

**University of Neuchâtel**  
**Discrete Mathematics and Applications - Fall 2025**  
**Problems - 5**

1. *Product rule - Sum rule*
  - (a) How many different car license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits?
  - (b) Suppose variable names in a programming language can be either a single lowercase letter or a lowercase letter followed by a digit. Find the number of possible variable names.
2. How many (non-empty) strings of length at most 5 can be formed from the 26 lowercase letters of the English alphabet?
3. How many bit strings of length 10 begin with three 0s or end with two 0s ?
4. How many integers in  $[1, 100]$  are not divisible by 2, 3 or 5 ?
5. There are 4369 students at the University of Neuchâtel. Show that there are at least 12 students with the same birthday. (There are 366 possible birthdays.)
6. A bowl contains 10 red balls and 10 green balls. Ouny selects balls at random without looking at them.
  - (a) What is the minimum number of balls she must select to be sure of having at least 3 balls of the same color?
  - (b) What is the minimum number of balls she must select to be sure of having at least 3 green balls?
7. Show that there exist two distinct powers of 2 that end with the same two decimal digits. Can you find two such powers explicitly?