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THE UNIVERSITY OF WARWICK

UNIVERSITY OF WARWICK	
Department	Computer Science
Module Code	CS929
Module Title	Algorithmic Game Theory
Exam Paper Code	CS9290_A
Duration	3 hours
Exam Paper Type	24-hour window

STUDENT INSTRUCTIONS

1. Read all instructions carefully. We recommend you read through the entire paper at least once before writing.

2. There are **6** questions. All candidates should **attempt 4 questions**.

3. You should not submit answers to more than the required number of questions.

4. You should handwrite your answers either with paper and pen or using an electronic device with a stylus (unless you have special arrangements for exams which allow the use of a computer). Start each question on a new page and clearly mark each page with the page number, your student id and the question number.
Handwritten notes must be scanned or photographed and all individual solutions collated into a single PDF with pages in the correct order.

5. Please ensure that all your handwritten answers are written legibly, preferably in dark blue or black ink. If you use a pencil ensure that it is not too faint to be captured by a scan or photograph.

6. Please check for legibility before uploading. It is your responsibility to ensure your work can be read.

7. Add your student number to all uploaded files.

8. You are permitted to access module materials, notes, resources, references and the internet during the online assessment.

9. You must not communicate with any other candidate during the assessment period or seek assistance from anyone else in completing your answers. The Computer Science Department expects the conduct of all students taking this assessment to conform to the stated requirements. Measures will be in operation to check for possible misconduct. These will include the use of similarity detection tools and the right to require live interviews with selected students following the assessment.

10. By starting this assessment, you are declaring yourself fit to undertake it. You are expected to make a reasonable attempt at the assessment by answering the questions in the paper.

IMPORTANT INFORMATION

- We strongly recommend you use Google Chrome or Mozilla Firefox to access the Alternative Exams Portal.
- You are granted an additional 45 minutes beyond the stated duration of this assessment to allow for downloading/uploading your assessment, your files and any technical delays.
- Students with approved Alternative Exam Arrangements (Reasonable Adjustments) that permit extra time and/or rest breaks will have this time added on to the stated duration.
- You must have completed and uploaded the assessment before the 24-hour assessment window closes.
- Late submissions are not accepted.
- If you are unable to submit your assessment, you must submit Mitigating Circumstances immediately, attaching supporting evidence and your assessment. The Mitigating Circumstances Panel will consider the case and make a recommendation based on the evidence to the Board of Examiners.

SUPPORT DURING THE ASSESSMENT

Operational Support

- Use the Alternative Exams Portal to **seek advice immediately if during the assessment period:**
    - you cannot access the online assessment
    - you believe you have been given access to the wrong online assessment

  Operational support will be available between 09:00 and 17:00 BST for each examination (excluding Sunday)

Technical Support

- If you experience any technical difficulties with the Alternative Exam Portal please contact helpdesk@warwick.ac.uk

Technical support will be available between 09:00 and 17:00 BST for each examination (excluding Sunday)

Academic Support

- If you have an academic query, contact the invigilator (using the 'Contact an Invigilator' tool in AEP) to raise your issue. Please be aware that two-way communication in AEP is not currently possible

  Academic support will normally be provided for the duration of the examination (i.e. for a 2 hour exam starting at 09:00 BST, academic support will normally be provided between 09:00 and 11:45 BST). Academic support beyond this time is at the discretion of the department.

Other Support

- **if you cannot complete your assessment for the following reasons submit Mitigating Circumstances immediately**:
    - you lose your internet connection
    - your device fails
    - you become unwell and are unable to continue
    - you are affected by circumstances beyond your control

1. (a) Consider the following three-player game:

$\mathcal{C}$	L	R
T	$0,0,3$	$0,0,0$
B	$1,0,0$	$0,0,0$

$\mathcal{D}$	L	R
T	$2,2,2$	$0,0,0$
B	$0,0,0$	$2,2,2$

$\mathcal{E}$	L	R
T	$0,0,0$	$0,0,0$
B	$0,1,0$	$0,0,3$

in which player 1 chooses one of the two rows (T or B), player 2 chooses one of the two columns (L or R), and player 3 chooses one of the three tables ($\mathcal{C}$, $\mathcal{D}$, or $\mathcal{E}$). For each strategy profile, there is a corresponding cell in one of the three tables, and the three numbers in the cell are the utilities of players 1, 2, and 3, respectively, for that strategy profile.

s=(B,L,C), (T,R,C), (B,L,D), (T,R,E)

Find all the pure Nash equilibria in this game. [6]

(b) Write down the constraints that define the set of correlated equilibria in the game from part (a). [6]

(c) Find a correlated equilibrium in the game from part (a) that maximizes the social welfare. [6]

(d) Consider a two-player zero-sum game in which (a, C) and (b, D) are pure Nash equilibria, where $a \neq b$ and $C \neq D$, and in which each of the two players has 2021 strategies. Prove that there are at least four pure Nash equilibria in the game. [7]

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2. Two lions are hunting for a little antelope and a big antelope. Each is guaranteed to catch whatever prey they chase, but if both go after the same antelope then each lion will only get half of the kill and the other antelope will escape. Assume that the caloric value to the lion of the little antelope is α and that of the big antelope is β, and that $\beta > \alpha > 0$.

   (a) Model this scenario formally as a two-player zero-sum game. [4]

   (b) Describe all possible pure Nash equilibria and for what relative values of α and β they occur.
   Fully describe the number of pure Nash equilibria as a function of α and β. [7]

   (c) Describe for what values of α and β a fully mixed Nash equilibrium exists. [7]

   (d) Can the expected utility in a fully mixed Nash equilibrium exceed the utility α of catching the little antelope in a pure Nash equilibrium? [7]

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3. A mountain village that consists of one long road has decided that, for road safety reasons, each of the two ice cream vans serving the village can only sell ice cream at a fixed location. The village council has identified k possible van locations along the road. All consecutive locations are equidistant. A single ice-cream eating family lives at each of the k locations, and each such family eats two ice creams per day on average.

   Each of the two ice cream vans must choose one of the k locations and they will be given a licence to serve ice cream from this location for the whole summer. It is allowed for the two vans to choose the same location.

   Families at each of the k locations will buy ice cream from the nearest van, or they will split their purchases evenly between the two vans if both are equally distant from the location where they live.

   (a) Model the above situation formally as a two-player game in which the two vans must decide which of the $k = 4$ locations to choose and they aim to maximize the number of ice creams they sell. [6]

   (b) Find all Nash equilibria in the game that you have formulated in part (a). [5]

   (c) Perform IESDS on the game for $k = 5$. At each step of IESDS, indicate which strategy is eliminated and what other strategy it is strictly dominated by. [7]

   (d) Describe the outcome of IESDS on the games for all positive numbers k. Indicate the order in which strategies are eliminated and what strategies they are strictly dominated by. [7]

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4. Every day, 100 cars travel from the suburb S to the city C. There are two lakes, connected by a river, between S and C, and there are two routes from S to C. One route goes on the left banks of the two lakes, first from S to village L, then from L to C; the other route goes on the right banks of the two lakes, first from S to village R, then from R to C.

   The individual cost of travel for each car on each road segment measures a combination of fuel used and time spent on driving. The individual costs of travel for the segments from S to R and from L to C are both equal to 1. The individual cost of travel for the segment from S to L is $(100 + x)/200$, where x is the number of cars that choose the left-banks route, and the individual cost of travel for the segment from R to C is $(100 + y)/200$, where y is the number of cars that choose the right-banks route.

   (a) Draw the network described above and label the edges with the corresponding cost-of-travel functions. [3]

   (b) Calculate all equilibria in the traffic game, that is an assigment of numbers of cars to routes, in which no car has an incentive to deviate to a different route. [4]

   (c) The local council has built a short **one-way** bridge over the river from village L to village R. The individual cost of travel on the bridge is 0. Calculate all equilibria in the modified traffic game. [6]

   (d) Calculate the largest total costs of travel of all cars in equilibria from parts (b) and (c). How has building the bridge affected the largest total cost of travel at equilibrium? [6]

   (e) The local council has been criticised for the traffic outcomes resulting from the bridge building project. Advised by the AGT Consultancy, the local council has introduced the following system of tolls and subsidies: cars driving from S to L pay a toll of $1/8$ and cars driving from S to R receive a subsidy of $1/8$.

       What is now the worst total cost of travel at equilibrium? Calculate and compare the amounts that the local council receives as tolls and pays out in subsidies. [6]

5. (a) Consider a Vickrey's Second Price Auction with three bidders whose valuations are $v_1 = 7$, $v_2 = 4$, and $v_3 = 2$, respectively. Give bids b_1 and b_3 of bidders 1 and 3, respectively, for which the false bid $b_2 = 2$ of bidder 2 yields strictly smaller utility to bidder 2 than the true bid $v_2 = 4$ would have. [4]

   (b) Consider a Vickrey's Second Price Auction with n bidders whose pairwise distinct valuations are $v_1, v_2, \ldots, v_n$, respectively. Let i be any of the n bidders and let b_i be a number such that $b_i \neq v_i$. Prove that there are bids b_j for all bidders j such that $j \neq i$, for which the false bid b_i yields strictly smaller utility to bidder i than the true bid v_i would have. [6]

   (c) Each of three firms 1, 2, and 3 uses water from a lake in their production. Each firm has two strategies: purify their sewage or dump their untreated sewage to the lake. The cost of purification is 1 for each firm that does purify. If at most one of the three firms dumps untreated sewage to the lake then the water in the lake remains pure. If at least two firms dump untreated sewage to the lake then the water in the lake is impure and each firm suffers a loss of 3 (for those firms that do purify, this loss is in addition to their cost of purification).

   Model this scenario formally as a three-player game. [5]

   (d) Describe all the behaviours of the three firms that correspond to pure Nash equilibria in the game from part (c). [4]

   (e) Calculate a fully mixed equilibrium in the game from part (c). [6]

6. Consider the following 2×3 two-player game:

	c	d	e
A	$1, 6$	$8, 2$	$4, 7$
B	$4, 4$	$6, 8$	$3, 0$

in which player 1 chooses one of the two rows (A or B) and player 2 chooses one of the two columns (c, d, or e). For each strategy profile, there is a corresponding cell in the table, and the two numbers in the cell are the utilities of players 1 and 2, respectively, in the strategy profile.

(a) Draw player 1's mixed strategy simplex and subdivide it into the best response regions of player 2. Calculate the coordinates of all the relevant points. [6]

(b) Draw player 2's mixed strategy simplex and subdivide it into the best response regions of player 1. Calculate the coordinates of all the relevant points. [6]

(c) Use the fully-labelled pairs of points criterion to identify all Nash equilibria of the game. [6]

(d) Using the diagrams from parts (a) and (b), illustrate the execution of the Lemke-Howson algorithm in which the first label dropped is B. Which Nash equilibrium does this execution of the algorithm find? [7]