Problem Set 2

COM SCI 188: Intro to Robotics — Winter 2026

Question:	1	2	3	4	5	6	7	Total
Points:	16	12	16	16	8	16	16	100

1. **Rigid body motion.**

(a) (4 points) In homogeneous coordinates, how is a 2D Cartesian point (x, y) represented if the scaling factor s is 1?

A. $[x, y, 0]^T$
B. $[1, x, y]^T$
C. $[x, y]^T$
D. $[x, y, 1]^T$

(b) (4 points) In the transformation notation ${}^{A}T_{B}$, what is the correct interpretation of the symbols?

A. It is a scalar multiplier for frames A and B.
B. It describes the motion starting from frame B and ending at A.
C. It changes the coordinate frame from B to A.
D. It represents the pose of frame A relative to frame B.

(c) (4 points) What is a key advantage of using quaternions to represent 3D rotation compared to rotation matrices?

A. Quaternions are more intuitive for humans to visualize and specify.
B. Quaternions are more compact, using 4 numbers instead of 9, and avoid discontinuous jumps.
C. Quaternion multiplication is computationally simpler than matrix multiplication.
D. Quaternions use only 3 numbers, making them the most compact representation.

(d) (4 points) According to Euler's rotation theorem, any sequence of rotations of a rigid body is equivalent to what?

A. A single translation along a fixed vector.
B. A 4x4 homogeneous transformation matrix.
C. A single rotation about a single fixed axis.
D. A series of three rotations about the x, y, and z axes (Euler angles).

2. **Rotation matrices.**

(a) (3 points) Which axis does the following matrix rotate around?

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

A. X-axis
B. Y-axis
C. Z-axis
D. Arbitrary axis

(b) (3 points) Which of the following is a key property of a valid 3D rotation matrix R?

A. $R^T = -R$
B. $R^T = R^{-1}$
C. $\det(R) = 0$
D. $R + R^T = I$

(c) (3 points) What is the result of multiplying two 3D rotation matrices $R_1 R_2$?

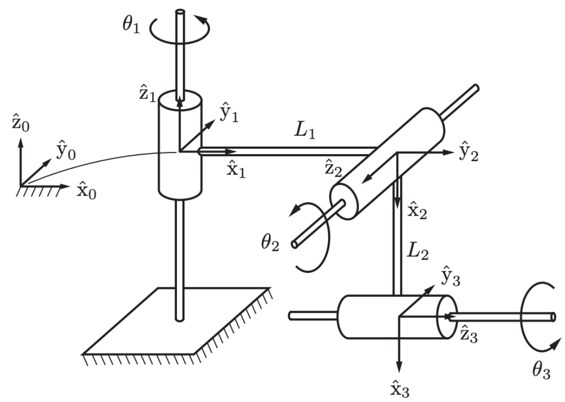
A. The product is commutative, meaning $R_1 R_2$ always equals $R_2 R_1$.
B. The resulting matrix is no longer orthogonal and must be re-normalized.
C. It results in a valid rotation matrix representing the composition of the two rotations.
D. The determinant of the resulting matrix becomes 0.

(d) (3 points) A rotation matrix must be:

A. Symmetric and non-singular
B. Orthogonal with a determinant of ± 1
C. Orthogonal with a determinant of 1
D. Skew-symmetric and invertible

3. **Denavit-Hartenberg (DH) Parameters**

Consider the following RRR manipulator:



(a) (4 points) According to the DH convention, how is the z-axis assigned?

A. It must point along the common normal between two links.

B. It represents the link offset distance d_i.

C. It is along the joint axis of rotation or translation.

D. It is always perpendicular to the direction of gravity.

(b) (6 points) Write down the DH Parameters for the RRR manipulator:

Solution: (note: the orders of first two columns are flipped)

i	α_{i-1}	a_{i-1}	d_i	ϕ_i
1	0	0	0	θ_1
2	90°	L_1	0	$\theta_2 - 90^\circ$
3	-90°	L_2	0	θ_3

(c) (6 points) Derive the forward kinematics for this manipulator.

Solution: ${}^0T_3 =^0 T_1^1 T_2^2 T_3$
(spell out each transformation using DH parameters with formula in equation sheet)

4. **Control**

(a) (4 points) In the context of control theory, which of the following best describes a 'maintenance goal'?

A. Driving to a specific delivery point.

B. Making a cup of coffee.

C. Assembling a product on a factory line.

D. Keeping a constant distance of 2 car-lengths from a vehicle ahead.

(b) (4 points) What is the primary role of the Integral (I) component when added to create a PI controller?

A. To increase damping and prevent oscillations.

B. To speed up the initial response of the system.

C. To predict future errors based on the current rate of change.

D. To eliminate the steady-state error by accumulating past errors.

(c) (4 points) Increasing the gain of which PID parameter is most likely to reduce the Rise Time but increase the Overshoot?

A. The damping ratio.

B. The proportional gain K_p.

C. The sampling rate.

D. The derivative gain K_d.

(d) (4 points) In Model Predictive Control (MPC), what does the term 'receding horizon' refer to?

A. Calculating only the final state and working backward to the start.

B. Reducing the distance the robot can see to save on computational power.

C. Solving an optimization over a future time window but only applying the first step.

D. The point where the camera's field of view meets the floor.

5. **Camera Calibration I.**

(a) (4 points) Given a camera image dimension d and a focal length f, what is the formula to calculate the Field of View (α)?

A. $\alpha = 2\sin[\frac{d}{f}]$

B. $\alpha = \frac{d \times f}{2}$

C. $\alpha = \frac{f}{d}$

D. $\alpha = 2\,\text{atan}[\frac{d}{2f}]$

(b) (4 points) While a full affine transformation accounts for rotation, translation, scale, and shear, what is the primary focus of the Kabsch algorithm?

A. Estimating the intrinsic focal length of a camera lens.

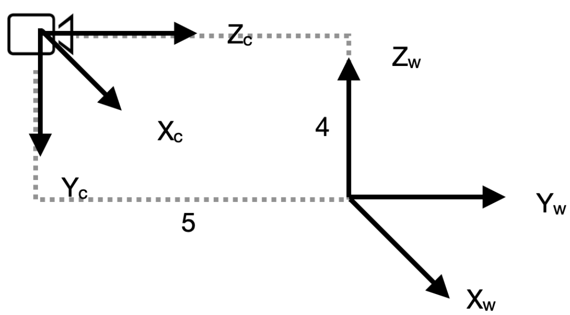
B. Downsampling high-resolution point clouds for faster processing.

C. Finding the optimal rotation matrix to align two sets of 3D points.

D. Predicting the future trajectory of a robot using sensor fusion.

6. **Camera Calibration II.**

Let's defined a world coordinate frame and a camera coordinate Frame, as shown in figure below. Camera's position is [0,-5,4] in world frame, pointing along the Y-axis of the coordinate.



(a) (4 points) What is the camera's extrinsic matrix?

Solution: Extrinsc matrix transforms points from world frame to camera frame and geometrically describes the motion from camera frame to world frame, which is: Rotate along x +90, Move (0,4,5)

$$^{c}T_w = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) (4 points) If ${}^{w}P = [-1, 5, 3]$ what is ${}^{c}P$

Solution:

$$
\begin{aligned}
{}^{c}P &= {}^{c}T_{w} * {}^{w}P \\
&= [-1, 1, 10]
\end{aligned}
$$

(c) (4 points) If camera's intrinsic matrix is:

$$
R = \begin{bmatrix} 100 & 0 & 200 \\ 0 & 100 & 100 \\ 0 & 0 & 1 \end{bmatrix}
$$

Compute P's 2D image [u,v] coordinate (aligned with camera).

Solution:

$$u = 100 * (-1)/10 + 200 = 190$$

$$v = 100 * (1)/10 + 100 = 110$$

$$[u, v] = [190, 110]$$

(d) (4 points) Assume the depth value for pixel [u,v] = 2.8. What is the projective signed distance value for location P? (Before truncation and normalization). Is this point P in occluded space or free space?

Solution: $d(u, v) - z = 2.8 - 10 = -7.2 < 0$, point P is occluded

7. **Particle Filter.**

(a) (4 points) What is the primary function of a particle filter (Sequential Monte Carlo method)?

A. To increase the spatial resolution of raw sensor data.

B. To smooth out high-frequency noise in static images.

C. To estimate the state of a dynamic system using a set of weighted samples.

D. To detect edges in an image using gradient-based operators.

(b) (4 points) What is the specific purpose of the “resampling” step in the particle filter algorithm?

A. To arbitrarily increase the total number of particles to improve accuracy.

B. To duplicate high-probability particles and eliminate low-weight ones to prevent degeneracy.

C. To convert the discrete particles into continuous analog measurements.

D. To filter out high-frequency noise directly from the raw sensor readings.

(c) (4 points) Which of the following is the most standard application of particle filters in robotics?

A. Encrypting communication channels between the robot and base station.

B. Detecting malware injections in the robot’s operating system.

C. Robot Localization (estimating pose) given a map and noisy sensor data.

D. Compressing LIDAR point clouds to reduce storage requirements.

(d) (4 points) What does the phenomenon of “particle degeneracy” refer to?

A. When the particles converge to the exact true state with zero error variance.

B. When the weight is concentrated in a negligible number of particles, leaving most with near-zero weight.

C. When the system dynamics become perfectly linear and Gaussian.

D. When the computational cost exceeds the available hardware resources.