Section 4-2 Node Voltage Analysis of Circuits with Current Sources

P 4.2-1 The node voltages in the circuit of Figure P 4.2-1 are

$v_1 = -4$ V and $v_2 = 2$ V.

Determine i, the current of the current source.
Answer: $i = 1.5$ A

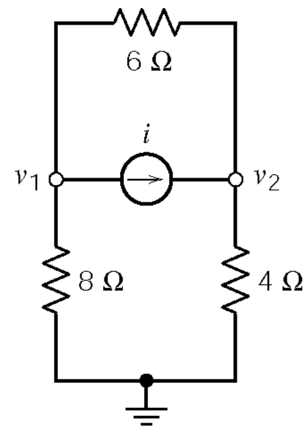


Figure P 4.2-1

Solution:

KCL at node 1:
$$0 = \frac{v_1}{8} + \frac{v_1 - v_2}{6} + i = \frac{-4}{8} + \frac{-4-2}{6} + i = -1.5 + i \quad \Rightarrow \quad i = 1.5 \text{ A}$$

(checked using LNAP 8/13/02)

P 4.2-2 Determine the node voltages for the circuit of Figure P 4.2-2.

Answer: $v_1 = 2$ V, $v_2 = 30$ V, and $v_3 = 24$ V

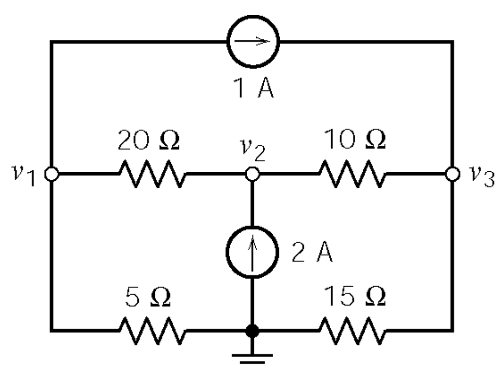


Figure P 4.2-2

Solution:

KCL at node 1:
$$\frac{v_1 - v_2}{20} + \frac{v_1}{5} + 1 = 0 \quad \Rightarrow \quad 5\,v_1 - v_2 = -20$$

KCL at node 2:
$$\frac{v_1 - v_2}{20} + 2 = \frac{v_2 - v_3}{10} \quad \Rightarrow \quad -v_1 + 3\,v_2 - 2\,v_3 = 40$$

KCL at node 3:
$$\frac{v_2 - v_3}{10} + 1 = \frac{v_3}{15} \quad \Rightarrow \quad -3\,v_2 + 5\,v_3 = 30$$

Solving gives $v_1 = 2$ V, $v_2 = 30$ V and $v_3 = 24$ V.

(checked using LNAP 8/13/02)

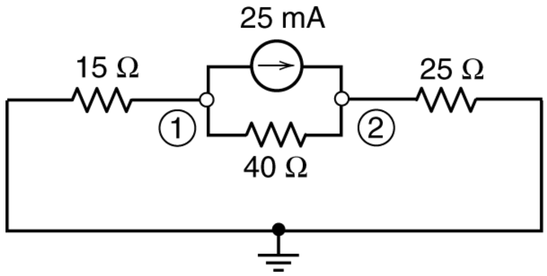
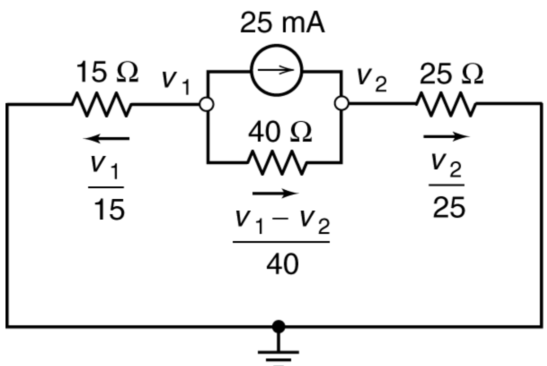


Figure P4.2-3

P4.2-3 The encircled numbers in the circuit shown Figure P4.2-3 are node numbers. Determine the values of the corresponding node voltages, v_1 and v_2.

Solution:
First, express the resistor currents in terms of the node voltages:



Apply KCL at node 1 to get $$\frac{v_1}{15} + 0.025 + \frac{v_1 - v_2}{40} = 0$$

Multiply both sides by 40 and simplify to get

$$\frac{11}{3} v_1 - v_2 = -1$$

Apply KCL at node 2 to get $$0.025 + \frac{v_1 - v_2}{40} = \frac{v_2}{25}$$

Multiply both sides by 40 and simplify to get

$$-v_1 + \frac{13}{5} v_2 = 1$$

Solving, we get $$v_1 = -0.1875 \text{ V} \quad \text{and} \quad v_2 = 0.3125 \text{ V}$$

P 4.2-4 Consider the circuit shown in Figure P 4.2-4. Find values of the resistances R_1 and R_2 that cause the voltages v_1 and v_2 to be $v_1 = 1$ V and $v_2 = 2$ V.

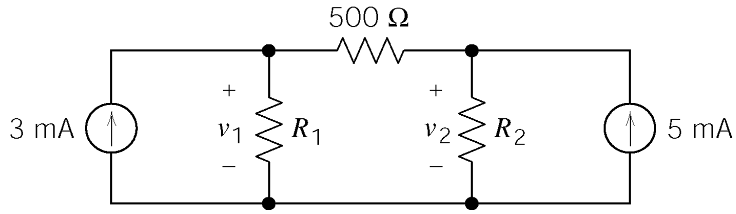


Figure P 4.2-4

Solution:

Write the node equations:

$$-.003 + \frac{v_1}{R_1} + \frac{v_1 - v_2}{500} = 0$$

$$-\frac{v_1 - v_2}{500} + \frac{v_2}{R_2} - .005 = 0$$

When $v_1 = 1$ V, $v_2 = 2$ V

$$-.003 + \frac{1}{R_1} + \frac{-1}{500} = 0 \Rightarrow R_1 = \frac{1}{.003 + \frac{1}{500}} = \underline{200\ \Omega}$$

$$-\frac{-1}{500} + \frac{2}{R_2} - .005 = 0 \Rightarrow R_2 = \frac{2}{.005 - \frac{1}{500}} = \underline{667\ \Omega}$$

(checked using LNAP 8/13/02)

P 4.2-5 Find the voltage v for the circuit shown in Figure P 4.2-5.
Answer: $v = 21.7$ mV

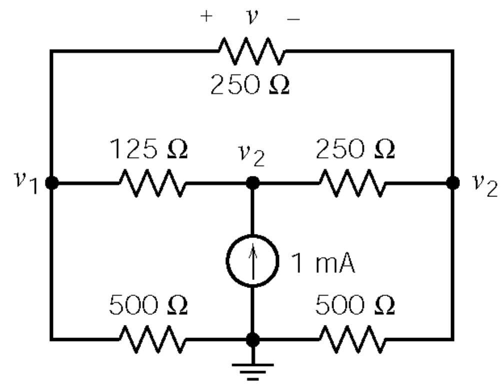


Figure P 4.2-5

Solution:

Write node equations:

$$\frac{v_1}{500} + \frac{v_1 - v_2}{125} + \frac{v_1 - v_3}{250} = 0$$

$$-\frac{v_1 - v_2}{125} - .001 + \frac{v_2 - v_3}{250} = 0$$

$$-\frac{v_2 - v_3}{250} - \frac{v_1 - v_3}{250} + \frac{v_3}{500} = 0$$

Solving gives: $v_1 = 0.261\text{ V},\quad v_2 = 0.337\text{ V},\quad v_3 = 0.239\text{ V}$

Finally: $v = v_1 - v_3 = \underline{0.022\text{ V}}$

(checked using LNAP 8/13/02)

P 4.2-6 Simplify the circuit shown in Figure P 4.2-6 by replacing series and parallel resistors with equivalent resistors; then analyze the simplified circuit by writing and solving node equations.

(a) Determine the power supplied by each current source.

(b) Determine the power received by the 12-Ω resistor.

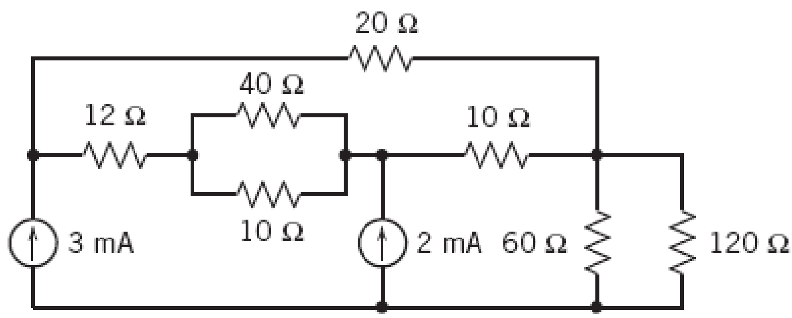
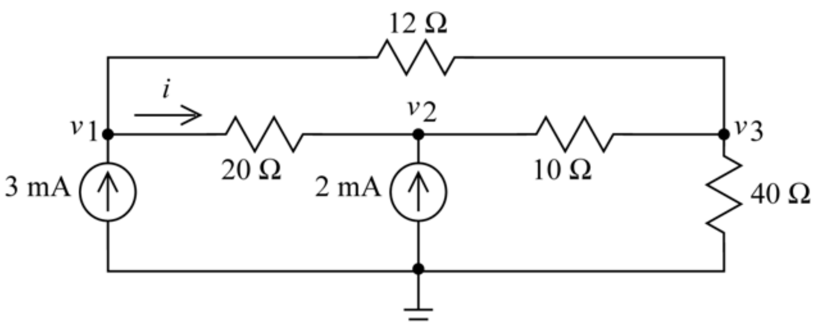


Figure P 4.2-6

Solution: Replacing series and parallel resistors with equivalent resistors we get



$$12\ \Omega + \left(40\ \Omega \,||\, 10\ \Omega\right) = 20\ \Omega$$

$$60\ \Omega \,||\, 120\ \Omega = 40\ \Omega$$

The node equations are

$$3\times10^{-3} = \frac{v_1 - v_2}{20} + \frac{v_1 - v_3}{20} \quad \Rightarrow \quad 0.06 = 2v_1 - \left(v_2 - v_3\right)$$

$$2\times10^{-3} + \frac{v_1 - v_2}{20} = \frac{v_2 - v_3}{10} \quad \Rightarrow \quad 0.04 = -v_1 + 3v_2 - 2v_3$$

$$\frac{v_2 - v_3}{10} + \frac{v_1 - v_3}{20} = \frac{v_3}{40} \quad \Rightarrow \quad 0 = -\left(2v_1 + 4v_2\right) + 7v_3$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 3 & -2 \\ -2 & -4 & +7 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} .06 \\ .04 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0.244 \\ 0.228 \\ 0.200 \end{bmatrix}$$

(a) The power supplied by the 3 mA current source is $\left(3\times10^{-3}\right)\left(0.244\right) = 0.732$ mW. The power supplied by the 2 mA source is $\left(2\times10^{-3}\right)\left(0.228\right) = 0.456$ mW.

(b) The current in the 12 Ω resistor is equal to the current $i = \dfrac{v_1 - v_2}{20} = \dfrac{0.244 - 0.228}{20} = 0.8\ \text{mA}$ so the power received by the 12 Ω resistor is $\left(0.8\times10^{-3}\right)^2\left(12\right) = 7.68\times10^{-b} = 7.68\ \mu\text{W}$.

(checked: LNAP and MATLAB 5/31/04)

P 4.2-7 The node voltages in the circuit shown in Figure P 4.2-7 are $v_a = 7$ V and $v_b = 10$ V. Determine values of the current source current, i_s, and the resistance, R.

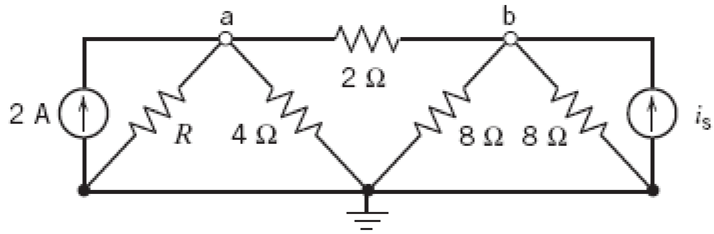


Figure P 4.2-7

Solution

Apply KCL at node a to get

$$2 = \frac{v_a}{R} + \frac{v_a}{4} + \frac{v_a - v_b}{2} = \frac{7}{R} + \frac{7}{4} + \frac{7-10}{2} = \frac{7}{R} + \frac{1}{4} \quad \Rightarrow \quad R = 4\ \Omega$$

Apply KCL at node b to get

$$i_s + \frac{v_a - v_b}{2} = \frac{v_b}{8} + \frac{v_b}{8} = i_s + \frac{7-10}{2} = \frac{10}{8} + \frac{10}{8} \quad \Rightarrow \quad i_s = 4\ \text{A}$$

(checked: LNAP 6/21/04)

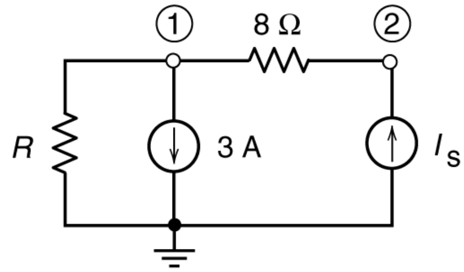


Figure P4.2-8

P4.2-8 The encircled numbers in the circuit shown Figure P4.2-8 are node numbers. The corresponding node voltages are v_1 and v_2.The node equation representing this circuit is

$$\begin{bmatrix} 0.225 & -0.125 \\ -0.125 & 0.125 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

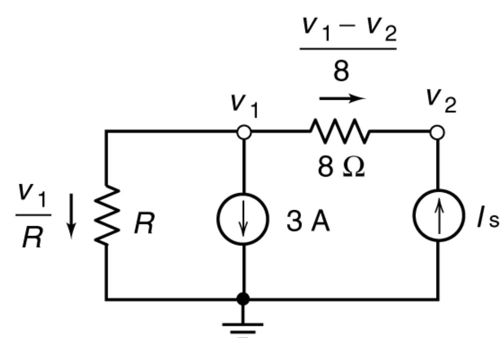
***a*)** Determine the values of R and I_s in Figure P4.2-8

***b*)** Determine the value of the power supplied by the 3 A current source.

Solution:

***a*)** The node equations representing the circuit are

$$\frac{v_1}{R} + \frac{v_1 - v_2}{8} + 3 = 0 \quad \text{and} \quad \frac{v_1 - v_2}{8} + I_s = 0$$



In matrix form, we have

$$\begin{bmatrix} 0.125 + \frac{1}{R} & -0.125 \\ 0.125 & -0.125 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -3 \\ -I_s \end{bmatrix}$$

After multiplying the 2nd row by -1, we have

$$\begin{bmatrix} 0.125 + \frac{1}{R} & -0.125 \\ -0.125 & 0.125 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -3 \\ I_s \end{bmatrix}$$

Comparing to the given node equation, we see that

$$\frac{1}{R} = 0.1 \quad \Rightarrow \quad R = 10 \ \Omega \quad \text{and} \quad I_s = 2 \ \text{A}$$

***b*)** Solving the given node equations, we get

$$v_1 = -10 \ \text{V} \quad \text{and} \quad v_2 = 6 \ \text{V}$$

The power supplied by the 3 A current source is given by

$$-v_1(3) = -(-10)(3) = 30 \ \text{W}$$

Section 4-3 Node Voltage Analysis of Circuits with Current and Voltage Sources

P 4.3-1 The voltmeter in Figure P 4.3-1 measures v_c, the node voltage at node c. Determine the value of v_c.

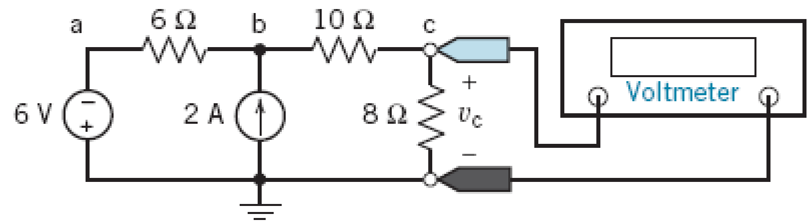
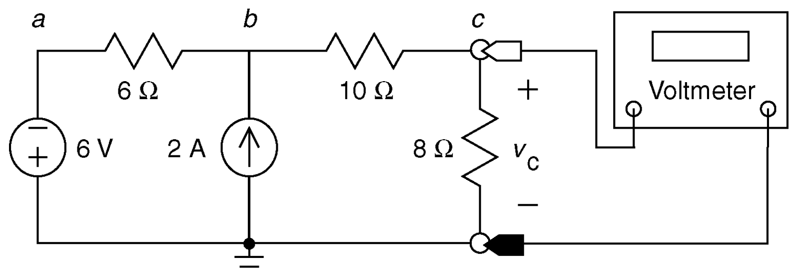


Figure P 4.3-1

Solution:



Express the voltage of the voltage source in terms of its node voltages:

$$0 - v_a = 6 \quad \Rightarrow \quad v_a = -6 \text{ V}$$

KCL at node *b*:

$$\frac{v_a - v_b}{6} + 2 = \frac{v_b - v_c}{10} \quad \Rightarrow \quad \frac{-6 - v_b}{6} + 2 = \frac{v_b - v_c}{10} \quad \Rightarrow \quad -1 - \frac{v_b}{6} + 2 = \frac{v_b - v_c}{10} \quad \Rightarrow \quad 30 = 8\,v_b - 3\,v_c$$

KCL at node *c*: $$\frac{v_b - v_c}{10} = \frac{v_c}{8} \quad \Rightarrow \quad 4\,v_b - 4\,v_c = 5\,v_c \quad \Rightarrow \quad v_b = \frac{9}{4}v_c$$

Finally: $$30 = 8\left(\frac{9}{4}v_c\right) - 3\,v_c \quad \Rightarrow \quad v_c = 2 \text{ V}$$

(checked using LNAP 8/13/02)

P 4.3-2 The voltages v_a, v_b, v_c, and v_d in Figure P 4.3-2 are the node voltages corresponding to nodes a, b, c, and d. The current i is the current in a short circuit connected between nodes b and c. Determine the values of v_a, v_b, v_c, and v_d and of i.

Answer: $v_a = -12$ V, $v_b = v_c = 4$ V, $v_d = -4$ V, $i = 2$ mA

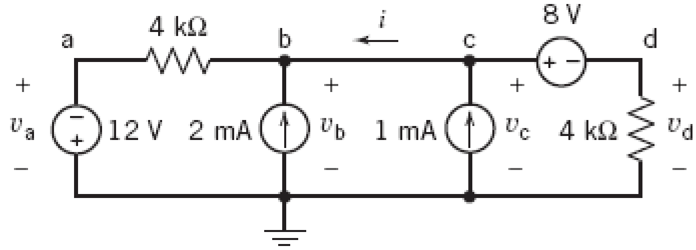
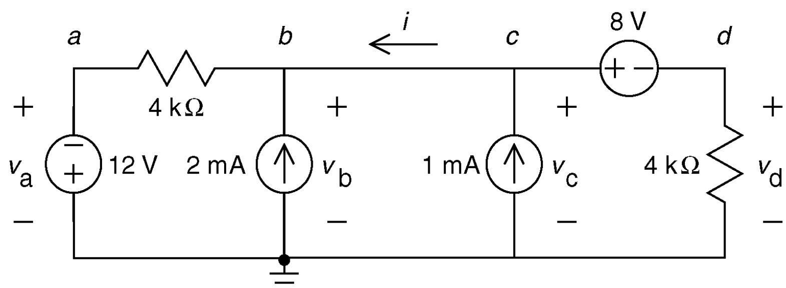


Figure P 4.3-2

Solution:



Express the branch voltage of each voltage source in terms of its node voltages to get:

$$v_a = -12 \text{ V}, \;\; v_b = v_c = v_d + 8$$

KCL at node b:

$$\frac{v_b - v_a}{4000} = 0.002 + i \;\;\Rightarrow\;\; \frac{v_b - (-12)}{4000} = 0.002 + i \;\;\Rightarrow\;\; v_b + 12 = 8 + 4000\, i$$

KCL at the supernode corresponding to the 8 V source:

$$0.001 = \frac{v_d}{4000} + i \;\;\Rightarrow\;\; 4 = v_d + 4000\, i$$

so $$v_b + 4 = 4 - v_d \;\;\Rightarrow\;\; (v_d + 8) + 4 = 4 - v_d \;\;\Rightarrow\;\; v_d = -4 \text{ V}$$

Consequently $v_b = v_c = v_d + 8 = 4$ V and $i = \dfrac{4 - v_d}{4000} = 2$ mA

(checked using LNAP 8/13/02)

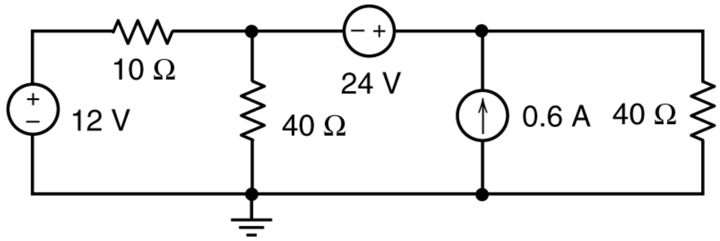
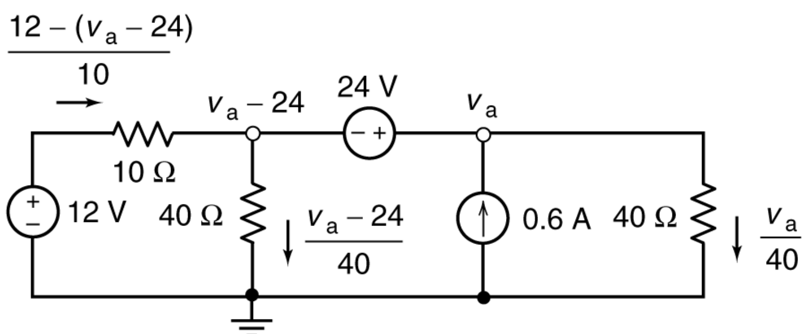


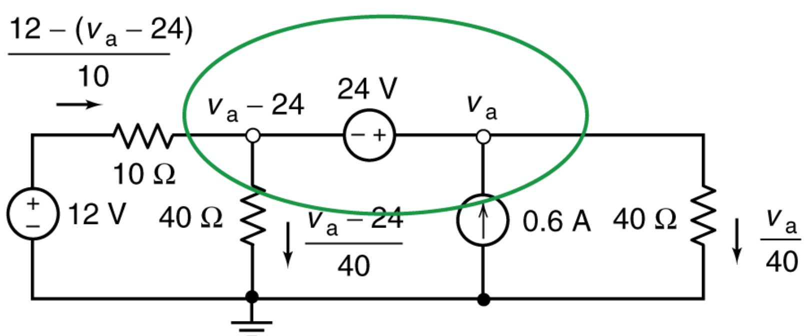
Figure P4.3-3

P4.3-3. Determine the values of the power supplied by each of the sources in the circuit shown in Figure P4.3-3.

Solution: First, label the node voltages. Next, express the resistor currents in terms of the node voltages.



Identify the supernode corresponding to the 24 V source



Apply KCL to the supernode to get

$$\frac{12-(v_a-24)}{10}+0.6=\frac{v_a-24}{40}+\frac{v_a}{40} \quad\Rightarrow\quad 196=6v_a \quad\Rightarrow\quad v_a=32\ \text{V}$$

The 12 V source supplies $12\left(\frac{12-(v_a-24)}{10}\right)=12\left(\frac{12-(32-24)}{10}\right)=4.8\ \text{W}$

The 24 V source supplies $24\left(-0.6+\frac{v_a}{40}\right)=24\left(-0.6+\frac{32}{40}\right)=4.8\ \text{W}$

The current source supplies $0.6v_a=0.6(32)=19.2\ \text{W}$

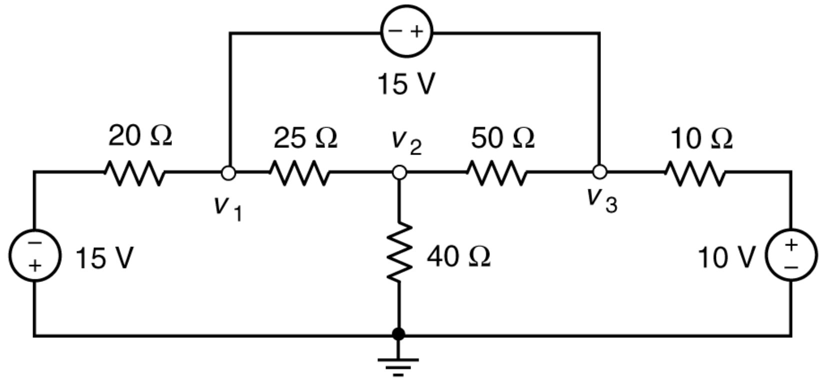
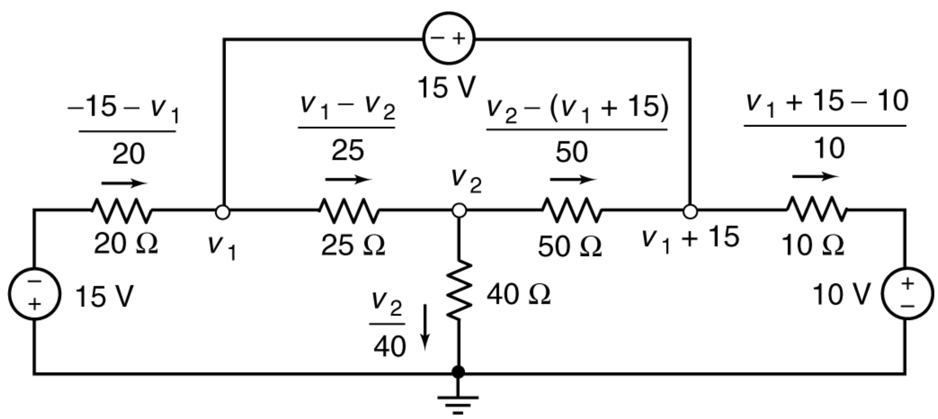


Figure P4.3-4

P4.3-4. Determine the values of the node voltages, v_1, v_2 and v_3 in the circuit shown in Figure P4.3-13.

Solution:

First, express the resistor currents in terms of the node voltages:



Apply KCL to the supernode to get

$$\frac{-15-v_1}{20}+\frac{v_2-(v_1+15)}{50}=\frac{v_1-v_2}{25}+\frac{v_1+5}{10} \quad \Rightarrow \quad 0.21v_1-0.06v_2=-1.55$$

Apply KCL at node 2 to get $\frac{v_1-v_2}{25}=\frac{v_2}{40}+\frac{v_2-(v_1+15)}{50} \quad \Rightarrow \quad -0.06v_1+0.085v_2=0.3$

In matrix form:

$$\begin{bmatrix} 0.21 & -0.06 \\ -0.06 & 0.085 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} -1.55 \\ 0.3 \end{bmatrix}$$

Solving using MATLAB: $v_1=-7.9825\ \text{V}$ and $v_2=-2.1053\ \text{V}$

P 4.3-5 The voltages v_a, v_b, and v_c in Figure P 4.3-5 are the node voltages corresponding to nodes a, b, and c. The values of these voltages are:

$$v_a = 12 \text{ V},\ v_b = 9.882 \text{ V}, \quad \text{and} \quad v_c = 5.294 \text{ V}$$

Determine the power supplied by the voltage source.

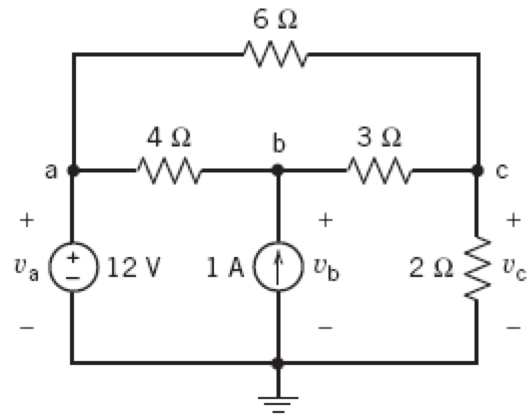
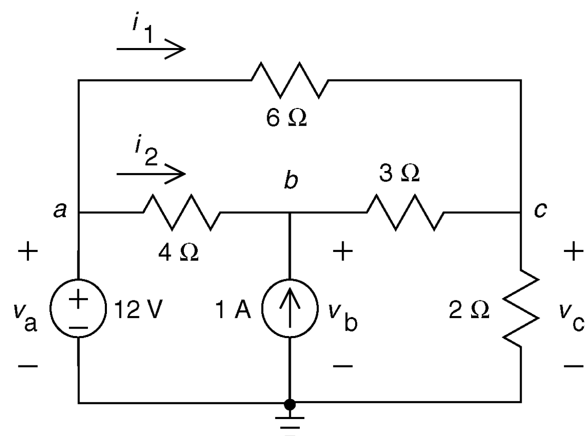


Figure P 4.3-5

Solution:



The power supplied by the voltage source is

$$v_a\left(i_1+i_2\right)=v_a\left(\frac{v_a-v_b}{4}+\frac{v_a-v_c}{6}\right)=12\left(\frac{12-9.882}{4}+\frac{12-5.294}{6}\right)$$

$$=12(0.5295+1.118)=12(1.648)=19.76 \text{ W}$$

(checked using LNAP 8/13/02)

P 4.3-6 The voltmeter in the circuit of Figure P 4.3-6 measures a node voltage. The value of that node voltage depends on the value of the resistance *R*.

(a) Determine the value of the resistance *R* that will cause the voltage measured by the voltmeter to be 4 V.

(b) Determine the voltage measured by the voltmeter when $R = 1.2\ \text{k}\Omega = 1200\ \Omega$.

Answer: (a) 6 kΩ (b) 2 V

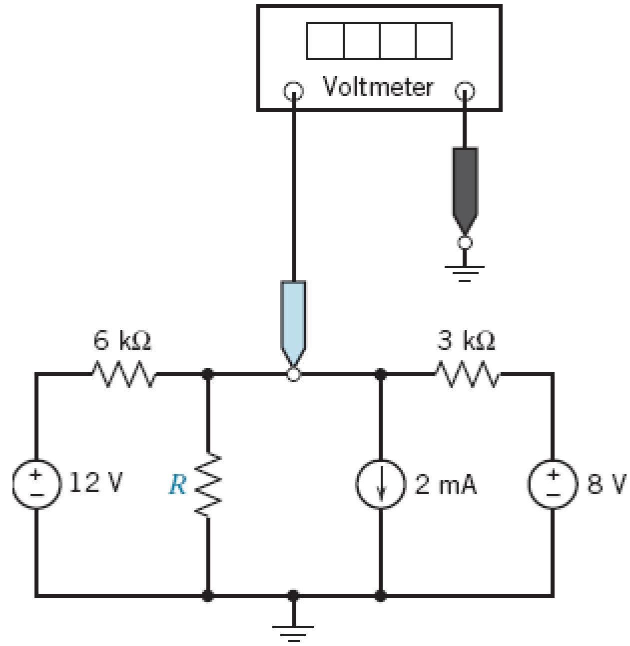
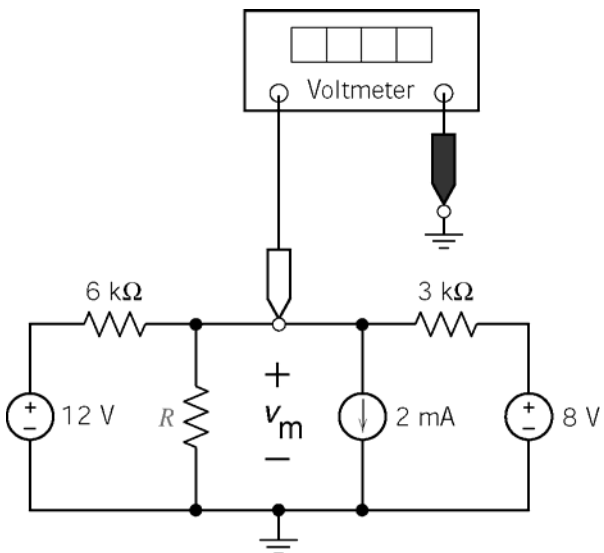


Figure P 4.3-6

Solution:
Label the voltage measured by the meter. Notice that this is a node voltage.



Write a node equation at the node at which the node voltage is measured.

$$-\left(\frac{12 - v_m}{6000}\right) + \frac{v_m}{R} + 0.002 + \frac{v_m - 8}{3000} = 0$$

That is

$$\left(3 + \frac{6000}{R}\right) v_m = 16 \quad \Rightarrow \quad R = \frac{6000}{\frac{16}{v_m} - 3}$$

(a) The voltage measured by the meter will be 4 volts when $R = 6\ \text{k}\Omega$.

(b) The voltage measured by the meter will be 2 volts when $R = 1.2\ \text{k}\Omega$.

P 4.3-7 Determine the values of the node voltages, v_1 and v_2, in Figure P 4.3-7. Determine the values of the currents i_a and i_b.

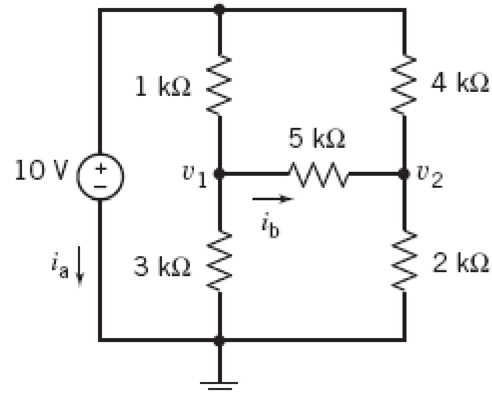
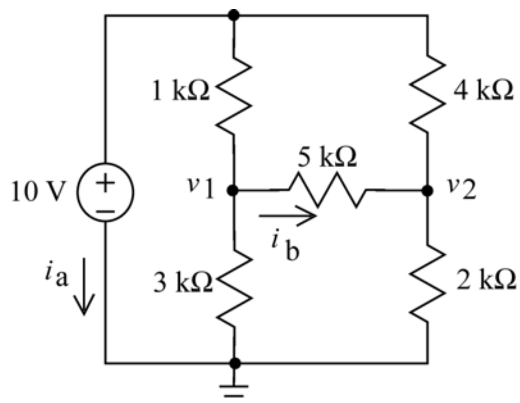


Figure P 4.3-7

Solution:



Apply KCL at nodes 1 and 2 to get

$$\frac{10-v_1}{1000}=\frac{v_1}{3000}+\frac{v_1-v_2}{5000} \quad\Rightarrow\quad 23v_1-3v_2=150$$

$$\frac{10-v_2}{4000}+\frac{v_1-v_3}{5000}=\frac{v_3}{2000} \quad\Rightarrow\quad \text{-}4v_1+19v_3=50$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 23 & -3 \\ -4 & 19 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} 150 \\ 50 \end{bmatrix} \quad\Rightarrow\quad v_1=7.06\text{ V and } v_1=4.12\text{ V}$$

Then
$$i_b=\frac{v_1-v_2}{5000}=\frac{7.06-4.12}{5000}=0.588\text{ mA}$$

Apply KCL at the top node to get

$$i_a=\frac{v_1-10}{1000}+\frac{v_2-10}{4000}=\frac{7.06-10}{1000}+\frac{4.12-10}{4000}=-4.41\text{ mA}$$

(checked: LNAP 5/31/04)

P 4.3-8 The circuit shown in Figure P 4.3-8 has two inputs, v_1 and v_2, and one output, v_o. The output is related to the input by the equation

$$v_o = av_1 + bv_2$$

where a and b are constants that depend on R_1, R_2 and R_3.

(a) Determine the values of the coefficients a and b when $R_1 = 10\ \Omega$, $R_2 = 40\ \Omega$ and $R_3 = 8\ \Omega$.
(b) Determine the values of the coefficients a and b when $R_1 = R_2$ and $R_3 = R_1 \parallel R_2$.

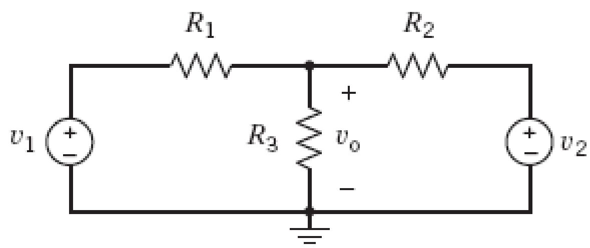


Figure P 4.3-8

Solution:

$$\frac{v_o}{R_3}+\frac{v_o-v_1}{R_1}+\frac{v_o-v_2}{R_2}=0 \quad\Rightarrow\quad v_o=\frac{v_1}{1+\frac{R_1}{R_2}+\frac{R_1}{R_3}}+\frac{v_2}{1+\frac{R_2}{R_1}+\frac{R_2}{R_3}}$$

(a) When $R_1 = 10\ \Omega,\ R_2 = 40\ \Omega$ and $R_3 = 8\ \Omega$

$$v_o=\frac{v_1}{1+\frac{1}{4}+\frac{5}{4}}+\frac{v_2}{1+4+5}=0.4v_1+0.1v_2$$

So $a = 0.4$ and $b = 0.1$.

(b) When $R_1 = R_2$ and $R_3 = R_1 \parallel R_2 = R_1/2$

$$v_o=\frac{v_1}{1+1+2}+\frac{v_2}{1+1+2}=0.25v_1+0.25v_2$$

So $a = 0.25$ and $b = 0.25$.

(checked: LNAP 5/31/04)

P 4.3-9 Determine the values of the node voltages of the circuit shown in Figure P 4.3-9.

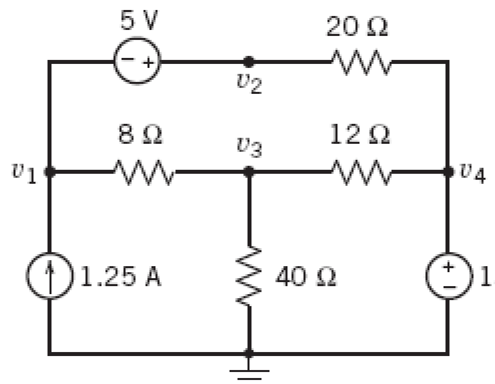


Figure P 4.3-9

Solution:
Express the voltage source voltages as functions of the node voltages to get

$$v_2 - v_1 = 5 \text{ and } v_4 = 15$$

Apply KCL to the supernode corresponding to the 5 V source to get

$$1.25 = \frac{v_1 - v_3}{8} + \frac{v_2 - 15}{20} = 0 \quad \Rightarrow \quad 80 = 5v_1 + 2v_2 - 5v_3$$

Apply KCL at node 3 to get

$$\frac{v_1 - v_3}{8} = \frac{v_3}{40} + \frac{v_3 - 15}{12} \quad \Rightarrow \quad -15v_1 + 28v_3 = 150$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} -1 & 1 & 0 \\ 5 & 2 & -5 \\ -15 & 0 & 28 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 80 \\ 150 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 22.4 \\ 27.4 \\ 17.4 \end{bmatrix}$$

So the node voltages are:

$$v_1 = 22.4 \text{ V}, v_2 = 27.4 \text{ V}, v_3 = 17.4 \text{ V, and } v_4 = 15$$

(checked: LNAP 6/9/04)

P 4.3-10 Figure P 4.3-10 shows a measurement made in the laboratory. Your lab partner forgot to record the values of R_1, R_2, and R_3. He thinks that the two resistors were 10-kΩ resistors and the other was a 5-kΩ resistor. Is this possible? Which resistor is the 5-kΩ resistor?

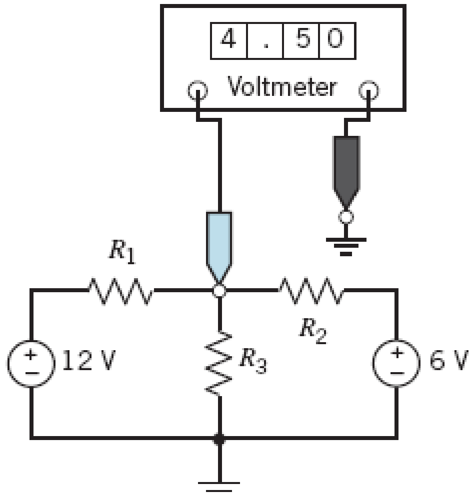


Figure P 4.3-10

Solution:
Write a node equation to get

$$-\left(\frac{12-4.5}{R_1}\right)+\frac{4.5}{R_3}+\frac{4.5-6}{R_2}=0 \quad\Rightarrow\quad -\frac{7.5}{R_1}+\frac{4.5}{R_3}-\frac{1.5}{R_2}=0$$

Notice that $\frac{7.5}{R_1}$ is either 0.75 mA or 1.5 mA depending on whether R_1 is 10 kΩ or 5 kΩ. Similarly, $\frac{4.5}{R_3}$ is either 0.45 mA or 0.9 mA and $\frac{1.5}{R_2}$ is either 0.15 mA or 0.3 mA. Suppose R_1 and R_2 are 10 kΩ resistors and R_3 is a 5 kΩ resistor. Then

$$-\frac{7.5}{R_1}+\frac{4.5}{R_3}-\frac{1.5}{R_2}=-0.75+0.9-0.15=0$$

It is possible that two of the resistors are 10 kΩ and the third is 5 kΩ. R_3 is the 5 kΩ resistor.

(checked: LNAP 6/9/04)

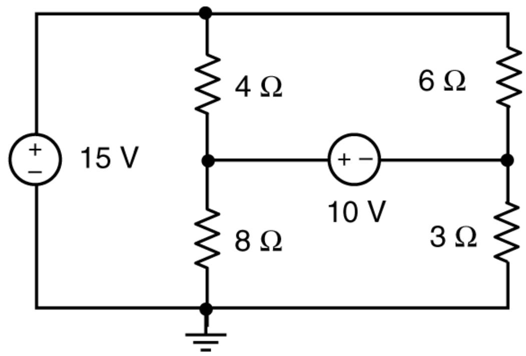
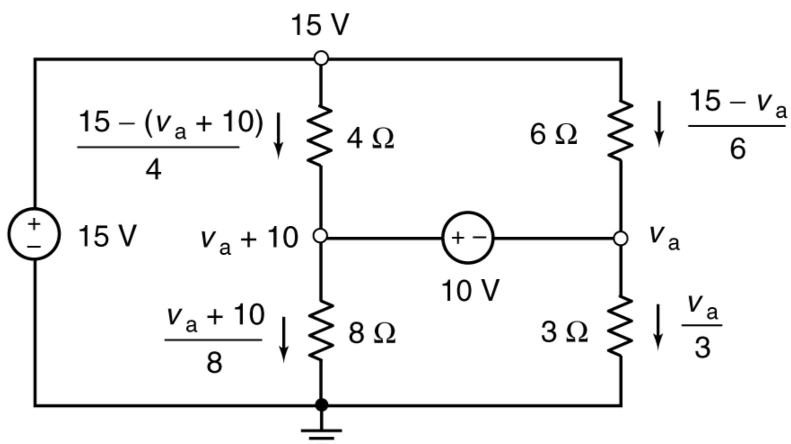


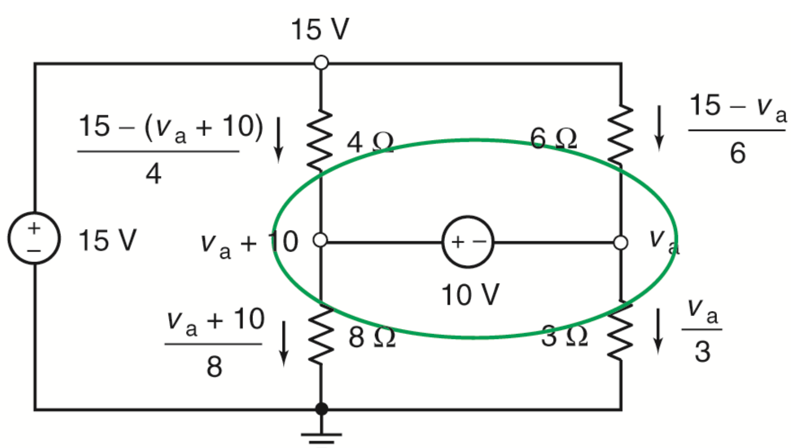
Figure P4.3-11

P4.3-11. Determine the values of the power supplied by each of the sources in the circuit shown in Figure P4.3-11.

Solution: First, label the node voltages. Next, express the resistor currents in terms of the node voltages.



Identify the supernode corresponding to the 10 V source



Apply KCL to the supernode to get

$$\frac{15-\left(v_a+10\right)}{4}+\frac{15-v_a}{6}=\frac{v_a+10}{8}+\frac{v_a}{3} \quad \Rightarrow \quad 60=21v_a \quad \Rightarrow \quad v_a=2.857\text{ V}$$

The 15 V source supplies

$$15\left(\frac{15-(v_a+10)}{4}+\frac{15-v_a}{6}\right)=15\left(\frac{15-12.857}{4}+\frac{15-2.857}{6}\right)=15(2.56)=38.4\ \text{ W}$$

The 10 V source supplies $\quad 10\left(\frac{15-v_a}{6}+\frac{v_a}{3}\right)=10\left(\frac{15-2.857}{6}+\frac{2.857}{3}\right)=10(1.071)=10.71\ \text{ W}$

P 4.3-12 Determine the values of the node voltages of the circuit shown in Figure P 4.3-12.

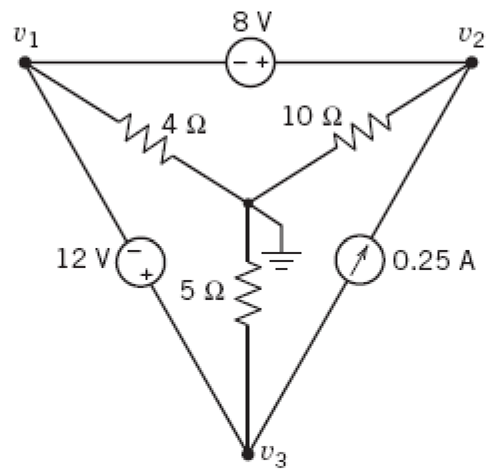


Figure P 4.3-12

Solution:

Express the voltage source voltages in terms of the node voltages:

$$v_2-v_1=8 \text{ and } v_3-v_1=12$$

Apply KVL to the supernode to get

$$\frac{v_2}{10}+\frac{v_1}{4}+\frac{v_3}{5}=0 \quad\Rightarrow\quad 2v_2+5v_1+4v_3=0$$

so $\quad 2(8+v_1)+5v_1+4(12+v_1)=0 \quad\Rightarrow\quad v_1=-\frac{64}{11}\text{ V}$

The node voltages are

$$v_1=-5.818\text{ V}$$
$$v_2=2.182\text{ V}$$
$$v_3=6.182\text{ V}$$

(checked: LNAP 6/21/04)

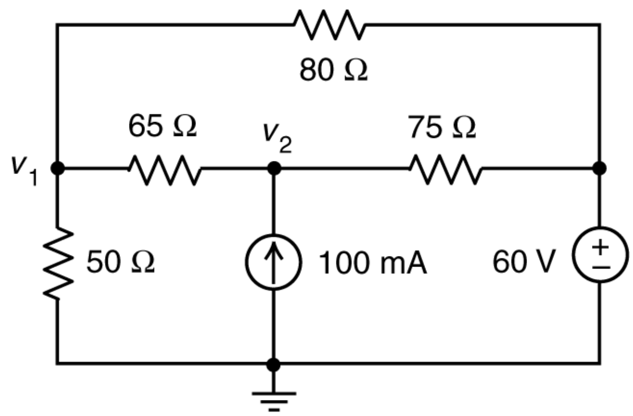


Figure P4.3-13

P4.3-13. Determine the values node voltages, v_1 and v_2, in the circuit shown in Figure P4.3-13.

Solution: Apply KCL at node 1 to get

$$\frac{v_1}{50}+\frac{v_1-v_2}{65}+\frac{v_1-60}{80}=0 \quad\Rightarrow\quad \left(\frac{1}{50}+\frac{1}{65}+\frac{1}{80}\right)v_1-\left(\frac{1}{65}\right)v_2=\frac{60}{80}$$

Apply KCL at node 2 to get

$$0.1=\frac{v_2-v_1}{65}+\frac{v_2-60}{75}= \quad\Rightarrow\quad -\left(\frac{1}{65}\right)v_1+\left(\frac{1}{65}+\frac{1}{75}\right)v_2=0.1$$

In matrix form
$$\begin{bmatrix} \frac{1}{50}+\frac{1}{65}+\frac{1}{80} & -\frac{1}{65} \\ -\frac{1}{65} & \frac{1}{65}+\frac{1}{75} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} \frac{60}{80} \\ 0.1 \end{bmatrix}$$

Solving, we get $v_1 = 13.2356$ V and $v_2 = 22.3456$ V

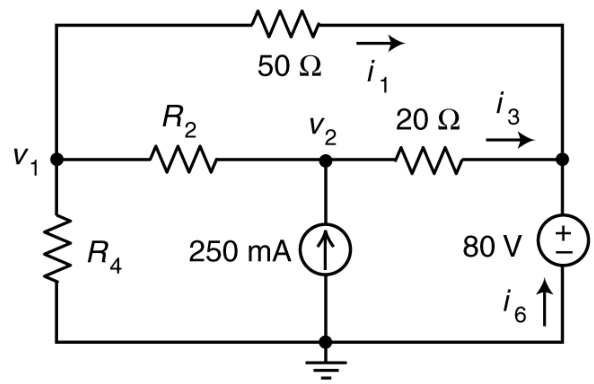


Figure P4.3-14

P4.3-14. The voltage source in the circuit shown in Figure P4.3-14 supplies 83.802 W. The current source supplies 17.572 W. Determine the values of the node voltages v_1 and v_2.

Solution: From the power supplied by the current source we calculate

$$17.572 = v_2(0.25) \quad \Rightarrow \quad v_2 = \frac{17.572}{0.25} = 70.288 \text{ V}$$

Using Ohm's law
$$i_3 = \frac{70.288 - 80}{20} = -0.4856 \text{ A}$$

From the power supplied by the voltage source we calculate

$$83.802 = 80\, i_6 \quad \Rightarrow \quad i_6 = \frac{83.802}{80} = 1.0475 \text{ V}$$

$$i_1 = -(i_3 + i_6) = -(-0.4856 + 1.0475) = -0.5619 \text{ A}$$

Using Ohm's law
$$-0.5619 = \frac{v_1 - 80}{50} \quad \Rightarrow \quad v_1 = 80 + 50(-0.5619) = 51.905 \text{ V}$$

In summary
$v_1 = 51.905$ V and $v_2 = 70.288$ V

Section 4-4 Node Voltage Analysis with Dependent Sources

P 4.4-1 The voltages v_a, v_b, and v_c in Figure P 4.4-1 are the node voltages corresponding to nodes a, b, and c. The values of these voltages are:

$$v_a = 8.667 \text{ V}, \; v_b = 2 \text{ V}, \quad \text{and} \quad v_c = 10 \text{ V}$$

Determine the value of A, the gain of the dependent source.

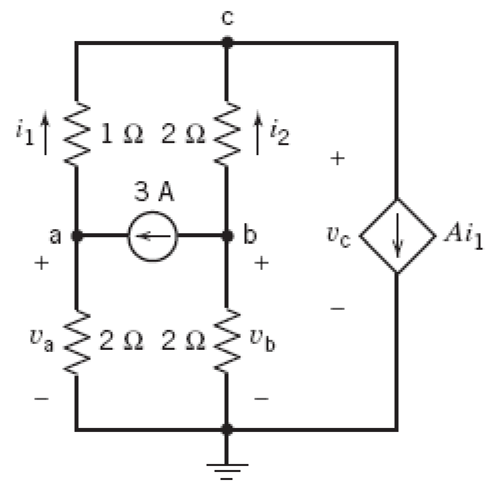
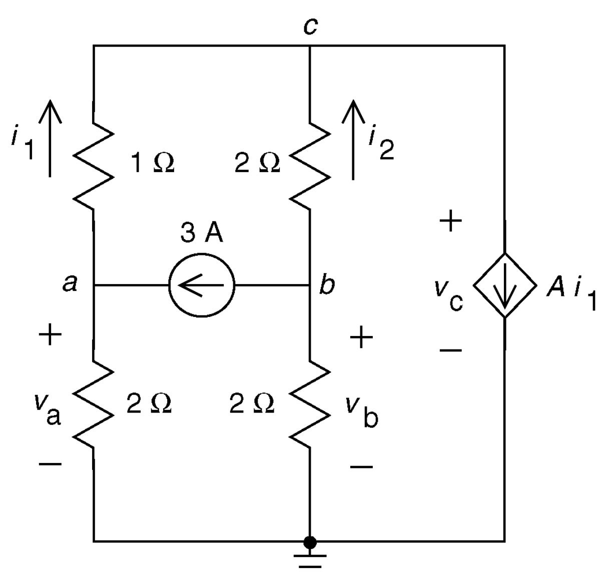


Figure P 4.4-1

Solution:



Express the resistor currents in terms of the node voltages:

$$i_1 = \frac{v_a - v_c}{1} = 8.667 - 10 = -1.333 \text{ A} \;\text{ and}$$

$$i_2 = \frac{v_b - v_c}{2} = \frac{2-10}{2} = -4 \text{ A}$$

Apply KCL at node c:

$$i_1 + i_2 = A\,i_1 \quad \Rightarrow \quad -1.333 + (-4) = A\,(-1.333)$$

$$\Rightarrow \quad A = \frac{-5.333}{-1.333} = 4$$

(checked using LNAP 8/13/02)

P 4.4-2 Find i_b for the circuit shown in Figure P 4.4-2.

Answer: $i_b = -12$ mA

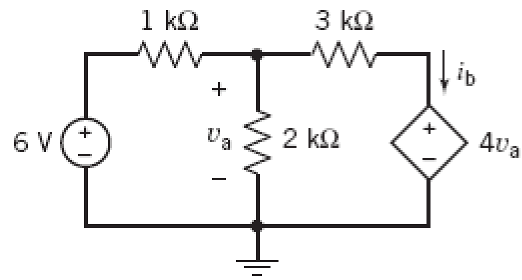
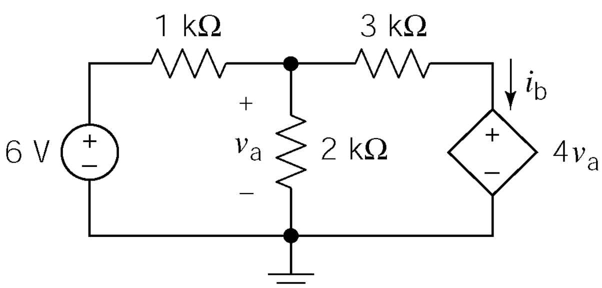


Figure P 4.4-2

Solution:



Write and solve a node equation:

$$\frac{v_a - 6}{1000} + \frac{v_a}{2000} + \frac{v_a - 4v_a}{3000} = 0 \quad \Rightarrow \quad v_a = 12 \text{ V}$$

$$i_b = \frac{v_a - 4v_a}{3000} = \underline{-12 \text{ mA}}$$

(checked using LNAP 8/13/02)

P 4.4-3 Determine the node voltage v_b for the circuit of Figure P 4.4-3.
Answer: $v_b = 1.5$ V

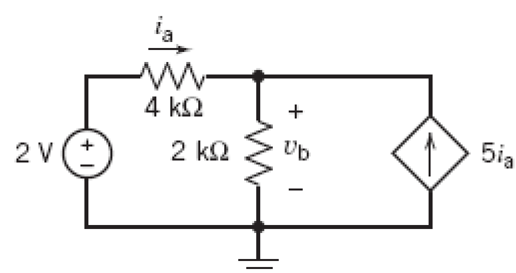
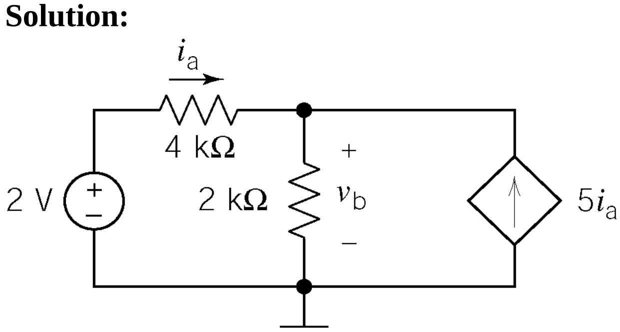


Figure P 4.4-3

Solution:



First express the controlling current in terms of the node voltages:

$$i_a = \frac{2 - v_b}{4000}$$

Write and solve a node equation:

$$-\frac{2 - v_b}{4000} + \frac{v_b}{2000} - 5\left(\frac{2 - v_b}{4000}\right) = 0 \quad \Rightarrow \quad \underline{v_b = 1.5 \text{ V}}$$

(checked using LNAP 8/14/02)

P 4.4-4 The circled numbers in Figure P 4.4-4 are node numbers. The node voltages of this circuit are v_1 = 10 V, v_2 = 14 V, and v_3 = 12 V.

(a) Determine the value of the current i_b.

(b) Determine the value of r, the gain of the CCVS.

Answers: (a) –2 A (b) 4 V/A

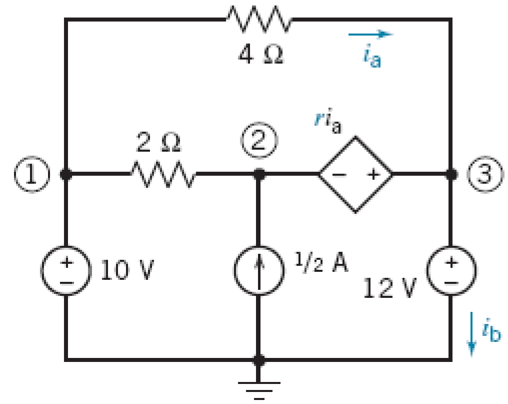
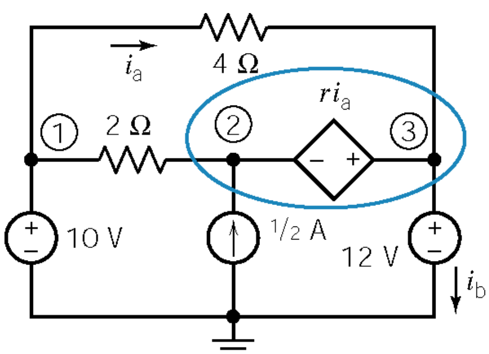


Figure P 4.4-4

Solution:



Apply KCL to the supernode of the CCVS to get

$$\frac{12-10}{4}+\frac{14-10}{2}-\frac{1}{2}+i_b=0 \quad \Rightarrow \quad i_b=-2 \text{ A}$$

Next

$$\left.\begin{array}{l} i_a=\dfrac{10-12}{4}=-\dfrac{1}{2} \\ r\, i_a=12-14 \end{array}\right\} \quad \Rightarrow \quad r=\frac{-2}{-\frac{1}{2}}=4\ \frac{\text{V}}{\text{A}}$$

(checked using LNAP 8/14/02)

P 4.4-5 Determine the value of the current i_x in the circuit of Figure P 4.4-5.

Answer: i_x = 2.4 A

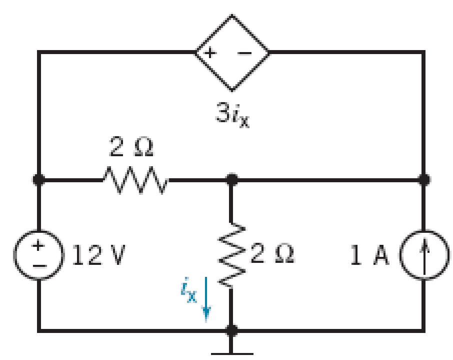
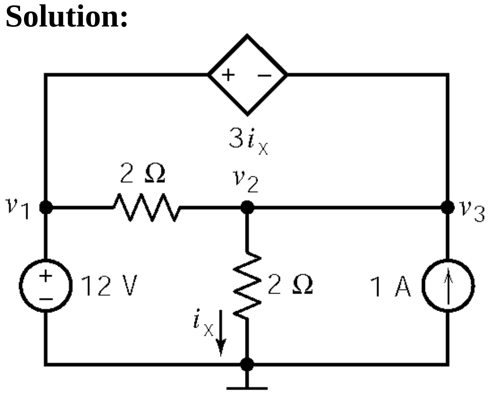


Figure P 4.4-5

Solution:



First, express the controlling current of the CCVS in terms of the node voltages: $i_x=\dfrac{v_2}{2}$

Next, express the controlled voltage in terms of the node voltages:

$$12-v_2=3i_x=3\frac{v_2}{2} \quad \Rightarrow v_2=\frac{24}{5}\text{ V}$$

so i_x = 12/5 A = 2.4 A.

(checked using ELab 9/5/02)

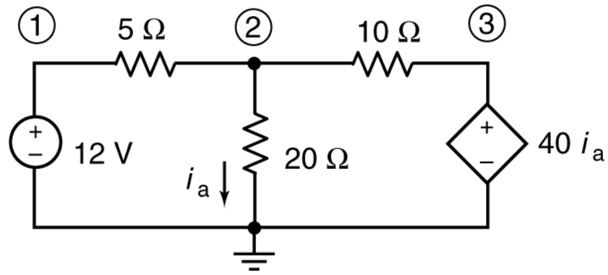
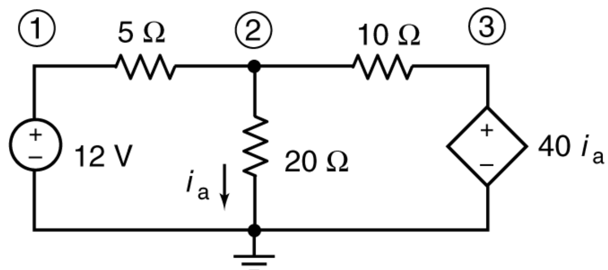


Figure P4.4-6

P4.4-6 The encircled numbers in the circuit shown Figure P4.4-6 are node numbers. Determine the value of the power supplied by the CCVS .

P4.4-6

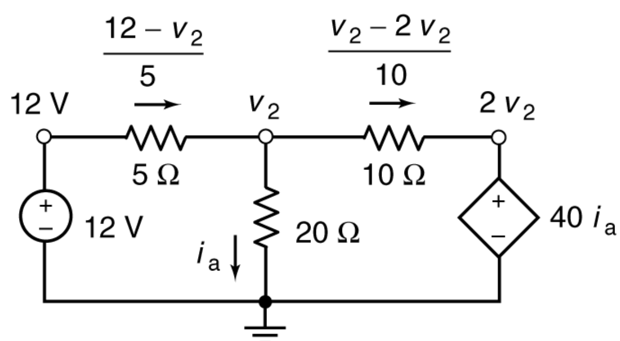


First, express the contolling current of the CCVS in terms of the node voltages:

$$i_a = \frac{v_2}{20}$$

Notice that $\qquad v_1 = 12 \text{ V} \quad \text{and} \quad v_3 = 40\, i_a = 40\left(\frac{v_2}{20}\right) = 2\, v_2$

Next, express the resistor currents in terms of the node voltages:



Apply KCL at node 2 to get

$$\frac{12 - v_2}{5} + \frac{v_2}{20} + \frac{v_2 - 2\, v_2}{10} = 0 \quad \Rightarrow \quad v_2 = 16 \text{ V}$$

Then $\qquad i_a = \frac{v_2}{20} = \frac{16}{20} = 0.8 \text{ A} \quad \text{and} \quad v_3 = 40\, i_a = 40(0.8) = 32 \text{ V}$

The CCVS supplies $\qquad v_3\left(\frac{v_3 - v_2}{10}\right) = 32\left(\frac{32 - 16}{10}\right) = 51.2 \text{ W}$

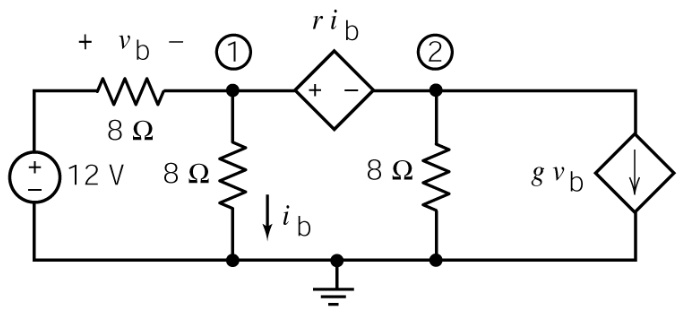


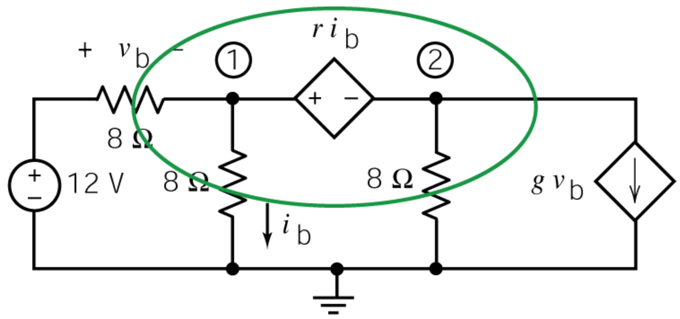
Figure P4.4-7

P4.4-7 The encircled numbers in the circuit shown Figure 4.4-27 are node numbers. The corresponding node voltages are:

$$v_1 = 9.74 \text{ V} \quad \text{and} \quad v_2 = 6.09 \text{ V}$$

Determine the values of the gains of the dependent sources, *r* and *g*.

Solution:



Using Ohm's law, $i_b = \dfrac{v_1}{8} = \dfrac{9.74}{8} = 1.2175 \text{ A}$. Using KVL, the voltage across the CCVS is

$$r\,i_b = v_1 - v_2 = 9.74 - 6.09 = 3.65 \text{ V}$$

Then
$$r = \frac{r\,i_b}{i_b} = \frac{3.65}{1.2175} = 2.9979 \text{ V/A}$$

Using KVL, $v_b = 12 - v_1 = 12 - 9.74 = 2.26 \text{ V}$. Apply KCL to the supernode corresponding to the CCVS to get

$$\frac{12 - v_1}{8} = \frac{v_1}{8} + \frac{v_2}{8} + g\,v_b \quad \Rightarrow \quad \frac{12 - 9.74}{8} = \frac{9.74}{8} + \frac{6.09}{8} + g\,v_b \quad \Rightarrow \quad g\,v_b = -1.6963 \text{ A}$$

Then
$$g = \frac{g\,v_b}{v_b} = \frac{-1.6963}{2.26} = -0.7506 \text{ A/V}$$

P 4.4-8 Determine the value of the power supplied by the dependent source in Figure P 4.4-8.

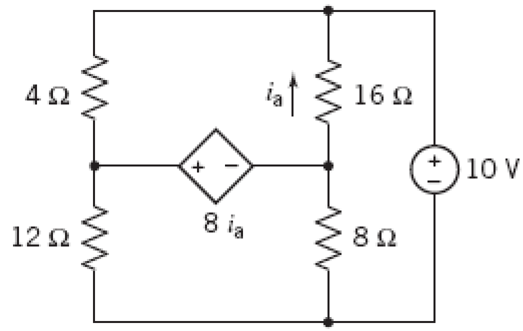
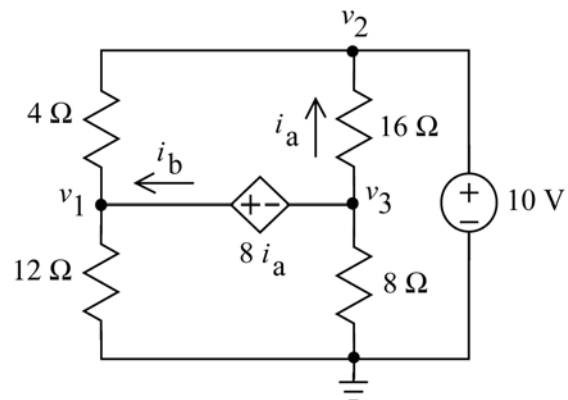


Figure P 4.4-8

Solution:

Label the node voltages.



First, v_2 = 10 V due to the independent voltage source. Next, express the controlling current of the dependent source in terms of the node voltages:

$$i_a = \frac{v_3 - v_2}{16} = \frac{v_3 - 10}{16}$$

Now the controlled voltage of the dependent source can be expressed as

$$v_1 - v_3 = 8\, i_a = 8\left(\frac{v_3 - 10}{16}\right) \quad \Rightarrow \quad v_1 = \frac{3}{2} v_3 - 5$$

Apply KCL to the supernode corresponding to the dependent source to get

$$\frac{v_1 - v_2}{4} + \frac{v_1}{12} + \frac{v_3 - v_2}{16} + \frac{v_3}{8} = 0$$

Multiplying by 48 and using v_2 = 10 V gives

$$16 v_1 + 9 v_3 = 150$$

Substituting the earlier expression for v_1

$$16\left(\frac{3}{2} v_3 - 5\right) + 9 v_3 = 150 \quad \Rightarrow \quad v_3 = 6.970 \text{ V}$$

Then v_1 = 5.455 V and i_a = -0.1894 A. Applying KCL at node 2 gives

$$\frac{v_1}{12} = i_b + \frac{10 - v_1}{4} \quad \Rightarrow \quad 12\, i_b = -3 + 4\, v_1 = -30 + 4(5.455)$$

So $i_b = -0.6817$ A.

Finally, the power supplied by the dependent source is

$$p = (8\, i_a)\, i_b = 8\,(-0.1894)\,(-0.6817) = 1.033 \text{ W}$$

(checked: LNAP 5/24/04)

P 4.4-9 The node voltages in the circuit shown in Figure P 4.4-9 are

$$v_1 = 4\text{ V},\ v_2 = 0\text{ V},\quad \text{and}\quad v_3 = -6\text{ V}$$

Determine the values of the resistance, R, and of the gain, b, of the CCCS.

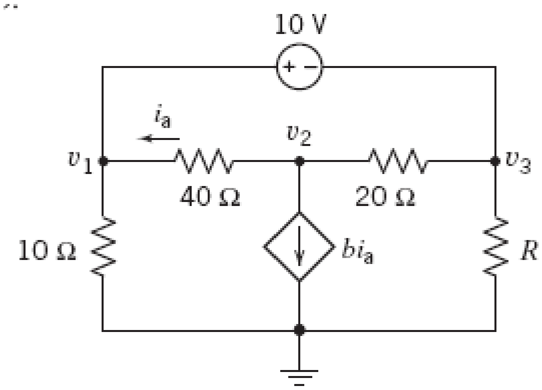
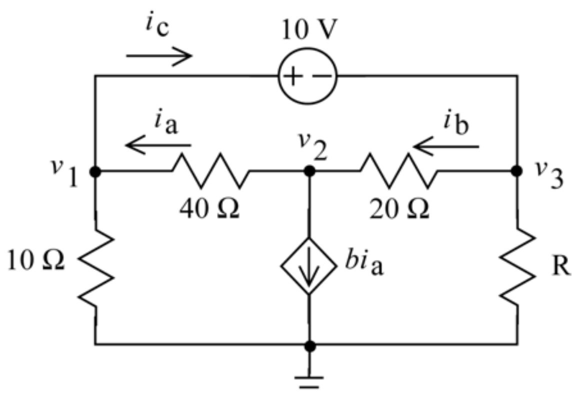


Figure P 4.4-9

Solution:



Apply KCL at node 2:

$$i_a + bi_a = i_b = \frac{v_3 - v_2}{20} = \frac{-6-(0)}{20} = -0.3\text{ A}$$

but

$$i_a = \frac{v_2 - v_1}{40} = \frac{0-4}{40} = -0.1$$

so

$$(1+b)(-0.1) = (-0.3) \quad \Rightarrow \quad b = 2\ \frac{\text{A}}{\text{A}}$$

Next apply KCL to the supernode corresponding to the voltage source.

$$\frac{v_1}{10} + 2\,i_a + \frac{v_3}{R} = 0 \quad \Rightarrow \quad \frac{4}{10} + 2(-0.1) + \frac{-6}{R} = 0 \quad \Rightarrow \quad R = \frac{6}{.2} = 30\ \Omega$$

(checked: LNAP 6/9/04)

P 4.4-10 The value of the node voltage at node *b* in the circuit shown in Figure P 4.4-10 is $v_b = 18$ V.

(a) Determine the value of *A*, the gain of the dependent source.

(b) Determine the power supplied by the dependent source.

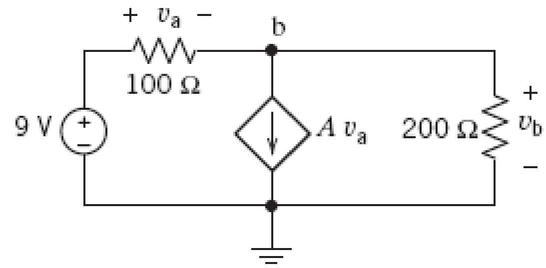


Figure P 4.4-10

Solution:

(a) Express the controlling voltage of the dependent source in terms of the node voltages:

$$v_a = 9 - v_b$$

Apply KCL at node b to get

$$\frac{9 - v_b}{100} = A(9 - v_b) + \frac{v_b}{200} \quad \Rightarrow \quad A = \frac{18 - 3v_b}{200(9 - v_b)} = 0.02$$

(b) The power supplied by the dependent source is

$$-(Av_a)v_b = -(0.02(9-18))(18) = 3.24 \text{ W}$$

(checked: LNAP 6/06/04)

P 4.4-11 Determine the power supplied by the dependent source in the circuit shown in Figure P 4.4-11.

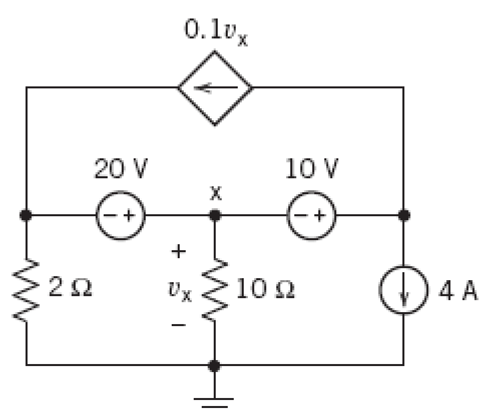


Figure P 4.4-11

Solution:

This circuit contains two ungrounded voltage sources, both incident to node x. In such a circuit it is necessary to merge the supernodes corresponding to the two ungrounded voltage sources into a single supernode. That single supernode separates the two voltage sources and their nodes from the rest of the circuit. It consists of the two resistors and the current source. Apply KCL to this supernode to get

$$\frac{v_x - 20}{2} + \frac{v_x}{10} + 4 = 0 \quad \Rightarrow \quad v_x = 10 \text{ V}.$$

The power supplied by the dependent source is

$$(0.1\,v_x)(-30) = -30 \text{ W}.$$

(checked: LNAP 6/6/04)

P 4.4-12 Determine values of the node voltages, v_1, v_2, v_3, v_4, and v_5, in the circuit shown in Figure P 4.4-12.

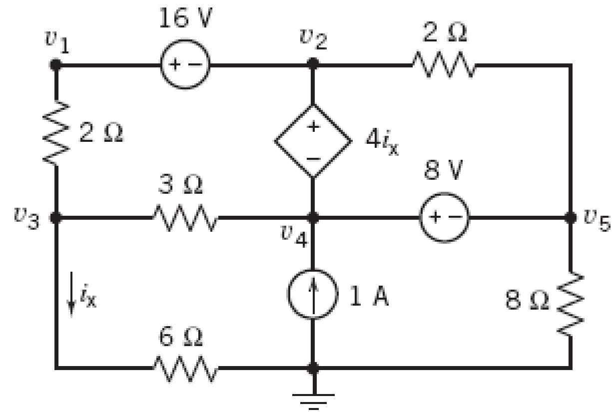
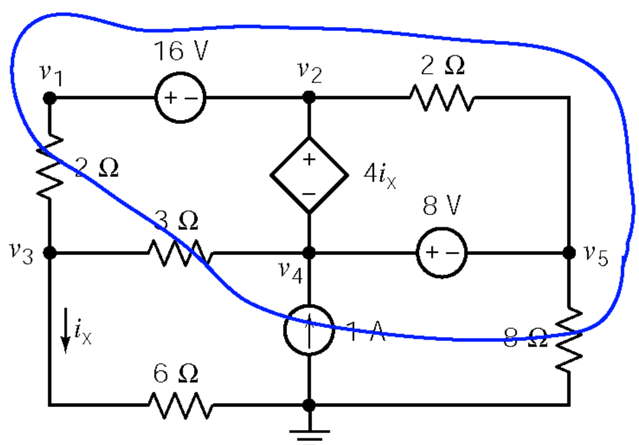


Figure P 4.4-12.

Solution:



Express the voltages of the independent voltage sources in terms of the node voltages

$$v_1 - v_2 = 16 \text{ and } v_4 - v_5 = 8$$

Express the controlling current of the dependent source in terms of the node voltages

$$i_x = \frac{v_3}{6}$$

Express the controlled voltage of the dependent source in terms of the node voltages

$$v_2 - v_4 = 4i_x = 4\left(\frac{v_3}{6}\right) \quad \Rightarrow \quad -6v_2 + 4v_3 + 6v_4 = 0$$

Apply KCL to the supernode to get

$$\frac{v_1 - v_3}{2} + \frac{v_4 - v_3}{3} + \frac{v_5}{8} = 1 \quad \Rightarrow \quad 12v_1 - 20v_3 + 8v_4 + 3v_5 = 24$$

Apply KCL at node 3 to get

$$\frac{v_3 - v_1}{2} + \frac{v_3}{6} + \frac{v_3 - v_4}{3} = 0 \quad \Rightarrow \quad -3v_1 + 6v_2 - 2v_4 = 0$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & -6 & 4 & 6 & 0 \\ 12 & 0 & -20 & 8 & 3 \\ -3 & 0 & 6 & -2 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 16 \\ 8 \\ 0 \\ 24 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 24 \\ 8 \\ 12 \\ 0 \\ -8 \end{bmatrix}$$

(checked: LNAP 6/13/04)

P 4.4-13 Determine values of the node voltages, v_1, v_2, v_3, v_4, and v_5, in the circuit shown in Figure P 4.4-13.

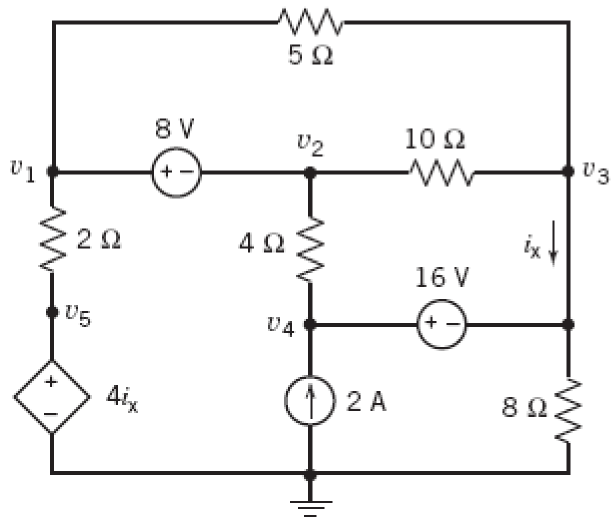


Figure P 4.4-13

Solution:

Express the voltage source voltages in terms of the node voltages:

$$v_1 - v_2 = 8 \;\text{ and }\; v_4 - v_3 = 16$$

Express the controlling current of the dependent source in terms of the node voltages:

$$i_x = \frac{v_2 - v_3}{10} + \frac{v_1 - v_3}{5} = 0.2v_1 + 0.1v_2 - 0.3v_3$$

Express the controlled voltage of the dependent source in terms of the node voltages:

$$v_5 = 4i_x = 0.8v_1 = 0.4v_2 - 1.2v_3 \qquad \Rightarrow \qquad 0.8v_1 + 0.4v_2 - 1.2v_3 - v_5 = 0$$

Apply KVL to the supernodes

$$\frac{v_1 - v_5}{2} + \frac{v_2 - v_4}{4} + \frac{v_2 - v_3}{10} + \frac{v_1 - v_3}{5} = 0 \qquad \Rightarrow \qquad 14v_1 + 7v_2 - 6v_3 - 5v_4 - 10v_5 = 0$$

$$\frac{v_4 - v_2}{4} + \frac{v_3}{8} + \frac{v_3 - v_2}{10} + \frac{v_3 - v_1}{5} = 2 \qquad \Rightarrow \qquad -8v_1 - 14v_2 + 17v_3 + 10v_4 = 80$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0.8 & 0.4 & -1.2 & 0 & -1 \\ 14 & 7 & -6 & -5 & -10 \\ -8 & -14 & 17 & 10 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 0 \\ 0 \\ 80 \end{bmatrix} \qquad \Rightarrow \qquad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 11.32 \\ 3.32 \\ 2.11 \\ 18.11 \\ 7.85 \end{bmatrix}$$

(checked: LNAP 6/13/04)

P 4.4-14 The voltages v_1, v_2, v_3, and v_4 are the node voltages corresponding to nodes 1, 2, 3, and 4 in Figure P 4.4-14. Determine the values of these node voltages.

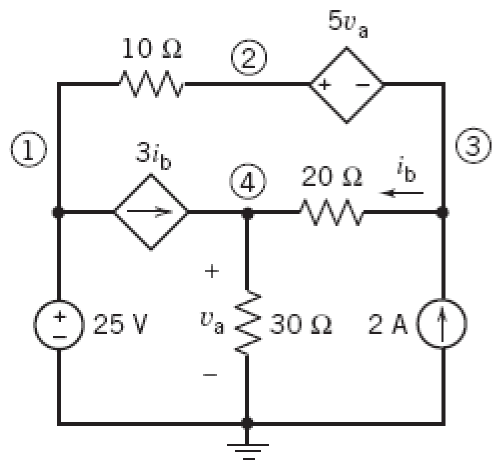
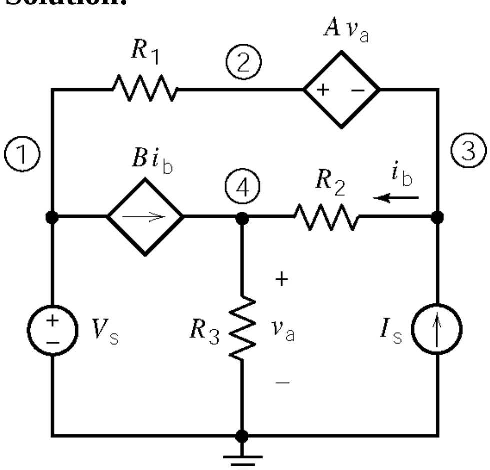


Figure P 4.4-14

Solution:



Express the controlling voltage and current of the dependent sources in terms of the node voltages:

$$v_a = v_4 \text{ and } i_b = \frac{v_3 - v_4}{R_2}$$

Express the voltage source voltages in terms of the node voltages:

$$v_1 = V_s \text{ and } v_2 - v_3 = A v_a = A v_4$$

Apply KCL to the supernode corresponding to the dependent voltage source

$$\frac{v_2 - v_1}{R_1} + \frac{v_3 - v_4}{R_2} = I_s \Rightarrow -R_2 v_1 + R_2 v_2 + R_1 v_3 - R_1 v_4 = R_1 R_2 I_s$$

Apply KCL at node 4:

$$B\frac{v_3 - v_4}{R_2} + \frac{v_3 - v_4}{R_2} = \frac{v_4}{R_3} \Rightarrow (B+1)v_3 - \left(B+1+\frac{R_2}{R_3}\right)v_4 = 0$$

Organizing these equations into matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -A \\ -R_2 & R_2 & R_1 & -R_1 \\ 0 & 0 & B+1 & -\left(B+1+\frac{R_2}{R_3}\right) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ R_1 R_2 I_s \\ 0 \end{bmatrix}$$

With the given values:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -5 \\ -20 & 20 & 10 & -10 \\ 0 & 0 & 4 & -4.667 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 25 \\ 0 \\ 400 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 25 \\ 44.4 \\ 8.4 \\ 7.2 \end{bmatrix}$$

(Checked using LNAP 9/29/04)

P 4.4-15 The voltages v_1, v_2, v_3, and v_4 in Figure P 4.4-15 are the node voltages corresponding to nodes 1, 2, 3, and 4. The values of these voltages are

$v_1 = 10$ V, $v_2 = 75$ V, $v_3 = -15$ V, and $v_4 = 22.5$ V

Determine the values of the gains of the dependent sources, A and B, and of the resistance R_1.

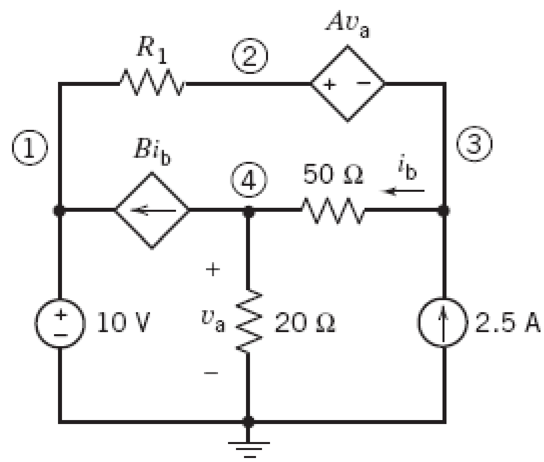


Figure P 4.4-15

Solution:

Express the controlling voltage and current of the dependent sources in terms of the node voltages:

$$v_a = v_4 = 22.5 \text{ V}$$

and
$$i_b = \frac{v_3 - v_4}{R_2} = \frac{-15 - 22.5}{50} = -0.75$$

Express the dependent voltage source voltage in terms of the node voltages:

$$v_2 - v_3 = A v_a = A v_4$$

so
$$A = \frac{v_2 - v_3}{v_4} = \frac{75 - (-15)}{22.5} = 4 \text{ V/V}$$

Apply KCL to the supernode corresponding to the dependent voltage source

$$\frac{v_2 - v_1}{R_1} + \frac{v_3 - v_4}{R_2} = I_s \quad \Rightarrow \quad \frac{75 - 10}{R_1} + \frac{-15 - 22.5}{50} = 2.5 \quad \Rightarrow \quad R_1 = 20 \ \Omega$$

Apply KCL at node 4:

$$\frac{v_3 - v_4}{R_2} = \frac{v_4}{R_3} + B\frac{v_3 - v_4}{R_2} \quad \Rightarrow \quad \frac{-15 - 22.5}{50} = \frac{22.5}{20} + B\frac{-15 - 22.5}{50} \quad \Rightarrow \quad B = 2.5 \text{ A/A}$$

(Checked using LNAP 9/29/04)

P 4.4-16 The voltages v_1, v_2, and v_3 in Figure P 4.4-16 are the node voltages corresponding to nodes 1, 2, and 3. The values of these voltages are

$$v_1 = 12 \text{ V}, \; v_2 = 21 \text{ V}, \quad \text{and} \quad v_3 = -3 \text{ V}$$

(a) Determine the values of the resistances R_1 and R_2.

(b) Determine the power supplied by each source.

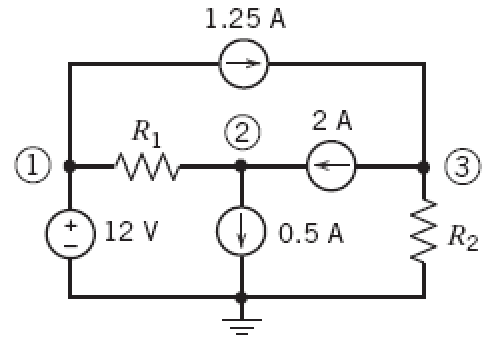


Figure P 4.4-16

Solution:

(a) $R_1 = \dfrac{v_2 - v_1}{2-0.5} = \dfrac{21-12}{1.5} = 6\ \Omega$ and $R_2 = \dfrac{v_2}{1.25-2} = \dfrac{-3}{-0.75} = 4\ \Omega$

(b) The power supplied by the voltage source is $12(0.5+1.25-2) = -3$ W. The power supplied by the 1.25-A current source is $1.25(-3-12) = -18.75$ W. The power supplied by the 0.5-A current source is $-0.5(21) = -10.5$ W. The power supplied by the 2-A current source is $2(21-(-3)) = 48$ W.

P 4.4-17 The voltages v_1, v_2, and v_3 in Figure P 4.4-17 are the node voltages corresponding to nodes 1, 2, and 3. The values of these voltages are

$$v_1 = 12 \text{ V}, \; v_2 = 9.6 \text{ V}, \quad \text{and} \quad v_3 = -1.33 \text{ V}$$

(a) Determine the values of the resistances R_1 and R_2.

(b) Determine the power supplied by each source.

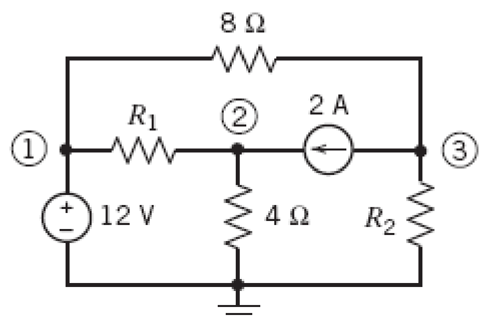
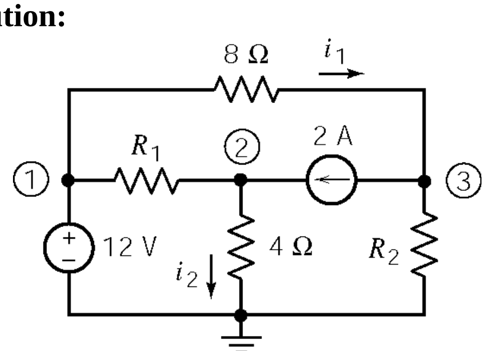


Figure P 4.4-18

Solution:



$$i_1 = \frac{12-(-1.33)}{8} = 1.666 \text{ A}$$

and

$$i_2 = \frac{9.6}{4} = 2.4 \text{ A}$$

(a) $R_1 = \dfrac{v_2 - v_1}{2-i_1} = \dfrac{9.6-12}{2-2.4} = 6\ \Omega$ and

$$R_2 = \frac{v_3}{i_1 - 2} = \frac{-1.33}{1.666-2} = 3.98 \simeq 4\ \Omega$$

(b) The power supplied by the voltage source is $12(2.4+1.66-2) = 24.7$ W. The power supplied by the current source is $2(9.6-(-1.33)) = 21.9$ W.

(Checked using LNAP 10/2/04)

P4.4-18

The voltages v_2, v_3 and v_4 for the circuit shown in Figure P4.4-18 are:

$$v_2 = 16 \text{ V}, \; v_3 = 8 \text{ V} \quad \text{and} \quad v_4 = 6 \text{ V}$$

Determine the values of the following:

(a) The gain, A, of the VCVS
(b) The resistance R_5
(c) The currents i_b and i_c
(d) The power received by resistor R_4

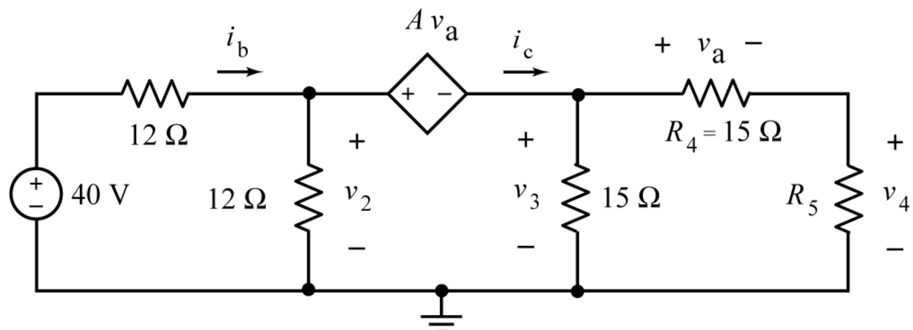


Figure P4.4-18

Solution:

Given the node voltages $\quad v_2 = 16 \text{ V}, \; v_3 = 8 \text{ V} \quad \text{and} \quad v_4 = 6 \text{ V}$

$$A = \frac{A v_a}{v_a} = \frac{16-8}{8-6} = 4 \; \frac{\text{V}}{\text{V}}$$

$$R_5 \left(\frac{v_3 - v_4}{15} \right) = v_4 \quad \Rightarrow \quad R_5 = \frac{15(6)}{8-6} = 45 \; \Omega,$$

$$i_b = \frac{40-24}{12} = 2 \text{ A} \;\; \text{and} \;\; i_c = \frac{40-16}{12} - \frac{16}{12} = 0.6667 \text{ A}$$

$$p_4 = \frac{{v_a}^2}{15} = \frac{2^2}{15} = 0.2667 \text{ W}$$

P4.4-19

Determine the values of the node voltages v_1 and v_2 for the circuit shown in Figure P4.4-19.

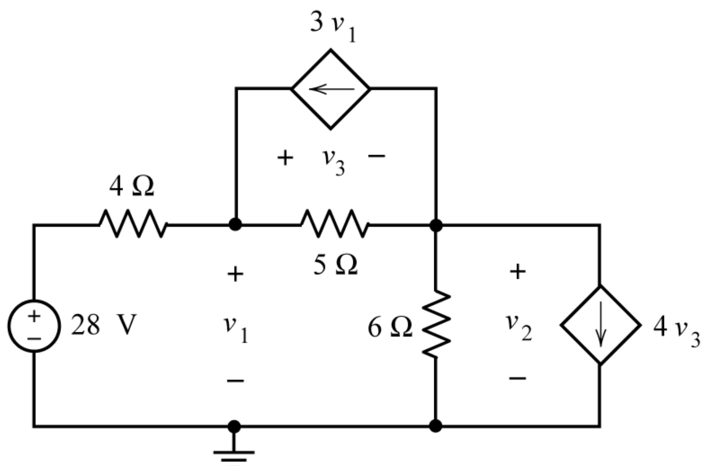


Figure P4.4-19

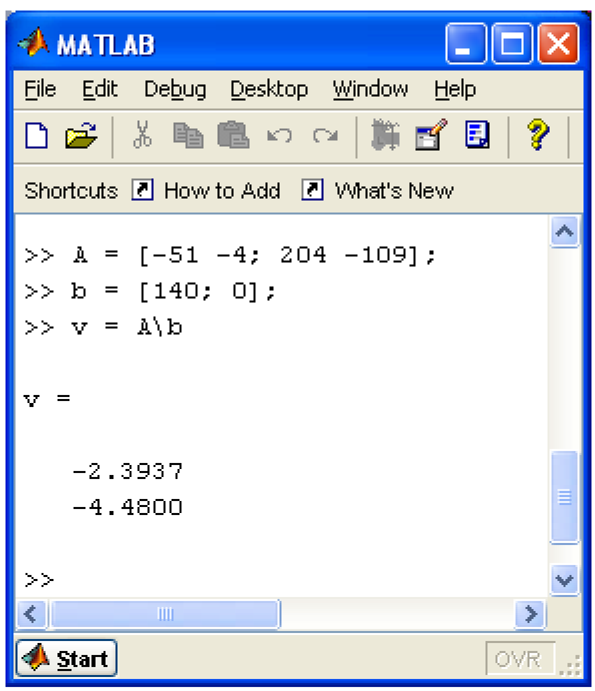
P4.4-19

The node equations are

$$\frac{28-v_1}{4}+3v_1=\frac{v_1-v_2}{5} \quad\Rightarrow\quad 5\left(28-v_1\right)+20\left(3v_1\right)=4\left(v_1-v_2\right) \quad\Rightarrow\quad 140=-51v_1-4v_2$$

and
$$\frac{v_1-v_2}{5}=3v_1+\frac{v_2}{6}+4v_3=3v_1+\frac{v_2}{6}+4\left(v_1-v_2\right) \quad\Rightarrow\quad 0=204v_1-109v_2$$

Using MATLAB to solve these equations:



Consequently $\quad v_1=-2.3937 \text{ V}$ and $v_2=-4.4800 \text{ V}$

P4.4-20

The encircled numbers in Figure P4.4-20 are node numbers. Determine the values of v_1, v_2 and v_3, the node voltages corresponding to nodes 1, 2 and 3.

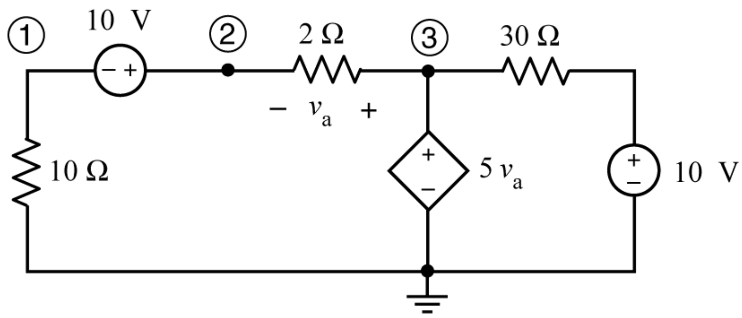


Figure P4.4-20

Solution:

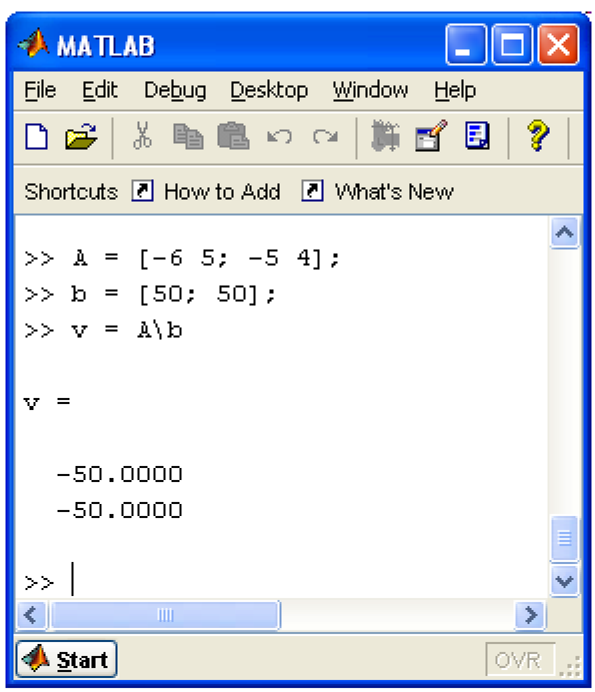
Apply KCL to the supernode corresponding to the horizontal voltage source to get

$$\frac{v_1}{10} = \frac{v_a}{2} = \frac{v_3 - v_2}{2} = \frac{v_3 - (v_1 + 10)}{2} \quad \Rightarrow \quad v_1 = 5\big(v_3 - (v_1 + 10)\big) \quad \Rightarrow \quad 50 = -6\,v_1 + 5\,v_3$$

Looking at the dependent source we notice that

$$v_3 = 5\,v_a = 5\big(v_3 - v_2\big) = 5\big(v_3 - (v_1 + 10)\big) \quad \Rightarrow \quad 50 = -5\,v_1 + 4\,v_3$$

Using MATLAB to solve these equations:



Consequently $\qquad v_1 = -50\text{ V}$ and $v_3 = -50\text{ V}$

Then $\qquad v_2 = v_1 + 10 = -40\text{ V}$

P4.4-21

Determine the values of the node voltages v_1, v_2 and v_3 for the circuit shown in Figure P4.4-21.

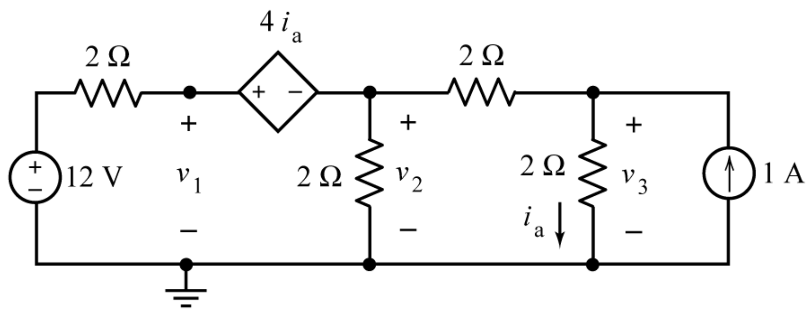


Figure P4.4-21

Solution:

Represent the controlling current of the dependent source in terms of the node voltages: $i_a = \dfrac{v_3}{2}$

Represent the controlled voltage of the dependent source in terms of the node voltages:

$$4i_a = v_1 - v_2 \quad \Rightarrow \quad = 4\left(\frac{v_3}{2}\right) = v_1 - v_2 \quad \Rightarrow \quad 0 = v_1 - v_2 - 2v_3$$

Apply KCL to the supernode corresponding to the dependent voltage source:

$$\frac{12 - v_1}{2} = \frac{v_2}{2} + \frac{v_2 - v_3}{2} \quad \Rightarrow \quad 12 - v_1 = v_2 + v_2 - v_3 \quad \Rightarrow \quad 12 = v_1 + 2v_2 - v_3$$

Apply KCL to top node of the current source:

$$\frac{v_2 - v_3}{2} + 1 = \frac{v_3}{2} \quad \Rightarrow \quad v_2 - v_3 + 2 = v_3 \quad \Rightarrow \quad 2 = -v_2 + 2v_3$$

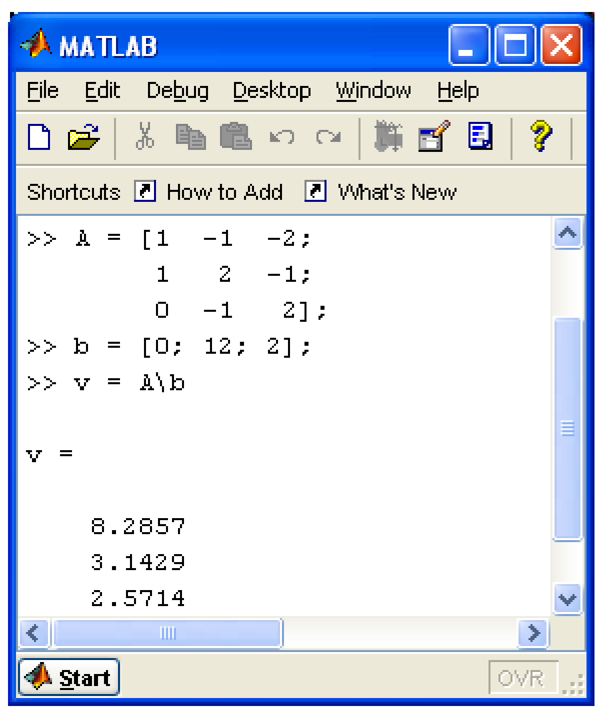
Solving these equations using MATLAB gives

$v_1 = 8.2857$ V,

$v_2 = 3.1459$ V

and

$v_3 = 2.5714$ V



P4.4-22

Determine the values of the node voltages v_1, v_2 and v_3 for the circuit shown in Figure P4.4-22.

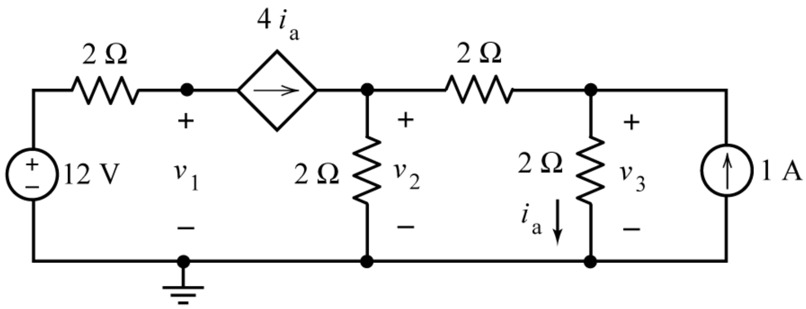


Figure P4.4-22

Solution:

The node equations are:

$$\frac{12-v_1}{2}=4i_a=4\left(\frac{v_3}{2}\right) \Rightarrow 12-v_1=4v_3 \Rightarrow 12=v_1+4v_3$$

$$4i_a=\frac{v_2}{2}+\frac{v_2-v_3}{2} \Rightarrow 4\left(\frac{v_3}{2}\right)=\frac{v_2}{2}+\frac{v_2-v_3}{2} \Rightarrow 0=2v_2-5v_3$$

$$\frac{v_2-v_3}{2}+1=\frac{v_3}{2} \Rightarrow v_2-v_3+2=v_3 \Rightarrow 2=-v_2+2v_3$$

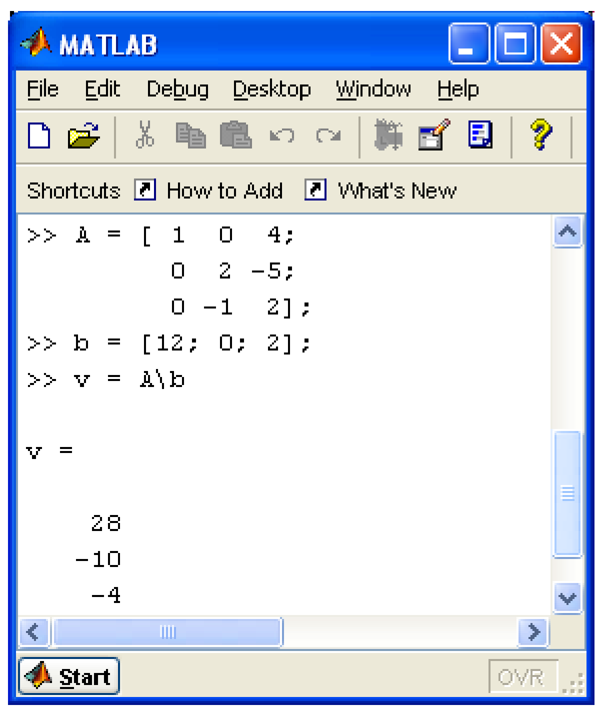
Solving these equations using MATLAB gives

$$v_1=28\text{ V},$$

$$v_2=-10\text{ V}$$

and

$$v_3=-4\text{ V}$$



Section 4-5 Mesh Current Analysis with Independent Voltage Sources

P 4.5-1 Determine the mesh currents, i_1, i_2, and i_3, for the circuit shown in Figure P 4.5-1.

Answers: $i_1 = 3$ A, $i_2 = 2$ A, and $i_3 = 4$ A

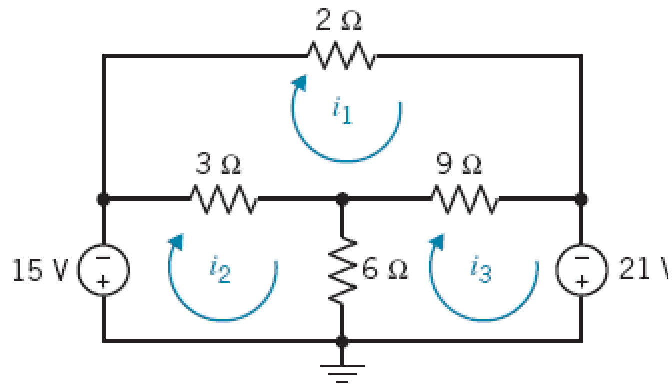


Figure P 4.5-1

Solution:
The mesh equations are

$$2\,i_1 + 9\,(i_1 - i_3) + 3\,(i_1 - i_2) = 0$$
$$15 - 3\,(i_1 - i_2) + 6\,(i_2 - i_3) = 0$$
$$-6\,(i_2 - i_3) - 9\,(i_1 - i_3) - 21 = 0$$

or

$$14\,i_1 - 3\,i_2 - 9\,i_3 = 0$$
$$-3\,i_1 + 9\,i_2 - 6\,i_3 = -15$$
$$-9\,i_1 - 6\,i_2 + 15\,i_3 = 21$$

so

$$i_1 = 3 \text{ A},\ \ i_2 = 2 \text{ A and } i_3 = 4 \text{ A}.$$

(checked using LNAP 8/14/02)

P 4.5-2 The values of the mesh currents in the circuit shown in Figure P 4.5-2 are

$$i_1 = 2 \text{ A},\ i_2 = 3 \text{ A, and } i_3 = 4 \text{ A}.$$

Determine the values of the resistance R and of the voltages v_1 and v_2 of the voltage sources.

Answers: $R = 12\ \Omega$, $v_1 = -4$ V, and $v_2 = -28$ V

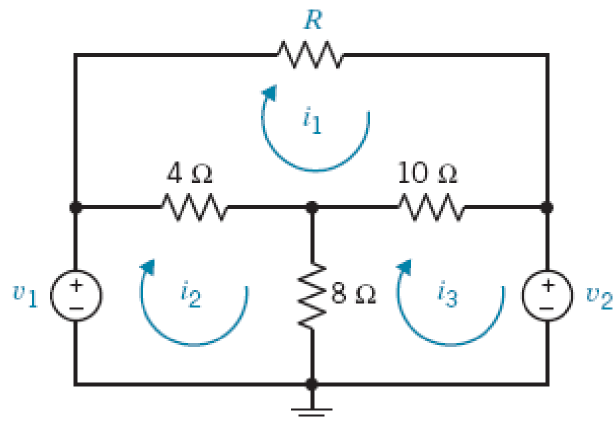


Figure P 4.5-2

Solution:

The mesh equations are:

Top mesh: $$4\,(2-3)+R(2)+10\,(2-4)=0$$

so $$R = 12\ \Omega.$$

Bottom, right mesh: $$8\,(4-3)+10\,(4-2)+v_2=0$$

so $$v_2 = -28 \text{ V}.$$

Bottom left mesh $$-v_1+4\,(3-2)+8\,(3-4)=0$$

so $$v_1 = -4 \text{ V}.$$

(checked using LNAP 8/14/02)

P 4.5-3 The currents i_1 and i_2 in Figure P 4.5-3 are the mesh currents. Determine the value of the resistance R required to cause $v_a = -6$ V.

Answer: $R = 4\ \Omega$

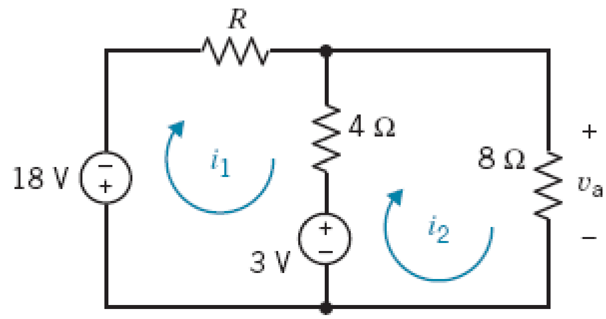
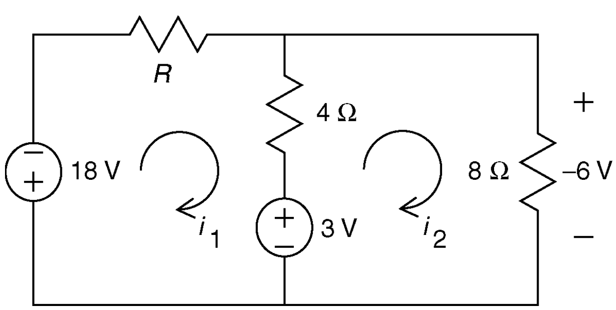


Figure P 4.5-3

Solution:



Ohm's Law: $i_2 = \frac{-6}{8} = -0.75$ A

KVL for loop 1:

$$R\,i_1 + 4\left(i_1 - i_2\right) + 3 + 18 = 0$$

KVL for loop 2

$$+(-6) - 3 - 4\left(i_1 - i_2\right) = 0$$

$$\Rightarrow \quad -9 - 4\left(i_1 - (-0.75)\right) = 0$$

$$\Rightarrow \quad i_1 = -3 \text{ A}$$

$$R(-3) + 4\left(-3 - (-0.75)\right) + 21 = 0 \quad \Rightarrow \quad R = 4\ \Omega$$

(checked using LNAP 8/14/02)

P 4.5-4 Determine the mesh currents, i_a and i_b, in the circuit shown in Figure P 4.5-4.

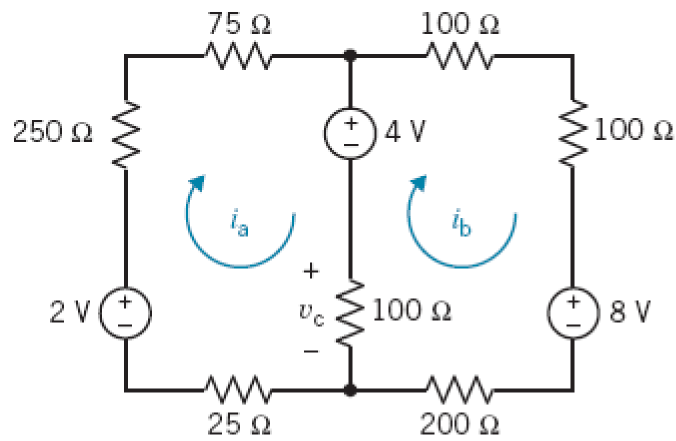


Figure P 4.5-4

Solution:

KVL loop 1:

$$25\, i_a - 2 + 250\, i_a + 75\, i_a + 4 + 100\, (i_a - i_b) = 0$$

$$450\, i_a - 100\, i_b = -2$$

KVL loop 2:

$$-100(i_a - i_b) - 4 + 100\, i_b + 100\, i_b + 8 + 200\, i_b = 0$$

$$-100\, i_a + 500\, i_b = -4$$

Solving these equations: $\underline{i_a = -6.5 \text{ mA}}$, $\underline{i_b = -9.3 \text{ mA}}$

(checked using LNAP 8/14/02)

P 4.5-5 Find the current i for the circuit of Figure P 4.5-5.

Hint: A short circuit can be treated as a 0-V voltage source.

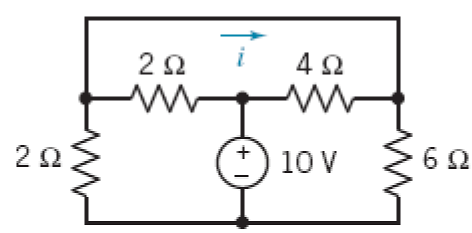
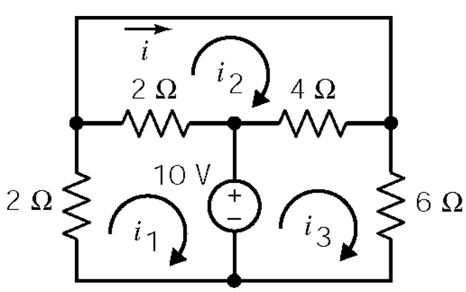


Figure P 4.5-5

Solution:



Mesh Equations:

$$\text{mesh 1}: \; 2i_1 + 2\,(i_1 - i_2) + 10 = 0$$

$$\text{mesh 2}: \; 2(i_2 - i_1) + 4\,(i_2 - i_3) = 0$$

$$\text{mesh 3}: \; -10 + 4\,(i_3 - i_2) + 6\, i_3 = 0$$

Solving:

$$i = i_2 \;\Rightarrow\; i = -\frac{5}{17} = -0.294 \text{ A}$$

(checked using LNAP 8/14/02)

P 4.5-6 Simplify the circuit shown in Figure P 4.5-6 by replacing series and parallel resistors by equivalent resistors. Next, analyze the simplified circuit by writing and solving mesh equations.

(a) Determine the power supplied by each source.

(b) Determine the power absorbed by the 30-Ω resistor.

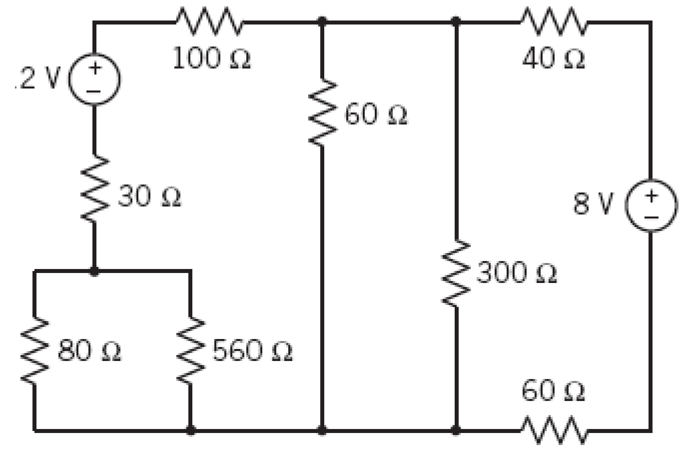
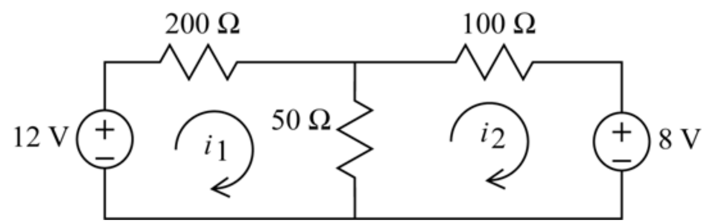


Figure P 4.5-6

Solution: Replace series and parallel resistors with equivalent resistors:

$$60\ \Omega \,||\, 300\ \Omega = 50\ \Omega,\quad 40\ \Omega + 60\ \Omega = 100\ \Omega \quad \text{and} \quad 100\ \Omega + 30\ \Omega + (80\ \Omega \,||\, 560\ \Omega) = 200\ \Omega$$

so the simplified circuit is



The mesh equations are

$$200 i_1 + 50(i_1 - i_2) - 12 = 0$$
$$100 i_2 + 8 - 50(i_1 - i_2) = 0$$

or

$$\begin{bmatrix} 250 & -50 \\ -50 & 150 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 12 \\ -8 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0.04 \\ -0.04 \end{bmatrix}$$

The power supplied by the 12 V source is $12 i_1 = 12(0.04) = 0.48\text{ W}$. The power supplied by the 8 V source is $-8 i_2 = -8(-0.04) = 0.32\text{ W}$. The power absorbed by the 30 Ω resistor is

$$i_1^2(30) = (0.04)^2(30) = 0.048\text{ W}.$$

(checked: LNAP 5/31/04)

Section 4-6 Mesh Current Analysis with Voltage and Current Sources

P 4.6-1 Find i_b for the circuit shown in Figure P 4.6-1.

Answer: $i_b = 0.6$ A

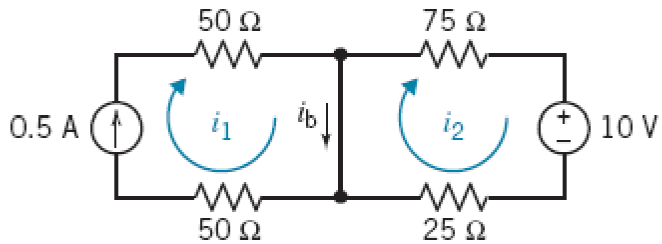


Figure P 4.6-1

Solution:

$$\text{mesh 1:} \quad i_1 = \frac{1}{2} \text{ A}$$

$$\text{mesh 2:} \quad 75\, i_2 + 10 + 25\, i_2 = 0 \quad \Rightarrow i_2 = -0.1 \text{ A}$$

$$i_b = i_1 - i_2 = \underline{0.6 \text{ A}}$$

(checked using LNAP 8/14/02)

P 4.6-2 Find v_c for the circuit shown in Figure P 4.6-2.

Answer: $v_c = 15$ V

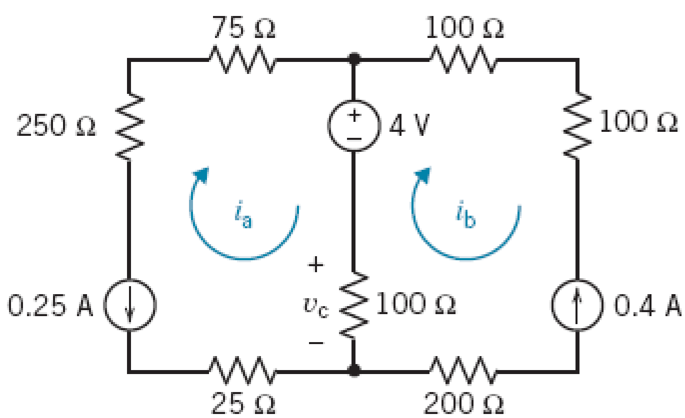


Figure P 4.6-2

Solution:

Mesh currents:

$$\text{mesh a:} \quad i_a = -0.25 \text{ A}$$

$$\text{mesh b:} \quad i_b = -0.4 \text{ A}$$

Ohm's Law:

$$v_c = 100(i_a - i_b) = 100(0.15) = \underline{15 \text{ V}}$$

(checked using LNAP 8/14/02)

P 4.6-3 Find v_2 for the circuit shown in Figure P 4.6-3.

Answer: $v_2 = 2$ V

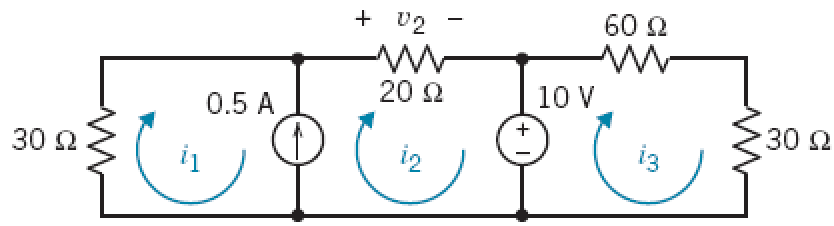
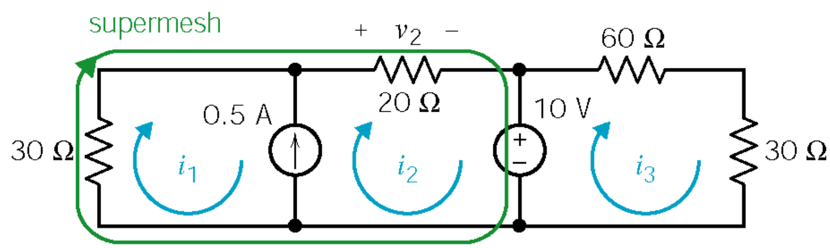


Figure P 4.6-3

Solution:



Express the current source current as a function of the mesh currents:

$$i_1 - i_2 = -0.5 \Rightarrow i_1 = i_2 - 0.5$$

Apply KVL to the supermesh:

$$30\, i_1 + 20\, i_2 + 10 = 0 \quad \Rightarrow \quad 30\,(i_2 - 0.5) + 20 i_2 = -10$$

$$50\, i_2 - 15 = -10 \quad \Rightarrow \quad i_2 = \frac{5}{50} = .1 \text{ A}$$

$$i_1 = -.4 \text{ A} \quad \text{and} \quad v_2 = 20\, i_2 = \underline{2 \text{ V}}$$

(checked using LNAP 8/14/02)

P 4.6-4 Find v_c for the circuit shown in Figure P 4.6-4.

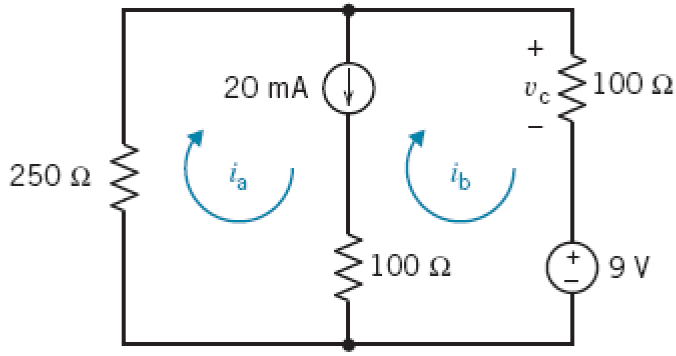


Figure P 4.6-4

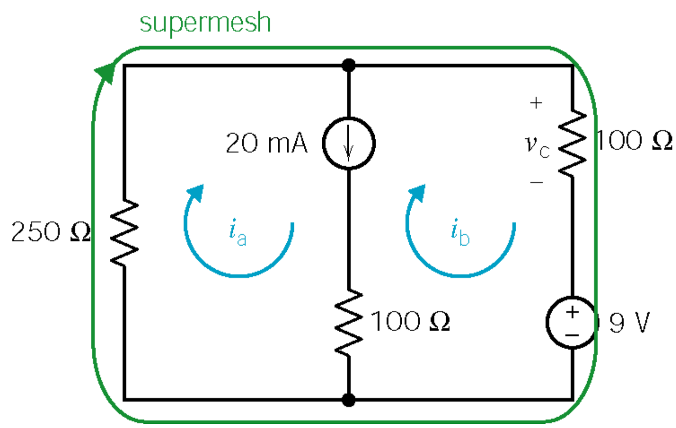
Solution:

Express the current source current in terms of the mesh currents:

$$i_b = i_a - 0.02$$

Apply KVL to the supermesh:

$$\begin{aligned} 250\, i_a + 100\,(i_a - 0.02) + 9 &= 0 \\ \therefore i_a &= -.02 \text{ A} = -20 \text{ mA} \\ v_c &= 100(i_a - 0.02) = \underline{-4 \text{ V}} \end{aligned}$$



(checked using LNAP 8/14/02)

P 4.6-5 Determine the value of the voltage measured by the voltmeter in Figure P 4.6-5.

Answer: 8 V

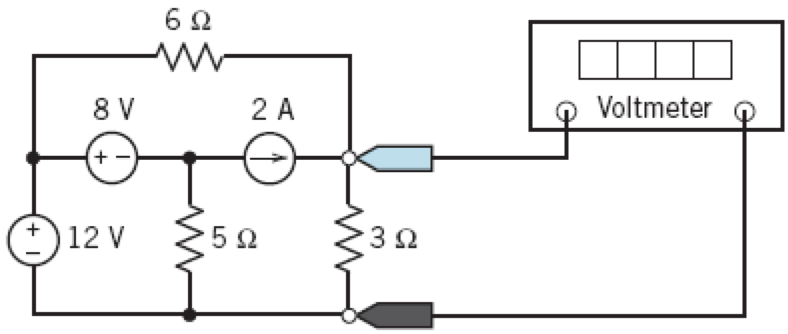
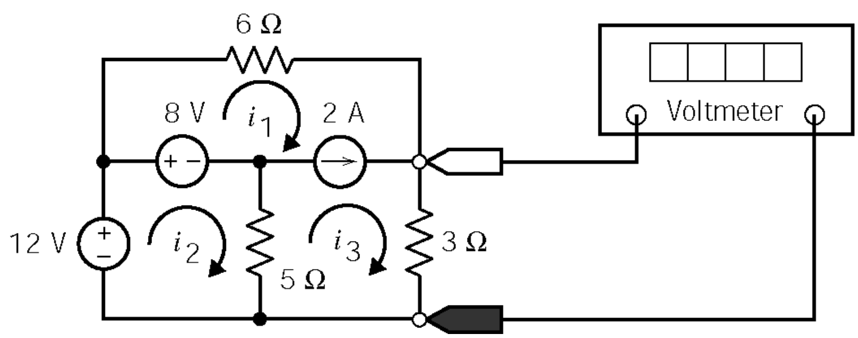


Figure P 4.6-5

Solution: Label the mesh currents:



Express the current source current in terms of the mesh currents:

$$i_3 - i_1 = 2 \quad \Rightarrow \quad i_1 = i_3 - 2$$

Supermesh: $$6\,i_1 + 3\,i_3 - 5\left(i_2 - i_3\right) - 8 = 0 \quad \Rightarrow \quad 6\,i_1 - 5\,i_2 + 8\,i_3 = 8$$

Lower, left mesh: $$-12 + 8 + 5\left(i_2 - i_3\right) = 0 \quad \Rightarrow \quad 5\,i_2 = 4 + 5\,i_3$$

Eliminating i_1 and i_2 from the supermesh equation:

$$6\left(i_3 - 2\right) - \left(4 + 5\,i_3\right) + 8\,i_3 = 8 \quad \Rightarrow \quad 9\,i_3 = 24$$

The voltage measured by the meter is: $3\,i_3 = 3\left(\frac{24}{9}\right) = 8 \text{ V}$

(checked using LNAP 8/14/02)

P 4.6-6 Determine the value of the current measured by the ammeter in Figure P 4.6-6.

Hint: Write and solve a single mesh equation.

Answer: –5/6 A

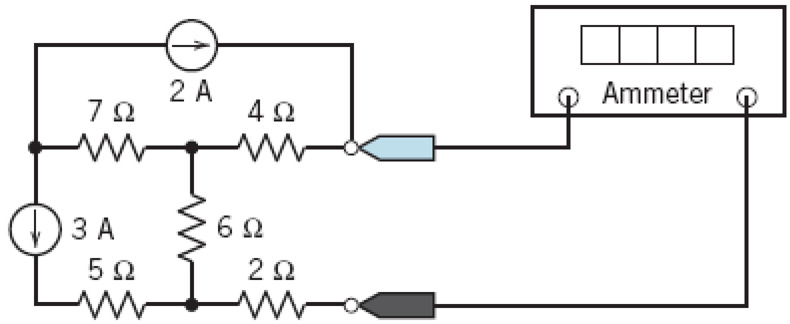
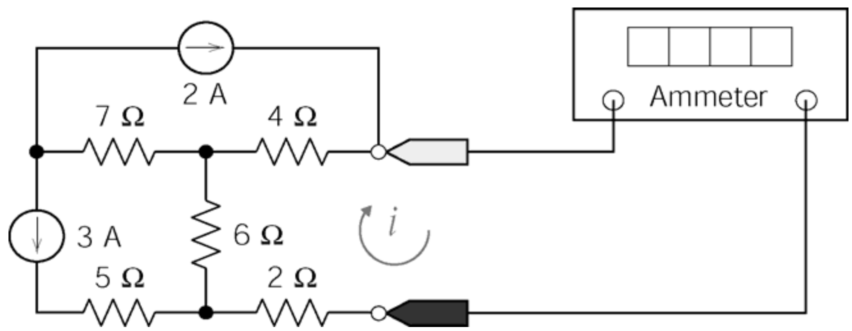


Figure P 4.6-6

Solution:



Mesh equation for right mesh:

$$4(i-2)+2\,i+6(i+3)=0 \quad\Rightarrow\quad 12\,i-8+18=0 \quad\Rightarrow\quad i=-\frac{10}{12}\text{ A}=-\frac{5}{6}\text{ A}$$

(checked using LNAP 8/14/02)

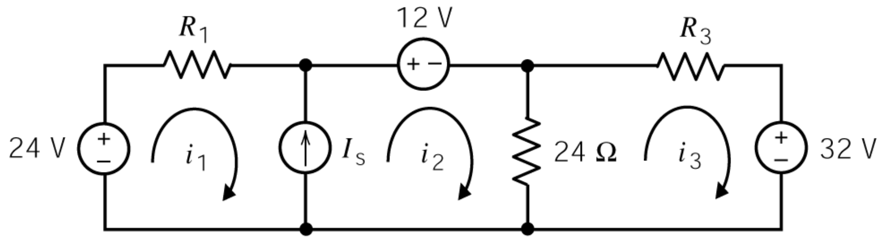


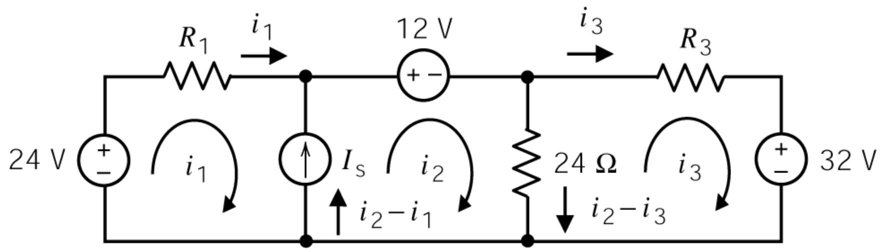
Figure P4.6-7

P4.6-7 The mesh currents are labeled in the circuit shown Figure 4.6-7. The value of these mesh currents are:

$$i_1 = -1.1014 \text{ A}, \; i_2 = 0.8986 \text{ A} \; \text{ and } \; i_3 = -0.2899 \text{ A}$$

a.) Determine the values of the resistances R_1 and R_3.
b.) Detemine the value of the current sourc current.
c.) Determine the value of the power supplied by the 12 V voltage source.

Solution: Label the resistor currents and the current source currrents in terms of the mesh currents:



a.) Apply KVL to the supermesh corresponding to the current source to get

$$R_1 i_1 + 12 + 24(i_2 - i_3) - 24 = 0 \quad \Rightarrow \quad R_1 = \frac{12 - 24(i_2 - i_3)}{i_1} = \frac{12 - 24(0.8986 - (-0.2899))}{-1.1014} = 15\ \Omega$$

Apply KVL to the rightmost mesh to get

$$R_3 i_3 + 32 - 24(i_2 - i_3) = 0 \quad \Rightarrow \quad R_3 = \frac{-32 + 24(i_2 - i_3)}{i_3} = \frac{-32 + 24(0.8986 - (-0.2899))}{-0.2899} = 12\ \Omega$$

b.)

$$I_s = i_2 - i_1 = 0.8986 - (-1.1014) = 2 \text{ A}$$

c.) Noticing that 12 V and i_2 adhere to the passive convention, the power supplied by the 12 V voltage source is

$$-12 i_2 = -12(0.8986) = -10.783 \text{ W}.$$

P 4.6-8 Determine values of the mesh currents, i_1, i_2, and i_3, in the circuit shown in Figure P 4.6-8.

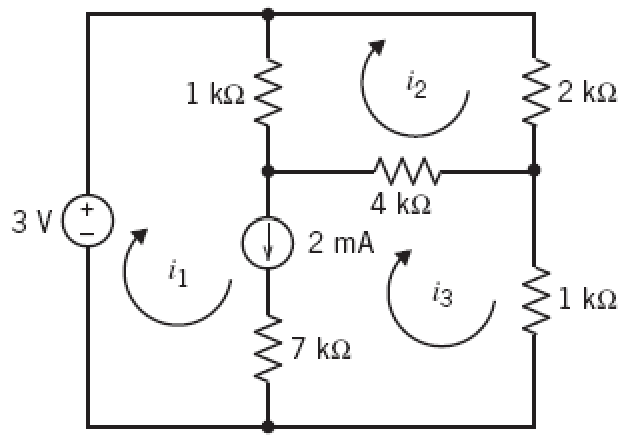


Figure P 4.6-8

Solution: Use units of V, mA and kΩ. Express the currents to the supermesh to get

$$i_1 - i_3 = 2$$

Apply KVL to the supermesh to get

$$4(i_3 - i_2) + (1)i_3 - 3 + (1)(i_1 - i_2) = 0 \quad \Rightarrow \quad i_1 - 5i_2 + 5i_3 = 3$$

Apply KVL to mesh 2 to get

$$2i_2 + 4(i_2 - i_3) + (1)(i_2 - i_1) = 0 \quad \Rightarrow \quad (-1)i_1 + 7i_2 - 4i_3 = 0$$

Solving, e.g. using MATLAB, gives

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & -5 & 5 \\ -1 & 7 & -4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$$

(checked: LNAP 6/21/04)

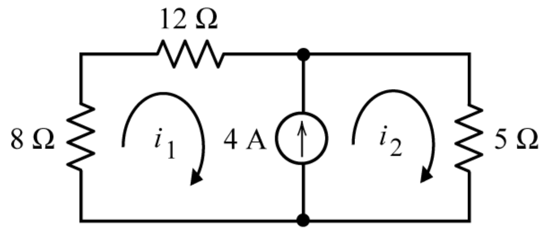


Figure P4.6-9

P4.6-9 The mesh currents are labeled in the circuit shown Figure 4.6-9. Determine the value of the mesh currents i_1 and i_2.

Solution: Determine the value of the mesh currents i_1 and i_2.

$$i_2 = i_1 + 4\text{ A},\ \ 8i_1 + 12i_1 + 5i_2 = 0 \quad \Rightarrow \quad 8i_1 + 12i_1 + 5\left(4 + i_1\right) = 0 \quad \Rightarrow \quad i_1 = -0.8\text{ A}$$

and

$$i_2 = i_1 + 4 = -0.8 + 4 = 3.2\text{ A}$$

P 4.6-10 The mesh currents in the circuit shown in Figure P 4.6-10 are

$i_1 = -2.2213$ A, $i_2 = 0.7787$ A, and $i_3 = 0.0770$ A

(a) Determine the values of the resistances R_1 and R_3.

(b) Determine the value of the power supplied by the current source.

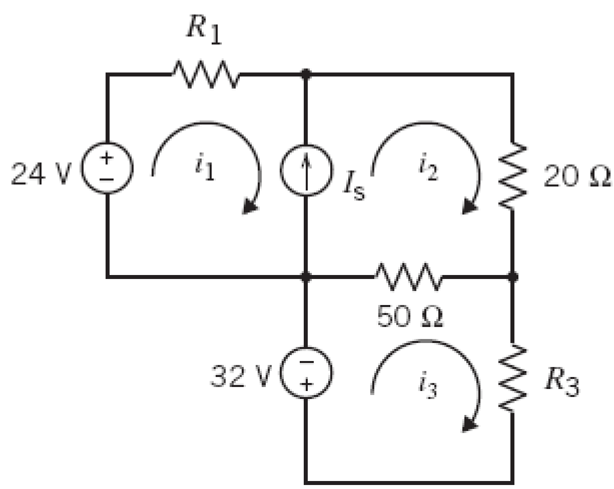


Figure P 4.6-10

Solution:

(a)

$$50\left(i_3 - i_2\right) + R_3 i_3 + 32 = 0 \quad \Rightarrow \quad 50\left(0.0770 - 0.7787\right) + R_3\left(0.0770\right) + 32 = 0$$

$$\Rightarrow \quad R_3 = 40\ \Omega$$

$$i_1 R_1 + 20i_2 + 50\left(i_2 - i_3\right) - 24 = 0 \quad \Rightarrow \quad R_1\left(-2.2213\right) + 20\left(0.7787\right) + 50\left(0.7787 - 0.0770\right) = 24$$

$$\Rightarrow \quad R_1 = 12\ \Omega$$

(b)

$$I_s = i_2 - i_1 = 0.7787 - \left(-2.2213\right) = 3\text{ A}$$

The power supplied by the current source is

$$p = I_s\left(24 - R_1 i_1\right) = 3\left(24 - 12\left(-2.2213\right)\right) = 152\text{ W}$$

(checked: LNAP 6/19/04)

P 4.6-11 Determine the value of the voltage measured by the voltmeter in Figure P 4.6-11.

Hint: Apply KVL to a supermesh to determine the current in the 2-Ω resistor.

Answer: 4/3 V

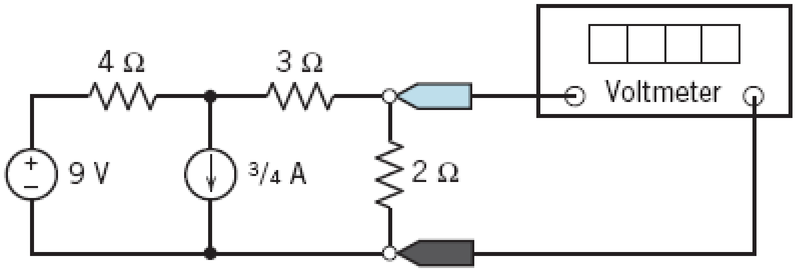
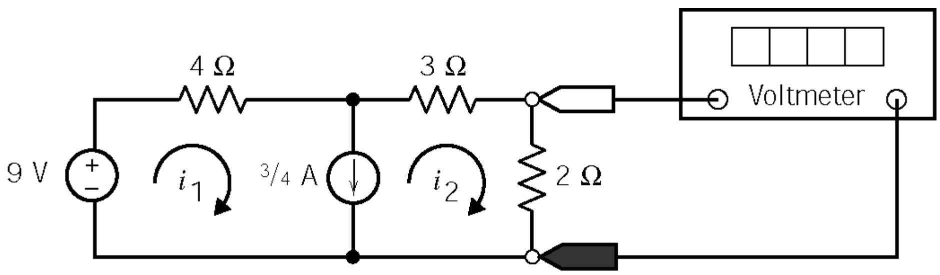


Figure P 4.6-11

Solution:



Express the current source current in terms of the mesh currents: $\frac{3}{4} = i_1 - i_2 \quad \Rightarrow \quad i_1 = \frac{3}{4} + i_2$.

Apply KVL to the supermesh: $-9 + 4i_1 + 3\,i_2 + 2\,i_2 = 0 \quad \Rightarrow \quad 4\left(\frac{3}{4} + i_2\right) + 5\,i_2 = 9 \quad \Rightarrow \quad 9\,i_2 = 6$

so $i_2 = \frac{2}{3}$ A and the voltmeter reading is $2\,i_2 = \frac{4}{3}$ V

P 4.6-12 Determine the value of the current measured by the ammeter in Figure P 4.6-12.

Hint: Apply KVL to a supermesh.

Answer: –0.333 A

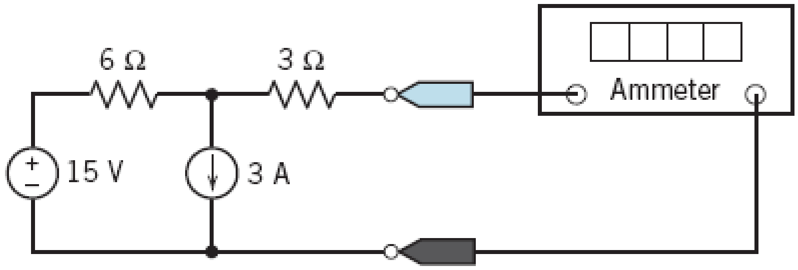
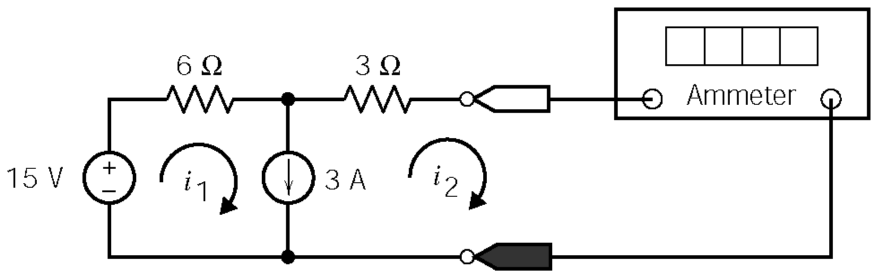


Figure P 4.6-12

Solution:



Express the current source current in terms of the mesh currents: $3 = i_1 - i_2 \quad \Rightarrow \quad i_1 = 3 + i_2$.

Apply KVL to the supermesh: $-15 + 6\, i_1 + 3\, i_2 = 0 \quad \Rightarrow \quad 6\left(3 + i_2\right) + 3\, i_2 = 15 \quad \Rightarrow \quad 9\, i_2 = -3$

Finally, $i_2 = -\dfrac{1}{3}$ A is the current measured by the ammeter.

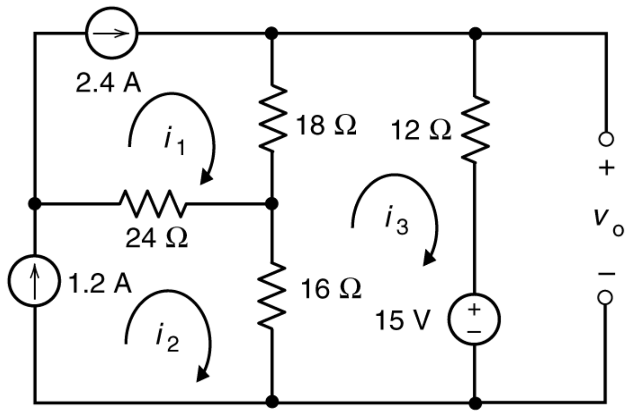
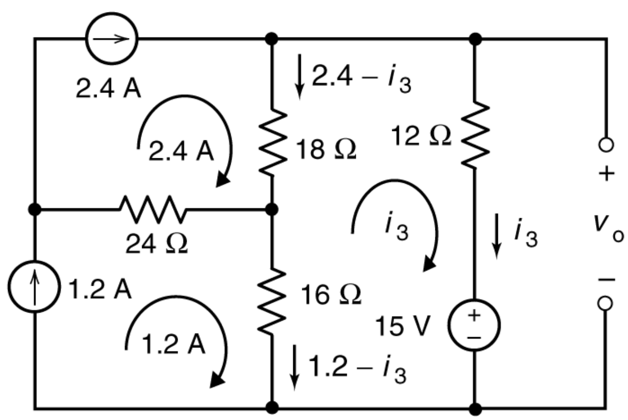


Figure P4.6-13

P4.6-13 Determine the values of the mesh currents i_1, i_2 and i_3 and the output voltage v_o in the circuit shown Figure 4.6-13.

Solution: Notice that the current source are each in a single mesh. Consequently, i_1 = 2.4 A and i_2 = 1.2 A. Label the resistor currents in terms of the mesh currents:



Apply KVL to mesh 3 to get

$$12 i_3 + 15 - 16\left(1.2 - i_3\right) - 18\left(2.4 - i_3\right) = 0 \quad \Rightarrow \quad 46 i_3 = 47.4 \quad \Rightarrow \quad i_3 = 1.0304 \text{ A}$$

Apply KVL to the rightmost mesh to get

$$v_o - 15 - 12 i_3 = 0 \quad \Rightarrow \quad v_o = 15 + 12\left(1.0304\right) = 27.3648 \text{ V}$$

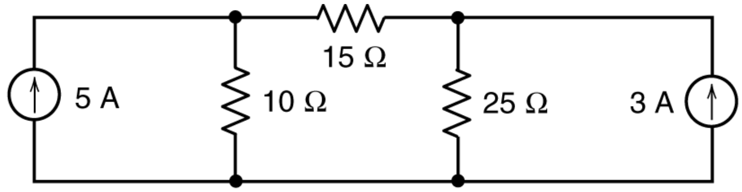
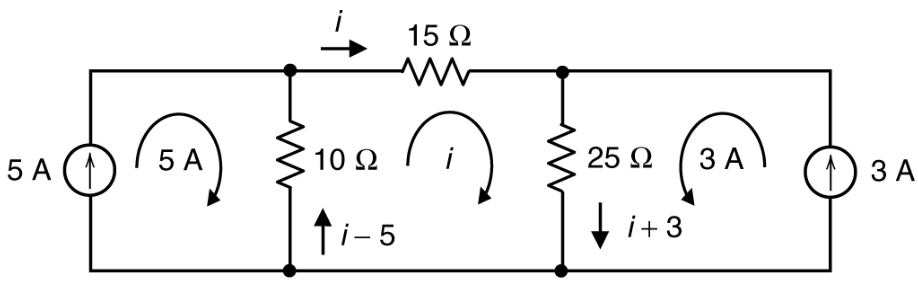


Figure P4.6-14

P4.6-14 Determine the values of the power supplied by the sources in the circuit shown Figure P4.6-14.

Solution: First, label the mesh currents, taking advantage of the current sources. Next, express the resistor currents in terms of the mesh currents:



Apply KVL to the middle mesh: $15i + 25(i+3) + 10(i-5) = 0 \quad \Rightarrow \quad i = -\frac{1}{2}$ A

The 5 A current source supplies $5(10)(i-5) = 5(10)\left(-\frac{1}{2} - 5\right) = 275$ W

The 3 A current source supplies $3(25)(i+3) = 3(25)\left(-\frac{1}{2} + 3\right) = 187.5$ W

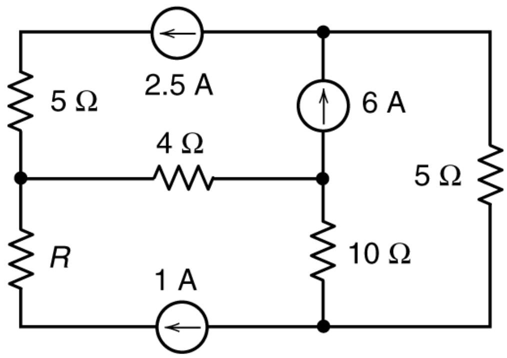
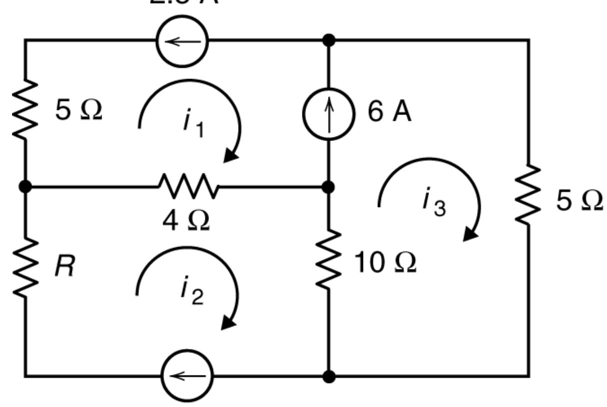


Figure P4.6-15

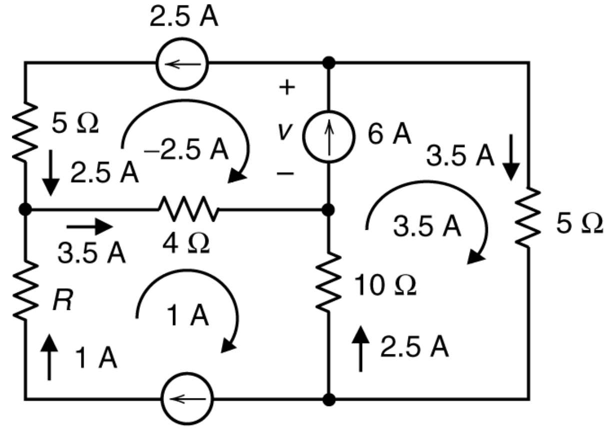
P4.6-15 Determine the values of the resistance *R* and of the power supplied by the 6 A current source in the circuit shown Figure P4.6-15.

Solution: First, label the mesh currents:



Notice that $i_1 = -2.5 \text{ A},\ i_2 = 1 \text{ A}$ and $6 = i_3 + 2.5 \quad \Rightarrow \quad i_3 = 3.5 \text{ A}$

Next, express the resistor currents in terms of the mesh currents:



Apply KVL to the bottom, left mesh: $4(3.5) - 10(2.5) + R(1) = 0 \quad \Rightarrow \quad R = 11\ \Omega$

Apply KVL to the right mesh $3.5(5) + 2.5(10) - v = 0 \quad \Rightarrow \quad v = 42.5 \text{ V}$

The 6 A current source supplies $6v = 6(42.5) = 255 \text{ W}$

Section 4-7 Mesh Current Analysis with Dependent Sources

P 4.7-1 Find v_2 for the circuit shown in Figure P 4.7-1.

Answer: $v_2 = 10$ V

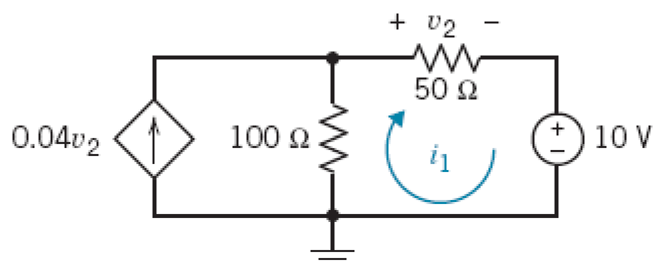


Figure P 4.7-1

Solution:
Express the controlling voltage of the dependent source as a function of the mesh current

$$v_2 = 50\, i_1$$

Apply KVL to the right mesh:

$$-100\,(0.04(50 i_1) - i_1) + 50 i_1 + 10 = 0 \Rightarrow i_1 = 0.2 \text{ A}$$
$$v_2 = 50\, i_1 = 10 \text{ V}$$

(checked using LNAP 8/14/02)

P4.7-2 Determine the values of the power supplied by the voltage source and by the CCCS in the circuit shown Figure P4.7-2

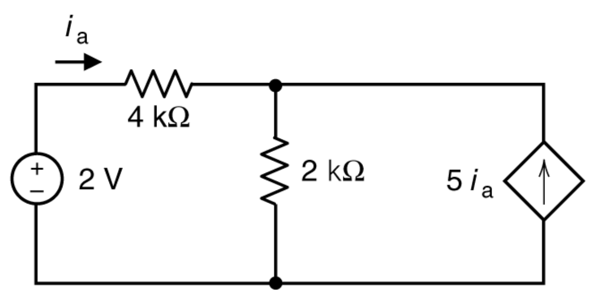
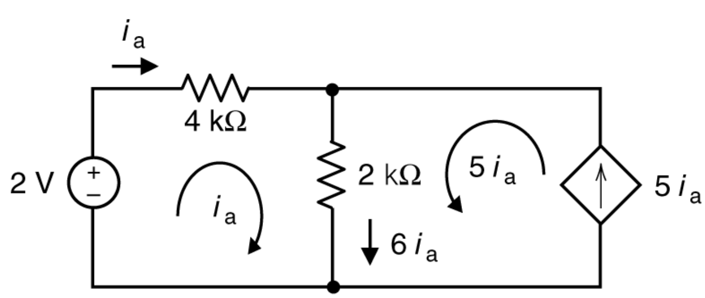


Figure P4.7-2

Solution: First, label the mesh currents, taking advantage of the current sources. Next, express the resistor currents in terms of the mesh currents:



Apply KVL to the left mesh: $4000\,i_a + 2000\left(6\,i_a\right) - 2 = 0 \quad \Rightarrow \quad i_a = \dfrac{1}{8} = 0.125$ mA

The 2 A voltage source supplies $\quad 2\,i_a = 2\left(0.125 \times 10^{-3}\right) = 0.25$ mW

The CCCS supplies $\quad \left(5\,i_a\right)\left[\left(2000\right)\left(6\,i_a\right)\right] = \left(60 \times 10^3\right)\left(0.125 \times 10^{-3}\right)^2 = 0.9375 \times 10^{-3} = 0.9375$ mW

P 4.7-3 Find v_o for the circuit shown in Figure P 4.7-3.

Answer: $v_o = 2.5$ V

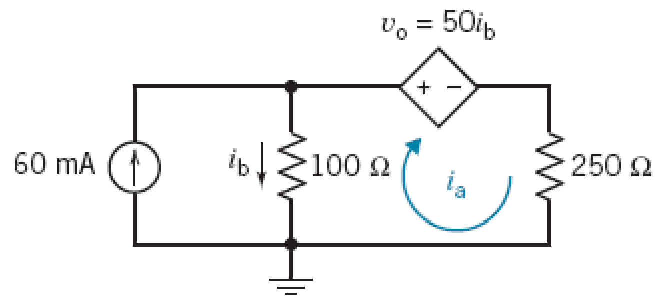


Figure P 4.7-3

Solution: Express the controlling current of the dependent source as a function of the mesh current:

$$i_b = .06 - i_a$$

Apply KVL to the right mesh:

$$-100\ (0.06 - i_a) + 50\ (0.06 - i_a) + 250\ i_a = 0 \quad \Rightarrow \quad i_a = 10 \text{ mA}$$

Finally:

$$v_o = 50\, i_b = 50\,(0.06 - 0.01) \ = 2.5 \text{ V}$$

(checked using LNAP 8/14/02)

P 4.7-4 Determine the mesh current i_a for the circuit shown in Figure P 4.7-4.

Answer: $i_a = -24$ mA

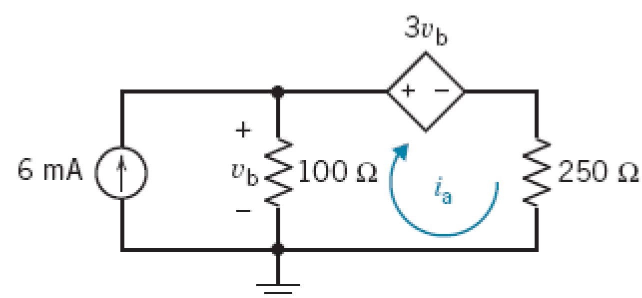


Figure P 4.7-4

Solution: Express the controlling voltage of the dependent source as a function of the mesh current:

$$v_b = 100\ (.006 - i_a)$$

Apply KVL to the right mesh:

$$-100\ (.006 - i_a) + 3\left[100(.006 - i_a)\right] + 250\, i_a = 0 \quad \Rightarrow \quad \underline{i_a = -24 \text{ mA}}$$

(checked using LNAP 8/14/02)

P 4.7-5 Although scientists continue to debate exactly why and how it works, the process of utilizing electricity to aid in the repair and growth of bones—which has been used mainly with fractures—may soon be extended to an array of other problems, ranging from osteoporosis and osteoarthritis to spinal fusions and skin ulcers.

An electric current is applied to bone fractures that have not healed in the normal period of time. The process seeks to imitate natural electrical forces within the body. It takes only a small amount of electric stimulation to accelerate bone recovery. The direct current method uses an electrode that is implanted at the bone. This method has a success rate approaching 80 percent.

The implant is shown in Figure P 4.7-5*a* and the circuit model is shown in Figure P 4.7-5*b*. Find the energy delivered to the cathode during a 24-hour period. The cathode is represented by the dependent voltage source and the 100-kΩ resistor.

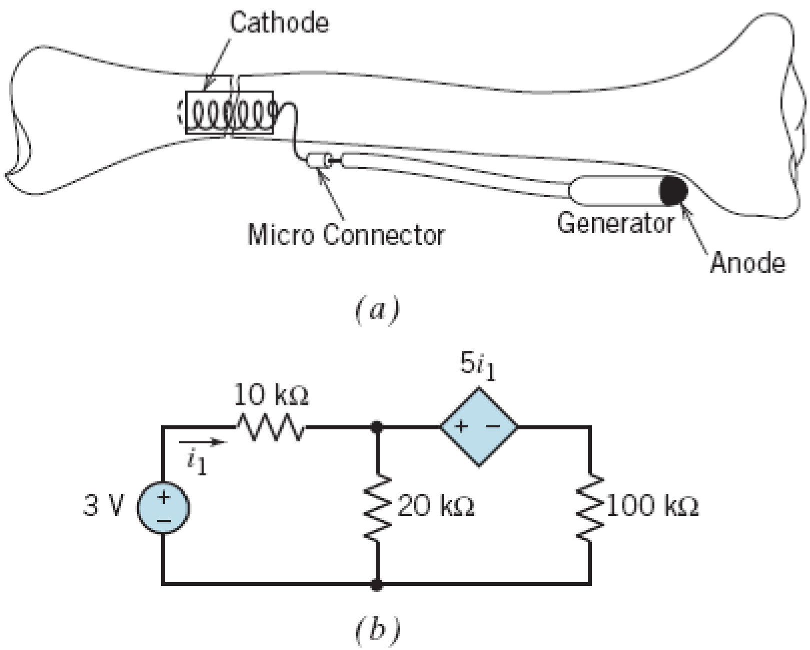
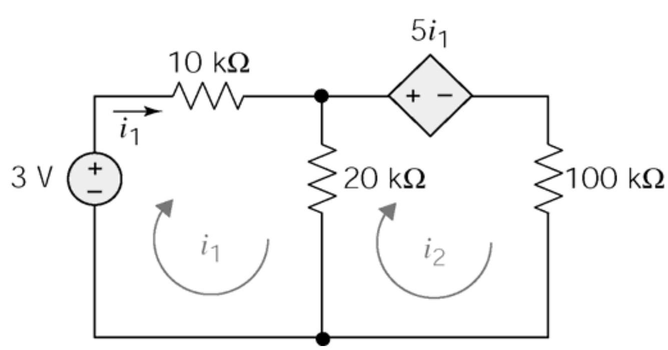


Figure P 4.7-5

Solution:



Apply KVL to left mesh : $-3+10\times10^3\, i_1+20\times10^3\left(i_1-i_2\right)= 0 \Rightarrow \underline{30\times10^3\, i_1-20\times10^3\, i_2=3}$ (1)

Apply KVL to right mesh : $5\times10^3\, i_1+100\times10^3\, i_2+20\times10^3\left(i_2-i_1\right)=0 \Rightarrow \underline{\;i_1 \;=\; 8i_2\;}$ (2)

Solving (1) & (2) simultaneously $\Rightarrow\; i_1=\dfrac{6}{55}\text{ mA},\; i_2=\dfrac{3}{220}\text{ mA}$

Power delivered to cathode $= (5i_1)\ (i_2) + 100(i_2)^2$

$$= 5\left(\frac{6}{55}\right)\left(\frac{3}{220}\right) + 100\left(\frac{3}{220}\right)^2 = 0.026 \text{ mW}$$

$\therefore$ Energy in 24 hr. $= \left(2.6\times10^{-5} \text{ W}\right)(24 \text{ hr})\left(3600 \text{ s/hr}\right) = 2.25 \text{ J}$

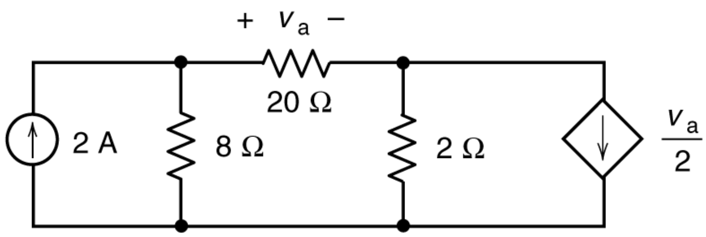
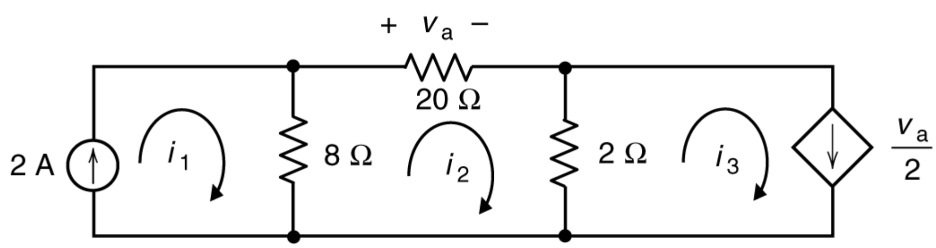


Figure P4.7-6

P4.7-6 Determine the value of the power supplied by the VCCS in the circuit shown Figure P4.7-6.

Solution: First, label the mesh currents.

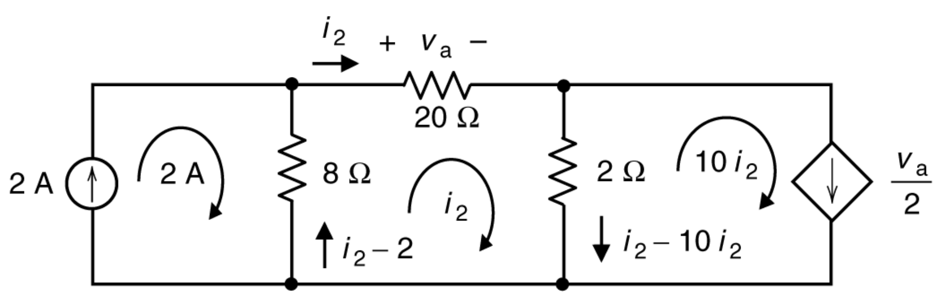


Next, express the controlling voltage of the VCCS in terms of the mesh currents:

$$v_a = 20 i_2$$

Notice that $\quad i_1 = 2 \text{ A} \quad \text{and} \quad i_3 = \dfrac{v_a}{2} = 10 i_2$

Next, express the resistor currents in terms of the mesh currents:



Apply KVL to the middle mesh: $\quad 20i_2 + 2(i_2 - 10i_2) + 8(i_2 - 2) = 0 \;\Rightarrow\; i_2 = 1.6 \text{ A}$

Consequently $\quad v_a = 20i_2 = 20(1.6) = 32 \text{ V} \quad \text{and} \quad i_3 = \dfrac{v_a}{2} = \dfrac{32}{2} = 16 \text{ A}$

The VCCS supplies $\quad \dfrac{v_a}{2}\left[2(i_3 - i_2)\right] = \dfrac{32}{2}(2)(16 - 1.6) = 460.8 \text{ W}$

P 4.7-7 The currents i_1, i_2 and i_3 are the mesh currents of the circuit shown in Figure P 4.7-7. Determine the values of i_1, i_2, and i_3.

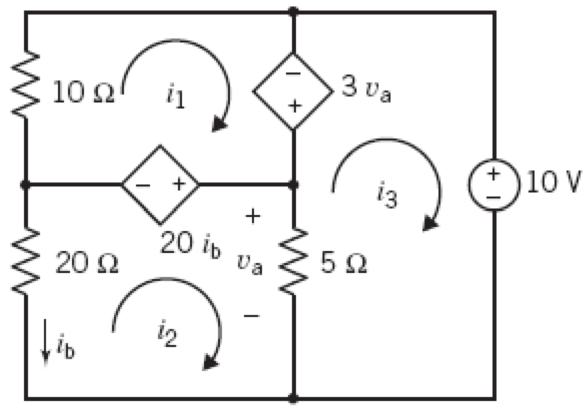


Figure P 4.7-7

Solution:

Express v_a and i_b, the controlling voltage and current of the dependent sources, in terms of the mesh currents

$$v_a = 5(i_2 - i_3) \text{ and } i_b = -i_2$$

Next express 20 i_b and 3 v_a, the controlled voltages of the dependent sources, in terms of the mesh currents

$$20\ i_b = -20\ i_2 \text{ and } 3\ v_a = 15(i_2 - i_3)$$

Apply KVL to the meshes

$$-15(i_2 - i_3) + (-20\ i_2) + 10\ i_1 = 0$$
$$-(-20\ i_2) + 5(i_2 - i_3) + 20\ i_2 = 0$$
$$10 - 5(i_2 - i_3) + 15\ (i_2 - i_3) = 0$$

These equations can be written in matrix form

$$\begin{bmatrix} 10 & -35 & 15 \\ 0 & 45 & -5 \\ 0 & 10 & -10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix}$$

Solving, e.g. using MATLAB, gives

$$i_1 = -1.25 \text{ A},\ i_2 = +0.125 \text{ A, and } i_3 = +1.125 \text{ A}$$

(checked: MATLAB & LNAP 5/19/04)

P 4.7-8 Determine the value of the power supplied by the dependent source in Figure P 4.7-8.

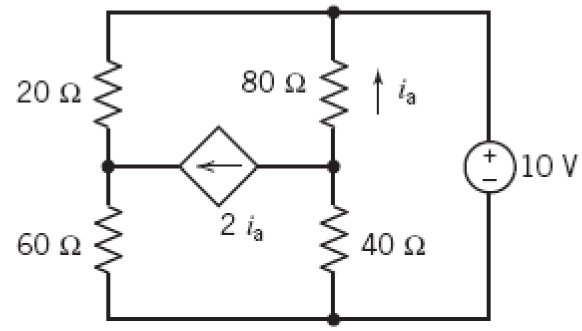
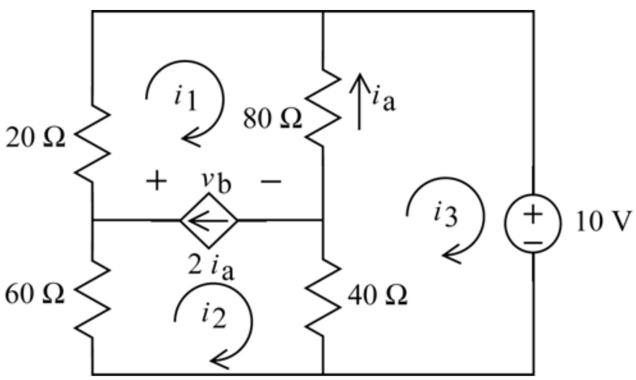


Figure P 4.7-8

Solution: Label the mesh currents:



Express i_a, the controlling current of the CCCS, in terms of the mesh currents

$$i_a = i_3 - i_1$$

Express 2 i_a, the controlled current of the CCCS, in terms of the mesh currents:

$$i_1 - i_2 = 2\, i_a = 2\left(i_3 - i_1\right) \quad \Rightarrow \quad 3\, i_1 - i_2 - 2\, i_3 = 0$$

Apply KVL to the supermesh corresponding to the CCCS:

$$80\left(i_1 - i_3\right) + 40\left(i_2 - i_3\right) + 60\, i_2 + 20\, i_1 = 0 \qquad \Rightarrow \qquad 100 i_1 + 100 i_2 - 120 i_3 = 0$$

Apply KVL to mesh 3

$$10 + 40\left(i_3 - i_2\right) + 80\left(i_3 - i_1\right) = 0 \qquad \Rightarrow \qquad \text{-}80\, i_1 - 40\, i_2 + 120\, i_3 = -10$$

These three equations can be written in matrix form

$$\begin{bmatrix} 3 & -1 & -2 \\ 100 & 100 & -120 \\ -80 & -40 & 120 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -10 \end{bmatrix}$$

Solving, e.g. using MATLAB, gives

$$i_1 = -0.2 \text{ A},\ i_2 = -0.1 \text{ A and } i_3 = -0.25 \text{ A}$$

Apply KVL to mesh 2 to get

$$v_b + 40\left(i_2 - i_3\right) + 60 i_2 = 0 \quad \Rightarrow \quad v_b = -40\left(-0.1 - \left(-0.25\right)\right) - 60\left(-0.1\right) = 0 \text{ V}$$

So the power supplied by the dependent source is $p = v_b\left(2 i_a\right) = 0 \text{ W}$.

(checked: LNAP 6/7/04)

P 4.7-9 Determine the value of the resistance R in the circuit shown in Figure P 4.7-9.

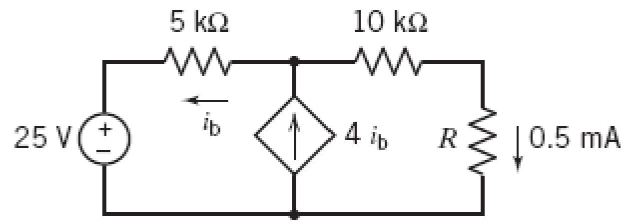


Figure P 4.7-9

Solution:

Notice that i_b and 0.5 mA are the mesh currents. Apply KCL at the top node of the dependent source to get

$$i_b + 0.5\times10^{-3} = 4\,i_b \quad \Rightarrow \quad i_b = \frac{1}{6}\ \text{mA}$$

Apply KVL to the supermesh corresponding to the dependent source to get

$$-5000\,i_b + (10000+R)\left(0.5\times10^{-3}\right) - 25 = 0$$

$$-5000\left(\frac{1}{6}\times10^{-3}\right) + (10000+R)\left(0.5\times10^{-3}\right) = 25$$

$$R = \frac{\frac{125}{6}}{0.5\times10^{-3}} = 41.67\ \text{k}\Omega$$

(checked: LNAP 6/21/04)

P 4.7-10 The circuit shown in Figure P 4.7-10 is the small signal model of an amplifier. The input to the amplifier is the voltage source voltage, v_s. The output of the amplifier is the voltage v_o.

(a) The ratio of the output to the input, v_o/v_s, is called the gain of the amplifier. Determine the gain of the amplifier.

(b) The ratio of the current of the input source to the input voltage, i_b/v_s, is called the input resistance of the amplifier. Determine the input resistance.

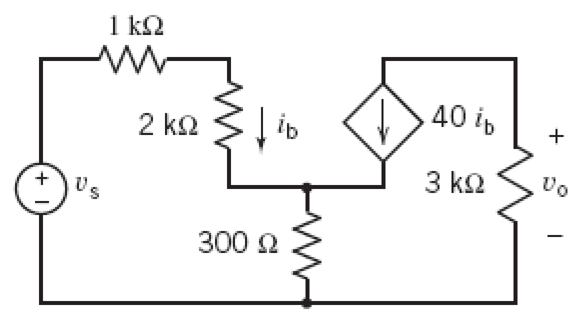


Figure P 4.7-10

Solution: The controlling and controlled currents of the CCCS, i_b and 40 i_b, are the mesh currents. Apply KVL to the left mesh to get

$$1000\,i_b + 2000\,i_b + 300\left(i_b + 40 i_b\right) - v_s = 0 \quad \Rightarrow \quad 15300 i_b = v_s$$

The output is given by $$v_o = -3000\left(40\,i_b\right) = -120000\,i_b$$

(a) The gain is $$\frac{v_o}{v_s} = -\frac{120000}{15300} = -7.84\ \text{V/V}$$

(b) The input resistance is $$\frac{v_s}{i_b} = 15300\ \Omega$$

(checked: LNAP 5/24/04)

P 4.7-11 Determine the values of the mesh currents of the circuit shown in Figure P 4.7-11.

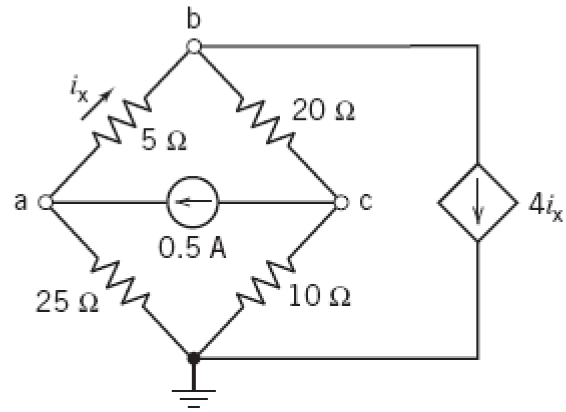
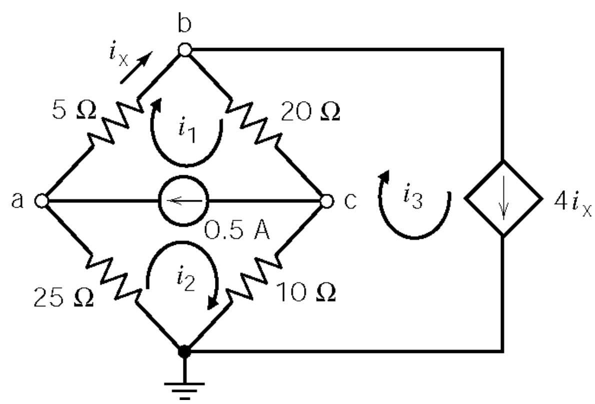


Figure P 4.7-11

Solution:
Label the mesh currents.



Express i_x in terms of the mesh currents:

$$i_x = i_1$$

Express $4i_x$ in terms of the mesh currents:

$$4i_x = i_3$$

Express the current source current in terms of the mesh currents to get:

$$0.5 = i_1 - i_2 \quad \Rightarrow \quad i_2 = i_x - 0.5$$

Apply KVL to supermesh corresponding to the current source to get

$$5i_1 + 20(i_1 - i_3) + 10(i_2 - i_3) + 25i_2 = 0$$

Substituting gives

$$5i_x + 20(-3i_x) + 10(i_x - 0.5 - 4i_x) + 25(i_x - 0.5) = 0 \quad \Rightarrow \quad i_x = -\frac{35}{120} = -0.29167$$

So the mesh currents are

$$i_1 = i_x = -0.29167 \quad \text{A}$$
$$i_2 = i_x - 0.5 = -0.79167 \quad \text{A}$$
$$i_3 = 4i_x = -1.1667 \quad \text{A}$$

(checked: LNAP 6/21/04)

P 4.7-12 The currents i_1, i_2, and i_3 are the mesh currents corresponding to meshes 1, 2, and 3 in Figure P 4.7-12. Determine the values of these mesh currents.

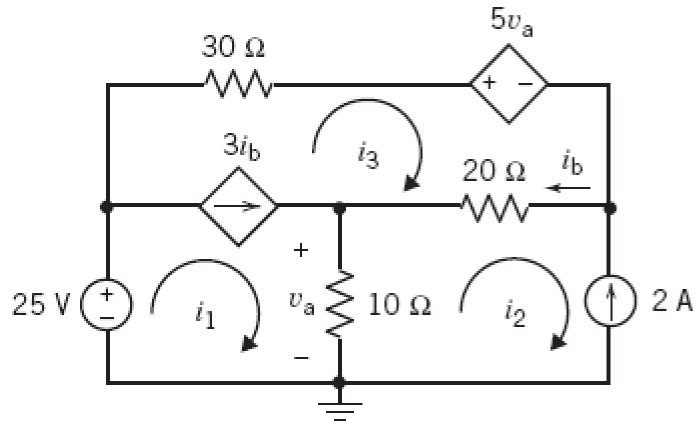
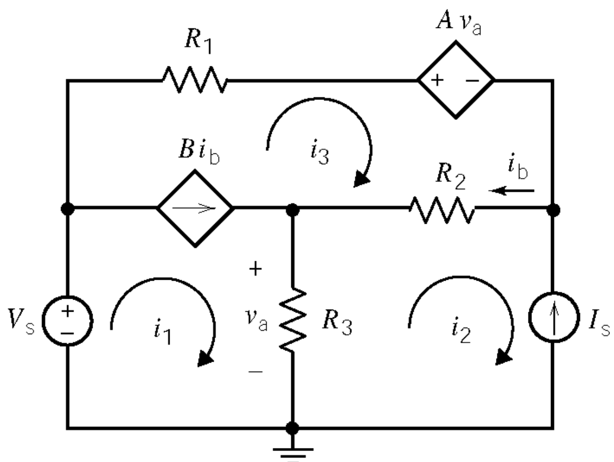


Figure P 4.7-12

Solution:



Express the controlling voltage and current of the dependent sources in terms of the mesh currents:

$$v_a = R_3\left(i_1 - i_2\right) \quad \text{and} \quad i_b = i_3 - i_2$$

Express the current source currents in terms of the mesh currents:

$$i_2 = -I_s \quad \text{and} \quad i_1 - i_3 = B\,i_b = B\left(i_3 - i_2\right)$$

Consequently

$$i_1 - \left(B+1\right)i_3 = B\,I_s$$

Apply KVL to the supermesh corresponding to the dependent current source

$$R_1\,i_3 + A\,R_3\left(i_1 - i_2\right) + R_2\left(i_3 - i_2\right) + R_3\left(i_1 - i_2\right) - V_s = 0$$

or

$$\left(A+1\right)R_3\,i_1 - \left(R_2 + \left(A+1\right)R_3\right)i_2 + \left(R_1 + R_2\right)i_3 = V_s$$

Organizing these equations into matrix form:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -\left(B+1\right) \\ \left(A+1\right)R_3 & -\left(R_2 + \left(A+1\right)R_3\right) & R_1 + R_2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -I_s \\ B\,I_s \\ V_s \end{bmatrix}$$

With the given values:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -4 \\ 60 & -80 & 50 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 6 \\ 25 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -0.8276 \\ -2 \\ -1.7069 \end{bmatrix} \text{ A}$$

(Checked using LNAP 9/29/04)

P 4.7-13 The currents i_1, i_2, and i_3 are the mesh currents corresponding to meshes 1, 2, and 3 in Figure P 4.7-13. The values of these currents are

$i_1 = -1.375$ A, $i_2 = -2.5$ A, and $i_3 = -3.25$ A

Determine the values of the gains of the dependent source, A and B.

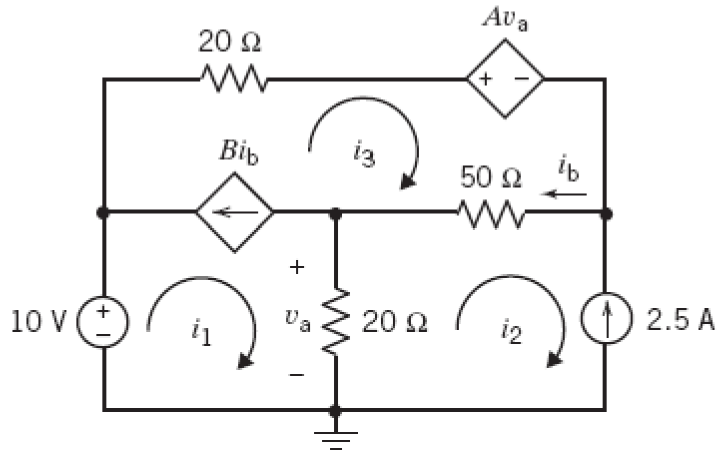


Figure P 4.7-13

Solution: Express the controlling voltage and current of the dependent sources in terms of the mesh currents:

$$v_a = 20(i_1 - i_2) = 20(-1.375 - (-2.5)) = 22.5$$

and

$$i_b = i_3 - i_2 = -3.25 - (-2.5) = -0.75 \text{ A}$$

Express the current source currents in terms of the mesh currents:

$$i_2 = -2.5 \text{ A}$$

and

$$i_3 - i_1 = B\, i_b \quad \Rightarrow \quad -1.375 - (-2.5) = B(-0.75) \quad \Rightarrow \quad B = 2.5 \text{ A/A}$$

Apply KVL to the supermesh corresponding to the dependent current source

$$0 = 20\, i_3 + A v_a + 50\, i_b + v_a - 10 = 20\,(-3.25) + A(22.5) + 50(-0.75) + 22.5 - 10 \quad \Rightarrow \quad A = 4 \text{ V/V}$$

(Checked using LNAP 9/29/04)

P 4.7-14 Determine the current *i* in the circuit shown in Figure P 4.7-14.

Answer: $i = 3$ A

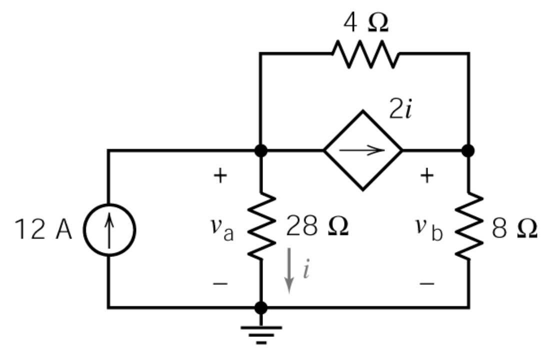


Figure P 4.7-14

Solution:

Label the node voltages as shown. The controlling currents of the CCCS is expressed as $i = \dfrac{v_a}{28}$.

The node equations are
$$12 = \frac{v_a}{28} + \frac{v_a - v_b}{4} + \frac{v_a}{14}$$

and
$$\frac{v_a - v_b}{4} + \frac{v_a}{14} = \frac{v_b}{8}$$

Solving the node equations gives $v_a = 84$ V and $v_b = 72$ V. Then $i = \dfrac{v_a}{28} = \dfrac{84}{28} = 3$ A.

(checked using LNAP 6/16/05)

P4.7-15 Determine the values of the mesh currents i_1 and i_2 for the circuit shown in Figure P4.7-15.

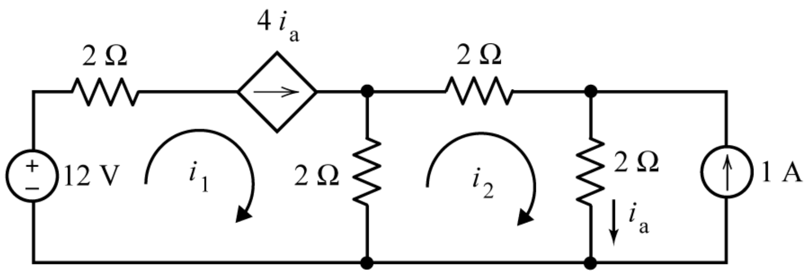


Figure P4.7-15

Solution: Expressing the dependent source currents in terms of the mesh currents we get:

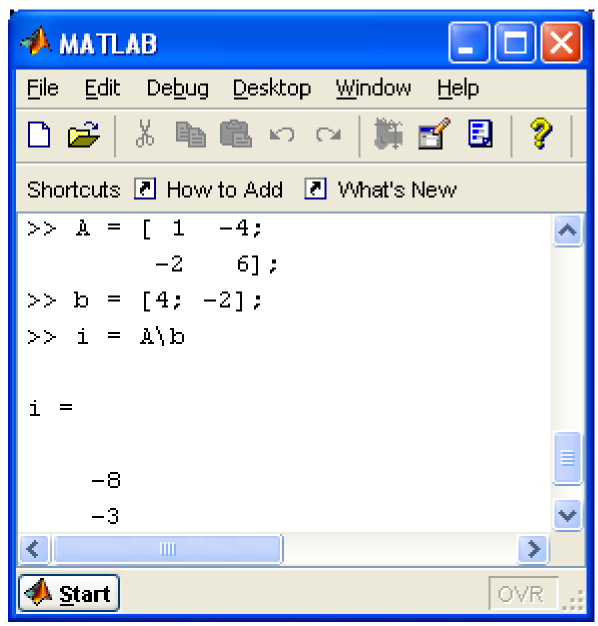
$$i_1 = 4\,i_a = 4\left(i_2 + 1\right) \quad \Rightarrow \quad 4 = i_1 - 4\,i_2$$

Apply KVL to mesh 2 to get

$$2\,i_2 + 2\left(i_2 + 1\right) - 2\left(i_1 - i_2\right) = 0 \quad \Rightarrow \quad -2 = -2\,i_1 + 6\,i_2$$

Solving these equations using MATLAB we get

$i_1 = -8$ A and $i_2 = -3$ A



P4.7-16 Determine the values of the mesh currents i_1 and i_2 for the circuit shown in Figure P4.7-16 **.**

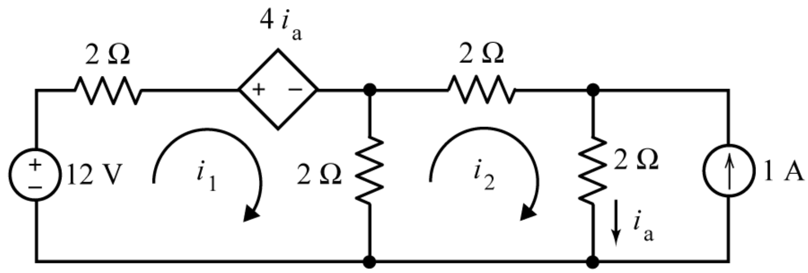


Figure P4.7-16

Apply KVL to mesh 1 to get

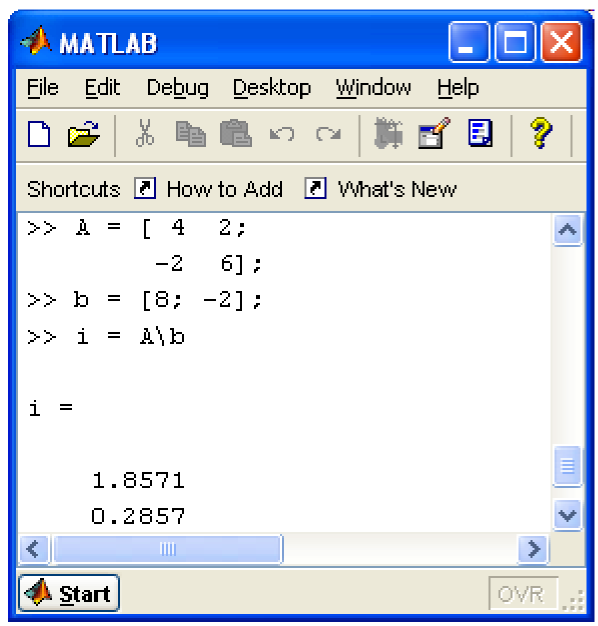
$$2i_1+4i_a+2(i_1-i_2)-12=0 \quad\Rightarrow\quad 2i_1+4(i_2+1)+2(i_1-i_2)-12=0 \quad\Rightarrow\quad 8=4i_1+2i_2$$

Apply KVL to mesh 2 to get

$$2i_2+2(i_2+1)-2(i_1-i_2)=0 \quad\Rightarrow\quad -2=-2i_1+6i_2$$

Solving these equations using MATLAB we get

$i_1 = 1.8571$ A and $i_2 = 0.2857$ A



Section 4.8 The Node Voltage Method and Mesh Current Method Compared

P 4.8-2 The circuit shown in Figure P 4.8-2 has two inputs, v_s and i_s, and one output v_o. The output is related to the inputs by the equation

$$v_o = a i_s + b v_s$$

where a and b are constants to be determined. Determine the values a and b by

(a) writing and solving mesh equations and

(b) writing and solving node equations.

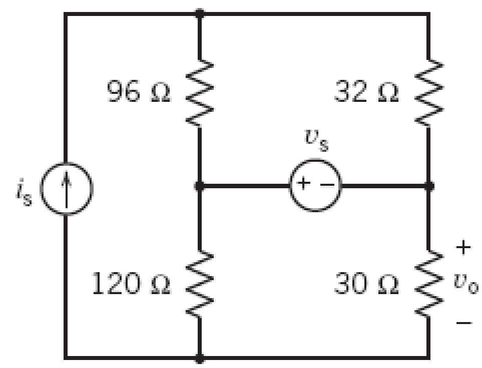
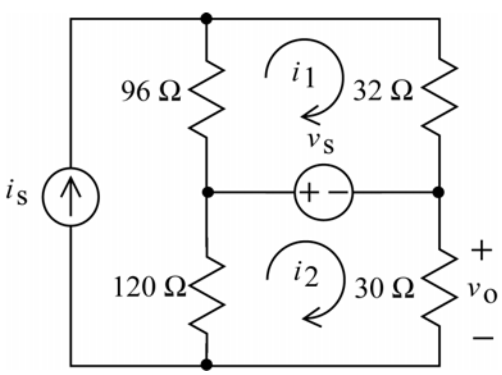


Figure P 4.8-2

Solution:

(a)



Apply KVL to meshes 1 and 2:

$$32 i_1 - v_s + 96\left(i_1 - i_s\right) = 0$$

$$v_s + 30 i_2 + 120\left(i_2 - i_s\right) = 0$$

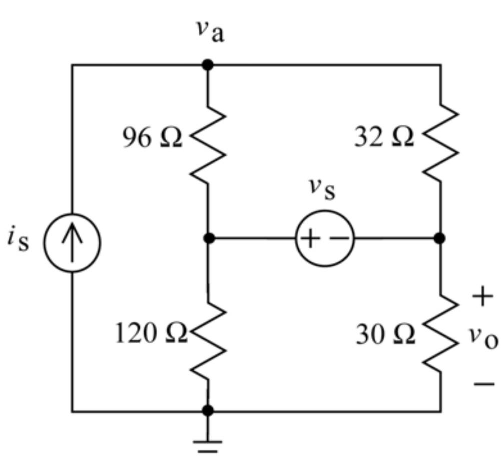
$$150 i_2 = +120 i_s - v_s$$

$$i_2 = \frac{4}{5} i_s - \frac{v_s}{150}$$

$$v_o = 30 i_2 = 24 i_s - \frac{1}{5} v_s$$

So a = 24 and b = -.02.

(b)



Apply KCL to the supernode corresponding to the voltage source to get

$$\frac{v_a - \left(v_s + v_o\right)}{96} + \frac{v_a - v_o}{32} = \frac{v_s + v_o}{120} + \frac{v_o}{30}$$

So

$$i_s = \frac{v_s + v_o}{120} + \frac{v_o}{30} = \frac{v_s}{120} + \frac{v_o}{24}$$

Then

$$v_o = 24 i_s - \frac{1}{5} v_s$$

So a = 24 and b = -0.2.

(checked: LNAP 5/24/04)

P 4.8-3 Determine the power supplied by the dependent source in the circuit shown in Figure P 4.8-3 by writing and solving

(a) node equations and

(b) mesh equations.

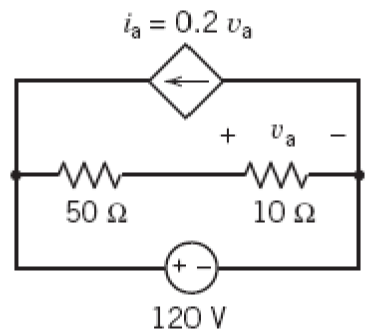
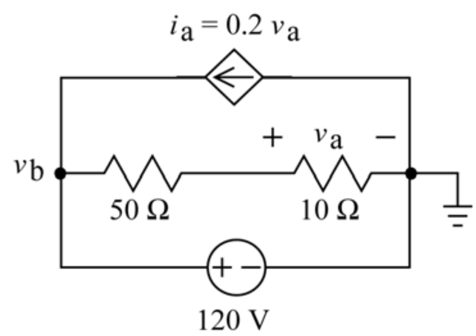


Figure P 4.8-3

Solution:

(a) Label the reference node and node voltages.



$$v_b = 120 \text{ V}$$

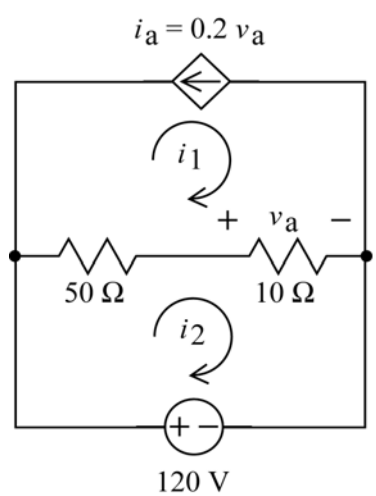
due to the voltage source.

Apply KCL at the node between the resistors to get

$$\frac{v_b - v_a}{50} = \frac{v_a}{10} \quad \Rightarrow \quad v_a = 20 \text{ V}$$

and the power supplied by the dependent source is

$$p = v_b i_a = (120)\left[0.2(20)\right] = 480 \text{ W}$$



(b) Label the mesh currents. Express the controlling voltage of the dependent source in terms of the mesh current to get

$$v_a = 10\left(i_2 - i_1\right)$$

Express the controlled current of the dependent source in terms of the mesh currents to get

$$-i_1 = i_a = 0.2\left[10\left(i_2 - i_1\right)\right] = 2i_2 - 2i_1 \quad \Rightarrow \quad i_1 = 2i_2$$

Apply KVL to the bottom mesh to get

$$50\left(i_2 - i_1\right) + 10\left(i_2 - i_1\right) - 120 = 0 \quad \Rightarrow \quad i_2 - i_1 = 2$$

So $\quad i_2 - 2i_2 = 2 \quad \Rightarrow \quad i_2 = -2 \text{ A} \quad \Rightarrow \quad i_1 = -4 \text{ A}$

Then $\quad v_a = 10\left(-2 - (-4)\right) = 20 \text{ V}$ and $i_a = 0.2(20) = 4 \text{ A}$

The power supplied by the dependent source is

$$p = 120\left(i_a\right) = 120(4) 480 \text{ W}$$

(checked: LNAP 6/21/04)

Section 4.8 The Node Voltage Method and Mesh Current Method Compared

P 4.8-2 The circuit shown in Figure P 4.8-2 has two inputs, v_s and i_s, and one output v_o. The output is related to the inputs by the equation

$$v_o = a i_s + b v_s$$

where a and b are constants to be determined. Determine the values a and b by

(a) writing and solving mesh equations and

(b) writing and solving node equations.

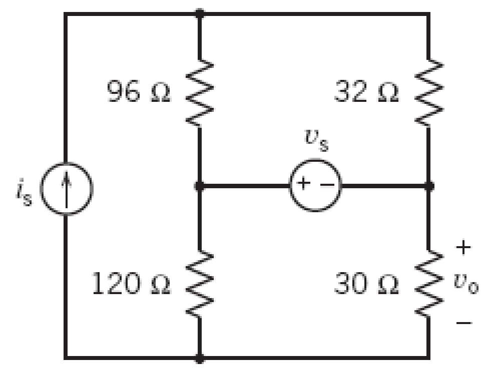
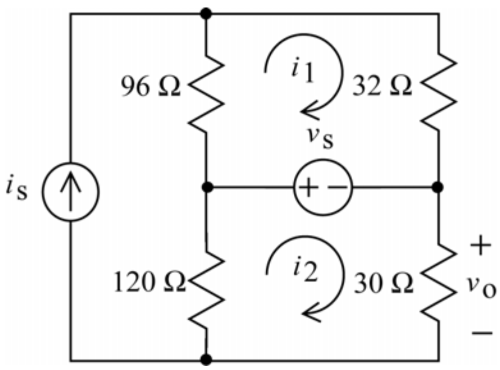


Figure P 4.8-2

Solution:

(a)



Apply KVL to meshes 1 and 2:

$$32 i_1 - v_s + 96(i_1 - i_s) = 0$$

$$v_s + 30 i_2 + 120(i_2 - i_s) = 0$$

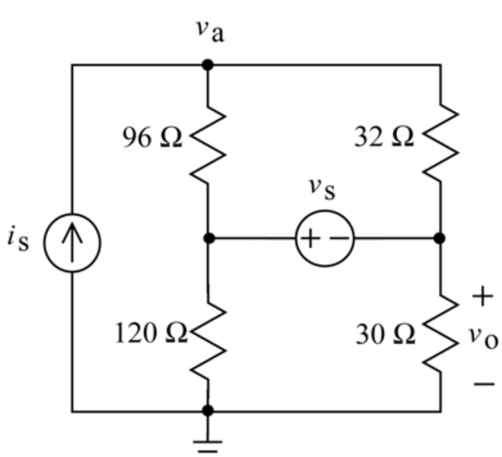
$$150 i_2 = +120 i_s - v_s$$

$$i_2 = \frac{4}{5} i_s - \frac{v_s}{150}$$

$$v_o = 30 i_2 = 24 i_s - \frac{1}{5} v_s$$

So a = 24 and b = -.02.

(b)



Apply KCL to the supernode corresponding to the voltage source to get

$$\frac{v_a - (v_s + v_o)}{96} + \frac{v_a - v_o}{32} = \frac{v_s + v_o}{120} + \frac{v_o}{30}$$

So

$$i_s = \frac{v_s + v_o}{120} + \frac{v_o}{30} = \frac{v_s}{120} + \frac{v_o}{24}$$

Then

$$v_o = 24 i_s - \frac{1}{5} v_s$$

So a = 24 and b = -0.2.

(checked: LNAP 5/24/04)

P 4.8-3 Determine the power supplied by the dependent source in the circuit shown in Figure P 4.8-3 by writing and solving

(a) node equations and

(b) mesh equations.

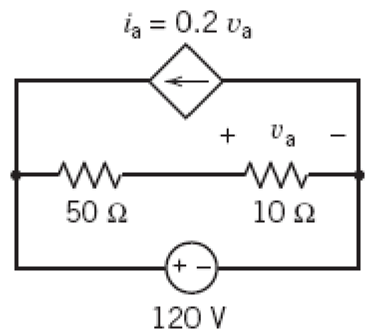


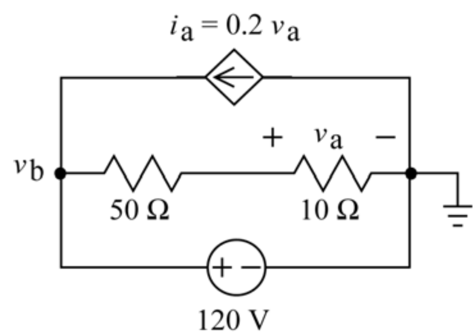
Figure P 4.8-3

Solution:

(a) Label the reference node and node voltages.

$$v_b = 120 \text{ V}$$

due to the voltage source.

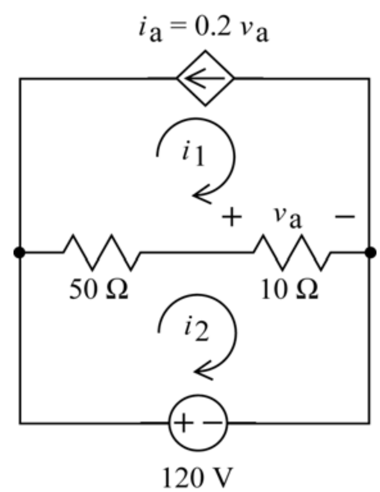


Apply KCL at the node between the resistors to get

$$\frac{v_b - v_a}{50} = \frac{v_a}{10} \quad \Rightarrow \quad v_a = 20 \text{ V}$$

and the power supplied by the dependent source is

$$p = v_b i_a = (120)\left[0.2(20)\right] = 480 \text{ W}$$



(b) Label the mesh currents. Express the controlling voltage of the dependent source in terms of the mesh current to get

$$v_a = 10(i_2 - i_1)$$

Express the controlled current of the dependent source in terms of the mesh currents to get

$$-i_1 = i_a = 0.2\left[10(i_2 - i_1)\right] = 2i_2 - 2i_1 \quad \Rightarrow \quad i_1 = 2i_2$$

Apply KVL to the bottom mesh to get

$$50(i_2 - i_1) + 10(i_2 - i_1) - 120 = 0 \quad \Rightarrow \quad i_2 - i_1 = 2$$

So $\qquad i_2 - 2i_2 = 2 \quad \Rightarrow \quad i_2 = -2 \text{ A} \quad \Rightarrow \quad i_1 = -4 \text{ A}$

Then $\qquad v_a = 10(-2 - (-4)) = 20 \text{ V}$ and $i_a = 0.2(20) = 4 \text{ A}$

The power supplied by the dependent source is

$$p = 120(i_a) = 120(4) 480 \text{ W}$$

(checked: LNAP 6/21/04)

Section 4.8 The Node Voltage Method and Mesh Current Method Compared

P 4.8-2 The circuit shown in Figure P 4.8-2 has two inputs, v_s and i_s, and one output v_o. The output is related to the inputs by the equation

$$v_o = a i_s + b v_s$$

where a and b are constants to be determined. Determine the values a and b by

(a) writing and solving mesh equations and

(b) writing and solving node equations.

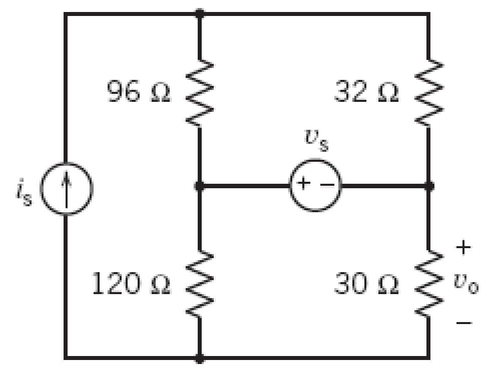
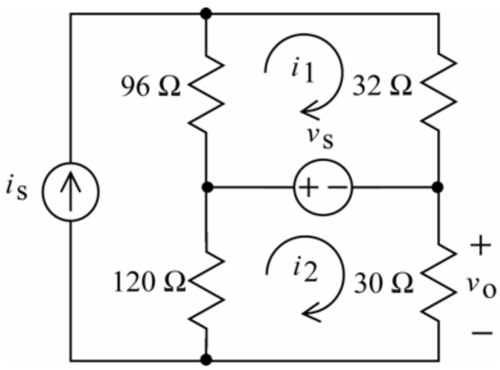


Figure P 4.8-2

Solution:

(a)



Apply KVL to meshes 1 and 2:

$$32 i_1 - v_s + 96\left(i_1 - i_s\right) = 0$$

$$v_s + 30 i_2 + 120\left(i_2 - i_s\right) = 0$$

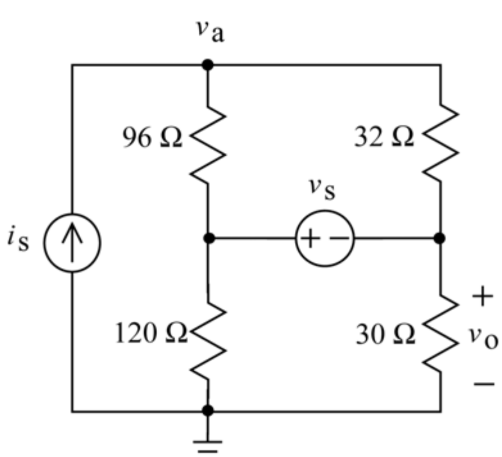
$$150 i_2 = +120 i_s - v_s$$

$$i_2 = \frac{4}{5} i_s - \frac{v_s}{150}$$

$$v_o = 30 i_2 = 24 i_s - \frac{1}{5} v_s$$

So a = 24 and b = -.02.

(b)



Apply KCL to the supernode corresponding to the voltage source to get

$$\frac{v_a - \left(v_s + v_o\right)}{96} + \frac{v_a - v_o}{32} = \frac{v_s + v_o}{120} + \frac{v_o}{30}$$

So

$$i_s = \frac{v_s + v_o}{120} + \frac{v_o}{30} = \frac{v_s}{120} + \frac{v_o}{24}$$

Then

$$v_o = 24 i_s - \frac{1}{5} v_s$$

So a = 24 and b = -0.2.

(checked: LNAP 5/24/04)

P 4.8-3 Determine the power supplied by the dependent source in the circuit shown in Figure P 4.8-3 by writing and solving

(a) node equations and

(b) mesh equations.

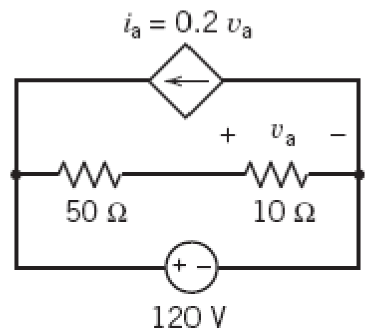
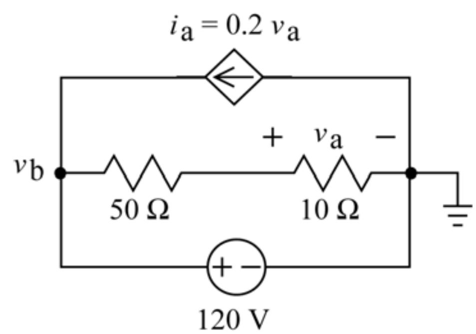


Figure P 4.8-3

Solution:

(a) Label the reference node and node voltages.



$$v_b = 120 \text{ V}$$

due to the voltage source.

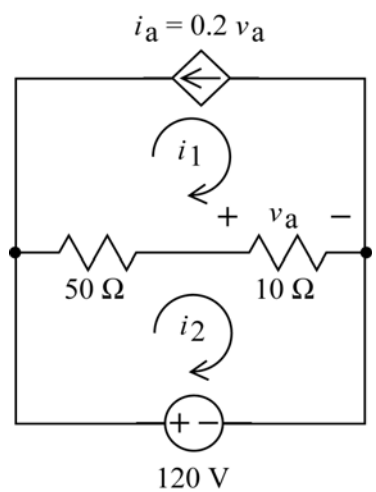
Apply KCL at the node between the resistors to get

$$\frac{v_b - v_a}{50} = \frac{v_a}{10} \quad \Rightarrow \quad v_a = 20 \text{ V}$$

and the power supplied by the dependent source is

$$p = v_b i_a = (120)\left[0.2(20)\right] = 480 \text{ W}$$

(b) Label the mesh currents. Express the controlling voltage of the dependent source in terms of the mesh current to get



$$v_a = 10\left(i_2 - i_1\right)$$

Express the controlled current of the dependent source in terms of the mesh currents to get

$$-i_1 = i_a = 0.2\left[10\left(i_2 - i_1\right)\right] = 2i_2 - 2i_1 \quad \Rightarrow \quad i_1 = 2i_2$$

Apply KVL to the bottom mesh to get

$$50\left(i_2 - i_1\right) + 10\left(i_2 - i_1\right) - 120 = 0 \quad \Rightarrow \quad i_2 - i_1 = 2$$

So $\quad i_2 - 2i_2 = 2 \quad \Rightarrow \quad i_2 = -2 \text{ A} \quad \Rightarrow \quad i_1 = -4 \text{ A}$

Then $\quad v_a = 10\left(-2 - (-4)\right) = 20 \text{ V}$ and $i_a = 0.2(20) = 4 \text{ A}$

The power supplied by the dependent source is

$$p = 120\left(i_a\right) = 120(4)\,480 \text{ W}$$

(checked: LNAP 6/21/04)

Section 4.8 The Node Voltage Method and Mesh Current Method Compared

P 4.8-2 The circuit shown in Figure P 4.8-2 has two inputs, v_s and i_s, and one output v_o. The output is related to the inputs by the equation

$$v_o = a i_s + b v_s$$

where a and b are constants to be determined. Determine the values a and b by

(a) writing and solving mesh equations and

(b) writing and solving node equations.

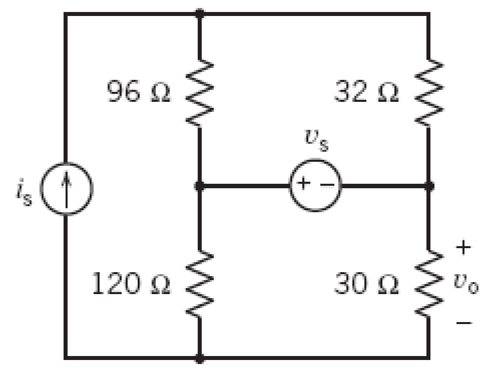
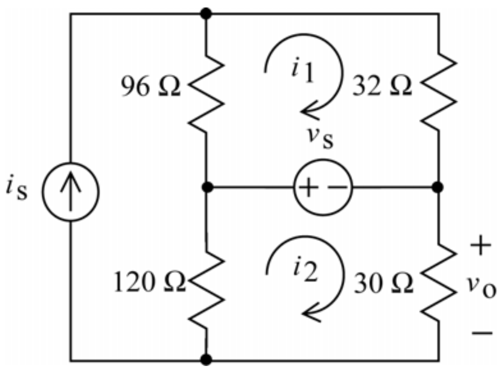


Figure P 4.8-2

Solution:

(a)



Apply KVL to meshes 1 and 2:

$$32 i_1 - v_s + 96\left(i_1 - i_s\right) = 0$$

$$v_s + 30 i_2 + 120\left(i_2 - i_s\right) = 0$$

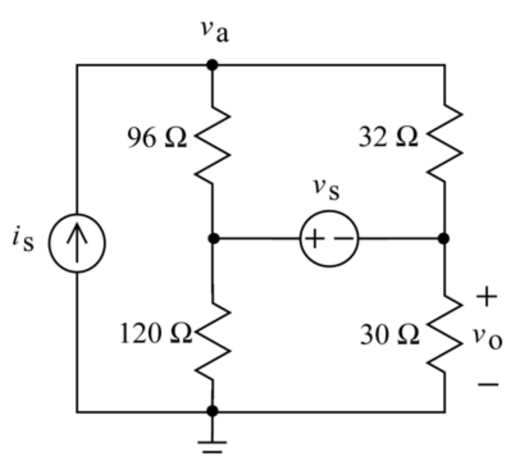
$$150 i_2 = +120 i_s - v_s$$

$$i_2 = \frac{4}{5} i_s - \frac{v_s}{150}$$

$$v_o = 30 i_2 = 24 i_s - \frac{1}{5} v_s$$

So a = 24 and b = -.02.

(b)



Apply KCL to the supernode corresponding to the voltage source to get

$$\frac{v_a - \left(v_s + v_o\right)}{96} + \frac{v_a - v_o}{32} = \frac{v_s + v_o}{120} + \frac{v_o}{30}$$

So

$$i_s = \frac{v_s + v_o}{120} + \frac{v_o}{30} = \frac{v_s}{120} + \frac{v_o}{24}$$

Then

$$v_o = 24 i_s - \frac{1}{5} v_s$$

So a = 24 and b = -0.2.

(checked: LNAP 5/24/04)

P 4.8-3 Determine the power supplied by the dependent source in the circuit shown in Figure P 4.8-3 by writing and solving

(a) node equations and

(b) mesh equations.

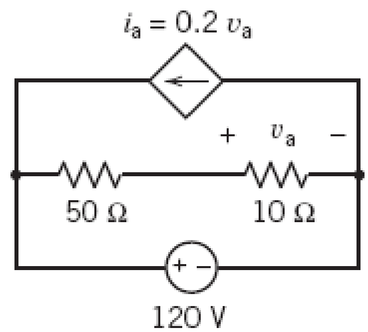
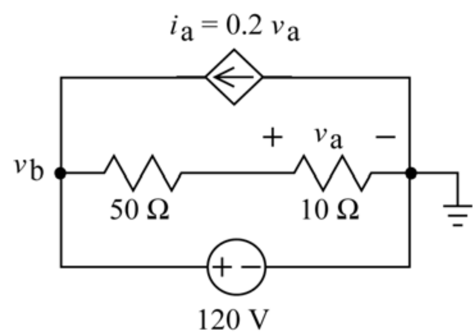


Figure P 4.8-3

Solution:

(a) Label the reference node and node voltages.



$$v_b = 120 \text{ V}$$

due to the voltage source.

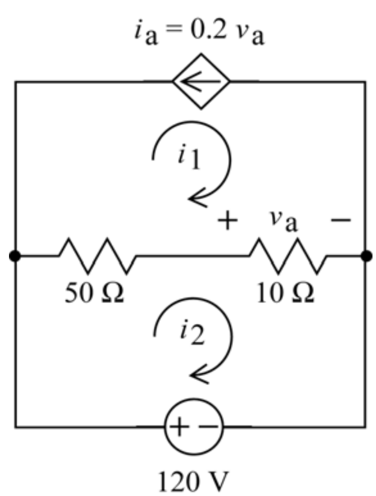
Apply KCL at the node between the resistors to get

$$\frac{v_b - v_a}{50} = \frac{v_a}{10} \quad \Rightarrow \quad v_a = 20 \text{ V}$$

and the power supplied by the dependent source is

$$p = v_b i_a = (120)\left[0.2(20)\right] = 480 \text{ W}$$

(b) Label the mesh currents. Express the controlling voltage of the dependent source in terms of the mesh current to get



$$v_a = 10(i_2 - i_1)$$

Express the controlled current of the dependent source in terms of the mesh currents to get

$$-i_1 = i_a = 0.2\left[10(i_2 - i_1)\right] = 2i_2 - 2i_1 \quad \Rightarrow \quad i_1 = 2i_2$$

Apply KVL to the bottom mesh to get

$$50(i_2 - i_1) + 10(i_2 - i_1) - 120 = 0 \quad \Rightarrow \quad i_2 - i_1 = 2$$

So $\quad i_2 - 2i_2 = 2 \quad \Rightarrow \quad i_2 = -2 \text{ A} \quad \Rightarrow \quad i_1 = -4 \text{ A}$

Then $\quad v_a = 10(-2 - (-4)) = 20 \text{ V}$ and $i_a = 0.2(20) = 4 \text{ A}$

The power supplied by the dependent source is

$$p = 120(i_a) = 120(4) 480 \text{ W}$$

(checked: LNAP 6/21/04)

Section 4.9 Circuit Analysis Using MATLAB

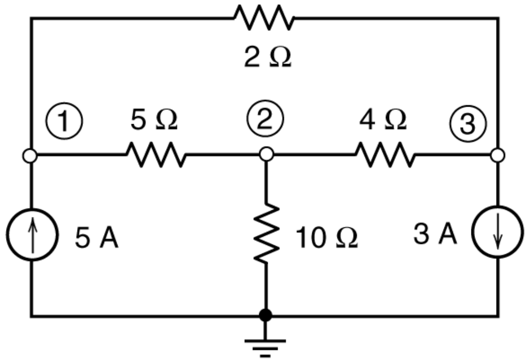
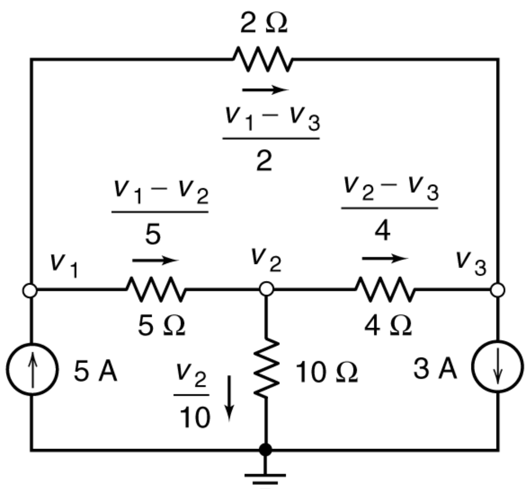


Figure P4.9-1

P4.9-1. The encircled numbers in the circuit shown Figure P4.9-1 are node numbers. Determine the values of the corresponding node voltages, v_1, v_2 and v_3.

Solution: First, express the resistor currents in terms of the node voltages:



Apply KCL at node 1 to get $5 = \dfrac{v_1 - v_2}{5} + \dfrac{v_1 - v_3}{2} \quad \Rightarrow \quad 0.7\,v_1 - 0.2\,v_2 - 0.5\,v_3 = 5$

Apply KCL at node 2 to get $\dfrac{v_1 - v_2}{5} = \dfrac{v_2}{10} + \dfrac{v_2 - v_3}{4} \quad \Rightarrow \quad -0.2\,v_1 + 0.55\,v_2 - 0.25\,v_3 = 0$

Apply KCL at node 3 to get $\dfrac{v_2 - v_3}{4} + \dfrac{v_1 - v_3}{2} = 3 \quad \Rightarrow \quad -0.5\,v_1 - 0.25\,v_2 + 0.75\,v_3 = -3$

In matrix form:

$$\begin{bmatrix} 0.7 & -0.2 & -0.5 \\ -0.2 & 0.55 & -0.25 \\ -0.5 & -0.25 & 0.75 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -3 \end{bmatrix}$$

Solving using MATLAB: $v_1 = 28.1818$ V, $v_2 = 20$ V and $v_3 = 21.4545$

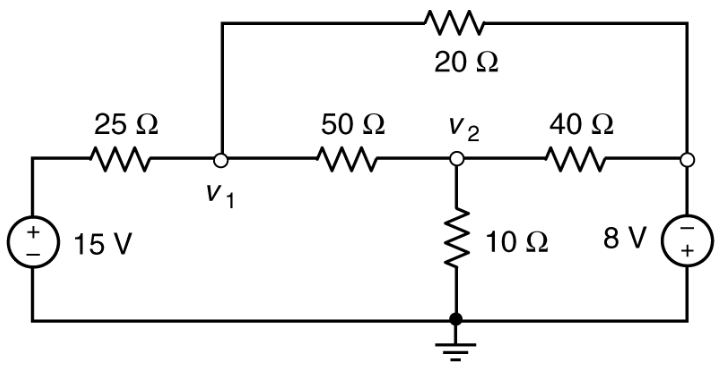
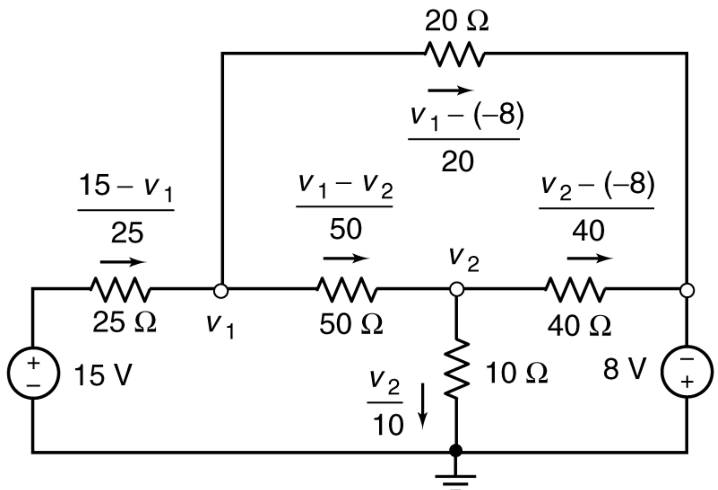


Figure P4.9-2

P4.9-2. Determine the values of the node voltages, v_1 and v_2, in the circuit shown Figure P4.9-2.

Solution: First, express the resistor currents in terms of the node voltages:



Apply KCL at node 1 to get $\frac{15-v_1}{25}=\frac{v_1-v_2}{50}+\frac{v_1+8}{20} \Rightarrow 0.11v_1-0.02v_2=0.2$

Apply KCL at node 2 to get $\frac{v_1-v_2}{50}=\frac{v_2}{10}+\frac{v_2+8}{40} \Rightarrow -0.02v_1+0.145v_2=-0.2$

In matrix form: $$\begin{bmatrix} 0.11 & -0.02 \\ -0.02 & 0.145 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}=\begin{bmatrix} 0.2 \\ -0.2 \end{bmatrix}$$

Solving using MATLAB: $v_1=1.6077\ \text{V}$ and $v_2=-1.1576\ \text{V}$

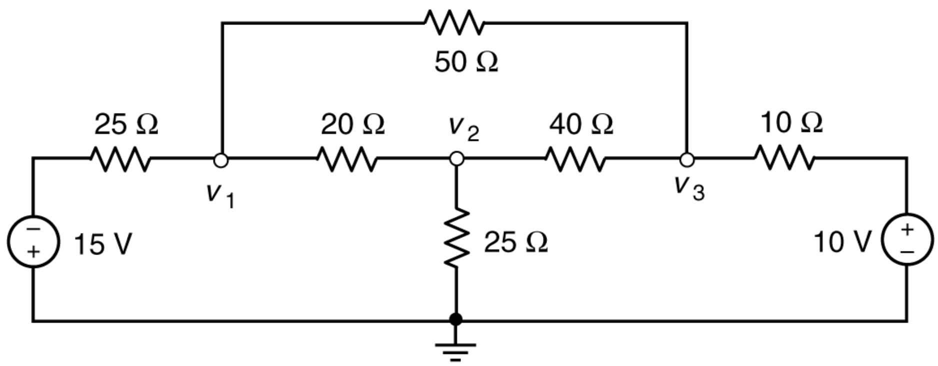
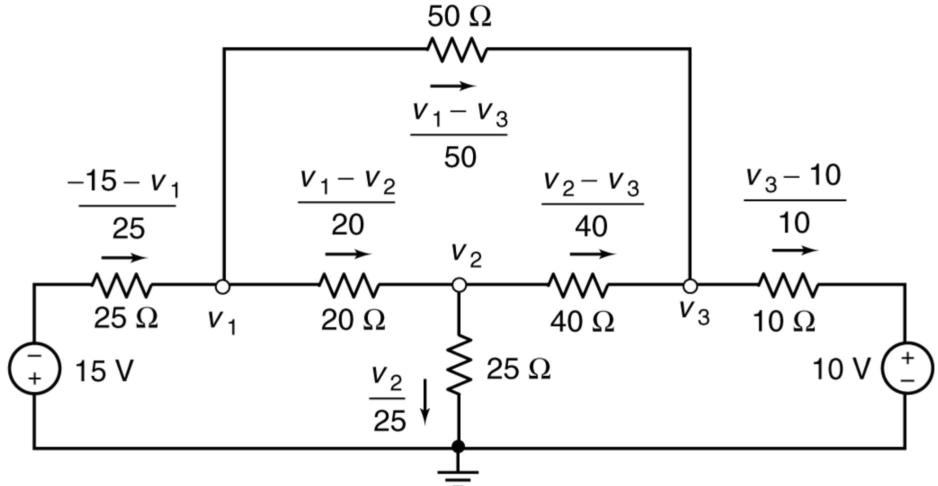


Figure P4.9-3

P4.9-3. Determine the values of the node voltages, v_1, v_2 and v_3 in the circuit shown in Figure P4.9-3.

Solution: First, express the resistor currents in terms of the node voltages:



Apply KCL at node 1 to get $\frac{-15-v_1}{25} = \frac{v_1 - v_2}{20} + \frac{v_1 - v_3}{50} \quad \Rightarrow \quad 0.11 v_1 - 0.05 v_2 - 0.02 v_3 = -0.6$

Apply KCL at node 2 to get $\frac{v_1 - v_2}{20} = \frac{v_2}{25} + \frac{v_2 - v_3}{40} \quad \Rightarrow \quad -0.05 v_1 + 0.115 v_2 - 0.025 v_3 = 0$

Apply KCL at node 3 to get $\frac{v_1 - v_2}{50} + \frac{v_2 - v_3}{40} = \frac{v_3 - 10}{10} \quad \Rightarrow \quad -0.02 v_1 - 0.025 v_2 + 0.145 v_3 = 1$

In matrix form:
$$\begin{bmatrix} 0.11 & -0.05 & -0.02 \\ -0.05 & 0.115 & -0.025 \\ -0.02 & -0.025 & 0.145 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -0.6 \\ 0 \\ 1 \end{bmatrix}$$

Solving using MATLAB: $v_1 = 1.6077 \text{ V}$ and $v_2 = -1.1576 \text{ V}$

P4.9-4 Determine the node voltages, v_1 and v_2, for the circuit shown in Figure P4.9-4.

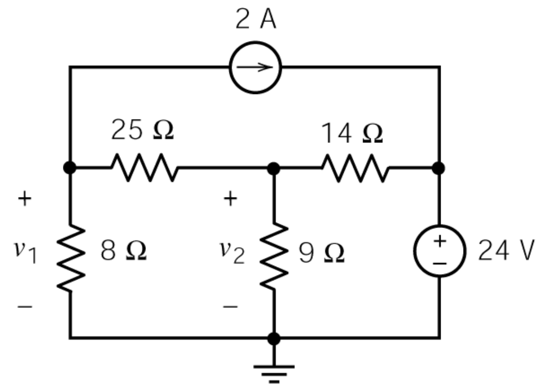
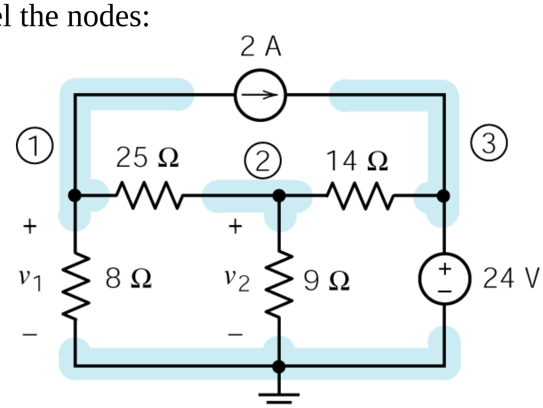


Figure P.4.9-4

Solution: Emphasize and label the nodes:



Notice the 24 V source connected between node 3 and the reference node. Consequently

$$v_3 = 24 \text{ V}$$

Apply KCL at node 1 to get

$$\frac{v_1}{8} + \frac{v_1 - v_2}{25} + 2 = 0$$

In this equation $\frac{v_1}{8}$ is the current directed downward in the 8 Ω resistor and $\frac{v_1 - v_2}{25}$ is the current directed from left to right in the 25 Ω resistor. We will simplify this equation by doing two things:

1. Multiplying each side by $8 \times 25 = 200$ to eliminate fractions.
2. Move the terms that don't involve node voltages to the right side of the equation.

The result is

$$33v_1 - 8v_2 = -400$$

Next, apply KCL at node 2 to get

$$\frac{v_2}{9} + \frac{v_2 - 24}{14} = \frac{v_1 - v_2}{25}$$

In this equation $\frac{v_2}{9}$ is the current directed downward in the 9 Ω resistor, $\frac{v_2 - 24}{14}$ is the current directed from left to right in the 14 Ω resistor and $\frac{v_1 - v_2}{25}$ is the current directed from left to right in the 25 Ω resistor. We will simplify this equation by doing two things:

1. Multiplying each side by $8 \times 25 \times 14 = 2800$ to eliminate fractions.

2. Move the terms that involve node voltages to the left side of the equation and move the terms that don't involve node voltages to the right side of the equation.

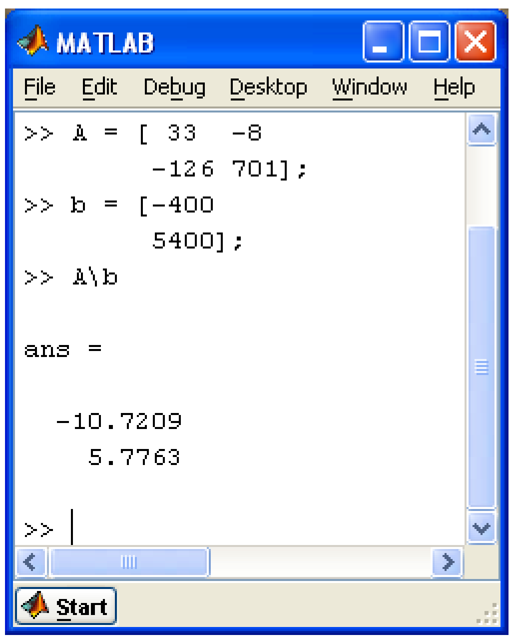
The result is

$$-(9\times 14)v_1 + (9\times 14 + 25\times 14 + 25\times 9)v_2 = 24\times 25\times 9 \quad \Rightarrow \quad -126 v_1 + 701 v_2 = 5400$$

The simultaneous equations can be written in matrix form

$$\begin{aligned} 33v_1 - 8v_2 &= -400 \\ -126v_1 + 701v_2 &= 5400 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 33 & -8 \\ -126 & 701 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -400 \\ 5400 \end{bmatrix}$$

We can use MATLAB to solve the matrix equation:



Then
$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -10.7209 \\ 5.7763 \end{bmatrix}$$

That is, the node voltages are $v_1 = -10.7209$ V and $v_2 = 5.7763$ V.

P4.9-5 Determine the mesh currents, i_1 and i_2, for the circuit shown in Figure P4.9-5.

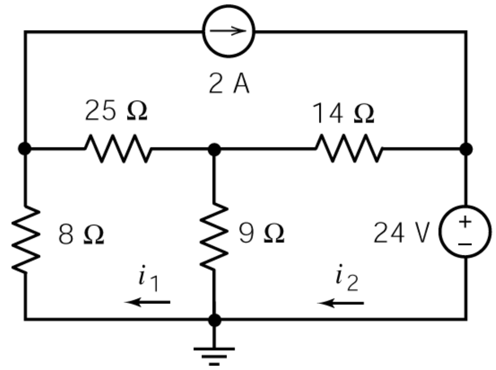
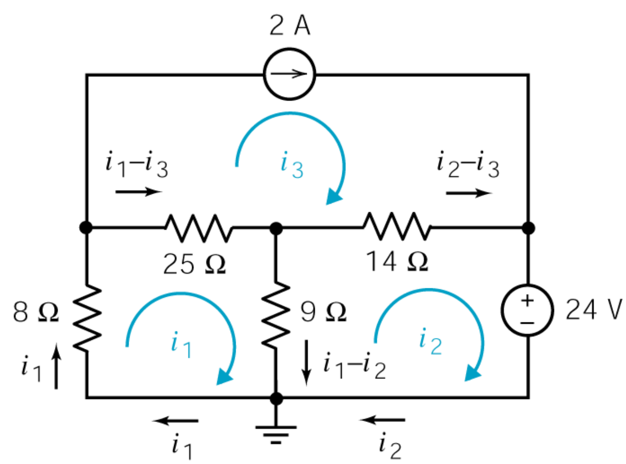


Figure P4.9.5

Solution: Label the label the mesh currents. Then, label the element currents in terms of the mesh currents:



Notice that the 2 A source on the outside of the circuit is in mesh 3 and that the currents 2 A and i_3 have the same direction. Consequently

$$i_3 = 2 \text{ A}$$

Apply KVL to mesh 1 to get

$$25(i_1 - i_3) + 9(i_1 - i_2) + 8i_1 = 0$$

In this equation $25(i_1 - i_3)$ is the voltage across the 25 Ω resistor (+ on the left), $9(i_1 - i_2)$ is the voltage across the 9 Ω resistor (+ on top) and $8i_1$ is the voltage across the 8 Ω resistor (+ on bottom). Substituting $i_3 = 2$ A and doing a little algebra gives

$$42i_1 - 9i_2 = 50$$

Next, apply KVL to mesh 2 to get

$$14(i_2 - i_3) + 24 - 9(i_1 - i_2) = 0$$

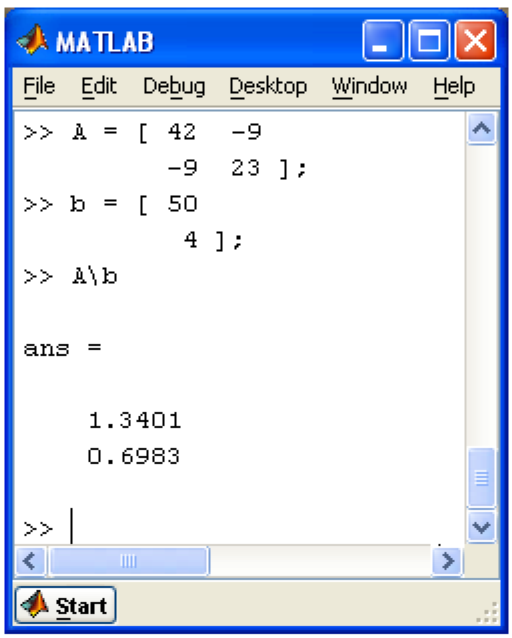
In this equation $14(i_2 - i_3)$ is the voltage across the 14 Ω resistor (+ on the left), 24 is the voltage source voltage and $9(i_1 - i_2)$ is the voltage across the 9 Ω resistor (+ on top). Substituting $i_3 = 2$ A and doing a little algebra gives

$$-9i_1 + 23i_2 = -24 + 14(2) = 4$$

The simultaneous equations can be written in matrix form

$$\begin{aligned} 42i_1 - 9i_2 &= 50 \\ -9i_1 + 23i_2 &= 4 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 42 & -9 \\ -9 & 23 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 4 \end{bmatrix}$$

We can use MATLAB to solve the matrix equation:



Then $$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1.3401 \\ 0.6983 \end{bmatrix}$$

That is, the mesh currents are $i_1 = 1.3401$ A and $i_2 = 0.6983$ A.

P4.9-6 Represent the circuit shown in Figure P4.9-6 by the matrix equation

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -40 \\ -228 \end{bmatrix}$$

Determine the values of the coefficients a_{11}, a_{12}, a_{21} and a_{22}.

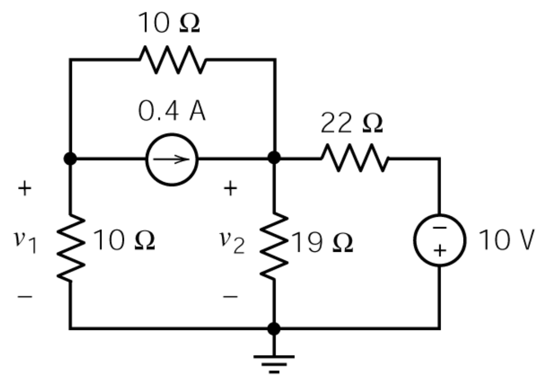
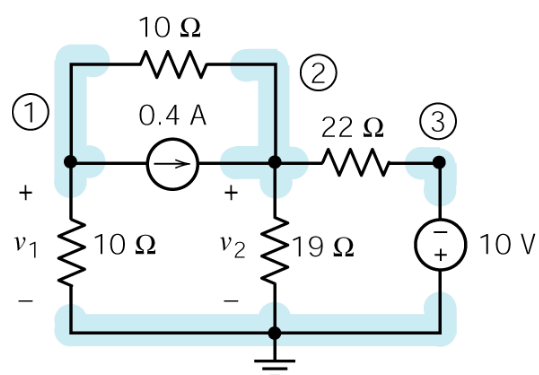


Figure P4.9-6

Solution: Emphasize and label the nodes:



Noticing the 10 V source connected between node 3 and the reference node, we determine that node voltage at node 3 is

$$v_3 = -10 \text{ V}$$

Apply KCL at node 1 to get $$\frac{v_1}{10} + 0.4 + \frac{v_1 - v_2}{10} = 0$$

In this equation $\frac{v_1}{10}$ is the current directed downward in the vertical 10 Ω resistor and $\frac{v_1 - v_2}{10}$ is the current directed from left to right in the horizontal 10 Ω resistor. We will simplify this equation by doing two things:

1. Multiplying each side by 10 to eliminate fractions.

2. Move the terms that don't involve node voltages to the right side of the equation.

The result is $$2v_1 - v_2 = -4$$

Next, apply KCL at node 2 to get

$$\frac{v_2}{19} + \frac{v_2 - (-10)}{22} = \frac{v_1 - v_2}{10} + 0.4$$

In this equation $\frac{v_2}{19}$ is the current directed downward in the 19 Ω resistor, $\frac{v_2 - (-10)}{22}$ is the current directed from left to right in the 22 Ω resistor and $\frac{v_1 - v_2}{10}$ is the current directed from left to right in the horizontal 10 Ω resistor. We will simplify this equation by doing two things:

1. Multiplying each side by $19 \times 22 \times 10 = 4180$ to eliminate fractions.

2. Move the terms that involve node voltages to the left side of the equation and move the terms that don't involve node voltages to the right side of the equation.

The result is

$$-(19 \times 22)v_1 + (19 \times 10 + 22 \times 10 + 19 \times 22)v_2 = -10 \times 10 \times 19 + 0.4 \times 19 \times 22 \times 10$$

$$\Rightarrow \quad -418v_1 + 828v_2 = -228$$

Comparing our equations to the given equations, we see that we need to multiply both sides of our first equation by 10. Then

$$\begin{aligned} 20v_1 - 10v_2 &= -40 \\ -418v_1 + 828v_2 &= -228 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} 20 & -10 \\ -418 & 828 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -40 \\ -228 \end{bmatrix}$$

Comparing coefficients gives

$$a_{11} = 20\,,\; a_{12} = -10\,,\; a_{21} = -418 \text{ and } a_{22} = 828.$$

P4.9-7 Represent the circuit shown in Figure P4.9-7 by the matrix equation

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

Determine the values of the coefficients a_{11}, a_{12}, a_{21} and a_{22}.

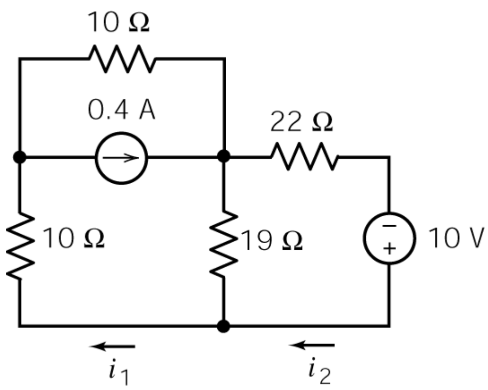
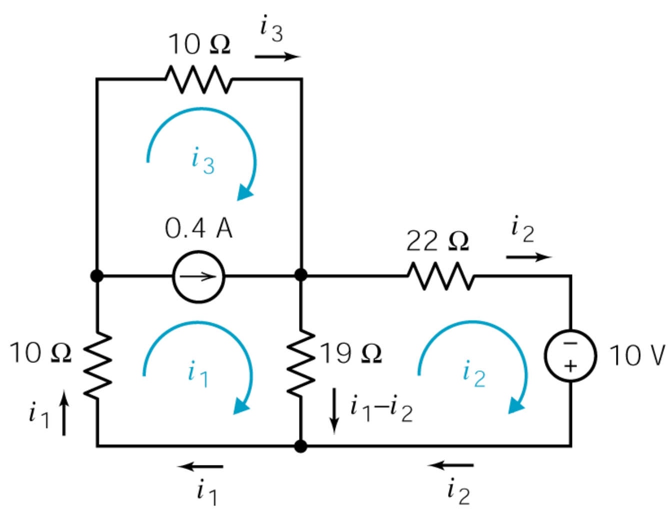


Figure P4.9-7

Solution: Label the label the mesh currents. Then, label the element currents in terms of the mesh currents:



Notice that the 0.4 A source on the inside of the circuit is in both mesh 1 and mesh 3. Mesh current i_1 is directed in the same way as current source current but the mesh current i_3 is directed opposite to the current source current. Consequently

$$i_1 - i_3 = 0.4 \text{ A}$$

The current source is in both mesh 1 and mesh 3 so we apply KVL to the supermesh corresponding to the current source (i.e. the perimeter of meshes 1 and 3). The result is

$$10\, i_3 + 19\left(i_1 - i_2\right) + 10\, i_1 = 0$$

In this equation $10 i_3$ is the voltage across the horizontal 10 Ω resistor (+ on the left), $19\left(i_1 - i_2\right)$ is the voltage across the 19 Ω resistor (+ on top) and $10 i_1$ is the voltage across the vertical 10 Ω resistor (+ on bottom). Substituting $i_3 = i_1 - 0.4$ and doing a little algebra gives

$$39 i_1 - 19 i_2 = 4$$

Next, apply KVL to mesh 2 to get

$$22 i_2 - 10 - 19\left(i_1 - i_2\right) = 0$$

In this equation $22 i_2$ is the voltage across the 22 Ω resistor (+ on the left), 10 is the voltage source voltage and $19\left(i_1 - i_2\right)$ is the voltage across the 19 Ω resistor (+ on top). Doing a little algebra gives

$$-19 i_1 + 41 i_2 = 10$$

To summarize, the circuit is represented by the simultaneous equations:

$$\begin{array}{c} 39 i_1 - 19 i_2 = 4 \\ -19 i_1 + 41 i_2 = 10 \end{array} \Rightarrow \begin{bmatrix} 39 & -19 \\ -19 & 41 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

Comparing these equations to the given equations shows

$$a_{11} = 39,\ a_{12} = -19,\ a_{21} = -19 \text{ and } a_{22} = 41.$$

P4.9-8 Determine the values of the power supplied by each of the sources for the circuit shown in Figure P4.9-8.

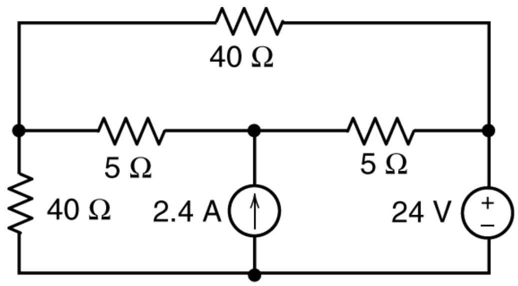
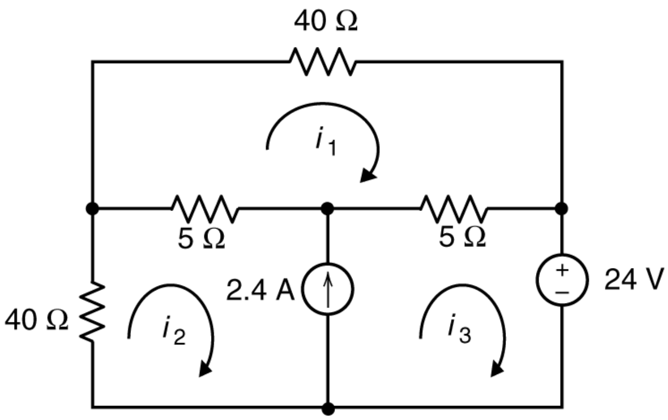
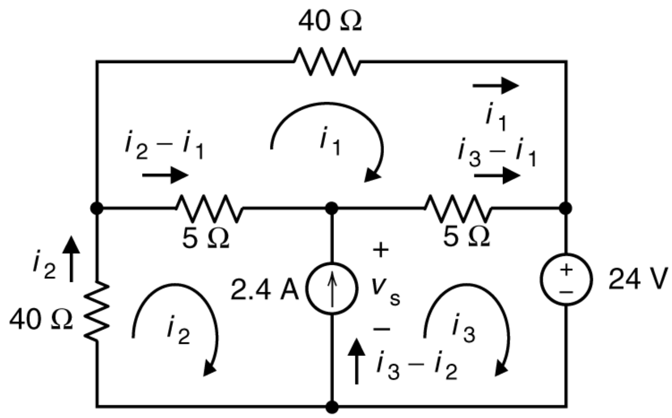


Figure P.4.9-8

Solution: First, label the mesh currents and then label the element currents:





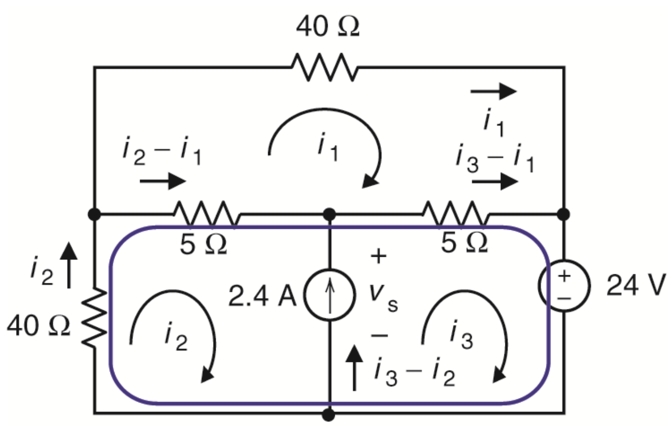
Notice the 2.4 A source in both mesh 2 and mesh 3. We have

$$i_3 - i_2 = 2.4 \text{ A}$$

Apply KVL to mesh 1 to get

$$40 i_1 - 5(i_3 - i_1) - 5(i_2 - i_1) = 0 \quad \Rightarrow \quad 50 i_1 - 5 i_2 - 5 i_3 = 0$$

Identify the supermesh corresponding to the 2.4 A current source:



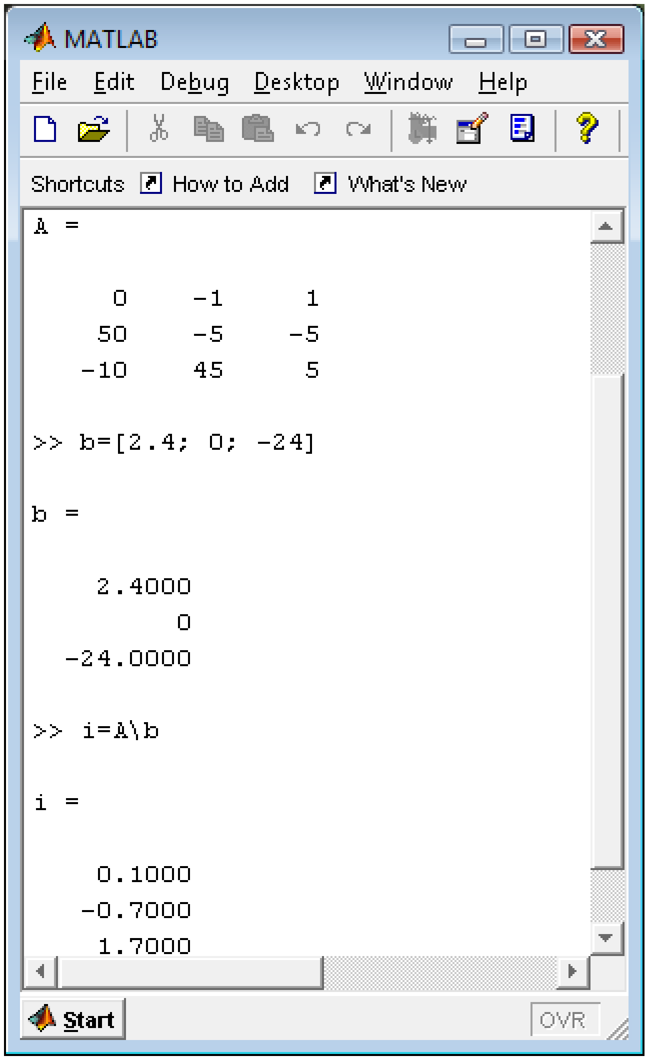
Apply KVL to the supermesh to get

$$5(i_2 - i_1) + 5(i_3 - i_1) + 24 + 40 i_2 = 0 \quad \Rightarrow \quad -10 i_1 + 45 i_2 + 5 i_3 = -24$$

Writing the mesh equations in matrix form gives

$$\begin{bmatrix} 0 & -1 & 1 \\ 50 & -5 & -5 \\ -10 & 45 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 2.4 \\ 0 \\ -24 \end{bmatrix}$$

Solving using MATLAB:



That is, the mesh currents are $i_1 = 0.1$ A, $i_2 = -0.7$ A and $i_3 = 1.7$ A.

The 24 V source supplies $\qquad -24\,i_3 = (-24)(1.7) = -40.8$ W

The power supplied by the current source depends on v_s, the voltage across the current source. Apply KVL to mesh 3 to get

$$5(i_3 - i_1) + 24 - v_s = 0 \quad \Rightarrow \quad v_s = 5(1.7 - 0.1) + 24 = 32 \text{ V}$$

The current source supplies $\qquad 2.4\,v_s = 2.4(32) = 76.8$ W

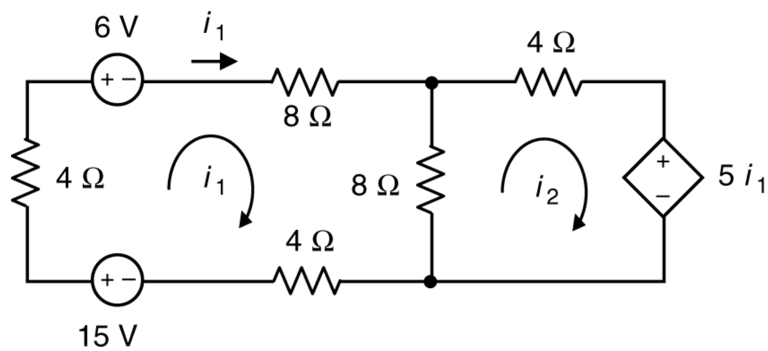
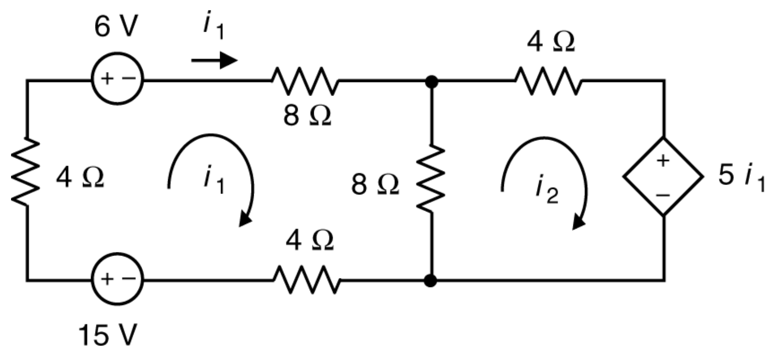


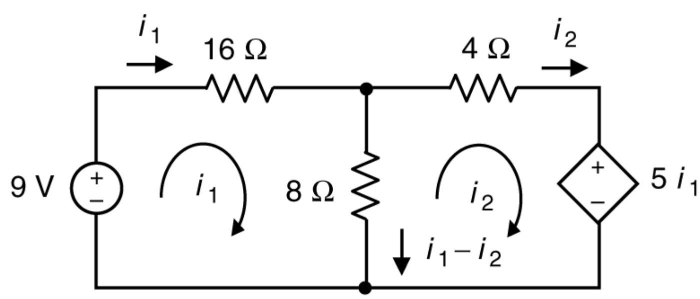
Figure P4.9-9

P4.9-9 The mesh currents are labeled in the circuit shown Figure 4.9-9. Determine the value of the mesh currents i_1 and i_2.

Solution: Determine the value of the mesh currents i_1 and i_2.



Replace series resistors with an equivalent resistor and series voltage sources with and equivalent voltage source to get

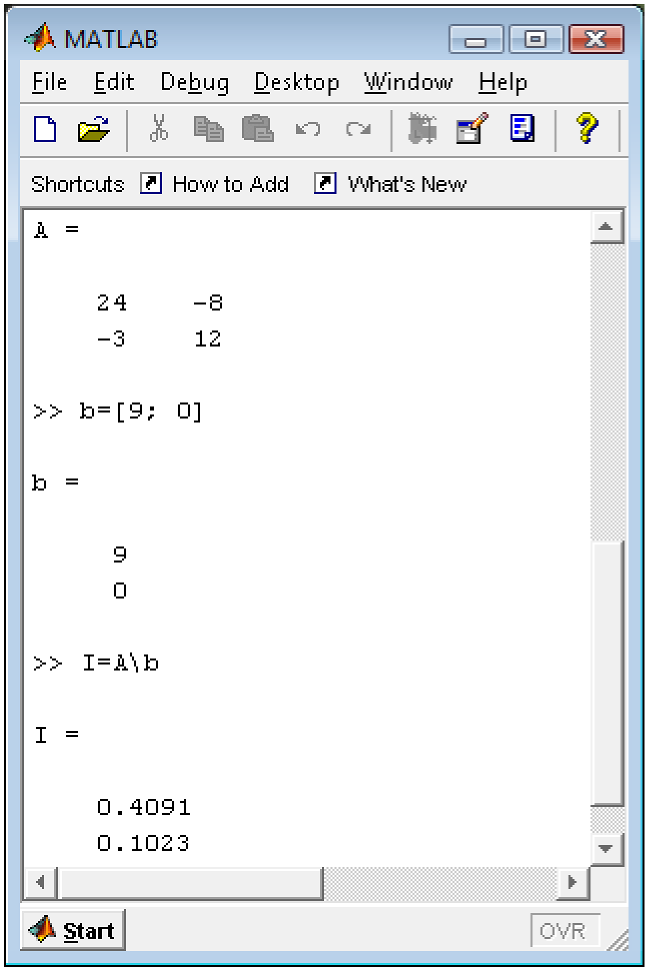


Apply KVL to mesh 1 $\quad 16i_1 + 8(i_1 - i_2) - 9 = 0 \;\Rightarrow\; 24i_1 - 8i_2 = 9$

Apply KVL to mesh 2 $\quad 4i_2 + 5i_1 - 8(i_1 - i_2) = 0 \;\Rightarrow\; -3i_1 + 12i_2 = 0$

In matrix form $\quad \begin{bmatrix} 24 & -8 \\ -3 & 12 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$

Solving using MATLAB



So the mesh currents are $i_1 = 0.4091$ A and $i_2 = 0.1023$ A

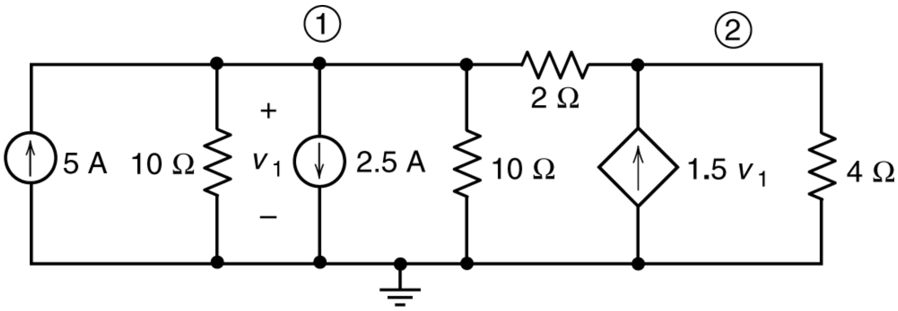
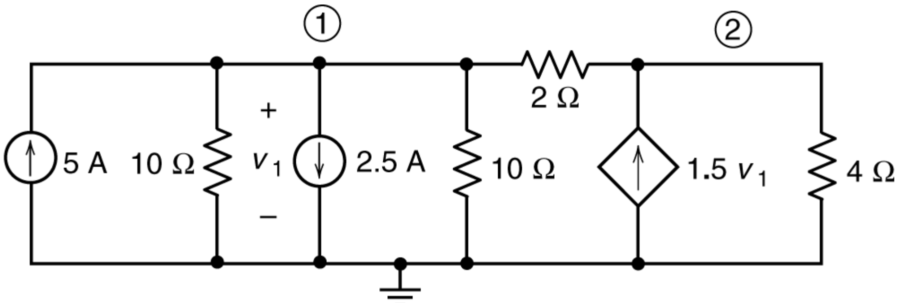


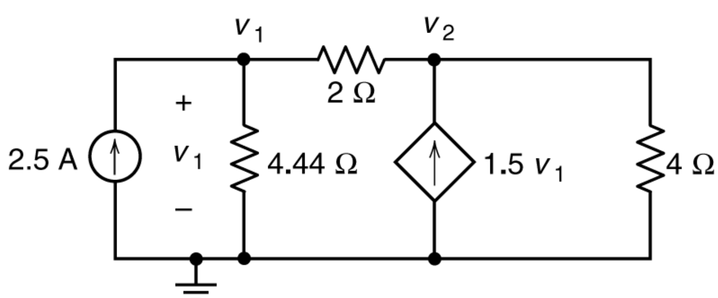
Figure P4.9-10

P4.9-10 The encircled numbers in the circuit shown Figure P4.9-10 are node numbers. Determine the values of the corresponding node voltages, v_1 and v_2.

Solution: Determine the value of the node voltages, v_1 and v_2.



Replace parallel resistors with an equivalent resistor and parallel sources with and equivalent current source to get

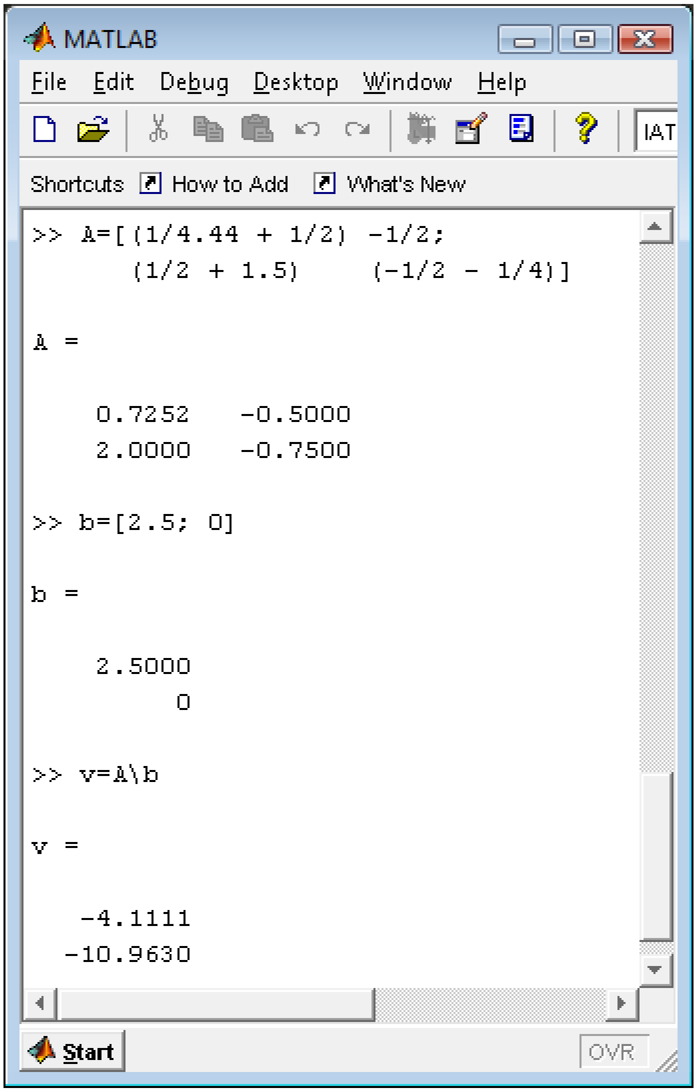


Apply KCL at node 1 $$2.5 = \frac{v_1}{4.44} + \frac{v_1 - v_2}{2} = 0$$

Apply KCL at node 2 $$\frac{v_1 - v_2}{2} + 1.5 v_1 = \frac{v_2}{4}$$

In matrix form $$\begin{bmatrix} \frac{1}{4.44} + \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} + 1.5 & -\frac{1}{2} - \frac{1}{4} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0 \end{bmatrix}$$

Solving using MATLAB



So the node voltages are $v_1 = -4.1111$ V and $v_2 = -10.9630$ V

Section 4.10 How Can We Check … ?

P 4.10-1 Computer analysis of the circuit shown in Figure P 4.10-1 indicates that the node voltages are

$v_a = 5.2$ V, $v_b = -4.8$ V, and $v_c = 3.0$ V.

Is this analysis correct?

Hint: Use the node voltages to calculate all the element currents. Check to see that KCL is satisfied at each node.

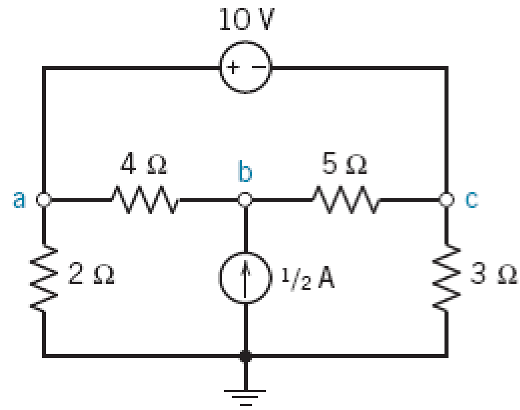


Figure P 4.10-1

Solution:

Apply KCL at node b:

$$\frac{v_b - v_a}{4} - \frac{1}{2} + \frac{v_b - v_c}{5} = 0$$

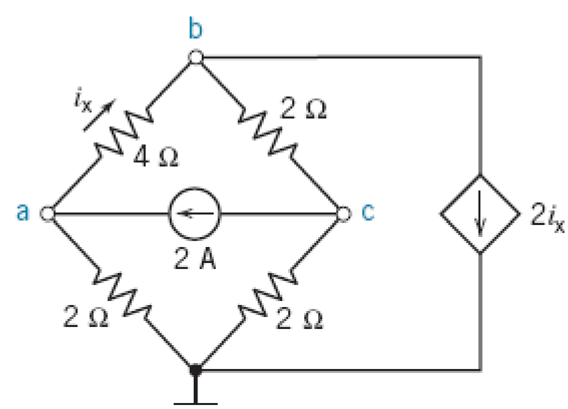
$$\frac{-4.8 - 5.2}{4} - \frac{1}{2} + \frac{-4.8 - 3.0}{5} \neq 0$$

The given voltages do not satisfy the KCL equation at node b. They are **not correct.**

P 4.10-2 An old lab report asserts that the node voltages of the circuit of Figure P 4.10-2 are

$v_a = 4$ V, $v_b = 20$ V, and $v_c = 12$ V.

Are these correct?



Solution:

Apply KCL at node a:

$$-\left(\frac{v_b - v_a}{4}\right) - 2 + \frac{v_a}{2} = 0$$

$$-\left(\frac{20 - 4}{4}\right) - 2 + \frac{4}{2} = -4 \neq 0$$

The given voltages do not satisfy the KCL equation at node a. They are **not correct.**

P 4.10-3 Your lab partner forgot to record the values of R_1, R_2, and R_3. He thinks that two of the resistors in Figure P 4.10-3 had values of 10 kΩ and that the other had a value of 5 kΩ. Is this possible? Which resistor is the 5-kΩ resistor?

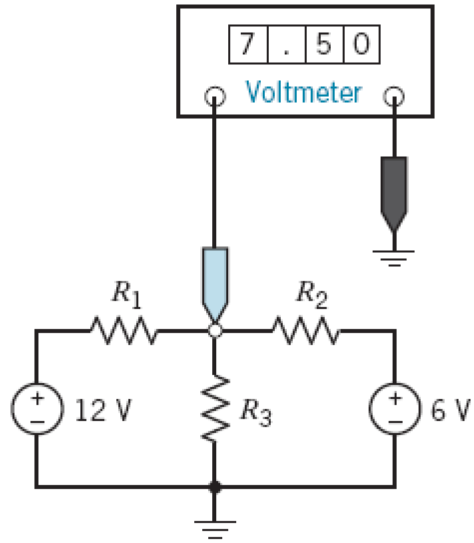


Figure P 4.10-3

Solution:

Writing a node equation:

$$-\left(\frac{12-7.5}{R_1}\right)+\frac{7.5}{R_3}+\frac{7.5-6}{R_2}=0$$

So

$$-\frac{4.5}{R_1}+\frac{7.5}{R_3}+\frac{1.5}{R_2}=0$$

There are only three cases to consider. Suppose $R_1 = 5\text{ k}\Omega$ and $R_2 = R_3 = 10\text{ k}\Omega$. Then

$$-\frac{4.5}{R_1}+\frac{7.5}{R_3}+\frac{1.5}{R_2}=\frac{-0.9+0.75+0.15}{1000}= 0$$

This choice of resistance values corresponds to branch currents that satisfy KCL. Therefore, it is indeed possible that two of the resistances are 10 kΩ and the other resistance is 5 kΩ. The 5 kΩ is R_1.

P 4.10-4 Computer analysis of the circuit shown in Figure P 4.10-4 indicates that the mesh currents are

$$i_1 = 2\text{ A},\ i_2 = 4\text{ A, and } i_3 = 3\text{ A}.$$

Verify that this analysis is correct.

Hint: Use the mesh currents to calculate the element voltages. Verify that KVL is satisfied for each mesh.

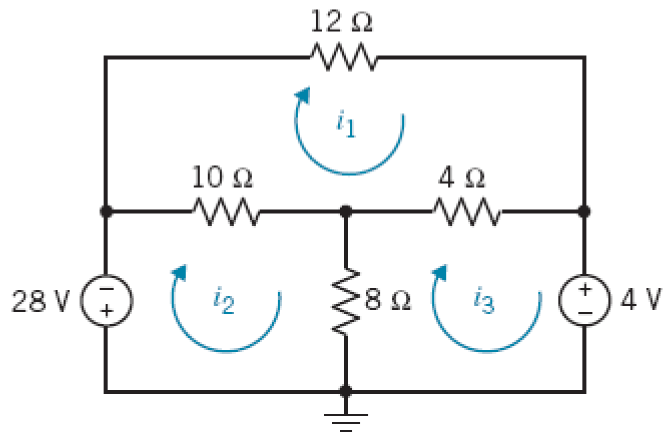


Figure P 4.10-4

Solution: Applying KVL to each mesh:

Top mesh: $$10\,(2-4)+12(2)+4\,(2-3)=0$$

Bottom right mesh $$8\,(3-4)+4\,(3-2)+4=0$$

Bottom, left mesh: $$28+10\,(4-2)+8\,(4-3)\neq 0$$
(Perhaps the polarity of the 28 V source was entered incorrectly.)

KVL is not satified for the bottom, left mesh so the computer analysis is **not correct**.

Design Problems

DP 4-1 An electronic instrument incorporates a 15-V power supply. A digital display is added that requires a 5-V power supply. Unfortunately, the project is over budget and you are instructed to use the existing power supply. Using a voltage divider, as shown in Figure DP 4-1, you are able to obtain 5 V. The specification sheet for the digital display shows that the display will operate properly over a supply voltage range of 4.8 V to 5.4 V. Furthermore, the display will draw 300 mA (I) when the display is active and 100 mA when quiescent (no activity).

(a) Select values of R_1 and R_2 so that the display will be supplied with 4.8 V to 5.4 V under all conditions of current I.

(b) Calculate the maximum power dissipated by each resistor, R_1 and R_2, and the maximum current drawn from the 15-V supply.

(c) Is the use of the voltage divider a good engineering solution? If not, why? What problems might arise?

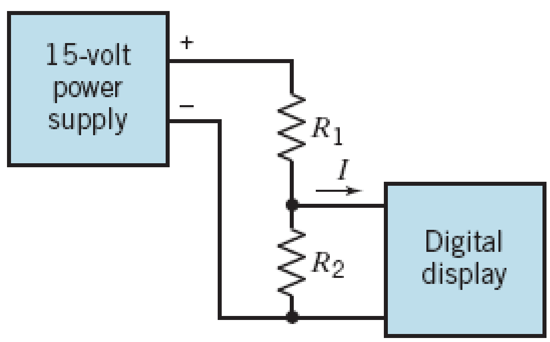
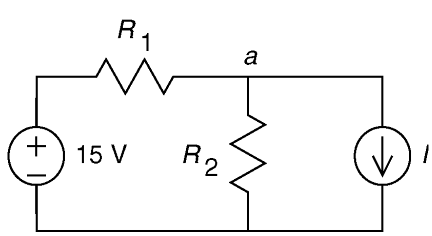


Figure DP 4-1

Solution:

Model the circuit as:



(a) We need to keep v_2 across R_2 in the range $4.8 \le v_2 \le 5.4$

$$\text{For } I = \begin{cases} 0.3 \text{ A} & \text{display is active} \\ 0.1 \text{ A} & \text{display is not active} \end{cases}$$

KCL at a: $\dfrac{v_2-15}{R_1}+\dfrac{v_2}{R_2}+I=0$

Assumed that maximum I results in minimum v_2 and visa-versa.

Then

$$v_2 = \begin{cases} 4.8\text{ V} & \text{when } I=0.3\text{ A} \\ 5.4\text{ V} & \text{when } I=0.1\text{ A} \end{cases}$$

Substitute these corresponding values of v_2 and I into the KCL equation and solve for the resistances

$$\frac{4.8-15}{R_1}+\frac{4.8}{R_2}+0.3=0$$

$$\frac{5.4-15}{R_1}+\frac{5.4}{R_2}+0.1=0$$

$$\Rightarrow \underline{R_1 = 7.89\ \Omega},\ \underline{R_2 = 4.83\ \Omega}$$

(b) $$I_{R_{1\max}} = \frac{15-4.8}{7.89} = 1.292\text{ A} \Rightarrow P_{R_{1\max}} = (1.292)^2(7.89) = 13.17\text{ W}$$

$$I_{R_{2\max}} = \frac{5.4}{4.83} = 1.118\text{ A} \Rightarrow P_{R_{2\max}} = \frac{(5.4)^2}{4.83} = 6.03\text{ W}$$

maximum supply current $= I_{R_{1\max}} = 1.292$ A

(c) No; if the supply voltage (15V) were to rise or drop, the voltage at the display would drop below 4.8V or rise above 5.4V.

The power dissipated in the resistors is excessive. Most of the power from the supply is dissipated in the resistors, not the display.

DP 4-2 For the circuit shown in Figure DP 4-2, it is desired to set the voltage at node a equal to 0 V in order to control an electric motor. Select voltages v_1 and v_2 in order to achieve $v_a = 0$ V when v_1 and v_2 are less than 20 V and greater than zero and $R = 2\ \Omega$.

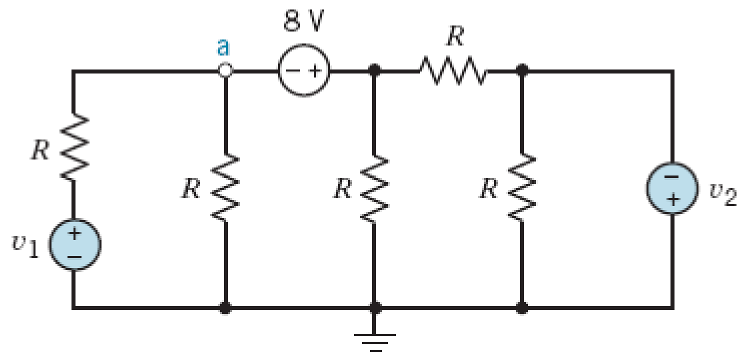


Figure DP 4-2

Solution:

Express the voltage of the 8 V source in terms of its node voltages to get $v_b - v_a = 8$. Apply KCL to the supernode corresponding to the 8 V source:

$$\frac{v_a - v_1}{R} + \frac{v_a}{R} + \frac{v_b}{R} + \frac{v_b - (-v_2)}{R} = 0 \quad \Rightarrow \quad 2\,v_a - v_1 + 2\,v_b + v_2 = 0$$

$$\Rightarrow \quad 2\,v_a - v_1 + 2\,(v_a + 8) + v_2 = 0$$

$$\Rightarrow \quad 4\,v_a - v_1 + v_2 + 16 = 0$$

$$\Rightarrow \quad v_a = \frac{v_1 - v_2}{4} - 4$$

Next set $v_a = 0$ to get

$$0 = \frac{v_1 - v_2}{4} - 4 \quad \Rightarrow \quad v_1 - v_2 = 16 \text{ V}$$

For example, $v_1 = 18$ V and $v_2 = 2$ V.

DP 4-3 A wiring circuit for a special lamp in a home is shown in Figure DP 4-3. The lamp has a resistance of 2 Ω, and the designer selects R = 100 Ω. The lamp will light when $I \geq 50$ mA but will burn out when $I > 75$ mA.

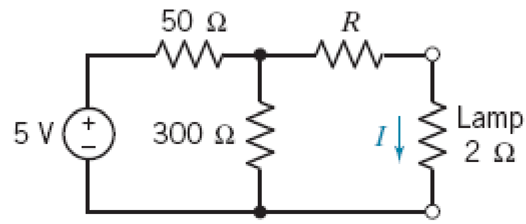


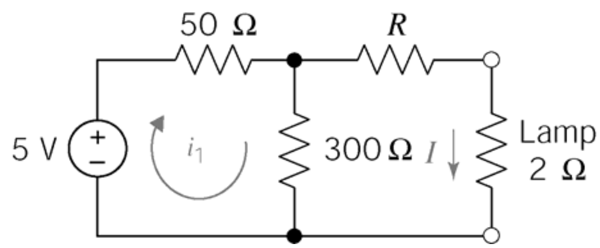
Figure DP 4-3

(a) Determine the current in the lamp and determine if it will light for R = 100 Ω.

(b) Select R so that the lamp will light but will not burn out if R changes by ± 10 percent because of temperature changes in the home.

Solution:

(a)



Apply KCL to left mesh: $-5+50i_1+300\,(i_1-I)=0$

Apply KCL to right mesh: $(R+2)\,I+300\,(I-i_1)=\ 0$

Solving for I: $I=\dfrac{150}{1570\ +\ 35\,R}$

We desire 50 mA ≤ I ≤ 75 mA so if R = 100 Ω, then I = 29.59 mA ⇒ l amp so the lamp will not light.

(b) From the equation for I, we see that decreasing R increases I:

$$\text{try } R = 50\ \Omega\ \Rightarrow\ I = 45 \text{ mA (won't light)}$$

$$\text{try } R = 25\Omega\ \Rightarrow\ I = 61 \text{ mA}\ \Rightarrow\ \underline{\text{will light}}$$

Now check R±10% to see if the lamp will light and not burn out:

$$\left.\begin{array}{l}-10\%\ \rightarrow\ 22.5\Omega\ \rightarrow\ I = 63.63\text{ mA}\\ +10\%\ \rightarrow\ 27.5\Omega\ \rightarrow\ I = 59.23\text{ mA}\end{array}\right\}\begin{array}{l}\text{lamp will}\\ \text{stay on}\end{array}$$

DP 4-4 In order to control a device using the circuit shown in Figure DP 4-4, it is necessary that v_{ab} = 10 V. Select the resistors when it is required that all resistors be greater than 1 Ω and R_3 + R_4 = 20 Ω.

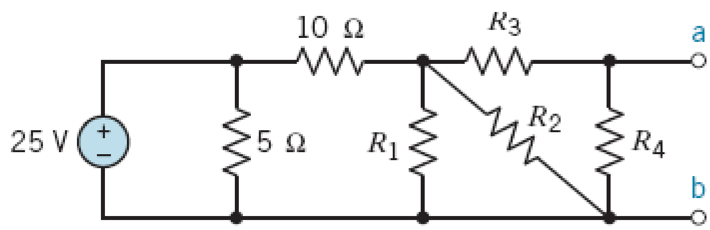
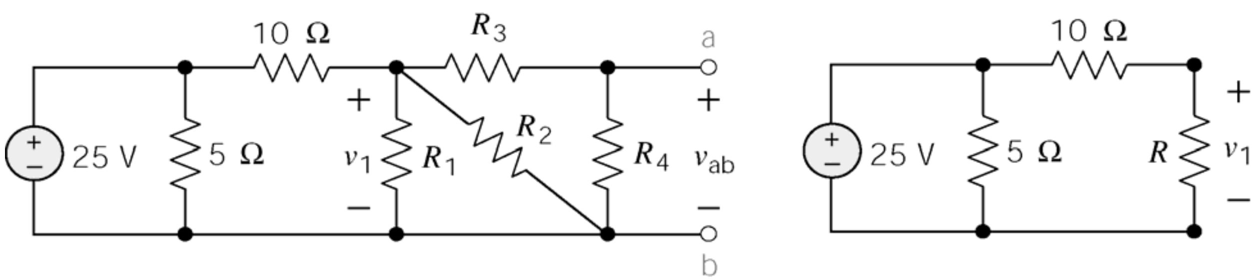


Figure DP 4-4

Solution:



Equivalent resistance: $R = R_1 \parallel R_2 \parallel (R_3 + R_4)$

Voltage division in the equivalent circuit: $v_1 = \dfrac{R}{10+R}(25)$

We require v_{ab} = 10 V. Apply the voltage division principle in the left circuit to get:

$$10 = \frac{R_4}{R_3 + R_4}\, v_1 = \frac{R_4}{R_3 + R_4} \times \frac{\left(R_1 \| R_2 \| \left(R_3 + R_4\right)\right)}{10 + \left(R_1 \| R_2 \| \left(R_3 + R_4\right)\right)} \times 25$$

This equation does not have a unique solution. Here's one solution:

$$\text{choose } R_1 = R_2 = 25\,\Omega \text{ and } R_3 + R_4 = 20\,\Omega$$

$$\text{then } 10 = \frac{R_4}{20} \times \frac{(12.5 \| 20)}{10 + (12.5 \| 20)} \times 25 \Rightarrow \underline{R_4 = 18.4\Omega}$$

$$\text{and } R_3 + R_4 = 20 \Rightarrow \underline{R_3 = 1.6\,\Omega}$$

DP 4-5 The current i shown in the circuit of Figure DP 4-5 is used to measure the stress between two sides of an earth fault line. Voltage v_1 is obtained from one side of the fault, and v_2 is obtained from the other side of the fault. Select the resistances R_1, R_2, and R_3 so that the magnitude of the current i will remain in the range between 0.5 mA and 2 mA when v_1 and v_2 may each vary independently between +1 V and +2 V ($1 \text{ V} \le v_n \le 2 \text{ V}$).

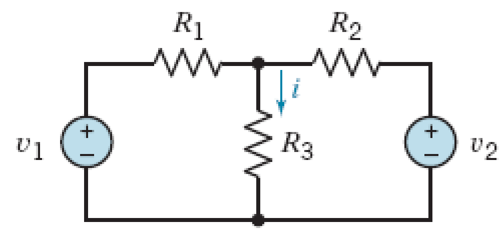
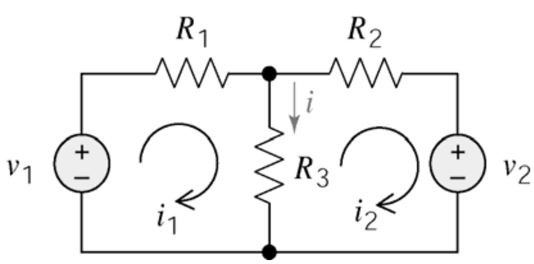


Figure DP 4-5 A circuit for earth fault-line stress measurement.

Solution:



Apply KCL to the left mesh:

$$\left(R_1+R_3\right) i_1 - R_3 i_2 - v_1 = 0$$

Apply KCL to the left mesh:

$$-R_3 i_1 + \left(R_2+R_3\right) i_2 + v_2 = 0$$

Solving for the mesh currents using Cramer's rule:

$$i_1 = \frac{\begin{bmatrix} v_1 & -R_3 \\ -v_2 & \left(R_2+R_3\right) \end{bmatrix}}{\Delta} \text{ and } i_2 = \frac{\begin{bmatrix} (R_1+R_3) & v_1 \\ -R_3 & -v_2 \end{bmatrix}}{\Delta}$$

$$\text{where } \Delta = \left(R_1+R_3\right)\left(R_2+R_3\right) - R_3^{\,2}$$

Try $R_1 = R_2 = R_3 = 1 \text{ k}\Omega = 1000\ \Omega$. Then $\Delta = 3\text{ M}\Omega$. The mesh currents will be given by

$$i_1 = \frac{\left[2v_1 - v_2\right] 1000}{3\times 10^6} \text{ and } i_2 = \frac{\left[-2v_2 + v_1\right] 1000}{3\times 10^6} \Rightarrow i = i_1 - i_2 = \frac{v_1+v_2}{3000}$$

Now check the extreme values of the source voltages:

$$\text{if } v_1 = v_2 = 1\text{ V} \Rightarrow i = 2/3 \text{ mA} \quad \textbf{okay}$$

$$\text{if } v_1 = v_2 = 2\text{ V} \Rightarrow i = 4/3 \text{ mA} \quad \textbf{okay}$$

Chapter 4 Exercises

Exercise 4.2-1 Determine the node voltages, v_a and v_b, for the circuit of Figure E 4.2-1.

Answer: $v_a = 3$ V and $v_b = 11$ V

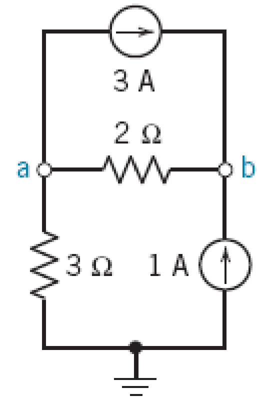


Figure E 4.2-1

Solution:

KCL at a: $$\frac{v_a}{3}+\frac{v_a-v_b}{2}+3=0 \;\Rightarrow\; 5v_a-3v_b=-18$$

KCL at b: $$\frac{v_b-v_a}{2}-3-1=0 \;\Rightarrow\; v_b-v_a=8$$

Solving these equations gives: $v_a = 3$ V and $v_b = 11$ V

Exercise 4.2-2 Determine the node voltages, v_a and v_b, for the circuit of Figure E 4.2-2.

Answer: $v_a = -4/3$ V and $v_b = 4$ V

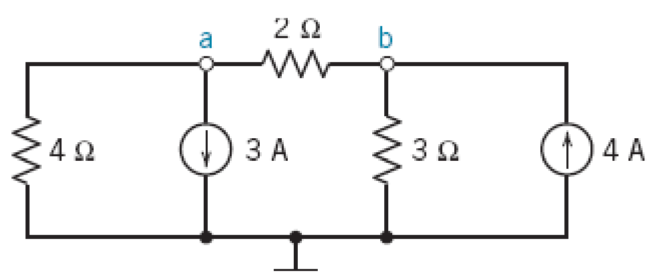


Figure E 4.2-2

Solution:

KCL at a: $$\frac{v_\mathrm{a}}{4}+\frac{v_\mathrm{a}-v_\mathrm{b}}{2}+3=0 \;\Rightarrow\; 3v_\mathrm{a}-2v_\mathrm{b}=-12$$

KCL at b: $$\frac{v_b}{3}-\frac{v_a-v_b}{2}-4=0 \;\Rightarrow\; -3v_a+5v_b=24$$

Solving: $v_a = -4/3$ V and $v_b = 4$ V

Exercise 4.3-1 Find the node voltages for the circuit of Figure E 4.3-1.

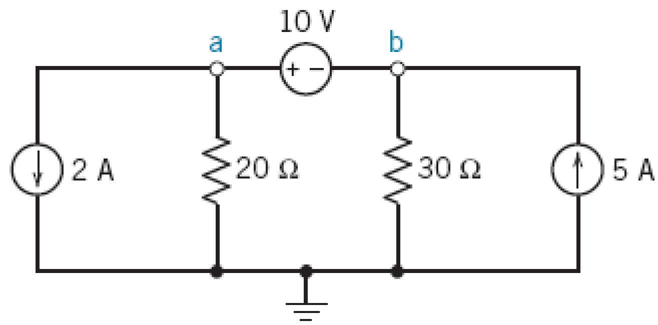


Figure E 4.3-1

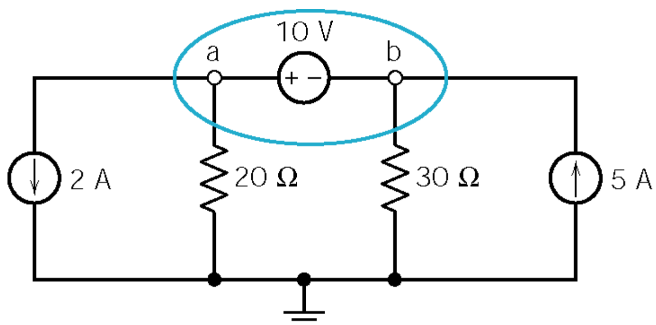
Hint: Write a KCL equation for the supernode corresponding to the 10-V voltage source.

Answer:

$$2+\frac{v_b+10}{20}+\frac{v_b}{30}=5 \quad \Rightarrow \quad v_b=30\text{ V} \quad \text{and} \quad v_a=40\text{ V}$$

Solution:

Apply KCL to the supernode to get



$$2+\frac{v_b+10}{20}+\frac{v_b}{30}=5$$

Solving:

$$v_b=30\text{ V and } v_a=v_b+10=40\text{ V}$$

Exercise 4.3-2 Find the voltages v_a and v_b for the circuit of Figure E 4.3-2.

Answer: $\dfrac{(v_b+8)-(-12)}{10}+\dfrac{v_b}{40}=3 \quad \Rightarrow \quad v_b=8\text{ V} \;\text{ and }\; v_a=16\text{ V}$

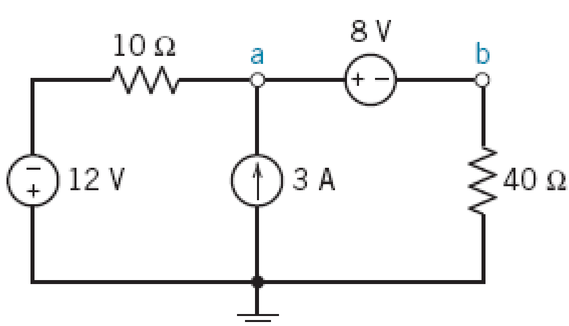


Figure E 4.3-2

Solution:

$$\frac{\left(v_b+8\right)-\left(-12\right)}{10}+\frac{v_b}{40}=3 \quad \Rightarrow \quad v_b=8\text{ V and } v_a=16\text{ V}$$

Exercise 4.4-1 Find the node voltage v_b for the circuit shown in Figure E 4.4-2.

Hint: Apply KCL at node a to express i_a as a function of the node voltages. Substitute the result into $v_b = 4i_a$ and solve for v_b.

Answer: $-\frac{6}{8}+\frac{v_b}{4}-\frac{v_b}{12}=0 \Rightarrow v_b = 4.5 \text{ V}$

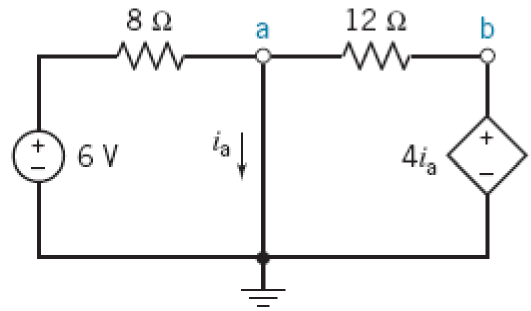


Figure E 4.4-2

Solution:

Apply KCL at node a to express i_a as a function of the node voltages. Substitute the result into $v_b = 4\, i_a$ and solve for v_b .

$$\frac{6}{8}+\frac{v_b}{12}=i_a \quad \Rightarrow \quad v_b = 4i_a = 4\left(\frac{9+v_b}{12}\right) \quad \Rightarrow \quad v_b = 4.5 \text{ V}$$

Exercise 4.4-2 Find the node voltages for the circuit shown in Figure E 4.4-2.

Hint: The controlling voltage of the dependent source is a node voltage, so it is already expressed as a function of the node voltages. Apply KCL at node a.

Answer: $\frac{v_a-6}{20}+\frac{v_a-4v_a}{15}=0 \Rightarrow v_a = -2 \text{ V}$

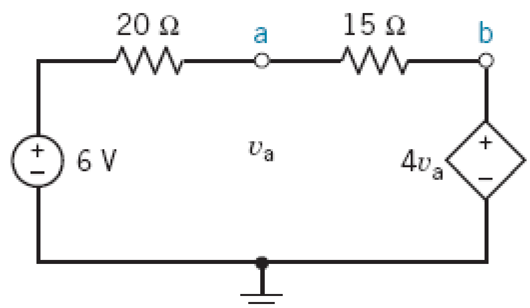


Figure E 4.4-2

Solution:

The controlling voltage of the dependent source is a node voltage so it is already expressed as a function of the node voltages. Apply KCL at node a.

$$\frac{v_a-6}{20}+\frac{v_a-4\,v_a}{15}=0 \quad \Rightarrow \quad v_a = -2 \text{ V}$$

Exercise 4.5-1 Determine the value of the voltage measured by the voltmeter in Figure E 4.5-1.

Answer: –1 V

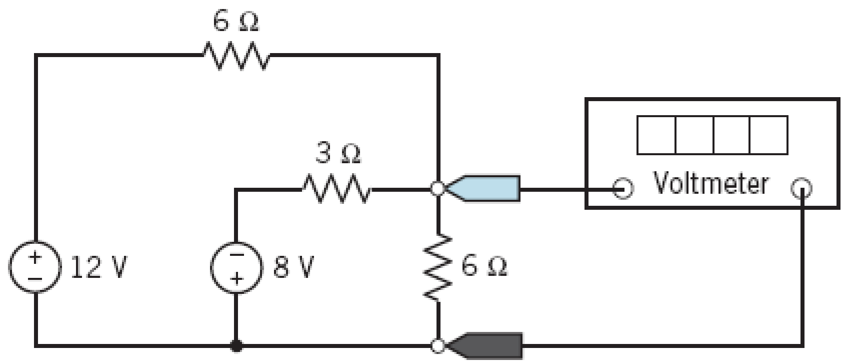
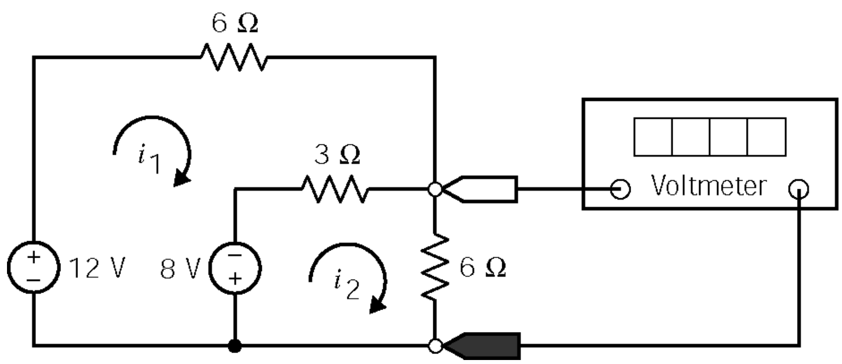


Figure E 4.5-1

Solution:



Mesh equations:

$$-12+6\,i_1+3\left(i_1-i_2\right)-8=0 \quad\Rightarrow\quad 9\,i_1-3\,i_2=20$$

$$8-3\left(i_1-i_2\right)+6\,i_2=0 \quad\Rightarrow\quad -3\,i_1+9\,i_2=-8$$

Solving these equations gives:

$$i_1=\frac{13}{6}\text{ A and } i_2=-\frac{1}{6}\text{ A}$$

The voltage measured by the meter is 6 i_2 = –1 V.

Exercise 4.6-1 Determine the value of the voltage measured by the voltmeter in Figure E 4.6-1.

Hint: Write and solve a single mesh equation to determine the current in the 3-Ω resistor.

Answer: –4 V

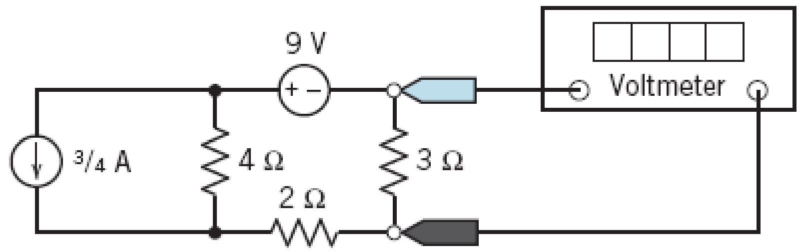
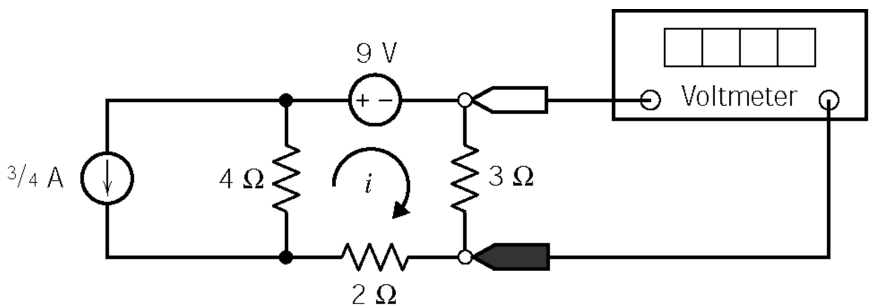


Figure E 4.6-1

Solution:



Mesh equation: $9+3\,i+2\,i+4\left(i+\frac{3}{4}\right)=0 \;\Rightarrow\; (3+2+4)\,i=-9-3 \;\Rightarrow\; i=\frac{-12}{9}$ A

The voltmeter measures $3\,i=-4$ V

Exercise 4.6-2 Determine the value of the current measured by the ammeter in Figure E 4.6-2.

Hint: Write and solve a single mesh equation.

Answer: –3.67 A

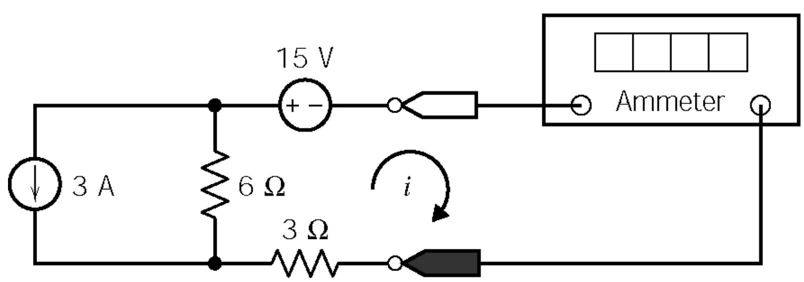


Figure E 4.6-2

Solution:

Mesh equation: $15+3\,i+6\,(i+3)=0 \;\Rightarrow\; (3+6)\,i=-15-6\,(3) \;\Rightarrow\; i=\frac{-33}{9}=-3\frac{2}{3}$ A

PSpice Problems

SP 4-1 Use PSpice to determine the node voltages of the circuit shown in Figure SP 4-1

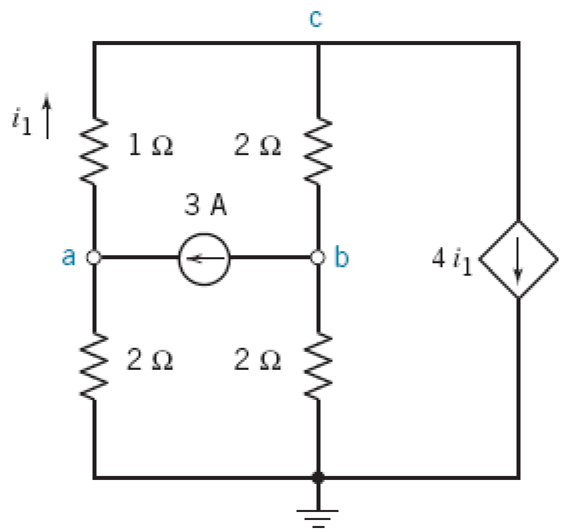
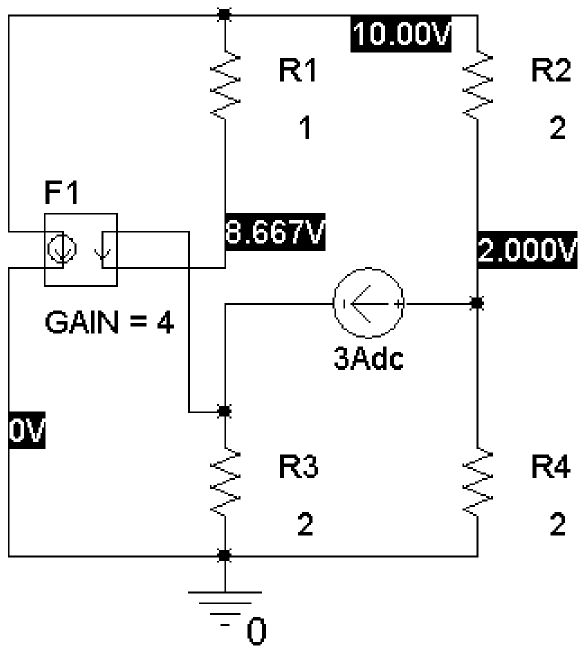


Figure SP 4-1

Solution: The PSpice schematic after running a “Bias Point” simulation:



SP 4-2 Use PSpice to determine the mesh currents of the circuit shown in Figure SP 4-2.

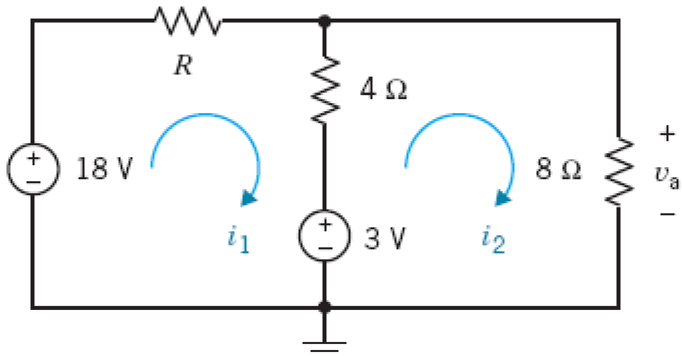
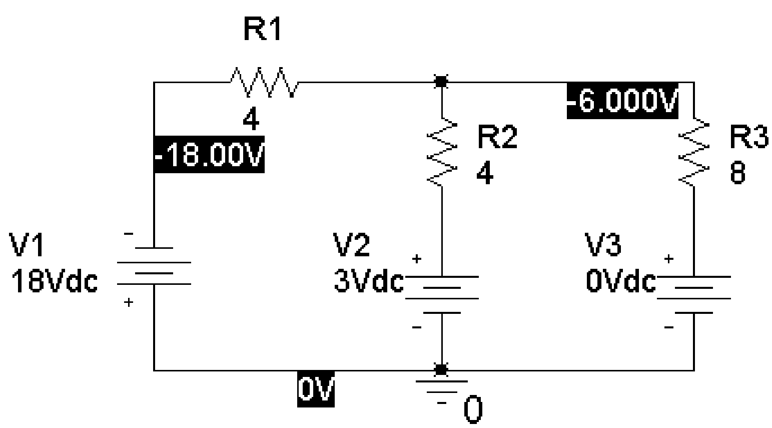


Figure SP 4-2

Solution: The PSpice schematic after running a "Bias Point" simulation:



From the PSpice output file:

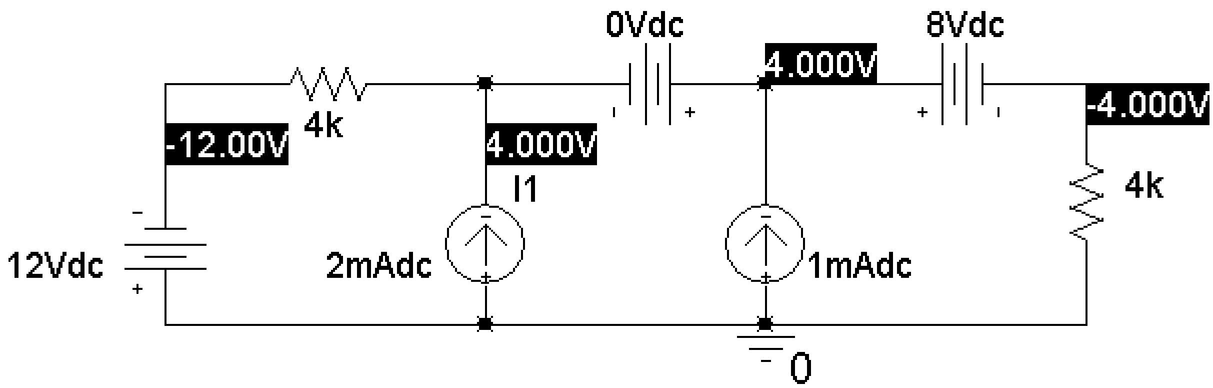
VOLTAGE SOURCE CURRENTS

NAME	CURRENT
V_V1	-3.000E+00
V_V2	-2.250E+00
V_V3	**-7.500E-01**

The voltage source labeled V3 is a short circuit used to measure the mesh current. The mesh currents are i_1 = –3 A (the current in the voltage source labeled V1) and i_2 = –0.75 A (the current in the voltage source labeled V3).

SP 4-3 The voltages v_a, v_b, v_c, and v_d in Figure SP 4-3 are the node voltages corresponding to nodes a, b, c and d. The current *i* is the current in a short circuit connected between nodes b and c. Use PSpice to determine the values of v_a, v_b, v_c, and v_d and of *i*.

Solution: The PSpice schematic after running a "Bias Point" simulation:



The PSpice output file:

```
**** INCLUDING sp4_2-SCHEMATIC1.net ****
* source SP4_2
V_V4       0 N01588 12Vdc
R_R4       N01588 N01565  4k
V_V5       N01542 N01565 0Vdc
R_R5       0 N01516  4k
V_V6       N01542 N01516 8Vdc
I_I1       0 N01565 DC 2mAdc
I_I2       0 N01542 DC 1mAdc

VOLTAGE SOURCE CURRENTS
   NAME       CURRENT

   V_V4      -4.000E-03
   V_V5       2.000E-03
   V_V6      -1.000E-03
```

From the PSpice schematic: $v_a = -12$ V, $v_b = v_c = 4$ V, $v_d = -4$ V. From the output file: $i = 2$ mA.

SP 4-4 Determine the current, i, shown in Figure SP 4-4.

Answer: i = 0.56 A

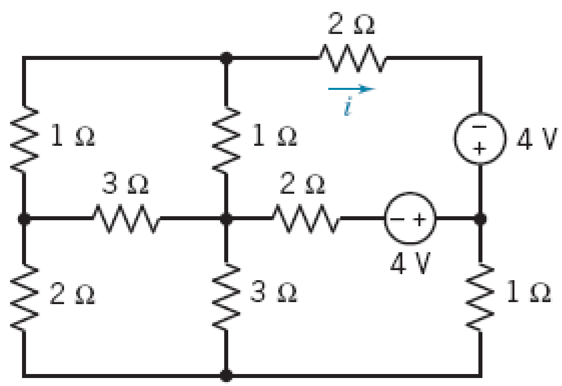
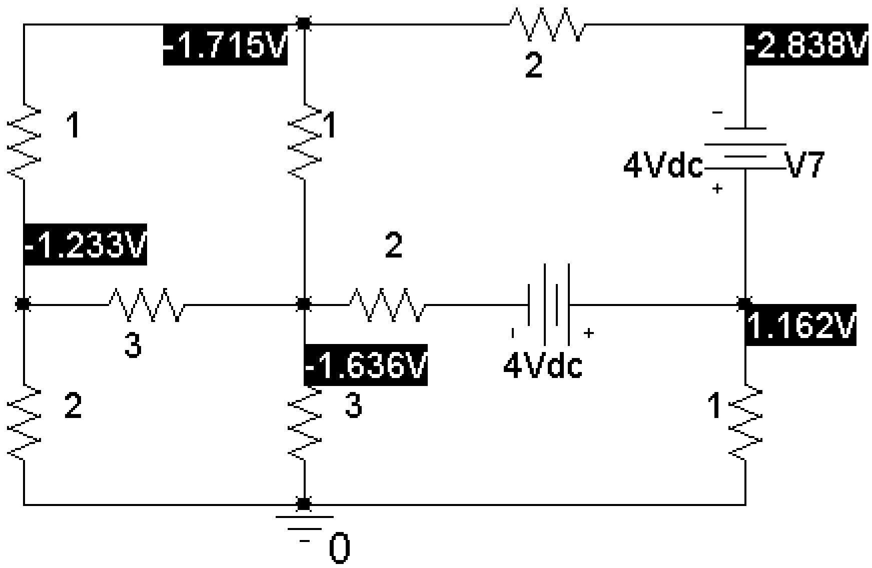


Figure SP 4-4

Solution: The PSpice schematic after running a "Bias Point" simulation:



The PSpice output file:

VOLTAGE SOURCE CURRENTS
NAME CURRENT

V_V7 -5.613E-01
V_V8 -6.008E-01

The current of the voltage source labeled V7 is also the current of the 2 Ω resistor at the top of the circuit. However this current is directed from right to left in the 2 Ω resistor while the current i is directed from left to right. Consequently, i = +5.613 A.