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## CS188: Introduction to Robotics

### Worksheet 2

#### Problem 1: Image Filters

(a) What will the following convolution kernels do when they are used on an image?

$$\begin{bmatrix} -1 & 0 & +1 \\ -1 & 0 & +1 \\ -1 & 0 & +1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$$

(b) What is gamma correction? Why do we need that?

#### Problem 2: PF vs. KF

##### Conceptual

- Why do we use particle filters and Kalman filters in robotics? Alternatively, how, or where do we use them?
- When might we want to use a Kalman filter over a particle filter?
- When might we want to use a particle filter over a Kalman filter?

## Problem 3: SLAM

### Conceptual

What makes the *Simultaneous Localization and Mapping* (SLAM) problem difficult? Describe a challenge that can arise when a robot tries to build a map of an unknown environment while simultaneously localizing itself within that map.

### Multiple Choice

In a SLAM system, why is it important to account for both the robot's motion and the sensor's measurement noise when constructing the map?

- A) To improve the map resolution and reduce the computational cost of mapping
- B) To minimize the effect of drift and inaccuracies in the robot's localization over time
- C) To ensure the robot can detect obstacles in unknown environments
- D) To increase the speed at which the map is generated

## Problem 4: PRM & RRT

### Conceptual

Consider the following conceptual questions.

- Describe, in words, how the PRM (Probabilistic Roadmap) algorithm finds a path from *start* to *goal*.
- What aspect of the RRT (Rapidly-Exploring Random Tree) algorithm allows it to efficiently search high-dimensional spaces for a path?

## Problem 5: EKF SLAM

### Part A: Conceptual - The Covariance Matrix

Consider an EKF SLAM system at time step  $t$ . The state vector consists of the robot's pose  $x_R = [x, y, \theta]^T$  and  $N$  static landmarks  $m = [m_1, \dots, m_N]^T$ , where each landmark  $m_i$  has coordinates  $(x_{L_i}, y_{L_i})$ .

**Q1.** Construct the full covariance matrix  $\Sigma_t$  in the space below.

- Explicitly show the sub-blocks for the Robot ( $3 \times 3$ ), the first Landmark ( $2 \times 2$ ), and the Cross-Covariances.
- Use indices to denote variances (e.g.,  $\sigma_{xx}, \sigma_{xy}$ ) and vertical/horizontal lines to visually separate the robot state from the map state.

$$\Sigma_t = \left[ \begin{array}{ccc|cc|c} \sigma_{xx} & \sigma_{xy} & \sigma_{x\theta} & \sigma_{xm_1x} & \sigma_{xm_1y} & \cdots \\ \sigma_{yx} & \sigma_{yy} & \sigma_{y\theta} & \sigma_{ym_1x} & \sigma_{ym_1y} & \cdots \\ \sigma_{\theta x} & \sigma_{\theta y} & \sigma_{\theta\theta} & \sigma_{\theta m_1x} & \sigma_{\theta m_1y} & \cdots \\ \hline \sigma_{m_1xx} & \sigma_{m_1xy} & \sigma_{m_1x\theta} & \sigma_{m_1xm_1x} & \sigma_{m_1xm_1y} & \cdots \\ \sigma_{m_1yx} & \sigma_{m_1yy} & \sigma_{m_1y\theta} & \sigma_{m_1ym_1x} & \sigma_{m_1ym_1y} & \cdots \\ \hline \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array} \right]$$

**Q2. Evolution of Uncertainty** For each step below, indicate which blocks of the matrix (Robot-Robot, Robot-Map, Map-Map) change values.

1. **Prediction Step (Motion):** The robot moves into unexplored space.
2. **Correction Step (Observation):** The robot observes Landmark 1 ( $m_1$ ), which has been mapped previously.

### Part B: 2D Numerical Update (Simplified)

Consider a robot moving in a 2D plane.

- **State:**  $\mu = [x, y, \theta, m_x, m_y]^T$ .
- **Initial Belief ( $\mu_{t-1}, \Sigma_{t-1}$ ):** The robot is at the origin with zero uncertainty. There is one landmark at  $(10, 0)$  with some initial uncertainty.

$$\mu_{t-1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 10 \\ 0 \end{bmatrix}, \quad \Sigma_{t-1} = \left[ \begin{array}{ccc|cc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{array} \right]$$

**Step 1: Motion (Prediction)** The robot moves forward by  $u_d = 2$  meters with no rotation ( $u_\theta = 0$ ). The motion noise covariance  $R_t$  is:

$$R_t = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$$

1. Calculate the predicted mean  $\bar{\mu}_t$ .
2. Calculate the predicted covariance  $\bar{\Sigma}_t$ .

**Step 2: Measurement (Correction)** The robot observes the landmark using a Range-Bearing sensor.

- **Measurement function:**  $z = \begin{bmatrix} r \\ \phi \end{bmatrix} = \begin{bmatrix} \sqrt{\Delta x^2 + \Delta y^2} \\ \tan(\Delta y / \Delta x) - \theta \end{bmatrix}$
- **Actual Measurement:** The sensor returns  $z_t = \begin{bmatrix} 7.5 \\ 0 \end{bmatrix}$ .
- **Measurement Noise:**  $Q_t = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.1 \end{bmatrix}$ .

**Calculate:**

1. The expected measurement  $\hat{z}_t$ .
2. The Innovation  $\nu_t$  (residual).

**Step 3: Adding New Landmarks** How will the state vector  $\mu_t$  and covariance matrix  $\Sigma_t$  change if a new, previously unseen landmark is identified at global coordinates (6,12)? Specify the new dimensions of the matrices.