Technology Driven Market Concentration through Idea Allocation*

Yueyuan Ma[†] Shaoshuang Yang[‡]

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Abstract

Using a newly-created measure of technology novelty, this paper identifies periods with and without technology breakthroughs from the 1980s to the 2020s in the US. It is found that market concentration decreases at the advent of revolutionary technologies. We establish a theory addressing inventors' decisions to establish new firms or join incumbents of selected sizes, yielding two key predictions: (1) A higher share of inventors opt for new firms during periods of heightened technology novelty. (2). There is positive assortative matching between idea quality and firm size if inventors join incumbents. Both predictions align with empirical findings and collectively contribute to a reduction in market concentration when groundbreaking technologies occur. Quantitative analysis shows the overall slowdown in technological breakthroughs can capture 95.9% of the rising trend in market concentration and the correlation between the model-generated and the actual detrended market concentration is 0.910.

Keywords: technological waves, HHI, startups, incumbent firms.

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[†]Affiliation: University of California, Santa Barbara. Email: yueyuanma@ucsb.edu.

[‡]Affiliation: The Chinese University of Hong Kong, Shenzhen. Email: yangshaoshuang@cuhk.edu.cn.

1 Introduction

The interplay between technological progress and market concentration plays a significant role in economic growth and resource allocation. Most of the existing studies focus on the impact of the firm size distribution on technology evolvement (e.g., Akcigit and Kerr (2018); Cunningham, Ederer and Ma (2021); Akcigit and Goldschlag (2023)), this paper provides empirical evidence and structural analysis showing that the reverse relationship is also important—technological novelty waves affects the market concentration by relocating innovative ideas between incumbent firms and new businesses.

Using a newly-created measure of the novelty of new technologies, this paper identifies periods in the US when technology breakthroughs occur and periods when most new technologies follow existing ones from the 1980s to the 2020s. There is a declining trend in technological novelty. Besides, technological novelty follows waves. At the peak, groundbreaking technologies replace existing ones, while at the trough, most technologies have entered a mature stage.

Surprisingly, we find a rising trend and a cyclical pattern in market concentration, as measured by the Herfindahl-Hirschman Index (HHI) of firm sales or employment, which exhibits a notable negative correlation with the technological novelty waves. This observation strongly suggests that the emergence and maturation of novel technologies may be influential factors in shaping market concentration dynamics.

How are technological waves and market concentration connected? A potential channel is through the allocation of ideas. Since firm size is to a large extent impacted by firm productivity and new ideas are important sources of productivity growth, where new ideas contribute their value will determine the firm size distribution, and therefore, market concentration. Combining the Longitudinal Business Database (LBD) from the Census Bureau and the patent information from the USPTO, this paper tracks the affiliation of patents at their formation. It is shown that at the peaks of the technological waves, a larger share of patents are forming in new businesses, while at the troughs, a larger share of patents come from incumbent firms. Besides, among patents from incumbent firms, there is a positive relationship between patent citations, a quality measure of the ideas behind them, and the size of the firm. These patterns indicate that technological waves affect the number of firm entries and the way new ideas combine with firms of different sizes.

Further patent-level regression analysis reveals that incumbent firm size positively affects the private economic value of patents, given their scientific value, indicating synergy between inventors and incumbent firms. The scientific value of patents has a positive impact on the economic value, while this impact decreases in the aggregate technological novelty.

Based on the empirical findings, this paper proposes a theory about inventors' choice of where to contribute the value of their ideas, and how it connects the technological waves and market concentration. The technological novelty level is assumed to be a random aggregate shock capturing the random arrival of ground-breaking innovations in a period. Each inventor is endowed with an idea of idiosyncratic quality. The inventor needs to choose between forming a new firm of a random size with a partner or joining an incumbent firm. In the case of the latter, she must also decide on the size of the incumbent firm to join. It is frictional for an incumbent firm to adopt new technology as in Greenwood and Yorukoglu (1997), and the friction increases in the aggregate technological novelty. Hence, occurrence of groundbreaking technologies leads more inventors to form startups. Inventors' decisions directly impact firm-level innovation intensity, technology improvement, and hence the firm size distribution. Simulation of the calibrated model shows that the evolvement of the market concentration generated by the technological waves captures 95.9% of the actual rising trend and has a correlation of 0.910 with the actual detrended fluctuations.

The model in this paper includes three key elements: novelty-related realization potential, commercialization synergy, and inventor-firm contracts. The novelty-related realization potential captures the value of innovation when an incumbent firm, already utilizing established technologies, incorporates new ideas into its production processes. This integration requires interaction between the idea and existing technologies, a process that becomes increasingly challenging as the novelty of the idea grows, highlighting the frictions inherent in adopting new technologies. During the peaks of technological waves, the realization potential of ideas often declines. Consequently, startups, free from these adoption frictions, emerge as more attractive platforms for innovation. Commercialization synergy pertains to the added value incumbent firms can provide through their production and commercialization capacities, which startups typically lack. Larger firms offer more synergy, especially to high-quality ideas. Inventor-firm contracts define the collaboration between inventors and firms, ultimately determining idea allocation. The R&D process is risky and success depends on the inventors' unobservable effort, necessitating contracts that incentivize optimal effort through a mix of equity and wages. Larger firms face greater incentive challenges due to larger unrelated shocks, making equity a weaker incentive for R&D efforts.

Inventors must consider the realization potential, synergy, and contract terms when choosing between startups and incumbent firms. Startups, while free from adoption friction and offering aligned incentives, lack the capacity to generate synergy. In contrast, incumbents face adoption friction, with larger firms providing weaker incentives but better synergy. These trade-offs guide inventors in their strategic decision-making regarding firm affiliation.

The model has two major predictions. First, a larger share of inventors choose to start new firms to develop their ideas during periods of high technological novelty since the realization potential at incumbent firms is lower. Second, among inventors that choose to do R&D in incumbent firms, there is positive assortative matching between idea quality and firm size. Therefore, firms already with a larger size attract ideas of higher value. These two predictions are consistent with observations in data and collectively contribute to a reduction in market concentration when the economy is closer to the peak of the technological waves. The upsurge in new startups leads to a proliferation of firms in the market. Given that new startups are less constrained by the positive matching between idea quality and firm size, they offer a counterbalance to the tendency of larger firms to further expand.

To quantify the impact of the technological novelty waves on market concentration through allocation of new ideas, we calibrate the model and then do simulations by changing the degree of novelty of new technologies in an economy. The model is calibrated to match the average data moments between 1982 and 2016. Key moments include patent novelty, average patent value, degree of positive matching between patent citation and firm size, the growth rates, etc. In the simulation exercise, we fix all the parameters except for the one related to patent novelty for each year following 1986, the first peak of the technological waves within our sample period. This variation serves to capture the evolving dynamics of realization potential within incumbent firms. Consequently, we generate paths of two essential data moments: (1) the ratio of the number of ideas in new firms relative to those in incumbent firms; (2) the HHI of firm sales or employment. The two paths are compared with the data.

The two generated paths of moments are consistent with the actual trend in general and nearly have simultaneous peaks and troughs with the actual time variations. In particular, the model-generated HHI captures 95.9% of the actual rising trend. The correlation between the detrended model-generated and the detrended actual HHI is 0.910; the correlation between the detrended model-generated and the detrended actual new-to-incumbent ratio is 0.825. These comparisons indicate that the technology waves is a strong driving force of idea allocation and market concentration.

To decompose the effect of the two channels, changes in firm numbers (extensive margin) and the positive assortative matching (intensive margin) between idea quality and firm size, on the evolvement of market concentration, we track the HHI change driven by firm numbers in the simulation process. The decomposition shows that while the extensive margin is the main driver of the rising trend in market concentration, the intensive margin reacts more swiftly to technological waves, aligning the simulated HHI's response timing more closely with the data.

Related Literature

This paper is closely related to the literature on the interplay between innovation and market concentration. On the one hand, innovation leads to technological advancement that creates monopoly rents and larger firm size (Aghion and Howitt, 1990; Grossman and Helpman, 1991; Klette and Kortum, 2004). On the other hand, firms of different sizes are shown to have different innovation intensities in the literature, indicating that the overall innovation intensity depends on both the firm size distribution (Akcigit and Kerr, 2018) and the market for ideas (Eaton and Kortum, 1996; Silveira and Wright, 2010; Chatterjee and Rossi-Hansberg, 2012; Cabral, 2018; Perla, Tonetti and Waugh, 2021; Fons-Rosen, Roldan-Blanco and Schmitz, 2021). Theories and empirical evidence in this aspect can be traced back to the Schumpeterian argument that large firms have a higher capacity to do R&D, to more recent findings that small firms are more inclined to engage in innovation activities due to the rise of the patent market (Cassiman and Veugelers, 2006; Bena and Li, 2014; Akcigit, Celik and Greenwood, 2016; Liu and Ma, 2021; Ma, 2022; Yang, 2023). Most of the existing studies focus on the relationship between innovation efforts and market structure (e.g., Cavenaile, Celik and Tian (2019)), while this paper finds novel patterns that the novelty of new technologies is closely correlated with the market concentration measure. To our knowledge, this is the first paper that uncovers the relationship between market concentration and the technological novelty waves.

Our empirical and theoretical analyses indicate that the degree of novelty associated with emerging technologies significantly influences where inventors choose to conduct their R&D. This perspective provides an alternative viewpoint on the relationship between the allocation of new ideas and market concentration. Existing research emphasizes the opposite relationship. Studies like Cunningham, Ederer and Ma (2021) and Akcigit and Goldschlag (2023) have posited that incumbent firms strategically acquire innovative startups or independent inventors only to subsequently abandon their ideas, thus preventing competition from new entrants and effectively stifling novel ideas. Therefore, the decrease in the novelty of new technologies is due to market concentration and the high monopoly power of incumbent firms. Our paper does not contradict these assertions. Instead, the analysis in this paper suggests that technological novelty and market concentration may have mutual effects and the mutual effects amplify each other in the negative correlation between the technological waves and market concentration.

This paper provides a new perspective on the causes of rising market concentration in the U.S. since the late 1990s. Notably, this increasing concentration has been accompanied by greater allocative efficiency and productivity growth (Autor et al. (2020) and Ganapati (2021)). A similar positive relationship has been observed in Europe (Bighelli et al. (2023)). However, literature also indicates that good ideas are becoming harder to find (Bloom et al. (2020)), the productivity gap between large and small firms is widening, and firm entry rates are declining (Akcigit and Ates (2023) and Olmstead-Rumsey (2019)). Our analysis offers an unexplored explanation for the rise in market concentration: the deceleration in the emergence of revolutionary technologies. This perspective reconciles the seemingly contradictory findings in the literature. The novelty metric for new technologies, defined in this paper, shows that the peaks of technological waves occurred in the mid-1980s and mid-1990s, with a significant 20-year gap before reemerging in the early 2010s. This extended period without significant technological breakthroughs led idea holders to gravitate toward incumbent firms, resulting in increased concentration among these larger firms and a decline in new firm entry. The lack of significant adoption frictions also increased realization potential of technologies by incumbent firms, thereby enhancing short-term productivity growth as it took time for groundbreaking technologies to materialize their economic benefits.

Finally, our analysis delves into the implications of the introduction of groundbreaking technologies. Bowen III, Frésard and Hoberg (2023) show empirically that in an era with rapid evolving technologies, more startups remain independent rather than being sold out. Dinlersoz, Dogan and Zolas (2024) discover a surge in AI business applications after 2016. Greenwood and Yorukoglu (1997) and Greenwood and Jovanovic (1999) establish that technological revolutions lead to deterioration in the stock value of existing firms. The adoption of the novel technologies is costly and requires skilled labor, therefore, slowing down economic growth and widening income inequality in the short run. Jovanovic and Rousseau (2014) shows that at the advent of new technologies, incumbent firms decrease investment due to lack of compatibility while new firms increase investment. This paper extends the existing literature by investigating how a leap in technological progress affects the distribution of firm sizes, primarily due to the frictions when integrating inventors' novel ideas into incumbent firms. It is shown that market concentration is another important outcome of technological revolutions. This paper demonstrates that apart from the high-frequency business cycle influenced by productivity fluctuations (Kydland and Prescott (1982)), the economy may also be susceptible to a low-frequency cycle driven by the waves of technological novelty.

The rest of the paper is organized as follows. Section 2 introduces measures of the technological waves, market concentration, and the allocation of ideas, and subsequently presents their patterns. Section 3 constructs a model where inventors make decisions between initiating new ventures or joining established incumbents at specific sizes. We derive predictions concerning the mapping between the quality of inventors' ideas and their optimal choices. Section 4 defines the balanced growth path, the aggregate growth rate, and market concentration. Section 5 calibrates the model. Section 6 simulates the model to evaluate the degree to which technological waves can account for changes in market concentration through the idea allocation channel. Section 7 concludes.

2 Empirical Patterns

This section exhibits empirical patterns of the technological waves, market concentration, and a potential channel that links the two—the choices of the inventors on where to develop their ideas.

2.1 Technological Waves

Technology waves capture the extent of new technology breakthroughs over time. At the peak of the technological waves, significantly highly novel technologies emerge that are often incompatible with existing technologies; at the trough of the waves, most of the technologies in the economy have reached a mature state, and the improvement over existing ones is incremental.

2.1.1 Measurement

To measure the technological waves, we create a "Novelty" Index of the new technologies in each year using the patent citation data. Specifically,

$$\text{Novelty}_t = \frac{\sum_{i \in I_t} \sum_{s=0}^{5} \text{Forward Citations}_{i,t+s}}{\sum_{i \in I_t} \sum_{s=0}^{5} \text{Forward Citations}_{i,t+s} + \sum_{i \in I_t} \sum_{s=0}^{5} \text{Backward Citations}_{i,t-s}}, \quad (1)$$

where I_t is the set of the new patents granted in year t. The numerator is a summation of the number of forward citations (citations by others) each new patent gets within the next five years. The denominator is a summation of the number of forward citations plus a summation of the number of backward citations (citation on others) each patent makes on other patents granted within the previous five years. The five-year window is to ensure every year in the sample is compared on the common ground, since more recent patents are more likely to receive fewer forward citations due to the right-censoring issue. The rationale for this measure is that groundbreaking innovations typically exhibit lower similarity to current technologies, but pave the way for subsequent patents to emulate them. Since the forward citations capture the overlap of future patents with the focal patent, while the backward citations capture the overlap of the focal patents with previous patents, the relative number of the former provides a measure of patent novelty. The "Novelty" index is in the range between zero and one. A higher index indicates that the year witnesses significant breakthroughs in new technologies; a lower index indicates that most of the technologies have evolved into a mature stage in that year.

The data used to generate the "Novelty" index comes from the USPTO patent and citation data. The USPTO records all patents granted after 1976 and all the patents they cite. To get a smoother trend, we take a three-year average for each observation,[1]

$$\text{Novelty\_avg}_t = \frac{1}{3}\sum_{h=-1}^{1} \text{Novelty}_{t+h}. \tag{2}$$

There are other measures of patent novelty. Bowen III, Frésard and Hoberg (2023) analyzes the text of all the US patents and defines patents as being revolutionary if the vocabulary they use is growing rapidly in the patent corpus overall. Their novelty measure is called "RETech". Kelly et al. (2021) also uses textual analysis and measures patent importance according to its similarity to previous work and subsequent innovations.

Figure 1 shows the technological waves defined in this paper and the "RETech" in the literature.[2] They are significantly positively correlated with nearly simultaneous peaks and troughs, indicating the robustness of different measures. The figure suggests that significant technological breakthroughs happened in the mid-1980s, the mid-1990s, and the beginning of the 2010s although the third peak is lower, while the period around 1990 and the mid-2000s are periods when most of the technologies have entered a mature stage. The "Novelty" index for different technology fields—the first digit of the International Patent Classification (IPC) defined by the The World Intellectual Property Organization (WIPO)—are shown in Figure 13 in Appendix B.2. There are both co-movements and heterogeneity across different fields.[3]

The alignment between the technological waves identified by the citation-based

[1] The smoother does not change the original pattern, as shown in figures without the smoothing techniques in Appendix B.1

[2] The "RETech" defined by Bowen III, Frésard and Hoberg (2023) has a similar meaning to our measure. Kelly et al. (2021) captures aggregate technology breakthroughs by counting the number of patents in the top 10 percent of the unconditional distribution of their importance measure. Because this measure is influenced by the total number of patents each year, its meaning differs from ours.

[3] The "Novelty" Index by field is defined in a similar way as the aggregate index, except that the patent set, I_t, now includes only patents in the corresponding technology field. The nine fields are respectively human necessities, performing operations and transportation, chemistry and metallurgy, textiles and paper, fixed constructions, mechanical engineering; lighting; heating; weapons; blasting, physics, electricity.

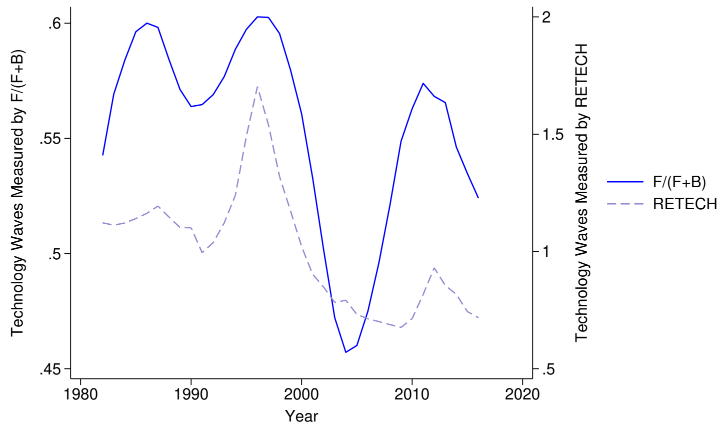

Figure 1: Two Measures of Technological Waves

Notes: This figure illustrates two measures of the technological waves. The blue solid curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations, while the gray dashed curve represents the "RETech" index, a measure of patent novelty from the literature, which assesses patent novelty by the prevalence of vocabularies that are growing in use in the patent description. The two curves have different y-axises, as shown on the left and right.
Sources: USPTO patent and citation data.

measure in this paper and those found through textual analysis in the literature enhances the credibility of our newly developed measure. However, our measure may underestimate the declining trend in technological novelty since the number of citations generally increases at a faster pace over time, particularly after the 1980s. The citation-based measure in this paper complements the text-based measure in the literature and offers several advantages. First, it does not rely on the digitization quality of patent abstracts, thereby avoiding issues of inaccuracy. Second, it is unaffected by strategic language use in patent abstracts or changes in language over time. Third, its definition is more transparent and not constrained by computational resources. We anticipate that this measure will be used more broadly to capture technological shifts over time.

2.1.2 Contributors to the Tech Waves

Which classes of technology contributed to the three peaks of the technological waves? Who were the major applicants for breakthrough patents—incumbents, startups, or public institutions?

To answer the first question, we decompose the "Novelty" index into the contribution of each three-digit IPC code using the following method,

$$\text{Novelty}_t = \frac{\sum_{i\in I_t}\sum_{s=0}^{5} F_{i,t+s}}{\sum_{i\in I_t}\sum_{s=0}^{5} F_{i,t+s} + \sum_{i\in I_t}\sum_{s=0}^{5} B_{i,t-s}}$$
$$= \sum_{j\in J} \frac{\sum_{i\in I_{jt}}\sum_{s=0}^{5} F_{ij,t+s}}{\sum_{i\in I_{jt}}\sum_{s=0}^{5} F_{ij,t+s} + \sum_{i\in I_{jt}}\sum_{s=0}^{5} B_{ij,t-s}} \frac{\sum_{i\in I_{jt}}\sum_{s=0}^{5} F_{ij,t+s} + \sum_{i\in I_{jt}}\sum_{s=0}^{5} B_{ij,t-s}}{\sum_{i\in I_t}\sum_{s=0}^{5} F_{i,t+s} + \sum_{i\in I_t}\sum_{s=0}^{5} B_{i,t-s}}, \tag{3}$$

where J is the set of 3-digit IPC code and I_{jt} is the set of patents belonging to the IPC code j granted in year t. Intuitively, the contribution of each technology class in a given year is determined by the IPC-specific Novelty Index multiplied by the share of forward and backward citations of that class. Table 1 lists the top three contributors at the three peaks of the technological novelty waves. Medical or Veterinary Science and Hygiene contribute most to the first peak, while Computing; Calculating or Counting is the leading contributor to the second and third peak.

Table 1: Major Contributors to the Technological Novelty Peaks

	First Peak (1985-1987)	Second Peak (1995-1997)	Third Peak (2010-2012)
1	Medical or Vet. Sci.; Hygiene	Computing; Calculating or Counting	Computing; Calculating or Counting
2	Electric Elements	Medical or Vet. Sci.; Hygiene	Medical or Vet. Sci.; Hygiene
3	Measuring; Testing	Electric Communication Technique	Electric Communication Technique

Notes: This table shows the major technological classes of the top three fields with the highest "Novelty" index at the technological novelty peaks in the period between 1981 and 2017.

To address the second question, we use the "historically significant patents" compiled by Kelly et al. (2021) from online lists. There are 54 breakthrough patents within the sample period of the Novelty Index. To determine if these patents significantly contribute to the aggregate novelty, we calculate their patent-level Novelty Index using a method similar to that used for IPC-level Index.[4] We then compute the percentile rank of these 54 breakthrough patents within the unconditional distribution for the sample period. The mean and median ranks are in the top 24% and top 7% of all patents, respectively, indicating significant overlap between the list and our novelty measure. We identify the applicants for the breakthrough patents and present their types of institutions in Figure 2. The results show that the sources of technological breakthroughs are quite diverse, suggesting that where highly novel technologies emerge is somewhat random.

[4]Namely, $\text{Novelty}_t = \sum_{i\in I_t} \frac{\sum_{s=0}^{5} F_{i,t+s}}{\sum_{i\in I_t}\sum_{s=0}^{5} F_{i,t+s}} \frac{\sum_{i\in I_t}\sum_{s=0}^{5} F_{i,t+s}}{\sum_{i\in I_t}\sum_{s=0}^{5} F_{i,t+s}+\sum_{i\in I_t}\sum_{s=0}^{5} B_{i,t-s}}$.

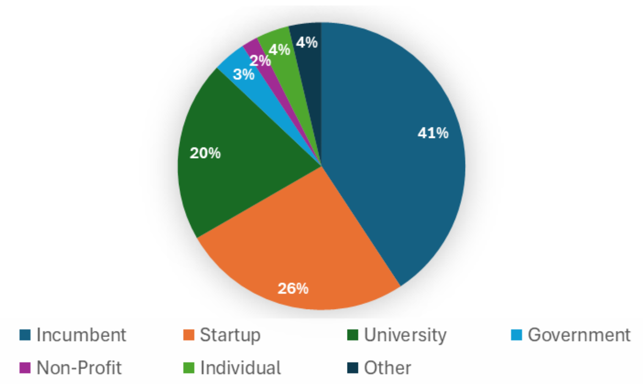

Figure 2: Composition of Applicants of Breakthrough Patents
Notes: The pie chart illustrates the share of applicants from each institutional group for the 54 breakthrough patents. Incumbents are defined as private firms that applied for a patent at least three years after their founding. Startups are private firms that applied within their first three years or before. Others include cases where the applicants are multiple institutions of different types.
Sources: The "historically significant patents" complied by Kelly et al. (2021).

2.1.3 Tech Waves in Europe

While this paper primarily focuses on analysis within the United States, we also calculate the Novelty Index for several European countries with the highest patenting activity. Figure 14 in Appendix B.3 illustrates the technological waves in six European countries with the most patent issuances during the sample period, based on PATSTAT data, respectively. The figure reveals a declining trend in technological novelty among all the six countries from the 1980s through the 2010s.

2.2 Market Concentration

The Herfindahl-Hirschman Index (HHI), a widely adopted measure of market concentration, serves as the primary metric. The analysis relies on two datasets: Compustat Fundamentals Annual and the Census Bureau's Longitudinal Business Database (LBD). Compustat documents sales information for publicly listed U.S. firms. We focus on industrial firms headquartered in the U.S. in Compustat. The LBD provides employment and payroll data for all employer businesses in the U.S. The HHI is constructed through several steps. First, in Compustat, the squared ratios of each firm's sales to total industry sales are calculated, defined by the 2-digit SIC code, for each year. In the LBD, the squared ratios of each firm's employment or payroll to the total industry employment or payroll are computed within each industry, defined by the 3-digit NAICS code, for each year. These squared ratios are then summed across firms in each industry to derive the annual industry-level HHIs. Each industry is weighted by its total sales (for

Compustat) or total employment (for the LBD), and a weighted average across industries is computed. To smooth the trend, a three-year average is applied to each observation.[5]

Panel A of Figure 3 displays the annual Herfindahl-Hirschman Index (HHI) for firm sales, employment, and payroll, all of which exhibit similar trends and fluctuations. The pairwise correlations among these measures are high: 0.86 between sales and payroll, 0.88 between sales and employment, and 0.99 between employment and payroll. Panel B illustrates the relationship between market concentration, measured by sales-based HHI, and the technological waves defined in the previous section. The two series are negatively correlated. The technological waves exhibit a downward linear trend, while the HHI shows an upward trend. The cross-correlation between the detrended HHI (x_t) and the detrended technological waves (y_{t+k}) reaches its maximum absolute value of -0.770 at $k = -2$. This suggests that changes in market concentration closely follow technological waves with a lag of approximately two years.[6]

To assess the robustness of market concentration patterns, the share of sales by top firms is calculated using the cleaned data series from Kwon, Ma and Zimmermann (2023), which is based on IRS data covering the entire population of U.S. corporations. Figure 15 in Appendix B.5 shows that the HHI exhibits similar upward trends and cyclical patterns to the top sales shares.

The negative correlation between technological waves and market concentration is evident across most major sectors, including mining and construction, manufacturing, transportation and utilities, wholesale and retail trade, and finance. By-sector graphs illustrating this relationship are presented in Figure 16 in Appendix B.6. A more granular analysis is conducted using regression analysis, as specified in Equation (4):

$$\text{HHI}_{st} = \beta_0 \text{Novelty Index}_{st} + \beta_1 \text{Size}_{st} + \theta_s + \mu_t + \epsilon_{st}. \tag{4}$$

The Herfindahl-Hirschman Index (HHI) at the 4-digit SIC industry level is regressed on the Novelty Index, controlling for total industry size, as well as industry and year fixed effects. The Novelty Index, calculated at the 4-digit IPC level, is mapped to the 4-digit SIC level using the concordance developed by Silverman (2002), which links IPC and SIC codes based on patent usage. Results based on sales HHI using Compustat are presented in Table 2, with similar findings for payroll HHI using Census data shown in Table 11 in Appendix B.7. In columns (1)–(4), the regressor is the Novelty Index for the concurrent year, while in columns (5)–(8), it is the Novelty Index from two years prior. Columns

[5]Figure 11 in Appendix B.1 shows the patterns of HHI (based on sales in Compustat) and the Novelty Index without smoothing. Their correlation is similar to the smoothed version.

[6]Cross-correlations for $k \in \{-3, -2, -1, 0, 1, 2, 3\}$ are reported in Table 10 in Appendix B.4. The strongest correlation, in absolute value, occurs at $k = -2$. Similar results are obtained when using employment or payroll to measure market concentration.

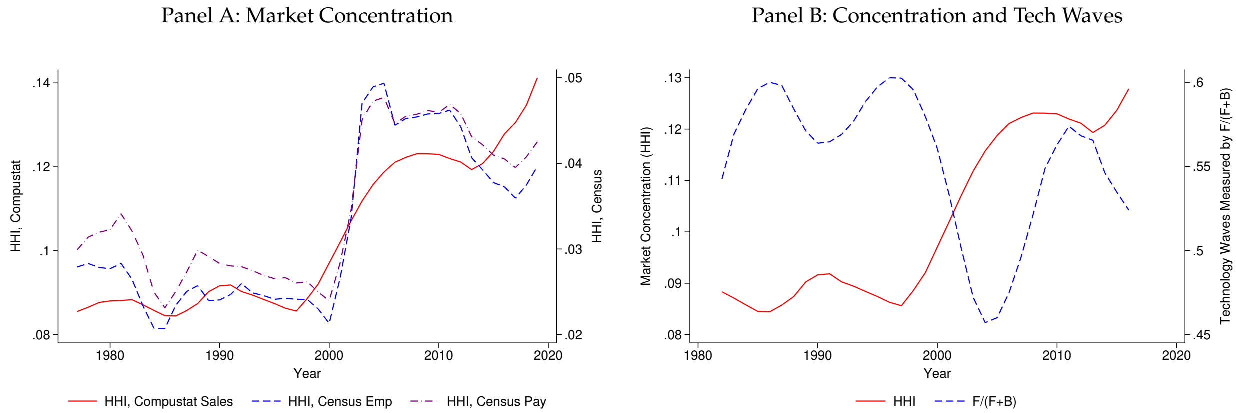

Figure 3: Technological Waves and Market Concentration

Notes: Panel A displays the annual HHI for sales (red solid curve), employment (blue dash curve), and payroll (purple dash-dot curve). The first is based on Compustat data, while the last two are derived from the LBD dataset from the Census. Panel B illustrates the technological waves alongside the trend of market concentration over time. The blue dashed curve, following the methodology defined in this paper, represents the relative ratio of forward citations to the sum of forward and backward citations. The red solid curve shows the HHI for sales. Each panel uses distinct y-axes, shown on the left and right, respectively.
Sources: Longitudinal Business Database (LBD), Compustat Fundamental Annuals, and USPTO patent data.

(1) and (5) include no fixed effects, capturing the aggregate correlation. Columns (2) and (6) control for industry fixed effects, highlighting the relationship within the same industry across different years. Columns (3) and (7) incorporate year fixed effects, capturing the cross-industry relationship within the same year. Finally, columns (4) and (8) control for both industry and year fixed effects, isolating within-industry and within-year variations. Across all specifications, we observe a significant negative relationship, with the coefficient being larger in absolute terms when using the Novelty Index lagged by two years. These findings confirm the robustness of the negative relationship, even when accounting for business cycle factors at the industry or year level.

The negative correlation between technological waves and market concentration is also evident in Europe, as shown by the declining trend of the Novelty Index in Appendix B.3 and the increasing market concentration across European countries, as measured by the Herfindahl-Hirschman Index (HHI) and top sales share in recent studies (e.g., Bighelli et al. (2023) and Ma, Zhang and Zimmermann (2024)).

2.3 Allocation of Ideas

One potential link between the technological waves and the market concentration is inventors' choices of where to do innovation. They can work independently and start their own businesses or contribute their innovation efforts to incumbent firms. In the latter case, they also choose the size of incumbent firms to work in. This section describes

Table 2: Relationship between HHI and Novelty Index at the 4-digit SIC code

	HHI							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Novelty Index	-0.491***	-0.432***	-0.215***	-0.0678*				
	(0.0242)	(0.0178)	(0.0546)	(0.0406)				
Novelty Index(Lag 2 yrs)					-0.639***	-0.556***	-0.285***	-0.150***
					(0.0348)	(0.0257)	(0.0554)	(0.0411)
Industry Sales	-0.0566***	-0.0243***	-0.0635***	-0.0481***	-0.0556***	-0.0200***	-0.0630***	-0.0468***
	(0.00114)	(0.00177)	(0.00115)	(0.00188)	(0.00119)	(0.00190)	(0.00119)	(0.00199)
Industry Fixed Effect	N	Y	N	Y	N	Y	N	Y
Year Fixed Effect	N	N	Y	Y	N	N	Y	Y
Observations	10,333	10,331	10,333	10,331	9,780	9,780	9,780	9,780
R-squared	0.205	0.613	0.256	0.660	0.196	0.614	0.252	0.667

Notes: Standard errors are clustered at the industry and year level. The HHI and industry size are measured by sales in Compustat. Columns (1) and (5) include no fixed effects. Columns (2) and (6) incorporate industry fixed effects. Columns (3) and (7) incorporate year fixed effects. Columns (4) and (8) include both industry and year fixed effects. *** Significant at the 1 percent level; ** Significant at the 5 percent level; * Significant at the 10 percent level.

the flow of the new ideas using the Census data.

2.3.1 Entrants or Incumbent Firms

Data on the affiliations of inventors when they initiate a research project is unavailable to us, but we can observe the age of the firm when a patent is granted to it or applied by it and infer inventors' affiliation. Specifically, if a patent is granted to a firm at age zero to five, it implies that the initial idea was developed independently or spun off from other firms five years ago; if a patent is granted to a firm at age above five, it implies that the initial idea was developed by the incumbent firm five years ago or the firm bought the idea from independent inventors or other firms. We choose the time window to be five years since the average time between patent applications and patent issuance is around two or three years according to the USPTO and we assume the average time to complete a research project to be also two to three years. We can compute the ratio between the number of ideas in new firms to the number of ideas absorbed in incumbent firms, i.e.,

$$\text{New-to-Incumbent Ratio}_t = \frac{\sum_{i \in I_{t+5}} \text{Granted in Firm(Age} \leq 5)_{i,t+5}}{\sum_{i \in I_{t+5}} \text{Granted in Firm(Age>5)}_{i,t+5}}, \tag{5}$$

where I_{t+5} denotes the set of patents granted five years after time t; "Granted in Firm(Age$\leq$5)" and "Granted in Firm(Age$>$5)" are dummy variables indicating whether patent i is issued to a firm above five years old. An alternative measure is to use the age of a firm when it applies for patents. If a patent is applied for in a firm at age zero to three, it implies the founding of a new firm with the idea in the past three years. Otherwise, it implies incumbent firms absorbing new ideas.[7]

Note that there may be discrepancies between the patent affiliations and inventors' affiliations due to spinoffs and patent sales. In the case of spinoffs, the "New-to-Incumbent Ratio" (N-to-I Ratio) based on patent affiliation is larger than the ratio based on inventors' affiliation. In the case of patent sales, the situation is reversed. However, if we want to capture where innovation ideas finally contribute its value, taking into account spinoffs and patent sales works towards the purpose.

The data used to observe patent affiliations is constructed by combining the Longitudinal Business Database (LBD) from the US Census Bureau and the USPTO patent data. The combined dataset can track the age of firms at patent application and issuance.

Since the two measures of the New-to-Incumbent Ratio have very similar trends, we only report the first measure. We take the three-year average for each observation year as before and show the result in Figure 4.[8] To compare it with the technological waves, the Novelty Index defined in this paper is also plotted. Notably, the New-to-Incumbent Ratio demonstrates prominent cyclicality, with zeniths and nadirs occurring in close proximity to the peaks and troughs of technological novelty waves. There is a slight declining trend of slope -0.001, similar to the declining trend of the technological waves, -0.002. The cross correlation between the detrended New-to-Incumbent Ratio (x_t) and the detrended technological waves (y_{t+k}) at different year gaps, $\mathrm{corr}(x_t, y_{t+k})$, has the highest absolute magnitude, 0.612, when $k = 0$, showing the two time series moves simultaneously.

2.3.2 Granular Relationship between Tech Waves and the N-I-Ratio

The positive correlation between technological waves and the N-I Ratio is evident across most major technological fields, including human necessities, performing operations, chemistry, textiles, fixed constructions, physics, and electricity. Graphs by field illustrating this relationship are presented in Figure 17 in Appendix B.8. Analysis at a more granular level is conducted using regression, as specified in Equation (7):

$$\text{New Firm Share}_{st} = \beta_0 \text{Novelty Index}_{st} + \beta_1 \text{Patent Number}_{st} + \theta_s + \mu_t + \epsilon_{st}. \tag{7}$$

The annual share of new firms among all patenting firms at the 4-digit IPC level is regressed on the Novelty Index, controlling for the total number of patents within each

[7] In this alternative measure, the "New-to-Incumbent Ratio" is defined as

$$\text{New-to-Incumbent Ratio}_t = \frac{\sum_{i \in I_{t+3}} \text{Applied in Firm}(\text{Age} \leq 3)_{i,t+3}}{\sum_{i \in I_{t+3}} \text{Applied in Firm}(\text{Age}>3)_{i,t+3}}, \tag{6}$$

[8] Figure 12 in Appendix B.1 shows the patterns of the New-to-Incumbent Ratio and the Novelty Index without smoothing. Their correlation is similar to the smoothed version.

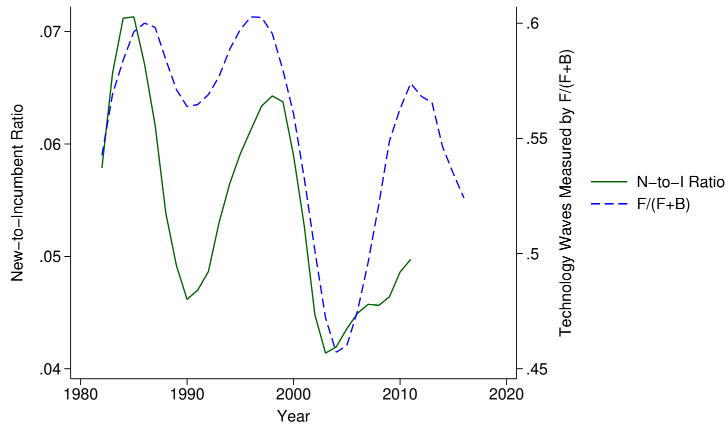

Figure 4: Technological Waves and Idea Allocation

Notes: This figure shows the technological waves and the idea allocation between new and incumbent firms over time. The blue dashed curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations. The green solid curve displays the "New-to-Incumbent Ratio" defined in the paper, capture where new ideas contribute their value. The two curves have different y-axises, which are shown respective on the left and right.
Sources: Longitudinal Business Database (LBD) and USPTO patent and citation data.

IPC, along with IPC and year fixed effects. Table 3 presents the results. Column (1) includes no fixed effects, capturing the aggregate correlation. Column (2) controls for 4-digit IPC fixed effects, highlighting the relationship within the same technological field over time. Column (3) incorporates year fixed effects, capturing cross-IPC relationships within the same year. Finally, Column (4) controls for both IPC and year fixed effects, isolating variations within fields and years. Across all specifications, a significant positive relationship is observed, confirming that the relationship holds even when accounting for business cycle factors at the IPC or year level.

2.3.3 Size of Incumbent Firms

When inventors opt to contribute their ideas to incumbent firms, they are also making a choice regarding the size of the firm, as it impacts the potential value that their innovations can attain. We establish a connection between the quality of inventors' ideas and the size of the incumbent firms they select by examining a subset of patents that have been granted to firms with a history of at least five years in operation. This subset serves as the denominator for calculating the "New-to-Incumbent Ratio," as described in Section 2.3.1. The quality of inventors' ideas is gauged by the number of forward citations each patent receives within the first five years following its issuance. We amalgamate data of all patents (issued to both new and incumbent firms) from various years and compute the quartiles for patent citations. Subsequently, we categorize patents into four distinct

Table 3: Relationship between N-I-Ratio and Novelty Index at the 4-digit IPC Level

	Share of New Firms			
	(1)	(2)	(3)	(4)
Novelty Index	0.0281***	0.0243***	0.0238***	0.0175***
	(0.00555)	(0.00558)	(0.00590)	(0.00597)
Ln(Patent Number)	-0.00258***	-0.00385***	-0.00245***	-0.00309**
	(0.000454)	(0.00146)	(0.000458)	(0.00156)
4-digit-IPC fixed effect	NO	YES	NO	YES
Year fixed effect	NO	NO	YES	YES
Observations	20,000	20,000	20,000	20,000
R-squared	0.003	0.160	0.006	0.164

Notes: Standard errors are clustered at the 4-digit IPC level. Columns (1) includes no fixed effects. Columns (2) incorporates IPC fixed effects. Columns (3) incorporates year fixed effects. Columns (4) include both IPC and year fixed effects. To comply with Census Bureau disclosure requirements, the number of observations is rounded to the nearest thousand. *** Significant at the 1 percent level; ** Significant at the 5 percent level; * Significant at the 10 percent level.

groups based on the quartile to which their citation count belongs. Then we calculate the average size of firms the patents in each quartile are granted to if they are granted to incumbent firms. The firm size is measured by the number of employees and the average employment in the first citation quartile is normalized to a value of one. The relative employment in each citation quartile is plotted in Figure 5. It is shown that there is positive assortative matching between idea quality and firm size when ideas are combined with incumbent firms. One potential concern is that the firm's employment at the patent's issuance may differ from the employment when the inventor chooses the firm. To address this concern, we track each firm's employment five years ago, using data from the LBD. The relationship between relative firm size and patent citation quartiles mirrors the mapping depicted in Figure 5.[9]

To check whether the positive relationship between idea quality and firm size exists for new firms, we calculate the average size of firms the patents in each quartile are granted to if they are granted to new firms—firms with less than five years of operation. It turns out the average firm sizes are similar across quartiles, suggesting the positive assortative matching only holds when ideas are contributed to incumbent firms.

2.4 Economic Value of Patents over Tech Waves

To explore the underlying channels driving the co-movement between technological waves and the allocation of ideas, as well as the positive assortative matching between idea quality and firm size, we perform patent-level regressions. We use an extended

[9]The figure with firms' employment five years ago is available upon request.

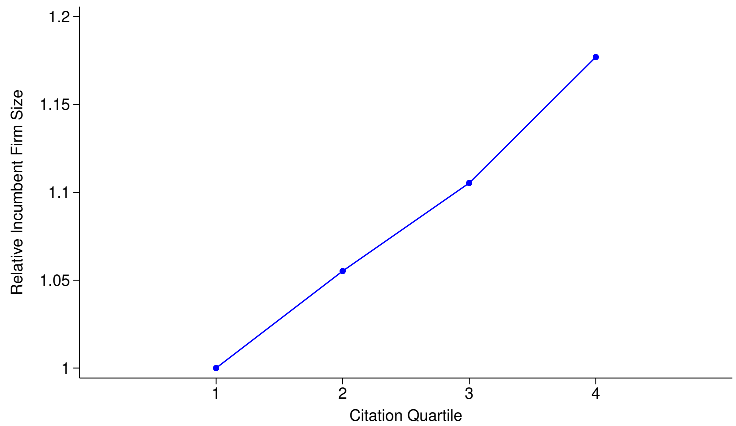

Figure 5: Mapping between Patent Citations and Incumbent Firm Size

Notes: This figure shows the mapping between inventors' idea quality and firm size if inventors opt to develop their ideas in incumbent firms. The idea quality is measured by the number of patent citations and is classified into four quartiles. The firm size is measured by the number of employees. The average employment of firms corresponding to the first citation quartile is normalized to be one.
Sources: Longitudinal Business Database (LBD) and USPTO patent and citation data.

version of the sample constructed by Kogan et al. (2017), which includes more recent years. Kogan et al. (2017) leverages the stock market's response to patent news to estimate the private economic value of patents. Since the sample encompasses all patents granted to publicly listed firms in the US, it provides valuable insights into factors affecting the economic value of patents in incumbent firms over technological waves. The following regression analysis is conducted,

$$\begin{aligned} \ln(\text{economic value}_{ijst}) =& b \ln(\text{Firm size}_{jst}) + \iota \ln(1 + \text{Citations}_{ijst}) + \\ & \phi \text{Novelty}_t \times \ln(1 + \text{Citations}_{ijst}) + \mu_t + \theta_{st} + \gamma_{jt} + \epsilon_{ijst}. \end{aligned} \tag{8}$$

where i, j, s, and t are respectively indexes for patents, firms, patent technology classes (the first-digit IPC), and years. The dependent variable corresponds to the economic value of the patents. Firm size is measured by either the employment or sales of the firm to which the patent belongs. The number of citations is used to measure the scientific value of patents, serving as a proxy for idea quality. The interaction term between the Novelty Index and the citations captures the impact of technological waves on the relationship between the scientific and economic value of patents. The model controls for year fixed effects, year by patent technology class fixed effects, and year by firm fixed effects.

Table 4 presents the results using firm employment as the measure of firm size. Similar regression results using firm sales as the measure of firm size are provided in Appendix B.9. Columns (1) and (2) exclude the technological wave measure, focusing solely on the properties of patents and firms. Columns (3) and (4) display results of Equation (8). In Columns (5) and (6), the yearly Novelty Index is replaced by the year-by-IPC Novelty Index.

Firm size has a significantly positive effect on the economic value of patents, given the idea quality. This suggests that the synergy between inventors and firms increases with firm size. Additionally, idea quality positively impacts the economic value of patents, but this impact diminishes with higher aggregate technological novelty, as indicated by the negative coefficients of the interaction terms. This finding highlights the adoption frictions of novel technologies on existing product lines.

Table 4: Factors of Patents' Economic Value for Incumbent Firms

	Ln(Patents' Economic Value)					
	(1)	(2)	(3)	(4)	(5)	(6)
Ln(1+Employment)	0.330***		0.330***		0.330***	
	(0.0262)		(0.0262)		(0.0262)	
Ln(1+Citations)	0.0732***	0.00277***	0.285***	0.0131**	0.231**	0.0115**
	(0.00561)	(0.000576)	(0.0730)	(0.00574)	(0.0907)	(0.00501)
Ln(1+Citations)$\times$Novelty$_t$			-0.390***	-0.0190*		
			(0.135)	(0.0107)		
Ln(1+Citations)$\times$Novelty$_{st}$					-0.291*	-0.0162*
					(0.162)	(0.00909)
Year Fixed Effect	Y	Y	Y	Y	Y	Y
Year$\times$IPC Fixed Effect	Y	Y	Y	Y	Y	Y
Year$\times$Firm Fixed Effect	N	Y	N	Y	N	Y
Observations	1,111,737	1,101,355	1,111,737	1,101,355	1,111,633	1,101,250
R-squared	0.295	0.882	0.295	0.882	0.295	0.882

Notes: Standard errors are clustered at the year level. Columns (1)-(2) exclude the technological wave measure and focus solely on the property of the patents and firms. Columns (3)-(4) show coefficients of the regression equation (8). Columns (5)-(6) replace the yearly Novelty Index by the year-by-IPC Novelty Index. The regressions control for year fixed effects and year by patent technology class fixed effects across all specifications. The year by firm fixed effects are controlled in columns (2), (4), and (6). *** Significant at the 1 percent level; ** Significant at the 5 percent level; * Significant at the 10 percent level.

3 Model

To clarify the mechanism through which technological waves influence market concentration, we develop a general equilibrium model featuring two groups of individuals (households and inventors) and two types of firms (intermediate goods producers and final goods producers). In this economy, there is an aggregate shock capturing the degree of novelty of new technologies in each period. This shock applies to all agents and determines the extent of friction when inventors' ideas combine with incumbent intermediate good producers. Inventors in each period receive ideas of idiosyncratic quality. They choose to start up new intermediate-good-producing firms or join incumbent ones of selected size based on the aggregate shock and their idea quality.

3.1 Preferences

There is a long-lived representative household in the economy. She works in the production sector, supplies one unit of labor to firms inelastically, and consumes final goods. She also owns all the firms in the economy. The household's utility function is

$$U_H = \int_0^\infty e^{-\rho t} log(C_H(t))dt, \tag{9}$$

where $\rho > 0$ is the discount rate and $C_H(t)$ is the consumption of the household.

Inventors are the ones who work in the R&D sector. In each period, there is a continuum of inventors of measure one. An inventor, with a short-lived lifespan of dt time periods, dedicates effort e_I to create innovations within either an incumbent firm or a new business. Simultaneously, they engage in consumption. Inventors are risk-averse and have a mean-variance utility:

$$U_I\left(c_I, e_I\right) = \mathbb{E}\left(c_I\right) - A\frac{var\left(c_I\right)}{\bar{q}} - R\left(e_I\right)\bar{q}, \tag{10}$$

where c_I is the consumption, e_I is the effort level, and $R\left(e_I\right)\bar{q}$ is the associated cost. $\bar{q}$ (defined below) is the average quality in the economy. The variance and cost are normalized by $\bar{q}$ to keep the problem stable over time. Denote the inventors' aggregate consumption using C_I, i.e., $C_I = \int_0^1 c_{Ii} di$.

3.2 Technology

The economy features two types of firms: intermediate goods producers and final goods producers. The setup is similar to Akcigit and Kerr (2018). Both types of firms are owned by the household. The former hires inventors to create innovations, and produce intermediate goods. The latter assembles intermediate goods and produces final goods.

The final good producers produce final goods using a continuum of intermediate goods $j \in [0, N_F]$:

$$Y(t) = \frac{1}{1-\beta}\int_0^{N_F} q_j^\beta(t) y_j^{1-\beta}(t) dj. \tag{11}$$

In this function, $q_j(t)$ is the quality of the intermediate good j, and $y_j(t)$ is its quantity. We normalize the price of the final good to be one in every period. The final good producers are perfectly competitive, taking the input prices as given. Henceforth, we will drop the time index t when it does not cause confusion.

The final goods are consumed by the household and inventors. The resource constraint of the economy is:

$$Y = C_H + C_I. \tag{12}$$

The intermediate goods producers are a continuum of risk neutral firms of measure N_F. Each firm produces one type of good, with a linear technology using only labor:

$$y_j = \bar{q} l_j, \tag{13}$$

where l_j is the labor input; $\bar{q} = \frac{1}{N_F}\int_0^{N_F} q_j dj$ is the average quality, meaning that improvement in q_j has positive externality (Romer, 1986). The cost is linear in wage w, which intermediate firms take as given. The labor market satisfies the constraint:

$$\int_0^{N_F} l_j dj \leq 1. \tag{14}$$

The production technologies, together with the market setting on innovation, ensure that an intermediate good producer's value $V(q_j)$ is linear in quality q_j (the proof is shown in the next section),

$$V(q_j) = \nu q_j, \tag{15}$$

where ν is endogenous.

This paper focuses on the balanced growth path. We normalize the variables using the average quality $\bar{q}$, and denote the normalized variables using tilde:

$$\tilde{q}_j \equiv \frac{q_j}{\bar{q}}, \tilde{Q} \equiv \frac{Q}{\bar{q}}, \tilde{V}\left(\tilde{q}\right) \equiv \frac{V\left(q_j\right)}{\bar{q}} = \nu \tilde{q}_j, \tag{16}$$

where $Q \equiv \int_0^{N_F} q_j dj$ is the total technology stock in the economy.

Within a given period, intermediate firms consist of a combination of established incumbents and new entrants. Incumbents hire inventors to improve their quality through innovations, while new entrants arise as a result of successful innovations by inventors collaborating with a partner. These innovations are generated by inventors exerting effort denoted as e_I. Given the level of effort e_I, the success rate of an innovation follows an instantaneous Poisson flow rate:

$$\lambda(e_I) = \lambda_0 e_I. \tag{17}$$

It is costly for inventors to dedicate effort, and the flow cost of choosing effort e_I is $R(e_I)\,\tilde{q}$, and $R(e_I) = \frac{1}{1+\delta} e_I^{\delta+1} dt$. This implies a linear cost in time dt at a rate of $\frac{1}{1+\delta} e_I^{\delta+1}$, which is an increasing and convex function of the effort taken.[10]

Inventors are directly responsible for the cost of their efforts, but their efforts cannot be observed by the partner or incumbent intermediate firms. In the absence of a performance-based incentive, an inventor, receiving a flat wage, would opt for $e_I = 0$. Consequently, the partner and the incumbent firms must incentivize inventors to take effort by implementing an innovation-dependent payment scheme. This paper adopts the assumption that firms utilize a common contract, which is a combination of wage and equity, to compensate inventors. The wage allows the partner and the firms to share risk with an inventor whereas equity aligns the inventor's interests with theirs.

Each inventor is born with one innovative idea characterized by an idea quality z_0. The inventor can choose to work either within an incumbent intermediate firm or start up a new intermediate firm with a partner. In the case of working in a startup, the inventor retains full control over the innovation process, and the innovation value is solely determined by the idea quality z_0. Following creation, the normalized innovation value, $\tilde{z}$, is a stochastic draw from a uniform distribution, $U\left((1-\phi)\,z_0\nu, (1+\phi)\,z_0\nu\right)$. While, on average, a higher quality idea yields a better outcome, the inclusion of ϕ allows for some randomness in the mapping between the innovation value and idea quality, with $\phi \in (0,1)$ capturing this variability.

In the alternative case where an inventor with idea quality z_0 chooses to work within an incumbent firm of quality $\tilde{q}$, the resulting innovation value, $\tilde{x}(z_0, \tilde{q})$, is a stochastic variable drawn from a uniform distribution:

$$U\left((1-\phi)x_0(z_0,\tilde{q})\nu,\ (1+\phi)x_0(z_0,\tilde{q})\nu\right),$$

where the mean innovation value $x_0(z_0,\tilde{q})$ depends positively on both idea quality and firm size, i.e.,

$$\frac{\partial x_0(z_0,\tilde{q})}{\partial z_0} > 0 \quad \text{and} \quad \frac{\partial x_0(z_0,\tilde{q})}{\partial \tilde{q}} > 0.$$

The function $x_0(z_0,\tilde{q})$ takes the form:

$$x_0(z_0,\tilde{q}) = \chi(\tilde{q})\gamma(z_0) = \left(\frac{\tilde{q}}{\tilde{q}_0}\right)^b \left(B^\eta + z_0^\eta\right)^{\frac{1}{\eta}}.$$

[10]The innovation production function and the cost functions are based on the growth theory literature (Romer, 1990; Klette and Kortum, 2004; Akcigit and Kerr, 2018). In the calibration, we choose $\delta = 1$ following the literature.

The first term, $\left(\frac{\tilde{q}}{\tilde{q}_0}\right)^b$, captures the synergistic benefits provided by the incumbent firm, with synergy increasing in firm quality. The second term, $\left(B^\eta + z_0^\eta\right)^{\frac{1}{\eta}}$ with $\eta < 0$, is a CES production function representing the interaction between the inventor's idea and existing technologies. The parameter B denotes the stock of backward citations, serving as a proxy for the maturity of the existing technology base.

The CES structure implies complementarity between the new idea z_0 and the technological stock B, meaning that the new idea cannot fully substitute for existing product lines within incumbent firms. Consequently, the realized innovation value is typically lower than if the idea were implemented in a startup, reflecting frictions in adopting and integrating new technologies.

The value of B evolves over time and is calibrated by matching the average $\frac{B}{B+z_0}$ to $1 -$ Novelty Index. When the economy approaches the peak of a technological wave, the stock of relevant previous innovations is lower, resulting in a smaller calibrated value of B. Due to the complementarity between new and existing technologies, outdated product lines within incumbent firms diminish the contribution of new ideas more. As a result, during such periods, ideas implemented within incumbent firms tend to have lower realized values, reflecting heightened adoption frictions of innovations.

3.3 Timeline

Upon an inventor's birth, she observes the quality z_0 of her idea. A potential partner observe z_0 and extends contracts to the inventor to jointly start a new intermediate firm. Concurrently, incumbent firms observe their corresponding $x_0(z_0, \tilde{q})$ and also extend employment contracts to the inventor. The contracts from the potential partner and incumbent firms are strategically designed to maximize their payoff, taking into account the competition with other firms, as well as the inventor's incentive problem. They possess two key components: a fixed wage $\tilde{T}$ and a stake in equity $a \in [0, 1]$.[11] After viewing all contracts, the inventor decides to either join her preferred incumbent firm of quality $\tilde{q}^*(z_0)$, or initiates a startup with the partner. In both cases, the matching process is frictional. When the inventor chooses to innovate in an incumbent firm, she joins the firm with the optimal size with probability h; alternatively, she is randomly assigned to another incumbent firm $\tilde{q}$, based on the incumbent firm size distribution $\tilde{F}(\tilde{q})$. Similarly, when the inventor prefers to start a new business, she initiates it with probability h_s; with probability $1 - h_s$, the inventor is randomly assigned to an incumbent firm. The frictions in the matching process are introduced to match the data, since the actual mapping

[11] It is worth noting that the level of effort e_I is unobservable and unverifiable. Consequently, contracts cannot be contingent on the effort level.

between idea quality and firm size is not perfect. After signing the contract, the inventor chooses an R&D effort, e_I.

3.4 Entry and Exit

A new intermediate firm enters the market upon successful innovation of an inventor who choose to work with a partner. Upon entry, the firm first draws a quality $\tilde{q}$ from the current incumbent firm size distribution $\tilde{F}(\tilde{q})$. Subsequently, the entrant incurs a cost equivalent to the firm value associated with the drawn quality $\tilde{q}$. Following this, the firm applies the innovation, enhancing its quality by incorporating the value of the innovation itself. The rate at which entrants join the market is represented by λ_I.

Intermediate firms face an exogenous exit rate τ, which is independent of their size and is a risk unrelated to innovation. We focus on a balanced growth path such that the number of entrants equals the number of firm exits,

$$\tau N_f = \lambda_I. \tag{18}$$

4 Equilibrium: Balanced Growth Path

This section characterizes the equilibrium of the economy in which aggregate variables $(Y, C, R, w, \bar{q})$ grow at a constant rate g.

4.1 Production

The final good producer chooses $\{y_j\}_j$ to maximize its profit using the technology described in Section 3.2, which yields the demand function faced by intermediate goods producers: $p_j = q_j^{\beta} y_j^{-\beta}$. The intermediate good producers engage in monopolistic competition.[12] Their FOC yields,

$$y_j = q_j \left(\frac{\bar{q}\,(1-\beta)}{w} \right)^{\frac{1}{\beta}}, l_j = y_j / \bar{q}, p_j = \frac{w}{\bar{q}\,(1-\beta)}. \tag{19}$$

In each period, the labor market clearing satisfies $\int_0^{N_F} l_j dj = 1$, which pins down the wage

$$w = N_F^{\beta}\,(1-\beta)\,\bar{q}. \tag{20}$$

[12] The profit maximization problem and the solution process of the final good and intermediate good producers are shown in Section C.1 in the Appendix.

Thus, both the production output y_j and profit π_j are linear in quality,

$$y_j = \frac{q_j}{N_F}, \pi_j = \frac{\beta q_j}{N_F^{1-\beta}}. \tag{21}$$

We drop the subscript j from the firm-level variable when it does not cause confusion. In this model, it is assumed that intermediate firms, responsible for hiring inventors to create innovation, operate in an environment where the competition ensures that the entire value from innovations is captured by inventors. The discounted value of being a firm of quality q is, therefore, the same as the net present value in the case where no innovation occurs. The value function of an intermediate firm q at time t is a linear function of firm size q.

$$V(q,t) = \nu q. \tag{22}$$

where $\nu = \frac{\beta}{(r+\tau)N_F^{1-\beta}}$. See appendix for proof.

The value function is linear in its quality, q, and does not depend on time. This result implies that for any firm, the value of the same quality improvement Δq is the same. We will use q to denote both firm quality and size in the following sections.

The aggregate production is linear in the average quality $\bar{q}$. The resource constraint of the economy is $Y = C_H + C_I$. The relationship between the growth rate, g, and the time discount factor can be derived from the household's maximization problem,

$$g = \frac{\dot{Y}}{Y} = \frac{\dot{C}_H}{C_H} = \frac{\dot{\bar{q}}}{\bar{q}} = r - \rho. \tag{23}$$

4.2 Joining Incumbent Firms

Incumbent (intermediate) firms engage in competition to attract inventors by offering a compensation package including equity a and wage $\tilde{T}$. The setup yields a principal-agent problem, where the interests of the risk-neutral firms, who benefit from innovation, and the risk-averse inventors, who dedicate effort to create innovations, are not aligned.

While firms derive value from the innovations, they are not able to monitor the effort exerted by inventors. Consequently, firms aim to incentivize the inventors to invest effort by offering equity, while concurrently share the risk with inventors through a fixed wage.

For an intermediate firm, the optimization problem is as follows:

$$\begin{aligned}
&\max_{a,\tilde{T}} (1-a)\left(\tilde{V}(\tilde{q}) + \lambda_0 e_I x_0(z_0,\tilde{q})\nu dt\right) - \tilde{T} \\
&\quad \text{st}\, e_I = \arg\max\left\{u\left(c_I\left(a,\tilde{q},\tilde{T}\right), e_I\right)\right\} \\
&\qquad u\left(c_I\left(a,\tilde{q},\tilde{T}\right), e_I\right) \geq \bar{u}(z_0) \\
&\qquad (1-a)\left(\tilde{V}(\tilde{q}) + \lambda_0 e_I x_0(z_0,\tilde{q})\nu dt\right) - \tilde{T} \geq \tilde{V}(\tilde{q})
\end{aligned} \tag{24}$$

In this problem, a firm $\tilde{q}$ chooses the optimal contract $\{a,\tilde{T}\}$ for an inventor z_0 to maximize its own payoff while taking three constraints into consideration. The firm's expected payoff is equal to the expected firm value owned by the original shareholders (all shareholders except the inventor), given by $(1-a)\left(\tilde{V}(\tilde{q}) + \lambda_0 e_I x_0(z_0,\tilde{q})\nu dt\right)$, minus the wage paid to the inventor $\tilde{T}$. Note that $\tilde{V}(\tilde{q})$ is the firm value prior to innovation, which is subject to exogenous exit shocks.

The first constraint is the inventor's incentive compatibility constraint, ensuring that when the inventor is employed by the firm, her actions align with utility maximization. Namely, when facing the firm-specific contract $\{a,\tilde{T}\}$, the inventor chooses an effort level e_I to maximize her expected utility, denoted as $u\left(c_I\left(a,\tilde{q},\tilde{T}\right), e_I\right)$. The second constraint describes the inventor's participation constraint, meaning the inventor prefers to accept this firm's employment offer over other alternatives. This condition implies that the firm needs to offer the inventor a utility level surpassing her outside option $\bar{u}(z_0)$. The outside option is endogenously determined within this model by the Betrand competition among firms in the inventor market. Lastly, the third constraint is the firm's participation constraint, guaranteeing the firm will not be worse off by hiring one inventor.

Though firms all have the same optimization problem in Equation 24, their optimal equity level a depends not only on the inventor's idea quality z_0, but also the firm size $\tilde{q}$. Firm sizes affect the composition of the risk profile in an inventor's utility function:

$$u\left(c_I\left(a,\tilde{q},\tilde{T}\right), e_I\right) = \mathbb{E}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right) - A\mathrm{Var}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right) - R(e_I).\text{[13]} \tag{25}$$

The consumption $c_I\left(a,\tilde{q},\tilde{T}\right)$ includes two components—the flat wage and the stochastic equity value, which is the product of the equity share and the sum of the original firm value and the value of innovation, i.e.,

$$c_I\left(a,\tilde{q},\tilde{T}\right) = a\left(\tilde{V}(\tilde{q}) + \tilde{x}(z_0,\tilde{q})\nu\mathbb{1}_{\mathcal{S}}\right) + \tilde{T}$$

where $\mathcal{S}$ denotes the event that the inventor successfully creates an innovation. The

[13] The utility and consumption have been normalized by the average firm quality, $\bar{q}$.

expected consumption is[14]

$$\mathbb{E}\left(c_I\right)=a\left(\mathbb{E}\tilde{V}\left(\tilde{q}\right)+\lambda_0 e_I x_0\left(z_0,\tilde{q}\right)\nu dt\right)+\tilde{T},$$

and the associated variance is[15]

$$\operatorname{Var}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right)=a^2\left(\underbrace{\tau\tilde{q}^2\nu^2 dt}_{\operatorname{Var}(\tilde{V}(\tilde{q}))}+\underbrace{\lambda_0 e_I\mathbb{E}\left(\tilde{x}\left(z_0,\tilde{q}\right)^2\right)\nu^2 dt}_{\operatorname{Var}(\text{innovation})}\right).$$

The variance comes from two sources: non-innovation-related firm value and the R&D process. Both terms increases in firm size $\tilde{q}$, but the former one increases in a faster speed, meaning in larger firms, shocks unrelated to R&D are stronger. Hence, larger firms are subject to larger incentive problems and the equity held by the inventor provides a weaker incentive for R&D efforts. Upon reviewing all available contracts, an inventor determines her preferred firm $\tilde{q}$.

Section 4.2.1 uses a simplified model to show the inventor's trade off in a closed form. Section 4.2.2 studies the inventor-firm matching in the full model.

4.2.1 A Closed Form Example

This section describes a simplified model which gives tractable results. We use it to illustrate the intuition. This simplified model adopt one additional assumption: the innovation value $\tilde{x}$ is drawn from a distribution with mean $x_0\left(z_0,\tilde{q}\right)\nu$ and second order moment $e_I^{-1}x_0\left(z_0,\tilde{q}\right)^2\nu^2$, instead of the uniform distribution $U\left(\left(1-\phi\right)x_0\left(z_0,\tilde{q}\right)\nu,\left(1+\phi\right)x_0\left(z_0,\tilde{q}\right)\nu\right)$. With this change, the innovation-related uncertainty is now

$$\operatorname{Var}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right)=\left(\underbrace{\tau\tilde{q}^2\nu^2 dt}_{\operatorname{Var}\tilde{V}(\tilde{q})}+\underbrace{\lambda_0\left(x_0\left(z_0,\tilde{q}\right)^2\right)\nu^2 dt}_{\operatorname{Var}(\text{innovation})}\right),$$

which does not depend on effort level any more.

Using backward induction, firms know the inventor would choose an effort level:[16]

$$e_I=\lambda_0 a x_0\left(z_0,\tilde{q}\right)\nu.$$

[14] The derivation of the expectation is shown in Appendix C.
[15] The derivation of the variance is shown in Appendix C.
[16] We use $\delta=1$, which is found to match the data in the literature (e.g., Akcigit and Kerr (2018)).

When an inventor owns a higher proportion of equity a or when the potential value of her innovation, $x_0(z_0, \tilde{q})$, is greater, she is inclined to invest more effort. This is because in both cases, the return of spending one more unit of effort is larger.[17]

The firm's problem in Equation 24 yields,

$$a^* = \frac{1}{1 + 2\frac{A}{\lambda_0}\left(\frac{\tau \tilde{q}^2}{\lambda_0 x_0(z_0,\tilde{q})^2} + 1\right)} \tag{26}$$

The optimal equity level, a^*, decreases in the firm size $\tilde{q}$ when $b < 1$. This is because a^* is determined jointly by two forces: the commercialization value $x_0(z_0, \tilde{q})$, and the non-innovation-related shock—the exit shock, $\tau \tilde{q}^2$. The firm size $\tilde{q}$ affects both factors but in opposite directions. Larger firms can provide the inventor with more synergy, leading to a greater commercialization value. This raises the equity share of the inventor, since it is more worthwhile to incentivize her effort. Meanwhile, larger firms face a higher exit risk. A lower equity share will expose inventors less to risks unrelated to innovation. The relationship between the equity a and the firm size $\tilde{q}$ depends on the relative strength of the two channels. When $b < 1$, which is the case in the model calibration, the second channel dominates. Therefore, larger firms optimally offer less equity to an inventor.

The optimal compensation scheme is $(a^*, \tilde{T}^*)$, where the wage $\tilde{T}^*$ is determined by the zero profit, due to Betrand competition. $\tilde{T}^* = -a^* \tilde{V}(\tilde{q}) + (1 - a^*)\lambda_0 e_I x_0(z_0, \tilde{q})\, \nu dt$.

Upon reviewing all contracts, an inventor with idea quality z_0 chooses the firm $\tilde{q}$ she would like to work for. The first-order-condition yields

$$\frac{\partial x_0(z_0, \tilde{q})}{\partial \tilde{q}} = \frac{2A\tau}{4A\tau \frac{\tilde{q}}{x_0(z_0,\tilde{q})} + \frac{2A\lambda_0 + \lambda_0^2}{\tilde{q}/x_0(z_0,\tilde{q})}} \tag{27}$$

The left-hand-side element is the benefit of joining a larger firm—higher synergy and hence better commercialization. The right-hand-side element is the cost—the inventor gets a lower equity share when combining her idea with a firm with a higher risk unrelated to innovation. The optimal firm size is

$$\tilde{q}^* = \left(\frac{\left(2A\lambda_0 + \lambda_0^2\right)(\gamma(z_0))^2 b}{2A\tau q_0^{2b}(1 - 2b)}\right)^{\frac{1}{2-2b}}.$$

Proposition 1. *When $b < 0.5$, $\frac{\partial \tilde{q}^*}{\partial z_0} > 0$.*[18]

When b is low, the synergy does not grow unboundedly with firm size and the

[17]The derivation of the contract solution is presented in Section C.2 in the Appendix.

[18]See Appendix C for proof.

balancing role of a lower equity share ensures the existence and uniqueness of a solution. The model predicts that among incumbent firms, better-quality innovations are more likely to be created in larger ones. The reasons are twofold. First, better ideas benefit more from synergy. Second, they generate relatively greater innovation-related uncertainty, making inventors less vulnerable to incentive problems.

Proposition 2. *When B goes up, i.e., technology becomes more mature, a larger share of inventors opt for joining incumbent firms.*[19]

The technology stock affects the adoption friction in the incumbent firms negatively, but not the innovation utilization process in new businesses. Therefore, when technology is more matured, innovations are worth more in incumbent firms whereas their values are unchanged in new firms, resulting in incumbent firms being more attractive.

Proposition 3. *Under certain parameter assumption ($b < \frac{\min\left(z_0^{-\eta}\right)}{\min\left(z_0^{-\eta}\right)+\max\left(B^{-\eta}\right)}$), there exists a cutoff $\bar{z}_0\left(B\right)$, such that all inventors with $z_0 < \bar{z}_0\left(B\right)$ opt for incumbent firms.*[20]

The impacts of both incumbents' adoption efficiency $\gamma(z_0)$ and synergy adjusted innovation value $x(z_0, \tilde{q})$ are determined by idea quality z_0. High quality ideas suffer more loss from adoption frictions and meanwhile enjoy more synergy when implemented by an incumbent firm. If the synergy does not increase too dramatically with firm size ($b < \frac{\min\left(z_0^{-\eta}\right)}{\min\left(z_0^{-\eta}\right)+\max\left(B^{-\eta}\right)}$), the adoption channel dominates and inventors with high-quality ideas opt for starting up new business.

4.2.2 The Full Model

This section describes the full model, when releasing the assumption that the second-order moment of the innovation value is inversely related to the effort e_I. A more detailed solution to the full model can be found in Section C.6 in the Appendix. Similar as in the closed-form case, the incentive constraint implies that the firm knows the optimal effort of the inventor,

$$e_I = \lambda_0 a x_0\left(z_0, \tilde{q}\right)\nu - Aa^2\lambda_0 \mathbb{E}\left(\tilde{x}\left(z_0, \tilde{q}\right)^2\right)\nu^2. \tag{28}$$

Exerting one more unit of effort has three effects: a larger likelihood of successful innovation, a greater disutility from the effort, and a larger variance of consumption. The last effect does not show up in the closed-form case, where the inventor's effort reduces

[19] See Appendix C for proof.
[20] See Appendix C for proof.

the variance of the innovation value. In the full model, an inventor strategically takes a lower level of effort e_I for any given contract.

The firm's problem is described in Equation 24. When b is relatively low, the synergy, $x_0(z_0, \tilde{q})$, increases mildly with $\tilde{q}$. It holds numerically that the optimal stock a decreases with firm size $\tilde{q}$. The optimal compensation scheme is $(a, \tilde{T})$, where $\tilde{T}$ is determined by the zero profit condition of the Betrand competition: $\tilde{T} = -a\tilde{V}(\tilde{q}) + (1-a)\lambda_0 e_I x_0(z_0, \tilde{q})\nu dt$.

Given the contracts, the inventor chooses the optimal firm size $\tilde{q}$ by maximizing her utility. In each firm, her optimal effort level is given in Equation 28. Larger firms provide better commercialization but worse incentives due to lower equity. The numerical solution shows that, among all inventors that choose to join incumbent firms, those with better ideas prefer bigger firms.

However, due to the friction in the inventor-firm matching process, only a fraction h of inventors can go to their ideal incumbent firm: the rest are assigned to firms with random size. Therefore, the innovations within a firm are composed of two distinct components: the directedly matched part and the frictionally matched part.

The Novelty Index matters for both the inventor-firm matching process and the utility obtained. At the peak of a technology wave, the technology tend to exhibit greater novelty, thereby escalating adoption costs. As a result, in incumbent firms, an innovation is worth less and the synergy effect is weaker since $\gamma(z_0)$ decreases. Inventors strategically move to smaller firms, as the advantages of larger firms are less silent. Meanwhile, inventors working within incumbent firms experience lower utility.

4.3 Starting up a New Business

In additional to joining an incumbent firm, an inventor can also start her own business. The inventor, who is risk-averse, works with a risk-neutral partner to share risk. Similarly, the inventor faces a compensation scheme $(a, \tilde{T})$. However, the inventor is in charge of the research direction by herself. Hence, the innovation value is solely determined by her idea quality z_0. Upon successful innovation, the normalized innovation value $\tilde{z}$ is a random draw from the distribution $U((1-\phi)z_0\nu, (1+\phi)z_0\nu)$. On average, a higher-quality idea yields a better outcome.

The partner's problem shares the same form as the incumbent firm's (Equation 24), with $\tilde{q} = 0$ and the average innovation value being z_0 instead of $x_0(z_0, \tilde{q})$. The partners are assumed to get zero profit. The inventor decides her effort by maximizing her utility.[21]

[21] The exposition of the problem of starting up a business is presented in Appendix Section C.7.

4.4 Inventor's Choice

Each inventor chooses between working in an incumbent firm (with h probability in the firm with optimal size and $1-h$ probability working in a firm of random size), and in a startup (with h_s probability starting up a new business and $1-h_s$ probability working in an incumbent firm of random size). The inventor's decision rule is:

$$u(z_0) = \max\{hu(c_I(z_0,\tilde{q}^*), e_I(z_0,\tilde{q}^*)) + (1-h)\int_{\tilde{q}} u(c_I(z_0,\tilde{q}), e_I(z_0,\tilde{q}))\tilde{f}(\tilde{q}), \\ h_s u(c_I(z_0,0), e_I(z_0,0)) + (1-h_s)\int_{\tilde{q}} u(c_I(z_0,\tilde{q}), e_I(z_0,\tilde{q}))\tilde{f}(\tilde{q})\}. \tag{29}$$

where $\tilde{f}(\tilde{q})$ is the firm size distribution endogenously determined in the equilibrium. The inventor joins a startup when it offers a higher expected utility.

4.5 Entry and Exit

A firm enters the market when it successfully creates an innovation as a startup. The amount of entry equals the amount of innovations in startups:

$$\lambda_I = \int_{z_0\in\{\tilde{q}^*=0\}} h_s\lambda_0 e_I(z_0,\tilde{q}=0)\,\psi(z_0)\,dz_0. \tag{30}$$

When it is stationary, the amount of firm that enters is the same as those who exit:

$$\tau N_f = \lambda_I. \tag{31}$$

4.6 Growth Rate

The growth is from a single source—innovation. The aggregate growth can be written as,

$$g = \frac{\bar{q}(t+\Delta t)-\bar{q}(t)}{\bar{q}(t)\Delta t} \\ = \frac{\int_{z_0\in\{z_0|\tilde{q}^*>0\}}\left(h\lambda_0 e_I(z_0,\tilde{q}^*)\,x_0(z_0,\tilde{q}^*) + (1-h)\lambda_0\int_{\tilde{q}} e_I(z_0,\tilde{q})\,x_0(z_0,\tilde{q})\tilde{f}(\tilde{q})\,d\tilde{q}\right)d\Psi(z_0)}{N_f} \\ + \frac{\int_{z_0\in\{z_0|\tilde{q}^*=0\}}\left(h_s\lambda_0 e_I(z_0,\tilde{q}^*=0)\,z_0 + (1-h_s)\lambda_0\int_{\tilde{q}} e_I(z_0,\tilde{q})\,x_0(z_0,\tilde{q})\tilde{f}(\tilde{q})\,d\tilde{q}\right)d\Psi(z_0)}{N_f}. \tag{32}$$

4.7 Equilibrium

We end this section by summarizing the equilibrium. The R&D expenditure of the economy, C_I, can be written as

$$C_I = \int_{z_0 \in \{z_0 | \tilde{q}^* > 0\}} \nu \left(h\lambda_0 e_I (z_0, \tilde{q}^*) \, x_0 (z_0, \tilde{q}^*) + (1-h) \lambda_0 \int_{\tilde{q}} e_I (z_0, \tilde{q}) \, x_0 (z_0, \tilde{q}) \, \tilde{f} (\tilde{q}) \, d\tilde{q} \right) \psi (z_0) \, dz_0$$
$$+ \int_{z_0 \in \{z_0 | \tilde{q}^* = 0\}} \nu \left(h_s \lambda_0 e_I (z_0, \tilde{q}^* = 0) \, z_0 + (1-h_s) \lambda_0 \int_{\tilde{q}} e_I (z_0, \tilde{q}) \, x_0 (z_0, \tilde{q}) \, \tilde{f} (\tilde{q}) \, d\tilde{q} \right) \psi (z_0) \, dz_0. \tag{33}$$

It captures all transfers made to inventors. Based on Equation (37), the equilibrium output level Y is linear in $\bar{q}$

$$Y = \frac{1}{1-\beta} N_F^{\beta} \bar{q}. \tag{34}$$

and the consumption level is

$$C_H = Y - C_I. \tag{35}$$

Definition A balanced growth path of this economy for any combination of (t, q) is the mapping between q and z_0, the allocation $\left(\left\{ y_j^* \right\}_j, Y^*, C_I^*, C_H^* \right)$ the prices $\left(w^*, r^*, \left\{ p_j^* \right\}_j \right)$, the growth rate g^*, the entry rate λ_I^*, and the measure of firms N_F^*, such that (1) for any $j \in [0,1]$, y_j^* and p_j^* satisfy Equation (19); (2) the wage w^* satisfies Equation (20); (3) the interest rate r^* satisfies Equation (23); (4) the measure of the intermediate producers N_F^* satisfies Equation (31); (5) the mapping between q and z_0 is the solution of Equation (29); (6) the entry rates λ_I^* satisfy Equation (30); (7) R&D spending C_I^* satisfies Equation (33); (8) the aggregate output Y^* satisfies Equation (34); (9) the aggregate consumption C_H^* satisfies Equation (35); and (10) the steady-state growth rate g^* satisfies Equation (32).

5 Calibration

We calibrate the model to target the average US economy from 1982 to 2016. Patents are used as a surrogate for innovations. An innovation's idea quality, denoted by z_0, and the realized value, x (z in the context of a startup), correspond to the patent's citation (scientific importance) and the patent's pecuniary value, respectively. Additionally, we assume that the idea quality z_0 follows the Pareto distribution characterized by a scale factor z_m and a shape factor α.

5.1 Identification

Parameters in the model are categorized into two groups. The first group is calibrated by a prior information from the aggregate statistics or the literature. The second group is calibrated by estimation from the micro-level data or through the model. Table 5 reports the parameters in the first group, $(\rho, \beta, \tau, A, \delta)$. The discount rate, ρ, is set to 0.02 to match the average interest rate in the sample period. The production function quality share, β, is 0.109, following Akcigit and Kerr (2018). The firm exit rate, τ, is 0.06, targeting the average exit rate of firms above 5 years old during our sample period based on the Business Dynamics Statistics (BDS).[22] The risk aversion parameter, A, and the effort cost elasticity, δ, are set to be 0.5 and 1, respectively, which are commonly used in the literature (Hall and Van Reenen, 2000).

Table 5: Parameter Values from a Priori Information

Parameter	Description	Value	Identification
ρ	Discount rate	0.02	Interest Rate
β	Production function quality share	0.109	Firm profitability
τ	Exo. exit rate	0.06	BDS
A	Risk aversion	0.5	Risk aversion
δ	Effort cost elasticity	1	Effort cost elasticity

Notes: This table shows parameter values from the literature or direct estimation.

We calibrate the ten remaining parameters in the second group, (λ_0, α, z_m, ϕ,η, B, b, q_0, h, h_s), using the minimum distance method, inspired by Lentz and Mortensen (2008). The parameters, along with their corresponding moments are in Table 6.

Table 6: Parameters from the Minimum Distance Estimation

Para.	Description	Identification
λ_0	Innovation arrival rate	Growth rate
α	Shape of idea quality distribution	S.d.-to-mean ratio of patent citations
z_m	Scale of idea quality distribution	Average pecuniary value of innovations
ϕ	Innovation value dispersion	S.d.-to-mean ratio of pecuniary value cond. on citations
η	Elasticity of substitution	MLE estimation
B	Maturity of technology	Technology "Novelty" index
b	Exponent of the synergy function	Regression coefficient of pecuniary value on firm size
q_0	Scale of the synergy function	New-to-incumbent ratio
h	Matching friction (incumbent)	Firm size ratio by fourth-to-first-quartile of citations
h_s	Matching friction (startup)	Citation ratio between new and incumbent firms

Notes: Parameters in this table are jointly calibrated to minimize the distance between the model and data moments.

[22]The BDS data is compiled from the Longitudinal Business Database (LBD) by the Census Bureau.

Growth Rate—Innovation is the sole driver of growth in this model. Consequently, the scale parameter of the innovation arrival rate, λ_0, plays a critical role in determining the aggregate growth rate. A higher arrival rate shortens the average time between innovations, thereby raising the overall growth rate. We calibrate λ_0 so that the model's implied aggregate growth rate matches 2.75%, consistent with the average annual growth rate observed in the U.S. economy between 1982 and 2016 after applying the HP filter.

The S.D.-to-Mean Ratio of Patent Citations—This ratio captures the dispersion in patent citations observed in the data, which reflects the underlying dispersion in inventors' idea quality. The parameter α is the primary driver of this dispersion. Specifically, the standard deviation-to-mean ratio of the idea distribution is given by $\frac{1}{\sqrt{\alpha(\alpha-2)}}$. Although the patents recorded in the USPTO data represent only successful innovations—a selected subset of all ideas—the dispersion in patent citations remains heavily influenced by α. We construct the citation distribution by pooling all patents granted since 1976 and their corresponding citations recorded by the USPTO, and compute the standard deviation-to-mean ratio, which is approximately 2.784.

Average Innovation Value—The pecuniary value of innovations is modeled as directly contributing to firm value. In the model, the pecuniary value, denoted by x (or z for startups), is assumed to follow a uniform distribution with its mean determined by the underlying idea quality, z_0. Given α, the average scientific value of ideas is governed by the scale parameter of the Pareto distribution, z_m. Accordingly, z_m is calibrated to match the average pecuniary value of patents. The estimation method proposed by Kogan et al. (2017) is adopted, in which the stock market's reaction to patent-related news is used to infer patent value. An extended version of their dataset, provided by the authors, is employed. It links patents issued to U.S. firms from 1926 to 2022 with stock returns from CRSP and firm-level data from Compustat. Since the distribution of public firms differs from that of the broader firm population, the statistical model developed by Yang (2023) is applied to estimate the average patent value across all firms based on public firm data. It is found that, on average, a patent is worth 0.0255 times the average firm value. The parameter z_m is then calibrated to replicate this value.

S.D.-to-Mean Ratio of Innovations' Pecuniary Value Conditional on Citations—The pecuniary value of innovations is based on the scientific value of ideas but is also subject to additional randomness. In the model, the degree of randomness is governed by the parameter ϕ. Specifically, conditional on idea quality, the standard deviation-to-mean ratio of the uniform distribution of innovation pecuniary value is given by $\frac{\phi}{\sqrt{3}}$. Using the same sample employed to calibrate z_m, the standard deviation-to-mean ratio of patent pecuniary value is estimated while controlling for the number of patent citations. In the data, this ratio is found to be approximately 0.416.

Elasticity of Substitution between past Knowledge Stock and New Ideas—In incumbent firms, the pecuniary value of innovation is determined by both the new idea and the existing technologies. The new idea and the existing technologies are integrated using a CES production function, governed by a parameter η in the model. In addition, there is some randomness in the realization of the pecuniary value, allowing us to employ the maximum likelihood estimation method to estimate the parameter. Appendix D specifies the estimation details.

Technological "Novelty" Index—Technological novelty is defined as the ratio of total forward citations to the sum of forward and backward citations for all patents granted in a given year. In the model, the past technological stock, B, which enters the realization potential of new ideas, $\gamma(z_0) = (z_0^\eta + B^\eta)^{1/\eta}$, corresponds to the stock of backward citations, representing the maturity of the existing technological base. The value of B is calibrated so that the model-generated average forward-to-backward citation ratio, $B/(B + \int z_0 \, d\Psi(z_0))$, matches $1 -$ the average "Novelty" Index between 1982 and 2016, where $\int z_0 \, d\Psi(z_0)$ corresponds to the total forward citations of available ideas.[23]

Regression Coefficient of Innovations' Pecuniary Value on Firm Size—The synergy provided by incumbent firms is governed by two parameters: b, which determines the elasticity of synergy with respect to firm size, and $\tilde{q}_0$, which sets the scale. To identify b, the innovation's average pecuniary value function is log-linearized as $\log\left(x_0\left(z_0, q\right)\right) = b\log\left(\frac{\tilde{q}}{\tilde{q}_0}\right) + \log\left(\gamma\left(z_0\right)\right) + \log\left(z_0\right)$. This expression maps directly to the regression specification in Equation (8) in Section 2.4. The coefficient on the firm size variable in the regression identifies the value of b.

New-to-Incumbent Ratio—The scale parameter in the synergy function, $\tilde{q}_0$, affects the benefit of contributing an idea to an incumbent firm compared to initiating a new venture. Therefore, it is related to inventors' choice between incumbent firms and startups. We use the "New-to-Incumbent Ratio" derived in Section 2.3.1 to calibrate $\tilde{q}_0$.

Firm Size Ratio by Fourth-to-First Quartile of Patent Citations—The model predicts that when inventors choose to join incumbent firms, the size of the firm they select should increase with the quality of their ideas. However, this positive sorting is hindered by matching frictions. Greater frictions weaken the relationship between idea quality and firm size. To calibrate the degree of frictions, h, the model generates the average firm size by patent citation quartiles, conditional on the patent being developed within an incumbent firm. The ratio of firm size between the fourth and first quartiles is then computed and matched to its empirical counterpart.

[23] The "Novelty" Index defined in Section 2.1 can be expressed as $\frac{F}{F+B}$, where F denotes all forward citations in a given period. Thus, $(B/(B + \int z_0 \, d\Psi(z_0)))$ corresponds to $\frac{B}{B+F} = 1 - \frac{F}{B+F} = 1 -$ the "Novelty" Index.

Citation Ratio Between New and Incumbent Firms—The model predicts the existence of a threshold in idea quality above which inventors prefer founding startups over joining incumbent firms. As a result, patents from startups should, on average, exhibit higher scientific value than those from incumbents. However, frictions in the startup decision, governed by h_s, attenuate this effect. The patent citation ratio between new and incumbent firms captures this relationship and is used to calibrate h_s.

5.2 Calibration Results

Table 7 reports the model-generated moments and their counterparts in the data. Overall, the model matches the targeted moments closely. The resulting parameter values are reported in Table 8.

Table 7: Moments

Identification Moment	Data	Model
Growth rate	0.0275	0.0290
S.d.-to-mean ratio of patent citations	2.784	2.659
Average innovation value	0.0255	0.0240
S.d.-to-mean ratio of innovation value cond. on citations	0.416	0.416
Elasticity of Substitution between past Knowledge Stock and New Ideas	-0.4	-0.4
Technology "Novelty" index	0.554	0.554
Regression coefficient of innovation value on firm size	0.33	0.33
New-to-incumbent ratio	0.054	0.059
Firm size ratio by fourth-to-first-quartile of citations	1.18	1.18
Citation ratio between new and incumbent firms	1.361	1.385

Notes: This table compares the moments generated from the calibrated model and the data. In general, the model generated moments match the data well.

Our estimates indicate that, relative to startups, incumbent firms face substantially lower realization potential when utilizing innovations, primarily due to frictions in adopting and integrating new technologies. Specifically, the ratio $\frac{\int \gamma(z_0)\, d\Psi(z_0)}{\int z_0\, d\Psi(z_0)} = 0.15$ is significantly below one. Moreover, synergy plays an important role in commercialization: the scale parameter $\tilde{q_0}$ in the synergy function is as small as 2.2×10^{-4}, and the elasticity parameter is 0.33. This implies that an incumbent firm of average size can generate approximately 16 times more value at commercialization than a startup due to the synergy effect. When combining the effects of synergy and adoption frictions, this advantage shrinks to about 2.4 times.

Panel A of Figure 6 illustrates the relationship between optimal firm size ($\tilde{q}^*$) and idea quality (z_0) when an inventor chooses to contribute her idea to an incumbent firm,

Table 8: Estimated Parameter Values

Parameter	Description	Value
λ_0	Innovation arrival rate	7.48
α	Shape of idea quality distribution	2.34
z_m	Scale of idea quality distribution	5.3E-3
ϕ	Innovation quality draw	0.72
η	CES elasticity of substitution	-0.4
B	Maturity of technology	0.0076
b	Exponent of the synergy function	0.33
q_0	Denominator of the synergy function	2.2E-4
h	Matching friction (incumbent)	0.18
h_s	Matching friction (startup)	0.54

Notes: Parameters in this table are jointly calibrated to minimize the distance between the model and data moments.

as observed in both the data and the calibrated model.[24] The positive relationship reflects assortative matching between inventors and firms—higher-quality ideas are matched with larger firms, enabling those firms to expand further.

Technological novelty waves influence both the extensive and intensive margins of idea allocation. The extensive margin governs the number of inventors starting new businesses, while the intensive margin determines the firm size chosen by inventors who join incumbents. Panel B of Figure 6 illustrates these dimensions by showing optimal firm size choices across idea qualities in a frictionless matching environment ($h = h_s = 1$). A positive firm size indicates joining an incumbent; zero implies starting a new business. The figure compares 1986 and 2005 by plotting optimal choices when the technological stock, B, adopts the value in the two years, respectively, such that the model-generated average forward-to-backward citation ratio ($\frac{B}{B+\int z_0 d\Psi(z_0)}$) matches 1-Novelty Index in the data. 1986 is a peak of technological novelty, while 2005 is a trough with mostly incremental innovations. In 2005, the threshold idea quality for forming a startup is higher, implying more inventors opt to join incumbents. Moreover, by absorbing higher-quality ideas, incumbents grow larger, enhancing the synergy they offer. This leads to stronger positive assortative matching between firm size and idea quality. Both margins contribute to greater market concentration in 2005.

6 Quantitative Analysis

Using the calibrated model, we simulate the model beginning from 1986, the first peak of the technology wave in the sample. In each year, we adjust the technological stock, B,

[24]The model counterpart without matching frictions ($h = 1$) is shown in Figure **??** in the Appendix.

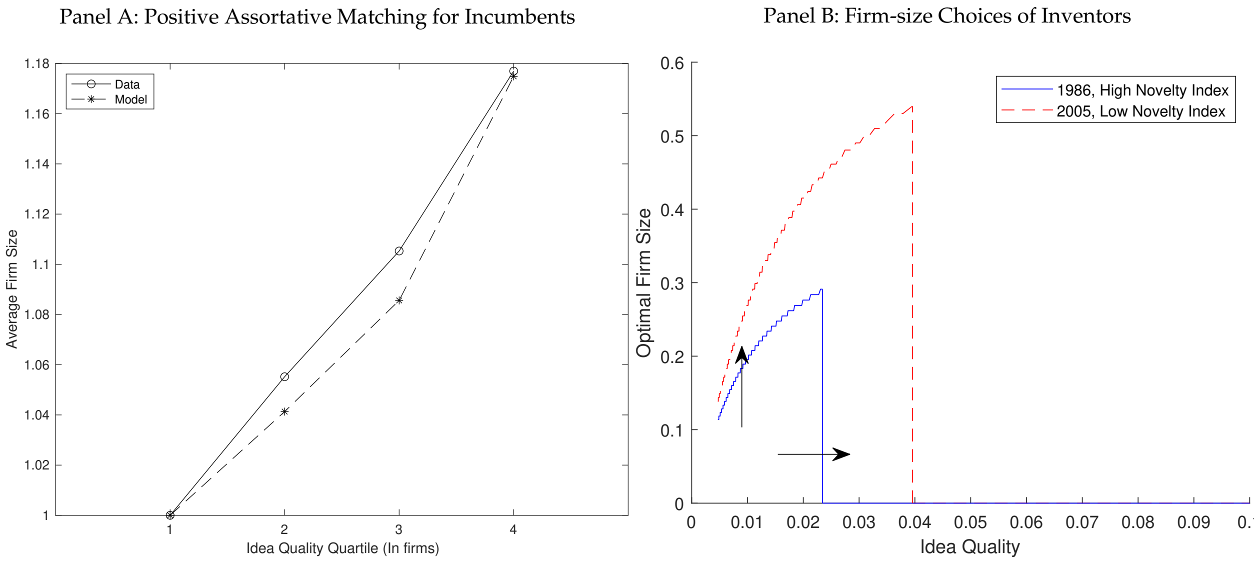

Figure 6: The Trend and Detrended Time Variations of the HHI

Notes: Panel A exhibits the mapping between inventors' idea quality and the firm sizes if inventors opt to develop their ideas in incumbent firms. The idea quality is measured by the number of patent citations and is classified into four quartiles. The firm size is measured by the number of employees. The average employment of firms corresponding to the first citation quartile is normalized to be one. The solid curve represents the model prediction and the dashed one is the actual data in Figure 5. Panel B shows the optimal firm size by idea quality. The blue solid line and the red dashed line represents 1986 and 2005, respectively.

to match 1-Novelty Index in the data. In another word, we fix all the parameters except the one that governs the aggregate technological novelty shocks. The simulation yields the model-generated market concentration and the innovation allocation in each year. We compare the model-generated results with the data and calculate their correlations.

In the simulation, each year must satisfy equilibrium conditions, though the system does not necessarily lie on a balanced growth path. In this non-stationary environment, the model state is characterized by two evolving state variables: the total number of firms, N_f, and the firm size distribution, $f(q)$. Unlike the balanced growth path, these variables are not stationary and adjust each year to satisfy equilibrium conditions. Consequently, the entry-exit equality in Equation (31) no longer holds, introducing a gap between entry and exit that drives the system's dynamics. The number of firms evolves according to:

$$N_f' = N_f(1-\tau) + \lambda_I, \tag{36}$$

where the number of entrants, λ_I, is endogenously determined by the model. If entry exceeds exit, the firm count rises the following year; if exit exceeds entry, it declines. Meanwhile, shifts in the inventor-firm matching process generate changes in the firm size distribution. Using the distribution from the previous year as a starting point, we

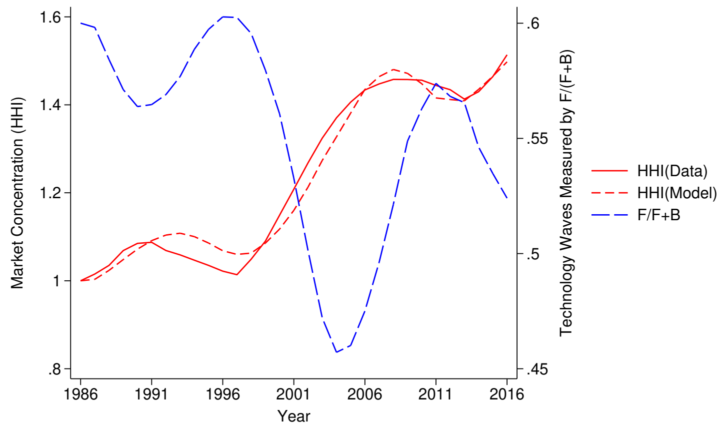

Figure 7: Technology Waves and Model Generated HHI

Notes: This figure shows the technological waves and the trend of model-generated market concentration over time. The blue curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations (same as Figure 1). The red solid curve displays the simulated HHI in each year, which is normalized by the HHI in 1986. The red dashed curve is the empirical HHI moved forward for three years, also normalized by 1986.

simulate firm dynamics over one period to obtain the updated distribution. Together, the evolving N_f and $f(q)$ shape the trajectory of market concentration over time.

6.1 Technology Waves and the Market Concentration

Figure 7 presents the simulated evolution of market concentration, measured by the Herfindahl-Hirschman Index (HHI), alongside its empirical counterpart and the technological novelty waves. The solid red curve depicts the HHI from the data, normalized to its 1986 level, while the dashed red curve shows the simulated HHI from the model, also normalized by the 1986 value. To illustrate the connection with technological waves, the figure also includes the relative ratio of forward citations to the sum of forward and backward citations-following the methodology defined in this paper, and consistent with Figure 1. The model-generated HHI closely mirrors its empirical counterpart and exhibits a negative relationship with technological waves.

Although the calibration does not explicitly target any measure of market concentration, the model successfully replicates both the upward trend and cyclical fluctuations observed in the data. To disentangle these components, we fit separate linear trends to the empirical and simulated HHI series and subtract them to obtain the detrended time variations, as shown in Figure 8. Summary statistics are reported in the first two rows of Panel A in Table 9.

The average HHI in the model over the 1986–2016 period is 1.236—above one and

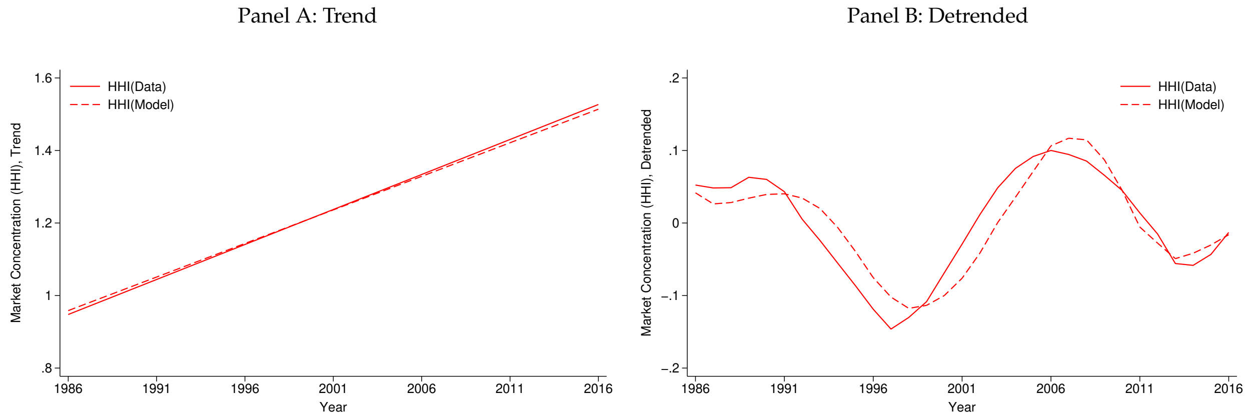

Figure 8: The Trend and Detrended Time Variations of the HHI

Table 9: Comparison between Model and Data

	No Detrend		Detrend		
	Mean	Time Trend	S.D.	Autocorr	Corr with Data
			Panel A. HHI		
Data	1.237	0.0193	0.072	0.926	1
Model	1.236	0.0185	0.066	0.922	0.910
			Panel B. N-I-Ratio		
Data	0.052	-5.05E-5	0.007	0.830	1
Model	0.072	-2.49E-3	0.031	0.883	0.825

Notes: This table shows the trend and detrended time variations of the HHI and New-to-Incumbent ratio in the data and the model.

comparable to the empirical counterpart. The estimated linear trend has a slope of 0.0185 in the model, closely matching the empirical slope of 0.0193. This implies that the technological novelty waves alone account for approximately 95.9% of the observed increase in market concentration over the sample period. The detrended HHI series from the model also closely tracks the data, with a correlation coefficient of 0.910. The standard deviation of the model's detrended series is 0.066, close to 0.072 in the data. The first-order autocorrelations are 0.922 in the model and 0.926 in the data. These results suggest that technological waves are an important driver of the fluctuations in market concentration.

To examine the relationship between detrended HHI and technological novelty waves in both the model and the data, we compute the cross-correlation between the former (x_t) and the latter (y_{t+k}) at various time lags, $\text{corr}(x_t, y_{t+k})$, following the method in Stock and Watson (1999). A negative value of k compares the HHI with past technological waves, while a positive k compares it with future waves. The results are presented in Panel A of Figure 9. In the data, the absolute value of the correlation is highest when k is negative, indicating that market concentration reacts to

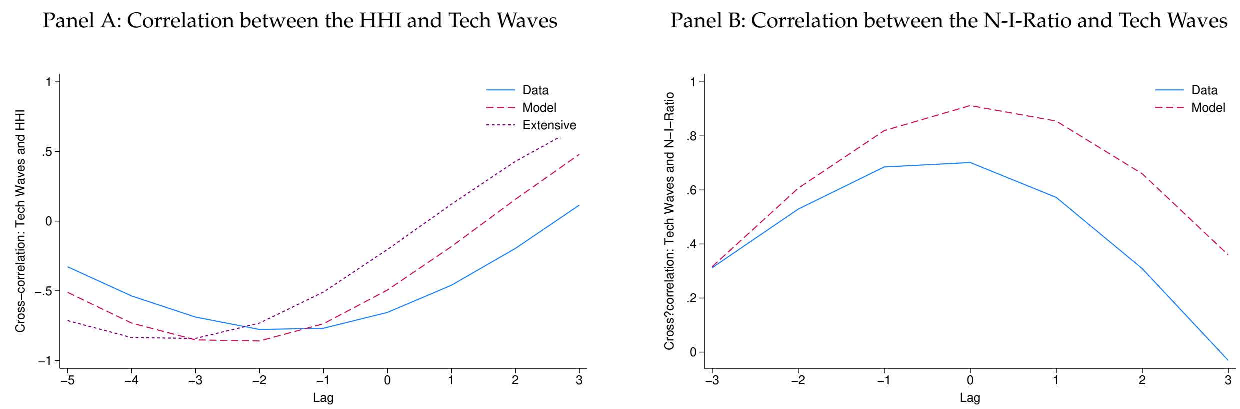

Figure 9: Cross Correlations with the Technological Waves

past technological waves. The peak correlation occurs at $k = -2$, suggesting a typical response lag of approximately two years. The model produces a similar response pattern, with the highest correlation also occurring at $k = -2$. This alignment indicates that the model not only captures the cyclical dynamics of market concentration, but also successfully replicates its lagged response to technological change.

6.2 Allocation of ideas

Empirically, this paper shows that inventors are more likely to form startups when revolutionary technologies appear and join incumbent firms when technologies mature. This is repeatedly shown by the solid curve in Panel A of Figure 10. The New-to-Incumbent ratio generated by the model is shown by the dashed curve in the same figure. They have nearly simultaneous waves. To further evaluate their relationship, we use linear trends to fit the two curves respectively, and then subtract them to get the detrended time variations. The correlation between the detrended model-generated and the detrended actual new-to-incumbent ratio is 0.825. Further summary statistics are diplayed in Panel B of Table 9.

The average New-to-Incumbent ratio in the model is 0.072, slightly higher than the corresponding value of 0.052 in the data. The slope of the linear trend is $-2.49E-3$, indicating a declining share of inventors launching new businesses. While this is qualitatively consistent with the data, the model exhibits a larger magnitude of decline. The detrended series also shows greater variability, with a standard deviation of 0.031 compared to 0.007 in the data, and a slightly higher first-order autocorrelation (0.883 vs. 0.830). The model's larger amplitude is analogous to the excessive volatility of model-predicted real gross investment per capita in real business cycle models—both stem from the absence of adjustment costs. In our model, inventors are short-lived and choose

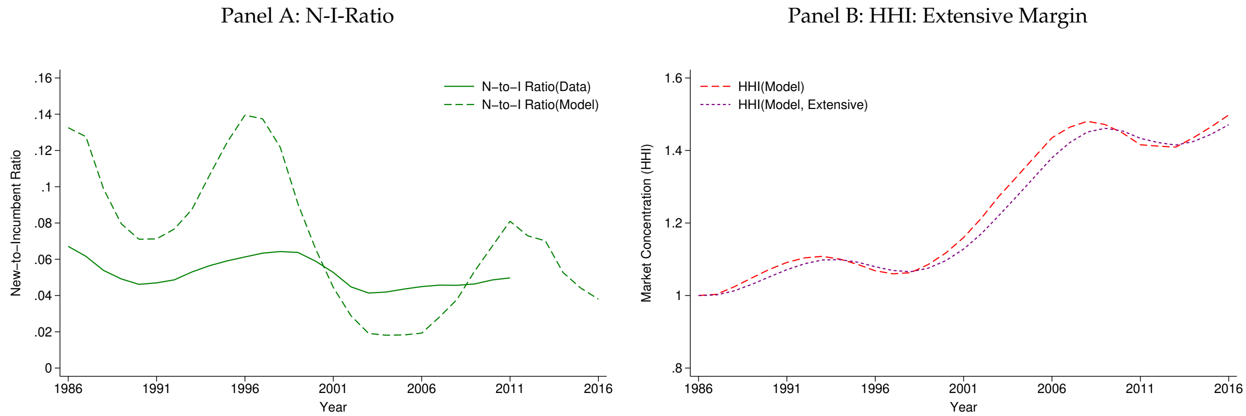

Figure 10: The Trend and Detrended Time Variations of the HHI
Notes: Panel A shows the technological waves and the trend of model-generated share of innovations in startups over time. The dashed curve displays the simulated New-to-Incumbent ratio in each year whereas the solid curve shows the New-to-Incumbent ratio in the data (same as Figure 4). Panel B shows the extensive margin of the model-generated market concentration over time. The dashed curve and short-dashed curve displays the simulated HHI and the HHI only considering extensive margin, respectively. Both are normalized by the model generated HHI level in 1986.

between joining startups or incumbent firms without regard to their previous affiliations. As a result, the New-to-Incumbent ratio reacts immediately to aggregate shocks. This dynamic is supported by the cross-correlation between the New-to-Incumbent ratio and waves of technological novelty, shown in Panel B of Figure 9. The correlation coefficient peaks at a zero lag in both the model and the data, but the model exhibits a stronger contemporaneous correlation (0.912 vs. 0.702).

6.3 Decomposition of the Intensive and Extensive Margins

To isolate the two margins, we simulate the HHI based on the evolution of firm numbers over time while holding the firm size distribution constant. In this setup, changes in HHI are driven solely by firm entry and exit, effectively removing the intensive margin by excluding resource reallocation among incumbents. The resulting HHI, capturing only the extensive margin, is shown as the short-dashed curve in Panel B of Figure 10, alongside the HHI generated from the full model with both margins (dashed curve). While the extensive margin alone reproduces the overall upward trend in market concentration, its response to technological shocks is slower. This suggests that the intensive margin plays a key role in accelerating the HHI's responsiveness. Figure 9 supports this interpretation: in both the model and the data, the peak cross-correlation occurs at $k = -2$, whereas under the extensive margin alone, it shifts to $k = -3$.

In summary, the extensive and intensive margins jointly affect the evolvement of market concentration. (1). The extensive margin is a main driver of the trend. (2). The intensive margin responds more swiftly to the technological waves.

6.4 Checking the Effect of Aggregate Novelty

The lower realization potential of new ideas within incumbent firms during periods of heightened technological breakthroughs is evidenced by the negative coefficient on the interaction between patent citations and the aggregate novelty index in Table 4. To assess whether the model captures a similar effect, we replicate column (3) of Table 4 using simulated data. The estimated interaction term in the regression with simulated data is -0.428, closely matching the empirical value of -0.390. Importantly, this regression was not targeted during the calibration process. Further details on the simulated regression are provided in Appendix E.1.

7 Conclusion

This paper studies how technological waves shape the market concentration, through the reallocation of inventors. It provides empirical evidence and structural analysis showing that market concentration is inversely related to and lagged behind the technological waves. This discovery suggests the presence of a low-frequency business cycle in the economy. We explore one potential channel behind this connection: the allocation of ideas. Using the data from the Longitudinal Business Database (LBD) from the Census Bureau and the patent information from the USPTO, this paper shows that the share of patents formed in new businesses co-move closely with the technological waves. At the peaks of the technological waves, a larger share of patents are forming in new businesses, while at the troughs, a larger share of patents come from existing businesses.

This paper proposes a theoretical framework that elucidates the decision-making process of inventors regarding their choice of innovation pathways, thus providing an explanation for the observed empirical patterns. Inventors are faced with a choice between forming a new business of a random size with a partner or joining an incumbent business of a selected size. This decision hinges on a trade-off: new businesses offer better incentives and adaptability in embracing novel technologies, while incumbents possess synergies and experience in commercialization. Our model effectively captures the relationship between technological waves and market concentration, primarily through the redistribution of innovative ideas. It implies that the deceleration in the emergence of groundbreaking technologies could be a significant contributing factor to the rise in market concentration after the 2000s.

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Appendix

A Data Description

The data used in this paper includes the Longitudinal Business Database (LBD), the USPTO patent data, and the Compustat Fundamentals Annual. This section provides details about the information of the datasets and the construction of key variables.

A.1 The USPTO Patent Data

The USPTO patent data contains information of all patents issued between 1976 and 2022. It can be downloaded from the PatentsView website. For each patent, the data documents the patent type (utility, design, plant, etc.), the IPC code indicating its technological class, the grant year, and the patents it cites and it is cited. We keep all the utility patents to focus our attention to the introduction of new products and processes.

Forward Citations Forward citations are citations a focal patent receives from others. It indicates how many patents follow the focal one. This paper calculates the number of forward citations each patent gets within five years after issuance.

Backward Citations Backward citations are citations that other patents receive from the focal patent. It indicates to what extent the focal patent follows the existing technology. This paper calculates the number of backward citations by counting the number of patents cited by the focal patent that were granted within the previous five years.

The Novelty Index According to the definition in the paper, we calculate this index by dividing the number of forward citations received by all the utility patents granted in a year by the summation of the forward and backward citations of those patents. The Novelty Index by IPC is derived in a similar way for each IPC class and each year.

A.2 The Compustat Fundamentals Annual

The Compustat Fundamentals Annual contains information of all the publicly listed firms in the US. It records the firms' net sales, the number of employees, the primary industry (4-digit SIC code), and the headquarter locations of each firm. We keep all the firms that are headquartered in the US.

Primary Industry The primary industry of each firm in Compustat is based on the 4-digit SIC code assigned to each firm in the Fundamentals Annual. The code can be aggregated to different levels. Manufacturing is corresponding to SIC codes 2000-3999; utility and transportation is corresponding to SIC codes 4000-4999; wholesale trade is corresponding to SIC codes 5000-5199; retail trade is corresponding to SIC codes 5200-5999; finance is corresponding to SIC codes 6000-6999; service is corresponding to SIC codes 7000-8999.

The Herfindahl-Hirschman Index (HHI) Following the methods in Grullon, Larkin and Michaely (2019), we first calculate the HHI of each 3-digit SIC code by the squared ratios of firm net sales to the total net sales in that 3-digit industry. To get the aggregate HHI, we sum up the HHIs of all the 3-digit SIC codes and weight them by their total net sales.

A.3 The Longitudinal Business Database (LBD)

The LBD is collected by the US Census Bureau and is an establishment-level data that covers the universe of US businesses with paid employees from 1976 to 2020. The dataset assigns a firm ID to all establishments belonging to the same firm. Using the Business Dynamics Statistics of Patenting Firms (BDS-PF) patent assignee-FIRMID crosswalk from the Census, this paper links the USPTO patent data with firms in the LBD, therefore, derives all utility patents in the US that were granted to employer businesses between 1976 and 2020.

New-to-Incumbent Ratio After merging the patent data with the LBD, this paper can identify the firm each patent was granted to. If the firm is less than or equal to five years old in the patent's grant year, we indicate that the idea behind the patent was absorbed by a new firm 5 years ago. Otherwise, we indicate that the idea was absorbed by an incumbent firm 5 year ago. Then we divide the number of ideas combined with new firms by the number of ideas combined with incumbent firms to get the New-to-Incumbent Ratio.

Firm Size The LBD documents the number of employees each firm hires in each year. We deriving the mapping between patent forward citations and incumbent firm size, we use the number of employees as a proxy for size.

B More Empirical Evidence

B.1 Empirical Patterns without Smoothing

Figure 11 and Figure 12 present the patterns of the technological waves, HHI, and the New-to-Incumbent ratio without smoothing techniques. The negative correlation between the HHI and the Novelty Index, as well as the positive correlation between the New-to-Incumbent ratio and the Novelty Index, remain prominent.

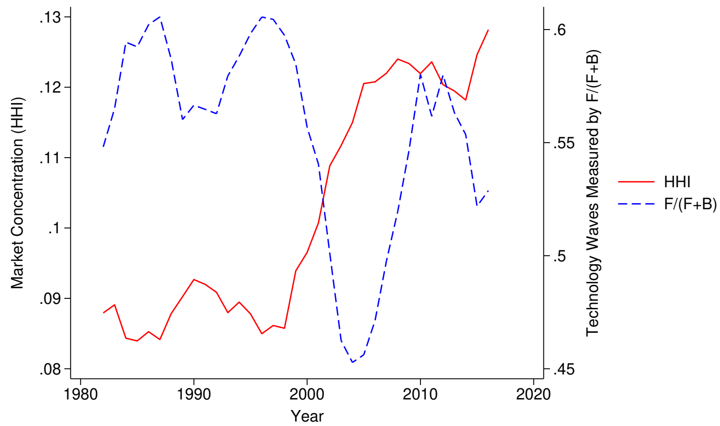

Figure 11: Technological Waves and Market Concentration without Smoothing
Notes: This figure shows the technological waves and the trend of market concentration over time. The blue dashed curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations. The red solid curve displays the HHI in each year, which is the weighted average of the industry-level HHI in each year. The weight is the total sales of firms in each industry. The two curves have different y-axises, which are shown respective on the left and right.
Sources: Compustat Fundamental Annuals and USPTO patent and citation data.

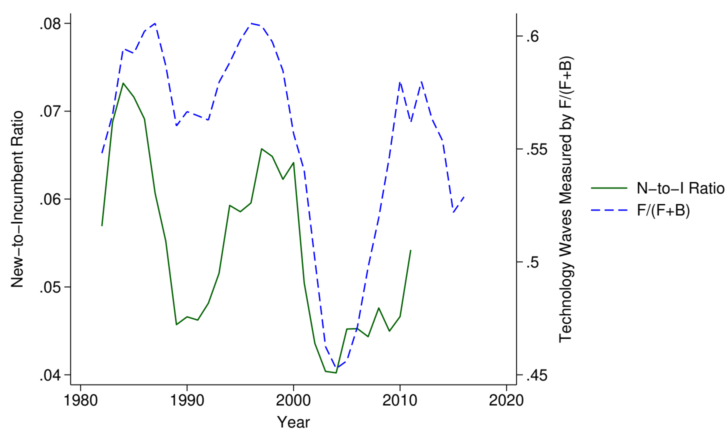

Figure 12: Technological Waves and Idea Allocation without Smoothing

Notes: This figure shows the technological waves and the idea allocation between new and incumbent firms over time. The blue dashed curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations. The green solid curve displays the "New-to-Incumbent Ratio" defined in the paper, capture where new ideas contribute their value. The two curves have different y-axises, which are shown respective on the left and right.
Sources: Longitudinal Business Database (LBD) and USPTO patent and citation data.

B.2 Technological Waves by Technological Field

The "Novelty" index across the nine technological fields is shown in Figure 13. The index is based on the same algorithm as in Equation 1 except that the forward and backward citations are aggregated across each of the 1-digit IPC code. The top three fields with the highest "Novelty" index are Human Necessities, Physics, and Electricity at the first peak; Electricity, Physics, and Human Necessities at the second peak; Human Necessities, Chemistry and Metallurgy, and Mechanical Engineering etc. at the third peak.

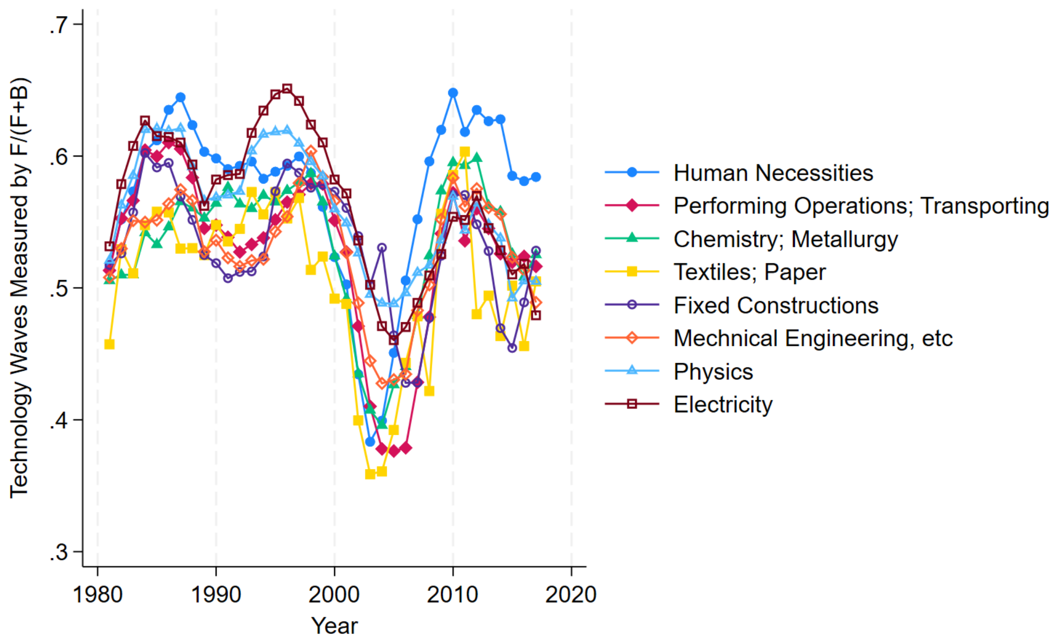

Figure 13: Technological Waves by Technological Fields

Notes: This figure shows the technological waves by the nine technological fields between 1981 and 2017. The nine fields are defined by the 1-digit IPC code. The technological waves are measured by the "Novelty" index as defined by Equation 1 in the paper.
Sources: USPTO patent and citation data.

B.3 Novelty Index in Europe

To calculate the Novelty Index for European countries with intensive patenting activities, we use data from PATSTAT (Patent Statistical Database), a comprehensive global dataset maintained by the European Patent Office (EPO). PATSTAT provides detailed bibliographic data on patents from various patent offices worldwide, with a particular focus on those filed through the EPO. We restrict our sample to patents with inventors based in European countries. The six countries with the highest number of patent issuances between 1982 and 2016 are Germany, France, the United Kingdom, Italy, Switzerland, and the Netherlands. Using the definition of the Novelty Index as outlined in Equation 1, we calculate the technological waves in these six countries and present them in Figure 14. Across all six countries, we observe an overall declining trend in technological novelty, with common peaks in the mid-1980s and early 2010s. Italy also experienced a distinct peak in the mid-1990s. In general, the technological trends in these European countries with the highest patenting activity mirror those observed in the U.S.[25]

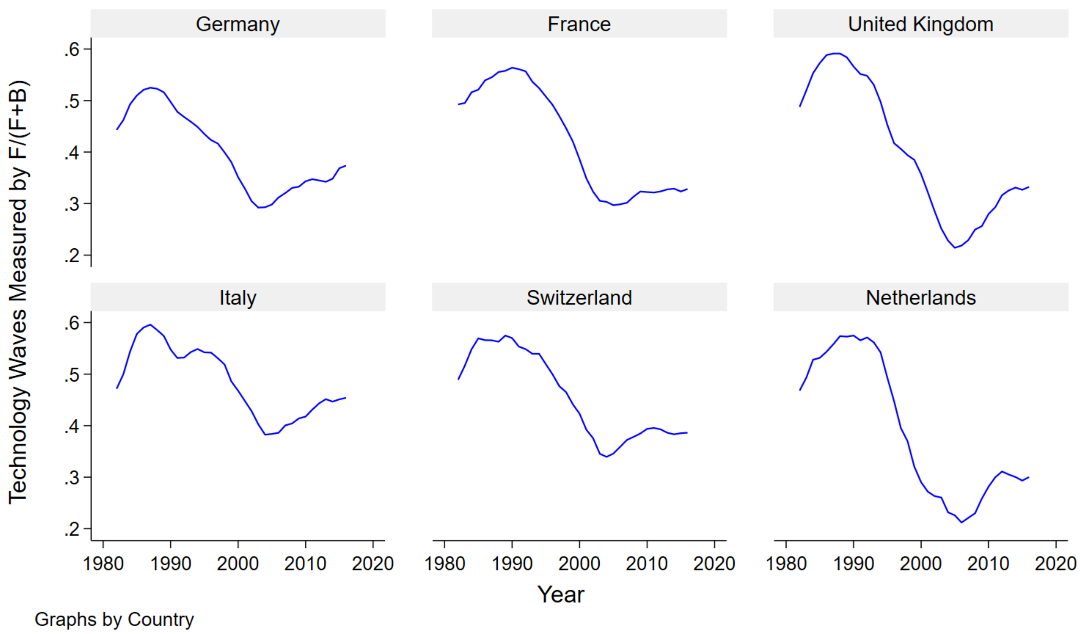

Figure 14: Technological Waves in European Countries

Notes: This figure shows the technological waves in six European countries with the highest number of patent issuances between 1982 and 2016. The technological waves are measured by the "Novelty" index as defined by Equation 1 in the paper.
Sources: PATSTAT (Patent Statistical Database).

B.4 Relationship with the Technology Waves

Table 10 exhibits the time trend of the technological novelty waves, the market concentration measured by the HHI, and the New-to-Incumbent Ratio of idea allocations (Panel A). It also displays the cross correlation of the two latter time series with the technological waves at different year gaps (Panel B). The time trend is derived by fitting

[25]Similar results can be obtained by calculating the Novelty Index using Google Patents data, as cleaned by Ayerst et al. (2023). These results are available upon request.

a linear trend to the focal time series and taking its slope. The cross correlations are obtained by calculating the correlation coefficients of the detrended time series when the year gaps of the two series are respectively -3, -2, -1, 0, 1, 2, 3. The detrending process subtracts the linear trend from the original time series. The cross correlations capture not only the co-movement of the different time series, but also the relative timing of their movements. The first row of each panel shows the statistics for the whole sample; the subsequent rows are statistics by major industries according to the Standard Industrial Classification (SIC) code or technological fields according to the International Patent Classification (IPC).

Table 10: Time Trend and Cross Correlation

	Time Trend		Detrended Cross Correlation						
			Panel A. HHI						
	Tech Wave	HHI	$k=-3$	$k=-2$	$k=-1$	$k=0$	$k=1$	$k=2$	$k=3$
All	-0.002	0.001	-0.683	-0.770	-0.763	-0.654	-0.424	-0.146	0.145
Mining & Construction	-0.002	0	-0.747	-0.782	-0.688	-0.500	-0.279	-0.046	0.189
Manufacturing	-0.002	0.001	-0.226	-0.475	-0.637	-0.692	-0.747	-0.663	-0.460
Transportation & Utilities	-0.001	0.001	-0.197	-0.132	-0.041	0.043	0.330	0.523	0.599
Wholesale & Retail Trade	-0.002	0.005	-0.495	-0.483	-0.432	-0.344	-0.195	-0.039	0.117
Finance	-0.003	0	-0.330	-0.339	-0.330	-0.272	-0.074	0.107	0.210
Services	-0.001	0.004	0.255	0.366	0.457	0.539	0.654	0.734	0.771
			Panel B. N-I-Ratio						
	Tech Wave	N-I-Ratio	$k=-3$	$k=-2$	$k=-1$	$k=0$	$k=1$	$k=2$	$k=3$
All	-0.002	-0.001	0.107	0.314	0.504	0.612	0.536	0.317	-0.001
Human Necessities	-0.001	-0.001	0.539	0.557	0.514	0.402	0.199	-0.063	-0.361
Performing Operations	-0.003	-0.001	0.117	0.210	0.283	0.301	0.191	0.017	-0.201
Chemistry; Metallurgy	-0.001	0.001	0.239	0.231	0.211	0.163	0.049	-0.128	-0.349
Textiles; Paper	-0.002	-0.001	0.458	0.451	0.462	0.417	0.371	0.270	0.189
Fixed Construction	-0.002	0	0.211	0.331	0.397	0.386	0.297	0.215	0.161
Mechanical Engineering	-0.001	0	-0.456	-0.505	-0.467	-0.358	-0.208	-0.061	0.059
Physics	-0.003	-0.001	-0.156	0.016	0.212	0.343	0.284	0.068	-0.207
Electricity	-0.003	-0.002	0.373	0.465	0.540	0.580	0.552	0.367	0.070

Notes: This table shows the trends of the technological waves, HHI, New-to-Incumbent ratio and the detrended cross correlations among them. The trend is derived by running linear regressions of the focal time series on year and taking the coefficient; the cross correlations are derived by computing the correlation coefficients at different year gaps of the detrended time series.

B.5 Alternative Measures of Market Concentration

The main text of this paper uses the Herfindahl-Hirschman Index to measure market concentration. It captures the whole distribution of firm sales in the economy, but the limitation is that it is based on only publicly listed firm. An alternative measure of market concentration is the share of sales by the top firms. This paper adopts the cleaned data series by Kwon, Ma and Zimmermann (2023) to calculate respectively the three-year

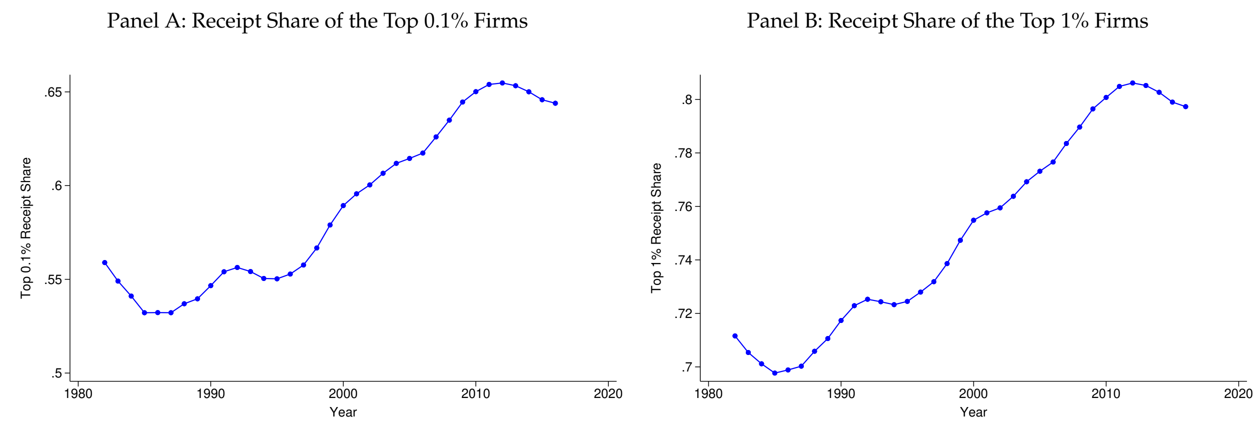

Figure 15: Receipt Shares by Top firms

Notes: This figure shows the three-year moving average of the receipt share of the top 0.1% (Panel A) and 1% firms (Panel B). The receipt shares are from the cleaned data series by Kwon, Ma and Zimmermann (2023), which is posted on `https://businessconcentration.com/`. The data source is the Statistics of Income (SOI) and the associated Corporation Source Book published annually by the IRS. Their statistics cover the whole population of US corporations.
Sources: `https://businessconcentration.com/`.

moving average of the receipt share of the top 0.1% and 1% firms. The top shares are generated by the IRS data, which covers a more comprehensive set of firms. So, it can be used as a complement to the HHI measure in the paper. As displayed in Figure 15, the top shares exhibit increasing trends in general but with fluctuations. The peaks and troughs of the fluctuations appear nearly simultaneously with the HHI measured in this paper, showing the robustness of the market concentration patterns shown in the paper.

B.6 Sector-Level Relationship between Tech Waves and HHI

To examine the sector-level relationship between the market concentration and technological waves, we calculate the HHI and the "Novelty" Index by major sectors defined by the SIC code—Mining and Construction, Manufacturing, Transportation and Utilities, Wholesale and Retail Trade, Finance, and Services.[26] Aggregating the HHIs within each major sector is a straightforward process, accomplished by computing a sales-weighted average of the HHIs at the 2-digit SIC level using Compustat. However, performing a similar aggregation for the "Novelty" Index presents a more complex challenge, since patents are classified by the technology class (as captured by the International Patent Classification (IPC)) instead of sectors. To map the technology classes

[26]The division is according to the U.S. department of Labor. Mining includes SIC 10-14; Construction includes SIC 15-17; Manufacturing includes SIC 20-39; Transportation and Utilities includes SIC 40-49; Wholesale Trade includes SIC 50-51; Retail Trade includes SIC 52-59; Finance includes SIC 60-67; Services includes SIC 70-89. To ensure sufficient observations, Mining and Constructions are combined; Wholesale and Retail Trade are combined.

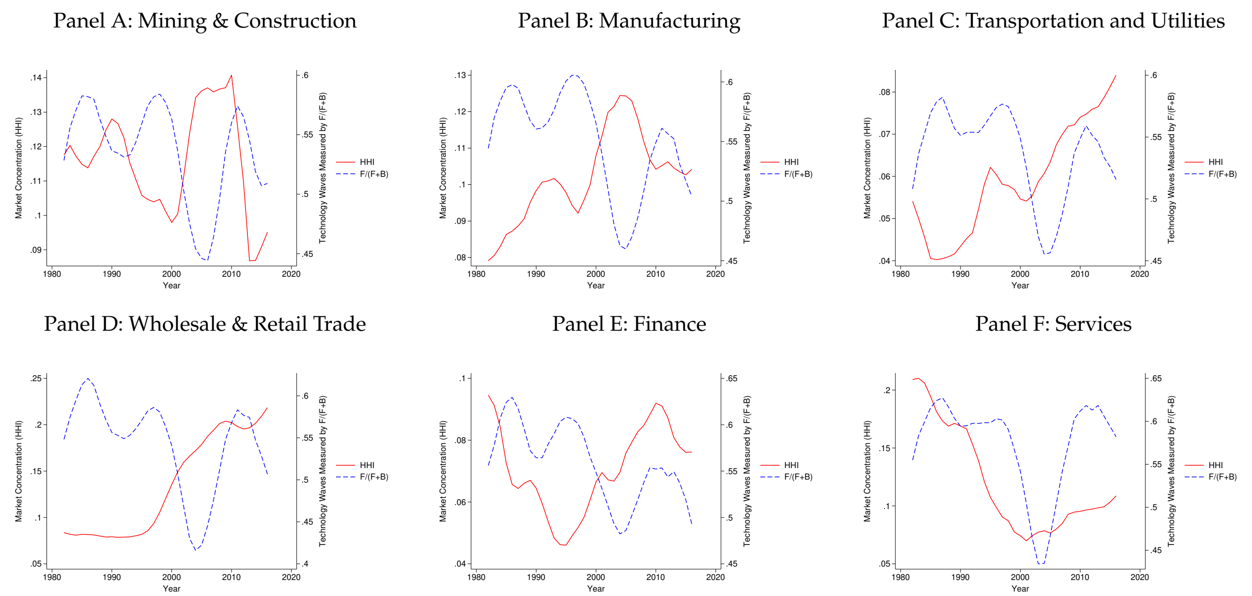

Figure 16: Technological Waves and Market Concentration by Industry

Notes: This figure shows the technological waves and the trend of market concentration over time by major sectors. The blue dashed curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations in each major sector. The red solid curve displays the HHI in each year, which is the weighted average of the 2-digit-SIC-level HHIs by major sectors and years. The weight is the total sales of firms in each 2-digit SIC industry. The two curves have different y-axises, which are shown respective on the left and right.
Sources: Compustat Fundamental Annuals and USPTO patent and citation data.

to sectors, we use the concordance developed by Silverman (2002) that links the 4-digit IPC code to the 4-digit SIC code according to usage. After applying this concordance, we obtain the counts of forward and backward citations at the 4-digit SIC level. These citation counts are then cumulatively summed up to the primary sector level, allowing us to calculate the "Novelty" Index for each sector. The visual representation of our findings can be observed in Figure 16.

Generally, a discernible negative relationship between technological waves and market concentration prevails across most major industries. The linear trend of the HHIs are non-negative, as opposed to negative trend of the technological waves. The detrended cross correlation between the two time series has the highest absolute magnitude at $k = -2$ in most of the sectors. $\text{corr}(x_t, y_{t-2})$ is respectively -0.782 for Mining and Construction; -0.475 for Manufacturing, -0.132 for Transportation and Utilities, -0.483 for Wholesale and Retail Trade, -0.339 for Finance, and 0.366 for Services. The cross correlations when $k \in \{-3, -2, -1, 0, 1, 2, 3\}$ for each sector are shown in Table 10 in Appendix B.4. These findings offer additional supporting evidence suggesting that market concentration may be influenced by the dynamics of technological novelty waves.

Table 11: Relationship between HHI and Novelty Index at the 3-digit NAICS code

	HHI							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Novelty Index	-0.0375***	-0.0102**	-0.0292**	-0.0121**				
	(0.0115)	(0.00483)	(0.0120)	(0.00501)				
Novelty Index(Lag 2 yrs)					-0.0409***	-0.0133***	-0.0326***	-0.0169***
					(0.0348)	(0.0257)	(0.0554)	(0.0411)
Size (Total Payroll)	-0.00258**	0.0138***	-0.00430***	0.0205***	-0.00242**	0.0177***	-0.00437***	0.0238***
	(0.00107)	(0.00116)	(0.00111)	(0.00197)	(0.00110)	(0.00116)	(0.00113)	(0.00193)
Industry Fixed Effect	N	Y	N	Y	N	Y	N	Y
Year Fixed Effect	N	N	Y	Y	N	N	Y	Y
Observations 3200	3200	3200	3200	3200	3200	3200	3200	
R-squared	0.005	0.844	0.023	0.848	0.005	0.859	0.025	0.862

Notes: Standard errors are clustered at the 3-digit-NAICS level. The HHI and industry size are measured by payroll in the LBD. Columns (1) and (5) include no fixed effects. Columns (2) and (6) incorporate industry fixed effects. Columns (3) and (7) incorporate year fixed effects. Columns (4) and (8) include both industry and year fixed effects. To comply with Census Bureau disclosure requirements, the number of observations is rounded to the nearest hundred. *** Significant at the 1 percent level; ** Significant at the 5 percent level; * Significant at the 10 percent level.

B.7 Regression Analysis—Tech Waves and HHI of Sales

Table 11 presents the regression results for the HHI and the Novelty Index. The HHI is measured by firm payroll using the Census data at the 3-digit NAICS industry level. The Novelty Index, calculated at the 4-digit IPC level, is mapped to the 3-digit NAICS level using the concordance developed by Silverman (2002), which links IPC and SIC codes based on patent usage, combined with a mapping between SIC and NAICS codes. A significant negative relationship is observed across all specifications, both with and without industry and year fixed effects. The coefficient is larger in absolute terms when the 2-year lagged Novelty Index is used.

B.8 IPC-Level Relationship between Tech Waves and N-I-Ratio

To assess the robustness of the relationship between idea allocation and technological waves, this paper compares the two trends by patent technological fields, categorized by the first digit of the patent IPC code. The IPC-level "Novelty" Index and "New-to-Incumbent Ratio" are computed using the same methodology as described in equations 1 and 5, with patent sets segregated according to their respective technology classes. Figure 17 illustrates that a positive correlation between idea allocation and technological waves is consistently observed across most technology classes. When a specific technology class experiences breakthroughs, there is an increase in the flow of ideas toward new startups. The contemporaneous correlation coefficients between the two curves are, respectively, 0.40 for Human Necessities, 0.30 for Performing Operations, 0.16 for Chemistry, 0.42 for

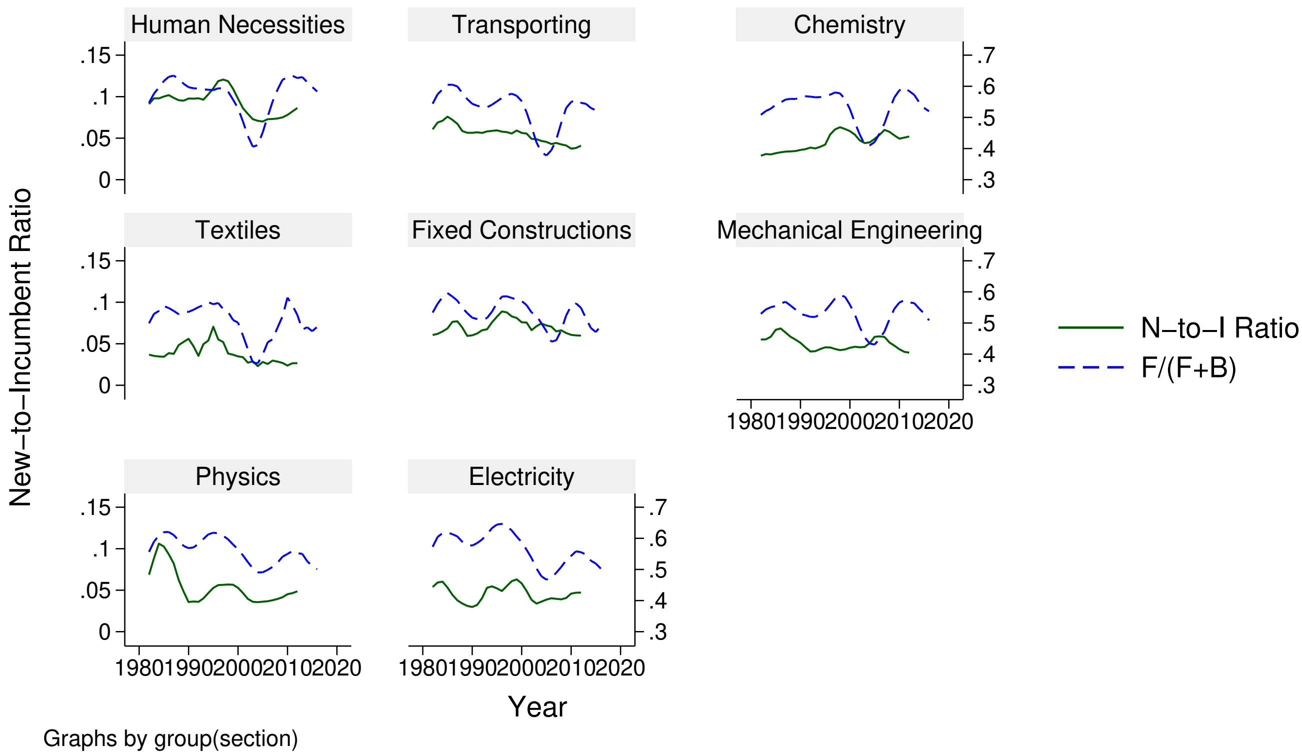

Figure 17: Technological Waves and Idea Allocation by Patent Technology Class

Notes: This figure shows the technological waves and the idea allocation between new and incumbent firms by patent technology class. The blue dashed curve, based on the methodology defined in this paper, calculates the relative ratio of forward citations to the sum of forward and backward citations. The green solid curve displays the "New-to-Incumbent Ratio" defined in the paper, capture where new ideas contribute their value. The two curves have different y-axises, which are shown respective on the left and right.
Sources: Longitudinal Business Database (LBD) and USPTO patent and citation data.

Textiles, 0.39 for Fixed Constructions, -0.47 for Mechanical Engineering, 0.34 for Physics, and 0.58 for Electricity. The cross correlations when $k \in \{-3, -2, -1, 0, 1, 2, 3\}$ for each technological field are shown in Table 10 in Appendix B.4.

B.9 Patents' Economic Value Regression

Table 12 uses firm sales as a proxy for size and shows the results of the regression on patents' economic value (Equation 8). The coefficients are close to those in Table 4, which uses employment as a measure of firm size. Notably, the coefficient for the firm size variable is nearly identical in both tables, indicating a robust estimation of the elasticity of synergy with respect to incumbent firm size in the calibration.

Table 12: Factors of Patents' Economic Value for Incumbent Firms

	Ln(Patent Economic Value)					
	(1)	(2)	(3)	(4)	(5)	(6)
Ln(1+Firm Sales)	0.331***		0.331***		0.331***	
	(0.0142)		(0.0142)		(0.0142)	
Ln(1+Citations)	0.0809***	0.00273***	0.236***	0.0131**	0.192**	0.0114**
	(0.00535)	(0.000583)	(0.0644)	(0.00570)	(0.0827)	(0.00499)
Ln(1+Citations)×FB Ratio			-0.285**	-0.0191*		
			(0.118)	(0.0106)		
Ln(1+Citations)×IPC FB Ratio					-0.205	-0.0161*
					(0.147)	(0.00906)
Year Fixed Effect	Y	Y	Y	Y	Y	Y
Year×IPC Fixed Effect	Y	Y	Y	Y	Y	Y
Year×Firm Fixed Effect	N	Y	N	Y	N	Y
Observations	1,118,163	1,107,618	1,118,163	1,107,618	1,118,059	1,107,513
R-squared	0.403	0.882	0.403	0.882	0.403	0.882

Notes: Standard errors are clustered at the year level. Columns (1)-(2) exclude the technological wave measure and focus solely on the property of the patents and firms. Columns (3)-(4) show coefficients of the regression equation (8). Columns (5)-(6) replace the yearly Novelty Index by the year-by-IPC Novelty Index. The regressions control for year fixed effects and year by patent technology class fixed effects across all specifications. The year by firm fixed effects are controlled in columns (2), (4), and (6). *** Significant at the 1 percent level; ** Significant at the 5 percent level; * Significant at the 10 percent level.

C Model and Proof

C.1 Production

The production sector features two types of firms: a representative final goods producer and intermediate goods producers. The final good producer assembles intermediate goods, denoted by j within the range $[0, N_F]$, to produce final goods. It chooses $\{y_j\}_j$ to maximize its profit using the technology described in Section 3.2. The final goods producer's problem can be written as:

$$\max_{\{y_j\}} \frac{1}{1-\beta} \int_0^{N_F} q_j^{\beta} y_j^{1-\beta} dj - \int_0^{N_F} y_j p_j dj. \tag{37}$$

The first-order condition

$$p_j = q_j^{\beta} y_j^{-\beta}$$

yields the demand function for goods produced by intermediate firms.

The intermediate goods are produced by their corresponding firm $j \in [0, N_F]$ using only labor $y_j = \bar{q} l_j$, where $\bar{q} = \frac{1}{N_F} \int_0^{N_F} q_j dj$ represents the average quality, and l_j is the labor input. Intermediate good producers engage in monopolistic competition, optimizing their profit by choosing l_j, p_j, y_j, given the wage level w:

$$\begin{aligned}
&\max_{l_j,p_j,y_j} y_j p_j - w l_j. \\
&s.t.\ y_j = \bar{q} l_j \\
&\quad\ p_j = q_j^{\beta} y_j^{-\beta}
\end{aligned} \tag{38}$$

The labor market clears, which derives that $\frac{\int_0^{N_F} q_j \left(\frac{\bar{q}(1-\beta)}{w}\right)^{\frac{1}{\beta}} dj}{\bar{q}} = 1$.

The value function of an intermediate firm q at time t is a linear function of firm size q.

$$V(q,t) = \nu q. \tag{39}$$

where $\nu = \frac{\beta}{(r+\tau)N_F^{1-\beta}}$.

Proof. The value function of an intermediate firm q at time t can be written as

$$\begin{aligned}
V(q,t) = &\int_t^{\infty} e^{-(r+\tau)(s-t)} \beta q / N_F^{1-\beta} ds \\
= &\nu q.
\end{aligned}$$

□

When working in firm $\tilde{q}$, inventor z_0 receives consumption:

$$c_I\left(a,\tilde{q},\tilde{T}\right) = a\left(\tilde{V}\left(\tilde{q}\right) + \tilde{x}\left(z_0,\tilde{q}\right)\nu \mathbb{1}_{\mathcal{S}}\right) + \tilde{T}$$

where $\mathcal{S}$ denotes the event that the inventor successfully creates an innovation. The expected consumption is

$$\mathbb{E}\left(c_I\right) = a\left(\mathbb{E}\tilde{V}\left(\tilde{q}\right) + \lambda_0 e_I x_0\left(z_0,\tilde{q}\right)\nu dt\right) + \tilde{T},$$

and the associated variance is

$$\operatorname{Var}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right) = a^2\left(\underbrace{\tau\tilde{q}^2\nu^2 dt}_{\operatorname{Var}(\tilde{V}(\tilde{q}))} + \underbrace{\lambda_0 e_I \mathbb{E}\left(\tilde{x}\left(z_0,\tilde{q}\right)^2\right)\nu^2 dt}_{\operatorname{Var}(\text{innovation})}\right).$$

Proof. Inventor's consumption is:

$$c_I\left(a,\tilde{q},\tilde{T}\right) = a\left(\tilde{V}\left(\tilde{q}\right) + \tilde{x}\left(z_0,\tilde{q}\right)\nu \mathbb{1}_{\mathcal{S}}\right) + \tilde{T}$$

where $\mathcal{S}$ denotes the event that the inventor successfully creates an innovation:

$$\mathbb{1}_{\mathcal{S}} = \begin{cases} 1, & \Pr = \lambda_0 e_I dt \\ 0, & \Pr = 1 - \lambda_0 e_I dt \end{cases}$$

The expected consumption is:

$$\begin{aligned} \mathbb{E}\left(c_I\right) =& a\mathbb{E}\left(\tilde{V}\left(\tilde{q}\right) + \tilde{x}\left(z_0, \tilde{q}\right)\nu\mathbb{1}_{\mathcal{S}}\right) + \mathbb{E}\left(\tilde{T}\right) \\ =& a\left(\mathbb{E}\left(\tilde{V}\left(\tilde{q}\right)\right) + \mathbb{E}\left(\tilde{x}\left(z_0, \tilde{q}\right)\nu\mathbb{1}_{\mathcal{S}}\right)\right) + \left(\tilde{T}\right) \end{aligned}$$

Upon the creation of an innovation, its value is a random draw from a distribution which is independent of its realization probability, yielding:

$$\begin{aligned} \mathbb{E}\left(\tilde{x}\left(z_0, \tilde{q}\right)\nu\mathbb{1}_{\mathcal{S}}\right) =& \mathbb{E}\left(\tilde{x}\left(z_0, \tilde{q}\right)\nu\right)\mathbb{E}\left(\mathbb{1}_{\mathcal{S}}\right) \\ =& \tilde{x_0}\left(z_0, \tilde{q}\right)\nu\lambda_0 e_I dt \end{aligned}$$

Therefore:

$$\mathbb{E}\left(c_I\right) = a\left(\mathbb{E}\tilde{V}\left(\tilde{q}\right) + \lambda_0 e_I x_0\left(z_0, \tilde{q}\right)\nu dt\right) + \tilde{T},$$

The variance in consumption is

$$\begin{aligned} \mathrm{Var}\left(c_I\left(a, \tilde{q}, \tilde{T}\right)\right) =& \mathrm{var}\left(a\left(\tilde{V}\left(\tilde{q}\right) + \tilde{x}\left(z_0, \tilde{q}\right)\nu\mathbb{1}_{\mathcal{S}}\right) + \tilde{T}\right) \\ =& a^2\left(\mathrm{var}\left(\tilde{V}\left(\tilde{q}\right)\right) + \mathrm{var}\left(\tilde{x}\left(z_0, \tilde{q}\right)\nu\mathbb{1}_{\mathcal{S}}\right)\right), \end{aligned}$$

because the firm value prior to innovation, $\tilde{V}\left(\tilde{q}\right)$, is independent of innovation and the wage $\tilde{T}$ is constant.

The uncertainty in the first component solely comes from exogenous exit:

$$\tilde{V}\left(\tilde{q}, t\right) = \begin{cases} 0, & \Pr = \tau dt \\ \tilde{V}\left(\tilde{q}, t - dt\right), & \Pr = 1 - \tau dt \end{cases}$$

So, the variance of the firm value can be rewritten as:

$$\begin{aligned} \mathrm{var}\left(\tilde{V}\left(\tilde{q}\right)\right) =& \tau\tilde{V}\left(\tilde{q}\right)^2 dt \\ =& \tau\tilde{q}^2\nu^2 dt \end{aligned}$$

The uncertainty in the innovation process is:

$$\begin{aligned}
\operatorname{var}\left(\tilde{x}\left(z_0,\tilde{q}\right)\nu\mathbb{1}_{\mathcal{S}}\right) =& \mathbb{E}\left(\left(\tilde{x}\left(z_0,\tilde{q}\right)\nu\mathbb{1}_{\mathcal{S}}\right)^2\right) - \mathbb{E}\left(\left(\tilde{x}\left(z_0,\tilde{q}\right)\nu\mathbb{1}_{\mathcal{S}}\right)\right)^2 \\
=& \lambda_0 e_I \nu^2 dt \mathbb{E}\left(x\left(z_0,\tilde{q}\right)^2\right) - \left(\lambda_0 e_I \nu dt \mathbb{E}\left(x\left(z_0,\tilde{q}\right)\right)\right)^2 \\
=& \lambda_0 e_I \nu^2 dt \mathbb{E}\left(x\left(z_0,\tilde{q}\right)^2\right)
\end{aligned}$$

when $dt \to 0$. Hence, the variance in consumption is:

$$\operatorname{Var}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right) = a^2\left(\underbrace{\tau\tilde{q}^2\nu^2 dt}_{\operatorname{Var}(\tilde{V}(\tilde{q}))} + \underbrace{\lambda_0 e_I \mathbb{E}\left(\tilde{x}\left(z_0,\tilde{q}\right)^2\right)\nu^2 dt}_{\operatorname{Var}(\text{innovation})}\right).$$

$\square$

C.2 Closed Form Model

The firm's problem in Equation 24 can be rewritten as:

$$\begin{aligned}
&\max_a \left(\tilde{V}\left(\tilde{q}\right) + \lambda_0 e_I x_0\left(z_0,\tilde{q}\right)\nu dt\right) - \bar{u}\left(z_0\right) \\
&\quad - Aa^2\left(\tau\tilde{q}^2\nu^2 dt + \lambda_0 x_0\left(z_0,\tilde{q}\right)^2\nu^2 dt\right) - \frac{1}{2}e_I^2 \\
&\text{st}\, e_I = \lambda_0 a x_0\left(z_0,\tilde{q}\right)\nu
\end{aligned} \tag{40}$$

Putting the expression of e_I into the maximization problem and taking the FOC with regard to the equity share, a, derives,

$$\begin{aligned}
a^* &= \frac{\lambda_0^2 x_0\left(z_0,\tilde{q}\right)^2\nu^2}{\lambda_0^2 x_0\left(z_0,\tilde{q}\right)^2\nu^2 + 2A\left(\tau\tilde{q}^2\nu^2 + \lambda_0 x_0\left(z_0,\tilde{q}\right)^2\nu^2\right)} \\
&= \frac{1}{1+2\frac{A}{\lambda_0}\left(\frac{\tau\tilde{q}^2}{\lambda_0 x_0\left(z_0,\tilde{q}\right)^2}+1\right)}
\end{aligned} \tag{41}$$

Upon reviewing all contracts, an inventor with idea quality z_0 chooses which firm $\tilde{q}$

to work for by maximizing her utility:

$$\begin{aligned}\max_{\tilde{q}} u\left(c_I\left(a,\tilde{q},\tilde{T}\right),e_I\right) =& \mathbb{E}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right) - A\mathrm{Var}\left(c_I\left(a,\tilde{q},\tilde{T}\right)\right) - R\left(e_I\right) \\ \mathrm{st}\, a &= a^*\left(\tilde{q}\right) \\ \tilde{T} &= \tilde{T}^*\left(\tilde{q}\right)\end{aligned} \tag{42}$$

Putting the expression of the optimal equity level, $a^*(\tilde{q})$, and $\tilde{T}^*(\tilde{q}) = -a^*\tilde{V}\left(\tilde{q}\right) + \left(1-a^*\right)\lambda_0 e_I x_0\left(z_0,\tilde{q}\right) v dt$ into the maximization problem and solving the first-order condition,

$$\frac{\partial x_0\left(z_0,\tilde{q}\right)}{\partial \tilde{q}} = \frac{2A\tau}{4A\tau\frac{\tilde{q}}{x_0(z_0,\tilde{q})} + \frac{2A\lambda_0+\lambda_0^2}{\tilde{q}/x_0(z_0,\tilde{q})}}, \tag{43}$$

derive the optimal firm size,

$$\tilde{q}^* = \left(\frac{\left(2A\lambda_0+\lambda_0^2\right)\left(\gamma\left(z_0\right)\right)^2 b}{2A\tau q_0^{2b}\left(1-2b\right)}\right)^{\frac{1}{2-2b}}.$$

The left-hand side and right-hand side of the first-order condition when $b < 0.5$ and $\eta = -1$ are shown in Figure 18.

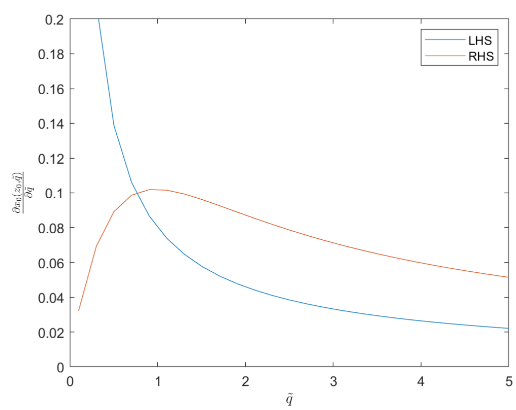

Figure 18: FOC Condition

Notes: This figure shows respectively the left-hand side (the blue curve) and the right-hand side (the red curve) of Equation 43. When $b < 0.5$, there exists a unique intersection.

C.3 Proof for Proposition 1

Proof.

$$\begin{aligned}\frac{\partial \tilde{q}^*}{\partial z_0} &= \frac{\tilde{q}^*}{\gamma(z_0)(1-b)} \frac{\partial(\gamma(z_0))}{\partial z_0} \\ &= \frac{\tilde{q}^*}{\gamma(z_0)(1-b)} \left(z_0^\eta + B^\eta\right)^{\frac{1}{\eta}-1} z_0^{\eta-1}\end{aligned}$$

where

$$\tilde{q}^* = \left(\frac{\left(2A\lambda_0 + \lambda_0^2\right)(\gamma(z_0))^2 b}{2A\tau q_0^{2b}(1-2b)}\right)^{\frac{1}{2-2b}}.$$

When $b < 0.5$, the optimal size increases in the inventor's idea quality, z_0. □

C.4 Proof for Proposition 2

Proof. The highest expected utility $u_N(z_0)$ an inventor z_0 can obtain when working in a new firm is

$$\begin{aligned}u_N(z_0) =& u\left(c_I(z_0, \tilde{q}), e_I(z_0, \tilde{q})\right) \\ =& \frac{1}{2}\frac{\lambda_0}{\lambda_0 + 2A}\lambda_0^2 z_0^2,\end{aligned}$$

which is unrelated to the technology wave indicator B. However, the highest expected utility $u_I(z_0)$ an inventor z_0 can obtain when working in an incumbent firm depends on B:

$$\begin{aligned}u_I(z_0) =& u\left(c_I(z_0, \tilde{q}^*), e_I(z_0, \tilde{q}^*)\right) \\ =& \frac{1}{2}\lambda_0^2 \frac{\left(\frac{2A\lambda_0 + \lambda_0^2}{2(1-2b)A\tau} b\right)^{\frac{b}{1-b}}}{\left(1 + \frac{2A(1-b)+\lambda_0 b}{\lambda_0(1-2b)}\right) q_0^{\frac{2b}{1-b}}} \gamma(z_0)^{\frac{2}{1-b}} \\ =& \frac{1}{2}\lambda_0^2 \hat{q}_0 \gamma(z_0)^{\frac{2}{1-b}},\end{aligned}$$

where $\hat{q}_0$ is a parameter ($\hat{q}_0 = \frac{\left(\frac{2A\lambda_0+\lambda_0^2}{2(1-2b)A\tau} b\right)^{\frac{b}{1-b}}}{\left(1+\frac{2A(1-b)+\lambda_0 b}{\lambda_0(1-2b)}\right) q_0^{\frac{2b}{1-b}}}$). The utility $u_I(z_0)$ is positively associated with $\gamma(z_0)$, which increases in B, implies that $\frac{\partial u_I(z_0)}{B} > 0$

An inventor decides whether to join a startup by comparing the incumbent-startup utility $u_I(z_0) - u_N(z_0)$ and zero. The utility gap increases in B, meaning that a larger share of inventors would choose incumbent firms when B goes up. $\square$

C.5 Proof for Proposition 3

Proof. An inventor decides whether to join a startup by comparing the highest utility offered by incumbents $u_I(z_0)$ and startups $u_N(z_0)$. When $\frac{\gamma(z_0)^{\frac{1}{1-b}}}{z_0} < \left(\frac{\lambda_0}{\hat{q}_0(\lambda_0+2A)}\right)^{\frac{1}{2}}$, $u_I(z_0) < u_N(z_0)$, inventor chooses to join a startup. When $\eta < 0$, if $b < \frac{\min\left(z_0^{-\eta}\right)}{\min\left(z_0^{-\eta}\right)+\max(B^{-\eta})}$, $b - \frac{B^\eta}{z_0^\eta+B^\eta} < 0$ always holds:

$$\frac{\partial\left(\gamma(z_0)^{\frac{1}{1-b}} z_0^{-1}\right)}{\partial z_0} = \frac{\gamma(z_0)^{\frac{1}{1-b}}}{z_0^2}\frac{1}{1-b}\left(b - \frac{B^\eta}{z_0^\eta + B^\eta}\right) < 0,$$

since $\gamma(z_0) = \left(z_0^\eta + B^\eta\right)^{\frac{1}{\eta}}$. $\gamma(z_0)^{\frac{1}{1-b}} z_0^{-1}$ monotonically decreases in z_0, when holding B constant. It implies there exists a cutoff $\bar{z}_0(B)$, when $z_0 > \bar{z}_0(B)$,

$$\frac{\gamma(z_0)^{\frac{1}{1-b}}}{z_0} < \left(\frac{\lambda_0}{\hat{q}_0(\lambda_0 + 2A)}\right)^{\frac{1}{2}}$$

always holds, and hence $u_I(z_0) < u_N(z_0)$, inventors opts in new businesses instead of incumbent firms. $\square$

C.6 Full Model

The firm's problem in Equation 24 becomes

$$\begin{aligned}
&\max_a \left(\tilde{V}(\tilde{q}) + \lambda_0 e_I x_0(z_0, \tilde{q})\, \nu dt\right) \\
&\qquad - Aa^2\left(\tau\tilde{q}^2\nu^2 dt + \lambda_0 e_I \mathbb{E}\left(\tilde{x}(z_0,\tilde{q})^2\right)\nu^2 dt\right) - R(e_I) \\
&\text{st}\, e_I = \lambda_0 a x_0(z_0,\tilde{q})\,\nu - Aa^2\lambda_0\mathbb{E}\left(\tilde{x}(z_0,\tilde{q})^2\right)\nu^2
\end{aligned} \tag{44}$$

Given the contracts, inventor chooses which firm $\tilde{q}$ to work for by maximizing her utility. In each firm, her optimal effort level in given in Equation 28.

$$\begin{aligned}\max_{\tilde{q}} u\left(c_I\left(a,\tilde{q},\tilde{T}\right),e_I\right) =& a\left(z_0,\tilde{q}\right)\left(\tilde{V}\left(\tilde{q}\right)+\lambda_0 e_I x_0\left(z_0,\tilde{q}\right)\nu dt\right)+\tilde{T}\\ &-Aa\left(z_0,\tilde{q}\right)^2\left(\tau\tilde{q}^2\nu^2 dt+\lambda_0 e_I\mathbb{E}\left(x\left(z_0,\tilde{q}\right)\right)^2\nu^2 dt\right)-R\left(e_I\right)\\ \text{st}\, e_I =& \lambda_0 a\left(z_0,\tilde{q}\right)x_0\left(z_0,\tilde{q}\right)\nu - Aa\left(z_0,\tilde{q}\right)^2\lambda_0\mathbb{E}\left(\tilde{x}\left(z_0,\tilde{q}\right)^2\right)\nu^2\end{aligned}$$

The firm-level innovation arrival rate can be written as:

$$\lambda_q\left(\tilde{q}\right)=\frac{h\lambda_0 e_I\left(z_0^*,\tilde{q}\right)\psi\left(z_0^*\right)dz_0^*+\left(1-h\right)\tilde{f}\left(\tilde{q}\right)dq\int_{z_0\in\{z_0|q^*(z_0)>0\}}\lambda_0 e_I\left(z_0,\tilde{q}\right)\psi\left(z_0\right)dz_0}{N_F\tilde{f}\left(\tilde{q}\right)dq} \tag{45}$$

where z_0^* is the inventor whose optimal choice is $\tilde{q}$.[27]

[27]If an inventor z_0 works in a firm $\tilde{q}$ when the novelty index is γ, the utility level is:

$$u\left(z_0,\tilde{q}\right)=\lambda_0 e_I x_0\left(z_0,\tilde{q}\right)\nu - a^2 A\left(\lambda_0 e_I k x_0^2\left(z_0,\tilde{q}\right)\nu^2+\tau\nu^2\tilde{q}^2\right)-e_I^2/2$$

Take derivative with respect to $x_0\left(z_0,\tilde{q}\right)$ yields:

$$\begin{aligned}\frac{\mathrm{d}u}{\mathrm{d}x_0} =& \frac{\partial u}{\partial x_0}+\frac{\partial u}{\partial e_I}\frac{\partial e_I}{\partial x_0}+\frac{\partial u}{\partial a}\frac{\partial a}{\partial x_0}\\ =& \lambda_0 e_I\nu-2a^2A\lambda_0 e_I k x_0\left(z_0,\tilde{q}\right)\nu^2+\left(1-a\right)\lambda_0 x_0\left(z_0,\tilde{q}\right)\nu\frac{\partial e_I}{\partial x_0}\\ =& \lambda_0^2 a x_0\left(z_0,\tilde{q}\right)\nu^2\left(1-2a^2Akx_0\left(z_0,\tilde{q}\right)\nu\right)\left(1-aAkx_0\left(z_0,\tilde{q}\right)\nu\right)\\ &+\left(1-a\right)a\lambda_0^2\nu^2\left(1-2Aakx_0\left(z_0,\tilde{q}\right)\nu\right)\\ =& \lambda_0^2 a x_0\left(z_0,\tilde{q}\right)\nu^2\left(2-3aAkx_0\left(z_0,\tilde{q}\right)\nu+2a^3A^2k^2x_0\left(z_0,\tilde{q}\right)^2\nu^2-a\right)\\ =& \lambda_0^2 a x_0\left(z_0,\tilde{q}\right)\nu^2\left[\left(1-a\right)+\left(1-aAkx_0\left(z_0,\tilde{q}\right)\nu\right)\right.\\ &\left.-2aAkx_0\left(z_0,\tilde{q}\right)\nu\left(1-a^2Akx_0\left(z_0,\tilde{q}\right)\nu\right)\right]\end{aligned}$$

As long as $aAkx_0\left(z_0,\tilde{q}\right)\nu<1$, the derivative is positive and the utility increases in x_0, and hence it increases in γ. It means that during the period when the technologies breakthroughs (B and γ are low), an inventor expects systematically less utility when working in an incumbent firm.

C.7 Starting up New Businesses

The partner's problem has the same form as the incumbent firm's, with $\tilde{q} = 0$ and the average innovation value being z_0 instead of $x_0(z_0, \tilde{q})$ (Equation 24).

$$\begin{aligned}
&\max_a (1-a)(\lambda_0 e_I z_0 \nu dt) - \tilde{T} \\
&\quad \text{st}\, e_I = \arg\max \left\{ u\left(c_I\left(a, 0, \tilde{T}\right), e_I\right)\right\} \\
&\quad\quad u\left(c_I\left(a, 0, \tilde{T}\right), e_I\right) \geq \bar{u}(z_0) \\
&\quad\quad (1-a)(\lambda_0 e_I x_0(z_0, \tilde{q}) \nu dt) - \tilde{T} \geq 0
\end{aligned} \tag{46}$$

The partners are assumed to get zero profit due to competition.

The inventor decides her effort level by maximizing her utility, which yields:

$$e_I = \lambda_0 a z_0 \nu - A a^2 \lambda_0 \mathbb{E}\left(\tilde{z}(z_0)^2\right)\nu^2 \tag{47}$$

The firm's problem in Equation 46 becomes

$$\begin{aligned}
&\max_a (\lambda_0 e_I z_0 \nu dt) - A a^2 \left(\lambda_0 e_I \mathbb{E}\left(\tilde{z}(z_0)^2\right)\nu^2 dt\right) - \frac{1}{2} e_I^2 \\
&\text{st}\, e_I = \lambda_0 a z_0 \nu - A a^2 \lambda_0 \mathbb{E}\left(\tilde{z}(z_0)^2\right)\nu^2
\end{aligned} \tag{48}$$

It gives the highest utility an inventor can obtain when working in a startup.

D Innovation value in an incumbent firm

D.1 Model

When inventor with idea quality z_0 works in an incumbent firm with quality $\tilde{q}$, the resulting innovation value $\tilde{x}(z_0, \tilde{q})$ is a random draw from a uniform distribution $U\left((1-\phi)\, x_0(z_0, \tilde{q})\, \nu, (1+\phi)\, x_0(z_0, \tilde{q})\, \nu\right)$. The mean value of the innovation depends on x_0, which takes the following functional form:

$$x_0(z_0, \tilde{q}) = \left(\frac{\tilde{q}}{\tilde{q}_0}\right)^b (B^\eta + z^\eta)^{\frac{1}{\eta}}$$

where the parameter η governs the elasticity of substitution between technology stock B and idea quality z. From the model, the economic value of a patent $\tilde{x}$ satisfied:

$$\frac{\tilde{x}}{\tilde{q}^b} = \varepsilon (B^\eta + z^\eta)^{\frac{1}{\eta}}$$

where $\varepsilon = \frac{\nu}{\tilde{q}_0^b}\epsilon$. ν is an equilibrium outcome, $\tilde{q}_0$ and b are parameters. ϵ is stochastic.

D.2 Data

The economic value of a patent is estimated by Kogan et al. (2017). The firm size is mapped to the employment size.[28] B and z are measured using average backward citations in a year and the number of forward citations received by each patent. [29]

D.3 Calibration

We estimate the parameter η using maximum likelihood estimation (MLE). To simplify calculation, assume that ε follows a log-normal distribution $\log(\varepsilon) \sim N(\mu_\varepsilon, \sigma_\varepsilon^2)$. [30]. For each observation, the probability to happen is:

$$Pr(\tilde{x}, \tilde{q}, b, B, z; \eta, \mu_\varepsilon, \sigma_\varepsilon^2) = \frac{1}{\sigma_\varepsilon\sqrt{2\pi}} \exp\left\{ -\frac{\left(\log\left(\frac{\tilde{x}}{\tilde{q}^b}\right) - \frac{1}{\eta}\left(B^\eta + z^\eta\right) - \mu_\varepsilon\right)^2}{2\sigma_\varepsilon^2} \right\}$$

Choose $\{\eta, \mu_\varepsilon, \sigma_\varepsilon^2\}$ to maximize the aggregate likelihood. The estimation result yields:

$$\eta = -0.4$$

Meaning that the elasticity of substitution is $\frac{1}{1+0.4} = 0.71$.

E Quantification Details

E.1 Checking the Effect of Aggregate Novelty—Details

We replicate the regressions presented in Table 4 using the simulated data. In year 1986, we randomly generate $40000 \times N_f$ of firms, which in turn create simulated observations that are of comparable magnitude as data. We then track these simulated firms over a 31-year period, documenting their innovation quality, innovation value, and firm size. Using this simulated dataset, the regression results are presented in Table 13.

The first three columns contain regression results derived from the actual data (same as Table 4), while the last three columns display results based on the simulated data. Employment is measured by $\tilde{q}$, and the variable $\text{Ln}(1 + \text{Employment})$ corresponds to

[28] The firm size can also be measured using sales. The estimation is similar.
[29] Both are truncated at 5-year window as before
[30] If we adopt the uniform assumption, the results will be similar

$\text{Ln}(\tilde{q})$ in the simulated dataset. In the model, idea quality z_0 determines citations; thus, the simulated variable $\text{Ln}(1 + \text{Citations})$ is calculated as $\text{Ln}(z_0)$. The simulated Novelty index is the same as the one used in the data.

Columns (1) and (3) exclude the technological wave measure, focusing solely on the properties of patents and firms. In contrast, Columns (2) and (4) present regression results based on Equation (8). Consistent with the empirical results, the model-generated regressions demonstrate that firm size has a significantly positive effect on the economic value of patents, controlling for idea quality. The coefficients for this relationship are closely aligned, as we explicitly calibrate this moment using the synergy parameter b. Additionally, both the simulated and empirical results suggests that idea quality positively impacts the economic value of patents. However, this impact diminishes as aggregate technological novelty increases, as reflected in the negative coefficients of the interaction terms. The untargeted coefficient for the interaction term is -0.428 in the model, compared to -0.390 in the data.

Table 13: Factors of Patents' Economic Value for Incumbent Firms

	Ln(Patents' Economic Value)			
	Data		Model	
	(1)	(2)	(3)	(4)
Ln(1+Employment)	0.330***	0.330***	0.319***	0.306***
	(0.0262)	(0.0262)	(0.000119)	(0.00011)
Ln(1+Citations)	0.0732***	0.285***	0.443***	0.673***
	(0.00561)	(0.073)	(3.37e-05)	(0.000405)
Ln(1+Citations)$\times$Novelty$_t$		-0.390***		-0.428***
		(0.135)		(0.000753)
Year Fixed Effect	Y	Y	Y	Y
Year$\times$IPC Fixed Effect	Y	Y	-	-
Observations	1,111,737	1,111,737	1,438,450	1,438,450
R-squared	0.295	0.295	0.995	0.996

Notes: Standard errors are clustered at the year level. Columns (1)-(2) are from data. Columns (3)-(4) are generated by simulated data. Columns (1) and (3) exclude the technological wave measure and focus solely on the property of the patents and firms. Columns (2) and (4) show coefficients of the regression equation (8). The regressions control for year fixed effects across all specifications. *** Significant at the 1 percent level; ** Significant at the 5 percent level; * Significant at the 10 percent level.