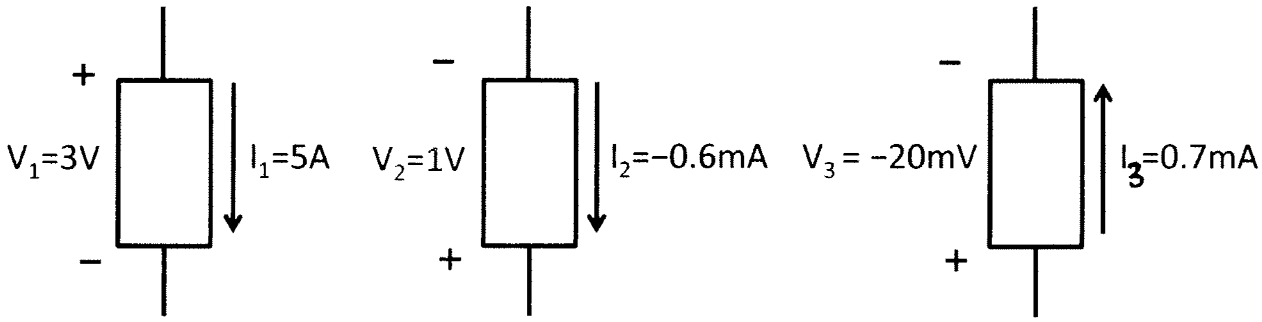
ECSE-200 Quiz #1 (10 Sept 2010)

NAME________________________________________ McGill ID#____________________

*READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).*

1. For each circuit element below, indicate the power that is being delivered or absorbed by the element. [ 2 pts / element x 3 elements = 6 pts ]




a) b) c)

a) $P_{abs} = V_1 \cdot I_1$ (passive sign convention) [+1]

$= (3V)(5A)$

$= 15W$ absorbed by element [+1]

b) $P_{del} = V_2 \cdot I_2$ (opposite to passive sign convention) [+1]

$= (1V)(-0.6mA)$

$= -0.6mW$ delivered by element [+1]

or $0.6mW$ absorbed by element

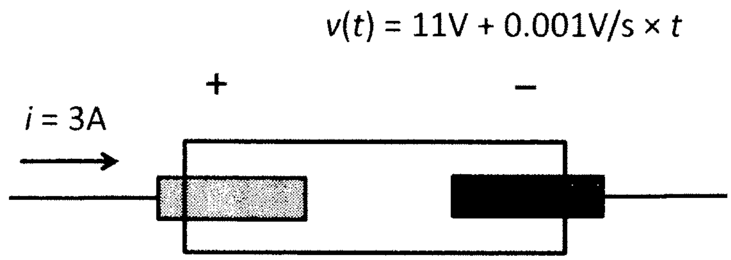
c) $P_{abs} = V_3 \cdot I_3$ (passive sign convention) [+1]

$= (-20mV) \cdot (0.7mA)$

$= -14\mu W$ absorbed by element [+1]

or $14\mu W$ delivered by element

2. Consider the electrochemical cell (battery) being recharged below with a constant current.




a) What is the total charge delivered to the cell over the time $0 < t < 1$ hour, in SI units? [2 pts]

b) What is the power delivered to the cell at the moment $t = 1200\text{s}$? [2 pts]

c) What is the total energy delivered to the cell over the time $0 < t < 1$ hour, in SI units? [3 pts]

a) $$q = \int_0^{3600s} 3A\, dt = 3A \cdot 3600s \quad [+1]$$

$$= 10.8\, kC \quad [+1]$$

b) $$P_{\substack{\text{delivered} \\ \text{to cell}}} = i \cdot v(1200s) \quad [+1]$$

$$= 3A \cdot \left(11V + \frac{0.001V}{s} \times 1200s\right)$$

$$= 36.6\, W \quad \text{delivered to cell} \quad [+1]$$

c) $$\Delta U_{\text{delivered}} = \int_0^{3600s} p(t)\, dt \quad [+1]$$

$$= \int_0^{3600s} 3A \cdot \left(11V + \frac{0.001V}{s} \times t\right) dt \quad [+1]$$

$$= 3A \cdot 11V \cdot t \Big]_0^{3600s} + \frac{1}{2} 3A \cdot 0.001\frac{V}{s}\, t^2 \Big]_0^{3600s}$$

$$= 118.8\, kJ + 19.44\, kJ$$

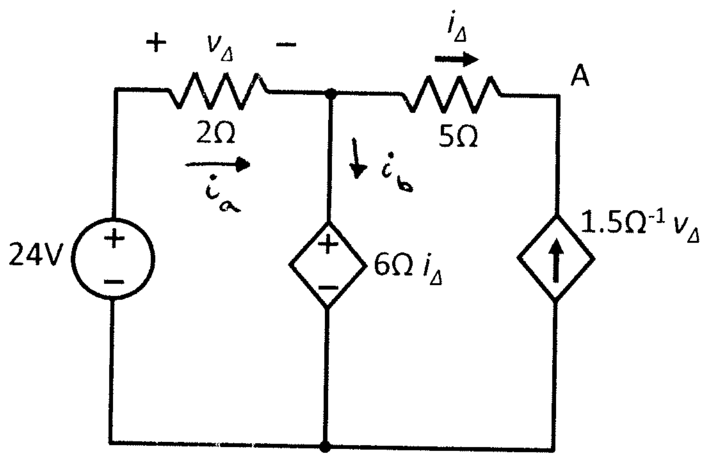
$$= 138.24\, kJ \quad \text{delivered to cell} \quad [+1]$$

ECSE-200 Quiz #2(17 Sept 2010)

NAME________________________________ McGill ID#________________

*READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).* **HINT**: *Use KCL, KVL and Ohm's Law. You may find it useful to add clearly defined circuit variables on your circuit diagrams to assist you in your solution.*

1. Consider the circuit below.




a) Write a KVL equation for the left loop in terms of the variables $v_\Delta$ and $i_\Delta$. [1 pt]

b) Write a KCL equation for node A in terms of the variables $v_\Delta$ and $i_\Delta$. [1 pt]

c) What is the power absorbed by the 2Ω resistor? [3 pts]

d) What is the power absorbed by the 5Ω resistor? [3 pts]

e) What is the power that is delivered or absorbed *by* the dependent voltage source? [4 pts]

a) $0 = -24V + v_\Delta + 6\Omega\, i_\Delta$ [+1]

b) $0 = -i_\Delta - 1.5\Omega^{-1} v_\Delta$ [+1]

c) Substitution: $i_\Delta = -1.5\Omega^{-1} v_\Delta$

$0 = -24V + v_\Delta + 6\Omega \cdot (-1.5\Omega^{-1})\, v_\Delta$

$v_\Delta = -3V$ [+1]

$$P_{abs} = \frac{v_\Delta^2}{2\Omega} \quad [+1]$$

$$= 4.5\ W \quad [+1]$$

d) $i_\Delta = -1.5\Omega^{-1}\, v_\Delta = 4.5\ A \quad [+1]$

$$P_{abs} = i_\Delta^2 \cdot 5\Omega \quad [+1]$$

$$= 101.25\ W \quad [+1]$$

e) Ohm's Law: $v_\Delta = i_a \cdot 2\Omega \rightarrow i_a = v_\Delta / 2\Omega \quad [+1]$

$$= -1.5\ A$$

KCL: $0 = -i_a + i_b + i_\Delta \quad [+1]$

$$i_b = i_a - i_\Delta$$

$$= -1.5A - 4.5A$$

$$= -6A$$

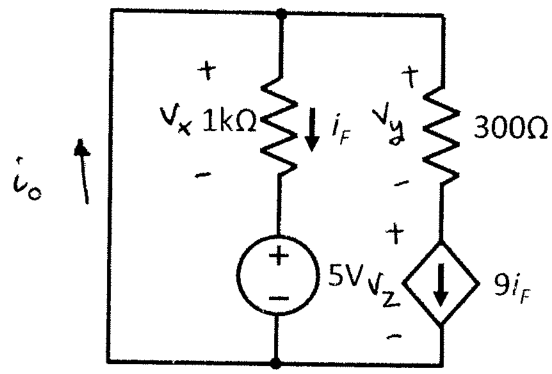
Power absorbed $= P_{abs} = i_b \cdot 6\Omega\, i_\Delta \quad [+1]$

$$= (-6A) \cdot (6\Omega \cdot 4.5A)$$

$$= -162\ W$$

<u>or</u> 162W is delivered by the source [+1]

2. Consider the circuit below.




a) What is the current $i_F$? Indicate the sign of your answer clearly. [3 pts]

b) What is the current flowing through the short circuit in the left-most branch? Indicate clearly both the value and direction of this current. [2 pts]

c) What is the power that is delivered or absorbed *by* the dependent source? [4 pts]

a) KVL on left loop: $0 = V_x + 5V$ [+1]

$V_x = -5V$

Ohm's Law: $i_F = \frac{V_x}{1k\Omega}$ [+1]

$= -5mA$ [+1]

b) KCL: $0 = -i_o + i_F + 9i_F$ [+1]

$i_o = 10\, i_F$

$= -50\, mA$ [+1]

c) Ohm's Law and KCL: $v_y = 9 i_F \cdot 300\,\Omega$ [+1]

$= -13.5\text{V}$

KVL: $0 = v_y + v_z$ [+1]

(exterior loop) $v_z = -v_y$

$= 13.5\text{V}$

Power absorbed by source $= P_{abs} = 9 i_F \cdot v_z$ [+1]

$= (9 \cdot -5\text{mA}) \cdot 13.5\text{V}$

$= -607.5\text{mW}$

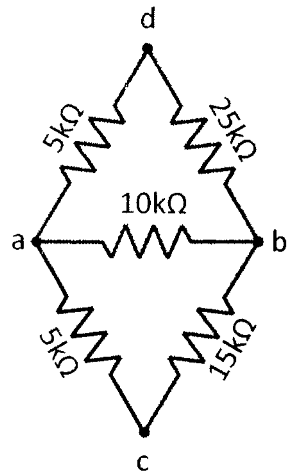
or 607.5mW is delivered by the dependent source. [+1]

ECSE-200 Quiz #3(24 Sept 2010)

NAME______________________________ McGill ID#____________________

*READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).*

1. Consider the circuit below.

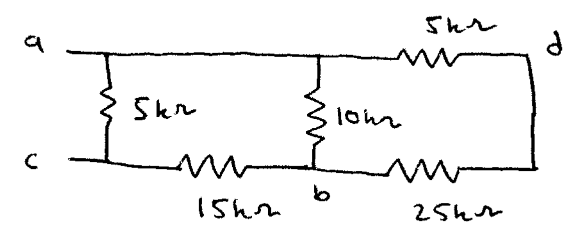



a) What is the equivalent resistance between nodes a and c? [3 pts]

b) What is the equivalent resistance between nodes a and b? [3 pts]

c) What is the equivalent resistance between nodes a and d? [3 pts]

a)




[+1]

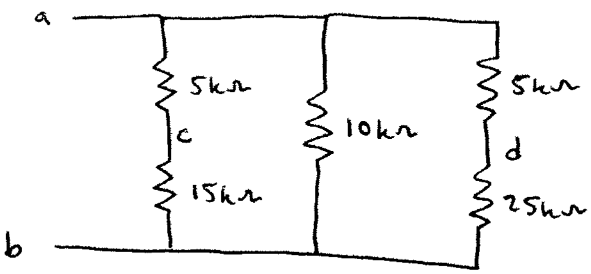
$$R_{ac} = 5k\Omega \,//\, (15k\Omega + 10k\Omega \,//\, (5k\Omega + 25k\Omega)) \quad [+1]$$

$$= 5k\Omega \,//\, \left(15k\Omega + \frac{10k\Omega \cdot 30k\Omega}{40k\Omega}\right)$$

$$= \frac{5k\Omega \cdot 22.5k\Omega}{27.5k\Omega}$$

$$= 4.091\ k\Omega \quad [+1]$$

b)




[+1]

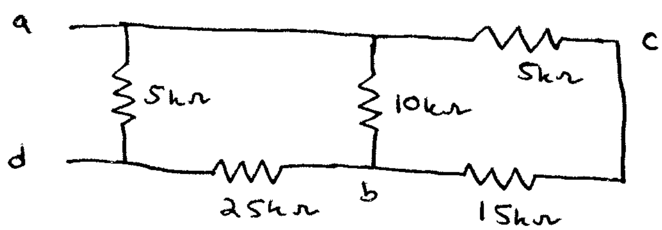
$$R_{ab} = (5+15)k\Omega \,//\, 10k\Omega \,//\, (5+25)k\Omega \qquad [+1]$$

$$= 20k\Omega // 10k\Omega // 30k\Omega$$

$$= \frac{1}{\left(\frac{1}{20k\Omega} + \frac{1}{10k\Omega} + \frac{1}{30k\Omega}\right)}$$

$$= 5.455\,k\Omega \qquad [+1]$$

c)



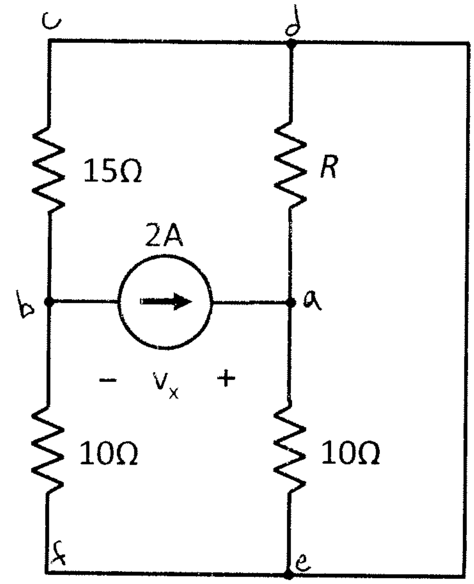

$$R_{ad} = 5k\Omega \,//\, (25k\Omega + 10k\Omega \,//\, (5k\Omega + 15k\Omega))$$

$$= 5k\Omega \,//\, \left(25k\Omega + \frac{10k\Omega \cdot 20k\Omega}{30k\Omega}\right)$$

$$= \frac{5k\Omega \cdot 31.667k\Omega}{36.667k\Omega}$$

$$= 4.318\,k\Omega$$

2. Consider the following circuit.

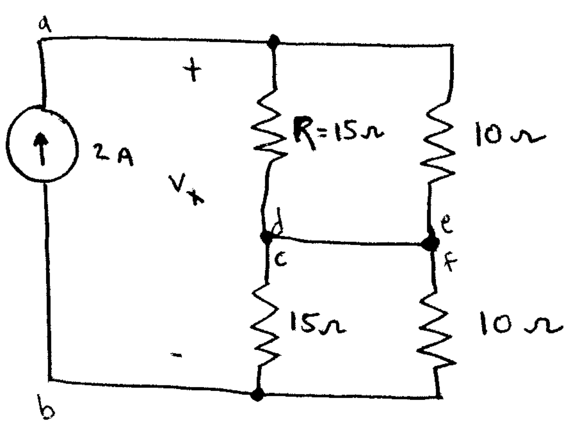



a) If $R = 15\Omega$, what is the voltage $v_x$? [4 pts]

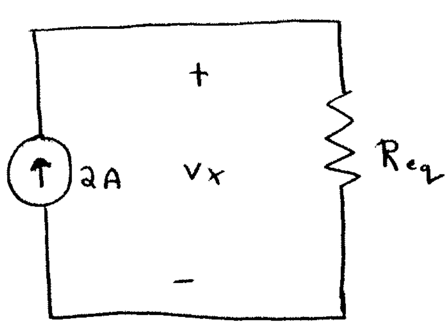
**HINT:** You may find it useful to redraw the circuit.

b) It is desired that the current source deliver 44W to the circuit. What should the value of $R$ be in order for this condition to be satisfied? [5 pts]

a)




[+1]




$$R_{eq} = 15\Omega // 10\Omega + 15\Omega // 10\Omega \quad [+1]$$

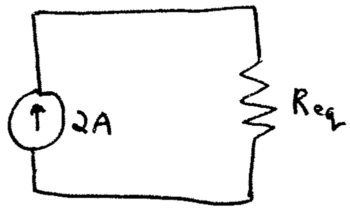
$$= \frac{15\Omega \cdot 10\Omega}{25\Omega} + \frac{15\Omega \cdot 10\Omega}{25\Omega}$$

$$= 12\Omega$$

$$V_x = 2A \cdot R_{eq} \quad [+1]$$

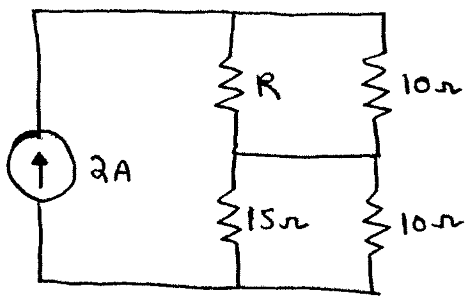
$$= 24\ V \quad [+1]$$

b)




$$P_{\substack{del \\ source}} = P_{\substack{abs \\ res}} = (2A)^2 \cdot R_{eq} \quad [+1]$$

$$R_{eq} = \frac{44W}{(2A)^2} = 11\Omega \quad [+1]$$




$$R_{eq} = R // 10\Omega + 15\Omega // 10\Omega \quad [+1]$$

$$R // 10\Omega = R_{eq} - 15\Omega // 10\Omega$$

$$= 11\Omega - 6\Omega$$

$$= 5\Omega$$

$$\frac{1}{R} + \frac{1}{10\Omega} = \frac{1}{5\Omega} \quad [+1]$$

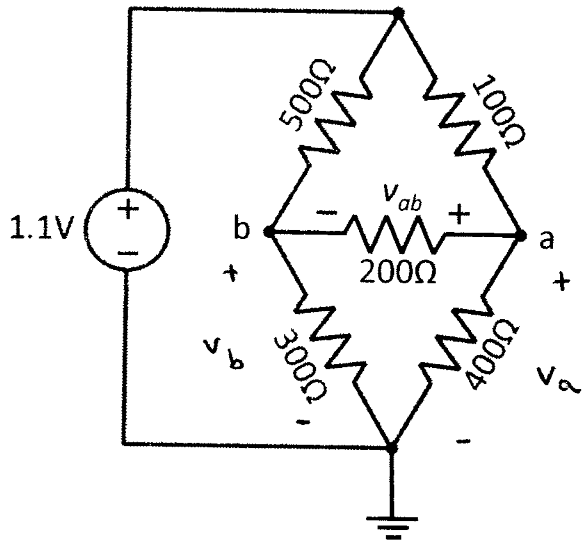
$$\frac{1}{R} = \frac{1}{5\Omega} - \frac{1}{10\Omega}$$

$$R = 10\Omega \quad [+1]$$

NAME_______________________________________ McGill ID#____________________

*READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).*

1. Consider the circuit below.




a) What are the node-voltages at the nodes a and b with respect to the identified reference node. [8 pts]
**HINT:** You may find it useful to apply the node-voltage technique.

b) What is the voltage $v_{ab}$? [2 pts]

a)

$$0 = \frac{V_a}{400\Omega} + \frac{V_a - V_b}{200\Omega} + \frac{V_a - 1.1V}{100\Omega}$$

[+1] [+1] [+1]

$$0 = \frac{V_b}{300\Omega} + \frac{V_b - V_a}{200\Omega} + \frac{V_b - 1.1V}{500\Omega}$$

[+1] [+1] [+1]

$$\frac{1.1}{100} = V_a\left(\frac{1}{100} + \frac{1}{200} + \frac{1}{400}\right) - V_b\left(\frac{1}{200}\right)$$

$$\frac{1.1}{500} = -V_a\left(\frac{1}{200}\right) + V_b\left(\frac{1}{300} + \frac{1}{200} + \frac{1}{500}\right)$$

work space

$$0.011 = 0.0175\,V_a - 0.005\,V_b$$

$$0.0022 = -0.005\,V_a + 0.010\bar{3}\,V_b$$

$$V_a = \frac{\begin{vmatrix} 0.011 & -0.005 \\ 0.0022 & 0.0103 \end{vmatrix}}{\begin{vmatrix} 0.0175 & -0.005 \\ -0.005 & 0.010\bar{3} \end{vmatrix}} = 0.800V \quad [+1]$$

$$V_b = \frac{\begin{vmatrix} 0.0175 & 0.011 \\ -0.005 & 0.0022 \end{vmatrix}}{\begin{vmatrix} 0.0175 & -0.005 \\ -0.005 & 0.010\bar{3} \end{vmatrix}} = 0.600V \quad [+1]$$

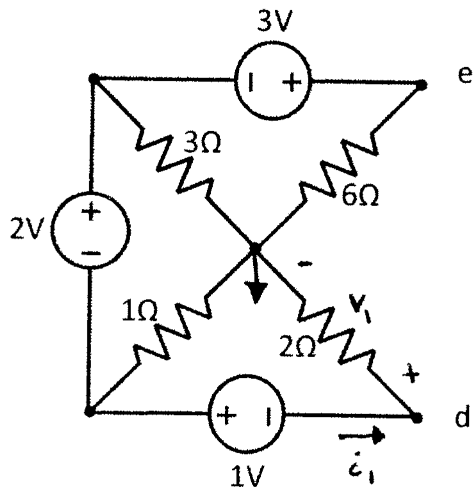
b) KVL: $0 = -V_b - V_{ab} + V_a \quad [+1]$

$$V_{ab} = V_a - V_b$$

$$= 0.800V - 0.600V$$

$$= 0.200V \quad [+1]$$

2. Consider the following circuit.




a) What is the voltage across the 2Ω resistor? Indicate clearly the definition and value of your voltage variable. [5 pts]
**HINT:** You may find it useful to apply the node-voltage technique.

b) How much power does the 1V source deliver or absorb? [3 pts]

c) If a 12Ω resistor is connected between nodes e and d, how would your answer to part a) change? Justify your answer with an equation or circuit diagram. [1 pt]

a) One supernode equation:

$$0 = \underset{[+1]}{\frac{v_1}{2\Omega}} + \underset{[+1]}{\frac{(v_1 + 1V)}{1\Omega}} + \underset{[+1]}{\frac{(v_1 + 1V + 2V)}{3\Omega}} + \underset{[+1]}{\frac{(v_1 + 1V + 2V + 3V)}{6\Omega}}$$

$$0 = v_1\left(\frac{1}{2} + 1 + \frac{1}{3} + \frac{1}{6}\right) + \frac{1}{1} + \frac{1+2}{3} + \frac{1+2+3}{6}$$

$$0 = 2v_1 + 3$$

$$v_1 = -1.5V \quad [+1]$$

b) $i_1 = \frac{v_1}{2\Omega}$ [+1]

Power absorbed by source:

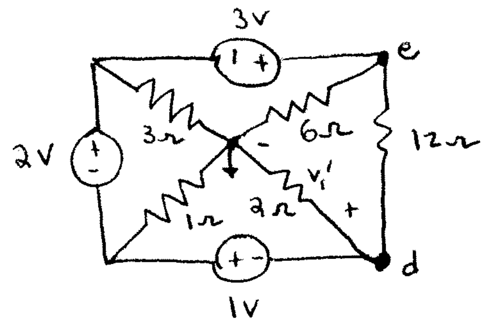
$$P_{abs} = 1V \cdot i_1 \quad [+1]$$

$$= 1V \cdot \frac{(-1.5V)}{2\Omega} = -0.75W \quad [+1]$$

or 0.75W is delivered by source.

work space

c)




$$0 = \frac{v_1'}{2\Omega} + \frac{(v_1' + 1V)}{1\Omega} + \frac{(v_1' + 1V + 2V)}{3\Omega} + \frac{(v_1' + 1V + 2V + 3V)}{6\Omega}$$

$$+ \underbrace{\frac{(v_1' - (v_1' + 6V))}{12\Omega}}_{i_{d \to e}} + \underbrace{\frac{((v_1' + 6V) - v_1')}{12\Omega}}_{i_{e \to d}}$$

The last two terms cancel, ie. $i_{d \to e} = - i_{e \to d}$,

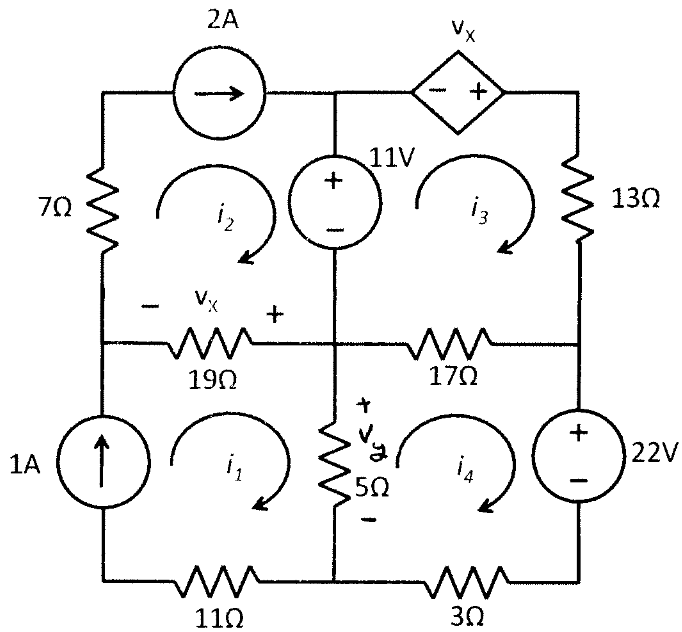
giving the exact same equation for $v_1'$ as for $v_1$.

Thus, the voltage across 2Ω resistor is unchanged. [+1]

NAME________________________________________ McGill ID#____________________

*READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).*

1. Consider the circuit below.




a) What is the mesh current $i_1$ ? [1pt]
b) What is the mesh current $i_2$ ? [1pt]
c) What are the mesh currents $i_3$ and $i_4$ ? [5pts]
d) Would the mesh current $i_4$ become more positive, more negative, or stay the same if the 5Ω resistor is replaced with a 5V source having positive terminal pointing towards the top of the page? Justify your answer. [2pts]

a) $i_1 = 1A$ [+1]

b) $i_2 = 2A$ [+1]

c) $0 = -11V - v_x + 13\Omega\, i_3 + 17\Omega(i_3 - i_4)$ [+1]

$0 = 5\Omega(i_4 - 1A) + 17\Omega(i_4 - i_3) + 22V + 3 i_4$ [+1]

$v_x = \underbrace{19\Omega(i_2 - i_1)} = 19\Omega(2A - 1A)$

$= 19V$ [+1]

Substitution of $v_x$:

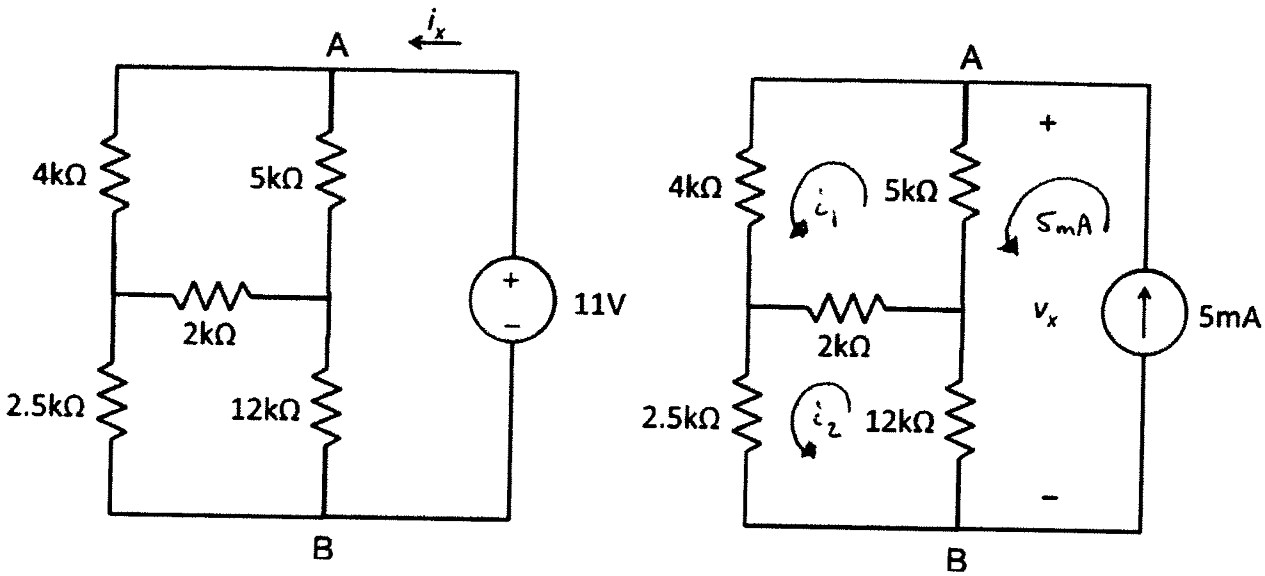
$$30 = 30 i_3 - 17 i_4$$
$$-17 = -17 i_3 + 25 i_4$$

$$i_3 = \frac{\begin{vmatrix} 30 & -17 \\ -17 & 2 \end{vmatrix}}{\begin{vmatrix} 30 & -17 \\ -17 & 25 \end{vmatrix}} = 1A \quad [+1] \qquad i_4 = \frac{\begin{vmatrix} 30 & 30 \\ -17 & -17 \end{vmatrix}}{\begin{vmatrix} 30 & -17 \\ -17 & 25 \end{vmatrix}} = 0A \quad [+1]$$

d) $v_y = 5\Omega\,(i_1 - i_4)$
$\quad = 5V \quad [+1]$

Replacing the $5\Omega$ resistor with a 5V source would leave $i_4$ unchanged, since the voltage drop appearing in the KVL equation for mesh 4 would be unchanged ($v_y = 5V$). [+1]

2. Consider the following two similar circuits.




a) If you are solving for the circuit variables in the **left** circuit, which method requires fewer equations, the node-voltage method or the mesh-current method? [1pt]
b) If you are solving for the circuit variables in the **right** circuit, which method requires fewer equations, the node-voltage method or the mesh-current method? [1pt]
c) In the **right** circuit, what is the voltage $v_x$? [4pts]
d) What is the equivalent resistance of the resistor network between nodes A and B? [2pts]
e) What is the current $i_x$? [2 pts]

a) 3 mesh variables, 2 node voltage variables → node voltage [+1]

b) 2 mesh variables, 3 node voltage variables → mesh current [+1]

c) $0 = 5k\Omega(i_1 - 5mA) + 4k\Omega\, i_1 + 2k\Omega\,(i_1 - i_2) = 0$ [+1]

$0 = 12k\Omega\,(i_2 - 5mA) + 2k\Omega\,(i_2 - i_1) + 2.5k\Omega\, i_2 = 0$ [+1]

$25V = 11k\Omega\, i_1 - 2k\Omega\, i_2$

$60V = -2k\Omega\, i_1 + 16.5k\Omega\, i_2$

$$i_1 = \frac{\begin{vmatrix} 25 & -2000 \\ 60 & 16500 \end{vmatrix}}{\begin{vmatrix} 11000 & -2000 \\ -2000 & 16500 \end{vmatrix}} = 3mA$$

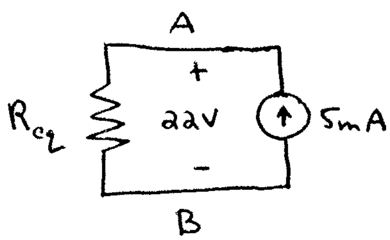
work space

$$i_2 = \frac{\begin{vmatrix} 11000 & 25 \\ -2000 & 60 \end{vmatrix}}{\begin{vmatrix} 11000 & -2000 \\ -2000 & 16500 \end{vmatrix}} = 4\,mA$$

$$v_x = 5k\Omega\,(5mA - i_1) + 12k\Omega\,(5mA - i_2) \quad [+1]$$

$$= 22V \quad [+1]$$

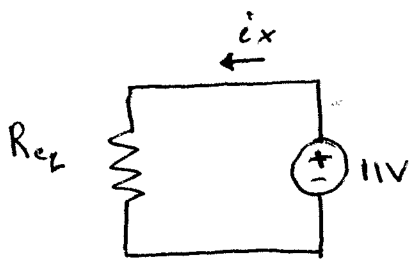
d)




$$R_{eq} = \frac{v_x}{5mA} \quad [+1]$$

$$= \frac{22V}{5mA} = 4.4\,k\Omega \quad [+1]$$

e)




$$i_x = \frac{11V}{R_{eq}} \quad [+1]$$

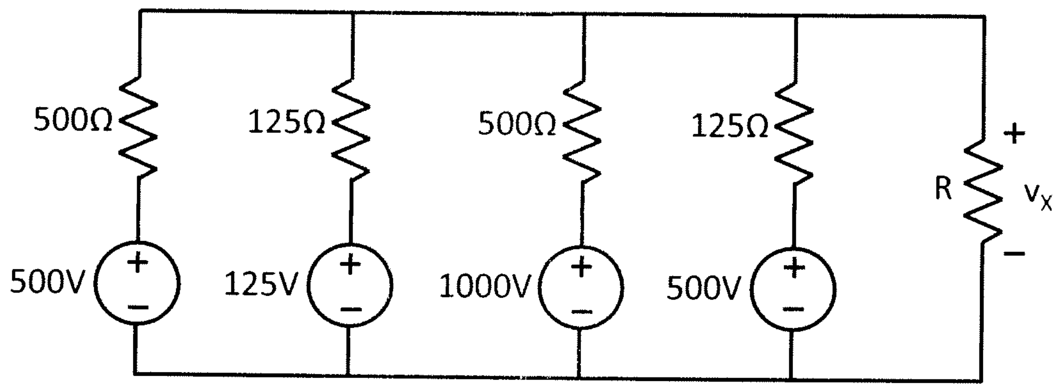
$$= \frac{11V}{4.4k\Omega} = 2.5\,mA \quad [+1]$$

ECSE-200 Quiz #6(15 Oct 2010)

NAME________________________________ McGill ID#____________________

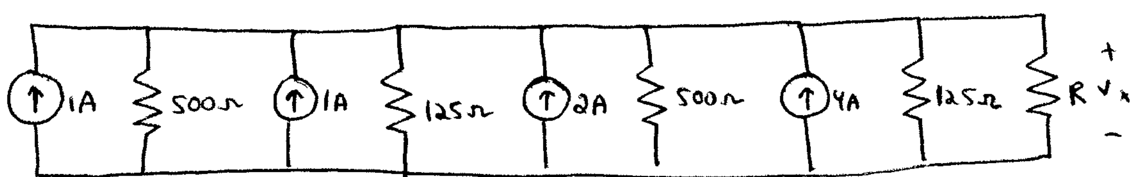
*READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).*

1. Consider the circuit below.




a) For an arbitrary resistance $R$, what is $v_X$? Your answer should include the resistance $R$. [4pts]
b) What value of $R$ will give $v_X = 50V$? [2pts]
c) For $1/R = 0\Omega^{-1}$, what is $v_X$? [2pts]

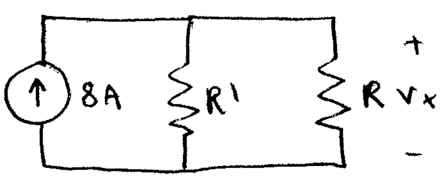
a)




[+1]

[+1]

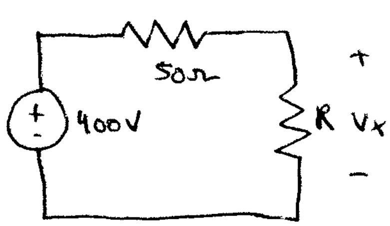
(8A source)




$R' = 500\Omega \,//\, 125\Omega \,//\, 500\Omega \,//\, 125\Omega$ [+1]

$$\frac{1}{R'} = \frac{1}{500} + \frac{1}{125} + \frac{1}{500} + \frac{1}{125}$$

$R' = 50\Omega$




$$v_x = 400V \cdot \frac{R}{R + 50\Omega}$$ [+1]

b) $$50V = 400V \frac{R}{R+50\Omega} \quad [+1]$$

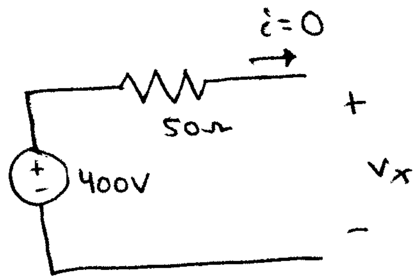
$$\frac{1}{8} = \frac{R}{R+50}$$

$$R + 50 = 8R$$

$$R = \frac{50}{7}\Omega \quad (+1)$$

$$= 7.143\Omega$$

c) $\frac{1}{R} = 0\Omega^{-1}$ is equivalent to an open circuit [+1]

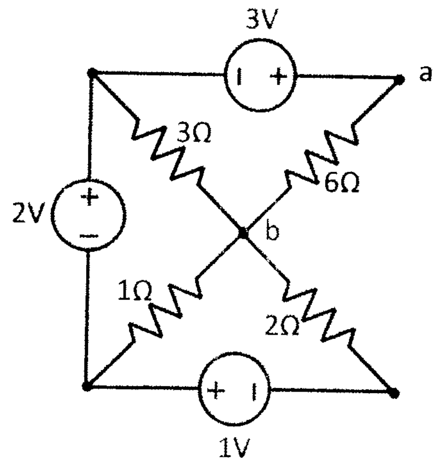



KVL:

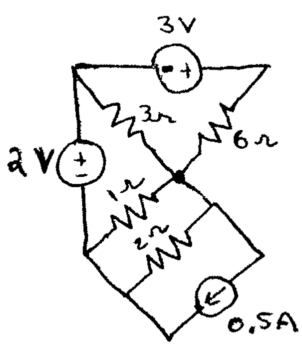
$$-400V + 0 \cdot 50\Omega + v_x = 0$$

$$v_x = 400V \quad [+1].$$

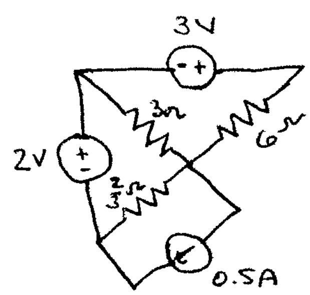
2. Consider the following circuit.




a) Use source transformations and equivalent resistance to find the Thévenin equivalent circuit between the nodes a and b. [8pts]
b) Consider a short circuit that is applied to nodes a and b; what current will flow through this short circuit? Be sure to indicate the direction of current flow. [2pts]

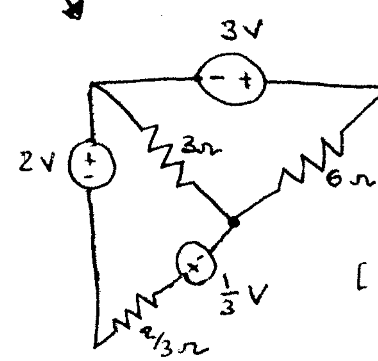



[+1/2]

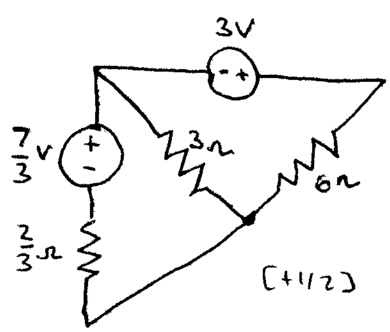



$R = \frac{1\Omega \cdot 2\Omega}{1\Omega + 2\Omega} = \frac{2}{3}\Omega$

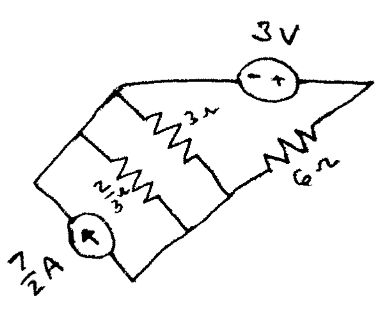
[+1/2]



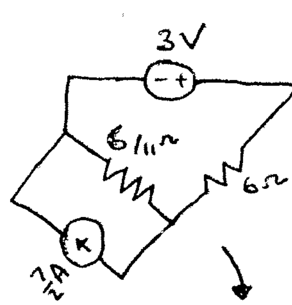

[+1/2]




[+1/2]



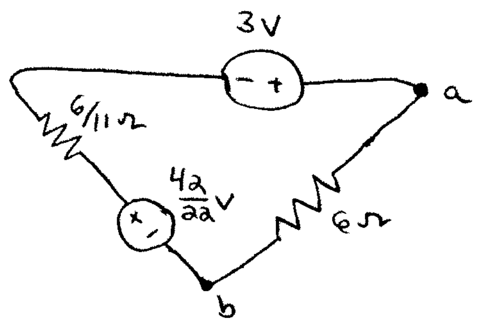

[+1/2]



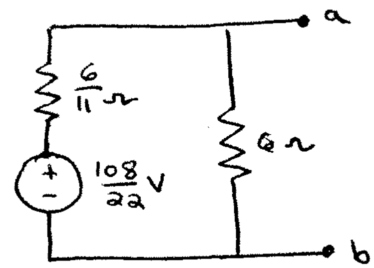

$R = \frac{\frac{2}{3}\Omega \cdot 3\Omega}{\frac{2}{3}\Omega + 3\Omega} = \frac{6}{11}\Omega$

[+1/2]

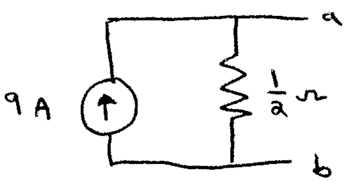
work space



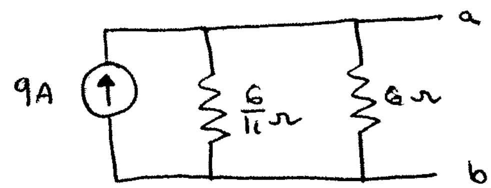

[+1/2]




[+1/2]




[+1/2]



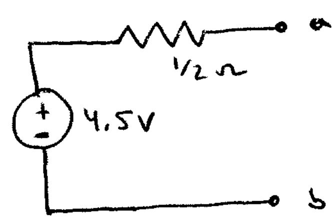

[+1/2]

$$R = \frac{6\,\Omega \cdot \frac{6}{11}\,\Omega}{6\,\Omega + \frac{6}{11}\,\Omega}$$

$$= \frac{36/11}{72/11}\,\Omega$$

$$= \frac{1}{2}\,\Omega$$

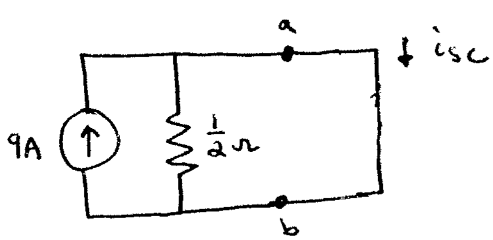
[+1]




[+1]

[+1 for voltage source and resistor in series]

b)




$$0 = -9A + \frac{0}{1/2\,\Omega} + i_{sc} \quad [+1]$$

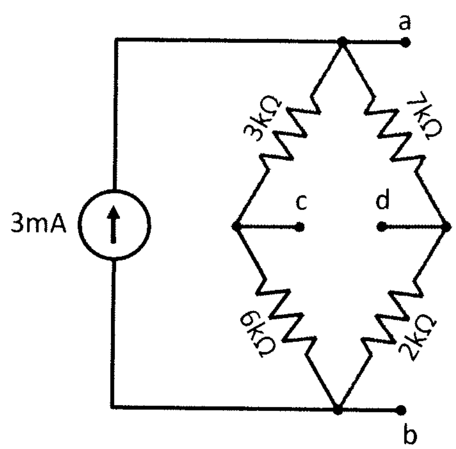
$$i_{sc} = 9A \quad [+1]$$

ECSE-200 Quiz #7(22 Oct 2010)

NAME________________________________ McGill ID#____________________

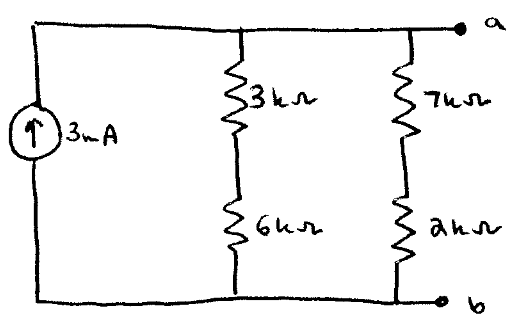
*READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).*

1. Consider the circuit below.

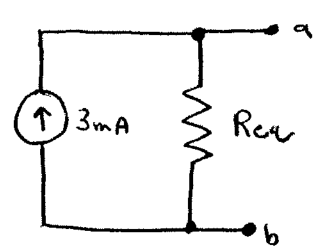



a) What is the Thévenin equivalent circuit with respect to the terminals a and b? Indicate clearly terminals a and b in your answer. [2pts]
b) What is the Thévenin equivalent circuit with respect to the terminals c and d? Indicate clearly terminals c and d in your answer. [5pts]
c) If a resistor R = 8.888kΩ is attached between nodes c and d, what voltage will develop across the resistor R? Indicate clearly the polarity of the voltage with respect to terminals c and d. [2 pts]

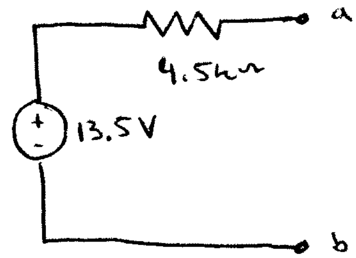
a)




→




$R_{eq} = (3k\Omega + 6k\Omega) \,//\, (7k\Omega + 2k\Omega)$ [+1]
$= 4.5k\Omega$

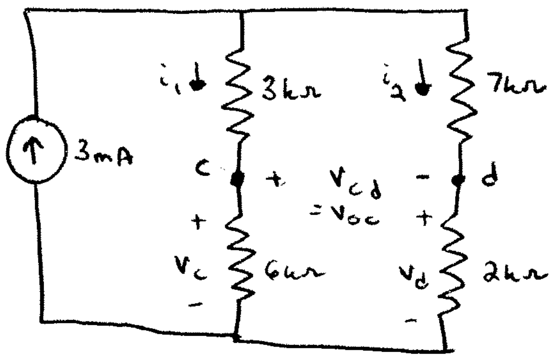



[+1]

Find Voc:

work space

b)




[+1/2]

$$i_1 = \frac{3mA \cdot (7+2)k\Omega}{((3+6)+(7+2))k\Omega} = 1.5mA$$

$$i_2 = \frac{3mA \cdot (3+6)k\Omega}{((3+6)+(7+2))k\Omega} = 1.5mA$$

[+1/2]

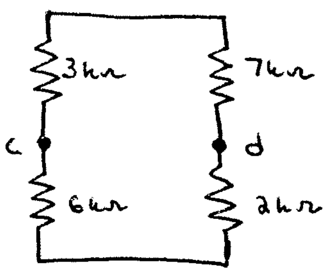
$V_c = i_1 \cdot 6k\Omega = 9V$ [+1/2]

$V_d = i_2 \cdot 2k\Omega = 3V$ [+1/2]

Find $R_T$:

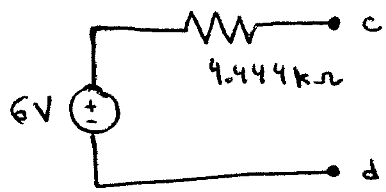
KVL: $V_{cd} = V_c - V_d = 6V$

[+1]



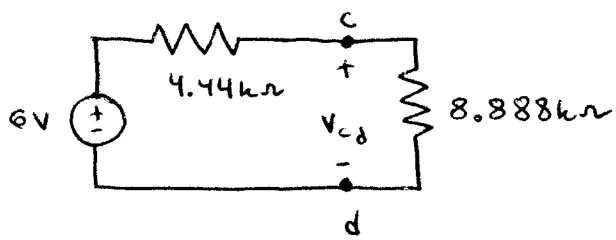

$R_T = (3k\Omega + 7k\Omega) \parallel (6k\Omega + 2k\Omega)$ [+1]

$$= \frac{10k\Omega \cdot 8k\Omega}{10k\Omega + 8k\Omega} = 4.444k\Omega$$




[+1]

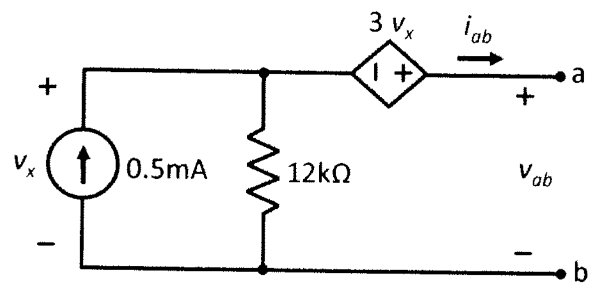
c)




$$V_{cd} = \frac{6V \cdot 8.888k\Omega}{4.444k\Omega + 8.888k\Omega}$$ [+1]

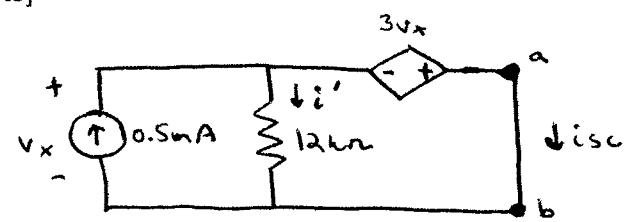
$= 4V$ [+1]

2. Consider the following circuit.




a) Find the Norton equivalent circuit with respect to the terminals a and b. [5pts]
b) Find the Norton equivalent circuit with respect to the terminals a and b, if the polarity of the dependent voltage source is reversed (equivalent to a change in the dependent source's controlling equation: $3v_x \rightarrow -3v_x$ ). [5pts]

a) short circuit:




KVL:

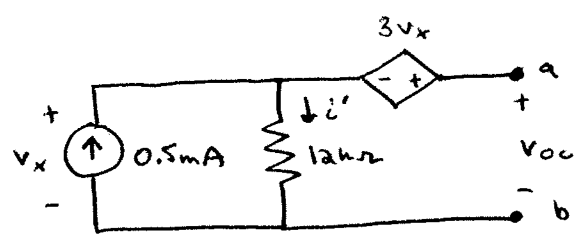
$0 = -v_x - 3v_x$ [+1]

$v_x = 0V$

Ohm: $i' = \frac{v_x}{12k\Omega} = 0mA$

KCL: $i_{sc} = 0.5mA$ [+1]

open circuit:



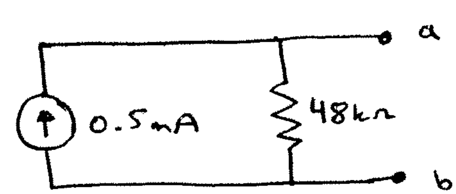

KCL: $i' = 0.5mA$ [+1]

Ohm: $v_x = 0.5mA \cdot 12k\Omega$
$= 6V$

KVL: $0 = -v_x - 3v_x + v_{oc}$

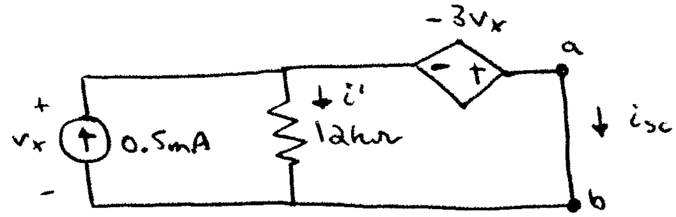
$v_{oc} = 4v_x = 24V$ [+1]

$R_T = \frac{24V}{0.5mA} = 48k\Omega$ [+1]



work space

b) short circuit:




KVL:

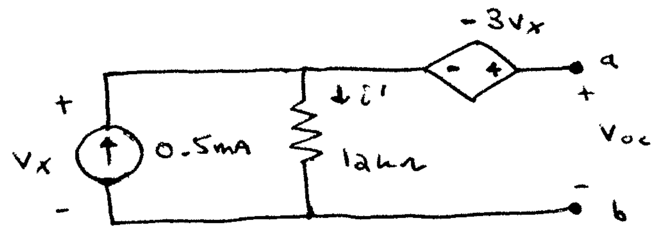
$0 = -v_x - (-3v_x)$ [+1]

$v_x = 0\,V$

Ohm: $i' = \dfrac{v_x}{12k\Omega} = 0\,mA$

KCL: $i_{sc} = 0.5\,mA$ [+1]

open circuit:




KCL: $i' = 0.5\,mA$ [+1]

Ohm: $v_x = 0.5mA \cdot 12k\Omega$

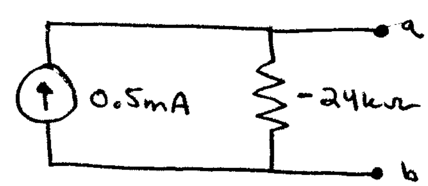
$= 6\,V$

KVL: $0 = -V_x - (-3v_x) + v_{oc}$

$V_{oc} = -2v_x$

$= -12\,V$ [+1]

$R_T = \dfrac{-12V}{0.5mA} = -24k\Omega$ [+1]

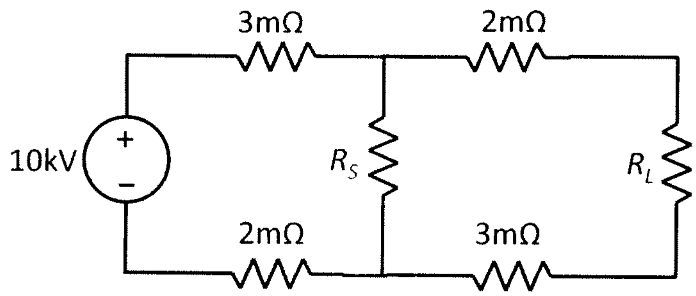


ECSE-200 Quiz #8(29 Oct 2010)

NAME________________________________ McGill ID#________________

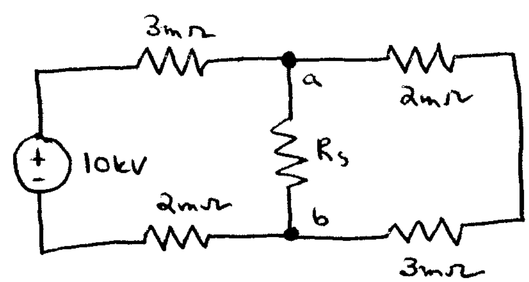
*READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).*

1. Consider the circuit below.

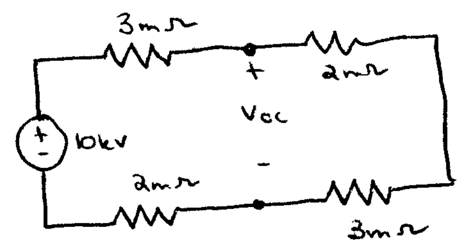



a) Under the condition $R_L = 0\Omega$, what value should $R_S$ have in order to maximize the power delivered to $R_S$ and what is the maximum power that can be delivered to $R_S$? [4pts]
b) Under the condition $1/R_S = 0\Omega^{-1}$, what value should $R_L$ have in order to maximize the power delivered to $R_L$, and what is the maximum power that can be delivered to $R_L$? [4pts]
c) Assume again that $1/R_S = 0\Omega^{-1}$. What value should $R_L$ have in order to maximize the power delivered to $R_L$, with the additional constraint that the current through $R_L$ must be at least 800 kA? [4pts]

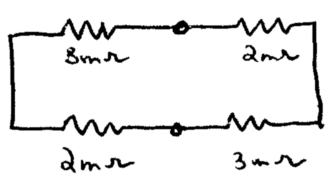
a)




open circuit voltage:




Thévenin resistance




$R_T = 5m\Omega \,||\, 5m\Omega$ [+1]

$= 2.5m\Omega$

$V_{oc} = 10kV \cdot \dfrac{5m\Omega}{5m\Omega + 5m\Omega}$ [+1]

$= 5kV$

Maximum power is delivered to $R_s$ when $R_s = R_T = 2.5m\Omega$ [+1]

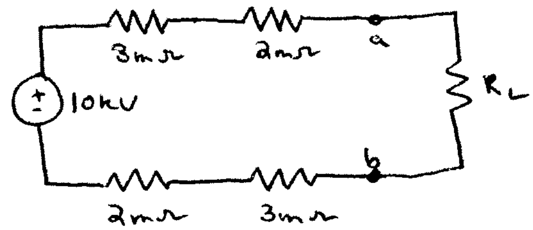
The maximum power delivered is:

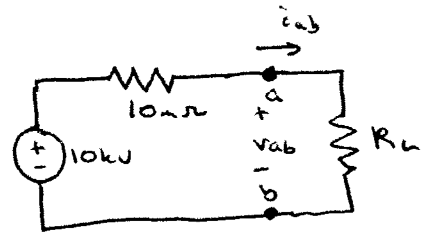
$P_{max} = \dfrac{V_{oc}}{2} \cdot \dfrac{i_{sc}}{2} = \dfrac{1}{4} V_{oc}^2 / R_T$

$= 2.5GW$ [+1]

work space

b)




$V_{oc} = 10\text{kV}$ [+1]

$R_T = 10\text{m}\Omega$ [+1]

Maximum power is delivered when:

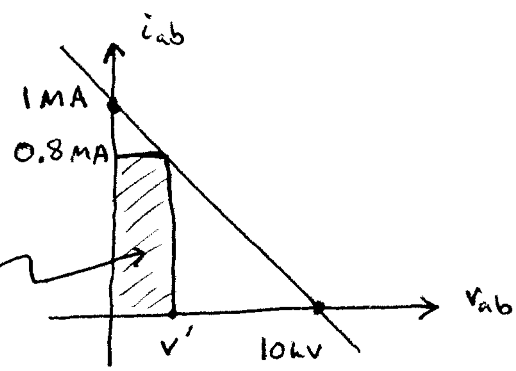
$R_L = R_T$

$= 10\text{m}\Omega$ [+1]

The maximum power delivered is:

$P = \frac{V_{oc}}{2} \cdot \frac{i_{sc}}{2} = \frac{1}{4} V_{oc}^2 / R_T = 2.5\text{ GW}$ [+1]

c)




max. area box with $i_{ab} \geq 0.8\text{MA}$

[+1] for diagram

$i_{sc} = \frac{V_{oc}}{R_T} = \frac{10\text{kV}}{10\text{m}\Omega} = 1\text{MA}$ [+1]

$\frac{v'}{10\text{kV}} = \frac{1\text{MA} - 0.8\text{MA}}{1\text{MA}}$

$v' = 2\text{kV}$ [+1]

$P_{delivered} = i' \times v'$

$= 0.8\text{MA} \times 2\text{kV}$

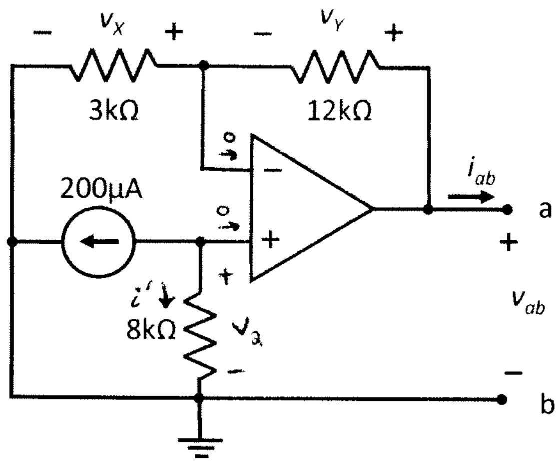
$= 1.6\text{ GW}$

$R_L = \frac{v'}{i'}$

$= \frac{2\text{kV}}{0.8\text{MA}}$

$= 2.5\text{m}\Omega$ [+1]

2. Consider the following circuit. Assume ideal op-amp behaviour.




a) What is the voltage $v_X$? [3pts]
b) What is the voltage $v_Y$? [2pts]
c) What is the voltage $v_{ab}$? [2pts]
d) Draw the $i_{ab}$-$v_{ab}$ diagram for the above circuit. [2pts]
f) With a 1kΩ resistor connected across the nodes a and b, what is the ***total*** power that is delivered by the op-amp to the rest of the circuit? [3 pts]

a) ideal op-amp → 0A current into input nodes.

KCL: $0 = 200\mu A + \frac{V_2}{8k\Omega}$ [+1]

$V_2 = -200\mu A \cdot 8k\Omega = -1.6V$

ideal op-amp → input node voltages are equal

$V_x = V_2$ [+1]

$V_x = -1.6V$ [+1]

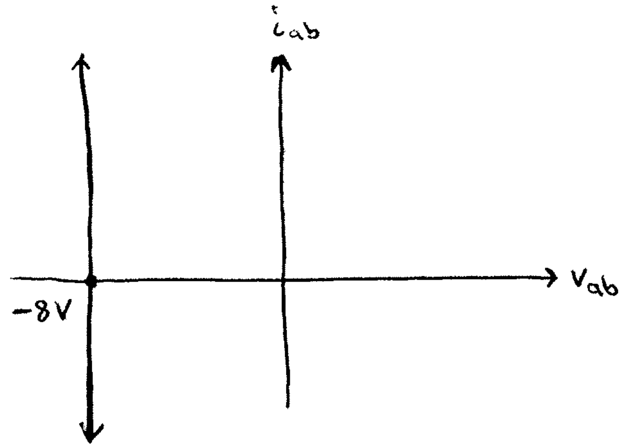
b) KCL: $0 = \frac{V_x}{3k\Omega} - \frac{V_y}{12k\Omega}$ [+1]

$V_y = V_x \cdot \frac{12k\Omega}{3k\Omega} = -1.6V \cdot 4 = -6.4V$ [+1]

c) KVL: $0 = -V_x - V_y + V_{ab}$ [+1]

$V_{ab} = V_x + V_y = -8.0V$ [+1]

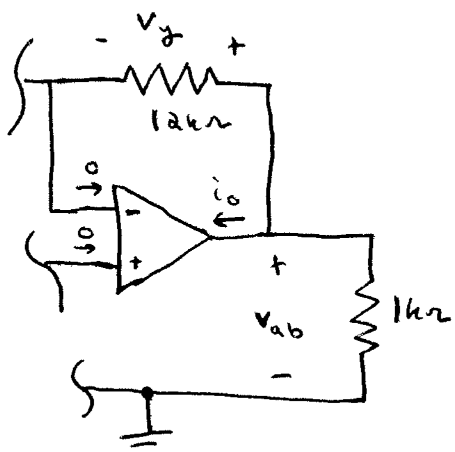
d)




[+1] for vertical line / voltage source behaviour

[+1] for −8V

e/f)




KCL: $0 = i_o + \frac{V_y}{12k\Omega} + \frac{V_{ab}}{1k\Omega}$ [+1]

$i_o = -\frac{V_y}{12k\Omega} - \frac{V_{ab}}{1k\Omega}$

$= \frac{6.4V}{12k\Omega} + \frac{8V}{1k\Omega}$

$= 8.533mA$

$P_{\substack{\text{delivered}\\ \text{by op-amp}}} = - P_{\substack{\text{absorbed}\\ \text{by op-amp}}}$

$= -(i_o \cdot V_{ab})$ [+1]

$= -(8.533mA \cdot -8.0V)$

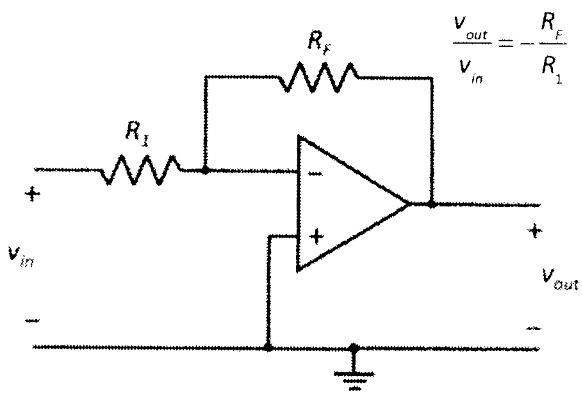
$= 68.267mW$ [+1]

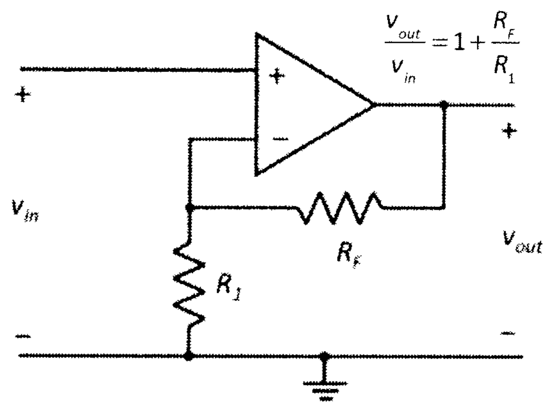
ECSE-200 Quiz #9(5 Nov 2010)

NAME______________________________ McGill ID#________________

*READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).*

1. The circuits below are common operational amplifier circuits. For each design question provide the circuit diagram with resistor values clearly identified. You may assume ideal op-amp behaviour.



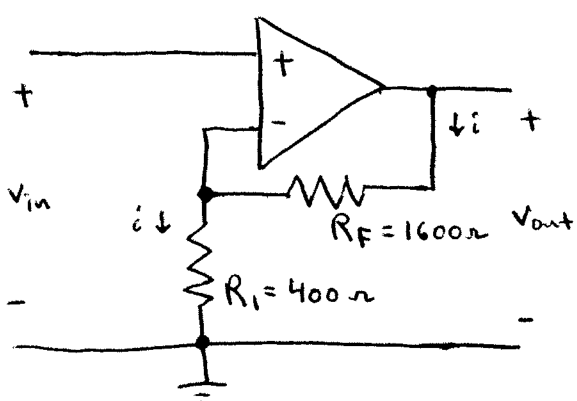




a) Design an op-amp circuit with a voltage gain of $v_{out}/v_{in}$ = +5 and a total power dissipation of 0.5mW when $v_{out}$ = +1V. [5pts]

b) Design an op-amp circuit with a voltage gain of $v_{out}/v_{in}$ = −5 and a total power dissipation of 0.5mW when $v_{out}$ = +1V. [5pts]

c) Your input signal $v_{in}$ is produced by a source that has a Thévenin equivalent circuit with $R_T$ = 500Ω. If you do not care about the sign of the voltage gain, which of your two op-amp circuits would you select, and why? [BONUS 2pts]

a)




[+2] for configuration

$$\frac{v_{out}}{v_{in}} = 1 + \frac{R_F}{R_1}$$

$$5 = 1 + \frac{R_F}{R_1} \quad [+1] \quad \therefore \frac{R_F}{R_1} = 4$$

$$P_{abs} = v_{out} \cdot i = \frac{v_{out}^2}{(R_1 + R_F)}$$

$$0.5\,mW = \frac{1V^2}{R_1 + R_F} \quad [+1]$$

$$\therefore R_1 + R_F = \frac{1V^2}{0.5\,mW} = 2\,k\Omega$$

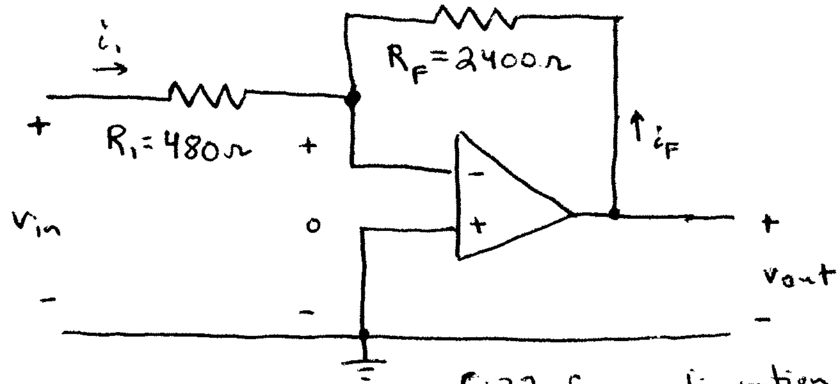
$R_1 + R_F = 2k\Omega$

$R_F / R_1 = 4$

$5R_1 = 2k\Omega \longrightarrow R_1 = 400\Omega$ [+1]

$R_F = 4R_1 \qquad R_F = 1600\Omega$

b)




[+2] for configuration

$$\frac{v_{out}}{v_{in}} = -\frac{R_F}{R_1}$$

$$-5 = -\frac{R_F}{R_1} \quad [+1]$$

$$P_{abs} = v_{out} \cdot i_F + v_{in} \cdot i_1$$

$$= \frac{v_{out}^2}{R_F} + \frac{v_{in}^2}{R_1}$$

$$0.5mW = \frac{(1V)^2}{R_F} + \frac{(1V/-5)^2}{R_1} \quad [+1]$$

$$\frac{0.5mW}{1V^2} = \frac{1}{R_F} + \frac{1}{25R_1}$$

$$\frac{1}{2k\Omega} = \frac{1}{5R_1} + \frac{1}{25R_1}$$

$$R_1 = 2k\Omega\left(\frac{1}{5} + \frac{1}{25}\right)$$

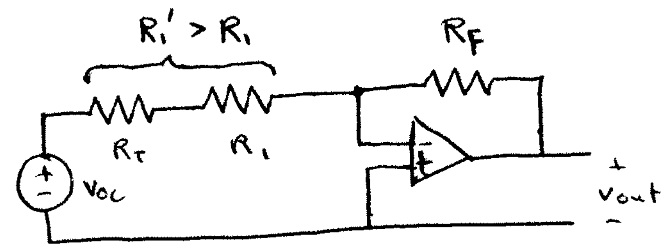
$$R_1 = 480\Omega$$

$$R_F = 5R_1 = 2400\Omega \quad [+1]$$

c) The non-inverting amplifier is preferable. [+1]

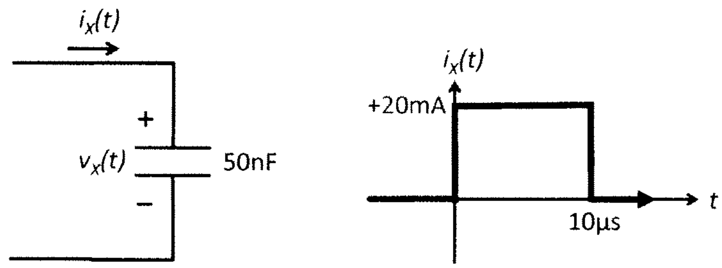
The inverting amplifier gain is effectively reduced by the presence of $R_T$. [+1]

$$\frac{v_{out}}{v_{in}} = -\frac{R_F}{R_1 + R_T}$$



work space

2. Consider the following capacitor and specified current pulse. The capacitor is uncharged for $t<0$.




a) What is the voltage $v_X$ for $t<0$? [1pt]
b) What is the voltage $v_X$ for $t>10\mu s$? [4pts]
c) Plot $v_X(t)$ versus $t$. [3pts]
d) How much energy is stored in the capacitor at $t=10\mu s$? [2pts]
e) How much power is delivered to the capacitor at $t=5\mu s$? [BONUS 2pts]

a) $V_x = 0V$ [+1]
for $t<0$

b) $q_{0\to 10\mu s} = \int_0^{10\mu s} i_x(t)\,dt$ [+1]

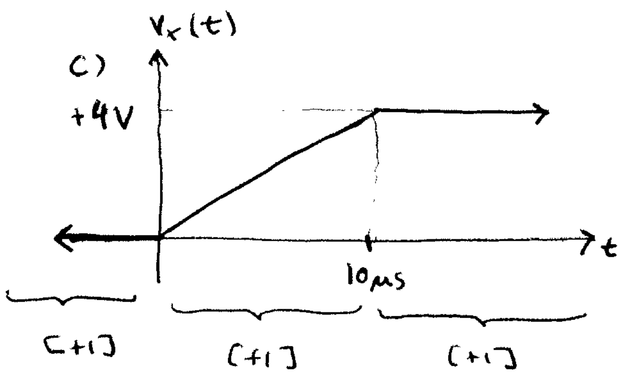
$= 20mA \cdot 10\mu s$

$= 200nC$ [+1]

$q_{0\to 10\mu s} = C \cdot V_x$ [+1]

$V_x = \frac{200nC}{50nF} = 4V$ [+1]

c)




d) $U = \frac{1}{2} C V_x^2$ [+1]

$= \frac{1}{2} \cdot 50nF \cdot (4V)^2 = 400nJ$ [+1]

e) $P_{\text{del. to cap.}} = V_x \cdot i_x \Big|_{t=5\mu s}$ [+1]

$= 2V \cdot 20mA$

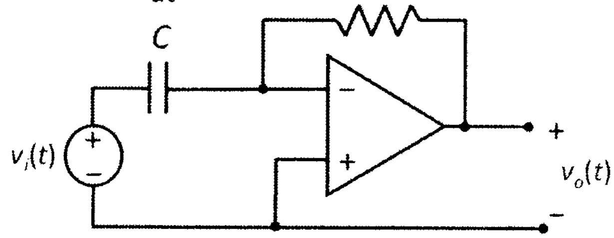
$= 40mW$ [+1]

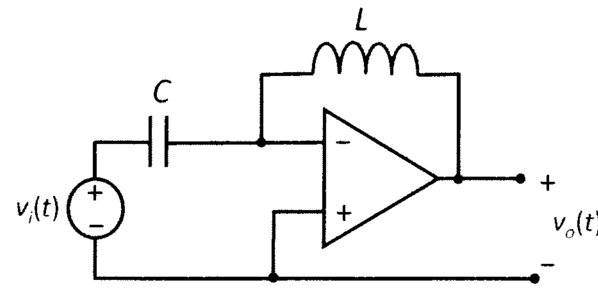
NAME______________________________________ McGill ID#____________________

*READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).*

1. Consider the circuits below. You may assume ideal op-amp behaviour.

$$v_o(t) = -RC\frac{d}{dt}v_i(t)$$




a) An ac input voltage $v_i(t) = A_i \cos(2\pi ft)$ is applied to the input of the left circuit, where the amplitude $A_i$ = 1V. At what frequency $f$ is the amplitude of the resulting ac output voltage $v_O(t)$ also equal to 1V? [3pts]

b) Design the op-amp circuit on the left, giving the value of $R$ and $C$, according to the following specifications. When the input voltage increases at 400mV/ms, the current drawn from the input voltage source should be 100μA and the output voltage should be $v_O$=−1V. [4pts]

c) For the circuit on the right, express the output voltage $v_O(t)$ in terms of the input voltage $v_i(t)$. [3pts]

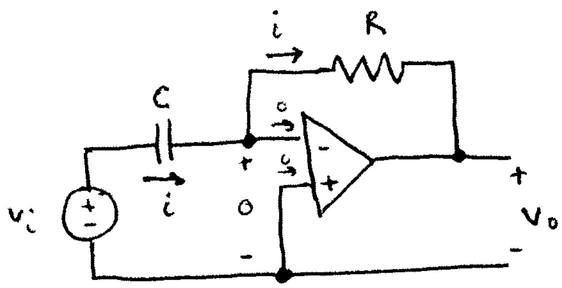
d) Design the op-amp circuit on the right, giving the value of $L$ and $C$, according to the following specifications. The current drawn from the input voltage source should be 100μA when the input voltage increases at 1V/ms. With an ac input voltage $v_i(t) = A_i \cos(2\pi ft)$, the amplitude of the ac output voltage should be equal to the input amplitude $A_i$ at a frequency $f$ = 10kHz = $10^4 s^{-1}$. [BONUS 4pts]

a) $V_o = -RC\frac{d}{dt}(A_i \cos(2\pi ft))$ [+1]

$= \underbrace{-RC A_i 2\pi f}_{A_o = RCA_i \cdot 2\pi f} \cdot \sin(2\pi ft)$ [+1]

$f = \frac{1}{2\pi RC}$ [+1]

b)




work space

$$i = C\,\frac{dv_i}{dt}$$

$$C = \frac{i}{dv_i/dt} = \frac{100\,\mu A}{400\,mV/ms} = 250\,nF$$

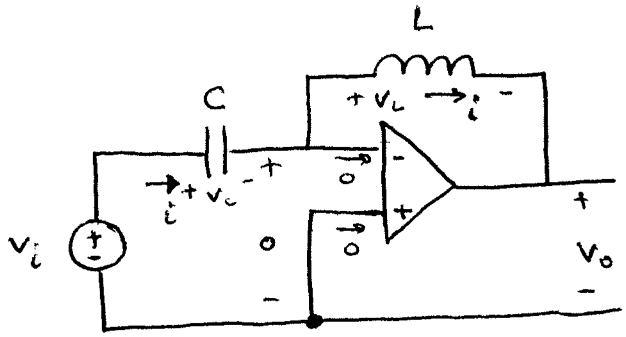
[+1] [+1]

$$v_o = -RC\,\frac{dv_i}{dt} = -R\,i$$

$$R = \frac{-v_o}{i} = \frac{-(-1V)}{100\,\mu A} = 10\,k\Omega$$

[+1] [+1]

c)




KVL: $v_c = v_i$ $\quad v_L = -v_o$

$$i = C\,\frac{dv_i}{dt} \quad [+1]$$

$$v_L = L\,\frac{di}{dt} \quad [+1]$$

$$\therefore\ v_o = -v_L = -L\,\frac{di}{dt}$$

$$= -L\,\frac{d}{dt}\left(C\,\frac{dv_i}{dt}\right)$$

$$= -LC\,\frac{d^2 v_i}{dt^2} \quad [+1]$$

d) $i = C \frac{dv_i}{dt}$

$$C = \frac{i}{dv_i/dt} = \frac{100\mu A}{1V/ms} = 100\ nF$$

[+1] [+1]

$$v_i = A_i \cos(2\pi f t)$$

$$V_o = -LC \frac{d^2}{dt^2}\left(A_i \cos(2\pi f t)\right)$$

$$= -LC \cdot A_i \cdot (2\pi f)^2 \left(-\cos(2\pi f t)\right)$$

$$= \underbrace{LC A_i (2\pi f)^2}_{A_o} \cos(2\pi f t) \quad [+1]$$

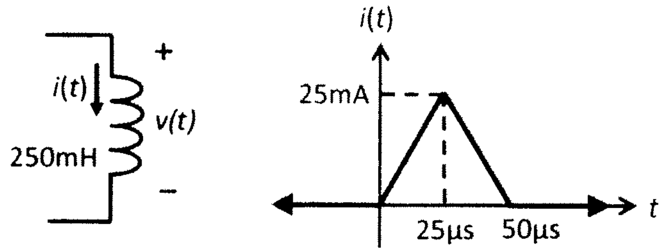
$$\frac{A_o}{A_i} = LC(2\pi f)^2 = 1$$

$$L = \frac{1}{C \cdot (2\pi f)^2}$$

$$= \frac{1}{100nF \cdot (2\pi \cdot 10^4 s^{-1})^2}$$

$$= 2.533\ mH \quad [+1]$$

2. Consider the following inductor and specified current pulse.




a) What is the voltage $v$ at $t$=10µs? [2pts]
b) What is the voltage $v$ at $t$=40µs? [2pts]
c) Plot $v(t)$ versus $t$. [4pts]
d) What is the energy stored by the inductor at t=40µs? [2pts]

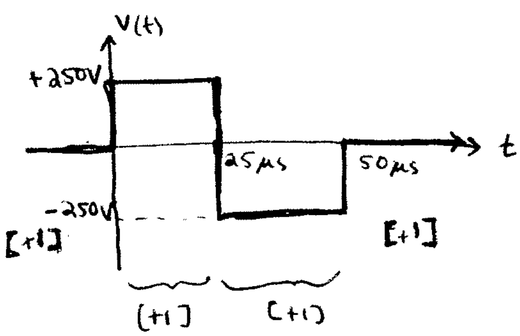
a) $v = L\,\dfrac{di}{dt} = 250\,\text{mH} \cdot \dfrac{25\,\text{mA}}{25\,\mu\text{s}} = +250\,\text{V}$

[+1] [+1]

b) $v = L\,\dfrac{di}{dt} = 250\,\text{mH} \cdot \dfrac{-25\,\text{mA}}{25\,\mu\text{s}} = -250\,\text{V}$

[+1] [+1]

c)




d) $i = \dfrac{10\,\mu\text{s}}{25\,\mu\text{s}} \times 25\,\text{mA} = 10\,\text{mA}$

$U = \frac{1}{2} L i^2 = \frac{1}{2} \cdot 250\,\text{mH} \cdot (10\,\text{mA})^2 = 12.5\,\mu\text{J}$

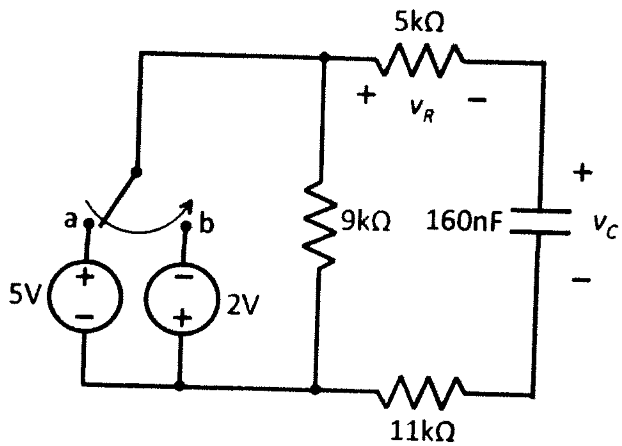
[+1] [+1]

ECSE-200 Quiz #11 (19 Nov 2010)

NAME________________________________ McGill ID#________________

*READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).*

1. Consider the circuit below. The switch moves from position a to position b instantaneously at $t$=0. The circuit is in dc steady state for $t$<0.

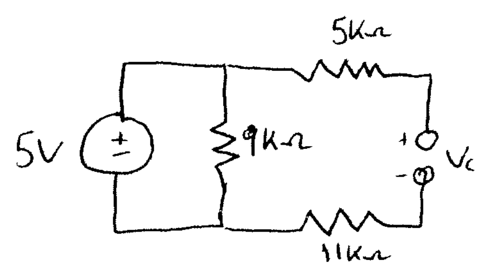



a) What is the voltage $v_c(t)$ for $t$>0 ? [6pts]
b) What is the voltage $v_R(t)$ for $t$>0? [4pts]
c) At what time $t$ is the energy stored by the capacitor a minimum? [BONUS 4pts]

a) $t<0$,

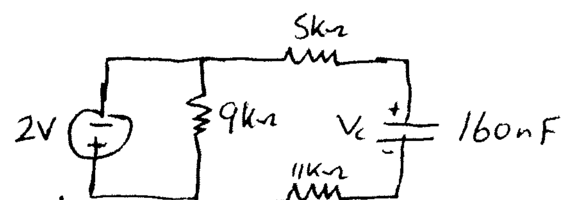
$V_c(0^-) = V_c(0^+) = 5V$

[+1]




$t>0$,

$V_c(\infty) = -2V$ [+1]




$V_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)]e^{-t/\tau}$ [+1]

$\tau = R_{eq}C$

$R_{eq} = 5K\Omega + 11K\Omega = 16K\Omega$ [+1]

$\Rightarrow \tau = 2.56\,ms$ [+1]

$\Rightarrow \boxed{V_c(t) = -2 + 7e^{-\frac{t}{2.56ms}}\ V,\ t>0}$ [+1]

work space

b) $i(t) = C \dfrac{dV_c(t)}{dt}$ [+1]

$= (160nF)\left[-\dfrac{1}{2.56\times10^{-3}} \cdot 7 \cdot e^{-\frac{t}{2.56\times10^{-3}}}\right]$

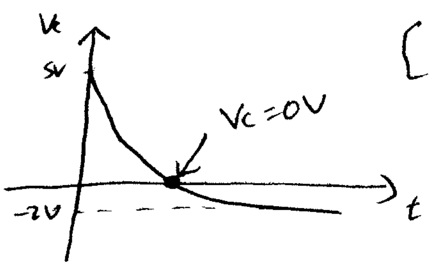
$= -437.5\, e^{-\frac{t}{2.56\times10^{-3}}}\ \mu A$ [+1]

$V_R(t) = i(t)\, R_{\text{-5k}\Omega\text{-}}$ [+1]

$= \boxed{-2.1875\, e^{-\frac{t}{2.56\times10^{-3}}}\ V,\ t > 0}$ [+1]

c) $U = \frac{1}{2} C V_c^2$ [+1]

minimum energy occurs at minimum $|V_c|$




[+1]

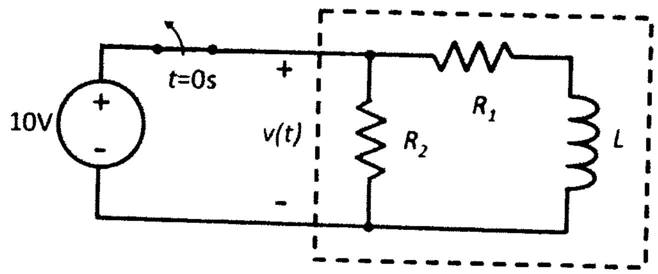
$-2 + 7e^{-\frac{t}{2.56ms}} = 0$

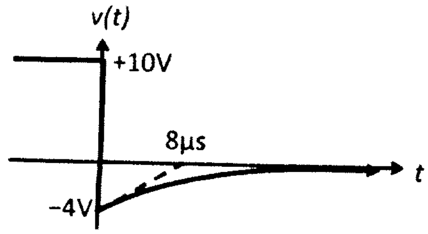
$e^{-\frac{t}{2.56ms}} = \frac{2}{7}$ [+1]

$\dfrac{-t}{2.56ms} = \ln\left[\frac{2}{7}\right]$

$\Rightarrow \boxed{t = 3.207ms}$ [+1]

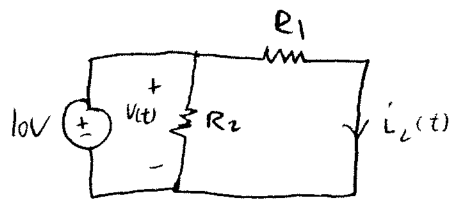
2. Consider the following circuit and voltage $v(t)$. The switch opens instantaneously at $t$=0. The circuit is in dc steady state for $t$<0. It is also known that 3.5mA is delivered by the independent source for $t$<0.




What are the values of $R_1$, $R_2$ and $L$? [10 pts]

$t<0$,



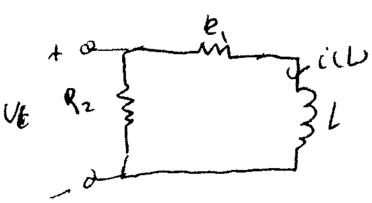

$$R_{eq} = \frac{10V}{3.5mA}$$

$$= R_1 // R_2 = 2857\Omega$$

$$i_L(0^-) = i_L(0^+) = \frac{10}{R_1} \quad [+2]$$

$t>0$,

$$V(0^+) = -4V \quad \text{(from } v(t) \text{ vs. } t \text{ plot)}$$




$$\frac{-4}{R_2} + i_L = 0 \quad [+2]$$

$$\Rightarrow i_L(0^+) = \frac{4}{R_2}$$

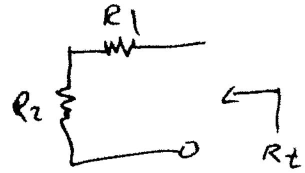
$$\Rightarrow \frac{10}{R_1} = \frac{4}{R_2} \quad \Rightarrow R_2 = 0.4R_1 \quad [+1]$$

using $R_1 // R_2 = 2857\Omega$ and $R_2 = 0.4 R_1$

$\Rightarrow R_1 = 10k\Omega$ , $R_2 = 4k\Omega$
[+1] [+1]

work space

$\tau = \frac{L}{R_t}$ [+1]




$\Rightarrow R_t = R_1 + R_2$ [+1]

$= 14k\Omega$

$\Rightarrow L = R_t \tau$

$= (14k\Omega)(8\mu s)$ (from V(t) vs. t plot)

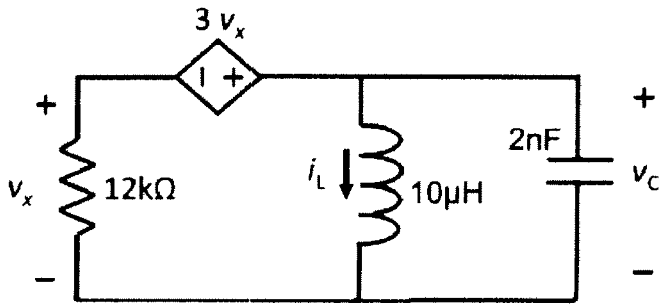
$= 0.112 H$ [+1]

ECSE-200 Quiz #12 (25 Nov 2010)

NAME Answer Key McGill ID#

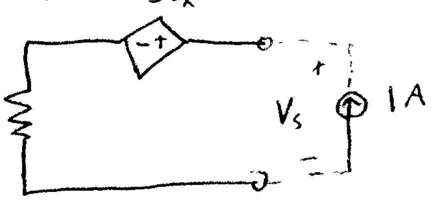
*READ each question and its parts carefully before starting. SHOW ALL YOUR WORK. Give units on your answers (where appropriate).*

1. Consider the circuit below. Recall that $\omega_0 = (LC)^{-1/2}$ and $\alpha = 1/(2RC)$ for a parallel RLC circuit.




a) Simplify the above circuit to a parallel *RLC* circuit. [3 pts]
b) Find the characteristic equation for the above circuit. [3 pts]
**HINT:** You may find it useful to apply the operator method.
c) What are the roots $s_1$ and $s_2$ of the characteristic equation? [2pts]
d) Is the above circuit over-damped, critically damped, or under-damped? [2 pts]
e) If it is known that $i_L(0)$ = 1mA and $v_C(0)$ = 0V, what is the natural response $i_L(t)$ for $t$>0.
[ BONUS 4pts ]

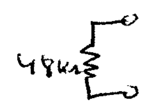
a)



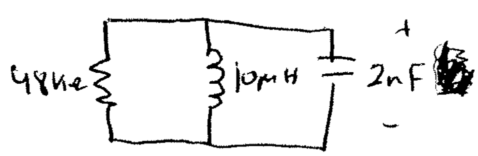

$V_x = 12KV$

$V_s = 3V_x + V_x = 4V_x = 48KV$

$\Rightarrow R_t = 48K\Omega$




--- [+2]




-- [+1]

b)

$$\frac{V_c}{48K\Omega} + i_L + C\frac{dv_c}{dt} = 0$$

$$V_c = L\frac{di_L}{dt} = 10\mu H \frac{di_L}{dt}$$

$$\frac{10\mu H}{48k\Omega}\frac{di_L}{dt} + i_L + 2nF \cdot 10\mu H \frac{d^2 i_L}{dt^2} = 0$$ [+1]

$s^2 + 10.42\cdot10^3 s + 50\cdot10^{12} = 0$ [+2]

c) $s_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow s_{1,2} = -5.21\cdot10^3 \pm j7.07\cdot10^6$ [+2]

d) $s_{1,2}$ are imaginary $\Rightarrow$ circuit is under-damped [+2]

e) $i_L(t>0) = e^{-\alpha t}\left[A_1\cos(\omega_d t) + A_2\sin(\omega_d t)\right]$

$\alpha = \dfrac{1}{2RC} = 5.21\cdot10^3$ [+1/2]

$\omega_d = 7.07\cdot10^6$ [+1/2]

$i_L(0) = 1mA = A_1$ [+1/2]

$V_c(0) = 0 = L\dfrac{di_L(0)}{dt}$

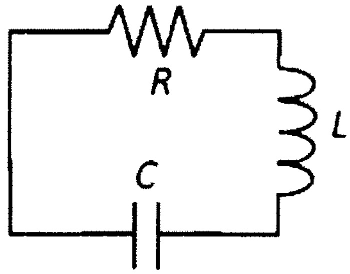
[+1/2]

$\Leftrightarrow \dfrac{di_L(0)}{dt} = 0 = -\alpha A_1 + \omega_d A_2 \Leftrightarrow A_2 = \dfrac{\alpha A_1}{\omega_d} = 737nA$

$\Rightarrow i_L(t>0) = e^{-5.21\cdot10^3 t}\left[1mA\cdot\cos(7.07\cdot10^6 t) + 737nA\cdot\sin(7.07\cdot10^6 t)\right]$

[+2]

2. Consider the following circuit. and voltage $v(t)$. Recall that $\omega_0 = (LC)^{-1/2}$ and $\alpha = R/(2L)$ for a series $RLC$ circuit.




a) Specify the value of $R$ and $L$ in the circuit above such that critically damped response with $s_1 = s_2 = -10^4$rad/s is obtained using a $C$=250nF capacitor. [4 pts]
b) Specify the value of $R$ and $C$ in the circuit above such that under-damped response is obtained with: undamped resonant frequency $\omega_0 = 10^6$rad/s, damping coefficient $\alpha = 10^3$rad/s, and an inductance $L$=600nH. [4 pts]
c) What is the damped resonant frequency $\omega_d$ for the circuit specified in part b)? Give your answer in rad/s. [2pts]

a) For a critically damped response,

$s_1 = s_2 = -\alpha$ [+1]

$\Rightarrow s_1 = s_2 = -\frac{R}{2L}$

$\Rightarrow -10^4 = \frac{-R}{2L}$ $\Rightarrow \frac{R}{L} = 2 \times 10^4$ [+1]

$\omega_0 = \frac{1}{\sqrt{LC}}$

we use $s^2 + 2\alpha s + \omega_0^2 = 0$ to find $\omega_0$

$\Rightarrow s^2 + 2\alpha s + \omega_0^2 = 0$

$(-10^4)^2 + 2(10^4)(-10^4) + \omega_0^2 = 0$

$\Rightarrow \omega_0^2 = 10^8$

$\Rightarrow \omega_0 = 10^4$ rad/s [+1]

$\omega_0 = \frac{1}{\sqrt{LC}}$ $\Rightarrow 10^8 = \frac{1}{LC}$

$\Rightarrow L = 0.04$ H [½] $\Rightarrow R = 800\ \Omega$ [½]

work space

b) $\omega_0 = \frac{1}{\sqrt{LC}}$

$10^6 = \frac{1}{\sqrt{LC}}$ $\Rightarrow$ $\frac{1}{LC} = 10^{12}$

$\Rightarrow C = 1.\bar{6}\,\mu F$ [+2 ]

$\alpha = \frac{R}{2L}$

$\Rightarrow 10^3 = \frac{R}{(2)(600nH)}$

$\Rightarrow R = 1.2 \times 10^{-3}\,\Omega$ [+2 ]

c) $\omega_d$ for under-damped response

$\omega_d = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$ [+1]

$= \sqrt{10^6 - 10^{12}}$

$\approx 10^6 j$ rad/s [+1]