Research Question: How does the cross-sectional area of a vibrating constant mass attached to a spring in a spring-mass system affect the time period and rate of damping?

Introduction: A spring[1] is a mechanical component that, when stretched or compressed, creates a restoring force. The restoring force, which is normally proportionate to displacement, acts to return the spring to its equilibrium position. Usually when a force is attached to a spring so that its length increases it causes a tension force to be created and this force springs it back to its original length. Thus, springs are also used majorly for practical's and experiments in school. The field that incentivized me to investigate was the variation of the period and the damping of springs. We often learn that expensive cars usually have a more efficient feature of comfort and driving experience because they have high-quality shock absorbers. Shock absorber's main usage is to reduce the vibrations of springs to reduce the damping and offer a smoother ride. Thus, investigating deeper into this led me to consider the topic of damping and over a period of time as I did further research on this phenomenon I came across on the study of as to how the surface area of the card attached to a constant mass in a spring-mass system affects it's rate of damping. Moreover, on learning the hypothesis of how the surface area of a parachute affects the rate of declination of a parachutist. I decided to investigate the effect of the air resistance on the rate of damping of the spring, thus I shortlisted that I wanted to verify this study myself by using methods at school.

Background Information: A spring[1] is a mechanical component when stretched or compressed, creates a restoring force which is normally proportional to displacement and acts to return the spring to its equilibrium position. Damped harmonic oscillators are vibrating systems for which the amplitude of vibration decreases (decays) over time. Almost all physical systems air resistance, friction, and intermolecular forces where energy in the system is lost to heat or sound. Thus, accounting for damping is very important in oscillatory systems. When an objects acceleration is proportional to its displacement from some equilibrium position and it is always oppositely directed and directed to a particular fixed point. In this case we say the object is moving with simple harmonic oscillation. When a mass is made to oscillate in a spring-mass system, the amplitude remains constant; but, due to opposing forces, damping influences the simple harmonic system. Damping is defined as the decay in the amplitude of vibrations due to dissipative forces. Additionally, mechanical energy reduces over time, and hence the motion is said to be damped. Three types of damping is introduced in physics: Critically Damped, Heavy Damped, and Underdamped[2]. In a critically damped simple harmonic system, the mass block returns to equilibrium rapidly and without oscillations. Heavy damping occurs when the resistive forces acting are extremely large. In a underdamped system the oscillatory behaviour of the spring mass system changes very slightly wherein the amplitude of the oscillation decreases overtime due to the effects of damping. Thus in case of a spring mass system standing vertically, the resistive force: air resistance causes the amplitude of the oscillations to decrease leading to damping. In this experiment, I will be particularly focusing on underdamped simple harmonic systems. Figure 1 below resonates a graphical comparison between a system experiencing no damping and another system experiencing damping that has a decaying amplitude.

Hypothesis: The rate of damping and time period increases as the surface area of the card attached to the constant mass in the spring mass system increases due to a greater resistive force of air resistance acting on it slowing down the motion.

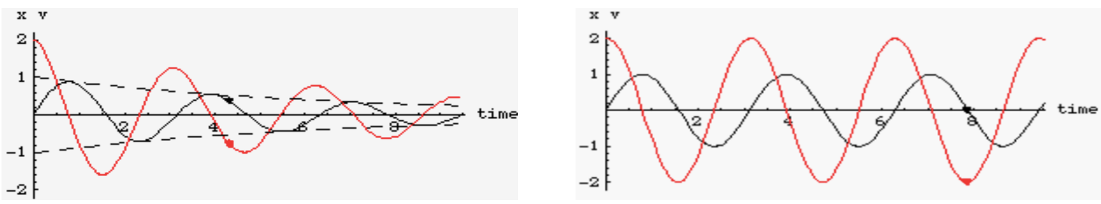

Figure 1 : Differentiating Undamped and Damped System Simple Harmonic Motion

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[1] "Mechanical Engineering Components: Springs." Mechanical Engineering Components: Springs, lo.unisa.edu.au/mod/book/view.php?id=424247&chapterid=69948#:~:text=Springs%20are%20components%20which%20deflect, eg%2C%20spring%20loaded%20safety%20valves). Accessed 16 Dec. 2022.

[2] [2] OpenStax. "16.7 Damped Harmonic Motion – College Physics." 16.7 Damped Harmonic Motion – College Physics, pressbooks.uiowa.edu/clonedbook/chapter/damped-harmonic-motion/#:~:text=An%20underdamped%20system%20will%20oscillate,one%20that%20is%20critically%20damped. Accessed 16 Dec. 2022.

Damping has many practical applications, such as the design of a vehicle's suspension system, such as a car or motorcycle. It would be highly unpleasant if a passenger experiences undulating up and down motion as the vehicle drove on an uneven road surface. Fortunately, the suspension system includes shock absorbers[3] such as those shown in Figure 2 and 3. They have an oil-mounted piston to keep the car's body from oscillating up and down after the wheels hit a bump on the road. Figure 2 shows the overall system of a shock absorber and as to how the suspension system works. Movement of an object back and forth within a fixed equilibrium position under influence of a restoring force proportional to its displacement is hence termed “simple harmonic motion”. With no friction acting, an object would continue to oscillate indefinitely in an ideal situation. Of course, objects in the real world do not oscillate indefinitely; instead, most oscillating particles are subject to damping, or dissipation of energy, mainly due to friction. With my investigation taken into consideration if the spring suspended from the ceiling is pulled to its maximum displacement and then released, it will, of course, behave dramatically at first. However, due to the damping effect, with time it’s movements will become increasingly slow and overall, as the surface area increases it leads to a greater drag force acting upwards, enhancing the effect of damping. My main area of focus would be the effect of air resistance on the rate of damping. A highly evident real life system example pertaining to the experiment are the shock absorber mechanical products in a vehicle. The applications of this amazing phenomenon of shock absorbers include the suspension of a car which is designed to introduce damping forces to increase comfort of an erratic ride. It does so by reducing and counteracting the vibrations that come with a bumpy ride converting the kinetic into thermal energy, thus acting like a form of external factor causing a greater rate of damping.

Moreover, the oscillation of the bungee jumper[4] is subject to damping. Maximum speed and rebounded height will decrease after a few oscillations, and the bungee jumper will eventually stop moving. The dissipation of kinetic and potential energy stored in the bungee cord and the jumper causes this. As the jumper oscillates against air movement, air resistance acts on him and the cord, converting kinetic energy of the bungee jumper and elastic potential energy in the cord into sound and heat energy. Energy in the bungee cord-bungee jumper system slowly decreases to zero, denoting that there will be no energy available for the jumper to oscillate, and thus the speed of the jumper gradually decreases to zero. Figure 4 is a demonstration of damping (decaying amplitude of vibrations) in bungee jumping system.[5]

Aim of the Experiment: The main objective of this experiment is to investigate the relationship between the surface area and rate of damping by attaching a card to the bottom of the mass attached to a spring in a spring-mass system. Once the objective of determining the relationship is achieved, the analysis pertaining to it will be depicted.

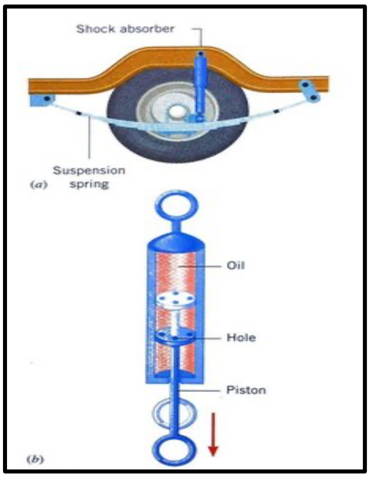

Figure 2: Shock Absorber

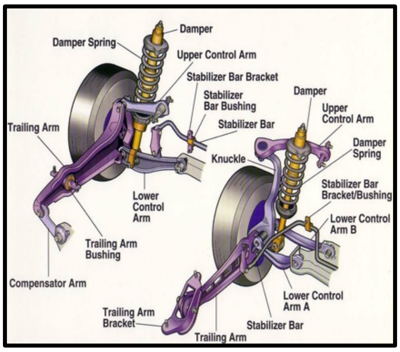

Figure 3: Shock Absorber

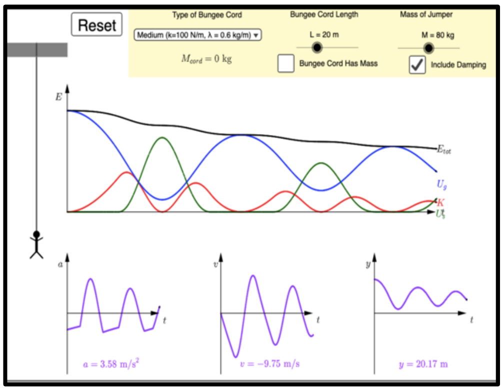

Figure 4: Simulation of Bungee Jumping with Damping

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[3]“Physics.” CAN THE ACADEMIC READING CENTER HELP YOU SUCCEED?, arcsubject.weebly.com, https://arcsubject.weebly.com/physics.html. Accessed 10 Aug. 2022.

[4] “Bungee Jumping and Rubber Bands - Simple Harmonic Motion.” Bungee Jumping and Rubber Bands - Simple Harmonic Motion, sites.google.com, https://sites.google.com/site/wkleowshm/bungee-1. Accessed 17 Aug. 2022.

[5] MECANICA AUTOMOTRIZ, INGENIERIA Y. “SUSPENSION SYSTEM: COMPONENTS, TYPES AND WORKING PRINCIPLE - www.ingenieriaymecanicaautomotriz.com, 27 July 2019, https://www.ingenieriaymecanicaautomotriz.com/suspension-system-components-types-and-working-principle/.

Variables

Independent Variable: Cross sectional Area of the Card **Range:** $0cm^2$, $25cm^2$, $49cm^2$, $81cm^2$, $121cm^2$ **Reason:** To analyze how air resistance affects the rate of damping. **Method:** Using cards with different surface areas	**Dependent Variable:** Rate of Damping **Reason:** This is the variable that is analyzed and investigated in the experiment. **Method:** Finding the change in amplitude and dividing by time to find the rate of damping for different surface areas of the card.

Controlled Variable	Material of the card	To keep the experiment fair and only have one independent variable	By using the same material to make cards of different surface areas.
	Mass hung on the spring Mass used: **200g / 0.200kg**	To obtain reliable results so that the only variable being changed is the surface area of the card which gives a change in the rate of damping and keeping the mass constant so that the experiment is fair and reliable.	Adding only a fixed weight to the spring. Weigh the mass using a digital balance to ensure that accurate weight is added.
	Initial Displacement	Since the rate of damping is being measured, having the spring released at different initial displacements will provide inaccurate results for rate of damping. Since the amount of energy transferred depends on the displacement from equilibrium. Thus, keeping displacement constant ensures that potential energy is the same for all trials.	By placing a straight standing ruler parallel and attaching a pointer to the bottom of the set-up and displacing the mass upto a particular point and making a mark at that point so the spring can be released from the same point whenever different trials are taken or the experiment is repeated using a different surface area card.
	Spring Constant	Ensure that other properties of the spring remain constant else it would affect the rate of damping.	Use the same spring for all the experiment trials with cards of different surface areas.

Apparatus

Name	Purpose
Vernier Motion Sensor	Measure position, velocity and acceleration
Vernier Data Logger	Create complex tables and graphs
Retort Clamp Stand	To hold the spring and 200g mass
Cards with different cross sectional area	Attach to mass, to investigate rate of damping
Metre Rule	To measure initial and final amplitude
Spring (spring constant calculated externally)	To add mass and carry out experiment
200 g of mass	Attach to spring and carry out experiment
Digital Balance	To measure the weight of the mass
Stop Watch	Time taken for set of oscillations

Experiment Carried out to measure the spring constant of the spring:

Since the spring that I used for my experiment was provided by the school there were no specifications or details pertaining to the spring given and hence to proceed with the experiment I had to determine certain variables about the spring itself. The most important was the spring constant of the spring for which I separately carried out an experiment physically to determine the value. Simulation has been shown to resonate the basic features of the spring mass system set up. To find the spring constant I used the Hooke's Law. Hooke's Law states that the extension of a spring is proportional to the load provided that the limit of proportionality or elastic limit is not exceeded. Resonated in the following equation.

$F = -kx$ [F = Force, k = springs constant, x = extension of the spring]

The experiment was carried out in the following steps[6]:

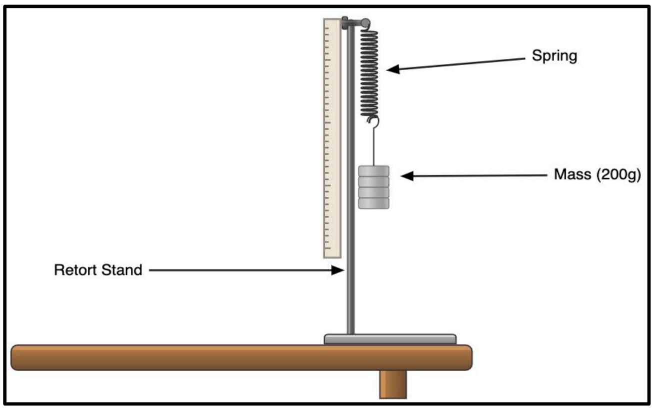
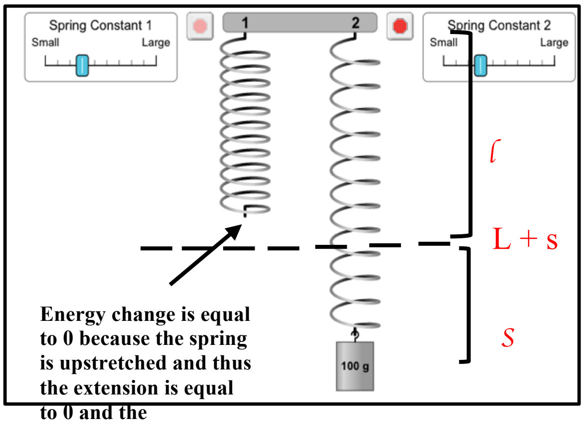


Figure 5: Simulation to show set up of experiment



Process for the calculation of the spring constant:
First step, the initial position of the spring was noted down from the ruler. Next a fixed mass of 200g was attached to the bottom hook of the spring and the new position of the spring was noted after its extension due to the weight attached. The change in position was calculated which is equivalent to the extension of the spring:

$$Extension^{7} = final\ displacement - initial\ displacement$$

OR

$$Extension\ (x) = stretched\ length - unstretched\ length$$

The Force (F) on the spring is equivalent to the weight[8] attached to the spring's hook. Since in this experiment the **Force = Weight** thus force was found out using the formula:

$$Weight = Mass \text{ x } Graviational\ Field\ Strength$$
$$W = mg$$

Lastly the formula **F = -kx** was derived and the equation was shuffled to make ***k*** **(spring constant)** the subject and calculate the spring constant.

$$k = \frac{F}{x}$$

Calculation:

Mass of the Weight Attached	200g / 0.200kg
Gravitational Field Strength	9.81 Ng^{-1}
Unstretched Spring Length;	13.0 cm
Stretched Spring Length	19.0 cm
Extension	Extension(x) = stretched – unstretched length ∴ 19.0cm – 13.0cm = 6.0cm 6.0cm → 0.06m
Weight	F = W → W = mg W = 0.200kg x 9.81Ng^{-1} = 1.962N

[6] https://chemix.org/
[7] Heather Duncan - Cambridge O Level Physics (2021, Hodder Education)
[8] Cambridge International AS and A Level Physics, 2nd edition (Mike Crundell, Geoff Goodwin, Chris Mee)

Calculation of Springs Constant	Force = Spring Constant x Extension F = k x $k = \frac{F}{x}$ $k = \frac{1.962N}{0.06m} = \mathbf{32.70Nm^{-1}}$

Procedure of the Experiment:

Set up the apparatus as shown in figure 6 below. Next, measure the dimensions of the cardboard square card and then multiply the length with the breadth in order to find out the area of the card. Next, attach the spring to the clamp retort stand. Using a digital balance accurately measure the weight of the mass being attached to the spring for oscillation (200g of mass used). Next step, find the un-extended length of the spring and attach the $200g$ ($\mathbf{0.200kg}$) of mass to the spring hook and find the extended length. In order to solve the uncertainty of the spring's nature use the formula $\boldsymbol{F = kx}$ to calculate the spring constant of the spring. Attach the damping card to the bottom of the mass and wait for the mass to decline till its final stage. Next, pull the mass or displace the mass with an amplitude of $\mathbf{4.0cm}$. Allow for the mass to oscillate in a spring-mass system for $\mathbf{60s}$. Using a motion sensor, the motion is tabulated and analyzed. Find the period of the oscillation by timing **60 oscillations** and finding the time period. Repeat the procedure at least 5 more times for the mass attached to the same card area **(5 trials)**. Finally, repeat all the steps for damping card with area ($\mathbf{25cm^2, 49cm^2, 81cm^2, 121cm^2}$) / ($\mathbf{0, 2.5 \times 10^{-3}, 4.9 \times 10^{-3}, 8.1 \times 10^{-3}, 1.21 \times 10^{-2}\ m^2}$).

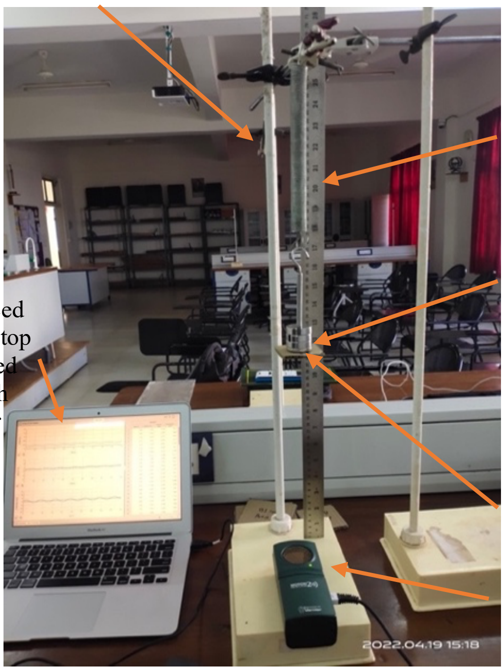


Figure 6: Experimental Set up to determine how surface area affects the damping ratio

Figure 7: Apparatus set up to determine how surface area affects the rate of damping

	Data Set 1			
	Time (s)	Position (m)	Velocity (m/s)	Acceleration (m/s²)
1	0.00	0.243	-0.121	1.220
2	0.05	0.235	-0.068	1.395
3	0.10	0.232	0.010	1.558
4	0.15	0.235	0.098	1.524
5	0.20	0.243	0.177	1.187
6	0.25	0.254	0.228	0.579
7	0.30	0.267	0.240	-0.170
8	0.35	0.280	0.209	-0.891
9	0.40	0.289	0.144	-1.440
10	0.45	0.295	0.056	-1.751
11	0.50	0.295	-0.042	-1.780
12	0.55	0.291	-0.134	-1.508
13	0.60	0.281	-0.205	-0.952
14	0.65	0.268	-0.237	-0.215
15	0.70	0.256	-0.226	0.529
16	0.75	0.244	-0.179	1.154
17	0.80	0.236	-0.104	1.589
18	0.85	0.233	-0.010	1.761
19	0.90	0.235	0.085	1.624
20	0.95	0.243	0.164	1.215
21	1.00	0.253	0.214	0.619
22	1.05	0.265	0.230	-0.079
23	1.10	0.277	0.206	-0.777

Figure 8: Raw Data Collected from the Motion Sensor

For each area of the card the acceleration, position and velocity was measured and tabulated for 60s with (20 samples per second).

Risk Assessment/ Precautions and Ethicality in Experiment:

The safety issues in this experiment are certainly minimal. While the spring was oscillating, immediately a step was taken back so as to prevent any injury. Pertaining to environmental issues or ethics no as such accountability was there. Another precaution was to ensure the clamp stand was tight so that it doesn't collapse and cause an injury. Any apparatus or material regarded as redundant was disposed of safely to evade any environmental issue maintaining the ethicality of the experiment. The experiment was conducted in an enclosed room with all the windows closed so that the air doesn't affect readings and cause erroneous data since the increase in air resistance would cause greater oscillation of the spring mass system leading to inaccurate calculations for the rate of damping of the system.

Investigation of How the surface area of the vibrating mass affects Time Period

Processed Data:

Area (m^2)	T1 (s)	T2 (s)	T3 (s)	T4 (s)	T5 (s)
0	14.32	14.30	14.35	14.45	14.30
2.50 x $10^{-3} \pm 0.00000503$	14.80	14.85	14.88	14.76	14.60
4.90 x $10^{-3} \pm 0.00000704$	15.30	15.45	15.53	15.38	15.47
8.10 x $10^{-3} \pm 0.00000904$	16.30	16.21	16.10	16.35	16.45
1.21 x $10^{-2} \pm 0.000110$	17.10	17.12	16.98	17.20	17.35

Area (m^2)	Average Time ($\pm 0.01s$)	Time Period ($\pm 0.01s$)
0	14.34	0.7170
2.50 x $10^{-3} \pm 0.00000503$	14.78	0.7390
4.90 x $10^{-3} \pm 0.00000704$	15.43	0.7715
8.10 x $10^{-3} \pm 0.00000904$	16.28	0.8140
1.21 x $10^{-2} \pm 0.000110$	17.15	0.8575

$$\boldsymbol{Error = \frac{Maximum\ Time\ Period - Minimum\ Time\ Period}{2}}$$

Area (m^2)	Maximum Time Period	Minimum Time Period	Time Period Error
0	14.45	14.30	0.075
2.50 x $10^{-3} \pm 0.00000503$	14.88	14.60	0.140
4.90 x $10^{-3} \pm 0.00000704$	15.53	15.30	0.115
8.10 x $10^{-3} \pm 0.00000904$	16.45	16.10	0.175
1.21 x $10^{-2} \pm 0.000110$	17.35	16.98	0.185

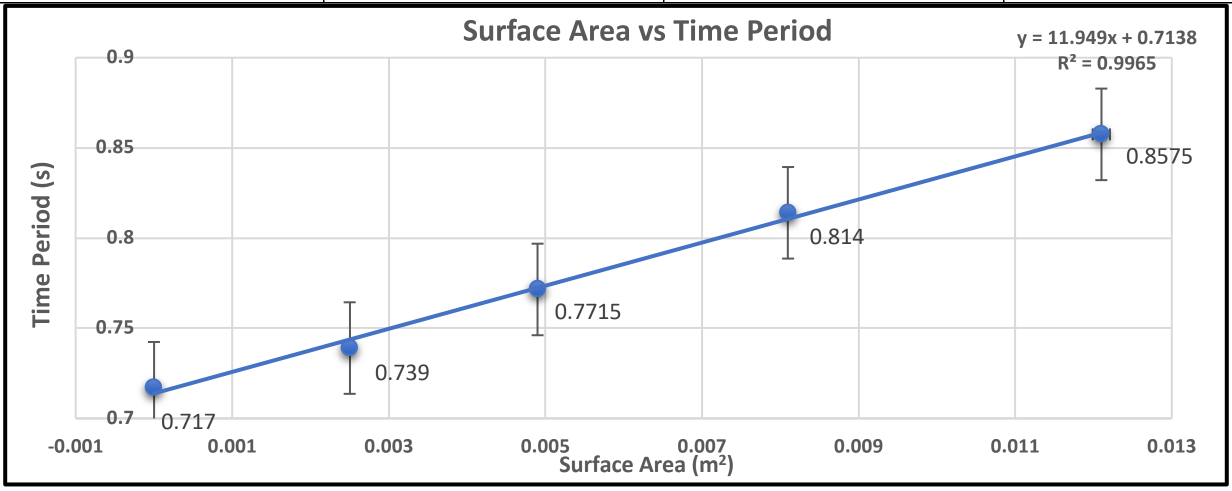


Figure 9: Graph of Surface Area vs Time Period of Spring Mass System

$$\boldsymbol{A \propto T}$$

Surface Area is directly proportional to the time period

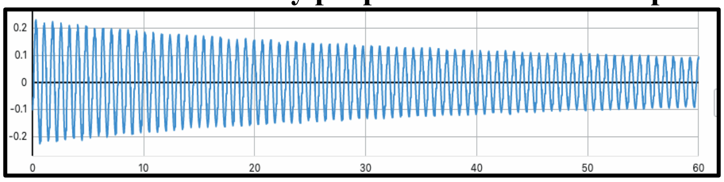
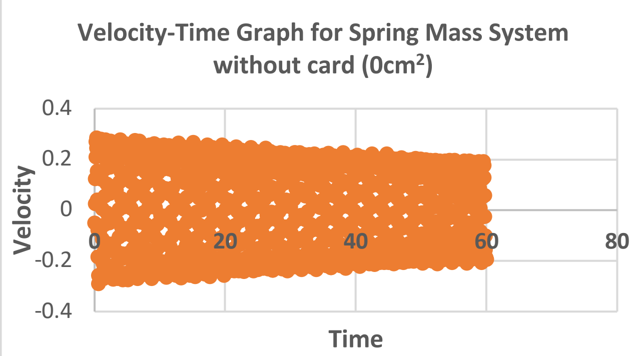
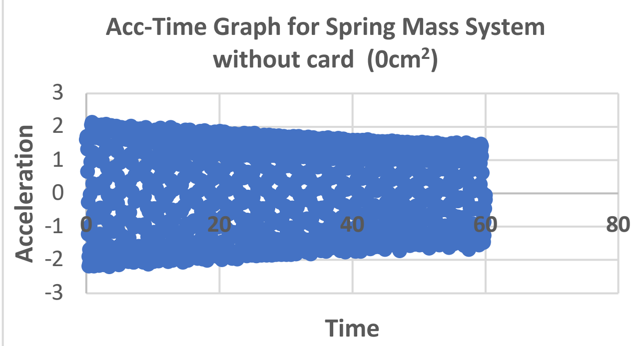
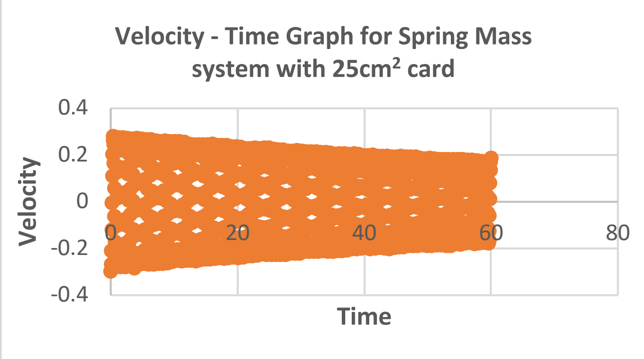
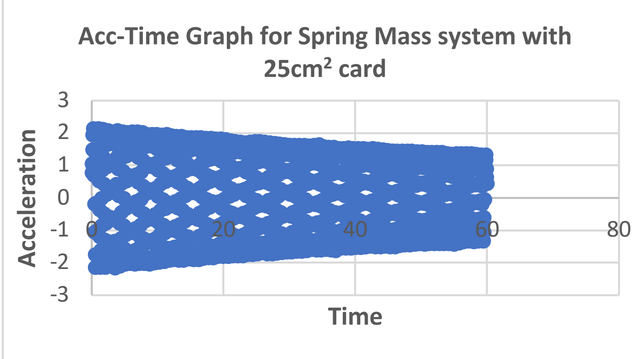
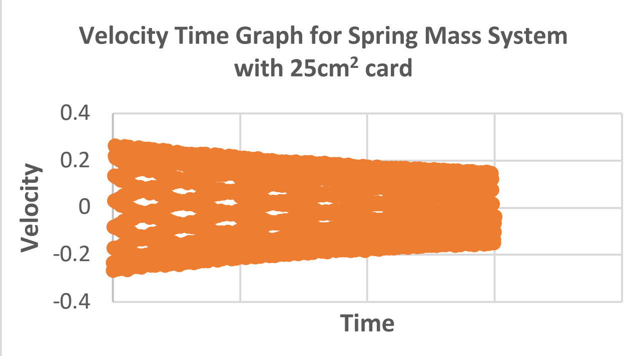
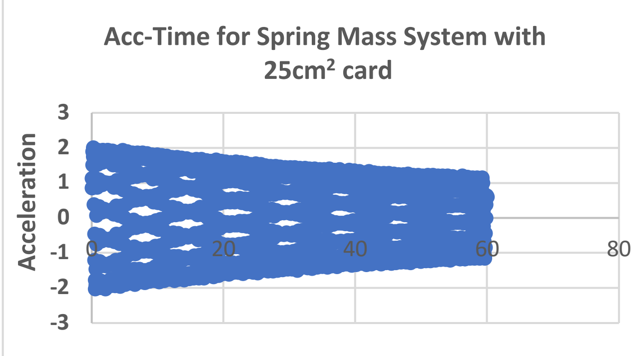
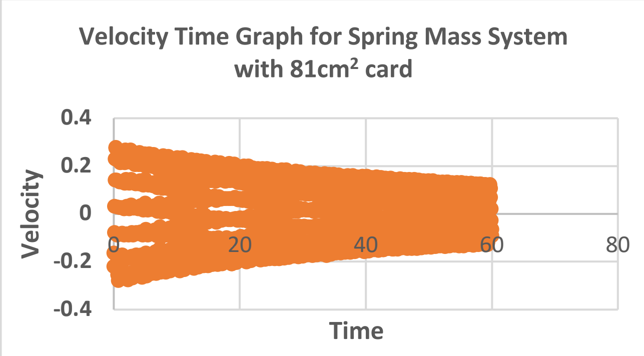
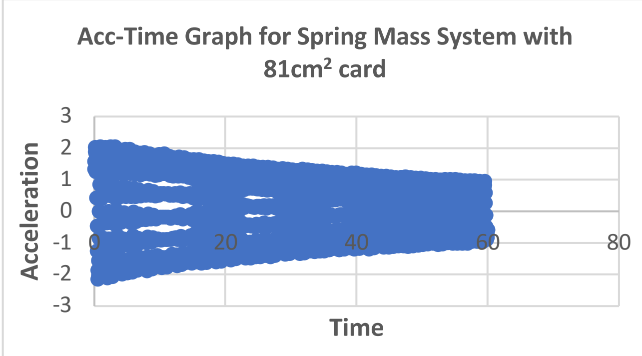
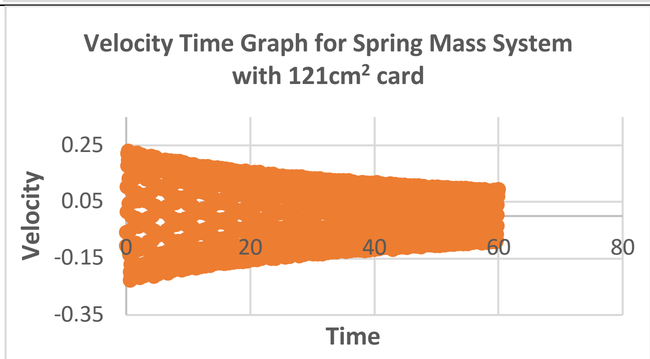
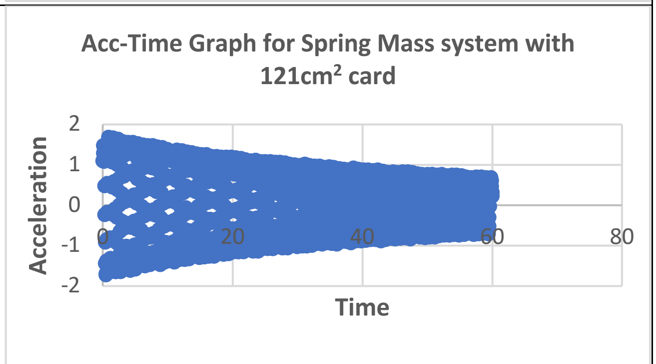
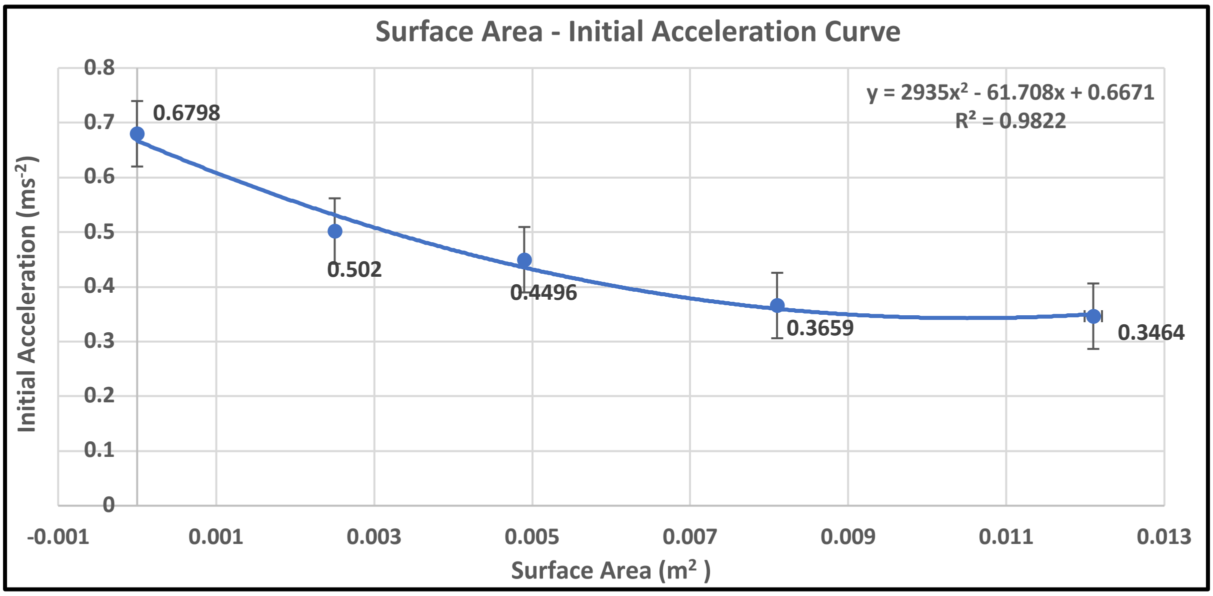


Figure 10: Graph of 60 oscillations recorded for time period

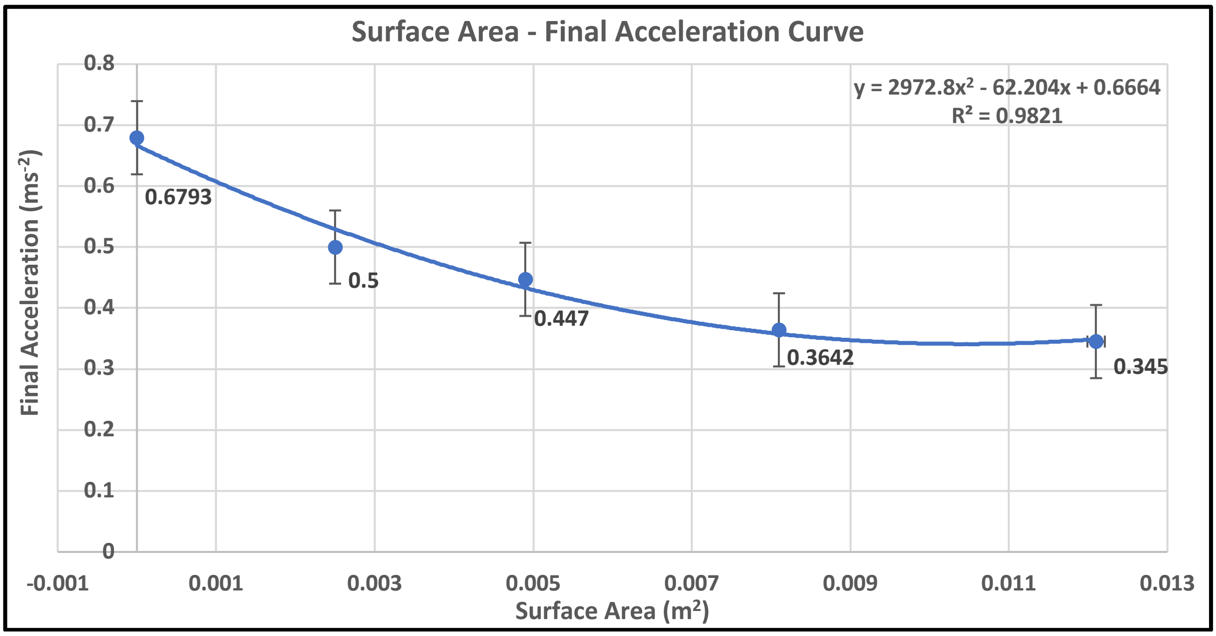
To determine the relationship between the surface area and the time period of the vibrating mass in a spring mass system, the time taken for a total of 60 oscillations was measured using a stop clock. Once accurate readings were obtained, the total time was divided by the number of oscillations to calculate the time period. A proportionality was observed in the relationship between the surface area and the time period of the vibrating mass in the spring mass system. For 25cm^2 or 2.5x10^{-3}m^2 of card attached to the spring mass system, a relatively smaller upward air resistance force opposing mobility acted on it and thus as a result, it had a time period of $\mathbf{0.717s}$. The card with area 121cm^2 or (1.21x10^{-2} m) almost 5 times greater than 25cm^2 card had the highest time period of $\mathbf{0.8575s}$ a **19.60% increment**, because the greatest force of air resistance acted on this bigger surface area (121cm^2).

Area (cm^2)	Velocity-Time Graph ms^{-1}	Acceleration-Time Graph ms^{-2}
0		
25		
49		
81		
121		

The graphs above, are collated data from the Motion Sensor Readings that are processed and graphically represented showing the damping occurs as the surface area of the card increases and tabulated from the narrowing shape of the curves approaching the x-axis. Velocity and Acceleration decrease due to greater resistive force "air resistance" acting on the mass attached in the spring mass system, leading to more damping.



Graph 1: Curve for Surface Area of Cards Vs Initial Acceleration



Graph 2: Curve for Surface Area of Cards vs Final Acceleration

Tabulating the following graphs:

Decrease in acceleration is denoted as damping. As the vibrating mass oscillates in the spring mass system, due to the buoyancy forces acting upwards, the frictional force acts against the motion of the mass, opposing it. Damping reduces the amount of vibrations and causes decay of amplitude thus as a result decreasing acceleration is evidence of damping. Due to a small surface area of 25cm^2 this damping card has an **initial acceleration of $\boldsymbol{a = 0.6798\, ms^{-2}}$ and final acceleration $\boldsymbol{a = 0.6793\, ms^{-2}}$**. There's a trend observed in the initial and final acceleration, wherein a decrease is tabulated. Increasing the surface area by ~5 times (from 25cm^2 to 121cm^2) causes initial acceleration to decrease to $\boldsymbol{a = 0.3464\, ms^{-2}}$ and final acceleration deteriorates to $\boldsymbol{a = 0.3450\, ms^{-2}}$, a **$\boldsymbol{49.04\%}$ reduction in initial acceleration and $\boldsymbol{49.21\%}$ reduction in final acceleration**. Thus, the smallest acceleration is noted for the 121cm^2 card with $\boldsymbol{a = 0.3464\, ms^{-2}}$ **initial acceleration and $\boldsymbol{a = 0.345 ms^{-2}}$ final acceleration** keeping the fact in mind that the greatest force of air resistance acts on this bigger surface area thus slowing down the motion of the spring mass system. Statistical analysis of the initial and final acceleration has led to the conclusion that as the surface area of the card is increased, greater damping occurs leading to a further decrease in acceleration, denoting a direct proportionality as to how damping increases as surface area increases.

$$\boldsymbol{Surface\ Area \propto Damping}$$

Data For Area $0.0\ m^2$				
Trial No.	Maximum Amplitude(m)	Minimum Amplitude(m)	ΔA (m)	ΔT (s)
1	3.0×10^{-1}	2.67×10^{-1}	3.3×10^{-2}	6.0×10^{-1}
2	3.0×10^{-1}	2.60×10^{-1}	4.0×10^{-2}	6.0×10^{-1}
3	3.0×10^{-1}	2.59×10^{-1}	4.1×10^{-2}	6.0×10^{-1}
4	3.0×10^{-1}	2.55×10^{-1}	4.5×10^{-2}	6.0×10^{-1}
5	3.0×10^{-1}	2.68×10^{-1}	3.2×10^{-2}	6.0×10^{-1}

Data For Area for $2.5 \times 10^{-3} m^2$				
Trial No.	Maximum Amplitude (m)	Minimum Amplitude (m)	ΔA (m)	ΔT (s)
1	3.0×10^{-1}	2.55×10^{-1}	4.5×10^{-2}	6.0×10^{-1}
2	3.0×10^{-1}	2.48×10^{-1}	5.2×10^{-2}	6.0×10^{-1}
3	3.0×10^{-1}	2.63×10^{-1}	3.7×10^{-2}	6.0×10^{-1}
4	3.0×10^{-1}	2.55×10^{-1}	4.5×10^{-2}	6.0×10^{-1}
5	3.0×10^{-1}	2.56×10^{-1}	4.4×10^{-2}	6.0×10^{-1}

Data For Area for $4.9 \times 10^{-3} m^2$				
Trial No.	Maximum Amplitude (m)	Minimum Amplitude (m)	ΔA (m)	ΔT (s)
1	3.0×10^{-1}	2.45×10^{-1}	5.5×10^{-2}	6.0×10^{-1}
2	3.0×10^{-1}	2.60×10^{-1}	4.0×10^{-2}	6.0×10^{-1}
3	3.0×10^{-1}	2.48×10^{-1}	5.2×10^{-2}	6.0×10^{-1}
4	3.0×10^{-1}	2.45×10^{-1}	5.5×10^{-2}	6.0×10^{-1}
5	3.0×10^{-1}	2.50×10^{-1}	5.0×10^{-2}	6.0×10^{-1}

Data For Area for $8.1 \times 10^{-3} m^2$				
Trial No.	Maximum Amplitude (m)	Minimum Amplitude (m)	ΔA (m)	ΔT (s)
1	3.0×10^{-1}	2.38×10^{-1}	6.2×10^{-2}	6.0×10^{-1}
2	3.0×10^{-1}	2.45×10^{-1}	5.5×10^{-2}	6.0×10^{-1}
3	3.0×10^{-1}	2.48×10^{-1}	5.2×10^{-2}	6.0×10^{-1}
4	3.0×10^{-1}	2.50×10^{-1}	5.0×10^{-2}	6.0×10^{-1}
5	3.0×10^{-1}	2.45×10^{-1}	5.5×10^{-2}	6.0×10^{-1}

Data For Area for $1.21 \times 10^{-2} m^2$				
Trial No.	Maximum Amplitude (m)	Minimum Amplitude (m)	ΔA (m)	ΔT (s)
1	3.0×10^{-1}	2.30×10^{-1}	7.0×10^{-2}	6.0×10^{-1}
2	3.0×10^{-1}	2.29×10^{-1}	7.1×10^{-2}	6.0×10^{-1}
3	3.0×10^{-1}	2.34×10^{-1}	6.6×10^{-2}	6.0×10^{-1}
4	3.0×10^{-1}	2.20×10^{-1}	8.0×10^{-2}	6.0×10^{-1}
5	3.0×10^{-1}	2.35×10^{-1}	6.5×10^{-2}	6.0×10^{-1}

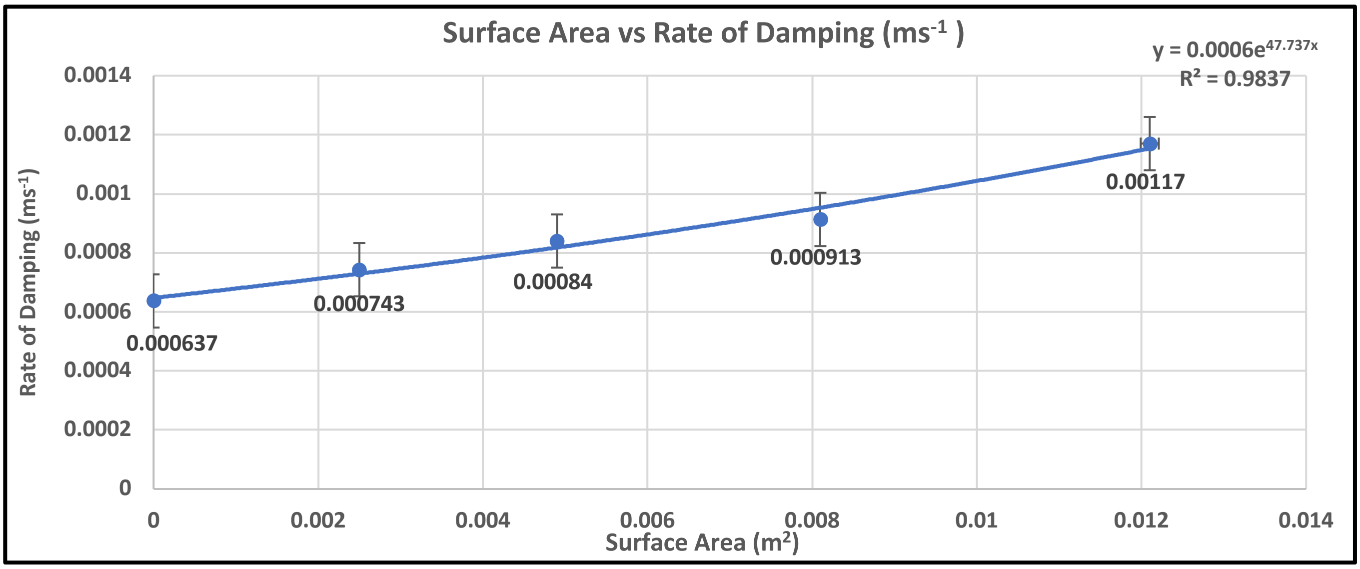
Processed Data for Surface Area vs Rate of Damping

Processed Data for Surface Area vs Rate of Damping			
Surface Area (m^2)	Average ΔA	Average ΔT	Rate of Damping (ms^{-1})
$\mathbf{0.0}$	3.82×10^{-2}	6.0×10^{1}	6.37×10^{-4}
$\mathbf{2.50 \times 10^{-3}}$	4.46×10^{-2}	6.0×10^{1}	7.43×10^{-4}
$\mathbf{4.90 \times 10^{-3}}$	5.04×10^{-2}	6.0×10^{1}	8.40×10^{-4}
$\mathbf{8.10 \times 10^{-3}}$	5.48×10^{-2}	6.0×10^{1}	9.13×10^{-4}
$\mathbf{1.21 \times 10^{-2}}$	7.04×10^{-2}	6.0×10^{1}	1.17×10^{-3}

$$\boldsymbol{Change\ in\ Amplitude = Maximum\ Amplitude - Minimum\ Amplitude}$$

$$\boldsymbol{Rate\ of\ Change\ in\ Amplitude\ or\ Rate\ of\ Damping = \frac{Maximum\ Amplitude - Minimum\ Amplitude}{Time}}$$

Graph 3: Surface Area vs Rate of Damping



The rate of change in amplitude is calculated by calculating the difference between the maximum and minimum amplitude and dividing it by the time for which the total number of oscillations of the mass oscillating in a spring mass system was taken. The rate of change in amplitude thus refers to the damping rate. The damping rate was investigated for different surface areas of cards attached to the mass in a spring mass system, wherein the surface area of the card was increased and the rate of change in amplitude was found. Tabulating the graph, its interpreted that the rate of damping increases as the surface area of the card increases: for a surface area of $\mathbf{2.50 \times 10^{-3} m^2}$ the rate of damping is $\mathbf{7.43 \times 10^{-4} ms^{-1}}$ however almost doubling the surface area to $\mathbf{4.90 \times 10^{-3} m^2}$ causes rate of damping to increase to $\mathbf{8.40 \times 10^{-4} ms^{-1}}$ increment by $\mathbf{9.70 \times 10^{-5} ms^{-1}}$, a $\mathbf{13.10}$% increment. Increasing surface area by 5 times from $\mathbf{2.50 \times 10^{-3} m^2}$ to $\mathbf{1.21 \times 10^{-2} m^2}$ causes rate of damping to increase from $\mathbf{7.43 \times 10^{-4} ms^{-1}}$ to $\mathbf{1.17 \times 10^{-3} ms^{-1}}$, which is a $\mathbf{57.47}$% increment.

Although the points on the graph do not demonstrate a perfect straight line, the coefficient of determination R^2 value is **0.9837** close to the value of 1, which is high enough to establish that the relationship between the surface area and rate of damping shown by the graph is accurate to a degree where appropriate conclusions can be made from it. The graph shows a direct proportionality between the surface area and rate of damping: as surface area increases the rate of damping increases. A positive relationship or trend is obtained in the results as, the rate of damping increases as the surface area is increased. As the surface area of the card is increased, a greater number of particles collide with the card thus a greater vertical component of buoyancy force acts causing the motion of the spring-mass system to slow down faster and thus enhancing the effect of damping. This investigation can be related to the declination of a parachutist jumping out of an aircraft. At first as the parachutist jumps of the plane, since the exposed surface area is relatively small, the parachutist is falling down with a high velocity due to a smaller drag force acting vertically, however, as the parachutist opens the parachute as they are about to reach the ground the sudden increase in the exposed surface area causes greater number of air particles to collide with object and a greater buoyancy force to act, furthermore slowing down the motion of the parachutist causing terminal velocity to be reached. Hence a bigger surface area allows more air particles to collide with it, causing a bigger resistive force opposing motion, leading to greater damping. This is also shown in the formula: [9]

$$\boldsymbol{F_D = \frac{1}{2}\rho v^2 C_D}$$

$\boldsymbol{where\ F_D\ is\ drag\ force(air\ resistance), \rho\ is\ the\ density, v\ is\ the\ speed\ of\ the\ object, C_D\ is}$
$\boldsymbol{the\ drag\ coefficient\ and\ A\ is\ cross-sectional\ area\ of\ the\ object_6}$

In the experiment, the air is the fluid and the density can be considered to be constant since all the trials are carried out in the same room. The initial speed can be considered to be constant since the initial amplitude and mass was the same for each trial ($3.0\text{x}10^{-1}$m / 0.200kg). The shape of the object was kept the same throughout the experiment since the drag coefficient depends on the shape of the object. The only variable changed in the experiment was the cross-sectional area of the card, which was affecting the air resistance.

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[9] "Drag Forces | Physics." Drag Forces | Physics, courses.lumenlearning.com/suny-physics/chapter/5-2-drag-forces/#:~:text=For%20larger%20objects%20(such%20as,%CF%81%20is%20the%20fluid%20density. Accessed 14 Dec. 2022.

Rate of Damping/ Change in Amplitude Error Calculation:

Surface Area of the Cards (m^2)	Error Calculation for Change in Amplitude					
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average Error
0.0	1.65×10^{-2}	2.00×10^{-2}	2.05×10^{-2}	2.25×10^{-2}	1.60×10^{-2}	1.46×10^{-2}
2.50×10^{-3}	2.25×10^{-2}	2.60×10^{-2}	1.85×10^{-2}	2.25×10^{-2}	2.20×10^{-2}	2.23×10^{-2}
4.90×10^{-3}	2.75×10^{-2}	2.00×10^{-2}	2.60×10^{-2}	2.75×10^{-2}	2.50×10^{-2}	2.52×10^{-2}
8.10×10^{-3}	3.10×10^{-2}	2.75×10^{-2}	2.60×10^{-2}	2.50×10^{-2}	2.75×10^{-2}	2.74×10^{-2}
1.21×10^{-2}	3.50×10^{-2}	3.55×10^{-2}	3.30×10^{-2}	4.00×10^{-2}	3.25×10^{-2}	3.52×10^{-2}

Surface Area Error Calculation:

Surface Area (m^2)	Side 1	Side 2	Percentage Error	Absolute Error	Final Value
$\mathbf{0.0}$	$\mathbf{0.0}$	$\mathbf{0.0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0 \pm 0}$
$\mathbf{2.50 \times 10^{-3}}$	$\mathbf{5.02 \times 10^{-2}}$	$\mathbf{5.04 \times 10^{-2}}$	$\mathbf{1.9881}$	$\mathbf{5.03 \times 10^{-3}}$	$\mathbf{2.50 \times 10^{-3} \pm 0.00000503}$
$\mathbf{4.90 \times 10^{-3}}$	$\mathbf{7.02 \times 10^{-2}}$	$\mathbf{7.06 \times 10^{-2}}$	$\mathbf{1.4205}$	$\mathbf{7.04 \times 10^{-3}}$	$\mathbf{4.90 \times 10^{-3} \pm 0.00000704}$
$\mathbf{8.10 \times 10^{-3}}$	$\mathbf{9.02 \times 10^{-2}}$	$\mathbf{9.04 \times 10^{-2}}$	$\mathbf{1.1074}$	$\mathbf{9.03 \times 10^{-3}}$	$\mathbf{8.10 \times 10^{-3} \pm 0.00000904}$
$\mathbf{1.21 \times 10^{-2}}$	$\mathbf{1.102 \times 10^{-1}}$	$\mathbf{1.102 \times 10^{-1}}$	$\mathbf{0.9074}$	$\mathbf{1.10 \times 10^{-2}}$	$\mathbf{1.10 \times 10^{-2} \pm 0.000110}$

Sample Calculation for 25 cm^2: $(5.02 \times 10^{-2} \pm 0.0005) \times (5.04 \times 10^{-2} \pm 0.0005)$

$$\frac{5.00 \times 10^{-4}}{5.02 \times 10^{-2}} \times 100 = 0.9960\% \longrightarrow \mathbf{5.02 \times 10^{-2} \pm 0.9960\%}$$

$$\frac{5.00 \times 10^{-4}}{5.04 \times 10^{-2}} \times 100 = 0.9921\%$$

$$\mathbf{5.04 \times 10^{-4} \pm 0.9921\%}$$

$$5.02 \times 10^{-2} \times 5.04 \times 10^{-2} = \mathbf{2.53008 \times 10^{-4}\ m^2}$$

$$0.9960 + 0.9921 = \mathbf{1.9881\%}$$

$$\frac{1.9881}{100} \times 2.53008 \times 10^{-4} = \mathbf{5.03 \times 10^{-6}}$$

$$2.53008 \times 10^{-3}\ m^2 \pm 5.03 \times 10^{-3} \rightarrow \mathbf{2.5 \times 10^{-3} cm^2 \pm 5.03 \times 10^{-6}}$$

Conclusion: Hence from our results, the conclusion can be made that the trend predicted in the hypothesis does exist between the surface area of card attached to mass in a spring mass system and the rate of damping. Although the trendline does not pass through the origin (0,0) there is a direct proportionality between the surface area of the card attached to the mass and the rate of damping. Given that the trendline was expected to pass through the origin, since there is a y-intercept It suggests the fact that there were slight systematic errors related to the experiment. The R^2 value for my graph is $\mathbf{0.9837}$ suggesting a strong correlation between my data and the curve line denoting my results are precise to an extent however not fully accurate. Through the tabulation of the results, the main conclusion made is that there is notably a defined relationship between the rate of damping and the surface area of the vibrating mass on a spring-mass system; as the surface area of the card attached to the mass in the spring mass system is increased, the rate of damping increases as a greater buoyancy force or force of air resistance acts on the mass thus causing the motion of the spring-mass system to slow down. There is a linear trend between the surface area and time period of the spring mass system as supported by the firm results wherein with a surface area of $25cm^2$, the time period was $\mathbf{0.7170s}$, almost doubling the surface area caused the time period to increase by $\mathbf{4.4\%}$. In terms of the relationship between the surface area and rate of damping, for the card with a surface area of $25cm^2$ the rate of damping was equal to $\mathbf{7.43 \times 10^{-4} ms^{-1}}$, almost doubling the surface area of the card causes rate of damping to increase significantly by $\mathbf{13.05\%}$. In terms of the investigation for the effect of exposed surface area of the card on the time period for an oscillating mass on a spring, the time period gradually increases and the amplitude of the oscillations decrease due to the damping caused by the air resistance acting on the spring-mass system due to the card with a specified surface area.

Further Scope of Investigation: It would be interesting to analyze how different shapes of cards attached to the constant mass in the spring mass system affects damping, not only focusing on the surface area but other factors. To extend the research, how damping is affected by different spring lengths and different spring materials with different spring constants could be examined.

Strengths

Strength	Significance/ Effect on Results
Motion Detector	Using this software technology to make measurements enabled accurate data to be obtained. Moreover, a range of accurate graphs were obtained, pertaining to velocity, position and acceleration to time. Since greater samples of data readings were taken by the motion detector it ensured more reliable results.
Digital Stopwatch to measure time taken for oscillations	Using a digital equipment to measure the time for oscillation of the spring mass system enables accurate readings to be obtained.
Tighten the retort stands	Since the apparatus has been utilized for the internal assessments and experiments of other candidates. Thus, overtime usage causes the equipment and apparatus to wear out and the screws to loosen up. Thus, by using a wench to tighten the retort stand it prevents it from oscillating or wobbling and affecting the readings. This way random error was minimized.
Closed Room	Preventing wind from distorting the results so that no other variable other than the surface area affects the rate of damping.

Limitation and Problems

Limitation	Significance / Effect on Results	Solution
Difficulty in displacing the mass attached to a spring-mass system, and ensuring it move vertically upwards, without horizontal displacement.	The 0.200kg mass hung on the spring was displaced to a particular point referred to as the maximum amplitude, which is the point of release to analyze the damping using the motion detector. Because the experiment was carried out individually, displacing the spring with one hand was relatively difficult. My line of sight was parallel to the spring's plane of oscillation, thus it was difficult to ensure that the mass moves vertically upwards.	As seen in different scientific videos, many of the professors have a vertical column of plastic or transparent glass around the spring mass system to ensure, movement, thus implementing the same in the experiment would help to avoid the unnecessary horizontal movement and achieve vertical motion to obtain accurate results for rate of damping.
Less number of Trials	Due to time constrains, only 5 trials were carried out and a mean value was calculated to define the trend and relationships. This can cause greater standard deviation in results to occur. Due to a complicated set up, it would take too long for the apparatus to be put up each day and conduct the experiment.	Carrying out greater number of trials (15 trials) for the experiment and calculate a mean value to increase result reliability. Take help of peers, classmates to increase productivity in setting up the apparatus.
Difficulty in determining the maximum amplitude because no pointer was used.	Because a motion detector was being used to analyze the damping, thus the mass was displaced to an initial amplitude and the reading was obtained by visual determination. However, this could cause the spring to be displaced not exactly at the same initial (maximum amplitude) for each trial. Using a pointer would cause the motion detector to also detect the pointers motion, leading to erroneous result being produced regarding the damping motion of the vibrating mass on the spring mass system.	Make sure to view the reading parallel, to avoid parallax error. Moreover, ensure that the metre rule is straight to avoid incorrect reading. In addition, the metre rule could be attached right next to the set up, and ensure that it is not slanting.
The wind in the room	Wind through the room can definitely distort the readings and influence the rate of damping.	To evade such a flaw in the experiment affecting the readings, ensure that the windows are closed and fans are switched off.

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Appendix: Raw Data Collected

Raw Data

For each area of the card the acceleration, position and velocity was measured and tabulated for 60s and with (20 samples per second) equivalent to 1200 samples. The following data shows an example of the raw data collected for Spring with a card of area $121cm^2$/ $1.21x10^{-2}m^2$ attached to it

	Data Set 1			
	Time (s)	Position (m)	Velocity (m/s)	Acceleration (m/s^2)
1	0.00	0.243	-0.121	1.220
2	0.05	0.235	-0.068	1.395
3	0.10	0.232	0.010	1.558
4	0.15	0.235	0.098	1.524
5	0.20	0.243	0.177	1.187
6	0.25	0.254	0.228	0.579
7	0.30	0.267	0.240	-0.170
8	0.35	0.280	0.209	-0.891
9	0.40	0.289	0.144	-1.440
10	0.45	0.295	0.056	-1.751
11	0.50	0.295	-0.042	-1.780
12	0.55	0.291	-0.134	-1.508
13	0.60	0.281	-0.205	-0.952
14	0.65	0.268	-0.237	-0.215
15	0.70	0.256	-0.226	0.529
16	0.75	0.244	-0.179	1.154
17	0.80	0.236	-0.104	1.589
18	0.85	0.233	-0.010	1.761
19	0.90	0.235	0.085	1.624
20	0.95	0.243	0.164	1.215
21	1.00	0.253	0.214	0.619
22	1.05	0.265	0.230	-0.079
23	1.10	0.277	0.206	-0.777

	Data Set 1			
	Time (s)	Position (m)	Velocity (m/s)	Acceleration (m/s^2)
23	1.10	0.277	0.206	-0.777
24	1.15	0.287	0.147	-1.351
25	1.20	0.293	0.062	-1.690
26	1.25	0.294	-0.035	-1.716
27	1.30	0.289	-0.122	-1.439
28	1.35	0.281	-0.187	-0.951
29	1.40	0.270	-0.223	-0.315
30	1.45	0.257	-0.222	0.396
31	1.50	0.246	-0.181	1.049
32	1.55	0.238	-0.109	1.519
33	1.60	0.234	-0.019	1.728
34	1.65	0.236	0.075	1.643
35	1.70	0.242	0.157	1.270
36	1.75	0.252	0.212	0.674
37	1.80	0.265	0.230	-0.044
38	1.85	0.277	0.206	-0.739
39	1.90	0.287	0.149	-1.285
40	1.95	0.293	0.070	-1.625
41	2.00	0.294	-0.023	-1.710
42	2.05	0.291	-0.114	-1.499
43	2.10	0.282	-0.184	-1.021
44	2.15	0.271	-0.223	-0.367
45	2.20	0.258	-0.223	0.341

	Data Set 1			
	Time (s)	Position (m)	Velocity (m/s)	Acceleration (m/s^2)
45	2.20	0.258	-0.223	0.341
46	2.25	0.247	-0.186	0.979
47	2.30	0.239	-0.119	1.451
48	2.35	0.234	-0.031	1.689
49	2.40	0.235	0.063	1.632
50	2.45	0.241	0.143	1.302
51	2.50	0.250	0.200	0.772
52	2.55	0.262	0.226	0.108
53	2.60	0.275	0.213	-0.594
54	2.65	0.285	0.163	-1.202
55	2.70	0.292	0.084	-1.597
56	2.75	0.294	-0.009	-1.704
57	2.80	0.291	-0.098	-1.524
58	2.85	0.284	-0.171	-1.097
59	2.90	0.273	-0.216	-0.476
60	2.95	0.261	-0.222	0.231
61	3.00	0.249	-0.190	0.881
62	3.05	0.240	-0.127	1.364
63	3.10	0.236	-0.045	1.632
64	3.15	0.235	0.046	1.644

	Data Set 1			
	Time (s)	Position (m)	Velocity (m/s)	Acceleration (m/s^2)
86	4.25	0.293	0.011	-1.663
87	4.30	0.291	-0.079	-1.562
88	4.35	0.285	-0.157	-1.176
89	4.40	0.274	-0.206	-0.576
90	4.45	0.263	-0.218	0.100
91	4.50	0.251	-0.194	0.741
92	4.55	0.242	-0.139	1.256
93	4.60	0.236	-0.061	1.567
94	4.65	0.235	0.028	1.626
95	4.70	0.239	0.113	1.401
96	4.75	0.248	0.179	0.924
97	4.80	0.259	0.212	0.294
98	4.85	0.270	0.209	-0.373
99	4.90	0.281	0.172	-0.979
100	4.95	0.289	0.105	-1.416
101	5.00	0.292	0.020	-1.600
102	5.05	0.291	-0.066	-1.517
103	5.10	0.285	-0.141	-1.187
104	5.15	0.276	-0.193	-0.659
105	5.20	0.264	-0.212	-0.018
106	5.25	0.253	-0.195	0.636
107	5.30	0.244	-0.144	1.184
108	5.35	0.238	-0.068	1.519