

Name: _____

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CS188: Introduction to Robotics

Worksheet 2

Problem 1: Image Filters

(a) What will the following convolution kernels do when they are used on an image?

$$\begin{bmatrix} -1 & 0 & +1 \\ -1 & 0 & +1 \\ -1 & 0 & +1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$$

(b) What is gamma correction? Why do we need that?

Problem 2: PF vs. KF

Conceptual

- Why do we use particle filters and Kalman filters in robotics? Alternatively, how, or where do we use them?
- When might we want to use a Kalman filter over a particle filter?
- When might we want to use a particle filter over a Kalman filter?

Problem 3: SLAM

Conceptual

What makes the *Simultaneous Localization and Mapping* (SLAM) problem difficult? Describe a challenge that can arise when a robot tries to build a map of an unknown environment while simultaneously localizing itself within that map.

Multiple Choice

In a SLAM system, why is it important to account for both the robot's motion and the sensor's measurement noise when constructing the map?

- A) To improve the map resolution and reduce the computational cost of mapping
- B) To minimize the effect of drift and inaccuracies in the robot's localization over time
- C) To ensure the robot can detect obstacles in unknown environments
- D) To increase the speed at which the map is generated

Problem 4: PRM & RRT

Conceptual

Consider the following conceptual questions.

- Describe, in words, how the PRM (Probabilistic Roadmap) algorithm finds a path from *start* to *goal*.
- What aspect of the RRT (Rapidly-Exploring Random Tree) algorithm allows it to efficiently search high-dimensional spaces for a path?

Problem 5: EKF SLAM

Part A: Conceptual - The Covariance Matrix

Consider an EKF SLAM system at time step t . The state vector consists of the robot's pose $x_R = [x, y, \theta]^T$ and N static landmarks $m = [m_1, \dots, m_N]^T$, where each landmark m_i has coordinates (x_{L_i}, y_{L_i}) .

Q1. Construct the full covariance matrix Σ_t in the space below.

- Explicitly show the sub-blocks for the Robot (3×3), the first Landmark (2×2), and the Cross-Covariances.
- Use indices to denote variances (e.g., σ_{xx}, σ_{xy}) and vertical/horizontal lines to visually separate the robot state from the map state.

$$\Sigma_t = \left[\begin{array}{ccc|cc|c} \sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xm_{1x}} & \sigma_{xm_{1y}} & \cdots \\ \sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} & \sigma_{ym_{1x}} & \sigma_{ym_{1y}} & \cdots \\ \sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} & \sigma_{\theta m_{1x}} & \sigma_{\theta m_{1y}} & \cdots \\ \hline \sigma_{m_{1x}x} & \sigma_{m_{1x}y} & \sigma_{m_{1x}\theta} & \sigma_{m_{1x}m_{1x}} & \sigma_{m_{1x}m_{1y}} & \cdots \\ \sigma_{m_{1y}x} & \sigma_{m_{1y}y} & \sigma_{m_{1y}\theta} & \sigma_{m_{1y}m_{1x}} & \sigma_{m_{1y}m_{1y}} & \cdots \\ \hline : & : & : & : & : & \ddots \end{array} \right]$$

Q2. Evolution of Uncertainty For each step below, indicate which blocks of the matrix (Robot-Robot, Robot-Map, Map-Map) change values.

1. **Prediction Step (Motion):** The robot moves into unexplored space.
2. **Correction Step (Observation):** The robot observes Landmark 1 (m_1), which has been mapped previously.

Part B: 2D Numerical Update (Simplified)

Consider a robot moving in a 2D plane.

- **State:** $\mu = [x, y, \theta, m_x, m_y]^T$.
- **Initial Belief (μ_{t-1}, Σ_{t-1}):** The robot is at the origin with zero uncertainty. There is one landmark at $(10, 0)$ with some initial uncertainty.

$$\mu_{t-1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \\ 0 \end{bmatrix}, \quad \Sigma_{t-1} = \left[\begin{array}{ccc|cc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{array} \right]$$

Step 1: Motion (Prediction) The robot moves forward by $u_d = 2$ meters with no rotation ($u_\theta = 0$). The motion noise covariance R_t is:

$$R_t = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$$

1. Calculate the predicted mean $\bar{\mu}_t$.
2. Calculate the predicted covariance $\bar{\Sigma}_t$.

Step 2: Measurement (Correction) The robot observes the landmark using a Range-Bearing sensor.

- **Measurement function:** $z = \begin{bmatrix} r \\ \phi \end{bmatrix} = \begin{bmatrix} \sqrt{\Delta x^2 + \Delta y^2} \\ \tan(\Delta y / \Delta x) - \theta \end{bmatrix}$
- **Actual Measurement:** The sensor returns $z_t = \begin{bmatrix} 7.5 \\ 0 \end{bmatrix}$.
- **Measurement Noise:** $Q_t = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix}$.

Calculate:

1. The expected measurement \hat{z}_t .
2. The Innovation ν_t (residual).

Step 3: Adding New Landmarks How will the state vector μ_t and covariance matrix Σ_t change if a new, previously unseen landmark is identified at global coordinates (6, 12)? Specify the new dimensions of the matrices.