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COM SCI 188

# Intro to Robotics

## Lecture 2

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Winter 2026



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# Agenda

- Announcements
- Recap: what is a robot?
- Configuration Space & Degrees of Freedom
- Sensors (overview)
- Actuators (motors & gears)

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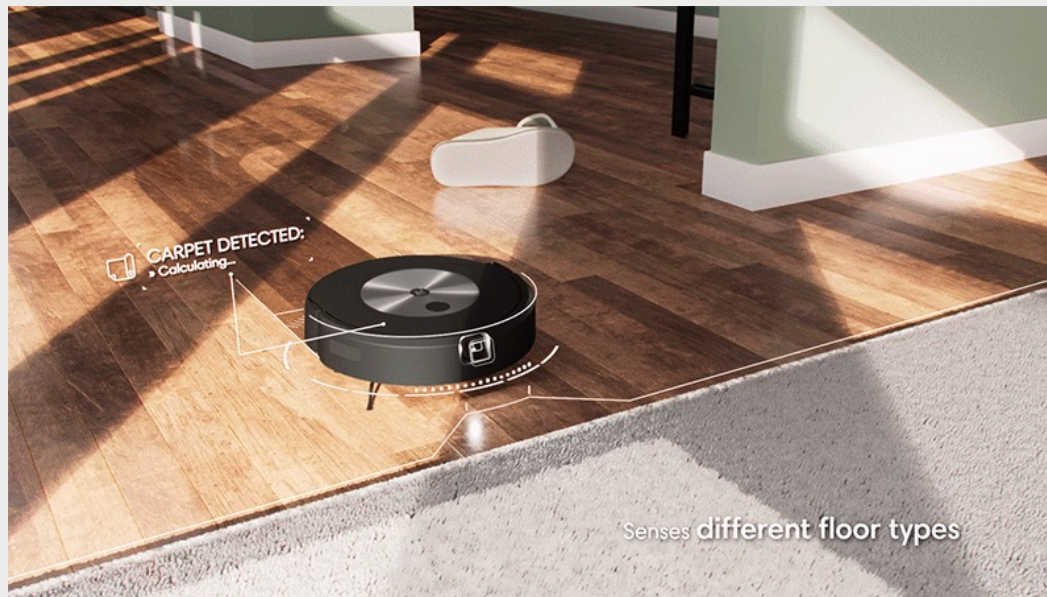
# Announcements

- This Week's Discussion Section: Robosuite Installation
  - Read the instructions on Piazza!
- Problem Set 1 is out, Due next Friday
- PTEs have been assigned

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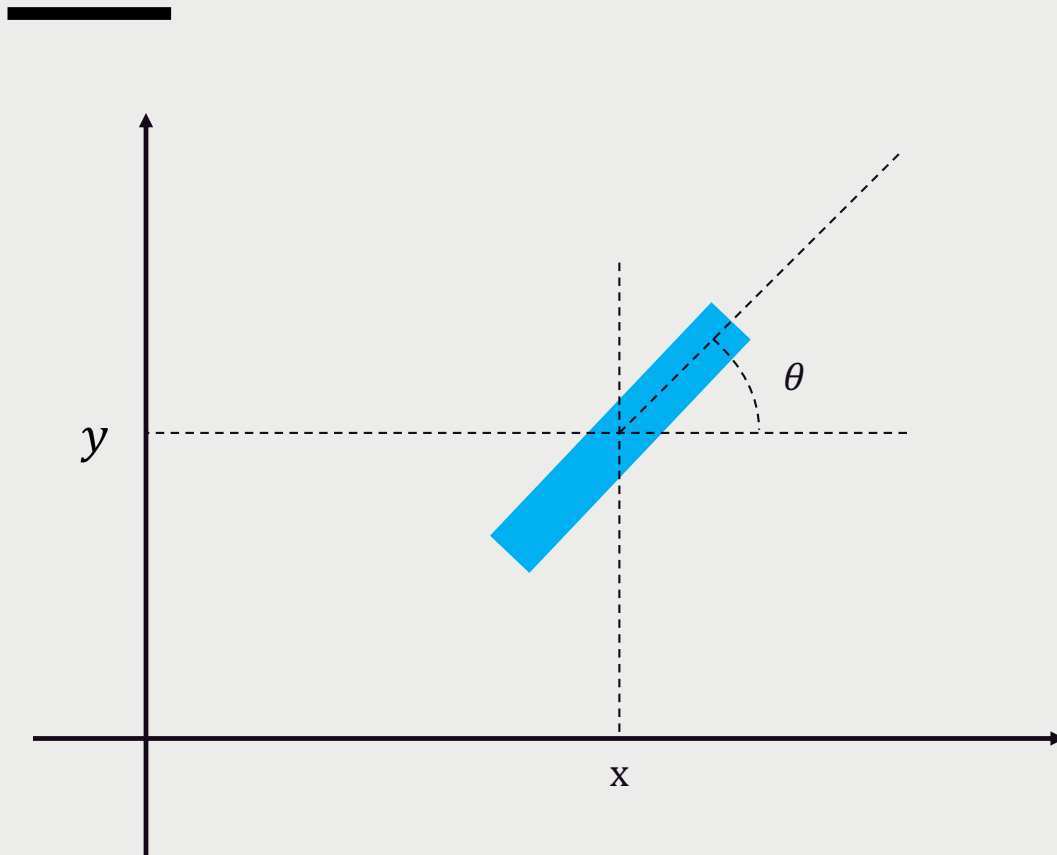
## Recap: Definition of Robots

# Sense, Plan, Act



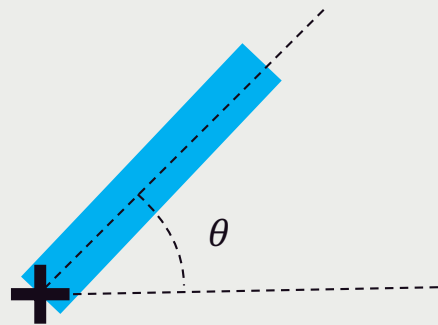
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How to describe the state of a robot?



How many variables do we need to fully describe the configuration of this **rigid body** in 2D?

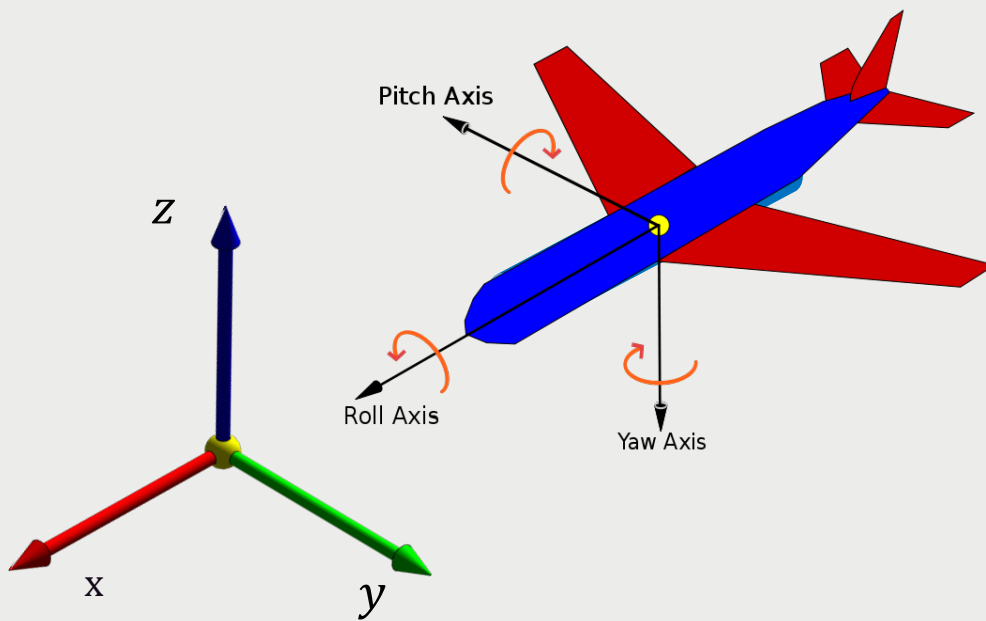
Answer:  $(x, y, \theta) = 3$



What if one of the end points is fixed?

Answer:  $(\theta) = 1$

This is called the **Degree of Freedom (DoF)** of a rigid body

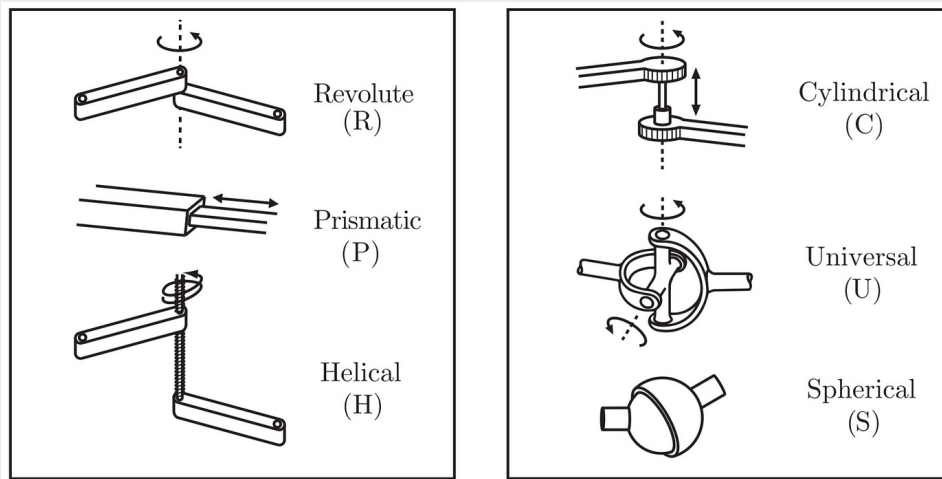


Rigid body in 3D space:  
How many DoF does it have?

$6 = 3$  for position +  $3$  for orientation

# Joints and DoF

## Common Joints

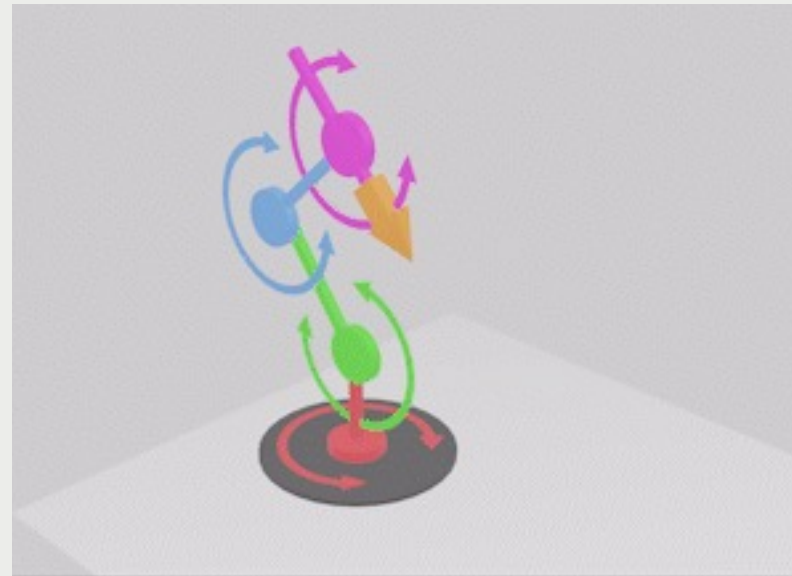
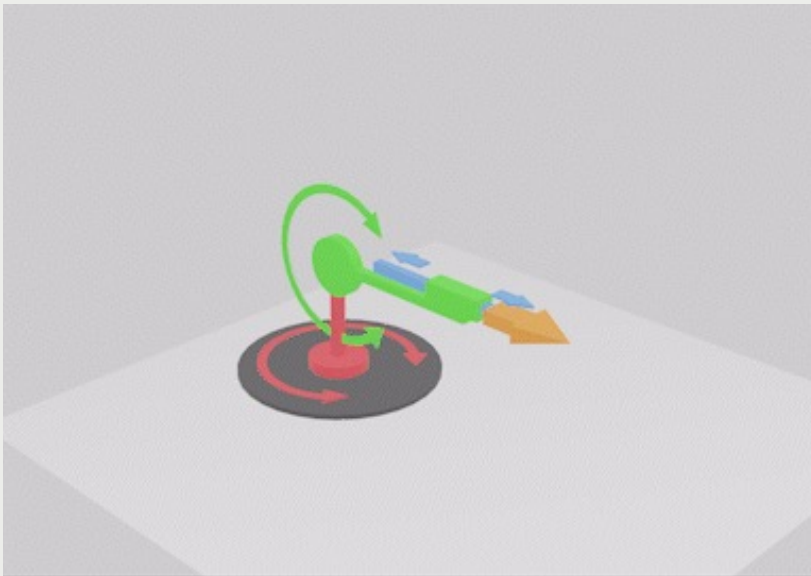


Joint type	dof $f$	2D robot	3D robot
		Constraints $c$ between two planar rigid bodies	Constraints $c$ between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

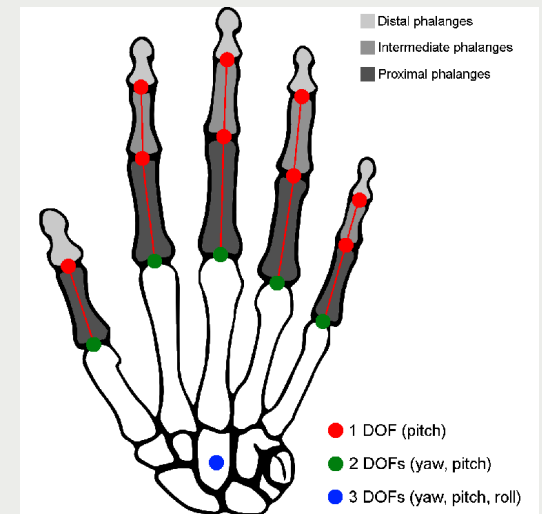
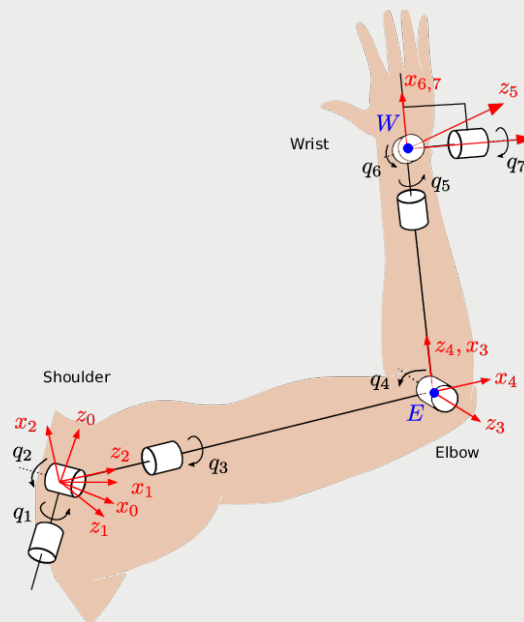
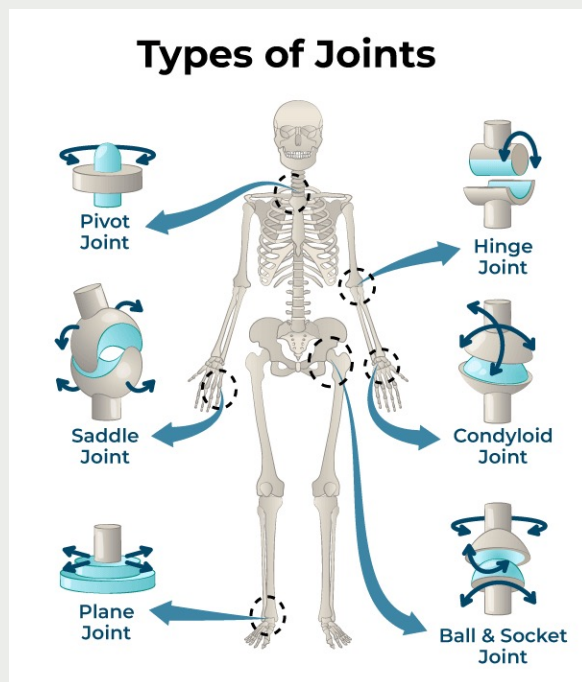
MODERN ROBOTICS

Kevin M. Lynch and Frank C. Park May 3, 2017

# Joints and DoF



# How many DoF do you have?



Whole body: ~30 DoF (major joints)

Images from Google search.

# Grübler's formula

$$\begin{aligned}
 \text{dof} &= \underbrace{m(N-1)}_{\text{rigid body freedoms}} - \underbrace{\sum_{i=1}^J c_i}_{\text{joint constraints}} \\
 &= m(N-1) - \sum_{i=1}^J (m - f_i) \\
 &= m(N-1-J) + \sum_{i=1}^J f_i.
 \end{aligned}$$

Joint type	dof $f$	Constraints $c$ between two planar rigid bodies	Constraints $c$ between two spatial rigid bodies
<del>Revolute (R)</del>	1	2	<del>5</del>
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

$m = 3$  for planar mechanism (2D)  
 $m = 6$  for a spatial mechanism (3D)

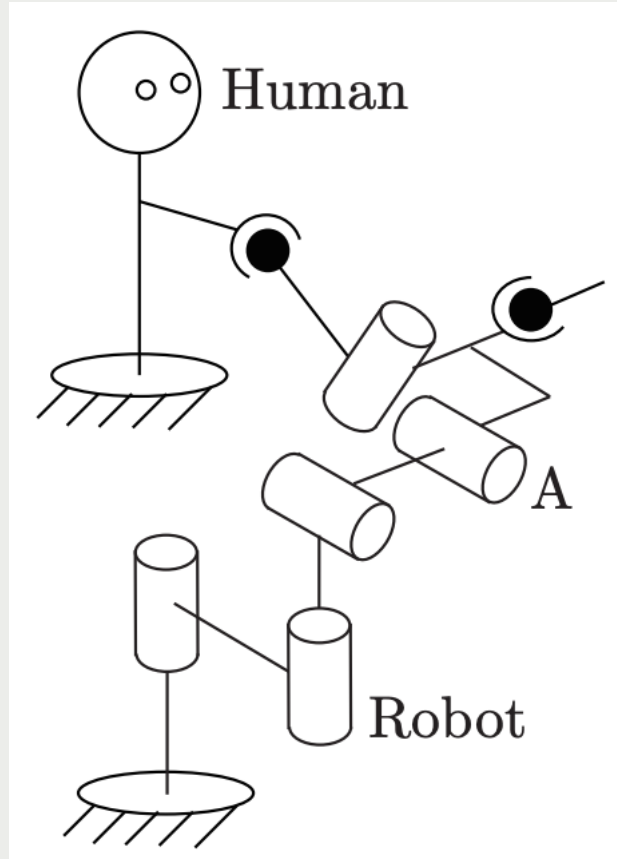
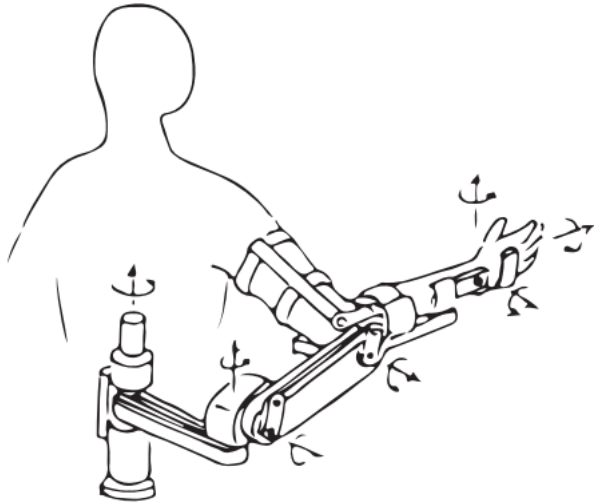
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How many DoF does this system have?

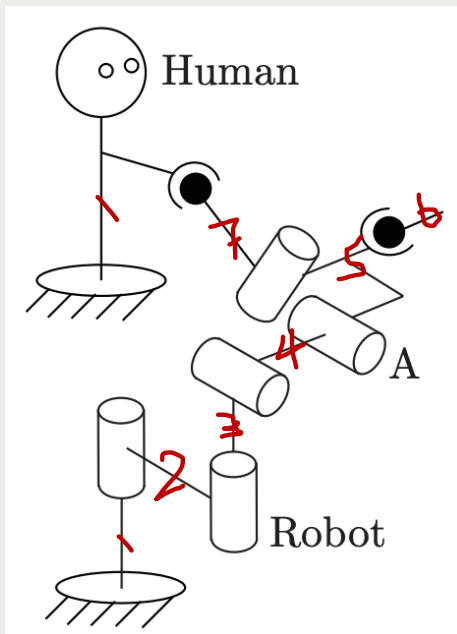


# How many DoF does this system have?

(assume the human is rigid except for the right arm)



# How many DoF does this system have?



$$\text{dof} = \underbrace{m(N - 1)}_{\text{rigid body freedoms}} - \underbrace{\sum_{i=1}^J c_i}_{\text{joint constraints}}$$

$N$  (#bodies) = 7

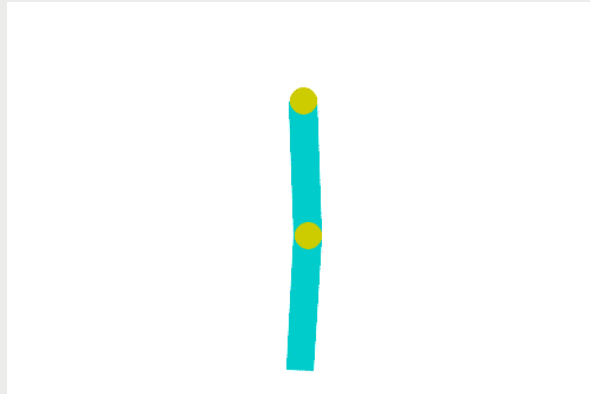
(ground counts as 1 rigid body)

5 Revolute Joints + 2 Spherical Joints

$$\begin{aligned} &= 6 * (7 - 1) - (5 \times 5 + 2 \times 3) \\ &= 36 - 31 = 5 \end{aligned}$$

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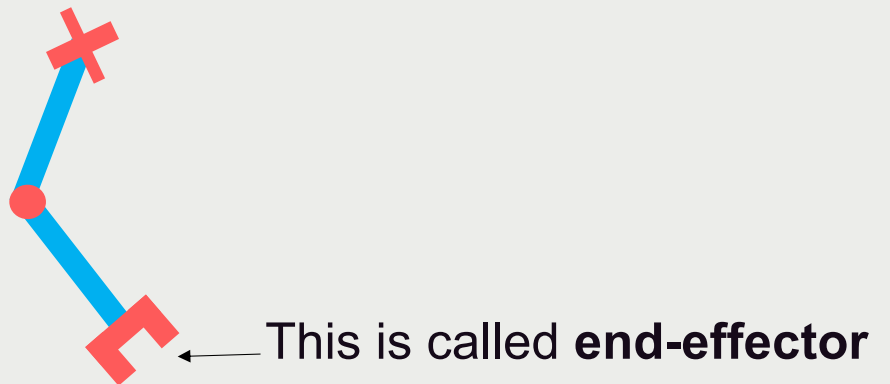
## Acrobot (Double Pendulum)



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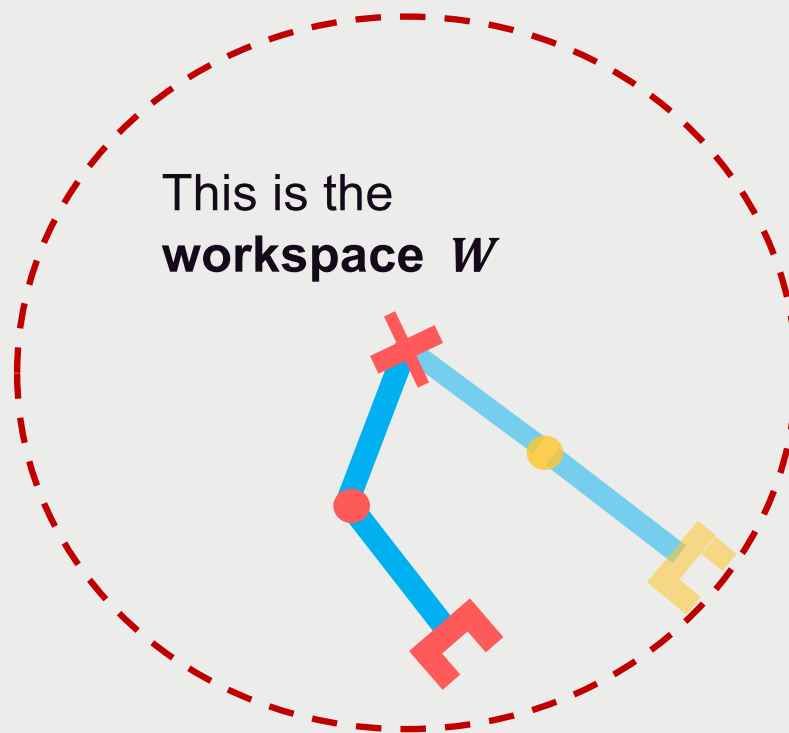
# Acrobot

The entire 2D plane is the **task space**



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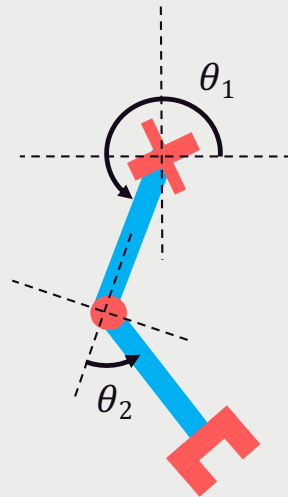
# Acrobot



# Acrobot

$$(\theta_1, \theta_2) \in \mathcal{C}$$

This denotes the  
**configuration space**



# Acrobot

configuration space  $\mathcal{C}$

$$(\theta_1, \theta_2) \in \mathcal{C}$$

**Forward Kinematics**

$$\text{FK}: \mathcal{C} \rightarrow \mathcal{W}$$

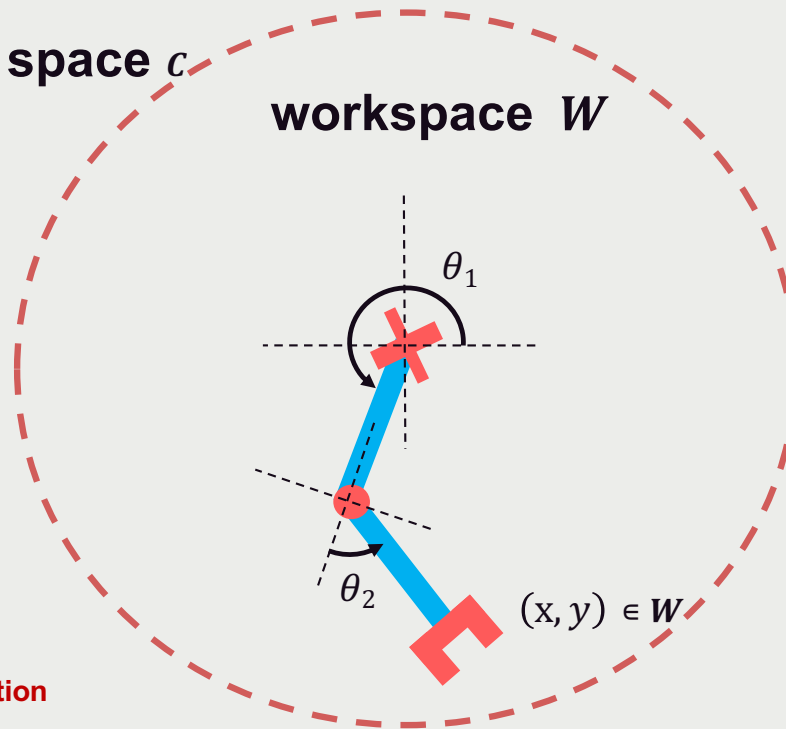
$$\text{FK}((\theta_1, \theta_2)) = (x, y)$$

**Inverse Kinematics**

$$\text{IK}: \mathcal{W} \rightarrow \mathcal{C}$$

$$\text{IK}((x, y)) = (\theta_1, \theta_2)$$

This is not a proper function. Many configuration can lead to the same end-effector pose.

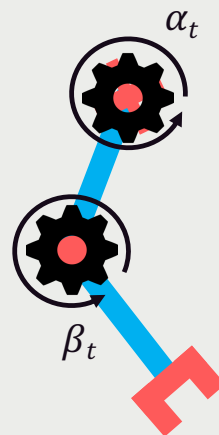


# Acrobot



## Action space

$$(\alpha_t, \beta_t) \in A$$



# Acrobot

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**configuration space  $\mathcal{C}$**

$$(\theta_1, \theta_2) \in \mathcal{C}$$

**Forward Kinematics**

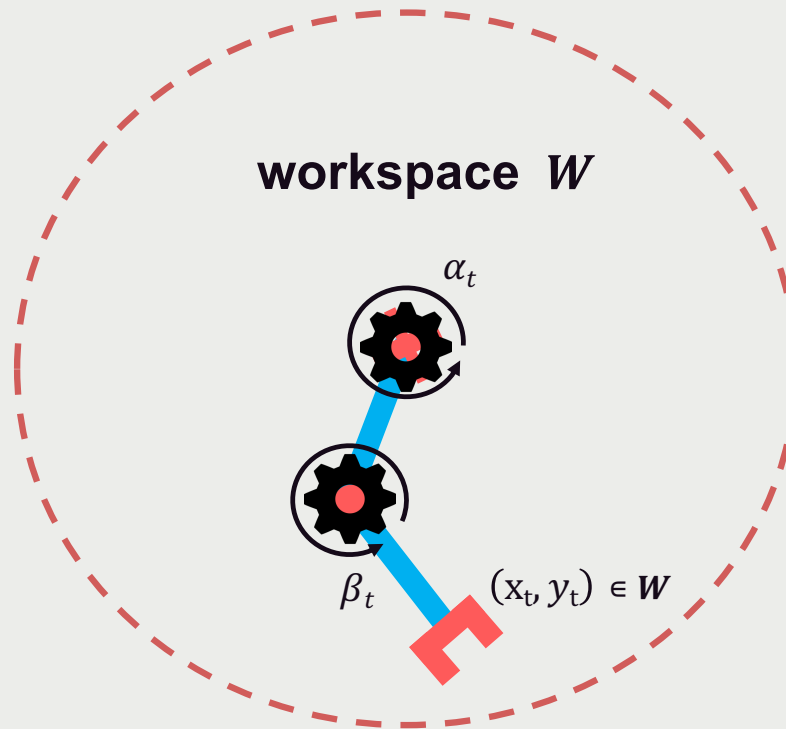
$$\text{FK}: \mathcal{C} \rightarrow \mathcal{W}$$

$$\text{FK}((\theta_1, \theta_2)) = (x, y)$$

**Inverse Kinematics**

$$\text{IK}: \mathcal{W} \rightarrow \mathcal{C}$$

$$\text{IK}((x, y)) = (\theta_1, \theta_2)$$



**Action space**

$$(\alpha_t, \beta_t) \in \mathcal{A}$$

**Forward Dynamics**

$$\text{FD}: \mathcal{A} \rightarrow \mathcal{W}$$

**Inverse Dynamics**

$$\text{ID}: \mathcal{W} \rightarrow \mathcal{A}$$

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How to detect the state of a robot?

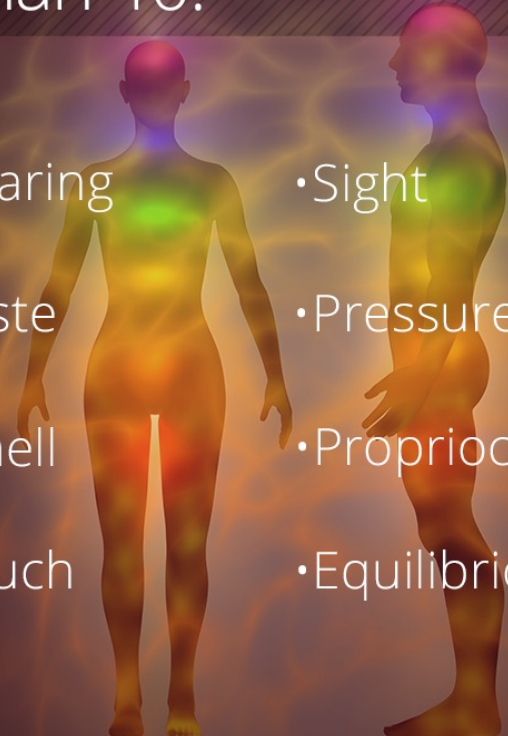
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# Sensors (overview)

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How many senses  
do you have?

Humans have **more than 5 senses**, and probably more than 10.

- 
- Hearing •Sight  
•Taste •Pressure  
•Smell •Proprioception  
•Touch •Equilibrioception

*...and more.*

Source: TestTube Plus, Today I Found Out

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## Common Robot Senses

- Proximity/distance sensors (infrared, ultrasonic)
- Positioning/navigation sensors (GPS, IMU, wheel encoder)
- Visual Sensors (RGB camera, LIDAR)
- Force & Torque sensors
- Temperature sensors
- ...

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**Sensors:** physical devices that provides information about the world

Internal states -> *Proprioceptive* sensors

External states -> *Exteroceptive* sensors

Perception = proprioception + exteroception

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**Sensors:** physical devices that provides information about the world

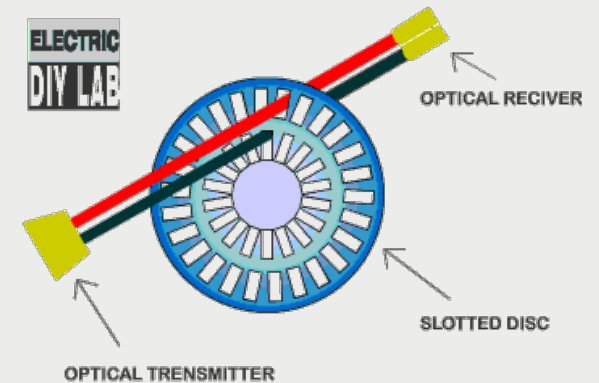
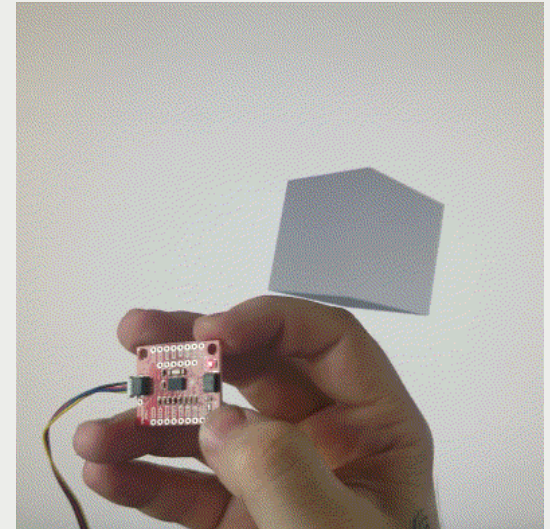
Passive sensors: detects natural energy

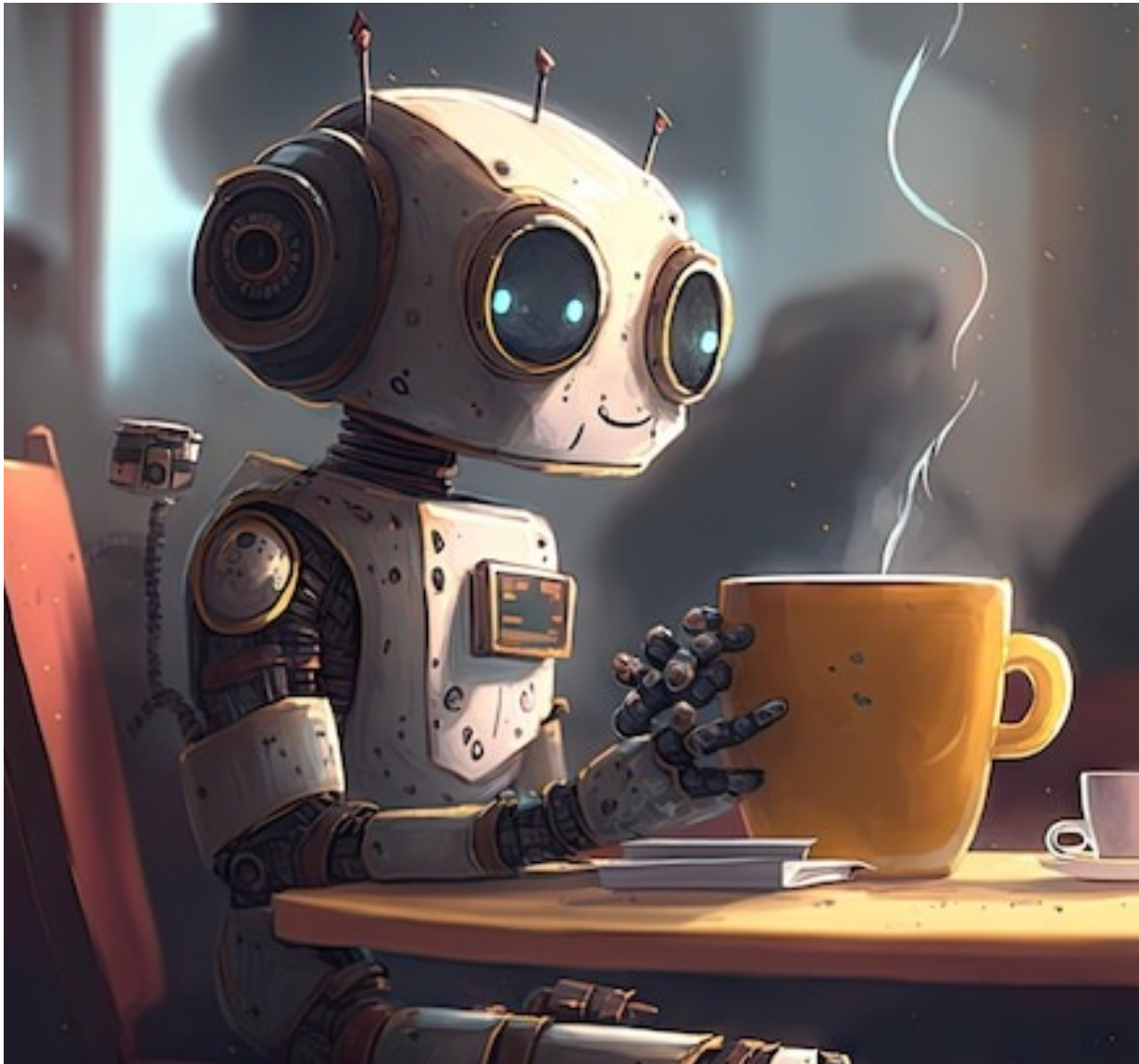
Active sensors: needs power, emits signals and detects feedback

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# Common Robot Sensors

- IMU (Inertial measurement unit)
  - Gyroscope + accelerometer + magnetometer
- Wheel encoder
- LIDAR
- RGB Camera
- Bumper switch
- ...





Break time!

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# Actuators (Motors & Gears)

slides are adapted from USC CSCI 445L

## Action/Actuation

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- A robot acts through its *actuators* (e.g., motors), which typically drive *effectors* (e.g., wheels, grippers)
- Robotic actuators are very different from biological ones, both are used for:
  - *locomotion* (moving around, going places)
  - *manipulation* (handling objects)
- This divides robotics into two areas
  - *mobile* robotics
  - *manipulator* robotics



## Definition of Effector

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- An effector is any device that has an effect on the environment.
- A robot's effectors are used to purposefully create an effect on the environment.
- E.g., legs, wheels, arms, fingers...
- *The role of the controller is to get the effectors to produce the desired effect on the environment, based on the robot's task.*

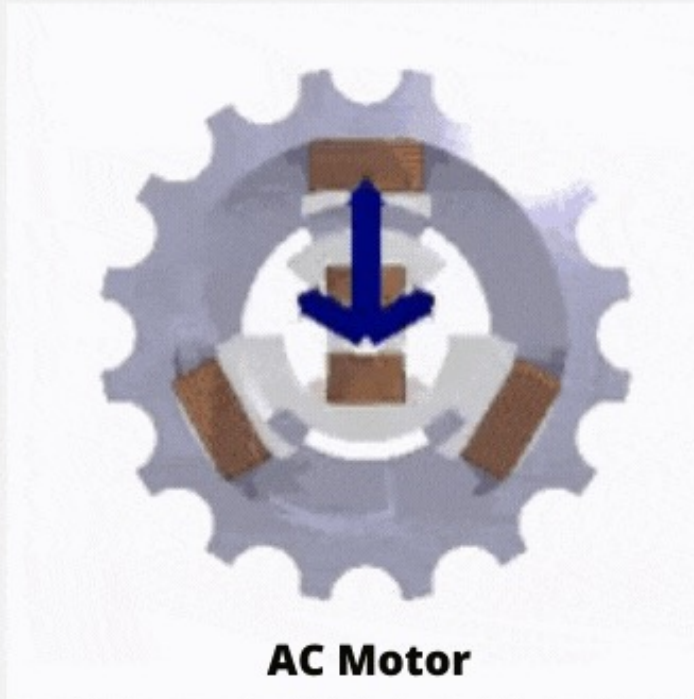
## Definition of Actuator

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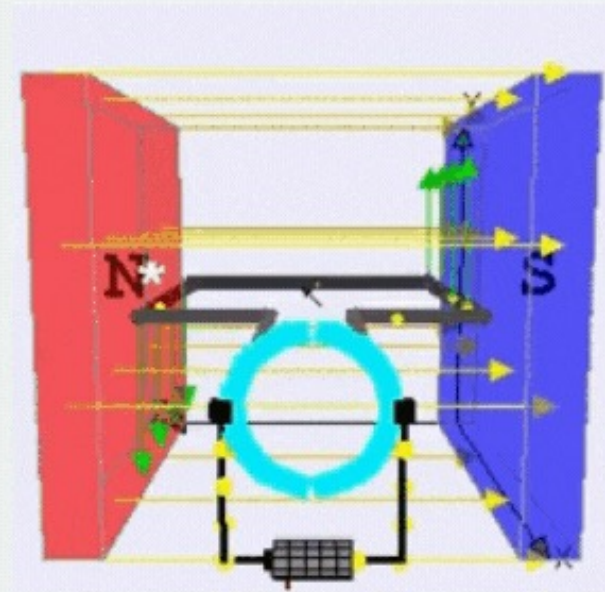
- An actuator is the mechanism that enables the effector to execute an action.
- E.g., electric motors, hydraulic or pneumatic cylinders, pumps...
- Actuators and effectors are not the same thing.

## Electric Motors

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Hard to control speed directly  
Cheaper and more durable -> Common in household appliances, HVAC, pumps, and fans

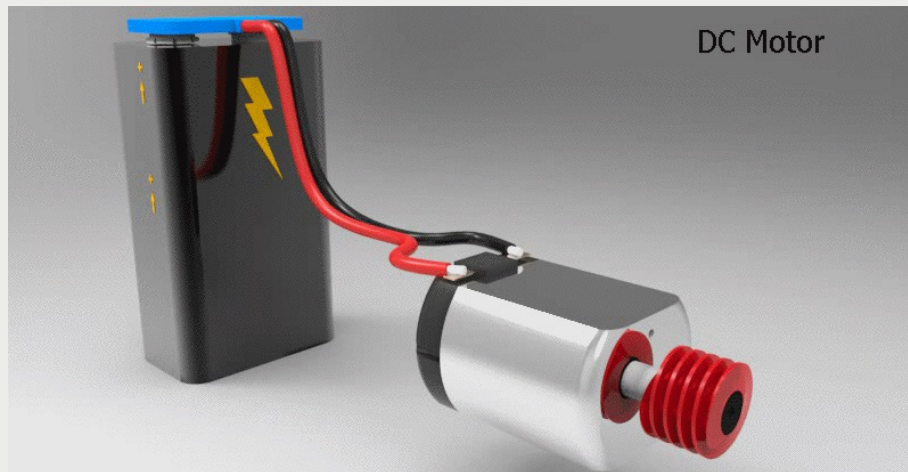


Speed is controlled by voltage  
Expensive and less durable, but better for precise control

## DC Motors

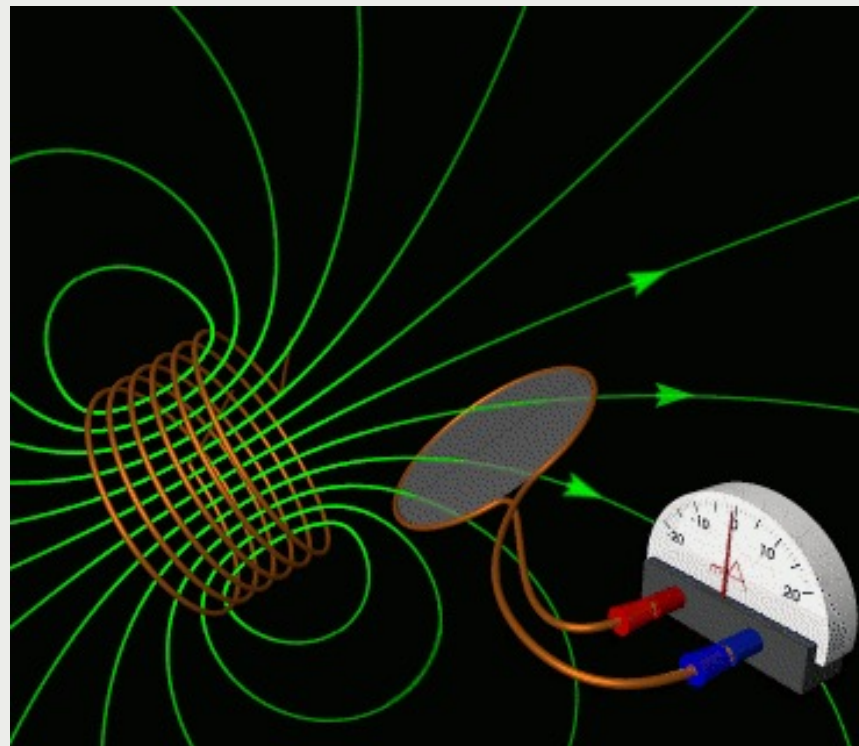
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- The most common actuator in mobile robotics is the direct current (DC) motor
- Advantages: simple, cheap, various sizes and packages, easy to interface, clean.
- DC motors convert electrical into mechanical energy



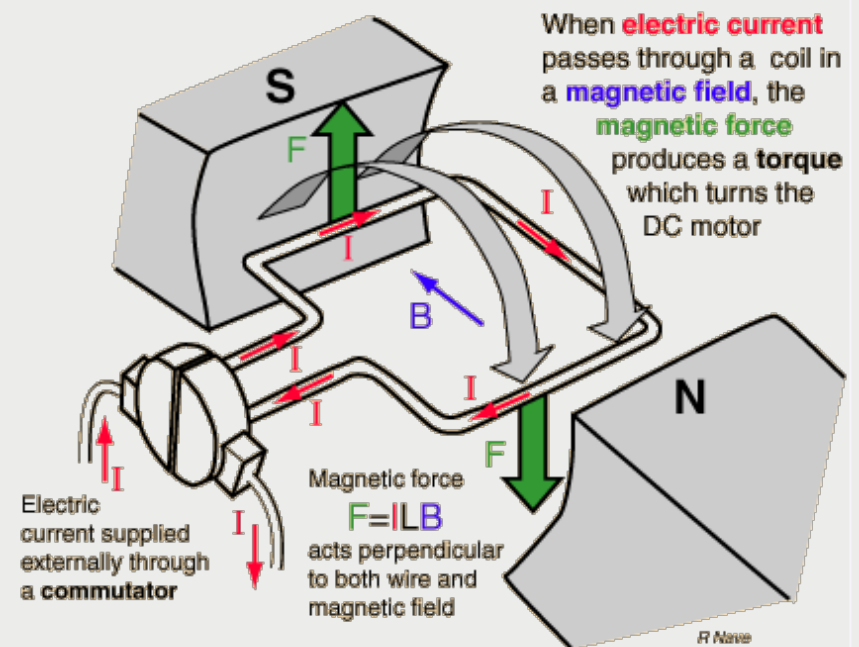
## How Motors Work

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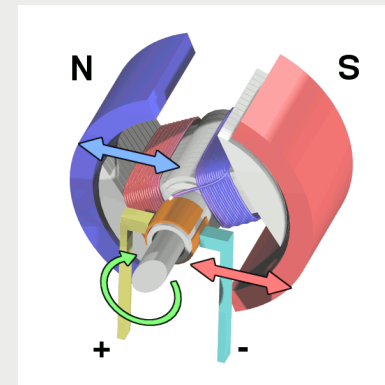
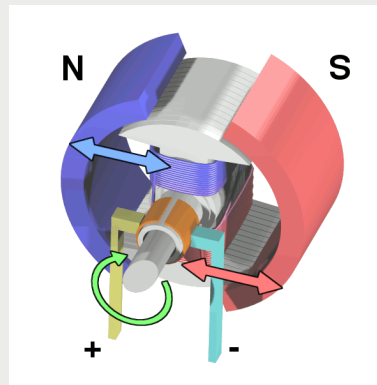
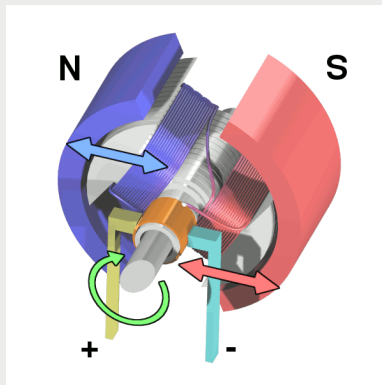
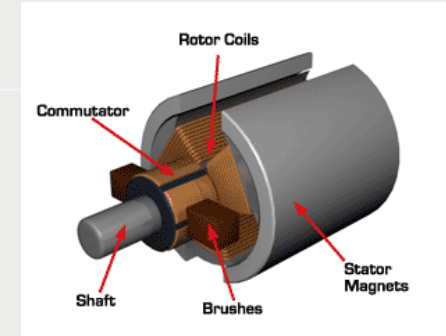
## How DC Motors Work

- DC motors consist of permanent magnets with loops of wire inside
- When current is applied, the wire loops generate a **magnetic field**, which reacts against the outside field of the static magnets
- The interaction of the fields produces the movement of the shaft/armature
- A **commutator** switches the direction of the current flow, yielding continuous motion



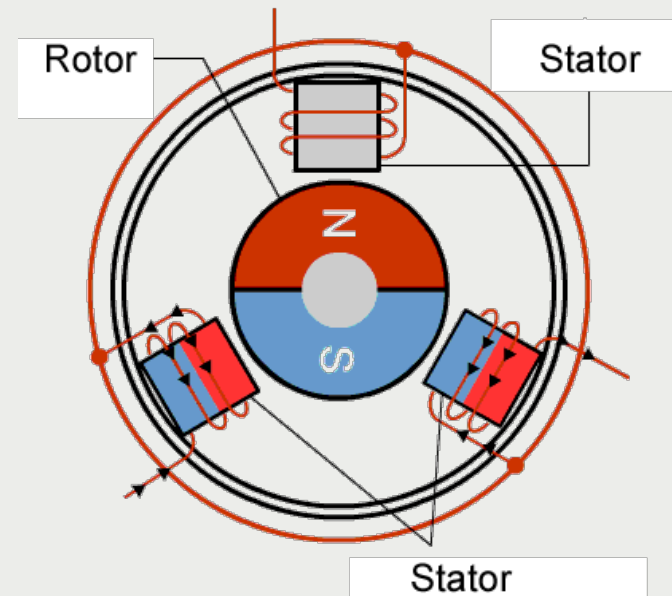
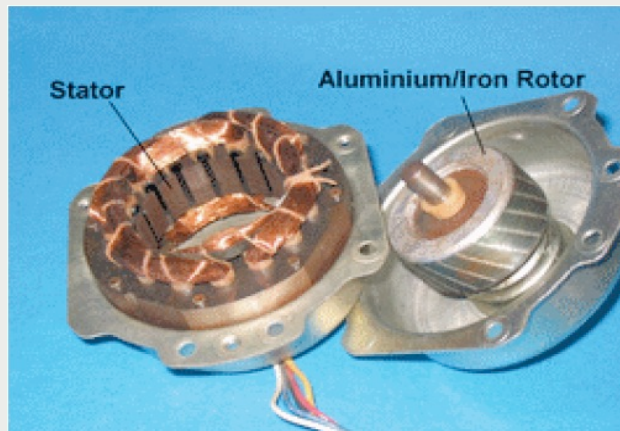
# Types of DC motors

- Brushed motors (mechanical commutation)
  - Low-voltage, low-torque, cheap



## Types of DC motors

- Brushless motors (electronic commutation)
  - High-voltage, high-torque, expensive
  - No friction / wear of brushes



## Motor Efficiency

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- As any physical system, DC motors are not perfectly efficient
- Energy is not converted perfectly; some is wasted as heat generated by winding resistance and friction
- Inefficiencies are minimized in well-designed (more expensive) motors, and their efficiency can be high.

## Motor Efficiency

- 
- Good DC motors can be made to have efficiencies in the 90<sup>th</sup> percentile
  - Cheap DC motors can be as low as 50%
  - Other types of effectors, such as miniature electrostatic motors, may have much lower efficiencies still

## Speed and Torque

- Motor speed  $\omega$  is proportional to induced voltage  $V$

$$\omega = k_v V$$

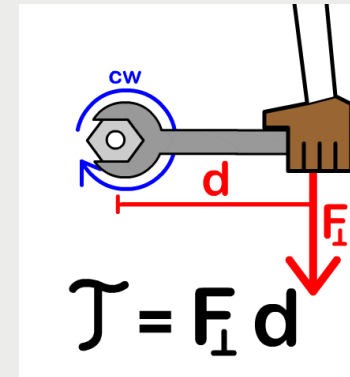
- Torque is a force that acts in a rotational manner

$$\mathbf{t} = \mathbf{r} \times \mathbf{F}$$

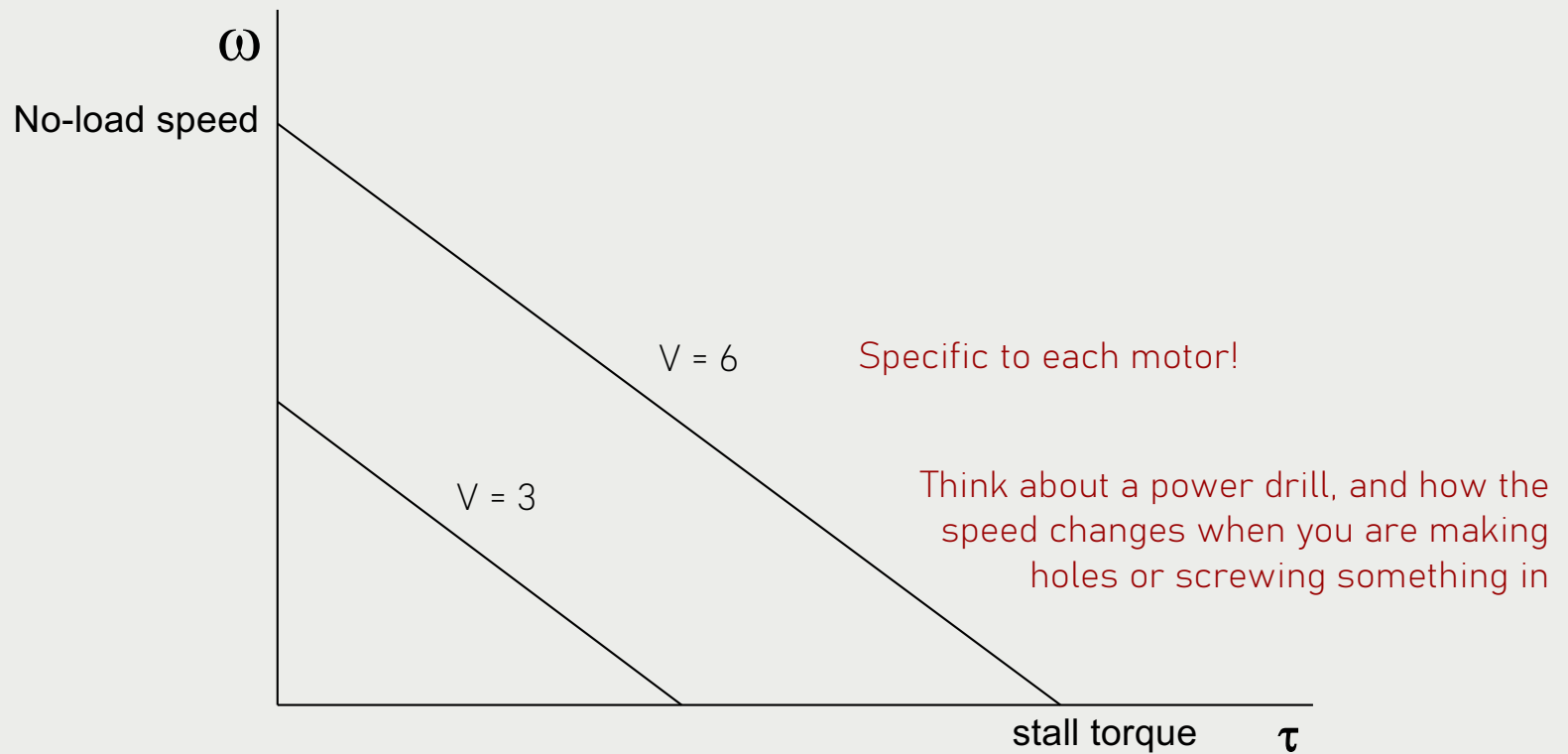
- Motor torque  $t$  is proportional to applied current  $I$ :

$$t = k_t I$$

- Motors have a maximum speed (no-load speed) and a maximum torque (stall torque)

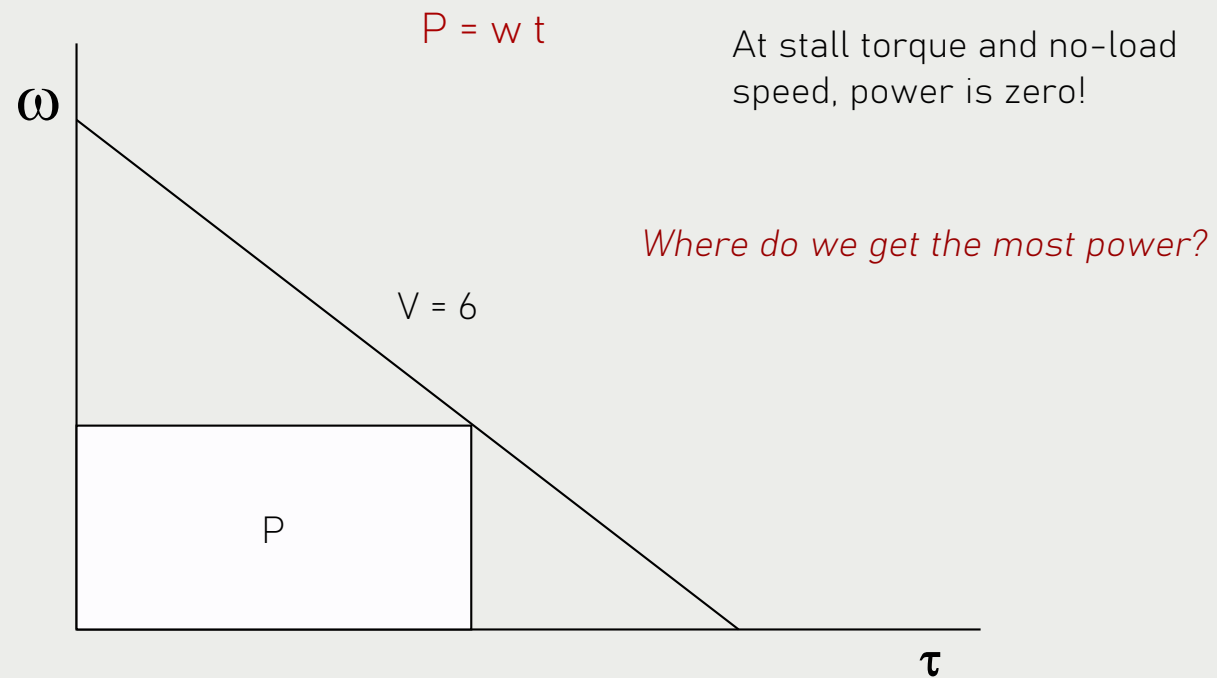


# Speed/Torque Relationship



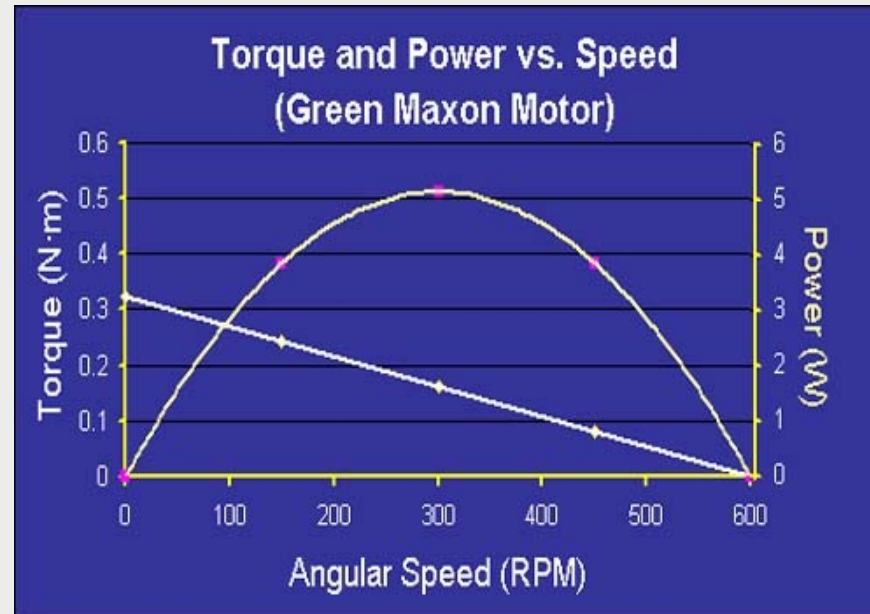
# Motor power

- Output power is the product of speed & torque:



## Power as a function of $\tau$ , $\omega$

$$P_{motor}(\omega) = -\frac{\tau_s}{\omega_n}\omega^2 + \tau_s\omega \quad P_{motor}(\tau) = -\frac{\omega_n}{\tau_s}\tau^2 + \omega_n\tau$$



## Operating voltage and speed

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- Motors have maximum voltage rating
  - Higher voltages may overheat windings
- Motors have maximum speed rating
  - Higher speeds may destroy bearings or commutator
- Operating motors at higher speeds/voltages will reduce their life expectancy

## DC motors and Robots

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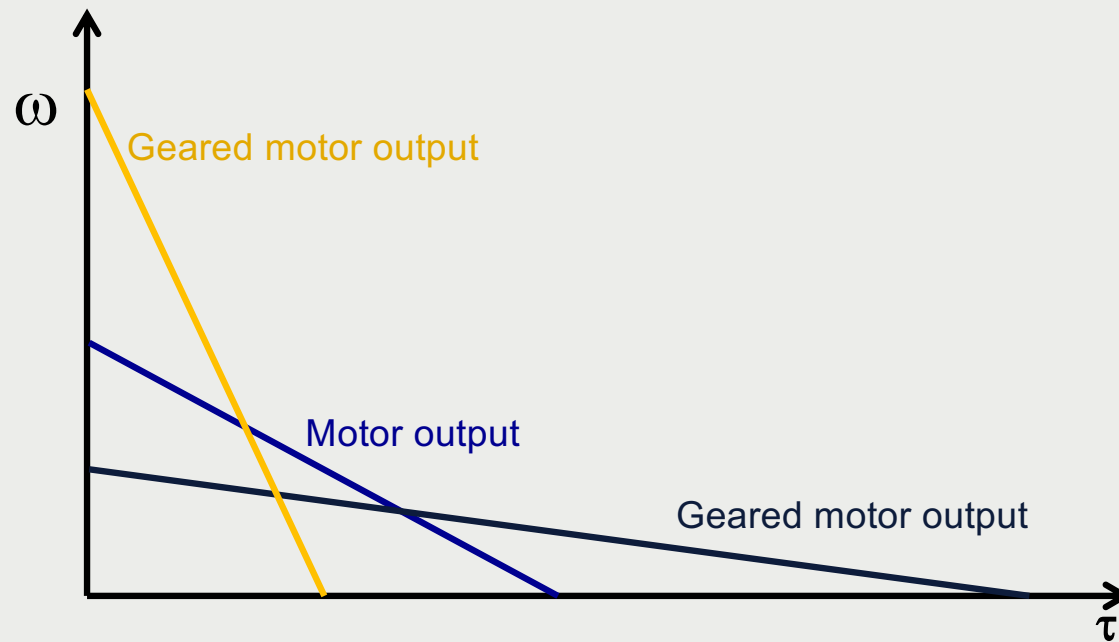
- DC motors have high-speed, low torque
- Typical speed range:
  - 9,000 to 12,000 RPM
  - 150 to 200 Hz
- Robots require low-speed, high torque

*What do we do about this?*

Motors must be geared!

## Gearing

- Gears are used to alter the output speed/torque of a motor (slope of speed/torque graph)



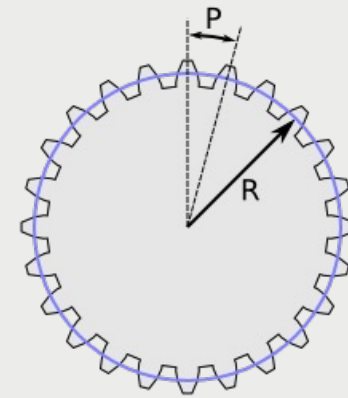
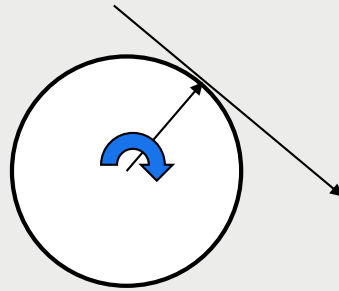
# Gear Fundamentals

- The force  $F$  at the edge of a gear of radius  $r$  is given by:

$$F = t / r$$

- The linear speed  $v$  at the edge of a gear of radius  $r$  is given by:

$$v = \omega r$$



## Combining Gears

- Meshing gears have equal linear speeds:

$$v_1 = v_2$$

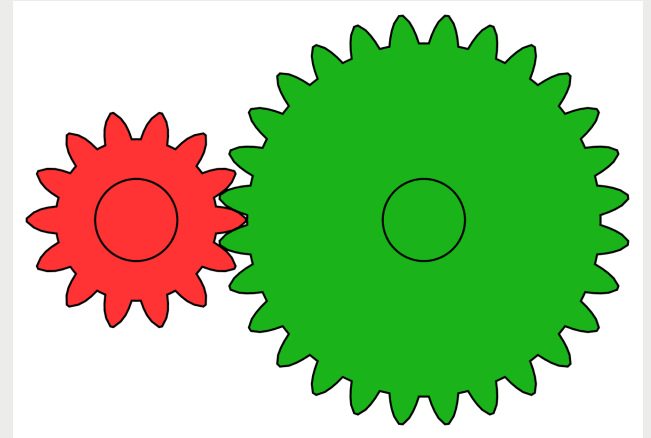
- Thus the output speed is:

$$v = \omega r \qquad \omega_2 = (r_1 / r_2) \omega_1$$

- And the output torque is:

$$F = \tau / r \qquad t_2 = (r_2 / r_1) t_1$$

- $r_2 / r_1$  is known as the *gear ratio*



## Examples

- Gearing down:

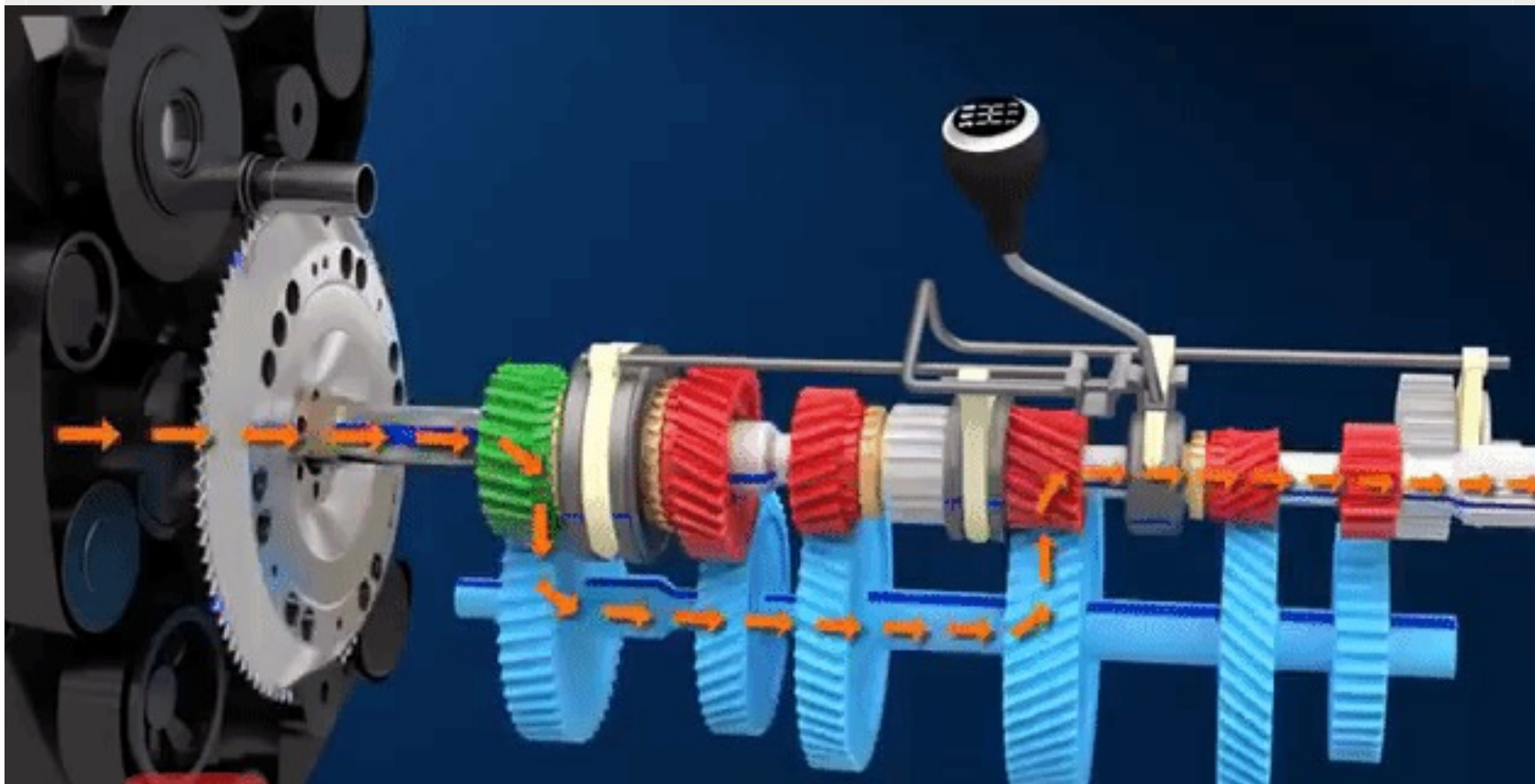
$$r_1 = 1, r_2 = 2$$

- 2:1 gear ratio doubles the torque and halves speed

- Gearing up:

$$r_1 = 2, r_2 = 1$$

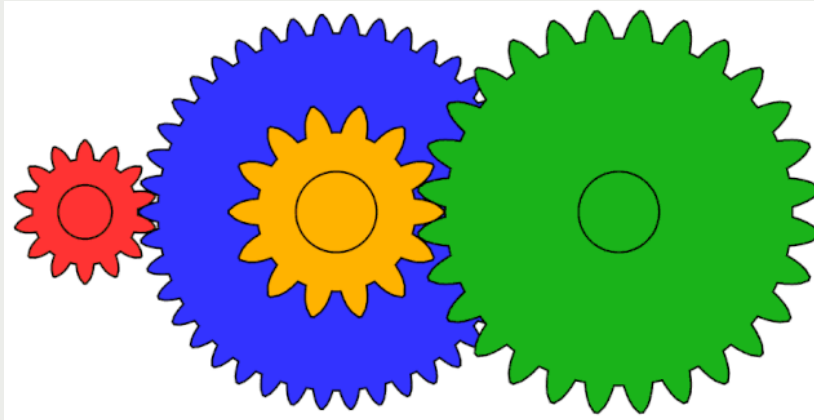
- 1:2 gear ratio halves torque and doubles speed



## Gear Stages

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- Usually, it is not possible to achieve a sufficient gear ratio with a single pair of gears
- Gears can be arranged *in stages*
- The total gear ratio is the product of gear ratios for each stage
  - E.g.:  $3:1 \times 3:1 = 9:1$

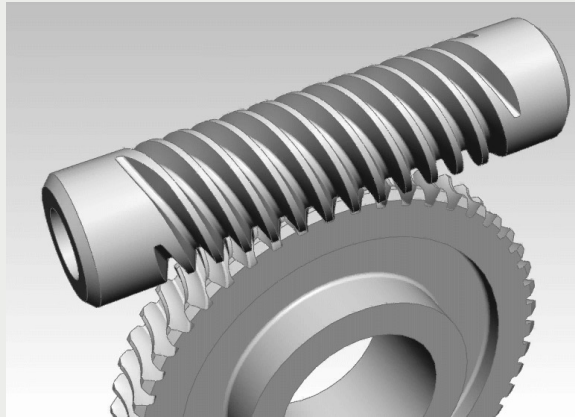


## Types of Gears

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Spur



Worm

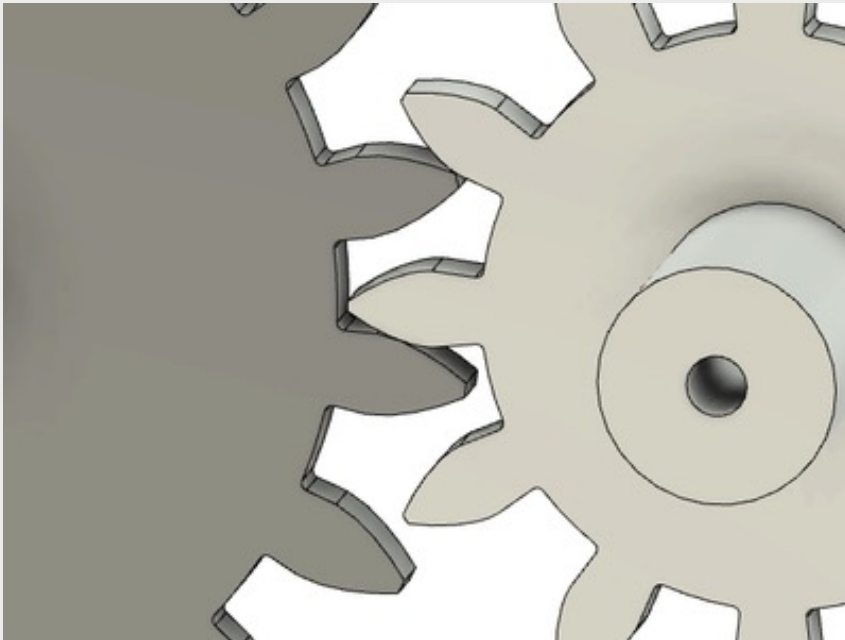


Planetary

## Backlash

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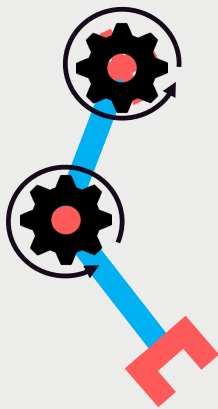
Simple gears suffer from *backlash* (teeth not meshing completely)



Backlash is the  
enemy of precision!

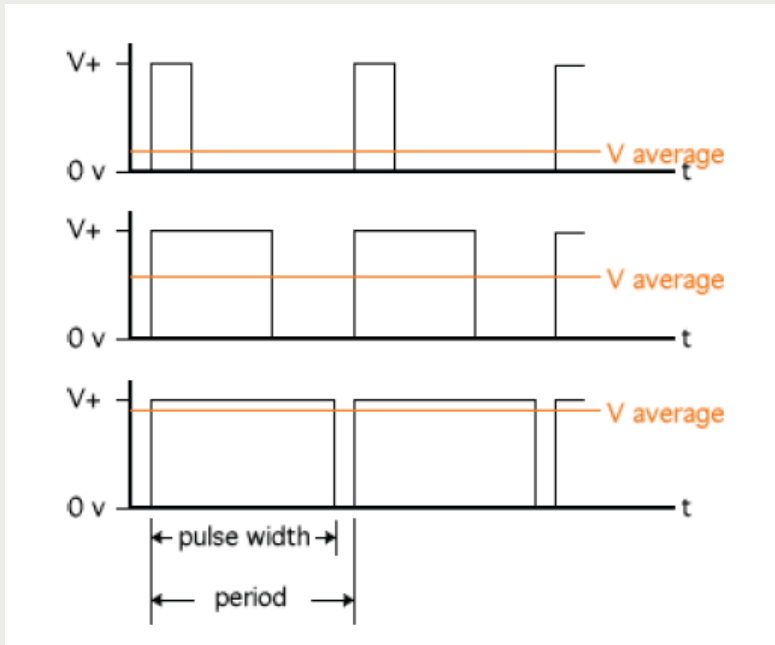
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## Control of Motors



## Controlling Speed: Pulse Width Modulation (PWM)

$$\omega = k_V V_{\text{average}}$$



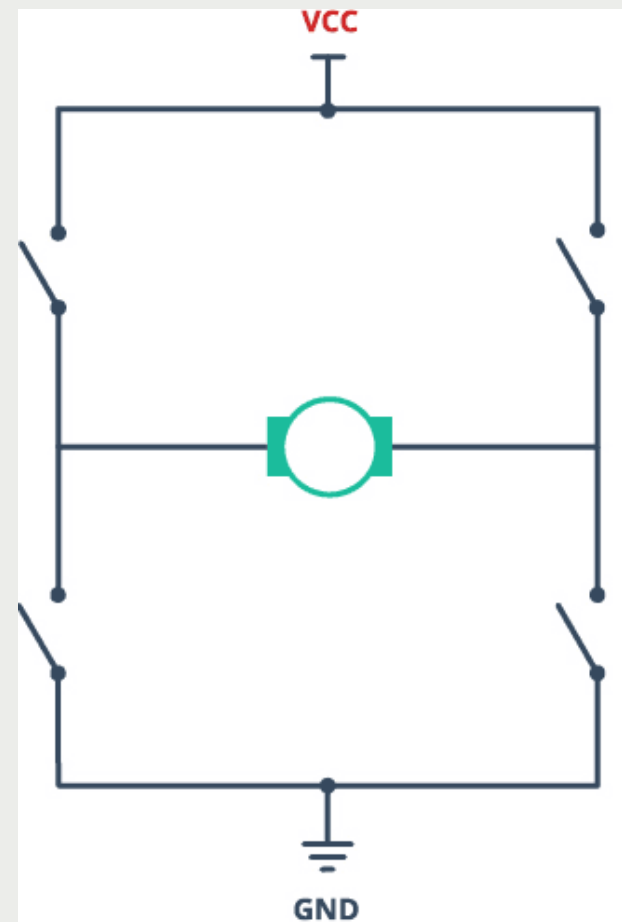
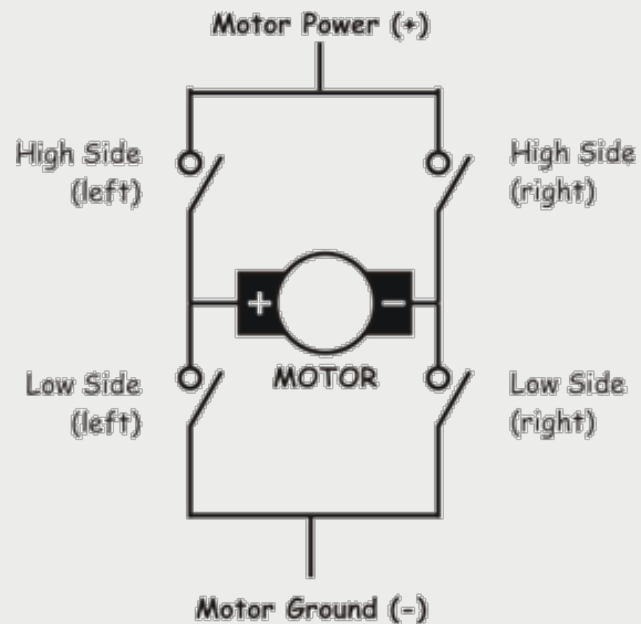
What is the duty cycle?

Percentage of one period in which a signal is active.

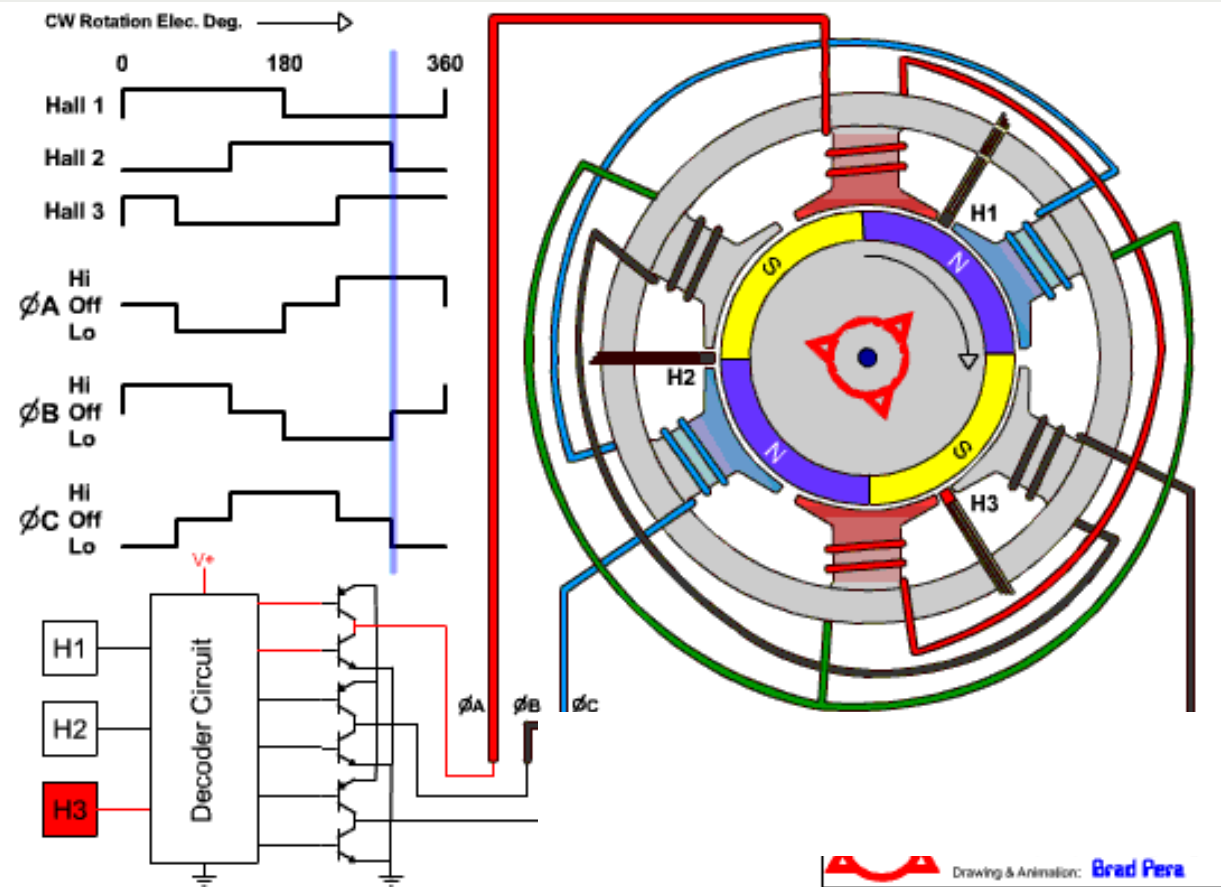
## Controlling Speed: Pulse Width Modulation (PWM)

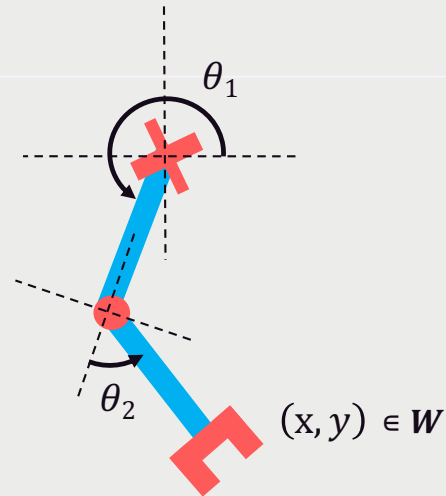


## Controlling direction: H-Bridge

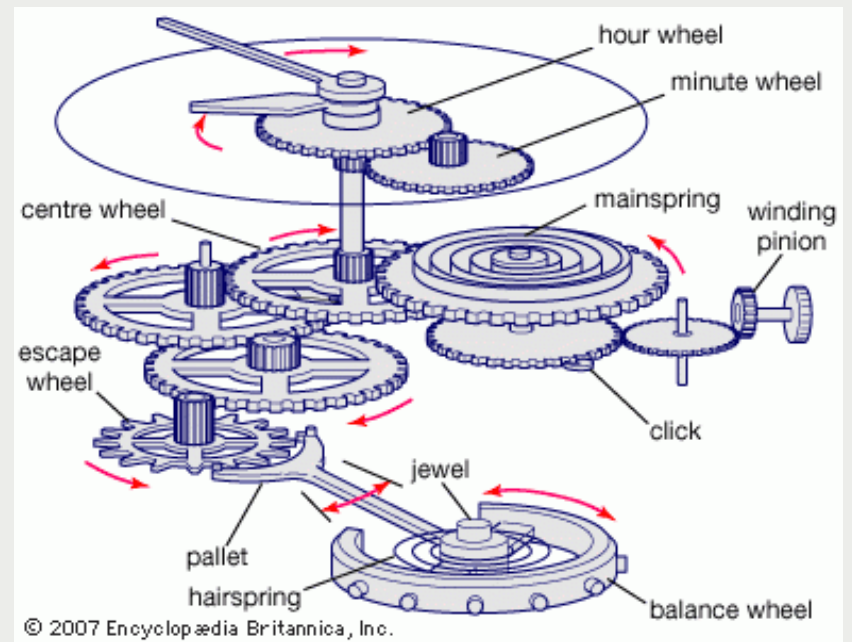
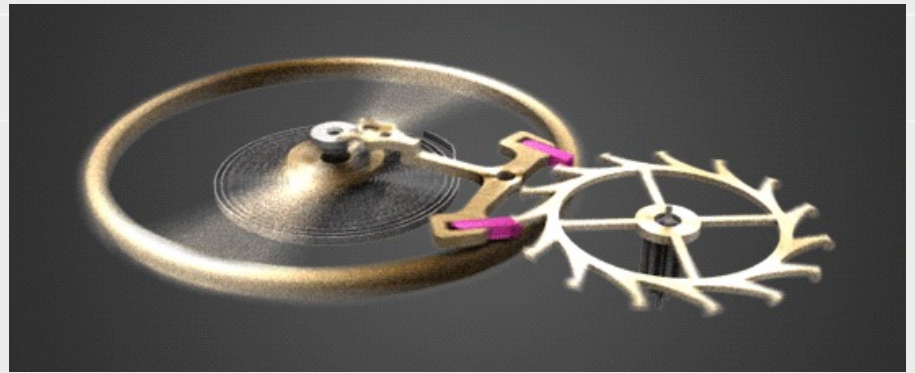


# Control of Motors





- For precise control, we want to move a joint to a specific position
- DC motors are not built for this



<https://www.youtube.com/watch?v=9Fn8X272m7w>

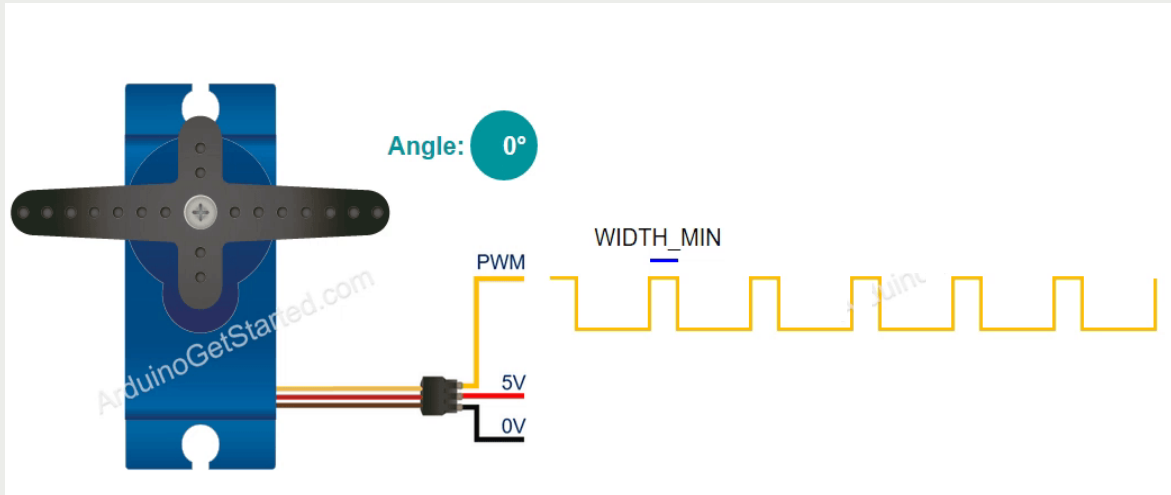
## Servo Motors

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- Servo motors are adapted DC motors:
  - gear reduction
  - position sensor (encoder, potentiometer)
  - electronic controller
- Range of at least 180 degrees



## PWM Position Control



Not defined by PWM duty cycle but only *duration* of the pulse!

Pulse width must be very accurate

Noise in width => noise in position

Pulse rate may be variable

Noise in rate => no change



That's it for today!

Questions?