ES 221: Mechanics of Solids
Project Report - Group: 05

ANALYSIS OF TRUSS SYSTEM

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1 Problem Statement

Trusses are essential structural systems composed of straight members connected at joints, commonly used to support loads in bridges, towers, and buildings. In such structures, members are subjected to axial forces—either tension or compression—depending on the loading conditions and support configuration. This problem involves analyzing a fixed truss to determine the internal force in each of its members and to classify the nature of these forces as either tensile or compressive.

2 Objective

This report aims to develop a computational tool for analyzing truss structures with varying dimensions. The tool is designed to accept user-defined inputs which includes support conditions and applied loads. By applying the principles of static equilibrium, the tool will compute the internal forces within each truss member and classify them as either tension or compression. Additionally, a graphical representation will be generated to visualize the truss structure, highlighting the magnitude and nature of internal forces. This approach provides both analytical insight and visual clarity into the behavior of truss systems under external loading.

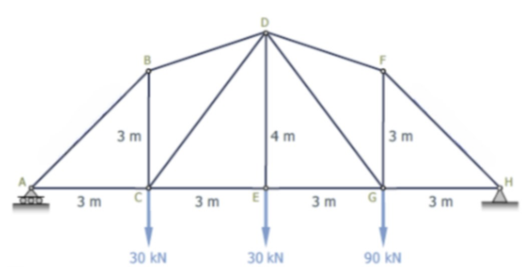
3 Methodology

This section explains the step-by-step process used to find the forces in each truss member. We begin by defining the truss geometry and the applied loads (as defined by the user). Then, using basic equilibrium principles, we calculate the support reactions and internal forces. The results are shown with color-coded diagrams to indicate which members are in tension and which are in compression.

To achieve our objective, we have utilized Wolfram Mathematica as the primary tool for performing all computations and visualizations. Mathematica was chosen for its powerful symbolic and numerical capabilities, which make it well-suited for structural analysis tasks. We developed a custom code that automates the entire process—from defining the truss geometry and applying loads, to solving equilibrium equations and visualizing the results.

3.1 Definition of Truss Geometry

To ensure computational accuracy and reduce the chance of errors from incorrect user inputs, the code is built for a specific truss configuration rather than a general one. This helps avoid mistakes related to node positions, member connections, or support definitions. The truss shown below has been selected for our analysis. Below is the detailed methodology for defining the truss geometry:



- Node Positions: The coordinates of each node in the truss are defined in a 2D plane. The positions are stored in an array for easy reference.
- Member Connectivity: Members are defined as straight links between two nodes, forming the truss framework.
- Support Conditions:
  - Roller Support at A: Restricts vertical displacement but allows horizontal movement R_{A_x}. Only provides a vertical reaction force R_{A_y}.
  - Fixed Support at H : Restricts both horizontal and vertical movement. Provides both horizontal R_{H_x} and vertical R_{H_y} reaction forces.

By hard-coding the truss dimensions, the code avoids user input errors such as incorrect node placements or invalid member connections. It also ensures that the equilibrium equations are solvable by tailoring them to a specific, well-defined truss. Additionally, the fixed geometry simplifies force calculations, making unit vector determination and equilibrium checks more straightforward.

3.2 Input of External Forces

- User Input Format: Forces are entered by the user.
- Force Assignment: These forces are stored in an associative array, ensuring correct application at each node.

3.3 Calculation of Support Reactions

The truss is designed to be statically determinate, meaning the supports provide exactly three reaction forces. Support A is a roller, which resists only vertical forces R_{A_y}, while support H is fixed and resists both horizontal R_{H_x} and vertical R_{H_y} forces. A consistent sign convention is followed throughout the analysis: positive reaction forces act in the positive x or y direction, while negative values indicate forces acting in opposite directions. The support reactions are determined using static equilibrium conditions:

- ($\sum F_x = 0$ (Equilibrium in X direction)
  - The sum of all horizontal forces must balance.
  - Since the roller support at A restricts only vertical movement, the horizontal reaction R_{H_x} is found by:

$$R_{H_x} = -\sum F_x$$

- ($\sum F_y = 0$ (Vertical Equilibrium)
  - The sum of all vertical forces must balance.
  - Reactions R_{A_y} (roller) and R_{H_y} (fixed support) satisfy:

$$R_{A_y} + R_{H_y} = -\sum F_y$$

- ($\sum M = 0$ (Moment Equilibrium about A)
  - Taking moments about A eliminates R_{A_y}.
  - The moment due to external forces and R_{H_y} must balance each other out.
  - $\sum$ Moments of all forces about A$+R_{H_y} \cdot L = 0$, where L is the horizontal distance from A to H.
  - Solves for R_{H_y}, and then RAY is found from vertical equilibrium

3.4 Method of Joints for Internal Forces

The following assumptions are made to calculate the internal forces in the truss members.
1.Members are weightless.
2. Loads act only at joints.
3. All connections are pinned (no moments).

- Unit Vectors: Each truss member is represented as a vector, normalized to compute directional components $(\cos\theta, \sin\theta)$
- Equilibrium Conditions: At every joint, forces must satisfy:
  - ($\sum F_x = 0$ (Sum of horizontal forces = 0)
- System of Equations: For n joints, 2 n equations are formed and solved as a matrix to determine member forces (tension/compression).
  - ($\sum F_y = 0$ (Sum of vertical forces = 0)
  - We begin at a joint with less than equal to 2 unknowns (e.g., support reactions) and solve till all the internal forces are determined.
- Force Resolution: Member forces are decomposed into x and y components using unit vectors.

3.5 Force Classification in Truss Members

The internal forces in truss members are categorized based on their mechanical behavior:

- Tension: Members with positive forces elongate (pulled apart).
- Compression : Members with negative forces shorten (pushed together).
- Zero Force: Members with negligible forces (near-zero magnitude) remain inactive.

3.6 Visualization of Results

The calculated member forces are presented through a color-coded truss diagram. Members in tension are shown in red, while those in compression appear in green. Members carrying negligible force are marked in black. Each member is labeled with its corresponding force value, and arrows indicate the support reactions along with their magnitudes. A legend is included to explain the color coding, enabling quick and clear interpretation. This visual representation allows for easy identification of critical members and helps verify that the structure satisfies equilibrium conditions.

4 Numerical implementation

In this project, we applied the **Method of Joints**, as taught in class, to analyze a truss structure. First, we verified the equilibrium equations and the general approach on paper, ensuring that each node satisfies the balance of forces in both x and y directions and that global equilibrium (including moments about support) is correctly applied to find reaction forces. After confirming the hand-calculated steps, we translated this procedure into the Mathematica code. The code allows the user to input external loads at various nodes and automatically computes internal member forces (tension or compression) and the support reactions, finally displaying a color-coded diagram.

The implementation allows the user to:

- Enter external loads in the format: `Node Fx Fy Node Fx Fy ...`
- Automatically compute all internal member forces and support reactions
- Display the final truss diagram with colored members:
  - Red for tension
  - Green for compression
  - Black if negligible force
- Include a legend and formatted numeric labels (in decimal form)

Below is a concise pseudocode representation of our program, explaining each variable and how the approach is implemented:

4.1 Pseudocode Overview

(1) Clear all previous symbols
  - Ensure clean workspace.

(2) Define variables
  - Declare internal force symbols and reaction components (E_x, E_y, A_y)

(3) Geometry setup
  - Define node positions (as 2D coordinates)
  - Define member connections (list of node pairs)

(4) Input external loads
  - Parse input string (Node Fx Fy ...)
  - Store forces in externalForces[node]

(5) Global equilibrium to find support reactions
  - Sum external forces: (F_x, (F_y
  - Sum moments about node A
  - Solve for (E_y using moment balance
  - Solve for (A_y using vertical force balance
  - (E_x is -(F_x (roller at A cannot resist horizontal force)

(6) Method of Joints
  - Define unit vectors for each member (Normalize[position differences])
  - For each node, apply:
    - sum((F_x) = 0
    - sum((F_y) = 0
  - Solve the resulting system for all internal member forces

(7) Visualization
  - Draw lines for each member, color-coded based on force sign
  - Force values are labelled in decimal, always in black
  - Arrows show reaction forces at supports
  - A legend is placed to indicate tension (red) vs compression (green)

4.2 Variable Definitions Table

- $F_{AB}, F_{BC}, \ldots$:
  Force in each truss member (from node X to node Y)
- A_y, E_x, E_y :
  Support reactions at node A (roller) and E (fixed)

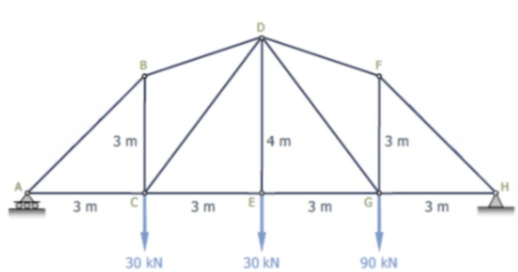
- `positions`:
  Association mapping node names to 2-D coordinates
- `connections`:
  List of edges (member pairs) connecting two nodes
- `externalForces`:
  Forces applied by the user at each node as 2-D vectors
- u_{XY} :
  Unit vector from node X to node Y
- $eq_A, eq_B, \ldots$:
  Equilibrium equations for each node in x and y directions
- `solVals`:
  Association of solved member force values
- `drawTrussSolution[]`:
  Function to generate the final annotated and color-coded truss diagram

In essence, the Method of Joints (applied node-by-node) and global equilibrium (applied to find support reactions) form the backbone of our code. By combining the symbolic solver in Mathematica with a straightforward approach for visualizing the geometry, we can efficiently compute and illustrate all the forces in the truss system.

5 Results and discussion

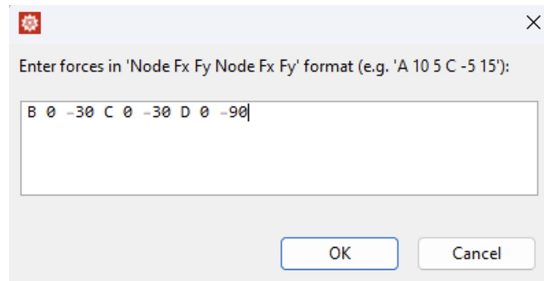
5.1 Joint Locations and Connectivity

We have taken the following truss for analysis.The truss is composed of eight nodes labeled A, B, C, D, E, F, G, and H. It features a lower chord formed by nodes A, B, C, D, and E, and an upper chord made up of nodes F, G, and H. These chords are interconnected by diagonal and vertical members that create a stable framework for distributing applied loads throughout the structure.



5.2 User Load Input

The analysis is designed to take load inputs from the user interactively. When the program is executed, the user is prompted to enter the loads in the following format:



This indicates that:

- At node B, a load of 0 kN in the horizontal direction and -30 kN (downward) in the vertical direction is applied.
- At node C, a load of 0 kN in the horizontal direction and -30 kN (downward) in the vertical direction is applied.
- At node D, a load of 0 kN in the horizontal direction and -90 kN (downward) in the vertical direction is applied.

After processing the input, the code confirms the applied loads by printing an output similar to:

```
Applied External Forces (kN):
A: {0, 0}
B: {0, -30}
C: {0, -30}
D: {0, -90}
E: {0, 0}
F: {0, 0}
G: {0, 0}
H: {0, 0}
```

5.3 Computation of Support Reactions

Once the load input is captured, the solver applies the principles of static equilibrium. The overall equilibrium equations ($\sum F_x = 0$, $\sum F_y = 0$, and $\sum M = 0$) are used to determine the support reactions at nodes A and E:

- **Support at A (Roller):**
  Since node A is a roller support, it only provides a vertical reaction A_y (with $A_x = 0$).

- **Support at E (Fixed in-plane):**
  Node E offers both a horizontal reaction E_x and a vertical reaction E_y.

```
Global Reaction Forces:
At A (roller):   (Ax=0, Ay) = {0, 60.000}  kN
At E (fixed):    (Ex, Ey) = {0, 90.000}  kN
```

5.4 Determination of Internal Member Forces

Next, using the Method of Joints, the solver establishes equilibrium equations for each node. Unit vectors for every truss member are computed based on the node coordinates, and the resulting simultaneous equations are solved symbolically for the axial forces in the individual members.

Member	Nature of Force	Force (kN)
F_{AB}	Tensile	60.000
F_{BC}	Tensile	67.500
F_{CD}	Tensile	67.500
F_{DE}	Tensile	90.000
F_{AF}	Compressive	84.853
F_{BF}	Tensile	40.000
F_{FG}	Compressive	63.246
F_{CG}	Tensile	30.000
F_{GH}	Compressive	94.868
F_{DH}	Tensile	60.000
F_{HE}	Compressive	127.280
F_{BG}	Compressive	12.500
F_{DG}	Tensile	37.500

5.5 Visualization: Truss Force Diagram

The final output of the solver is a detailed truss force diagram that uses color coding to distinguish between tension and compression in each member:

- **Red Lines**: Indicate members in tension.
- **Green Lines**: Indicate members in compression.
- **Black Lines**: Indicate members with forces near zero (within a defined tolerance).

Additionally, the reaction forces at nodes A and E are depicted with arrows and their numerical values are labeled directly on the diagram.

5.6 Discussion

Load Distribution and Reaction Forces

- **Load Distribution**: The user-specified loads at nodes B, C, and D are transmitted through the truss. The load at node D (90 kN) is the highest, resulting in significant internal forces in the members closest to that node.
- **Support Reactions**: The computed support reactions confirm that node A (roller) provides only a vertical force A_y, while node E (fixed) reacts with both horizontal E_x and vertical E_y components. These reactions balance the externally applied loads, ensuring static equilibrium.

Internal Forces and Structural Behavior

- **Tension vs. Compression**: The analysis reveals that certain members (e.g., CD and FG) are primarily in tension, while the majority of the other members experience compression. The negative sign in the computed force value indicates compression; a positive value indicates tension.
- **Implications for Design**:
  - Compression members should be checked for buckling risks, particularly those showing high compressive values.
  - Tension members require validation against yield strength to avoid excessive elongation or failure under load.

Visual Insights from the Diagram

- The color-coded diagram helps us easily see how the load moves through the truss.
- It clearly shows the important parts that carry more force, using red to show tension and green to show compression. This makes it easier to understand the structure and make improvements if needed.

6 Learning outcomes

Through the process of analyzing truss systems in this project, we learned to apply the fundamental principles of static equilibrium to determine both support reactions and internal member forces , gaining practical experience with the Methods of Joints. The project also helped to enhance our computational skills particularly in using Mathematica to automate the analysis , solve systems of equations and visualize results. Moreover ,we became skilled at classifying truss members as being in tension, compression, or carrying negligible force and also learned to interpret color-coded diagrams to assess structural behaviour. Additionally, working together collaboratively improved our teamwork and communication skills, enabling us to approach complex problems more effectively.

[1]

References

[1] Jhun Vert. Problem 414: Truss by method of joints. https://mathalino.com/reviewer/engineering-mechanics/problem-414-truss-method-joints, 2025. Accessed: April 15, 2025.