

Robot Mapping

Grid Maps

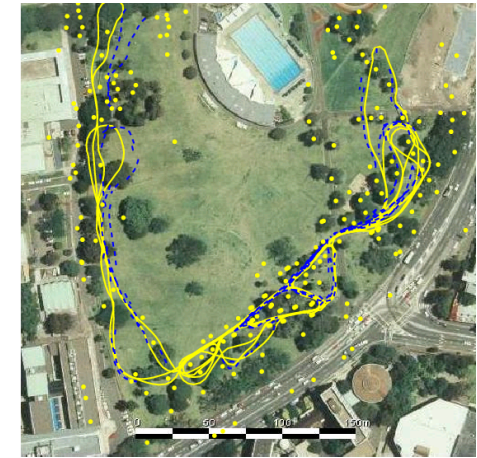
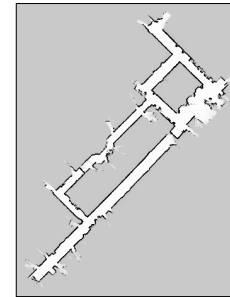
Cyrill Stachniss



AIS Autonomous
Intelligent
Systems

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Features vs. Volumetric Maps



Courtesy by E. Nebot

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Features

- So far, we only used feature maps
- Natural choice for Kalman filter-based SLAM systems
- Compact representation
- Multiple feature observations improve the position estimate (EKF)

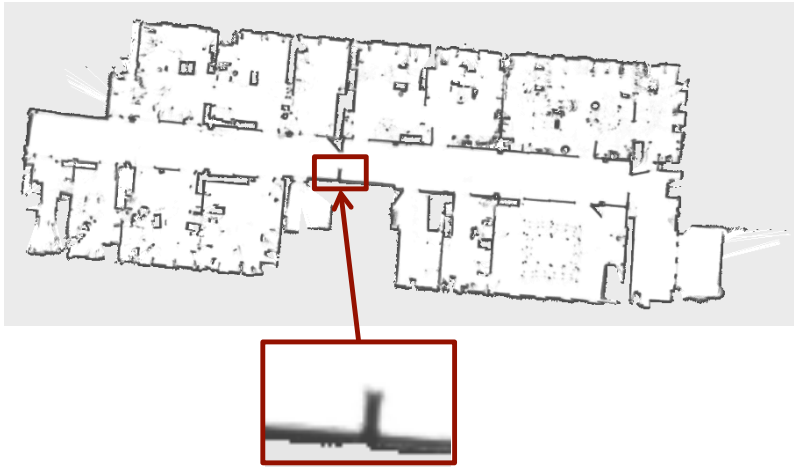
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Grid Maps

- Discretize the world into cells
- Grid structure is rigid
- Each cell is assumed to be occupied or free space
- Non-parametric model
- Require substantial memory resources
- Does not rely on a feature detector

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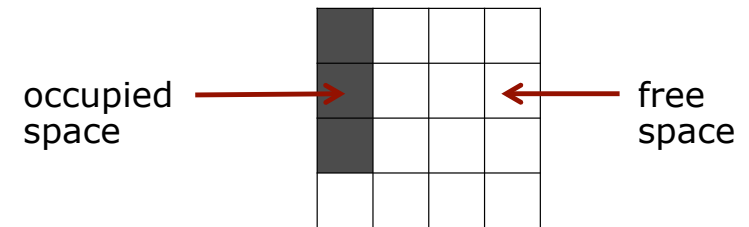
Example



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Assumption 1

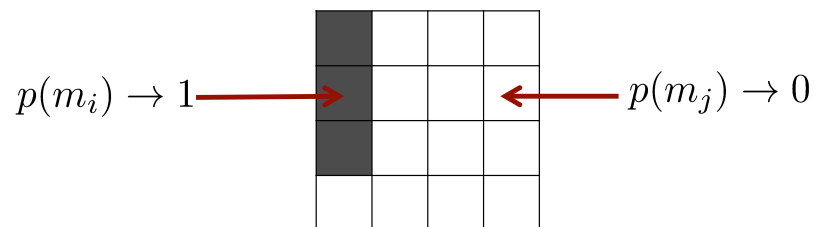
- The area that corresponds to a cell is either completely free or occupied



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Representation

- Each cell is a **binary random variable** that models the occupancy



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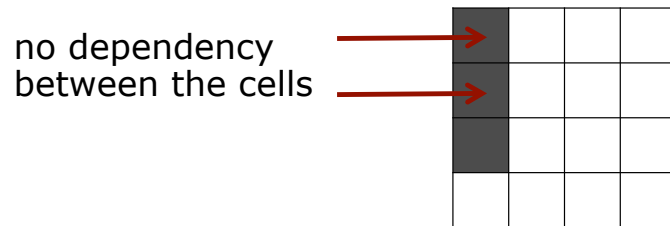
Occupancy Probability

- Each cell is a **binary random variable** that models the occupancy
- Cell is occupied $p(m_i) = 1$
- Cell is not occupied $p(m_i) = 0$
- No knowledge $p(m_i) = 0.5$
- The **state** is assumed to be **static**

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Assumption 2

- The cells (the random variables) are **independent** of each other



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Representation

- The probability distribution of the map is given by the product over the cells

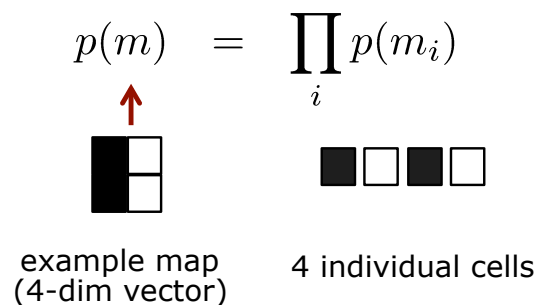
$$p(m) = \prod_i p(m_i)$$

↑map
 ↑cell

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Representation

- The probability distribution of the map is given by the product over the cells



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Estimating a Map From Data

- Given sensor data $z_{1:t}$ and the poses $x_{1:t}$ of the sensor, estimate the map

$$p(m \mid z_{1:t}, x_{1:t}) = \prod_i p(m_i \mid z_{1:t}, x_{1:t})$$

↑
↑

binary random variable

➡ Binary Bayes filter
(for a static state)

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Static State Binary Bayes Filter

$$p(m_i \mid z_{1:t}, x_{1:t}) \stackrel{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t}) p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

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Static State Binary Bayes Filter

$$p(m_i \mid z_{1:t}, x_{1:t}) \stackrel{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t}) p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\stackrel{\text{Markov}}{=} \frac{p(z_t \mid m_i, x_t) p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

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Static State Binary Bayes Filter

$$p(m_i \mid z_{1:t}, x_{1:t}) \stackrel{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t}) p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\stackrel{\text{Markov}}{=} \frac{p(z_t \mid m_i, x_t) p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$p(z_t \mid m_i, x_t) \stackrel{\text{Bayes rule}}{=} \frac{p(m_i \mid z_t, x_t) p(z_t \mid x_t)}{p(m_i \mid x_t)}$$

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Static State Binary Bayes Filter

$$p(m_i \mid z_{1:t}, x_{1:t}) \stackrel{\text{Bayes rule}}{=} \frac{p(z_t \mid m_i, z_{1:t-1}, x_{1:t}) p(m_i \mid z_{1:t-1}, x_{1:t})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\stackrel{\text{Markov}}{=} \frac{p(z_t \mid m_i, x_t) p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(z_t \mid z_{1:t-1}, x_{1:t})}$$

$$\stackrel{\text{Bayes rule}}{=} \frac{p(m_i \mid z_t, x_t) p(z_t \mid x_t) p(m_i \mid z_{1:t-1}, x_{1:t-1})}{p(m_i \mid x_t) p(z_t \mid z_{1:t-1}, x_{1:t})}$$

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Static State Binary Bayes Filter

$$\begin{aligned}
 p(m_i | z_{1:t}, x_{1:t}) & \stackrel{\text{Bayes rule}}{=} \frac{p(z_t | m_i, z_{1:t-1}, x_{1:t}) p(m_i | z_{1:t-1}, x_{1:t})}{p(z_t | z_{1:t-1}, x_{1:t})} \\
 & \stackrel{\text{Markov}}{=} \frac{p(z_t | m_i, x_t) p(m_i | z_{1:t-1}, x_{1:t-1})}{p(z_t | z_{1:t-1}, x_{1:t})} \\
 & \stackrel{\text{Bayes rule}}{=} \frac{p(m_i | z_t, x_t) p(z_t | x_t) p(m_i | z_{1:t-1}, x_{1:t-1})}{p(m_i | x_t) p(z_t | z_{1:t-1}, x_{1:t})} \\
 & \stackrel{\text{Markov}}{=} \frac{p(m_i | z_t, x_t) p(z_t | x_t) p(m_i | z_{1:t-1}, x_{1:t-1})}{p(m_i) p(z_t | z_{1:t-1}, x_{1:t})}
 \end{aligned}$$

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Static State Binary Bayes Filter

$$\begin{aligned}
 p(m_i | z_{1:t}, x_{1:t}) & \stackrel{\text{Bayes rule}}{=} \frac{p(z_t | m_i, z_{1:t-1}, x_{1:t}) p(m_i | z_{1:t-1}, x_{1:t})}{p(z_t | z_{1:t-1}, x_{1:t})} \\
 & \stackrel{\text{Markov}}{=} \frac{p(z_t | m_i, x_t) p(m_i | z_{1:t-1}, x_{1:t-1})}{p(z_t | z_{1:t-1}, x_{1:t})} \\
 & \stackrel{\text{Bayes rule}}{=} \frac{p(m_i | z_t, x_t) p(z_t | x_t) p(m_i | z_{1:t-1}, x_{1:t-1})}{p(m_i | x_t) p(z_t | z_{1:t-1}, x_{1:t})} \\
 & \stackrel{\text{Markov}}{=} \frac{p(m_i | z_t, x_t) p(z_t | x_t) p(m_i | z_{1:t-1}, x_{1:t-1})}{p(m_i) p(z_t | z_{1:t-1}, x_{1:t})}
 \end{aligned}$$

Do exactly the same for the opposite event:

$$p(\neg m_i | z_{1:t}, x_{1:t}) \stackrel{\text{the same}}{=} \frac{p(\neg m_i | z_t, x_t) p(z_t | x_t) p(\neg m_i | z_{1:t-1}, x_{1:t-1})}{p(\neg m_i) p(z_t | z_{1:t-1}, x_{1:t})}$$

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Static State Binary Bayes Filter

- By computing the ratio of both probabilities, we obtain:

$$\frac{p(m_i | z_{1:t}, x_{1:t})}{p(\neg m_i | z_{1:t}, x_{1:t})} = \frac{\frac{p(m_i | z_t, x_t) p(z_t | x_t) p(m_i | z_{1:t-1}, x_{1:t-1})}{p(m_i) p(z_t | z_{1:t-1}, x_{1:t})}}{\frac{p(\neg m_i | z_t, x_t) p(z_t | x_t) p(\neg m_i | z_{1:t-1}, x_{1:t-1})}{p(\neg m_i) p(z_t | z_{1:t-1}, x_{1:t})}}$$

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Static State Binary Bayes Filter

- By computing the ratio of both probabilities, we obtain:

$$\begin{aligned}
 & \frac{p(m_i | z_{1:t}, x_{1:t})}{p(\neg m_i | z_{1:t}, x_{1:t})} \\
 &= \frac{p(m_i | z_t, x_t) p(m_i | z_{1:t-1}, x_{1:t-1}) p(\neg m_i)}{p(\neg m_i | z_t, x_t) p(\neg m_i | z_{1:t-1}, x_{1:t-1}) p(m_i)} \\
 &= \frac{p(m_i | z_t, x_t)}{1 - p(m_i | z_t, x_t)} \frac{p(m_i | z_{1:t-1}, x_{1:t-1})}{1 - p(m_i | z_{1:t-1}, x_{1:t-1})} \frac{1 - p(m_i)}{p(m_i)}
 \end{aligned}$$

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Static State Binary Bayes Filter

- By computing the ratio of both probabilities, we obtain:

$$\begin{aligned}
 & \frac{p(m_i \mid z_{1:t}, x_{1:t})}{1 - p(m_i \mid z_{1:t}, x_{1:t})} \\
 &= \frac{p(m_i \mid z_t, x_t) p(m_i \mid z_{1:t-1}, x_{1:t-1}) p(\neg m_i)}{p(\neg m_i \mid z_t, x_t) p(\neg m_i \mid z_{1:t-1}, x_{1:t-1}) p(m_i)} \\
 &= \underbrace{\frac{p(m_i \mid z_t, x_t)}{1 - p(m_i \mid z_t, x_t)}}_{\text{uses } z_t} \underbrace{\frac{p(m_i \mid z_{1:t-1}, x_{1:t-1})}{1 - p(m_i \mid z_{1:t-1}, x_{1:t-1})}}_{\text{recursive term}} \underbrace{\frac{1 - p(m_i)}{p(m_i)}}_{\text{prior}}
 \end{aligned}$$

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Log Odds Notation

- Log odds ratio is defined as

$$l(x) = \log \frac{p(x)}{1 - p(x)}$$

- and with the ability to retrieve $p(x)$

$$p(x) = \frac{1}{1 + \exp l(x)}$$

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Occupancy Mapping in Log Odds Form

- The product turns into a sum

$$\begin{aligned}
 l(m_i \mid z_{1:t}, x_{1:t}) \\
 &= \underbrace{l(m_i \mid z_t, x_t)}_{\text{inverse sensor model}} + \underbrace{l(m_i \mid z_{1:t-1}, x_{1:t-1})}_{\text{recursive term}} - \underbrace{l(m_i)}_{\text{prior}}
 \end{aligned}$$

- or in short

$$l_{t,i} = \text{inv_sensor_model}(m_i, x_t, z_t) + l_{t-1,i} - l_0$$

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Occupancy Mapping Algorithm

occupancy_grid_mapping($\{l_{t-1,i}\}, x_t, z_t$):

```

1:   for all cells  $m_i$  do
2:     if  $m_i$  in perceptual field of  $z_t$  then
3:        $l_{t,i} = l_{t-1,i} + \text{inv\_sensor\_model}(m_i, x_t, z_t) - l_0$ 
4:     else
5:        $l_{t,i} = l_{t-1,i}$ 
6:     endif
7:   endfor
8:   return  $\{l_{t,i}\}$ 

```

highly efficient, only requires to compute sums

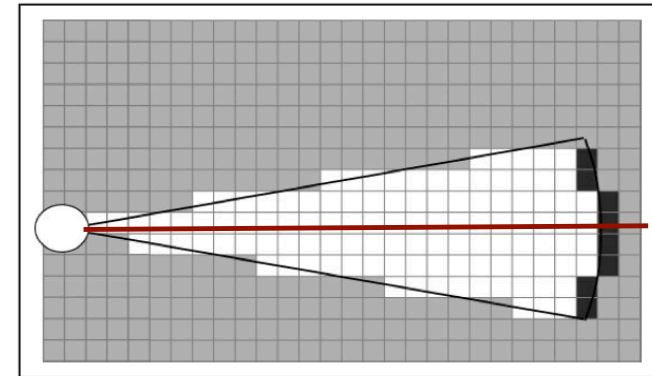
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Occupancy Grid Mapping

- Developed in the mid 80'ies by Moravec and Elfes
- Originally developed for noisy sonars
- Also called "mapping with know poses"

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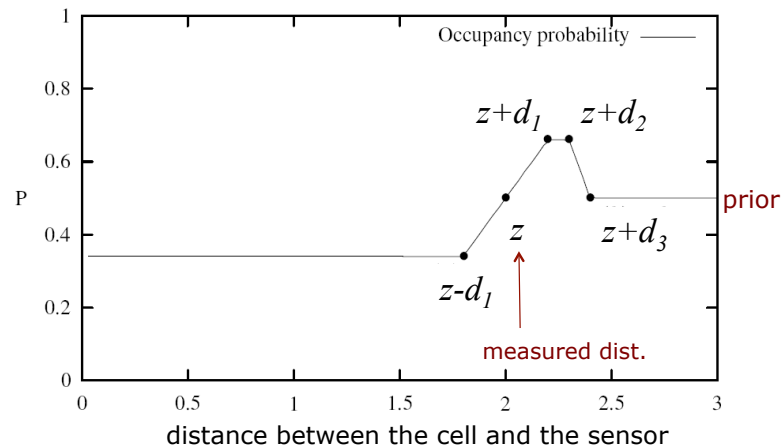
Inverse Sensor Model for Sonars Range Sensors



In the following, consider the cells along the optical axis (red line)

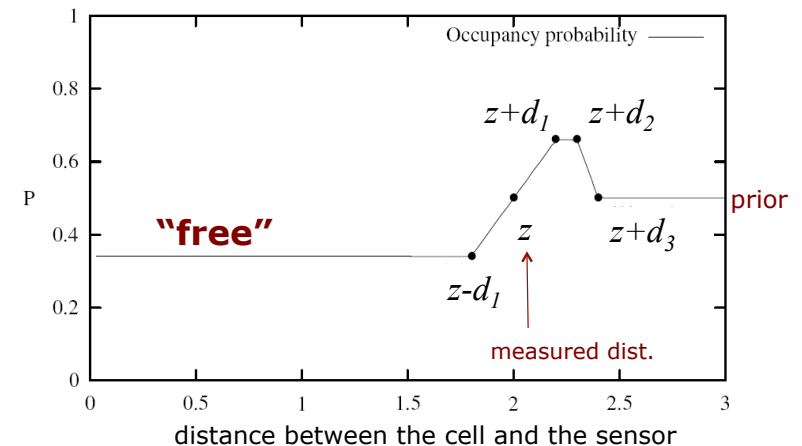
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Occupancy Value Depending on the Measured Distance



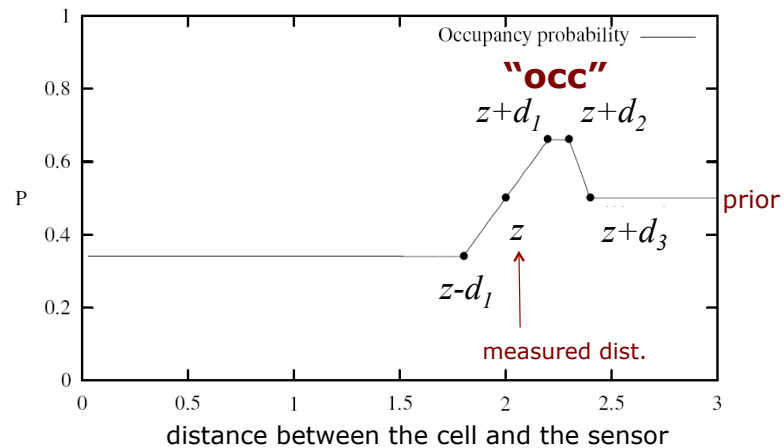
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Occupancy Value Depending on the Measured Distance



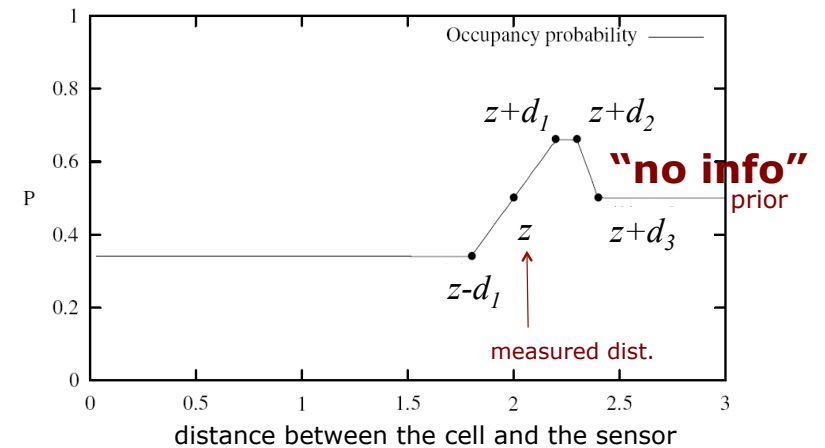
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Occupancy Value Depending on the Measured Distance



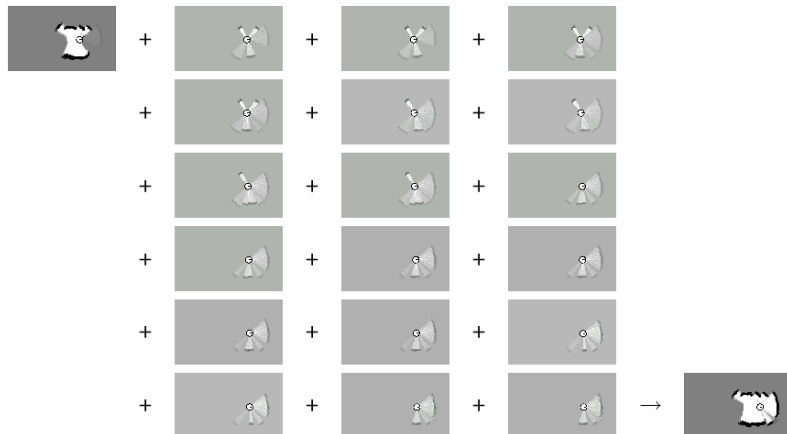
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Occupancy Value Depending on the Measured Distance



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Example: Incremental Updating of Occupancy Grids



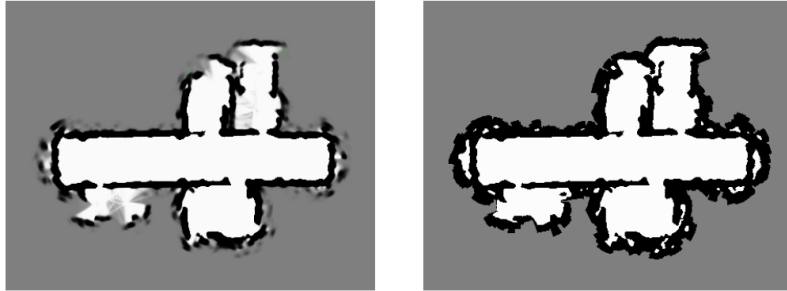
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Resulting Map Obtained with Ultrasound Sensors



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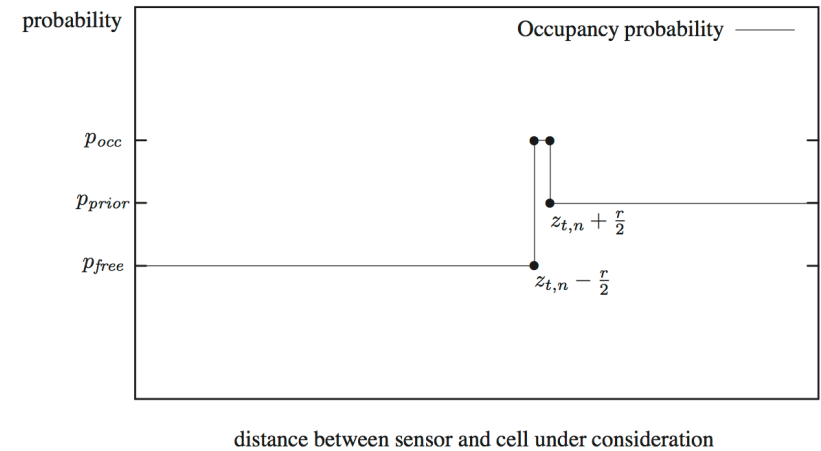
Resulting Occupancy and Maximum Likelihood Map



The maximum likelihood map is obtained by rounding the probability for each cell to 0 or 1.

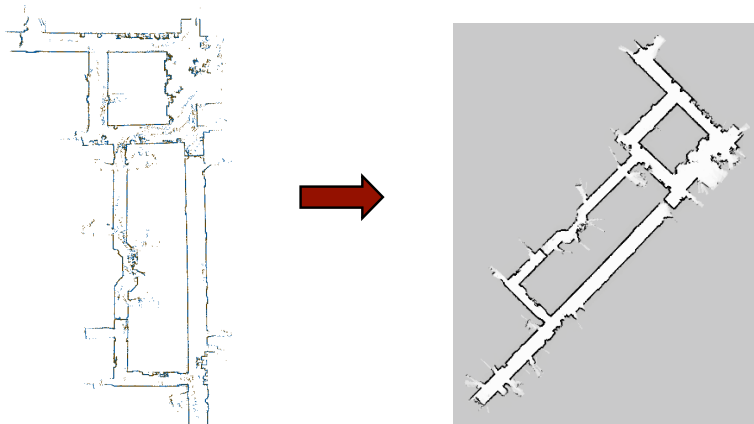
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Inverse Sensor Model for Laser Range Finders



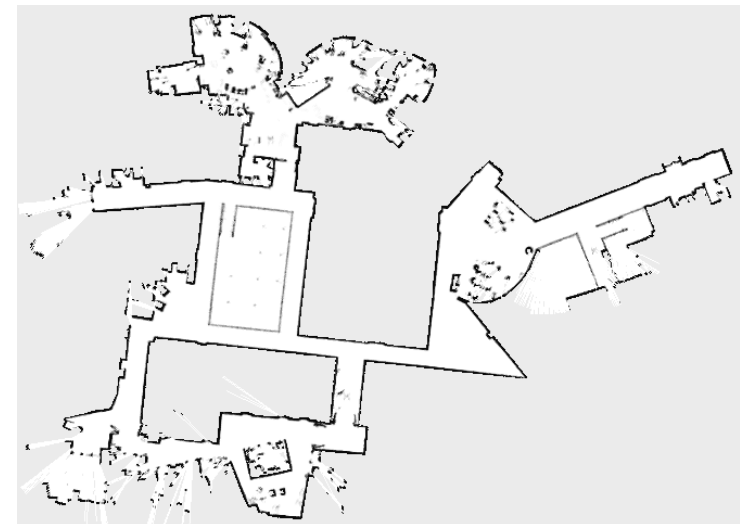
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Occupancy Grids From Laser Scans to Maps



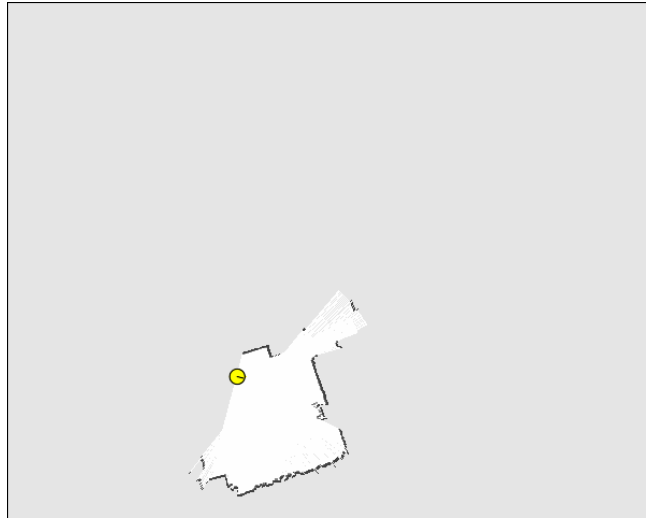
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Example: MIT CSAIL 3rd Floor



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Uni Freiburg Building 106



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Summary

- Occupancy grid maps discretize the space into independent cells
- Each cell is a binary random variable estimating if the cell is occupied
- Static state binary Bayes filter per cell
- Mapping with known poses is easy
- Log odds model is fast to compute
- No need for predefined features

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Literature

Static state binary Bayes filter

- Thrun et al.: "Probabilistic Robotics", Chapter 4.2

Occupancy Grid Mapping

- Thrun et al.: "Probabilistic Robotics", Chapter 9.1+9.2

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