C2 S4: Monte Carlo Methdod for Estimating Heat Content

I. Monte Carlo Integration

   A. Definition and application

   B. Heat content Q_Ω

      i. Golbal solution $u(\boldsymbol{s}, t)$: integration over the initial position: conditional probability multiply initial condition

         a. Estimating the local solution based on the Random Walk Method (RWM)

         b. Generating initial positions distrbuted uniformly in the domain, and run RWM to estimate $u(\boldsymbol{s}, t)$.

      ii. Intergation $u(\boldsymbol{s}, t)$ over the space domain

II. Brownian Motion (BM)

   A. General description and history of BM (easier for non-mathematican to understand)

      i. irregualer, continuous, and permanent random motion found by Brownion: microscopic pollen grains suspend in the water

      ii. Einstein's explaination for BM and the solution to the heat equation

         a. Becaue of the continual collision from the surrounding water molecules, pollen grains have the same average kinetic energy as the molecules.

         b. Einstein's proof: BM provides a solution to the Fourier's heat equation

   B. Mathematical Prospective

      i. Formula

      ii. BM's propertities

   C. Random Walk

      i. Definition and history

      ii. Connection with BM