CMPUT 414: Winter 2016

Lab2, due date Jan 23, 2016, 23:59 PM

1. **3D Transformations**

a) What is the homogenous transformation matrix for:

i) Translation of (2, -3, 2)

$$M_1 = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii) Followed by Scaling with $(2\sqrt{2}, 2\sqrt{2}, 2)$

$$M_2 = \begin{bmatrix} 2\sqrt{2} & 0 & 0 & 0 \\ 0 & 2\sqrt{2} & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2\sqrt{2} & 0 & 0 & 4\sqrt{2} \\ 0 & 2\sqrt{2} & 0 & -6\sqrt{2} \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii) Followed by Rotation of 45 degrees around Z-axis

$$M_3 = \begin{bmatrix} cos(\frac{\pi}{4}) & sin(\frac{\pi}{4}) & 0 & 0 \\ -sin(\frac{\pi}{4}) & cos(\frac{\pi}{4}) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2\sqrt{2} & 0 & 0 & 4\sqrt{2} \\ 0 & 2\sqrt{2} & 0 & -6\sqrt{2} \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 & -2 \\ -2 & 2 & 0 & -10 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

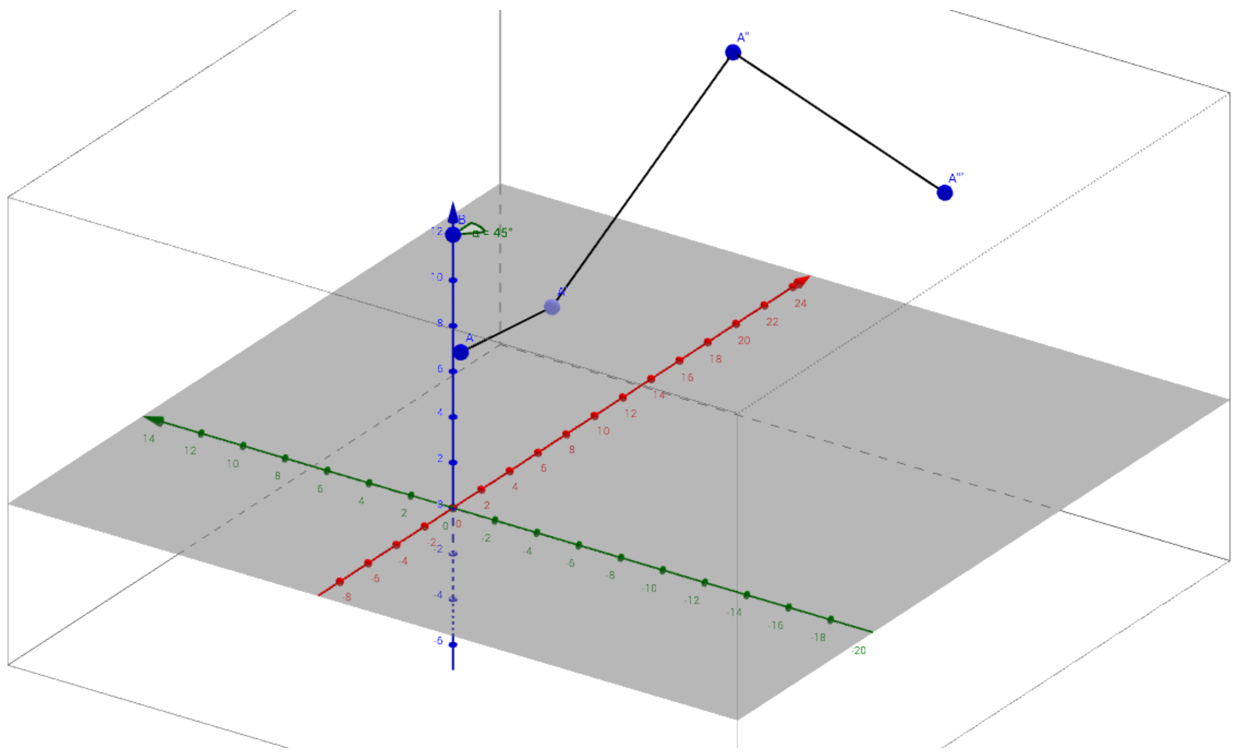
b) Apply the above final transformation matrix to a 3D point at (5, 3, 4).

$$M_{Final} = \begin{bmatrix} 2 & 2 & 0 & -2 \\ -2 & 2 & 0 & -10 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ -14 \\ 12 \\ 1 \end{bmatrix}$$

c) Illustrate the transformations applied in (b). You can either draw them manually, or build them by using a geometry software package (e.g. Geogebra http://app.geogebra.org/3d/), and copying its output. Make sure to illustrate rotation angles.

I did translation first, and then scaling, then rotation.
The answers are as follows (starting from (5, 3, 4)):

$$\begin{bmatrix}5\\3\\4\end{bmatrix} \rightarrow \begin{bmatrix}7\\0\\6\end{bmatrix} \rightarrow \begin{bmatrix}14\sqrt{2}\\0\\12\end{bmatrix} \rightarrow \begin{bmatrix}14\\-14\\12\end{bmatrix}$$



d) If after the same transformations as in (a), we get the resulting 3D point (6, 1, 4), what is the original 3D point?

$$\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix} = \begin{bmatrix}2 & 2 & 0 & -2\\-2 & 2 & 0 & -10\\0 & 0 & 2 & 4\\0 & 0 & 0 & 1\end{bmatrix}^{-1} * \begin{bmatrix}6\\1\\4\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{4} & \frac{-1}{4} & 0 & -2\\\frac{1}{4} & \frac{1}{4} & 0 & 3\\0 & 0 & \frac{1}{2} & -2\\0 & 0 & 0 & 1\end{bmatrix}^{-1} * \begin{bmatrix}6\\1\\4\\1\end{bmatrix} = \begin{bmatrix}\frac{-3}{4}\\\frac{19}{4}\\0\\1\end{bmatrix}$$

So, the initial 3D point is $(\frac{-3}{4}, \frac{19}{4}, 0)$

e) Rotate the vector from the origin to $(-\sqrt{2}, 1/\sqrt{2}, 2)$ by an angle 60 around direction (-1, 1, 1).

Normalized direction: $[-1, 1, 1] \rightarrow [\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}]$

$$Arbitray\ Rotation = \begin{bmatrix} \frac{2}{3} & \frac{-2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\ \frac{-2}{3} & \frac{-1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{A}_{Final} = \begin{bmatrix} \frac{2}{3} & \frac{-2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & 0 \\ \frac{-2}{3} & \frac{-1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\sqrt{2} \\ \frac{1}{\sqrt{2}} \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2-3\sqrt{2}}{3} \\ \frac{4}{3} \\ \frac{8+3\sqrt{2}}{6} \\ 1 \end{bmatrix}$$

Therefore, the final vector is $(\frac{2-3\sqrt{2}}{3}, \frac{4}{3}, \frac{8+3\sqrt{2}}{6})$