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Problem Set 6

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Question 1

Prove that if $f(x)$ is differentiable at a , then it is continuous at a .

Proof:

Suppose $f(x)$ is differentiable at a . Then, $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = L$ and $f(a)$ exist. Since h is constant $\lim_{h \rightarrow 0} h$ exist. Then,

$$\begin{aligned} \lim_{h \rightarrow 0} h \cdot \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} &= \lim_{h \rightarrow 0} h \cdot L \\ &= 0 \cdot L \\ &= 0 \\ &= \lim_{h \rightarrow 0} (f(a+h) - f(a)) \end{aligned}$$

Then we know $\lim_{h \rightarrow 0} f(a+h) = \lim_{h \rightarrow 0} f(a)$. Thus, $\lim_{x \rightarrow a} f(x) = f(a)$.

Question 2

Prove if $\forall x, f(x) = c, c \in \mathbb{R}$, then $\forall a : f'(a) = 0$.

Proof:

Suppose a is any number, then $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$.

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} &= \lim_{h \rightarrow 0} \frac{c - c}{h} \\ &= \lim_{h \rightarrow 0} 0 \\ &= 0 \end{aligned}$$

Therefore, $f'(a) = 0$.

Question 3

Prove if $f(x) = x$, then $\forall a : f'(a) = 1$.

Question 4

Prove that if $f(x)$ and $g(x)$ are differentiable at a , then $f(x) + g(x)$ is differentiable at a and $(f+g)'(a) = f'(a) + g'(a)$.

Question 5

Prove that if $f(x)$ and $g(x)$ are differentiable at a , then $f(x) \cdot g(x)$ is differentiable at a and $(f \cdot g)'(a) = f'(a) \cdot g(a) + f(a) \cdot g'(a)$.