

University of Neuchâtel
Discrete Mathematics and Applications - Fall 2025
Problems - 2

1. In electronics, a multiplexer is a device that selects one of several analog or digital input signals and forwards the selected input into a single line.

A 2-1 binary multiplexer has two boolean input variables I_1 and I_2 , and a binary selector S . If the selector is set to 1, then the output of the multiplexer is I_1 , and if the selector is set to 0, then the output of the multiplexer is I_2 .

- (a) Construct a truth table for the 2-1 multiplexer.
 - (b) Suppose $M(S, I_1, I_2)$ is the resulting boolean function. Express M in *full disjunctive normal form* (full DNF) *i.e.* a sum-of-products of minterms.
 - (c) Simplify M if possible, and construct a boolean circuit for M with NOT, AND, and OR gates only.
 - (d) Redo the above circuit with NAND gates only.
 - (e) Express M in *full conjunctive normal form* (full CNF) *i.e.* a product-of-sums of maxterms.
 - (f) Using the above expression, construct a boolean circuit for M with NOR gates (with two or more inputs) only.
2. Consider the 2-bit binary integers $x = x_1x_2$ and $y = y_1y_2$.
- (a) Derive a statement $F(x, y)$ in full DNF that returns True (1) if and only if $x > y$ and False (0) otherwise.
 - (b) Minimize the statement form above with the Quine-McCluskey method and draw the resulting circuit.
 - (c) Redo the circuit with NAND gates only.
3. The squares of a 4×4 chessboard are numbered as follows:

| | | | |
|------|------|------|------|
| 0000 | 0001 | 0010 | 0011 |
| 0100 | 0101 | 0110 | 0111 |
| 1000 | 1001 | 1010 | 1011 |
| 1100 | 1101 | 1110 | 1111 |

- (a) Find a boolean function of a, b, c , and d that returns “true” if a knight can reach the square $abcd$ parting from the square 0110 and “false” otherwise.
NB: In chess, a knight moves two squares in one direction (horizontally or vertically) and then one square perpendicular to that direction.
- (b) Use the Quine-McCluskey method to minimize the function found in (a).

NB: For a boolean function F with n input variables:

- A *minterm* is a *product (AND)* of n *literals* in which each variable appears exactly once.
- A *maxterm* is a *sum (OR)* of n *literals* in which each variable appears exactly once.