CS282K: Numerical Methods for Scientific Computing and Machine Learning

Homework 3

Issued: **September 11**
Due: **September 22 (11:59PM, Beijing time)**

Please submit the PDF file of your solution to the "Drop Box" on Sakai.

Problem 1: Convex Function, Convex Set and Convex Optimization

Are the following functions convex? Why?

$$f(x) = e^x \tag{1}$$

$$f(x_1, x_2) = x_1 x_2 \tag{2}$$

Are the following sets convex? Why?

$$\{x \mid \log(x) \le 0\} \tag{3}$$

$$\left\{(x_1, x_2) \mid \sqrt{x_1^2 + x_2^2} + |x_1| + |x_2| \le 1\right\} \tag{4}$$

Are the following optimization problems convex? Why?

$$\begin{aligned} \max_x \quad & \log(x) \\ \text{S.T.} \quad & x \le 2 \end{aligned} \tag{5}$$

$$\begin{aligned} \max_x \quad & x \\ \text{S.T.} \quad & |x| \ge 1 \end{aligned} \tag{6}$$

Problem 2: QR decomposition

Find the QR decomposition for the following matrix by hand calculation. Show all steps in detail.

$$S = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \tag{7}$$

Problem 3: Linear Regression with Regularization

Build a 4th-order polynomial model for a one-dimensional function $f(x) = \exp(x)$ where $x \in [0, 1]$. The 4th-order polynomial model is in the form of:

$$f(x) = \exp(x) \approx \alpha_4 x^4 + \alpha_3 x^3 + \alpha_2 x^2 + \alpha_1 x + \alpha_0 . \quad (8)$$

The following table lists five sampling points (including measurement error) that you should use to fit the polynomial model.

x	0.00	0.25	0.50	0.75	1.00
$f(x)$	1.53	3.11	–0.61	2.97	3.03

When solving the model coefficients, add a penalty term into the cost function by using L_2-norm regularization. In this case, the cost function should be in the form of $||A \cdot \alpha - B||_2^2 + \lambda \cdot ||\alpha||_2^2$. Write your MATLAB code to build the over-determined linear equation that takes the regularization term into account. Solve the model coefficients by using the backslash "\" in MATALB. Try the following three different weight values: $\lambda = 10^{-6}$, $\lambda = 1$ and $\lambda = 10^6$. You should get three sets of model coefficients corresponding to these three different weight values. Plot the function $\exp(x)$ and the fitted models, and print out the model coefficients for all three cases. Submit them along with your MATLAB code.