ELEN90051 Advanced Communication Systems
2018 Semester 1 Tutorial 1

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Instructions:

Answer all tutorial questions. Do not use any solution material that you happen to have, thus simulating a genuine exam environment.

1 Consider a DMS X that takes values in $\{a_1, a_2, \ldots, a_N\}$, uniformly distributed. Compute the entropy $H(X)$. $H(X) = -\sum_{i=1}^{N} P_i \log_2 P_i = -\sum_{i=1}^{N} \frac{1}{N} \log_2 \frac{1}{N} = \sum_{i=1}^{N} \frac{1}{N} \log_2 N = \log_2 N$

2 (*Biased Dice*) Consider a DMS X that takes values in $\{a_1, a_2, a_3, a_4, a_5, a_6\}$, with corresponding probabilities $p_1 = 0.30$, $p_2 = 0.20$, $p_3 = 0.20$, $p_4 = 0.15$, $p_5 = 0.10$ and $p_6 = 0.05$. Compute the entropy and compare with the entropy of a fair dice. $H(X) = -(0.3 \cdot \log_2 0.3 + 0.2 \log_2 0.2 \times 2 + 0.15 \log_2 0.15 + 0.1 (\log_2 0.1) + 0.05 (\log_2 0.05)) = 2.408$

The fair dice has more uncertenity
So has more entropy.

Fair $H(x) = \log_2 6 = 2.585$

3 Let X and Y be independent r.v.'s. Show that

$$H(XY) = H(X) + H(Y)$$

$= -\sum_{i=1}^{N} P_{XY} \log_2 P_{XY}$, Because X & Y are independence,
$P_{XY} = P_X \cdot P_Y$, Hence $H(x) = -\sum_{i=1}^{N} P_X \cdot P_Y (\log P_X + \log P_Y)$
$= -\sum_{i=1} P_Y P_Y \log P_X + -\sum_{i=1} P_Y \log P_Y$

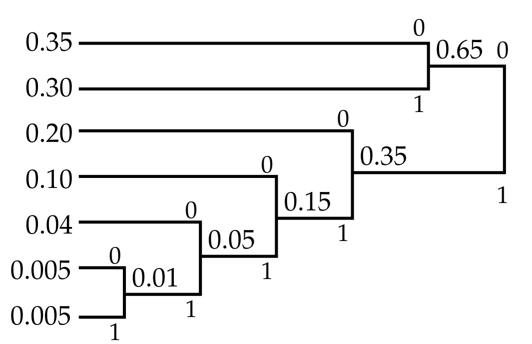
4 Let X be a random variable and let n be a positive integer. Then X^n is the socalled ***n*'th extension** of X. Show that

why

$$H(X^n) = nH(X)$$

$$H(X^n) = \sum_{i=1}^{N} P_i^n \log P_i^n = n \cdot \sum_{i=1}^{N} P_i^n \log_2 P_i$$

5 (*Huffman Coding*)



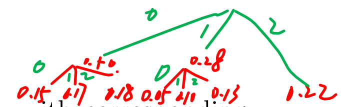
(a) Is the Huffman coding shown above optimal?

(b) Can you think of a DMS Y with 4 symbols for which the Huffman code is optimal?

(c) Let N be a positive integer. Can you think of a DMS Z with N symbols for which the Huffman code is optimal?

6 (*Advanced*) Consider a DMS X that takes values in $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$, with corresponding probabilities $p_1 = 0.22$, $p_2 = 0.18$, $p_3 = 0.17$, $p_4 = 0.15$, $p_5 = 0.13$, $p_6 = 0.10$ and $p_7 = 0.05$. Design a **ternary** Huffman code, using 0, 1 and 2 as letters. What is the resulting average codeword length? Compare the average codeword length with the entropy in a meaningful way.

有什么好处吗？



7 Consider the DMS X that takes values in the alphabet $\{-5, -3, -1, 0, 1, 3, 5\}$, with corresponding probabilities $0.05, 0.10, 0.10, 0.15, 0.05, 0.25, 0.30$.

(a) Calculate $H(X)$. $H(x) =$

(b) Assume that the source is quantized as So there are 3 symbol

$$\begin{aligned} q(-5) &= q(-3) = -4 \\ q(-1) &= q(0) = q(1) = 0 \\ q(3) &= q(5) = 4 \end{aligned}$$

$-4 : 0.15$
$0 : 0.30$
$4 : 0.55$

Calculate the entropy of the quantized source.

8 Consider a DMS X that takes values in $\{0, 1\}$ with corresponding probabilities 0.9 and 0.1 Recall that $H(X) = 0.47$. How to approach $H(X)$ by J'th extension Huffman coding? Construct a table for $J = 1$, $J = 2$ and $J = 3$.

9 Consider a DMS X that takes values in $\{a_1, a_2, a_3, a_4\}$, with corresponding probabilities $p_1 = 0.4$, $p_2 = 0.3$, $p_3 = 0.2$ and $p_4 = 0.1$.

(a) Compute the entropy $H(X)$. $\simeq 1.846$

(b) What is the minimum required average codeword length to represent this source for error-free reconstruction? $2^2 = 4 \Rightarrow 2$ bits

(c) Design a (symbol-by-symbol) Huffman code and compare its average codeword length with $H(X)$.

(d) Design a Huffman code for the 2nd extension of the source ($J = 2$, taking two symbols at a time). What is the average codeword length? What is the average number of bits per source symbol?

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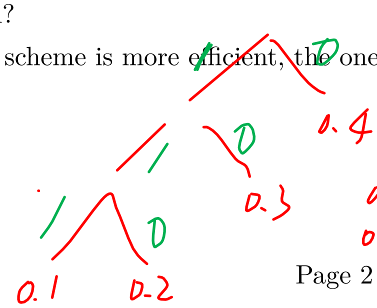
(e) Which scheme is more efficient, the one in (c) or the one in (d)? of course (d)



$0.4 \times 1 + 0.3 \times 2 + 0.2 \times 3 + 0.1 \times 3 = 1.9$

10 Perform LZ78 encoding on the binary source sequence

0001001000000110000100000001000000010100001000000110100000001100

Hint: you require two passes through the sequence to decide on the size of the dictionary