Chapter 1: Real Number Axioms

My problems. Here are some problems that I wrote. I tried to put them all in logical order, to develop the things we need systematically. Some of these were written on the board last class. (Note that I changed the order of #5 and #6 to put them in correct logical order.) Some of these problems are solved in Chapter 1; if you are stuck, you can refer back there. You can skip #1 and #2, but can refer to them.

I would like you to write up all of them carefully, ideally. But I would like to single out the following to write up particularly carefully, to talk about in feedback meetings: #6, 7, 9, 10, 13, 14, 16, 22, 24, 26.

All variables are real numbers. The problem in every case is to prove the statement.

1. $\forall a, b, c$: if $a = b$, then $a + c = b + c$.
2. $\forall a, b, c$: if $a = b$, then $a \cdot c = b \cdot c$.
3. $\forall a, b, c$: if $a + c = b + c$, then $a = b$.
4. $\forall a, b, c$: if $a \cdot c = b \cdot c$ and $c \neq 0$, then $a = b$.
5. $\forall a : a \cdot 0 = 0$.
6. $\forall a, b$: if $a \cdot b = 0$, then either $a = 0$ or $b = 0$.[1]
7. $\forall a, b : (a + b)^2 = a^2 + 2ab + b^2$. (How is the exponent 2 defined? How is the number 2 in $2ab$ defined?)
8. $\forall a, b, c, d : (a + b)(c + d) = ac + ad + bc + bd$.
9. $\forall a : (-1) \cdot a = -a$.
10. $\forall a, b : (-a) \cdot (-b) = a \cdot b$.
11. $\forall a, b : -(a + b) = (-a) + (-b)$.
12. $\forall a, b, c$: if $a < b$, then $a + c < b + c$. (Be sure to use the definition of $<$ from the text!)
13. $\forall a, b, c$: if $a + c < b + c$, then $a < b$.
14. $\forall a, b$: if $a < 0$ and $b < 0$, then $a \cdot b > 0$.
15. $\forall a, b, c$: if $a < 0$ and $b > 0$, then $a \cdot b < 0$.
16. $\forall a, b$: if $a \cdot b > 0$, then either $a > 0$ and $b > 0$, or $a < 0$ and $b < 0$.
17. $\forall a, b, c$: if $a \cdot b < 0$, then either $a > 0$ and $b < 0$, or $a < 0$ and $b > 0$.
18. $\forall a : a \cdot a > 0$.
19. $1 > 0$.
20. $\forall a$: if $a > 0$, then $a^{-1} > 0$.
21. $\forall a$: if $a < 0$, then $a^{-1} < 0$.
22. $\forall a, b, c$: if $a < b$, and $c > 0$, then $a \cdot c < b \cdot c$.
23. $\forall a, b, c$: if $a \cdot c < b \cdot c$, and $c > 0$, then $a < b$.
24. $\forall a, b, c$: if $a < b$, and $c < 0$, then $a \cdot c < b \cdot c$.
25. $\forall a, b, c$: if $a \cdot c < b \cdot c$, and $c < 0$, then $a < b$.
26. $\forall a, b : |a + b| \leq |a| + |b|$.

Textbook problems. From the textbook, I would like you to do the following:

- Chapter 1, problems #1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 14.

Note that there is some overlap between the list of book problems and the list of my problems above; you can just refer to previous solutions when appropriate. Ideally I would like you to write these all up fairly carefully, but I would like you to pay particular attention to 3(ii), 3(iii), 3(v), 4(iv), 4(xiii), 7, 11 (all parts), 12 (all parts).

[1] Note that in mathematics, the statement "P or Q is true" always includes the possibility that P and Q are *both* true.