McGill

2. Resistive Circuits

- Circuit Topology
- Kirchhoff's Current and Voltage Laws
- Equivalent Circuits
- Series and Parallel Resistors
- Equivalent Resistance and Circuit Analysis
- Ammeters, Voltmeters, Ohmmeters

Reference book Chapter 3, sections 3.1 to 3.6

Circuit Topology

Precise language is needed to describe the parts of a circuits:

Element: a model for a physical component

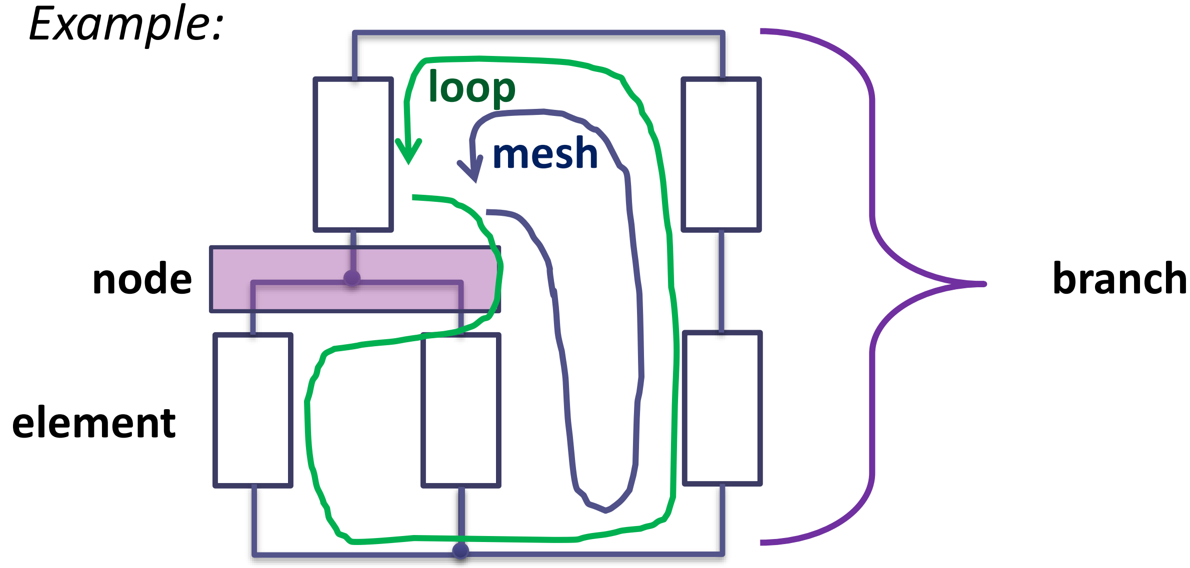
Node: a point where multiple elements meet

Branch: a path that connect two nodes

Loop: a closed path through a circuit with no node passed more than once

Mesh: a loop that cannot be broken up into smaller loops

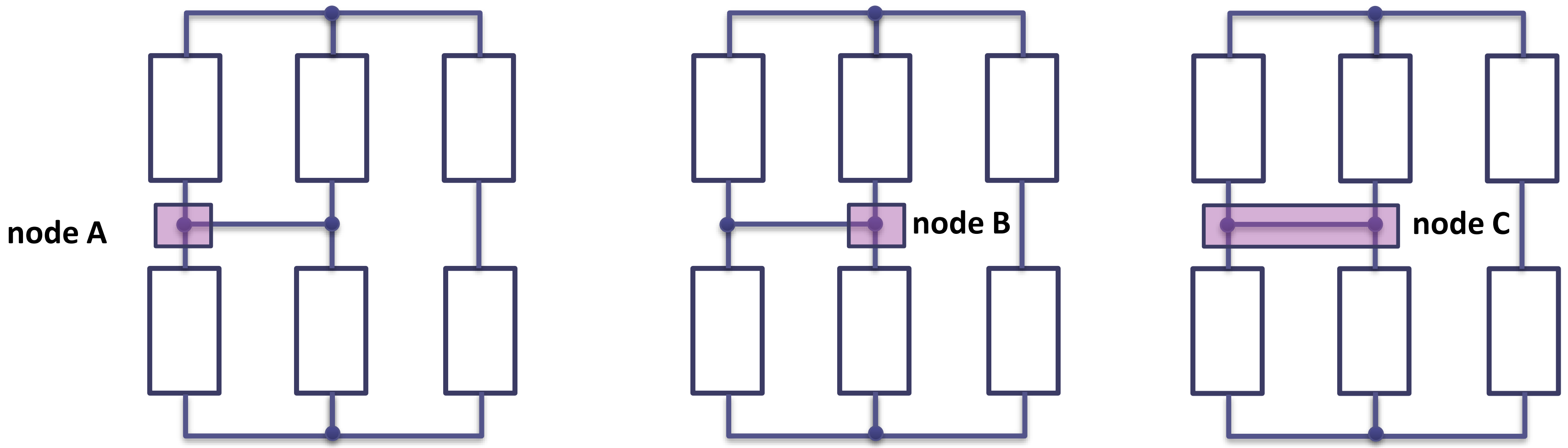
Example:



a note on nodes

A **node**, being the point where multiple elements meet, can be identified in several ways on the same portion of a circuit.

Consider the example below:



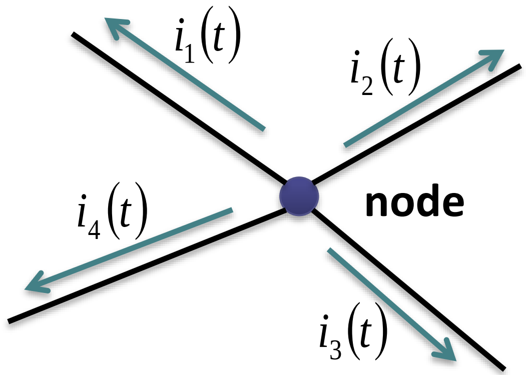
For node C, a *short circuit* connects node A and node B.

It is important to be clear about precisely which node one is discussing.

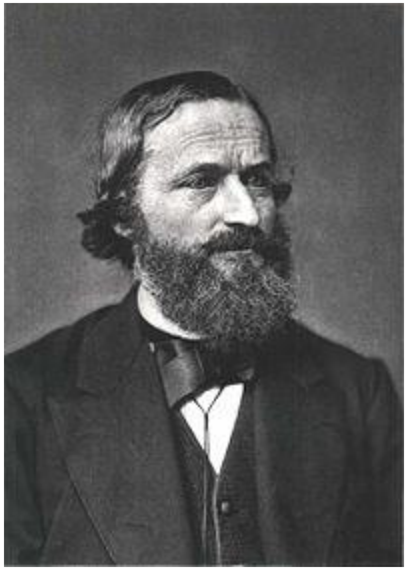
Kirchhoff's Current Law (KCL)

Kirchhoff's Current Law: The algebraic sum of currents leaving a *node* is zero.

$$\sum_m i_m(t) = 0$$



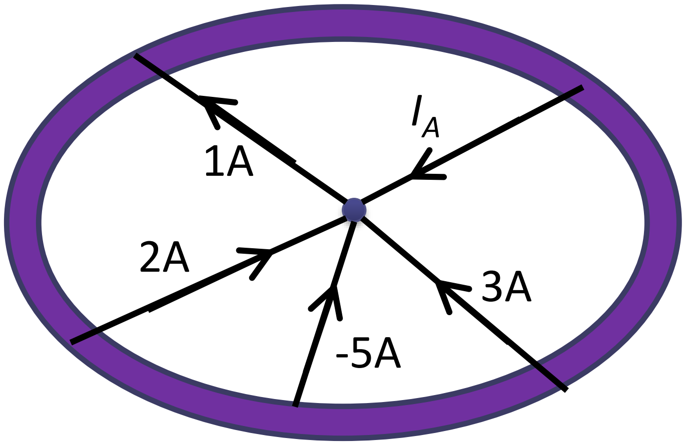
The physical basis for KCL is **conservation of charge** (charge cannot be created or destroyed at a node).



Gustav Robert Kirchhoff (1824-1887)
Photo: Wikipedia

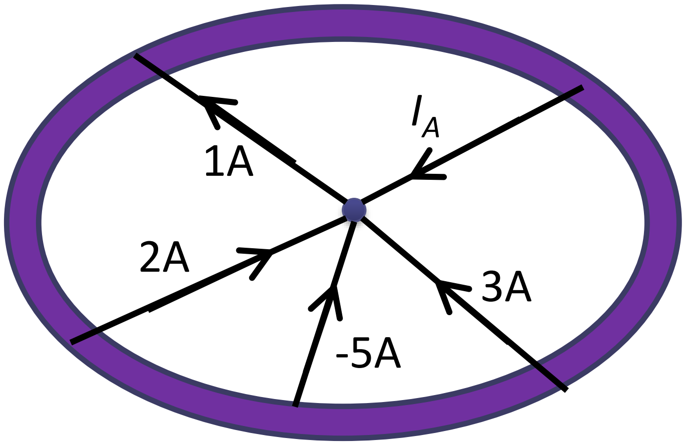
Example 1

Use KCL to find the unknown current I_A below.



Example 1

Use KCL to find the unknown current I_A below.



We sum the currents leaving the node:

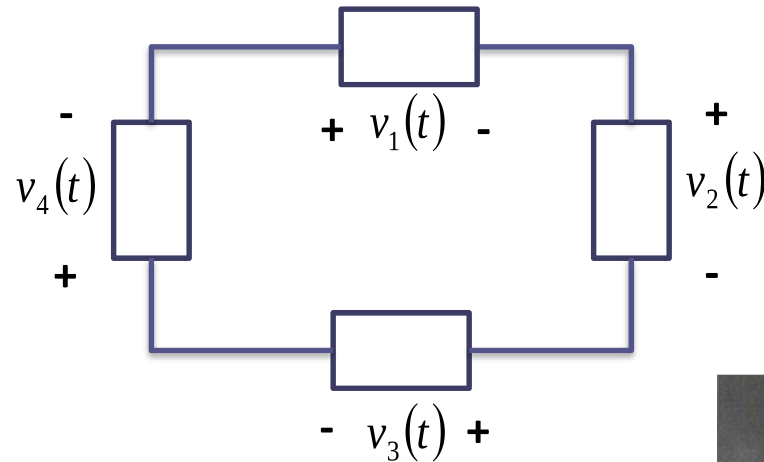
$0 = 1A + (-2A) + 5A + (-3A) + (-I_A)$

$I_A = 1A$

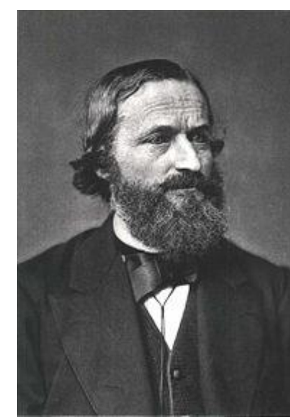
Kirchoff's Voltage Law (KVL)

Kirchhoff's Voltage Law: The algrebraic sum of voltage drops around a loop is zero,

$$\sum_m v_m(t) = 0$$



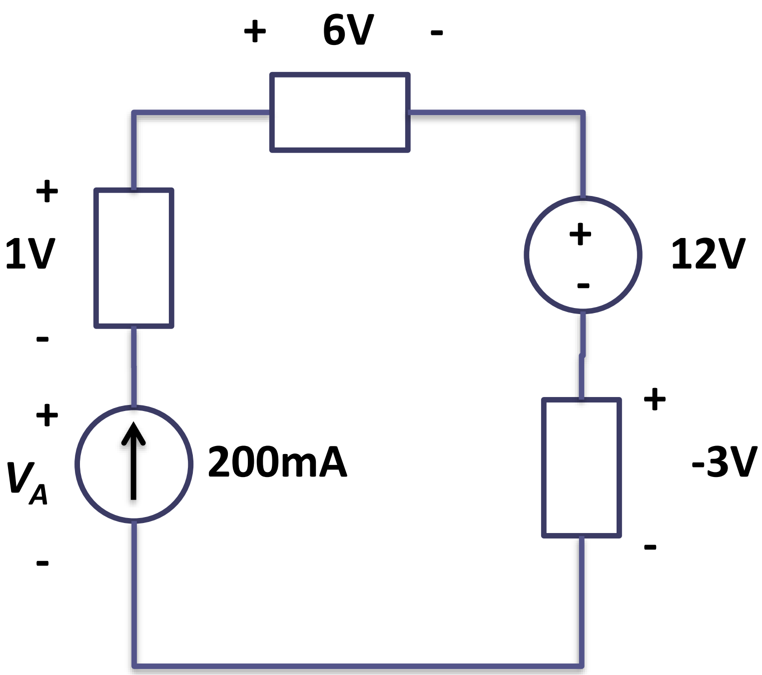
The physical basis for KVL is also the **conservation of energy** (the potential energy of a particle cannot be increased by traversing a closed loop).



Gustav Robert Kirchhoff
(1824-1887)
Photo: Wikipedia

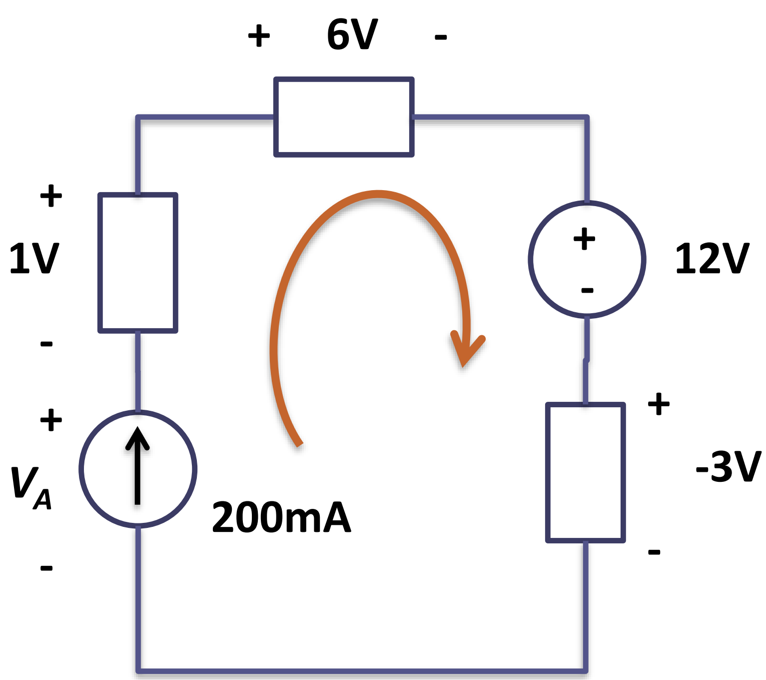
Example 2

Use KVL to find the unknown voltage V_A below.



Example 2

Use KVL to find the unknown voltage V_A below.



We sum the voltage drops across the loop, choosing a clockwise direction:

$$0 = (-V_A) + (-1V) + 6V + 12V + (-3V)$$

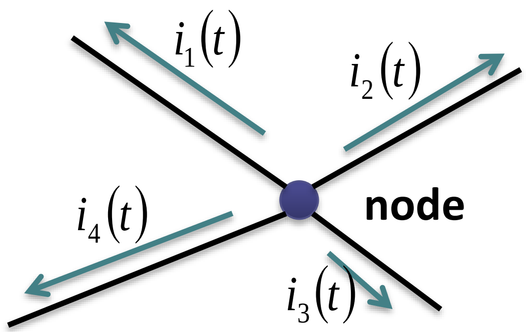
$$V_A = 14V$$

Note that the same answer will be found by taking the loop in counter-clockwise direction.

Equivalent Forms of KCL and KVL

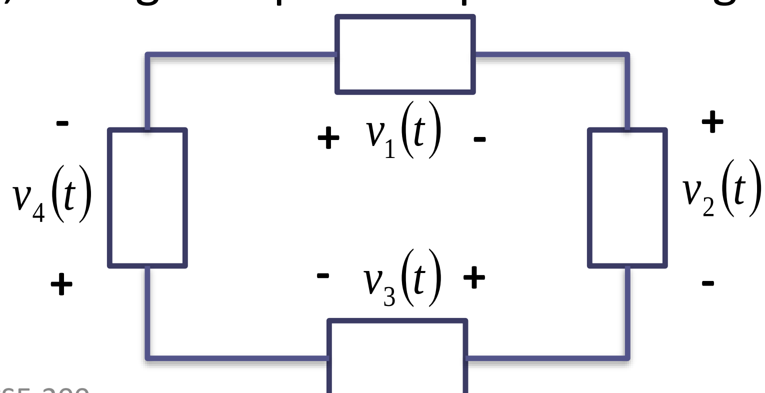
KCL: The algebraic sum of currents leaving a node is zero, as is the algebraic sum of currents entering the node. Also, currents leaving are equal to currents entering.

$$\boxed{\sum_m i_m(t) = 0}$$



KVL: The algebraic sum of voltage drops around a loop is zero, as is the sum of voltage rises around the loop. Also, voltage drops are equal to voltage rises.

$$\boxed{\sum_m v_m(t) = 0}$$



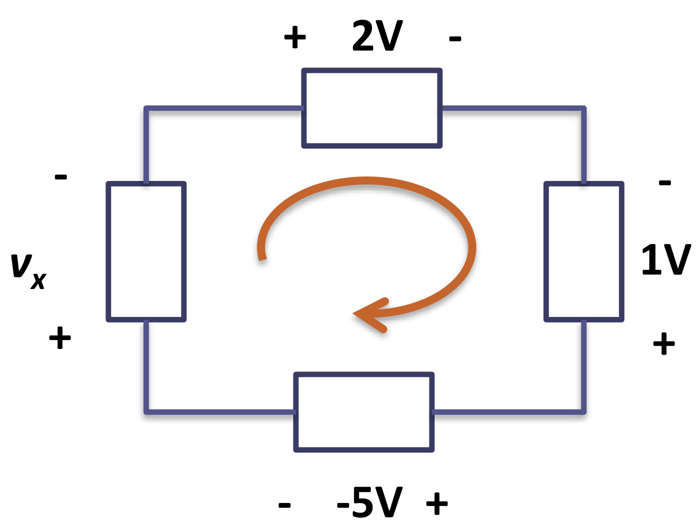
Keeping it Simple

To keep equations simple, we will use the following explicit forms:

KCL: The algebraic sum of currents leaving a node is zero.

KVL: The algebraic sum of voltage drops around a loop is zero.

Example: KVL can be written in multiple ways for the following, single-loop circuit. Note that it is simpler to express everything as a voltage drop.



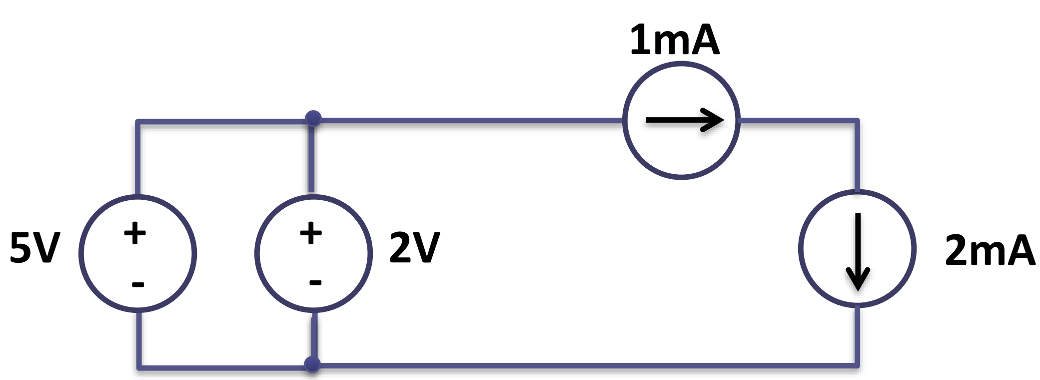
KVL: $0 = v_x + 2V - (1V) + (-5V)$

$v_x = 5V + 1V – 2V = 4V$

Consistency with KCL and KVL

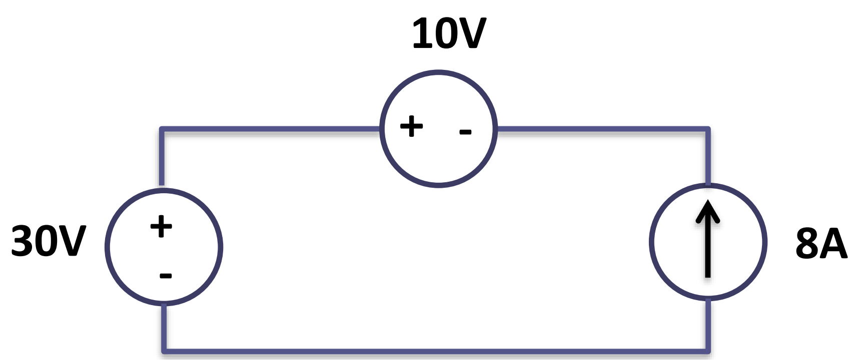
- KCL and KVL are based on the ***fundamental physical laws*** of conservation of charge and energy
- All physical circuits obey KCL and KVL
- There are connections of elements that are inconsistent with KCL and KVL, and thus represent unphysical situations (nonsense)?

Example: what is wrong with the circuit below?



Example

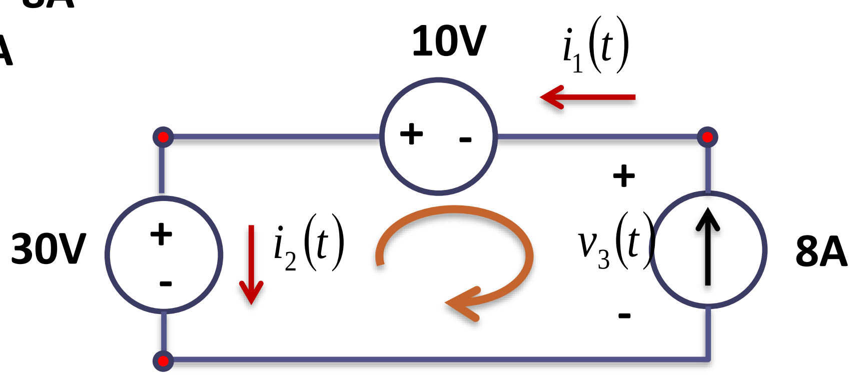
For the circuit below, how much power is being delivered or absorbed by each source?



Example

Define the necessary variables, and apply KCL and KVL.

KCL: $0 = i_2 - 8A$
$i_2 = 8A$



KCL: $0 = i_1 - 8A$
$i_1 = 8A$

KCL: $0 = -i_2 + 8A$
$i_2 = 8A$

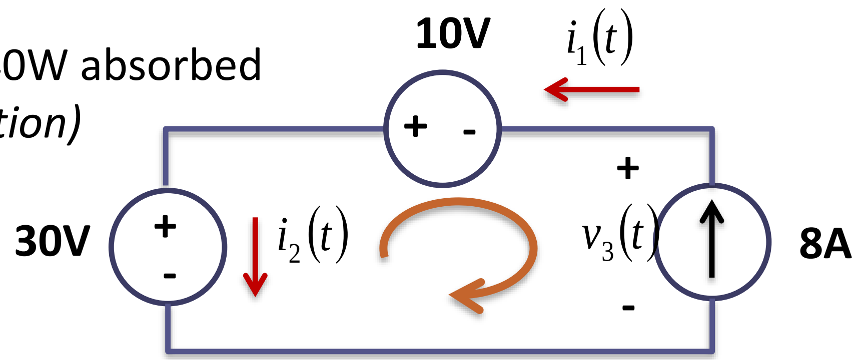
KVL: $0 = -30V + 10V + v_3$
$v_3 = 20V$

Example

For the circuit below, how much power is being delivered or absorbed by each source?

P_{10V} = (10V)(8A) = 80W delivered
(opposite to passive sign convention)

P_{30V} = (30V)(8A) = 240W absorbed
(passive sign convention)

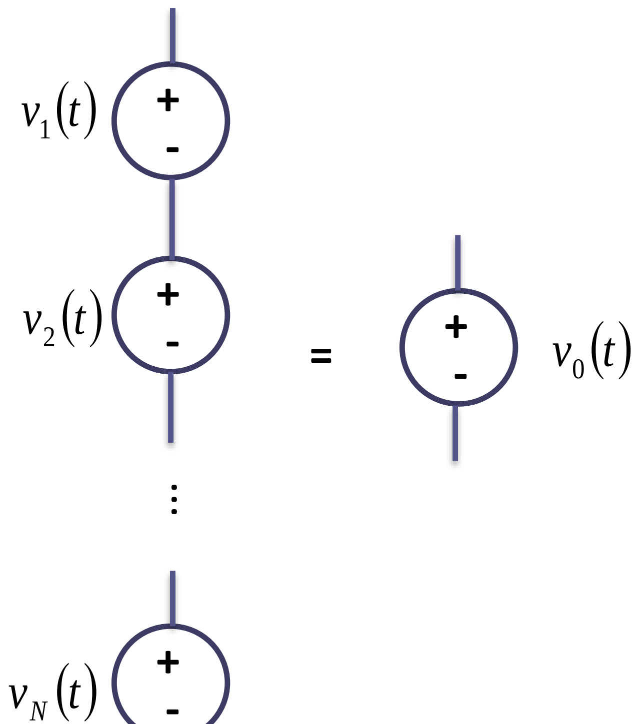


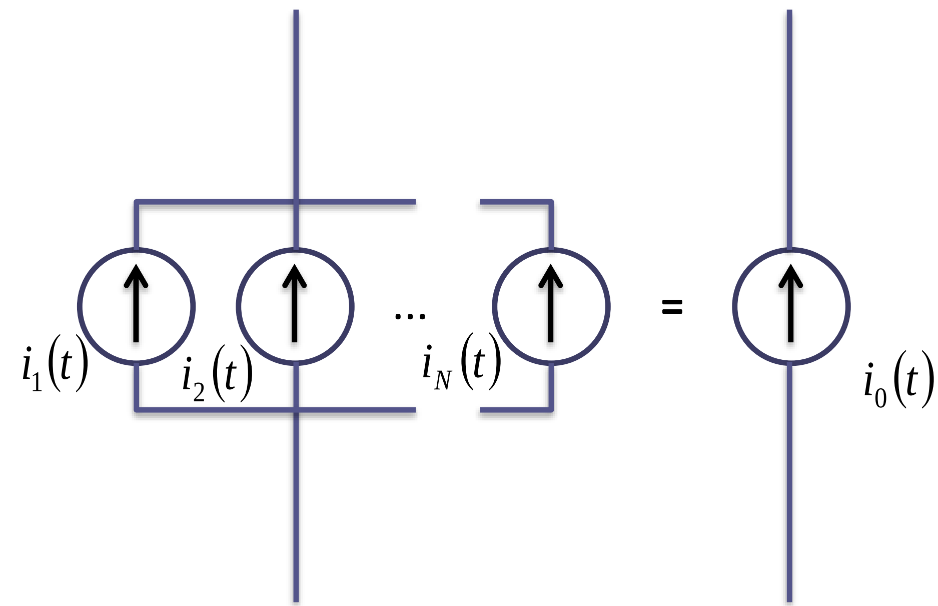
P_{8A} = (20V)(8A) = 160W delivered
(opposite to passive sign convention)

Note: The total power delivered by circuit elements (80W + 160W) to the flowing charges is equal to total power absorbed (240W) by circuit elements from the flowing charges

Exercise

Use KCL, KVL and the definitions of independent voltage sources and independent current sources to show the two following equivalences.





$$v_0(t) = v_1(t) + v_2(t) + \ldots + v_N(t)$$

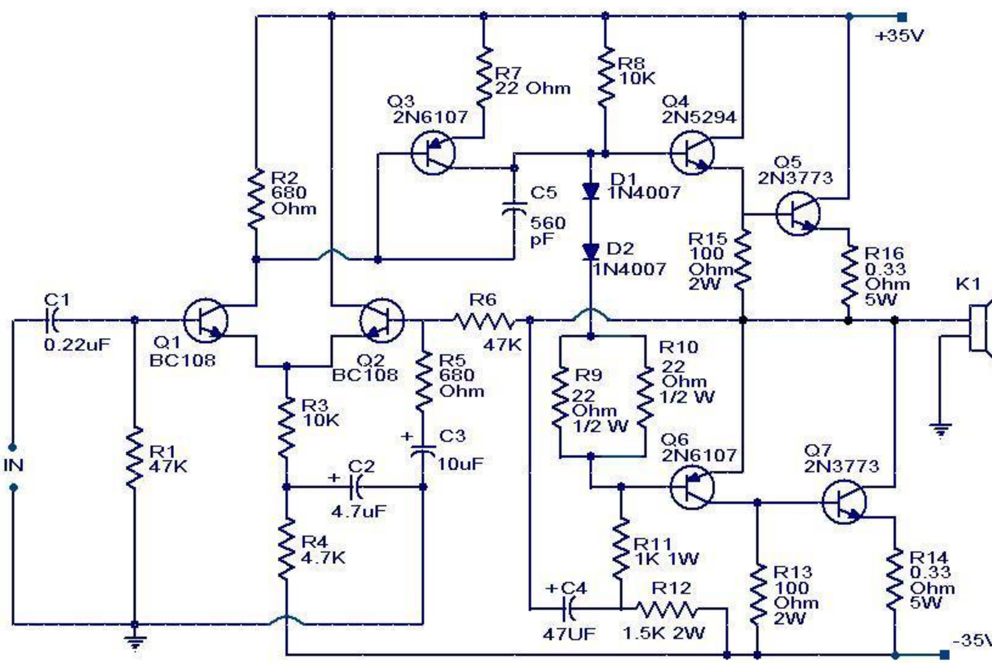
$$i_0(t) = i_1(t) + i_2(t) + \ldots + i_N(t)$$

Equivalent Circuits

In many situations, we only care about the relation between *i(t)* and *v(t)* at two terminals of a specific circuit branch of particular interest

(sometimes referred to as a **sub-circuit**).

Example: When connecting a pair of speakers to an audio amplifier (circuit shown below), we might only want to know the voltage and current at the amplifier terminals. We <u>need not</u> know every voltage and current within the audio amplifiers.



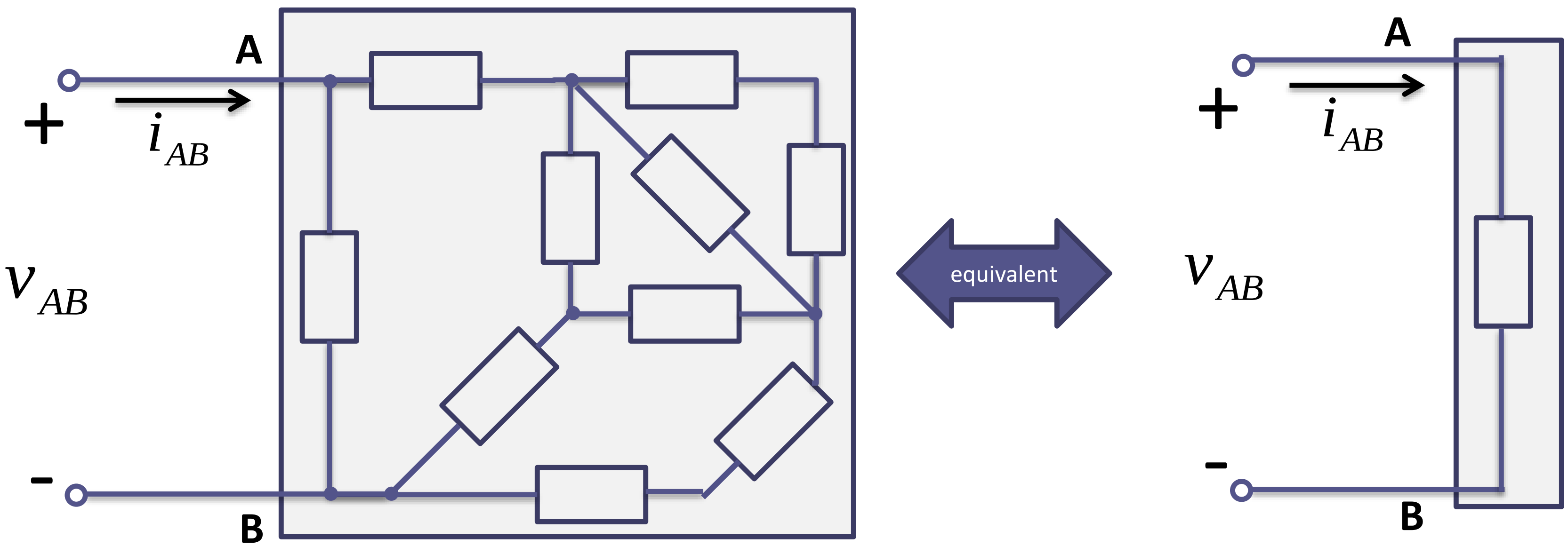
100W Subwoofer power amplifier www.circuitstoday.com

just want to know *v(t)* and *i(t)* here

Equivalent Circuits

Equivalent Circuits: Two two-terminal circuits that have the same **terminal law**, i.e., the same mathematical relationship between i_{AB} and v_{AB}

- equivalent circuits produce identical terminal voltages and currents when connected to other circuits
- replacing circuits with simpler equivalent circuits is a very powerful circuit analysis technique



$$\boxed{i_{AB} = F\left(v_{AB}\right)} \qquad \boxed{i_{AB} = F\left(v_{AB}\right)}$$

2. Resistive Circuits

- Circuit Topology
- Kirchhoff's Current and Voltage Laws
- Equivalent Circuits
- Series and Parallel Resistors
- Equivalent Resistance and Circuit Analysis
- Ammeters, Voltmeters, Ohmmeters

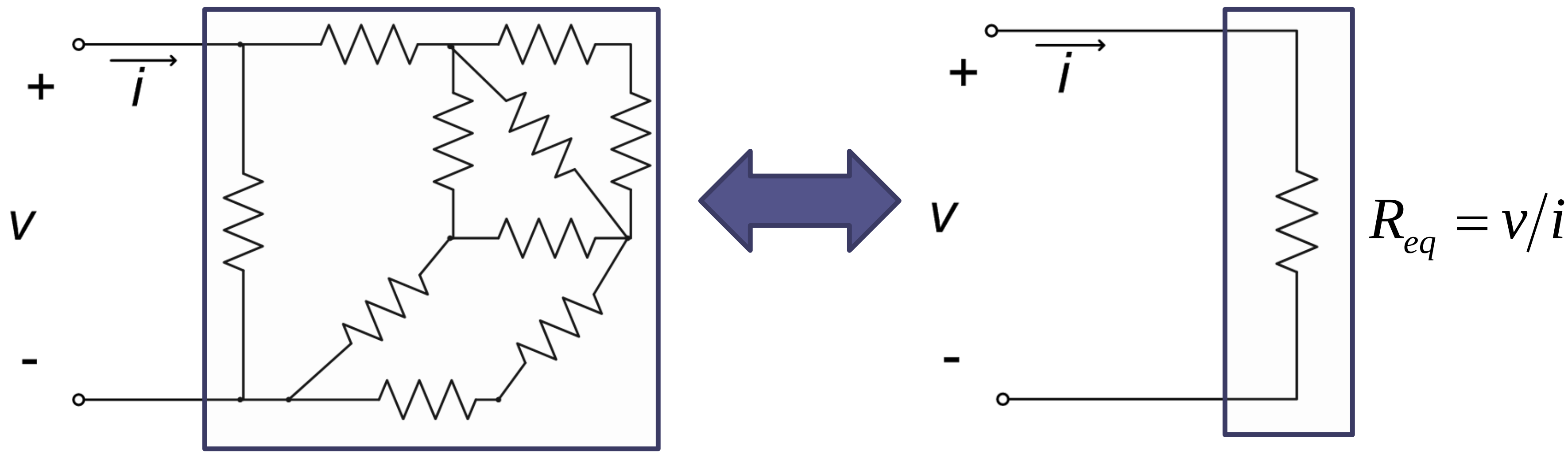
Reference book Chapter 3, sections 3.1 to 3.6

Problem Set #3

Equivalent Resistance

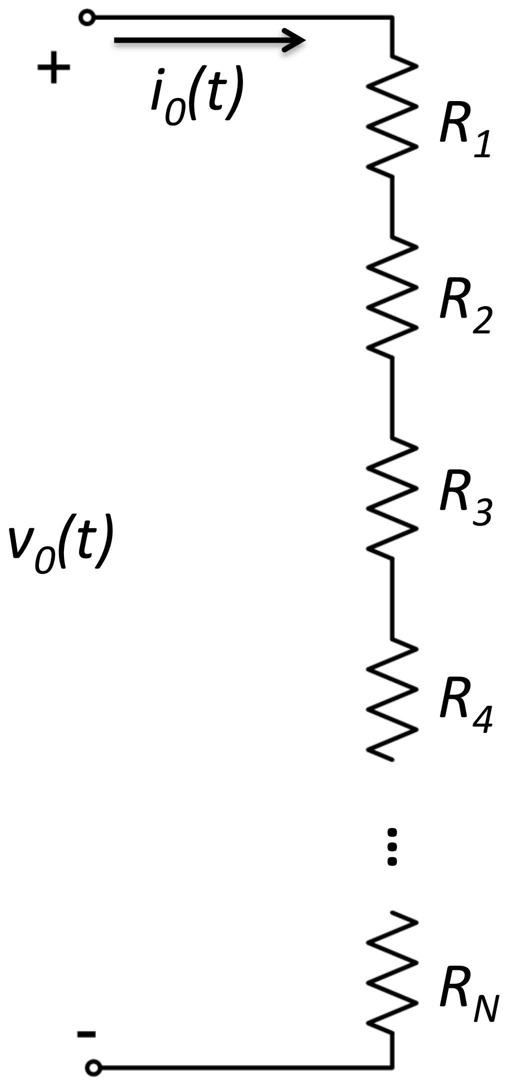
Equivalent Resistance: a single ideal resistor that is equivalent to a two-terminal circuit composed ***only*** of ideal resistors.

- *any* two-terminal circuit that is composed of ideal resistors alone is equivalent to a single resistor.



McGill

Series Equivalent Resistance



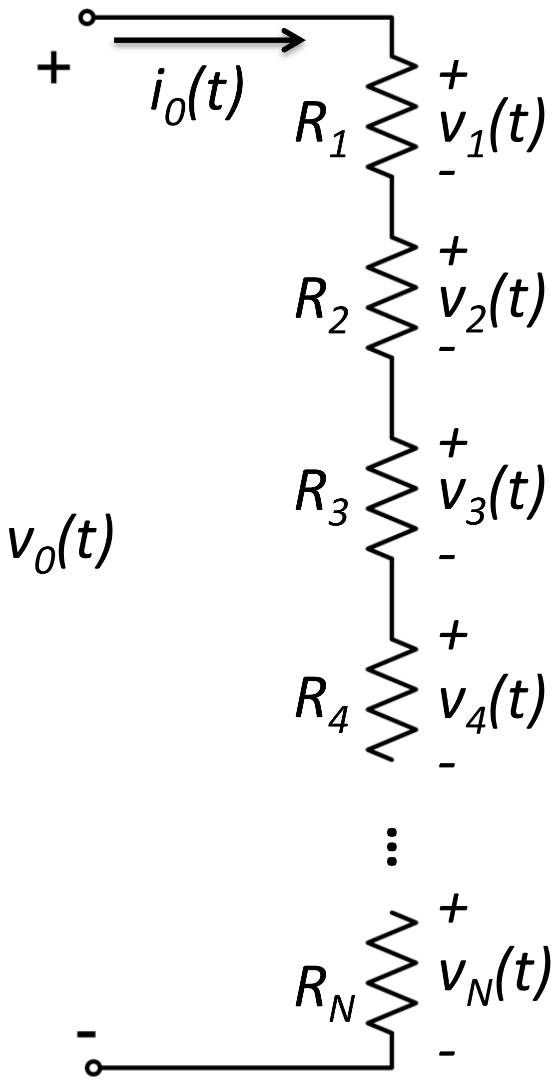
In a **series** connection, each element conducts the same current (by KCL).

To find the equivalent resistance of several resistors in series:

1. we assume unknown terminal voltage $v_0(t)$ and terminal current $i_0(t)$
2. we find the relationhip between $v_0(t)$ and $i_0(t)$, by applying KCL, KVL, and Ohm's Law to the network of resistors
3. we apply the definition of equivalent resistance, $R_{eq} = v_0(t) / i_0(t)$

McGill

Series Equivalent Resistance



By KVL:

$$0 = - v_0(t) + v_1(t) + v_2(t) + \ldots + v_N(t)$$

By KCL and Ohm's Law: $v_m(t) = i_0(t)\, R_m$

Combining the above:

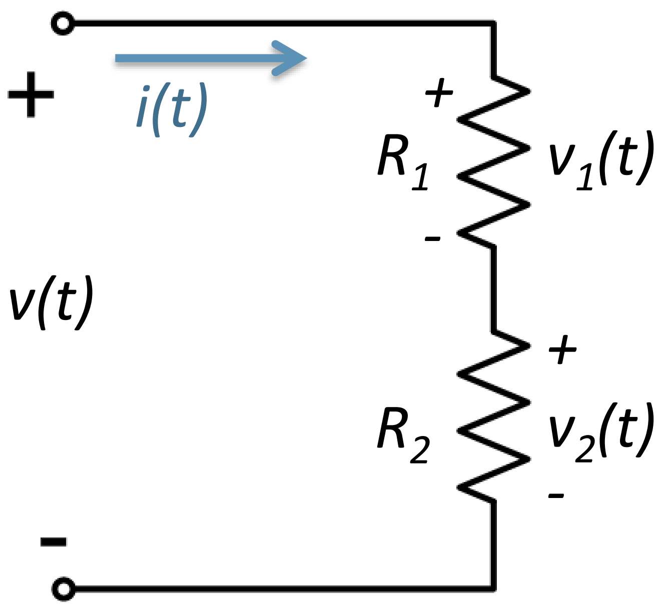
$$0 = - v_0(t) + i_0(t)\,(R_1 + R_2 + \ldots + R_N)$$

By definition $R_{eq} = v_0(t) / i_0(t)$, thus:

$$\boxed{R_{eq} = R_1 + R_2 + \ldots + R_N}$$

McGill

Voltage Divider with 2 Resistors



Consider a series combination of two resistors.

By KCL and Ohm's Law:

$$v_1(t) = i(t)\, R_1$$

$$v_2(t) = i(t)\, R_2$$

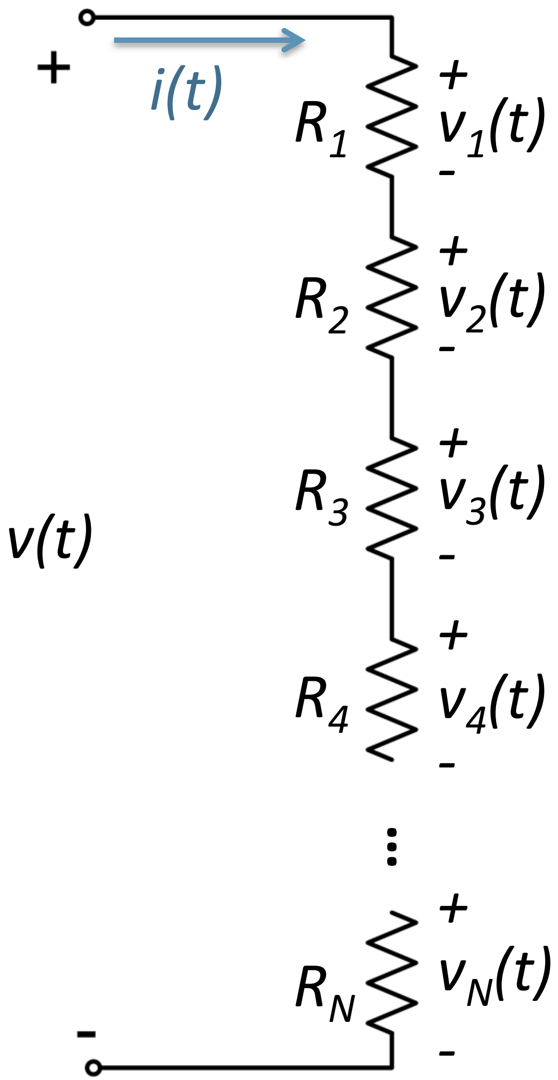
From the series equivalent circuit:

$$v(t) = i(t)\, R_{eq} = i(t)\, (R_1+R_2)$$

We, thus, have a ***voltage divider:***

$$\frac{v_1(t)}{v(t)} = \frac{R_1}{R_1 + R_2} \qquad \frac{v_2(t)}{v(t)} = \frac{R_2}{R_1 + R_2}$$

Voltage Divider with N Resistors



A series combination of N resistors also acts as a voltage divider.

By KCL and Ohm's Law: $v_m(t) = i(t)\, R_m$

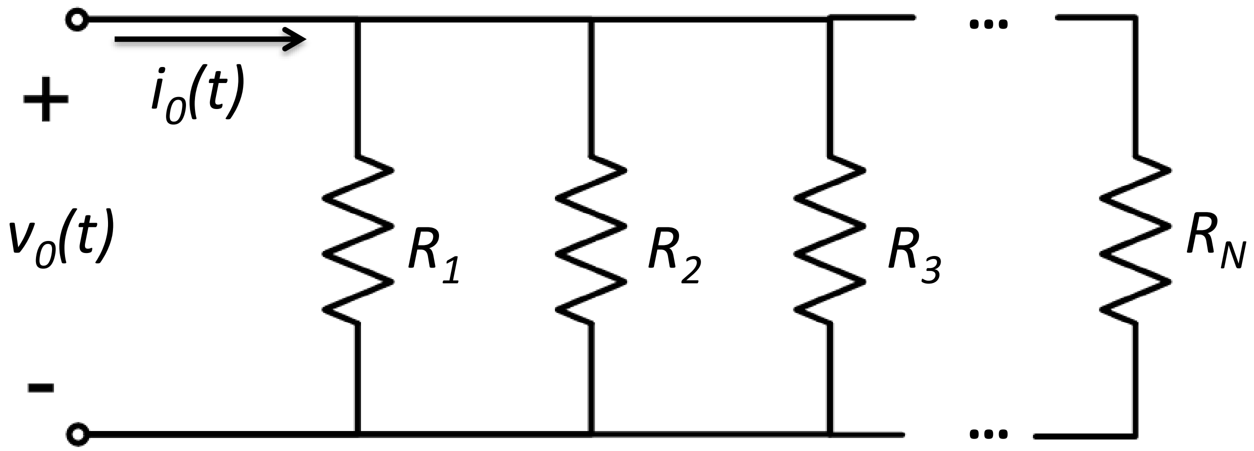
From the series equivalent circuit:

$v(t) = i(t)\, R_{eq} = i(t)\, (R_1 + R_2 + \ldots + R_N)$

We thus have:

$$\boxed{\frac{v_m(t)}{v(t)} = \frac{R_m}{R_1 + R_2 + \ldots + R_N}}$$

Parallel Equivalent Resistance

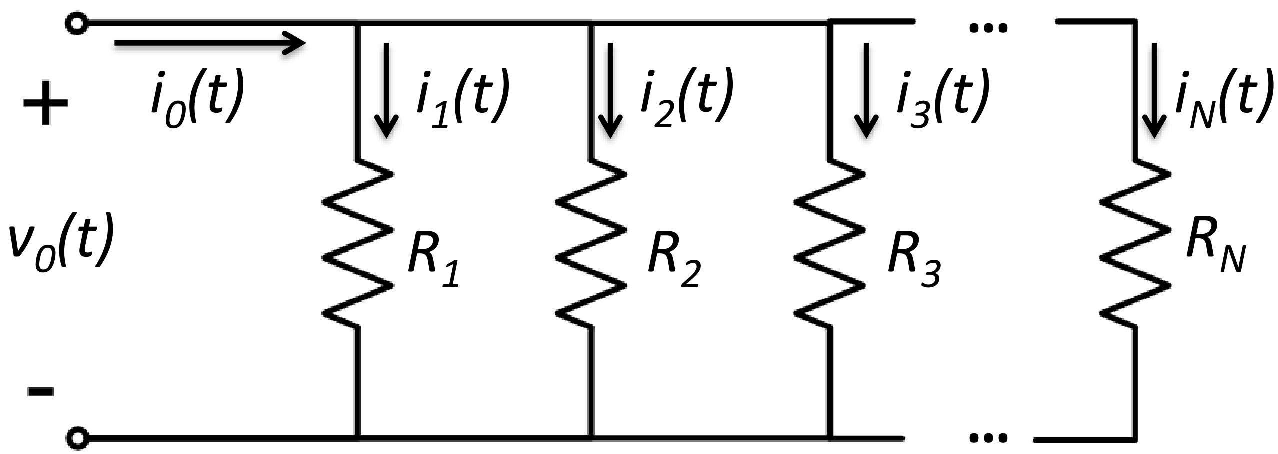


In a **parallel** connection, each element carries the same voltage (by KVL).

To find the equivalent resistance of several resistors in parallel:

1. we assume unknown terminal voltage $v_0(t)$ and terminal current $i_0(t)$
2. we find the relationship betwen $v_0(t)$ and $i_0(t)$, by applying KCL, KVL and Ohm's Law to the network of resistors
3. we apply the definition of equivalent resistance, $R_{eq} = v_0(t) / i_0(t)$

Parallel Equivalent Resistance



By KCL: $0 = - i_0(t) + i_1(t) + i_2(t) + \ldots + i_N(t)$

By KVL and Ohm's Law: $i_m(t) = v_0(t) / R_m$

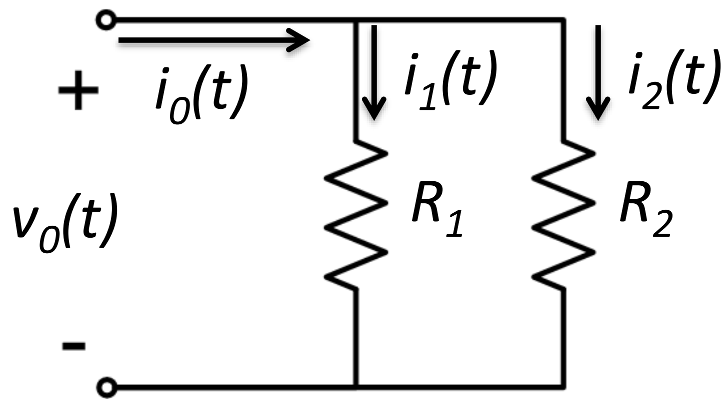
Combining the above: $0 = - i_0(t) + v_0(t) \,(1/ R_1 + 1/ R_2 + \ldots + 1/R_N)$

By definition $R_{eq} = v_0(t) / i_0(t)$, thus:

$\boxed{1/ R_{eq} = 1/ R_1 + 1/ R_2 + \ldots + 1/R_N}$ or $\boxed{G_{eq} = G_1 + G_2 + \ldots + G_N}$

McGill

Two Parallel Resistors



The parallel combination of two resistors appears frequently enough that it is worth remembering the specific case:

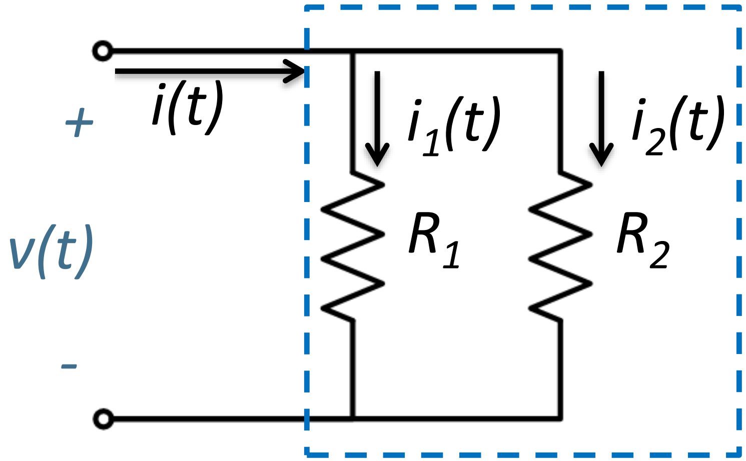
$$1/R_{eq} = 1/R_1 + 1/R_2$$

$$R_{eq} = 1/(1/R_1 + 1/R_2)$$

$$\boxed{R_{eq} = R_1 R_2 / (R_1 + R_2)}$$

A short hand notation is often used, $R_{eq} = R_1 \,||\, R_2$, which reads as "R_1 in parallel with R_2".

Current Divider with 2 Resistors



$$R_{eq} = \frac{1}{1/R_1 + 1/R_2} \qquad or \qquad G_{eq} = G_1 + G_2$$

Consider a parallel combination of two resistors.

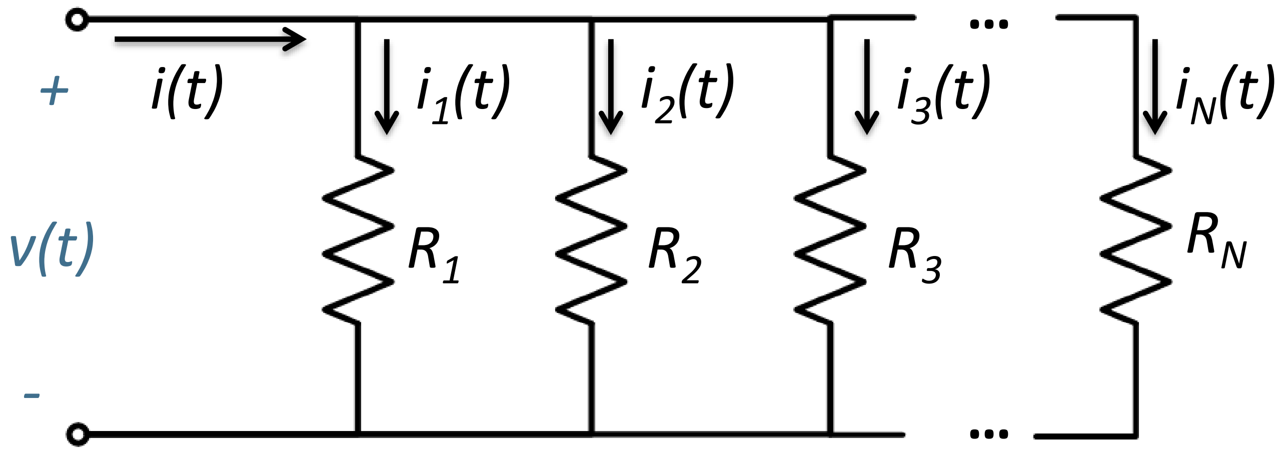
By KVL and Ohm's Law: $i_1 = v(t)/R_1 = v(t)G_1 \quad i_2 = v(t)/R_2 = v(t)G_2$

From the parallel equivalent: $i = v(t)/R_{eq} = v(t)G_{eq} = v(t)(G_1 + G_2)$

We thus have a **current divider**:

$$\boxed{\frac{i_1(t)}{i(t)} = \frac{G_1}{G_1 + G_2} = \frac{1/R_1}{1/R_1 + 1/R_2} = \frac{R_2}{R_1 + R_2}} \quad \boxed{\frac{i_2(t)}{i(t)} = \frac{G_2}{G_1 + G_2} = \frac{1/R_2}{1/R_1 + 1/R_2} = \frac{R_1}{R_1 + R_2}}$$

Current Divider with *N* Resistors



A parallel combination of *N* resistors also acts as a current divider.

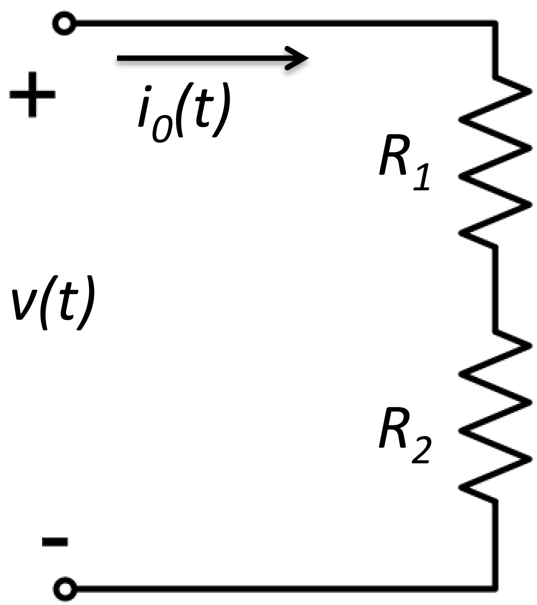
By KVL and Ohm's Law: $i_m = v(t)G_m$

From the parallel equivalent: $i = v(t)G_{eq} = v(t)(G_1 + G_2 + \ldots + G_N)$

We thus have:

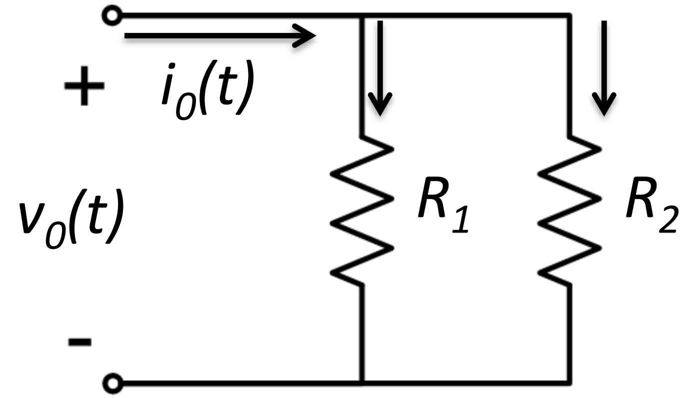
$$\boxed{\frac{i_m(t)}{i(t)} = \frac{G_m}{G_1 + G_2 + \ldots + G_N} = \frac{1/R_m}{1/R_1 + 1/R_2 + \ldots + 1/R_N}}$$

Series Resistors



$$\frac{v_0(t)}{i_0(t)} = R_{eq} = R_1 + R_2$$

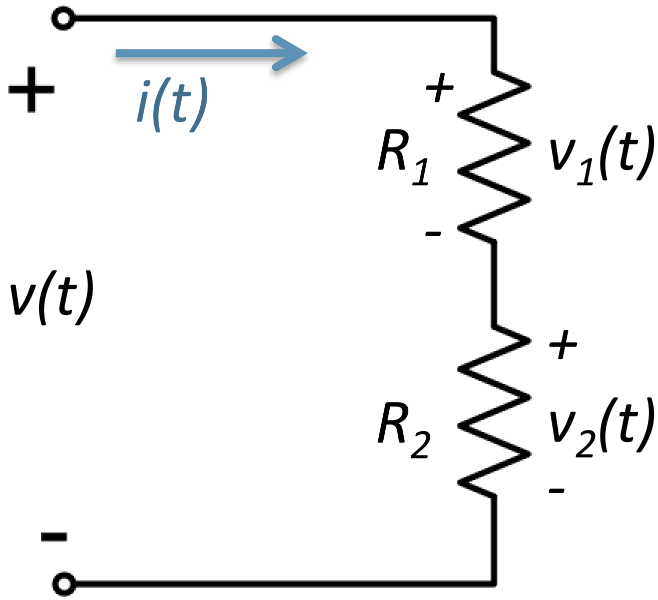
Parallel Resistors



$$\frac{v_0(t)}{i_0(t)} = R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{i_0(t)}{v_0(t)} = G_{eq} = G_1 + G_2$$

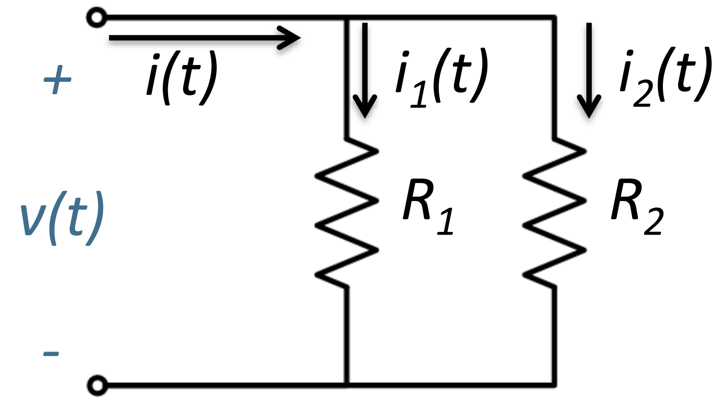
Voltage Divider



$$\boxed{\frac{v_1(t)}{v(t)} = \frac{R_1}{R_1 + R_2}}$$

$$\frac{v_2(t)}{v(t)} = \frac{R_2}{R_1 + R_2}$$

Current Divider

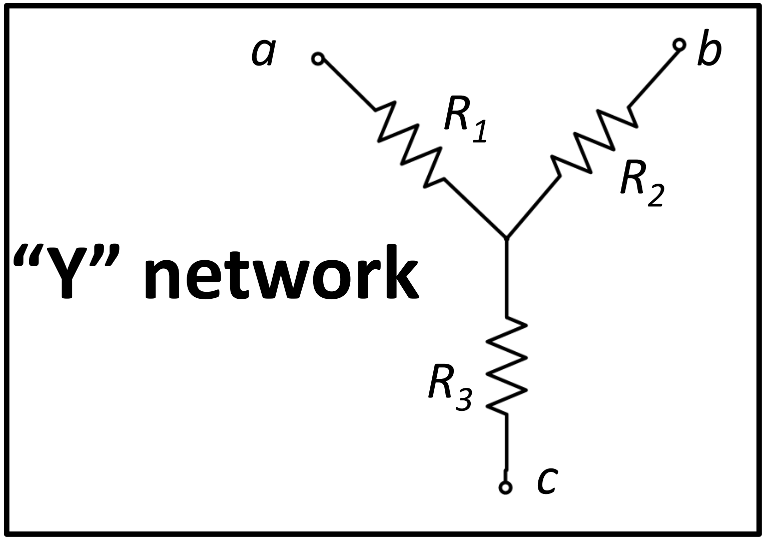


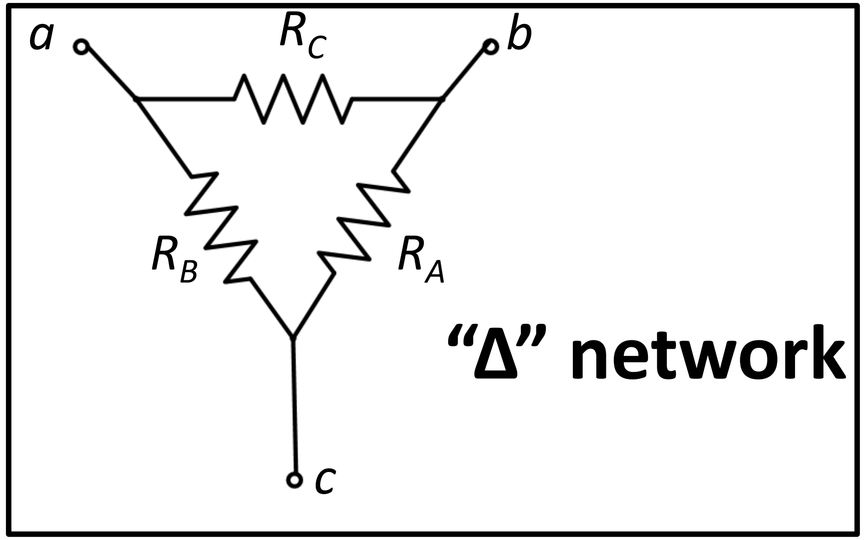
$$\boxed{\frac{i_1(t)}{i(t)} = \frac{R_2}{R_1 + R_2}}$$

$$\frac{i_2(t)}{i(t)} = \frac{R_1}{R_1 + R_2}$$

Example

What value should R_1, R_2, and R_3 have so that the **Y network** (the Wye-network) has the <u>same equivalent resistances</u> between node pairs as the **Δ network** (the delta-network)?

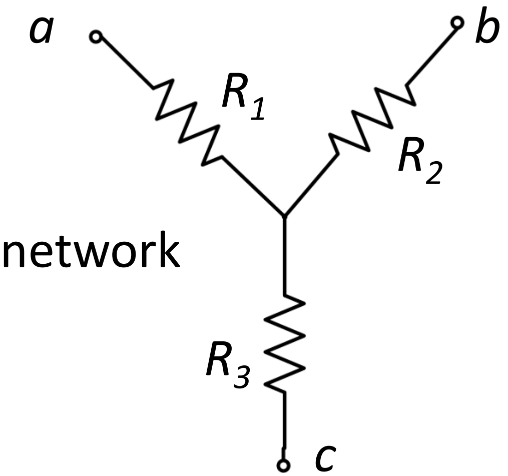




Strategy:

1. *Find equivalent resistance for all terminal pairs (ab,bc,ca) in Y and Δ.*
2. *Equate terminal resistances.*
3. *Solve for R_1, R_2, and R_3.*

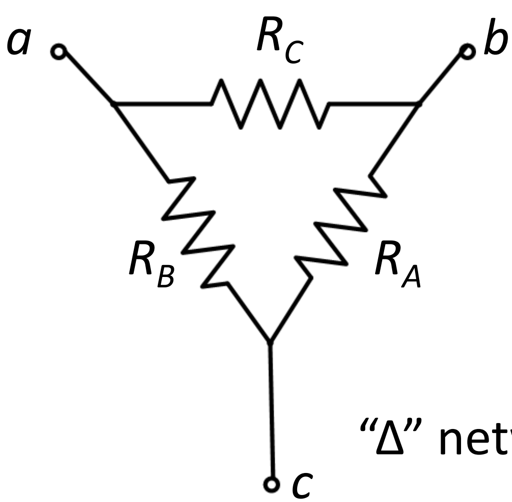
Example



"Y" network

Equivalent resistance for Y network:

$$R_{ab} = R_1 + R_2$$
$$R_{bc} = R_2 + R_3$$
$$R_{ca} = R_3 + R_1$$

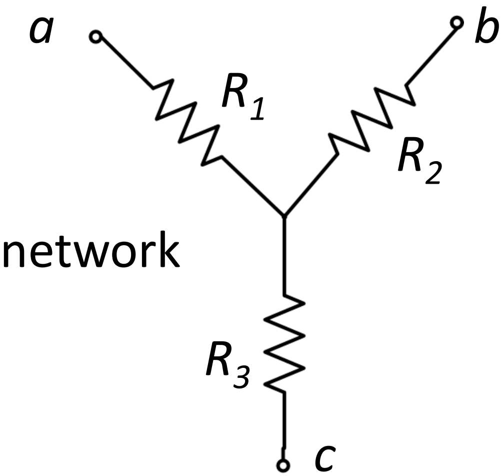


"Δ" network

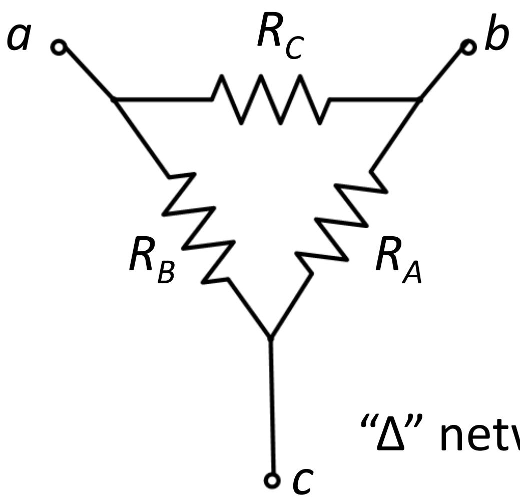
Equivalent resistances for Δ network:

$$R_{ab} = R_C \parallel (R_B + R_A) = R_C(R_B + R_A)/(R_A + R_B + R_C)$$
$$R_{bc} = R_A \parallel (R_C + R_B) = R_A(R_C + R_B)/(R_A + R_B + R_C)$$
$$R_{ca} = R_B \parallel (R_A + R_C) = R_B(R_A + R_C)/(R_A + R_B + R_C)$$

Example



“Y” network



“Δ” network

Equate R_{eq}:

$$R_{ab} = R_1 + R_2 = R_C\left(R_B + R_A\right)/\left(R_A + R_B + R_c\right)$$

$$R_{bc} = R_2 + R_3 = R_A\left(R_C + R_B\right)/\left(R_A + R_B + R_c\right)$$

$$R_{ca} = R_3 + R_1 = R_B\left(R_A + R_C\right)/\left(R_A + R_B + R_c\right)$$

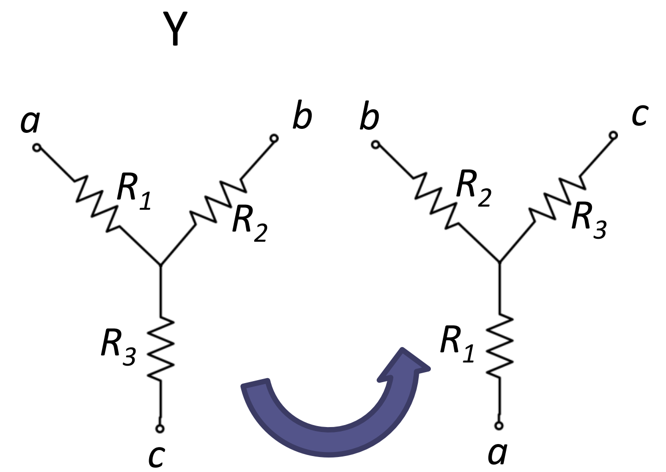
Solve for R_1:

$$R_1 = \left(R_{ab} + R_{ca} - R_{bc}\right)/2 \qquad \text{(by inspection, or substitution)}$$

$$R_1 = 1/2 \cdot \left(R_C R_B + R_C R_A + R_B R_A + R_B R_C - R_A R_C - R_A R_B\right)/\left(R_A + R_B + R_C\right)$$

$$\boxed{R_1 = R_C R_B/\left(R_A + R_B + R_C\right)}$$

Example

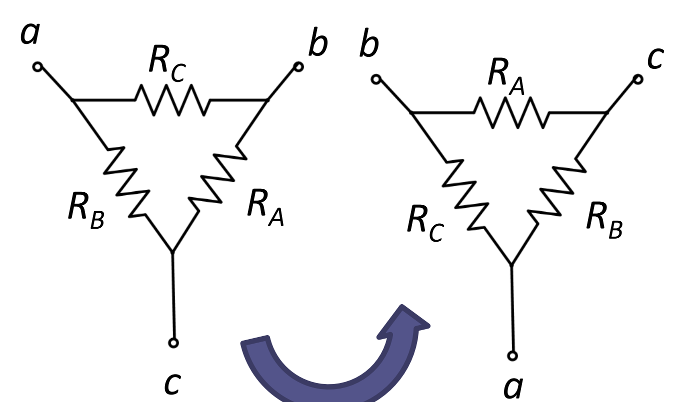


<u>For R_2 and R_3, use symmetry:</u>

Rotate either network counter-clockwise:

$$\left(R_1 \to R_2; R_C \to R_A; R_A \to R_B; R_B \to R_C\right)$$

$$R_1 = R_C R_B/\left(R_A + R_B + R_c\right) \to \boxed{R_2 = R_A R_C/\left(R_A + R_B + R_c\right)}$$



Rotate either networks counter-clockwise again:

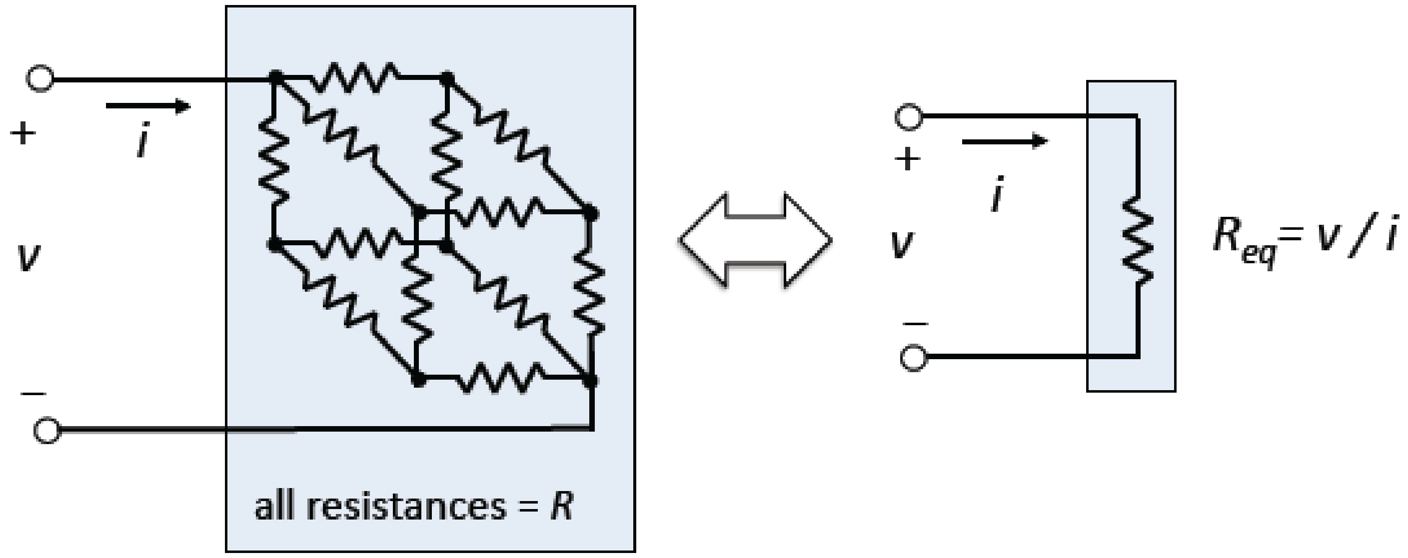
$$\left(R_2 \to R_3; R_C \to R_A; R_A \to R_B; R_B \to R_C\right)$$

$$R_2 = R_A R_C/\left(R_A + R_B + R_c\right) \to \boxed{R_3 = R_B R_A/\left(R_A + R_B + R_c\right)}$$

Equivalent Resistance

There are some circuits that *cannot* be reduced to R_{eq} by identifying a sequence of series and parallel resistance reductions, such as the cube of identical resistors (below).

In such cases, the circuit must be solved. Assume that v is known, and then express i in terms of v using KCL, KVL and Ohm's law (or assume i is known…), as we did in deriving the series equivalent resistance rule.

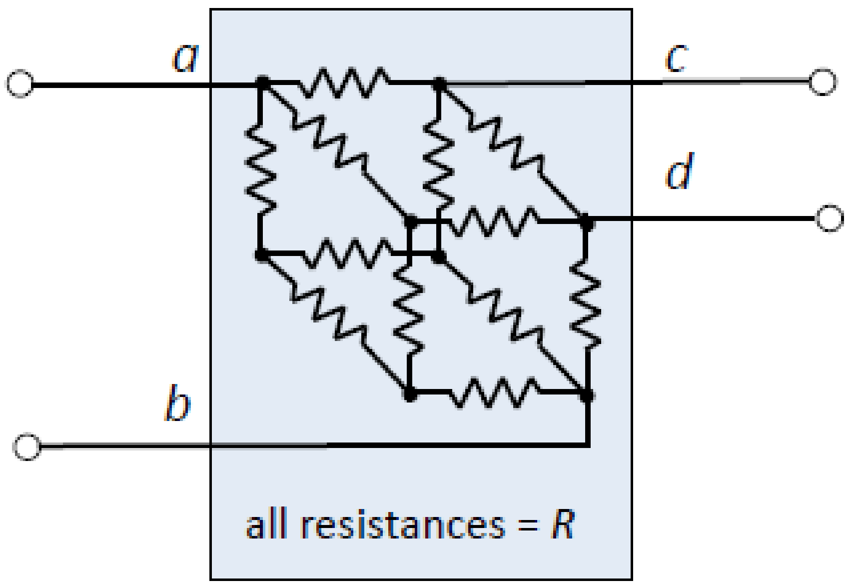


McGill

Equivalent Resistance

Equivalent resistance is always defined with respect to **two terminals**.

A network with multiple terminal pairs can have more than one equivalent resistance between those pairs.



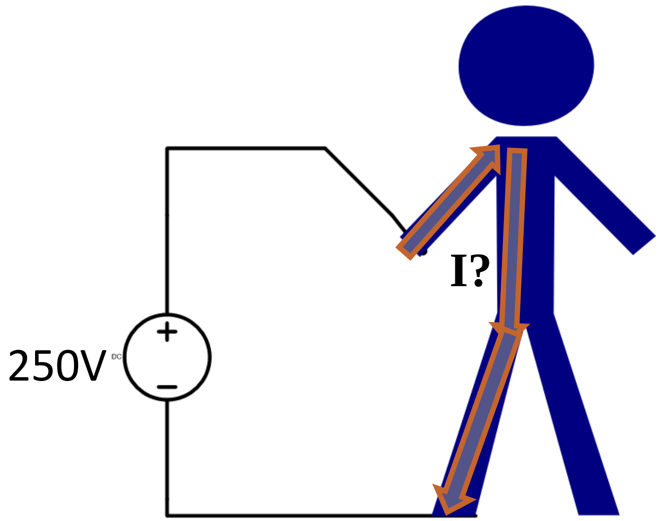
$R_{ab} \neq R_{cd}$

Equivalent Circuit – Example 1

Should there be any concern about the following situation?

Physiological Reaction	**Current (I)**
Barely Perceptible	3-5mA
Extreme Pain	35-50mA
Muscle Paralysis	50-70mA
Heart Stoppage	500mA

W.F Cooper, *Electrical Safety Engineering,* 2nd ed., Butterworth, 1986

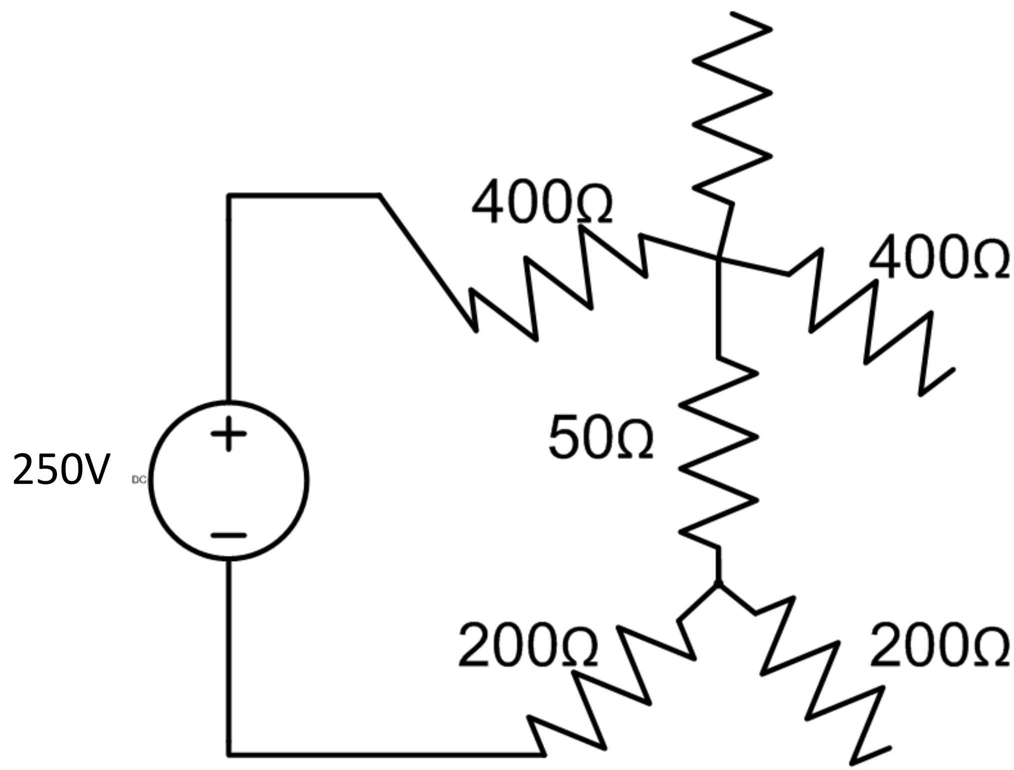


To answer this question, we need an electrical model.

Do not try this at home, nor on your little brother!

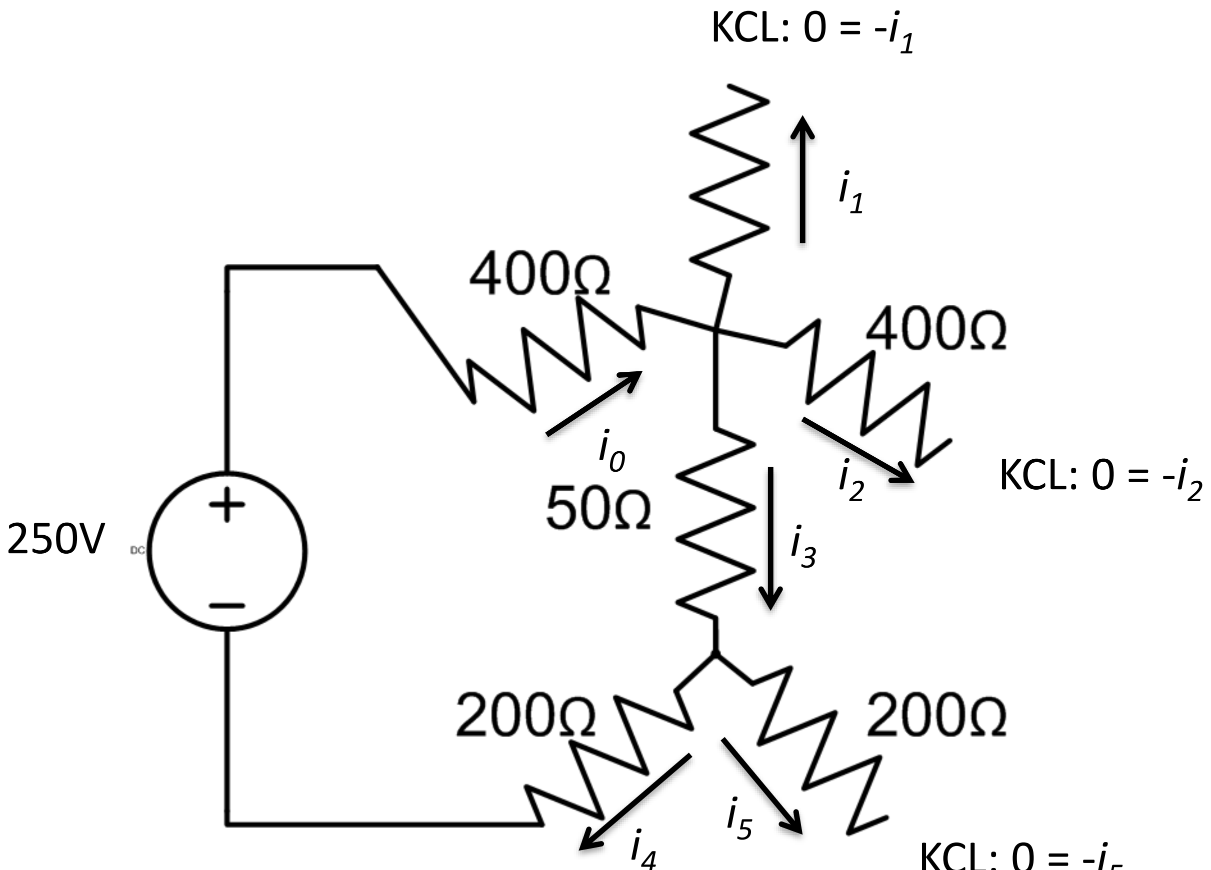
Example 1

An approximate electrical model is given by a resistor network (from experimental data), and the current can be easily found.

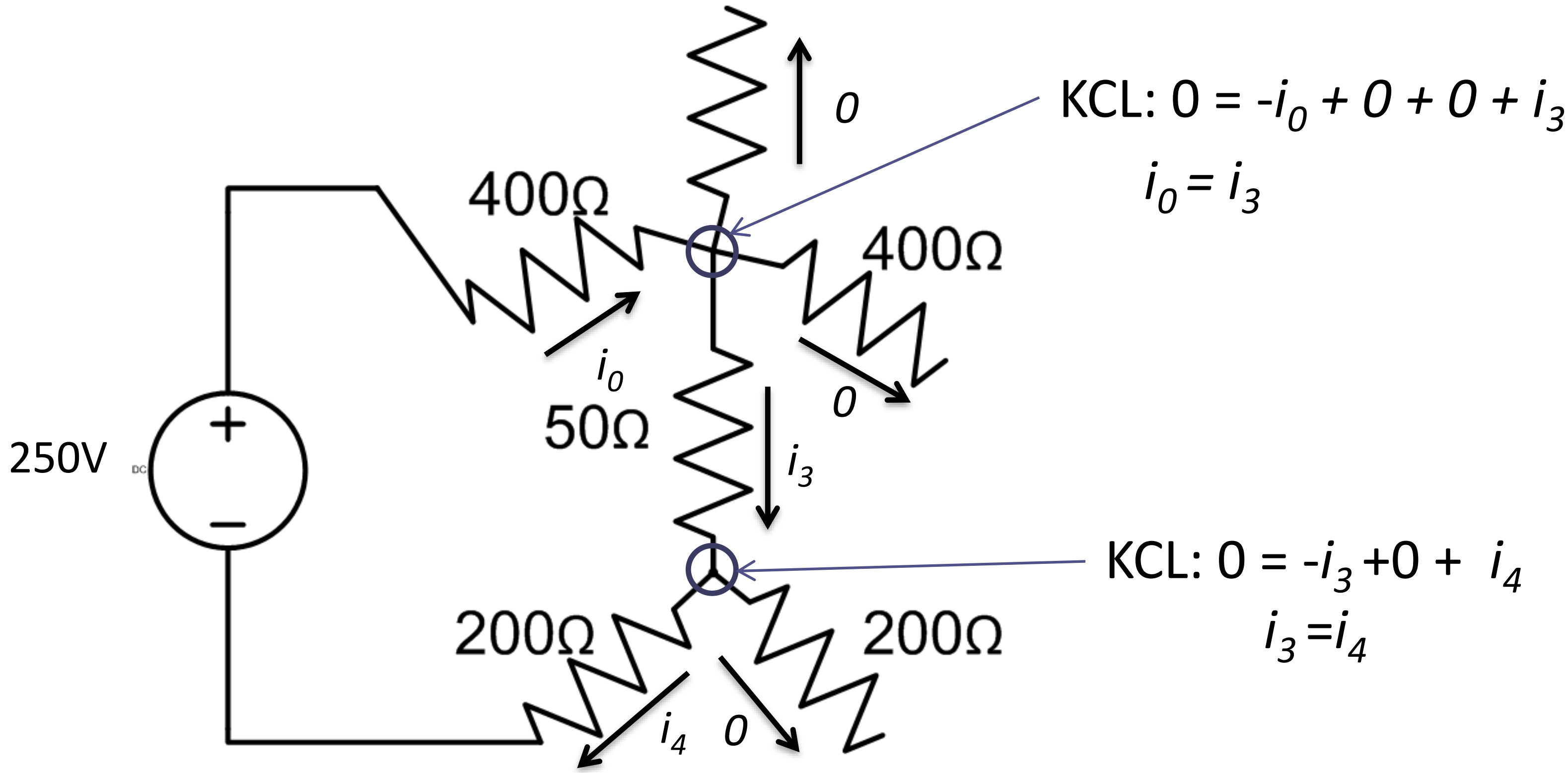


Example 1

Define your algebraic variables and use KCL



Example 1

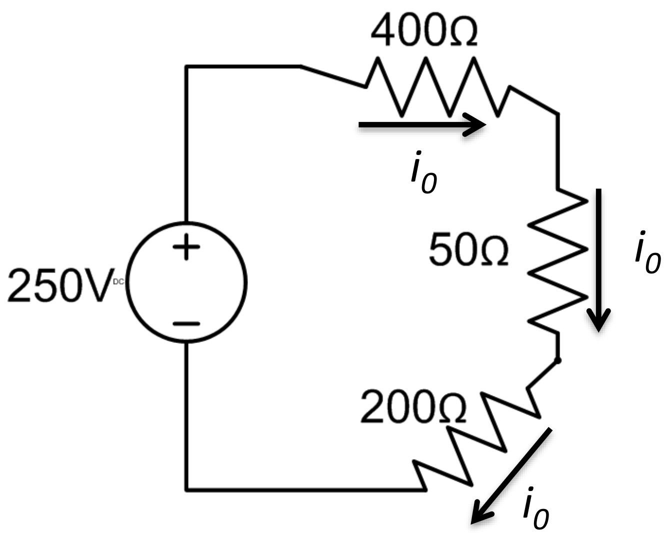


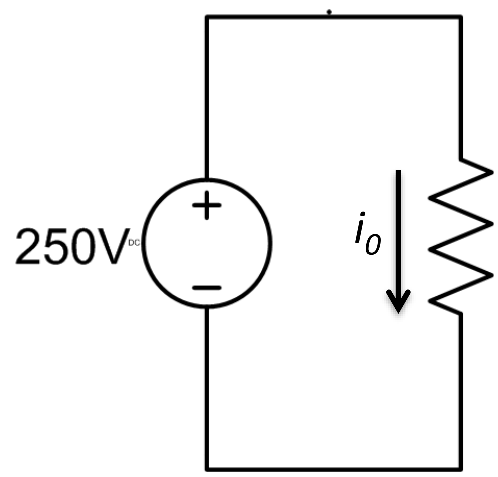
From KCL, the "dangling" resistors do not conduct current.

We therefore have a current $i_0 = i_3 = i_4$ flowing through the series combination of 400Ω, 50Ω and 200Ω resistors.

Recall that resistors in series carry the same current.

Example 1





R_{eq} = 400Ω+50Ω+200Ω
R_{eq} = 650 Ω

Ohm's Law: i_0 = 250V/650 Ω = 385mA

The current flow is very dangerous

Physiological Reaction	**Current**
Barely Perceptible	3-5mA
Extreme Pain	35-50mA
Muscle Paralysis	50-70mA
Heart Stoppage	500mA

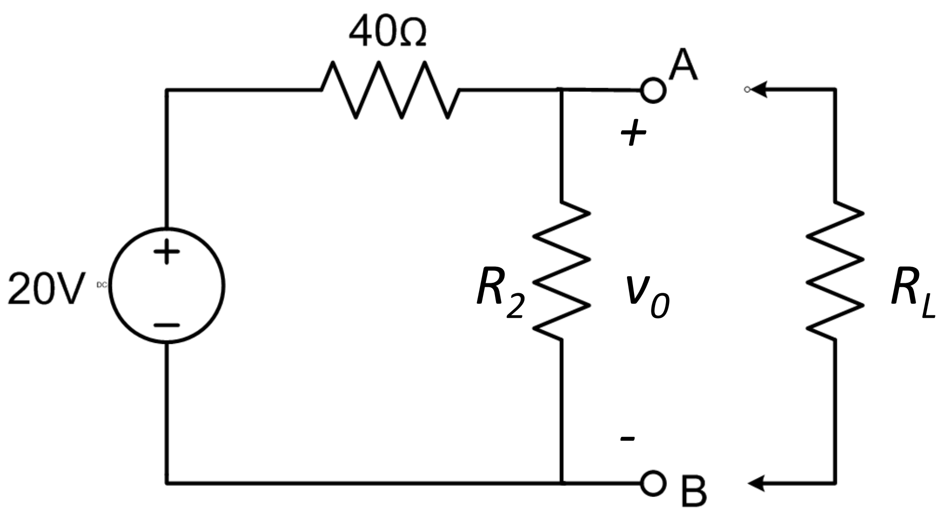
W.F Cooper, *Electrical Safety Engineering,* 2nd ed., Butterworth, 1986

Circuit Analysis - Example 2

In the absence of the load resistor R_L, the voltage divider produces v_0 = 4V.

When the load resistor is attached to the terminals A B, the voltage is v_0 = 3V.

What is the value of R_L?

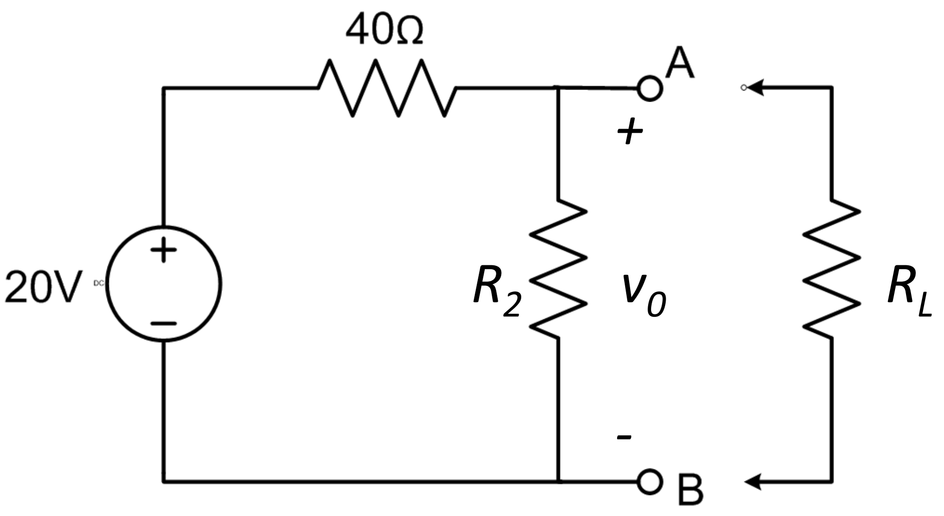


Strategy:

- find R_2 using the voltage divider equation
- find R_2 // R_L using the voltage divider equation, and thus find R_L

Example 2

Find R_2 using the voltage divider equation.



v_0 = 4V

$$\frac{4V}{20V} = \frac{R_2}{40\Omega + R_2}$$

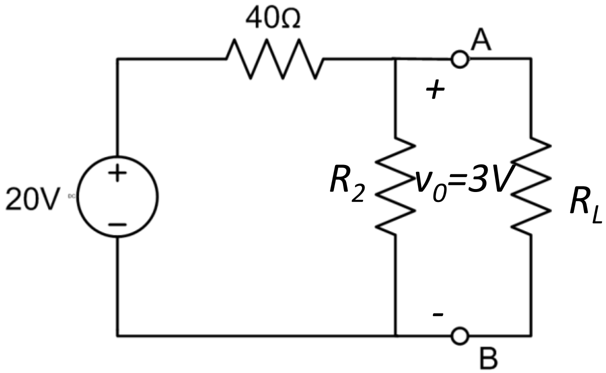
$$\frac{1}{5}(40\Omega + R_2) = R_2$$

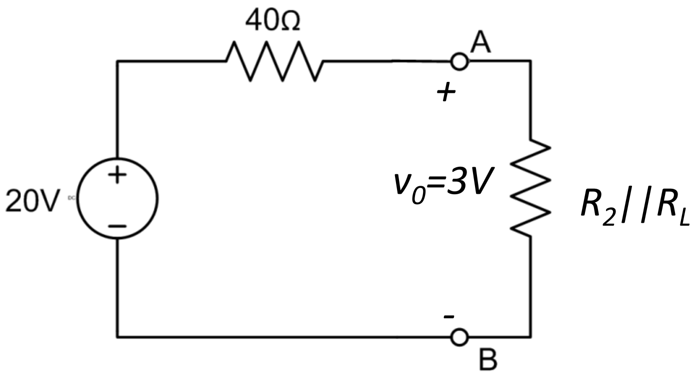
$$8\Omega = R_2\left(1 - \frac{1}{5}\right)$$

$$R_2 = \frac{5}{4}\cdot 8\Omega = 10\Omega$$

Example 2

Find $R_2\ //\ R_L$ using the voltage equation





$$\frac{3V}{20V} = \frac{R_2 \parallel R_L}{40\Omega + R_2 \parallel R_L}$$

$$\frac{3}{20}\left(40\Omega + R_2 \parallel R_L\right) = R_2 \parallel R_L$$

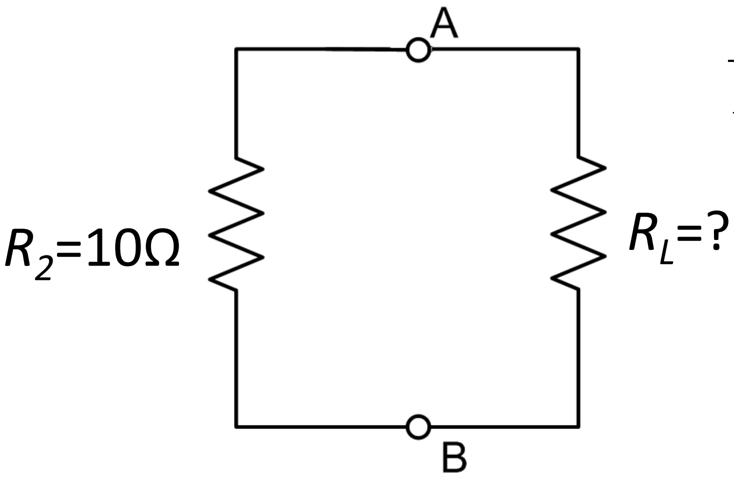
$$6\Omega = R_2 \parallel R_L \left(1 - \frac{3}{20}\right)$$

$$\begin{aligned} R_2 \parallel R_L &= \frac{20}{17} \cdot 6\Omega \\ &= \frac{120}{17}\Omega \\ &= 7.059\Omega \end{aligned}$$

Example 2

Find R_L from the value of R_2 and $R_2 \,//\, R_L$.

For parallel combinations, we can add conductances (G=1/R).



$$\frac{1}{R_2 \parallel R_L} = \frac{1}{R_2} + \frac{1}{R_L}$$

$$\frac{17}{120\Omega} = \frac{1}{10\Omega} + \frac{1}{R_L}$$

$$\frac{1}{R_L} = \frac{17}{120\Omega} - \frac{1}{10\Omega} = \frac{17}{120\Omega} - \frac{12}{120\Omega}$$

$$= \frac{5}{120\Omega}$$

$$R_L = 24\Omega$$

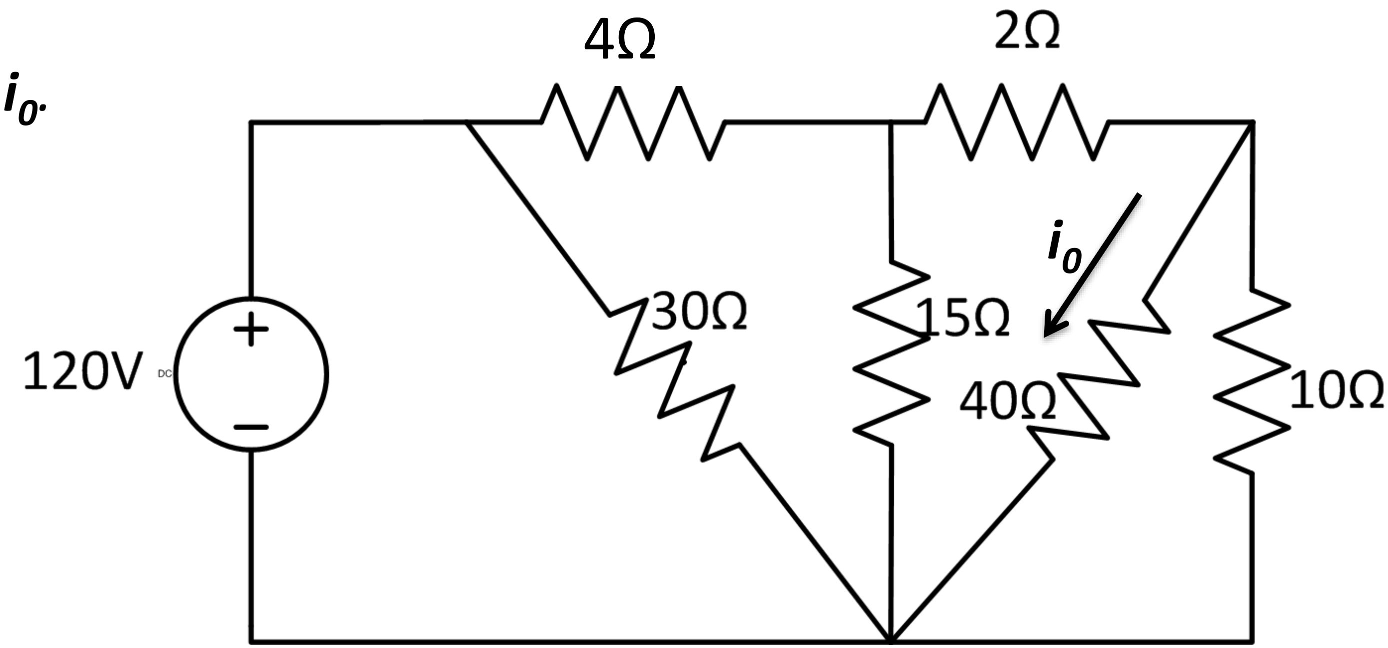
$$R_2 \parallel R_L = (120/17)\Omega$$

We could of course use decimals or fractions for this calculation.

If you elect to use decimals, use 4 significant digits (A.BCD).

Circuit Analysis - Example 3

Find the current $\boldsymbol{i_0}$.



Strategy:

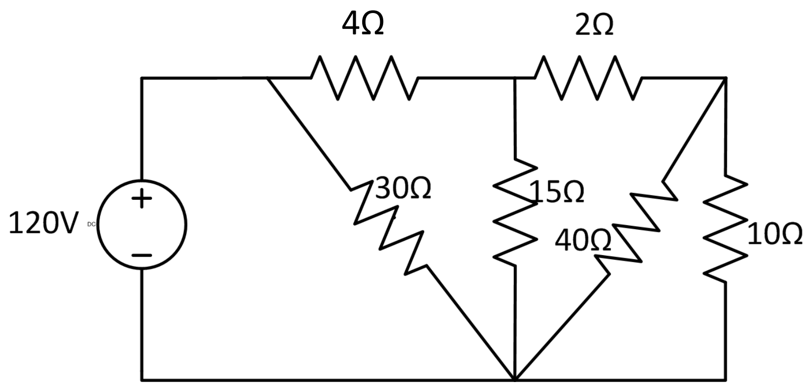
- reduce the circuit to a source and single equivalent resistor
- work through the equivalent circuits to find i_0

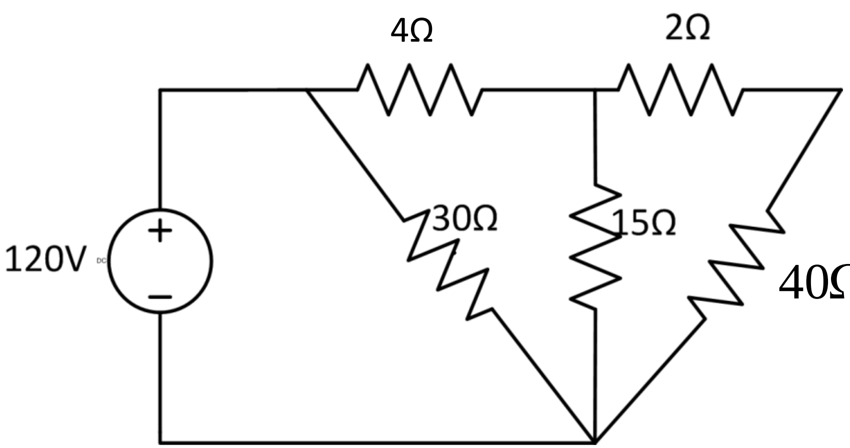
Note that we could also use KVL and KCL directly, but this example shows how creative one can be in using resistor equivalence and voltage/current dividers to solve a problem

McGill

Example 3

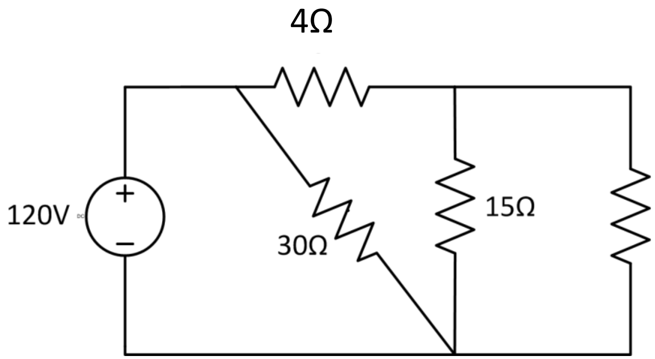
Reduce the circuit to a single equivalent resistors.



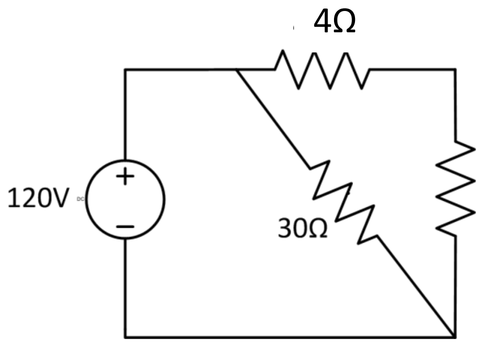


$$40\Omega \,||\, 10\Omega = \frac{10\Omega \cdot 40\Omega}{10\Omega + 40\Omega} = 8\Omega$$

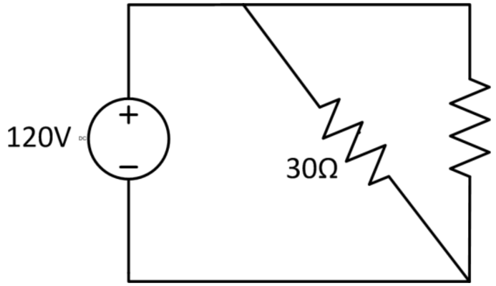
Example 3



$$8\Omega + 2\Omega = 10\Omega$$

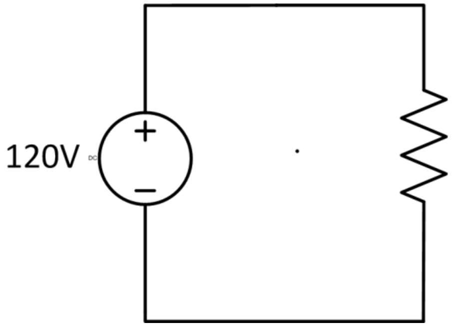


$$10\Omega \,||\, 15\Omega = \frac{10\Omega \cdot 15\Omega}{10\Omega + 15\Omega} = 6\Omega$$

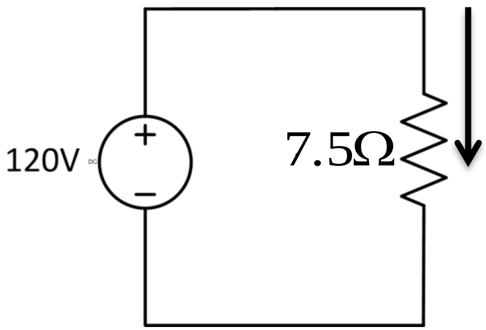


$$6\Omega + 4\Omega = 10\Omega$$

Example 3



$$10\Omega \,||\, 30\Omega = \frac{10\Omega \cdot 30\Omega}{10\Omega + 30\Omega} = 7.5\Omega$$

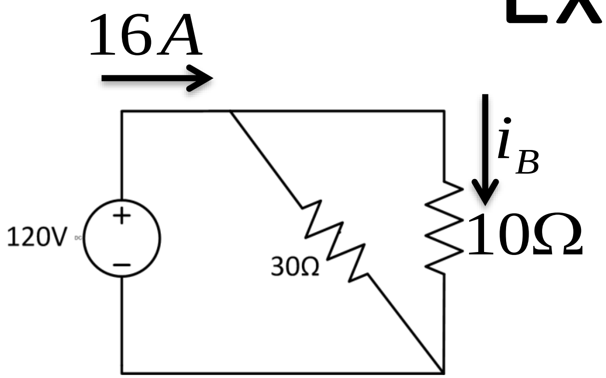


$$i_A = 120V / 7.5\Omega = 16A$$

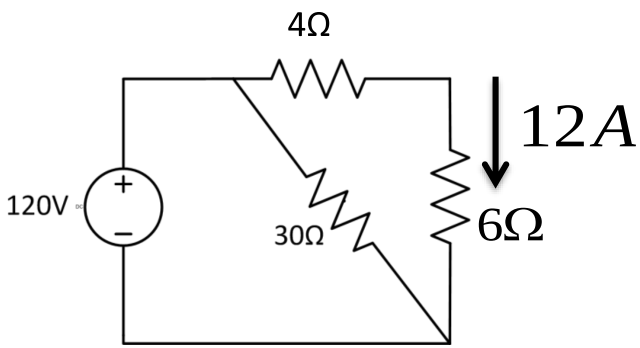
Note that the entire network of *resistors* is equivalent to a single 7.5Ω resistor across the terminals of the 120V source.

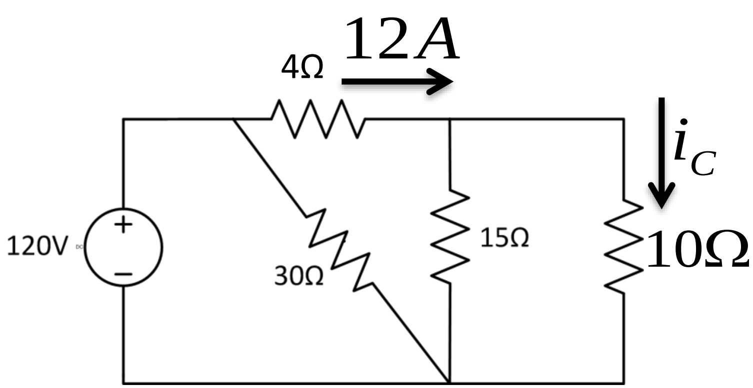
We now work forward through the equivalent circuits, solving for currents along the way.

Example 3



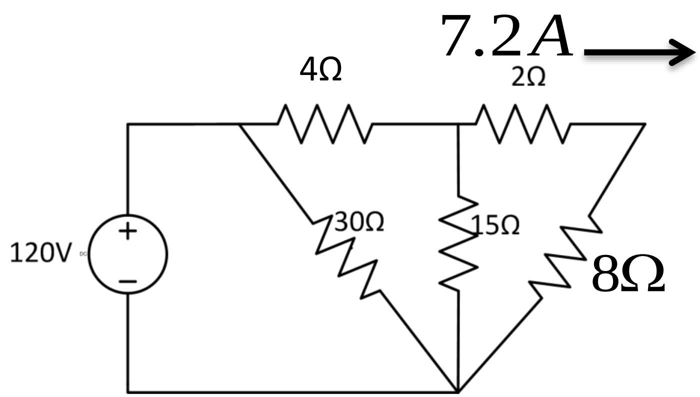
$$i_B = 16A\frac{30\Omega}{10\Omega + 30\Omega} = 12A$$

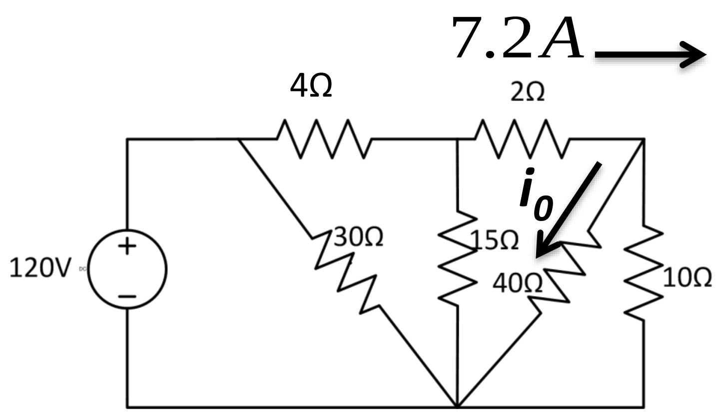




$$i_C = 12A\frac{15\Omega}{10\Omega + 15\Omega} = 7.2A$$

Example 3





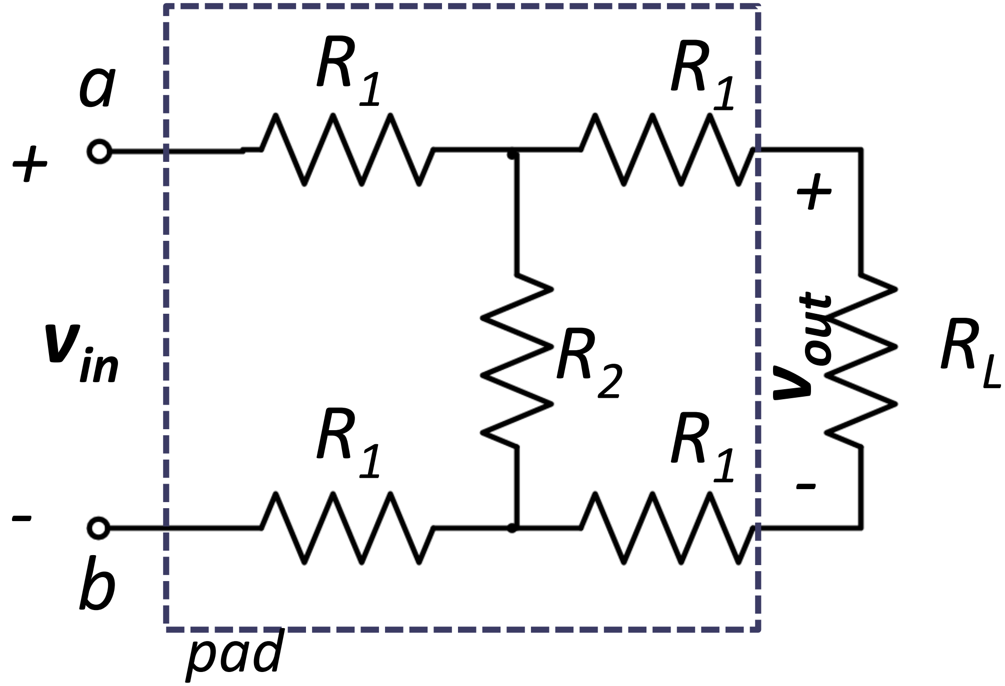
$$i_0 = 7.2A\frac{10\Omega}{10\Omega + 40\Omega} = 1.44A$$

Note that we have reduced the “large” circuit problem to a series of small, more easily solvable problems.

Circuit Analysis - Example 4

The circuit below is an **attenuator pad**, often used in volume control circuits.

What is the equivalent resistance at terminals a and b? What is the ratio of v_{out} to v_{in}?

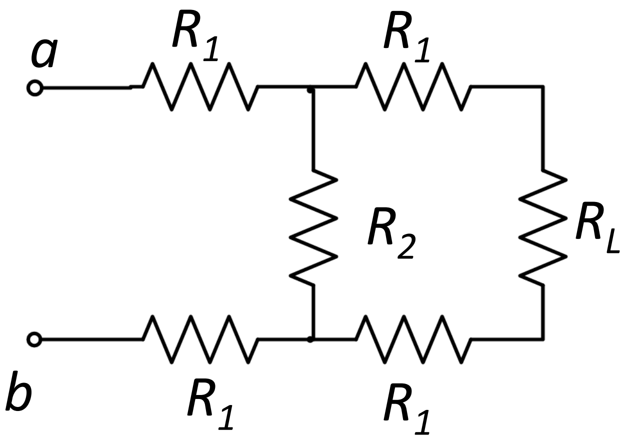


Strategy:

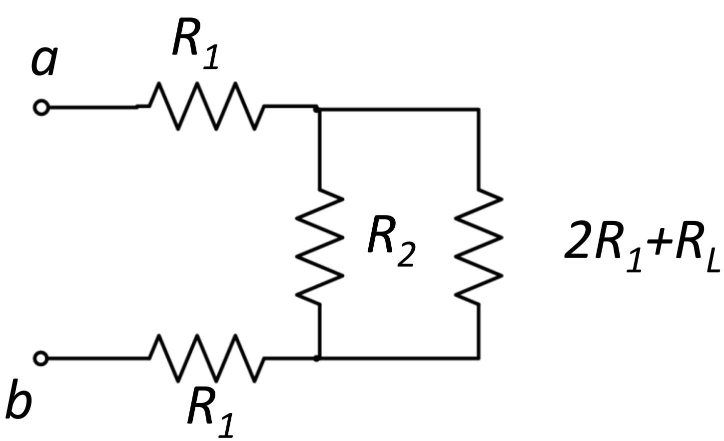
- apply equivalent resistance rules for series and parallel
- if that doesn't work, revert to definition of equivalent resistance $i=v/R_{eq}$

Example 4 → R_{ab}

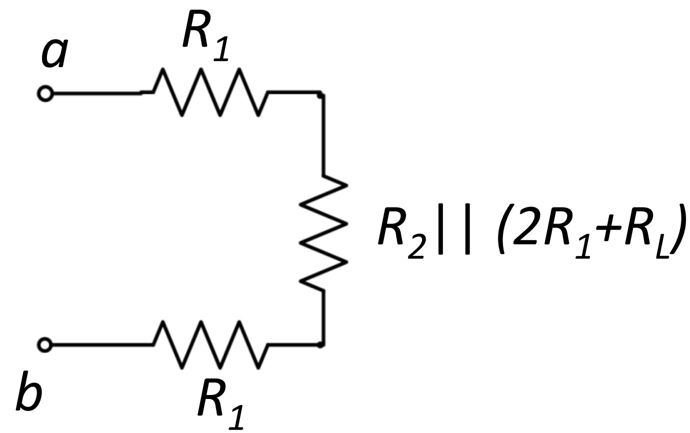
Step #1



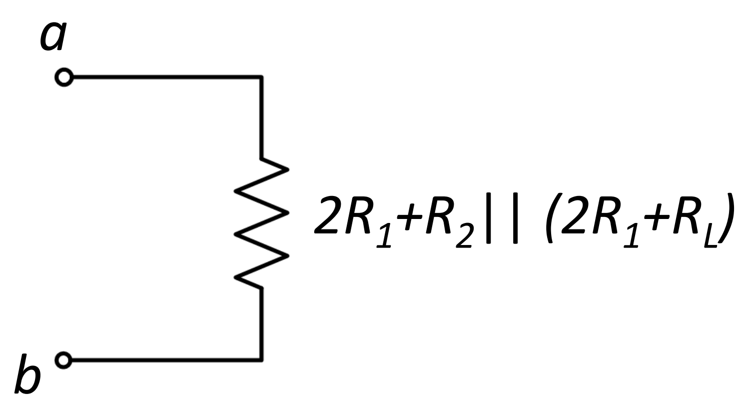
Step #2



Step #3

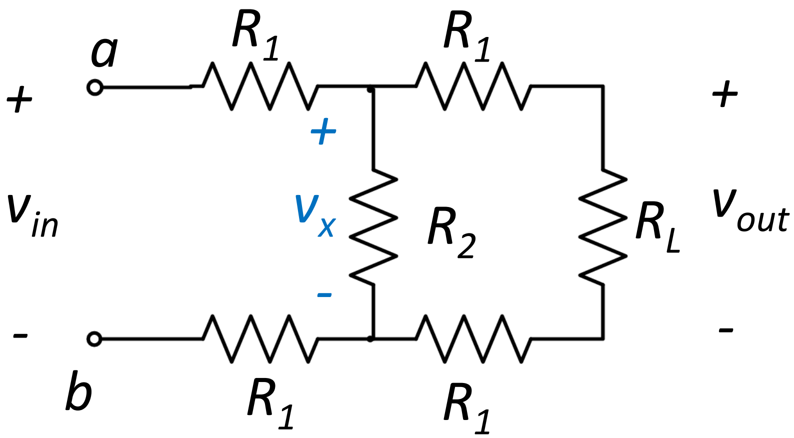


Step #4



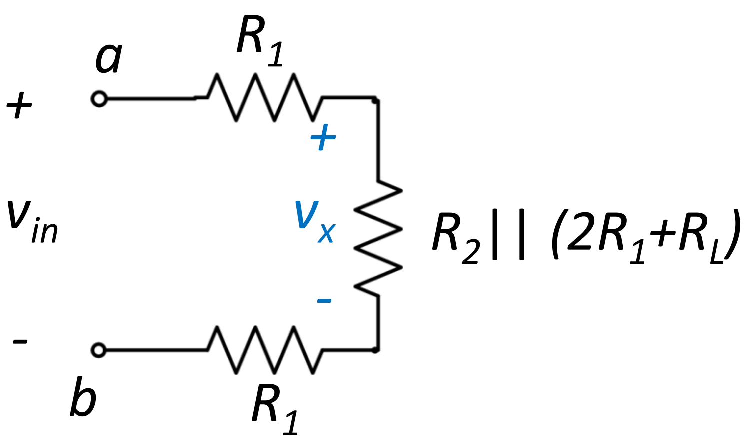
$$\boxed{R_{ab}=2R_1+R_2 || (2R_1+R_L)}$$

Example 4 → v_{out}/v_{in}



Applying a voltage divider equation:

$$v_{out} = v_x \frac{R_L}{2R_1 + R_L}$$

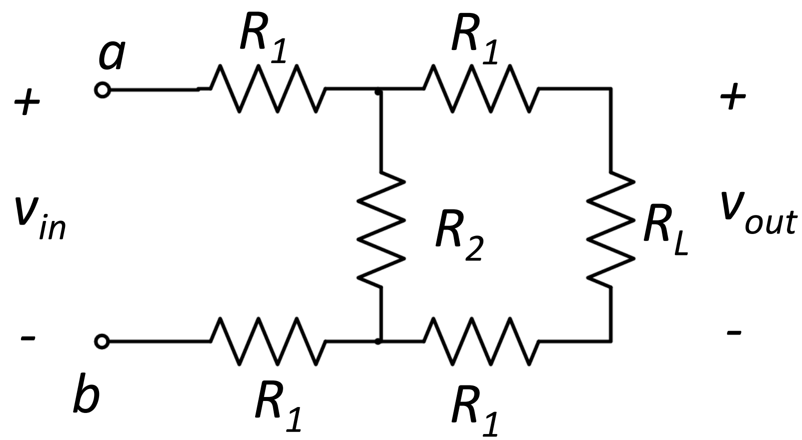


Applying another voltage divider equation:

$$v_x = v_{in} \frac{R_2 \,||\, (2R_1 + R_L)}{2R_1 + R_2 \,||\, (2R_1 + R_L)}$$

Combining results: $$\frac{v_{out}}{v_{in}} = \frac{v_{out}}{v_x}\frac{v_x}{v_{in}} = \frac{R_L}{2R_1 + R_L} \cdot \frac{R_2 \,||\, (2R_1 + R_L)}{2R_1 + R_2 \,||\, (2R_1 + R_L)}$$

Example 4



The equivalent resistance at terminals a and b (R_{ab}), and the voltage ratio v_{out}/v_{in} are given by:

$$R_{ab} = 2R_1 + R_2 \,||\, (2R_1 + R_L) \qquad \frac{v_{out}}{v_{in}} = \frac{R_L}{2R_1 + R_L} \cdot \frac{R_2 \,||\, (2R_1 + R_L)}{2R_1 + R_2 \,||\, (2R_1 + R_L)}$$

In practice, one specifies the load resistance R_L, the equivalent resistance R_{ab} and the voltage ratio v_{out}/v_{in} to determine the required R_1 and R_2 of the pad.

Note that there are many other pad geometries.

Measurements

- Precise measurement of current, voltage or resistance requires the use of a properly chosen/designed instrument

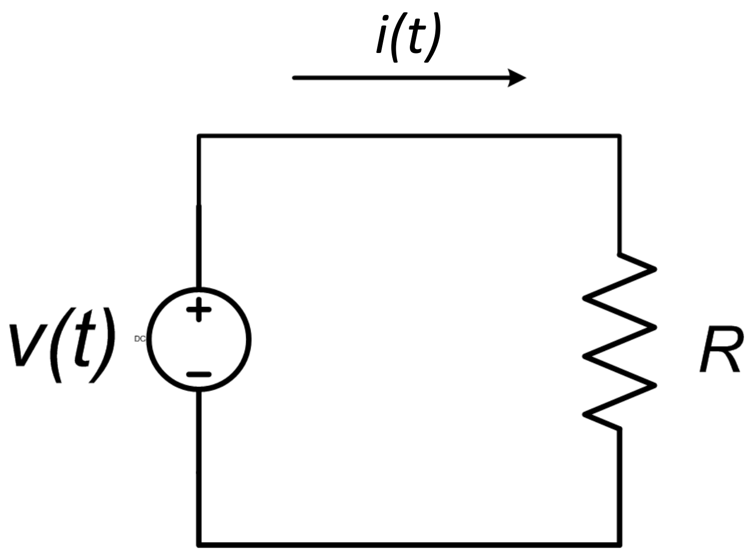
- **Ideal** ammeters, voltmeters, ohmmeters are elements (ideal models) permitting the perfect measurement of current, voltage or resistance

- **Real (in practice)**, physical instruments introduce errors; choosing the correct instrument in the laboratory requires an understanding of physical measurement instruments

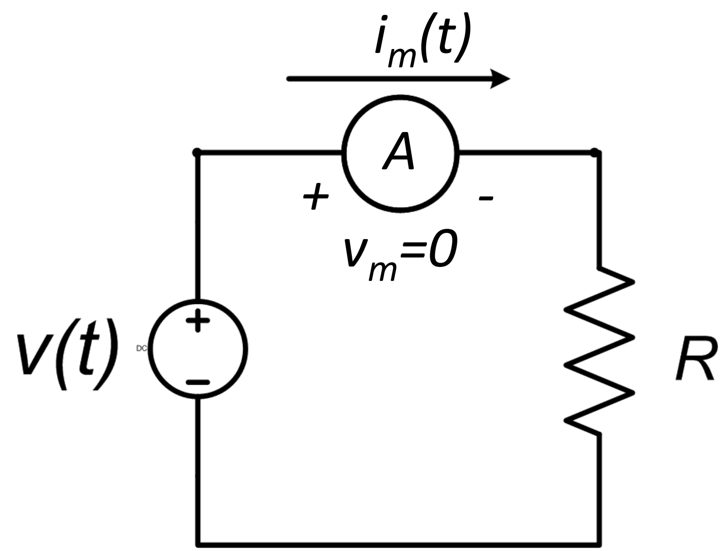
Ideal Ammeter

Ideal ammeter: an element that measures the current flowing through its terminals with zero power absorbed, meaning the ammeter voltage drop $v_m = 0$, equivalent to a short-circuit

- ammeter is placed in series with branch current, $i(t)$ to be measured
- the measured current $i_m(t) = i(t)$ because the ideal ammeter is equivalent to a short circuit in series, leaving circuit voltages unchanged



circuit

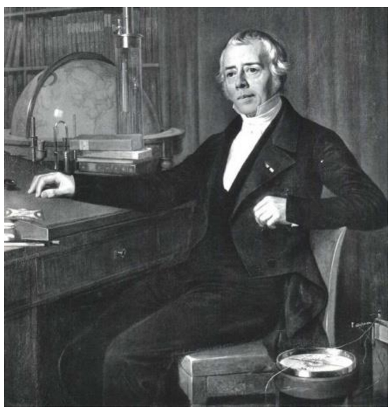


circuit with ideal ammeter

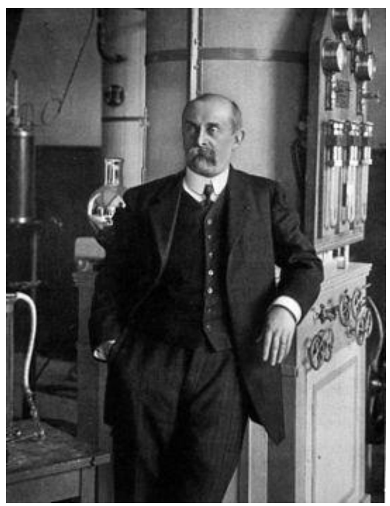
D’Arsonval Galvanometer

In practice, how can a current be “reported”?

A coil carrying a current in the presence of a magnetic field can be arranged to produce an observable movement



Hans Christian Ørsted (1777-1851)
Photo: makezine.com



Jacques-Arsène D’arsonval (1851-1940) *Photo*: *wikipedia*

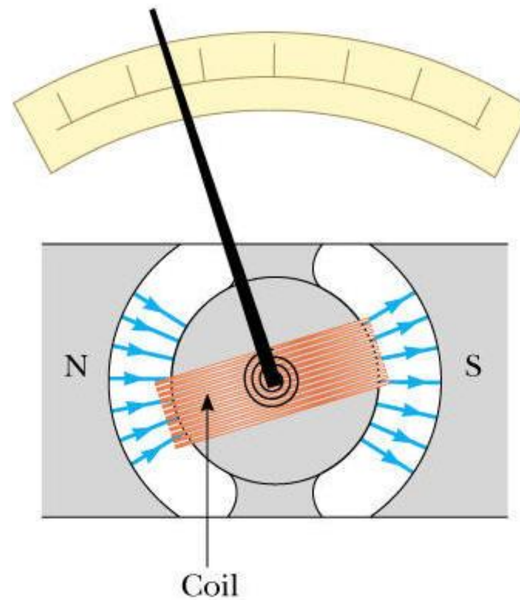


Photo: www.physics.byu.edu

$$I = \frac{2r}{\mu_0 n} \cdot B$$

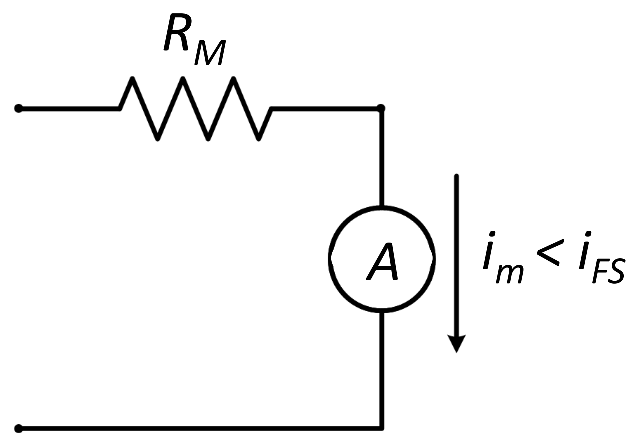
B: magnetic field
n: number of turns of coil
r: radius of coil
I: current

Physical Ammeter

A physical ammeter is characterized by:

- an **internal resistance** R_M (due to wire loop resistance for a galvanometer)
- a **full-scale current** limit i_{FS} (due to the limit of mechanical deflection for a galvanometer)

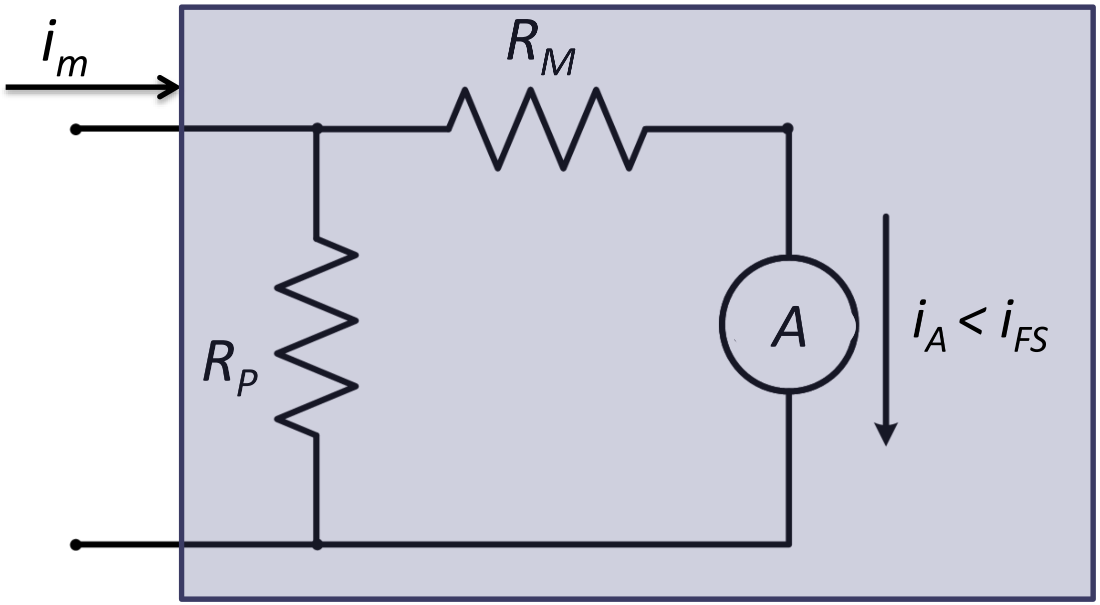
Modern ammeters based on microelectronic circuits have internal resistance and a full-scale current limit arising from other physical reasons



physical ammeter modeled
with a resistor and ideal ammeter

Practical Ammeter

To measure a current i_m greater than the full-scale current i_{FS} of a galvanometer, a *current divider* is used to divert a known fraction of the current from the galvanometer



internal resistance:

$$R_{int} = R_P \,||\, R_M$$

ratio of i_m to i_A:

$$i_A = i_m \frac{R_P}{R_P + R_M}$$

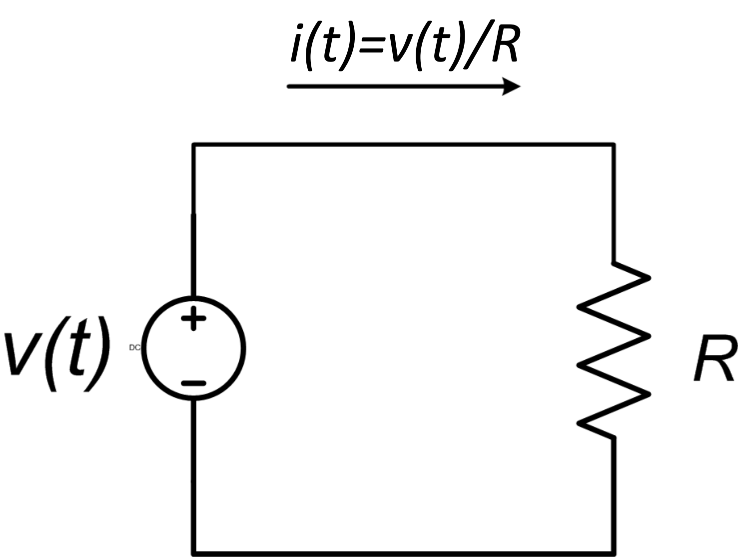
$$i_m = i_A \frac{R_P + R_M}{R_P}$$

$$= i_A \left(1 + R_M / R_P\right)$$

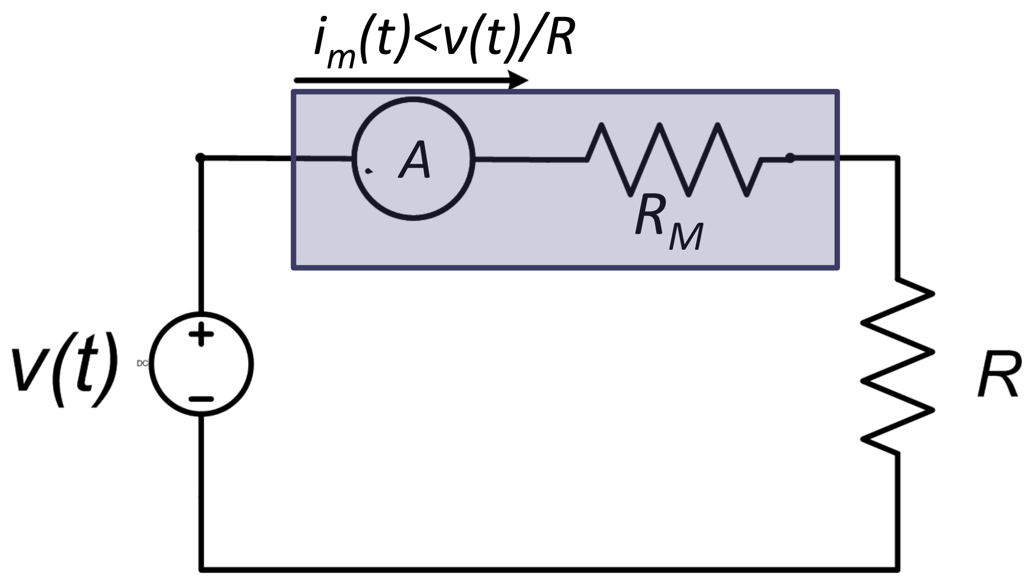
Measurement Error

Practical meters perturb the circuit being measured, causing a change in the flow of current and distribution of voltage, leading to **measurement error.**

In the case of the circuit below, the finite resistance of the ammeter clearly modifies the current flow in the circuit.



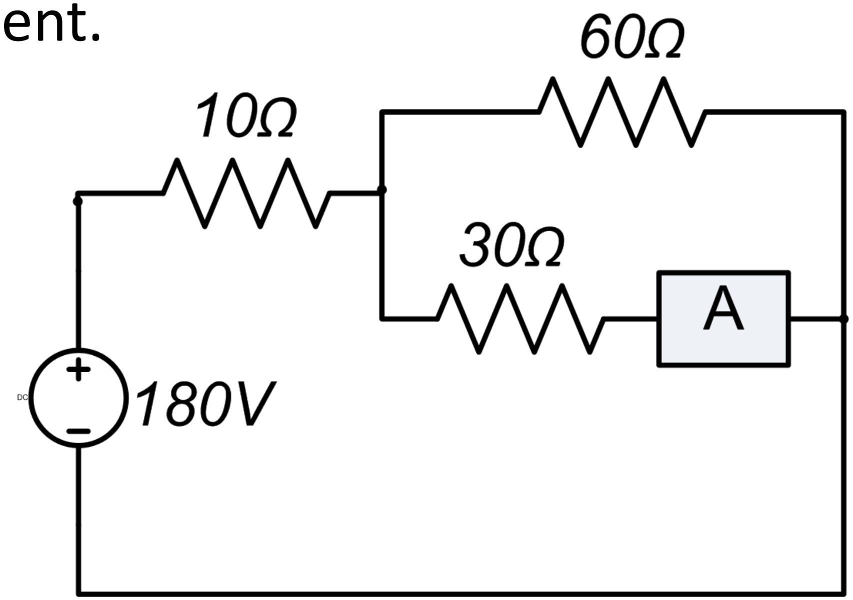
circuit



circuit with practical ammeter

Practical Ammeter Example

The ammeter below (box labeled A) has a total internal resistance of 0.5Ω. Find the percentage error in the ammeter measurement.

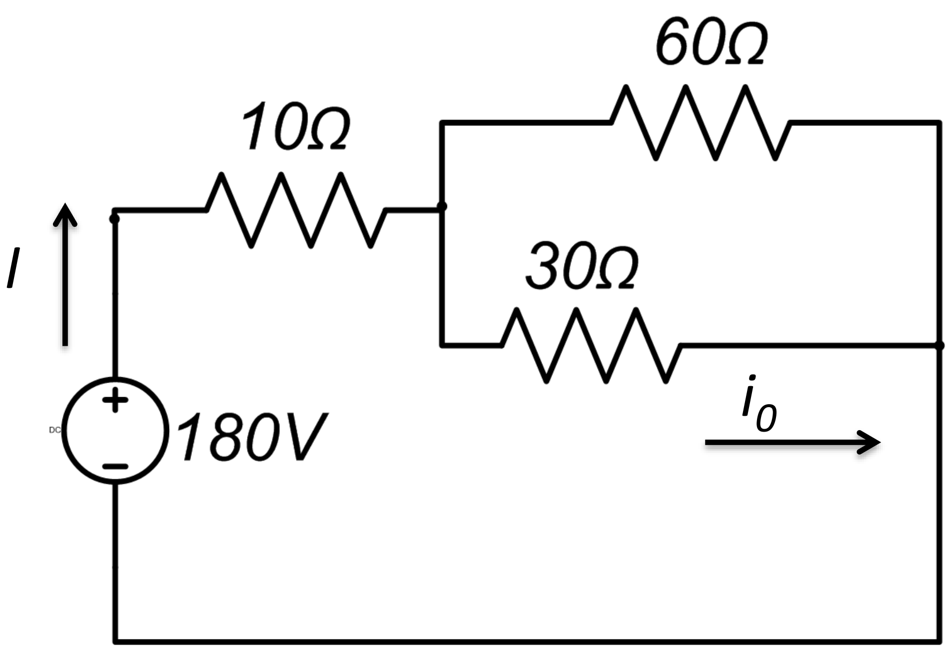


Strategy:

- find the ideal current and the actual measured current

Example

First, find the ideal current



The equivalent resistance "seen" by the source:

$$R_{eq} = 10\Omega + 30\Omega \,||\, 60\Omega$$
$$= 10\Omega + 20\Omega$$
$$= 30\Omega$$

Find the total current drawn:

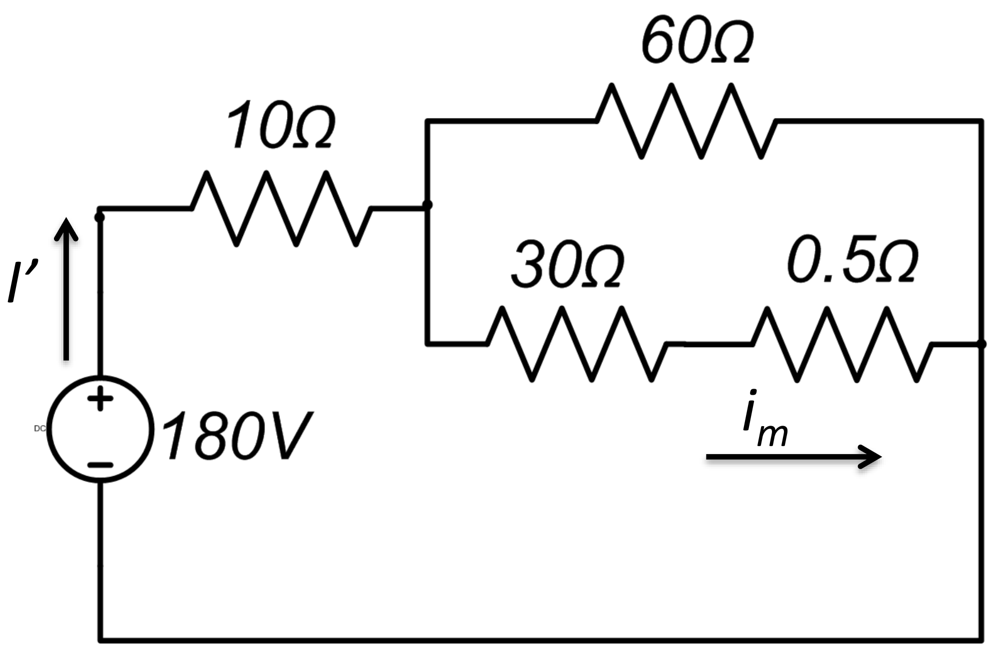
$$I = 180V / 30\Omega = 6A$$

Use the current divider:

$$i_0 = 6A \frac{60\Omega}{30\Omega + 60\Omega} = 4A$$

Example

Then, find the measured current



The equivalent resistance "seen" by the source:

$$R_{eq} = 10\Omega + (30\Omega + 0.5\Omega) \,||\, 60\Omega$$
$$= 10\Omega + 20.22\Omega$$
$$= 30.22\Omega$$

Find the total current drawn:

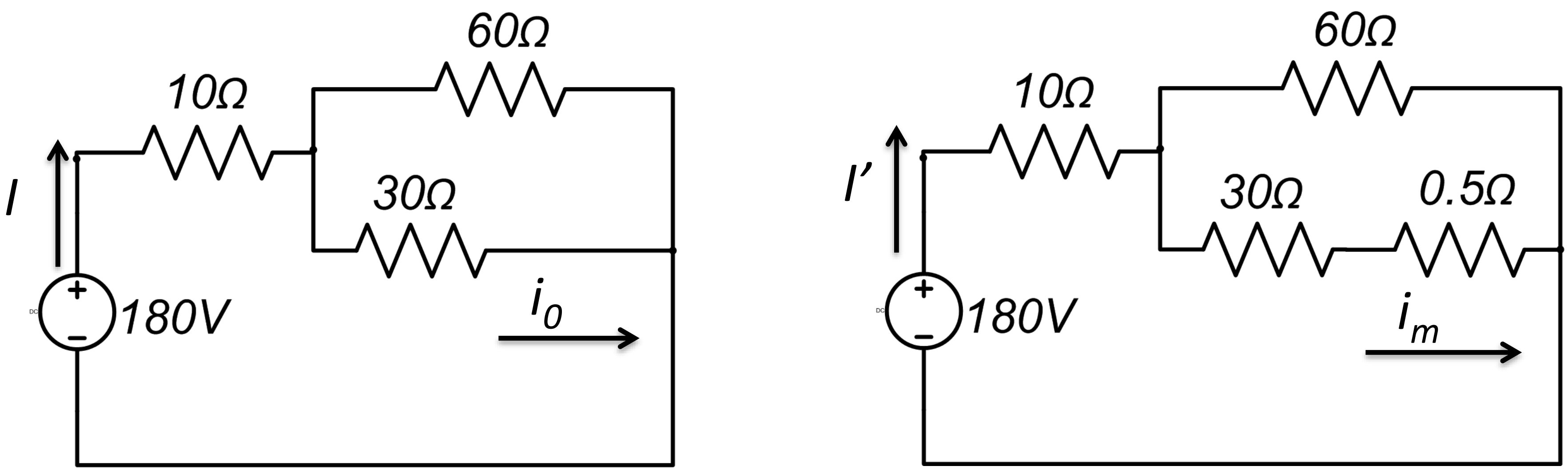
$$I' = 180V/30.22\Omega = 5.956A$$

Use the current divider:

$$i_m = 5.956A \frac{60\Omega}{30.5\Omega + 60\Omega} = 3.949A$$

Example

Compare the ideal current and the actual measured current.



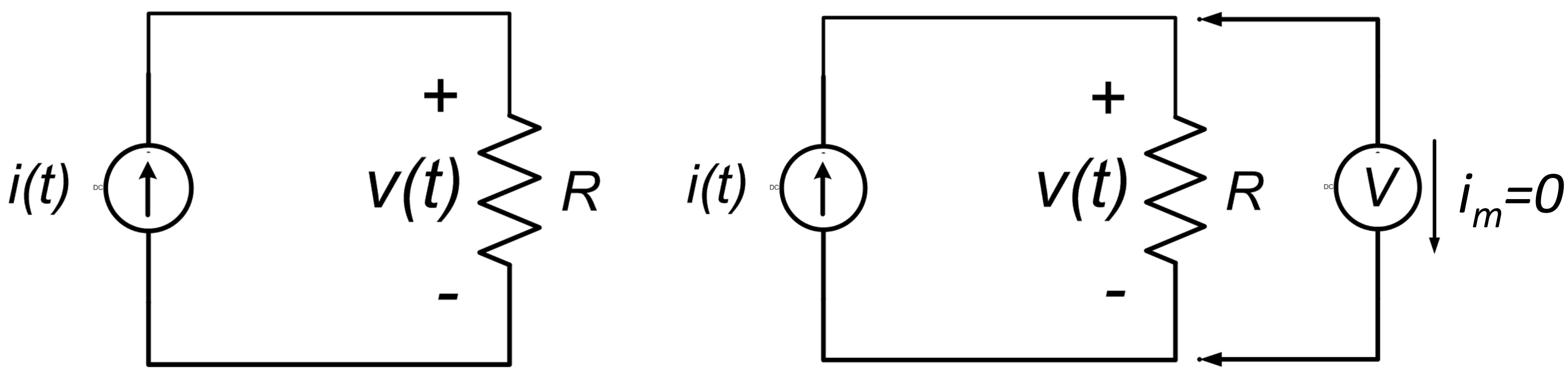
Percentage error = $\dfrac{3.949A-4A}{4A}\times 100\% = -1.30\%$

McGill

Ideal Voltmeter

Ideal voltmeter: an element that measures the voltage across its terminals with zero power absorbed, meaning the voltmeter current drawn $i_m = 0$, equivalent to an open-circuit.

- voltmeter is placed in parallel with branch voltage, *v(t)* to be measured
- the measured voltage $v_m(t) = v(t)$ because the ideal voltmeter is equivalent to an open circuit in parallel, leaving circuit currents unchanged.

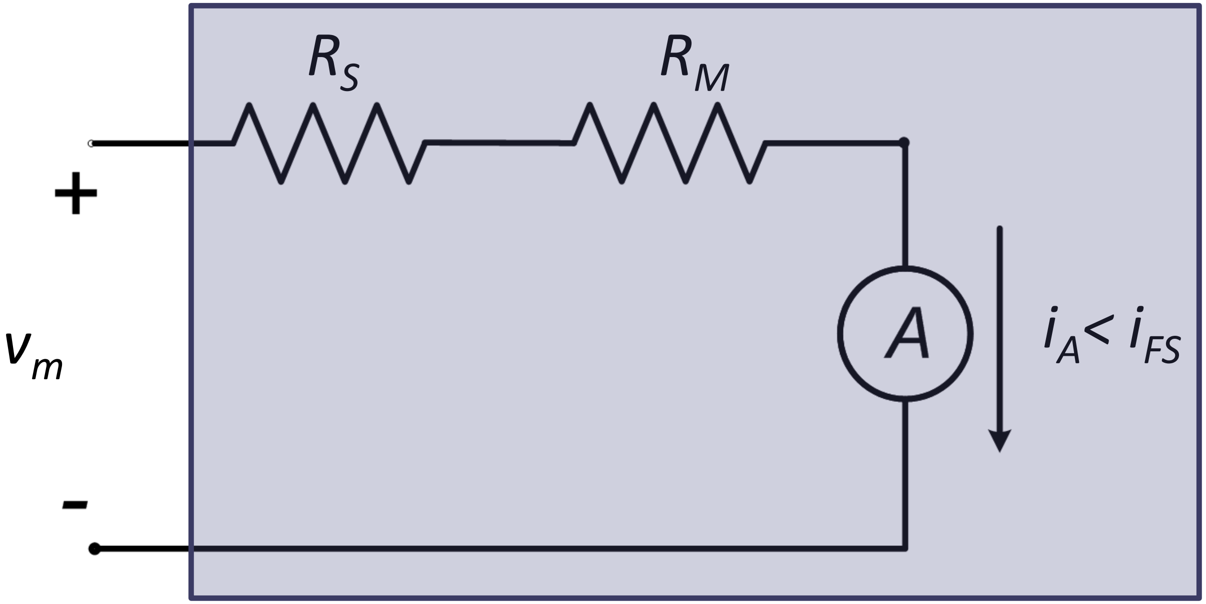


circuit

circuit with ideal voltmeter

Practical Voltmeter

- To measure a voltage with a galvanometer, a *potential divider* using a large series resistor R_s is used to reduce the potential, and thus current flow, through the galvanometer
- A practical voltmeter thus has a finite internal resistance



internal resistance:

$$R_{int} = R_S + R_M$$

ratio of v_m to i_A:

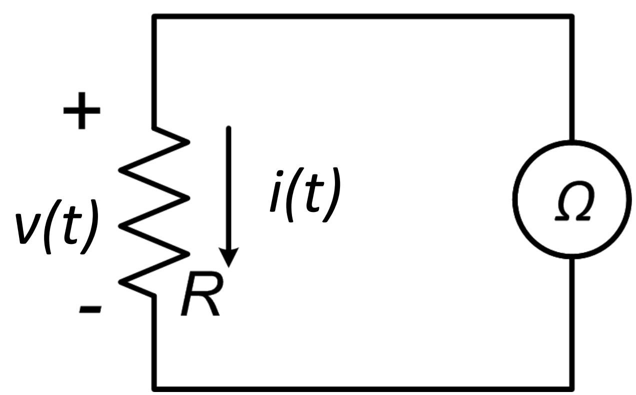
$$v_m = i_A\left(R_S + R_M\right)$$

ideal Ohmmeter

Ideal ohmmeter: an element that measures the resistance across its terminals with zero power delivered, meaning the voltage and current at the ohmmeter terminals is zero

- ohmmeter is placed across the resistor terminals

circuit equivalent resistor

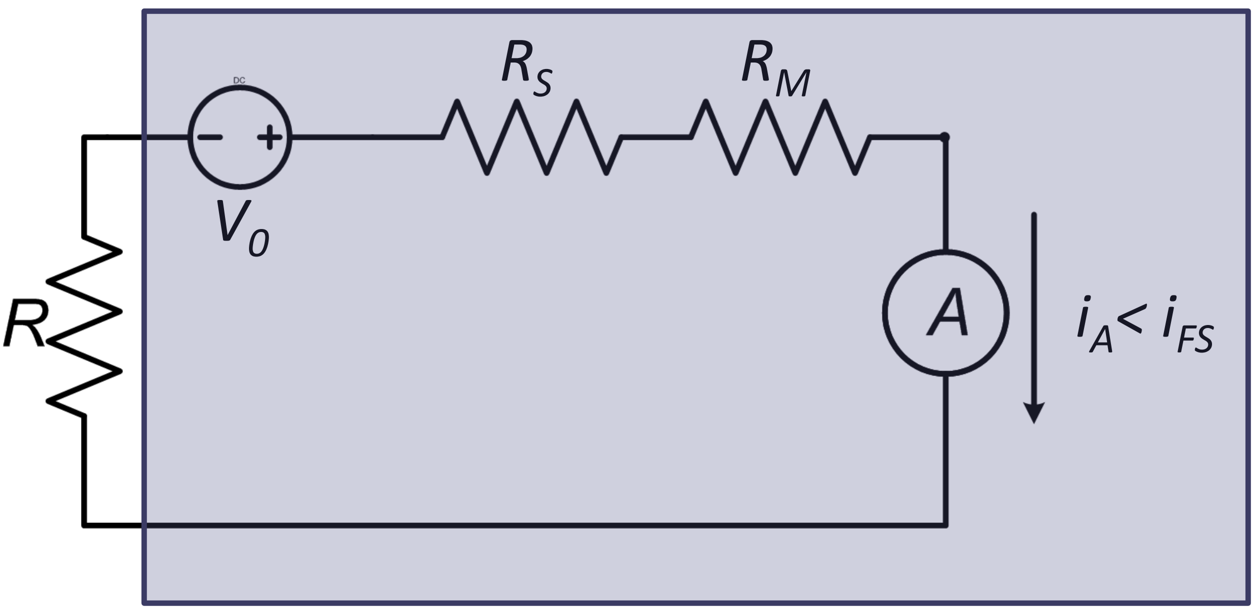


$$p(t) = i^2(t)R = v^2(t)/R = 0$$

equivalent resistor with ideal ohmmeter

Practical Ohmmeter

- To measure a resistance with a galvanometer, a *voltage source* is used along with a series resistor
- A practical ohmmeter thus applies a finite voltage to the resistance being tested and measure the current



practical ohmmeter incorporating a voltage source, series resistor and galvanometer

<u>ratio of R to i_A:</u>

$$i_A = \frac{V_0}{R_S + R_M + R}$$

$$R = \frac{V_0}{i_A} - R_M - R_S$$

Module 2 Summary

Kirchhoff's Current Law and Kirchhoff's Voltage Law: Arising from physical conservation of charge and energy, these laws allow us to write equations relating algebraic current and voltage variables. These laws are the basis of all circuit analysis.

Equivalent Resistance: Any two-terminal circuit composed of ideal resistors has identical terminal characteristics to that of a single equivalent resistor, a fact that can be used to simplify circuit analysis problems. The equivalent resistance for series and parallel combinations of resistors are particularly useful.

Voltage Division and Current Division: Series resistors divide voltage as the ratio of resistance. Parallel resistors divide current as the ratio of conductance.

Ammeters, Voltmeters, Ohmmeters: ideal meters do not transfer any power to/from the circuit under test, but practical meters do. The finite resistance of ammeters and voltmeters perturbs the circuit under test.