Yufei Lin
Problem Set 4
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Problem Set 4

Chapter 5 #12

(a) Proof

Suppose $\forall x, f(x) \leq g(x)$, and both $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$. Then we know, $\lim_{x\to a} g(x) - \lim_{x\to a} f(x) = \lim_{x\to a}(g(x) - f(x)) = M - L$. Since $(g(x) - f(x)) \geq 0$, then $\lim_{x\to a}(g(x) - f(x))$ is a limit of a positive number or 0. Thus, $\lim_{x\to a} g(x) - \lim_{x\to a} f(x) = \lim_{x\to a}(g(x) - f(x)) \geq 0$. Therefore, $\lim_{x\to a} g(x) \geq \lim_{x\to a} f(x)$.

(b) Answer

If $\exists d > 0$ such that $\forall x, |x - a| < d$, $f(x) \leq g(x)$ and both limit exists at a, then $\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x)$.

(c) Counter Example

$$f(x) = \begin{cases} |x| & x \neq 0 \\ 1 & x = 0 \end{cases}$$

$$g(x) = \begin{cases} -|x| & x \neq 0 \\ -1 & x = 0 \end{cases}$$

Thus, $f(x) < g(x)$ and $\lim_{x\to a} g(x) = \lim_{x\to a} f(x) = 0$.

Chapter 5 #13

Chapter 5 #17

Chapter 5 #19