Technology Driven Market Concentration through Idea Allocation

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Abstract

Using a measure of technology novelty, this paper identifies periods with and without technology breakthroughs from the 1980s to the 2020s in the US. It is found that market concentration decreases at the advent of revolutionary technologies. We establish a theory addressing inventors' decisions to establish new firms or join incumbents of selected sizes, yielding two key predictions: (1) A higher share of inventors opt for new firms during periods of heightened technology novelty. (2). There is positive assortative matching between idea quality and firm size if inventors join incumbents. Both predictions align with empirical findings and collectively contribute to a reduction in market concentration when groundbreaking technologies occur. Quantitative analysis shows the channel proposed generates 89% of the correlation between technology novelty waves and market concentration.

Keywords: technological waves, HHI, startups, incumbent firms.

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1 Introduction

The interplay between technological progress and market concentration plays a significant role in economic growth and resource allocation. Most of the existing studies focus on the impact of the firm size distribution on technology evolvement (e.g., Akcigit and Kerr (2018); Cunningham, Ederer and Ma (2021*a*); Akcigit and Goldschlag (2023)), this paper provides empirical evidence and structural analysis showing that the reverse relationship is also important—technological novelty waves affects the market concentration by relocating innovative ideas between incumbent firms and new businesses.

Using a measure of the novelty of new technologies, this paper identifies periods in the US when technology breakthroughs occur and periods when most new technologies follow existing ones from the 1980s to the 2020s. In other words, the trend of technological novelty follows waves. At the peak, groundbreaking technologies replace existing ones, while at the trough, most technologies have entered a mature stage.

Surprisingly, we find a cyclical pattern in market concentration, as measured by the Herfindahl-Hirschman Index (HHI) of firm sales, which lags behind technological waves by approximately three years and exhibits a notable negative correlation with these waves. This observation strongly suggests that the emergence and maturation of novel technologies may be influential factors in shaping market concentration dynamics.

How are technological waves and market concentration connected? A potential channel is through the allocation of ideas. Since firm size is to a large extent impacted by firm productivity and new ideas are important sources of productivity growth, where new ideas contribute their value will determine the firm size distribution, and therefore, market concentration. Combining the Longitudinal Business Database (LBD) from the Census Bureau and the patent information from the USPTO, this paper tracks the affiliation of patents at their formation. It is shown that at the peaks of the technological waves, a larger share of patents are forming in new businesses, while at the troughs, a larger share of patents come from incumbent firms. Besides, among patents from incumbent firms, there is a positive relationship between patent citations, a quality measure of the ideas behind them, and the size of the firm. These patterns indicate that technological waves affect the number of firm entries and the way new ideas combine with firms of different sizes.

Based on the empirical findings, this paper proposes a theory about inventors' choice of where to contribute the value of their ideas, and how it connects the technological waves and market concentration. The technological novelty level is assumed to be a random aggregate shock capturing the random arrival of ground-breaking innovations in a period. Each inventor is endowed with an idea of idiosyncratic quality. The inventor needs to choose between forming a new firm of random size with a partner or joining an incumbent firm. In the case of the latter, she must also decide on the size of the incumbent firm to join. It is frictional for an incumbent firm to adopt new technology due to creative destruction (Greenwood and Yorukoglu, 1997), and the friction decreases when technologies enter a mature stage. Hence, higher aggregate technological novelty leads more inventors to form startups. Inventors' decisions directly impact firm-level innovation intensity, technology improvement, and hence the firm size distribution. This channel provides new insights into market concentration.

There are three key elements in our model: novelty-related adoption cost, commercialization synergy, and inventor-firm contracts. The first element, novelty-related adoption cost, refers to the learning friction that arises when new ideas are integrated into established incumbent firms. It impedes ideas from reaching their full value upon completion. In contrast, new firms do not suffer from such frictions. The learning frictions capture the cost incumbent firms have to pay to adapt to the new technology as in Greenwood and Yorukoglu (1997). The extent of the friction is more pronounced at the peaks of the technological waves since new technologies are more distinct from existing ones in these periods. Hence, startups become more attractive to inventors.

The second element is commercialization synergy between ideas and incumbent firms. This is another major difference, in addition to the adoption cost, between new and incumbent firms. Incumbent firms can provide the idea with synergy through its production or commercialization. Conversely, new firms lack the capacity to provide such synergies. The size of incumbent firms and inventors' idiosyncratic idea quality also matter. Larger firms offer more synergy due to better production or commercialization capacity and idea of higher quality is worth more and benefits more from synergy. This interaction in synergy value between firm size and idea quality leads to a positive assortative sorting.

The third element is inventor-firm contracts. It characterizes how inventors and firms collaborate and ultimately determines the idea allocation. Ideas are developed under the research and development (R&D) process into useful technologies. The process is risky and the success rate depends on the inventors' effort, which is unobservable to either the partner or the owner of the incumbent firm. To incentivize the inventor to devote her optimal effort, a contract is signed between the inventor and the other party through a combination of equity and wages. Firm sizes matter for the contracts. Larger firms are subject to larger incentive problems since shocks unrelated to R&D are stronger and the equity held by the inventor provides a weaker incentive for R&D efforts.

The contract, together with the adoption friction and synergy, are the three dimensions an inventor needs to consider when choosing the optimal firm. Startups do not suffer from adoption friction, and provide more aligned incentives, but are not capable of generating synergy. All incumbents encounter adoption friction while larger firms offer weaker incentives yet better synergy. In light of these trade-offs, inventors make strategic decisions regarding whether to establish their own firms or join incumbent firms of a specific size.

The model has two major predictions. First, a larger share of inventors choose to start new firms to develop their ideas during periods of high technological novelty since the learning friction at incumbent firms is larger. Second, among inventors that choose to do R&D in incumbent firms, there is positive assortative matching between idea quality and firm size. Therefore, firms already with larger size attract ideas of higher value. These two predictions are consistent with observations in data and collectively contribute to a reduction in market concentration when the economy is closer to the peak of the technological waves. The upsurge in new startups leads to a proliferation of firms in the market. Given that new startups are less constrained by the positive matching between idea quality and firm size, they offer a counterbalance to the tendency of larger firms to further expand.

To quantify the impact of the technological novelty waves on market concentration through allocation of new ideas, we calibrate the model and then do simulations by changing the degree of novelty of new technologies in an economy. The model is calibrated to match data moments in 1986, the first peak of the technological waves within our sample period. Key moments include patent novelty, average patent value, degree of positive matching between patent citation and firm size, the growth rates, etc. In the simulation exercise, we fix all the parameters except for the one related to patent novelty for each year following 1986. This variation serves to capture the evolving dynamics of learning friction within incumbent firms. Consequently, we generate paths of two essential data moments: (1) the ratio of the number of ideas in new firms relative to those in incumbent firms; (2) the HHI of firm sales. The two paths of moments are compared with actual trends after 1986.

The two generated paths of moments nearly have simultaneous peaks and troughs with the actual trends. The correlation between the technological novelty waves and the new-to-incumbent ratio is 0.95, which is 127% of the actual correlation. The correlation between the technological waves and the HHI cycles is -0.70, which is 89% of the actual correlation.

To decompose the effect of the two channels where technological waves play a role in market concentration, we respectively shut down the change in firm numbers and the positive assortative matching between idea quality and firm size in the simulation process. These two scenarios respectively represent isolating the effects of the intensive margin and the extensive margin. The correlations between technological waves and the trend in the HHI are similar in the two cases, with a correlation coefficient of approximately -0.71 in the former case and -0.70 in the latter. However, they exhibit distinct patterns. When considering only the effect of the intensive margin, the HHI responds more rapidly to technological waves but with a smaller magnitude of change. Conversely, when focusing solely on the extensive margin, market concentration lags further behind the technological waves.

Related Literature

This paper is closely related to the literature on the interplay between innovation and market concentration. On the one hand, innovation leads to technological advancement that creates monopoly rents and larger firm size (Aghion and Howitt, 1990; Grossman and Helpman, 1991; Klette and Kortum, 2004). On the other hand, firms of different sizes are shown to have different innovation intensities in the literature, indicating that the overall innovation intensity depends on both the firm size distribution (Akcigit and Kerr, 2018; Akcigit and Goldschlag, 2023) and the market for ideas (Eaton and Kortum, 1996; Silveira and Wright, 2010; Chatterjee and Rossi-Hansberg, 2012; Chiu, Meh and Wright, 2017; Cabral, 2018; Cunningham, Ederer and Ma, 2021*b*; Perla, Tonetti and Waugh, 2021; Fons-Rosen, Roldan-Blanco and Schmitz, 2021). Theories and empirical evidence in this aspect can be traced back to the Schumpeterian argument that large firms have a higher capacity to do R&D, to more recent findings that small firms are more inclined to engage in innovation activities due to the rise of the patent market (Cassiman and Veugelers, 2006; Higgins and Rodriguez, 2006; Phillips and Zhdanov, 2013; Bena and Li, 2014; Akcigit, Celik and Greenwood, 2016; Liu and Ma, 2021; Ma, 2022; Yang, 2023). Most of the existing studies focus on the relationship between innovation efforts and market structure, while this paper finds novel patterns that the novelty of new technologies is closely correlated with the market concentration measure. To our knowledge, this is the first paper that uncovers the cyclicality of market concentration and explains it by the technological novelty waves.

Our empirical and theoretical analyses indicate that the degree of novelty associated with emerging technologies significantly influences where inventors choose to conduct their R&D.[1] This perspective provides an alternative viewpoint on the relationship between the allocation of new ideas and market concentration. Existing research emphasizes the opposite relationship. Studies like Cunningham, Ederer and Ma (2021*a*) and Akcigit and Goldschlag (2023) have posited that incumbent firms strategically acquire innovative startups or independent inventors only to subsequently abandon their ideas, thus preventing competition from new entrants and effectively stifling novel ideas. Therefore, the decrease in the novelty of new technologies is due to market concentration and the high monopoly power of incumbent firms. Our paper does not contradict these assertions. Instead, the analysis in

[1]It adds to the literature on the boundary of the firm, going back to Coase (1937), important examples of which include Grossman and Hart (1986), Hart and Moore (1990), and Hart and Moore (2008). Closest are Aghion and Tirole (1994) and Schmitz (2005) who analyze the implications of the innovations' ownership. Most of the studies focuses on how ownership affects outcomes, while this paper casts these ideas in a general economic framework to think about how novelty interacts with ownership.

this paper suggests that technological novelty and market concentration may have mutual effects and the mutual effects amplify each other in the negative correlation between the technological waves and market concentration.

This paper also sheds light on the concerns about the rising market concentration after the 2000s. Akcigit and Ates (2023) and Olmstead-Rumsey (2019) observe a rising trend of concentration and attribute it to the lack of knowledge diffusion between leading and laggard firms. Our analysis, however, suggests an additional factor contributing to the rise in market concentration—the deceleration in the emergence of revolutionary technologies. The novelty metric for new technologies defined in this paper indicates that the zeniths of technological waves occurred in the mid-1980s and mid-1990s, with a significant 20-year gap before resurfacing in the early 2010s. This extended period without significant technological breakthroughs has led inventors to gravitate toward incumbent firms, resulting in an augmentation of concentration among these larger firms.

Finally, our analysis delves into the implications of the introduction of groundbreaking technologies. Bowen III, Frésard and Hoberg (2023) show empirically that in an era with rapid evolving technologies, more startups remain independent rather than being sold out. Greenwood and Yorukoglu (1997) and Greenwood and Jovanovic (1999) establish that technological revolutions lead to deterioration in the stock value of existing firms due to concerns about the prospect of creative destruction. The learning process of the novel technologies is costly and requires skilled labor, therefore, slowing down economic growth and widening income inequality in the short run. This paper extends the existing literature by investigating how a leap in technological progress affects the distribution of firm sizes, primarily due to the learning frictions when integrating novel ideas into incumbent firms. It is shown that market concentration is another important outcome of technological revolutions. This paper demonstrates that apart from the high-frequency business cycle influenced by productivity fluctuations (Kydland and Prescott (1982)), the economy may also be susceptible to a low-frequency cycle driven by the waves of technological novelty.

The rest of the paper is organized as follows. Section 2 introduces measures of the technological waves, market concentration, and the allocation of ideas, and subsequently presents their patterns. Section 3 constructs a model where inventors make decisions between initiating new ventures or joining established incumbents at specific sizes. We derive predictions concerning the mapping between the quality of inventors' ideas and their optimal choices. Section 4 defines the balanced growth path, the aggregate growth rate, and market concentration. Section 5 calibrates the model. Section 6 simulates the model to evaluate the degree to which technological waves can account for changes in market concentration through the idea allocation channel. Section 7 concludes.

2 Empirical Patterns

This section exhibits empirical patterns of the technological waves, market concentration, and a potential channel that links the two—the choices of the inventors on where to invent.

2.1 Technological Waves

Technology waves capture the extent of new technology breakthroughs over time. At the peak of the technological waves, significantly innovative new technology emerges and substitutes existing technologies; at the trough of the waves, most of the technologies in the economy have reached a mature state, and the extent of creative destruction of new technology over existing ones is smaller.

To measure the technological waves, we define a "Novelty" Index of the new technologies in each year using the patent citation data. Specifically,

$$\text{Novelty}_t = \frac{\sum_{i\in I_t}\sum_{s=0}^{5}\text{Forward Citations}_{i,t+s}}{\sum_{i\in I_t}\sum_{s=0}^{5}\text{Forward Citations}_{i,t+s} + \sum_{i\in I_t}\sum_{s=0}^{5}\text{Backward Citations}_{i,t-s}}, \quad (1)$$

where I_t is the set of the new patents granted in year t. The numerator is a summation of the number of forward citations (citations by others) each new patent gets within the next five years. The denominator is a summation of the number of forward citations plus a summation of the number of backward citations (citation on others) each patent makes on other patents granted within the previous five years. The five-year window is to ensure every year in the sample is compared on the common ground, since more recent patents are more likely to receive fewer forward citations due to the right-censoring issue. The rationale for this measure is that groundbreaking innovations typically exhibit lower similarity to current technologies, but pave the way for subsequent patents to emulate them. Since the forward citations capture the overlap of future patents with the focal patent, while the backward citations capture the overlap of the focal patents with previous patents, the relative number of the former provides a measure of patent novelty. The "Novelty" index sums the number of forward and backward citations across all the patents granted in a year and is in the range between zero and one. A higher index indicates that the year witnesses significant breakthroughs in new technologies; a lower index indicates that most of the technologies have evolved into a mature stage in that year.

The data used to generate the "Novelty" index comes from the USPTO patent and citation data.[2] The USPTO records all patents granted after 1976 and all the patents they

[2] The data can be found on PatentsView.

Figure 1: Two Measures of Technological Waves

Notes: This figure illustrates two measures of technological waves over time. The blue curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations, while the red curve represents the "RETech" index, a measure of patent novelty from the literature, which assesses patent novelty by the prevalence of vocabularies that are growing in use in the patent description. The two curves have different y-axises, which are shown respective on the left and right. *Sources:* USPTO patent and citation data.

cite. To get a smoother trend, we take a three-year average for each observation,

$$\text{Novelty\_avg}_t = \frac{1}{3}\sum_{h=-1}^{1} \text{Novelty}_{t+h}. \tag{2}$$

There are other measures of patent novelty. Bowen III, Frésard and Hoberg (2023) analyzes the text of all the US patents and defines patents as being revolutionary if the vocabulary they use is growing rapidly in the patent corpus overall. Their novelty measure is called "RETech". Kelly et al. (2021) also uses textual analysis and measures patent novelty according to its similarity to previous work and subsequent innovations.

Figure 1 shows the technological waves defined in this paper and the "RETech" in the literature. They are significantly positively correlated with nearly simultaneous peaks and troughs, indicating the robustness of different measures. The figure suggests that significant technological breakthroughs happened in the mid-1980s, the mid-1990s, and the beginning of the 2010s, while the period around 1990 and the mid-2000s are periods when most of the technologies have entered a mature stage.

2.2 Market Concentration

We adopt the most commonly used measure, the HHI, to capture market concentration. The dataset used is Compustat Fundamentals Annual due to its comprehensive coverage of firms' sales. It contains information on all publicly listed firms in the U.S. The construction

Figure 2: Technological Waves and Market Concentration

Notes: This figure shows the technological waves and the trend of market concentration over time. The blue curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations. The red curve displays the HHI in each year, which is the weighted average of the industry-level HHI in each year. The weight is the number of firms in each industry. The two curves have different y-axises, which are shown respective on the left and right.
Sources: Compustat Fundamental Annuals and USPTO patent and citation data.

process is the following. First, we calculate the squared ratios of firm sales to total industry sales within each industry defined by the 4-digit SIC code in each year. Second, we sum up the ratios across all firms in each industry to get the industry-level HHIs in each year. Third, we weight each industry by the number of firms in it and take a weighted average of all industry-level HHIs. To smooth the trend, we also take the three-year average for each observation point.

The trend of the yearly HHI is shown in Figure 2. To illustrate its relationship with the technological waves, the "Novelty" index defined in the previous section is also plotted in the figure. The two curves exhibit negative correlations with the HHI lagging the technological waves around three years. This suggests that the technological waves may be a driving force of market concentration. If we assume the time lag between the two reflects the responding time of the market concentration to technological waves and shifts the HHI trend left for three years. The correlation between the two curves is -0.76.

To examine the sector-level relationship between the market concentration and technological waves, we calculate the HHI and the "Novelty" Index by major sectors defined by the SIC code—Manufacturing, Transportation and Utilities, Wholesale Trade, Retail Trade, Finance, and Services.[3] Aggregating the HHIs within each major sector is a straightforward process, accomplished by computing a weighted average based on the number of firms at the 4-digit SIC level. However, performing a similar aggregation for the "Novelty" Index presents a more complex challenge, since patents are classified by the technology class

[3]The division is according to the U.S. department of Labor.

(as captured by the International Patent Classification (IPC)) instead of sectors. To map the technology classes to sectors, we use the concordance developed by Silverman (2002) that links the 4-digit IPC code to the 4-digit SIC code according to usage. After applying this concordance, we obtain the counts of forward and backward citations at the 4-digit SIC level. These citation counts are then cumulatively summed up to the primary sector level, allowing us to calculate the "Novelty" Index for each sector. The visual representation of our findings can be observed in Figure 3.

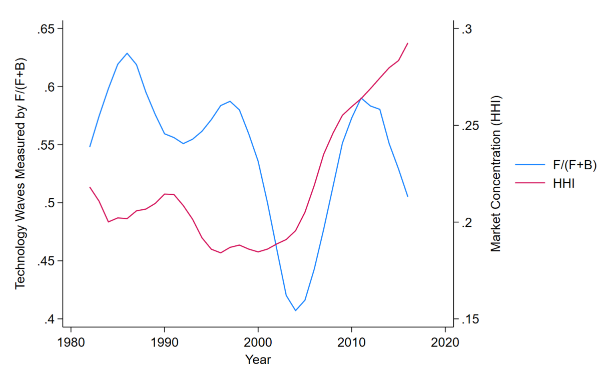
Generally, a discernible negative relationship between technological waves and market concentration prevails across most major industries, with the latter trailing behind the former by approximately 3 years. When we align the HHI curve with a 3-year shift to the left in each major sector, the resulting correlations between the two curves are as follows: -0.22 for Manufacturing, -0.42 for Transportation and Utilities, -0.63 for Wholesale Trade, -0.20 for Retail Trade, -0.91 for Finance, and 0.18 for Services. Therefore, the negative correlation exists for all major sectors except for Servies. These findings offer additional supporting evidence suggesting that market concentration may be influenced by the dynamics of technological novelty waves.

Panel A: Manufacturing

Panel B: Transportation and Utilities

Panel C: Wholesale Trade

Panel D: Retail Trade



Panel E: Finance

Panel F: Services

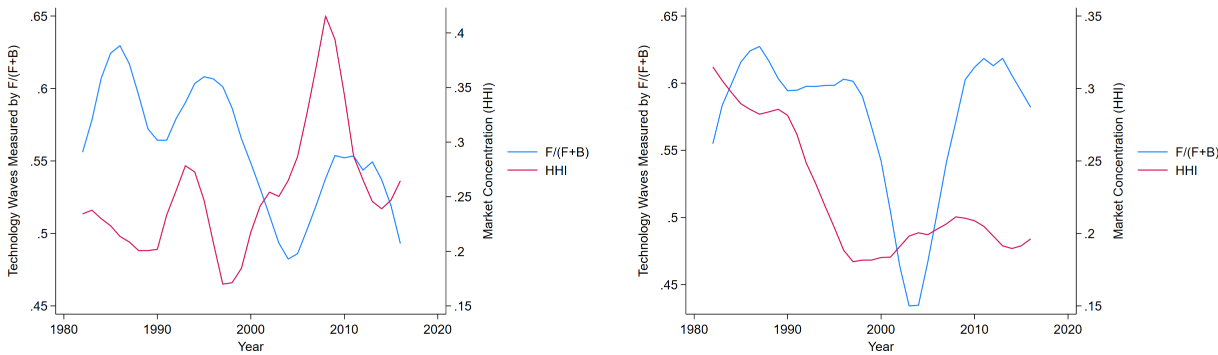


Figure 3: Technological Waves and Market Concentration by Industry

Notes: This figure shows the technological waves and the trend of market concentration over time by major sectors. The blue curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations in each major sector. The red curve displays the HHI in each year, which is the weighted average of the 4-digit-SIC-level HHIs by major sectors and years. The weight is the number of firms in each 4-digit SIC industry. The two curves have different y-axises, which are shown respective on the left and right.

Sources: Compustat Fundamental Annuals and USPTO patent and citation data.

2.3 Allocation of Ideas

One potential link between the technological waves and the market concentration is inventors' choices of where to do innovation. They can work independently and start their own businesses or contribute their innovation efforts to incumbent firms. In the latter case, they also choose the size of incumbent firms to work in. This section describes the flow of the new ideas using the Census data.

2.3.1 Entrants or Incumbent Firms

Data on the affiliations of inventors when they initiate a research project is unavailable to us, but we can observe the age of the firm when a patent is granted to it or applied by it and infer inventors' affiliation. Specifically, if a patent is granted to a firm at age zero to five, it implies that the initial idea was developed independently or spun off from other firms five years ago; if a patent is granted to a firm at age above five, it implies that the initial idea was developed by the incumbent firm five years ago or the firm bought the idea from independent inventors or other firms. We choose the time window to be five years since the average time between patent applications and patent issuance is around two or three years according to the USPTO and we assume the average time to complete a research project to be also two to three years. We can compute the ratio between the number of ideas in new firms to the number of ideas absorbed in incumbent firms, i.e.,

$$\text{New-to-Incumbent Ratio}_t = \frac{\sum_{i \in I_{t+5}} \text{Granted in Firm(Age} \leq 5)_{i,t+5}}{\sum_{i \in I_{t+5}} \text{Granted in Firm(Age}>5)_{i,t+5}}, \tag{3}$$

where I_{t+5} denotes the set of patents granted five years later; "Granted in Firm(Age≤5)" and "Granted in Firm(Age>5)" are dummy variables indicating whether patent i' is issued to a firm above five years old. An alternative measure is to use the age of a firm when it applies for patents. If a patent is applied for in a firm at age zero to three, it implies the founding of a new firm with the idea in the past three years. Otherwise, it implies incumbent firms absorbing new ideas.[4]

Note that there may be discrepancies between the patent affiliations and inventors' affiliations due to spinoffs and patent sales. In the case of spinoffs, the "New-to-Incumbent Ratio" based on patent affiliation is larger than the ratio based on inventors' affiliation. In the case of patent sales, the situation is reversed. However, if we want to capture where innovation ideas finally contribute its value, taking into account spinoffs and patent sales works towards the purpose.

The data used to observe patent affiliations is constructed by combining the Longitudinal Business Database (LBD) from the US Census Bureau and the USPTO patent data. The LBD covers all the employer businesses in the US and documents the age of each firm. The combined dataset can track the age of firms at patent application and issuance.

Since the two measures of the "New-to-Incumbent Ratio" have very similar trends, we only report the first measure. We take the three-year average for each observation year as before and show the result in Figure 4. To compare it with the technological waves, the Novelty Index defined in this paper is also plotted. Notably, the 'New-to-Incumbent Ratio' demonstrates prominent cyclicality, with zeniths and nadirs occurring in close proximity to the peaks and troughs of technological novelty waves. This strong correspondence is underscored by a correlation coefficient of 0.75.

To assess the robustness of the relationship between idea allocation and technological waves, this paper compares the two trends by patent technology classes, categorized by the first digit of the patent IPC code. The IPC-level "Novelty" Index and "New-to-Incumbent Ratio" are computed using the same methodology as described in equations 1 and 4, with

[4] In this alternative measure, the "New-to-Incumbent Ratio" is defined as

$$\text{New-to-Incumbent Ratio}_t = \frac{\sum_{i \in I_{t+3}} \text{Applied in Firm(Age} \leq 3)_{i,t+3}}{\sum_{i \in I_{t+3}} \text{Applied in Firm(Age}>3)_{i,t+3}}, \tag{4}$$

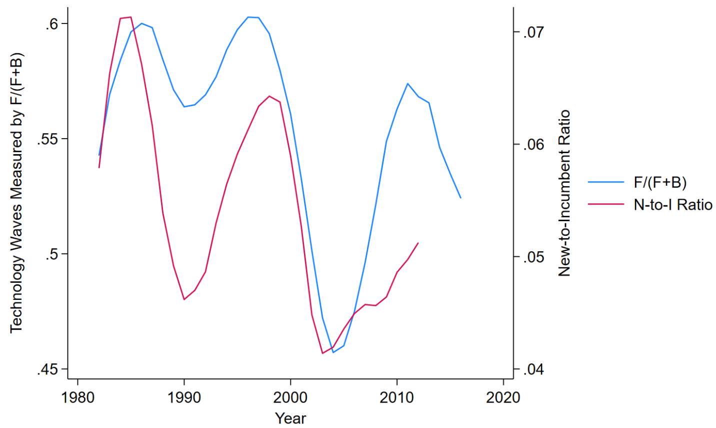


Figure 4: Technological Waves and Idea Allocation

Notes: This figure shows the technological waves and the idea allocation between new and incumbent firms over time. The blue curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations. The red curve displays the "New-to-Incumbent Ratio" defined in the paper, capture where new ideas contribute their value. The two curves have different y-axises, which are shown respective on the left and right.
Sources: Longitudinal Business Database (LBD) and USPTO patent and citation data.

patent sets segregated according to their respective technology classes. Figure 5 illustrates that a positive correlation between idea allocation and technological waves is consistently observed across most technology classes. When a specific technology class experiences breakthroughs, there is an increase in the flow of ideas toward new startups. The correlation coefficients between the two curves are, respectively, 0.48 for Human Necessities, 0.62 for Transporting, -0.05 for Chemistry, 0.53 for Textiles, 0.37 for Fixed Constructions, -0.17 for Mechanical Engineering, 0.64 for Physics, and 0.55 for Electricity.

2.3.2 Size of Incumbent Firms

When inventors opt to contribute their ideas to incumbent firms, they are also making a choice regarding the size of the firm, as it impacts the potential value that their innovations can attain. We establish a connection between the quality of inventors' ideas and the size of the incumbent firms they select by examining a subset of patents that have been granted to firms with a history of at least five years in operation. This subset serves as the basis for calculating the "New-to-Incumbent Ratio," as described in Section 2.3.1. The quality of inventors' ideas is gauged by the number of forward citations each patent receives within the first five years following its issuance. We amalgamate data from various years and compute the quartiles for patent citations. Subsequently, we categorize patents into four distinct groups based on the quartile to which their citation count belongs. On the other hand, we

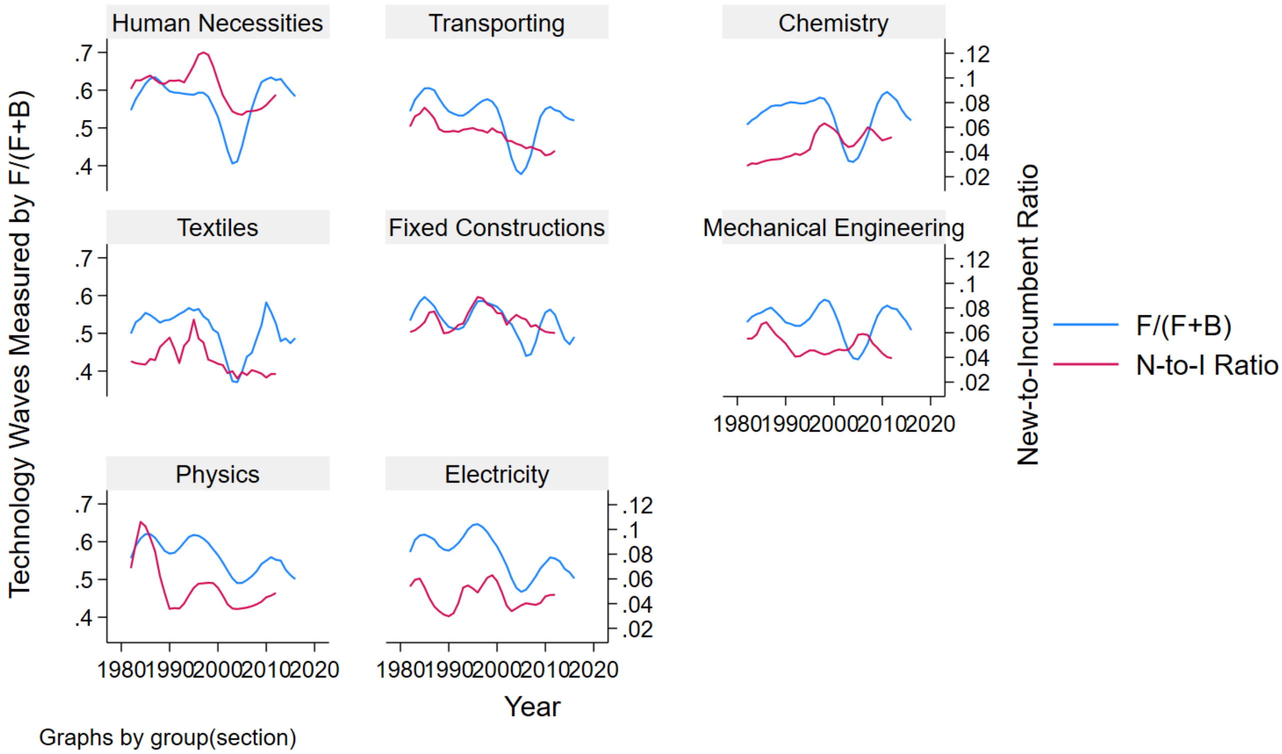


Figure 5: Technological Waves and Idea Allocation by Patent Technology Class

Notes: This figure shows the technological waves and the idea allocation between new and incumbent firms by patent technology class. The blue curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations. The red curve displays the "New-to-Incumbent Ratio" defined in the paper, capture where new ideas contribute their value. The two curves have different y-axises, which are shown respective on the left and right.
Sources: Longitudinal Business Database (LBD) and USPTO patent and citation data.

measure the size of incumbent firms in terms of their number of employees, normalizing the average employment in the first citation quartile to a value of one. The relative employment in each citation quartile is plotted in Figure 6

It is shown that there is positive assortative matching between idea quality and firm size when ideas combine with incumbent firms. One potential concern is that the firm's employment at the patent's issuance may differ from the employment when the inventor chooses the firm. To address this concern, we track each firm's employment five years ago, using data from the LBD. Subsequently, we compute the average number of employees for firms falling into each of the four citation quartiles.[5] The relationship between relative firm size and patent citation quartiles closely mirrors the mapping depicted in Figure 6.

[5]Note that the citation quartiles are based on citations of patents granted five years later.

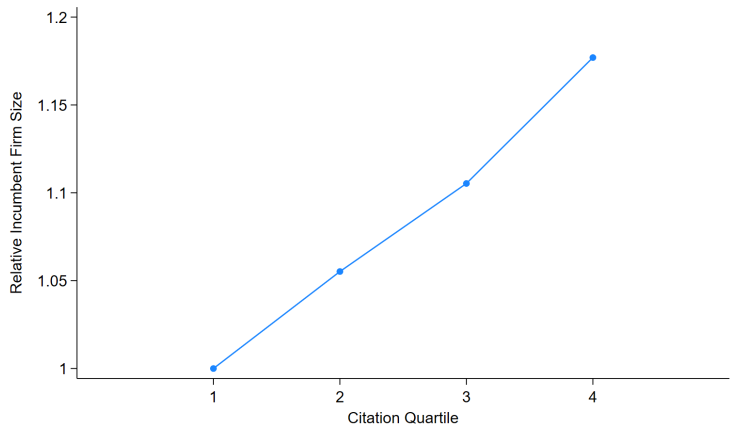


Figure 6: Mapping between Patent Citations and Incumbent Firm Size

Notes: This figure shows the mapping between inventors' idea quality and firm size if inventors opt to develop their ideas in incumbent firms. The idea quality is measured by the number of patent citations and is classified into four quartiles. The firm size is measured by the number of employees. The average employment of firms corresponding to the first citation quartile is normalized to be one.
Sources: Longitudinal Business Database (LBD) and USPTO patent and citation data.

3 Model

We have constructed a macroeconomic framework to elucidate the influence of technological novelty on market concentration by examining the allocation of innovative ideas. This outcome is propelled by two mechanisms. First, there is an increase in the number of firms after the emergence of technological breakthroughs. Second, when an inventor operates within an established firm, the firm size she chooses increases in her idea quality.

The two channels are underpinned by three key model features: novelty-related adoption friction, commercialization synergy, and inventor-firm contracts. The novelty-related adoption friction directly speaks to the first channel by affecting inventor's choice between incumbents and new businesses. It captures the creative destruction caused by new technologies on incumbents as in Greenwood and Yorukoglu (1997), and the magnitude of the destruction increases in the aggregate technological novelty. New businesses, since they do not own any production line beforehand, are immune to this particular friction.

The remaining two elements together contribute to the emergence of the second channel. On one hand, synergy enhances the value of innovations, with the effect increasing in firm size and idea quality. On the other hand, given the unobservability of inventors' efforts, firms adopt a common contract that combines equity and wages to motivate inventors. This contract features incentive problems, which are more pronounced in larger firms due to greater R&D-unrelated shocks. Inventors optimally choose the size of the firm they join, taking joint consideration of synergy and the incentive problems. Inventors with better ideas get more synergy from larger firms and face fewer incentive problems, given their larger stake in the overall firm value. Consequently, high-quality inventors prefer larger firms, establishing a positive sorting relationship between firm size and ideas.

This model is embedded in a general equilibrium with two categories of individuals (households and inventors) alongside two types of firms (intermediate goods producers and final goods producers). In this economy, there is an aggregate shock capturing the degree of novelty of new technologies in each period. This shock applies to all agents and determines the extent of friction when inventors' idea combine with incumbent intermediate goods producers. Inventors in each period receive ideas of idiosyncratic quality. They choose to start up new intermediate firms or join incumbent ones of selected size based on the aggregate shock and their idea quality. During periods of high aggregate technological novelty, a broader spectrum of ideas are developed within new firms, leading to a surge in the number of entrants and a decline in innovations among incumbents. Besides, the positive assortative nature of the firm-inventor matching suggests that when there fewer ideas are developed in incumbents, less sorting happens between idea quality and firm size. The resulting weaker positive assortative matching, together with the increment in firm numbers, collectively contributes to a reduction in the market concentration.

3.1 Preferences

There is a long-lived representative household in the economy. She works in the production sector, supplies one unit of labor to firms inelastically, and consumes final goods. The household's utility function is

$$U_H = \int_0^\infty e^{-\rho t} log(C_H(t))dt, \tag{5}$$

where $\rho > 0$ is the discount rate and $C_H(t)$ is the consumption of the household.

Inventors are the ones who work in the R&D sector. In each period, there is a continuum of inventors of measure one. An inventor, with a short-lived lifespan of dt time periods, dedicates effort e_I to create innovations within either an incumbent firm or a new business. Simultaneously, they engage in consumption. Inventors are risk-averse and have a mean-variance utility, similar to Yang (2023):

$$U_I\left(c_I, e_I\right) = \mathbb{E}\left(c_I\right) - A\frac{var\left(c_I\right)}{\bar{q}} - R\left(e_I\right)\bar{q}, \tag{6}$$

where c_I is the consumption, e_I is the effort level, and $R\left(e_I\right)\bar{q}$ is the associated cost. $\bar{q}$

(defined below) is the average quality in the economy. The variance and cost are normalized by $\bar{q}$ to keep the problem stable over time. Denote the inventors' aggregate consumption using C_I, i.e., $C_I = \int_0^1 c_{Ii} di$.

3.2 Technology

The economy features two types of firms: intermediate goods producers and final goods producers. The setup is similar to Akcigit and Kerr (2018). Both types of firms are owned by the household. The former hires inventors to create innovations, and produce intermediate goods. The latter assembles intermediate goods and produces final goods.

The final good producers produce final goods using a continuum of intermediate goods $j \in [0, N_F]$:

$$Y(t) = \frac{1}{1-\beta} \int_0^{N_F} q_j^{\beta}(t) y_j^{1-\beta}(t) dj. \tag{7}$$

In this function, $q_j(t)$ is the quality of the intermediate good j, and $y_j(t)$ is its quantity. We normalize the price of the final good to be one in every period. The final good producers are perfectly competitive, taking the input prices as given. Henceforth, we will drop the time index t when it does not cause confusion.

The final goods are consumed by the household and inventors. The resource constraint of the economy is:

$$Y = C_H + C_I. \tag{8}$$

The intermediate goods producers are a continuum of risk neutral firms of measure N_F. Each firm produces one type of good, with a linear technology using only labor:

$$y_j = \bar{q} l_j, \tag{9}$$

where l_j is the labor input; $\bar{q} = \frac{1}{N_F} \int_0^{N_F} q_j dj$ is the average quality, meaning that innovations have positive externality (Romer, 1986). The cost is linear in wage w, which intermediate firms take as given. The labor market satisfies the constraint:

$$\int_0^{N_F} l_j dj \leq 1. \tag{10}$$

The production technologies, together with the market setting on innovation, ensure that a firm's value $V(q_j)$ is linear in quality q_j (the proof is shown in the next section),

$$V(q_j) = \nu q_j, \tag{11}$$

where ν is endogenous.

This paper focuses on the balanced growth path. We normalize the variables using the average quality $\bar{q}$, and denote the normalized variables using tilde:

$$\tilde{q}_j \equiv \frac{q_j}{\bar{q}}, \tilde{Q} \equiv \frac{Q}{\bar{q}}, \tilde{V}\left(\tilde{q}\right) \equiv \frac{V\left(q_j\right)}{\bar{q}} = \nu \tilde{q}_j, \tag{12}$$

where $Q \equiv \int_0^{N_F} q_j dj$ is the total technology stock in the economy.

Within a given period, intermediate firms consist of a combination of established incumbents and new entrants. Incumbents hire inventors to improve their quality through innovations, while new entrants arise as a result of successful innovations by inventors collaborating with a partner. These innovations are generated by inventors exerting effort denoted as e_I. Given the level of effort e_I, the success rate of an innovation follows an instantaneous Poisson flow rate:

$$\lambda(e_I) = \lambda_0 e_I. \tag{13}$$

It is costly for inventors to dedicate effort, and the flow cost of choosing effort e_I is $R\left(e_I\right)\bar{q}$, and $R\left(e_I\right) = \frac{1}{1+\delta} e_I^{\delta+1} dt$. This implies a linear cost in time dt at a rate of $\frac{1}{1+\delta} e_I^{\delta+1}$, which is an increasing and convex function of the effort taken.[6]

Inventors are directly responsible for the cost of their efforts, but their efforts cannot be observed by the partner or incumbent intermediate firms. In the absence of a performance-based incentive, an inventor, receiving a flat wage, would opt for $e_I = 0$. Consequently, the partner and the incumbent firms must incentivize inventors to take effort by implementing an innovation-dependent payment scheme. This paper adopts the assumption that firms utilize a common contract, which is a combination of wage and equity, to compensate inventors (Yang, 2023). The wage allow the partner and the firms to share risk with an inventor whereas equity aligns the inventor's interests with theirs.

Each inventor is born with one innovative idea characterized by an idea quality z_0. The inventor can choose to work either within an incumbent intermediate firm or start up a new intermediate firm with a partner. In the case of working in a startup, the inventor retains full control over the innovation process, and the innovation value is solely determined by the idea quality z_0. Following creation, the normalized innovation value $\tilde{z}$ is a stochastic draw from the distribution, $U\left(\left(1-\phi\right)z_0\nu, \left(1+\phi\right)z_0\nu\right)$. While, on average, a higher quality idea yields a better outcome, the inclusion of ϕ allows for some randomness in the mapping between the innovation value and idea quality, with $\phi \in (0,1)$ capturing this variability.

[6] The innovation production function and the cost functions are based on the growth theory literature (Romer, 1990; Klette and Kortum, 2004; Akcigit and Kerr, 2018). In the calibration, we choose $\delta = 1$ following the literature.

In the other case, when the inventor with idea quality z_0 works in an incumbent firm with quality $\tilde{q}$, the resulting innovation value becomes a stochastic variable drawn from another uniform distribution, $U\left(\left(1-\phi\right)x_0\left(z_0,\tilde{q}\right)\nu, \left(1+\phi\right)x_0\left(z_0,\tilde{q}\right)\nu\right)$, where $\frac{\partial x_0(z_0,\tilde{q})}{\partial z_0} > 0$ and $\frac{\partial x_0(z_0,\tilde{q})}{\partial \tilde{q}} > 0$. The mean value of the innovation, $x_0\left(z_0,\tilde{q}\right)$, now depends not only on the idea quality z_0, but also on the firm size $\tilde{q}$. This reflects that the incumbent firm provides the inventor's idea with synergy and the synergy increases in the firm's quality.

The mean value of the innovation is also subject to adoption friction that increases in the aggregate technological novelty of the economy. The friction captures the learning cost when incumbent firms combine with new ideas. We assume that $x_0\left(z_0,q\right)$ takes the following functional form: $x_0\left(z_0,q\right) = \left(\frac{\tilde{q}}{\tilde{q}_0}\right)^b \gamma\left(z_0\right)z_0$. The first term, $\left(\frac{\tilde{q}}{\tilde{q}_0}\right)^b$, denotes the synergy between an incumbent firm and an innovation, where $\tilde{q}_0$ is a parameter. The second term, $\gamma\left(z_0\right) = \frac{B}{B+z_0}$, captures the influence of the technology waves. The function form is inspired by the Novelty Index defined in the empirical section. The parameter B corresponds to the backward citation stock in a certain period, representing the maturity of the technology to which the inventor's idea contributes. The value of B varies over time and is pinned down by mapping the average $\gamma\left(z_0\right)$ to $(1 -$ the Novelty Index). When the economy is closer to the peak of the technological waves, past innovations are less influential, leading to a smaller calibrated value of B. In such case, incumbent firms are subject to larger learning friction when integrating innovations, as revolutionary technologies cause more creatively destruction to the current production line (Greenwood and Yorukoglu (1997)).

3.3 Timeline

Upon an inventor's birth, she observes the quality z_0 of her idea. A potential partner observe z_0 and extends contracts to the inventor to jointly start a new intermediate firm. Concurrently, incumbent firms observe their corresponding $x_0\left(z_0,\tilde{q}\right)$ and also extend employment contracts to the inventor. The contracts from the potential partner and incumbent firms are strategically designed to maximize their payoff, taking into account the competition with other firms, as well as the inventor's incentive problem. They possess two key components: a fixed wage $\tilde{T}$ and a stake in equity $a \in [0,1]$.[7] After viewing all contracts, the inventor decides to either join her preferred incumbent firm of quality $\tilde{q}^*\left(z_0\right)$, or initiates a startup with the partner. In the event of selecting an incumbent firm, the matching process is frictional. With a probability of h, the inventor joins the optimal firm; alternatively, she is randomly assigned to another firm $\tilde{q}$, with the assignment determined by

[7] It is worth noting that the level of effort e_I is unobservable and unverifiable. Consequently, contracts cannot be contingent on the effort level.

the incumbent firm size distribution $\tilde{F}(\tilde{q})$. The friction in the matching process is introduced to match the data, since the actual mapping between idea quality and firm size is not perfect. After signing the contract, the inventor chooses an R&D effort, e_I.

3.4 Entry and Exit

An new intermediate firm enters the market upon successfully innovation of an inventor who choose to work with a partner. Upon entry, the firm first draws a quality $\tilde{q}$ from the current incumbent firm size distribution $\tilde{F}(\tilde{q})$. Subsequently, the entrant incurs a cost equivalent to the firm value associated with the drawn quality $\tilde{q}$. Following this, the firm applies the innovation, enhancing its quality by incorporating the value of the innovation itself. The rate at which entrants join the market is represented by λ_I.

Intermediate firms face an exogenous exit rate τ, which is independent of their size and is a risk unrelated to innovation. We focus on a balanced growth path such that the number of entrants equals the number of firm exits,

$$\tau N_f = \lambda_I. \tag{14}$$

4 Equilibrium: Balanced Growth Path

This section characterizes the equilibrium of the economy in which aggregate variables $(Y, C, R, w, \bar{q})$ grow at a constant rate g.

4.1 Production

The production sector features two types of firms: a representative final goods producer and intermediate goods producers. The final good producer assembles intermediate goods, denoted by j within the range $[0, N_F]$, to produce final goods. It chooses $\{y_j\}_j$ to maximize its profit using the technology described in Section 3.2. The final goods producer's problem can be written as:

$$\max_{\{y_j\}} \frac{1}{1-\beta} \int_0^{N_F} q_j^{\beta} y_j^{1-\beta} dj - \int_0^{N_F} y_j p_j dj. \tag{15}$$

The first-order condition

$$p_j = q_j^{\beta} y_j^{\beta}$$

yields the demand function for goods produced by intermediate firms.

The intermediate goods are produced by their corresponding firm $j \in [0, N_F]$ using only labor $y_j = \bar{q} l_j$, where $\bar{q} = \frac{1}{N_F} \int_0^{N_F} q_j dj$ represents the average quality, and l_j is the labor input. Intermediate good producers engage in monopolistic competition, optimizing their profit by choosing l_j, p_j, y_j, given the wage level w:

$$\begin{aligned} &\max_{l_j,p_j,y_j} y_j p_j - w l_j. \\ &s.t.\ y_j = \bar{q} l_j \\ &\quad\ \ p_j = q_j^{\beta} y_j^{-\beta} \end{aligned} \tag{16}$$

Therefore, the FOC yields

$$y_j = q_j \left(\frac{\bar{q}\,(1-\beta)}{w}\right)^{\frac{1}{\beta}}, l_j = y_j/\bar{q}, p_j = \frac{w}{\bar{q}\,(1-\beta)}. \tag{17}$$

In each period, the labor market clearing satisfies $\int_0^{N_F} l_j dj = 1$, which gives that $\frac{\int_0^{N_F} q_j \left(\frac{\bar{q}(1-\beta)}{w}\right)^{\frac{1}{\beta}} dj}{\bar{q}} = 1$. The wage w can then be solved,

$$w = N_F^{\beta}\,(1-\beta)\,\bar{q}. \tag{18}$$

Plugging it back into the intermediate firm's problem derives that both the production output y_j and profit π_j are linear in quality,

$$y_j = \frac{q_j}{N_F}, \pi_j = \frac{\beta q_j}{N_F^{1-\beta}}. \tag{19}$$

We drop the subscript j from the firm-level variable when it does not cause confusion. In this model, it is assumed that intermediate firms, responsible for hiring inventors to create innovation, operate in an environment where the competition ensures that the entire value from innovations is captured by inventors. The discounted value of being a firm of quality q is, therefore, the same as the net present value in the case where no innovation occurs. Thus, the value function of intermediate firm q at time t can be written as

$$V(q,t) = \int_t^{\infty} e^{-(r+\tau)(s-t)} \beta q / N_F^{1-\beta} ds = \nu q. \tag{20}$$

where $\nu = \frac{\beta}{(r+\tau) N_F^{1-\beta}}$. The value function is linear in its quality, q, and does not depend on time. This result implies that for any firm, the value of the same quality improvement Δq is the same. We will use q to denote both firm quality and size in the following sections.

The aggregate production is linear in the average quality $\bar{q}$. The resource constraint of the economy is $Y = C_H + C_I$. The relationship between the growth rate, g, and the time discount factor can be derived from the household's maximization problem,

$$g = \frac{\dot{Y}}{Y} = \frac{\dot{C}_H}{C_H} = \frac{\dot{\bar{q}}}{\bar{q}} = r - \rho. \tag{21}$$

4.2 Hiring Inventors

Incumbent (intermediate) firms engage in competition to attract inventors by offering a compensation package including equity a and wage $\tilde{T}$. The setup yields a principal-agent problem, where the interests of the risk-neutral firms, who benefit from innovation, and the risk-averse inventors, who dedicate effort to create innovations, are not aligned.

While firms derive value from the innovations, they are not able to monitor the effort exerted by inventors. Consequently, firms aim to incentivize the inventors to invest effort by offering equity, while concurrently share the risk with inventors through a fixed wage. Firms enjoy the innovations produced by inventors, but it is costly for inventors to work and effort is impossible to monitor. Thus, firms want to split the surplus with the inventor by offering a constant wage; meanwhile, they need to incentivize the inventor to exert effort by offering equity. For an intermediate firm, the optimization problem is as follows:

$$\begin{aligned}
&\max_{a,\tilde{T}} (1-a)\left(\tilde{V}(\tilde{q}) + \lambda_0 e_I x_0 (z_0, \tilde{q}) \nu dt\right) - \tilde{T} \\
&\quad \text{st}\, e_I = \arg\max \left\{ u\left(c_I\left(a, \tilde{q}, \tilde{T}\right), e_I\right)\right\} \\
&\quad u\left(c_I\left(a, \tilde{q}, \tilde{T}\right), e_I\right) \geq \bar{u}(z_0) \\
&\quad (1-a)\left(\tilde{V}(\tilde{q}) + \lambda_0 e_I x_0 (z_0, \tilde{q}) \nu dt\right) - \tilde{T} \geq \tilde{V}(\tilde{q})
\end{aligned} \tag{22}$$

In this problem, a firm $\tilde{q}$ chooses the optimal contract $\left\{a, \tilde{T}\right\}$ for an inventor z_0 to maximize its own payoff while taking three constraints into consideration. The firm's expected payoff consists of two components: the wage paid to the inventor $\tilde{T}$, and the expected firm value owned by the original shareholders (all shareholders except the inventor), given by $(1-a)\left(\tilde{V}(\tilde{q}) + \lambda_0 e_I x_0 (z_0, \tilde{q}) \nu dt\right)$, where $\tilde{V}(\tilde{q})$ is the firm value prior to innovation.

The first constraint is the inventor's incentive compatibility constraint, ensuring that when the inventor is employed by the firm, her actions align with utility maximization. Namely, when facing the firm-specific contract $\left\{a, \tilde{T}\right\}$, the inventor chooses an effort level e_I to maximize her expected utility, denoted as $u\left(c_I\left(a, \tilde{q}, \tilde{T}\right), e_I\right)$. The second constraint describes the inventor's participation constraint, meaning the inventor prefers to accept this firm's employment offer over other alternatives. This condition implies that the firm needs to offer the inventor a utility level surpassing her outside option $\bar{u}((z_0))$. The outside option is endogenously determined within this model by the Betrand competition among firms in the inventor market. Lastly, the third constraint is the firm's participation constraint, guaranteeing the firm will not be worse off by hiring one inventor.

Though firms all have the same optimization problem in Equation 22, their optimal equity level a depends not only on the inventor's idea quality z_0, but also the firm size $\tilde{q}$. Firm sizes affect the composition of the risk profile in an inventor's utility function:

$$u\left(c_I\left(a, \tilde{q}, \tilde{T}\right), e_I\right) = \mathbb{E}\left(c_I\left(a, \tilde{q}, \tilde{T}\right)\right) - A\mathrm{Var}\left(c_I\left(a, \tilde{q}, \tilde{T}\right)\right) - R\left(e_I\right). \tag{23}$$

The consumption $c_I\left(a, \tilde{q}, \tilde{T}\right)$ depends on the contract terms $a, \tilde{T}$ and the firm size $\tilde{q}$. The expected consumption includes two components—the flat wage and the equity value, which is the sum of the original firm value, and the value of innovation:

$$\mathbb{E}\left(c_I\right) = a\left(\tilde{V}\left(\tilde{q}\right) + \lambda_0 e_I x_0\left(z_0, \tilde{q}\right)\nu dt\right) + \tilde{T}$$

Similarly, the variance also comes from two sources: the variance in non-innovation-related firm value[8] and the variance in the R&D process. It can be written as:

$$\mathrm{Var}\left(c_I\left(a, \tilde{q}, \tilde{T}\right)\right) = a^2\left(\underbrace{\sigma_0^2\left(\tilde{q}\right)dt}_{\mathrm{Var}\tilde{V}(\tilde{q})} + \underbrace{\lambda_0 e_I \mathbb{E}\left(\tilde{x}\left(z_0, \tilde{q}\right)^2\right)\nu^2 dt}_{\mathrm{Var(innovation)}}\right)$$

Both terms increases in firm size $\tilde{q}$, but the former one increases in a faster speed, meaning in larger firms, shocks unrelated to R&D are stronger. Hence, larger firms are subject to larger incentive problems and the equity held by the inventor provides a weaker incentive for R&D efforts. Upon reviewing all available contracts, an inventor determines her preferred firm $\tilde{q}$.

Section 4.2.1 uses a simplified model to show the inventor's trade off in a closed form. With the same intuition, section 4.2.2 studies the inventor-firm matching using the full model.

4.2.1 A Closed Form Example

This section describes a simplified model which gives tractable results. We use it to illustrate the intuition. This simplified model adopt one additional assumption: the innovation value $\tilde{x}$ is drawn from a distribution with mean $x_0\left(z_0, \tilde{q}\right)\nu$ and second order moment $e_I^{-1}x_0\left(z_0, \tilde{q}\right)^2\nu^2$, instead of the uniform distribution $U\left(\left(1-\phi\right)x_0\left(z_0, \tilde{q}\right)\nu, \left(1+\phi\right)x_0\left(z_0, \tilde{q}\right)\nu\right)$. With this

[8] $\mathrm{Var}\tilde{V}\left(\tilde{q}\right) = \sigma_0^2\left(\tilde{q}\right)dt$. The risk is from exogenous exit, meaning $\sigma_0^2\left(\tilde{q}\right) = \tau\tilde{q}^2\nu^2$.

change, the innovation-related uncertainty is now

$$\operatorname{Var}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right)=\left(\underbrace{\sigma_0^2\left(\tilde{q}\right)dt}_{\operatorname{Var}\tilde{V}(\tilde{q})}+\underbrace{\lambda_0\left(x_0\left(z_0,\tilde{q}\right)^2\right)\nu^2dt}_{\text{Var(var innovation)}}\right),$$

which does not depend on effort level any more.

Use backward induction. Firm knows the inventor would choose an effort level [9]:

$$e_I=\lambda_0 a x_0\left(z_0,\tilde{q}\right)\nu$$

Both the equity a and the innovation value $x_0\left(z_0,\tilde{q}\right)$ positively influence the exertion of effort. When an inventor owns a higher proportion of equity or when the potential value of her innovation is greater, she is inclined to invest more effort. This is because that, in both scenarios, given the cost function, the return of spending one more unit of effort is larger.

The firm's problem in Equation 22 can be rewritten as:

$$\begin{aligned}\max_a &\left(\tilde{V}\left(\tilde{q}\right)+\lambda_0 e_I x_0\left(z_0,\tilde{q}\right)\nu dt\right)-\bar{u}\left(z_0\right)\\ &-Aa^2\left(\sigma_0^2\left(\tilde{q}\right)dt+\lambda_0 x_0\left(z_0,\tilde{q}\right)^2\nu^2dt\right)-\frac{1}{2}e_I^2\\ \text{st}\, &e_I=\lambda_0 a x_0\left(z_0,\tilde{q}\right)\end{aligned}\tag{24}$$

The first order condition yields:

$$a^*=\frac{\lambda_0^2 x_0\left(z_0,\tilde{q}\right)^2\nu^2}{\lambda_0^2 x_0\left(z_0,\tilde{q}\right)^2\nu^2+2A\left(\sigma_0^2\left(\tilde{q}\right)+\lambda_0 x_0\left(z_0,\tilde{q}\right)^2\nu^2\right)}\tag{25}$$

The equity level a decreases with the firm size $\tilde{q}$ when $b<1$. This is because the optimal equity a is determined jointly by two forces: the commercialization value $x_0\left(z_0,\tilde{q}\right)$, and the non-innovation-related shocks $\sigma_0^2\left(\tilde{q}\right)$. The grater commercialization value leads to a higher equity choice, since it is more worthwhile to incentivize inventors. Meanwhile, the non-innovation-related shocks is negatively related to the equity level, for that firms want to avoid exposing inventors to unrelated risks. The firm size $\tilde{q}$ affects both factors but in opposite directions. The innovation value increases in the firm size, causing larger incumbent firms to offer a higher amount equity. It has a positive influence on the former force, , and has a negative impact on the latter one. The relationship between the equity a and the firm size $\tilde{q}$ depends on the relative strength of the two channels. Under our functional form assumption, the second channel dominates, meaning larger firms optimally offer less equity to

[9] We use $\delta=1$.

an inventor. The optimal compensation scheme is $(a^*, \tilde{T}^*)$, where the wage $\tilde{T}^*$ is determined by the zero profit, due to Betrand competition. $\tilde{T}^* = -a^*\tilde{V}(\tilde{q}) + (1-a^*)\lambda_0 e_I x_0(z_0, \tilde{q})\nu dt$.

Upon reviewing all contracts, an inventor z_0 chooses which firm $\tilde{q}$ to work for by maximizing her utility:

$$\begin{aligned}\max_{\tilde{q}} u\left(c_I\left(a, \tilde{q}, \tilde{T}\right), e_I\right) =& \mathbb{E}\left(c_I\left(a, \tilde{q}, \tilde{T}\right)\right) - A\text{Var}\left(c_I\left(a, \tilde{q}, \tilde{T}\right)\right) - R(e_I) \\ \text{st } a &= a^*(\tilde{q}) \\ \tilde{T} &= \tilde{T}^*(\tilde{q})\end{aligned}$$

The FOC yields

$$\frac{\partial x_0(z_0, \tilde{q})}{\partial \tilde{q}} = \frac{2A\sigma_0^2}{4A\sigma_0^2 \frac{\tilde{q}}{x_0(z_0,\tilde{q})} + \frac{2A\lambda_0 + \lambda_0^2}{\tilde{q}/x_0(z_0,\tilde{q})}} \tag{26}$$

The left-hand-side element is the benefit of joining a larger firm—higher synergy and hence better commercialization. The right-hand-side element is the cost—the innovation contributes to a smaller share of firm uncertainty. The two forces and the solution of the optimization problem is shown in Figure 7.

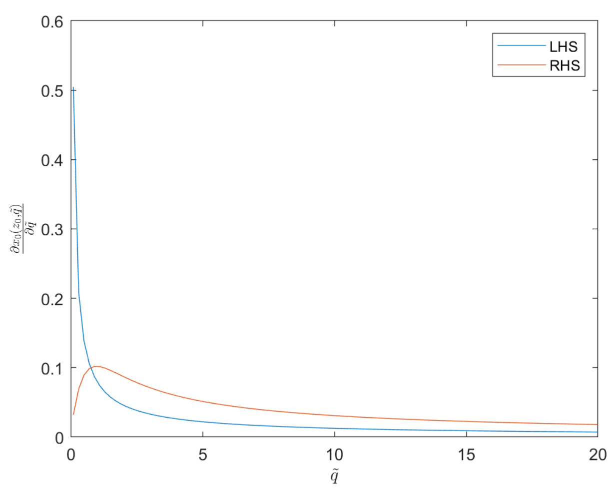


Figure 7: FOC condition - the optimal size of firm to work for

For inventor z_0, the optimal firm size is $\tilde{q}^* = \left(\frac{\left(2A\lambda_0 + \lambda_0^2\right)(\gamma(z_0)z_0)^{2b}}{2A\sigma_0^2 q_0^{2b}(1-2b)}\right)^{\frac{1}{2-2b}}$. When $b < 0.5$, which puts a upper bound on how fast the synergy can change with firm size, the second force dominates. The model predicts that better quality innovations are more likely to be created in larger firms, as long as they are developed in an incumbent.

4.2.2 The Full Model

This section describes the full model, releasing the assumption that the innovation value's second order moment is inversely related to the effort e_I. Similar as before, use backward induction, firm knows the inventor would choose an effort level:

$$e_I = \lambda_0 a x_0 (z_0, \tilde{q}) \nu - A a^2 \lambda_0 \mathbb{E} \left(\tilde{x} (z_0, \tilde{q})^2 \right) \nu^2 \tag{27}$$

Exerting one more unit of effort has three effects: a heightened likelihood of generating innovation, an increased probability of experiencing a positive shock and greater cost. Comparing with the closed-form example, an inventor strategically opts for a lower level of effort e_I, since the effort introduces uncertainty in this case.

The firm's problem in Equation 22 becomes

$$\begin{aligned} &\max_a \left(\tilde{V} (\tilde{q}) + \lambda_0 e_I x_0 (z_0, \tilde{q}) \nu dt \right) \\ &\quad - A a^2 \left(\sigma_0^2 (\tilde{q}) dt + \lambda_0 e_I \mathbb{E} \left(\tilde{x} (z_0, \tilde{q})^2 \right) \nu^2 dt \right) - R (e_I) \\ &\text{st } e_I = \lambda_0 a x_0 (z_0, \tilde{q}) \nu - A a^2 \lambda_0 \mathbb{E} \left(\tilde{x} (z_0, \tilde{q})^2 \right) \nu^2 \end{aligned} \tag{28}$$

When $x_0 (z_0, \tilde{q})$ increases mildly with $\tilde{q}$, the optimal stock a decreases with firm size $\tilde{q}$. The optimal compensation scheme is $(a, \tilde{T})$, where $\tilde{T}$ is determined by the zero profit, due to Betrand competition. $\tilde{T} = -a\tilde{V} (\tilde{q}) + (1 - a) \lambda_0 e_I x_0 (z_0, \tilde{q}) \nu dt$.

Given contracts, inventor chooses which firm $\tilde{q}$ to work for by maximizing her utility. In each firm, her optimal effort level in given in Equation 27.

$$\begin{aligned} \max_{\tilde{q}} u \left(c_I \left(a, \tilde{q}, \tilde{T} \right), e_I \right) =& a (z_0, \tilde{q}) \left(\tilde{V} (\tilde{q}) + \lambda_0 e_I x_0 (z_0, \tilde{q}) \nu dt \right) + \tilde{T} \\ & - A a (z_0, \tilde{q})^2 \left(\sigma_0^2 (\tilde{q}) dt + \lambda_0 e_I \mathbb{E} \left(x (z_0, \tilde{q}) \right)^2 \nu^2 dt \right) - R (e_I) \\ \text{st } e_I =& \lambda_0 a (z_0, \tilde{q}) x_0 (z_0, \tilde{q}) \nu - A a (z_0, \tilde{q})^2 \lambda_0 \mathbb{E} \left(\tilde{x} (z_0, \tilde{q})^2 \right) \nu^2 \end{aligned}$$

The trade off is the same as before: larger firms provide better commercialization but worse incentives. The numerical solution shows that, among all inventors that are in firms, inventors with better ideas prefer bigger firms.

However, due to the friction in the inventor-firm matching market, only a fraction \$h\$ of inventors can go to their ideal firm: the rest are assigned randomly. The innovations within a firm are composed of two distinct components: the directed matched part and the frictional matched part. The firm-level innovation arrival rate can be written as:

$$\lambda_q(\tilde{q}) = \frac{h\lambda_0 e_I(z_0^*, \tilde{q})\,\psi(z_0^*)\,dz_0^* + (1-h)\,\tilde{f}(\tilde{q})\,dq\int_{z_0\in\{z_0|q^*(z_0)>0\}} \lambda_0 e_I(z_0,\tilde{q})\,\psi(z_0)\,dz_0}{N_F\tilde{f}(\tilde{q})\,dq} \quad (29)$$

where z_0^* is the inventor whose optimal choice is $\tilde{q}$.

The Novelty Index matters for both the inventor-firm matching process and the utility obtained. At the peak of a technology wave, the technology tend to exhibit greater novelty, thereby escalating adoption costs. As a result, in incumbent firms, an innovation is worth less and the synergy effect is weaker. Inventors strategically move to smaller firms, for that the advantages of big firms are less silent. Meanwhile, systematically, inventors working within firms experience lower utility[10].

4.3 Independent Inventors

In additional to joining an incumbent, an inventor starts her own company. The inventor, who is risk-averse, work with some risk-neutral partner to share risk. Similar as before, the inventor faces a compensation scheme $\left(a, \tilde{T}\right)$. However, the inventor, instead of others, is in charge of the research direction. Hence, the innovation value is solely determined by the idea quality z_0. Upon the creation, the normalized innovation value $\tilde{z}$ is a random draw from the distribution $U((1-\phi)\,z_0\nu, (1+\phi)\,z_0\nu)$. On average, a higher quality idea yields a better outcome.

The partner's problem is in the same form as the intermediate firm's, with $\tilde{q} = 0$ and

[10] If an inventor z_0 works in a firm $\tilde{q}$ when the novelty index is γ, the utility level is:

$$u(z_0,\tilde{q}) = \lambda_0 e_I x_0(z_0,\tilde{q}) - a^2 A\left(\lambda_0 e_I k x_0^2(z_0,\tilde{q}) + \sigma_0^2(\tilde{q})\right) - e_I^2/2$$

Take derivative with respect to $x_0(z_0,\tilde{q})$ yields:

$$\frac{\mathrm{d}u}{\mathrm{d}x_0} = 2A\sigma_0^2(\tilde{q}) + (1 - aAkx_0(z_0,\tilde{q}))\,\lambda_0^2 x_0^2(z_0,\tilde{q})$$

As long as comparing with the optimal firm size $\tilde{q}$, x_0 is not too big, the derivative is positive and the utility increases in x_0, and hence it increases in γ. It means that during the period when the technologies breakthroughs (B and γ are low), an inventor expects systematically less utility when working in an incumbent firm.

the innovation value is z_0 instead of $x_0(z_0, \tilde{q})$:

$$\begin{aligned} &\max_a (1-a)(\lambda_0 e_I z_0 \nu dt) - \tilde{T} \\ &\quad \text{st}\, e_I = \arg\max \left\{ u\left(c_I\left(a, 0, \tilde{T}\right), e_I\right)\right\} \\ &\quad\quad u\left(c_I\left(a, 0, \tilde{T}\right), e_I\right) \geq \bar{u}(z_0) \\ &\quad\quad (1-a)(\lambda_0 e_I x_0(z_0, \tilde{q})\nu dt) - \tilde{T} \geq 0 \end{aligned} \tag{30}$$

The partners still have zero profit.

The inventor decides her effort level by maximizing her utility level, which yields:

$$e_I = \lambda_0 a z_0 \nu - A a^2 \lambda_0 \mathbb{E}\left(\tilde{z}(z_0)^2\right)\nu^2 \tag{31}$$

The firm's problem in Equation 30 becomes

$$\begin{aligned} &\max_a (\lambda_0 e_I z_0 \nu dt) \\ &\quad\quad - A a^2 \left(\lambda_0 e_I \mathbb{E}\left(\tilde{z}(z_0)^2\right)\nu^2 dt\right) - \frac{1}{2}e_I^2 \\ &\quad \text{st}\, e_I = \lambda_0 a z_0 \nu - A a^2 \lambda_0 \mathbb{E}\left(\tilde{z}(z_0)^2\right)\nu^2 \end{aligned} \tag{32}$$

It gives the highest utility an inventor can obtain when working in a startup.

For each inventor, she can choose between working in a firm (with h probability in the optimal firm and $1-h$ probability working in a random firm), and in a startup. The inventor's decision rule is:

$$u(z_0) = \max\left(u\left(c_I(z_0, \tilde{q}^*), e_I(z_0, \tilde{q}^*)\right), u\left(c_I(z_0, 0), e_I(z_0, 0)\right)\right). \tag{33}$$

The inventor joins a startup when it offers higher expected utility.

4.4 Entry and Exit

A firm enters the market when it successfully creates an innovation as a startup. The amount of entry equals the amount of innovations in startups:

$$\lambda_I = \int_{z_0 \in \{\tilde{q}^*=0\}} \lambda_0 e_I(z_0, \tilde{q}=0)\psi(z_0)dz_0 \tag{34}$$

When it is stationary, the amount of firm that enters is the same as exits:

$$\tau N_f = \lambda_I. \tag{35}$$

4.5 Growth Rate

The growth is from one source: innovation. The aggregate growth can be written as:

$$\begin{aligned}
g &= \frac{\bar{q}(t+\Delta t) - \bar{q}(t)}{\bar{q}(t)\,\Delta t} \\
&= \frac{\int_{z_0 \in \{z_0 | \tilde{q}^* > 0\}} \left(h\lambda_0 e_I(z_0, \tilde{q}^*)\, x_0(z_0, \tilde{q}^*) + (1-h)\,\lambda_0 \int_{\tilde{q}} e_I(z_0, \tilde{q})\, x_0(z_0, \tilde{q})\, \tilde{f}(\tilde{q})\, d\tilde{q} \right) d\Psi(z_0)}{N_f} \\
&+ \frac{\int_{z_0 \in \{z_0 | \tilde{q}^* = 0\}} \lambda_0 e_I(z_0, \tilde{q}^* = 0)\, z_0\, d\Psi(z_0)}{N_f}
\end{aligned} \tag{36}$$

4.6 Market Concentration

The novelty level affects the market concentration through both intensive margin and extensive margin. On the one hand, there is positive assortative matching between idea quality and firm sizes: better ideas tend to be developed in bigger firms. When there is a technology breakthrough, the technology is less mature. It implies that the adoption cost is high, and inventors systematically shift to smaller firms, which weaker the sorting between firms and ideas. On the other hand, since more inventors move to startups, there are more entries and drives down the market concentration through the extensive margin.

4.7 Equilibrium

We end this section by summarizing the equilibrium. The R&D expenditure C_I of the economy can be written as

$$\begin{aligned}
C_I = &\int_{z_0 \in \{z_0 | \tilde{q}^* > 0\}} \left(h\lambda_0 e_I(z_0, \tilde{q}^*)\, x_0(z_0, \tilde{q}^*) + (1-h)\,\lambda_0 \int_{\tilde{q}} e_I(z_0, \tilde{q})\, x_0(z_0, \tilde{q})\, \tilde{f}(\tilde{q})\, d\tilde{q} \right) \psi(z_0)\, dz_0 \\
&+ \int_{z_0 \in \{z_0 | \tilde{q}^* = 0\}} \lambda_0 e_I(z_0, \tilde{q}^* = 0)\, z_0 \psi(z_0)\, dz_0
\end{aligned} \tag{37}$$

It captures all transfers made to inventors. Based on Equation (15), the equilibrium out put level Y is linear in $\bar{q}$

$$Y = \frac{1}{1-\beta} \frac{\bar{q}}{N_F^{1-\beta}}. \tag{38}$$

and the consumption level is

$$C_H = Y - C_I. \tag{39}$$

Definition A balanced growth path of this economy for any combination of t, q is the mapping between q and z_0, the allocation $\left(\left\{y_j^*\right\}_j, Y^*, C_I^*, C_H^*\right)$ the prices $\left(w^*, \left\{p_j^*\right\}_j\right)$, the growth rate g^*, the entry rate λ_I^*, and the measure of firms N_F^*, such that (1) for any $j \in [0,1]$, y_j^* and p_j^* satisfy Equation (17); (2) wage w^* satisfies Equation (18); (3) measure of the intermediate producers N_F^* satisfies Equation (35); (4) the mapping is the solution of Equation (33); (5) the entry rates λ_I^* satisfy Equation (34); (6) R&D spending C_I^* satisfies Equation (37); (8) aggregate output Y^* satisfies Equation (38); (9) aggregate consumption C_H^* satisfies Equation (39); and (10) steady-state growth rate g^* satisfies Equation (36).

5 Calibration

We calibrate the model to target the US economy in 1986. We use patents as a surrogate for innovations. An innovation's idea quality, denoted by z_0, and the realized value, x (z in the context of a startup), correspond to the patent's citation (scientific importance) and patent's pecuniary value, respectively. Additionally, we assume that the idea quality z_0 follows Pareto distribution characterized by a scale factor z_m and a shape factor α.

5.1 Identification

Table 1 reports the parameters we obtained from the literature, which we subsequently hold constant. We calibrate the eight remaining parameters, (λ_0, α, z_m, ϕ, B, b, q_0, h), using the minimum distance method, inspired by Lentz and Mortensen (2008). The parameters, along with their corresponding moments are in Table 2.

Growth Rate—Innovation is the only driver of growth in this model. Therefore, the scale factor of the innovation arrival rate is an important determinant of the aggregate growth rate. A higher arrival rate implies a shorter average time for innovation creation, leading to a subsequently elevated aggregate growth rate. We match the aggregate growth rate generated by the model to 2%, the long-run steady-state growth rate in the US.

The S.D.-to-Mean Ratio of Patent Citations—This ratio measures the dispersion in patent citations in the data. The number of patent citations captures the scientific/nonpecuniary value of patents, which reflects the idea quality z_0. We assume the idea quality follows the Pareto distribution with the shape parameter, α, and the scale parameter, z_m. α governs how dispersed the distribution is. Specifically, the s.d.-to-mean ratio of the distribution can be expressed as $\frac{1}{\sqrt{\alpha(\alpha-2)}}$. Although patents observed in the USPTO data are successful innovations, which is a subset of all the ideas, the s.d.-to-mean ratio of the patent citation distribution is still significantly affected by α. We derived the citation distribution by pooling all granted patents from 1976 with their citations recorded by the USPTO and calculate the s.d.-to-mean ratio. The ratio turns out to be around 2.

Innovation Value—The pecuniary value of innovations directly contributes to the value of firms. In the model, the pecuniary value of innovations, $x(z$ when in a startup), is a uniform distribution with its mean depending on the underlying scientific value of the idea. Given α, the average scientific value of ideas is governed by the scale parameter of the Pareto distribution, z_m. Therefore, we can use the average pecuniary value of patents to calibrate z_m. We adopt the same estimation method as in Kogan et al. (2017) that uses the stock market response to news about patents. The sample used extends the one in their paper and is provided by the authors. It combines patents issued to US firms from 1926 to 2022 with the stock market information from the CRSP and firm-level information from the Compustat. Based on our calculation, a patent is, on average, worth 0.035 times the average firm value. z_m is calibrated to match this number.

S.D.-to-Mean Ratio of Innovation Value conditional on Citations—The pecuniary value of innovations is based on the scientific value of ideas but is subject to some randomnesses. The degree of randomnesses is governed by ϕ in the model. Specifically, the s.d.-to-mean ratio of the uniform distribution of the innovation pecuniary value is $\frac{\phi}{\sqrt{3}}$. Exploiting the same sample used to pin down z_m, we estimate the s.d.-to-mean ratio of patent pecuniary value when controlling the number of citations of the patents. In the data, this ratio is 0.50.

Technology "Novelty" Index—The technology novelty is defined as the total forward citations over the sum of backward and forward citations of all patents granted in a year. The adoption frictions ($\gamma(z_0) = \frac{B}{B+z_0}$) of new ideas in an incumbent firm is determined by B. B corresponds to the backward citation stock in a certain period, representing the maturity of the technology to which the inventor's idea contributes. We calibrate the value of B such that the model-generated average adoption frictions ($\frac{B}{B+\int z_0 d\Psi(z_0)}$) equals to 1 − the "Novelty" Index, since $\int z_0 d\Psi(z_0)$ is corresponding to the total forward citations of all patents issued in the period.

Regression Coefficient of Innovation Value on Firm Size—Synergy provided by incumbent firms is governed by two parameters, b and $\tilde{q}_0$, with the former determining the elasticity of synergy with regard to the incumbent firm size and the latter determining the scale. To derive b, we take natural logarithm on both sides of the innovation value function, $x_0(z_0, q) = \left(\frac{\tilde{q}}{\tilde{q}_0}\right)^b \gamma(z_0) z_0$. Then we run the following regression in the extended sample of Kogan et al. (2017),

$$\ln(x(z_0, q)_{ist}) = b\ln(\tilde{q}_{ist}) + \gamma\ln(z_{0,ist}) + \theta_s + \mu_t + \epsilon_{ist},$$

where i, s, and t are respectively indexes for patents, technology classes, and years. The dependent variable is corresponding to the patent pecuniary value; the firm size, $\tilde{q}_{ist}$, is measured by the employment of the firm the patent belongs to; the idea quality, $z_{0,ist}$, is measured by the number of patent citations; θ_s and μ_t capture the fixed effects of patent technology classes and years. The coefficient of firm size pins down b.

Entrant-to-Incumbent Ratio—The scale parameter in the synergy function, q_0, affects the benefit of contributing an idea to an incumbent firm compared to initiating a new venture. Therefore, it is related to inventors' choice between incumbent firms and startups. We use the "Entrant-to-Incumbent Ratio" derived in Section 2.3.1 to calibrate q_0.

Firm Size Ratio by Fourth-to-First-Quartile of Patent Citations—The model predicts that, if inventors choose to join incumbent firms, ideally the firm size they choose increases in their idea quality. This positive sorting, nevertheless, is subject to matching frictions. When the friction is larger, the matching between inventors' idea quality and incumbent firm size is closer to random sorting, and their relationship is vaguer. To calibrate the degree of frictions, h, we generate in the model the average firm size by patent citation quartiles given the patent is developed by incumbent firms. Then we calculate the ratio of firm size in the fourth to first quartile and match it the the data counterpart as shown in Figure 8.

Table 1: Parameter Values from a Priori Information

Parameter	Description	Value	Identification
ρ	Discount rate	0.02	Interest Rate
β	Production function quality share	0.109	Firm profitability
τ	Exo. exit rate	0.04	BDS
A	Risk aversion	0.5	Risk aversion
δ	Effort cost elasticity	1	Effort cost elasticity

Notes: This table shows parameter values from the literature of direct estimation.

5.2 Estimation Results

Table 3 reports the empirical and model-generated moments using the model. Overall, the model matches the targeted moments closely. The resulting parameters are reported in table 4. Our estimates find that compared with startups, in terms of utilizing innovations,

Table 2: Parameter from the Minimum Distance Estimation

Parameter	Description	Identification
λ_0	Innovation arrival rate	Growth rate
α	Shape of idea quality distribution	S.d.-to-mean ratio of patent citations
z_m	Scale of idea quality distribution	Average innovation value
ϕ	Innovation value dispersion	S.d.-to-mean ratio of innovation value cond. on citations
B	Maturity of technology	Technology "Novelty" index
b	Exponent of the synergy function	Regression coefficient of innovation value on firm size
q_0	Scale of the synergy function	Entrant-to-incumbent ratio
h	Matching friction	Firm size ratio by fourth-to-first-quartile of citations

Notes: Parameters in this table are jointly calibrated to minimize the distance between the model and data moments.

Table 3: Moments

Identification Moment	Data	Model
Growth rate	0.02	0.028
S.d.-to-mean ratio of patent citations	2	2
Average innovation value	0.035	0.033
S.d.-to-mean ratio of innovation value cond. on citations	0.50	0.50
Technology "Novelty" index	0.6	0.6
Regression coefficient of innovation value on firm size	0.45	0.45
Entrant-to-incumbent ratio	0.06	0.06
Firm size ratio by fourth-to-first-quartile of citations	1.18	1.23

Notes: This table compares the moments generated from the calibrated model and the data. In general, the model generated moments match the data well.

incumbents have a non-negligible cost as captured by the discount factor $\gamma = 0.6$. Nevertheless, synergy plays a considerable important role in commercialization, since the denominator of the synergy function q_0 is as low as 0.002, and the exponent is 0.45; it means that a firm of the average size (about 2,000 employees) can generate 15 times more value at commercialization than a startup due to the synergy effect, and the difference shrinks to 9 times because of the cost of creative destruction.

Among the other results, the matching friction h suggests that there are significant friction in the inventor-firm matching market: only 14% inventors go to their optimal firms and the rest are assigned to sub-optimal firms. Figure 8 shows that the average firm size increases across idea quality quartiles. It implies that there is positive sorting between firms and inventors: better quality ideas tend to match with larger firms, which in turn causes concentration.

The sorting is more silent without the friction. The mapping between the optimal firm size and idea quality is in Figure 9. Compared with the frictional results, the differences are larger. Another implication is that when an idea exceeds some quality threshold, instead

Table 4: Estimated Parameter Values

Parameter	Description	Value
λ_0	Innovation arrival rate	1.167
α	Shape of idea quality distribution	2.300
z_m	Scale of idea quality distribution	0.005
ϕ	Innovation quality draw	0.632
B	Discount factor of idea commercialization	0.006
b	Exponent of the synergy function	0.450
q_0	Denominator of the synergy function	0.002
h	Matching friction	0.140

Notes: Parameters in this table are jointly calibrated to minimize the distance between the model and data moments.

of choosing a larger firm, the inventor would rather join a startup. This is consistent with the empirical observation that patents from startups on average receive more citations than patents from incumbents.

6 Quantitative Analysis

We use the model to think about how technology waves can shape market concentration through the perspectives of inventor's choice. Specifically, our analysis spans the period from 1986 to 2016, encompassing three distinct technology waves illustrated in Figure 1. Assuming the year 1986 as a steady state, we calibrate the model to align with the available data, as described in the previous section. Subsequently, we simulate the model for 20 years. Each year, we adjust the patent novelty level and the corresponding adoption friction to match the data. The simulation yields the model-generated market concentration and the innovation allocation. We compare the model-generated results with the actual data and assess to what extend our model can generate the time trend.

In the simulation exercise, we relax the constraint of stationarity except for the benchmark year (1986). Each year, we adjust B to ensure that commercialization discount factor of the average idea $\bar{z}_0$ consistently matches the patent novelty level. While we maintain the assumption of an always-equilibrium economy, it is no longer required to be stationary. In this non-stationary setting, the model state is jointly determined by two sets of state variables: the measure of firms N_f and the firm size distribution $f(q)$. Unlike the balanced growth path scenario, these variables are not stationary and adapt annually according to the equilibrium. As a result, the entry-exit equality in Equation 35 no longer holds, causing a gap between entry and exit, which introduces dynamism into the system. The corresponding

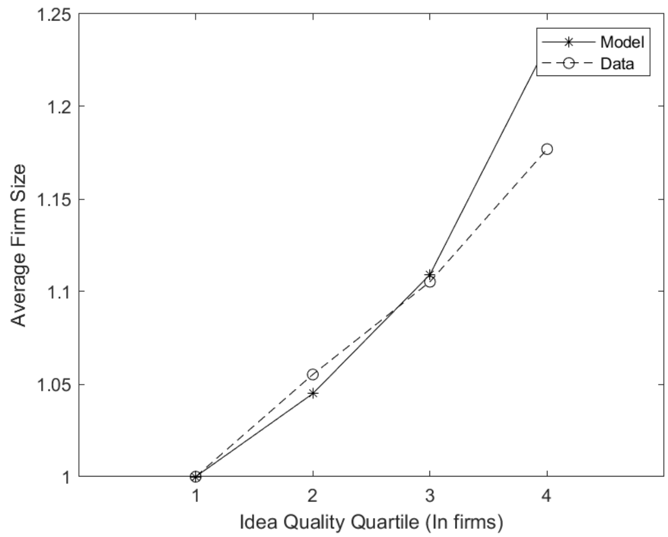


Figure 8: Estimated Mapping between Patent Citations and Incumbent Firm Size

Notes: This figure exhibits the mapping between inventors' idea quality and the firm sizes if inventors opt to develop their ideas in incumbent firms. The idea quality is measured by the number of patent citations and is classfied into four quartiles. The firm size is measured by the number of employees. The average employment of firms corresponding to the first citation quartile is normalized to be one. The solid curve represents the model prediction and the dashed one is the actual data in Figure 6.

firm numbers fluctuate each year according to:

$$N_f' = N_f(1-\tau) + \lambda_I. \tag{40}$$

When entry exceeds exit, the subsequent year begins with a higher count of firms, and conversely, if exit exceeds entry, fewer firms start the next year. Similarly, the firm size distribution shifts due to inventor-firm mapping. Simultaneously, the dynamics of inventor-firm mapping cause shifts in the firm size distribution. Starting with the firm size distribution from the previous year, we conduct a one-year simulation of firm size dynamics, ultimately arriving at the distribution for the subsequent year. The measure of firms, N_f, and the firm size distribution,$f(q)$, jointly determine the inventor-firm mapping, consequently shaping the market concentration each year.

6.1 Technology Waves and Market Concentration

We use to model to study to what extend, this channel can explain how technology waves influence market concentration. The results are presented in Figure 12. Despite not explicitly targeting any concentration measure during model calibration, our model

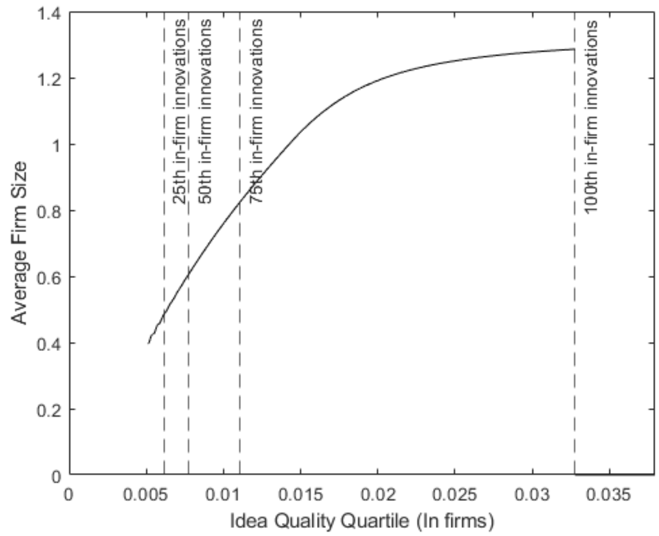


Figure 9: Estimated Mapping between Patent Citations and Incumbent Firm Size, frictionless

Notes: This figure exhibits the estimated mapping between inventors' idea quality and the firm sizes if inventors opt to develop their ideas in incumbent firms. The idea quality is measured by the number of patent citations and is classfied into four quartiles. The firm size is measured by the number of employees. The average employment size is normalized to be one. .

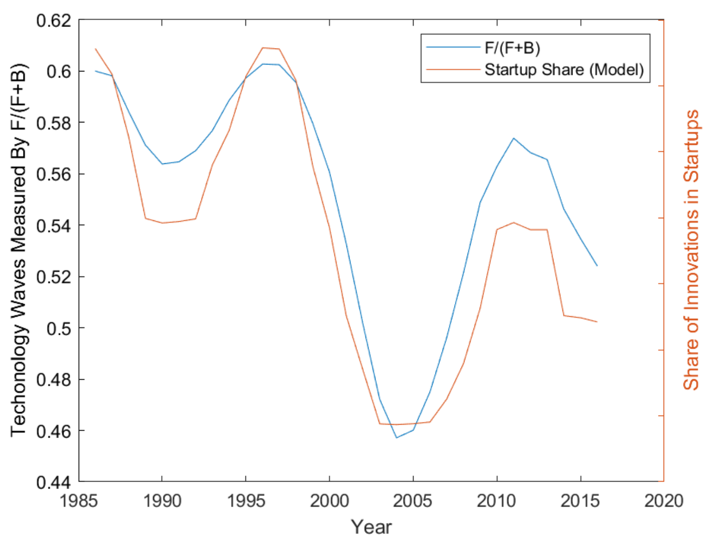


Figure 10: Technology Waves and the Share of Innovations in Startups

Notes: This figure shows the technological waves and the trend of model-generated share of innovations in startups over time. The blue curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations (same as Figure 1). The red curve displays the simulated startups' share of innovations in each year.

successfully replicates empirical concentration changes. The blue curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations (same as 1). The red curve displays the simulated HHI in each year, normalized by the HHI in 1986. The red dashed curve represents the empirical HHI, also normalized by 1986. Our model-generated HHI not only follows a similar pattern which inversely related to the data, but also lags behind the technological waves by approximately three years.

The simulated curve, however, tends to be higher than the empirical data, and exhibits a slight upward trend. This divergence is attributed to the duration of the technology waves. The second downward wave (1995-2005) and the third (2010-2016) take longer to unfold than the upward wave, which induces the seemingly upward trend in the model-generated HHI. Quantitatively, our model performs reasonably well, with a correlation of -0.70 between the technological wave and the 3-year-lagged HHI, compared to -0.79 in the empirical data.

6.2 Allocation of ideas

Empirically, we show that one potential link between the technological waves and the market concentration is inventors' choices of where to do innovation. They choose between starting a new business and working for an incumbent firm. In the second case, the inventor needs to decide the optimal firm size to work in. This section describes the idea flow and how it connects the technological novelty level and market concentration within the framework of our model.

The model predicts the co-movement between technological waves and the inventor's choice. As depicted in Figure 10, the model-generated share of innovations in startups move closely with the technological wave, measured by citations. The blue curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations (same as 1). The red curve displays the simulated startups' share of innovations in each year. Our pattern is consistent with the data but without the time lag. This is because in our model, citations and the technological waves are realized immediately, whereas in the data, it can take about five years for patents to accumulate citations and acceptance from others. This alignment is consistent with the empirical observations, whereas the correlation is higher than the data. One possible reason is that novelty is the only shock in our model, whereas there are more sources of time-varying shocks in the data.

The impact of technological novelty extends to both the intensive margin and the extensive margin of idea allocation. The former measures the number of inventors opting

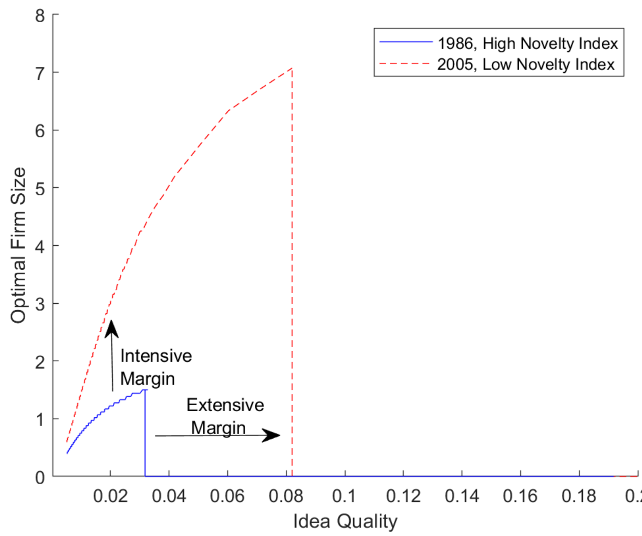


Figure 11: The Optimal Firm Size Comparison

Notes: This figure shows the optimal firm size by idea quality. The blue solid line and the red dashed line represents 1986 and 2006, respectively.

for new businesses, while the latter examines, among those working in firms, the selection of a particular firm. Figure 11 illustrates these two dimensions of idea allocation, drawing a comparison between 1986 and 2006. Specifically, 1986 wits technology breakthroughs, while technologies are mature in 2005. Comparing with 1986 (peak), in 2005 (trough), inventors displayed a tendency to choose larger firms in the latter period.

The extensive margin reflects the portion of inventors transitioning between incumbent firms and new businesses, while the intensive margin captures the reallocation within incumbent firms. Although operating through difference channels, both margins influence the market concentration. The extensive margin affects the number of new businesses. New businesses are not constraint by the positive assortative matching between firms and idea qualities, also increase the number of firms in the economy. These dual forces collectively shape market concentration dynamics.

Meanwhile, the intensive margin influences the market concentration by altering the positive assortative matching between firms and ideas. As depicted in Figure 11, ideas are developed in larger firms, augmenting the positive sorting effect.

We proceed to decompose the time-trend results. The extensive margin focuses on the changes in market concentration due to the entry of new businesses. In this context, inventors are restricted to altering their decision between new businesses and incumbents, without reallocating among incumbent firms. In other words, instead of selecting from a full spectrum of firm sizes, each inventor's decision set comprises solely a startup and one

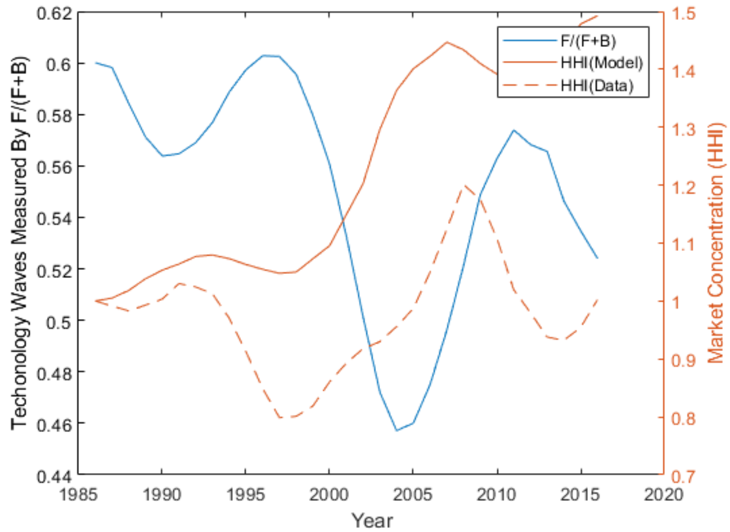


Figure 12: Technology Waves and Model Generated HHI

Notes: This figure shows the technological waves and the trend of model-generated market concentration over time. The blue curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations (same as 1). The red curve displays the simulated HHI in each year, which is normalized by the HHI in 1986. The red dashed curve is the empirical HHI moved forward for three years, also normalized by 1986.

incumbent firm that the inventor chooses in 1986. This restriction raises a question for inventors originally employed in new businesses: what is the optimal firm size if they decide to transition to an incumbent firm? Following the spirit of Heckman (1979), we define optimal incumbent firms for all inventors. For those initially working in incumbent firms, the optimal incumbent firm size corresponds to their current workplace. Conversely, for inventors in startups, it is the incumbent firm offering the highest utility, representing their second-best choiceâessentially, the optimal incumbent firm is the one they would have chosen in 1986 if selecting an incumbent.

The results generated by extensive margin itself are reported in Figure 13. Notably, the extensive margin is able to generate a substantial portion of the model effect. It is less silent at the peaks and even more lagged behind. This is because the extensive margin directly affect firm number, a predetermined factor in each period. By impacting this state variable, the extensive margin exerts a persistent influence on market concentration. In terms of correlation, the model is still doing a reasonable job in fitting data: the correlation is -0.70, almost 100% of the benchmark result.

In the second case, we control the extensive margin and adjust the intensive margin, meaning that ideas can be reallocated among incumbent firms, not between incumbent firms and new businesses. Namely, inventors choosing startups in 1986 can only select

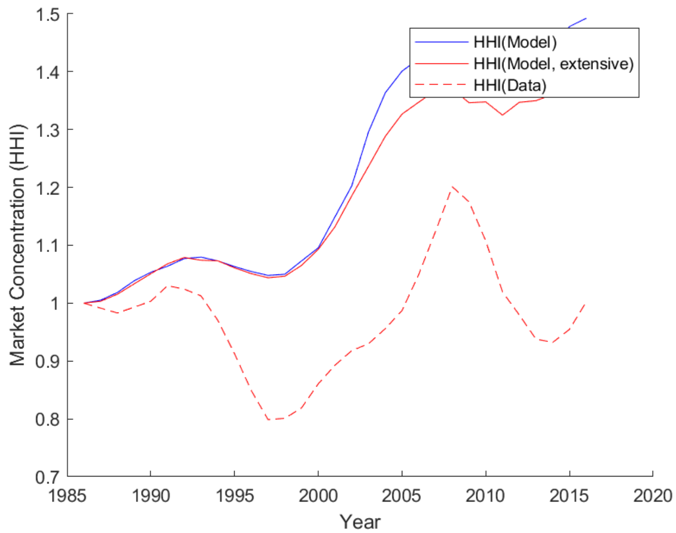


Figure 13: Technology Waves and Model Generated HHI, Extensive Margin

Notes: This figure shows the extensive margin of the model-generated market concentration over time. The blue curve and solid red curve displays the simulated HHI and the HHI only considering extensive margin, respectively. The red dashed curve is the empirical HHI moved forward for three years, also normalized by 1986.

startups in subsequent years. Conversely, those opting for incumbent firms in 1986 have the flexibility to choose any incumbent business each year. The results, depicted in Figure 14, demonstrate the elimination of trends and significantly smaller magnitudes of changes. The correlation remains similar at -0.71. This analysis underscores the importance of both extensive and intensive margins in shaping the dynamics between technological waves and market concentration.

7 Conclusion

This paper studies how technological waves shape the market concentration, through the reallocation of inventors. This study provides empirical evidence and structural analysis showing that market concentration, measured by HHI, is inversely related to and lagged behind the technological waves. This discovery suggests the presence of a low-frequency business cycle in the economy. We explore one potential channel behind this connection: the allocation of ideas. Using the data from the Longitudinal Business Database (LBD) from the Census Bureau and the patent information from the USPTO, this paper shows that the share of patents formed in new businesses co-move closely with the technological waves. At the peaks of the technological waves, a larger share of patents are forming in new businesses,

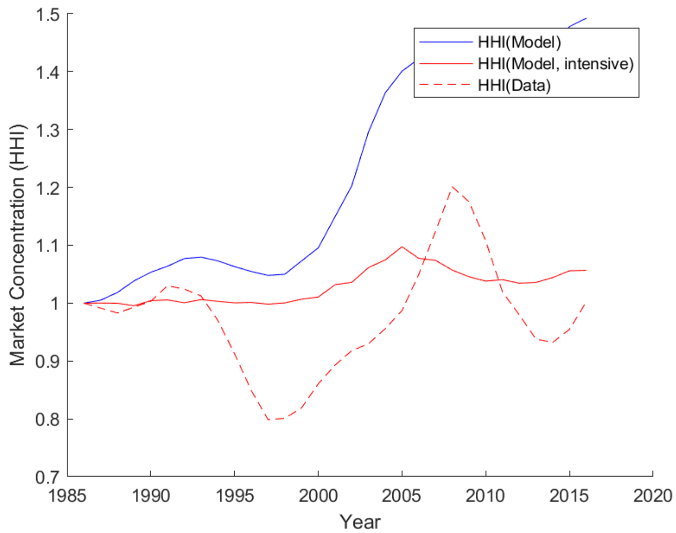


Figure 14: Technology Waves and Model Generated HHI, Intensive Margin

Notes: This figure shows the extensive margin of the model-generated market concentration over time. The blue curve and solid red curve displays the simulated HHI and the HHI only considering intensive margin, respectively. The red dashed curve is the empirical HHI moved forward for three years, also normalized by 1986.

while at the troughs, a larger share of patents come from existing businesses.

This paper proposes a theoretical framework that elucidates the decision-making process of inventors regarding their choice of innovation pathways, thus providing an explanation for the observed empirical patterns. Inventors are faced with a choice between forming a new business of a random size with a partner or joining an incumbent business of a selected size. This decision hinges on a trade-off: new businesses offer better incentives and adaptability in embracing novel technologies, while incumbents possess synergies and experience in commercialization. Our model effectively captures the relationship between technological waves and market concentration, primarily through the redistribution of innovative ideas. It implies that the deceleration in the emergence of groundbreaking technologies could be a significant contributing factor to the rise in market concentration after the 2000s.

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