Name: ______________________ **UID:** ____________

CS188: Introduction to Robotics

Worksheet 1

Problem 1: DH Parameters

Note that on exams, the axes will be given to you, so the more important part of these types of problems is translating from the given diagram and the axes to the DH parameter table. For this class, we will use the definition in *Modern Robotics*, which has been reproduced below:

- a_{i-1} (link length): distance along the common normal from z_{i-1} to z_i.
- α_{i-1} (link twist): twist angle from z_{i-1} to z_i, measured about x_{i-1}.
- d_i (link offset): displacement along z_i; distance from the intersection of x_{i-1} and z_i to the origin of the link-i frame.
- θ_i (joint angle): the angle from x_{i-1} to x_i, measured about the z_i-axis.

We have assigned frames for the robot manipulator shown below. Furthermore, in this diagram, the slider bar is parallel to the ground and $\theta_1 = 0, \theta_2 = 90$. There are many other ways to assign frames which will affect the final table. Using the following diagram, please fill in the DH table.

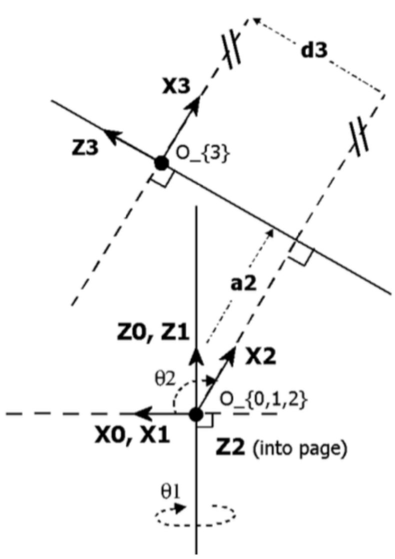

Figure 1: RRP manipulator with frames

i	a_{i-1}	α_{i-1}	d_i	θ_i
1				
2				
3				

Figure 2: DH parameters (fill in!)

Problem 2: DoF

Three identical SRS open-chain arms are grasping a common object. Find the number of degrees of freedom of this system. You may find the following table useful.

Joint type	dof f	Constraints c between two spatial rigid bodies
Revolute (R)	1	5
Spherical (S)	3	3

Additionally provided to you is *Grübler's formula*.

$$\begin{aligned}\text{DoF} &= m(N-1) - \sum_{i=1}^{J} c_i \\ &= m(N-1) - \sum_{i=1}^{J} (m - f_i) \\ &= m(N-1-J) + \sum_{i=1}^{J} f_i\end{aligned}$$

This exercise is taken from *Modern Robotics*, exercise 2.7. There are more examples and exercises involving calculating degrees of freedom in the textbook.

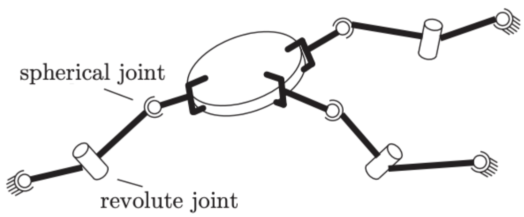

Figure 2.17: Three cooperating SRS arms grasping a common object.

DoF of the system =

Problem 3: Cartesian and Homogeneous Coordinates

You have a point $\mathbf{p} = (8, -2, 5)$ in 3D **Cartesian** coordinates.

1. Convert $\mathbf{p}$ into **homogeneous** coordinates.
2. Suppose instead you are given the same point in homogeneous form $\tilde{p} = (8, -2, 5, w)$. If $w \neq 1$, derive the corresponding Cartesian coordinates in terms of w.

Problem 4: Rotation about an axis

You have a rotation of 90° about the Y-axis in 3D space. Write the standard rotation matrix $\mathbf{R}_y(90^\circ)$ that implements this transformation. Then show what happens to the point $\mathbf{p} = (1, 0, 0)$ under this rotation.

Problem 5: Extrinsic Matrix from Camera Pose

A camera in the world frame is located at $\mathbf{C}_W = (2, -3, 5)$. It's **optical axis** (the camera's Z-axis) is aligned so that it points directly **along** the negative Z-direction of the world frame. The camera's Y-axis points along the world's positive Y direction, and the camera's X-axis follows the right-hand rule (it will align with $-X_W$ if Z flips sign and Y remains the same). Construct the **extrinsic matrix** ${}^{C}T_W$ that transforms a point from the **world** frame to the camera's frame.

Hint: first write the rotation that aligns the world axes with the camera axes, then add translation.

Second hint: for translation, note the following formula: $\mathbf{p}_C = \mathbf{R}_{CW}(\mathbf{p}_W - \mathbf{C}_W)$

Problem 6: Projecting a Point onto an Image Plane

A camera has the following **intrinsic** matrix:

$$K = \begin{bmatrix} 200 & 0 & 100 \\ 0 & 200 & 80 \\ 0 & 0 & 1 \end{bmatrix}$$

A point in camera coordinates is $\mathbf{p}_C = (0.5, -0.2, 1.0)$. Use K to compute the **2D** pixel coordinates (u, v) of this point on the image plane.