ELEN90051 Advanced Communication Systems
2018 Semester 1 Tutorial 7
Channel Coding—Linear block codes

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Instructions:

Answer all tutorial questions. Do not use any solution material that you happen to have, thus simulating a genuine exam environment.

1 How many codewords are there in a (n, k) binary linear block code?

2 Consider a linear code C with generator matrix

$$G = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

(a) Give a systematic generator matrix for C.

(b) Determine a parity check matrix H for C.

3 Consider the $(k+1, k)$ binary parity check code

$$C = \{(p, u_0, u_1, \ldots, u_{k-1}) | p = 0 \text{ if } \sum_{i=0}^{i=k-1} u_i = 0 \text{ or } p = 1 \text{ if } \sum_{i=0}^{i=k-1} u_i = 1\}$$

Give the corresponding generator matrix G and write down a parity check matrix H.

4 What is the dual code of the $(k+1, k)$ binary parity check code of

$$C = \{(p, u_0, u_1, \ldots, u_{k-1}) | p = 0 \text{ if } \sum_{i=0}^{i=k-1} u_i = 0 \text{ or } p = 1 \text{ if } \sum_{i=0}^{i=k-1} u_i = 1\}$$

5 A code is **self-dual** if $C = C^{\perp}$. Show that for a (n, k) self-dual code we must have n even and the rate k/n equal to 1/2.

6 Consider the $(7,4)$ Hamming code.

(a) Show that the syndrome decoding method, as described in the lecture notes, is single-error correcting.

(b) Let the probability of a bit transmission error be denoted by p. Express the decoder error probability in terms of p.

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(a) Show that a maximum-length code is a $(2^r - 1, r)$ code (with r any integer ≥ 2).

(b) (Advanced!) Show that its nonzero codewords all have the same weight, namely 2^{r-1}.

8 Construct the standard array for the $(7,3)$ binary code with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Also determine the correctable error patterns, their corresponding syndromes and then construct a syndrome table.

9 Show that for a linear code C with parity check matrix H we have

$$d_{\min}(C) = \min \ \#\text{columns of } H \text{ that are linearly dependent.}$$

Also show that from this it follows that $d_{\min}(C) \leq n - k + 1 (=$ Singleton bound$)$.

10 Consider a binary MDS code, that is, an MDS codes over GF(2). List at least two different types of binary MDS codes. Is the binary Hamming code a MDS code?

11 The ISBN code is an example of non-binary code. It is a (10,9) code over the field $\mathbb{Z}_{11} = \{0, 1, 2, \ldots, 9, X\}$ with parity-check matrix

$$H = \begin{bmatrix} 1 & 2 & 3 & \cdots & 9 & X \end{bmatrix}$$

Is this a single-error detecting code? Is this a single-error correcting code? Is this code MDS?

12 Consider the $(10,8)$ code over $\mathbb{Z}_{11}$, given by the parity-check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 2 & 3 & \cdots & 9 & X \end{bmatrix}$$

Is this a single-error detecting code? Is this a single-error correcting code? Is this code MDS?

13 Can $d_{\min}(C)$ decrease when a code is extended?

14 Consider the (5,2) code C with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

What is $d_{\min}(C)$?

15 Explore the ISBN-13 code (use internet search). What is its definition, what are its properties, how does it compare to the ISBN-10 code that was presented in lectures?

16 Consider the $(10, 8)$ code over $\mathbb{Z}_{11}$, given by the parity-check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 2 & 3 & \cdots & 9 & X \end{bmatrix}$$

Assume that the received word r equals $r = (F, 1, F, 1, 3, 1, 6, X, 2, 1)$ with two erasures, indicated by "F". Here you may assume that the non-erased components of r are correct.

(a) Recover the values of the two erasures.

(b) In general, up to how many erasures are guaranteed to be recoverable for this code?

17 Consider the $(10, 8)$ code over $\mathbb{Z}_{11}$, given by the parity-check matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 2 & 3 & \cdots & 9 & X \end{bmatrix}$$

Suppose that the received word r equals $r = (3, 2, 2, 2, 2, 3, 2, 2, 2, 3)$.

Compute its syndrome and use it to perform error correction.

End of Questions