Homework3

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Problem1 and 2

1. Yes.
$f''(x) = e^x > 0$.

2. No.
$f(x,y) = xy$. $P_1(x_1, y_1)$ $P_2(x_2, y_2)$.
$[\alpha x_1 + (1-\alpha)x_2][\alpha y_1 + (1-\alpha)y_2] - \alpha x_1 y_1 - (1-\alpha)x_2 y_2$
assume $\alpha = \frac{1}{2}$
$\Rightarrow \frac{1}{4}(x_2 y_1 + x_1 y_2 - x_1 y_1 - x_2 y_2) = \frac{1}{4}(x_1 - x_2)(y_2 - y_1)$.
The result could be positive.

3. Yes.
$\log(x) \le 0 \Rightarrow 0 < x \le 1$

4. Yes.
$P_1(x_1, y_1) \in D$. $P_2(x_2, y_2) \in D$. $\alpha < 1$.
$$\sqrt{(\alpha x_1 + (1-\alpha)x_2)^2 + (\alpha y_1 + (1-\alpha)y_2)^2} + |\alpha x_1 + (1-\alpha)x_2| + |\alpha y_1 + (1-\alpha)y_2| \le$$
$$\sqrt{\alpha^2(x_1^2 + y_1^2) + (1-\alpha)^2(x_2^2 + y_2^2) + 2\alpha(1-\alpha)(x_1 x_2 + y_1 y_2)} + \alpha(|x_1| + |y_1|) + (1-\alpha)(|x_2| + |y_2|) \le$$
$$\sqrt{\alpha^2(x_1^2 + y_1^2) + (1-\alpha)^2(x_2^2 + y_2^2) + 2\alpha(1-\alpha)\sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}} + \alpha(|x_1| + |y_1|) + (1-\alpha)(|x_2| + |y_2|) =$$
$$\alpha\sqrt{x_1^2 + y_1^2} + \alpha(|x_1| + |y_1|) + (1-\alpha)\sqrt{x_2^2 + y_2^2} + (1-\alpha)(|x_2| + |y_2|) \le \alpha + (1-\alpha) = 1.$$

5. No.
$(\log(x))'' = -\frac{1}{x^2} < 0$.

6. No
$|x| > 1$ is not a convex set.

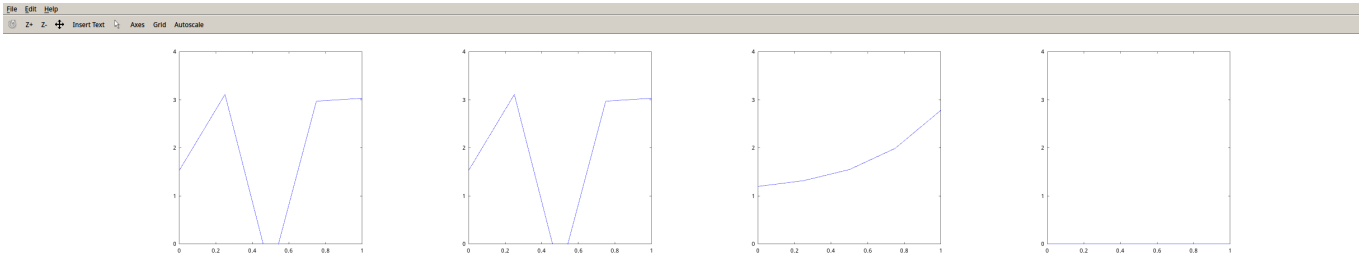
7. $$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} & & \end{bmatrix}\begin{bmatrix} 2 & & \end{bmatrix} = \begin{bmatrix} 1 & \\ 0 & \\ 0 & \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem3

source code

```
lambda1 = 1e-6;
lambda2 = 1;
lambda3 = 1e6;

x = [0, 0.25, 0.5, 0.75, 1];
y = [1.53, 3.11, -0.61, 2.97, 3.03];

A = [x'.^4, x'.^3, x'.^2, x', ones(5,1)];
reg1 = lambda1*diag(ones(5,1));
reg2 = lambda2*diag(ones(5,1));
reg3 = lambda3*diag(ones(5,1));


A_reg1 = [A;reg1];
A_reg2 = [A;reg2];
A_reg3 = [A;reg3];
B = [y';zeros(5,1)];

alpha1=A_reg1\B;
alpha2=A_reg2\B;
alpha3=A_reg3\B;
subplot(1,4,1);
plot(x, y); axis([0,1,0,4]);
subplot(1,4,2);
plot(x, A*alpha1); axis([0,1,0,4]);
subplot(1,4,3);
plot(x, A*alpha2); axis([0,1,0,4]);
subplot(1,4,4);
plot(x, A*alpha3); axis([0,1,0,4]);
```

Plot



coefficients1 =

-249.8133 509.1200 -314.9466 57.1400 1.5300

coefficients2 =

0.43225 0.42187 0.37412 0.35462 1.19751

coefficients3 =

3.9437e-12 4.2553e-12 4.7425e-12 5.7300e-12 1.0030e-11