

Homework2

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Problem1

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 15 \\ 25 \end{bmatrix}$$

① Gaussian Elimination:

$r_2 - 4r_1$ and $r_3 - 7r_1$:

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -3 & -6 & -9 \\ 0 & -6 & -11 & -17 \end{array} \right]$$

$r_3 - 2r_2$:

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -3 & -6 & -9 \\ 0 & 0 & 1 & 1 \end{array} \right] \Rightarrow \begin{cases} x_3 = 1 \\ x_2 = 1 \\ x_1 = 1 \end{cases}$$

② LU Factorization:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix} = \begin{bmatrix} L_{11} & & \\ L_{21} & L_{22} & \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \cdot \begin{bmatrix} 1 & U_{12} & U_{13} \\ & 1 & U_{23} \\ & & 1 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{c|cc} 1 & & \\ 4 & L_{22} & \\ 7 & L_{32} & L_{33} \end{array} \right] \cdot \begin{bmatrix} 1 & U_{12} & U_{13} \\ & 1 & U_{23} \\ & & 1 \end{bmatrix} \Rightarrow \left[\begin{array}{c|cc} 1 & & \\ 4 & L_{22} & \\ 7 & L_{32} & L_{33} \end{array} \right] \cdot \left[\begin{array}{c|cc} 1 & 2 & 3 \\ & 1 & U_{23} \\ & & 1 \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{c|cc} 1 & & \\ 4 & -3 & \\ 7 & -6 & L_{33} \end{array} \right] \cdot \begin{bmatrix} 1 & 2 & 3 \\ & 1 & U_{23} \\ & & 1 \end{bmatrix} \Rightarrow \left[\begin{array}{c|cc} 1 & & \\ 4 & -3 & \\ 7 & -6 & L_{33} \end{array} \right] \cdot \begin{bmatrix} 1 & 2 & 3 \\ & 1 & 2 \\ & & 1 \end{bmatrix}$$

$$\Rightarrow \left[\begin{array}{c|cc} 1 & & \\ 4 & -3 & \end{array} \right] \cdot \begin{bmatrix} 1 & 2 & 3 \\ & 1 & 2 \end{bmatrix} \Rightarrow \begin{matrix} A \cdot X = B \\ \Rightarrow G \end{matrix} \quad (L \cdot U = B)$$

$$L \cdot V = B \quad \text{where } L = \begin{bmatrix} 1 & 7 & -6 & 1 \\ 4 & -3 & & \\ 7 & -6 & 1 & \end{bmatrix}, V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}, B = \begin{bmatrix} 6 \\ 15 \\ 25 \end{bmatrix}$$

$$\Rightarrow \begin{cases} V_1 = 6 \\ V_2 = 3 \\ V_3 = 1 \end{cases}$$

$$U \cdot X = V \quad \text{where } U = \begin{bmatrix} 1 & 2 & 3 \\ & 1 & 2 \\ & & 1 \end{bmatrix}, X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}, V = \begin{bmatrix} 6 \\ 3 \\ 1 \end{bmatrix}$$

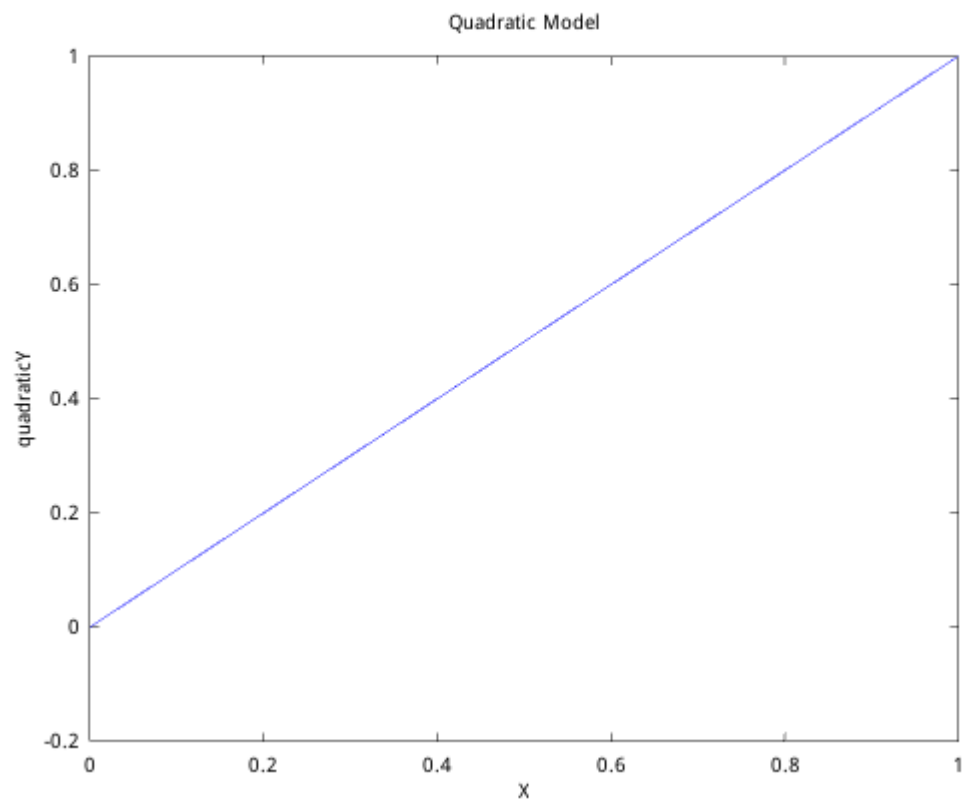
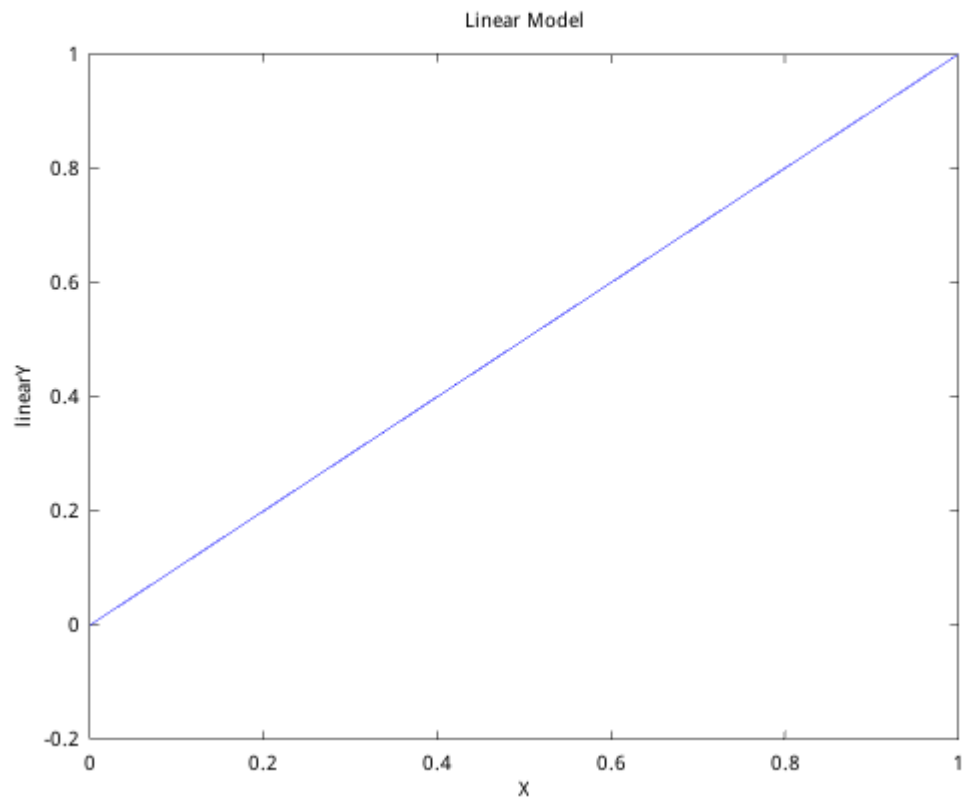
$$\Rightarrow \begin{cases} X_3 = 1 \\ X_2 = 1 \\ X_1 = 1 \end{cases}$$

Problem2

Source Code

```
x = [0 : 0.25 : 1]';
y = [0 : 0.25 : 1]';
%rewrite the x matrix
x1 = [x, ones(5,1)];
x2 = [x.^2, x, ones(5,1)];
%solve the coefficients
c1 = x1 \ y;
c2 = x2 \ y;
%generate predicted y
y1 = x1 * c1;
y2 = x2 * c2;
%plot figure of linear model
figure(1);
plot(x, y1), xlabel('X'), ylabel('linearY'), title('Linear Model');
%plot figure of quadratic model
figure(2);
plot(x, y2), xlabel('X'), ylabel('quadraticY'), title('Quadratic Model');
```

Plot



Coefficient

```
>> c1
```

```
c1 =
```

```
1.0000e+00
```

```
-2.2044e-16
```

```
>> c2
```

```
c2 =
```

```
-1.7659e-15
```

```
1.0000e+00
```

```
-1.1102e-16
```