + R$$

$$= \frac{5}{3} R \quad [+1]$$

f)

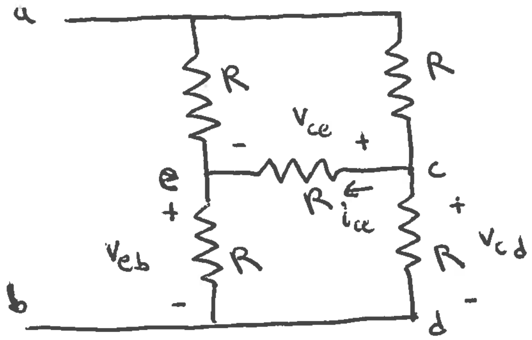



$$R_{ab} = (R + R//R) // R \quad [+1]$$

$$= \frac{\frac{3}{2} R \cdot R}{\frac{3}{2} R + R}$$

$$= \frac{3}{5} R \quad [+1]$$

g)



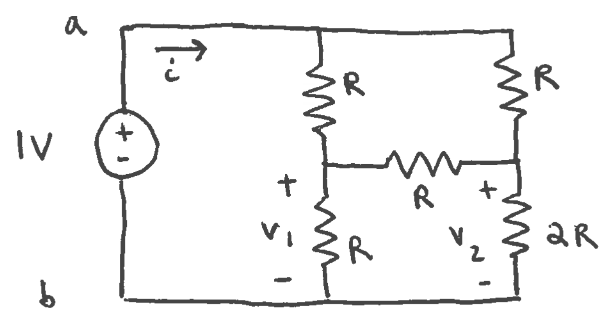

By the symmetry of the network, $v_{cd} = v_{eb}$.

Thus, $v_{ce} = 0\,V$ by KVL. [+1]

Thus, $i_{ce} = 0\,A$ by Ohm.

$$R_{ab} = (R+R) \,//\, (R+R)$$
$$= 2R \,//\, 2R$$
$$= R \quad [+1]$$

h)




Apply test source. [+1]

$$0 = \frac{v_1}{R} + \frac{v_1 - v_2}{R} + \frac{v_1 - 1V}{R}$$

$$0 = \frac{v_2}{2R} + \frac{v_2 - v_1}{R} + \frac{v_2 - 1V}{R}$$

$$1 = 3v_1 - v_2$$
$$1 = -v_1 + 2.5 v_2$$

$$v_1 = \frac{\begin{vmatrix} 1 & -1 \\ 1 & 2.5 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ -1 & 2.5 \end{vmatrix}} = \frac{7}{13} V$$

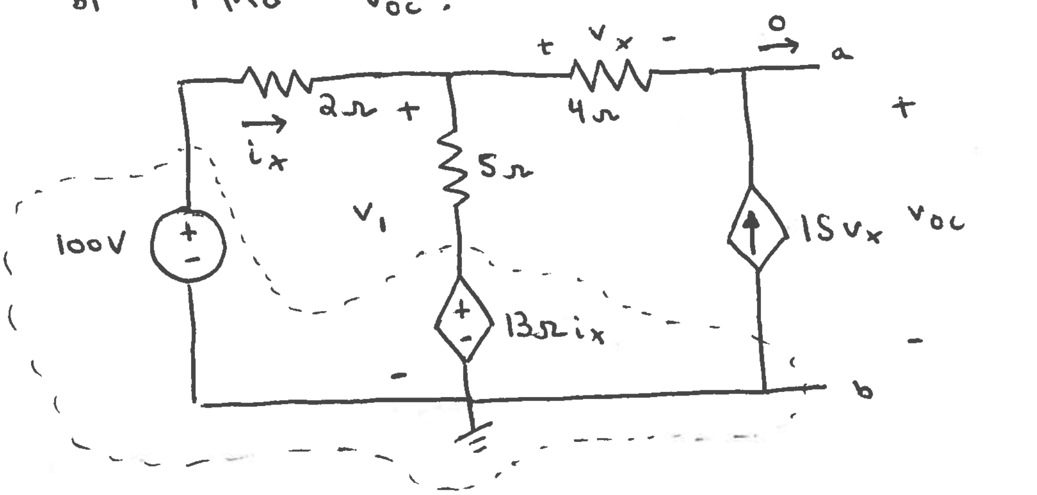
$$v_2 = \frac{\begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ -1 & 2.5 \end{vmatrix}} = \frac{8}{13} V$$

$$i = \frac{v_1}{R} + \frac{v_2}{2R} = \frac{11/13\ V}{R}$$

$$R_{ab} = \frac{1V}{i} = \frac{13}{11} R \quad [+1]$$

2 a) Any circuit with two terminals composed of resistors, dependent sources and independent sources is equivalent to a series combination of one resistor and one independent voltage source. [+1]

b) Find $V_{oc}$. [+1]




$$0 = \frac{v_1 - 100V}{2\Omega} + \frac{v_1 - 13\Omega i_x}{5\Omega} - 15 v_x$$

$$i_x = \frac{100V - v_1}{2\Omega}$$

$$\frac{v_x}{4\Omega} = -15 v_x \rightarrow v_x = 0V$$

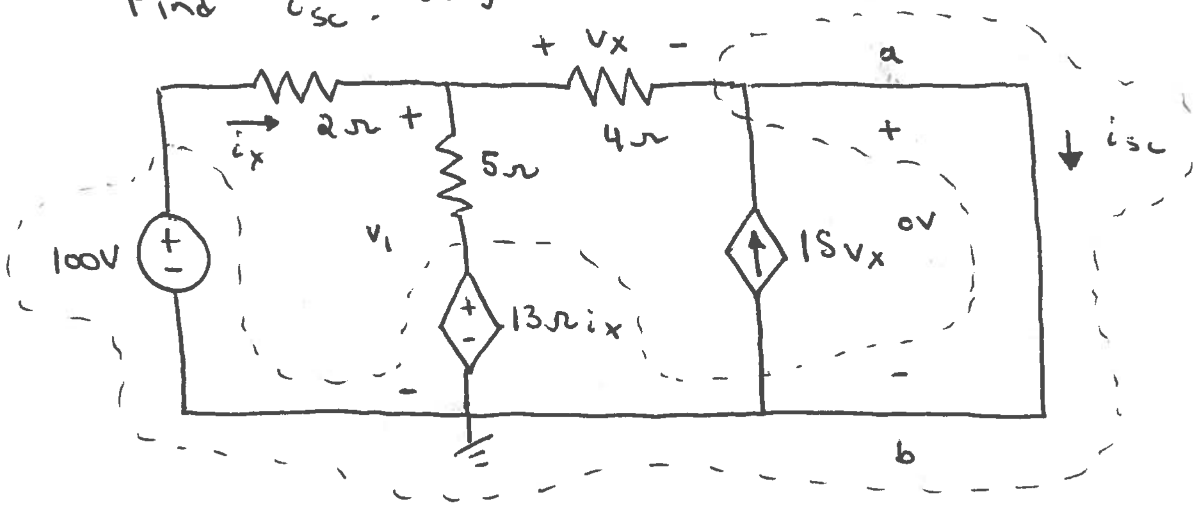
$$\therefore \quad 0 = \frac{v_1 - 100V}{2\Omega} + \frac{v_1 - 13\Omega\left(\frac{100V - v_1}{2\Omega}\right)}{5\Omega} + 0$$

$$\frac{100}{2} + \frac{13 \cdot 100}{2 \cdot 5} = \left(\frac{1}{2} + \frac{1}{5} + \frac{13}{2 \cdot 5}\right) v_1$$

$$v_1 = 90V$$

$$V_{oc} = v_1 - v_x = 90V$$

Find $i_{sc}$. [+1]




$$0 = \frac{v_1 - 100V}{2\Omega} + \frac{v_1 - 13\Omega i_x}{5\Omega} + \frac{v_1}{4\Omega}$$

$$i_x = \frac{100V - v_1}{2\Omega}$$

$$\therefore \quad 0 = \frac{v_1 - 100V}{2\Omega} + \frac{v_1 - 13\Omega\left(\frac{100V - v_1}{2\Omega}\right)}{5\Omega} + \frac{v_1}{4\Omega}$$

$$\frac{100}{2} + \frac{13 \cdot 100}{2 \cdot 5} = \left(\frac{1}{2} + \frac{1}{5} + \frac{13}{2 \cdot 5} + \frac{1}{4}\right) v_1$$

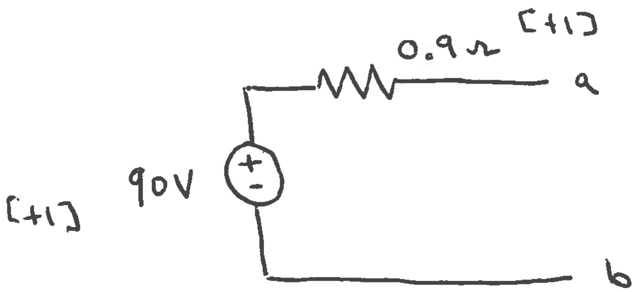
$$v_1 = 80V$$

$$V_x = v_1 = 80V$$

$$i_{sc} = \frac{V_x}{4\Omega} + 15\,V_x = 100\,A$$

$$R_T = \frac{V_{oc}}{i_{sc}} \quad [+1]$$

$$= \frac{90V}{100A} = 0.9\Omega$$

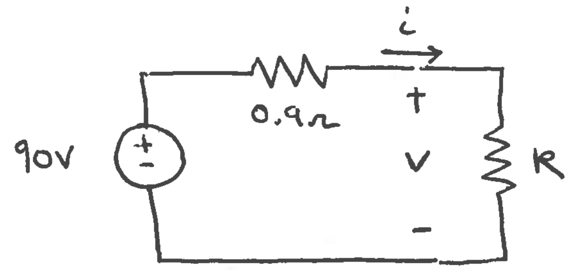



c) $$P_{max} = \frac{V_{oc}}{2} \cdot \frac{i_{sc}}{2} \quad [+1]$$

$$= \frac{90V \cdot 100A}{4}$$

$$= 2.250\ kW \quad [+1]$$

d)




[+1] for diagram

$$v = 90V - 0.9\Omega \cdot i$$

$$vi = 1000\ W \quad [+1]$$

$$\therefore \quad v = 90 - 0.9\left(\frac{1000}{v}\right)$$

$$v^2 = 90v - 900$$

$$0 = v^2 - 90v + 900$$

$$v = \frac{90 \pm \sqrt{(90)^2 - 4 \cdot 1 \cdot 900}}{2} = 78.54\ V,\ 11.46\ V$$

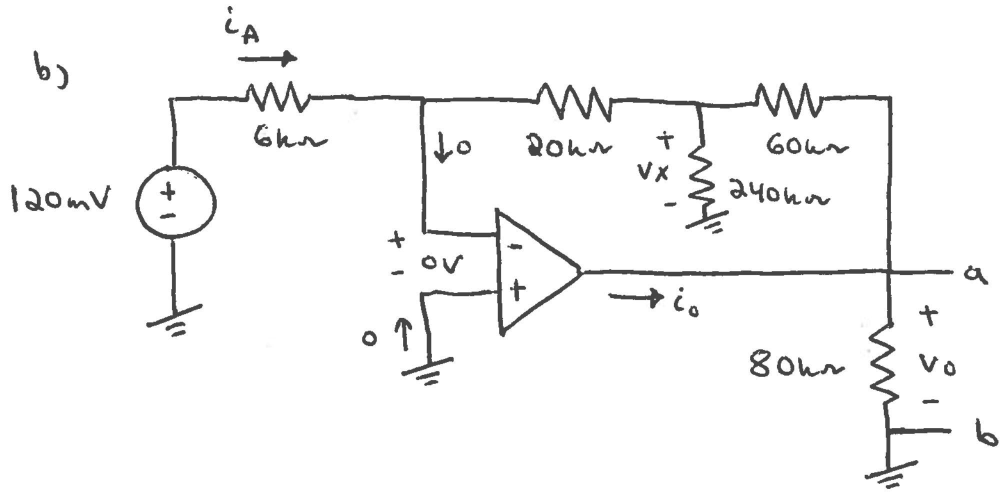
$$i = 1000W / v = 12.73\ A,\ 87.27\ A$$

$$R = v/i = 6.17\ \Omega,\ 0.131\ \Omega \quad [+2]$$

a) stability [+1] for any acceptable answer

programmable gain

b)




$$i_A = \frac{120\,mV}{6\,k\Omega} = 20\,\mu A \quad [+2]$$

c)

$$0 = -i_A + 0 + \frac{0 - v_x}{20\,k\Omega} \quad [+1]$$

$$v_x = -20\,k\Omega \cdot i_A$$

$$= -400\,mV \quad [+1]$$

d)

$$0 = \frac{v_x}{20\,k\Omega} + \frac{v_x}{240\,k\Omega} + \frac{v_x - v_o}{60\,k\Omega}$$

$$v_o = 60\,k\Omega \cdot \left( \frac{1}{20\,k\Omega} + \frac{1}{240\,k\Omega} + \frac{1}{60\,k\Omega} \right) v_x$$

$$= -1.7\ V \quad [+1]$$

e) $$0 = -i_o + \frac{v_o}{80k\Omega} + \frac{v_o - v_x}{60k\Omega}$$

$i_o = -42.92\ \mu A$ [+1]

f) $P_{del} = v_o \cdot i_o$ [+1]

$= (-1.7\,V)\cdot(-42.92\,\mu A)$

$= 72.96\ \mu W$ [+1]
delivered

g) $v_o' = -1.7\,V$ [+1]

h) $$0 = -i_o' + \frac{v_o'}{80k\Omega} + \frac{v_o'}{80k\Omega} + \frac{v_o' - v_x'}{60k\Omega}$$

$v_o' = v_o \qquad v_x' = v_x$

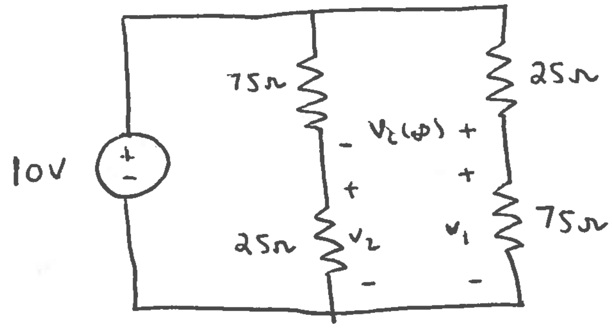
$i_o' = -64.17\ \mu A$

$P_{del}' = v_o' \cdot i_o'$

$= 109.1\ \mu W$ [+1]

4 a) $t = 0$ $\qquad V_c(0+) = V_c(0-) = 0\,V$ [+1]

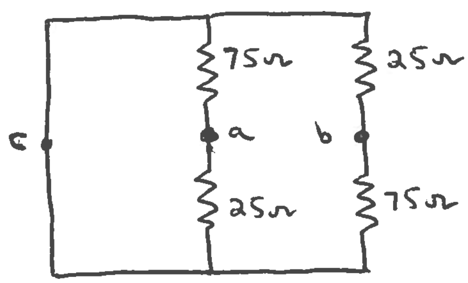
$t \to \infty$

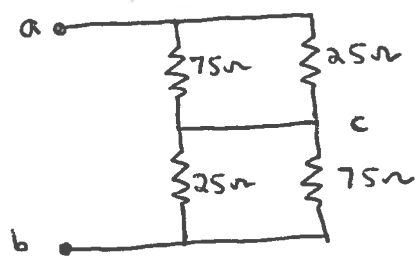



$$v_1 = \frac{10V \cdot 75\Omega}{100\Omega} = 7.5V \qquad v_2 = \frac{10V \cdot 25\Omega}{100\Omega} = 2.5V$$

$$V_c(\infty) = v_1 - v_2 = 5\,V \quad [+1]$$

Find $\tau$.



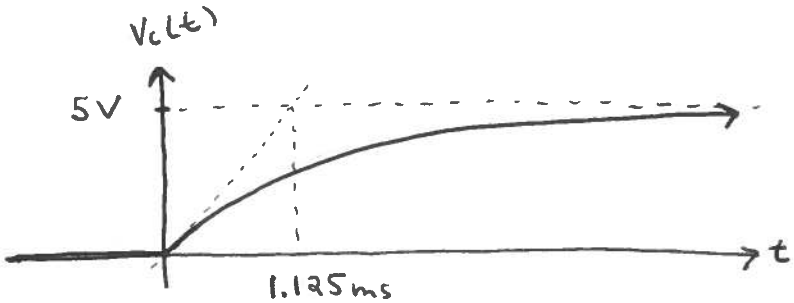




$$R_{TH} = 25\Omega \| 75\Omega + 25\Omega \| 75\Omega$$
$$= 37.5\Omega$$

$$\tau = R_{TH} \cdot C = 37.5\Omega \cdot 30\mu F = 1.125\ ms \quad [+1]$$

$t > 0$ $\qquad V_c(t) = V_c(\infty) + [V_c(0+) - V_c(\infty)]\exp(-t/\tau)$

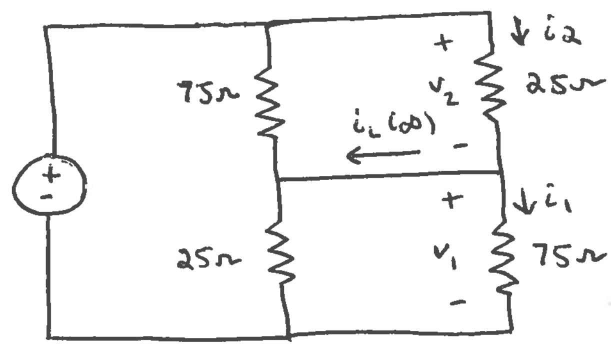
$$= 5V\,[1 - \exp(-t/1.125ms)] \quad [+1]$$




[+1]

b) $t=0$ $\quad i_L(0+) = i_L(0-) = 0$ A [+1]

$t \to \infty$




$$v_1 = \frac{10V \cdot 25\Omega \| 75\Omega}{25\Omega\|75\Omega + 25\Omega\|75\Omega} = 5V$$

$$v_2 = \frac{10V \cdot 25\Omega \| 75\Omega}{25\Omega\|75\Omega + 25\Omega\|75\Omega} = 5V$$

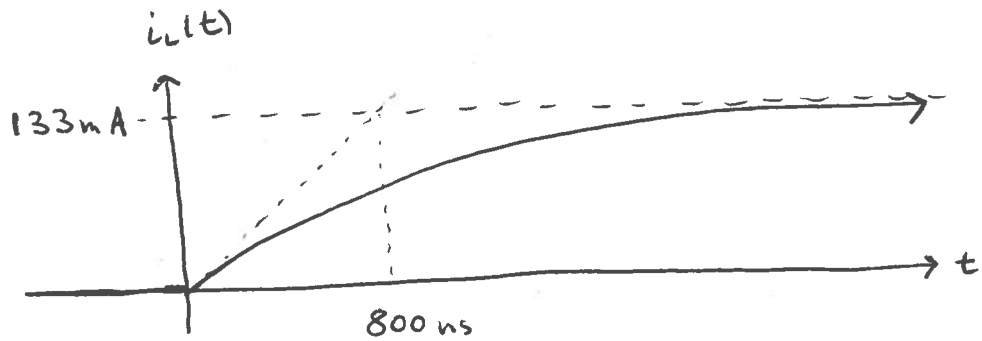
$i_2 = 5V/25\Omega = 0.2A$ $\qquad i_1 = 5V/75\Omega = 0.0667$ A

$i_L(\infty) = i_2 - i_1 = 0.1333$ A [+1]

$\tau = L / R_{TH} = 30\mu H / 37.5\Omega = 800$ ns [+1]

$t>0$ $\quad i_L(t) = i_L(\infty) + [i_L(0+) - i_L(\infty)] \exp(-t/\tau)$

$= 133mA\,[1 - \exp(-t/800ns)]$ [+1]

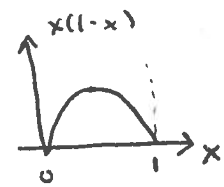



c) $$i_c = C\frac{dv_c}{dt} = 30\mu F \cdot (-5V)\left(\frac{-1}{1.125ms}\right)\exp(-t/1.125ms)$$

$$= 133\ mA\ \exp(-t/1.125ms)$$

$$p_{abs} = v_c \cdot i_c$$
$$= 5V \cdot 133mA\ (1-x)\cdot x \qquad x = \exp(-t/1.125ms)$$

Maximum of $p_{abs}$ occurs when $x = 1/2$.




$$\max(p_{abs}) = 5V \cdot 133mA \cdot 1/4$$
$$= 167\ mW \quad [+1]$$

d) $$v_L = L\frac{di_L}{dt} = 30\mu H \cdot (-133mA)\cdot\left(\frac{-1}{800ns}\right)\exp(-t/800ns)$$

$$= 5V\ \exp(-t/800ns)$$

$$p_{abs} = v_L \cdot i_L = 5V \cdot 133mA \cdot (1-x)\cdot x \qquad x = \exp(-t/800ns)$$

Maximum of $p_{abs}$ occurs when $x = 1/2$.

$$\max(p_{abs}) = 5V \cdot 133mA \cdot 1/4 = 167\ mW \quad [+1]$$