ELEN90051 Advanced Communication Systems
2018 Semester 1 Tutorial 6
Channel Capacity

Department of Electrical and Electronic Engineering
University of Melbourne

1/05/2018

Instructions:

Answer all tutorial questions. Do not use any solution material that you happen to have, thus simulating a genuine exam environment.

1 Let X and Y be two binary random variables, distributed according to the joint distributions

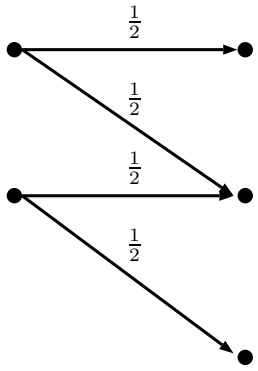
$$P(X = Y = 0) = P(X = 0, Y = 1) = P(X = Y = 1) = \frac{1}{3}.$$

Compute $H(X), H(Y), H(X|Y), H(Y|X), H(XY)$ and $I(X, Y)$.

2 Compute the capacity of the BSC.

3 Compute the capacity of the BEC.

4 Consider a discrete memoryless channel given by the figure below. Determine the channel's capacity.



5 Let X and Y be random variables. Show that

$$H(XY) \leq H(X) + H(Y).$$

Also show that equality holds if and only if X and Y are independent random variables.
Hint 1: First show that

$$H(X) = -\sum_{x,y} P(x,y) \log_2 P(x).$$

Hint 2: Then use the inequality $\ln w \leq w - 1$.

6 Let X and Y be random variables. Show that

$$H(XY) = H(X) + H(Y|X)$$

7 Let X and Y be random variables. Show that

$$H(Y|X) \leq H(Y)$$

When does equality hold?

End of Questions