ADVANCED COMMUNICATION SYSTEMS

ELEN90051 (LECTURER: MARGRETA KUIJPER)

Digital Modulation

1st Semester 2018

Material based on Chapters 2 & 3 of "Digital Communications" by Proakis & Salehi, 2008, and Chapter 7 of "Communication Systems Engineering" by Proakis & Salehi, 2002, and Chapter 5 of "Digital Communications—a Discrete-Time Approach" by Michael Rice, 2009

Digital Communication System

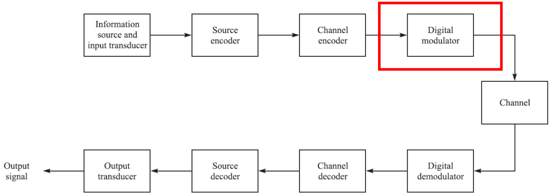


FIGURE 1.1–1
Basic elements of a digital communication system.

Digital modulation for AWGN channels

- Signal Space Representation
  - Baseband-bandpass signals
  - Signal space properties
  - Orthonormal bases
- Digital Modulation Schemes
  - One-dimensional modulation
  - Two-dimensional modulation
  - Multi-dimensional modulation

Digital Modulation

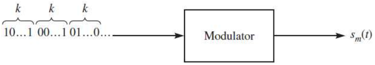
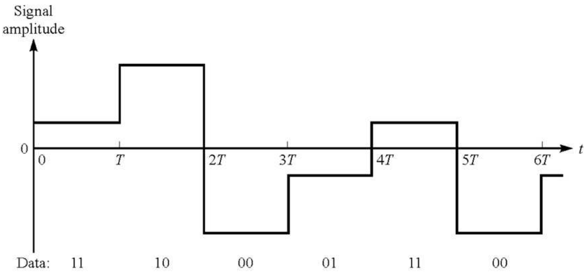


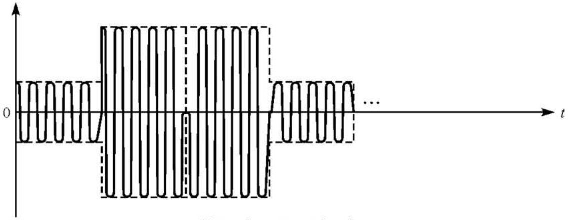
FIGURE 3.1–1
Block diagram of a memoryless digital modulation scheme.

- Modulation: mapping *digital* information into *analog* signals suitable for transmission over physical channels.
  - requires parsing of the incoming bit sequence into a sequence of binary words of length k
  - each binary word of length k corresponds to a symbol; there are $M = 2^k$ possible symbols
  - each symbol corresponds to a signaling interval of length T
  - $1/T$ is the symbol rate; k/T is the bit rate
- Examples of channels: Deep space, earth atmosphere, underwater acoustic channels, fibre optic cables, optical discs, etc.
- Different *channels* cause different types of *impairments* and require different types of modulation.

Baseband – Bandpass Signals



(a) Baseband PAM signal



(b) Bandpass PAM signal

BASEBAND – BANDPASS SIGNALS

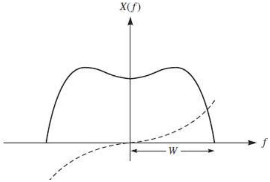


FIGURE 2.1–1
The spectrum of a real-valued lowpass (baseband) signal.

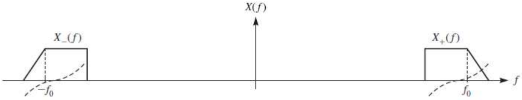


FIGURE 2.1–2
The spectrum of a real-valued bandpass signal.

- Baseband channels: Have frequency passbands that include zero frequency ($f = 0$). No need to use carrier waveforms to transmit source information.
- Bandpass channels: Have frequency passbands far removed from f=0. Digital information is impressed on higher frequency sinusoidal waveforms, i.e., carrier modulation.

Signal Space

- To simplify analysis, use geometric vector representation for baseband and bandpass signals $\Rightarrow$ signals have same properties as vectors. *Further Reading: Principles of Communication Engineering (1965) by Wozencraft and Jacobs*
- Vector Space: A linear space or vector space L over a field F (usually $F = \mathbb{R}$ or $F = \mathbb{C}$) is a set that is closed under two operations of addition and scalar multiplication, and satisfies the following axioms:
  - Associative $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$
  - Commutative: $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
  - Distributive: $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$, $(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}$
  - Addition: $\mathbf{v} + (-\mathbf{v}) = -\mathbf{v} + \mathbf{v} = \mathbf{0}$, $\mathbf{0} + \mathbf{v} = \mathbf{v} + \mathbf{0} = \mathbf{v}$
  - Multiplication: $(ab)\mathbf{v} = a(b\mathbf{v})$, $1\mathbf{v} = \mathbf{v}$, $0\mathbf{v} = \mathbf{0}$,
- Examples of vector spaces: $\mathbb{R}^n$, $\mathbb{C}^n$, function spaces, etc.

Signal Space

A signal space is a vector space consisting of functions $x(t)$ defined on a time set T; addition and scalar multiplication are defined by $(x_1 + x_2)(t) = x_1(t) + x_2(t)$ and $(\alpha x)(t) = \alpha x(t)$ where $\alpha \in \mathbb{C}$.

Main idea:

- represent a signal function by a vector
- represent this vector as a point in the signal space
- the modulation scheme is then visualized as a finite set of points, called the signal space diagram or signal constellation
- this approach enables a geometric interpretation, also it allows us to treat bandpass modulation in a similar way as baseband modulation

Signal Space

Properties of signal functions:(here α^* denotes the conjugate of a complex number α)

- The inner product of two complex-valued signals is

$$\langle x_1(t), x_2(t) \rangle := \int_{-\infty}^{\infty} x_1(t) x_2^*(t) dt.$$

- Two signals $x_1(t)$ and $x_2(t)$ are called orthogonal if $\langle x_1(t), x_2(t) \rangle = 0$.
- The norm of $x(t)$ is

$$||x(t)|| := \sqrt{\langle x(t), x(t) \rangle} = \sqrt{\int_{-\infty}^{\infty} |x(t)|^2 dt} = \sqrt{\mathcal{E}_x},$$

where $\mathcal{E}_x$ is the energy in $x(t)$.

SIGNAL SPACE

- Distance between two signals is

$$d(x_1(t), x_2(t)) = ||x_1(t) - x_2(t)||.$$

- Cauchy-Schwarz inequality for two signals is

$$|\langle x_1(t), x_2(t)\rangle| \leq ||x_1(t)|| \cdot ||x_2(t)|| = \sqrt{\mathcal{E}_{x_1}\mathcal{E}_{x_2}}$$

  with equality when $x_1(t) = \alpha x_2(t)$, where α is any complex number. Useful inequality which we will use in the topic "demodulation".

- Triangle inequality is

$$||x_1(t) + x_2(t)|| \leq ||x_1(t)|| + ||x_2(t)||.$$

Signal Space

- A set of N signals $\{\phi_j(t), j = 1, 2, \ldots, N\}$ spans a subspace S if any signal $s(t) \in S$ can be written as a linear combination of the N signals

$$s(t) = \sum_{j=1}^{N} s_j \phi_j(t),$$

where s_j are scalar-valued coefficients.

- A set of signals is linearly independent if *no signal in the set* can be represented as a linear combination of *the other signals in the set*.
- A basis for S is any linearly independent set that spans it.
- The dimension of S is the number of elements in *any basis for S*.

Orthonormal Bases

Orthonormal Bases: A basis $\{\phi_j(t), j = 1, 2, \ldots, N\}$ for a subspace S is an orthonormal basis if

$$\langle \phi_j(t), \phi_n(t) \rangle = \int_{-\infty}^{\infty} \phi_j(t)\phi_n^*(t)dt = \begin{cases} 1 & j = n \\ 0 & j \neq n \end{cases}$$

An orthonormal basis is a convenient way to represent any $s(t) \in S$ as

$$s(t) = \sum_{j=1}^{N} s_j \phi_j(t)$$

where s_j is the *projection* of $s(t)$ onto the basis vector $\phi_j(t)$, i.e.,

$$s_j = \langle s(t), \phi_j(t) \rangle = \int_{-\infty}^{\infty} s(t)\phi_j(t)dt.$$

Orthonormal Bases

- Recall that modulation involves a signal set $\{s_1(t), \ldots, s_M(t)\}$
- Each signal $s_m(t)$ (for $m = 1, \ldots, M$) is represented in terms of the N basis signals as a vector given by

$$\mathbf{s}_m = [s_{m1}, s_{m2}, \ldots, s_{mN}]^T$$

or equivalently as a point in the N-dimensional signal space.
- The plot of these M points is called a signal space diagram or a signal constellation, the following plots are $N = 2$ examples:

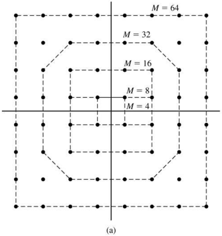


Figure 7.23
(a) Rectangular signal-space constellations for QAM.

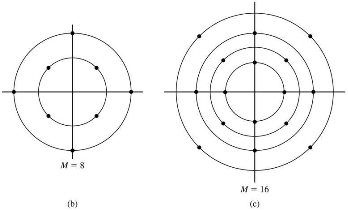


Figure 7.23
(b, c) Examples of combined PAM-PSK signal-space constellations.

Orthonormal Bases

- Each point corresponds to $k = \log_2 M$ bits of information.
- The square of the Euclidean distance of a point to the origin equals the energy of the corresponding signal

$$\mathcal{E}_{s_m} = s_{m1}^2 + s_{m2}^2 + \cdots + s_{mN}^2.$$

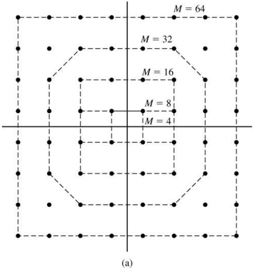


Figure 7.23

(a) Rectangular signal-space constellations for QAM.

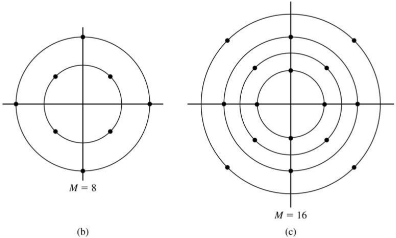


Figure 7.23

(b, c) Examples of combined PAM-PSK signal-space constellations.

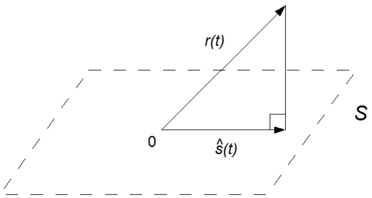
Orthonormal Bases

Question:(useful for later topic of "demodulation")

- Given a received signal $r(t)$ outside a subspace S with orthonormal basis $\{\phi_j(t), j = 1, 2, \dots, N\}$, which $\widehat{s}(t) = [s_1, \dots, s_N]$ in S is closest to $r(t)$? In other words, which $\widehat{s}(t) \in S$ minimizes the distance $||r(t) - \widehat{s}(t)||$?

Answer:

- $\widehat{s}(t)$ is the projection of $r(t)$ onto the signal space S. More specifically, $\widehat{s}(t) = [s_1, \dots, s_N]$ where $s_j = \langle r(t), \phi_j(t) \rangle$, is the projection of $r(t)$ onto each basis function $\phi_j(t)$ (for $j = 1, \dots, N$).



Orthonormal Bases

- We can construct a set of orthonormal basis signals using the Gram-Schmidt procedure, starting from a set of signals $\{s_m(t), m = 1, 2, \ldots, M\}$. The first orthonormal signal is simply

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{\mathcal{E}_1}},$$

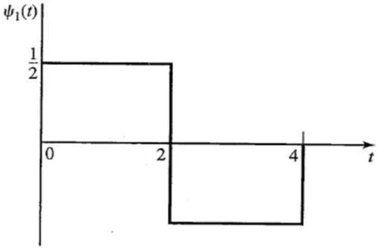
- The j-th basis signal can be found as $\phi_j(t) = \frac{\gamma_j(t)}{\sqrt{\mathcal{E}_j}}$ where $\gamma_j(t) = s_j(t) - \sum_{i=1}^{j-1} c_{ji}\phi_i(t)$, $c_{ji} = \langle s_j(t), \phi_i(t)\rangle$, and $\mathcal{E}_j = \int_{-\infty}^{\infty} \gamma_j^2(t)dt$. *(See textbook Ch 2.2-4 for further details.)*
- The procedure is continued until all the M signals have been exhausted and $N \leq M$ orthonormal signals have been constructed $\Rightarrow$ Dimension of signal space is N.

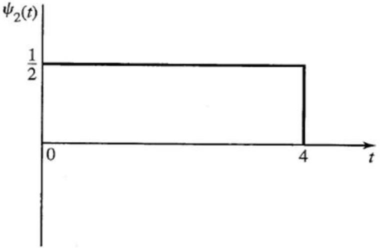
N.B. In most cases we will be able to recognize orthonormal bases fairly easily, so that Gram-Schmidt is not needed.

Signal Space

Questions:

- What are the norms of the two signals below?
- Are they orthogonal?
- What is the distance between them?

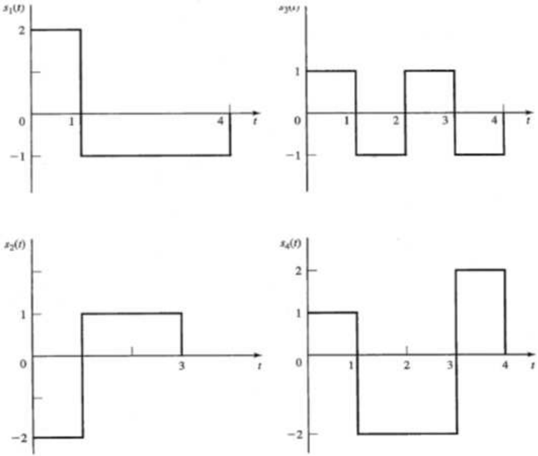




Signal Space

Question:

- What is the **dimension** of the subspace spanned by the signals below?



DIGITAL MODULATION

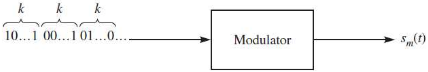


FIGURE 3.1–1
Block diagram of a memoryless digital modulation scheme.

- Symbol generator: maps a sequence of blocks of k bit values into a sequence of symbols. There are $M = 2^k$ possible symbols.
- Modulator: maps a symbol sequence into a continuous time signal $s(t)$.
    - Binary modulation: $k = 1$, $M = 2$ symbols
    - 4-ary modulation: $k = 2$, $M = 4$ symbols
    - M-ary modulation: $k > 1$, $M = 2^k$ symbols

One-dimensional Modulation: OOK

One of the simplest binary modulation schemes is On Off Keying (OOK).

- a baseband OOK modulator maps a binary symbol sequence $a(n)$ to a continuous time signal $s(t)$ given by

$$s(t) = \sum_{n \in \mathbb{Z}} a(n) p(t - nT) \qquad t \in \mathbb{R},$$

where $1/T$ is the symbol rate and $p(t)$ is a pulse signal, for example:

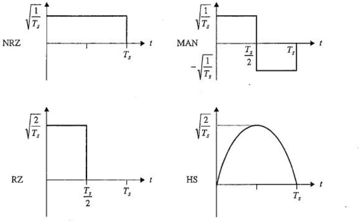


Figure A.1.1 The non-return-to-zero (NRZ), the return-to-zero (RZ), Manchester (MAN), and half-sine (HS) pulse shapes as shown.

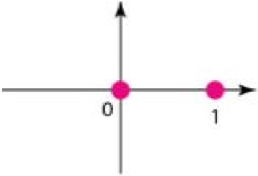
Thus on the signaling interval $(0, T]$ baseband OOK modulation produces a signal of the form

$$s_m(t) = A_m p(t), \quad \text{for } m = 1, 2, \quad A_1 = 1, A_2 = 0.$$

Bandpass OOK employs a carrier frequency f_c—on the signaling interval $(0, T]$ it produces a signal of the form

$$s_m(t) = A_m g(t) \cos(2\pi f_c t), \quad \text{for } m = 1, 2, \quad A_1 = 1, A_2 = 0.$$

where the pulse signal is now denoted by $g(t)$. The OOK signal constellation is one-dimensional and given by

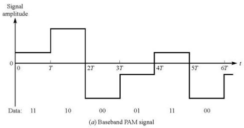


ONE-DIMENSIONAL MODULATION: PAM

Similar as OOK, a baseband PAM modulator maps a symbol sequence $a(n)$ to a continuous time signal $s(t)$ given by

$$s(t) = \sum_{n \in \mathbb{Z}} a(n) p(t - nT) \qquad t \in \mathbb{R},$$

for example



On the signaling interval $(0, T]$ baseband PAM modulation produces a signal of the form

$$s_m(t) = A_m p(t), \quad 1 \leq m \leq M,$$

where $\{A_1, A_2, \ldots, A_M\}$ is the set of possible amplitudes.

Some example constellations are shown below:

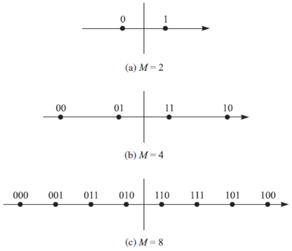


FIGURE 3.2–1
Constellation for PAM signaling.

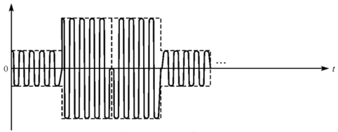
- The energy per constellation point equals

$$\mathcal{E}_m = ||s_m(t)||^2 = \int_0^T A_m^2 p^2(t)dt = A_m^2 \mathcal{E}_p$$

where $\mathcal{E}_p$ is the energy in $p(t)$.

- An orthonormal basis vector for PAM is given by

$$\phi(t) = \frac{p(t)}{\sqrt{\mathcal{E}_p}}$$



(b) Bandpass PAM signal

- For carrier-modulated PAM signals, we have

$$s_m(t) = A_m g(t)\cos(2\pi f_c t), \quad 1 \le m \le M, \quad 0 \le t < T$$

- the bandpass PAM energy per constellation point equals

$$\begin{aligned}\mathcal{E}_m = ||s_m(t)||^2 &= \int_0^T A_m^2 g^2(t)\cos^2(2\pi f_c t)dt \\ &= \frac{A_m^2}{2}\int_0^T g^2(t)dt + \frac{A_m^2}{2}\underbrace{\int_0^T g^2(t)\cos(4\pi f_c t)dt}_{\approx 0 \text{ for large } f_c} \approx \frac{A_m^2}{2}\mathcal{E}_g\end{aligned}$$

- binary bandpass PAM is also called Binary Phase Shift Keying (**BPSK**) because the symbol values inform the phase of $s_m(t)$. More precisely: $A_1 = 1 \leftrightarrow$ phase 0 and $A_2 = -1 \leftrightarrow$ phase π.

RECALL FROM EARLIER SUBJECTS (SIGNALS & SYSTEMS):

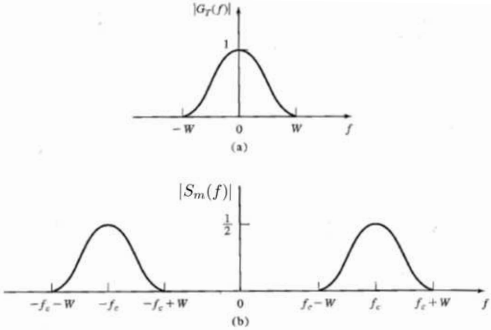


Figure 7.9 Spectra of (a) baseband and (b) amplitude-modulated signals.

- Modulating the signal waveform $s_m(t)$ with carrier $\cos(2\pi f_c t)$ shifts the spectrum of the baseband signal by f_c, i.e.,

$$S_m(f) = \frac{A_m}{2}\left(G_T(f-f_c) + G_T(f+f_c)\right)$$

- For bandpass PAM signaling, the orthonormal basis vector is given by

$$\phi(t) = \sqrt{\frac{2}{\mathcal{E}_g}}g(t)\cos(2\pi f_c t)$$

which results in $s_m(t) = A_m\sqrt{\frac{\mathcal{E}_g}{2}}\phi(t)$.

- Bandpass PAM has the same signal space diagram as baseband PAM but with a different basis vector

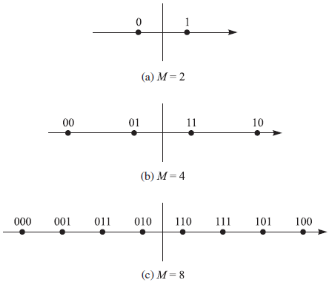
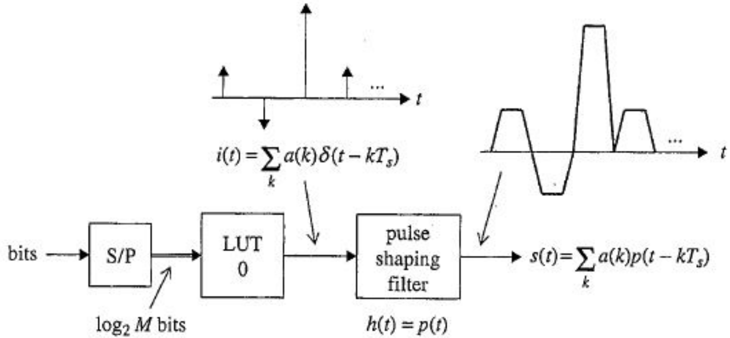


FIGURE 3.2–1
Constellation for PAM signaling.

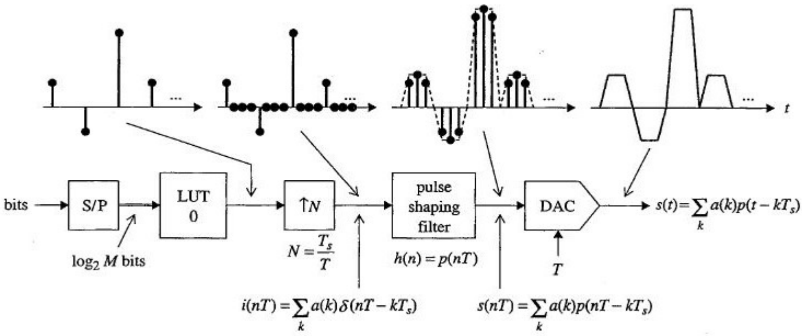
PAM MODULATOR IMPLEMENTATION

(in the next two figures the signaling interval is not denoted by T but by T_s)

PAM modulation, using a continuous-time pulse shaping filter:



PAM modulation, using a discrete-time FIR pulse shaping filter:



we see that this involves an upsampler as well as a digital-to-analog converter.

Two-dimensional Modulation

- Orthogonal signaling: Modulation using two signals $\phi_1(t)$ and $\phi_2(t)$ that are orthonormal, i.e.,

$$s(t) = s_1\phi_1(t) + s_2\phi_2(t)$$

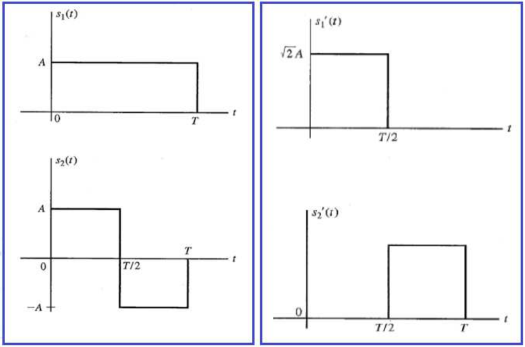
- We can denote the modulated signals as a vector $s(t) = (s_1, s_2)$ where s_1 and s_2 are the *projections* of $s(t)$ onto the basis vectors $\phi_1(t)$ and $\phi_2(t)$, i.e.,

$$s_j = \langle s(t), \phi_j(t) \rangle = \int_{-\infty}^{\infty} s(t)\phi_j(t)dt, \quad j \in \{1, 2\}.$$

Question: What are the geometric vector representations for the two sets of signals below given orthonormal basis functions

$$\phi_1(t) = \begin{cases} \sqrt{2/T} & 0 \le t < T/2 \\ 0 & T/2 \le t < T \end{cases}$$

$$\phi_2(t) = \begin{cases} 0 & 0 \le t < T/2 \\ \sqrt{2/T} & T/2 \le t < T \end{cases}$$



Question: What are the signals for $M = 4$ bi-orthogonal signal constellation below using the same basis functions $\{\phi_1(t), \phi_2(t)\}$?

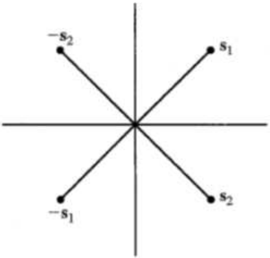
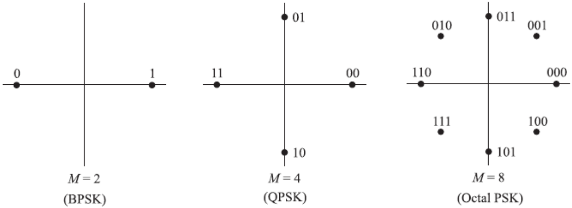


Figure 7.15 Signal constellation for $M = 4$ biorthogonal signals.

Two-dimensional Bandpass Modulation: MPSK

- *M*-ary phase-shift keying (PSK): All M bandpass signals are constrained to have the same energy, i.e., signal constellation points lie on a circle
- Examples:



- Gray encoding: Adjacent phases differ by only 1 bit. This leads to a better average bit error rate (BER). Indeed, in case the demodulator mistakes a symbol for its neighbour then this results in only 1 bit error. And this is the most likely demodulator error scenario.

- Signals with M-PSK modulation can be represented, for $m = 1, 2, \ldots, M$, as

$$\begin{aligned} s_m(t) &= g(t)\cos\left(2\pi f_c t + \theta_m\right), \\ &= \mathrm{Re}\left(g(t)e^{j\theta_m} e^{j2\pi f_c t}\right), \\ &= g(t)\cos\left(\theta_m\right)\cos(2\pi f_c t) - g(t)\sin\left(\theta_m\right)\sin(2\pi f_c t) \end{aligned}$$

where $g(t)$ is the signal pulse shape and $\theta_m = \frac{2\pi}{M}(m-1)$ is the phase that conveys the transmitted information.

- An orthonormal basis for the signal space is

$$\{\phi_1(t), \phi_2(t)\} = \left\{\sqrt{\frac{2}{\mathcal{E}_g}}g(t)\cos(2\pi f_c t), -\sqrt{\frac{2}{\mathcal{E}_g}}g(t)\sin(2\pi f_c t)\right\}$$

where basis functions are unit norm, i.e., $||\phi_1(t)|| = ||\phi_2(t)|| = 1$

Interpretation:

- the transmitted information is impressed on 2 orthogonal carrier signals, namely the in-phase carrier signal $\cos(2\pi f_c t)$ and the quadrature carrier signal $\sin(2\pi f_c t)$

$$s_m(t) = g(t)\cos(\theta_m)\cos(2\pi f_c t) - g(t)\sin(\theta_m)\sin(2\pi f_c t)$$

- its lowpass equivalent signal equals

$$s_m^{lowpass}(t) = g(t)e^{j\Theta_m} = I(t) + jQ(t),$$

where $I(t) := g(t)\cos\theta_m$ is the in-phase component and $Q(t) := g(t)\sin\theta_m$ is the quadrature component of $s_m(t)$.

QPSK

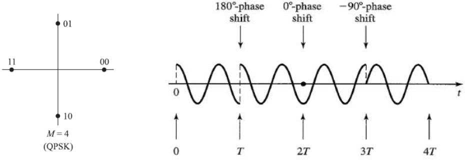


Figure 7.18 Example of a four PSK signal.

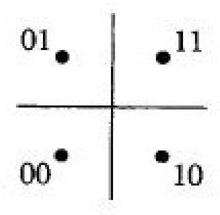
In quadrature/quaternary PSK (QPSK), the $M = 4$ signal points are differentiated via phase shifts by multiples of $\pi/2$, i.e., the corresponding bandpass signals are, for $t \in (0, T]$ given by

$$s_m(t) = g(t) \cos\left(2\pi f_c t + \frac{\pi}{2}(m-1)\right), \quad m = 1, 2, 3, 4$$

In the above figure we see how a QPSK modulator maps a sequence $a(n)$ of dibits into a continuous-time waveform $s(t)$.

QPSK

Equivalently, the signal constellation can be rotated:

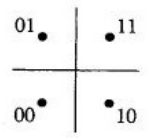


Denote the first bit of the dibit $a(n)$ by $a_1(n)$; denote the second bit of the dibit $a(n)$ by $a_2(n)$. Then the corresponding QPSK modulator maps the two binary symbol sequences $a_1(n)$ and $a_2(n)$ to a continuous-time waveform $s(t)$ given on the next page.

QPSK

$$s(t) = I(t)\sqrt{2}\cos 2\pi f_c t - Q(t)\sqrt{2}\sin 2\pi f_c t,$$

- $I(t) = \sum_n a_1(n)g(t - nT)$ is the in-phase component of $s(t)$
- $Q(t) = \sum_n a_2(n)g(t - nT)$ is the quadrature component of $s(t)$
- Note that $I(t)$ and $Q(t)$ are binary PAM pulse trains. In fact, this QPSK signal constellation can be interpreted as two PAM signal constellations, one on the in-phase component (ϕ_1-axis), the other on the quadrature component (ϕ_2-axis).



QPSK IMPLEMENTATION

Using a continuous-time pulse shaping filter:

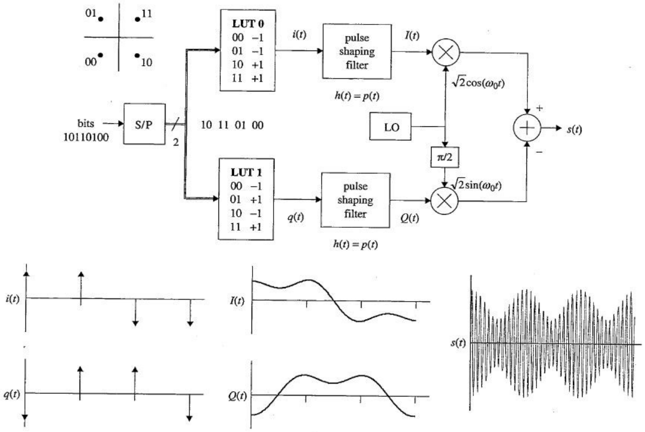


Figure 5.3.12 An example of a QPSK signal corresponding to the bit sequence 10110100.

DMPSK

Differential MPSK (DMPSK) modulation involves precoding of the information symbol sequence $b(n)$ into a symbol sequence $\delta(n)$ that is then input to a MPSK modulator.

- in MPSK each symbol value determines the actual value of the phase, but in DMPSK it determines the phase change from the previous signalling interval's symbol
- thus DMPSK is modulation with memory
- in which situations is differential modulation needed? If the MPSK demodulator's errors are always of the same type, determined by a fixed (or slowly varying) phase mismatch ϕ, the value of which we do not know (it may even be zero).
- how does differential modulation help? It leads to a better symbol error performance since it is robust against a fixed unknown phase mismatch.

EXAMPLE: DBPSK MODULATION

- Example: a symbol value of 0 causes a 0° phase change and a symbol value of 1 causes a 180° phase change

Data Bit		1	0	1	0	0	1
BPSK phase		0°	180°	0°	180°	180°	0°
DBPSK phase	180°	0°	0°	180°	180°	180°	0°

- *Exercise:* Assume that the BPSK demodulator is able to reconstruct the DBPSK values without error. How to retrieve the original data bit sequence?
- *Exercise:* Assume that the BPSK demodulator's phase mismatch $\phi = 180^\circ$ (so it always picks 1 when it should have picked 0 and vice versa, in other words all the DBPSK values are demodulated in error). Convince yourself that your data retrieval method of the previous exercise still works.

Two-dimensional Bandpass Modulation: QAM

- Quadrature amplitude modulation (QAM): Allow signals to have different amplitudes; impress separate information bits on each of the **quadrature carriers**
- Important performance parameters are average energy and minimum distance in the signal constellation

N.B. Note that averaging is needed because the symbols to be modulated are uncertain, one needs a probabilistic attitude

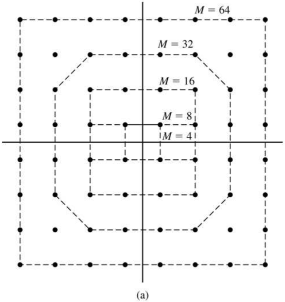


Figure 7.23

(a) Rectangular signal-space constellations for QAM.

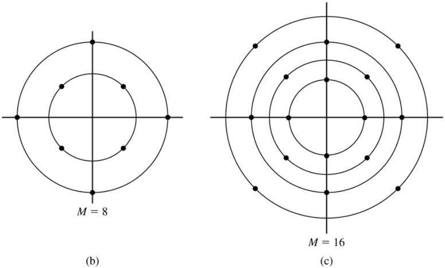


Figure 7.23

(b, c) Examples of combined PAM-PSK signal-space constellations.

Comparison of PAM, PSK and QAM

TABLE 3.2–1
Comparison of PAM, PSK, and QAM

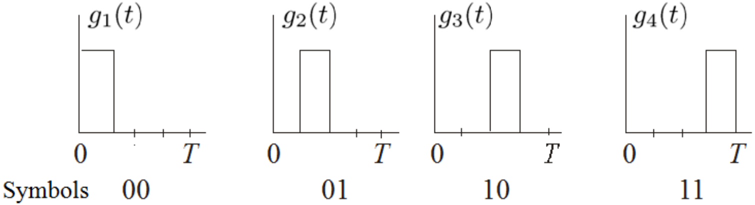
Signaling Scheme	$s_m(t)$	s_m	E_{avg}	E_{bavg}	$d_{\min}$
Baseband PAM	$A_m p(t)$	$A_m\sqrt{\mathcal{E}_p}$	$\frac{2(M^2-1)}{3}\mathcal{E}_p$	$\frac{2(M^2-1)}{3\log_2 M}\mathcal{E}_p$	$\sqrt{\frac{6\log_2 M}{M^2-1}\mathcal{E}_{\text{bavg}}}$
Bandpass PAM	$A_m g(t)\cos 2\pi f_c t$	$A_m\sqrt{\frac{\mathcal{E}_g}{2}}$	$\frac{M^2-1}{3}\mathcal{E}_g$	$\frac{M^2-1}{3\log_2 M}\mathcal{E}_g$	$\sqrt{\frac{6\log_2 M}{M^2-1}\mathcal{E}_{\text{bavg}}}$
PSK	$g(t)\cos\left[2\pi f_c t + \frac{2\pi}{M}(m-1)\right]$	$\sqrt{\frac{\mathcal{E}_g}{2}}\left(\cos\frac{2\pi}{M}(m-1), \sin\frac{2\pi}{M}(m-1)\right)$	$\frac{1}{2}\mathcal{E}_g$	$\frac{1}{2\log_2 M}\mathcal{E}_g$	$2\sqrt{\log_2 M \ \sin^2\left(\frac{\pi}{M}\right)\ \mathcal{E}_{\text{bavg}}}$
QAM	$A_{mi}\, g(t)\cos 2\pi f_c t - A_{mq}\, g(t)\sin 2\pi f_c t$	$\sqrt{\frac{\mathcal{E}_g}{2}}(A_{mi}, A_{mq})$	$\frac{M-1}{3}\mathcal{E}_g$	$\frac{M-1}{3\log_2 M}\mathcal{E}_g$	$\sqrt{\frac{6\log_2 M}{M-1}\mathcal{E}_{\text{bavg}}}$

Multi-dimensional Modulation

- Can use time domain and/or frequency domain to increase the number of dimensions
- Orthogonal Signaling (Baseband): e.g. Pulse position modulation (**PPM**)

$$s_m(t) = A_m g(t - \tau_m), \qquad \tau_m \leq t < \tau_{m+1}$$

where $1 \leq m \leq M$ and $g(t)$ is a pulse of duration T/M.



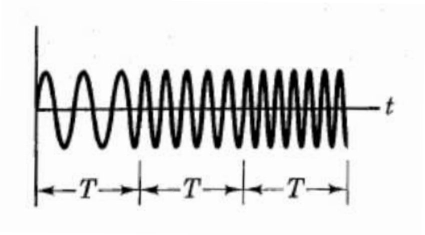
Four-dimensional PPM

- Orthogonal Signaling (Bandpass): e.g. Frequency shift-keying **(FSK)**

$$s_m(t) = \sqrt{\frac{2\mathcal{E}}{T}} \cos(2\pi(f_c + m\Delta f)t), \quad 0 \leq m \leq M-1, \quad 0 \leq t \leq T$$

  where Δf is the frequency separation between successive frequencies.

- For orthogonality, we need to have minimal frequency separation of $\Delta f = 1/(2T)$ (see Tutorial problem)



- Bi-Orthogonal Signaling: A set of $M = 2^k$ bi-orthogonal signals is constructed from $\frac{1}{2}M$ orthogonal signals and their negatives

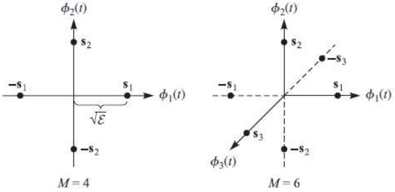


FIGURE 3.2–8
Signal space diagram for $M = 4$ and $M = 6$ biorthogonal signals.

N.B. Many other possible modulation schemes. See textbook Chapter 3 for further details.

QUIZ

Question: Which set of signals corresponds to the following modulations?

1. Orthogonal signalling
2. PAM
3. Bi-orthogonal signalling