Specialization in a Knowledge Economy

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Abstract

Using firm-level data from the US Census Longitudinal Business Database (LBD), this paper exhibits novel evidence about a wave of specialization experienced by US firms in the 1980s and 1990s. Specifically: 1) Firms, especially innovating ones, decreased production scope, i.e., the number of industries in which they produce. 2) Small firms increased innovation relative to production while large firms increased production relative to innovation. A new hypothesis is proposed to explain these phenomena. Pro-patent reforms in the 1980s and 1990s make firms' innovations more commodified and tradable. Trading provides another channel for firms to monetize their innovation besides production, especially when innovations are mismatched with a firm's production. Production scope then contributes less to the value of a firm's innovation, enabling innovation to shift to small firms with limited production scope. To gauge the importance of the new hypothesis, an endogenous growth model is developed with potential mismatches between innovation and production. Calibrating the model to data suggests that increasing tradability of innovation output can explain 25% of the observed production scope decrease and 58% of the reallocation of innovation activities. It results in a 0.64 percent point increase in the annual economic growth rate. Using regional and firm-level differences in exposure to patent policies, difference-in-difference analysis confirms causality from the pro-patent reforms to firms' production scope shrinkage.

Keywords: specialization, production scope, R&D, intellectual property rights, patent trading, basic research, applied research, endogenous growth

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1 Introduction

Profiting from innovation is vital for the survival of innovating firms and therefore economic growth. However, it is not always easy to monetize innovation using a firm's own production. First, ideas are random and are not always matched with a firm's production.[1] Second, the firm may lack the ability to mass-produce its innovation output.[2] Strategies to solve these problems within the firm include: spanning a large number of industries to raise the opportunity of utilizing new inventions; doing innovation only when the firm can produce and commercialize new inventions.

Surprisingly, this paper finds deviations from the aforementioned strategies among US firms in the 1980s and 1990s using the Longitudinal Business Database (LBD) from the Census Bureau–there is novel evidence of specialization trends.[3] Specifically,

1) US firms narrowed their production scopes, i.e., the number of industries in which they produce. The scope shrinkage was driven mainly by innovating firms.

2) Innovation shifted from large firms (firms with mass production) to small firms.

In other words, innovation and production became separated and were specialized in by different firms. This study then asks: What are the driving forces of the observed specialization, and how do they affect economic growth?

This paper proposes a new hypothesis to answer the questions above, highlighting the role of the pro-patent reforms in the 1980s and 1990s that allowed the trading of innovation output between firms on the patent market. To assess the new hypothesis in explaining the specialization choices of US firms and economic growth, an endogenous growth model is built with potential mismatches between innovation and production and firm heterogeneity in the ability to monetize new inventions through production. Then, the model is calibrated to rich firm-level data from the LBD, R&D data from the Survey of Industrial Research and Development (SIRD), and patent data from the US Patent and

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[1]Akcigit et al. (2016) provides evidence that firms may generate new inventions that are far away from the firms' primary line of business. In this case, the inventions have less value to the firms.

[2]For example, RC Cola was a small beverage company that introduced the first cola in a can and the first diet cola. However, it quickly lost the advantage to Coca-Cola and Pepsi. De Havilland, the world's first commercial jet airliner, invented the Comet I jet 2 years before Boeing introduced the 707. However, de Havilland was not able to capitalize its early invention. For more examples, please see Teece (1986)

[3]The LBD covers all US firms with paid employees.

Trademark Office (USPTO). The model suggests that increasing tradability of innovation output can explain 25% of the contraction in production scope and 58% of the shift of innovation activities. It leads to a 0.64 percent point increase in the annual economic growth rate. Finally, exploiting regional and firm-level differences in exposure to the reforms, difference-in-difference and triple-difference analyses confirm causality from the pro-patent reforms to firms' production scope shrinkage.

Here is a complete summary of the hypothesis. The pro-patent reforms in the United States in the 1980s and 1990s strengthened intellectual property rights protection and therefore made firms' innovation output more commodified and tradable. Trading of innovation output on the patent market allowed firms to sell the new inventions that fell outside of their production scope and buy inventions that could be utilized by their production; thus, making firms' production scope contribute less to the value of their innovation. This explains why innovating firms sharply decreased production scope in the 1980s and 1990s (Fact 1). Small firms often have more limited ability to monetize innovation through their own production. The increased chances of selling innovation output on the patent market benefited them more and incentivized them to increase innovation effort. Large firms could rely on small firms' innovation by purchasing patents on the market and therefore decreased innovation effort. This explains why innovation activities shifted to small firms (Fact 2).

The pro-patent reforms are part of the major policy changes implemented by the US government to confront the intensified global competition in the 1970s and early 1980s. The reforms consist of several pro-patent policies that include, but are not limited, to: an extension of patentability to genetic engineering and software, which became two of the most patented fields afterward; the creation of the Court of Appeals for the Federal Circuit (CAFC) that vastly increased the success rate of patent holders in legal disputes; longer effective patent terms for a majority of inventions. These policies affect the tradability of innovation output in two ways. First, stronger intellectual property rights protection incentivizes firms to patent their inventions instead of hiding them as secrets. Trading of patents is subject to fewer information frictions compared to trading of secrets. Second, firms with new inventions can extract more value in the trading process since stronger patent protection reduces the probability of other firms stealing the inventions. Evidence of the effect of these policy changes is rising volumes of patent trading activities. Using the Patent Assignment Dataset (PAD) from the USPTO, the share of patents ever traded

increased from 30.9% at the beginning of the 1980s to 44.1% at the end of the 1990s.

Other possible explanations are also considered for the observed specialization patterns. First, the US government introduced a R&D tax credit in 1981 as part of the strategies to increase the competency of US firms in the global market. The effective federal subsidy rate increased from 5% before the 1980s to 24% in the 1990s, as documented in Akcigit et al. (2018). Combined with the booming patent trading market, the R&D tax credit may have benefited small firms more as their R&D expense to domestic sales ratio grew to be higher than large firms' after 1985. Therefore, the credit may have amplified the shift of innovation to small firms. Second, the cost structure of production may have changed over time that directly affected firms' production scope. Recent papers like Hsieh and Rossi-Hansberg (2019) and De Ridder (2019) argue that the rise of information technology increases the fixed cost for firms to enter new industries but decreases the marginal production cost after entry. This may explain the observed shrinkage of production scope. Third, good ideas may be getting harder to find, as argued by Bloom et al. (2020). This may have pushed innovating firms to focus efforts on narrower fields of research and therefore production.

To evaluate the roles of the new hypothesis, as well as the aforementioned possible explanations in the specialization patterns and economic growth, a structural model is built with endogenous decisions of production scope and innovation effort. Distinct from existing theories about specialization, the model in this paper takes into account potential mismatches between innovation output and production. A key tradeoff that an innovating firm faces when choosing its production scope is that larger scope raises the probability that the firm's innovation output is better matched with its production, and therefore, increases the firm's ability to monetize its inventions; but at the same time, larger scope increases the management cost of the firm. The patent trading market provides another channel for firms to benefit from their innovation besides production, but is subject to search frictions. The pro-patent policies increase the matching efficiency and bargaining power of patent sellers on this market, and therefore, change the relative importance of production and trading in monetizing innovation. The effects are heterogeneous for small and large firms. Small firms have limited production scope and benefit more from the market channel by being patent sellers; large firms have broader scope and benefit from buying patents from other firms. The model entertains other explanations. The R&D tax credit rate, production cost structure, and the importance of new ideas are incorporated

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into the analysis.

The developed model is first calibrated to fit the period when the patent reforms just began (1981-1985) using the LBD, the SIRD, and the USPTO patent datasets. Key calibration targets include production scope, the R&D expense-to-domestic sales ratio of large and small firms, the share of patents traded, and the economic growth rate. Then, the model is recalibrated to fit the post-reform period (1996-2000), allowing changes in parameters relevant to the new hypothesis and the three alternative explanations. The direction of changes in these parameters is not constrained in the recalibration process, except for the R&D tax credit rate, which is taken from the US data. Comparison of parameter values in the two periods confirms the prior notions. There was an increase in the matching efficiency of the patent trading market, an increase in patent sellers' bargaining power, higher costs of producing in multiple industries, and less importance of new ideas. A decomposition exercise is conducted to explore the contribution of each possible explanation by looking at the changes in the key moments due to each relevant parameter. The decomposition shows that increasing tradability of innovations can explain 25% of the observed production scope decrease and 58% of the reallocation of R&D activities. The remaining part of specialization is primarily due to changes in production cost structure. Increasing tradability of innovations results in a 0.64 percent point increase in the annual growth rate, which makes it the main driver of economic growth in the 1980s and 1990s.

A firm's growth can depend on either its own innovation or other firms' innovation through patent purchasing. To explore the relative importance of these two channels, their contributions to growth are calculated for both the large and small firms. The result shows that the share of growth due to own innovation decreased sharply from 57.6% to 31.3% for large firms, while slightly from 2.9% to 2.3% for small firms. These changes reflect the fact that the growth of firms increasingly relied upon other firms' innovation, and this reliance was more salient for large firms.

Besides adjusting production scope, firms may also target their innovation to their production to improve matching between the two. One measure of the targeting behaviors of the innovation process is the share of basic research in total R&D spending. Since basic research is defined as "an activity aimed at acquiring new knowledge or understanding without specific immediate commercial application or use," lower basic research

share implies more targeted innovation.[4] Using the Survey of Industrial Research and Development (SIRD) collected by the Census Bureau and the NSF, this paper finds that basic research's share increased in the period when firms' production scope narrowed, implying that firms' innovation activities became less targeted. To check whether the new hypothesis can explain this trend, the baseline model is extended to include two types of innovation, basic and applied research, that differ in R&D costs, the probability of matching a firm's own production scope, and the importance of their output. Similar decomposition exercises are undertaken for the extended model. The result shows that increasing tradability of innovations can explain nearly all (90%) of the increase in the share of basic research. The intuition is that basic research benefits more from patent trading as its output is harder to be utilized by the firm's own production. So, firms are willing to make their innovation process less targeted when trading is available without worrying about their inability to monetize innovation.

Finally, this study uses regional and firm-level differences in exposure to the pro-patent policies to test whether the reforms are causes of the contraction in firms' production scope. The fraction of lawsuits invalidating the patents involved in legal disputes varied much across the twelve regional circuit courts before the establishment of the CAFC in 1982, as pointed out by [Henry and Turner (2006) and Han (2018)]. This implies a large variance in attitudes towards intellectual property rights across different regions. The establishment of the CAFC significantly lowered the regional invalidation rates and made them more uniform. So, regions with a higher invalidation rate before the CAFC experienced a larger increase in the strength of patent protection. Using a difference-in-difference (DiD) approach, firms in regions with a higher pre-CAFC invalidation rate decreased production scope more. Furthermore, genetic engineering and software were two of the most controversial fields of patentability in the 1970s. However, shortly before the establishment of the CAFC, the Supreme Court approved patentability in these two fields in two landmark cases, setting precedents for future cases. Therefore, these two fields experienced the most increase in patent protection strength and consistency in regional decisions afterward. The share of firms' employment in these two fields before 1982 is used as a proxy for the exposure to the change in patent protection. With a Triple-Difference (DDD) approach, a finding is that firms with higher exposure were more likely to shrink production scope. These empirical results confirm causality from the patent reforms to more focused production.

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[4]This is the definition of basic research in the Survey of Industrial Research and Development (SIRD).

Related Literature

This paper is closely related to the literature on the impacts of patent trading and intellectual property rights (IPR) protection. The structural model in this study is based upon Akcigit et al. (2016), which analyzes how the propinquity between a firm's and its patent's technology class affects the value of the patent to the firm, and how a patent trading market alleviates the problem of technology mismatch. This paper extends this work in a variety of directions to address firms' specialization decisions. First, production scope is introduced in the model, which is endogenously chosen by a firm according to the tradability of its innovation output. Second, firm size matters for the impact of market trading. Size affects the expected value the firm can extract from new ideas through production and determines whether a firm benefits more from buying or selling patents. Third, the model is calibrated to production scope and innovation effort choices of large and small firms in the data to explore the role of the pro-patent policies in the specialization wave in the 1980s and 1990s. Other literature about the trading of knowledge [Eaton and Kortum (1996), Cabral (2018), Perla et al. (2021)] studies the impact of technology adoption on firms' innovation and growth but not the effect on firm boundaries. Most discussions about the influence of IPR protection focus on the tradeoff between innovation incentives and inventors' monopoly power [Romer (1990), Aghion and Howitt (1990), Mukoyama (2003), Acemoglu and Akcigit (2012)]. Some empirical studies suggest that the strength of the patent system facilitates the disintegration of the innovation industries by allowing trade in knowledge [Hall and Ziedonis (2001), Arora and Merges (2004), Arora and Ceccagnoli (2006), Gans et al. (2008), Han et al. (2020)].[5] However, as mentioned by Hall and Harhoff (2012), research in this area is still limited. There have been few systematic theoretical and quantitative analyses about the role of IPR protection in firms' specialization decisions.

Theoretically, this paper contributes to the specialization literature by incorporating a new form of friction that determines firm boundaries between innovation and production. Following the wisdom of Coase (1937), a comparison between market transaction costs and firms' internal organization costs determines the scope of a firm. The literature about specialization has studied various forms of external and internal costs. Williamson (1985) considers problems of incomplete contracts. Klein et al. (1978) and Grout (1984) raise the "hold-up problem" in relation-specific investment. Grossman and Hart (1986) and Hart and Moore (1990) emphasize the role of ownership in determining firms' bound-

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[5]A summary of the relationship between patents and innovation can be found in Moser (2013).

ary.[6] Menzio (2021) explains product specialization by buyers' ability to locate sellers. De-Canio and Frech III (1993), Atalay et al. (2014) and Braguinsky et al. (2020) study the determinants and effect of vertical integration and diversification. Grossman and Helpman (2002), Boehm and Oberfield (2020), and Bostanci (2021) discuss factors that affects firms' outsourcing decisions. Some papers [Chiu et al. (2017), Baslandze (2016), Han (2018)] focus on frictions in the innovating sectors, but none of these papers considers how mismatches between innovation and production affects firms' specialization choices.

Empirically, this research is related to the recent debates about US business dynamism. Hsieh and Rossi-Hansberg (2019) find that the gap between the number of industries of a top firm and that of an average firm is smaller in 2013 compared to 1977. They explain these changes by introducing a new technology that raises the fixed costs but lowers the marginal costs of production in the service industry. Related arguments about technological changes are in Aghion et al. (2019), De Ridder (2019) and Autor et al. (2020). Inspired by their research, the current study explores the specialization patterns more thoroughly by looking at the number of industries per firm for all years from 1978 to 2016. Findings are that all firms experienced a drop in the number of industries, and this drop was mostly driven by firms that performed R&D activities. Furthermore, the drop started in the early 1980s but stopped before 2000. These observations direct the attention of this paper to policy reforms regarding intellectual property rights. The quantitative analysis of this paper supports the roles of both the increasing tradability of intellectual properties and the change in production cost structure. Besides, the observation of scope shrinkage complements the findings that the aggregate concentration of US firms was stable (White (2002)), but the within-industry concentration increased (Autor et al. (2020)).

The rest of the paper is organized as follows. Section 2 presents the specialization patterns observed in the data. Section 3 introduces the pro-patent policies in the 1980s and 1990s. It also shows evidence of a rising patent trading market. Section 4 constructs an endogenous growth model that includes potential mismatches between innovation output and production scope. Section 5 calibrates the model to key data moments and decomposes the specialization phenomena to effects of possible explanations. Section 6 extends the model to include basic and applied research. Section 7 exploits the changes of patent policies in the early 1980s to empirically examine the causality from the pro-patent reforms to firms' shrinkage of production scope. Section 8 concludes.

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[6]A summary of the literature on firms' boundary can be found in Holmstrom and Roberts (1998).

2   Specialization Patterns

This section exhibits the trends of production scope and R&D intensity of US firms. The datasets involved are the Longitudinal Business Database (LBD) constructed by the US Census Bureau;[7] the Survey of Industrial Research and Development (SIRD) collected by the US Census Bureau and the National Science Foundation (NSF); the Patent Data Project (PDP) collected and cleaned by the NBER.

The LBD covers the universe of business establishments with paid employees in the U.S. It has a consistent 6-digit NAICS code constructed by Fort et al. (2016) for each establishment and each year. This study uses the firm ID variable that identifies the ownership of each establishment to aggregate the number of the 6-digit NAICS codes of each firm and defines it as the production scope of a firm. Information about firms' patenting activities comes from the PDP. It records all patents issued by the U.S. Patent and Trademark Office from 1976 to 2006. A firm is classified as an innovating firm if it has ever been granted a patent between 1976 and 2006. The SIRD provides detailed industrial R&D information of a nationally representative sample of non-farm firms. Using the sample weights in the survey, the Census Bureau and the NSF calculate countrywide statistics each year and publish them on the Industrial Research and Development Information System (IRIS).[8] The ratio of a firm's R&D cost (excluding the federal funded part) to its net domestic sales is used by this study as a proxy for the firm's R&D intensity.

2.1   Production Scope

Figure 1 shows the average production scope of US firms with paid employees from 1978 to 2006 by whether they have ever issued a patent recorded by the PDP (innovating firms vs. others).[9] The scale for innovating firms is shown on the left y-axis, while the scale for other firms is shown on the right. Innovating firms produced in 3.07 6-digit NAICS industries on average at the beginning of the 1980s. This number experienced a sharp decrease by one-third to around 2.05 at the end of 1990s and then rebounded slightly

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[7]Description of this dataset can be found in Jarmin and Miranda (2002).

[8]https://wayback.archive-it.org/5902/20181004030303/https://www.nsf.gov/statistics/iris/start.cfm

[9]The data point for the year 2002 is omitted because, in the version of the LBD data available to the author of this paper, there is a problem in the scope statistics in 2002. Economists from the Census Bureau confirm that the newest version does not have the problem.

after 2000. Other firms' production scope also decreased, but to a much lesser extent.
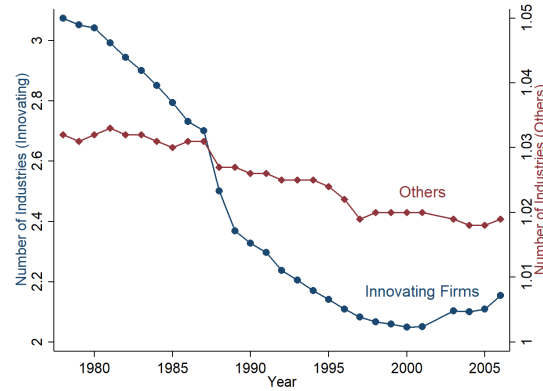
Figure 1: Trend of Production Scope by Innovating Activities

One potential reason for the more significant drop in the scope of innovating firms is that they are usually larger. To control the firm size, a regression of firms' production scope is run each year on a dummy variable of whether the firm is innovating or not, employment, and their interaction. Then the predicted production scope of innovating and other firms is calculated based on the estimated parameters when fixing the employment level at 20 and 1000, respectively. As shown in the two panels of Figure 2, at both employment levels, innovating firms shrank production scope more than other firms. Therefore, the decrease in production scope is more salient for innovating firms even when firm size is controlled.
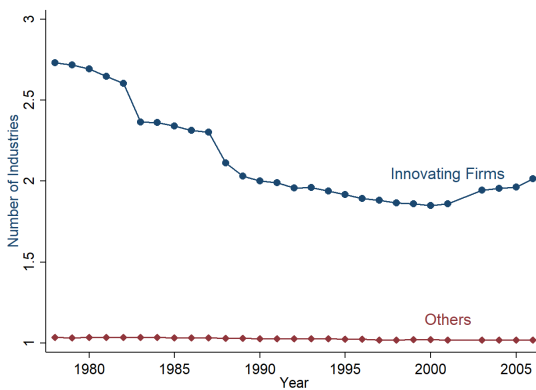
(a) 20 Employees
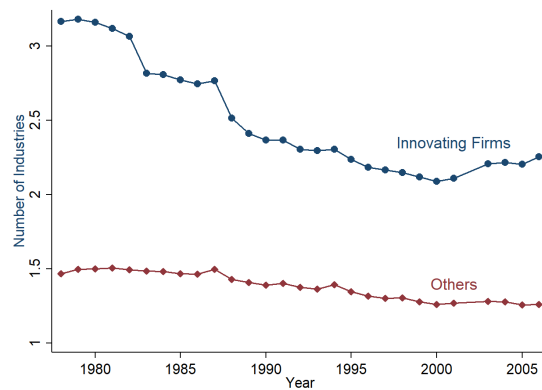
(b) 1000 Employees

Figure 2: Trends of Production Scope with Fixed Firm Size

2.2 Innovation Intensity

Figure 3a displays the R&D intensity of US firms by size. Here, a firm is defined as a small or medium if it has no more than 999 employees, while a large firm has at least 1000 employees. As shown in Figure 3a, the R&D expense-to-domestic sales ratio of small and medium firms started to surge after 1980, and the rising trend stopped after 2000. On the other hand, the ratio of large firms slightly decreased at the same period.[10] These diverging trends suggest that small and medium firms became more focused on innovation, while large firm became more focused on non-innovation activities. To exclude the impact of changes in domestic sales of the two groups of firms, the ratio of total R&D spending by large firms to total R&D spending by small and medium firms is plotted, as shown in Figure 3b. This ratio started to drop after the early 1980s and stabilized after 2000, indicating that small and medium firms conducted a larger share of R&D activities. As firm size is an essential determinant of firms' production and commercialization, but not R&D activities (Adams (1970), Moen (1999)), the two panels of Figure 3 suggest that firms spent more efforts on areas where they had comparative advantage.[11]

(a) R&D Intensity by Size   (b) Total R&D Expense Ratio
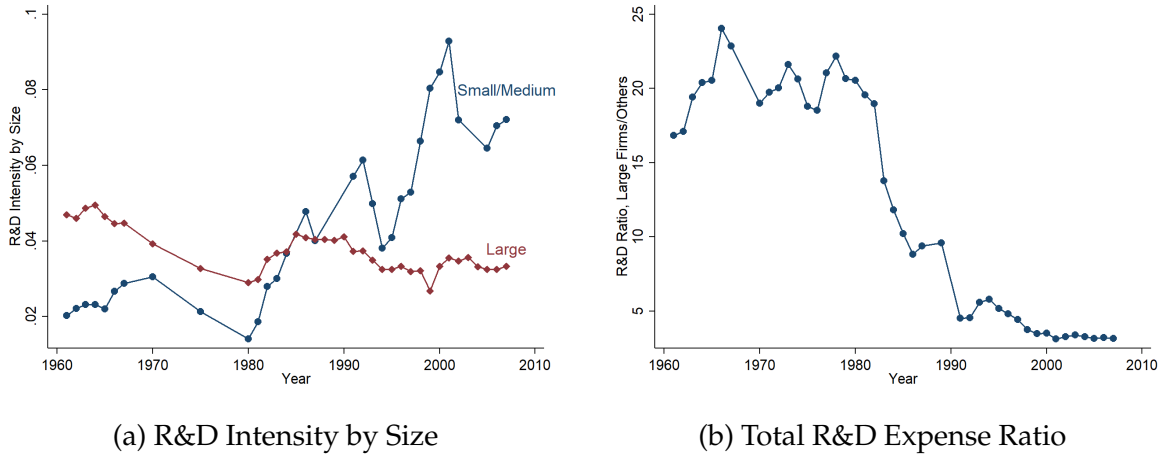
Figure 3: Trends of R&D Activities

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[10]As a robustness check, I also plot the number of citation-weighted patent numbers per employee for the two groups of firms, as shown in Figure 8 in the Appendix.

[11]In the following sections, this paper will call all the non-innovation activities as production. Therefore, production indicates all activities that are complementary to innovation.

3 Policy Reforms and the Patent Market

The two decades (the 1980s and 1990s) that witnessed the specialization wave described in the previous section also experienced important policy reforms in the United States. In the 1970s, the innovation activities in the U.S. were thought to fall behind other industrialized countries (Meador (1992)), so a series of policies were adopted to stimulate innovation and boost economic growth. Besides introducing the R&D tax credits at the federal level in 1981, the US government adopted a series of pro-patent reforms starting at the beginning of the 1980s that strengthened the protection of intellectual property rights. The US legal environment towards patents became increasingly positive in the following two decades until some counterbalancing new policy developments at the end of the 1990s. This paper will focus on three major pro-patent policies in the 1980s and 1990s, as summarized by Gallini (2002), and discuss how they are related to the trading of innovations.

3.1 Patent Policies

Extension of Patentability to Genetic Engineering and Software. The US Supreme Court's decision in 1980 in the case between *Diamond* and *Chakrabarty* approved the patentability of genetically engineered bacteria. The 1981 decision in *Diamond v. Diehr* affirmed patent protection of software. Bioengineering and software became two heavily patented areas afterward. The overall patent applications and issuances both doubled between 1980 and 2000 after a long stable phase before 1980.

Creation of the Court of Appeals for the Federal Circuit. Before 1982, legal disputes of patents were heard at district courts or regional appellate courts, which did not have consistent enforcement of the patent law across regions. The establishment of the Court of Appeals for the Federal Circuit (CAFC) in 1982 provided centralized patent jurisdiction. More importantly, it largely increased the success rate of patent holders in legal disputes (Henry and Turner (2006), Han (2018)). As shown by Figure 4, the fraction of lawsuits that invalidated the patents involved plummeted after the change in the court system.

Longer Patent Terms for Most Inventions. In 1984, The US government passed the Drug Price Competition and Patent Term Restoration Act, which allowed time spent on premar-
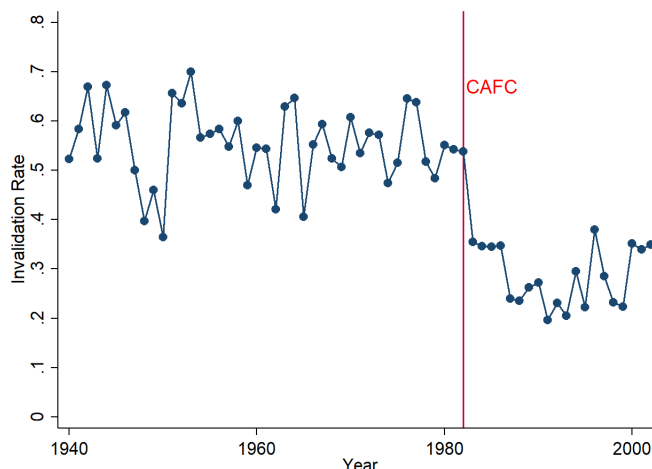
Figure 4: Fraction of Lawsuits Invalidating the Patents

ket testing and the Food and Drug Administration (FDA) approval process to be restored from the term of the patent for up to five years. In 1994, the patent law replaced a 17-year patent term from the issuance date with a 20-year term from the application date. Since the gap between application and issuance is generally less than three years, this policy change increased the duration of a majority of patents.

3.2 Relationship to Trading of Innovation Output

The pro-patent policies affect the trading of innovations along two dimensions. First, stronger intellectual property rights protection encourages inventors to patent their inventions and, at the same time, disclose the information of their inventions, as required by the patent law. Information disclosure helps to overcome a major friction in the patent trading process–a lack of information about the value of the transacted products. Therefore, the trading efficiency increases. Second, the pro-patent policies reduce the chance of stealing and ensure patent holders extract justified values from their innovations.

Evidence of increasing tradability of innovations is a rapid growth of the patent trading market in the 1980s and 1990s. Combining the Patent Assignment Dataset (PAD) with the LBD,[12] this paper calculates the citation weighted fraction of patents granted to

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[12]The PAD is collected by the USPTO. It maintains as much as possible a complete history of claimed interests in a patent. Marco et al. (2015) has a complete introduction and shows various statistics of this dataset.

US firms in each year that have ever been traded through sales or merger & acquisitions (M&A).[13] As shown in Figure 5, in the early 1980s, only about 30.9% of patents had been transacted. This fraction climbed to around 44.1% at the end of the 1990s and plateaued after 2000.[14] This rise was primarily due to early transactions, as shown by Figure 9 in the Appendix. Besides patent transactions, patent licensing activities also ballooned after 1980, as indicated by the rising trends of licensing fees and royalties presented in Arora and Gambardella (2010). Therefore, the increase in patent transactions shown in Figure 5 should be viewed as a lower bound of the estimation for the increase in patent trading activities.
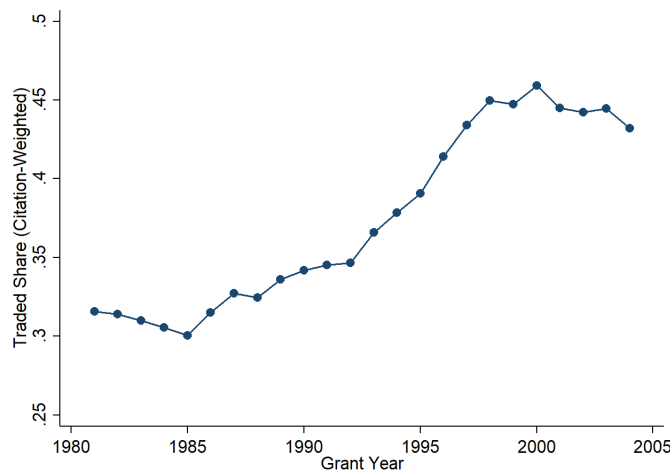
Figure 5: Fraction of Patents Involved in Trading

4 Model

To explore the driving forces of the observed specialization phenomena and their effects on economic growth, a model is constructed in this section. In the model, there are possible mismatches between a firm's innovations and its production. Firms endogenously choose their production scope, R&D intensity, and whether to buy or sell innovation output on the patent market. The patent market is subject to search frictions, the efficiency of which and the bargaining power between buyers and sellers depend on the legal environment towards patents. There are two types of production ability, which reflect firms'

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[13]The graph of the fraction not weighted by citations delivers very similar patterns.
[14]The trends are very similar for patent sales and M&A.

comparative advantage in innovation or production. Firms with a high production ability can extract higher value from new inventions through production and, on average, have larger size. Decisions of different types of firms are affected differently by the strength of patent protection, R&D tax credit rates, production cost structure, as well as the importance of new ideas.

4.1 Setup

There is a unit measure of firms in this economy, and each is exogenously and uniformly centered at a point on the industry circle shown in Figure 6. The industry circle contains all the industries in the economy and is assumed to have a radius of $\frac{1}{2\pi}$. At the beginning of each period, a firm chooses its production scope (ω)–the set of industries in which it will produce goods and sevices. Figure 1 shows an example of a firm that is centered at the top of the circle and chooses the arc ω as its production scope[15]. The absolute value of ω, $|\omega|$, stands for the number of industries the firm produces in and will be used in the following analysis. As the model only focuses on the symmetric equilibrium, the location of the center turns out to be irrelevant to firms' decisions.
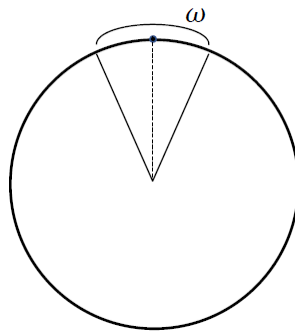
Figure 6: Schematic Diagram

A firm goes through two major stages of operation after the scope is determined: innovation and production. The key assumptions of the model are twofold. First, the location of the innovation output cannot be entirely controlled by the firm, and therefore it may not necessarily fall inside the firm's production scope. The firm cannot adjust its scope after the innovation stage. Second, the firm can only utilize the innovation output

[15]Whether the set of industries is connected is not assumed ex-ante, but will be solved from the model based on assumptions that will be unfolded later.

that matches its production scope. Between the two stages, firms can trade innovations on the patent market subject to a search and matching process. They can sell the innovation that is useless to them and buy patents that match their production scope. The pro-patent policies have two impacts on patent trading. 1) They increase the matching efficiency of the patent trading market by improving information disclosure. 2) They raise the bargaining power of the patent sellers by enhancing protection for patent holders.[16]

Each firm in this economy is characterized by production ability (m) and an innovation level (z). The production ability has two statuses, high (m_H) and low (m_L). The transition of statuses across periods is subject to a Markov process, $Q_{mm'} = \begin{bmatrix} q_{HH} & q_{HL} \\ q_{LH} & q_{LL} \end{bmatrix}$. In the stationary distribution, the shares of firms that have high and low production ability are respectively α_H and α_L. The innovation level is updated in each period according to the law of motion,

$$z' = z + \gamma(\mathbb{1}_{(RD \in \omega)} + \mathbb{B})\mathbf{z}, \tag{1}$$

where $\mathbb{1}_{(RD \in \omega)}$ is an indicator of whether the firm's innovation output falls inside of its production scope; $\mathbb{B}$ is an indicator of whether the firm buys a patent that matches its scope. It is assumed that: 1) At most one idea can be implemented in each period, so, $\mathbb{1}_{(RD \in \omega)}$ and $\mathbb{B}$ are exclusive. 2) Whenever a firm has an in-scope innovation, the firm uses it in its own production.[17] γ is a constant lock-step growth of the innovation level. $\mathbf{z}$ is the employment-weighted average innovation level of the economy, defined by the following equation,

$$\mathbf{z} = \frac{\int \int mz \, dF(m, z; \mathbf{z})}{\alpha_H m_H + \alpha_L m_L}, \tag{2}$$

where $F(m, z; \mathbf{z})$ is the joint distribution of production ability and innovation levels among all firms at the end of the previous period.

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[16]In this paper, patenting an innovation is free, so all new inventions are patented. The strength of the exclusive legal rights embodied in a patent is subject to policy changes.

[17]This is equivalent to an assumption that the value of applying an in-scope innovation in a firm's own production is higher than that of selling it to other firms, regardless of the firm's production ability. The model calibration result justifies the equivalent assumption, an explicit expression of which is shown in equation 55 in the Appendix.

The timing of events in each period is shown as follows:

m, z $\mathbb{1}_{(RD\in\omega)}$ realizes z' realizes

Choose ω R&D with i Search ideas   Production
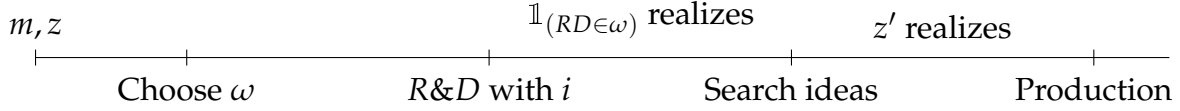
A firm starts a period with newly realized production ability and the innovation level inherited from the end of the previous period. It chooses the production scope ω according to an increasing and convex management cost function in the number of industries,

$$C^e(\omega; \mathbf{z}) = \mu|\omega|^{1+\iota}\mathbf{z}^{\zeta/(\zeta+\lambda)}/(1+\iota), \ \iota > 0. \tag{3}$$

After the scope is chosen, the firm begins to do R&D. This innovation process has a success rate of i, which is endogenously determined by the firm and also subject to an increasing and convex cost function,

$$C^i(i; \mathbf{z}) = \chi i^{1+\rho}\mathbf{z}^{\zeta/(\zeta+\lambda)}/(1+\rho), \ \rho > 0. \tag{4}$$

Both the management and innovation cost functions rise with the economy-wide innovation level, $\mathbf{z}$.

Whether the innovation process succeeds realizes then, together with the location of the output. The output is useful to the firm's own production only if it locates inside the scope. Firms that fail to innovate search on the patent market as potential buyers. At the same time, firms that successfully innovate, but the innovation output is useless, also search on the market, as both potential buyers and sellers. They want to sell the useless patent at hand and buy a patent that matches their production scope. It is assumed that each seller and buyer have one unit of search effort. Sellers spend their whole effort searching at the location of their patents; buyers evenly distribute their effort over their production scope. For any arc, d, on the industry circle, this paper denotes the total search effort on the arc by sellers and by buyers respectively as $n_s(d)$ and $n_b(d)$. The total number of matches on the arc is subject to the function,

$$M(n_s(d), n_b(d)) = \phi n_s(d)^\nu n_b(d)^{1-\nu}, \tag{5}$$

where ϕ represents the matching efficiency of the market and is exogenously affected by

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patent policies. The odds of a successful match for a potential seller can be expressed as

$$s = \lim_{|d_0| \to 0} \phi \left(\frac{n_b(d_0)}{n_s(d_0)} \right)^{1-\nu}. \tag{6}$$

where d_0 is the neighborhood that spans symmetrically around the location of the seller's patent. Since the model will only focus on the symmetric equilibrium, the location of the patent is not tracked. The odds of a successful match for a potential buyer depend on a function of the arc it searches over,

$$b(\omega) = \phi \left(\frac{n_s(\omega)}{n_b(\omega)} \right)^{\nu}. \tag{7}$$

Finally, the new innovation level of the firm realizes according to the law of motion in (1). At the production stage, a firm maximizes its overall profit by choosing capital and labor in each industry within its production scope. The production function exhibits decreasing return-to-scale with regard to capital and labor. Capital is hired at the rental rate $\tilde{r}$, and labor is hired at the wage rate w. It is assumed that goods in different industries are perfect substitutes and industries are symmetric. Denote the capital and labor in each industry respectively as k and l. The firm's optimization problem at the production stage is

$$\pi(\omega, m, z'; \mathbf{z}) = \max_{k,l} (mz')^{\zeta} (|\omega|k)^{\eta} (|\omega|l)^{\lambda} - \tilde{r}(|\omega|k) - w(|\omega|l). \tag{8}$$

4.2 Consumer Preference

A representative household in this economy maximizes the following lifetime utility,

$$\sum_{t=0}^{\infty} \beta^t \frac{c(t)^{1-\epsilon}}{1-\epsilon}.$$

where $c(t)$ is consumption in period t, β is the discount rate of the future, and ϵ is the degree of risk aversion of the household. The household owns and rents capital to all the firms in this economy, which generates both a profit and a risk-free rate of capital return, $\frac{1}{r}$, in each period. The depreciation rate of capital is δ. So, the rental rate of capital, $\tilde{r}$, is $\frac{1}{r} - 1 + \delta$. The household also provides one unit of labor to firms, from which it earns a wage rate $w(t)$.

4.3 Firm Decisions

This section solves firms' decisions in backward order. At the final production stage, the first-order condition derives

$$k(\omega, m, z'; \mathbf{z}) = \frac{mz'}{|\omega|}\left(\frac{\eta}{\tilde{r}}\right)^{1+\frac{\eta}{\zeta}}\left(\frac{\lambda}{w}\right)^{\frac{\lambda}{\zeta}}; \tag{9}$$

$$l(\omega, m, z'; \mathbf{z}) = \frac{mz'}{|\omega|}\left(\frac{\eta}{\tilde{r}}\right)^{\frac{\eta}{\zeta}}\left(\frac{\lambda}{w}\right)^{1+\frac{\lambda}{\zeta}}. \tag{10}$$

It is straightforward that the total amount of capital $|\omega|k(\omega, m, z'; \mathbf{z})$ and the total amount of labor $|\omega|l(\omega, m, z'; \mathbf{z})$ a firm hires do not depend on the production scope. So does the total profit, which equals to

$$\pi(m, z'; \mathbf{z}) = mz'(1 - \eta - \lambda)\left(\frac{\eta}{\tilde{r}}\right)^{\frac{\eta}{\zeta}}\left(\frac{\lambda}{w}\right)^{\frac{\lambda}{\zeta}}. \tag{11}$$

The independence of total input and output on the production scope implies that firms either span a wide range of industries but only touch on each of them, or focus on a narrow range of industries and deepen production in them. This independence is consistent with observations in the data, that US firms deepened production in fewer industries without changing much the total employment. The average employment of US firms was similar between the beginning of the 1980s and the end of the 1990s, even though the average number of industries was much lower at the latter period.[18]

The decision of R&D expenses is equivalent to determining the success rate (i) of innovation, as there is a one-to-one mapping between the two. Denote the value of a firm before the R&D decision as $D(\omega, m, z; \mathbf{z})$, taking the production scope as given. This value

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[18]To be more specific, the average employment of US firms first decreased in the 1980s and then rebounded in the 1990s. The levels at the start and the end were similar.

can be expressed as

$$D(\omega, m, z; \mathbf{z}) = \max_i \{ iX(\omega) \underbrace{[\pi(m, z'; \mathbf{z}) + r\mathbb{E}V(m', z'; \mathbf{z}')]}_{\text{Innovate within } \omega} +$$

$$(1 - iX(\omega)) \underbrace{[b(\omega)(\pi(m, z'; \mathbf{z}) - p_b(m, z; \mathbf{z}) + r\mathbb{E}V(m', z'; \mathbf{z}'))]}_{\text{Buy an idea within } \omega} +$$

$$(1 - iX(\omega)) \underbrace{[(1 - b(\omega))(\pi(m, z; \mathbf{z}) + r\mathbb{E}V(m', z; \mathbf{z}'))]}_{\text{No idea within } \omega} +$$

$$i(1 - X(\omega)) \underbrace{sp_s}_{\text{Sell an idea}} - (1 - \sigma)C^i(i; \mathbf{z})\},$$

(12)

where the function $X(\omega)$ is the probability that the firm's innovation output falls inside its production scope, ω. It is assumed that the closer an industry is to the firm's core business (center), the larger the probability the firm's inventions match that industry and generate values to the firm.[19] As will be shown in the proof of Proposition 4.1 in the Appendix, it is optimal for firms to span symmetrically around their center. It is also assumed that $X(|\omega|) = \xi|\omega|^\psi$ with $\xi > 0$ and $0 < \psi < 1$ if ω spans symmetrically around the firm's center.[20] In the following analysis, $X(|\omega|)$ will denote the relationship between the within-scope probability and the length of the production arc, given that the arc is symmetric around the firm's center.

$D(\omega, m, z; \mathbf{z})$ consists of five components, the first four of which describe the benefit of innovation in four different scenarios, while the last one of which is the innovation cost when the R&D tax credit rate equals to σ. The first scenario happens when the firm's innovation is successful, and the output falls within the firms' production scope. So, the probability of this scenario is $iX(\omega)$. The firm then updates its innovation level according to the law of motion described in (1). $\pi(m, z'; \mathbf{z})$ is the profit in the current period with the updated innovation level (z'). $r\mathbb{E}V(m', z'; \mathbf{z}')$ is the discounted future value of the firm at the beginning of the next period. The second and third scenarios happen when the firm does not develop useful innovation output through its own R&D process, either because the innovation fails or because the innovation output does not match the firm's produc-

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[19]This assumption is supported by the empirical findings in Akcigit et al. (2016) that the propinquity between a patent's technology class and the firm's main line of business positively affects the value of the patent to the firm.

[20]As will be shown in Table 3 in the quantitative analysis, the empirical estimation of $X(|\omega|)$ is consistent with this assumption.

tion scope. The firm then searches on the patent market as a potential buyer. With probability $b(\omega)$, the firm matches with a patent seller. It buys the patent at a price $p_b(m, z; \mathbf{z})$ and updates its innovation level with the patent. With probability, $1 - b(\omega)$, the firm cannot find a seller, and therefore, its innovations level remains the same as at the beginning of the period. The fourth scenario happens when the firm's R&D process succeeds, but the output falls outside the firm's own production scope. In this case, the firm searches on the patent market as a potential seller. With probability s, the firm matches with a buyer and sells its innovation output at a selling price denoted as p_s. Since the model assumes that the patent expires in one period, the firm that does not manage to sell its patent in the current period will have to dump it and earn nothing from its innovation.

The determination of the buying price of a patent, also the transaction price, is through Nash bargaining, which can be described as follows,

$$p_b(m, z; \mathbf{z}) = arg \max_{p_b} \ p_b^\theta [\pi(m, z'; \mathbf{z}) + r \mathbb{E} V(m', z'; \mathbf{z}') - p_b - (\pi(m, z; \mathbf{z}) + r \mathbb{E} V(m', z; \mathbf{z}'))]^{1-\theta}.$$

(13)

The buyer and seller choose the transaction price (p_b) to maximize the product of their surplus. The surplus of the seller is simply the price, as the seller will earn nothing if the patent is not sold. The surplus of the buyer is the difference between the firm value with updated innovation level minus the payment and the value with the original innovation level. θ denotes the bargaining power of the seller in the transaction. Note that the surplus of the buyer depends on the buyer's types, and therefore, so does the transaction price.

At the R&D stage, firms do not know what type of buyers they will meet if they have a useless innovation to themselves and want to sell it on the market. So, p_s should be the expected price in transactions with all potential buyers. The distribution of types of potential buyers on the market is denoted as $G(m, z; \mathbf{z}')$ and will be determined endogenously in the equilibrium. The selling price can be expressed as

$$p_s(\mathbf{z}) = \int \int p_b(m, z; \mathbf{z}) dG(m, z; \mathbf{z}).$$

(14)

The decision of the production scope at the beginning of each period is based on the tradeoff between the benefit and cost. The production scope, on the one hand, affects the ability that a firm monetizes its innovation output, and on the other hand, determines the

management difficulty. The optimal scope solves the following problem,

$$V(m, z; \mathbf{z}) = \max_{\omega} D(\omega, m, z; \mathbf{z}) - C^e(\omega; \mathbf{z}), \tag{15}$$

where $C^e(\omega; \mathbf{z})$ is the management cost function as introduced in the model setup.

4.4 Equilibrium

This paper focuses on a symmetric-balanced-growth-path (SBGP) equilibrium, where the employment-weighted average growth rate of the innovation level in the economy, g, is a constant, and the ratio of the average innovation level of firms with high production ability to that of firms with low production ability is a constant. The variables in this equilibrium can be expressed as functions of the model parameters and are displayed in the following proposition. The proof of the proposition is unfolded in Section 9.3 in the Appendix.

Proposition 4.1 (Symmetric Balanced Growth Path). *There exists a symmetric balanced growth path of the following form*

1. The employment-weighted growth rate of the aggregate productivity defined by,

$$g = \frac{\int \int m'z''dF(m', z') / \int \int m'dF(m', z')}{\int \int mz'dF(m, z) / \int \int mdF(m, z)}, \tag{16}$$

is a constant.

2. The interest factor $r = \beta / g^{\epsilon\zeta/(\zeta+\lambda)}$; the rental rate on capital $\tilde{r} = g^{\epsilon\zeta/(\zeta+\lambda)}/\beta - 1 + \delta$.

3. The odds of a successful match for a potential buyer, $b(\omega)$, and for a potential seller, s, only depend on the total number of patent buyers and sellers, i.e., $b(\omega) = \phi(\frac{n_s}{n_b})^\nu$, $s = \phi(\frac{n_b}{n_s})^{1-\nu}$.

4. The production scope of each firm spans symmetrically around the center, and the length of the scope depends only on the production ability of the firm, i.e., $|\omega(m, z; \mathbf{z})| = \Omega(m)$.

5. The R&D success rate does not depend on the firm's innovation level, z, or the economy-wide innovation level, $\mathbf{z}$, i.e., $i(\omega, m, z; \mathbf{z}) = i(\omega, m)$.

6. The value function $V(m, z; \mathbf{z})$ is linear in $\tilde{z}$ and $\tilde{\mathbf{z}}$, i.e., $V(m, z; \mathbf{z}) = v_1(m)\tilde{z} + v_2(m)\tilde{\mathbf{z}}$, where $\tilde{z} = z/\mathbf{z}^{\lambda/(\zeta+\lambda)}, \tilde{\mathbf{z}} = \mathbf{z}^{\zeta/(\zeta+\lambda)}$.

7. The number of buyers of both types (n_{bH}, n_{bL}) and the number of sellers (n_s) are

$$n_{bH} = \alpha_H(1 - i^*(\omega^*(m_H), m_H)X(\omega^*(m_H))), \ n_{bL} = \alpha_H(1 - i^*(\omega^*(m_L), m_L)X(\omega^*(m_L)));$$
(17)

$$n_s = \alpha_H i^*(\omega^*(m_H), m_H)(1 - X(\omega^*(m_H))) + \alpha_L i^*(\omega^*(m_L), m_L)(1 - X(\omega^*(m_L))). \quad (18)$$

8. The buying price and the expected selling price of a patent is

$$p_b(m, z; \mathbf{z}) = \theta(Am + \frac{r}{g^{\lambda/(\lambda+\zeta)}} \mathbb{E}[v_1(m')|m])\gamma\tilde{\mathbf{z}}; \quad (19)$$

$$p_s(\mathbf{z}) = \frac{n_{bH}}{n_b} p_b(m_H, z; \mathbf{z}) + \frac{n_{bL}}{n_b} p_b(m_L, z; \mathbf{z}), \quad (20)$$

where A is a constant.

The intuition of the matching rate of a potential buyer only depending on the total number of buyers and sellers is that firms are endowed with the same unit of search effort and have to dilute their effort at each point of the arc they search over. Therefore, although firms with different production scope have different chances of getting an in-scope idea if their innovation succeeds, they have equal opportunity to get an idea on the market. On the other hand, the matching rate of a potential seller is also the same for all firms, as on each point of the industry circle, there are equal number of buyers and sellers.

The R&D success rate does not rely on individual and aggregate innovation levels because both the benefit and the cost of R&D depend only on the aggregate innovation level of the economy and the aggregate level cancels out in the calculation. The irrelevance of the R&D success rate with the innovation levels results in the production scope only relying on firms' production ability.

4.5   Relevant Parameters for Specialization

According to the analysis in the previous section, changes in the patent trading environment, the R&D tax credit rate, production cost structure, and the difficulty to find good ideas may be potential reasons for the observed specialization patterns. Parameters in the model that correspond to these changes are listed here.

The matching efficiency of the patent market, ϕ, reflects information frictions in the trading process. Policies that make inventions more commodified and visible on the market are predicted to raise the matching efficiency. The bargaining power of patent sellers, θ, reflects protection towards patent holders. In an extreme case where patents are not protected, inventions can be easily stolen by other firms. Then sellers have zero bargaining power and get nothing from trade, while buyers get all the surplus. Stronger patent protection results in higher bargaining power of the sellers.[21]

The R&D tax credit is directly captured by σ. A higher fixed cost of entering new industries corresponds to a larger scale and elasticity parameters in the management cost function (μ and ι in equation (3)). As the production function at the final stage is DRS to total production factors and the total factors is the product of the number of industries and factors in each industry, decreasing the number of industries raises the marginal benefit of scaling production in each industry. This indirectly captures the decreasing marginal cost of production in a industry after entry, as proposed in the previous literature. Finally, the difficulty to find good ideas can be captured by the step size of new inventions (γ) and the parameters (χ and ρ) in the R&D cost function.

How do the two parameters related to the new hypothesis, matching efficiency (ϕ) and sellers' bargaining power (θ), affect firms' R&D intensity and production scope? Partial equilibrium analysis of the model sheds some light on the direction and channels.

4.5.1 Impacts of the Matching Efficiency

According to the model solution (see proof of Proposition 4.1 in the Appendix), the success rate of innovation given production scope can be expressed as

$$i^*(\omega, m) = \Big\{ \frac{\gamma}{(1-\sigma)\chi} [X(|\omega|) \underbrace{(1 - (1-\theta)b)B(m)}_{-} + (1 - X(|\omega|)) \underbrace{s\theta(\sigma_H B(m_H) + \sigma_L B(m_L))}_{+}] \Big\}^{\frac{1}{\rho}}.$$

(21)

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[21]More formally, there is a mapping between the invalidation rate of patents and sellers' bargaining power. Denote the invalidation rate as d, which captures the probability that "buyers" get a patent for free from "sellers". Then the expected transaction price of a patent becomes $(1-d)p_b$. Since in the solution of the Nash bargaining problem, p_b is θ times the surplus of the buyer, the expected transaction price with invalidation rate, d, is then $(1-d)\theta$ times the surplus. Therefore, a lower invalidation rate is equivalent to higher bargaining power of the sellers.

where $B(.)$ is a function of production ability with constants and aggregate variables. An increase in the matching efficiency (ϕ) increases both the matching rate of potential buyers (b) and potential sellers (s). Easier trading of patents for buyers decreases firms' R&D intensity as they can rely more upon other firms to do R&D (the first term in Equation (21)). On the other hand, firms can better monetize their innovation output when it falls outside of their own scope and therefore have a stronger incentive to do R&D (the second term in Equation (21)). The final direction of the effect will depend on which force dominates.

Firms' production scope is determined by the following equation,

$$X'(|\omega|) \underbrace{i^*(\omega, m)}_{+/-}\underbrace{[(1 - (1-\theta)b)B(m) - s\theta(\sigma_H B(m_H) + \sigma_L B(m_L))]}_{-}\gamma = \mu|\omega|^\iota. \qquad (22)$$

The right-hand side is the marginal cost of production scope, which is not affected by the efficiency change. The left-hand side is the marginal benefit of production scope, which is a product of marginal within-scope probability, the R&D success rate, and the difference in the values of a within-scope and an out-of-scope successful R&D output. An increase in ϕ has a direct negative effect through rises in b and s, capturing that easier patent trading makes scope less relevant in determining the value of a firm's successful invention (the second term in Equation (22)). On the other hand, ϕ also indirectly affects the marginal benefit by changing the success rate of invention. The direction of this indirect effect is ambiguous according to the discussion on R&D intensity (the first term in Equation (22). So, the overall effect of the market efficiency on production scope is ambiguous but is positive only if there is a large increase in the R&D intensity.

4.5.2 Impacts of Sellers' Bargaining Power

Unlike the matching efficiency, an increase in sellers' bargaining power benefits the seller at the cost of the buyer. Higher bargaining power of the seller increases the value of in-scope innovation output (the first term in Equation (23)) because it becomes more costly to buy patents from other firms. At the same time, it also increases the value of out-of-scope innovation output (the second term in Equation (23)) because it is more rewarding

to sell patents to other firms. In both cases, firms are more encouraged to do R&D.

$$i^*(\omega, m) = \{\frac{\gamma}{(1-\sigma)\chi}[X(|\omega|)\underbrace{(1-(1-\theta)b)B(m)}_{+}+(1-X(|\omega|))\underbrace{s\theta(\sigma_H B(m_H)+\sigma_L B(m_L))}_{+}]\}^{\frac{1}{\rho}}$$

(23)

Higher bargaining power of sellers leads to higher transaction prices of patents, which has an ambiguous direct effect on the production scope (the second term in Equation (24)). On the one hand, firms want to increase the likelihood that their innovation output matches their production as buying patents is expensive. On the other hand, having a smaller scope is beneficial as out-of-scope innovation output can be sold at a higher price. As for the indirect effect, the increase in the R&D success rate due to higher bargaining power of the seller raises the benefit of having a larger scope (the first term in Equation (24)). The overall effect is ambiguous, depending on which force dominates.

$$X'(|\omega|)\underbrace{i^*(\omega, m)}_{+}[\underbrace{(1-(1-\theta)b)B(m)-s\theta(\sigma_H B(m_H)+\sigma_L B(m_L))}_{+/-}]\gamma = \mu|\omega|^{\iota} \qquad (24)$$

5   Quantitative Analysis

The main goal of this section is to quantify the relative importance of the key drivers of the specialization patterns and their effects on economic growth. In particular, this study focuses on the four possible explanations: increasing tradability of innovations, the rise in the R&D tax credit rate, changes in production cost structure, and changes in the scarcity of good ideas. The quantitative analysis is undertaken in the following steps. First, the parameters in the model are set to fit data moments in the initial balanced growth path period, 1981-1985, when the policy changes just began. Then, the relevant parameters as analyzed in Section 4.5 are changed to make the model fit the moments in the ending balanced growth path period, 1996-2000, with other parameters fixed in this process. Finally, changes in firms' specialization decisions and the economic growth rate are decomposed to the contribution of each relevant explanation.

5.1 Calibration

There are eighteen parameters, $\{\eta, \lambda, \epsilon, \beta, \delta, \alpha_H, \alpha_L, \chi, \sigma, m_H, m_L, \nu, \gamma, \rho, \theta, \mu, \iota, \phi\}$, a transition matrix $Q_{mm'}$, and a function, $X(\omega)$, to be calibrated in the model. They are grouped into three categories. The first category comes directly from a priori information, as shown in Table 1. The capital and labor share (η and λ) are set respectively to be 0.28 and 0.57 (1/3 and 2/3 multiplied by a return to scale factor of 0.85). The profit share (ζ) is then 15%, which is consistent with the discussion in Guner et al. (2008). The degree of risk aversion for households (ϵ) is taken to be 2, a standard value in the literature. The discount factor (β) is set as 0.99, such that the interest rate of the model economy is 7.5%, a reasonable estimate for the early 1980s in the United States. The depreciation rate of capital (δ) is chosen to be 0.07, consistent with the US National Income and Product Accounts. The paper defines firms of high production ability as those at the top 10% of the production ability distribution; firms of low production ability as the rest. This division is to make the two types of firms respectively represent the large and small firms defined earlier. Among all innovating firms between 1981 and 2000, around 9.1% are large firms (firms with more than 1000 employees). 55.1% of large firms turned out to be of high production ability, while only 5.5% of small firms (firms with less than 1000 employees) have high production ability. Therefore, in the following analysis, firms of high and low production ability largely correspond to large and small firms. The scale parameter in the R&D cost function (χ) is normalized to be 1, which is irrelevant to the quantitative results. The reason of this normalization is that the step size of innovation (γ), which will be calibrated to match data moments, will adjust to any changes in χ. The R&D tax credit rate (σ) is set at the effective level before 1980 as calculated by Akcigit et al. (2018).

Parameter	Description	Value	Identification
η	Capital share	0.25	Guner et al. (2008)
λ	Labor share	0.60	Guner et al. (2008)
ϵ	CRRA parameter	2.00	Standard
β	Discount factor	0.99	Interest Rate
δ	Depreciation rate	0.07	NIPA
α_H	Share of high type	0.10	Imposed
α_L	Share of low type	0.90	Imposed
χ	R&D cost, scale	1.00	Normalization
σ	R&D tax credit rate	0.05	Akcigit et al. (2018)

Table 1: Parameter Values from a Priori Information

Parameters in the second category are pinned down by direct estimation from the data, as presented in Table 2. Estimation of firms' production ability is based upon the solution of employment decisions in the model, $l(m, z') = mz'[(\alpha_h m_h + \alpha_l m_l)\mathbf{z}']^{-1}$. By taking the natural logarithm of both sides, it can be shown that the logarithm of a firm's employment equals the summation of the logarithm of its production ability, the logarithm of the innovation level, and aggregate factors. This study uses the accumulated citation-weighted patent stock as a proxy for a firm's innovation level and uses the time and industry fixed effects as proxies for the aggregate factors. Then, the firm's production ability can be backed out from the residual term of the following regression,

$$ln(emp_{ijt}) = \beta_1 \underbrace{ln(patentstock_{ijt})}_{ln(z')} + \beta_0 + u_t + v_j + \underbrace{residual_{ijt}}_{ln(m)}. \tag{25}$$

The regression sample is all the firms in the Longitudinal Business Database (LBD) that have ever been granted a patent recorded in the Patent Data Project (PDP). Therefore, it is all the innovating firms. It spans from 1981 to 2000, covering both the initial and ending balanced growth paths. The production ability of the high type (m_H) and low type (m_L) are respectively estimated by the average production ability of firms at the top 10% and bottom 90% of the distribution. The transition matrix of production ability ($Q_{mm'}$) is derived from a maximum likelihood estimation.

Parameter	Description	Value	Identification
m_H	Prod. ability of high type	24.43	Regression
m_L	Prod. ability of low type	0.70	Regression
$Q_{mm'}$	Type transition matrix	$\begin{bmatrix} 0.872 & 0.128 \\ 0.017 & 0.983 \end{bmatrix}$	MLE
ν	Matching function, exponent	0.70	Regression
$X(\omega)$	Within-scope Probability	$e^{-4.443} * \|\omega\|^{0.7643}$	Regression

Table 2: Parameter Values from Direct Estimations

The elasticity parameter (ν) in the matching function is estimated by running panel regressions of the number of patent transactions on the number of potential sellers and potential buyers in different layers of industries (i.e., different numbers of digits of the NAICS code). Taking the natural logarithm of both sides of the matching function derives

$$ln(match\_num_{jt}) = \alpha_0 + \nu ln(seller\_num_{jt}) + (1 - \nu)ln(buyer\_num_{jt}) + u_t + v_j + e_{jt}, \tag{26}$$

where *seller_num* is the number of firms whose patent has a technology class that does not match any of the firm's 6-digit NAICS industries. *buyer_sum* is the number of firms that do not have an in-scope patent. Whether the technology class of a patent matches the firm's industries is based on the concordance developed by Silverman (2002).[22] The results are shown in Table 15 in the Appendix. The value of ν is taken to be the average of the estimates.

The within-scope probability function $(X(\omega))$ is estimated as follows. Since it is optimal for firms to produce in industries close to its main line of business (center), this paper assumes all firms do so and only estimates the relationship between a patent's within-scope probability and the number of industries of its inventor. The function form is assumed to be $X(|\omega|) = \xi|\omega|^{\psi}$. This paper groups firms with patents in the LBD by the number of industries and regress the logarithm of the average fraction of patents that match their firms' production scope in each group on the logarithm of the industry number. As shown by the following results in Table 3, ξ and ψ are respectively $e^{-4.443}$ and 0.7643. The high R-squared confirms that the function form assumed is appropriate.

VARIABLE	Log(within-scope prob.)
Ln(num. of Industries)	0.7643***
	(0.0134)
Constant	-4.443***
	(0.0370)
Observations	150[23]
R-squared	0.9547

*** p<0.01, ** p<0.05, * p<0.1

Table 3: Relationship between Within-Scope Probability and the Number of Industries

The third group of parameters is disciplined by minimizing the sum of squares between key moments in the data and the model predicted values jointly in the initial balanced growth path (1981-1985). The economic growth rate is primarily affected by the step size of growth driven by innovations (γ). The R&D cost-to-domestic sales ratio of firms with high and low production ability are informative of both the elasticity of the R&D cost function $(1 + \rho)$ and the bargaining power of sellers on the patent transaction market (θ). The average industry numbers of firms with high and low production ability

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[22]Silverman's concordance links the International Patent Classification (IPC) system to the U.S. Standard Industrial Classification (SIC) system. This study further links the SIC with the North American Industry Classification System (NAICS).

[23]The number of observations is rounded to the nearest 50 to comply with the disclosure requirement of the Census Bureau.

are directly determined by the scale (μ) and elasticity ($1 + \iota$) parameters in the management cost function. They are also indirectly influenced by sellers' bargaining power (θ). The share of patents ever transacted is closely linked with the scale parameter (ϕ) in the matching function. The estimated values of the relevant parameters are shown in Table 4. It is worth noting that both the R&D cost and management cost functions are convex, as assumed by the model, although no restrictions are imposed in the estimation process. The model predicted moments are almost the same as in the data, as shown by Table 5, attesting the model fits the economy well in the initial balanced growth path.

Parameter	Description	Value	Identification
γ	Step size of innovation	1.72	Growth rate
$1 + \rho$	R&D cost elasticity	1.79	R&D cost/sales
θ	Bargaining power	0.16	ratio (H and L)
μ	Management cost, scale	1.5E-4	Avg. number of
$1 + \iota$	Management cost, elasticity	3.31	industries (H and L)
ϕ	Matching function, scale	0.19	Patent traded share

Table 4: Parameter Values from the Minimum Distance Estimation

Targets	Data	Model
Economic growth rate(1981-1985)	3.05%	3.05%
R&D cost/sales of H firms(1981-1985)	3.62%	3.62%
R&D cost/sales of L firms(1981-1985)	2.83%	2.83%
Avg. number of industries of H firms(1981-1985)	11.81	11.81
Avg. number of industries of L firms(1981-1985)	1.92	1.92
The share of patents transacted(1981-1985)	30.9%	30.9%

Table 5: Model Fit for Key Moments in the Initial Balanced Growth Path

5.2 Recalibration to the Post-Reform Balanced Growth Path

As pointed out in Section 4.5, the set of parameters, $\{\phi, \theta, \sigma, \mu, \iota, \gamma, \rho\}$, corresponds to the possible explanations for the specialization patterns. To match the post-reform (ending) balanced growth path, this paper sets the new R&D tax credit rate as the actual effective rate, 24%, in the 1990s. Other parameters in this set are recalibrated to make the

model fit the economic growth rate, R&D cost-to-domestic sales ratio, the average industry numbers of firms with high and low production ability, and the fraction of patents ever transacted in 1996-2000. The value of parameters out of this set is fixed in the recalibration process. The performance of the recalibration is displayed in Table 6, showing a good match between the model and data.

Targets	Data	Model
Economic growth rate(1996-2000)	3.34%	3.34%
R&D cost/sales of H firms(1996-2000)	3.15%	3.15%
R&D cost/sales of L firms(1996-2000)	6.71%	6.71%
Avg. number of industries of H firms(1996-2000)	6.31	6.31
Avg. number of industries of L firms(1996-2000)	1.61	1.61
The share of patents transacted(1996-2000)	44.1%	44.1%

Table 6: Model Fit for Key Moments in Recalibration

Comparison between the initial and ending values of the parameters are displayed in Table 7. Although the direction of changes of these parameters is not restricted in the recalibration process, it turns out to be consistent with the original predictions. There is an increase in the matching efficiency, ϕ, of the patent market and the bargaining power, θ, of patent sellers, confirming decreasing market frictions and stronger protection towards patent holders. The scale and elasticity parameters in the management cost function (μ and ι) are larger, implying that the cost of producing in multiple industries is higher. The very slight change in γ shows the value of a successful R&D output remains nearly the same, while the decrease in ρ suggests that the success rate of R&D relies more on investment, reflecting that the generation of good ideas is more investment intensive. [24]

	Old Values	New Values	Interpretation
ϕ	0.19	0.27	Matching efficiency increase
θ	0.16	0.22	Sellers' bargaining power increase
μ	1.5E-4	1.7E-4	Higher costs of large scope
$1 + \iota$	3.31	3.93	More DRS to scope
γ	1.72	1.73	Goods ideas rely more on
$1 + \rho$	1.79	1.56	R&D investment

Table 7: Changes of Parameter Values

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[24]The elasticity of the R&D success rate with respect to investment can be expressed as $\frac{1}{1+\rho}$.

5.3 Decomposition

To gauge the contribution of each possible explanation, this paper sets the parameters that govern each explanation at the ending steady-state value while others at the initial steady-state value. Hypothetical moments about specialization and economic growth are derived in each case. Then the paper compares the hypothetical moments with the moments in the initial balanced growth path. The difference between them measures the effect of each mechanism. The decomposition process uses the following formula,

$$\frac{M_i(\Theta_{81-85}, \kappa_{96-00}) - M_i(\Theta_{81-85}, \kappa_{81-85})}{D_{i,96-00} - D_{i,81-85}}, \tag{27}$$

where M_i is the ith moment in the model and D_i is the corresponding value in the data. κ is the set of key parameters that correspond to each explanation. Θ represents all parameters in the model except for κ. This formula isolates the contributions of the key parameters.

Table 8 presents the decomposition results. The first row displays the direction of changes in the data regarding the average production scope, the R&D intensity of firms with high and low production ability, the share of patents traded, and the economic growth rate. The direction of changes predicted by the new hypothesis, increasing tradability of innovations, is consistent with the direction of all the real changes. Quantitatively, the new hypothesis can explain 25% of the decrease in production scope; 232% of the decrease in R&D intensity for firms with high production ability and 58% of the increase in R&D intensity for firms with low production ability. It is responsible for the bulk of (90%) the rise in the trading share of patents and 221% of the rise in economic growth. Since the annual economic growth rate increases by 0.29 percentage points between the two periods, changes in invention tradability lead to a 0.64 percentage points increase in growth. This study lists the respective contribution of the matching efficiency and sellers' bargaining power, finding that the former is the main driving force. The R&D tax credit has little explanatory power for the specialization patterns but significantly contributes to a higher growth rate. Most of the remaining part of specialization is explained by the change in production cost structure, although it has little effect on the patent trading activities and contributes negatively to growth. Rarer good ideas contribute to a significant part of the decrease in firms' scope but are muted in explaining other dimensions of specialization. The subsections below will discuss the effects of each mechanism in detail.

	Prod. Scope	$R\&D$(H)	$R\&D$(L)	Patent Trade	Growth
Data	-	-	+	+	+
Patent Market (ϕ, θ)	25%	232%	58%	90%	221%
Efficiency (ϕ)	26%	220%	15%	100%	151%
Bargaining Power (θ)	-8%	11%	37%	-6%	45%
Tax Credit (σ)	7%	-265%	12%	-8%	137%
Production Cost (μ, ι)	63%	287%	25%	3%	-180%
Rare Good Ideas (γ, ρ)	43%	-273%	-32%	12%	-27%

Table 8: Effects of Key Parameters

5.3.1   Increasing Tradability of Innovations

The effect of this mechanism on the specialization patterns is mostly driven by the rise in the matching efficiency of the patent trading market. As analyzed in Section 4.5.1, both the buyers and sellers get higher matching rates on the market. Higher chances of trading decrease R&D incentives for potential buyers while increase R&D incentives for potential sellers. Since firms with high production ability benefit more from buying patents on the market, the force that decreases R&D intensity dominates. Firms with low production ability benefit more from selling patents to other firms. Therefore, the force that increases R&D intensity dominates. The effect of the matching efficiency on production scope also has two sides. On the one hand, the trading channel makes the scope less critical in determining the value of a firm's innovation output, so there is a tendency to narrow the scope. On the other hand, the scope becomes more important if the firm increases R&D intensity due to the efficiency change. For firms with high production ability, the overall effect is unambiguous because they decrease R&D intensity. For firms with low production ability, as it turns out, the former force dominates. The fraction of patents traded is directly linked to the matching efficiency. Therefore it can be explained to a large extent.

Higher economic growth comes from three sources. First, there are fewer wasted ideas as innovation output that falls out of its inventor's scope can be utilized through

trading. Second, reduction in firms' production scope decreases management cost. Third, innovation activities are reallocated to firms with a comparative advantage.

The contribution of higher bargaining power mainly lies in the increase in the R&D intensity of firms with low production ability. This is consistent with the partial-equilibrium analysis in Section 4.5.2 that higher transaction prices of patents raise the value of both in-scope and out-of-scope innovation output and therefore increase R&D incentives. The slight decrease of R&D intensity of firm with high production ability is mainly due to a general-equilibrium effect.

5.3.2 R&D Tax Subsidy

An increase in the R&D tax credit boosts the R&D intensity of both types of firms since the innovation cost is lower. The effect on firms with high production ability turns out to be more significant because these firms can better monetize innovation output through their own production. Higher R&D intensity has a strong positive effect on economic growth.

5.3.3 Changes in Production Cost Structure

Changes in production cost structure can mainly explain the remaining part of the specialization patterns. A higher cost of producing in multiple industries directly shrinks firms' production scope. Smaller production scope reduces the likelihood of matches between innovation and production, thus disincentivizing firms to do R&D. This explains the decline in high-type firms' R&D intensity. The slight increase in low-type firms' R&D intensity is mainly due to the general-equilibrium effect. This mechanism alone has minimal effects on patent trading activities. It negatively affects growth as mismatches between innovation and production increase, and more inventions are wasted.

5.3.4 Good Ideas are Harder to Find

As innovation becomes more investment intensive, there is a direct decrease in the incentive to do R&D. Then, successful R&D output becomes scarcer and more valuable. So, firms with higher production ability (the ones that benefit more from R&D output) invest

more in innovation. This predicts a shift of R&D activities to firms good at production, contradictory to the trend in the 1980s and 1990s. The decrease in firms' production scope is mostly driven by a significant decrease in the scope of firms with low production ability. This is because those firms sharply reduce their R&D effort and get lower benefits from expanding production scope. The change in the R&D cost function contributes negatively to growth as idea generation is more costly than before.

5.4 Implications for Growth

Firms' growth has two engines–own innovation and others' innovation. The patent trading market changes the relative importance of the two in the two balanced growth paths. To gauge this change, this study calculates the share of contribution of each engine for both types of firms. The results are shown in Table 9. In the initial balanced growth path, 57.6% of the growth of firms with high production ability comes from utilizing their own innovations. The share decreased to 31.3% in the ending balanced growth path, implying that these firms rely more on other firms' R&D activities through buying patents on the market. For firms with low production ability, the contribution of their own innovation decreased slightly from 2.9% to 2.3%. In sum, all firms depended more on the market for new ideas, especially for large firms.

| | Decomposition | |
	Own R&D	Others' R&D
Old Eq. (81-85)		
High-Type (Large Firms)	57.6%	42.3%
Low-Type (Small Firms)	2.9%	97.1%
New Eq. (96-00)		
High-Type (Large Firms)	31.3%	68.7%
Low-Type (Small Firms)	2.3%	97.7%

Table 9: Decomposition of Growth

6 Discussion and Extension

Quantification of the baseline model shows that increasing tradability of innovations can explain a sizable share of the decrease in production scope and the reallocation of R&D

activities. However, this new hypothesis may be subject to several challenges. First, the direction of causality is not clear. It is possible that the more vibrant patent trading activities are the result of narrower production scope of firms, i.e., firms produce in fewer industries due to changes in the cost structure and then have to depend on the market for monetizing innovation as it becomes harder to match their innovation output with own production. Second, the potential mismatch between innovation output and production may not play an important role. Intellectual products may be similar to other goods in the sense that the production process requires ingredients from other industries. Alleviation of the incomplete contract problem in the ingredient trading process may also lead to more transactions and contraction in production scope.

To check whether the new hypothesis holds in front of these challenges, this paper looks at changes in the targeting behaviors of firms' R&D activities. If the reverse causality is true, it should be predicted that R&D becomes more targeted as the firm spans fewer industries. If there is no mismatch between innovation and production, but only the incomplete contract problem in the ingredient trading process for new inventions, the targeting behaviors in the innovation process will experience no change. On the contrary, the new hypothesis in this study predicts the R&D activities to be less targeted, as the type of R&D that is less likely to match the firm's own production benefits more from easier trading of intellectual properties.

6.1   Data Patterns

The targeting behavior of the innovation process can be measured by the expense shares of different R&D types-basis research, applied research, and development. They differ in the probability of being applied to a specific production process. In the Survey of Industrial Research and Development (SIRD), basic research is defined as "the activity aimed at acquiring new knowledge or understanding without specific immediate commercial application or use"; applied research is "the activity aimed at solving a specific problem or meeting a specific commercial objective"; development is "the systematic use of research and practical experience to produce new or significantly improved goods, services, or processes". Therefore, basic research has the broadest targets. This study uses the ratio of basic research expense to basic plus applied research expense and the ratio of basic research expense to total R&D expense as proxies for firms' targeting behaviors in R&D.

A higher share implies less targeting and broader R&D scope. Figure 7 shows the two ratios over the years[25]. They both picked up at the beginning of the 1980s, and the rising trends continued in the following two decades–the same period when the patent market grew. The pattern of widening R&D scope in the 1980s and 1990s is also supported by Akcigit and Ates (2019), in which the authors use the average length of patent claims as a measurement of the R&D scope. This pattern suggests that the reverse causality and the ingredient trading theory are insufficient to address the newly found specialization wave in the knowledge economy.
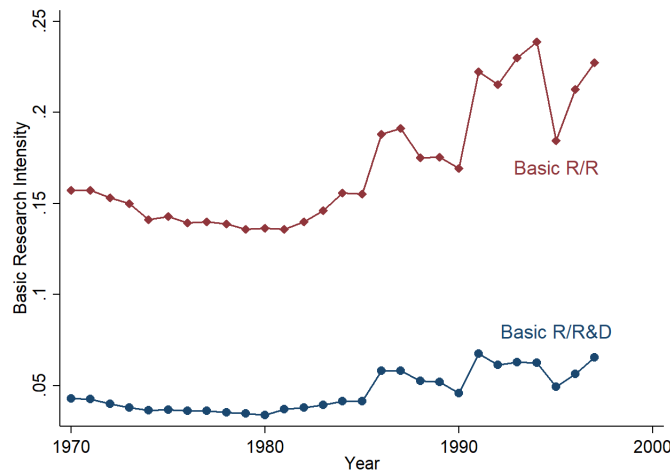
Figure 7: Share of Research Spending

6.2 Model Extension

The baseline model is extended to study the impact of the new hypothesis on firms' targeting behaviors in the innovation process. Now, firms choose the success rates (equivalent to expense) of two types of research at the innovation stage–(a)pplied and (b)asic research. The two types of research differ in three dimensions: 1) the scale and elasticity parameters in the R&D cost function. (i.e., $\chi^b \neq \chi^a$, $\rho^b \neq \rho^a$), 2) the probability of the innovation output falling inside the firm's own production scope (i.e., $X^b(.) \neq X^a(.)$), and 3) the step size of successful inventions coming from basic research and from applied research (i.e., $\gamma^b \neq \gamma^a$). Each firm is endowed with two units of search effort–one for basic

[25]Only data before 1998 is shown because statistics for 1998 and later years are not directly comparable to statistics for 1997 and earlier years, according to the statement made by the SIRD.

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research output and the other for applied research output. The innovation level of a firm is updated in each period according to the following law of motion,

$$z' = z + \sum_{j \in \{a,b\}} \gamma^j (\mathbb{1}^j_{(RD \in \omega)} + \mathbb{B}^j) \mathbf{z}, \tag{28}$$

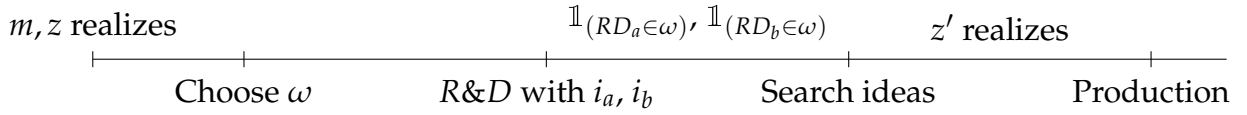where $\mathbb{1}^j_{(RD \in \omega)}$ is an indicator of whether the firm's type-j (applied or basic) research output falls inside its production scope. $\mathbb{B}^j$ is an indicator of whether the firm can buy a type-j (applied or basic) patent that matches its scope.

The new timeline is shown as follows.

m, z realizes			$\mathbb{1}_{(RD_a \in \omega)}, \mathbb{1}_{(RD_b \in \omega)}$		z' realizes	
	Choose ω		R&D with i_a, i_b	Search ideas		Production

The following proposition holds. Characterization and proof of Proposition 6.1 are presented in the Appendix.

Proposition 6.1 (Symmetric Balanced Growth Path)**.** *There exists a symmetric balanced growth path in the extended model.*

6.3   Quantification of the Extended Model

To quantitatively evaluate the effects of the new hypothesis and the three other mechanisms on firm's R&D scope, this paper calibrates the newly added parameters, $\{\chi^j, \rho^j, \gamma^j\}$, and the two probability functions, $X^j(.)$, where j is an indicator of basic or applied research, in the following way. The ratio of the step sizes, $\frac{\gamma_b}{\gamma_a}$, is set to be consistent with Akcigit et al. (2021). The probability functions are estimated by the same method as the estimation of $X(.)$ in the baseline model, except that the regression is run on two separate samples–patents from basic research and patents from applied research or development in the SIRD. The scale parameter of the applied research cost function (χ^a) is normalized to be 1. The scale parameter of the basic research cost function (χ^b), the step size of applied research (γ_a), and the two elasticities (ρ^a, ρ^b) are pinned down together with $\{\phi, \theta, \mu, \iota\}$ in the original calibration. Two additional moments are added–the share of basic research expense in total R&D expense respectively for firms with high and low production ability. All the other parameters are disciplined by the method used to calibrate the baseline

model, and the decomposition method is the same as before. A more detailed description of the calibration process and performance is shown in Section 9.7 of the Appendix.

	Basic R	Prod. Scope	$R\&D$(H)	$R\&D$(L)	Patent Trade	Growth
Data	+	-	-	+	+	+
Patent Market (ϕ, θ)	101%	29%	205%	55%	93%	227%
Efficiency (ϕ)	26%	29%	194%	10%	102%	149%
Bargaining Power (θ)	56%	-6%	7%	39%	-5%	51%
Tax Credit (σ)	32%	7%	-257%	12%	-6%	132%
Production Cost (μ, ι)	-17%	66%	223%	20%	2%	-155%
Rare Good Ideas ($\{\gamma^j, \rho^j\}_{j\in\{a,b\}}$)	-35%	23%	-168%	-17%	9%	-82%

Table 10: Effects of Key Parameters

Table 10 presents the explanatory power of the four mechanisms in the targeting behaviors of innovation and the other moments shown in the baseline calibration. As shown by the first column, increasing tradability of innovations is responsible for all (101%) of the increase in the share of basic research. The R&D tax credit also contributes to part of the increase. In contrast, changes in production cost structure make innovation more targeted. The reason is that when firms span fewer industries due to higher fixed costs, they also narrow R&D scope to improve matching between innovation and production. The increasing difficulty to find good ideas also leads to a contraction in R&D scope. The impacts of the mechanisms on other moments are very similar to the results in the baseline model, confirming the robustness of the previous conclusions.

In sum, the rise in the share of basic research spending provides evidence of the important role of potential mismatches between innovation and production in explaining the observed specialization wave.

7 Empirical Analysis

This section empirically tests whether there is causality from the pro-patent reform to US firms' shrinkage of production scope. The main idea is to exploit the regional and

firm-level differences in the exposure to policy changes and check whether they lead to different extents of the drop in scope. Two policies are used in the empirical analysis. 1) The establishment of the Court of Appeals for the Federal Circuit (CAFC) in 1982. 2) Extension of patentability to genetic engineering and software in 1980 and 1981. More details about these policies and the empirical strategy will be unfolded, then the results will follow.

7.1 Institutional Background

The US federal court system has three main layers: district courts, circuit courts, and the Supreme Court of the United States. All patent-related cases are heard initially at one of the ninety-four district courts across the country. If there are challenges to the decisions, the case can be appealed to one of the circuit courts. Since the Supreme Court rarely hears patent-related cases, the circuit courts usually have the final say on those cases.

Before 1982, twelve circuit courts divided the country into different regions. Attitudes towards patents in the circuit courts had a significant discrepancy. Therefore, decisions of district courts under different circuit courts varied much in the first place. The second column of Table 11 shows the fraction of lawsuits invalidating the involved patents in district courts of different regions from 1940 to before 1982. The legal environment towards patents was stable in this period. For example, the Third Circuit is headquartered in Philadelphia and has appellate jurisdiction over district courts in Delaware, New Jersey, and Pennsylvania. On average, 74% of the cases heard at these district courts invalidated the patents, suggesting weak protection towards patent holders in the region. In contrast, district courts under the Tenth Circuit Court located in Denver had an average of 27% invalidation rates, implying the patent protection was stronger in this region.

In 1982, Congress created the Court of Appeals for the Federal Circuit (CAFC). It has nationwide jurisdiction to hear appeals involving patent laws. So, decisions of district courts can be appealed to not only the twelve regional circuit courts but also the CAFC. The CAFC held a more positive view towards patents and had a much lower invalidation rate in its final decisions. Therefore, the decisions of district courts became lower and more uniform across different regions in the first place, as shown in the third column of Table 11.

Circuit Court	Invalidation rate	
	Before	After
Boston	0.64	0.18
New York	0.58	0.28
Philadelphia	0.74	0.32
Richmond	0.47	0.26
New Orleans	0.36	0.20
Cincinnati	0.60	0.30
Chicago	0.54	0.30
St.Louis	0.49	0.33
San Francisco	0.51	0.29
Denver	0.27	0.22
Atlanta	0.41	0.28
DC	0.59	—[26]

Table 11: Patent Invalidation Rates in District Courts under Different Circuit Courts

The establishment of the CAFC was exogenous to the district courts and resulted in stronger protection of patents, as shown in Figure 4 in Section 3.1. More importantly, the impact on the decisions of district courts varied across regions. Regions that had a higher patent invalidation rate before 1982 were affected more strongly by the CAFC. Although there are forum shopping behaviors, firms are more likely to bring their lawsuits to the district court that is in charge of the area in which they are located due to home-field advantage (Moore (2001)). Therefore, the regional differences in the change of patent protection in courts can be transmitted to firms located in different regions. If patent protection is one of the causes of the contraction in production scope, then it is predicted that firms in regions with higher invalidation rates before 1982 experienced a larger decrease in production scope.

Precedents of court decisions in patent-related legal disputes often determine the patentability of similar objects afterward. Genetic engineering and software are two of the most controversial fields of patentability in the 70s. In 1980, the Supreme Court ruled in the case between Diamond and Chakrabarty that genetically engineered bacteria involved in the case could be patented. This ruling was viewed as a turning point for the biotechnology industry in the following decades. In 1981, the decision of the Supreme Court in the dispute between Diamond and Diehr that software was not precluded from patentability also had a profound impact on court decisions afterward. These two landmark cases happened just before the establishment of the CAFC, making these two used-

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[26]The circuit court of DC has too few observations after the CAFC era.

to-be controversial fields experience the most reduction of inconsistency among different regions. This leads to another dimension of difference in firms' exposure to policy shocks. Firms with a higher share of production in these two fields were more likely to be affected by the increasing uniformity of court decisions. It is predicted that the establishment of the CAFC led to a more significant shrinkage of production scope for these firms.

7.2 Estimation Strategy

The following Difference-in-Difference (DiD) regression explores whether regional differences in the change of patent protection led to different extents of contraction in firms' production scope,

$$ln(ind_{ist}) = \alpha_i + \beta * inval_{c,pre} * post_t + \gamma X_{ist} + \mu_t + \epsilon_{ist}, \tag{29}$$

where the dependent variable, ind_{ist}, is the number of 6-digit NAICS industries of the firm i in the LBD. s is the state of its headquarters before the year of the CAFC establishment. The headquarter is measured by the state where the firm has the most employment. t is the year of the observation. The main explanatory variable is an interaction between $inval_{c,pre}$, the patent invalidation rate of the circuit court, c, that the state, s, belongs to prior to the CAFC era, and a dummy variable, $post_t$, that indicates whether the year is before or after the establishment of the CAFC. The control variables, X_{ist}, include the log of firm's employment, the effective federal and state corporate income tax rates, and R&D tax credit rates calculated by Wilson (2009), and the log of state-level real GDP. Firm-fixed effects, α_i, and year-fixed effects, μ_t, are also included in the regression to exclude permanent cross-firm and time differences. The coefficient of interest, β, captures the relationship between the differential changes in firms' production scope and the differential changes in the strength of patent protection across regions.

Firm-level differences add another dimension of difference in the exposure to patent protection. The following Triple-Difference (DDD) regression tests whether firms with a higher exposure decreased production scope more,

$$
\begin{aligned}
ln(ind_{ist}) =\; & \alpha_i + \beta_1 * high\_treat_i * inval_{c,pre} * post_t + \beta_2 * inval_{c,pre} * post_t + \\
& \beta_3 * high\_treat_i * post_t + \gamma X_{ist} + \mu_t + \epsilon_{ist},
\end{aligned}
\tag{30}
$$

where $high\_treat_i$ is the firm's share of employment in the NAICS code 541710 (Research and Development in the Physical, Engineering, and Life Sciences)[27] and 511210 (Software Publishers) prior to the CAFC. The rest of the variables are the same as defined earlier. The other interaction terms are omitted in the fixed effects. β_1 captures the differential impact of the change in patent protection for firms in the most controversial industries versus others; β_2 shows whether the influence of the CAFC only concentrates on the two industries or stretches to more general industries.

The standard errors are clustered at the circuit court region by the post dummy level in both specifications.

7.3 Sample Description

The sample of the regression analysis is the innovating firms in the LBD that existed before or in 1982, the year of the establishment of the CAFC. The requirement of existence before the reform is to avoid endogeneity issues induced by changes of firms' headquarters due to the policy change. The sample period is from 1976 to 1989, 7 years before and after the reform.[28] Summary statistics of the main variables are presented in Table 12.[29]

	Mean			Standard Deviation		
Sample	All	Post=0	Post=1	All	Post=0	Post=1
Observations	268000	131000	136000	268000	131000	136000
Number of Industries	3.066	3.074	3.058	6.722	6.952	6.494
Employment	1187	1187	1187	9670	10780	8467
Emp. Share (Highly Treated)	0.02101	0.01987	0.0221	0.1337	0.129	0.138
Pre-CAFC Invalidation Rate	0.5375	0.5381	0.5369	0.1082	0.1082	0.1083
Real GDP	144000	127200	160200	115000	95460	129100
Effective Federal Tax Rate	0.4105	0.4335	0.3883	0.0434	0.01645	0.04934
Effective State Tax Rate	0.07406	0.07325	0.07484	0.02676	0.0279	0.02558
Federal R&D Tax Credits	0.01443	0.004603	0.02388	0.01145	0.007372	0.004747
State R&D Tax Credits	0.0006073	0.0001753	0.001023	0.003604	0.002553	0.004343

Table 12: Summary Statistics of the Regression Sample

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[27]Bioengeering is embodied in this code.

[28]1976 is the earliest year of the LBD, so the longest period this study can explore before the establishment of the CAFC is seven years. This study also runs the same regressions on the samples of six years and five years before and after the reform. The results are very similar.

[29]The number of observations is rounded to the nearest 1000 to comply with the disclosure requirement of the Census Bureau.

The number of industries per firm experiences a decrease before (Post=0) and after (Post=1) the CAFC, while the average employment remains at nearly the same level. The average share of employment in the two highly treated industries is around 2%. The average invalidation rate across different regions is 53.75%.[30] There is a drop in the federal corporate income tax rate and a rise in both the federal and state-level R&D tax credits.

7.4 Regression Results

The regression results of Equation 29 that exploits regional differences in the change of patent invalidation rates are displayed in Table 13. The first two columns insert the *post* dummy in the regression instead of the year-fixed effects; the last two columns control the year-fixed effects. Columns (2) and (4) control the state-level characteristics compared to columns (1) and (3). In all of the columns, there are negative and significant coefficients of the interaction term, implying that firms located in regions with a larger change in patent protection strength experience a larger drop in production scope. The magnitude of this coefficient is quite stable across the columns with different control variables.

Estimation of Equation 30 that includes firm-level differences in exposure to the policy change is displayed in Table 14. Again, The first two columns insert the *post* dummy in the regression instead of the year-fixed effects; the last two columns control the year-fixed effects. Columns (2) and (4) control the state-level characteristics compared to columns (1) and (3). The negative and significant coefficient of the triple interaction term suggests that firms in the highly treated industries (bioengineering and software) are more affected by the establishment of the CAFC. It is worth noticing that the coefficient of the interaction term between the invalidation rate and the post dummy is still significantly negative, although the absolute magnitude is slightly lower than in Table 13. This shows that the impact of the CAFC is not concentrated in only the two highly treated industries but instead covers more general industries.

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[30]There is very little change in this rate before and after the CAFC because both of them are at the pre-CAFC level.

Dependent Variable	Ln(Number of Industries)			
	(1)	(2)	(3)	(4)
Invalidation Rate*Post	-0.0326**	-0.0326**	-0.0332**	-0.0281**
	(0.014)	(0.014)	(0.014)	(0.013)
Ln(Employment)	0.0888***	0.0899***	0.0893***	0.0894***
	(0.007)	(0.007)	(0.007)	(0.007)
Post	0.0196**	0.0218***		
	(0.008)	(0.007)		
Ln(Real GDP)		-0.00487		-0.0332**
		(0.012)		(0.014)
Effective Federal Tax Rate		0.272***		-1.462***
		(0.036)		(0.508)
Effective State Tax Rate		0.234*		-0.689**
		(0.132)		(0.278)
Federal R&D Tax Credits		0.532***		-10.27**
		(0.137)		(3.697)
State R&D Tax Credits		-0.068		-0.0807
		(0.141)		(0.127)
Year-fixed Effects	NO	NO	YES	YES
Firm-fixed Effects	YES	YES	YES	YES
Observations	268000	268000	268000	268000
R-squared	0.944	0.944	0.944	0.944

Standard errors are clustered in the circuit court region by the post dummy level, shown in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 13: DiD Regression Results

7.5 Placebo Tests

It is possible that the differential changes in the number of industries across regions and firms are due to some pre-trends instead of the policy impact. To check whether there are pre-existing trends, this study runs the same regressions in Equation 29 and 30 on the pre-CAFC sample (1976-1982). All variables are defined as the same as before, except the *post* dummy. Now, the *post* dummy equals zero if the observation year is before or in 1979; equals one if after 1979.[31] If there are pre-trends, β in Equation 29 and β_1 and β_2 in Equation 30 should still be significantly negative. However, as shown in Table in the Appendix, they are either positive or tiny in absolute magnitude. None of them is significant, showing that the differential changes in production scope are not due to pre-existing trends. Therefore, the empirical results in this section can be viewed as proof for causality from stronger protection of intellectual property rights to firms' shrinkage of

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[31]This study also tries other ways of segmenting the pre-CAFC sample. The results are similar.

Dependent Variable	Ln(Number of Industries)			
	(1)	(2)	(3)	(4)
High_treat*Invalidation Rate*Post	-0.134*	-0.132*	-0.132*	-0.128*
	(0.069)	(0.069)	(0.069)	(0.069)
Invalidation Rate*Post	-0.0301**	-0.0301**	-0.0307**	-0.0257*
	(0.014)	(0.014)	(0.014)	(0.013)
High_treat*Post	0.0840**	0.0833*	0.0833**	0.0829*
	(0.040)	(0.041)	(0.040)	(0.040)
Ln(Employment)	0.0888***	0.0899***	0.0893***	0.0894***
	(0.007)	(0.007)	(0.007)	(0.007)
Post	0.0180**	0.0202***		
	(0.008)	(0.007)		
Ln(Real GDP)		-0.00514		-0.0337**
		(0.012)		(0.014)
Effective Federal Tax Rate		0.271***		-1.464***
		(0.037)		(0.509)
Effective State Tax Rate		0.235*		-0.688**
		(0.132)		(0.279)
Federal R&D Tax Credits		0.534***		-10.35**
		(0.136)		(3.697)
State R&D Tax Credits		-0.0677		-0.0808
		(0.142)		(0.128)
Year-fixed Effects	NO	NO	YES	YES
Firm-fixed Effects	YES	YES	YES	YES
Observations	268000	268000	268000	268000
R-squared	0.944	0.944	0.944	0.944

Standard errors are clustered in the circuit court region by the post dummy level, shown in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 14: DDD Regression Results

production scope.

8   Conclusion

This study finds novel patterns of firm specialization in the US Census data that started in the early 1980s and ended in the late 1990s. 1) Firms narrowed down their production scope. This phenomenon was more pronounced for innovating firms. 2) Small firms increased R&D intensity while large firms decreased it. The same period also witnessed a growing patent trading market after a series of pro-patent policy reforms that strengthened the protection of intellectual property rights.

A new hypothesis is proposed to explain the observed phenomena–increasing tradability of intellectual properties makes production scope less critical in determining the value of a firm's innovation output. An endogenous growth model is developed to examine the roles of the new hypothesis and three other possible explanations–the increase in R&D tax credit, changes in production cost structure, and rising scarcity of good ideas. The developed model is calibrated to small and large firms' production scope and innovation intensity, the fraction of patents traded, and the economic growth rate. Four major conclusions can be drawn from the quantitative results. First, increasing tradability of innovations accounts for at least 25% of the production scope decrease; 58% of the reallocation of innovation activities. Second, the remaining part of specialization is mostly due to changes in the production cost structure. Third, increasing tradability of innovations leads to a 0.64 percent point increase in growth rates, which makes it a primary driving force of the economic growth in the 1980s and 1990s. Fourth, large firms' growth relies more on R&D activities of other firms.

This paper also finds in the data that the R&D activities of US firms became less targeted in the 1980s and 1990s. The baseline model is then extended to include two types of research that differ in the probability of matching the inventor's production scope. Quantitative results of the extended model show increasing tradability of innovations can explain 101% of the decrease in the R&D targeting behavior.

Using the regional and firm-level differences in the exposure to patent policy changes in the early 1980s, this paper provides empirical support for causality from patent protection to contraction in firms' production scope.

The findings of this paper suggest that innovation and production become more separate when protection towards intellectual property rights is stronger. Firms with high production ability used to be also innovation-intensive. Now, innovation activities depend less on the production side. A potential extension is to allow firms to endogenously choose their production ability at some costs. Mirroring the result that firms with high production ability choose to do less innovation, it is predicted that firms with high innovation levels will spend fewer resources in improving their production ability. This may provide a new explanation for the phenomenon found in Pugsley et al. (2019) that high-growth startups ("gazelles") have grown less rapidly in size since the mid-1980s.

An important policy implication of this paper is that strengthening intellectual prop-

erty rights protection has an impact that is often neglected–reducing mismatches between innovation and production through a market approach. This approach provides a strong engine for economic growth, as evidenced by the pro-patent reform adopted by the US government to deal with global competition in the 1970s. Therefore, increasing patent protection can be viewed as a potential policy tool to boost the economy in the future.

An increasing amount of literature tries to explain the slowdown in the US business dynamism after the 2000s. Although this paper focuses on the 1980s and 1990s, it may provide some possible explanations for what happened in the 2000s. First, the counterbalancing patent policies in the 2000s may have stifled the trading of innovation output between firms. This is partly evidenced by the findings in Akcigit and Ates (2021) and Olmstead-Rumsey (2019) that knowledge diffusion slowed down in the 2000s. Second, the monopoly power of big buyers and big sellers on the patent market may also have grown when the patent protection became stronger. Since firms depended more on the market for innovation in the early 2000s compared to the early 1980s, the increasing monopoly power may be more harmful to economic growth. It will be interesting to include market power in the analysis and study the optimal strength of patent protection when considering patent trading.

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9 Appendix

9.1 Another Measure of Innovation Intensity

The following figure shows the (citation-weighted) number of patents per employee for small/medium firms and large firms. They both increased starting from the early 1980s, but the increase was more salient for small/medium firms. The rising trends are partly due to the extension of patentability, but the different slops of them reflect that small-/medium firms engaged in more R&D activities.
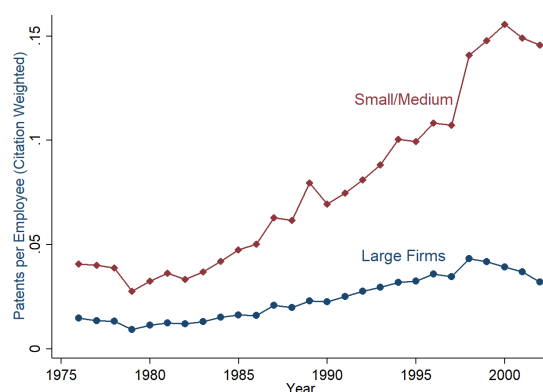

Figure 8: Patents per Employee by Size

9.2 Timing of Patent Trading

Figure 9 shows the timing of the patent trading. The blue, red, green, and yellow curves display respectively the fraction of patents traded within four years before issuance, one to five years after issuance, six to ten years after issuance, and more than ten years after issuance. It should be noted that the descending trend of the yellow curve after 2000 is due to the right censoring issue. Comparison of the four curves suggests that most of the increase happened between 1980 and 2000, consistent with the timing of the pro-patent policy reforms; earlier transactions occurred more often, evidence that the patent trading market has become more efficient.
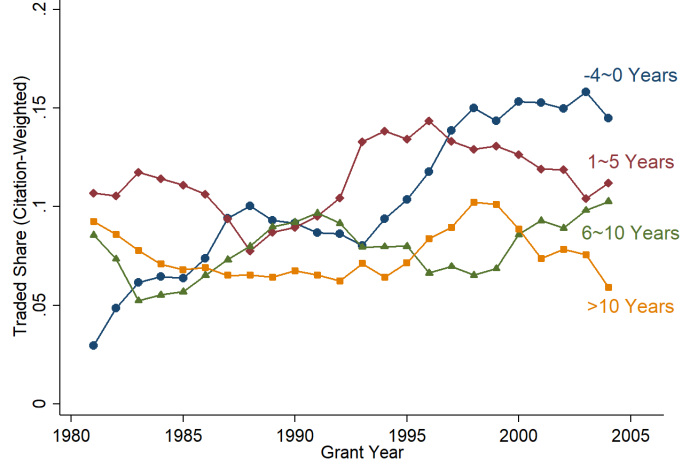
Figure 9: Fraction of Patent Traded by Gaps from the Grant Year

9.3 Proof of Proposition 4.1

Denote the distribution of production ability and innovation levels among all firms at the end of the current period as $F(m, z'; \mathbf{z})$. Equation (10) implies that the labor market clearing condition can be written as

$$(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}} (\frac{\lambda}{w})^{1+\frac{\lambda}{\zeta}} \int \int m z' dF(m, z'; \mathbf{z}) = 1. \tag{31}$$

Equation (31) can be transformed to

$$(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}} (\frac{\lambda}{w})^{1+\frac{\lambda}{\zeta}} (\alpha_H m_H \mathbf{z}'_H + \alpha_L m_L \mathbf{z}'_L) = 1, \tag{32}$$

where $\mathbf{z}'_H$ and $\mathbf{z}'_L$ are respectively the average innovation level of firms with high production ability and that of firms with low production ability at the end of this period. They are defined by

$$\mathbf{z}'_H = \frac{1}{\alpha_H} \int z' dF(m_H, z'; \mathbf{z}); \tag{33}$$

$$\mathbf{z}'_L = \frac{1}{\alpha_L} \int z' dF(m_L, z'; \mathbf{z}). \tag{34}$$

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The economy-wide average innovation level at the end of the previous period, $\mathbf{z}$, can then be expressed as

$$\mathbf{z} = \frac{\alpha_H m_H \mathbf{z}_H + \alpha_L m_L \mathbf{z}_L}{\alpha_H m_H + \alpha_L m_L}. \tag{35}$$

Assume $\mathbf{z}$ grows at a constant rate, g, across periods. Then, the labor market clearing condition can be further transformed to

$$(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{1+\frac{\lambda}{\zeta}}(\alpha_H m_H + \alpha_L m_L)g\mathbf{z} = 1. \tag{36}$$

The wage rate, w, can then be expressed as

$$w = \lambda(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta+\lambda}}[(\alpha_H m_H + \alpha_L m_L)g\mathbf{z}]^{\frac{\zeta}{\zeta+\lambda}}, \tag{37}$$

which implies that it grows at a rate of $g^{\frac{\zeta}{\zeta+\lambda}}$. The total output and capital of the economy also grow at $g^{\frac{\zeta}{\zeta+\lambda}}$, since

$$\int\int Y(m,z';\mathbf{z})dF(m,z';\mathbf{z}) = (\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{\frac{\lambda}{\zeta}}(\alpha_H m_H + \alpha_L m_L)g\mathbf{z}; \tag{38}$$

$$\int\int K(m,z';\mathbf{z})dF(m,z';\mathbf{z}) = (\frac{\eta}{\tilde{r}})^{1+\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{\frac{\lambda}{\zeta}}(\alpha_H m_H + \alpha_L m_L)g\mathbf{z}, \tag{39}$$

where w grows at the rate $g^{\frac{\zeta}{\zeta+\lambda}}$, $\mathbf{z}$ grows at the rate, g, and all the other parameters are fixed.

A firm with production ability m and an innovation level z at the beginning of the period may or may not update its innovation level through R&D or trade. If it updates the innovation level, the profit of the current period is

$$\pi(m,z';\mathbf{z}) = \zeta m(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{\frac{\lambda}{\zeta}}(z + \gamma\mathbf{z}). \tag{40}$$

Otherwise, the profit is

$$\pi(m,z;\mathbf{z}) = \zeta m(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta}}(\frac{\lambda}{w})^{\frac{\lambda}{\zeta}}z. \tag{41}$$

Denote $\tilde{z} = \frac{z}{\mathbf{z}^{\frac{\lambda}{\zeta+\lambda}}}$, $\tilde{\mathbf{z}} = \frac{\mathbf{z}}{\mathbf{z}^{\frac{\lambda}{\zeta+\lambda}}}$. Plugging the expression of w in (37) into (40) and (41) derives

$$\pi(m,z';\mathbf{z}) = Am(\tilde{z} + \gamma\tilde{\mathbf{z}}), \quad \pi(m,z;\mathbf{z}) = Am\tilde{z}, \tag{42}$$

where $A = \zeta(\frac{\eta}{\bar{r}})^{\frac{\eta}{\zeta+\lambda}}[(\alpha_H m_H + \alpha_L m_L)g]^{-\frac{\lambda}{\zeta+\lambda}}$. So, the difference of firm profit with the updated and non-updated innovation levels is $Am\gamma\tilde{\mathbf{z}}$, which is not a function of the firm's current innovation level, z.

Next, a guess-and-verify procedure is used to derive the value of the firm at the beginning of the period, $V(m,z;\mathbf{z})$. Conjecture

$$V(m,z;\mathbf{z}) = v_1(m)\tilde{z} + v_2(m)\tilde{\mathbf{z}}. \tag{43}$$

Then, the surplus of the firm if being a buyer in the Nash bargaining problem (13) is

$$[\pi(m,z';\mathbf{z}) + r\mathbb{E}V(m',z';\mathbf{z}')] - [\pi(m,z;\mathbf{z}) + r\mathbb{E}V(m',z;\mathbf{z}')] = [Am + r\mathbb{E}(v_1(m'))g^{-\frac{\lambda}{\zeta+\lambda}}]\gamma\tilde{\mathbf{z}}, \tag{44}$$

which is not a function of the firm's innovation level, z, either. Denote this surplus as $\Delta\psi(m;\mathbf{z})$ and use $B(m)$ as an abbreviation for $[Am + r\mathbb{E}(v_1(m'))g^{-\frac{\lambda}{\zeta+\lambda}}]$. We have

$$\Delta\psi(m;\mathbf{z}) = B(m)\gamma\tilde{\mathbf{z}}. \tag{45}$$

The price this firm has to pay if buying a patent from others can be expressed as (Point 8)

$$p_b(m;\mathbf{z}) = \theta\Delta\psi(m;\mathbf{z}) = \theta B(m)\gamma\tilde{\mathbf{z}}, \tag{46}$$

i.e., the buying price is the bargaining power of the seller times the trading surplus of the buyer. It only depends on the production ability of the buyer and the aggregate innovation level. The expected price this firm gets if selling a patent on the market depends on the shares of searching effort from high-type buyers and low-type buyers. Since we focus on a symmetric equilibrium, the shares are constants on any arc of the technology circle, i.e.,

$$\frac{n_{bH}(d)}{n_b(d)} = \frac{n_{bH}}{n_b}, \quad \forall d, \tag{47}$$

where $\frac{n_{bH}}{n_b}$ and $\frac{n_{bL}}{n_b}$ are the share of potential buyers with high and low production ability. The expected selling price can be expressed as

$$p_s = \theta \int \int \Delta \psi(m; \mathbf{z}) dG(m, z; \mathbf{z}) = [\frac{n_{bH}}{n_b} B(m_H) + \frac{n_{bL}}{n_b} B(m_L)] \theta \gamma \tilde{z}. \qquad (48)$$

To solve firms' optimal innovation intensity, it is necessary to derive the expressions of s and $b(\omega)$ in problem (12). Consider any arc on the circle. Without loss of generality, Figure 10 shows an arc d with length $|d|$. The total search effort by potential sellers on d equals to the number of potential sellers that have a patent located inside d. On a symmetric balanced growth path, sellers' patents are evenly distributed on the circle. So, $n_s(d) = |d|n_s$.
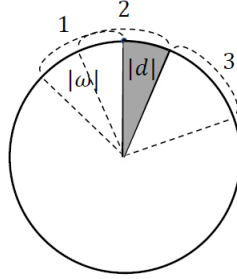

Figure 10: Schematic Diagram

Potential buyers that spend effort searching on d may have various scope. I classify these buyers according to the length of their scope. For potential buyers with scope length equal to $|\omega|$, their locations may span from 1 to 3. Buyers at location 1 or 3 spend measure 0 of search effort on d, while buyers at location 2 spend measure $\frac{|d|}{|\omega|}$ of search effort on d. The total measure of search effort on d conditional on the buyer having $|\omega|$ as the scope length is an integral of effort from location 1 to 3, which can be expressed as

$$\int_0^{|d|} \frac{i}{|\omega|} di + \int_{|d|}^{|\omega|} \frac{|d|}{|\omega|} di + \int_{|\omega|}^{|d|+|\omega|} \frac{|d| + |\omega| - i}{|\omega|} di = |d|, \ \forall |\omega|, |d|. \qquad (49)$$

This conditional measure does not rely on the scope length. So, the unconditional total measure of search effort on d is d times the total number of potential buyers, i.e., $n_b(d) = |d|n_b$.

The number of matches on the arc d equals to

$$M(s(d), b(d)) = |d|\phi n_s^\nu n_b^{1-\nu}, \forall d. \qquad (50)$$

Potential buyers with scope ω will only search within its scope, so, the probability of meeting a seller is

$$b(\omega) = \frac{M(n_s(\omega), n_b(\omega))}{n_b(\omega)} = \phi\left(\frac{n_s}{n_b}\right)^\nu \equiv b, \tag{51}$$

which is a constant and does not depend on the scope of the buyer (Point 3). The probability for a potential seller to meet a buyer is

$$s = \lim_{|d_0| \to 0} \frac{M(n_s(d_0), n_b(d_0))}{n_d(d_0)} = \phi\left(\frac{n_b}{n_s}\right)^{1-\nu}, \tag{52}$$

which is also a constant. Plugging the matching probabilities $b(\omega)$ and s into problem (12) derives the solution of firms' R&D success rate.

$$i^*(\omega, m) = \left\{ \frac{\gamma}{(1-\sigma)\chi} [X(\omega)(1 - (1-\theta)b)B(m) + (1 - X(\omega))s\theta(\sigma_H B(m_H) + \sigma_L B(m_L))] \right\}^{\frac{1}{\rho}}, \tag{53}$$

which only depends on the firm's production scope and production ability (Point 5).

The firm's value at the innovation stage, $D(\omega, m, z; \mathbf{z})$ is then

$$D(\omega, m, z; \mathbf{z}) = B(m)\tilde{z} + \left[\frac{(1-\sigma)\rho}{1+\rho}\chi i^*(\omega, m)^{1+\rho} + b(1-\theta)B(m)\gamma + r\mathbb{E}v_2(m')g^{\frac{\zeta}{\zeta+\lambda}}\right]\tilde{\mathbf{z}}. \tag{54}$$

$D(\omega, m, z; \mathbf{z})$ is larger when ω is closer to the center for any given length of ω if the following condition is fulfilled

$$(1 - (1-\theta)b)B(m) - s\theta(\sigma_H B(m_H) + \sigma_L B(m_L)) > 0, \tag{55}$$

i.e., the value of a within-scope patent is larger than an out-of-scope patent.[32] firms will always choose to span symmetrically around its center. The length of the firm's production scope ($|\omega|$) is determined by problem (15),

$$i^*(\omega, m)X'(|\omega|)[(1 - (1-\theta)b)B(m) - s\theta(\sigma_H B(m_H) + \sigma_L B(m_L))]\gamma = \mu|\omega|^\iota. \tag{56}$$

The solution to equation (56) is just a function of m, i.e, $|\omega^*(m, z; \mathbf{z})| = \Omega(m)$ (Point 4).

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[32]This condition is easily satisfied. The calibrated model confirms this condition holds true.

The number of buyers of each type, (n_{bH}, n_{bL}), are the share of firms in each type that do not get an innovation output matching their production scope. The total number of buyers is the summation of the buyers of the two types. They can be expressed as (Point 7)

$$n_{bH} = \alpha_H(1 - i^*(\omega^*(m_H), m_H)X(\omega^*(m_H))); \tag{57}$$

$$n_{bL} = \alpha_H(1 - i^*(\omega^*(m_L), m_L)X(\omega^*(m_L))); \tag{58}$$

$$b_b = n_{bH} + n_{bL}. \tag{59}$$

The number of sellers are the share of firms that successfully innovate, but the output falls outside of their own production scope

$$n_s = \alpha_H i^*(\omega^*(m_H), m_H)(1 - X(\omega^*(m_H))) + \alpha_L i^*(\omega^*(m_L), m_L)(1 - X(\omega^*(m_L))). \tag{60}$$

The value of the firm at the beginning of the period, $V(m, z; \mathbf{z})$, can be expressed as

$$V(m, z; \mathbf{z}) = D(\Omega(m), m, z; \mathbf{z}) - C^e(\omega; \mathbf{z}) \equiv v_1(m)\tilde{z} + v_2(m)\tilde{z}, \tag{61}$$

where

$$v_1(m) = B(m); \tag{62}$$

$$v_2(m) = [\frac{\rho}{1 + \rho}\chi i^*(\Omega(m), m)^{1+\rho} + b(1 - \theta)B(m)\gamma + r\mathbb{E}v_2(m')g^{\frac{\zeta}{\zeta + \lambda}} - \frac{\mu|\Omega(m)|^{1+\iota}}{1 + \iota}]. \tag{63}$$

Since both $v_1(m)$ and $v_2(m)$ are only functions of m, the value function, $V(m, z; \mathbf{z})$, is consistent with the conjecture (Point 6).

The representative household's problem can be expressed as

$$W(a; \mathbf{z}) = \max_{c, a'} u(c) + \beta W(a'; \mathbf{z})$$

$$s.t., c + a' = \frac{1}{r}a + \Pi,$$

where a is the asset holding of the household in the current period; $\frac{1}{r}$ is the capital return rate, where its relationship with the capital cost, $\tilde{r}$, is $\tilde{r} = \frac{1}{r} - 1 + \delta$; Π is the total profit of firms in this economy. Because all firms are owned by the household, the total profit is a part of the household's income. Solving the problem derives the following relationship

on consumption across periods,

$$\frac{c'}{c} = \left(\frac{\beta}{r}\right)^{\frac{1}{\epsilon}}. \tag{64}$$

Since consumption grows at the same rate, $g^{\frac{\zeta}{\zeta+\lambda}}$, as the total output, and the interest rate is fixed over time, we have (Point 2)

$$r = \frac{\beta}{g^{\epsilon\zeta/(\zeta+\lambda)}}. \tag{65}$$

The growth rate of the employment-weighted-average innovation level of the economy, g, can be expressed by the following equation according to the definition,

$$g \equiv \frac{\alpha_H m_H \mathbf{z_H}' + \alpha_L m_L \mathbf{z_L}'}{\alpha_H m_H \mathbf{z_H} + \alpha_L m_L \mathbf{z_L}}. \tag{66}$$

In the balanced growth path equilibrium, the ratio of the innovation level of firms with high production ability to that of the firms with low production ability should be stable across periods, i.e.,

$$\frac{\mathbf{z_H}'}{\mathbf{z_L}'} = \frac{\mathbf{z_H}}{\mathbf{z_L}} \equiv o, \tag{67}$$

where o is a constant. Then (66) implies that

$$g = \frac{\mathbf{z_H}'}{\mathbf{z_H}} = \frac{\mathbf{z_L}'}{\mathbf{z_L}}. \tag{68}$$

Equations in (68) show that the growth rate in the innovation level of the aggregate economy is the same as the growth rate of firms across types.

The change in the average innovation levels of high- and low-type firms consists of two components.
1) There is a reshuffling of firms at the beginning of each period because of the transition of production ability.
2) Firms update their innovation level through R&D or trade of patents.

The average innovation level of each type of firms after the transition of production

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ability but before the innovation stage in this period can be expressed as follows,

$$z_{Hr} \equiv \frac{\alpha_H q_{HH} z_H + \alpha_L q_{LH} z_L}{\alpha_H q_{HH} + \alpha_L q_{LH}};\tag{69}$$

$$z_{Lr} \equiv \frac{\alpha_L q_{LL} z_L + \alpha_H q_{HL} z_H}{\alpha_L q_{LL} + \alpha_H q_{HL}}.\tag{70}$$

Firms update their innovation level in the R&D or trading process following the law of motion described in (1). So, the growth rate of each type of firms in this process (denoted as g_H and g_L) depends on the share of them that successfully create an intellectual output that matches their scope and the share that successfully buy a patent on the market.

$$g_H \equiv \frac{z_H{}'}{z_{Hr}} = 1 + [i^*(\omega^*(m_H), m_H) X(\omega^*(m_H)) + (1 - i^*(\omega^*(m_H), m_H) X(\omega^*(m_H)) m_b)] \gamma \frac{z}{z_{Hr}};\tag{71}$$

$$g_L \equiv \frac{z_L{}'}{z_{Lr}} = 1 + [i^*(\omega^*(m_L), m_L) X(\omega^*(m_L)) + (1 - i^*(\omega^*(m_L), m_L) X(\omega^*(m_L)) m_b)] \gamma \frac{z}{z_{Lr}}.\tag{72}$$

Using the relationship $z_H{}' = g_H z_{Hr}$ and plugging equations (67), (69), (70), (71) and (72) into the first equation in (68) derives the solutions for g and o through the following system of equations,

$$g = \frac{g_H(\alpha_H q_{HH} + \alpha_L q_{LH} \frac{1}{o})}{\alpha_H q_{HH} + \alpha_L q_{LH}};\tag{73}$$

$$o = \frac{g_H(\alpha_H q_{HH} o + \alpha_L q_{LH})}{\alpha_H q_{HH} + \alpha_L q_{LH}} \frac{\alpha_L q_{LL} + \alpha_H q_{HL}}{g_L(\alpha_L q_{LL} + \alpha_H q_{HL} o)}.\tag{74}$$

Since all of the other variables and parameters are fixed in the equation system, the solutions of g and o are indeed both constants (Point 1).

9.4   Characterization of Proposition 6.1

There exists a symmetric balanced growth path of the form
1. The employment-weighted growth rate of the aggregate productivity defined by,

$$g = \frac{\int \int m'z''dF(m',z')/\int \int m'dF(m',z')}{\int \int mz'dF(m,z)/\int \int mdF(m,z)},\tag{75}$$

is a constant.

2. The interest factor $r = \beta / g^{\epsilon \zeta / (\zeta + \lambda)}$; the rental rate on capital $\tilde{r} = g^{\epsilon \zeta / (\zeta + \lambda)} / \beta - 1 + \delta$.

3. The odds of a successful match for a potential buyer, $b^j(\omega)$, and for a potential seller, s^j, on the market of each type (basic or applied) of patents, only depend on the total number of patent buyers and sellers on that market, i.e., $b^j(\omega) = \phi(\frac{n_s^j}{n_b^j})^\nu$, $s^j = \phi(\frac{n_b^j}{n_s^j})^{1-\nu}$, where $j \in \{a, b\}$.

4. The production scope of each firm spans symmetrically around the center, and the length of the scope depends only on the production ability of the firm, i.e., $|\omega(m, z; \mathbf{z})| = \Omega(m)$.

5. The success rates of applied and basic research do not depend on the firm's innovation level, z, or the economy-wide innovation level, $\mathbf{z}$, i.e., $i^j(\omega, m, z; \mathbf{z}) = i^j(\omega, m)$, $j \in \{a, b\}$.

6. The value function $V(m, z; \mathbf{z})$ is linear in $\tilde{z}$ and $\tilde{\mathbf{z}}$, i.e., $V(m, z; \mathbf{z}) = v_1(m)\tilde{z} + v_2(m)\tilde{\mathbf{z}}$, where $\tilde{z} = z / \mathbf{z}^{\lambda / (\zeta + \lambda)}$, $\tilde{\mathbf{z}} = \mathbf{z}^{\zeta / (\zeta + \lambda)}$.

7. The number of buyers of both types (n_{bH}^j, n_{bL}^j) and the number of sellers (n_s^j) for j $(j \in \{a, b\})$ type of patents are

$$n_{bH}^j = \alpha_H (1 - i^{j*}(\omega^*(m_H), m_H) X^j(\omega^*(m_H))), \ n_{bL}^j = \alpha_H (1 - i^{j*}(\omega^*(m_L), m_L) X^j(\omega^*(m_L))); \tag{76}$$

$$n_s^j = \alpha_H i^{j*}(\omega^*(m_H), m_H)(1 - X^j(\omega^*(m_H))) + \alpha_L i^{j*}(\omega^*(m_L), m_L)(1 - X^j(\omega^*(m_L))). \tag{77}$$

8. The buying price and the expected selling price of a j-type $(j \in \{a, b\})$ patent is

$$p_b^j(m, z; \mathbf{z}) = \theta \left(Am + \frac{r}{g^{\lambda / (\lambda + \zeta)}} \mathbb{E}[v_1(m') | m] \right) \gamma^j \tilde{\mathbf{z}}; \tag{78}$$

$$p_s(\mathbf{z}) = \frac{n_{bH}^j}{n_b^j} p_b^j(m_H, z; \mathbf{z}) + \frac{n_{bL}^j}{n_b^j} p_b^j(m_L, z; \mathbf{z}), \tag{79}$$

where A is a constant.

9.5 Proof of Proposition 6.1

The proof is very similar to that of Proposition 4.1. One difference is that the profit of each type of firms now have four possible cases. 1) The firm gets both applied and basic R&D output (either through own innovation or purchasing them from the market). The profit in this case is $\pi(m, z^{ab}; \mathbf{z}) = Am(\tilde{z} + \gamma^a \tilde{\mathbf{z}} + \gamma^b \tilde{\mathbf{z}})$. 2) The firm gets only applied R&D

output. The profit is $\pi(m, z^a; \mathbf{z}) = Am(\tilde{z} + \gamma^a \tilde{\mathbf{z}})$. 3) The firm gets only basic R&D output. The profit is $\pi(m, z^b; \mathbf{z}) = Am(\tilde{z} + \gamma^b \tilde{\mathbf{z}})$. (4). The firm gets neither R&D output. The profit is $\pi(m, z; \mathbf{z}) = Am(\tilde{z})$. $A = \zeta(\frac{\eta}{\tilde{r}})^{\frac{\eta}{\zeta + \lambda}} [(\alpha_H m_H + \alpha_L m_L)g]^{-\frac{\lambda}{\zeta + \lambda}}$. for all the four cases.

Then, from the Nash bargaining problem between the buyer and the seller, it can be derived that for a j-type patent ($j \in \{a, b\}$), the buying price can be expressed as

$$p_b^j(m; \mathbf{z}) = \theta B(m) \gamma^j \tilde{\mathbf{z}}, \tag{80}$$

where $B(m) = [Am + r\mathbb{E}(v_1(m'))g^{-\frac{\lambda}{\zeta + \lambda}}]$. The selling price is then

$$p_s^j = [\frac{n_{bH}^j}{n_b^j} B(m_H) + \frac{n_{bL}^j}{n_b^j} B(m_L)] \theta \gamma^j \tilde{\mathbf{z}}. \tag{81}$$

The optimal success rate of j-type R&D ($j \in \{a, b\}$)

$$i^{j*}(\omega, m) = \{\frac{\gamma^j}{(1-\sigma)\chi^j} [X^j(\omega)(1 - (1-\theta)b^j)B(m) + (1 - X^j(\omega))s^j\theta(\sigma_H^j B(m_H) + \sigma_L^j B(m_L))]\}^{\frac{1}{\rho}}, \tag{82}$$

which also only depends on the firm's production scope and production ability.

The length of the firm's production scope is determined by the following equation,

$$\sum_{j \in \{a, b\}} i^{j*}(\omega, m)X^{j'}(|\omega|)[(1 - (1-\theta)b^j)B(m) - s^j\theta(\sigma_H^j B(m_H) + \sigma_L^j B(m_L))]\gamma^j = \mu|\omega|^\iota. \tag{83}$$

The solution to the equation above is still just a function of m.

The growth rates of each type of firms in the R&D and search and matching stages are respectively

$$g_H \equiv \frac{\mathbf{z_H}'}{\mathbf{z_{Hr}}} = 1 + \sum_{j \in \{a, b\}} [i^{j*}(\omega^*(m_H), m_H)X^j(\omega^*(m_H)) + (1 - i^{j*}(\omega^*(m_H), m_H)X^j(\omega^*(m_H))m_b^j)]\gamma^j \frac{\mathbf{z}}{\mathbf{z_{Hr}}}; \tag{84}$$

$$g_L \equiv \frac{\mathbf{z_L}'}{\mathbf{z_{Lr}}} = 1 + \sum_{j \in \{a, b\}} [i^{j*}(\omega^*(m_L), m_L)X^j(\omega^*(m_L)) + (1 - i^{j*}(\omega^*(m_L), m_L)X^j(\omega^*(m_L))m_b^j)]\gamma^j \frac{\mathbf{z}}{\mathbf{z_{Lr}}}. \tag{85}$$

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Still, the growth rate in the social innovation level and the ratio of the innovation levels between high- and low-type firms are constants and equal to

$$g = \frac{g_H(\alpha_H q_{HH} + \alpha_L q_{LH} \frac{1}{o})}{\alpha_H q_{HH} + \alpha_L q_{LH}}; \tag{86}$$

$$o = \frac{g_H(\alpha_H q_{HH} o + \alpha_L q_{LH})}{\alpha_H q_{HH} + \alpha_L q_{LH}} \frac{\alpha_L q_{LL} + \alpha_H q_{HL}}{g_L(\alpha_L q_{LL} + \alpha_H q_{HL} o)}. \tag{87}$$

9.6 Estimation of the Matching Elasticity

This section displays the estimation results of the elasticity in the matching function of the patent trading market. The first three columns use raw numbers, while the last three columns use patent citation-weighted numbers. The numbers are summed at the 6-digit NAICS code level in columns (1) and (4); at the 4-digit NAICS code level in columns (2) and (5); at the 2-digit NAICS code level in columns (3) and (6). In all columns, the summation of the two coefficients is close to 1, suggesting that the matching function is close to being constant-return-to-scale. The coefficient of the number of sellers, which corresponds to the matching elasticity(ν), is in the range of 0.598-0.821. The calibration then sets the value of ν as 0.70.

	Ln(Number of Matches)					
	(1)	(2)	(3)	(4)	(5)	(6)
	Raw			Citation-Weighted		
Ln(Num. of Sellers)	0.598***	0.693***	0.780***	0.604***	0.694***	0.821***
	(0.006)	(0.012)	(0.049)	(0.006)	(0.012)	(0.050)
Ln(Num. of Buyers)	0.0713***	0.105***	0.291***	0.0698***	0.102***	0.222**
	(0.008)	(0.018)	(0.089)	(0.008)	(0.018)	(0.090)
Observations	20000	5700	500	20000	5700	500
R-squared	0.873	0.936	0.984	0.871	0.935	0.983

*** p<0.01, ** p<0.05, * p<0.1.

Table 15: Estimation of the Elasticity in the Matching Function

9.7 Calibration of the Extended Model

This section describes the calibration process and performance of the extended model. The step size and the cost function parameters of basic and applied research, $\{\gamma^j, \chi^j, \rho^j\}$,

and the two probability functions, $X^j(.)$ ($j \in \{a, b\}$), are calibrated together with other parameters in the baseline model. Table 16 shows the results. The step size ratio of the two types of research adopts the value in Akcigit et al. (2021); the scale parameter of applied research cost is normalized to be 1. The within-scope probability function for basic and applied research are estimated by the method described in the baseline model respectively on a sample of patents from basic research and a sample of patents from applied research or development. The estimated functions suggest that when the industry number of a firms is not too large, it is harder for basic research output to match the firm's production compared to applied research. The step size of applied research, γ_a, the scale of basic research cost function, χ_b, and the elasticity of the cost function of the two types of research, $1 + \rho_a$ and $1 + \rho_b$, are pinned down by matching the moments of the model with the moments of data in the initial balanced growth path (1981-1985), together with the parameters of the management cost function, the matching efficiency, and sellers' bargaining power, $\{\mu, \iota, \phi, \theta\}$. In the calibration, the annual growth rate is mostly affected by γ_a; the basic research share and R&D cost to domestic sales ratio of firms with high and low production ability are mostly governed by χ_b, $1 + \rho_a$, and $1 + \rho_b$.

Parameter	Description	Value	Identification
Priori Information			
$\frac{\gamma_b}{\gamma_a}$	Step size ratio	1.6	Akcigit et al. (2021)
χ_a	Applied R cost, scale	1	Normalization
Estimation			
$X^a(\omega)$	Applied R, within-scope Prob.	$e^{-3.837} * |\omega|^{0.602}$	Regression
$X^b(\omega)$	Basic R, within-scope Prob.	$e^{-4.944} * |\omega|^{0.932}$	Regression
Model			
γ_a	Applied R step size	1.46	Growth rate
χ_b	Basic R cost, scale	5.33	Basic research share,
$1 + \rho_a$	Applied R cost, elasticity	1.90	R&D cost/sales
$1 + \rho_b$	Basic R cost, elasticity	1.29	ratio (H and L)

Table 16: Parameter Values of the Extended Model

The extended model is then recalibrated to the ending balanced growth path. In this process, parameters corresponding to the four mechanisms, $\{\phi, \theta, \sigma, \mu, \iota, \gamma_a, \chi_b, \rho_a, \rho_b\}$, are changed to match the data moments in 1996-2000.

The model fit of the two balanced growth paths are shown respectively in Table 17

and Table 18. Overall, the model matches the data well.

Targets	Data	Model
Economic growth rate(1981-1985)	3.05%	3.05%
R&D cost/sales of H firms(1981-1985)	3.62%	3.62%
R&D cost/sales of L firms(1981-1985)	2.83%	2.83%
Basic R Share of H firms(1981-1985)	4.20%	4.20%
Basic R Share of L firms(1981-1985)	3.73%	3.73%
Avg. number of industries of H firms(1981-1985)	11.81	11.81
Avg. number of industries of L firms(1981-1985)	1.92	1.92
The share of patents transacted(1981-1985)	30.9%	30.9%

Table 17: Model Fit for Key Moments in the Initial Balanced Growth Path

Targets	Data	Model
Economic growth rate(1996-2000)	3.34%	3.34%
R&D cost/sales of H firms(1996-2000)	3.15%	3.15%
R&D cost/sales of L firms(1996-2000)	6.71%	6.71%
Basic R Share of H firms(1996-2000)	4.61%	4.61%
Basic R Share of L firms(1996-2000)	11.46%	11.46%
Avg. number of industries of H firms(1996-2000)	6.31	6.31
Avg. number of industries of L firms(1996-2000)	1.61	1.61
The share of patents transacted(1996-2000)	44.1%	44.1%

Table 18: Model Fit for Key Moments in the Ending Balanced Growth Path

9.8 Placebo Tests–Results

This section presents the regression results of the placebo tests described in Section 7.5. The goal is to check whether there are pre-trends in firms' production scope. The new dummy that segments the pre-CAFC sample is defined as $post2$. Table 19 is corresponding to the DiD regression in Equation 29 running on the pre-CAFC sample. The first two columns insert the $post2$ dummy in the regression instead of the year-fixed effects; the last two columns control the year-fixed effects. Columns (2) and (4) control the state-level characteristics compared to columns (1) and (3). The coefficient of the interaction term is either positive or very small in absolute magnitude. None of the coefficients is statistically significant. Table 20 is corresponding to the DDD regression in Equation 30 running on the pre-CAFC sample. The state and time controls in each column are the same as Table 19. Both the coefficients of the triple interaction and the interaction between the invalida-

tion rate and the *post*2 dummy are insignificant, confirming there is no pre-existing trend.

Dependent Variable	Ln(Number of Industries)			
	(1)	(2)	(3)	(4)
Invalidation Rate*Post2	0.00196	0.0206	0.000678	-0.00194
	(0.013)	(0.014)	(0.013)	(0.012)
Ln(Employment)	0.0539***	0.0529***	0.0526***	0.0527***
	(0.003)	(0.003)	(0.003)	(0.003)
Post2	0.0134*	-0.0690*		
	(0.007)	(0.040)		
Ln(Real GDP)		0.0321		-0.0194
		(0.020)		(0.023)
Effective Federal Tax Rate		-3.567*		-2.193
		(1.918)		(1.504)
Effective State Tax Rate		-1.747*		-1.156
		(0.914)		(0.716)
Federal R&D Tax Credits		0.665***		-0.219
		(0.133)		(1.616)
State R&D Tax Credits		-0.276**		-0.298***
		(0.121)		(0.083)
Year-fixed Effects	NO	NO	YES	YES
Firm-fixed Effects	YES	YES	YES	YES
Observations	131000	131000	131000	131000
R-squared	0.97	0.97	0.97	0.97

Standard errors are clustered in the circuit court region by the post dummy level, shown in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 19: Placebo Test-DiD Regression Results

Dependent Variable	Ln(Number of Industries)			
	(1)	(2)	(3)	(4)
High_treat*Invalidation Rate*Post2	0.00965	0.00894	0.011	0.0121
	(0.038)	(0.038)	(0.038)	(0.039)
Invalidation Rate*Post2	0.00177	0.0204	0.000479	-0.00213
	(0.012)	(0.014)	(0.012)	(0.012)
High_treat*Post2	-0.0094	-0.00877	-0.00935	-0.00908
	(0.020)	(0.020)	(0.020)	(0.020)
Ln(Employment)	0.0539***	0.0530***	0.0526***	0.0527***
	(0.003)	(0.003)	(0.003)	(0.003)
Post2	0.0136*	-0.0686		
	(0.007)	(0.040)		
Ln(Real GDP)		0.0321		-0.0193
		(0.020)		(0.023)
Effective Federal Tax Rate		-3.555*		-2.186
		(1.915)		(1.506)
Effective State Tax Rate		-1.741*		-1.152
		(0.912)		(0.716)
Federal R&D Tax Credits		0.665***		-0.206
		(0.133)		(1.615)
State R&D Tax Credits		-0.276**		-0.298***
		(0.121)		(0.083)
Year-fixed Effects	NO	NO	YES	YES
Firm-fixed Effects	YES	YES	YES	YES
Observations	131000	131000	131000	131000
R-squared	0.97	0.97	0.97	0.97

Standard errors are clustered in the circuit court region by the post dummy level, shown in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 20: Placebo Test-DDD Regression Results