Convolutional Coding

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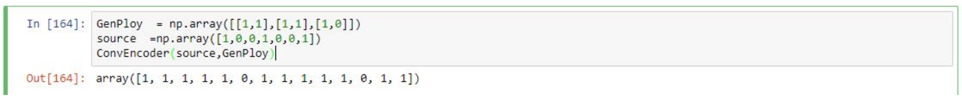
Task 1

The Python code of ConvEncoder are shown below:

```
# implement the encoder
def ConvEncoder(SourceSequence,GenPoly):
    # The SourceSequence is a stream of bits with {0,1} in string type
    # The GenPoly is the matrix.In this case GenPoly[0] = g1 = [1,1,1]
        GenPoly[1] = g2 = [1,1,0]
    # matrix multiply are using for calculation efficiency
    EncodedCodeSequence = list()
    reg = np.zeros((1,3)) # The initial state of register are [0,0,0]
    #print(reg.shape)
    for curBit in SourceSequence:# for every bit in the sequency
        # Insert the curBit into the reg
        print('curbit␣{}'.format(curBit))
        reg = np.array([curBit,reg[0][0],reg[0][1]])
        print('Registeris␣:{}'.format(reg))
        print(reg.shape)
        y = np.dot(reg,GenPoly)  # y = [x0+x1+x2,x0+x1]
        y = np.mod(y,2)
        EncodedCodeSequence = EncodedCodeSequence + [int(y[0][0]),int(y
            [0][1])]
    return np.array(EncodedCodeSequence)
```

With source code to be 1001001 ,the output are shown below,which is correct.



```
In [164]: GenPloy  = np.array([[1,1],[1,1],[1,0]])
          source  =np.array([1,0,0,1,0,0,1])
          ConvEncoder(source,GenPloy)

Out[164]: array([1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 1])
```

Figure 1: Output of CovEncoder

Task 2

1.

We assume the initial state is 00,and the source sequence is of length 5 using the encoder above.Also,the nosise of the channel is no greater than the signals. There are $2^5 = 32$ possible inputs.The most possible one is **11000**,which has only 1 bits error.

2.

The Figure are shown below of the value of path and branch of the optimized path indicated in yellow pen.We can see the optimum path is **11000** the same as previous question.

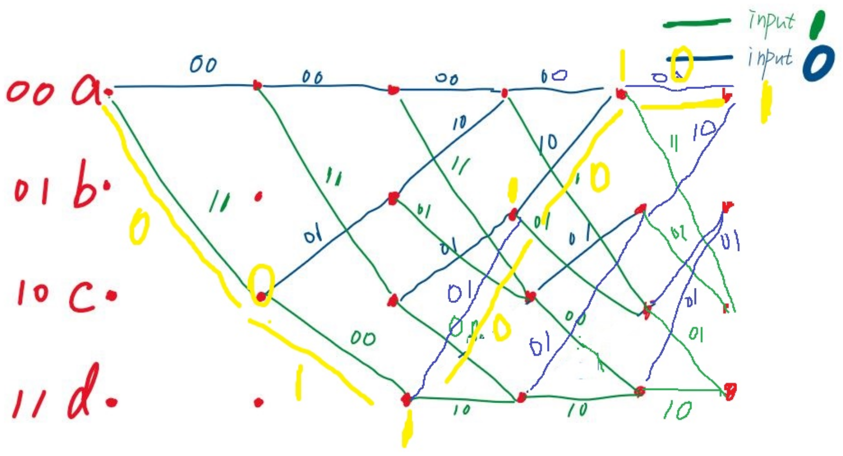


Figure 2: Vertb with Trellis Diagram

3.

When applied Soft Decision,the value of the branch and path should be changed from the hamming distance(# of different bits) to the Eucid Distance $\sqrt[n]{\sum^{n}(y_i - y1_i)^2}$

Task 3

1.

The Python code are shown below:

```
def ConvDecoder(ReceivedSequence,Type):
F = np.ones([4,ReceivedSequence.shape[0]//2+2])*0xFFFF # for state's F[00]
    F[01] F[10]  F[11] set the large value for init;
G = np.zeros([4,ReceivedSequence.shape[0]//2+2,2])  # to store the decision
     make. to recover the path to get the decode code.
#Reg = np.zeros(4)
F[0,0] = 0
for i in range(1,(ReceivedSequence.shape[0]//2)+1,1):
        for cur_state in range(4):
                for pre_state in range(4): # find the min value for
                    previous state to current state
                        trans = str(pre_state)+'to'+str(cur_state)
                        if(trans in stateTrs.keys()):
                        #if the state transform is exists()
                                predOut = np.array(stateTrs[trans][1])
                                realOut = ReceivedSequence[(i-1)*2:((i-1)
                                    *2+1)+1]
                                # The state diagram are stored in a
                                    stateTrs dictionary
                                dis = Distance(predOut,realOut,Type)
                                # determine the distance between  predict
                                    sequence and received sequence

                                if dis+F[pre_state,i-1]<=F[cur_state,i]:
                                    F[cur_state,i] = dis+F[pre_state,i
                                        -1]
                                # update the min Value.
                                    G[cur_state,i][1] = pre_state
                                    G[cur_state,i][0] = stateTrs[trans
                                        ][0]
                                    # store the cur_state is from
                                        pre_state and input.

# Find the min values of the final node.
minState = 0
minVal = 0xFFFF
for item in range(4):
        if(F[item,i]<minVal):
                minState = item
                minVal = F[item,i]

EncodedCodeSequence = list()
# use a recursive function to generate the predicted input sequence using G
     array
findCode(G,minState,i,EncodedCodeSequence)

return np.array(EncodedCodeSequence[::-1])
```

2.

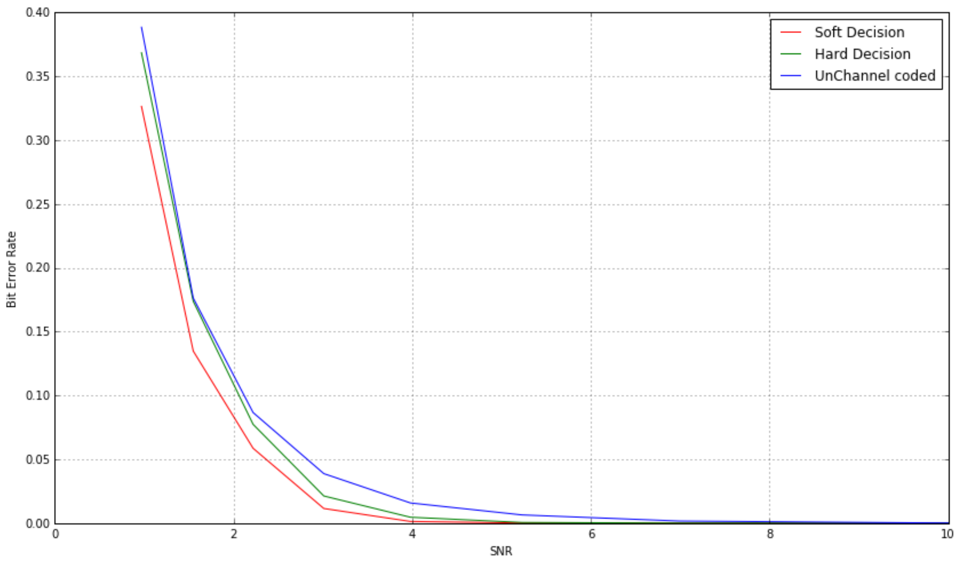


Figure 3: BER vs SNR Performance

3.

We could see from the Fig.3 that with high SNR,even with no channel coding,the ML detection with bpsk has a good error performance to be nearly 0 which is same as the one with channel coding.However,with noise increasing,the channel coding shows it better error performance compared with un-channel coding. The Soft and Hard decision method shows little error performance in high SNR,with no error.However,in low SNR,the soft decision method shows better error performance which is consistent with the theory.

Reflections and Comparisons

1.

Error Correcting codes are used to provide efficient transmission of data over the noisy channel. Convolutional codes are one of the FEC (Forward error correction) codes which are being used in wireless communication from early 1970s[2].
Convolutional codes were introduced in 1955 by Peter Elias. It was thought that convolutional codes could be decoded with arbitrary quality at the expense of computation and delay.[1]Convolutional codes transform a whole sequence of information bits into a sequence of encoded bits by convolving the information bits with a set of generator coefficients.[2] In 1967 Andrew Viterbi determined that convolutional codes could be maximum-likelihood decoded with reasonable complexity.[1].The Viterbi algorithm is just a dynamic programming method which reduced the time complexity from exponential to polynomial.
In computer science,the dynamic programming algorithm has the property of **optimal substructure** and **overlapping sub-problems**.With these two properties,the algorithm can determine the global optimal value in polynomial time complexity with more space consumptions.Therefore,it is a time and space trade-off.

2.

If we need to chnage a Viterbi equalizer to the Viterbi decoder,the branch's metrics need to be changed from the conditional probability to the distance.For Hard decision,the distance is hamming distance while for Soft decision,the distance is Eucild distance.
Also the state of the optimized function need to be changed from only two state0,1 in equalizer to more states depending on the coding method.

References

[1] Convolutional code. Website, 2018. `https://en.wikipedia.org/wiki/Convolutional_code#History`.

[2] Geetanjali Baghel and Ms Mamta Arora. Review paper convolutional codes in a nutshell. *IJRIT International Journal of Research in Information Technology, Volume 3, Issue 3, March 2015, Pg. 77-83*, 2015.