Vv186 Honors Mathematics II

Functions of a Single Variable

Assignment 1

Date Due: 8:00 AM, Thursday, the 24^{th} of September 2020

Mathematics is learned by doing it, not by watching other people do it.

Michael Reed and Barry Simon, *Methods of Modern Mathematical Physics*

I hear and I forget; I see and I remember; I do and I understand.

Attributed to Confucius[1]

A word of introduction

This is the first assignment for your first Calculus course at university. You will find the style of the exercises and the requirements of the solutions quite different from what you are used to from school. Some of the exercises on this set (and all further assignments) will be easy, some will be quite difficult. As with many problems in real life, you are not told in advance which are simple and which are more involved, but are supposed to find out for yourself. You will find it easier to obtain full marks for the exercises if you follow a few simple principles:

i) You are asked to *hand in the exercises on time.*

ii) You are required to compose your solutions in *neat and legible handwriting.* 10% of the total score will be awarded solely for the appearance and legibility of your writing and your use of the English language (see next point).

iii) In order to obtain the highest possible score, make sure that you explain your reasoning. Often, simple formulae are not enough to answer a question. *Explain what you are doing!* This will also ensure that you get a large fraction of the total points even if you make a mistake in your calculations. In short, write simple, whole grammatical sentences that include a subject, verb and object.

iv) You are **forbidden to use the symbols $\therefore$ and $\because$** in your writing. Write English sentences instead. You will **lose all points** for an exercise that includes these two symbols, no matter whether the solution is correct or not! On this subject, I would like to quote the great mathematician Serge Lang:

> It seems to me essential that students be required to write their mathematics papers in full and coherent sentences. A large portion of their difficulties with mathematics stems from their slapping down mathematical symbols and formulas isolated from a meaningful sentence and appropriate quantifiers. Papers should also be required to be neat and legible. They should not look as if a stoned fly had just crawled out of an inkwell. Insisting on reasonable standards of expression will result in drastic improvements of mathematical performance.

Serge Lang, *A First Course in Calculus*

v) If you have any problems, questions or comments regarding the exercises, the lecture or mathematics, please *visit me during my office hours.* This time has been set aside specifically for you, and you should make as much use fo it as you can. If you wish for comments to reach me anonymously, please talk to the Teaching Assistants.

With these words, I now let you commence your first exercises. Good luck!

Horst Hohberger

[1] This quote probably stems from Xun Kuang (荀况): "不闻不若闻之，闻之不若见之，见之不若知之，知之不若行之。学至于行之而止矣。" in 荀子·儒效 See http://ctext.org/xunzi/ru-xiao/zh.

This assignment has a total of **(20 Marks)**.

Exercise 1.1
Make a two-column table: in the left column, write out the letters of the Greek alphabet (lowercase and uppercase) in the correct order and in the right column write out the English names for the letters. It should start like this:

Greek letter	English name
α, A	alpha
β, B	beta
γ, $\mathit{\Gamma}$	gamma
$\vdots$	$\vdots$

You are allowed to use whatever sources you wish (e.g., Wikipedia) but you should cite them properly.
(2 Marks)

Exercise 1.2

i) Let a, b be statements. Write out the truth tables to prove *de Morgan's rules*:

$$\neg(a \wedge b) \equiv \neg a \vee \neg b, \qquad \neg(a \vee b) \equiv \neg a \wedge \neg b.$$

(2 Marks)

ii) Let M be a set and $A, B \subset M$. Use i) to prove the following equalities:

$$(A \cap B)^{\mathrm{c}} = A^{\mathrm{c}} \cup B^{\mathrm{c}}, \qquad (A \cup B)^{\mathrm{c}} = A^{\mathrm{c}} \cap B^{\mathrm{c}}.$$

(2 Marks)

Exercise 1.3
Explain in your own words the difference between the statements

$$\underset{0\in\mathbb{Q}}{\exists}\ \underset{a\in\mathbb{Q}}{\forall}\ a + 0 = 0 + a = a \qquad \text{and} \qquad \underset{a\in\mathbb{Q}}{\forall}\ \underset{0\in\mathbb{Q}}{\exists}\ a + 0 = 0 + a = a.$$

(4 Marks)

Exercise 1.4
Suppose that a truth table in n propositional variables is specified. Show that a compound proposition with this truth table can be formed by taking the disjunction of conjunctions of the variables or their negations, with one conjunction for each combination of values for which the compound proposition is true. The resulting compound proposition is said to be in *disjunctive normal form*.

For example, the table with the two propositional variables A and B,

A	B	$f(A,B)$
T	T	T
T	F	F
F	T	T
F	F	F

has the disjunctive normal form $f(A, B) = (A \wedge B) \vee (\neg A \wedge B)$.
(2 Marks)

Exercise 1.5

A collection of logical operators is called *functionally complete* if every compound proposition is logically equivalent to a compound proposition involving only these logical operators.

i) Show that $\{\wedge, \vee, \neg\}$ is a functionally complete collection of logical operators. (Hint: use the disjunctive normal form.)
(1 Mark)

ii) Show that $\{\wedge, \neg\}$ is a functionally complete collection of logical operators. (Hint: use a de Morgan law.)
(1 Mark)

iii) The *Scheffer stroke* $\mid$ is a logical operation defined by

$$A \mid B :\equiv \neg(A \wedge B).$$

(In computer science, it is known as the NAND operation.[2]) Write down the truth table for the Scheffer stroke.
(1 Mark)

iv) Prove that $\{\mid\}$ is a functionally complete collection of logical operators.
(3 Marks)

v) Show that $\mid$ is commutative but not associative, i.e., $A \mid B \equiv B \mid A$ but $(A \mid B) \mid C \not\equiv A \mid (B \mid C)$.
(2 Marks)

[2] See, for example, https://en.wikipedia.org/wiki/Flash_memory#NAND_flash for an example where this is applied.