I. Introduction

This document includes two appendixes of the paper called Balancing Hidden-node and Exposed-node Problems in Full-duplex Enabled CSMA Networks. Appendix A provides the proof of Theorem 1, and Appendix B provides the proof of Theorem 2. All the contents of this supplementary document are for reference only. If you have any questions or suggestions after reading, you are welcome to contact us by email.

Appendix A
Proof for Theorem 1

Theorem 1. *The hidden-node mean contention region for a bidirectional FD link is given by*

$$V_{HN}= 2V_{HN1} + 2V_{HN2}, \tag{1}$$

$$V_{HN1} = \frac{2}{\pi}\int_{r_{cs}}^{r_i+d} (\pi-\theta_2-\theta_3)\theta_1 r dr, \tag{2}$$

$$V_{HN2} = \int_{\delta_1}^{d}\int_{\theta_4-\theta_3}^{\pi-\theta_4+\theta_3} (\frac{\varphi_1+\varphi_2+\varphi_3}{2\pi}) r dr d\theta, \tag{3}$$

where θ_1 to θ_4, φ_1 to φ_3 and δ_1 are intermediate parameters, whose detailed expressions are

$$\theta_1 = \arccos(\frac{d^2+r^2-r_i^2}{2dr}), \theta_2 = \arccos(\frac{r^2+r_i^2-d^2}{2rr_i}),$$

$$\theta_3 = \arccos(\frac{d}{2r_i}), \theta_4 = \arccos(\frac{r^2+{r_i}^2-r_{cs}^2}{2rr_i}),$$

$$\varphi_1 = \arccos(\frac{\delta_2^2+d^2-r_i^2}{2\delta_2 d}), \varphi_2 = \arccos(\frac{\delta_2^2+\delta_3^2-d^2}{2\delta_2\delta_3}),$$

$$\varphi_3 = \arccos(\frac{\delta_3^2+d^2-r_i^2}{2\delta_3 d}),$$

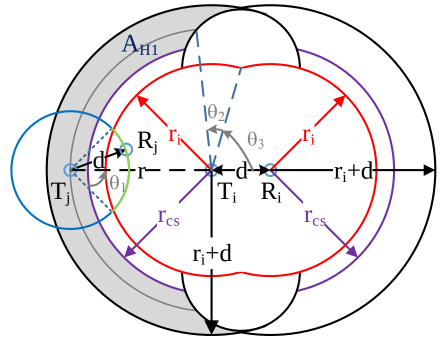
$$\delta_1 = \sqrt{r_{cs}^2-\frac{d^2}{4}} - \sqrt{{r_i}^2-\frac{d^2}{4}},$$

$$\delta_2 = \sqrt{r^2+r_i^2+2rr_i\cos(\theta-\theta_3)},$$

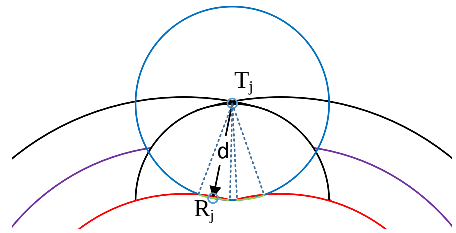
$$\delta_3 = \sqrt{r^2+r_i^2-2rr_i\cos(\theta+\theta_3)}.$$

Proof: We divide the hidden-node (HN) region into four regions, the left-hand side (LHS) HN region, the right-hand side (RHS) HN region, the upper side HN region and the lower side HN region. According to the symmetry principle, we only consider two cases to calculate the hidden-node mean contention region for a bidirectional FD link l_i.

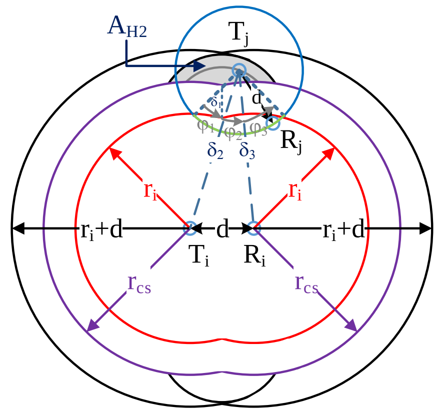
Case 1: Consider the transmitter T_j locates in the LHS HN region denoted by A_{H1} as shown in Fig. 1 (a). If the associated receiver R_j lies within the bicircle interference region of l_i (green arc), the hidden-node problem will occur. Otherwise, l_j is not interfered by l_i. Since R_j is randomly distributed around T_j with a distance d, the probability of R_j lying within the bicircle interference region is the ratio of $2\theta_1$ to 2π. Clearly, the probability $p(X)$ depends on the location of T_j. Thus, we integrate the probability throughout the A_{H1} region to obtain the hidden-node mean contention region (V_{HN1} in Theorem 1). The symmetric HN region at the right-hand side can be calculated in the same way.



(a) The left-hand side HN region in case 1.



(b) The overlaying HN region of left-hand side HN region and right-hand side HN region.



(c) The upper side HN region in case 2.

Fig. 1: Different cases in the HN region.

In Fig. 1, we observe an overlaying HN region where T_j lies both the LHS HN region and the RHS HN region, and the position of R_j (green arc) is discontinuous. Therefore, the overlaying HN region needs special consideration. For the LHS HN region, we only consider R_j is located within T_i's interference region; and similarly the RHS HN region for R_i's interference region.

Case 2: Consider the transmitter T_j lying in the upper HN region denoted by A_{H2} in Fig. 1 (c). The associated receiver R_j may locate within both T_i and R_i's interference region. This is different from case 1 as the potential location of R_j is continuous, which corresponds to V_{HN2}. The symmetric lower side HN region can be calculated in the same way. ∎

Appendix B
Proof for Theorem 2

Theorem 2. *The exposed-node mean reuse region for a bidirectional FD link is given by*

$$V_{EN} = 2V_{EN1} + 2V_{EN2} + 2V_{EN3}, \quad (4)$$

$$V_{EN1} = \frac{2}{\pi}\int_{r_i}^{r_{cs}} (\pi - \theta_2 - \theta_3)\cdot(\pi - \theta_1) r dr, \quad (5)$$

$$V_{EN2} = \int_0^{\delta_1}\int_{\frac{2\theta_6+\theta_5-\pi}{2}}^{\frac{3\pi-2\theta_6-\theta_5}{2}} (1 - \frac{\varphi_1+\varphi_2+\varphi_3}{2\pi}) r dr d\theta, \quad (6)$$

$$V_{EN3} = 2\int_{\delta_1}^{d}\int_{\frac{2\theta_6+\theta_5-\pi}{2}}^{\theta_4-\theta_3} (1 - \frac{\varphi_1+\varphi_2+\varphi_3}{2\pi}) r dr d\theta, \quad (7)$$

where θ_1 to θ_4, φ_1 to φ_3 and δ_1 are provided in Theorem 1; θ_5 and θ_6 are given by

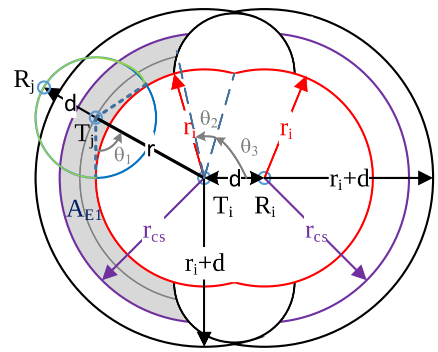
$$\theta_5 = \arccos(\frac{2{r_i}^2 - d^2}{2{r_i}^2}), \theta_6 = \arccos(\frac{r}{2r_i}).$$

Proof: We divide the exposed-node (EN) region into six regions, the LHS EN region, the RHS EN region, the continuous upper side EN region, the continuous lower side EN region, the discontinuous upper side EN region and the discontinuous lower side EN region. Similar to Theorem 1, we only consider three cases to calculate the exposed-node mean reuse region for a bidirectional FD link l_i.

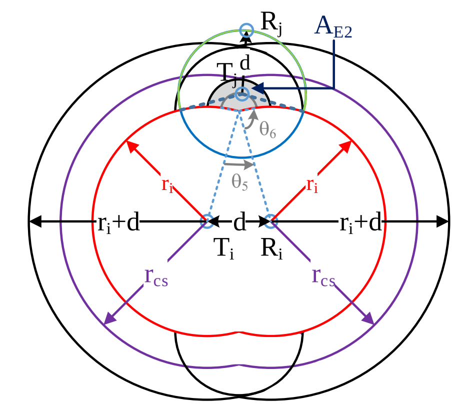
Case 1: Consider the transmitter T_j locates in the LHS EN region denoted by A_{E1} as shown in Fig. 2 (a). If the associated receiver R_j lies outside the bicircle interference region of l_i (green arc), the exposed-node problem will occur. Otherwise, R_j is interfered by l_i. Since R_j is randomly distributed around T_j with a distance d, the probability of R_j lying outside the bicircle interference region of l_i is the ratio of $2\pi - 2\theta_1$ to 2π. Clearly, the probability $p(X)$ depends on the location of T_j. Thus, we integrate the probability throughout the A_{E1} region to obtain the exposed-node mean reuse region (V_{EN1} in Theorem 2). The symmetric EN region at the right-hand side can be calculated in the same way.

Case 2: In Fig. 2 (b), T_j is inside the continuous upper side EN region denoted by A_{E2}, whose center is the upper intersection of the two interference circles and radius is δ_3. Symmetrically, the continuous lower side EN region is the same as V_{EN2}.

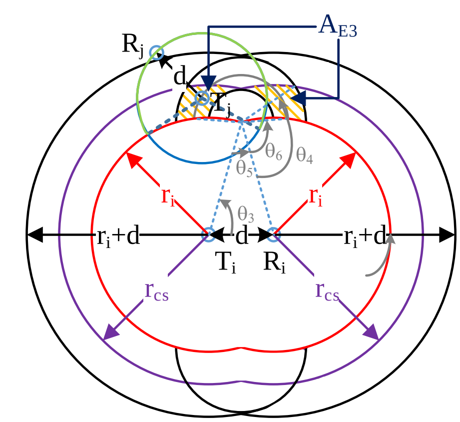
Case 3: In Fig. 2 (c), there is a discontinuous upper side EN region denoted by A_{E3}. In this case, we can only calculate the half part because the other half can be obtained by symmetry. Besides, the corresponding discontinuous lower side EN region is similar to V_{EN3}. ■



(a) The left-hand side EN region in case 1.



(b) The continuous upper side EN region in case 2.



(c) The discontinuous upper side EN region in case 3.

Fig. 2: Different cases in the EN region.