ELEN90051 Advanced Communication Systems
2018 Semester 1 Tutorial 4
Demodulation and Error Calculations

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26/03/2018

Instructions:

Answer all tutorial questions. Do not use any solution material that you happen to have, thus simulating a genuine exam environment.

1 Consider two orthonormal basis functions $\phi_1(t)$ and $\phi_2(t)$ over the interval $(0, T]$. Also consider a sample function $n(t)$ of a zero-mean white noise process, meaning that $\mathbb{E}[n^2(t)] = N_0/2$, where $\mathbb{E}[\cdot]$ denotes the expected value. Cross correlating $n(t)$ with $\phi_1(t)$ and $\phi_2(t)$ yields the two random variables

$$n_1 = \int_0^T \phi_1(t)n(t)\mathrm{d}t, \quad n_2 = \int_0^T \phi_2(t)n(t)\mathrm{d}t$$

Show that $\mathbb{E}[n_1 \cdot n_2] = 0$. What does this tell us about n_1 and n_2 in terms of correlation/independence?

2 A binary digital communication system employs the signals

$$s_0(t) = 0, \quad 0 \le t \le T$$
$$s_1(t) = A, \quad 0 \le t \le T$$

for transmitting the information. Here $A \in \mathbb{R}_+$ is a nonzero amplitude level. This is also known as on-off signalling or on-off keying (OOK). Let $f(t) := s_1(t)/||s_1(t)||$. The demodulator cross-correlates the received signal $r(t)$ with $f(t)$ and samples the output of the correlator at $t = T$.

(a) Determine the optimal detector for an AWGN channel and the optimal threshold, assuming that the signals are equally probable.

(b) Determine the probability of error as a function of the SNR, where the SNR is defined as $\frac{\mathcal{E}_b}{N_0/2}$, with $\mathcal{E}_b$ being the average bit energy.

(c) How does on-off signalling compare with antipodal signalling? Assume ML detection.

3

(a) Find the impulse response of the filter matched to the pulse

$$s_1(t) = \begin{cases} \frac{t}{T}, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

(b) Determine and sketch the output of the matched filter when the input is $s_1(t)$.

4 Binary antipodal signals are used to transmit information over an AWGN channel of power spectral density $N_0/2$. The prior probabilities for the two input symbols $s_1(t)$ and $s_2(t)$ are 1/3 and 2/3 respectively. Assume that the signals take the form

$$s_1(t) = \sqrt{\mathcal{E}_b}\phi(t), \quad s_2(t) = -\sqrt{\mathcal{E}_b}\phi(t), \quad 0 \le t \le T,$$

where $\phi(t)$ has unit energy.

(a) What is the ML detector decision rule for this signalling scheme?

(b) Express the bit error probability P_b^{ML} of the ML detector in terms of $\mathcal{E}_b/N_0$.

(c) Show that the MAP detector decides on s_1 if $r > \frac{N_0}{4}\frac{\ln 2}{\sqrt{\mathcal{E}_b}}$, and s_2 otherwise, where $r = \langle r(t), \phi(t) \rangle$.

(d) Let P_b^{MAP} be the bit error probability of the MAP detector. Is it smaller, larger or the same as your answer in (b)?

(e) Express P_b^{MAP} in terms of $\mathcal{E}_b/N_0$.

(f) Using Matlab or some other numerical method, compare the error probabilities of the ML and MAP detectors for $\mathcal{E}_b/N_0 = 1, 5, 10$.

5 Three messages $s_1(t)$, $s_2(t)$, and $s_3(t)$ are to be transmitted over an AWGN channel with noise power spectral density $\frac{N_0}{2}$. The messages are

$$s_1(t) = \begin{cases} 1, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

$$s_2(t) = \begin{cases} 1, & 0 \le t < \frac{T}{2} \\ -1, & \frac{T}{2} \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

$$s_3(t) = -s_2(t)$$

(a) What is the dimension of the signal space spanned by $\{s_1(t), s_2(t), s_3(t)\}$?

(b) Find an orthonormal basis for the signal space.

(c) Sketch the signal constellation.

(d) Sketch the optimal decision regions D_1, D_2, D_3, assuming that the messages are equally likely to be transmitted.

6 Consider a signal detector with an input $r = \pm\sqrt{\mathcal{E}_b} + n$, where $+\sqrt{\mathcal{E}_b}$ and $-\sqrt{\mathcal{E}_b}$ occur with equal probability, and the noise variable n is distributed with a Laplacian p.d.f.

$$p(n) = \frac{\lambda}{2} e^{-\lambda|n|}$$

(a) Determine the optimal detector. (Note that since the noise is not Gaussian, the mimimum distance interpretation of ML detection won't necessarily hold, or else needs to be proven.)

(b) Show that the average bit error probability P_b equals $\frac{1}{2}e^{-\lambda\sqrt{\mathcal{E}_b}}$.

(c) Define the SNR as $\frac{\mathcal{E}_b}{\mathrm{Var}[n]}$. Determine the SNR required to achieve an error probability of 10^{-5}. You may use the fact that $\mathrm{Var}[n] = \frac{2}{\lambda^2}$ for a Laplacian random variable.

(d) How does the SNR compare with the result for a Gaussian p.d.f with variance $N_0/2$?

End of Questions