THE UNIVERSITY OF MELBOURNE

Semester 1 Assessment

June, 2015

Department of Electrical and Electronic Engineering

ELEN90051 ADVANCED COMMUNICATION SYSTEMS

Time allowed: 180 minutes

Reading time: 15 minutes

This paper has 7 pages.

Authorised materials:
Melbourne School of Engineering approved calculators only.
Drawing instruments are permitted.

No computers or communicating devices are allowed.
This is a CLOSED BOOK exam; no other books, papers, written material of any kind, or other aids are permitted.

Instructions to invigilators:
All script books and exam papers should be collected at the end of the examination.

Instructions to students:
Students should attempt **ALL 9 QUESTIONS**.
The maximum total marks of the examination paper is 100.
All script books and exam papers will be collected at the end of the three hour examination period.

Question 1 $(3+2+3+2 = 10$ marks)

The bandpass signals of a M-ary pulse amplitude modulation (M-PAM) scheme are defined as

$$s_m(t) = A_m g(t) \cos(2\pi f_c t),$$

where the amplitudes are

$$A_m = 2m - 1 - M, \quad m \in \{1, 2, \ldots, M\},$$

f_c is the carrier frequency and $g(t)$ is the signal pulse with energy $\mathcal{E}_g$.

(a) Determine the dimension of the signal space spanned by the above signals and a set of orthonormal basis functions.

(b) Use your basis functions from (a) to represent the signals as vectors when $M = 4$.

(c) Sketch the corresponding signal space diagram, clearly labelling the axes and signal points.

(d) Describe one approach to increase the dimension of this signal space with multi-dimensional modulation.

Question 2 $(4+5+3 = 12$ marks)

The received signal for an additive white Gaussian noise (AWGN) channel is given by

$$r(t) = s_m(t) + n(t)$$

where $s_m(t)$ is the transmitted signal waveform and $n(t)$ is an AWGN random process to model thermal noise.

(a) Assume that M-ary phase-shift keying (M-PSK) modulation is used at the transmitter with orthonormal basis functions $\{\phi_1(t), \phi_2(t)\}$. Sketch the corresponding matched filter-type demodulator for the receiver. Clearly label the input waveform $r(t)$, output vector $\mathbf{r}$, and the impulse response of the matched filters.

(b) Recall that the maximum-likelihood (ML) detector decides which of M possible signal waveforms was transmitted based on the criterion of

$$\hat{m} = \arg\max_m p(\mathbf{r}|\mathbf{s_m}),$$

where $p(\mathbf{r}|\mathbf{s_m})$ is the conditional probability density function (pdf) of $\mathbf{r}$ given that $\mathbf{s_m}$ was transmitted. Show that the ML criterion simplifies to the minimum distance criterion of

$$\hat{m} = \arg\min_m ||\mathbf{r} - \mathbf{s_m}||,$$

when the demodulated noise vector components of $\mathbf{n} = (n_1, \ldots, n_N)$ are random variables that follow a Gaussian distribution of

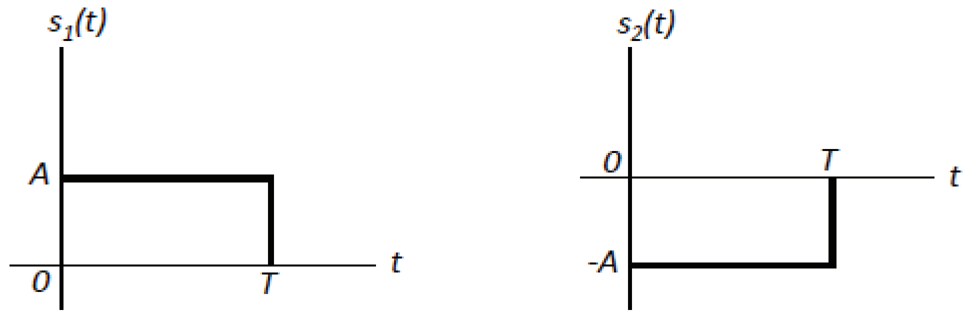
$$p(n_k) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(n_k - \mu)^2}{2\sigma^2}\right),$$

with zero mean and variance $N_0/2$.

Question 2 continues over the page

(c) Describe one advantage of differential phase-shift keying (DPSK) modulation compared with M-PSK modulation.

$\boxed{\textbf{Question 3}}$ $(6 + 2 + 4 = 12 \text{ marks})$



The two baseband signals shown above are used to transmit a binary phase-shift keying (BPSK) sequence over an AWGN channel of power spectral density $N_0/2$. Assume that the signals are equally likely to be transmitted and coherently demodulated at the receiver with a ML detector.

(a) Compute the probability of error $P(e)$ of this transmission scheme.

(b) Sketch two additional baseband signals to extend the signalling scheme to quadrature phase-shift keying (QPSK) modulation.

(c) Calculate the probability of error of QPSK based on your answer in (a).

$\boxed{\textbf{Question 4}}$ $(5 + (2 + 1 + 1) + 4 + 3 = 16 \text{ marks})$

The receive filter output of a band-limited channel with inter symbol interference (ISI) can be described as

$$y(kT) = \sum_{n=0}^{\infty} I_n x((k-n)T) + \nu(kT), \qquad k = 0, 1, 2, \ldots$$

where T is the symbol period, I_n is the transmitted symbol, $\nu(t)$ is the filtered noise, and $x(t)$ is the impulse response of the cascade of filters from the transmitter to the receiver.

(a) Explain (with the aid of diagrams) the Nyquist pulse-shaping criterion given by

$$\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T,$$

in terms of the baseband bandwidth of $X(f)$ and the symbol rate $R = 1/T$ Hz.

Question 4 continues over the page

(b) For each of the following modulation schemes, determine the maximum bit rate for zero ISI that can be transmitted through a 3 kHz voice-band telephone channel with a raised cosine spectrum pulse shape where $\beta = 1/4$. Recall that the baseband bandwidth of the raised cosine spectrum is $W = \frac{1+\beta}{2T}$ Hz.

(i) BPSK,

(ii) QPSK,

(iii) 8-PAM.

(c) To recover the transmitted symbols, the sampled output of the receive filter is passed through a noise whitening filter which results in

$$u_k = f_0 I_k + \sum_{n=1}^{\infty} f_n I_{k-n} + \eta_k, \qquad k = 0, 1, 2, \ldots$$

where η_k is white noise and f_n are coefficients for the cascade of filters. Calculate the filter taps for a three-tap zero-forcing linear equalizer when the channel impulse response is

$$f_k = \begin{cases} 0.5 & k = \pm 1, \\ 1 & k = 0, \\ 0 & \text{otherwise.} \end{cases}$$

Recall that the equalizer outputs are $\hat{I}_k = \sum_{j=-K}^{K} c_j u_{k-j}$ where c_j are the filter taps and $q_n = \sum_{j=-K}^{K} c_j f_{n-j}$. The following matrix inverses may be used:

$$\begin{bmatrix} 0 & 0.5 & 1 \\ 0.5 & 1 & 0.5 \\ 1 & 0.5 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0.5 & -1 & 1.5 \\ -1 & 2 & -1 \\ 1.5 & -1 & 0.5 \end{bmatrix};$$

$$\begin{bmatrix} 0 & 1 & 0.5 \\ 1 & 0.5 & 1 \\ 0.5 & 1 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} -1.1429 & 0.5714 & 0.8571 \\ 0.5714 & -0.2857 & 0.5714 \\ 0.8571 & 0.5714 & -1.1429 \end{bmatrix};$$

$$\begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0.5 \\ 0 & 0.5 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1.5 & -1 & 0.5 \\ -1 & 2 & -1 \\ 0.5 & -1 & 1.5 \end{bmatrix}.$$

(d) Describe one approach to improve the error performance of the zero-forcing equalizer.

Questions continue over the page

Question 5 $(5 + 3 = 8 \text{ marks})$

(a) Consider the following terms:

Reed-Solomon hard decoder, AWGN channel, ML detector, FSK, correlator-type demodulator, Lempel-Ziv encoder, FIR equalizer.

Draw a block diagram of a digital communication system, indicating where each of these terms goes. Note that your block diagram has to be detailed enough in order to be able to do this. Also make sure that, for every encoder block, you also include a corresponding decoder block and vice versa.

(b) Describe the difference between convolutional coding and block coding. Include an example of each type of code to illustrate your point.

Question 6 $(1 + 3 + 1 + 1 + 2 + 2 = 10 \text{ marks})$

Consider a binary Discrete Memoryless Source (DMS) X with $P(X = 0) = \frac{1}{7}$.

(a) Compute the entropy of X.

(b) Design a **Huffman code** for the second extension of X (recall that the "second extension" of X consists of pairs of bits from X). Clearly specify a codeword for each pair of bits.

(c) Determine the average code length per bit of the code that you derived in (b).

(d) Is the Huffman code of (b) optimal for X? Explain your answer.

(e) Consider a Discrete Memoryless Source (DMS) Y that takes values in $\{a_1, a_2, a_3, a_4, a_5\}$ with corresponding probabilities p_1, p_2, p_3, p_4, p_5.
Is it possible that the code $\{0, 100, 110, 101, 111\}$ is generated as a Huffman code for Y? If your answer is yes, then give values for p_1, p_2, p_3, p_4, p_5 that lead to this code. If your answer is no, then explain why not.

(f) Repeat (e) for the code $\{11, 00, 01, 10, 110\}$.

Questions continues over the page

Question 7 $((2+2+1+2+1+2)+(1+3+1+1+1)=17$ marks)

(a) Consider an $(8,4)$ binary code with parity check equations:

$$\begin{aligned} v_0 &= u_1+u_2+u_3 \\ v_1 &= u_0+u_1+u_2 \\ v_2 &= u_0+u_1+u_3 \\ v_3 &= u_0+u_2+u_3, \end{aligned}$$

where u_0, u_1, u_2, u_3 are the information bits and v_0, v_1, v_2, v_3 are the (redundant) check bits.

- **(i)** Determine a parity check matrix H for this code.
- **(ii)** Determine a generator matrix G for this code.
- **(iii)** Is your generator matrix in (ii) systematic?
- **(iv)** Determine the minimum distance of this code. Explain your answer.
- **(v)** What is the code's error-correcting capability?
- **(vi)** Draw a Tanner graph for this code.

(b) Recall that GF(8) = $\{0,1,\alpha,\alpha^2,\ldots,\alpha^6\}$. Repeat parts (i), (ii), (iii), (iv) and (v) of (a) for a $(7,3)$ Reed-Solomon code over GF(8). For part (i) and (ii) you do **not** need to simplify powers of α. To answer part (ii), you may use that

$$\phi_\alpha{}^{-1} = \phi_\beta, \text{ where } \beta=\alpha^{-1} \text{ and } \phi_\alpha = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \alpha & \alpha^2 & \cdots & \alpha^6 \\ 1 & \alpha^2 & (\alpha^2)^2 & \cdots & (\alpha^6)^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \alpha^6 & (\alpha^2)^6 & \cdots & (\alpha^6)^6 \end{bmatrix}.$$

Questions continues over the page

Question 8 $(2 + (3 + 1) = 6$ marks)

(a) Explain why the compression rate (output file size / input file size) is much greater for video coding than for still image coding.

(b) Consider the following table, which stems from the JPEG image compression standard.

17	18	24	47	99	99	99	99
18	21	26	66	99	99	99	99
24	26	56	99	99	99	99	99
47	66	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99
99	99	99	99	99	99	99	99

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(i) Explain the relevance of this table for JPEG image compression (in as much detail as you can).

(ii) Explain why not all numbers in the table are the same.

Question 9 $(2 + 4 + 3 = 9$ marks)

Consider the binary symmetric channel with X the binary source to be transmitted and Y the received binary source; denote the cross-over probability by p and assume that $p < \frac{1}{2}$. As usual, denote the binary entropy function by H_b. Denote $P(X = 1)$ by q.

(a) Express $H(X|Y = 0)$ in terms of the function H_b and p and/or q.

(b) Now let $q = \frac{1}{2}$. Express the mutual information $I(X, Y)$ in terms of the function H_b and p **by using (a)**.

(c) Check your answer in (b) by computing $I(X, Y)$ in a different way, not using (a). Show your workings.

END OF EXAMINATION QUESTIONS