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Problem Set 4

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Problem Set 4

Chapter 5 #12

(a) Proof

Suppose $\forall x, f(x) \leq g(x)$, and both $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$. Then we know, $\lim_{x\to a} g(x) - \lim_{x\to a} f(x) = \lim_{x\to a}(g(x) - f(x)) = M - L$. Since $(g(x) - f(x)) \geq 0$, then $\lim_{x\to a}(g(x) - f(x))$ is a limit of a positive number or 0. Thus, $\lim_{x\to a} g(x) - \lim_{x\to a} f(x) = \lim_{x\to a}(g(x) - f(x)) \geq 0$. Therefore, $\lim_{x\to a} g(x) \geq \lim_{x\to a} f(x)$.

(b) Answer

If $\exists d > 0$ such that $\forall x, |x - a| < d$, $f(x) \leq g(x)$ and both limit exists at a, then $\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x)$.

(c) Counter Example

$$f(x) = \begin{cases} |x| & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$$g(x) = \begin{cases} -|x| & x \neq 0 \\ -1 & x = 0 \end{cases}$$

Thus, $f(x) < g(x)$ and $\lim_{x\to a} g(x) = \lim_{x\to a} f(x) = 0$.

Chapter 5 #13

Proof:

Let $\lim_{x\to a} f(x) = l$. Thus,$\lim_{x\to a} h(x) = \lim_{x\to a} f(x) = l$. Then, $\forall\epsilon > 0$, $\exists\delta > 0$ such that if $|x - a| < \delta$, then $|f(x) - l| < \epsilon$ and $|h(x) - l| < \epsilon$. Therefore, $l - \epsilon < f(x) < l + \epsilon$ and $l - \epsilon < h(x) < l + \epsilon$. Also, because $f(x) \leq g(x) \leq h(x)$, $l - \epsilon < f(x) \leq g(x) \leq h(x) < l + \epsilon$. Therefore, $l - \epsilon < g(x) < l + \epsilon$. Then, $|g(x) - l| < \epsilon$ when $|x - a| < \delta$. $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = \lim_{x\to a} h(x) = l$.

Chapter 5 #17

(a)Proof:

Suppose $\lim_{x\to 0} \dfrac{1}{x} = l$. Therefore, $\forall\epsilon > 0, \exists\delta > 0$ such that if $|x - 0| < \delta$, then $|\frac{1}{x} - l| < \epsilon$. However, we have if $x = 0$, then $\frac{1}{0}$ does not exist. Then we cannot have $|\frac{1}{x} - l| < \epsilon$, since we cannot make the calculation.

(b)Proof:

Similar to the above proof, we cannot have $\frac{1}{1-1}$ although $|x-1|<\delta$. Therefore, this limit does not exist.

Chapter 5 #19

Proof:

Suppose $\lim\limits_{x \to a} f(x) = l$. Then, $\forall \epsilon > 0, \exists \delta > 0$ such that if $|x-a|<\delta$, then $|f(x)-l|<\epsilon$. However, since $f(x)$ is not certain within the range $|x-a|<\delta$, then the range for $|f(x)-l|$ is not set. Thus, the limit does not exist.