# Problem Set 3

COM SCI 188: Intro to Robotics — Spring 2025

| Question: | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| Points: | 33 | 20 | 18 | 20 | 9 | 100 |

1. **Particle Filer & Localization.**

(a) (3 points) What is the primary purpose of *Resampling* in a Particle Filter algorithm?

[✓ A)] To concentrate computational resources on high-probability particles by discarding low-weight particles and duplicating high-weight ones.

B) To increase the total number of particles in order to improve computational accuracy as uncertainty grows.

C) To inject additional Gaussian noise to increase computational diversity in the particle set.

D) To reduce computational complexity by converting the belief distribution from Gaussian to uniform.

(b) (4 points) When calculating the *Posterior Probability* of a particle after a sensor reading, which formula is used?

A) $P(\text{location} \mid \text{reading}) = P(\text{reading} \mid \text{location}) + P(\text{location})$

B) $P(\text{location} \mid \text{reading}) = \mu \pm \sigma^2$

[✓ C)] $P(\text{location} \mid \text{reading}) = \dfrac{P(\text{reading} \mid \text{location})P(\text{location})}{P(\text{reading})}$

D) $P(\text{location} \mid \text{reading}) = \dfrac{P(\text{reading} \mid \text{location})}{P(\text{reading})P(\text{location})}$

(c) (4 points) In robot localization, what does the measurement model represent?

A) The probability of the robot moving to a new location given a control input.

[✓ B)] The probability of receiving a sensor measurement given the robot's current location.

C) The probability of the robot's initial position.

D) The probability of a landmark being detected at any point on the map.

(d) (4 points) Which of the following is a specific advantage of a Particle Filter over a standard Kalman Filter for robotics applications?

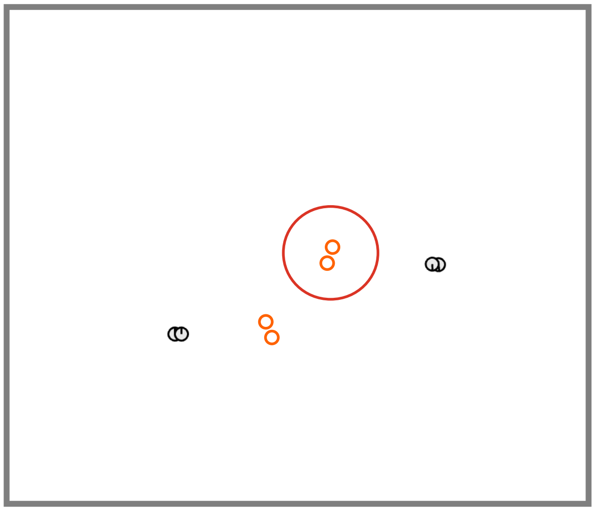
[✓ A)] It can naturally handle non-linear systems and non-Gaussian noise.

B) It is significantly more computationally efficient for high-dimensional state vectors.

C) It provides a single mean and covariance matrix as the final output.

D) It does not require any sensor noise model to function.

(e) (4 points) Recall the robot that navigates the 2D square room with just 2 range sensors (left and right). The figure below shows 4 particles of the robot at time step $t-1$. At timestep $t$, the robot receives a movement command $(\theta = -\pi/2, d = 1)$. Draw a possible configuration of the particles after the prediction step. The robot turns (couterclockwise is positive) before it moves.



(f) (4 points) After executing the movement, the robot receives sensor reading $(Left = 1.7m, Right = 1.8m)$. Circle the particle(s) with highest posterior probability.

(g) (10 points) **Particle Filter vs Kalman Filter.**

Both Particle Filters and Kalman Filters are built around Bayesian filtering:

- Prediction: predict where you might be after motion.
- Update: correct the prediction using sensor measurements.
- Belief representation: (particles for PF, mean/covariance for KF).

For each concept below, choose whether it applies to Particle filter, Kalman filter, or both.

| **Concept** | **PF** | **KF** |
|---|---|---|
| Represents belief with multiple samples (particles) | ● | ○ |
| Assumes Gaussian noise and linear system | ○ | ● |
| Uses motion models and measurement models | ● | ● |
| Handles non-linear and non-Gaussian models naturally | ● | ○ |
| Uses a single mean and covariance matrix | ○ | ● |
| Performs prediction-update (Bayes filtering cycle) | ● | ● |
| May require resampling to focus on high-probability particles | ● | ○ |
| Uses an optimal gain to balance prediction and measurement | ○ | ● |
| Requires an initial estimate (initial belief) | ● | ● |
| Can be used for robot localization and SLAM | ● | ● |

2. **Simultaneous Localization and Mapping.**

(a) (4 points) What is the primary purpose of SLAM in robotics?

A) To control robot movements and avoid obstacles

B) To detect and track moving objects in the environment

[✓ C)] To map an unknown environment while tracking the robot's position

D) To optimize battery usage during navigation

(b) (4 points) Why can't a basic Kalman Filter be used directly for SLAM?

A) SLAM systems have no noise.

[✓ B)] SLAM motion and measurement models are nonlinear.

C) Landmark motions are not Gaussian.

D) Kalman Filters require extensive compute.

(c) (4 points) Which of the following is true about the Extended Kalman Filter?

A) It requires the system dynamics and measurement models to be linear.

[✓ B)] It uses Jacobians to locally linearize motion and measurement models.

C) It represents uncertainty using a set of weighted particles.

D) It produces exact state estimates regardless of noise or model error.

(d) (4 points) In SLAM, *Loop Closure* is a critical concept. What happens when a robot successfully closes a loop?

A) The robot terminates exploration after completing a full traversal of the environment.

B) Previously collected sensor measurements are ignored to avoid duplicating mapped features.

[✓ C)] The robot recognizes a previously visited location, reducing uncertainty in both its pose and the map.

D) The SLAM algorithm becomes unstable and must be restarted.

(e) (4 points) What is the computational complexity of the *motion update* (prediction) step in EKF-SLAM? Why? Use the O( ) notation.

O(n^2) where n is the number of landmarks

For each movement of the robot, one needs to update the covariance matrix with size (n+2)x(n+2)

3. **Motion Planning.**

(a) (3 points) What is the primary objective of motion planning?

A) To find a sequence of control inputs that maximizes the robot's speed.
[✓ B)] To compute a collision-free path from a start to a goal position.
C) To minimize the robot's battery consumption during navigation.
D) To detect dynamic objects and reclassify the environment.

(b) (3 points) What is the main idea behind PRM?

A) Grow a tree from the start to the goal position
[✓ B)] Randomly sample free space and connect samples to build a graph
C) Search the entire configuration space exhaustively
D) Maximize the robot's velocity during planning

(c) (3 points) In PRM, which of the following is true about the roadmap?

A) It only connects samples in collision
[✓ B)] It connects nearby collision-free samples
C) It only connects samples after reaching the goal
D) It requires a full map before planning starts

(d) (3 points) What is the main advantage of RRT over PRM?

A) RRT builds a global roadmap in advance
[✓ B)] RRT grows a tree quickly toward unexplored areas
C) RRT guarantees finding the shortest path
D) RRT only works in static environments

(e) (3 points) In RRT, what happens when a random sample is generated?

A) It is ignored unless it is at the goal
[✓ B)] It is connected to the nearest node in the tree if collision-free
C) It replaces the previous nodes in the tree
D) It deletes earlier branches of the tree

(f) (3 points) Which of the following statements about PRM and RRT is correct?

A) Both PRM and RRT guarantee an optimal path.
[✓ B)] PRM builds a graph first; RRT grows a tree during search.
C) RRT is slower than PRM in open environments.
D) PRM is only used for real-time dynamic environments.

4. **Markov Decision Processes.**

(a) (3 points) What is the Markov property?

A) The future depends only on the current state.
[✓ B)] The future depends on all past states.
C) The current state depends on future observations.
D) Observations are independent of the state.

(b) (3 points) What does a *policy* represent in an MDP?

A) A probability distribution over states.
B) A model of state transition dynamics.
[✓ C)] A mapping from states to actions.
D) A function that assigns rewards to states.

(c) (3 points) What is the goal when solving an MDP?

A) To minimize the number of states in the model.
[✓ B)] To find a policy that maximizes expected cumulative reward.
C) To exactly predict all future state transitions.
D) To eliminate uncertainty in the environment.

(d) (3 points) In MDPs, what is "discounting" used for?

[✓ A)] Making immediate rewards more important than distant rewards.
B) Increasing the future rewards' value.
C) Ignoring all rewards beyond a fixed number of steps.
D) Penalizing the robot for collisions.

(e) (4 points) Write the Bellman equation for the value of a state $V(s)$. What is the main idea behind the Bellman equation (1 sentence)?

V*(s) = max_a[R(s,a) + \gamma* \sum_s’ (P(s’|s,a)* V*(s’))]
(principle of optimality) the value of a state is the maximum expected reward achievable by choosing the best action and then acting optimally from the resulting next state.

(f) (4 points) How does "policy iteration" differ from "value iteration"?

Policy iteration alternates between fully evaluating and improving a policy and typically converges in fewer but more expensive iterations, while value iteration repeatedly updates the value function directly with cheaper but usually more numerous iterations.

5. **Imitation Learning.**

(a) (3 points) What is a major challenge in imitation learning related to *compounding errors*?

[✓ A)] Small mistakes early in execution can lead the agent into unseen states, causing errors to accumulate over time.

B) The agent cannot observe the expert's actions directly.

C) The reward function becomes non-stationary during training.

D) The state and action spaces must be fully observable and discrete.

(b) (3 points) What is a key difference between *Behavior Cloning* and *Inverse Reinforcement Learning*?

A) Behavior Cloning requires an explicit reward function, while Inverse Reinforcement Learning does not use rewards.

B) Behavior Cloning models environment dynamics, while Inverse Reinforcement Learning assumes known dynamics.

C) Behavior Cloning is model-based, while Inverse Reinforcement Learning is purely model-free.

[✓ D)] Behavior Cloning learns a policy directly from expert actions, while Inverse Reinforcement Learning infers a reward function from expert behavior.

(c) (3 points) What assumption is commonly made by Behavior Cloning but relaxed by Inverse Reinforcement Learning?

A) That the reward function is unknown.

B) That the state space is continuous.

[✓ C)] That expert actions are optimal in demonstrations.

D) That the policy must be deterministic.