Practical Part: Linear Regression

October 14, 2018

In [2]:
```
#!/usr/bin/env python
import sys, os
import numpy as np
from random import uniform
import math
get_ipython().run_line_magic('pylab', 'inline')
import matplotlib.pyplot as plt

class regression_gradient:
    def __init__(self, lamb=100, step_size=1, n_steps=100):
        self.lamb = lamb
        self.step_size = step_size
        self.n_steps = n_steps

    def train(self, X, y):
        """Question 1: ridge regression with gradient descent"""

        # if X is a vector, add dummy dimension
        if np.ndim(X) == 1:
            X = np.expand_dims(X, axis=1)

        old_grad = 0
        stopping_tolerance = 1e-10

        self.n = np.shape(X)[0]
        self.d = np.shape(X)[1]
        self.w = np.random.uniform(low=-0.01, high=0.01, size=self.d + 1) # includes b

        # include column of 1s for b
        X = np.hstack((np.ones((self.n, 1)), X))

        empirical_risk = np.zeros((self.d + 1))
        for i in range(self.n_steps):
            empirical_risk = X.T.dot(X).dot(self.w) - X.T.dot(y)
            regularization = self.lamb * 2 * self.w
            gradient = (empirical_risk + regularization) #/ self.n
```

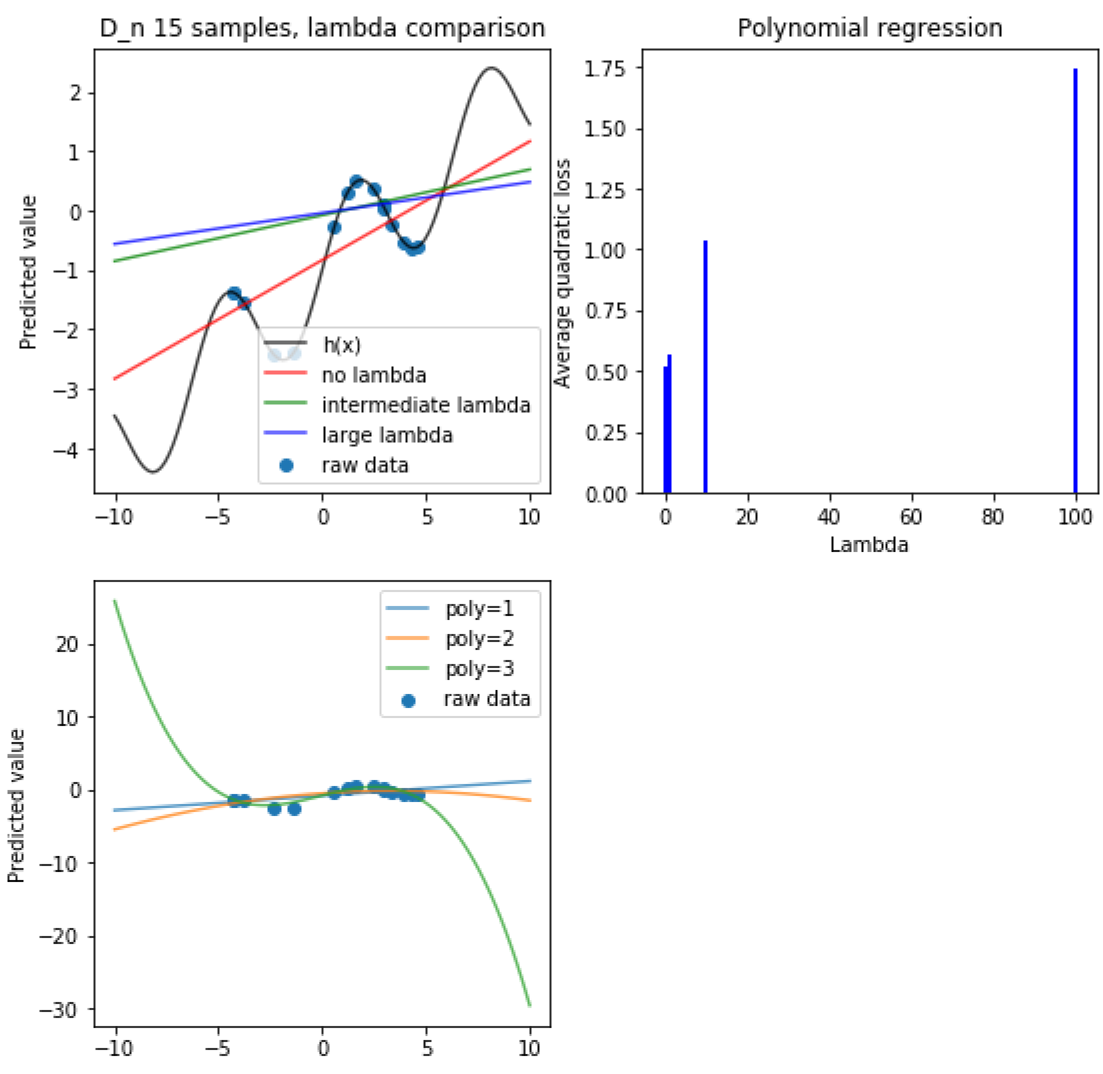
```python
            ## stopping criteria
            #if np.sum(np.abs(gradient - old_grad)) < stopping_tolerance:
            #    break

            self.w -= self.step_size  * gradient



    def predict(self, X):
        n = X.shape[0]
        y = np.zeros(n)
        w = self.w[1:]
        b = self.w[0]

        for j in range(n):
            y[j] = w.T.dot(x[j]) + b

        return(y)


def sample_h(n):
    """
    Question 2.
    h(x) = sin(x) + 0.3x -1
    Returns dataset D (x, h(x)) with n points. x in [-5, 5].
    """
    D = np.zeros((n,2))
    for i in range(n):
        D[i,0] = uniform(-5,5)
        D[i,1] = math.sin(D[i,0]) + 0.3*(D[i,0]) - 1
    return D


# test
iris= np.loadtxt('iris.txt')
train_X = iris[:, :-1]
train_Y = iris[:, -1]
mdl = regression_gradient(lamb=0.005, step_size=0.01, n_steps=10)
mdl.train(train_X, train_Y)
print(mdl.w)

# plottng options
n_bins = 100
axes_min = -10
axes_max = 10
alpha = 0.75
x = np.atleast_2d(np.linspace(axes_min, axes_max, n_bins)).T
```

```python
fig, axs = plt.subplots(2, 2, figsize=(9, 9))

# question 3
# plot data
D = sample_h(15)
axs[0][0].scatter(D[:,0], D[:,1]) # raw data

# plot h(x)
y1 = np.zeros(n_bins)
for i in range(n_bins):
    y1[i] = math.sin(x[i]) + 0.3*(x[i]) - 1
axs[0][0].plot(x, y1, color='black', alpha=alpha)

# training settings
step_size = 5e-5
n_steps = 10000

# plot regression_gradient --lambda 0
mdl_1 = regression_gradient(lamb=0, step_size=step_size, n_steps=n_steps)
mdl_1.train(D[:,0], D[:,1])
axs[0][0].plot(x, mdl_1.predict(x), color='red', alpha=alpha)

# question 4
# plot regression_gradient - lambda intermediate
mdl_2 = regression_gradient(lamb=62.5, step_size=step_size, n_steps=n_steps)
mdl_2.train(D[:,0], D[:,1])
axs[0][0].plot(x, mdl_2.predict(x), color='green', alpha=alpha)

# plot regression_gradient - lambda large
mdl_3 = regression_gradient(lamb=125, step_size=step_size, n_steps=n_steps)
mdl_3.train(D[:,0], D[:,1])
axs[0][0].plot(x, mdl_3.predict(x), color='blue', alpha=alpha)

axs[0][0].legend(['h(x)',
                  'no lambda',
                  'intermediate lambda',
                  'large lambda',
                  'raw data'])

axs[0][0].set_ylabel('Predicted value')
axs[0][0].set_title('D_n 15 samples, lambda comparison')

# question 5
D_test = sample_h(100)
lambs = [0.0001, 0.001, 0.01, 0.1, 1, 10, 100]

avg_loss = np.zeros(len(lambs))
for i, lamb in enumerate(lambs):
```

```
    mdl = regression_gradient(lamb=lamb , step_size=step_size, n_steps=n_steps)
    mdl.train(D[:,0], D[:,1])

    b= mdl.w[0]
    w= mdl.w[1:]

    loss = 0
    for j in range(len(D_test)):
        loss += (np.take(((w.T.dot(D_test[j,0]) + b) - D_test[j,1]), 0))**2
    avg_loss[i] = loss/len(D_test)

# bar plot of average quadratic loss per lambda
width = 1.0
axs[0][1].bar(lambs, avg_loss, width, color='b' )
axs[0][1].set_ylabel('Average quadratic loss')
axs[0][1].set_xlabel('Lambda')
axs[0][1].set_title('Average quadratic loss per lambda')

# question 6
orders = [1, 2, 3]
for i, l in enumerate(orders):

    # polynomial preprocessing
    poly = np.zeros((np.shape(D)[0], i+1))
    for j in range(np.shape(D)[0]):
        poly[j] = np.array([D[j,0]**exp for exp in orders[0:i+1]])

    # ridge regression
    mdl_4 = regression_gradient(lamb=0.01, step_size=step_size, n_steps=n_steps)
    mdl_4.train(poly, D[:,1])
    b = mdl_4.w[0]
    w = mdl_4.w[1:]

    # ploting f learned with ridge regression
    y = np.zeros(n_bins)
    for z in range(n_bins):
        processed_x = np.array([x[z]**exp for exp in orders[:i+1]])
        y[z] = np.dot(w.T, processed_x) + b
    axs[1][0].plot(x, y , alpha=alpha)

#Raw data
axs[1][0].scatter(D[:,0], D[:,1])
axs[1][0].legend(['poly=1', 'poly=2', 'poly=3', 'raw data'])
axs[1][0].set_ylabel('Predicted value')
axs[0][1].set_title('Polynomial regression')

fig.delaxes(axs[1][1])
```

```
plt.show()

plt.savefig('report.jpg')
```

```
Populating the interactive namespace from numpy and matplotlib
[-1.58191754e+18 -9.49306545e+18 -4.80645172e+18 -6.47767158e+18
 -2.11933527e+18]


/opt/python/sci_36/lib/python3.6/site-packages/IPython/core/magics/pylab.py:160: UserWarning: 
`%matplotlib` prevents importing * from pylab and numpy
  "\n`%matplotlib` prevents importing * from pylab and numpy"
```




```
<matplotlib.figure.Figure at 0x7fb99e00b9b0>
```