Homework4

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Problem1 and 2

1. $F(x) = x^3 = 0 \qquad x^{(0)} = 1 \Rightarrow F[x^{(0)}] = 1 \quad F'[x^{(0)}] = 3.$

$F'(x) = 3x^2$

$F[x^{(k+1)}] \approx F[x^{(k)}] + F'[x^{(k)}] \cdot [x^{(k+1)} - x^{(k)}] = 0$

iteration 1: $1 + 3[x^{(1)} - 1] = 0 \Rightarrow x^{(1)} = \frac{2}{3}$

$\Rightarrow F[x^{(1)}] = \frac{8}{27} \quad F'[x^{(1)}] = \frac{4}{3}$

iteration 2: $\frac{8}{27} + \frac{4}{3}[x^{(2)} - \frac{2}{3}] = 0 \Rightarrow x^{(2)} = \frac{4}{9}.$

$\Rightarrow F[x^{(2)}] = \frac{64}{729} \quad F'[x^{(2)}] = \frac{16}{27}$

iteration 3: $\frac{64}{729} + \frac{16}{27}[x^{(3)} - \frac{4}{9}] = 0 \Rightarrow x^{(3)} = \frac{8}{27}$

2. $F(x) = e^x - 1 \qquad x^{(0)} = 1 \Rightarrow F[x^{(0)}] = e - 1, \quad F'[x^{(0)}] = e$

$F'(x) = e^x$

$F[x^{(k+1)}] \approx F[x^{(k)}] + F'[x^{(k)}] \cdot [x^{(k+1)} - x^{(k)}] = 0$

iteration 1: $e - 1 + e[x^{(1)} - 1] = 0 \Rightarrow x^{(1)} \approx 0.37$

$\Rightarrow F[x^{(1)}] \approx 0.45 \quad F'[x^{(1)}] \approx 1.45$

iteration 2: $0.45 + 1.45[x^{(2)} - 0.37] = 0 \Rightarrow x^{(2)} \approx 0.06$

$\Rightarrow F[x^{(2)}] \approx 0.06. \quad F'[x^{(2)}] \approx 1.06$

iteration 3: $0.06 + 1.06[x^{(3)} - 1] = 0 \Rightarrow x^{(3)} = 0.003.$

3. $F(x) = x^4 \qquad x^{(0)} = 1 \Rightarrow F'(x) = 4x^3 \quad F''(x) = 12x^2$

$x^{(k+1)} = x^{(k)} - \left.\frac{d^2f}{dx^2}\right|_{x^{(k)}}^{-1} \cdot \left.\frac{df}{dx}\right|_{x^{(k)}}$

$\Rightarrow x^{(k+1)} = x^{(k)} - \frac{4x^3}{12x^2} = x^{(k)} - \frac{1}{3}x^{(k)}$

iteration 1: $x^{(1)} = 1 - \frac{1}{3} = \frac{2}{3}$

iteration 2: $x^{(2)} = \frac{2}{3} - \frac{2}{9} = \frac{4}{9}$

iteration 3: $x^{(3)} = \frac{4}{9} - \frac{4}{27} = \frac{8}{27}$

Problem 3

4. $\min_{x,y} f(X,Y)$

S.T. $A[X,Y]=B$

$$\Rightarrow \begin{bmatrix} \nabla^2 f[x^{(k)}, y^{(k)}] & A^T \\ A & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta[X,Y] \\ V \end{bmatrix} = \begin{bmatrix} -\nabla f[X^{(k)}, Y^{(k)}] \\ B - A[X^{(k)}, Y^{(k)}] \end{bmatrix}$$

$f(X,Y) = X^4 + y^4 \quad A = [1, 1] \quad B = 1$

$$\nabla^2 f(X^{(k)}, Y^{(k)}) = \begin{bmatrix} 12 X^{(k)2} & 0 \\ 0 & 12 y^{(k)2} \end{bmatrix} \quad \nabla f(x,y) = \begin{bmatrix} 4 X^{(k)3} \\ 4 y^{(k)3} \end{bmatrix}$$

iteration 1: $\begin{bmatrix} 12 & 0 & 1 \\ 0 & 12 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ V \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \\ -1 \end{bmatrix}$

$\Rightarrow \Delta X = -\frac{1}{2}, \Delta Y = -\frac{1}{2}, V = 0 \Rightarrow X^{(1)} = \frac{1}{2}, y^{(1)} = \frac{1}{2}$

iteration 2: $\begin{bmatrix} 3 & 0 & 1 \\ 0 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta X \\ \Delta Y \\ V \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{bmatrix}$

$\Rightarrow \Delta X = 0, \Delta Y = 0, V = -\frac{1}{2} \Rightarrow X^{(2)} = \frac{1}{2}, y^{(2)} = \frac{1}{2}.$

iteration 3: $\begin{bmatrix} 3 & 0 & 1 \\ 0 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ V \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{bmatrix}$

$\Rightarrow \Delta X = 0, \Delta Y = 0, V = -\frac{1}{2}, \Rightarrow X^{(3)} = \frac{1}{2}, y^{(3)} = \frac{1}{2}$