McGill

December 2017
Final Examination

Electric Circuits 1

ECSE 200A - 001

Monday, December 18, 2017, at 2:00 PM

EXAMINER: Prof. Odile Liboiron-Ladouceur ASSOC. EXAMINER: Prof. Sharmistha Bhadra

STUDENT NAME:	SOLUTIONS	McGILL ID:									

INSTRUCTIONS:

EXAM:	CLOSED BOOK ☒ OPEN BOOK ☐
	SINGLE-SIDED ☐ PRINTED ON BOTH SIDES OF THE PAGE ☐
	MULTIPLE CHOICE ANSWER SHEETS ☐ ANSWER IN BOOKLET ☒ EXTRA BOOKLETS PERMITTED: YES ☒ NO ☐ ANSWER ON EXAM ☐
	SHOULD THE EXAM BE: RETURNED ☒ KEPT BY STUDENT ☐
CRIB SHEETS:	NOT PERMITTED ☒ PERMITTED ☐ e.g. one 8 1/2X11 handwritten double-sided sheet Specifications:
DICTIONARIES:	TRANSLATION ONLY ☐ REGULAR ☐ NONE ☒
CALCULATORS:	NOT PERMITTED ☐ PERMITTED (Faculty Standard Only) ☒
ANY SPECIAL INSTRUCTIONS: e.g. molecular models	

Carefully read each question. There is a total of five questions. Write your answers in the exam booklet. Write your name on the exam booklet. Each question has multiple parts (a, b, c, d ...) to answer. Show the steps you take to find your answer and clearly indicate your answer. Write your answers using SI units and SI multipliers. Each question is worth 20 points for a total of 100 points.

Question 1

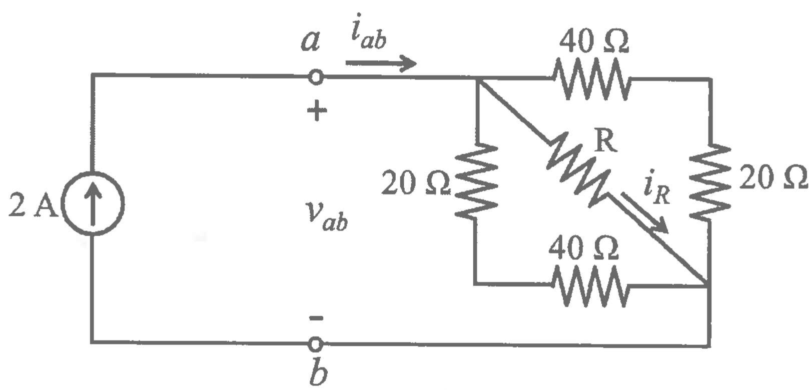
Consider the circuit shown below. Answer the following questions.

a) What is the voltage v_{ab} if the resistance R is infinite ($R \rightarrow \infty$)?

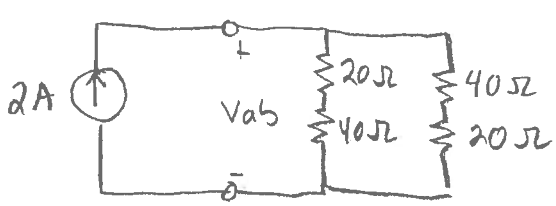
b) What is the current i_R through the resistor R if its resistance is zero ($R = 0\ \Omega$)?

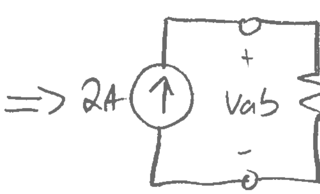
c) What is the power delivered by the current supply if the resistance R is 60 Ω ($R = 60\ \Omega$)?

d) Illustrate the relationship between the current i_{ab} and the voltage v_{ab} by plotting the i_{ab}-v_{ab} diagram of the circuit on the right side of the a-b terminals (i.e., disconnect the current supply). Clearly label your diagram. Assume the resistor R has a resistance R of 30 Ω ($R = 30\ \Omega$).



1a)

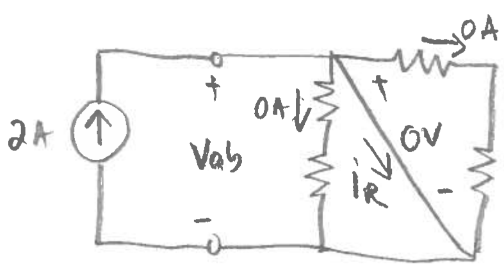




$R_{eq} = 60\Omega // 60\Omega = 30\Omega$

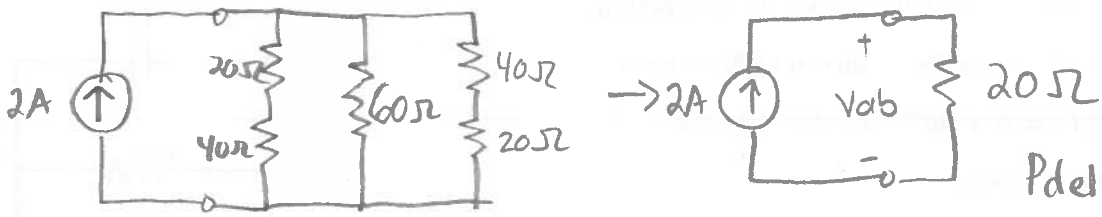
$\boxed{V_{ab} = 60V}$

b)



$\boxed{i_R = 2A}$

1c)

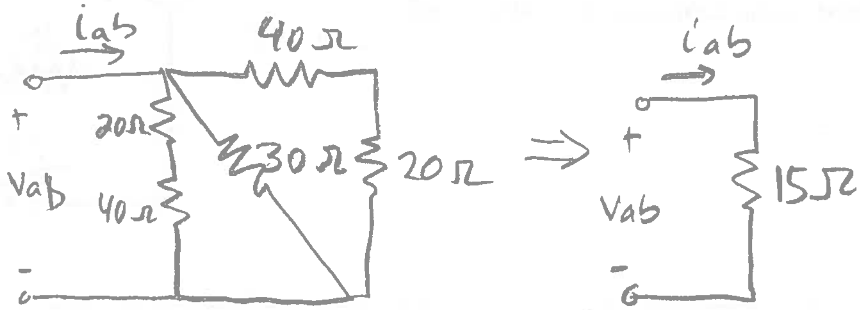


$P_{del} = 2A \cdot V_{ab}$

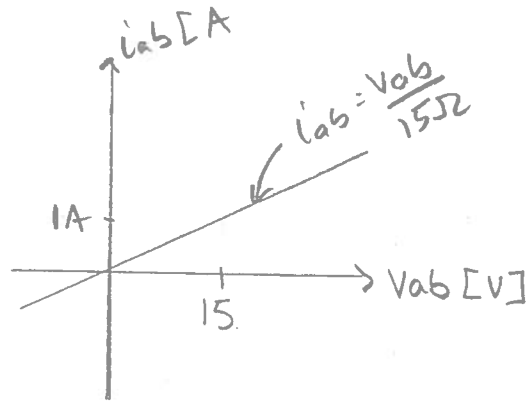
$= 2A \cdot 2A \cdot 20\Omega$

$\boxed{P_{del} = 80W}$

1d)



$i_{ab} = \dfrac{V_{ab}}{15\Omega}$



Question 2

Consider the circuit shown below. Answer the following questions.

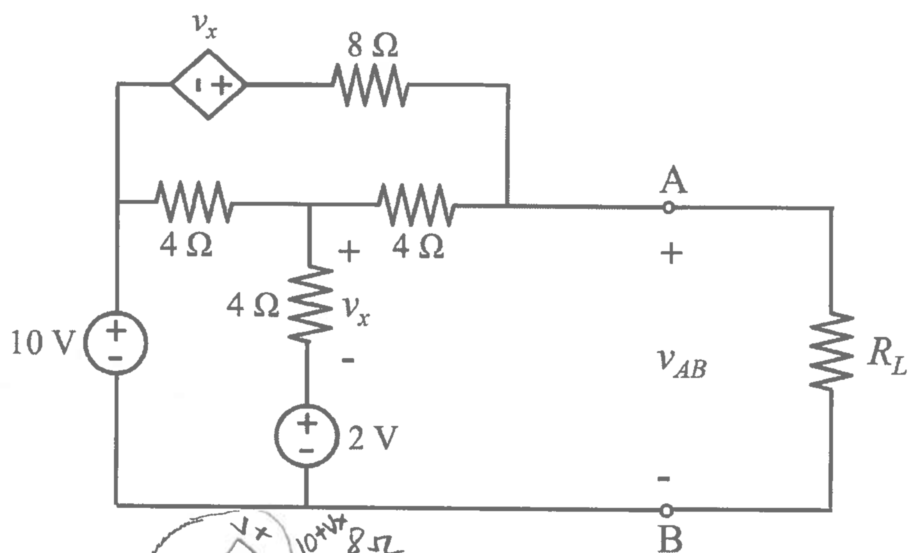
a) What is the value of v_x when an open circuit is applied to terminals A and B ($R_L \rightarrow \infty$)?

b) What is the value of v_x when a short circuit is applied to terminals A and B ($R_L \rightarrow 0$)?

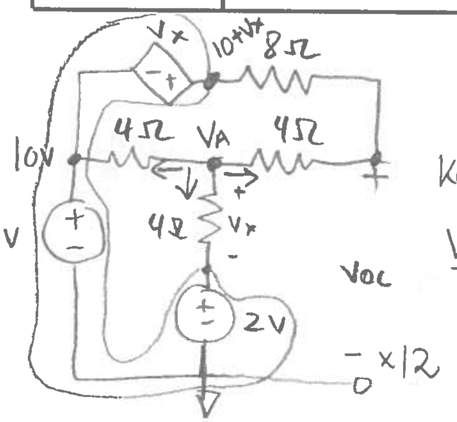
c) What is the Norton equivalent circuit with respect to the terminals A and B?

d) What is the maximum power that the circuit can deliver to an optimally chosen load resistor R_L?

e) What should R_L be for maximum power transfer if the current i_{AB} is 1 A?



a)



using nodal analysis

KCL to find node voltage VA

Control variable

$V_x = V_A - 2V$

$$\frac{V_A - 2V}{4\Omega} + \frac{V_A - 10V}{4\Omega} + \frac{V_A - (10 + V_x)}{4\Omega + 8\Omega} = 0$$

$\times 12$

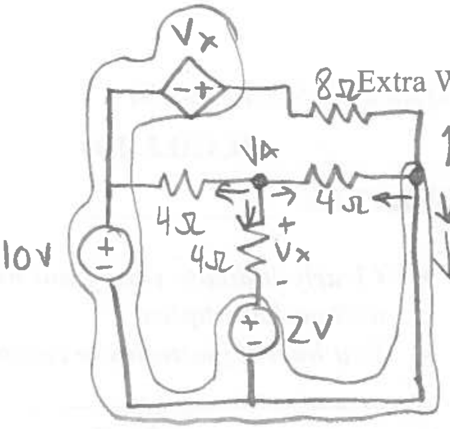
$$3V_A - 6V + 3V_A - 30V + V_A - 10V - V_A + 2V = 0$$

$$6V_A - 44V = 0 \qquad V_A = \frac{22}{3}V$$

$$\boxed{V_x = \frac{22}{3} - \frac{6}{3} = \frac{16}{3}V}$$

Extra Working Space

b) $R_L = 0$



KCL to find VA

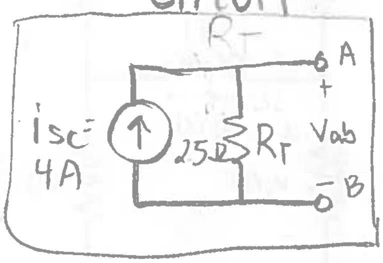
$$\frac{V_A - 10V}{4\Omega} + \frac{V_A - 2V}{4\Omega} + \frac{V_A}{4\Omega} = 0$$

$$V_A - 10V + V_A - 2V + V_A = 0$$

$$3V_A = 12V \quad V_A = 4V$$

$$\boxed{V_x = V_A - 2V = 2V}$$

c) Norton equiv circuit:

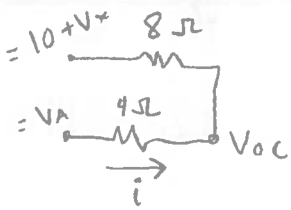


from conf. in b), find isc

KCL at terminal A: $\frac{0V - 4V}{4\Omega} + i_{sc} + \frac{0 - 12V}{4\Omega} = 0$

$$i_{sc} = \frac{16V}{4\Omega} = 4A$$

for R_T, find voc from part a)



$$i = \frac{V_A - (10 + V_x)}{4\Omega + 8\Omega} = \frac{V_A - (10 + V_A - 2)}{12} = \frac{-10+2}{12} = -\frac{8}{12}A$$

$$i = -\frac{2}{3}A$$

$$\frac{V_A - V_{oc}}{4\Omega} = i \rightarrow V_A - 4i = V_{oc}$$

$$\frac{22}{3} + 4 \cdot \frac{2}{3} = V_{oc} \qquad V_{oc} = \frac{30}{3}V = 10V$$

$$R_T = \frac{V_{oc}}{i_{sc}} = \frac{10V}{4A} = \frac{5}{2}\Omega = 2.5\Omega$$

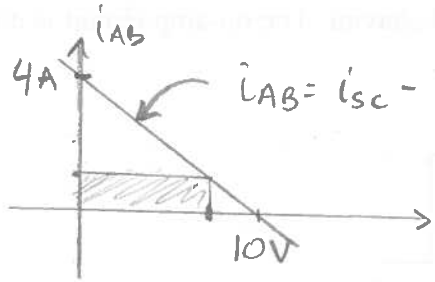
d) $R_L = R_T = 2.5\Omega$

$\uparrow$ from part c)

$$Pwr_{MAX} = \frac{V_{OC}}{2} \cdot \frac{I_{SC}}{2} = \frac{i_{SC} \cdot R_T \cdot i_{SC}}{4} = \frac{i_{SC}^2 \cdot R_T}{4} = \frac{4^2}{4} \cdot 2.5$$

$$\boxed{Pwr_{MAX} = 10\,W}$$

e)



$$i_{AB} = i_{SC} - \frac{V_{AB}}{R_T} \rightarrow 1A = 4A - \frac{V_{AB}}{2.5\Omega}$$

$$3A = \frac{V_{AB}}{2.5\Omega}$$

$$\boxed{V_{AB} = 7.5V}$$

Question 3

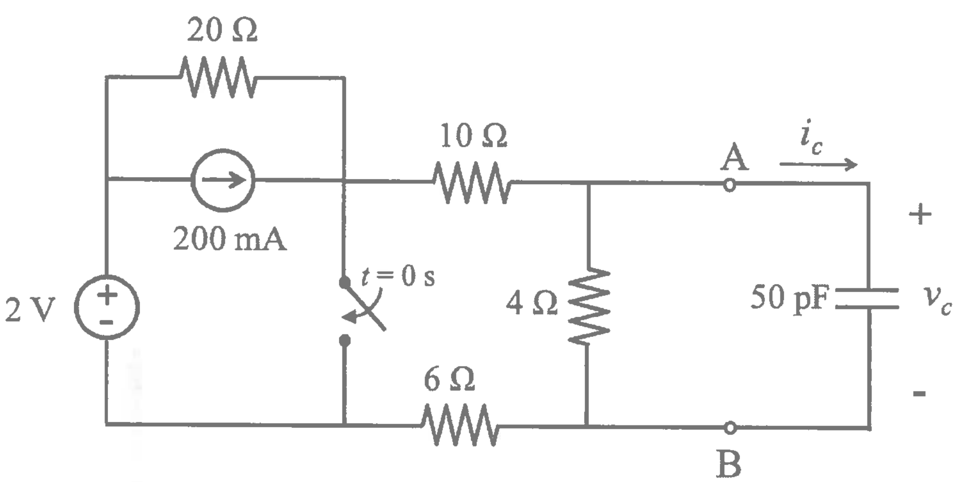
Consider the circuit shown below. The circuit is in dc steady state for $t < 0$ s. The switch closes at $t = 0$ s. Answer the following questions.

a) Draw the Thevenin equivalent circuit connected to the capacitor for $t < 0$ s.

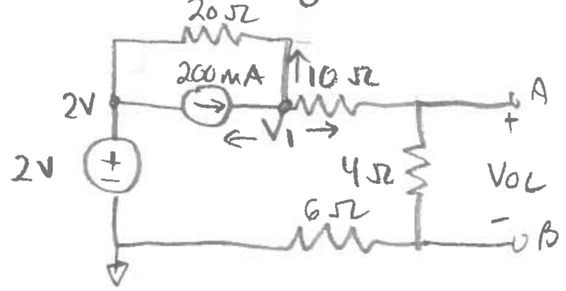
b) Draw the Thevenin equivalent circuit connected to the capacitor for $t > 0$ s.

c) What is $v_c(t)$ for $t > 0$?

d) What is $i_c(t)$ for $t > 0$?



3a) $t<0s$ steady state



only ind. src, so find Voc, then R_T

$$V_{oc} = V_1 \cdot \frac{4\Omega}{6\Omega + 4\Omega + 10\Omega}$$

find V_1 with nodal analysis

$$-0.2A + \frac{V_1 - 2V}{20\Omega} + \frac{V_1}{20\Omega} = 0$$

$$\times 20 \rightarrow -4V + V_1 - 2V + V_1 = 0$$

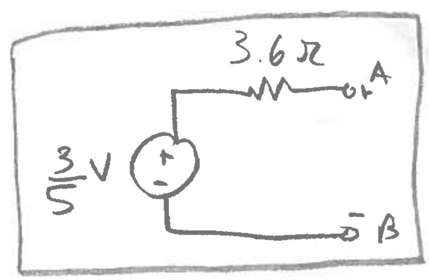
$$2V_1 = 6V \rightarrow V_1 = 3V$$

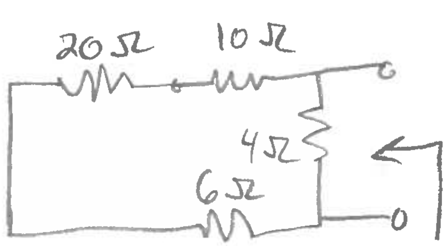
$$V_{oc}: 3V \cdot \frac{4\Omega}{20\Omega} = \frac{3}{5}\ V$$

Find R_T by shutting off supplies.

200mA → 0A → open

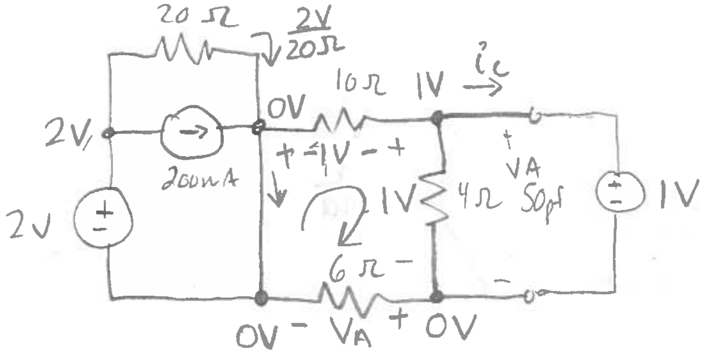
2V → 0V → short





$R_T: 4\Omega \,//\, (20\Omega + 10\Omega + 6\Omega) = 3.6\Omega$

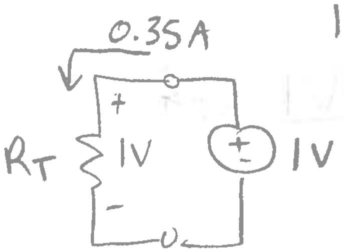
b) $t > 0s$



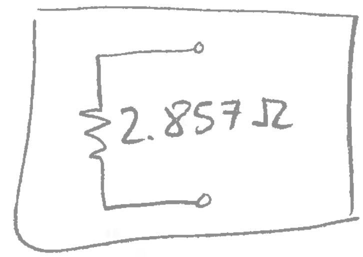
$V_{oc} = 0V$
$i_{sc} = 0A$
for a src of 1V.

KVL: $-1V + 1V + V_A = 0 \rightarrow V_A = 0V$

KCL: $\frac{1V}{10\Omega} + \frac{1V}{4\Omega} + i_c = 0 \quad i_c = -0.1 - 0.25 = -0.35A$



$R_T = \frac{1V}{0.35A} = 2.857\Omega$

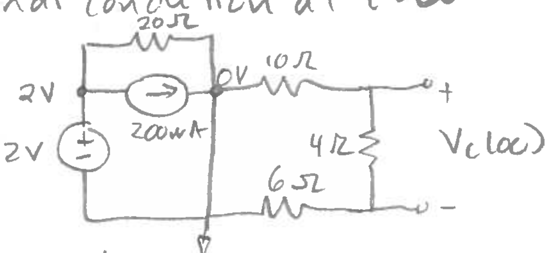


c) Find initial condition using voltage continuity $V_c(0^-) = V_c(0^+)$

from part a) $V_c(0^-) = V_c(0^+) = \frac{3}{5}V$

Find final condition at $t \rightarrow \infty$

$t \rightarrow \infty$



No current thru 4Ω → $V_c(\infty) = 0V$

time constant

$\tau = R_T C : \frac{100}{\cancel{35}\,7} \cdot \cancel{50}^{10} \times 10^{-12} = \frac{1000}{7} \times 10^{-12} = \frac{1}{7} ns = 142.86\ ps.$

$v_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)] e^{-t/\tau}\ V,\ t > 0$

$$\boxed{V_c(t) = \frac{3}{5} e^{-\frac{7 \cdot t}{ns}}\ V,\ t > 0}$$

d) $i = C\frac{dv}{dt}$ $\quad i_c(t) = 50 \times 10^{-12} F \frac{d}{dt}\left[\frac{3}{5} e^{-\frac{7 \cdot t}{ns}}\right]$

$= 30 \times 10^{-12} \cdot \left(\frac{-7}{10^{-9}}\right) e^{-\frac{7 \cdot t}{ns}} = 210 \times 10^{-3} e^{-7t/ns}$

$$\boxed{i_c(t) = 0.21 e^{-\frac{7 \cdot t}{ns}}\ A \quad t > 0}$$

Question 4

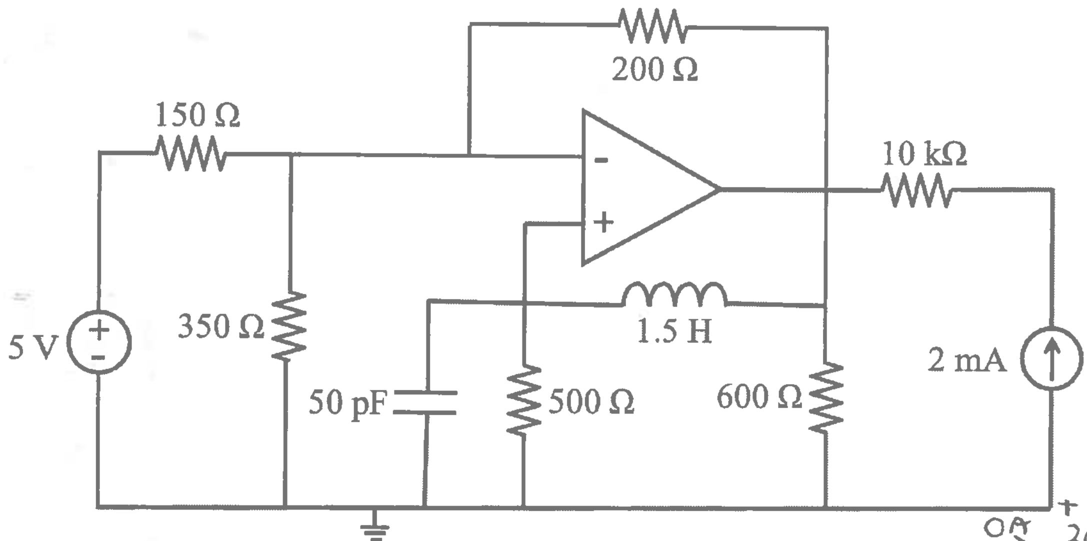
Consider the circuit shown below. Assume that the operational amplifier is ideal and that the circuit is in dc steady state. Answer the following questions.

a) What is the power supplied by the current source and the voltage source?

b) What is the resistor that dissipates the most power?

c) What is the electrical energy absorbed by the 500 Ω in one second?

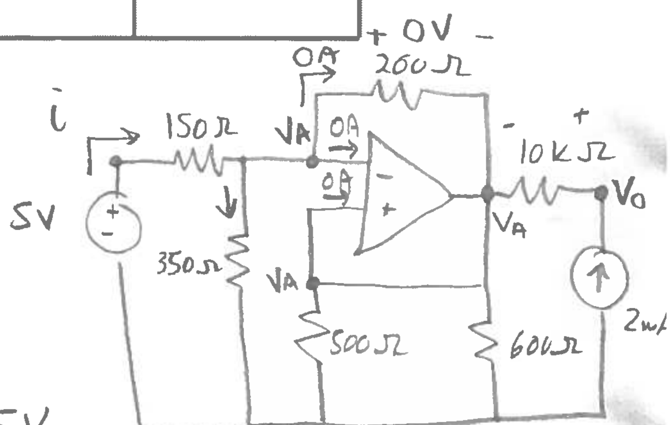
d) What is the energy stored by the capacitor and by the inductor?



a) Redraw for steady stat

$i = \frac{5V}{(150+350)\Omega} = 10mA$

$\boxed{\text{Power supplied by 5V supply: } 50mW}$



$V_A: 10mA \cdot 350\Omega = 3.5V$

$V_o - V_A = 2mA \cdot 10k\Omega = 20V \rightarrow V_o = 23.5V$

$\boxed{\text{Power supplied by current source: } 47mW}$

b) Pwr: $V \cdot I = I^2 R$ or $\frac{V^2}{R}$

$\boxed{10k\Omega}$

$150\Omega: (10mA)^2 \cdot 150\Omega = 15mW$

$350\Omega: (10mA)^2 \cdot 350\Omega = 35mW$

$500\Omega: \frac{(3.5V)^2}{500\Omega} = 24.5mW$

$10k\Omega: \frac{(20V)^2}{10k\Omega} = 40mW$

$600\Omega: \frac{(3.5V)^2}{600\Omega} = 20.42mW$

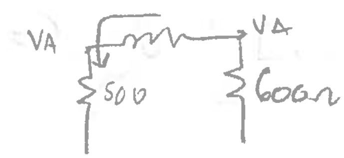
Extra Working Space

c) $\frac{dU}{dt} = \text{power} \rightarrow U(t) = \int_0^{1s} p(t)\,dt = \int_0^{1} 24.5\text{mW}\,dt$

constant in steady start

$\boxed{U = 24.5\text{mJ}}$

d) $U = \frac{1}{2}CV^2 = \frac{1}{2}\,50\times10^{-12}(3.5V)^2 = \boxed{306.25\,\text{pJ}}$ cap



$U = \frac{1}{2}Li^2 = \frac{1}{2}\,1.5H\left(\frac{3.5V}{500\Omega}\right)^2 = \boxed{36.75\,\mu\text{J}}$ inductor