An Alternative Method for Characterization and Comparison of Plant Root Shapes

A thesis submitted to the
College of Graduate and Postdoctoral Studies
in partial pulfillment of the requirements
for the degree of Master of Science
in the Department of School of Environment and Sustainability
University of Saskatchewan
Saskatoon

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Contents

Existing Morphological Descriptors for Root Systems

An Alternative Mathematical Method for Shape Description

LRWs in Artificial Images

3.1 Circle and Rectangle

3.1.1 Image Description

- Image size: 1200×1000 pixels
- Surface area of shapes: 90000 pixels
- The centroid of the shape is located at the center of the image

3.1.2 Output Analysis

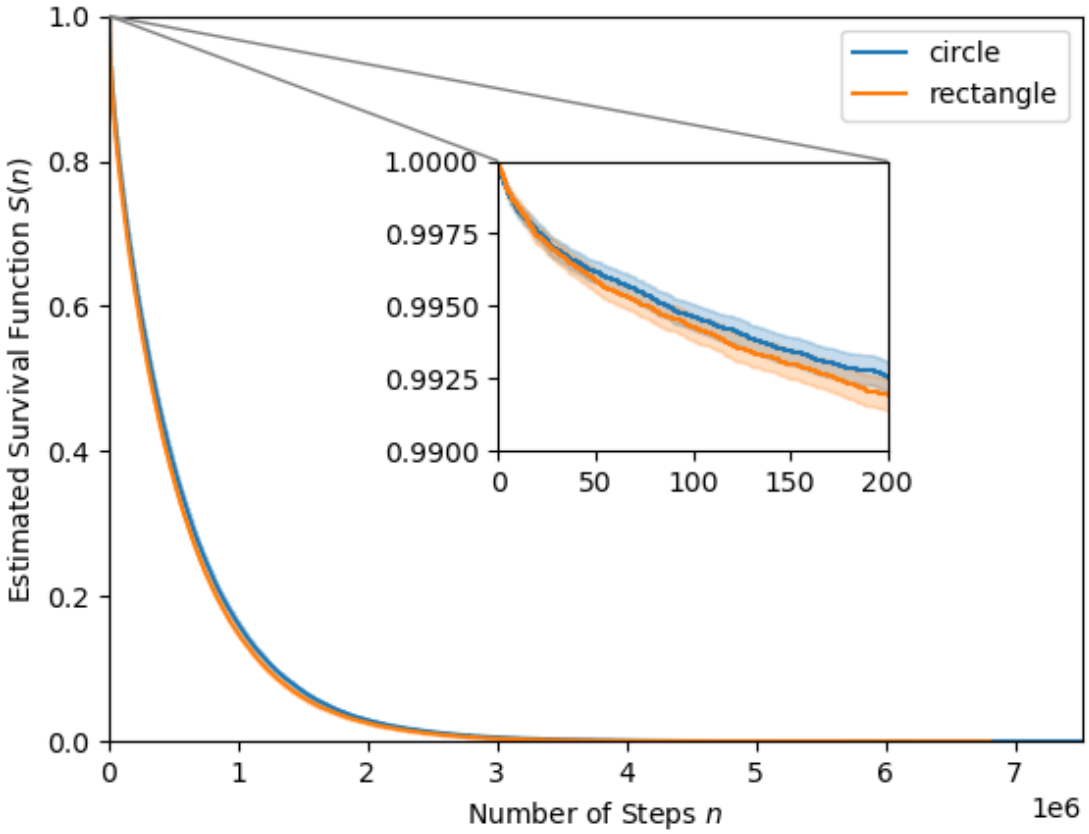


Figure 3.1

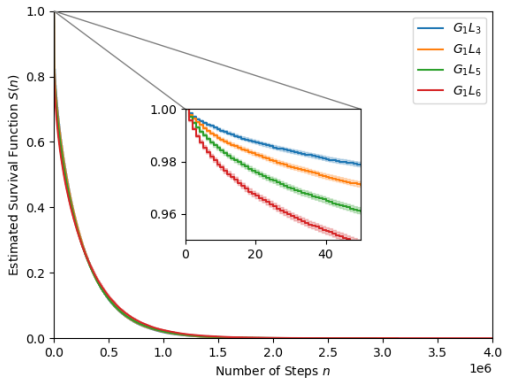
3.1.3 Conclusion

Given two distinct convex geometries

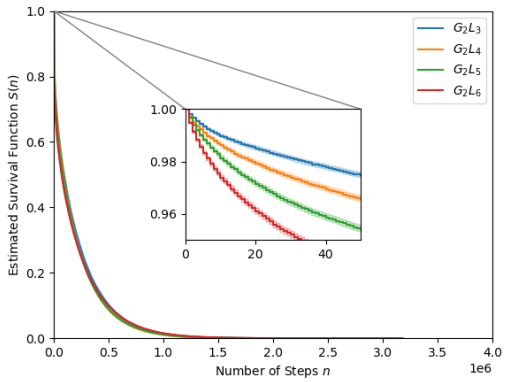
	test_statistic	p
Peto	137.23	0.0
Logrank	137.23	0.0
Tarone-Ware	134.31	0.0
Gehan-Breslow	123.83	0.0
Fleming-Harrington	123.83	0.0

Table 3.1

- the behaviours of the survival function of LRWs are consistent with the theoretical results.
- survival curves can be used to describe and distinguish them.



(a)



(b)

Figure 3.2

3.2 Complicated Branching Structures

3.2.1 Image Description

- Image size: 1200×1000 pixels
- Surface area of shapes: 90000 pixels
- Iterate the template $3, 4, 5, 6$ times to produce the targeted branching geometries labelled as L_3, L_4, L_5, L_6.
- Two groups of images labelled as G_1, G_2
  - G_1: the target object G_1L_i $(i = 3, 4, 5, 6)$ is equidistant to the edges of an image.
  - G_2: the template of G_2L_i $(i = 3, 4, 5, 6)$ is distinct from G_1 (thickness and aspect ratio).

3.2.2 Output Analysis

3.2.2.1 $S(n)$

3.2.2.1.1 Estimated Survival Curves

3.2.2.1.2 Non-Parametric Statistical Tests

3.2.2.1.3 Mesurement of Dissimilarities for $S(n)$

		p			
		Logrank	TW	GB	FH
$G_1\ L_3$	$G_1\ L_4$	0.4393	0.0285	0.0005	0.0005
	$G_1\ L_5$	0.0	0.0	0.0	0.0
	$G_1\ L_6$	0.0	0.0	0.0	0.0
$G_1\ L_4$	$G_1\ L_5$	0.0007	0.0	0.0	0.0
	$G_1\ L_6$	0.0002	0.0	0.0	0.0
$G_1\ L_5$	$G_1\ L_6$	0.7223	0.0	0.0	0.0

Table 3.2

		p			
		Logrank	TW	GB	FH
$G_2\ L_3$	$G_2\ L_4$	0.0	0.0	0.0	0.0
	$G_2\ L_5$	0.0	0.0	0.0	0.0
	$G_2\ L_6$	0.0	0.0	0.0	0.0
$G_2\ L_4$	$G_2\ L_5$	0.0016	0.0	0.0	0.0
	$G_2\ L_6$	0.0004	0.0	0.0	0.0
$G_2\ L_5$	$G_2\ L_6$	0.7199	0.0	0.0	0.0

Table 3.3

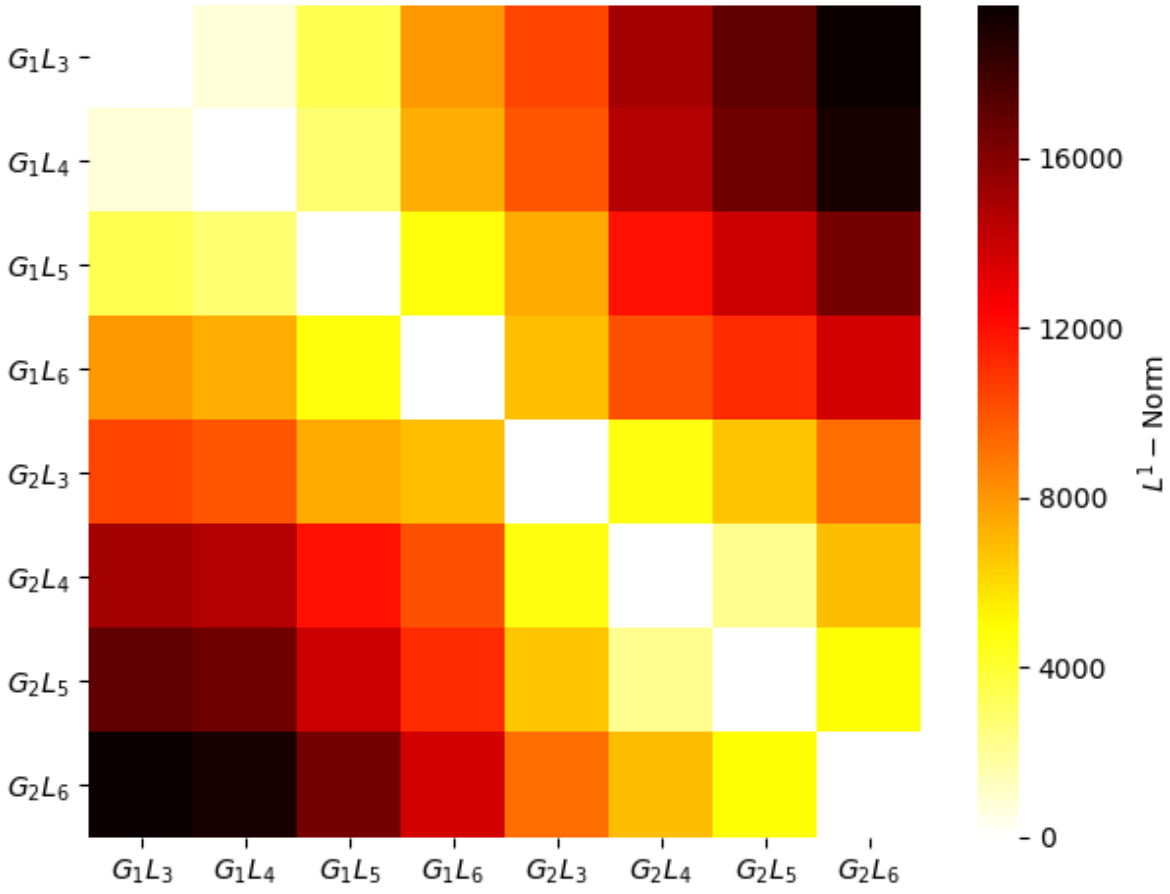

G_1L_3
G_1L_4
G_1L_5
G_1L_6
G_2L_3
G_2L_4
G_2L_5
G_2L_6
16000
12000
8000
4000
0
L^1- Norm

Figure 3.3

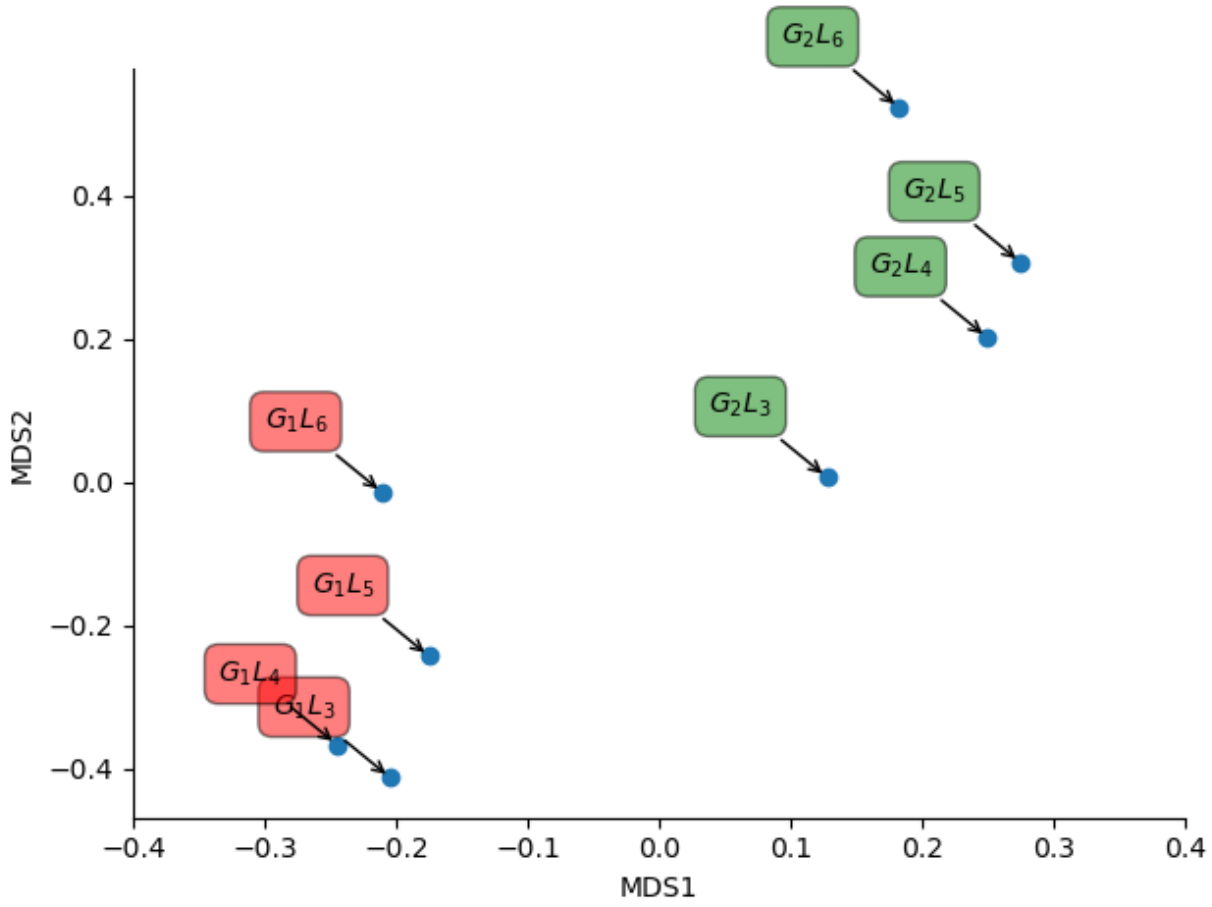


Figure 3.4

3.2.2.1.4 Visualize $S(n)$

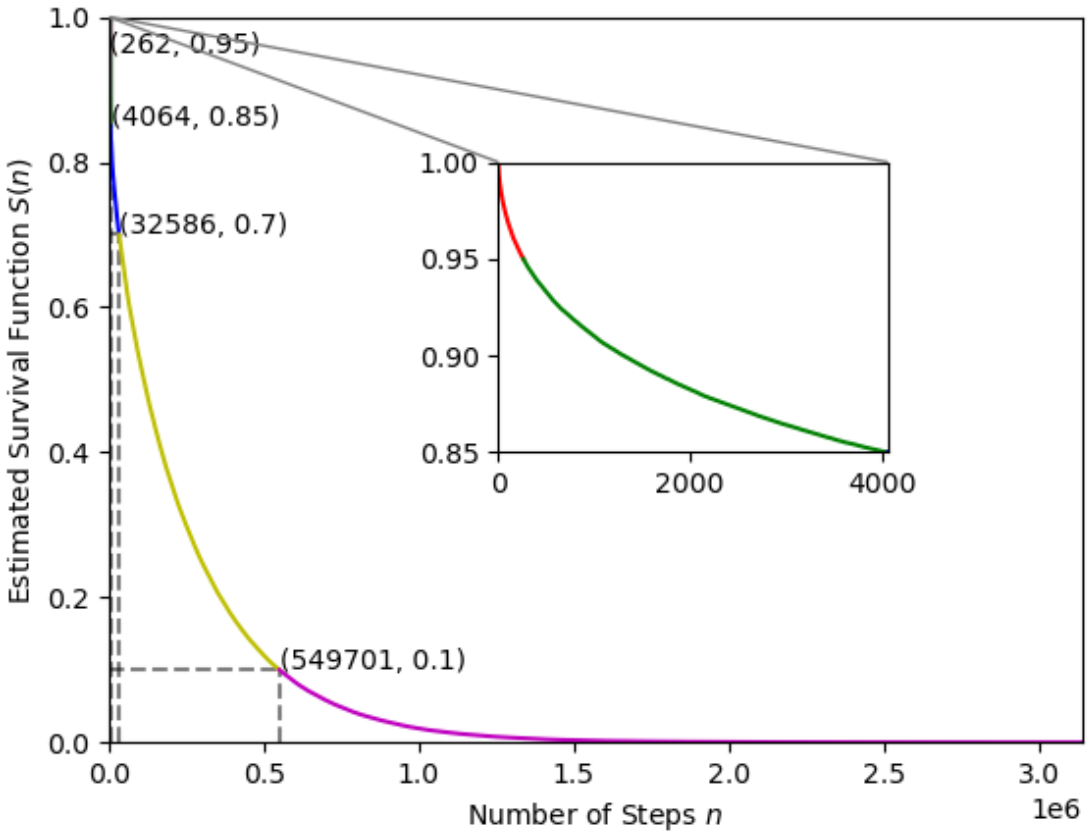

(262, 0.95)
(4064, 0.85)
(32586, 0.7)
(549701, 0.1)
Estimated Survival Function $S(n)$
Number of Steps n
1e6
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
1.00
0.95
0.90
0.85
0
2000
4000

Figure 3.5

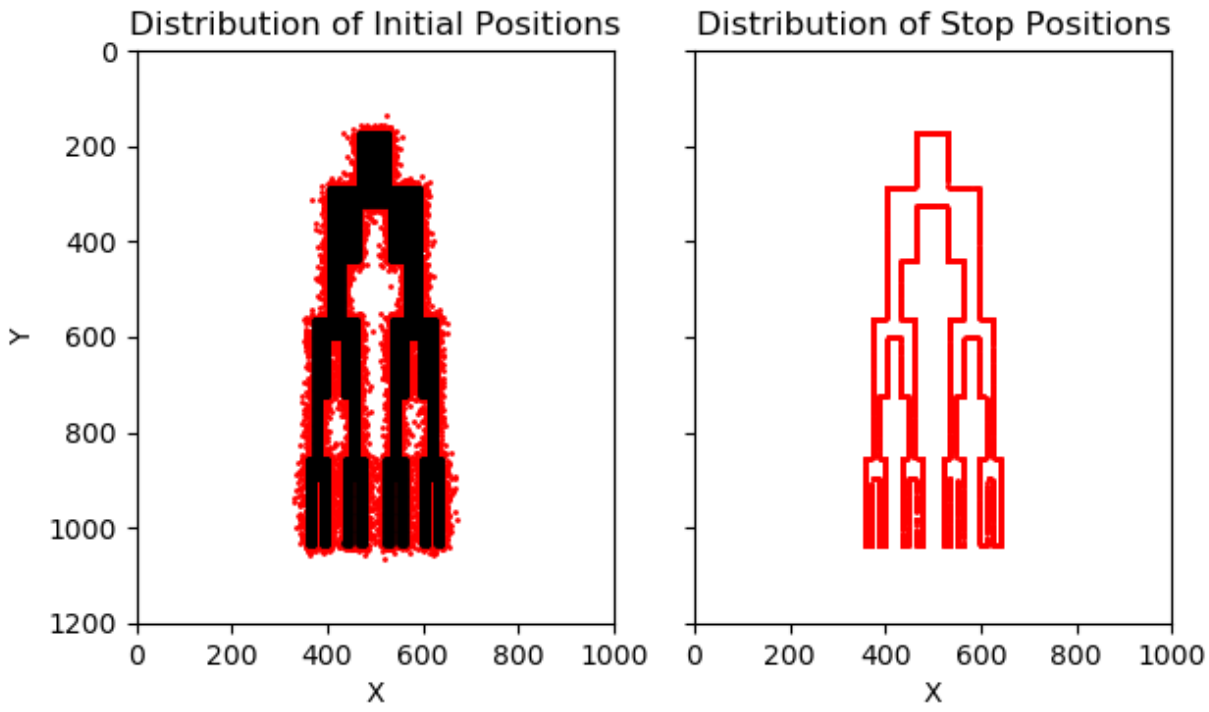

Distribution of Initial Positions
Distribution of Stop Positions
Y
X
X
0
200
400
600
800
1000
1200

Figure 3.6

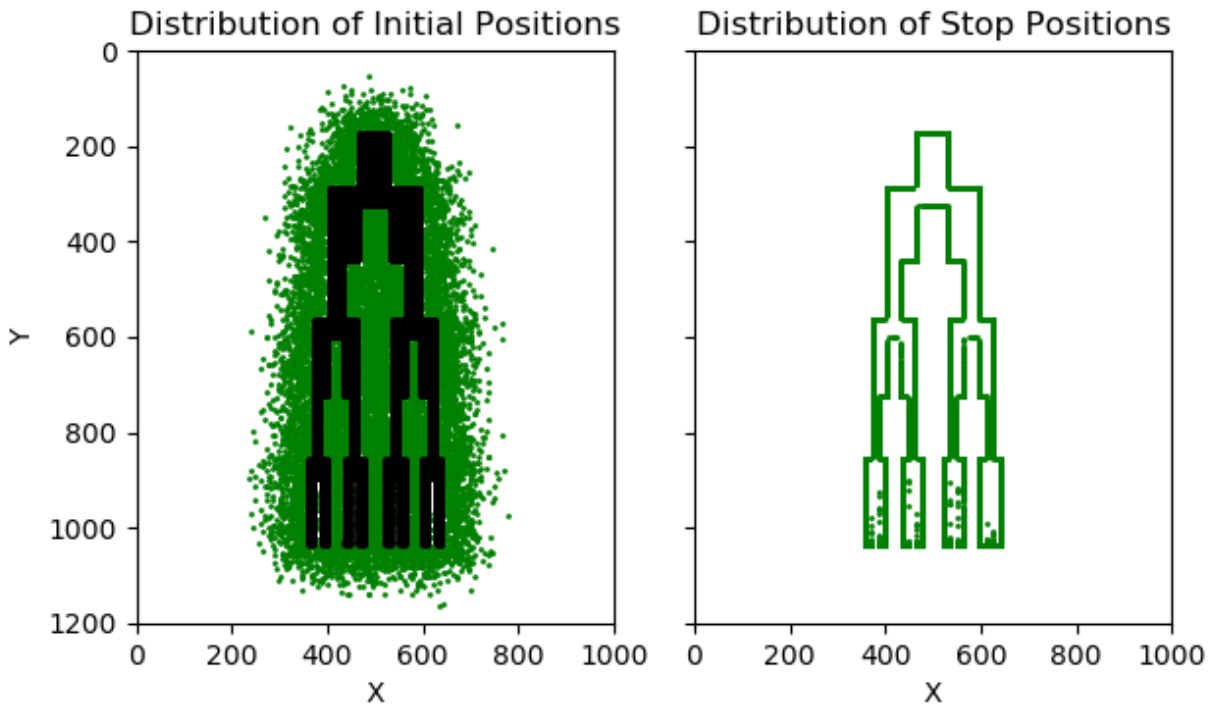

Distribution of Initial Positions
Distribution of Stop Positions
Y
X
X
0
200
400
600
800
1000
1200

Figure 3.7

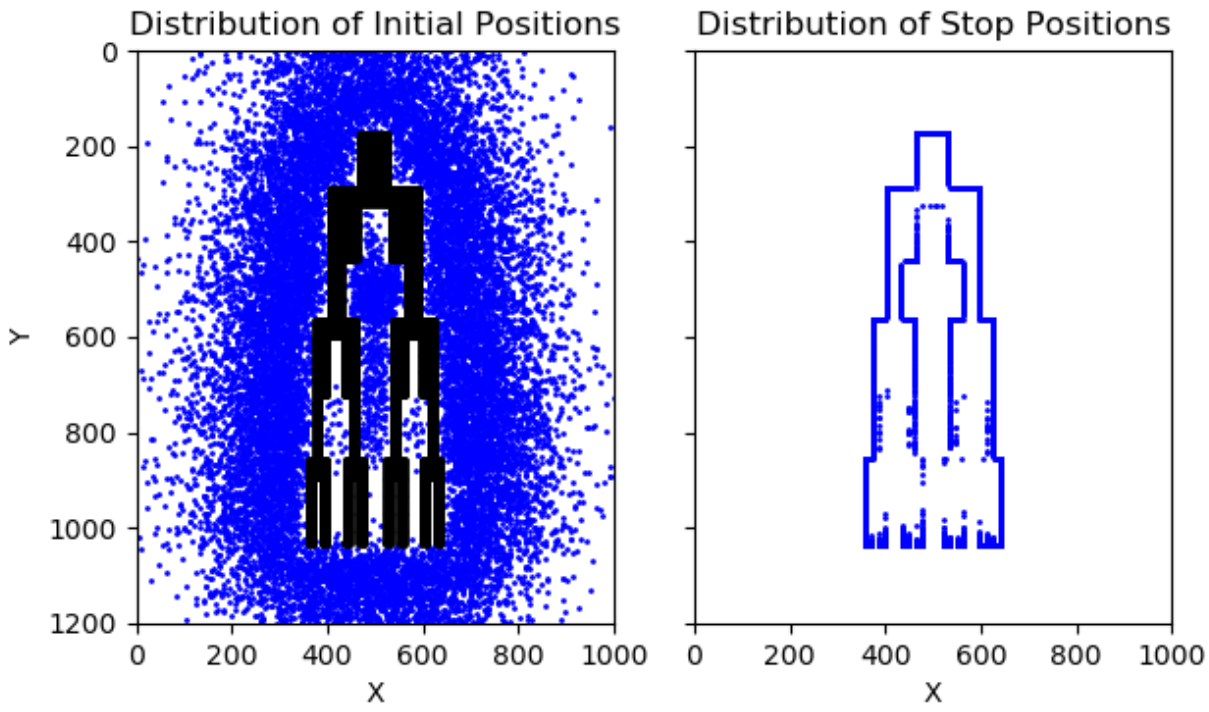

Distribution of Initial Positions
Distribution of Stop Positions
Y
X
X
0
200
400
600
800
1000
1200

Figure 3.8

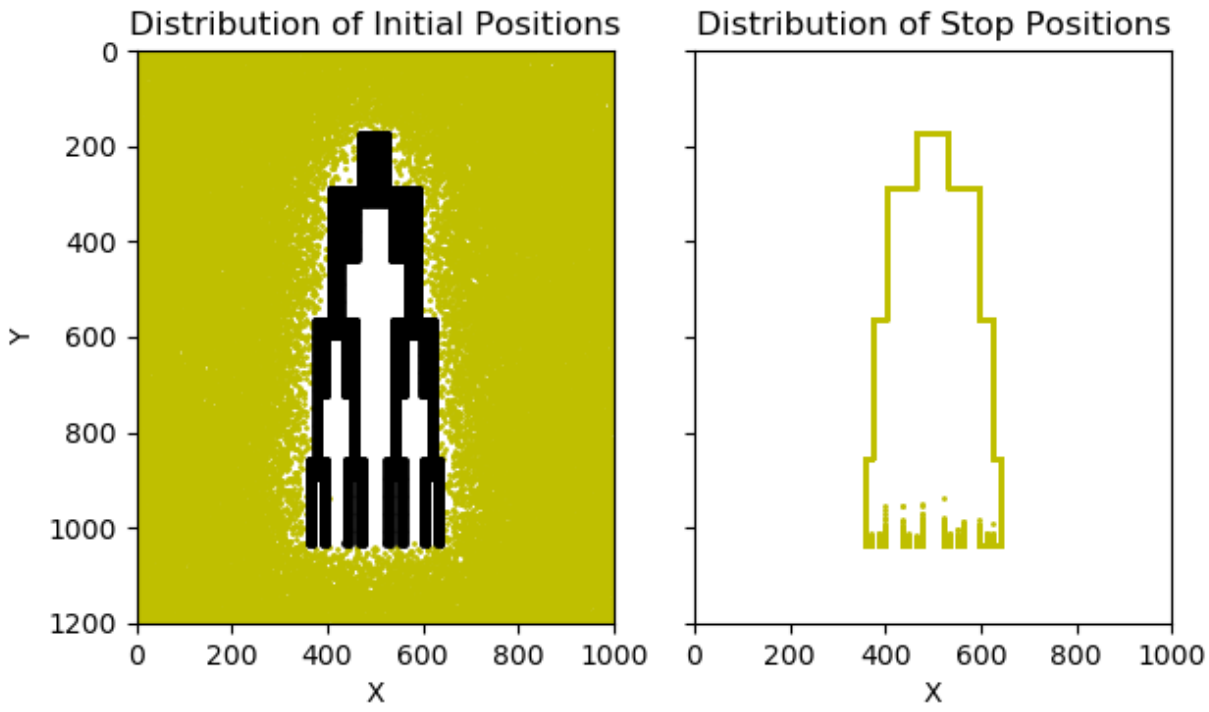

Distribution of Initial Positions
Distribution of Stop Positions
Y
X
X
0
200
400
600
800
1000
1200

Figure 3.9

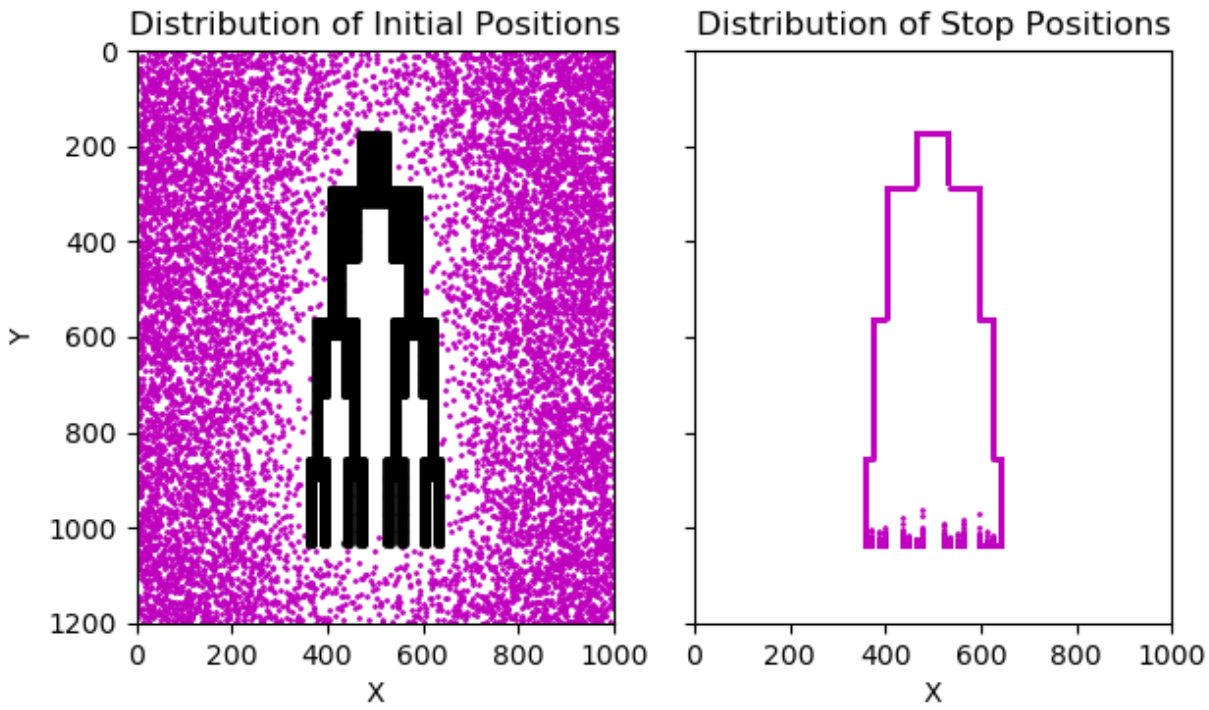
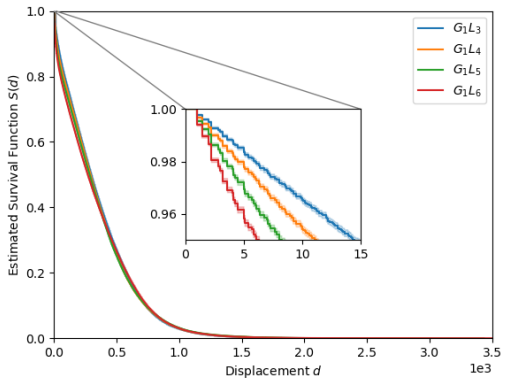
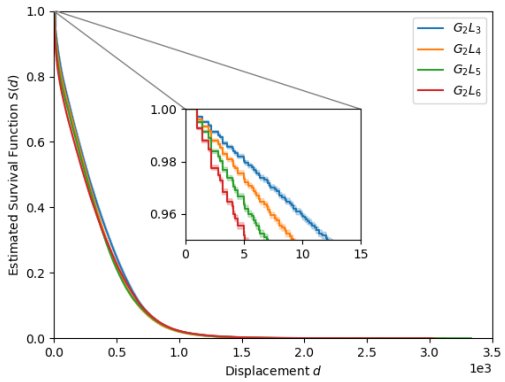

Distribution of Initial Positions
Distribution of Stop Positions
Y
X
X
0
200
400
600
800
1000
1200

Figure 3.10



(a)



(b)

Figure 3.11

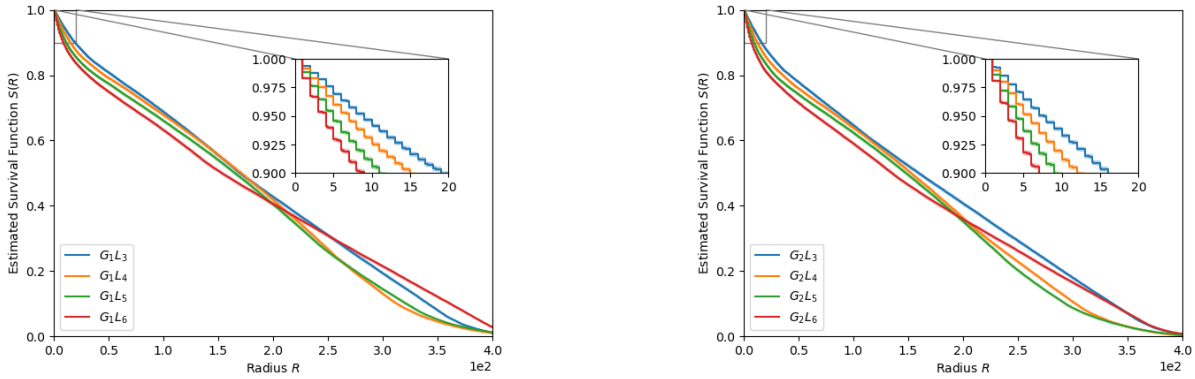
		p			
		Logrank	TW	GB	FH
$G_1\ L_3$	$G_1\ L_4$	0.0	0.0	0.0	0.0
	$G_1\ L_5$	0.0	0.0	0.0	0.0
	$G_1\ L_6$	0.0	0.0	0.0	0.0
$G_1\ L_4$	$G_1\ L_5$	0.0072	0.0	0.0	0.0
	$G_1\ L_6$	0.0003	0.0	0.0	0.0
$G_1\ L_5$	$G_1\ L_6$	0.2883	0.0	0.0	0.0

Table 3.4

3.2.2.2 $S(d)$

		p			
		Logrank	TW	GB	FH
$G_2\ L_3$	$G_2\ L_4$	0.0	0.0	0.0	0.0
	$G_2\ L_5$	0.0	0.0	0.0	0.0
	$G_2\ L_6$	0.0	0.0	0.0	0.0
$G_2\ L_4$	$G_2\ L_5$	0.0001	0.0	0.0	0.0
	$G_2\ L_6$	0.0015	0.0	0.0	0.0
$G_2\ L_5$	$G_2\ L_6$	0.7019	0.0	0.0	0.0

Table 3.5



(a) **(b)**

Figure 3.12

3.2.2.3 $S(R)$

		p			
		Logrank	TW	GB	FH
$G_1\ L_3$	$G_1\ L_4$	0.0	0.0	0.0	0.0
	$G_1\ L_5$	0.0	0.0	0.0	0.0
	$G_1\ L_6$	0.0	0.0	0.0	0.0
$G_1\ L_4$	$G_1\ L_5$	0.1773	0.0	0.0	0.0
	$G_1\ L_6$	0.0	0.0	0.0	0.0
$G_1\ L_5$	$G_1\ L_6$	0.0	0.0	0.0	0.0

Table 3.6

		p			
		Logrank	TW	GB	FH
$G_2\ L_3$	$G_2\ L_4$	0.0	0.0	0.0	0.0
	$G_2\ L_5$	0.0	0.0	0.0	0.0
	$G_2\ L_6$	0.0	0.0	0.0	0.0
$G_2\ L_4$	$G_2\ L_5$	0.0	0.0	0.0	0.0
	$G_2\ L_6$	0.0	0.0	0.0	0.0
$G_2\ L_5$	$G_2\ L_6$	0.0	0.0	0.0253	0.0253

Table 3.7

3.2.3 Conclusion

- In a short time, the survival function of rectangle decays faster than the circle, which conforms to the analytical results.
- The differences of estimated survival functions between circle and rectangle are statistically significant, which coincides with the real shape dissimilarities.
- Within a same group, when t is small, the more branching the object is, the faster the survival function decays.
- Within a same group, the pairwise survival functions are statistically different.
- The corresponding target structures in G_1 and G_3 are invariant shapes under translation since their survival function are not statistically different. In other words, periodic boundary conditions of the image can eliminate the effect of the locations.
- LRWs can describe and classify the geometries, their spatial configurations, and the unoccupied area in the image.

LRWs in Real Root Images

Conclusion

Future Work

Appendix A

Numerical Methods for Solving Parabolic Partial Differential Equations

A.1 Introduction

A.2 Summary of Commonly Used Numerical Techniques

A.3 Limitation in Practice

Appendix B

Method Validation in Annulus

B.1 Analytical Results

B.2 Numerical Approximation

B.3 Comparison of Numerical and Analytical Results

B.4 Conclusion

References