CS282K: Numerical Methods for Scientific Computing and Machine Learning

Homework 1

Issued: **August 28**
Due: **September 8 (11:59PM, Beijing time)**

Please submit the PDF file of your solution to the "Drop Box" on Sakai.

Problem 1: Ordinary Differential Equation (ODE)

Use backward Euler to solve the following ordinary differential equation (ODE):

$$\begin{aligned} \dot{x} &= -x \\ x(t=0) &= 1 \end{aligned} . \tag{1}$$

Assume that the step size is Δt for numerical integration. Derive the closed-form expression for the solution $x(t_n)$, i.e., the response at $t = t_n$. Set $\Delta t = 0.01$ and plot the response $x(t)$ using MATLAB. Print out your MATLAB code and plot and submit them as part of your solution.

Hint: We already show the closed-form expression for $x(t_n)$ in Lecture. In this homework, we expect you to show the mathematical equations to derive that answer and then plot the response.

Problem 2: Partial Differential Equation (PDE)

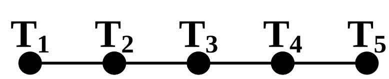

Fig. 1 A 1-D rod discretized into 4 segments

Consider the 1-D rod that is discretized into 4 segments, as shown in Fig. 1. We are interested in the temperature at different locations of the rod, i.e., T_1, T_2, T_3, T_4, and T_5. The 1-D thermal equation and the boundary condition are:

$$\begin{aligned} \rho \cdot C_p \cdot \frac{\partial T(x,t)}{\partial t} &= \kappa \cdot \frac{\partial^2 T(x,t)}{\partial x^2} \\ T_1 &= 30 \\ T_5 &= 100 \end{aligned} \tag{2}$$

where we assume no heat source in the system. Given the above setup, derive a set of linear equations for steady-state thermal analysis. Once you derive the linear equations, use the backslash "\" in

MATLAB to solve these equations and find the unknown temperatures T_2, T_3 and T_4. Report the values of T_2, T_3 and T_4.