Technology Driven Market Concentration through Idea Allocation*

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Abstract

Using a newly-created measure of technology novelty, this paper identifies periods with and without technology breakthroughs from the 1980s to the 2020s in the US. It is found that market concentration decreases at the advent of revolutionary technologies. We establish a theory addressing inventors' decisions to establish new firms or join incumbents of selected sizes, yielding two key predictions: (1) A higher share of inventors opt for new firms during periods of heightened technology novelty. (2). There is positive assortative matching between idea quality and firm size if inventors join incumbents. Both predictions align with empirical findings and collectively contribute to a reduction in market concentration when groundbreaking technologies occur. Quantitative analysis shows the overall slowdown in technological breakthroughs can capture 47.4% of the rising trend in market concentration and the correlation between the model-generated and the actual detrended market concentration is 0.932.

Keywords: technological waves, HHI, startups, incumbent firms.

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1 Introduction

The interplay between technological progress and market concentration plays a significant role in economic growth and resource allocation. Most of the existing studies focus on the impact of the firm size distribution on technology evolvement (e.g., Akcigit and Kerr (2018); Cunningham, Ederer and Ma (2021); Akcigit and Goldschlag (2023)), this paper provides empirical evidence and structural analysis showing that the reverse relationship is also important—technological novelty waves affects the market concentration by relocating innovative ideas between incumbent firms and new businesses.

Using a newly-created measure of the novelty of new technologies, this paper identifies periods in the US when technology breakthroughs occur and periods when most new technologies follow existing ones from the 1980s to the 2020s. There is a declining trend in technological novelty. Besides, technological novelty follows waves. At the peak, groundbreaking technologies replace existing ones, while at the trough, most technologies have entered a mature stage.

Surprisingly, we find a rising trend and a cyclical pattern in market concentration, as measured by the Herfindahl-Hirschman Index (HHI) of firm sales, which exhibits a notable negative correlation with the technological novelty waves. This observation strongly suggests that the emergence and maturation of novel technologies may be influential factors in shaping market concentration dynamics.

How are technological waves and market concentration connected? A potential channel is through the allocation of ideas. Since firm size is to a large extent impacted by firm productivity and new ideas are important sources of productivity growth, where new ideas contribute their value will determine the firm size distribution, and therefore, market concentration. Combining the Longitudinal Business Database (LBD) from the Census Bureau and the patent information from the USPTO, this paper tracks the affiliation of patents at their formation. It is shown that at the peaks of the technological waves, a larger share of patents are forming in new businesses, while at the troughs, a larger share of patents come from incumbent firms. Besides, among patents from incumbent firms, there is a positive relationship between patent citations, a quality measure of the ideas behind them, and the size of the firm. These patterns indicate that technological waves affect the number of firm entries and the way new ideas combine with firms of different sizes.

Further patent-level regression analysis reveals that incumbent firm size positively affects the private economic value of patents, given their scientific value, indicating synergy between inventors and incumbent firms. The scientific value of patents has a positive impact on the economic value, while this impact decreases in the aggregate technological novelty.

Based on the empirical findings, this paper proposes a theory about inventors' choice of where to contribute the value of their ideas, and how it connects the technological waves and market concentration. The technological novelty level is assumed to be a random aggregate shock capturing the random arrival of ground-breaking innovations in a period. Each inventor is endowed with an idea of idiosyncratic quality. The inventor needs to choose between forming a new firm of a random size with a partner or joining an incumbent firm. In the case of the latter, she must also decide on the size of the incumbent firm to join. It is frictional for an incumbent firm to adopt new technology due to creative destruction as in Greenwood and Yorukoglu (1997), and the friction decreases when technologies enter a mature stage. Hence, higher aggregate technological novelty leads more inventors to form startups. Inventors' decisions directly impact firm-level innovation intensity, technology improvement, and hence the firm size distribution. Simulation of the calibrated model shows that the evolvement of the market concentration generated by the technological waves captures 47.4% of the actual rising trend and has a correlation of 0.932 with the actual detrended fluctuations.

The model in this paper includes three key elements: novelty-related adoption cost, commercialization synergy, and inventor-firm contracts. The novelty-related adoption cost refers to the learning friction encountered by incumbent firms when integrating new ideas, preventing these ideas from reaching their full value. This friction is more pronounced during technological wave peaks, making startups more attractive to inventors due to the absence of such frictions. Commercialization synergy pertains to the added value incumbent firms can provide through their production and commercialization capacities, which startups typically lack. Larger firms offer more synergy, especially to high-quality ideas. Inventor-firm contracts define the collaboration between inventors and firms, ultimately determining idea allocation. The R&D process is risky and success depends on the inventors' unobservable effort, necessitating contracts that incentivize optimal effort through a mix of equity and wages. Larger firms face greater incentive challenges due to larger unrelated shocks, making equity a weaker incentive for R&D efforts.

Inventors must consider adoption friction, synergy, and contract terms when choosing between startups and incumbent firms. Startups, while free from adoption friction and offering aligned incentives, lack the capacity to generate synergy. In contrast, incumbents face adoption friction, with larger firms providing weaker incentives but better synergy. These trade-offs guide inventors in their strategic decision-making regarding firm affiliation.

The model has two major predictions. First, a larger share of inventors choose to start new firms to develop their ideas during periods of high technological novelty since the learning friction at incumbent firms is larger. Second, among inventors that choose to do R&D in incumbent firms, there is positive assortative matching between idea quality and firm size. Therefore, firms already with a larger size attract ideas of higher value. These two predictions are consistent with observations in data and collectively contribute to a reduction in market concentration when the economy is closer to the peak of the technological waves. The upsurge in new startups leads to a proliferation of firms in the market. Given that new startups are less constrained by the positive matching between idea quality and firm size, they offer a counterbalance to the tendency of larger firms to further expand.

To quantify the impact of the technological novelty waves on market concentration through allocation of new ideas, we calibrate the model and then do simulations by changing the degree of novelty of new technologies in an economy. The model is calibrated to match the average data moments between 1982 and 2016. Key moments include patent novelty, average patent value, degree of positive matching between patent citation and firm size, the growth rates, etc. In the simulation exercise, we fix all the parameters except for the one related to patent novelty for each year following 1986, the first peak of the technological waves within our sample period. This variation serves to capture the evolving dynamics of learning friction within incumbent firms. Consequently, we generate paths of two essential data moments: (1) the ratio of the number of ideas in new firms relative to those in incumbent firms; (2) the HHI of firm sales. The two paths are compared with the data.

The two generated paths of moments are consistent with the actual trend in general and nearly have simultaneous peaks and troughs with the actual time variations. In particular, the model-generated HHI captures 47.4% of the actual rising trend. The correlation between the detrended model-generated and the detrended actual HHI is 0.932; the correlation between the detrended model-generated and the detrended actual new-to-incumbent ratio is 0.810. These comparisons indicate that the technology waves is a strong driving force of idea allocation and market concentration.

To decompose the effect of the two channels, firm number changes (extensive margin) and the positive assortative matching (intensive margin) between idea quality and firm size, on the evolvement of market concentration, we shut down the intensive margin in the simulation process. The decomposition shows that both margins contribute to the rising trend in HHI, with the extensive margin explaining 36.8% and the intensive margin explaining 10.5%; they lead to fluctuations in HHI in the same directions in response to the technological novelty waves; the intensive margin responds to the technological waves more quickly and reduces the overall response time of the HHI.

Related Literature

This paper is closely related to the literature on the interplay between innovation and market concentration. On the one hand, innovation leads to technological advancement that creates monopoly rents and larger firm size (Aghion and Howitt, 1990; Grossman and Helpman, 1991; Klette and Kortum, 2004). On the other hand, firms of different sizes are shown to have different innovation intensities in the literature, indicating that the overall innovation intensity depends on both the firm size distribution (Akcigit and Kerr, 2018) and the market for ideas (Eaton and Kortum, 1996; Silveira and Wright, 2010; Chatterjee and Rossi-Hansberg, 2012; Cabral, 2018; Perla, Tonetti and Waugh, 2021; Fons-Rosen, Roldan-Blanco and Schmitz, 2021). Theories and empirical evidence in this aspect can be traced back to the Schumpeterian argument that large firms have a higher capacity to do R&D, to more recent findings that small firms are more inclined to engage in innovation activities due to the rise of the patent market (Cassiman and Veugelers, 2006; Bena and Li, 2014; Akcigit, Celik and Greenwood, 2016; Liu and Ma, 2021; Ma, 2022; Yang, 2023). Most of the existing studies focus on the relationship between innovation efforts and market structure, while this paper finds novel patterns that the novelty of new technologies is closely correlated with the market concentration measure. To our knowledge, this is the first paper that uncovers the relationship between market concentration and the technological novelty waves.

Our empirical and theoretical analyses indicate that the degree of novelty associated with emerging technologies significantly influences where inventors choose to conduct their R&D. This perspective provides an alternative viewpoint on the relationship between the allocation of new ideas and market concentration. Existing research emphasizes the opposite relationship. Studies like Cunningham, Ederer and Ma (2021) and Akcigit and Goldschlag (2023) have posited that incumbent firms strategically acquire innovative startups or independent inventors only to subsequently abandon their ideas, thus preventing competition from new entrants and effectively stifling novel ideas. Therefore, the decrease in the novelty of new technologies is due to market concentration and the high monopoly power of incumbent firms. Our paper does not contradict these assertions. Instead, the analysis in this paper suggests that technological novelty and market concentration may have mutual effects and the mutual effects amplify each other in the negative correlation between the technological waves and market concentration.

This paper provides a new perspective on the causes of rising market concentration in the U.S. since the late 1990s. Notably, this increasing concentration has been accompanied by greater allocative efficiency and productivity growth (Autor et al. (2020) and Ganapati (2021)). A similar positive relationship has been observed in Europe (Bighelli et al. (2023)). However, literature also indicates that good ideas are becoming harder to find (Bloom et al. (2020)), the productivity gap between large and small firms is widening, and firm entry rates are declining (Akcigit and Ates (2023) and Olmstead-Rumsey (2019)). Our analysis offers an unexplored explanation for the rise in market concentration: the deceleration in the emergence of revolutionary technologies. This perspective reconciles the seemingly contradictory findings in the literature. The novelty metric for new technologies, defined in this paper, shows that the peaks of technological waves occurred in the mid-1980s and mid-1990s, with a significant 20-year gap before reemerging in the early 2010s. This extended period without significant technological breakthroughs led idea holders to gravitate toward incumbent firms, resulting in increased concentration among these larger firms and a decline in new firm entry. The lack of significant creative destruction also reduced the adoption frictions of technologies by incumbent firms, thereby enhancing short-term productivity growth as it took time for groundbreaking technologies to materialize their economic benefits.

Finally, our analysis delves into the implications of the introduction of groundbreaking technologies. Bowen III, Frésard and Hoberg (2023) show empirically that in an era with rapid evolving technologies, more startups remain independent rather than being sold out. Dinlersoz, Dogan and Zolas (2024) discover a surge in AI business applications after 2016. Greenwood and Yorukoglu (1997) and Greenwood and Jovanovic (1999) establish that technological revolutions lead to deterioration in the stock value of existing firms. The adoption of the novel technologies is costly and requires skilled labor, therefore, slowing down economic growth and widening income inequality in the short run. Jovanovic and Rousseau (2014) shows that at the advent of new technologies, incumbent firms decrease investment due to lack of compatibility while new firms increase investment. This paper extends the existing literature by investigating how a leap in technological progress affects the distribution of firm sizes, primarily due to the frictions when integrating inventors' novel ideas into incumbent firms. It is shown that market concentration is another important outcome of technological revolutions. This paper demonstrates that apart from the high-frequency business cycle influenced by productivity fluctuations (Kydland and Prescott (1982)), the economy may also be susceptible to a low-frequency cycle driven by the waves of technological novelty.

The rest of the paper is organized as follows. Section 2 introduces measures of the technological waves, market concentration, and the allocation of ideas, and subsequently presents their patterns. Section 3 constructs a model where inventors make decisions between initiating new ventures or joining established incumbents at specific sizes. We derive predictions concerning the mapping between the quality of inventors' ideas and their optimal choices. Section 4 defines the balanced growth path, the aggregate growth rate, and market concentration. Section 5 calibrates the model. Section 6 simulates the model to evaluate the degree to which technological waves can account for changes in market concentration through the idea allocation channel. Section 7 concludes.

2 Empirical Patterns

This section exhibits empirical patterns of the technological waves, market concentration, and a potential channel that links the two—the choices of the inventors on where to invent.

2.1 Technological Waves

Technology waves capture the extent of new technology breakthroughs over time. At the peak of the technological waves, significantly innovative new technology emerges and substitutes existing technologies; at the trough of the waves, most of the technologies in the economy have reached a mature state, and the extent of creative destruction of new technology over existing ones is smaller.

2.1.1 Measurement

To measure the technological waves, we create a "Novelty" Index of the new technologies in each year using the patent citation data. Specifically,

$$\text{Novelty}_t = \frac{\sum_{i \in I_t} \sum_{s=0}^{5} \text{Forward Citations}_{i,t+s}}{\sum_{i \in I_t} \sum_{s=0}^{5} \text{Forward Citations}_{i,t+s} + \sum_{i \in I_t} \sum_{s=0}^{5} \text{Backward Citations}_{i,t-s}}, \quad (1)$$

where I_t is the set of the new patents granted in year t. The numerator is a summation of the number of forward citations (citations by others) each new patent gets within the next five years. The denominator is a summation of the number of forward citations plus a summation of the number of backward citations (citation on others) each patent makes on other patents granted within the previous five years. The five-year window is to ensure every year in the sample is compared on the common ground, since more recent patents are more likely to receive fewer forward citations due to the right-censoring issue. The rationale for this measure is that groundbreaking innovations typically exhibit lower similarity to current technologies, but pave the way for subsequent patents to emulate them. Since the forward citations capture the overlap of future patents with the focal patent, while the backward citations capture the overlap of the focal patents with previous patents, the relative number of the former provides a measure of patent novelty. The "Novelty" index sums the number of forward and backward citations across all the patents granted in a year and is in the range between zero and one. A higher index indicates that the year witnesses significant breakthroughs in new technologies; a lower index indicates that most of the technologies have evolved into a mature stage in that year.

The data used to generate the "Novelty" index comes from the USPTO patent and citation data. The USPTO records all patents granted after 1976 and all the patents they cite. To get a smoother trend, we take a three-year average for each observation,[1]

$$\text{Novelty\_avg}_t = \frac{1}{3}\sum_{h=-1}^{1} \text{Novelty}_{t+h}. \tag{2}$$

There are other measures of patent novelty. Bowen III, Frésard and Hoberg (2023) analyzes the text of all the US patents and defines patents as being revolutionary if the vocabulary they use is growing rapidly in the patent corpus overall. Their novelty measure is called "RETech". Kelly et al. (2021) also uses textual analysis and measures patent importance according to its similarity to previous work and subsequent innovations.

Figure 1 shows the technological waves defined in this paper and the "RETech" in the literature.[2] They are significantly positively correlated with nearly simultaneous peaks and troughs, indicating the robustness of different measures. The figure suggests that significant technological breakthroughs happened in the mid-1980s, the mid-1990s, and the beginning of the 2010s although the third peak is lower, while the period around 1990 and the mid-2000s are periods when most of the technologies have entered a mature stage. The "Novelty" index for different technology fields—the first digit of the International Patent Classification (IPC) defined by the The World Intellectual Property Organization (WIPO)—are shown in Figure 18 in Appendix B.2. There are both co-movements and heterogeneity across different fields.[3]

The alignment between the technological waves identified by the citation-based measure in this paper and those found through textual analysis in the literature enhances the credibility of our newly developed measure. However, our measure may underestimate the declining trend in technological novelty since the number of citations generally increases at a faster pace over time, particularly after the 1980s. The citation-based measure in this paper complements the text-based measure in the literature and offers several advantages. First, it does not rely on the digitization quality of patent abstracts, thereby avoiding issues of inaccuracy. Second, it is unaffected by strategic language use in patent abstracts or changes in language over time. Third, its definition is more transparent and not constrained by

[1] The smoother does not change the original pattern, as shown in figures without the smoothing techniques in Appendix B.1

[2] The "RETech" defined by Bowen III, Frésard and Hoberg (2023) has a similar meaning to our measure. Kelly et al. (2021) captures aggregate technology breakthroughs by counting the number of patents in the top 10 percent of the unconditional distribution of their importance measure. Because this measure is influenced by the total number of patents each year, its meaning differs from ours.

[3] The "Novelty" Index by field is defined in a similar way as the aggregate index, except that the patent set, I_t, now includes only patents in the corresponding technology field. The nine fields are respectively human necessities, performing operations and transportation, chemistry and metallurgy, textiles and paper, fixed constructions, mechanical engineering; lighting; heating; weapons; blasting, physics, electricity.

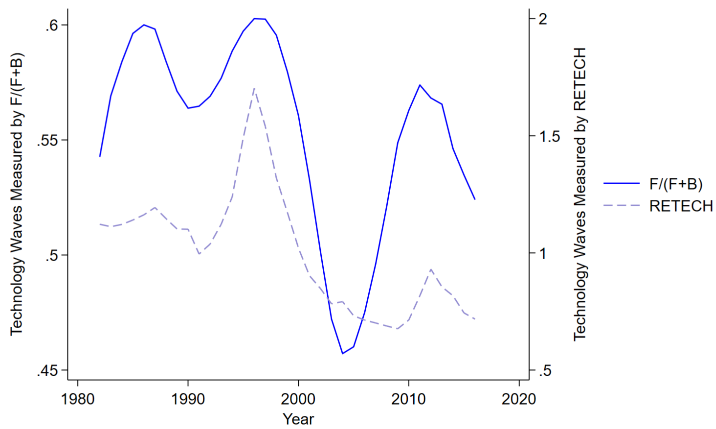

Figure 1: Two Measures of Technological Waves

Notes: This figure illustrates two measures of the technological waves. The blue solid curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations, while the gray dashed curve represents the "RETech" index, a measure of patent novelty from the literature, which assesses patent novelty by the prevalence of vocabularies that are growing in use in the patent description. The two curves have different y-axises, as shown on the left and right.
Sources: USPTO patent and citation data.

computational resources. We anticipate that this measure will be used more broadly to capture technological shifts over time.

2.1.2 Contributors to the Tech Waves

Which classes of technology contributed to the three peaks of the technological waves? Who were the major applicants for breakthrough patents—incumbents, startups, or public institutions?

To answer the first question, we decompose the "Novelty" index into the contribution of each three-digit IPC code using the following method,

$$\begin{aligned} \text{Novelty}_t &= \frac{\sum_{i\in I_t}\sum_{s=0}^{5} F_{i,t+s}}{\sum_{i\in I_t}\sum_{s=0}^{5} F_{i,t+s} + \sum_{i\in I_t}\sum_{s=0}^{5} B_{i,t-s}} \\ &= \sum_{j\in J} \frac{\sum_{i\in I_{jt}}\sum_{s=0}^{5} F_{ij,t+s}}{\sum_{i\in I_t}\sum_{s=0}^{5} F_{i,t+s}} \frac{\sum_{i\in I_t}\sum_{s=0}^{5} F_{i,t+s}}{\sum_{i\in I_t}\sum_{s=0}^{5} F_{i,t+s} + \sum_{i\in I_t}\sum_{s=0}^{5} B_{i,t-s}}, \end{aligned} \tag{3}$$

where J is the set of 3-digit IPC code. Intuitively, the contribution of each technology class in a given year is determined by the share of forward citations of that class multiplied by the aggregate Novelty Index. Table 1 lists the top three contributors at the three peaks of the technological novelty waves. Medical or Veterinary Science and Hygiene contribute most to the first peak, while Computing; Calculating or Counting is the leading contributor to the second and third peak.

Table 1: Major Contributors to the Technological Novelty Peaks

	First Peak (1985-1987)	Second Peak (1995-1997)	Third Peak (2010-2012)
1	Medical or Vet. Sci.; Hygiene	Computing; Calculating or Counting	Computing; Calculating or Counting
2	Electric Elements	Medical or Vet. Sci.; Hygiene	Medical or Vet. Sci.; Hygiene
3	Measuring; Testing	Electric Communication Technique	Electric Communication Technique

Notes: This table shows the major technological classes of the top three fields with the highest "Novelty" index at the technological novelty peaks in the period between 1981 and 2017.

To address the second question, we use the "historically significant patents" compiled by Kelly et al. (2021) from online lists. There are 54 breakthrough patents within the sample period of the Novelty Index. To determine if these patents significantly contribute to the aggregate novelty defined in this paper, we calculate their contribution using a method similar to that used for IPC-level contributions.[4] We then compute the percentile rank of these 54 breakthrough patents within the unconditional distribution for the sample period. The mean and median ranks are in the top 24% and top 7% of all patents, respectively, indicating significant overlap between the list and our novelty measure. We identify the applicants for the breakthrough patents and present their types of institutions in Figure 2. The results show that the sources of technological breakthroughs are quite diverse, suggesting that where highly novel technologies emerge is somewhat random.

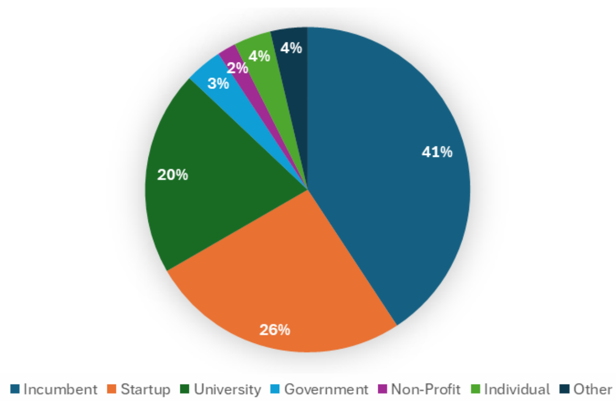

Figure 2: Composition of Applicants of Breakthrough Patents
Notes: The pie chart illustrates the share of applicants from each institutional group for the 54 breakthrough patents. Incumbents are defined as private firms that applied for a patent at least three years after their founding. Startups are private firms that applied within their first three years or before. Others include cases where the applicants are multiple institutions of different types.
Sources: The "historically significant patents" complied by Kelly et al. (2021).

[4]Namely, $\text{Novelty}_t = \sum_{i \in I_t} \frac{\sum_{s=0}^{5} F_{i,t+s}}{\sum_{i \in I_t} \sum_{s=0}^{5} F_{i,t+s}} \frac{\sum_{i \in I_t} \sum_{s=0}^{5} F_{i,t+s}}{\sum_{i \in I_t} \sum_{s=0}^{5} F_{i,t+s} + \sum_{i \in I_t} \sum_{s=0}^{5} B_{i,t-s}}$.

2.1.3 Tech Waves in Europe

While this paper primarily focuses on analysis within the United States, we also calculate the Novelty Index for several European countries with the highest patenting activity. Figure 19 in Appendix B.3 illustrates the technological waves in six European countries with the most patent issuances during the sample period, based on PATSTAT data, respectively. The figure reveals a declining trend in patent activity among all the six countries from the 1980s through the 2010s.

2.2 Market Concentration

We adopt the most commonly used measure, the HHI, to capture market concentration. The dataset used is Compustat Fundamentals Annual due to its comprehensive coverage of firms' sales. It contains all publicly listed firms in the U.S. The sample used in this paper keeps all industrial firms headquartered in the US from Compustat. To assess the representativeness of our sample relative to all U.S. firms, we calculate the share of sales by the top firms using the cleaned data series from Kwon, Ma and Zimmermann (2023), which is based on IRS data covering the entire population of U.S. corporations. Figure 20 in Appendix B.5 shows that the HHI derived from publicly listed firms exhibits similar upward trends and cyclicality to the top sales shares from the population data.

The HHI construction process is the following. First, we calculate the squared ratios of firm sales to total industry sales within each industry defined by the 2-digit SIC code in each year. Second, we sum up the ratios across all firms in each industry to get the industry-level HHIs in each year. Third, we weight each industry by its total sales and take a weighted average of all industry-level HHIs. To smooth the trend, we also take the three-year average for each observation point.[5]

The pattern of the yearly HHI is shown in Figure 3. To illustrate its relationship with the technological waves, the "Novelty" index defined in the previous section is also plotted in the figure. The two curves exhibit a negative correlation. The linear trend of the technological waves has a negative slope of -0.002, while the linear trend of the HHI has a positive slope of 0.002. The cross correlation between the detrended HHI (x_t) and the detrended technological waves (y_{t+k}) at different year gaps, $\text{corr}(x_t, y_{t+k})$, has the highest absolute magnitude, -0.770 when $k = -2$. This indicates that the evolvement of the market concentration measured by HHI closely follows the technological waves and lags the waves for about two years.[6]

[5] Figure 16 in Appendix B.1 shows the patterns of HHI and the Novelty Index without smoothing. Their correlation is similar to the smoothed version.

[6] The cross correlations when $k \in \{-3, -2, -1, 0, 1, 2, 3\}$ are shown in Table 8 in Appendix B.4. The lowest point (the highest in absolute value) appears when $k = -2$.

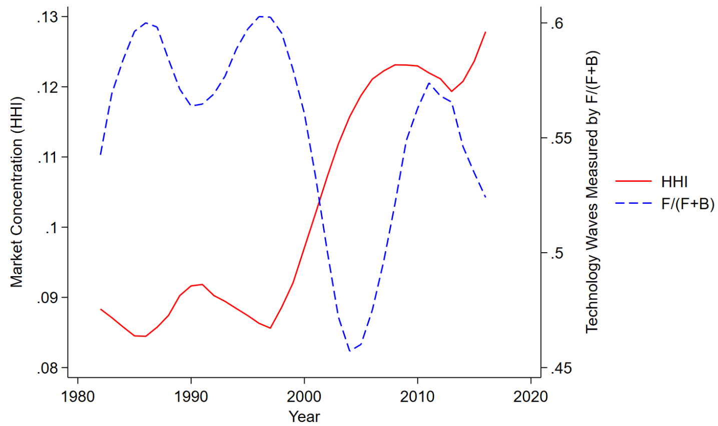

Figure 3: Technological Waves and Market Concentration

Notes: This figure shows the technological waves and the trend of market concentration over time. The blue dashed curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations. The red solid curve displays the HHI in each year, which is the weighted average of the industry-level HHI in each year. The weight is the total sales of firms in each industry. The two curves have different y-axises, which are shown respective on the left and right.
Sources: Compustat Fundamental Annuals and USPTO patent and citation data.

To examine the sector-level relationship between the market concentration and technological waves, we calculate the HHI and the "Novelty" Index by major sectors defined by the SIC code—Mining and Construction, Manufacturing, Transportation and Utilities, Wholesale and Retail Trade, Finance, and Services.[7] Aggregating the HHIs within each major sector is a straightforward process, accomplished by computing a sales-weighted average of the HHIs at the 2-digit SIC level. However, performing a similar aggregation for the "Novelty" Index presents a more complex challenge, since patents are classified by the technology class (as captured by the International Patent Classification (IPC)) instead of sectors. To map the technology classes to sectors, we use the concordance developed by Silverman (2002) that links the 4-digit IPC code to the 4-digit SIC code according to usage. After applying this concordance, we obtain the counts of forward and backward citations at the 4-digit SIC level. These citation counts are then cumulatively summed up to the primary sector level, allowing us to calculate the "Novelty" Index for each sector. The visual representation of our findings can be observed in Figure 4.

Generally, a discernible negative relationship between technological waves and market concentration prevails across most major industries. The linear trend of the HHIs are non-

[7]The division is according to the U.S. department of Labor. Mining includes SIC 10-14; Construction includes SIC 15-17; Manufacturing includes SIC 20-39; Transportation and Utilities includes SIC 40-49; Wholesale Trade includes SIC 50-51; Retail Trade includes SIC 52-59; Finance includes SIC 60-67; Services includes SIC 70-89. To ensure sufficient observations, Mining and Constructions are combined; Wholesale and Retail Trade are combined.

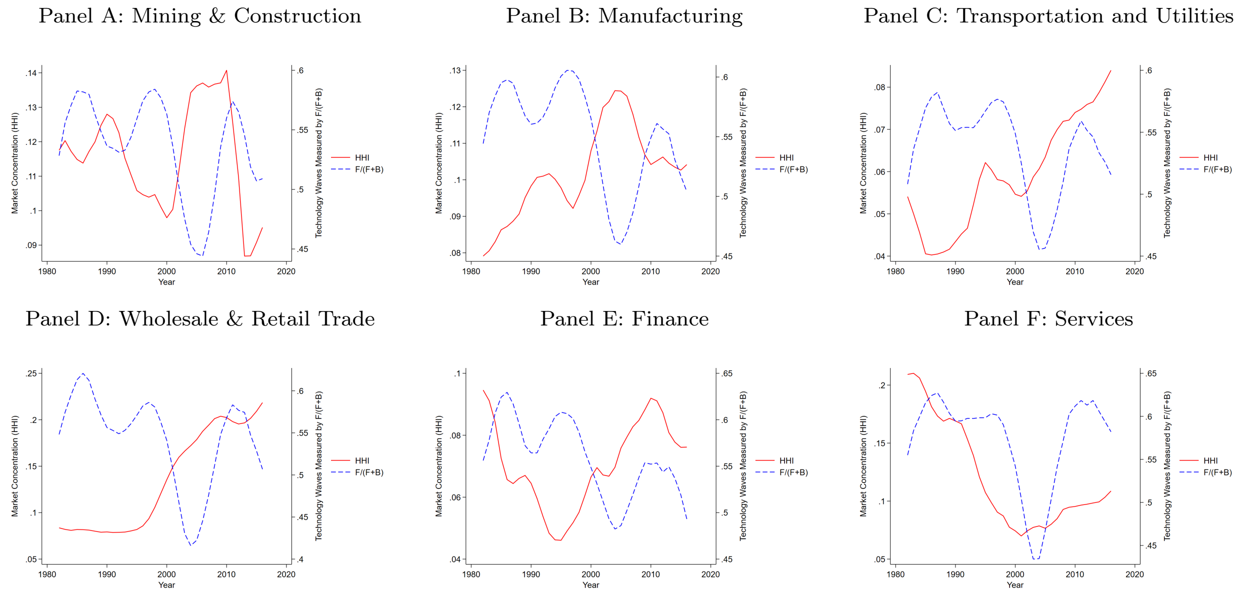

Figure 4: Technological Waves and Market Concentration by Industry

Notes: This figure shows the technological waves and the trend of market concentration over time by major sectors. The blue dashed curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations in each major sector. The red solid curve displays the HHI in each year, which is the weighted average of the 2-digit-SIC-level HHIs by major sectors and years. The weight is the total sales of firms in each 2-digit SIC industry. The two curves have different y-axises, which are shown respective on the left and right.

Sources: Compustat Fundamental Annuals and USPTO patent and citation data.

negative, as opposed to negative trend of the technological waves. The detrended cross correlation between the two time series has the highest absolute magnitude at $k = -2$ in most of the sectors. $\text{corr}(x_t, y_{t-2})$ is respectively -0.782 for Mining and Construction; -0.475 for Manufacturing, -0.132 for Transportation and Utilities, -0.483 for Wholesale and Retail Trade, -0.339 for Finance, and 0.366 for Services. The cross correlations when $k \in \{-3, -2, -1, 0, 1, 2, 3\}$ for each sector are shown in Table 8 in Appendix B.4. These findings offer additional supporting evidence suggesting that market concentration may be influenced by the dynamics of technological novelty waves.

The negative correlation between technological waves and market concentration is also evident in Europe, as shown by the declining trend of the Novelty Index in Appendix B.3 and the increasing market concentration across European countries, as measured by the Herfindahl-Hirschman Index (HHI) and top sales share in recent studies (e.g., Bighelli et al. (2023) and Ma, Zhang and Zimmermann (2024)).

2.3 Allocation of Ideas

One potential link between the technological waves and the market concentration is inventors' choices of where to do innovation. They can work independently and start their own businesses or contribute their innovation efforts to incumbent firms. In the latter case, they also choose the size of incumbent firms to work in. This section describes the flow of the new ideas using the Census data.

2.3.1 Entrants or Incumbent Firms

Data on the affiliations of inventors when they initiate a research project is unavailable to us, but we can observe the age of the firm when a patent is granted to it or applied by it and infer inventors' affiliation. Specifically, if a patent is granted to a firm at age zero to five, it implies that the initial idea was developed independently or spun off from other firms five years ago; if a patent is granted to a firm at age above five, it implies that the initial idea was developed by the incumbent firm five years ago or the firm bought the idea from independent inventors or other firms. We choose the time window to be five years since the average time between patent applications and patent issuance is around two or three years according to the USPTO and we assume the average time to complete a research project to be also two to three years. We can compute the ratio between the number of ideas in new firms to the number of ideas absorbed in incumbent firms, i.e.,

$$\text{New-to-Incumbent Ratio}_t = \frac{\sum_{i \in I_{t+5}} \text{Granted in Firm}(\text{Age} \leq 5)_{i,t+5}}{\sum_{i \in I_{t+5}} \text{Granted in Firm}(\text{Age}{>}5)_{i,t+5}}, \tag{4}$$

where I_{t+5} denotes the set of patents granted five years after time t; "Granted in Firm(Age$\leq$5)" and "Granted in Firm(Age>5)" are dummy variables indicating whether patent i is issued to a firm above five years old. An alternative measure is to use the age of a firm when it applies for patents. If a patent is applied for in a firm at age zero to three, it implies the founding of a new firm with the idea in the past three years. Otherwise, it implies incumbent firms absorbing new ideas.[8]

Note that there may be discrepancies between the patent affiliations and inventors' affiliations due to spinoffs and patent sales. In the case of spinoffs, the "New-to-Incumbent Ratio" (N-to-I Ratio) based on patent affiliation is larger than the ratio based on inventors'

[8] In this alternative measure, the "New-to-Incumbent Ratio" is defined as

$$\text{New-to-Incumbent Ratio}_t = \frac{\sum_{i \in I_{t+3}} \text{Applied in Firm}(\text{Age} \leq 3)_{i,t+3}}{\sum_{i \in I_{t+3}} \text{Applied in Firm}(\text{Age}{>}3)_{i,t+3}}, \tag{5}$$

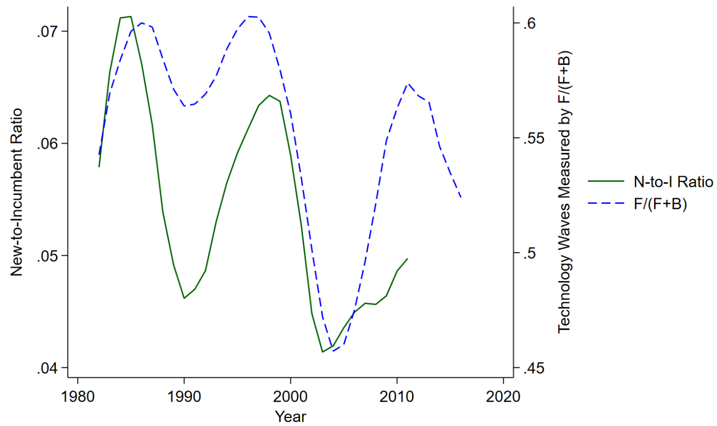

Figure 5: Technological Waves and Idea Allocation

Notes: This figure shows the technological waves and the idea allocation between new and incumbent firms over time. The blue dashed curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations. The green solid curve displays the "New-to-Incumbent Ratio" defined in the paper, capture where new ideas contribute their value. The two curves have different y-axises, which are shown respective on the left and right.
Sources: Longitudinal Business Database (LBD) and USPTO patent and citation data.

affiliation. In the case of patent sales, the situation is reversed. However, if we want to capture where innovation ideas finally contribute its value, taking into account spinoffs and patent sales works towards the purpose.

The data used to observe patent affiliations is constructed by combining the Longitudinal Business Database (LBD) from the US Census Bureau and the USPTO patent data. The LBD covers all the employer businesses in the US and documents the age of each firm. The combined dataset can track the age of firms at patent application and issuance.

Since the two measures of the New-to-Incumbent Ratio have very similar trends, we only report the first measure. We take the three-year average for each observation year as before and show the result in Figure 5.[9] To compare it with the technological waves, the Novelty Index defined in this paper is also plotted. Notably, the New-to-Incumbent Ratio demonstrates prominent cyclicality, with zeniths and nadirs occurring in close proximity to the peaks and troughs of technological novelty waves. There is a slight declining trend of slope -0.001, similar to the declining trend of the technological waves, -0.002. The cross correlation between the detrended New-to-Incumbent Ratio (x_t) and the detrended technological waves (y_{t+k}) at different year gaps, $\text{corr}(x_t, y_{t+k})$, has the highest absolute magnitude, 0.612, when $k = 0$, showing the two time series moves simultaneously.

[9]Figure 17 in Appendix B.1 shows the patterns of the New-to-Incumbent Ratio and the Novelty Index without smoothing. Their correlation is similar to the smoothed version.

To assess the robustness of the relationship between idea allocation and technological waves, this paper compares the two trends by patent technological fields, categorized by the first digit of the patent IPC code. The IPC-level "Novelty" Index and "New-to-Incumbent Ratio" are computed using the same methodology as described in equations 1 and 4, with patent sets segregated according to their respective technology classes. Figure 6 illustrates that a positive correlation between idea allocation and technological waves is consistently observed across most technology classes. When a specific technology class experiences breakthroughs, there is an increase in the flow of ideas toward new startups. The contemporaneous correlation coefficients between the two curves are, respectively, 0.40 for Human Necessities, 0.30 for Performing Operations, 0.16 for Chemistry, 0.42 for Textiles, 0.39 for Fixed Constructions, -0.47 for Mechanical Engineering, 0.34 for Physics, and 0.58 for Electricity. The cross correlations when $k \in \{-3, -2, -1, 0, 1, 2, 3\}$ for each technological field are shown in Table 8 in Appendix B.4.

2.3.2 Size of Incumbent Firms

When inventors opt to contribute their ideas to incumbent firms, they are also making a choice regarding the size of the firm, as it impacts the potential value that their innovations can attain. We establish a connection between the quality of inventors' ideas and the size of the incumbent firms they select by examining a subset of patents that have been granted to firms with a history of at least five years in operation. This subset serves as the denominator for calculating the "New-to-Incumbent Ratio," as described in Section 2.3.1. The quality of inventors' ideas is gauged by the number of forward citations each patent receives within the first five years following its issuance. We amalgamate data of all patents (issued to both new and incumbent firms) from various years and compute the quartiles for patent citations. Subsequently, we categorize patents into four distinct groups based on the quartile to which their citation count belongs. Then we calculate the average size of firms the patents in each quartile are granted to if they are granted to incumbent firms. The firm size is measured by the number of employees and the average employment in the first citation quartile is normalized to a value of one. The relative employment in each citation quartile is plotted in Figure 7. It is shown that there is positive assortative matching between idea quality and firm size when ideas are combined with incumbent firms. One potential concern is that the firm's employment at the patent's issuance may differ from the employment when the inventor chooses the firm. To address this concern, we track each firm's employment five years ago, using data from the LBD. The relationship between relative firm size and patent

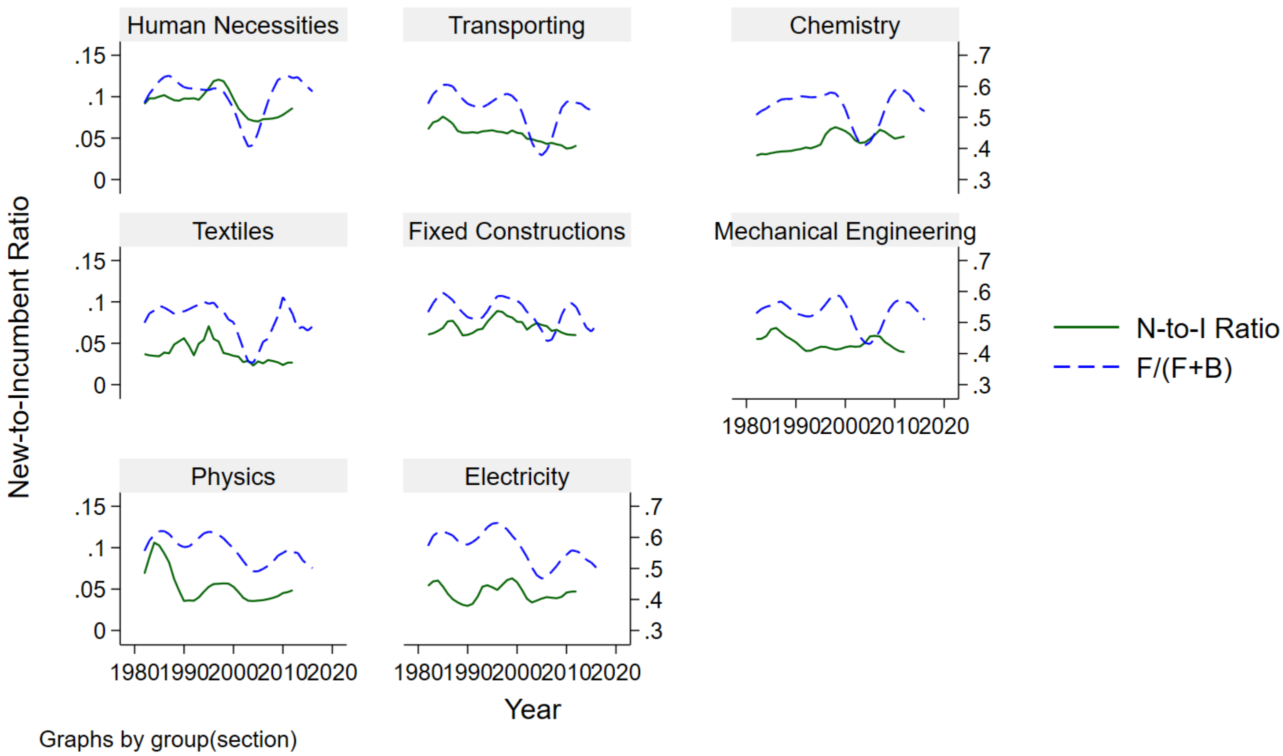

Figure 6: Technological Waves and Idea Allocation by Patent Technology Class

Notes: This figure shows the technological waves and the idea allocation between new and incumbent firms by patent technology class. The blue dashed curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations. The green solid curve displays the "New-to-Incumbent Ratio" defined in the paper, capture where new ideas contribute their value. The two curves have different y-axises, which are shown respective on the left and right.
Sources: Longitudinal Business Database (LBD) and USPTO patent and citation data.

citation quartiles mirrors the mapping depicted in Figure 7.[10]

To check whether the positive relationship between idea quality and firm size exists for new firms, we calculate the average size of firms the patents in each quartile are granted to if they are granted to new firms—firms with less than five years of operation. It turns out the average firm sizes are similar across quartiles, suggesting the positive assortative matching only holds when ideas are contributed to incumbent firms.

2.4 Economic Value of Patents over Tech Waves

To explore the underlying channels driving the co-movement between technological waves and the allocation of ideas, as well as the positive assortative matching between idea quality

[10] The figure with firms' employment five years ago is available upon request.

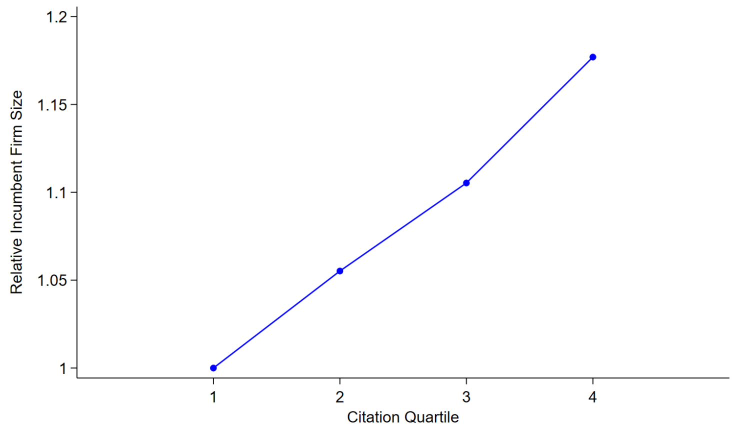

Figure 7: Mapping between Patent Citations and Incumbent Firm Size

Notes: This figure shows the mapping between inventors' idea quality and firm size if inventors opt to develop their ideas in incumbent firms. The idea quality is measured by the number of patent citations and is classified into four quartiles. The firm size is measured by the number of employees. The average employment of firms corresponding to the first citation quartile is normalized to be one.
Sources: Longitudinal Business Database (LBD) and USPTO patent and citation data.

and firm size, we perform patent-level regressions. We use an extended version of the sample constructed by Kogan et al. (2017), which includes more recent years. Kogan et al. (2017) leverages the stock market's response to patent news to estimate the private economic value of patents. Since the sample encompasses all patents granted to publicly listed firms in the US, it provides valuable insights into factors affecting the economic value of patents in incumbent firms over technological waves. The following regression analysis is conducted,

$$\begin{aligned} \ln(\text{economic value}_{ijst}) =& b \ln(\text{Firm size}_{jst}) + \iota \ln(1 + \text{Citations}_{ijst}) + \\ & \phi \text{Novelty}_t \times \ln(1 + \text{Citations}_{ijst}) + \mu_t + \theta_{st} + \gamma_{jt} + \epsilon_{ijst}. \end{aligned} \tag{6}$$

where i, j, s, and t are respectively indexes for patents, firms, patent technology classes (the first-digit IPC), and years. The dependent variable corresponds to the economic value of the patents. Firm size is measured by either the employment or sales of the firm to which the patent belongs. The number of citations is used to measure the scientific value of patents, serving as a proxy for idea quality. The interaction term between the Novelty Index and the citations captures the impact of technological waves on the relationship between the scientific and economic value of patents. The model controls for year fixed effects, year by patent technology class fixed effects, and year by firm fixed effects.

Table 2 presents the results using firm employment as the measure of firm size. Similar regression results using firm sales as the measure of firm size are provided in Appendix

B.6. Columns (1) and (2) exclude the technological wave measure, focusing solely on the properties of patents and firms. Columns (3) and (4) display results of Equation (6). In Columns (5) and (6), the yearly Novelty Index is replaced by the year-by-IPC Novelty Index.

Firm size has a significantly positive effect on the economic value of patents, given the idea quality. This suggests that the synergy between inventors and firms increases with firm size. Additionally, idea quality positively impacts the economic value of patents, but this impact diminishes with higher aggregate technological novelty, as indicated by the negative coefficients of the interaction terms. This finding highlights the creative destruction effect of novel technologies on existing product lines.

	Ln(Patents' Economic Value)					
	(1)	(2)	(3)	(4)	(5)	(6)
Ln(1+Employment)	0.330***		0.330***		0.330***	
	(0.0262)		(0.0262)		(0.0262)	
Ln(1+Citations)	0.0732***	0.00277***	0.285***	0.0131**	0.231**	0.0115**
	(0.00561)	(0.000576)	(0.0730)	(0.00574)	(0.0907)	(0.00501)
Ln(1+Citations)×Novelty$_t$			-0.390***	-0.0190*		
			(0.135)	(0.0107)		
Ln(1+Citations)×Novelty$_{st}$					-0.291*	-0.0162*
					(0.162)	(0.00909)
Year Fixed Effect	Y	Y	Y	Y	Y	Y
Year×IPC Fixed Effect	Y	Y	Y	Y	Y	Y
Year×Firm Fixed Effect	N	Y	N	Y	N	Y
Observations	1,111,737	1,101,355	1,111,737	1,101,355	1,111,633	1,101,250
R-squared	0.295	0.882	0.295	0.882	0.295	0.882

Notes: Standard errors are clustered at the year level. Columns (1)-(2) exclude the technological wave measure and focus solely on the property of the patents and firms. Columns (3)-(4) show coefficients of the regression equation (6). Columns (5)-(6) replace the yearly Novelty Index by the year-by-IPC Novelty Index. The regressions control for year fixed effects and year by patent technology class fixed effects across all specifications. The year by firm fixed effects are controlled in columns (2), (4), and (6). *** Significant at the 1 percent level; ** Significant at the 5 percent level; * Significant at the 10 percent level.

Table 2: Factors of Patents' Economic Value for Incumbent Firms

3 Model

To clarify the mechanism through which technological waves influence market concentration, we develop a general equilibrium model featuring two groups of individuals (households and inventors) and two types of firms (intermediate goods producers and final goods producers). In this economy, there is an aggregate shock capturing the degree of novelty of new technologies in each period. This shock applies to all agents and determines the extent of friction when inventors' ideas combine with incumbent intermediate good producers. Inventors in each period receive ideas of idiosyncratic quality. They choose to start up new intermediate-good-producing firms or join incumbent ones of selected size based on the aggregate shock and their idea quality.

3.1 Preferences

There is a long-lived representative household in the economy. She works in the production sector, supplies one unit of labor to firms inelastically, and consumes final goods. She also owns all the firms in the economy. The household's utility function is

$$U_H = \int_0^{\infty} e^{-\rho t} log(C_H(t))dt, \tag{7}$$

where $\rho > 0$ is the discount rate and $C_H(t)$ is the consumption of the household.

Inventors are the ones who work in the R&D sector. In each period, there is a continuum of inventors of measure one. An inventor, with a short-lived lifespan of dt time periods, dedicates effort e_I to create innovations within either an incumbent firm or a new business. Simultaneously, they engage in consumption. Inventors are risk-averse and have a mean-variance utility:

$$U_I(c_I, e_I) = \mathbb{E}(c_I) - A\frac{var(c_I)}{\bar{q}} - R(e_I)\bar{q}, \tag{8}$$

where c_I is the consumption, e_I is the effort level, and $R(e_I)\bar{q}$ is the associated cost. $\bar{q}$ (defined below) is the average quality in the economy. The variance and cost are normalized by $\bar{q}$ to keep the problem stable over time. Denote the inventors' aggregate consumption using C_I, i.e., $C_I = \int_0^1 c_{Ii} di$.

3.2 Technology

The economy features two types of firms: intermediate goods producers and final goods producers. The setup is similar to Akcigit and Kerr (2018). Both types of firms are owned by the household. The former hires inventors to create innovations, and produce intermediate goods. The latter assembles intermediate goods and produces final goods.

The final good producers produce final goods using a continuum of intermediate goods $j \in [0, N_F]$:

$$Y(t) = \frac{1}{1-\beta}\int_0^{N_F} q_j^{\beta}(t) y_j^{1-\beta}(t) dj. \tag{9}$$

In this function, $q_j(t)$ is the quality of the intermediate good j, and $y_j(t)$ is its quantity. We normalize the price of the final good to be one in every period. The final good producers are perfectly competitive, taking the input prices as given. Henceforth, we will drop the time index t when it does not cause confusion.

The final goods are consumed by the household and inventors. The resource constraint of the economy is:

$$Y = C_H + C_I. \tag{10}$$

The intermediate goods producers are a continuum of risk neutral firms of measure N_F. Each firm produces one type of good, with a linear technology using only labor:

$$y_j = \bar{q} l_j, \tag{11}$$

where l_j is the labor input; $\bar{q} = \frac{1}{N_F} \int_0^{N_F} q_j dj$ is the average quality, meaning that improvement in q_j has positive externality (Romer, 1986). The cost is linear in wage w, which intermediate firms take as given. The labor market satisfies the constraint:

$$\int_0^{N_F} l_j dj \leq 1. \tag{12}$$

The production technologies, together with the market setting on innovation, ensure that an intermediate good producer's value $V(q_j)$ is linear in quality q_j (the proof is shown in the next section),

$$V(q_j) = \nu q_j, \tag{13}$$

where ν is endogenous.

This paper focuses on the balanced growth path. We normalize the variables using the average quality $\bar{q}$, and denote the normalized variables using tilde:

$$\tilde{q_j} \equiv \frac{q_j}{\bar{q}}, \tilde{Q} \equiv \frac{Q}{\bar{q}}, \tilde{V}\left(\tilde{q}\right) \equiv \frac{V\left(q_j\right)}{\bar{q}} = \nu \tilde{q_j}, \tag{14}$$

where $Q \equiv \int_0^{N_F} q_j dj$ is the total technology stock in the economy.

Within a given period, intermediate firms consist of a combination of established incumbents and new entrants. Incumbents hire inventors to improve their quality through innovations, while new entrants arise as a result of successful innovations by inventors collaborating with a partner. These innovations are generated by inventors exerting effort denoted as e_I. Given the level of effort e_I, the success rate of an innovation follows an instantaneous Poisson flow rate:

$$\lambda(e_I) = \lambda_0 e_I. \tag{15}$$

It is costly for inventors to dedicate effort, and the flow cost of choosing effort e_I is $R(e_I)\bar{q}$, and $R(e_I) = \frac{1}{1+\delta} e_I^{\delta+1} dt$. This implies a linear cost in time dt at a rate of $\frac{1}{1+\delta} e_I^{\delta+1}$, which is an increasing and convex function of the effort taken.[11]

Inventors are directly responsible for the cost of their efforts, but their efforts cannot be observed by the partner or incumbent intermediate firms. In the absence of a performance-based incentive, an inventor, receiving a flat wage, would opt for $e_I = 0$. Consequently, the partner and the incumbent firms must incentivize inventors to take effort by implementing an innovation-dependent payment scheme. This paper adopts the assumption that firms utilize a common contract, which is a combination of wage and equity, to compensate inventors. The wage allows the partner and the firms to share risk with an inventor whereas equity aligns the inventor's interests with theirs.

Each inventor is born with one innovative idea characterized by an idea quality z_0. The inventor can choose to work either within an incumbent intermediate firm or start up a new intermediate firm with a partner. In the case of working in a startup, the inventor retains full control over the innovation process, and the innovation value is solely determined by the idea quality z_0. Following creation, the normalized innovation value, $\tilde{z}$, is a stochastic draw from a uniform distribution, $U\left((1-\phi) z_0 \nu, (1+\phi) z_0 \nu\right)$. While, on average, a higher quality idea yields a better outcome, the inclusion of ϕ allows for some randomness in the mapping between the innovation value and idea quality, with $\phi \in (0,1)$ capturing this variability.

In the other case, when the inventor with idea quality z_0 works in an incumbent firm with quality $\tilde{q}$, the resulting innovation value, $\tilde{x}(z_0, \tilde{q})$, becomes a stochastic variable drawn from another uniform distribution, $U\left((1-\phi) x_0(z_0,\tilde{q}) \nu, (1+\phi) x_0(z_0,\tilde{q}) \nu\right)$, where $\frac{\partial x_0(z_0,\tilde{q})}{\partial z_0} > 0$ and $\frac{\partial x_0(z_0,\tilde{q})}{\partial \tilde{q}} > 0$. The mean value of the innovation, $x_0(z_0,\tilde{q})$, now depends not only on the idea quality z_0, but also on the firm size $\tilde{q}$. This reflects that the incumbent firm provides the inventor's idea with synergy and the synergy increases in the firm's quality.

The mean value of the innovation is also subject to adoption friction that increases in the aggregate technological novelty of the economy. The friction captures the learning cost when incumbent firms combine with new ideas. We assume that $x_0(z_0,\tilde{q})$ takes the following functional form: $x_0(z_0,\tilde{q}) = \left(\frac{\tilde{q}}{\tilde{q}_0}\right)^b \gamma(z_0) z_0$. The first term, $\left(\frac{\tilde{q}}{\tilde{q}_0}\right)^b$, denotes the synergy between an incumbent firm and an innovation, where $\tilde{q}_0$ is a parameter. The second term, $\gamma(z_0) = \frac{B}{B+z_0}$, captures the influence of the technology waves. The function form is inspired by the Novelty Index defined in the empirical section. The parameter B corresponds to the backward citation stock in a certain period, representing the maturity of the technology to

[11] The innovation production function and the cost functions are based on the growth theory literature (Romer, 1990; Klette and Kortum, 2004; Akcigit and Kerr, 2018). In the calibration, we choose $\delta = 1$ following the literature.

which the inventor's idea contributes. The value of B varies over time and is pinned down by mapping the average $\gamma(z_0)$ to $(1 - \text{Novelty Index})$. When the economy is closer to the peak of the technological waves, past innovations are less influential, leading to a smaller calibrated value of B. In such case, incumbent firms are subject to larger learning friction when integrating innovations, as revolutionary technologies cause more creatively destruction to the current production line.

3.3 Timeline

Upon an inventor's birth, she observes the quality z_0 of her idea. A potential partner observe z_0 and extends contracts to the inventor to jointly start a new intermediate firm. Concurrently, incumbent firms observe their corresponding $x_0(z_0, \tilde{q})$ and also extend employment contracts to the inventor. The contracts from the potential partner and incumbent firms are strategically designed to maximize their payoff, taking into account the competition with other firms, as well as the inventor's incentive problem. They possess two key components: a fixed wage $\tilde{T}$ and a stake in equity $a \in [0, 1]$.[12] After viewing all contracts, the inventor decides to either join her preferred incumbent firm of quality $\tilde{q}^*(z_0)$, or initiates a startup with the partner. In both cases, the matching process is frictional. When the inventor chooses to innovate in an incumbent firm, she joins the firm with the optimal size with probability h; alternatively, she is randomly assigned to another incumbent firm $\tilde{q}$, based on the incumbent firm size distribution $\tilde{F}(\tilde{q})$. Similarly, when the inventor prefers to start a new business, she initiates it with probability h_s; with probability $1-h_s$, the inventor is randomly assigned to an incumbent firm. The frictions in the matching process are introduced to match the data, since the actual mapping between idea quality and firm size is not perfect. After signing the contract, the inventor chooses an R&D effort, e_I.

3.4 Entry and Exit

A new intermediate firm enters the market upon successful innovation of an inventor who choose to work with a partner. Upon entry, the firm first draws a quality $\tilde{q}$ from the current incumbent firm size distribution $\tilde{F}(\tilde{q})$. Subsequently, the entrant incurs a cost equivalent to the firm value associated with the drawn quality $\tilde{q}$. Following this, the firm applies the innovation, enhancing its quality by incorporating the value of the innovation itself. The rate at which entrants join the market is represented by λ_I.

[12] It is worth noting that the level of effort e_I is unobservable and unverifiable. Consequently, contracts cannot be contingent on the effort level.

Intermediate firms face an exogenous exit rate τ, which is independent of their size and is a risk unrelated to innovation. We focus on a balanced growth path such that the number of entrants equals the number of firm exits,

$$\tau N_f = \lambda_I. \tag{16}$$

4 Equilibrium: Balanced Growth Path

This section characterizes the equilibrium of the economy in which aggregate variables $(Y, C, R, w, \bar{q})$ grow at a constant rate g.

4.1 Production

The final good producer chooses $\{y_j\}_j$ to maximize its profit using the technology described in Section 3.2, which yields the demand function faced by intermediate goods producers: $p_j = q_j^{\beta} y_j^{-\beta}$. The intermediate good producers engage in monopolistic competition.[13] Their FOC yields,

$$y_j = q_j \left(\frac{\bar{q}(1-\beta)}{w} \right)^{\frac{1}{\beta}}, l_j = y_j/\bar{q}, p_j = \frac{w}{\bar{q}(1-\beta)}. \tag{17}$$

In each period, the labor market clearing satisfies $\int_0^{N_F} l_j dj = 1$, which pins down the wage

$$w = N_F^{\beta} (1-\beta) \bar{q}. \tag{18}$$

Thus, both the production output y_j and profit π_j are linear in quality,

$$y_j = \frac{q_j}{N_F}, \pi_j = \frac{\beta q_j}{N_F^{1-\beta}}. \tag{19}$$

We drop the subscript j from the firm-level variable when it does not cause confusion. In this model, it is assumed that intermediate firms, responsible for hiring inventors to create innovation, operate in an environment where the competition ensures that the entire value from innovations is captured by inventors. The discounted value of being a firm of quality q is, therefore, the same as the net present value in the case where no innovation occurs. The value function of an intermediate firm q at time t is a linear function of firm size q.

$$V(q, t) = \nu q. \tag{20}$$

[13] The profit maximization problem and the solution process of the final good and intermediate good producers are shown in Section C.1 in the Appendix.

where $\nu = \frac{\beta}{(r+\tau)N_F^{1-\beta}}$. See appendix for proof.

The value function is linear in its quality, q, and does not depend on time. This result implies that for any firm, the value of the same quality improvement Δq is the same. We will use q to denote both firm quality and size in the following sections.

The aggregate production is linear in the average quality $\bar{q}$. The resource constraint of the economy is $Y = C_H + C_I$. The relationship between the growth rate, g, and the time discount factor can be derived from the household's maximization problem,

$$g = \frac{\dot{Y}}{Y} = \frac{\dot{C}_H}{C_H} = \frac{\dot{\bar{q}}}{\bar{q}} = r - \rho. \tag{21}$$

4.2 Joining Incumbent Firms

Incumbent (intermediate) firms engage in competition to attract inventors by offering a compensation package including equity a and wage $\tilde{T}$. The setup yields a principal-agent problem, where the interests of the risk-neutral firms, who benefit from innovation, and the risk-averse inventors, who dedicate effort to create innovations, are not aligned.

While firms derive value from the innovations, they are not able to monitor the effort exerted by inventors. Consequently, firms aim to incentivize the inventors to invest effort by offering equity, while concurrently share the risk with inventors through a fixed wage. For an intermediate firm, the optimization problem is as follows:

$$\begin{aligned} &\max_{a,\tilde{T}} (1-a)\left(\tilde{V}(\tilde{q}) + \lambda_0 e_I x_0(z_0,\tilde{q})\,\nu dt\right) - \tilde{T} \\ &\quad \text{st } e_I = \arg\max\left\{u\left(c_I\left(a,\tilde{q},\tilde{T}\right), e_I\right)\right\} \\ &\quad\quad u\left(c_I\left(a,\tilde{q},\tilde{T}\right), e_I\right) \geq \bar{u}(z_0) \\ &\quad\quad (1-a)\left(\tilde{V}(\tilde{q}) + \lambda_0 e_I x_0(z_0,\tilde{q})\,\nu dt\right) - \tilde{T} \geq \tilde{V}(\tilde{q}) \end{aligned} \tag{22}$$

In this problem, a firm $\tilde{q}$ chooses the optimal contract $\left\{a,\tilde{T}\right\}$ for an inventor z_0 to maximize its own payoff while taking three constraints into consideration. The firm's expected payoff is equal to the expected firm value owned by the original shareholders (all shareholders except the inventor), given by $(1-a)\left(\tilde{V}(\tilde{q}) + \lambda_0 e_I x_0(z_0,\tilde{q})\,\nu dt\right)$, minus the wage paid to the inventor $\tilde{T}$. Note that $\tilde{V}(\tilde{q})$ is the firm value prior to innovation.

The first constraint is the inventor's incentive compatibility constraint, ensuring that when the inventor is employed by the firm, her actions align with utility maximization. Namely, when facing the firm-specific contract $\left\{a,\tilde{T}\right\}$, the inventor chooses an effort level

e_I to maximize her expected utility, denoted as $u\left(c_I\left(a,\tilde{q},\tilde{T}\right),e_I\right)$. The second constraint describes the inventor's participation constraint, meaning the inventor prefers to accept this firm's employment offer over other alternatives. This condition implies that the firm needs to offer the inventor a utility level surpassing her outside option $\bar{u}(z_0)$. The outside option is endogenously determined within this model by the Betrand competition among firms in the inventor market. Lastly, the third constraint is the firm's participation constraint, guaranteeing the firm will not be worse off by hiring one inventor.

Though firms all have the same optimization problem in Equation 22, their optimal equity level a depends not only on the inventor's idea quality z_0, but also the firm size $\tilde{q}$. Firm sizes affect the composition of the risk profile in an inventor's utility function:

$$u\left(c_I\left(a,\tilde{q},\tilde{T}\right),e_I\right)=\mathbb{E}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right)-A\mathrm{Var}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right)-R\left(e_I\right).^{14} \tag{23}$$

The consumption $c_I\left(a,\tilde{q},\tilde{T}\right)$ includes two components—the flat wage and the stochastic equity value, which is the product of the equity share and the sum of the original firm value and the value of innovation, i.e.,

$$c_I\left(a,\tilde{q},\tilde{T}\right)=a\left(\tilde{V}\left(\tilde{q}\right)+\tilde{x}\left(z_0,\tilde{q}\right)\nu\mathbb{1}_{\mathcal{S}}\right)+\tilde{T}$$

where $\mathcal{S}$ denotes the event that the inventor successfully creates an innovation. The expected consumption is[15]

$$\mathbb{E}\left(c_I\right)=a\left(\mathbb{E}\tilde{V}\left(\tilde{q}\right)+\lambda_0 e_I x_0\left(z_0,\tilde{q}\right)\nu dt\right)+\tilde{T},$$

and the associated variance is[16]

$$\mathrm{Var}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right)=a^2\left(\underbrace{\tau\tilde{q}^2\nu^2 dt}_{\mathrm{Var}(\tilde{V}(\tilde{q}))}+\underbrace{\lambda_0 e_I\mathbb{E}\left(\tilde{x}\left(z_0,\tilde{q}\right)^2\right)\nu^2 dt}_{\mathrm{Var(innovation)}}\right).$$

The variance comes from two sources: non-innovation-related firm value and the R&D process. Both terms increases in firm size $\tilde{q}$, but the former one increases in a faster speed, meaning in larger firms, shocks unrelated to R&D are stronger. Hence, larger firms are subject to larger incentive problems and the equity held by the inventor provides a weaker incentive for R&D efforts. Upon reviewing all available contracts, an inventor determines her preferred firm $\tilde{q}$.

[14] The utility and consumption have been normalized by the average firm quality, $\bar{q}$.
[15] The derivation of the expectation is shown in Appendix C.
[16] The derivation of the variance is shown in Appendix C.

Section 4.2.1 uses a simplified model to show the inventor's trade off in a closed form. With the same intuition, section 4.2.2 studies the inventor-firm matching in the full model.

4.2.1 A Closed Form Example

This section describes a simplified model which gives tractable results. We use it to illustrate the intuition. This simplified model adopt one additional assumption: the innovation value $\tilde{x}$ is drawn from a distribution with mean $x_0(z_0,\tilde{q})\,\nu$ and second order moment $e_I^{-1}x_0(z_0,\tilde{q})^2\,\nu^2$, instead of the uniform distribution $U\left((1-\phi)\,x_0(z_0,\tilde{q})\,\nu,(1+\phi)\,x_0(z_0,\tilde{q})\,\nu\right)$. With this change, the innovation-related uncertainty is now

$$\mathrm{Var}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right)=\left(\underbrace{\tau\tilde{q}^2\nu^2dt}_{\mathrm{Var}\tilde{V}(\tilde{q})}+\underbrace{\lambda_0\left(x_0\left(z_0,\tilde{q}\right)^2\right)\nu^2dt}_{\mathrm{Var(innovation)}}\right),$$

which does not depend on effort level any more.

Using backward induction, firms know the inventor would choose an effort level:[17]

$$e_I=\lambda_0 a x_0\left(z_0,\tilde{q}\right)\nu.$$

When an inventor owns a higher proportion of equity a or when the potential value of her innovation, $x_0(z_0,\tilde{q})$, is greater, she is inclined to invest more effort. This is because in both cases, given the cost function, the return of spending one more unit of effort is larger.[18]

The firm's problem in Equation 22 yields,

$$a^*=\frac{1}{1+2\frac{A}{\lambda_0}\left(\frac{\tau\tilde{q}^2}{\lambda_0 x_0(z_0,\tilde{q})^2}+1\right)} \tag{24}$$

The optimal equity level, a^*, decreases in the firm size $\tilde{q}$ when $b<1$. This is because a^* is determined jointly by two forces: the commercialization value $x_0(z_0,\tilde{q})$, and the non-innovation-related shock—the exit shock, $\tau\tilde{q}^2$. The firm size $\tilde{q}$ affects both factors but in opposite directions. Larger firms can provide the inventor with more synergy, leading to a greater commercialization value. This raises the equity share of the inventor, since it is more worthwhile to incentivize her effort. Meanwhile, larger firms face a higher exit risk. A lower equity share will expose inventors less to risks unrelated to innovation. The relationship between the equity a and the firm size $\tilde{q}$ depends on the relative strength of the two channels. When $b<1$, which is the case in the model calibration, the second channel

[17] We use $\delta=1$, which is found to match the data in the literature (e.g., Akcigit and Kerr (2018)).

[18] The derivation of the contract solution is presented in Section C.2 in the Appendix.

dominates. Therefore, larger firms optimally offer less equity to an inventor.

The optimal compensation scheme is $(a^*, \tilde{T}^*)$, where the wage $\tilde{T}^*$ is determined by the zero profit, due to Betrand competition. $\tilde{T}^* = -a^*\tilde{V}(\tilde{q}) + (1-a^*)\lambda_0 e_I x_0(z_0,\tilde{q})\nu dt$.

Upon reviewing all contracts, an inventor with idea quality z_0 chooses the firm $\tilde{q}$ she would like to work for. The first-order-condition yields

$$\frac{\partial x_0(z_0,\tilde{q})}{\partial \tilde{q}} = \frac{2A\sigma_0^2}{4A\sigma_0^2\frac{\tilde{q}}{x_0(z_0,\tilde{q})} + \frac{2A\lambda_0+\lambda_0^2}{\tilde{q}/x_0(z_0,\tilde{q})}} \tag{25}$$

The left-hand-side element is the benefit of joining a larger firm—higher synergy and hence better commercialization. The right-hand-side element is the cost—the inventor gets a lower equity share when combining her idea with a firm with a higher risk unrelated to innovation. The optimal firm size is

$$\tilde{q}^* = \left(\frac{(2A\lambda_0+\lambda_0^2)(\gamma(z_0)z_0)^2 b}{2A\sigma_0^2 q_0^{2b}(1-2b)}\right)^{\frac{1}{2-2b}}.$$

Proposition 1. *When $b<0.5$, $\frac{\partial\tilde{q}^*}{\partial z_0}>0$.*[19]

When b is low, the synergy does not grow unboundedly with firm size and the balancing role of a lower equity share ensures the existence and uniqueness of a solution. The model predicts that among incumbent firms, better-quality innovations are more likely to be created in larger ones. The reasons are twofold. First, better ideas benefit more from synergy. Second, they generate relatively greater innovation-related uncertainty, making inventors less vulnerable to incentive problems.

Proposition 2. *When B goes up, i.e., technology becomes more mature, a larger share of inventors opt for joining incumbent firms.*[20]

The technology stock affects the adoption friction in the incumbent firms negatively, but not the innovation utilization process in new businesses. Therefore, when technology is more matured, innovations are worth more in incumbent firms whereas their values are unchanged in new firms, resulting in incumbent firms being more attractive.

Proposition 3. *Under certain parameter assumption ($b<\frac{\min(z_0)}{\min(z_0)+\max(B)}$), there exists a cutoff $\bar{z}_0(B)$, such that all inventors with $z_0<\bar{z}_0(B)$ opt for incumbent firms.*[21]

[19] See Appendix C for proof.
[20] See Appendix C for proof.
[21] See Appendix C for proof.

The impacts of both incumbents' adoption efficiency $\gamma(z_0)$ and synergy adjusted innovation value $x(z_0, \tilde{q})$ are determined by idea quality z_0. High quality ideas suffer more loss from adoption frictions and meanwhile enjoy more synergy when implemented by an incumbent firm. If the synergy does not increase too dramatically with firm size ($b < \frac{\min(z_0)}{\min(z_0)+\max(B)}$), the adoption channel dominates and inventors with high-quality ideas opt for starting up new business.

4.2.2 The Full Model

This section describes the full model, when releasing the assumption that the second-order moment of the innovation value is inversely related to the effort e_I. A more detailed solution to the full model can be found in Section C.6 in the Appendix. Similar as in the closed-form case, the incentive constraint implies that the firm knows the optimal effort of the inventor,

$$e_I = \lambda_0 a x_0 \left(z_0, \tilde{q}\right) \nu - A a^2 \lambda_0 \mathbb{E}\left(\tilde{x}\left(z_0, \tilde{q}\right)^2\right) \nu^2. \tag{26}$$

Exerting one more unit of effort has three effects: a larger likelihood of successful innovation, a greater disutility from the effort, and a larger variance of consumption. The last effect does not show up in the closed-form case, where the inventor's effort reduces the variance of the innovation value. In the full model, an inventor strategically takes a lower level of effort e_I for any given contract.

The firm's problem is described in Equation 22. When b is relatively low, the synergy, $x_0\left(z_0, \tilde{q}\right)$, increases mildly with $\tilde{q}$. It holds numerically that the optimal stock a decreases with firm size $\tilde{q}$. The optimal compensation scheme is $(a, \tilde{T})$, where $\tilde{T}$ is determined by the zero profit condition of the Betrand competition: $\tilde{T} = -a\tilde{V}\left(\tilde{q}\right) + (1-a)\,\lambda_0 e_I x_0\left(z_0, \tilde{q}\right) \nu dt$.

Given the contracts, the inventor chooses the optimal firm size $\tilde{q}$ by maximizing her utility. In each firm, her optimal effort level is given in Equation 26. Larger firms provide better commercialization but worse incentives due to lower equity. The numerical solution shows that, among all inventors that choose to join incumbent firms, those with better ideas prefer bigger firms.

However, due to the friction in the inventor-firm matching process, only a fraction h of inventors can go to their ideal incumbent firm: the rest are assigned to firms with random size. Therefore, the innovations within a firm are composed of two distinct components: the directedly matched part and the frictionally matched part.

The Novelty Index matters for both the inventor-firm matching process and the utility obtained. At the peak of a technology wave, the technology tend to exhibit greater novelty, thereby escalating adoption costs. As a result, in incumbent firms, an innovation is worth less and the synergy effect is weaker since $\gamma(z_0)$ decreases. Inventors strategically move to smaller firms, as the advantages of larger firms are less silent. Meanwhile, systematically, inventors working within incumbent firms experience lower utility.

4.3 Starting up a New Business

In additional to joining an incumbent firm, an inventor can also start her own business. The inventor, who is risk-averse, works with a risk-neutral partner to share risk. Similarly, the inventor faces a compensation scheme $\left(a, \tilde{T}\right)$. However, the inventor is in charge of the research direction by herself. Hence, the innovation value is solely determined by her idea quality z_0. Upon successful innovation, the normalized innovation value $\tilde{z}$ is a random draw from the distribution $U\left(\left(1-\phi\right) z_0 \nu, \left(1+\phi\right) z_0 \nu\right)$. On average, a higher-quality idea yields a better outcome.

The partner's problem shares the same form as the incumbent firm's (Equation 22), with $\tilde{q} = 0$ and the average innovation value being z_0 instead of $x_0\left(z_0, \tilde{q}\right)$. The partners are assumed to get zero profit. The inventor decides her effort level by maximizing her utility.[22]

4.4 Inventor's Choice

Each inventor chooses between working in an incumbent firm (with h probability in the firm with optimal size and $1-h$ probability working in a firm of random size), and in a startup (with h_s probability starting up a new business and $1-h_s$ probability working in an incumbent firm of random size). The inventor's decision rule is:

$$\begin{aligned} u\left(z_0\right) = \max\{ & h_s u\left(c_I\left(z_0, \tilde{q}^*\right), e_I\left(z_0, \tilde{q}^*\right)\right) + \left(1-h_s\right) \int_{\tilde{q}} u\left(c_I\left(z_0, \tilde{q}\right), e_I\left(z_0, \tilde{q}\right)\right) \tilde{f}\left(\tilde{q}\right), \\ & h u\left(c_I\left(z_0, 0\right), e_I\left(z_0, 0\right)\right) + \left(1-h\right) \int_{\tilde{q}} u\left(c_I\left(z_0, \tilde{q}\right), e_I\left(z_0, \tilde{q}\right)\right) \tilde{f}\left(\tilde{q}\right)\}. \end{aligned} \tag{27}$$

where $\tilde{f}\left(\tilde{q}\right)$ is the firm size distribution endogenously determined in the equilibrium. The inventor joins a startup when it offers a higher expected utility.

[22]The exposition of the problem of starting up a business is presented in Appendix Section C.7.

4.5 Entry and Exit

A firm enters the market when it successfully creates an innovation as a startup. The amount of entry equals the amount of innovations in startups:

$$\lambda_I = \int_{z_0 \in \{\tilde{q}^* = 0\}} \lambda_0 e_I \left(z_0, \tilde{q} = 0\right) \psi \left(z_0\right) dz_0. \tag{28}$$

When it is stationary, the amount of firm that enters is the same as those who exit:

$$\tau N_f = \lambda_I. \tag{29}$$

4.6 Growth Rate

The growth is from a single source—innovation. The aggregate growth can be written as,

$$\begin{aligned} g &= \frac{\bar{q}\left(t + \Delta t\right) - \bar{q}\left(t\right)}{\bar{q}\left(t\right) \Delta t} \\ &= \frac{\int_{z_0 \in \{z_0 | \tilde{q}^* > 0\}} \left(h \lambda_0 e_I \left(z_0, \tilde{q}^*\right) x_0 \left(z_0, \tilde{q}^*\right) + \left(1 - h\right) \lambda_0 \int_{\tilde{q}} e_I \left(z_0, \tilde{q}\right) x_0 \left(z_0, \tilde{q}\right) \tilde{f}\left(\tilde{q}\right) d\tilde{q}\right) d\Psi \left(z_0\right)}{N_f} \\ &+ \frac{\int_{z_0 \in \{z_0 | \tilde{q}^* = 0\}} \left(h_s \lambda_0 e_I \left(z_0, \tilde{q}^* = 0\right) z_0 + \left(1 - h_s\right) \lambda_0 \int_{\tilde{q}} e_I \left(z_0, \tilde{q}\right) x_0 \left(z_0, \tilde{q}\right) \tilde{f}\left(\tilde{q}\right) d\tilde{q}\right) d\Psi \left(z_0\right)}{N_f}. \end{aligned} \tag{30}$$

4.7 Equilibrium

We end this section by summarizing the equilibrium. The R&D expenditure of the economy, C_I, can be written as

$$\begin{aligned} C_I = &\int_{z_0 \in \{z_0 | \tilde{q}^* > 0\}} \nu \left(h \lambda_0 e_I \left(z_0, \tilde{q}^*\right) x_0 \left(z_0, \tilde{q}^*\right) + \left(1 - h\right) \lambda_0 \int_{\tilde{q}} e_I \left(z_0, \tilde{q}\right) x_0 \left(z_0, \tilde{q}\right) \tilde{f}\left(\tilde{q}\right) d\tilde{q}\right) \psi \left(z_0\right) dz_0 \\ &+ \int_{z_0 \in \{z_0 | \tilde{q}^* = 0\}} \nu \left(h_s \lambda_0 e_I \left(z_0, \tilde{q}^* = 0\right) z_0 + \left(1 - h_s\right) \lambda_0 \int_{\tilde{q}} e_I \left(z_0, \tilde{q}\right) x_0 \left(z_0, \tilde{q}\right) \tilde{f}\left(\tilde{q}\right) d\tilde{q}\right) \psi \left(z_0\right) dz_0. \end{aligned} \tag{31}$$

It captures all transfers made to inventors. Based on Equation (35), the equilibrium output level Y is linear in $\bar{q}$

$$Y = \frac{1}{1 - \beta} N_F^{\beta} \bar{q}. \tag{32}$$

and the consumption level is

$$C_H = Y - C_I. \tag{33}$$

Definition A balanced growth path of this economy for any combination of (t, q) is the mapping between q and z_0, the allocation $\left(\left\{y_j^*\right\}_j, Y^*, C_I^*, C_H^*\right)$ the prices $\left(w^*, r^*, \left\{p_j^*\right\}_j\right)$, the growth rate g^*, the entry rate λ_I^*, and the measure of firms N_F^*, such that (1) for any $j \in [0, 1]$, y_j^* and p_j^* satisfy Equation (17); (2) the wage w^* satisfies Equation (18); (3) the interest rate r^* satisfies Equation (21); (4) the measure of the intermediate producers N_F^* satisfies Equation (29); (5) the mapping between q and z_0 is the solution of Equation (27); (6) the entry rates λ_I^* satisfy Equation (28); (7) R&D spending C_I^* satisfies Equation (31); (8) the aggregate output Y^* satisfies Equation (32); (9) the aggregate consumption C_H^* satisfies Equation (33); and (10) the steady-state growth rate g^* satisfies Equation (30).

4.8 Market Concentration

The novelty level affects the market concentration through both intensive margin and extensive margin. On the one hand, there is positive assortative matching between idea quality and firm sizes: better ideas tend to be developed in bigger firms. When there is a technology breakthrough, the technology is less mature. It implies that the adoption cost is high, and inventors systematically shift to smaller firms, which weakens the sorting between firms and ideas. On the other hand, since more inventors move to startups, there are more entries and drives down the market concentration through the extensive margin.

5 Calibration

We calibrate the model to target the average US economy from 1982 to 2016. Patents are used as a surrogate for innovations. An innovation's idea quality, denoted by z_0, and the realized value, x (z in the context of a startup), correspond to the patent's citation (scientific importance) and the patent's pecuniary value, respectively. Additionally, we assume that the idea quality z_0 follows the Pareto distribution characterized by a scale factor z_m and a shape factor α.

5.1 Identification

Parameters in the model are categorized into two groups. The first group is calibrated by a prior information from the aggregate statistics or the literature. The second group is calibrated by estimation from the micro-level data or through the model. Table 3 reports the parameters in the first group, $(\rho, \beta, \tau, A, \delta)$. The discount rate, ρ, is set to 0.02 to match the average interest rate in the sample period. The production function quality share, β, is

0.109, following Akcigit and Kerr (2018). The firm exit rate, τ, is 0.08, targeting the average exit rate in the United States during our sample period based on the Business Dynamics Statistics (BDS). The BDS data are compiled from the Longitudinal Business Database (LBD) by the Census Bureau. The risk aversion parameter, A, and the effort cost elasticity, δ, are set to be 0.5 and 1, respectively, which are commonly used in the literature (Hall and Van Reenen, 2000).

We calibrate the eight remaining parameters in the second group, (λ_0, α, z_m, ϕ, B, b, q_0, h, h_s), using the minimum distance method, inspired by Lentz and Mortensen (2008). The parameters, along with their corresponding moments are in Table 4.

Growth Rate—Innovation is the only driver of growth in this model. Therefore, the scale factor of the innovation arrival rate, λ_0, is an important determinant of the aggregate growth rate. A higher arrival rate implies a shorter average time for innovation creation, leading to a subsequently elevated aggregate growth rate. We match the aggregate growth rate generated by the model to 2.75%, the average annual growth rate in the US between 1982 and 2016.

The S.D.-to-Mean Ratio of Patent Citations—This ratio measures the dispersion in patent citations in the data. The number of patent citations captures the scientific/non-pecuniary value of patents, which reflects the idea quality z_0. We assume the idea quality follows the Pareto distribution with the shape parameter, α, and the scale parameter, z_m. α governs how dispersed the distribution is. Specifically, the s.d.-to-mean ratio of the idea distribution can be expressed as $\frac{1}{\sqrt{\alpha(\alpha-2)}}$. Although patents observed in the USPTO data are successful innovations, which is a subset of all the ideas, the s.d.-to-mean ratio of the patent citation distribution is still significantly affected by α. We derived the citation distribution by pooling all granted patents from 1976 with their citations recorded by the USPTO and calculate the s.d.-to-mean ratio. The ratio turns out to be around 2.784.

Innovation Value—The pecuniary value of innovations directly contributes to the value of firms. In the model, the pecuniary value of innovations, x (z when in a startup), is a uniform distribution with its mean depending on the underlying scientific value of the idea (i.e. idea quality), z_0. Given α, the average scientific value of ideas is governed by the scale parameter of the Pareto distribution, z_m. Therefore, we can use the average pecuniary value of patents to calibrate z_m. We adopt the same estimation method as in Kogan et al. (2017) that uses the stock market response to news about patents. The sample used extends the one in their paper and is provided by the authors. It combines patents issued to US firms from 1926 to 2022 with the stock market information from the CRSP and firm-level information from the Compustat. Admittedly, the public firm distribution is different from the rest. We use a statistical model developed in Yang (2023) to estimate the average patent value among all firms using patent value in public firms. Based on our calculation, a patent is, on average, worth 0.0255 times the average firm value. z_m is calibrated to match this number.

S.D.-to-Mean Ratio of Innovation Value conditional on Citations—The pecuniary value of innovations is based on the scientific value of ideas but is also subject to some randomnesses. The degree of randomnesses is governed by ϕ in the model. Specifically, the s.d.-to-mean ratio of the uniform distribution of the innovation pecuniary value is $\frac{\phi}{\sqrt{3}}$, conditional on the firm size. Across different firms, ϕ still significantly impact the dispersion of patent value. Exploiting the same sample used to pin down z_m, we estimate the s.d.-to-mean ratio of patent pecuniary value when controlling the number of citations of the patents. In the data, this ratio is 0.416.

Technology "Novelty" Index—The technology novelty is defined as the total forward citations over the sum of backward and forward citations of all patents granted in a year. The adoption frictions ($\gamma(z_0) = \frac{B}{B+z_0}$) of new ideas in an incumbent firm is determined by B. B corresponds to the backward citation stock in a certain period, representing the maturity of the technology to which the inventor's idea contributes. We calibrate the value of B such that the model-generated average adoption frictions ($\frac{B}{B+\int z_0 d\Psi(z_0)}$) equals to $1-$ the average "Novelty" Index between 1982 and 2016, since $\int z_0 d\Psi(z_0)$ is corresponding to the total forward citations of all ideas available in the period.[23]

Regression Coefficient of Innovation Value on Firm Size—Synergy provided by incumbent firms is governed by two parameters, b and $\tilde{q}_0$, with the former determining the elasticity of synergy with regard to the incumbent firm size and the latter determining the scale. To derive b, we take natural logarithm on both sides of the innovation value function, $\log(x_0(z_0, q)) = b\log\left(\frac{\tilde{q}}{\tilde{q}_0}\right) + \log(\gamma(z_0)) + \log(z_0)$. This equation can be mapped to the regression (Equation (6)) in Section 2.4. The coefficient of the firm size variable pins down the value of b.

New-to-Incumbent Ratio—The scale parameter in the synergy function, q_0, affects the benefit of contributing an idea to an incumbent firm compared to initiating a new venture. Therefore, it is related to inventors' choice between incumbent firms and startups. We use the "New-to-Incumbent Ratio" derived in Section 2.3.1 to calibrate q_0.

Firm Size Ratio by Fourth-to-First-Quartile of Patent Citations—The model predicts that, if inventors choose to join incumbent firms, ideally the firm size they choose increases in their idea quality. This positive sorting, nevertheless, is subject to matching frictions. When the friction is larger, the matching between inventors' idea quality and incumbent firm size is closer to random sorting, and their relationship is vaguer. To calibrate the degree

[23]The "Novelty" Index defined in Section 2.1 can be expressed as $\frac{F}{F+B}$, where F represents all the forward citations in a period. So, ($\frac{B}{B+\int z_0 d\Psi(z_0)}$) is corresponding to $\frac{B}{B+F} = 1 - \frac{F}{B+F} = 1 -$ the "Novelty" Index.

of frictions, h, we generate in the model the average firm size by patent citation quartiles given the patent is developed by incumbent firms. Then we calculate the ratio of firm size in the fourth to first quartile and match it the the data counterpart. Figure 9 shows the average firm size by the patent citation quartiles in the model and the data with the firm size normalized to 1 in the first quartile. The average firm size in the second and third quartiles are not targeted, but it turns out that they match the data well.

Citation ratio between new and incumbent firms—The model predicts that there is a threshold of idea quality above which the inventors would choose startups over incumbent firms. This implies that, on average, patent quality in startups should have higher scientific values than those in incumbents. The frictions in choosing startups, h_s, mitigates this effect. The patent citation ratio between new and incumbent firms reflects this effect.

Table 3: Parameter Values from a Priori Information

Parameter	Description	Value	Identification
ρ	Discount rate	0.02	Interest Rate
β	Production function quality share	0.109	Firm profitability
τ	Exo. exit rate	0.06	BDS
A	Risk aversion	0.5	Risk aversion
δ	Effort cost elasticity	1	Effort cost elasticity

Notes: This table shows parameter values from the literature of direct estimation.

Table 4: Parameter from the Minimum Distance Estimation

Parameter	Description	Identification
λ_0	Innovation arrival rate	Growth rate
α	Shape of idea quality distribution	S.d.-to-mean ratio of patent citations
z_m	Scale of idea quality distribution	Average innovation value
ϕ	Innovation value dispersion	S.d.-to-mean ratio of innovation value cond. on citations
B	Maturity of technology	Technology "Novelty" index
b	Exponent of the synergy function	Regression coefficient of innovation value on firm size
q_0	Scale of the synergy function	New-to-incumbent ratio
h	Matching friction (incumbent)	Firm size ratio by fourth-to-first-quartile of citations
h_s	Matching friction (startup)	Citation ratio between new and incumbent firms

Notes: Parameters in this table are jointly calibrated to minimize the distance between the model and data moments.

5.2 Estimation Results

Table 5 reports the model-generated moments and their counterparts in the data. Overall, the model matches the targeted moments closely. The resulting parameter values are reported in Table 6. Our estimates suggest that compared with startups, in terms of

Table 5: Moments

Identification Moment	Data	Model
Growth rate	0.0275	0.0293
S.d.-to-mean ratio of patent citations	2.784	2.753
Average innovation value	0.0255	0.0222
S.d.-to-mean ratio of innovation value cond. on citations	0.416	0.416
Technology "Novelty" index	0.554	0.554
Regression coefficient of innovation value on firm size	0.33	0.33
New-to-incumbent ratio	0.054	0.052
Firm size ratio by fourth-to-first-quartile of citations	1.18	1.20
Citation ratio between new and incumbent firms	1.361	1.369

Notes: This table compares the moments generated from the calibrated model and the data. In general, the model generated moments match the data well.

Table 6: Estimated Parameter Values

Parameter	Description	Value
λ_0	Innovation arrival rate	1.5
α	Shape of idea quality distribution	2.176
z_m	Scale of idea quality distribution	5.6E-3
ϕ	Innovation quality draw	0.201
B	Discount factor of idea commercialization	0.06
b	Exponent of the synergy function	0.33
q_0	Denominator of the synergy function	7.6E-4
h	Matching friction (incumbent)	0.0693
h_s	Matching friction (startup)	0.55

Notes: Parameters in this table are jointly calibrated to minimize the distance between the model and data moments.

utilizing innovations, incumbents have a non-negligible cost as the average adoption friction, $\int \gamma(z_0)dz_0 = 0.6$, is significantly below 1. In addition, synergy plays a considerable role in commercialization, since the denominator of the synergy function q_0 is as low as $7.6E-4$, and the exponent is 0.33—it means that a firm of the average size can generate 11 times value at commercialization than a startup due to the synergy effect. If we combine synergy with the adoption friction, it shrinks to 7 times.

Figure 8 shows the relationship between the optimal firm size ($\tilde{q}^*$) and the idea quality (z_0) when the inventor with the corresponding idea quality chooses to contribute her idea to an incumbent firm in an environment without matching frictions ($h = 1$). The positive relationship implies positive sorting between firms and inventors—better ideas contribute to larger firms, which allows larger firms to expand further. When the idea quality is above a threshold, the inventor would rather start a new firm. This pattern is consistent with the empirical observation that patents from startups, on average, receive more citations than

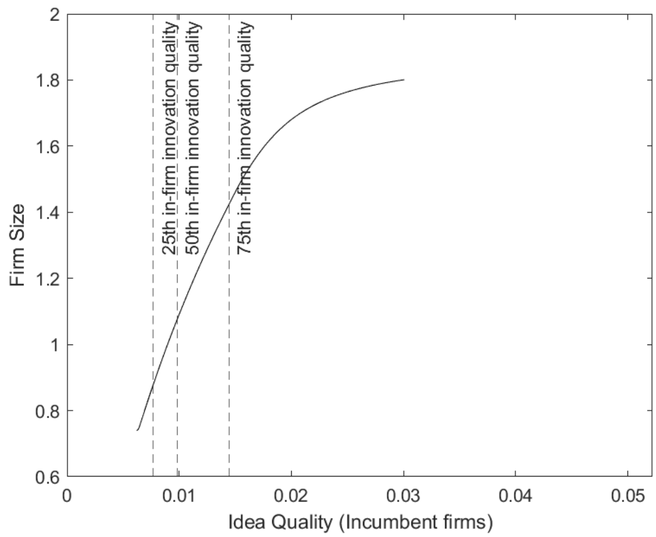

Figure 8: Estimated Mapping between Patent Citations and Incumbent Firm Size, Frictionless Matching between Inventors and Firms

Notes: This figure exhibits the estimated mapping between inventors' idea quality along the whole distribution of the idea quality when $h = 0$. The x-axis is idea quality of the inventors, and the y-axis is the corresponding optimal firm size. The optimal firm size is assumed to be 0 if the inventor chooses to form a startup. The average employment size is normalized to be one.

patents from incumbents.

The actual mapping between the firm size and idea quality if the inventor join incumbent firms is flatter than the optimal one due to the matching friction, h. In the calibrated model, only 19% inventors join firms of optimal size and the rest are assigned to incumbent firms of a random size. However, despite the matching friction, the overall relationship between the incumbent firm size and inventors' idea quality is still positive, as displayed in Figure 9.

6 Quantitative Analysis

We use the model to study the extent to which technological novelty waves shapes the market concentration through the flow of inventors' ideas. Our analysis spans the period from 1982 to 2016, encompassing three distinct technology waves illustrated in Figure 1. First, we calibrate the model to align with the average data moments between 1982 and 2016, as described in the previous section. Subsequently, we simulate the model beginning from 1986. In each year, we adjust the patent novelty stock, B, and therefore, the adoption friction such that the model-generated average adoption frictions ($\frac{B}{B+\int z_0 d\Psi(z_0)}$) matches 1-Novelty Index in the data. In another word, we fix all the parameters except the one that governs the

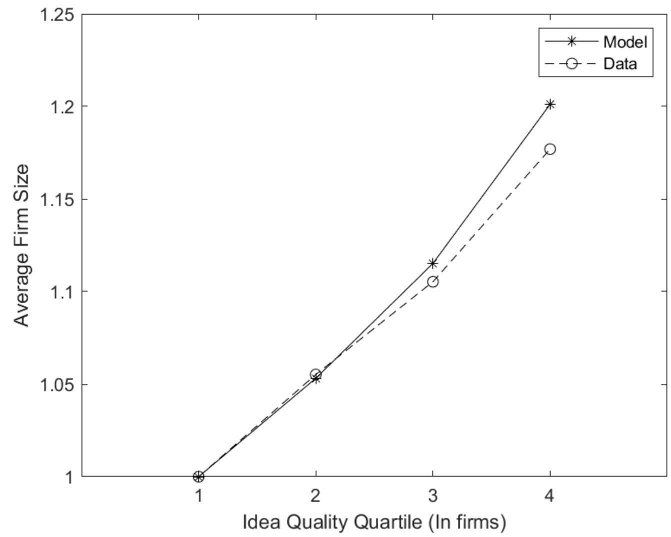

Figure 9: Estimated Mapping between Patent Citations and Incumbent Firm Size

Notes: This figure exhibits the mapping between inventors' idea quality and the firm sizes if inventors opt to develop their ideas in incumbent firms. The idea quality is measured by the number of patent citations and is classified into four quartiles. The firm size is measured by the number of employees. The average employment of firms corresponding to the first citation quartile is normalized to be one. The solid curve represents the model prediction and the dashed one is the actual data in Figure 7.

aggregate technological novelty shocks. The simulation yields the model-generated market concentration and the innovation allocation in each year. We compare the model-generated results with the actual data and calculate the similarity of them.

In the simulation exercise, we take 1986 as the benchmark year, which is the first peak of the technology wave in our sample. Each year is required to be in equilibrium but the equilibrium is not necessarily on a balanced-growth-path. In this non-stationary setting, the model state is jointly determined by two sets of state variables: the measure of firms N_f and the firm size distribution $f(q)$. Unlike the balanced-growth-path scenario, these state variables are not stationary and adapt annually according to the equilibrium. As a result, the entry-exit equality in Equation 29 no longer holds, causing a gap between entry and exit, which introduces dynamism into the system. Since even the benchmark year is not in a balanced-growth-path equilibrium, we use the net entry rate in 1986, 0.03 (from BDS), to pin down the initial measure of firms and the firm size distribution. In the following years, the corresponding firm numbers fluctuate in each year according to:

$$N_f' = N_f (1 - \tau) + \lambda_I, \tag{34}$$

where the number of entrants, λ_I, is endogenously determined by the model. When entry

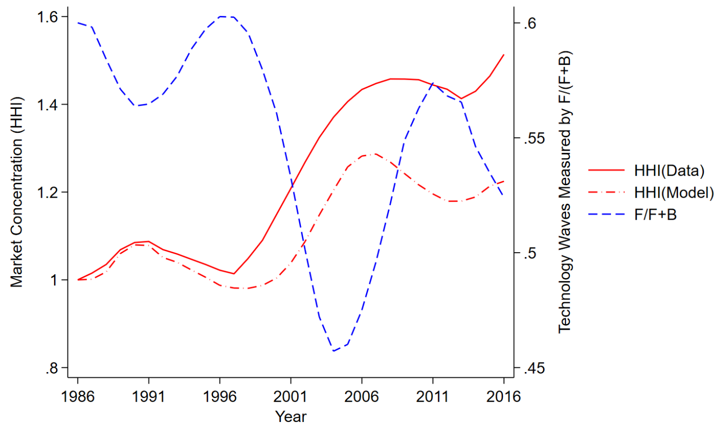

Figure 10: Technology Waves and Model Generated HHI

Notes: This figure shows the technological waves and the trend of model-generated market concentration over time. The blue curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations (same as Figure 1). The red solid curve displays the simulated HHI in each year, which is normalized by the HHI in 1986. The red dashed curve is the empirical HHI moved forward for three years, also normalized by 1986.

exceeds exit, the subsequent year starts with a higher count of firms, and conversely, if exit exceeds entry, the next year starts with fewer firms. Simultaneously, the dynamics of the inventor-firm mapping cause shifts in the firm size distribution. Starting with the firm size distribution in the previous year, we conduct a one-year simulation of firm dynamics, ultimately arriving at the firm size distribution in the subsequent year. The measure of firms, N_f, and the firm size distribution, $f(q)$, evolve over time, and consequently shape the dynamics of market concentration.

6.1 Technology Waves and the Market Concentration

The simulated evolvement of the market concentration measured by the HHI, its data counterpart, and the technological novelty waves are presented in Figure 10. The red solid curve represents the HHI measured in the data, normalized by the HHI in 1986. The red dashed curve displays the simulated HHI in each year from the model, also normalized by 1986. To show the relationship with the technological waves, the figure also plots the relative ratio of forward citations to the sum of forward and backward citations (the same as Figure1), based on the methodology defined in this paper. Our model-generated HHI follows a similar pattern to data. It is negatively correlated with the technological waves and lags behind the waves by approximately two years.

Although the calibration process does not explicitly target any market concentration

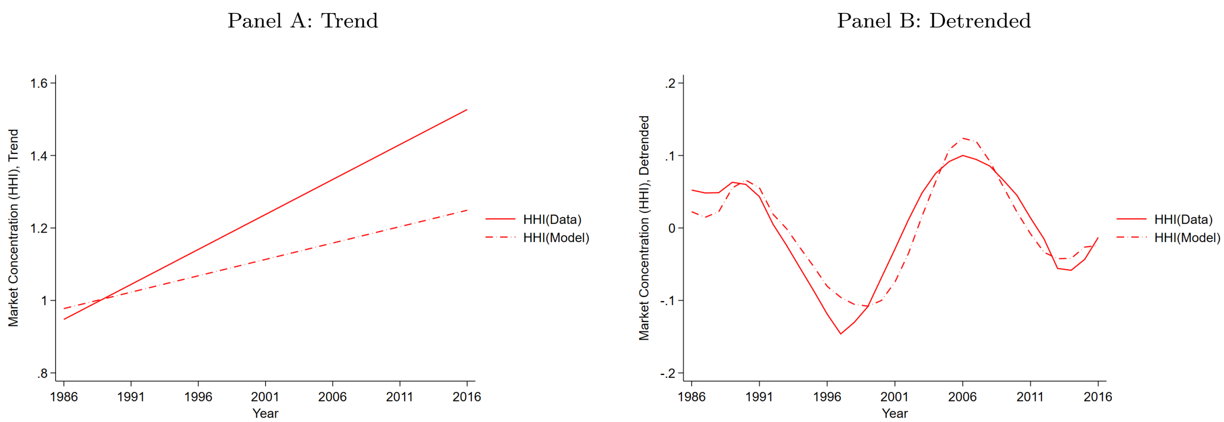

Figure 11: The Trend and Detrended Time Variations of the HHI

Table 7: Comparison between Model and Data

	No Detrend		Detrend	
	Mean	Time Trend	S.D.	Autocorr
		Panel A. HHI		
Data	1.237	0.019	0.072	0.926
Model	1.113	0.009	0.067	0.921
Model (No PAM)	1.054	0.007	0.051	0.923
		Panel B. N-I-Ratio		
Data	0.052	-5.05E-5	0.007	0.830
Model	0.060	-1.12E-4	0.019	0.914

Notes: This table shows the trend and detrended time variations of the HHI and New-to-Incumbent ratio in the data and the model.

measure, our model successfully replicates the rising trend and the waves as in the data. To separate the two, we use linear trends to fit respectively the data and the model and then subtract the linear trend to get the detrended time variations, as shown in Figure 11. Their statistics are displayed in the first two rows of the Panel A in Table 7. The average HHI in the model from 1986 to 2016 is 1.113, which is above one but a bit lower than that in the data. The linear trend has a slope of 0.009 in the model and the counterpart in the data is 0.019. This suggests that the technological novelty wave alone can generate 47.4% of the rise in market concentration in the sample period. The detrended time variations of the model closely match the data, with a correlation coefficient of 0.932. The standard deviations in the model is 0.067, close to 0.072 in the data. The first-order autocorrelation in the model and the data are respectively 0.921 and 0.926. The results imply that the technological waves is an important driving force of the detrended time variations of market concentration.

To explore the relationship between the detrended HHI and the technological novelty waves in both the model and data, this paper calculates the cross correlation between

Panel A: Correlation between the HHI and Tech Waves

Panel B: Correlation between the N-I-Ratio and Tech Waves

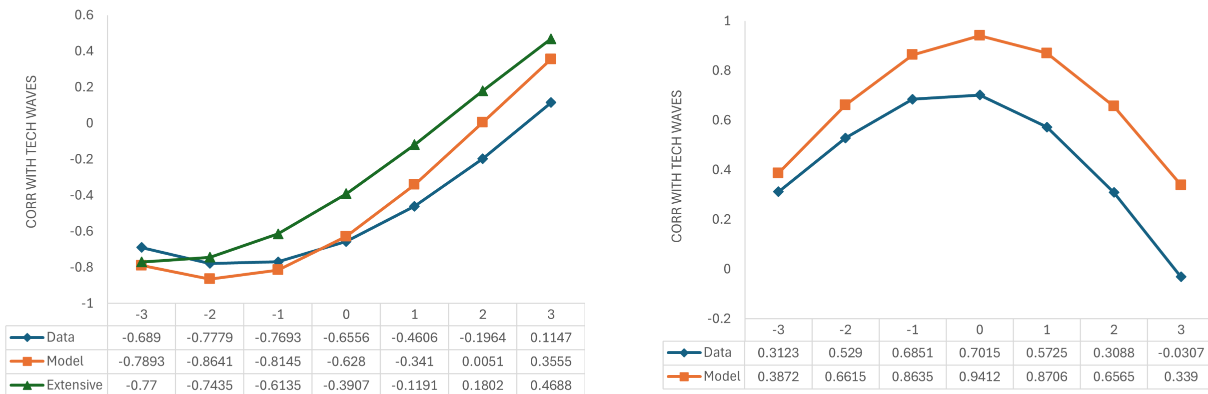

Figure 12: Cross Correlation between the Market Concentration and the Technological Waves

the former (x_t) and the latter (y_{t+k}) at different year gaps, $\text{corr}(x_t, y_{t+k})$, following the method in Stock and Watson (1999). When k is negative, the HHI is compared with the technological waves in previous years; when k is positive, the HHI is compared with technolocal waves afterwards. The results are shown in Panel A of Figure 12. In the data, the absolute magnitude of the correlation is high when k is negative, indicating that the market concentration responds to the technological waves. The highest value of the correlation occurs when $k = -2$, suggesting the responding time is around two years. The suggested responding time in the model is consistent with the data, showing the channel proposed in the model captures not only the time variations of the market concentration, but also its time lag to the technological change.

6.2 Allocation of ideas

Empirically, this paper shows that inventors are more likely to form startups when revolutionary technologies appear and join incumbent firms when technologies mature. This is repeatedly shown by the solid curve in Figure 13. The New-to-Incumbent ratio generated by the model is shown by the dashed curve in the same figure. They have nearly simultaneous waves. To further evaluate their relationship, we use linear trends to fit the two curves respectively, and then subtract them to get the detrended time variations. The correlation between the detrended model-generated and the detrended actual new-to-incumbent ratio is 0.810. Further summary statistics are diplayed in Panel B of Table 7.

The average New-to-Incumbent ratio in the model is 0.060, close to 0.052 in the data. The slope of the linear trend is $-1.12E-4$, showing a declining share of inventors starting up new businesses. This is consistent with the data qualitatively, but is larger in magnitude.

Figure 13: Technology Waves and the Share of Innovations in Startups

Notes: This figure shows the technological waves and the trend of model-generated share of innovations in startups over time. The dashed curve displays the simulated New-to-Incumbent ratio in each year whereas the solid curve shows the New-to-Incumbent ratio in the data (same as Figure 5).

The detrended variations have a larger standard deviation (0.019 compared with 0.007), and a slightly larger first-order autocorrelation (0.914 compared with 0.830). The reason of a larger amplitude of the model-generated ratio is similar to the cause of the excessive volatility of the model-predicted real gross investment per capita in real business cycle models—lack of adjustment costs. In our model, inventors are short-lived and make the choice between startups and incumbent firms without considering affiliations in the previous period. Therefore, the New-to-Incumbent ratio immediately responds to aggregate shocks. This can be supported by the cross correlation between the New-to-Incumbent ratio and the technological novelty waves, shown in Panel B of Figure 12. The correlation coefficient is the largest when the time lag, k, equals zero for both the data and the model, but the magnitude in the model, 0.9412, is much larger than that in the data, 0.7015.

6.3 Decomposition of the Intensive and Extensive Margins

The impact of the technological novelty waves extends to both the intensive margin and the extensive margin of idea allocation. The former determines the number of inventors opting for new businesses, while the latter affects, among those working in incumbent firms, the selection of firm size. Figure 14 illustrates these two dimensions of idea allocation by the optimal firm size choice at different levels of idea quality in the model when there is no frictions in the inventor-firm matching process ($h = h_s = 1$). A positive size suggests choosing an incumbent firm with that size, while zero means forming a new business. The figure draws a comparison between 1986 and 2005. Specifically, 1986 witnesses aggregate technological

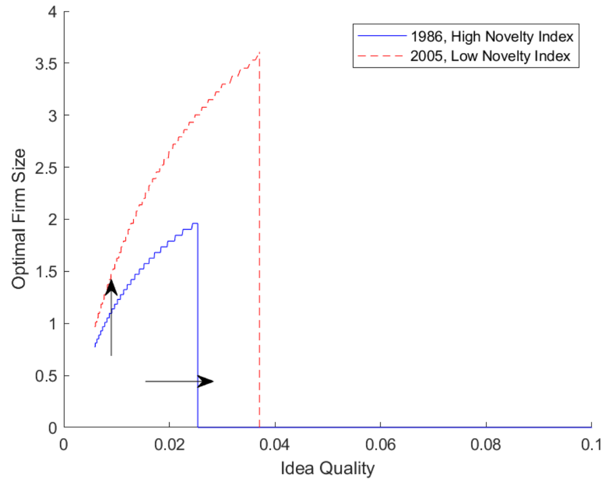

Figure 14: The Optimal Firm Size Comparison

Notes: This figure shows the optimal firm size by idea quality. The blue solid line and the red dashed line represents 1986 and 2006, respectively.

novelty, while technologies are mostly followers of existing ones in 2005. Compared with 1986 (peak), in 2005 (trough), the threshold of the idea quality for startups is higher, suggesting a larger share of inventors joining incumbent firms. Besides, among those who choose incumbent firms, there is a stronger positive assortative matching between firm size and idea quality. Both margins contribute to a higher market concentration in 2005.

To separate the two margins, we set the friction in the size selection of incumbent firms, h, to be 0.[24] In this context, inventors are restricted to altering their decision between new businesses and incumbents, without reallocating among incumbent firms. In other words, instead of choosing between a startup and an incumbent firm with a certain size, each inventor's decision set comprises solely a startup and one incumbent firm of a random size. Therefore, the intensive margin (positive assortative matching) is excluded. Following the same method as in the baseline simulation, we simulate the path when $h = 0$ starting from 1986. The model-generated HHI is shown as the dotted curve in Figure 15, together with the HHI generated by the model with both margins (the dashed curve) and the data (the solid curve). Without the intensive margin, the model-generated HHI still captures the actual waves, but has a smaller rising trend. The gap between the dashed and dotted curves represent the effect of positive assortative matching between idea quality and incumbent firm size. The figure indicates that both the extensive and intensive margins contribute to the trend and waves of the market concentration in the data.

[24]The friction when inventors choosing startups, h_s, adopts the same value as in the baseline calibrated model.

Figure 15: Technology Waves and Model Generated HHI, Extensive Margin

Notes: This figure shows the extensive margin of the model-generated market concentration over time. The dashed curve and dotted curve displays the simulated HHI and the HHI only considering extensive margin, respectively. The solid curve is the empirical HHI moved forward for three years, also normalized by 1986.

We fit a linear trend to the model-generated HHI without the intensive margin and get the detrended time variations. The summary statistics are displayed by the third row in Panel A of Table 7. The average HHI without positive assortative matching is 1.054, lower than 1.113 in the full model with both margins. The slope of the time trend is 0.007, indicating that the extensive margin explains 36.8% of the rising trend in the data, while the intensive margin explains 10.5%. The standard deviations is 0.051%, lower than the full model, indicating their fluctuations are in the same direction. The first-order autocorrelation is similar to the full model.

The cross correlation between the model-generated HHI with only the extensive margin and the technological wave is also shown in Panel A of Figure 12. The largest magnitude of the correlation appears when $k = -3$ instead of $k = -2$ as in the full model and the data. This implies that the extensive margin responds more slowly to the aggregate shocks compared to the intensive margin. This is because firm numbers are accumulated. Correspondingly, positive assortative matching between firm size and idea quality raises the speed at which market concentration responds to the technological waves.

In summary, the extensive and intensive margins jointly affect the evolvement of market concentration. (1). They both have a rising trend. (2). They fluctuates in the same directions at the advent of the technological shocks. (3). The intensive margin reduces the response time to shocks.

7 Conclusion

This paper studies how technological waves shape the market concentration, through the reallocation of inventors. This study provides empirical evidence and structural analysis showing that market concentration, measured by HHI, is inversely related to and lagged behind the technological waves. This discovery suggests the presence of a low-frequency business cycle in the economy. We explore one potential channel behind this connection: the allocation of ideas. Using the data from the Longitudinal Business Database (LBD) from the Census Bureau and the patent information from the USPTO, this paper shows that the share of patents formed in new businesses co-move closely with the technological waves. At the peaks of the technological waves, a larger share of patents are forming in new businesses, while at the troughs, a larger share of patents come from existing businesses.

This paper proposes a theoretical framework that elucidates the decision-making process of inventors regarding their choice of innovation pathways, thus providing an explanation for the observed empirical patterns. Inventors are faced with a choice between forming a new business of a random size with a partner or joining an incumbent business of a selected size. This decision hinges on a trade-off: new businesses offer better incentives and adaptability in embracing novel technologies, while incumbents possess synergies and experience in commercialization. Our model effectively captures the relationship between technological waves and market concentration, primarily through the redistribution of innovative ideas. It implies that the deceleration in the emergence of groundbreaking technologies could be a significant contributing factor to the rise in market concentration after the 2000s.

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Appendix

A Data Description

The data used in this paper includes the Longitudinal Business Database (LBD), the USPTO patent data, and the Compustat Fundamentals Annual. This section provides details about the information of the datasets and the construction of key variables.

A.1 The USPTO Patent Data

The USPTO patent data contains information of all patents issued between 1976 and 2022. It can be downloaded from the PatentsView website. For each patent, the data documents the patent type (utility, design, plant, etc.), the IPC code indicating its technological class, the grant year, and the patents it cites and it is cited. We keep all the utility patents to focus our attention to the introduction of new products and processes.

Forward Citations Forward citations are citations a focal patent receives from others. It indicates how many patents follow the focal one. This paper calculates the number of forward citations each patent gets within five years after issuance.

Backward Citations Backward citations are citations that other patents receive from the focal patent. It indicates to what extent the focal patent follows the existing technology. This paper calculates the number of backward citations by counting the number of patents cited by the focal patent that were granted within the previous five years.

The Novelty Index According to the definition in the paper, we calculate this index by dividing the number of forward citations received by all the utility patents granted in a year by the summation of the forward and backward citations of those patents. The Novelty Index by IPC is derived in a similar way for each IPC class and each year.

A.2 The Compustat Fundamentals Annual

The Compustat Fundamentals Annual contains information of all the publicly listed firms in the US. It records the firms' net sales, the number of employees, the primary industry (4-digit SIC code), and the headquarter locations of each firm. We keep all the firms that are headquartered in the US.

Primary Industry The primary industry of each firm in Compustat is based on the 4-digit SIC code assigned to each firm in the Fundamentals Annual. The code can be aggregated to different levels. Manufacturing is corresponding to SIC codes 2000-3999; utility and transportation is corresponding to SIC codes 4000-4999; wholesale trade is corresponding to SIC codes 5000-5199; retail trade is corresponding to SIC codes 5200-5999; finance is corresponding to SIC codes 6000-6999; service is corresponding to SIC codes 7000-8999.

The Herfindahl-Hirschman Index (HHI) Following the methods in Grullon, Larkin and Michaely (2019), we first calculate the HHI of each 3-digit SIC code by the squared ratios of firm net sales to the total net sales in that 3-digit industry. To get the aggregate HHI, we sum up the HHIs of all the 3-digit SIC codes and weight them by their total net sales.

A.3 The Longitudinal Business Database (LBD)

The LBD is collected by the US Census Bureau and is an establishment-level data that covers the universe of US businesses with paid employees from 1976 to 2020. The dataset assigns a firm ID to all establishments belonging to the same firm. Using the Business Dynamics Statistics of Patenting Firms (BDS-PF) patent assignee-FIRMID crosswalk from the Census, this paper links the USPTO patent data with firms in the LBD, therefore, derives all utility patents in the US that were granted to employer businesses between 1976 and 2020.

New-to-Incumbent Ratio After merging the patent data with the LBD, this paper can identify the firm each patent was granted to. If the firm is less than or equal to five years old in the patent's grant year, we indicate that the idea behind the patent was absorbed by a new firm 5 years ago. Otherwise, we indicate that the idea was absorbed by an incumbent firm 5 year ago. Then we divide the number of ideas combined with new firms by the number of ideas combined with incumbent firms to get the New-to-Incumbent Ratio.

Firm Size The LBD documents the number of employees each firm hires in each year. We deriving the mapping between patent forward citations and incumbent firm size, we use the number of employees as a proxy for size.

B More Empirical Evidence

B.1 Empirical Patterns without Smoothing

Figure 16 and Figure 17 present the patterns of the technological waves, HHI, and the New-to-Incumbent ratio without smoothing techniques. The negative correlation between the HHI and the Novelty Index, as well as the positive correlation between the New-to-Incumbent ratio and the Novelty Index, remain prominent.

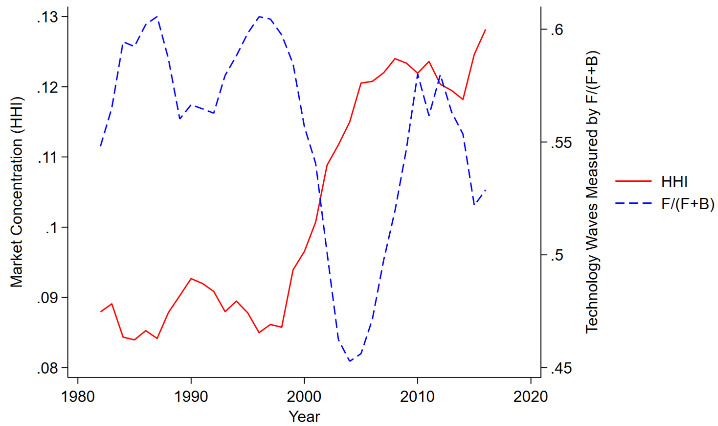

Figure 16: Technological Waves and Market Concentration without Smoothing

Notes: This figure shows the technological waves and the trend of market concentration over time. The blue dashed curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations. The red solid curve displays the HHI in each year, which is the weighted average of the industry-level HHI in each year. The weight is the total sales of firms in each industry. The two curves have different y-axises, which are shown respective on the left and right.
Sources: Compustat Fundamental Annuals and USPTO patent and citation data.

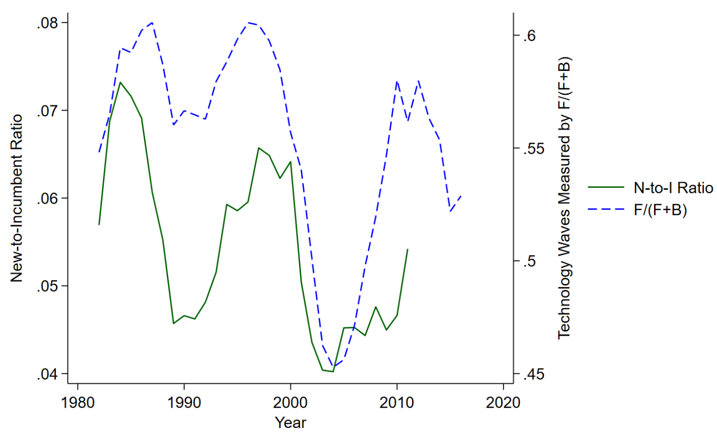

Figure 17: Technological Waves and Idea Allocation without Smoothing

Notes: This figure shows the technological waves and the idea allocation between new and incumbent firms over time. The blue dashed curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations. The green solid curve displays the "New-to-Incumbent Ratio" defined in the paper, capture where new ideas contribute their value. The two curves have different y-axises, which are shown respective on the left and right.
Sources: Longitudinal Business Database (LBD) and USPTO patent and citation data.

B.2 Technological Waves by Technological Field

The "Novelty" index across the nine technological fields is shown in Figure 18. The index is based on the same algorithm as in Equation 1 except that the forward and backward citations are aggregated across each of the 1-digit IPC code. The top three fields with the highest "Novelty" index are Human Necessities, Physics, and Electricity at the first peak; Electricity, Physics, and Human Necessities at the second peak; Human Necessities, Chemistry and Metallurgy, and Mechanical Engineering etc. at the third peak.

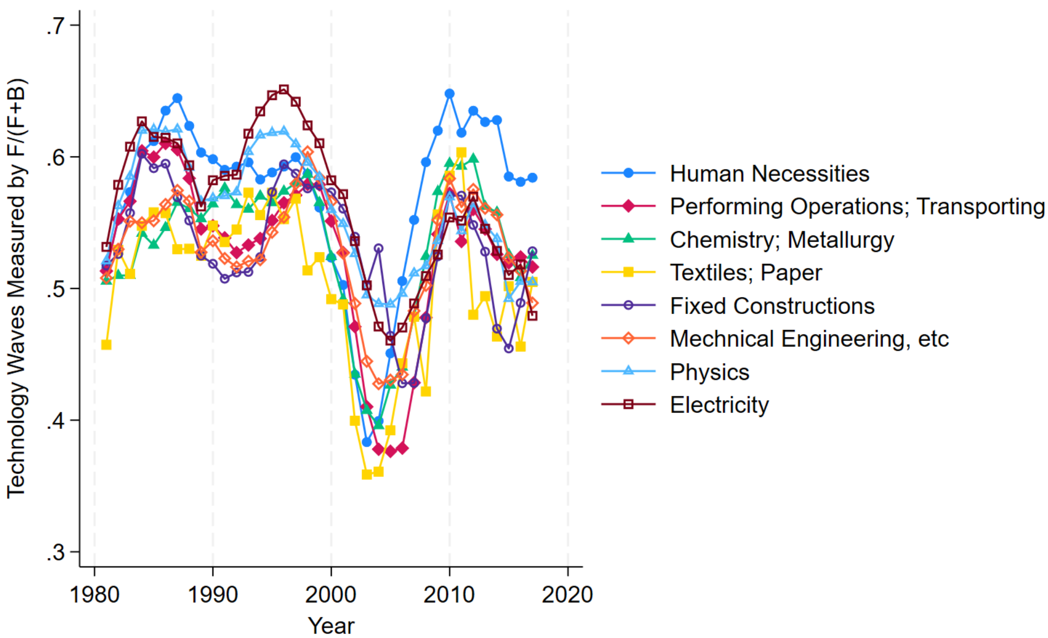

Figure 18: Technological Waves by Technological Fields

Notes: This figure shows the technological waves by the nine technological fields between 1981 and 2017. The nine fields are defined by the 1-digit IPC code. The technological waves are measured by the "Novelty" index as defined by Equation 1 in the paper.
Sources: USPTO patent and citation data.

B.3 Novelty Index in Europe

To calculate the Novelty Index for European countries with intensive patenting activities, we use data from PATSTAT (Patent Statistical Database), a comprehensive global dataset maintained by the European Patent Office (EPO). PATSTAT provides detailed bibliographic data on patents from various patent offices worldwide, with a particular focus on those filed through the EPO. We restrict our sample to patents with inventors based in European countries. The six countries with the highest number of patent issuances between 1982 and

2016 are Germany, France, the United Kingdom, Italy, Switzerland, and the Netherlands. Using the definition of the Novelty Index as outlined in Equation 1, we calculate the technological waves in these six countries and present them in Figure 19. Across all six countries, we observe an overall declining trend in technological novelty, with common peaks in the mid-1980s and early 2010s. Italy also experienced a distinct peak in the mid-1990s. In general, the technological trends in these European countries with the highest patenting activity mirror those observed in the U.S.[25]

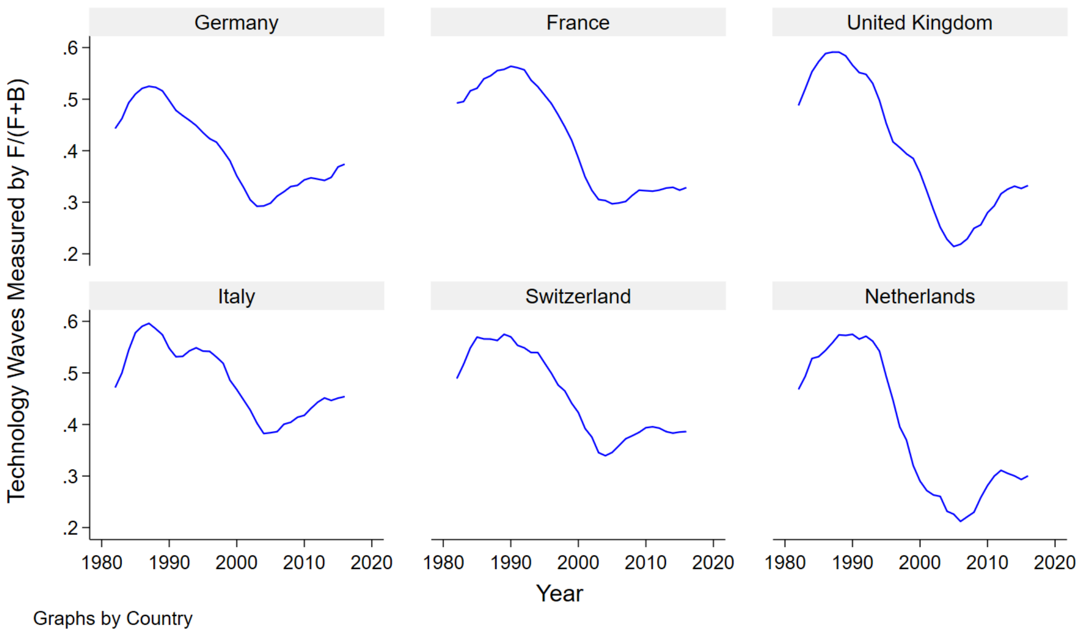

Figure 19: Technological Waves in European Countries

Notes: This figure shows the technological waves in six European countries with the highest number of patent issuances between 1982 and 2016. The technological waves are measured by the “Novelty” index as defined by Equation 1 in the paper.
Sources: PATSTAT (Patent Statistical Database).

B.4 Relationship with the Technology Waves

Table 8 exhibits the time trend of the technological novelty waves, the market concentration measured by the HHI, and the New-to-Incumbent Ratio of idea allocations (Panel A). It also displays the cross correlation of the two latter time series with the technological waves at different year gaps (Panel B). The time trend is derived by fitting a linear trend to the

[25] Similar results can be obtained by calculating the Novelty Index using Google Patents data, as cleaned by Ayerst et al. (2023). These results are available upon request.

focal time series and taking its slope. The cross correlations are obtained by calculating the correlation coefficients of the detrended time series when the year gaps of the two series are respectively -3, -2, -1, 0, 1, 2, 3. The detrending process subtracts the linear trend from the original time series. The cross correlations capture not only the co-movement of the different time series, but also the relative timing of their movements. The first row of each panel shows the statistics for the whole sample; the subsequent rows are statistics by major industries according to the Standard Industrial Classification (SIC) code or technological fields according to the International Patent Classification (IPC).

Table 8: Time Trend and Cross Correlation

	Time Trend		Detrended Cross Correlation						
						Panel A. HHI			
	Tech Wave	HHI	$k=-3$	$k=-2$	$k=-1$	$k=0$	$k=1$	$k=2$	$k=3$
All	-0.002	0.001	-0.683	-0.770	-0.763	-0.654	-0.424	-0.146	0.145
Mining & Construction	-0.002	0	-0.747	-0.782	-0.688	-0.500	-0.279	-0.046	0.189
Manufacturing	-0.002	0.001	-0.226	-0.475	-0.637	-0.692	-0.747	-0.663	-0.460
Transportation & Utilities	-0.001	0.001	-0.197	-0.132	-0.041	0.043	0.330	0.523	0.599
Wholesale & Retail Trade	-0.002	0.005	-0.495	-0.483	-0.432	-0.344	-0.195	-0.039	0.117
Finance	-0.003	0	-0.330	-0.339	-0.330	-0.272	-0.074	0.107	0.210
Services	-0.001	0.004	0.255	0.366	0.457	0.539	0.654	0.734	0.771
						Panel B. N-I-Ratio			
	Tech Wave	N-I-Ratio	$k=-3$	$k=-2$	$k=-1$	$k=0$	$k=1$	$k=2$	$k=3$
All	-0.002	-0.001	0.107	0.314	0.504	0.612	0.536	0.317	-0.001
Human Necessities	-0.001	-0.001	0.539	0.557	0.514	0.402	0.199	-0.063	-0.361
Performing Operations	-0.003	-0.001	0.117	0.210	0.283	0.301	0.191	0.017	-0.201
Chemistry; Metallurgy	-0.001	0.001	0.239	0.231	0.211	0.163	0.049	-0.128	-0.349
Textiles; Paper	-0.002	-0.001	0.458	0.451	0.462	0.417	0.371	0.270	0.189
Fixed Construction	-0.002	0	0.211	0.331	0.397	0.386	0.297	0.215	0.161
Mechanical Engineering	-0.001	0	-0.456	-0.505	-0.467	-0.358	-0.208	-0.061	0.059
Physics	-0.003	-0.001	-0.156	0.016	0.212	0.343	0.284	0.068	-0.207
Electricity	-0.003	-0.002	0.373	0.465	0.540	0.580	0.552	0.367	0.070

Notes: This table shows the trends of the technological waves, HHI, New-to-Incumbent ratio and the detrended cross correlations among them. The trend is derived by running linear regressions of the focal time series on year and taking the coefficient; the cross correlations are derived by computing the correlation coefficients at different year gaps of the detrended time series.

B.5 Alternative Measures of Market Concentration

The main text of this paper uses the Herfindahl-Hirschman Index to measure market concentration. It captures the whole distribution of firm sales in the economy, but the limitation is that it is based on only publicly listed firm. An alternative measure of market

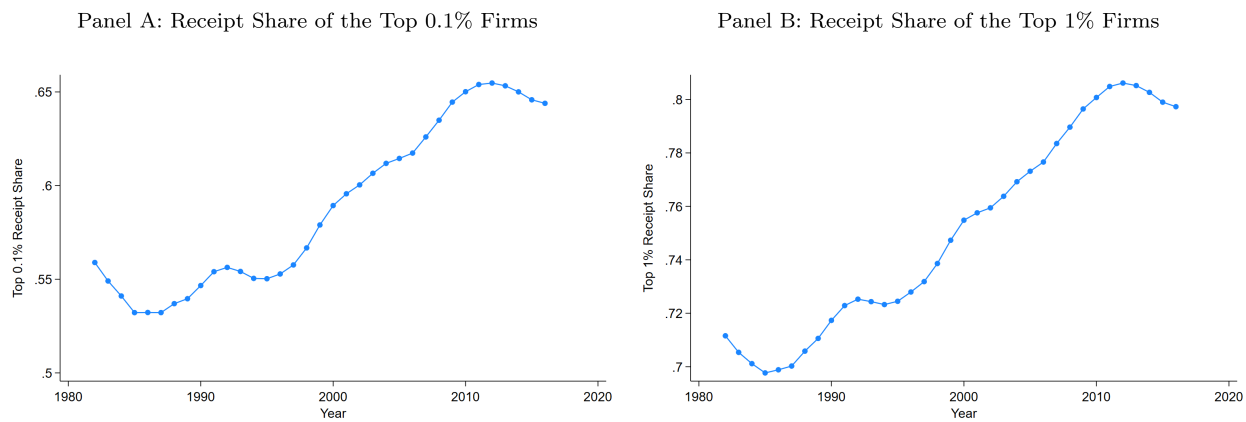

Figure 20: Receipt Shares by Top firms

Notes: This figure shows the three-year moving average of the receipt share of the top 0.1% (Panel A) and 1% firms (Panel B). The receipt shares are from the cleaned data series by Kwon, Ma and Zimmermann (2023), which is posted on `https://businessconcentration.com/`. The data source is the Statistics of Income (SOI) and the associated Corporation Source Book published annually by the IRS. Their statistics cover the whole population of US corporations.
Sources: `https://businessconcentration.com/`.

concentration is the share of sales by the top firms. This paper adopts the cleaned data series by Kwon, Ma and Zimmermann (2023) to calculate respectively the three-year moving average of the receipt share of the top 0.1% and 1% firms. The top shares are generated by the IRS data, which covers a more comprehensive set of firms. So, it can be used as a complement to the HHI measure in the paper. As displayed in Figure 20, the top shares exhibit increasing trends in general but with fluctuations. The peaks and troughs of the fluctuations appear nearly simultaneously with the HHI measured in this paper, showing the robustness of the market concentration patterns shown in the paper.

B.6 Patents' Economic Value Regression

Table 9 uses firm sales as a proxy for size and shows the results of the regression on patents' economic value (Equation 6). The coefficients are close to those in Table 2, which uses employment as a measure of firm size. Notably, the coefficient for the firm size variable is nearly identical in both tables, indicating a robust estimation of the elasticity of synergy with respect to incumbent firm size in the calibration.

	Ln(Patent Economic Value)					
	(1)	(2)	(3)	(4)	(5)	(6)
Ln(1+Firm Sales)	0.331***		0.331***		0.331***	
	(0.0142)		(0.0142)		(0.0142)	
Ln(1+Citations)	0.0809***	0.00273***	0.236***	0.0131**	0.192**	0.0114**
	(0.00535)	(0.000583)	(0.0644)	(0.00570)	(0.0827)	(0.00499)
Ln(1+Citations)×FB Ratio			-0.285**	-0.0191*		
			(0.118)	(0.0106)		
Ln(1+Citations)×IPC FB Ratio					-0.205	-0.0161*
					(0.147)	(0.00906)
Year Fixed Effect	Y	Y	Y	Y	Y	Y
Year×IPC Fixed Effect	Y	Y	Y	Y	Y	Y
Year×Firm Fixed Effect	N	Y	N	Y	N	Y
Observations	1,118,163	1,107,618	1,118,163	1,107,618	1,118,059	1,107,513
R-squared	0.403	0.882	0.403	0.882	0.403	0.882

Notes: Standard errors are clustered at the year level. Columns (1)-(2) exclude the technological wave measure and focus solely on the property of the patents and firms. Columns (3)-(4) show coefficients of the regression equation (6). Columns (5)-(6) replace the yearly Novelty Index by the year-by-IPC Novelty Index. The regressions control for year fixed effects and year by patent technology class fixed effects across all specifications. The year by firm fixed effects are controlled in columns (2), (4), and (6). *** Significant at the 1 percent level; ** Significant at the 5 percent level; * Significant at the 10 percent level.

Table 9: Factors of Patents' Economic Value for Incumbent Firms

C Model and Proof

C.1 Production

The production sector features two types of firms: a representative final goods producer and intermediate goods producers. The final good producer assembles intermediate goods, denoted by j within the range $[0, N_F]$, to produce final goods. It chooses $\{y_j\}_j$ to maximize its profit using the technology described in Section 3.2. The final goods producer's problem can be written as:

$$\max_{\{y_j\}} \frac{1}{1-\beta} \int_0^{N_F} q_j^{\beta} y_j^{1-\beta} dj - \int_0^{N_F} y_j p_j dj. \tag{35}$$

The first-order condition

$$p_j = q_j^{\beta} y_j^{\beta}$$

yields the demand function for goods produced by intermediate firms.

The intermediate goods are produced by their corresponding firm $j \in [0, N_F]$ using only labor $y_j = \bar{q} l_j$, where $\bar{q} = \frac{1}{N_F} \int_0^{N_F} q_j dj$ represents the average quality, and l_j is the labor input. Intermediate good producers engage in monopolistic competition, optimizing their profit by choosing l_j, p_j, y_j, given the wage level w:

$$\begin{aligned} &\max_{l_j,p_j,y_j} y_j p_j - w l_j. \\ &s.t.\ y_j = \bar{q} l_j \\ &\quad\quad p_j = q_j^{\beta} y_j^{-\beta} \end{aligned} \tag{36}$$

The labor market clears, which derives that $\frac{\int_0^{N_F} q_j \left(\frac{\bar{q}(1-\beta)}{w}\right)^{\frac{1}{\beta}} dj}{\bar{q}} = 1$.

The value function of an intermediate firm q at time t is a linear function of firm size q.

$$V(q,t) = \nu q. \tag{37}$$

where $\nu = \frac{\beta}{(r+\tau)N_F^{1-\beta}}$.

Proof. The value function of an intermediate firm q at time t can be written as

$$\begin{aligned} V(q,t) = &\int_t^{\infty} e^{-(r+\tau)(s-t)} \beta q / N_F^{1-\beta} ds \\ = &\nu q. \end{aligned}$$

□

When working in firm $\tilde{q}$, inventor z_0 receives consumption:

$$c_I\left(a, \tilde{q}, \tilde{T}\right) = a\left(\tilde{V}\left(\tilde{q}\right) + \tilde{x}\left(z_0, \tilde{q}\right)\nu \mathbb{1}_{\mathcal{S}}\right) + \tilde{T}$$

where $\mathcal{S}$ denotes the event that the inventor successfully creates an innovation. The expected consumption is

$$\mathbb{E}\left(c_I\right) = a\left(\mathbb{E}\tilde{V}\left(\tilde{q}\right) + \lambda_0 e_I x_0\left(z_0, \tilde{q}\right)\nu dt\right) + \tilde{T},$$

and the associated variance is

$$\text{Var}\left(c_I\left(a, \tilde{q}, \tilde{T}\right)\right) = a^2\left(\underbrace{\tau \tilde{q}^2 \nu^2 dt}_{\text{Var}(\tilde{V}(\tilde{q}))} + \underbrace{\lambda_0 e_I \mathbb{E}\left(\tilde{x}\left(z_0, \tilde{q}\right)^2\right)\nu^2 dt}_{\text{Var(innovation)}}\right).$$

Proof. Inventor's consumption is:

$$c_I\left(a, \tilde{q}, \tilde{T}\right) = a\left(\tilde{V}\left(\tilde{q}\right) + \tilde{x}\left(z_0, \tilde{q}\right)\nu \mathbb{1}_{\mathcal{S}}\right) + \tilde{T}$$

where $\mathcal{S}$ denotes the event that the inventor successfully creates an innovation:

$$\mathbb{1}_{\mathcal{S}} = \begin{cases} 1, & \Pr = \lambda_0 e_I dt \\ 0, & \Pr = 1 - \lambda_0 e_I dt \end{cases}$$

The expected consumption is:

$$\begin{aligned} \mathbb{E}\left(c_I\right) =& a\mathbb{E}\left(\tilde{V}\left(\tilde{q}\right) + \tilde{x}\left(z_0, \tilde{q}\right)\nu \mathbb{1}_{\mathcal{S}}\right) + \mathbb{E}\left(\tilde{T}\right) \\ =& a\left(\mathbb{E}\left(\tilde{V}\left(\tilde{q}\right)\right) + \mathbb{E}\left(\tilde{x}\left(z_0, \tilde{q}\right)\nu \mathbb{1}_{\mathcal{S}}\right)\right) + \left(\tilde{T}\right) \end{aligned}$$

Upon the creation of an innovation, its value is a random draw from a distribution which is independent of its realization probability, yielding:

$$\begin{aligned} \mathbb{E}\left(\tilde{x}\left(z_0, \tilde{q}\right)\nu \mathbb{1}_{\mathcal{S}}\right) =& \mathbb{E}\left(\tilde{x}\left(z_0, \tilde{q}\right)\nu\right)\mathbb{E}\left(\mathbb{1}_{\mathcal{S}}\right) \\ =& \tilde{x_0}\left(z_0, \tilde{q}\right)\nu\lambda_0 e_I dt \end{aligned}$$

Therefore:

$$\mathbb{E}\left(c_I\right) = a\left(\mathbb{E}\tilde{V}\left(\tilde{q}\right) + \lambda_0 e_I x_0\left(z_0, \tilde{q}\right)\nu dt\right) + \tilde{T},$$

The variance in consumption is

$$\begin{aligned} \operatorname{Var}\left(c_I\left(a, \tilde{q}, \tilde{T}\right)\right) =& \operatorname{var}\left(a\left(\tilde{V}\left(\tilde{q}\right) + \tilde{x}\left(z_0, \tilde{q}\right)\nu \mathbb{1}_{\mathcal{S}}\right) + \tilde{T}\right) \\ =& a^2\left(\operatorname{var}\left(\tilde{V}\left(\tilde{q}\right)\right) + \operatorname{var}\left(\tilde{x}\left(z_0, \tilde{q}\right)\nu \mathbb{1}_{\mathcal{S}}\right)\right), \end{aligned}$$

because the firm value prior to innovation, $\tilde{V}\left(\tilde{q}\right)$, is independent of innovation and the wage $\tilde{T}$ is constant.

The uncertainty in the first component solely comes from exogenous exit:

$$\tilde{V}\left(\tilde{q}, t\right) = \begin{cases} 0, & \Pr = \tau dt \\ \tilde{V}\left(\tilde{q}, t - dt\right), & \Pr = 1 - \tau dt \end{cases}$$

So, the variance of the firm value can be rewritten as:

$$\begin{aligned} \operatorname{var}\left(\tilde{V}\left(\tilde{q}\right)\right) =& \tau\tilde{V}\left(\tilde{q}\right)^2 dt \\ =& \tau\tilde{q}^2\nu^2 dt \end{aligned}$$

The uncertainty in the innovation process is:

$$\begin{aligned}\text{var}\left(\tilde{x}\left(z_0,\tilde{q}\right)\nu\mathbb{1}_{\mathcal{S}}\right) =&\mathbb{E}\left(\left(\tilde{x}\left(z_0,\tilde{q}\right)\nu\mathbb{1}_{\mathcal{S}}\right)^2\right)-\mathbb{E}\left(\left(\tilde{x}\left(z_0,\tilde{q}\right)\nu\mathbb{1}_{\mathcal{S}}\right)\right)^2\\ =&\lambda_0 e_I\nu^2 dt\mathbb{E}\left(x\left(z_0,\tilde{q}\right)^2\right)-\left(\lambda_0 e_I\nu^2 dt\mathbb{E}\left(x\left(z_0,\tilde{q}\right)\right)\right)^2\\ =&\lambda_0 e_I\nu^2 dt\mathbb{E}\left(x\left(z_0,\tilde{q}\right)^2\right)\end{aligned}$$

when $dt\to 0$. Hence, the variance in consumption is:

$$\text{Var}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right)=a^2\left(\underbrace{\tau\tilde{q}^2\nu^2 dt}_{\text{Var}(\tilde{V}(\tilde{q}))}+\underbrace{\lambda_0 e_I\mathbb{E}\left(\tilde{x}\left(z_0,\tilde{q}\right)^2\right)\nu^2 dt}_{\text{Var(innovation)}}\right).$$

□

C.2 Closed Form Model

The firm's problem in Equation 22 can be rewritten as:

$$\begin{aligned}&\max_a\left(\tilde{V}\left(\tilde{q}\right)+\lambda_0 e_I x_0\left(z_0,\tilde{q}\right)\nu dt\right)-\bar{u}\left(z_0\right)\\ &\quad-Aa^2\left(\sigma_0^2\left(\tilde{q}\right)dt+\lambda_0 x_0\left(z_0,\tilde{q}\right)^2\nu^2 dt\right)-\frac{1}{2}e_I^2\\ &\text{st}\, e_I=\lambda_0 a x_0\left(z_0,\tilde{q}\right)\nu\end{aligned}\tag{38}$$

Putting the expression of e_I into the maximization problem and taking the FOC with regard to the equity share, a, derives,

$$\begin{aligned}a^*=&\frac{\lambda_0^2 x_0\left(z_0,\tilde{q}\right)^2\nu^2}{\lambda_0^2 x_0\left(z_0,\tilde{q}\right)^2\nu^2+2A\left(\sigma_0^2\left(\tilde{q}\right)+\lambda_0 x_0\left(z_0,\tilde{q}\right)^2\nu^2\right)}\\ =&\frac{1}{1+2\frac{A}{\lambda_0}\left(\frac{\tau\tilde{q}^2}{\lambda_0 x_0(z_0,\tilde{q})^2}+1\right)}\end{aligned}\tag{39}$$

Upon reviewing all contracts, an inventor with idea quality z_0 chooses which firm $\tilde{q}$ to work for by maximizing her utility:

$$\begin{aligned}\max_{\tilde{q}} u\left(c_I\left(a,\tilde{q},\tilde{T}\right),e_I\right)=&\mathbb{E}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right)-A\text{Var}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right)-R\left(e_I\right)\\ &\text{st}\, a=a^*\left(\tilde{q}\right)\\ &\tilde{T}=\tilde{T}^*\left(\tilde{q}\right)\end{aligned}\tag{40}$$

Putting the expression of the optimal equity level, $a^*(\tilde{q})$, and $\tilde{T}^*(\tilde{q}) = -a^*\tilde{V}(\tilde{q}) + (1-a^*)\lambda_0 e_I x_0(z_0,\tilde{q})\nu dt$ into the maximization problem and solving the first-order condition,

$$\frac{\partial x_0(z_0,\tilde{q})}{\partial \tilde{q}} = \frac{2A\sigma_0^2}{4A\sigma_0^2\frac{\tilde{q}}{x_0(z_0,\tilde{q})} + \frac{2A\lambda_0+\lambda_0^2}{\tilde{q}/x_0(z_0,\tilde{q})}}, \tag{41}$$

derive the optimal firm size,

$$\tilde{q}^* = \left(\frac{(2A\lambda_0+\lambda_0^2)(\gamma(z_0)z_0)^2 b}{2A\sigma_0^2 q_0^{2b}(1-2b)}\right)^{\frac{1}{2-2b}}.$$

The left-hand side and right-hand side of the first-order condition when $b < 0.5$ are shown in Figure 21.

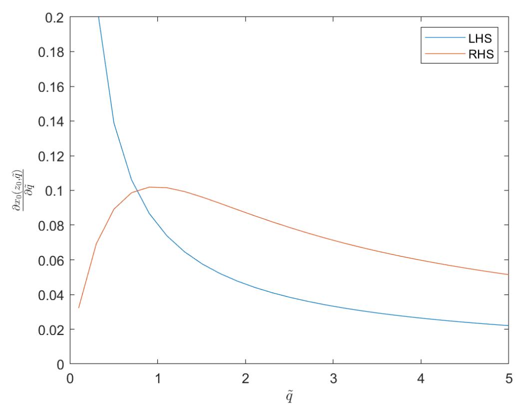

Figure 21: FOC Condition

Notes: This figure shows respectively the left-hand side (the blue curve) and the right-hand side (the red curve) of Equation 41. When $b < 0.5$, there exists a unique intersection.

C.3 Proof for Proposition 1

Proof.

$$\frac{\partial \tilde{q}^*}{\partial z_0} = \frac{\tilde{q}^*}{\gamma(z_0)z_0(1-b)}\frac{\partial(\gamma(z_0)z_0)}{\partial z_0}$$
$$= \frac{\tilde{q}^*}{\gamma(z_0)z_0(1-b)}\frac{B^2}{(B+z_0)^2}$$

When $b < 0.5$, the optimal size increases in the inventor's idea quality, z_0. $\square$

C.4 Proof for Proposition 2

Proof. The highest expected utility $u_N(z_0)$ an inventor z_0 can obtain when working in a new firm is

$$\begin{aligned} u_N(z_0) =& u\left(c_I(z_0,\tilde{q}), e_I(z_0,\tilde{q})\right) \\ =& \frac{1}{2}\frac{\lambda_0}{\lambda_0+2A}\lambda_0^2 z_0^2, \end{aligned}$$

which is unrelated to the technology wave indicator B. However, the highest expected utility $u_I(z_0)$ an inventor z_0 can obtain when working in an incumbent firm depends on B:

$$\begin{aligned} u_I(z_0) =& u\left(c_I(z_0,q^*), e_I(z_0,q^*)\right) \\ =& \frac{1}{2}\lambda_0^2 \frac{\left(\frac{2A\lambda_0+\lambda_0^2}{2(1-2b)A\sigma_0^2}b\right)^{\frac{b}{1-b}}}{\left(1+\frac{2A(1-b)+\lambda_0 b}{\lambda_0(1-2b)}\right)q_0^{\frac{2b}{1-b}}}\gamma(z_0)^{\frac{2}{1-b}} z_0^{\frac{2b}{1-b}} z_0^2 \\ =& \frac{1}{2}\lambda_0^2\hat{q}_0\gamma(z_0)^{\frac{2}{1-b}} z_0^{\frac{2b}{1-b}} z_0^2, \end{aligned}$$

where $\hat{q}_0$ is a parameter ($\hat{q}_0 = \frac{\left(\frac{2A\lambda_0+\lambda_0^2}{2(1-2b)A\sigma_0^2}b\right)^{\frac{b}{1-b}}}{\left(1+\frac{2A(1-b)+\lambda_0 b}{\lambda_0(1-2b)}\right)q_0^{\frac{2b}{1-b}}}$). The utility $u_I(z_0)$ is positively associated with $\gamma(z_0)$, which increases in B, implies that $\frac{\partial u_I(z_0)}{B} > 0$

An inventor decides whether to join a startup by comparing the incumbent-startup utility $u_I(z_0) - u_N(z_0)$ and zero. The utility gap increases in B, meaning that a larger share of inventors would choose incumbent firms when B goes up. □

C.5 Proof for Proposition 3

Proof. An inventor decides whether to join a startup by comparing the highest utility offered by incumbents $u_I(z_0)$ and startups $u_N(z_0)$. When $\gamma(z_0)^{\frac{2}{1-b}} z_0^{\frac{2b}{1-b}} < \frac{\lambda_0}{\hat{q}_0(\lambda_0+2A)}$, $u_I(z_0) < u_N(z_0)$, inventor chooses to join an incumbent firm. If $b < \frac{\min(z_0)}{\min(z_0)+\max(B)}$, $b(z_0+B)-z_0 < 0$ always holds:

$$\frac{\partial\left(\gamma(z_0) z_0^b\right)^{\frac{2}{1-b}}}{\partial z_0} = \frac{2Bz_0^{b-1}}{1-b}\left(\gamma(z_0) z_0^b\right)^{\frac{1+b}{1-b}}\frac{b(z_0+B)-z_0}{z_0+B} < 0,$$

since $\gamma(z_0) = \frac{B}{B+z_0}$. $\gamma(z_0)^{\frac{2}{1-b}} z_0^{\frac{2b}{1-b}}$ monotonically decreases in z_0, when holding B constant. It implies there exists a cutoff $\bar{z}_0(B)$, when $z_0 > \bar{z}_0(B)$,

$$\gamma(z_0)^{\frac{2}{1-b}} z_0^{\frac{2b}{1-b}} < \frac{\lambda_0}{\hat{q}_0(\lambda_0 + 2A)}$$

always holds, and hence $u_I(z_0) < u_N(z_0)$, inventors opts in new businesses instead of incumbent firms. □

C.6 Full Model

The firm's problem in Equation 22 becomes

$$\begin{aligned} \max_a & \left(\tilde{V}(\tilde{q}) + \lambda_0 e_I x_0(z_0, \tilde{q})\nu dt\right) \\ & - Aa^2\left(\sigma_0^2(\tilde{q})\,dt + \lambda_0 e_I \mathbb{E}\left(\tilde{x}(z_0, \tilde{q})^2\right)\nu^2 dt\right) - R(e_I) \\ \text{st } & e_I = \lambda_0 a x_0(z_0, \tilde{q})\nu - Aa^2\lambda_0\mathbb{E}\left(\tilde{x}(z_0, \tilde{q})^2\right)\nu^2 \end{aligned} \tag{42}$$

Given the contracts, inventor chooses which firm $\tilde{q}$ to work for by maximizing her utility. In each firm, her optimal effort level in given in Equation 26.

$$\begin{aligned} \max_{\tilde{q}} u\left(c_I\left(a, \tilde{q}, \tilde{T}\right), e_I\right) =& a(z_0, \tilde{q})\left(\tilde{V}(\tilde{q}) + \lambda_0 e_I x_0(z_0, \tilde{q})\nu dt\right) + \tilde{T} \\ & - Aa(z_0, \tilde{q})^2\left(\sigma_0^2(\tilde{q})\,dt + \lambda_0 e_I \mathbb{E}\left(x(z_0, \tilde{q})\right)^2 \nu^2 dt\right) - R(e_I) \\ \text{st } e_I =& \lambda_0 a(z_0, \tilde{q})\, x_0(z_0, \tilde{q})\nu - Aa(z_0, \tilde{q})^2 \lambda_0 \mathbb{E}\left(\tilde{x}(z_0, \tilde{q})^2\right)\nu^2 \end{aligned}$$

The firm-level innovation arrival rate can be written as:

$$\lambda_q(\tilde{q}) = \frac{h\lambda_0 e_I(z_0^*, \tilde{q})\,\psi(z_0^*)\,dz_0^* + (1-h)\,\tilde{f}(\tilde{q})\,dq \int_{z_0 \in \{z_0 | q^*(z_0) > 0\}} \lambda_0 e_I(z_0, \tilde{q})\,\psi(z_0)\,dz_0}{N_F \tilde{f}(\tilde{q})\,dq} \tag{43}$$

where z_0^* is the inventor whose optimal choice is $\tilde{q}$.[26]

[26] If an inventor z_0 works in a firm $\tilde{q}$ when the novelty index is γ, the utility level is:

$$u(z_0, \tilde{q}) = \lambda_0 e_I x_0(z_0, \tilde{q}) - a^2 A\left(\lambda_0 e_I k x_0^2(z_0, \tilde{q}) + \sigma_0^2(\tilde{q})\right) - e_I^2/2$$

Take derivative with respect to $x_0(z_0, \tilde{q})$ yields:

$$\frac{\mathrm{d}u}{\mathrm{d}x_0} = 2A\sigma_0^2(\tilde{q}) + (1 - aAkx_0(z_0, \tilde{q}))\,\lambda_0^2 x_0^2(z_0, \tilde{q})$$

C.7 Starting up New Businesses

The partner's problem has the same form as the incumbent firm's, with $\tilde{q} = 0$ and the average innovation value being z_0 instead of $x_0(z_0, \tilde{q})$ (Equation 22).

$$\begin{aligned}
&\max_a (1-a)(\lambda_0 e_I z_0 \nu dt) - \tilde{T} \\
&\text{st } e_I = \arg\max \left\{ u\left(c_I\left(a, 0, \tilde{T}\right), e_I\right)\right\} \\
&\quad u\left(c_I\left(a, 0, \tilde{T}\right), e_I\right) \geq \bar{u}(z_0) \\
&\quad (1-a)(\lambda_0 e_I x_0(z_0, \tilde{q}) \nu dt) - \tilde{T} \geq 0
\end{aligned} \tag{44}$$

The partners are assumed to get zero profit due to competition.

The inventor decides her effort level by maximizing her utility, which yields:

$$e_I = \lambda_0 a z_0 \nu - A a^2 \lambda_0 \mathbb{E}\left(\tilde{z}(z_0)^2\right) \nu^2 \tag{45}$$

The firm's problem in Equation 44 becomes

$$\begin{aligned}
&\max_a (\lambda_0 e_I z_0 \nu dt) - A a^2 \left(\lambda_0 e_I \mathbb{E}\left(\tilde{z}(z_0)^2\right) \nu^2 dt\right) - \frac{1}{2} e_I^2 \\
&\text{st } e_I = \lambda_0 a z_0 \nu - A a^2 \lambda_0 \mathbb{E}\left(\tilde{z}(z_0)^2\right) \nu^2
\end{aligned} \tag{46}$$

It gives the highest utility an inventor can obtain when working in a startup.

As long as comparing with the optimal firm size $\tilde{q}$, x_0 is not too big, the derivative is positive and the utility increases in x_0, and hence it increases in γ. It means that during the period when the technologies breakthroughs (B and γ are low), an inventor expects systematically less utility when working in an incumbent firm.