Self Normalized Method for Change Point Detection

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Objectives

Change point detection has wide applications in finance, economy, quality control and other objects. However, all existing methods are not universally applied and can not have higher power and smaller size in the same time. In this summer I work with Prof Chan kin wei to try to find a better method. For simplicity we focus on change in mean case first.

Introduction

The most common class of test statistics is based on the CUSUM process (cumulative sum):

$$T_n(\lfloor nr \rfloor) = n^{-1/2} \sum_{t=1}^{\lfloor nr \rfloor} \left(X_t - \overline{X}_n\right), r \in [0,1]$$

and the KS test

$$\mathrm{KS}_n = \sup_{r\in[0,1]} |T_n(\lfloor nr \rfloor)| / s_n = \sup_{k=1,\dots,n} |T_n(k)| / s_n$$

Based on the **FCLT**(functional Central limit theorem) and continuous theorem, we know T will converge to σ IB in the null case. Therefore, we can simulate Brownian Bridge to find the critical value for the KS test.

Key Challenge

The variance is unknown. Different form the i.i.d case, variance is hard to estimate for correlated time series. The most common way is to use **Kernel method**.

$$s_n^2 = \sum_{k=1-n}^{n-1} K\left(k/l_n\right) \hat{\gamma}_n(k)$$

However, the size for test with kernel method is not accurate, and there is no universal criteria to choose the bandwidth parameter. Self normalizition method can overcome such problems. Prof Shao put forward the self normalized method in 2010, and Zhang & Lavitas improved it to adapt to multiple change point case

What is Self Normalization?

Self-normalization is method with a long history in statistics that can be traced back to the student's test(T test). For T test statistic, the numerator divided by true standard deviation is a standard normal and the denominator divided by standard deviation is a $\chi^2(1)$ distribution, therefore the ratio is a distribution unrelated with variance, which we called t distribution. Below is some self normlizers for KS test (Shao, 2015)

$$W_{1n}^2 = n^{-2} \Sigma_{t=1}^n t^2 \left(\hat{\theta}_{n-t+1,n} - \hat{\theta}_{1,n}\right)^2$$

$$W_{2n}^2 = n^{-3} \Sigma_{i=1}^n \Sigma_{j=i}^n (j-i+1)^2 \left(\hat{\theta}_{i,j} - \hat{\theta}_{1,n}\right)^2$$

$$W_{3n}^2 = \left\{W_n^2 + W_{1n}^2\right\}/2$$

Methods

We have put forward some new normalizer, and some of them works well. For example, my mentor advise the T itself as normalizer: find k that maximize T denote the maximal value as T_k and divide the the time series into two parts(the first k data and the other) and find the maximal value in each part, denote as T_1 and T_2, then define the test statistic as $\frac{T_k}{T_1+T_2}$.

We have also tried localized method. Suppose there are several CP, then the process of testing one CP may be affected by other CP. So we want to test in smaller interval. To be specific,we want to test in each interval [1:l],[1:l+1]...[1:n-l],where l is a trimming parameters. However, the limiting distribution is difficult to find.

Important Result

The self normalized method has accurate size but less power. Our test has even more accurate size and the power is larger than traditional self- normalized method but smaller than kernel method

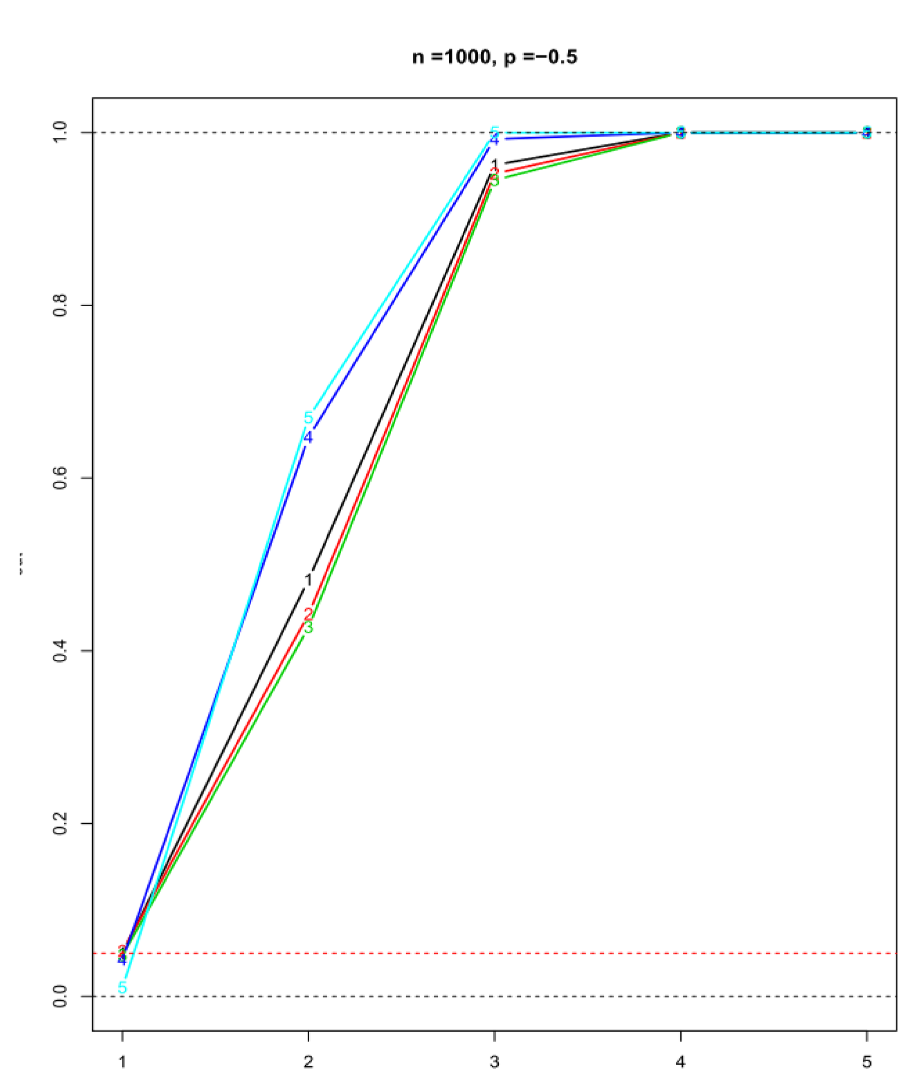

Figure 1: Here the forth and fifth curve represent the power of kernel method, and others are Zhang & Lavitas 's test with different trimming parameters, and the time series is an AR1 model with p=-0.5

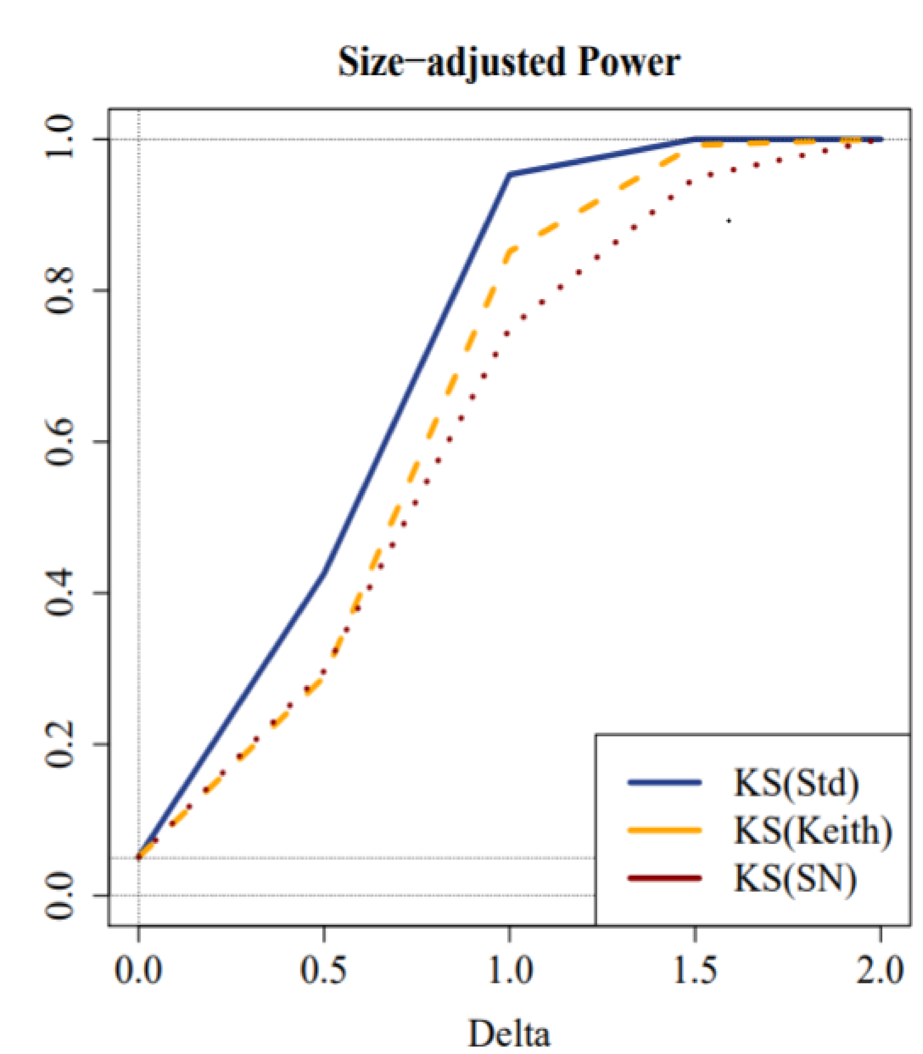

Figure 2: Comparison of power between kernel method, our new test and Zhang & Lavitas 's test

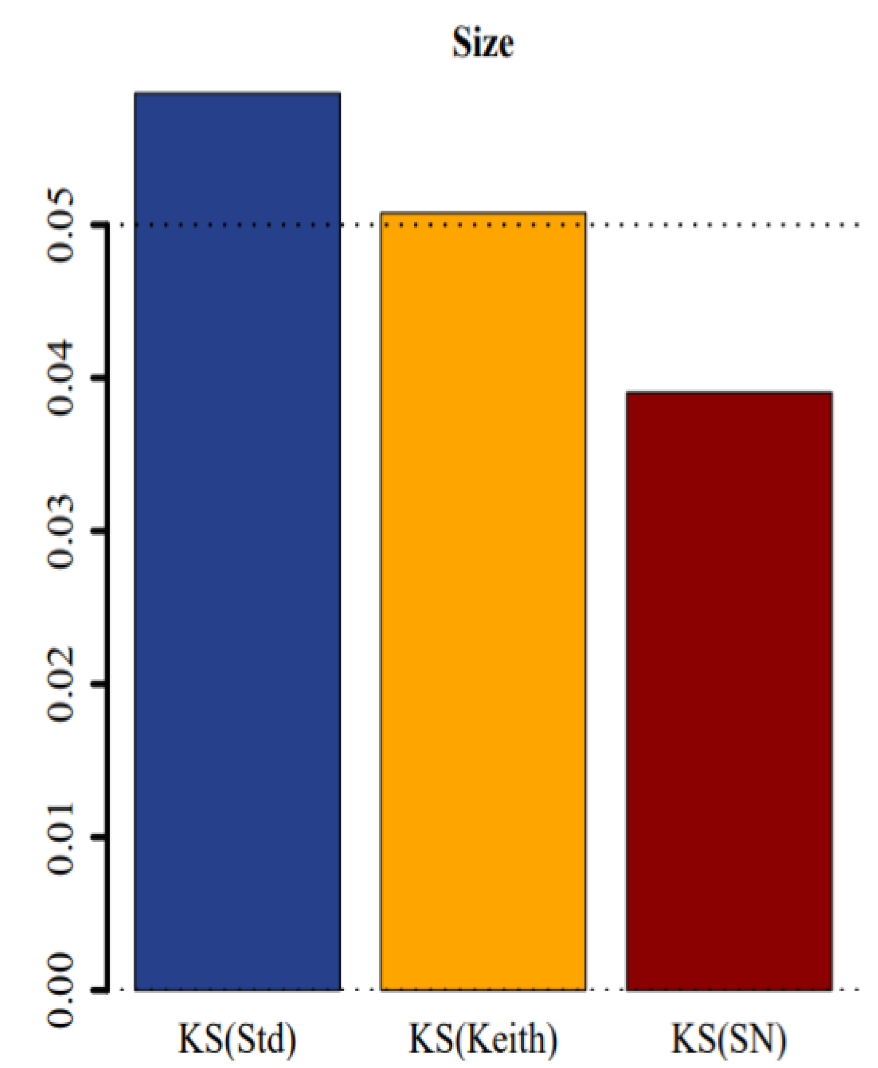

Figure 3: Although our test is not most powerful, it does has most accurate power; in fact, the size is very close to 0.05 in most simulations

Conclusion

Up to now, there is no perfect method and each method has space for improvement.It is like a trade of between power and size.

Our test has surprisingly accurate power and only suffer moderate power loss. It also avoids the non-monotone power problem. Currently my focus is to generalize the new test to handle multiple CP cases and find the form of limiting distribution.

References

Xiaofeng Shao (2015) Self-Normalization for Time Series: A Review of Recent Developments, Journal of the American Statistical Association, 110:512, 1797-1817

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