
COM SCI 188

Intro to Robotics

Lecture 2

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Winter 2026

Agenda

- Announcements
- Recap: what is a robot?
- Configuration Space & Degrees of Freedom
- Sensors (overview)
- Actuators (motors & gears)

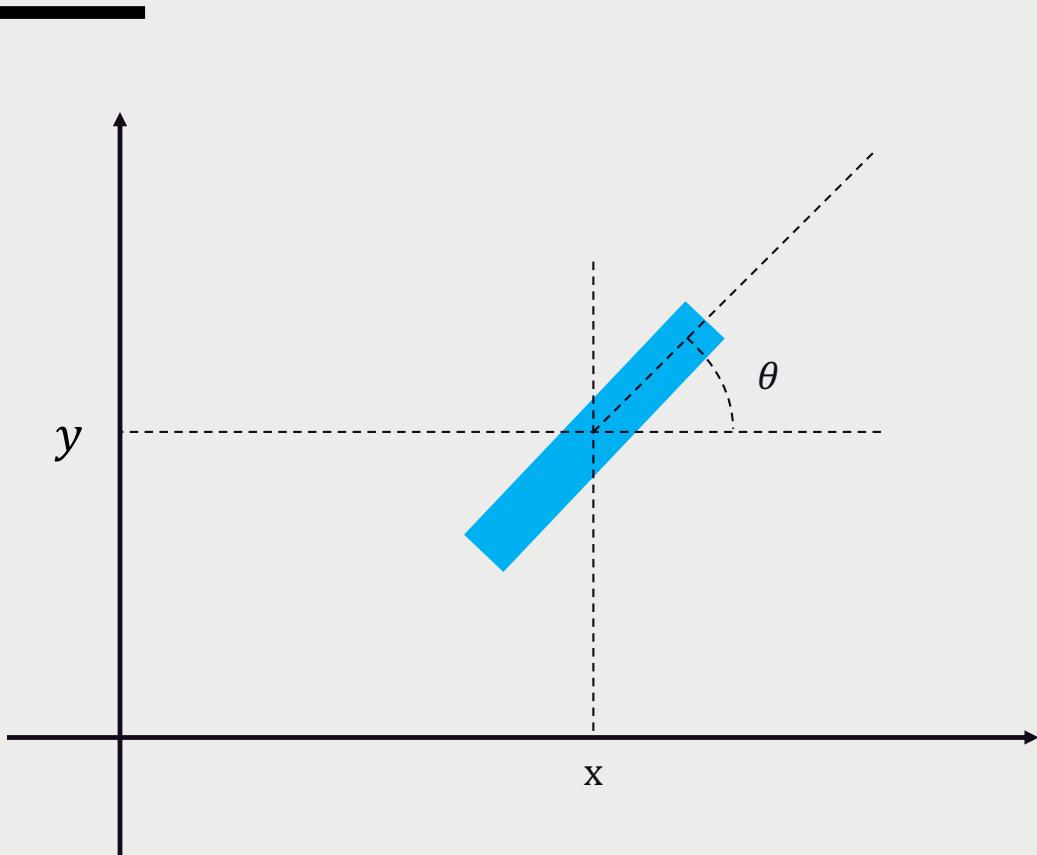
Announcements

- This Week's Discussion Section: Robosuite Installation
 - Read the instructions on Piazza!
- Problem Set 1 is out, Due next Friday
 - PTEs have been assigned

Recap: Definition of Robots Sense, Plan, Act

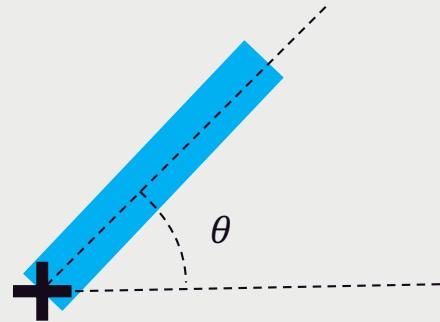


How to describe the state of a robot?



How many variables do we need to fully describe the configuration of this **rigid body** in 2D?

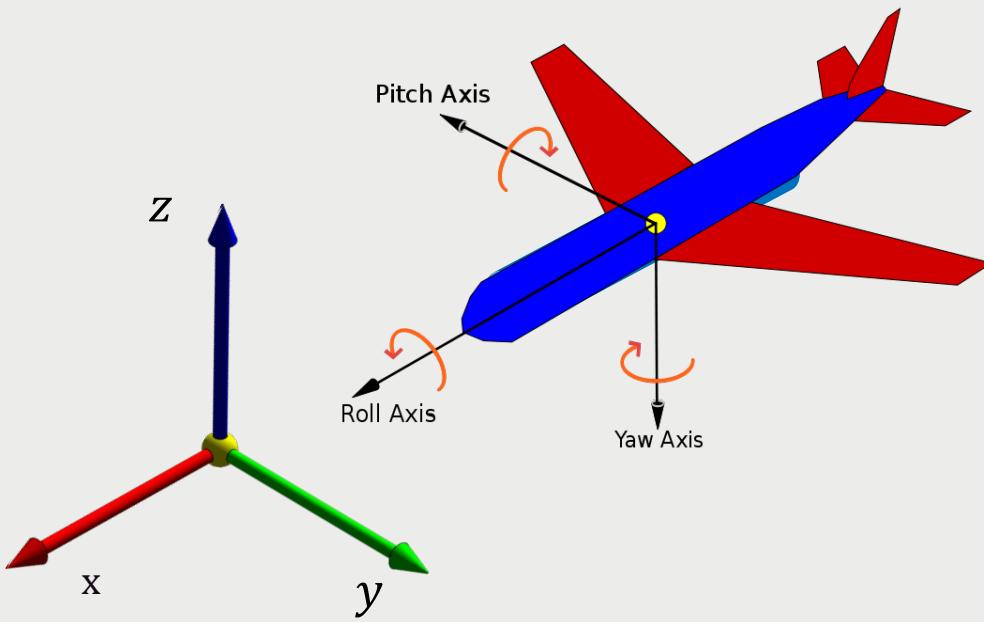
Answer: $(x, y, \theta) = 3$



What if one of the end points is fixed?

Answer: $(\theta) = 1$

This is called the **Degree of Freedom (DoF)** of a rigid body

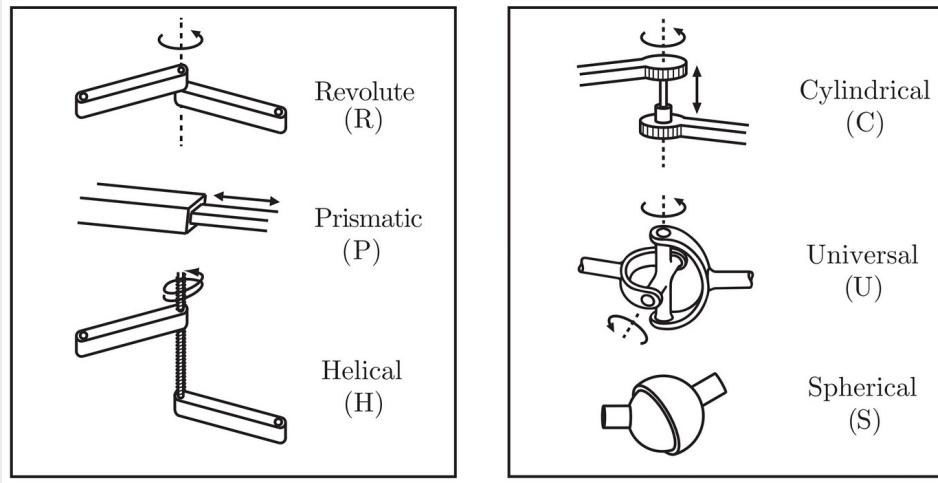


Rigid body in 3D space:
How many DoF does it have?

$6 = 3$ for position + 3 for orientation

Joints and DoF

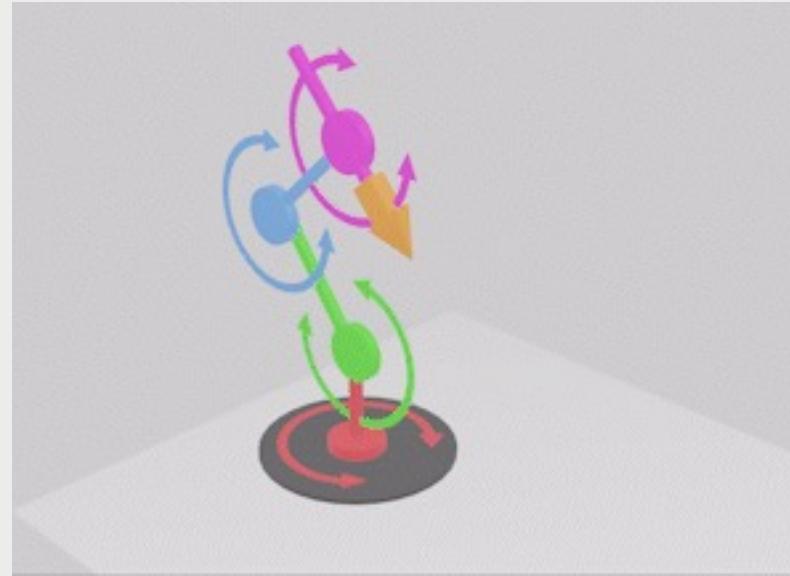
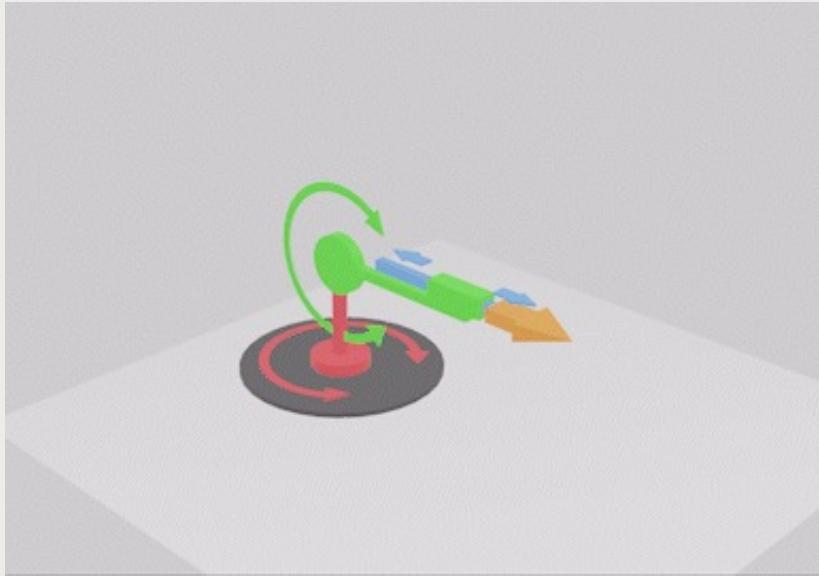
Common Joints



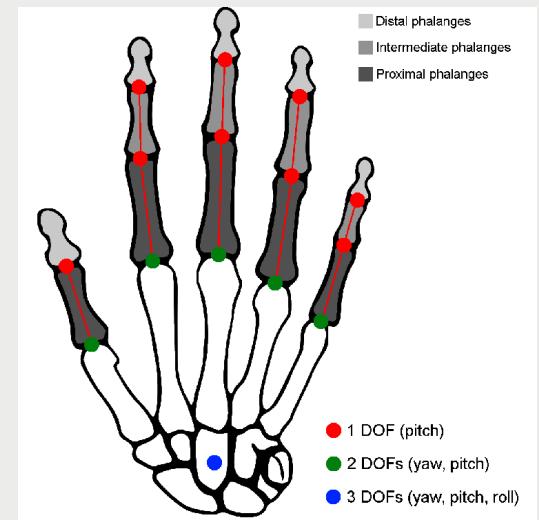
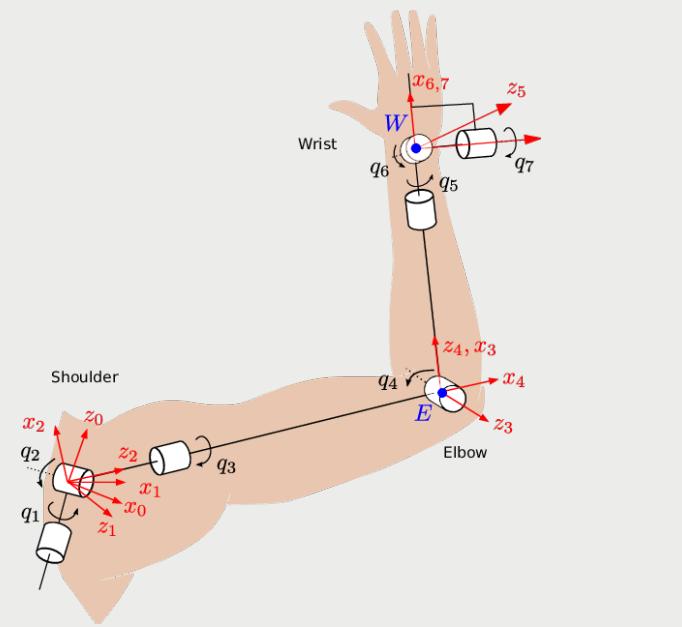
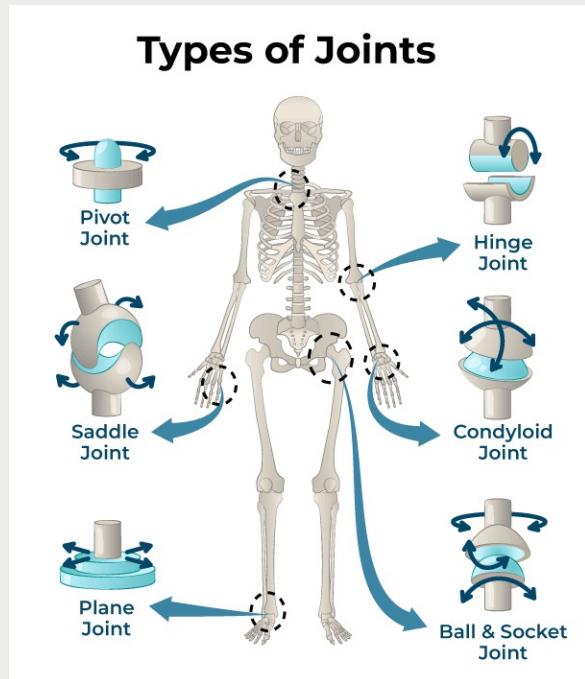
Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

MODERN ROBOTICS
Kevin M. Lynch and Frank C. Park May 3, 2017

Joints and DoF



How many DoF do you have?



Whole body: ~30 DoF (major joints)

Images from Google search.

Grübler's formula

$$\begin{aligned}
 \text{dof} &= \underbrace{m(N - 1)}_{\text{rigid body freedoms}} - \underbrace{\sum_{i=1}^J c_i}_{\text{joint constraints}} \\
 &= m(N - 1) - \sum_{i=1}^J (m - f_i) \\
 &= m(N - 1 - J) + \sum_{i=1}^J f_i.
 \end{aligned}$$

Joint type	dof f	Constraints c between two planar rigid bodies	Constraints c between two spatial rigid bodies
Revolute (R)	1	2	5
Prismatic (P)	1	2	5
Helical (H)	1	N/A	5
Cylindrical (C)	2	N/A	4
Universal (U)	2	N/A	4
Spherical (S)	3	N/A	3

$m = 3$ for planar mechanism (2D)

$m = 6$ for a spatial mechanism (3D)

MODERN ROBOTICS

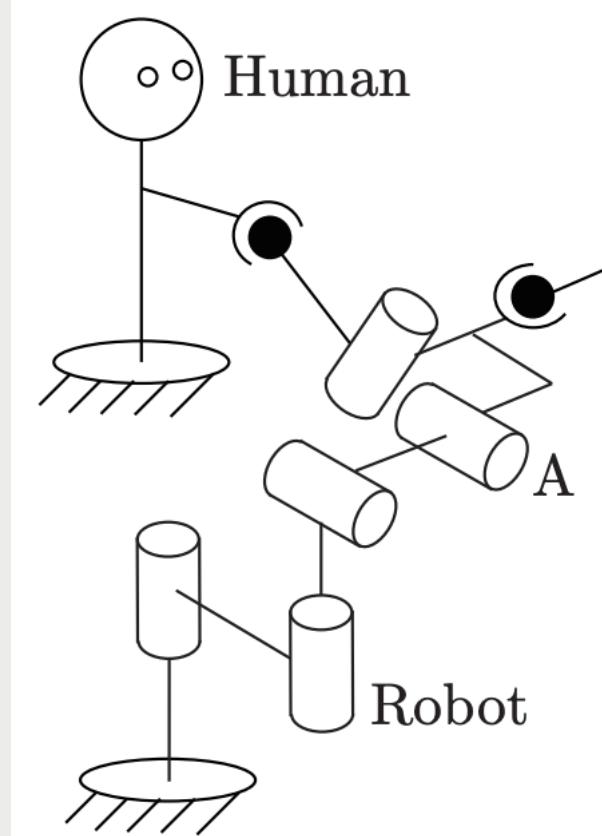
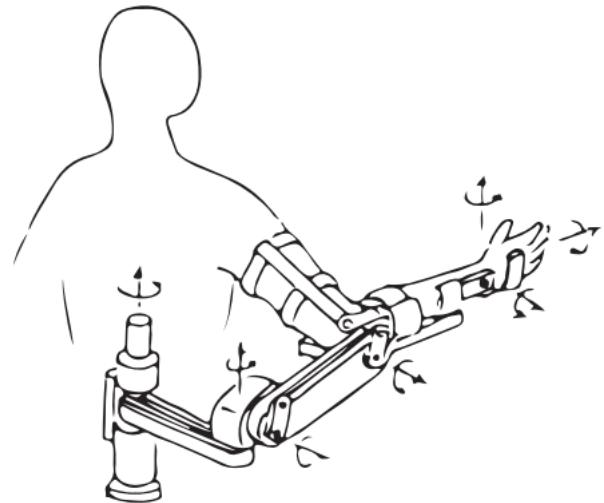
Kevin M. Lynch and Frank C. Park May 3, 2017

How many DoF does this system have?

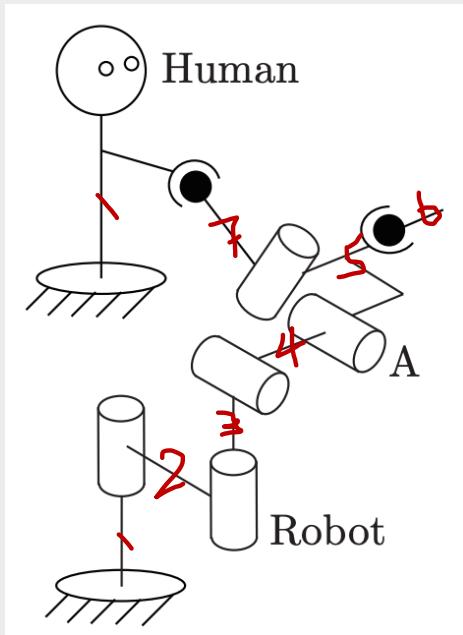


How many DoF does this system have?

(assume the human is rigid except for the right arm)



How many DoF does this system have?



$$\text{dof} = \underbrace{m(N - 1)}_{\text{rigid body freedoms}} - \underbrace{\sum_{i=1}^J c_i}_{\text{joint constraints}}$$

$$N (\# \text{bodies}) = 7$$

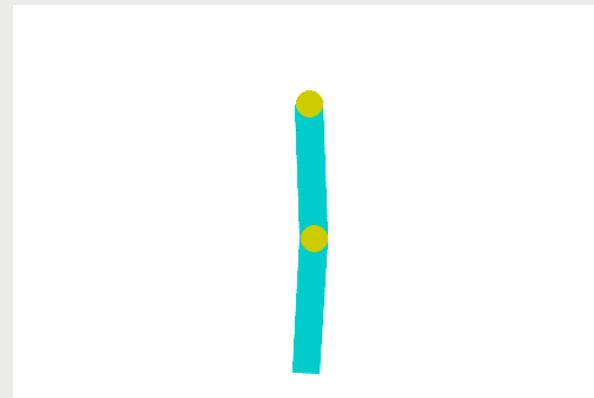
(ground counts as 1 rigid body)

5 Revolute Joints + 2 Spherical Joints

$$= 6 * (7 - 1) - (5 * 5 + 2 * 3)$$

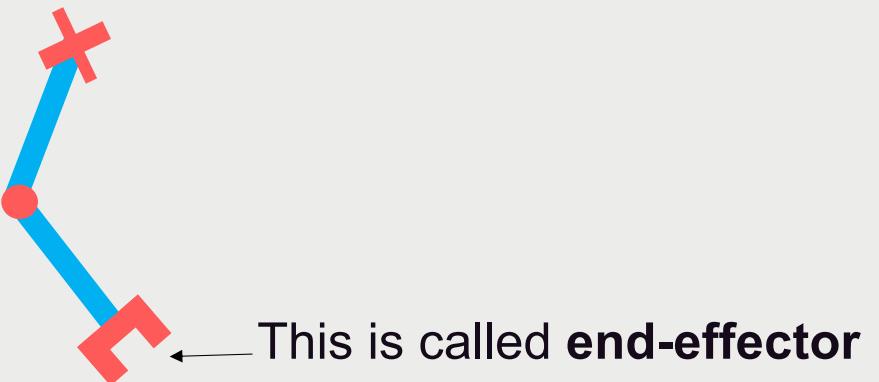
$$= 36 - 31 = 5$$

Acrobot (Double Pendulum)

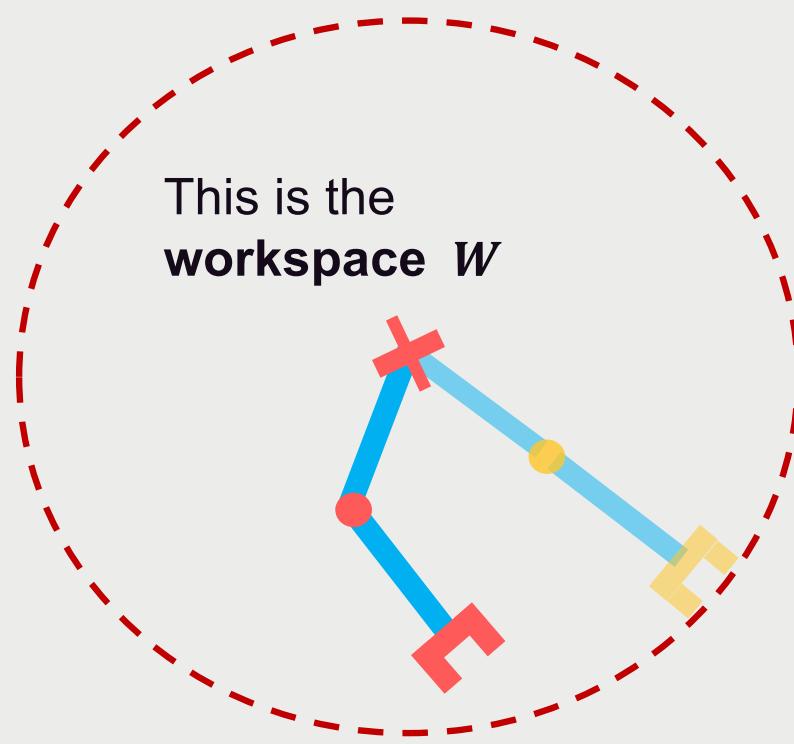


Acrobot

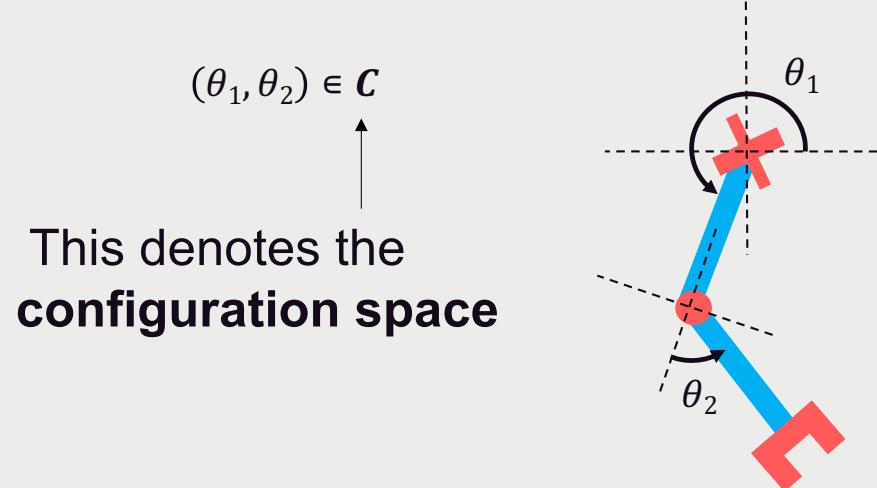
The entire 2D plane is the **task space**



Acrobot



Acrobot



This denotes the
configuration space

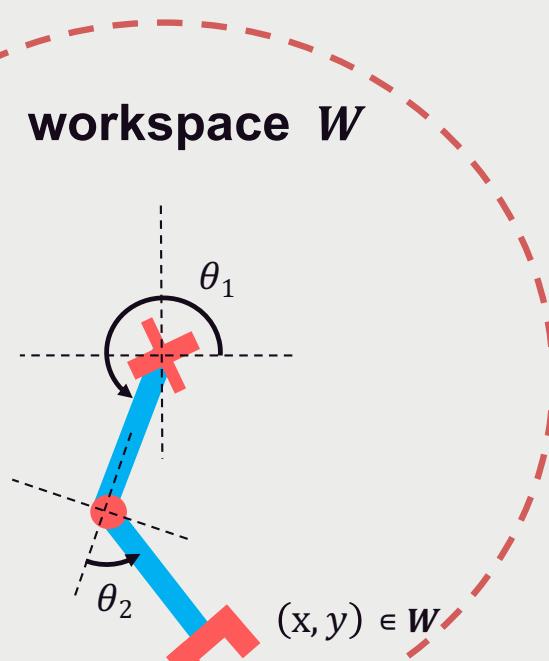
Acrobot

configuration space C
 $(\theta_1, \theta_2) \in C$

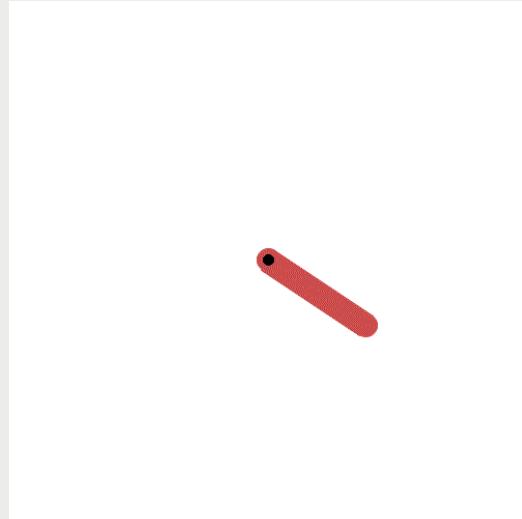
Forward Kinematics
FK: $C \rightarrow W$
 $FK((\theta_1, \theta_2)) = (x, y)$

Inverse Kinematics
IK: $W \rightarrow C$
 $IK((x, y)) = (\theta_1, \theta_2)$

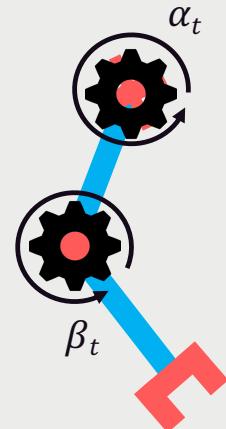
This is not a proper function. Many configuration
can lead to the same end-effector pose.



Acrobot



Action space
 $(\alpha_t, \beta_t) \in A$



Acrobot

configuration space C

$$(\theta_1, \theta_2) \in C$$

Forward Kinematics

$$\text{FK}: C \rightarrow W$$

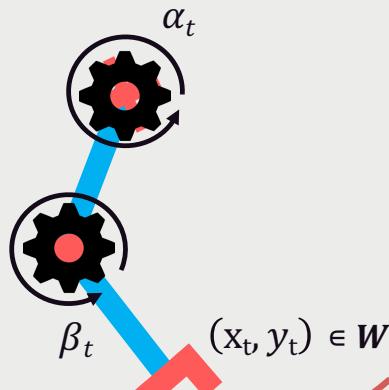
$$\text{FK}((\theta_1, \theta_2)) = (x, y)$$

Inverse Kinematics

$$\text{IK}: W \rightarrow C$$

$$\text{IK}(x, y) = (\theta_1, \theta_2)$$

workspace W



Action space

$$(\alpha_t, \beta_t) \in A$$

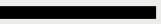
Forward Dynamics

$$\text{FD}: A \rightarrow W$$

Inverse Dynamics

$$\text{ID}: W \rightarrow A$$

How to detect the state of a robot?



Sensors (overview)

How many senses do you have?

Humans have **more than 5 senses**, and probably more than 10.

- 
- Hearing
 - Sight
 - Taste
 - Pressure
 - Smell
 - Proprioception
 - Touch
 - Equilibrioception

...and more.

Source: TestTube Plus, Today I Found Out

Common Robot Senses

- Proximity/distance sensors (infrared, ultrasonic)
- Positioning/navigation sensors (GPS, IMU, wheel encoder)
- Visual Sensors (RGB camera, LIDAR)
- Force & Torque sensors
- Temperature sensors
- ...

Sensors: physical devices that provides information about the world

Internal states → *Proprioceptive* sensors
External states → *Exteroceptive* sensors

Perception = proprioception + exteroception

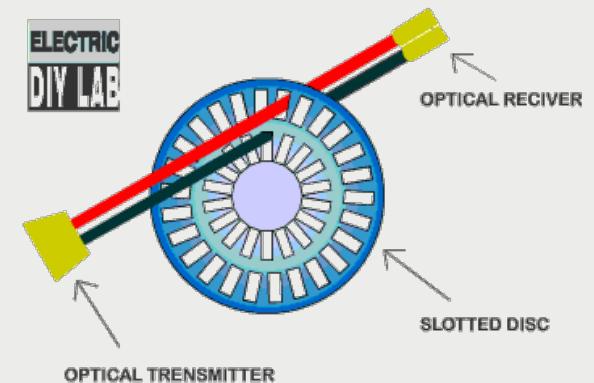
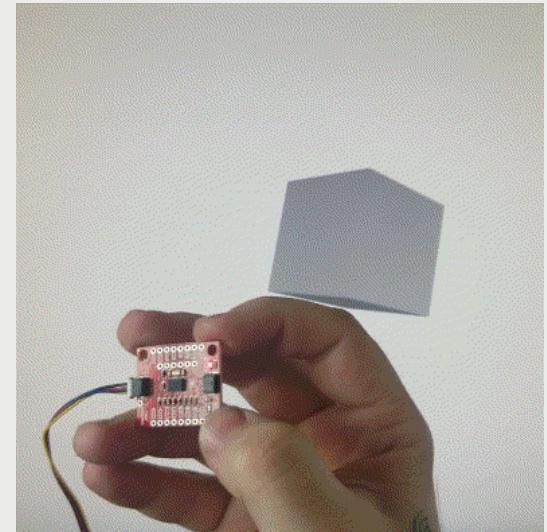
Sensors: physical devices that provides information about the world

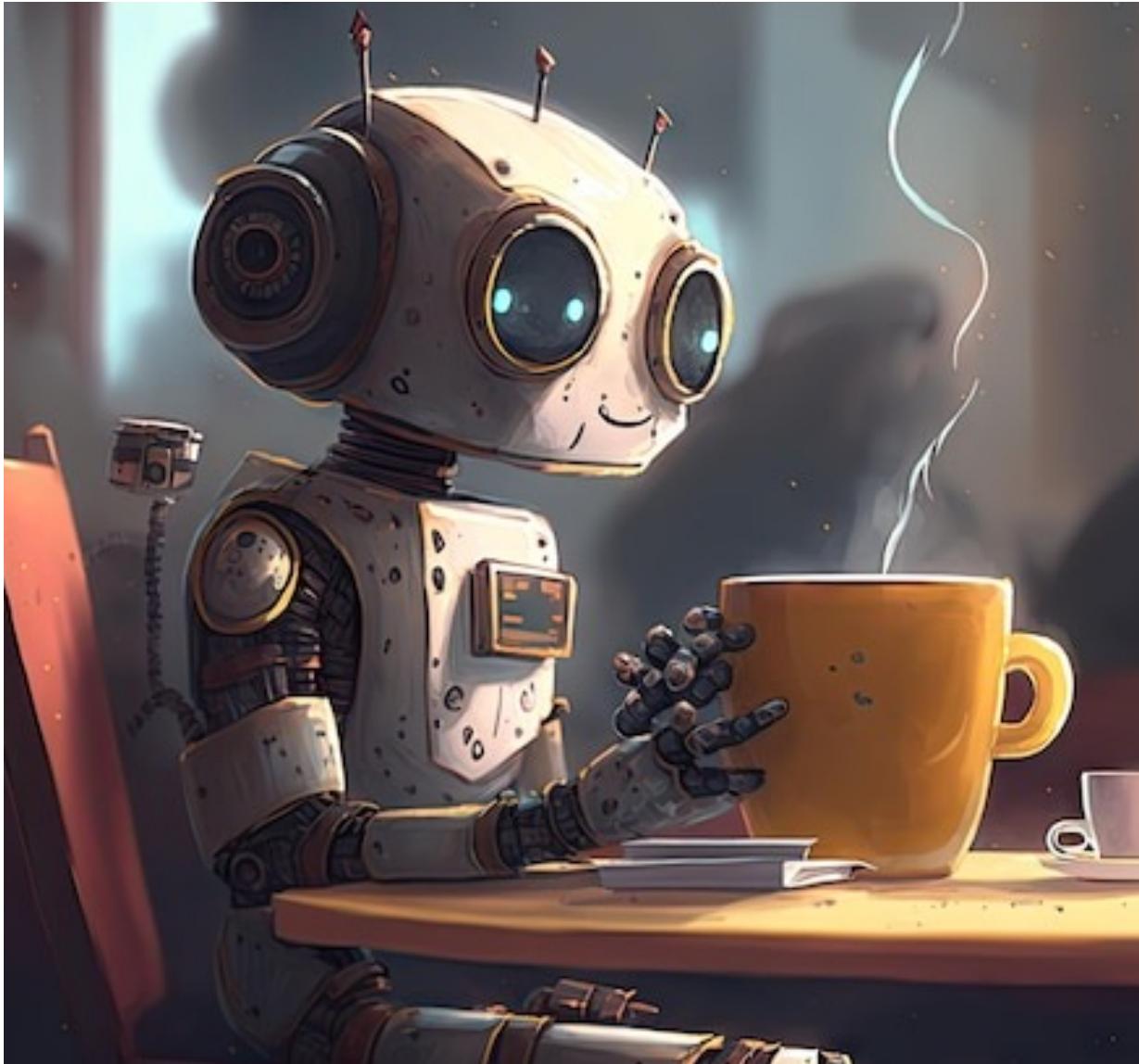
Passive sensors: detects natural energy

Active sensors: needs power, emits signals and detects feedback

Common Robot Sensors

- IMU (Inertial measurement unit)
 - Gyroscope + accelerometer + magnetometer
- Wheel encoder
- LIDAR
- RGB Camera
- Bumper switch
- ...





Break time!

Actuators (Motors & Gears)

slides are adapted from USC CSCI 445L

Action/Actuation

- A robot acts through its *actuators* (e.g., motors), which typically drive *effectors* (e.g., wheels, grippers)
- Robotic actuators are very different from biological ones, both are used for:
 - *locomotion* (moving around, going places)
 - *manipulation* (handling objects)
- This divides robotics into two areas
 - **mobile** robotics
 - **manipulator** robotics



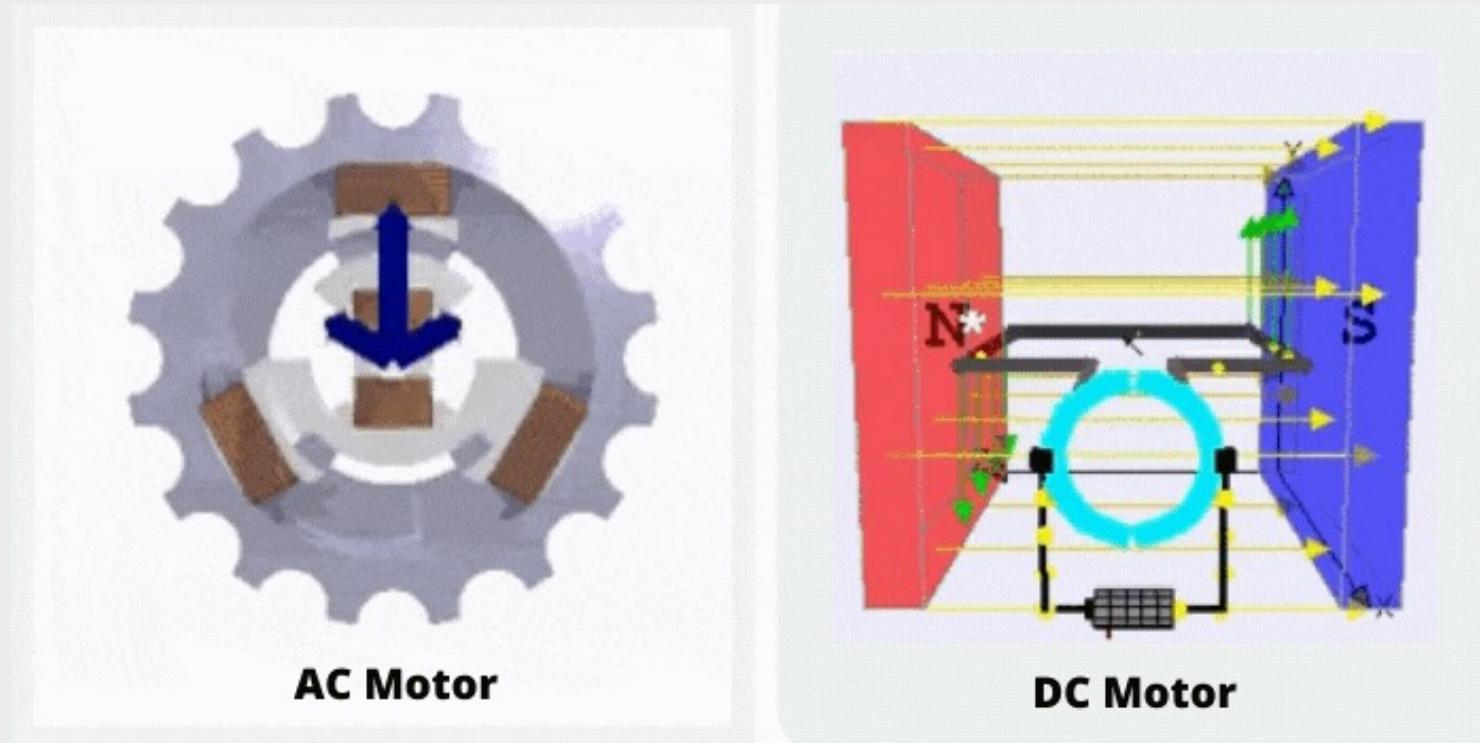
Definition of Effector

- An effector is any device that has an effect on the environment.
- A robot's effectors are used to purposefully create an effect on the environment.
- E.g., legs, wheels, arms, fingers...
- *The role of the controller is to get the effectors to produce the desired effect on the environment, based on the robot's task.*

Definition of Actuator

- An actuator is the mechanism that enables the effector to execute an action.
- E.g., electric motors, hydraulic or pneumatic cylinders, pumps...
- Actuators and effectors are not the same thing.

Electric Motors



AC Motor

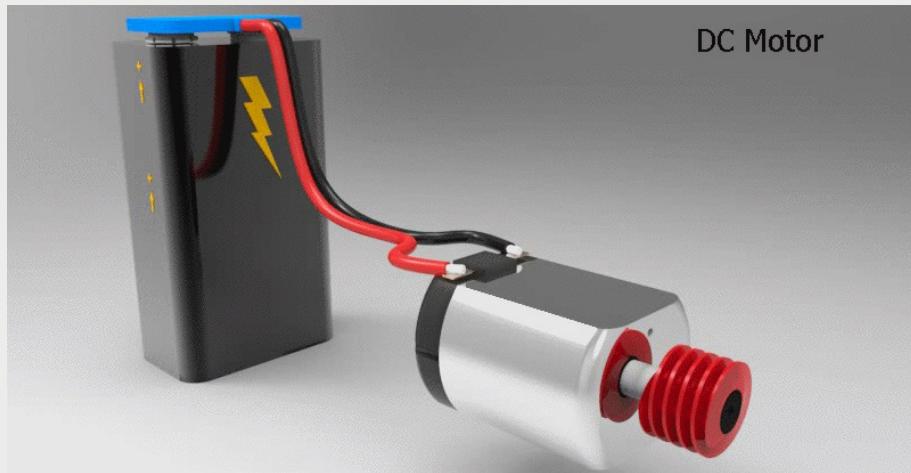
DC Motor

Hard to control speed directly
Cheaper and more durable -> Common in
household appliances, HVAC, pumps, and fans

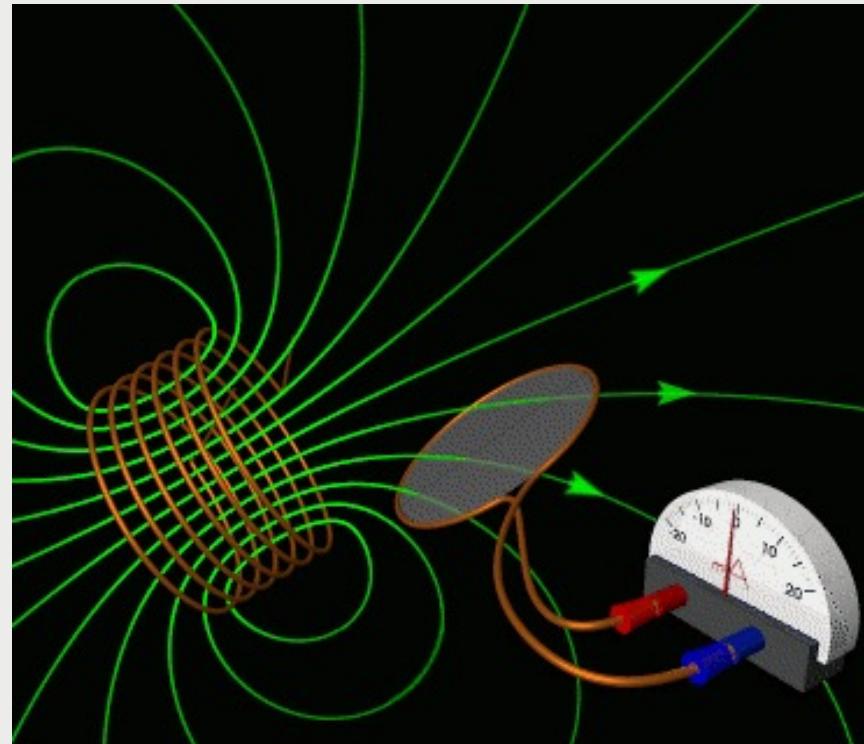
Speed is controlled by voltage
Expensive and less durable, but
better for precise control

DC Motors

- The most common actuator in mobile robotics is the direct current (DC) motor
- Advantages: simple, cheap, various sizes and packages, easy to interface, clean.
- DC motors convert electrical into mechanical energy

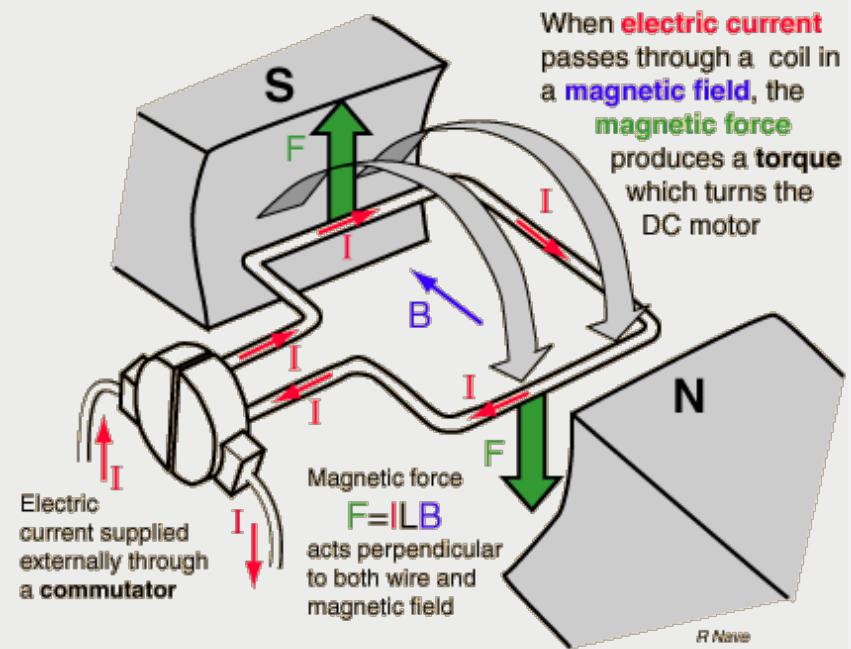


How Motors Work



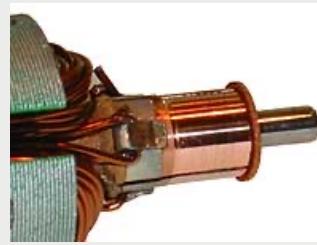
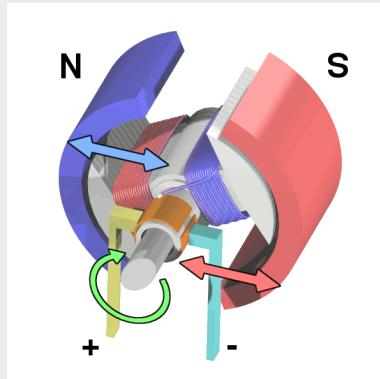
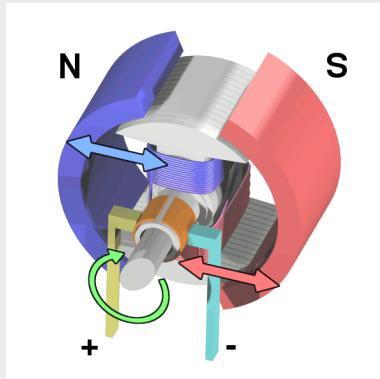
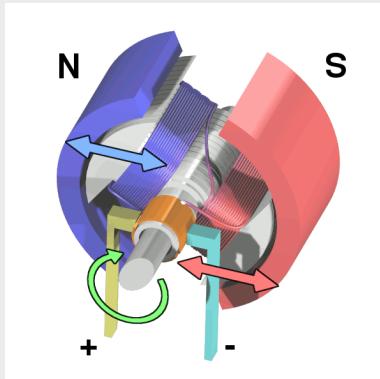
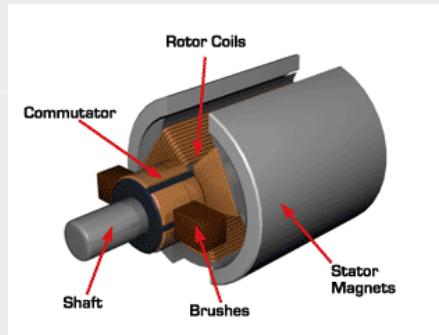
How DC Motors Work

- DC motors consist of permanent magnets with loops of wire inside
- When current is applied, the wire loops generate a **magnetic field**, which reacts against the outside field of the static magnets
- The interaction of the fields produces the movement of the shaft/armature
- A **commutator** switches the direction of the current flow, yielding continuous motion



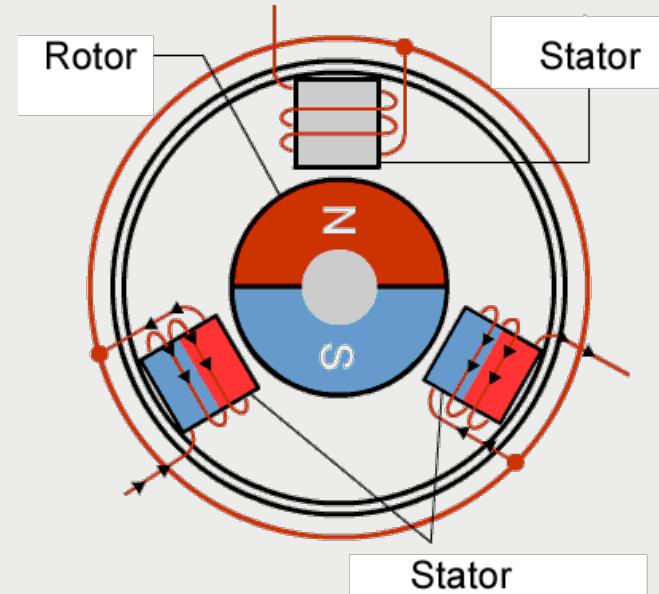
Types of DC motors

- Brushed motors (mechanical commutation)
 - Low-voltage, low-torque, cheap



Types of DC motors

- Brushless motors (electronic commutation)
 - High-voltage, high-torque, expensive
 - No friction / wear of brushes



Motor Efficiency

- As any physical system, DC motors are not perfectly efficient
- Energy is not converted perfectly; some is wasted as heat generated by winding resistance and friction
- Inefficiencies are minimized in well-designed (more expensive) motors, and their efficiency can be high.

Motor Efficiency

- Good DC motors can be made to have efficiencies in the 90th percentile
- Cheap DC motors can be as low as 50%
- Other types of effectors, such as miniature electrostatic motors, may have much lower efficiencies still

Speed and Torque

- Motor speed w is proportional to induced voltage V

$$w = k_v V$$

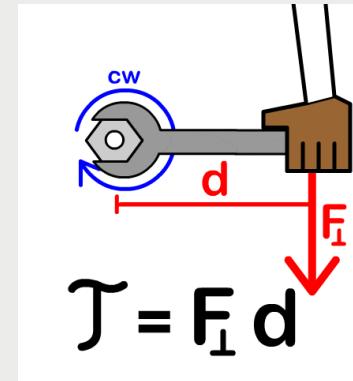
- Torque is a force that acts in a rotational manner

$$t = r \times F$$

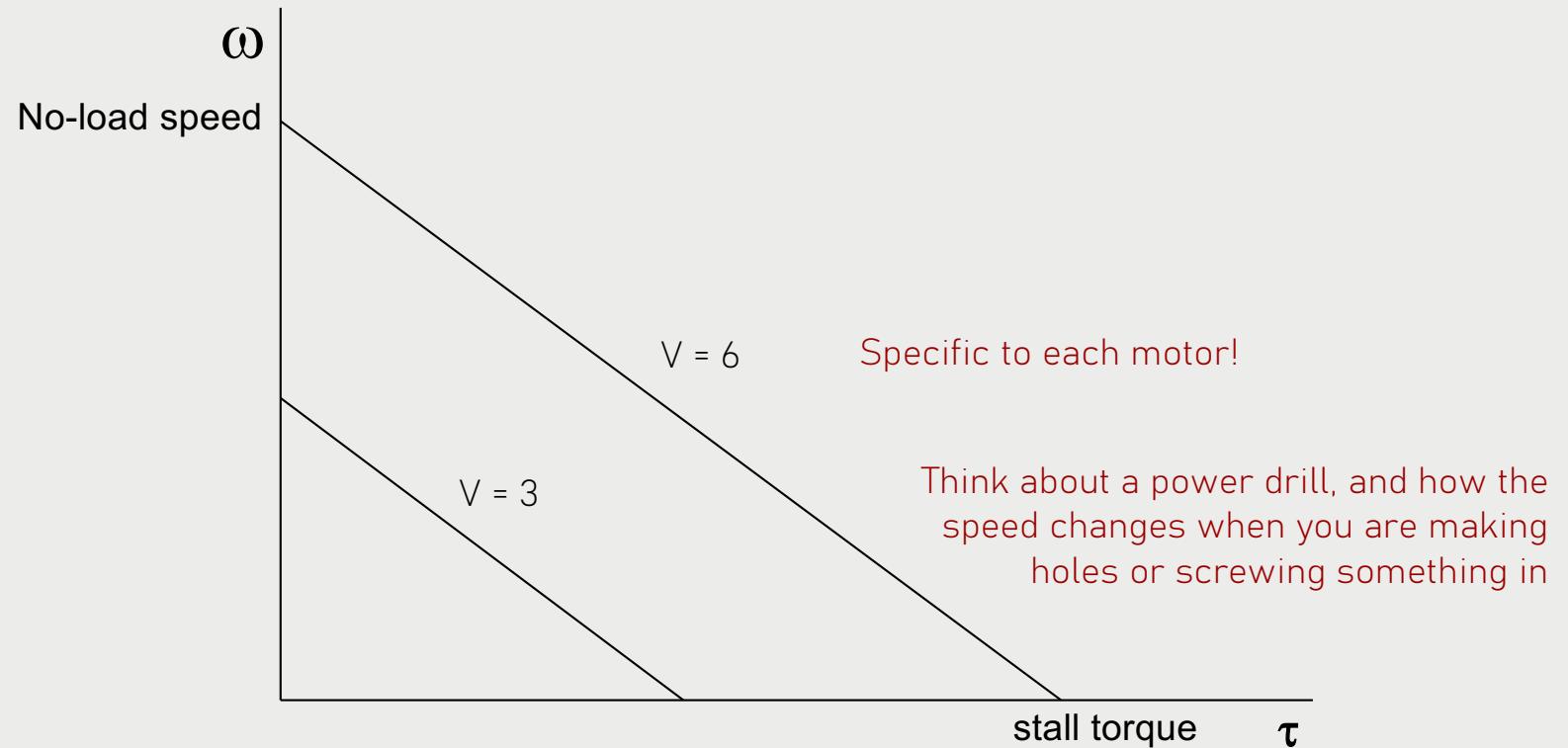
- Motor torque t is proportional to applied current I :

$$t = k_t I$$

- Motors have a maximum speed (no-load speed) and a maximum torque (stall torque)

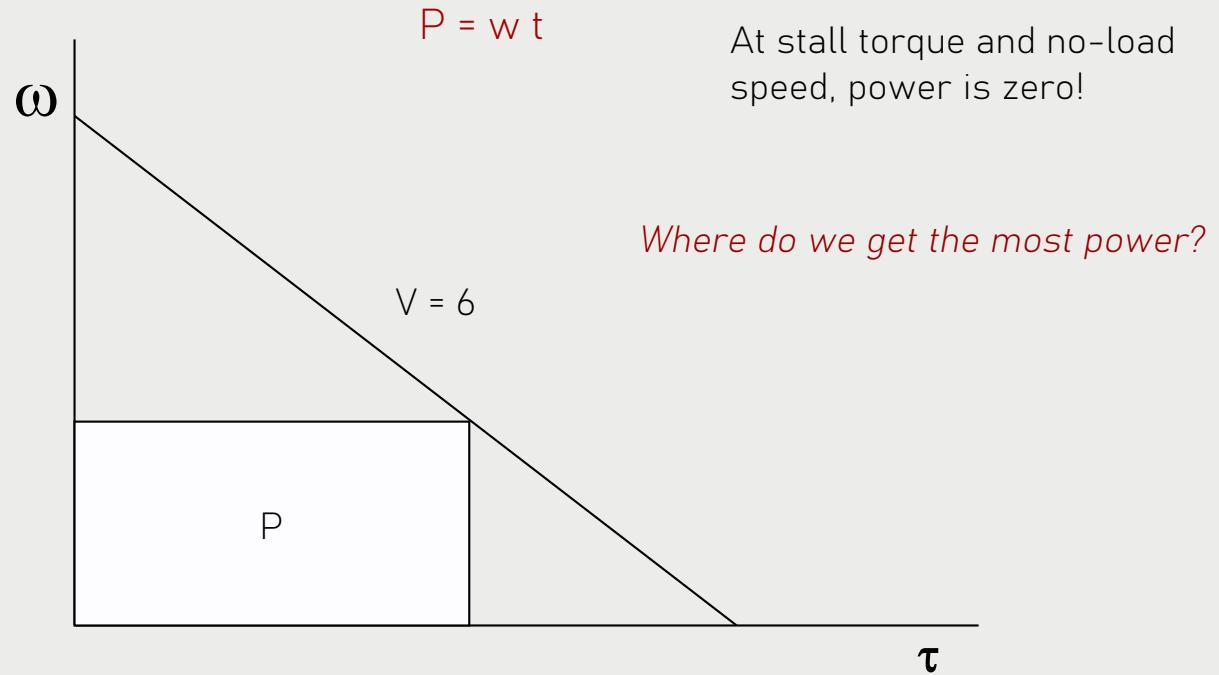


Speed/Torque Relationship



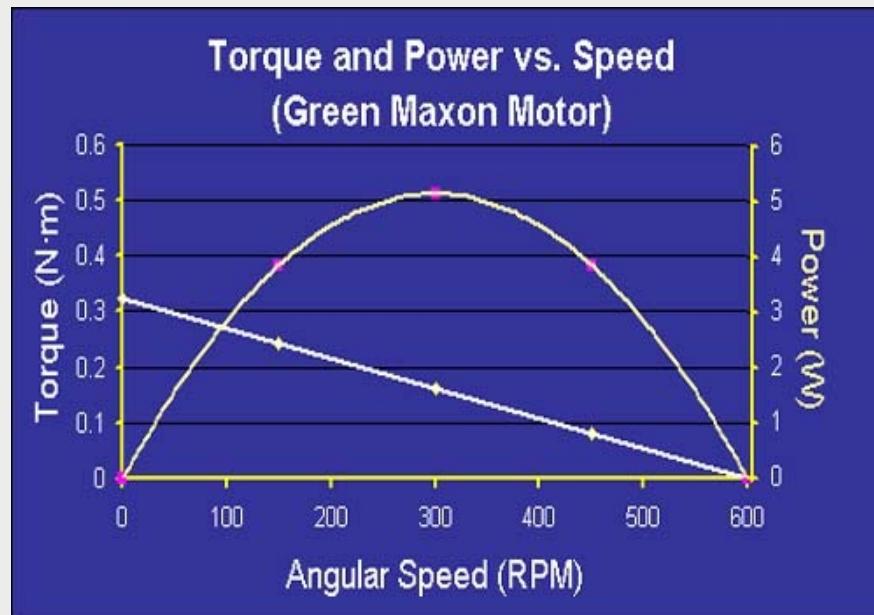
Motor power

- Output power is the product of speed & torque:



Power as a function of τ , ω

$$P_{motor}(\omega) = -\frac{\tau_s}{\omega_n} \omega^2 + \tau_s \omega \quad P_{motor}(\tau) = -\frac{\omega_n}{\tau_s} \tau^2 + \omega_n \tau$$



Operating voltage and speed

- Motors have maximum voltage rating
 - Higher voltages may overheat windings
- Motors have maximum speed rating
 - Higher speeds may destroy bearings or commutator
- Operating motors at higher speeds/voltages will reduce their life expectancy

DC motors and Robots

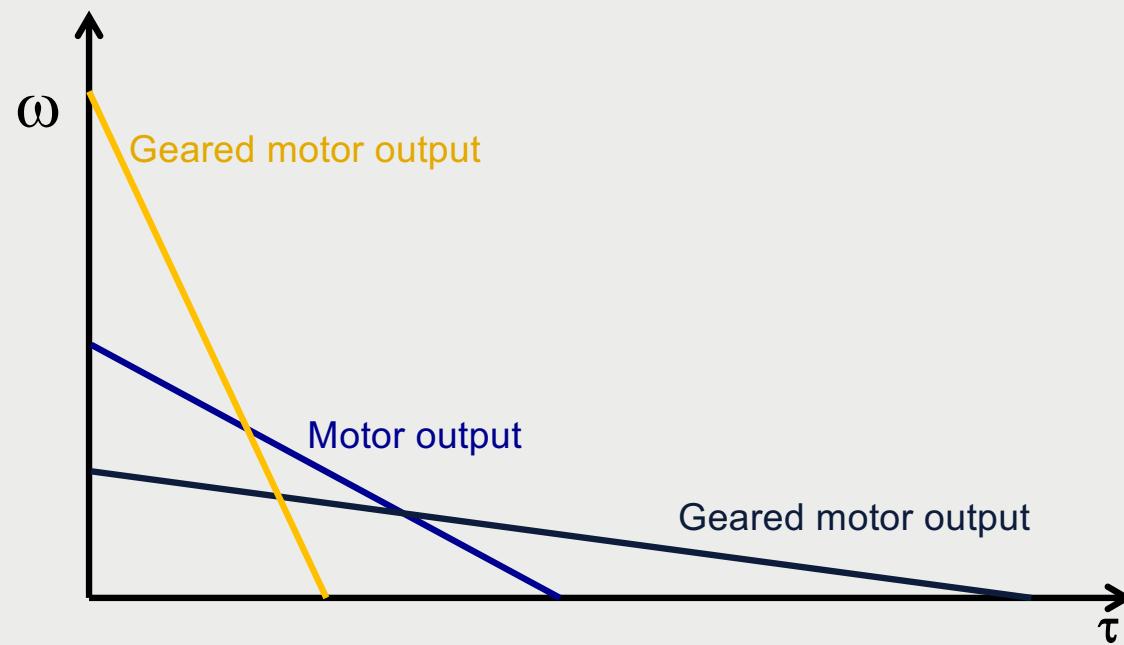
- DC motors have high-speed, low torque
- Typical speed range:
 - 9,000 to 12,000 RPM
 - 150 to 200 Hz
- Robots require low-speed, high torque

What do we do about this?

Motors must be geared!

Gearing

- Gears are used to alter the output speed/torque of a motor (slope of speed/torque graph)



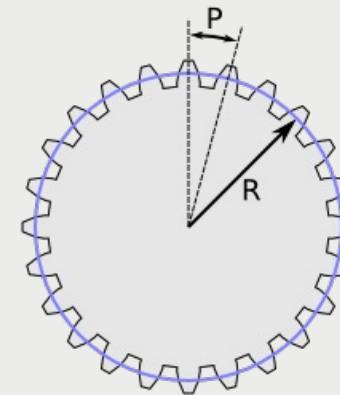
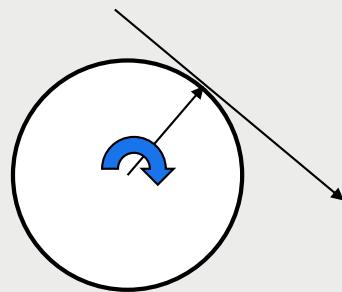
Gear Fundamentals

- The force F at the edge of a gear of radius r is given by:

$$F = t / r$$

- The linear speed v at the edge of a gear of radius r is given by:

$$v = \omega r$$



Combining Gears

- Meshing gears have equal linear speeds:

$$v_1 = v_2$$

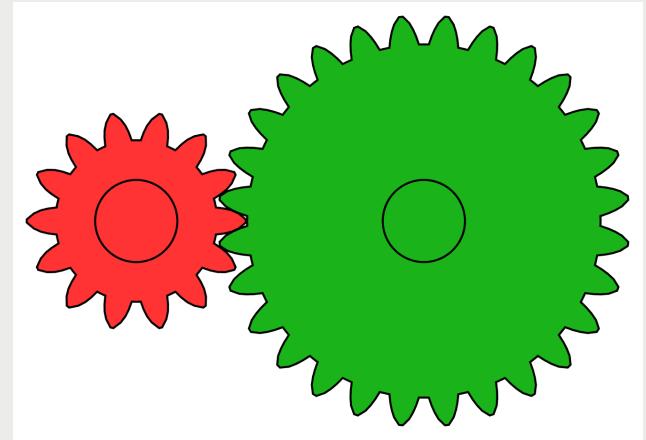
- Thus the output speed is:

$$v = \omega r \quad \omega_2 = (r_1 / r_2) \omega_1$$

- And the output torque is:

$$F = \tau / r \quad t_2 = (r_2 / r_1) t_1$$

- r_2 / r_1 is known as the *gear ratio*



Examples

- Gearing down:

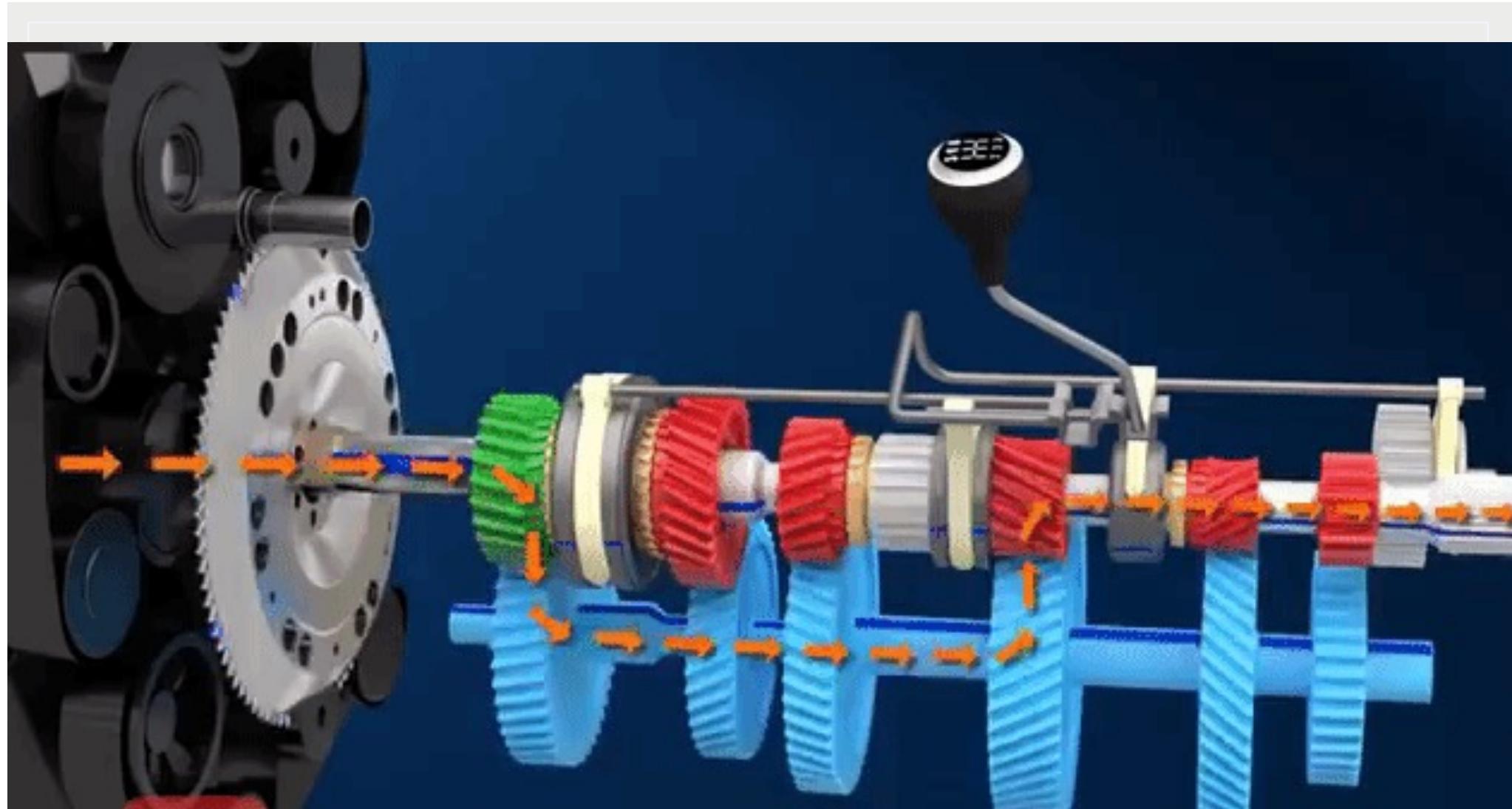
$$r_1 = 1, r_2 = 2$$

- 2:1 gear ratio doubles the torque and halves speed

- Gearing up:

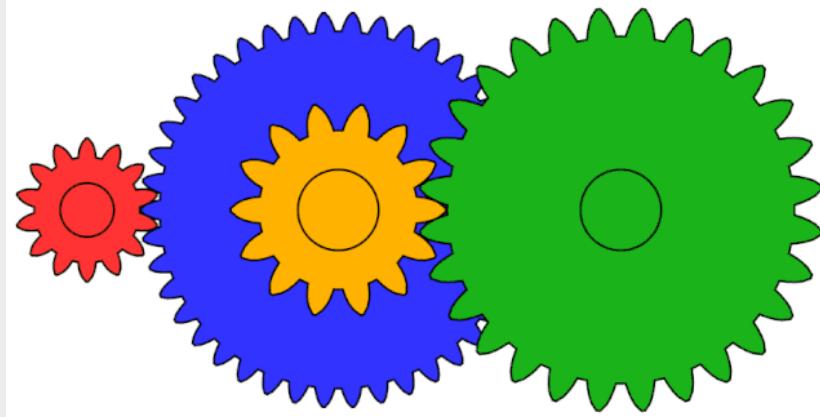
$$r_1 = 2, r_2 = 1$$

- 1:2 gear ratio halves torque and doubles speed



Gear Stages

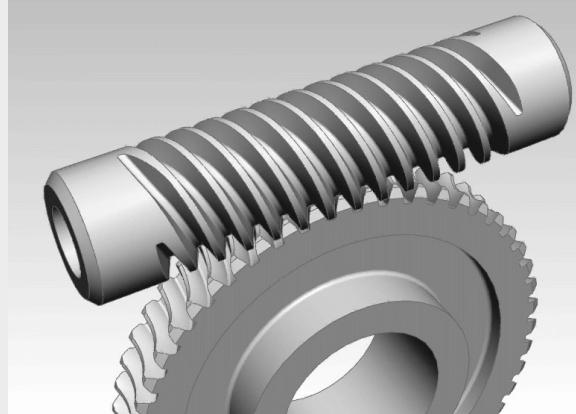
- Usually, it is not possible to achieve a sufficient gear ratio with a single pair of gears
- Gears can be arranged *in stages*
- The total gear ratio is the product of gear ratios for each stage
 - E.g.: $3:1 \times 3:1 = 9:1$



Types of Gears



Spur



Worm

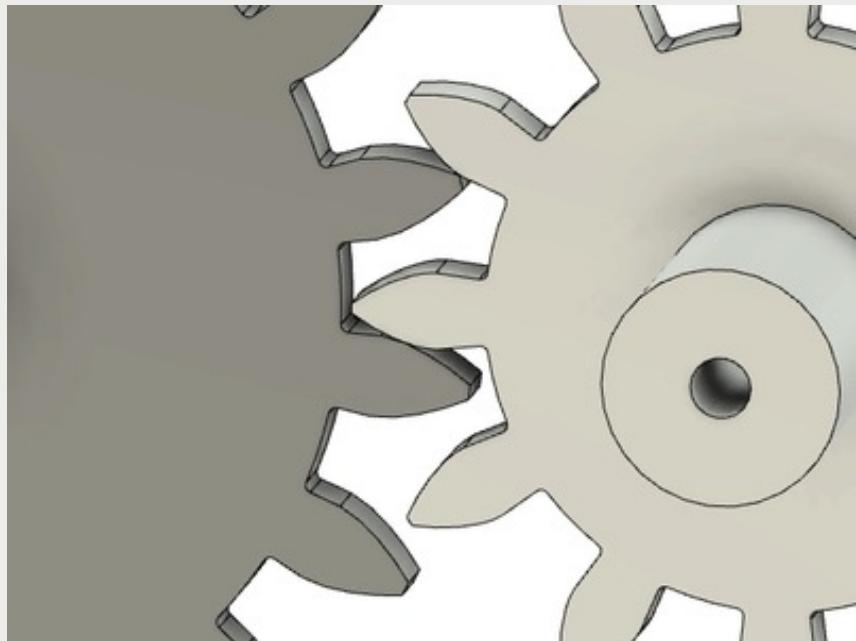


Planetary

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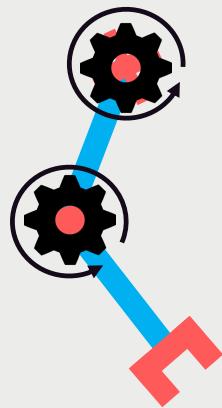
Backlash

Simple gears suffer from *backlash* (teeth not meshing completely)



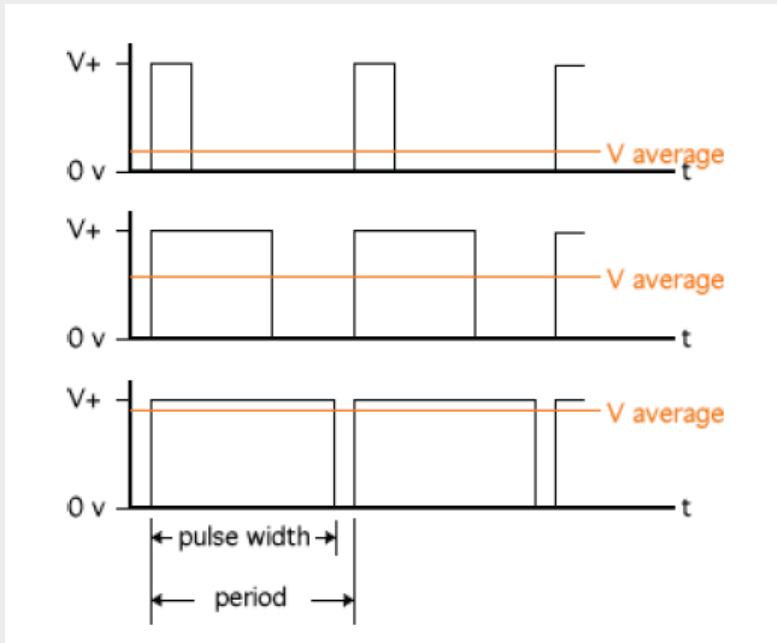
Backlash is the
enemy of precision!

Control of Motors



Controlling Speed: Pulse Width Modulation (PWM)

$$\omega = k_v V_{\text{average}}$$



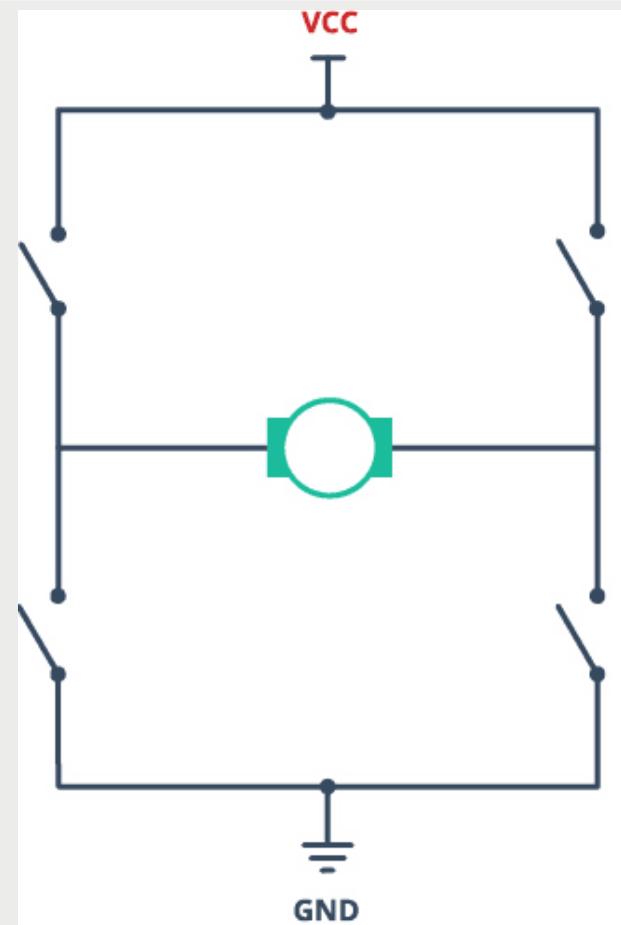
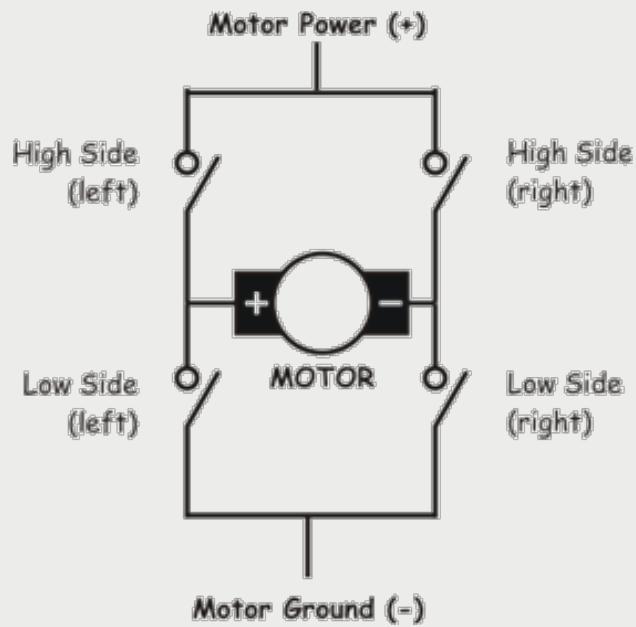
What is the duty cycle?

Percentage of one period in which a signal is active.

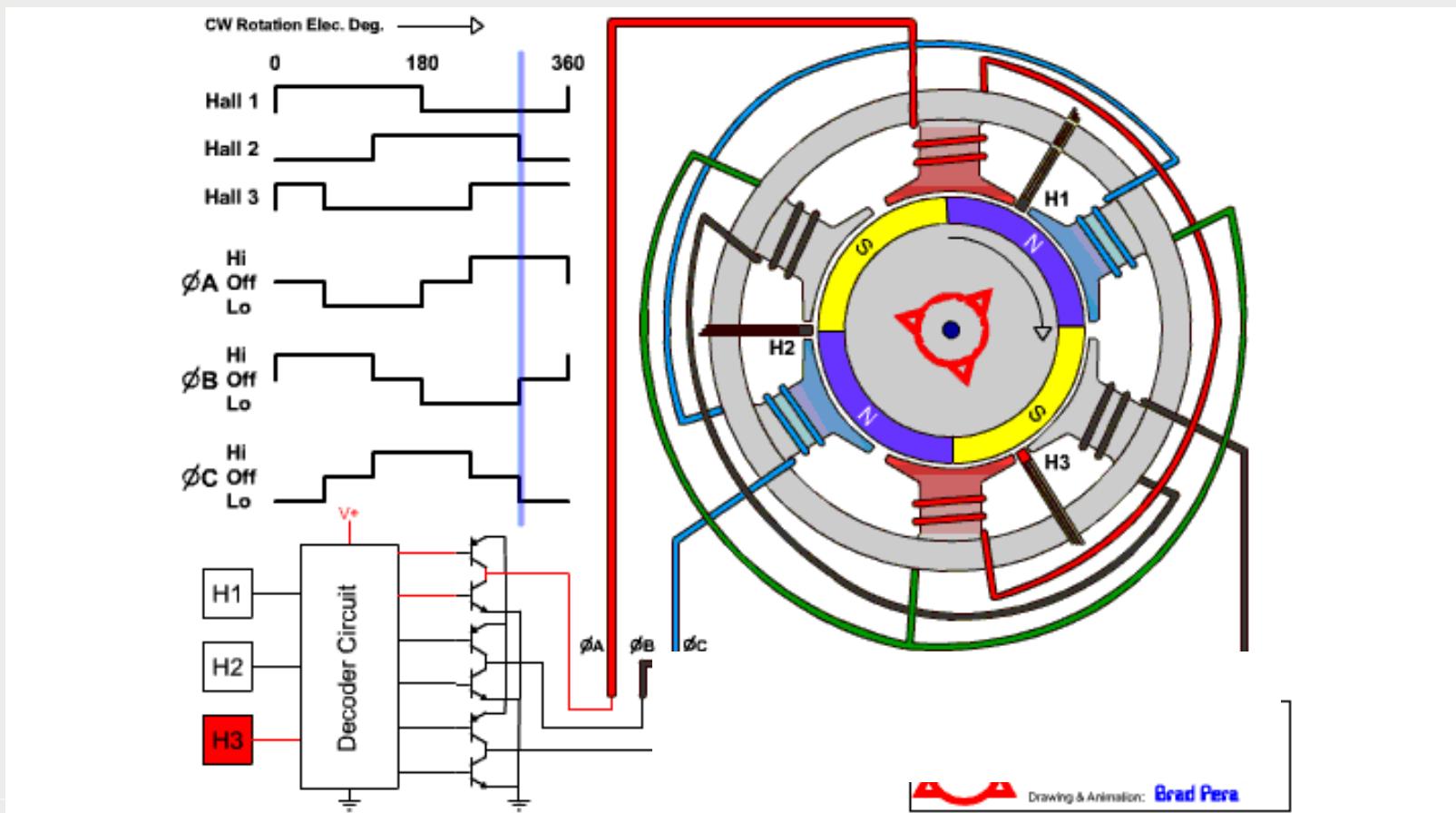
Controlling Speed: Pulse Width Modulation (PWM)

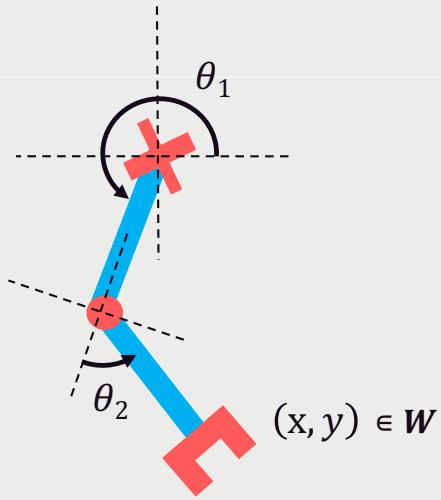


Controlling direction: H-Bridge

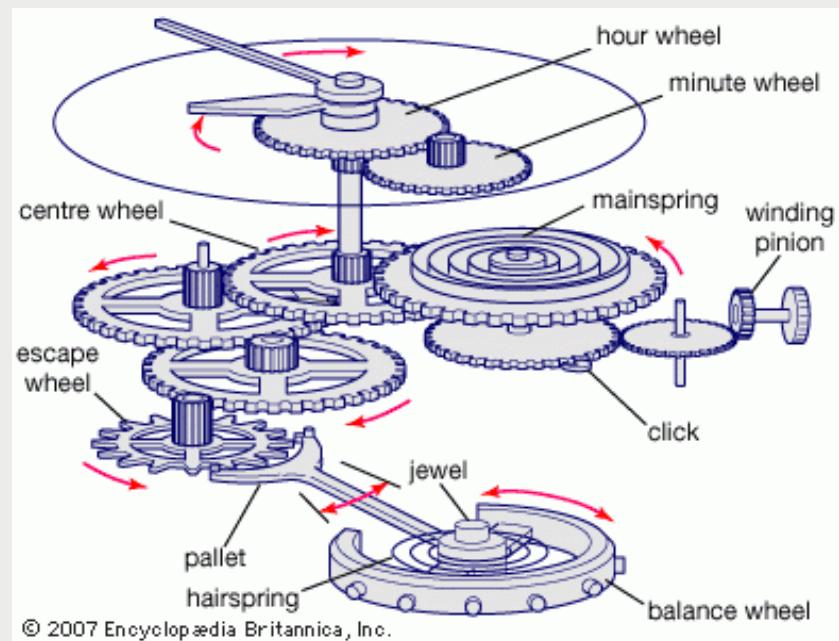
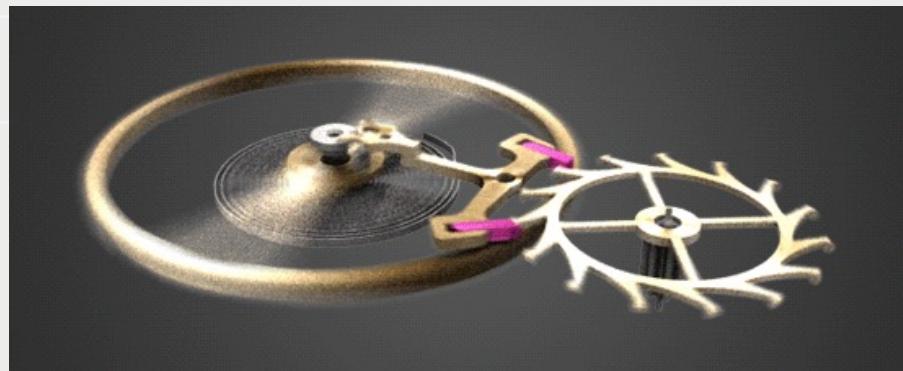


Control of Motors





- For precise control, we want to move a joint to a specific position
- DC motors are not built for this



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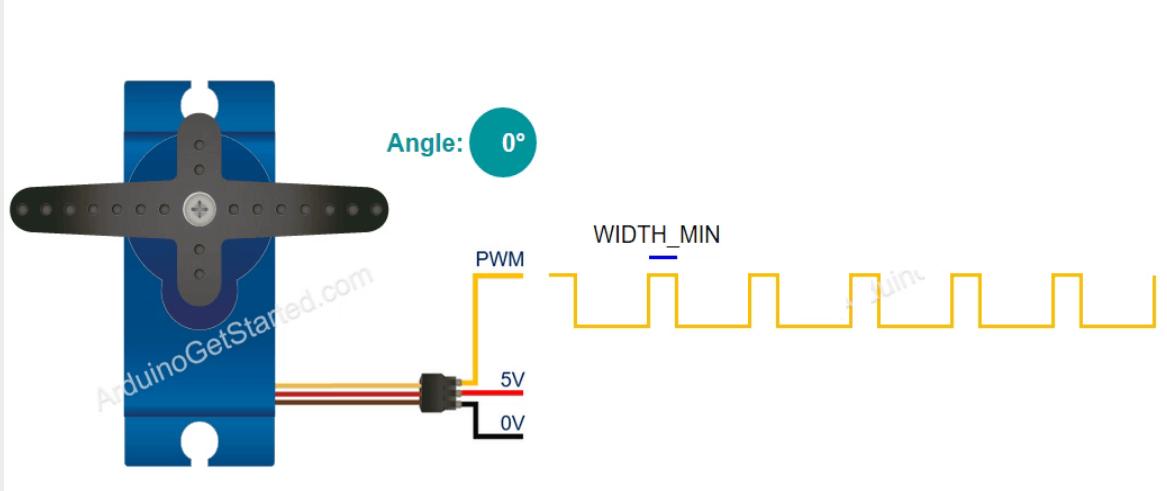
<https://www.youtube.com/watch?v=9Fn8X272m7w>

Servo Motors

- Servo motors are adapted DC motors:
 - gear reduction
 - position sensor (encoder, potentiometer)
 - electronic controller
- Range of at least 180 degrees



PWM Position Control



Not defined by PWM duty cycle but only *duration* of the pulse!

Pulse width must be very accurate

Noise in width => noise in position

Pulse rate may be variable

Noise in rate => no change



That's it for today!
Questions?