Mathématiques discrètes et applications
Discrete Mathematics and Applications

Problem Session 2

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Exercise 1: 2-1 multiplexer

S	I_1	I_2	M
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

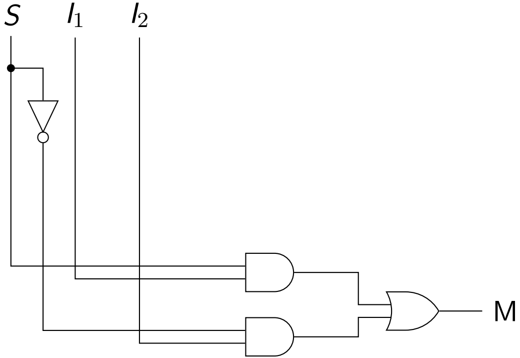
$$\boxed{M(S, I_1, I_2)_{DNF} = \bar{S}\overline{I_1}I_2 + \overline{S}I_1 I_2 + SI_1\overline{I_2} + SI_1 I_2}$$

Note: some documents suggest the DNF form does not necessarily have to contain all the boolean variables. However, the expression in **full DNF form** has all the input variables appearing exactly once.

Exercise 1: Circuit for 2-1 multiplexer

Simplification of M:

$$
\begin{aligned}
M(S, I_1, I_2) &= \bar{S}\overline{I_1}I_2 + \overline{S}I_1I_2 + SI_1\overline{I_2} + SI_1I_2 \\
&\equiv \bar{S}I_2 \cdot (\overline{I_1} + I_1) + SI_1 \cdot (\overline{I_2} + I_2) && \textbf{Distributive law} \\
&\equiv SI_1 + \bar{S}I_2 && \textbf{Negation law}
\end{aligned}
$$

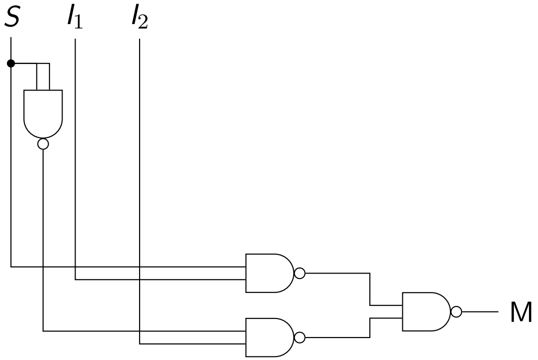


Exercise 1: 2-1 multiplexer with NAND gates only

$$M(S, I_1, I_2) = SI_1 + \bar{S}I_2$$

$$\equiv \overline{\overline{SI_1 + \bar{S}I_2}} \qquad \textbf{Double negative law}$$

$$\equiv \overline{(\overline{SI_1} \cdot \overline{\bar{S}I_2})} \qquad \textbf{De Morgan's law}$$



Exercise 1: M in conjunctive normal form (CNF)

Hint: To obtain the (full) CNF, we look at the rows of the truth table where $\overline{M} = 1$, i.e., where $M = 0$.
We find the DNF for $\overline{M}$ and then use the property $\overline{\overline{M}} \equiv M$ to obtain M in conjunctive normal form.

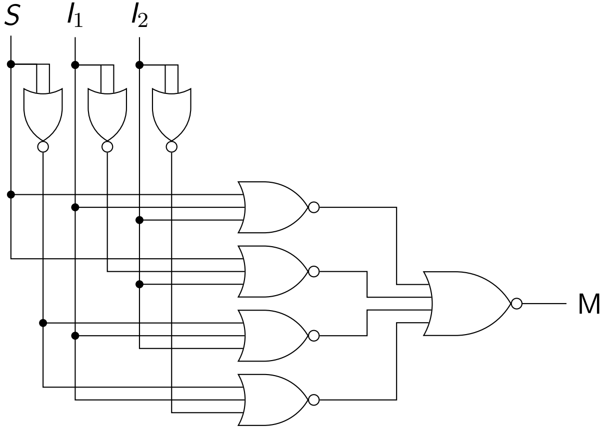
$$
\begin{aligned}
\overline{M}(S, I_1, I_2) &= \bar{S}\overline{I_1}\,\overline{I_2} + \bar{S}I_1\overline{I_2} + S\overline{I_1}\,\overline{I_2} + S\overline{I_1}I_2 \\
\rightarrow M(S, I_1, I_2) &= \overline{\bar{S}\overline{I_1}\,\overline{I_2} + \bar{S}I_1\overline{I_2} + S\overline{I_1}\,\overline{I_2} + S\overline{I_1}I_2} \\
&\equiv \overline{(\bar{S}\overline{I_1}\,\overline{I_2})} \cdot \overline{(\bar{S}I_1\overline{I_2})} \cdot \overline{(S\overline{I_1}\,\overline{I_2})} \cdot \overline{(S\overline{I_1}I_2)} \\
&\equiv (S + I_1 + I_2) \cdot (S + \overline{I_1} + I_2) \cdot (\bar{S} + I_1 + I_2) \cdot (\bar{S} + I_1 + \overline{I_2})
\end{aligned}
$$

$$\boxed{M(S, I_1, I_2)_{CNF} = (S + I_1 + I_2) \cdot (S + \overline{I_1} + I_2) \cdot (\bar{S} + I_1 + I_2) \cdot (\bar{S} + I_1 + \overline{I_2})}$$

Exercise 1: M with NOR gates only

$$M(S, I_1, I_2) = \overline{\overline{(S + I_1 + I_2) \cdot (S + \overline{I_1} + I_2) \cdot (\bar{S} + I_1 + I_2) \cdot (\bar{S} + I_1 + \overline{I_2})}}$$

$$\equiv \overline{\overline{(S + I_1 + I_2)} + \overline{(S + \overline{I_1} + I_2)} + \overline{(\bar{S} + I_1 + I_2)} + \overline{(\bar{S} + I_1 + \overline{I_2})}}$$



Ex-2: useful terms

· An **implicant** (I) is a conjunction of literals (in sum-of-product form) or disjunction of literals (in product-of-sums form) in the expression of a boolean function F. That is, $I\ true \rightarrow F\ true$. Two or more implicants can be combined to obtain implicants with fewer literals.

· A **prime implicant** (PI) is one which cannot be combined further to obtain another implicant.

· An **essential prime implicant** (EPI) is a prime implicant which covers a minterm alone. That is, no other PI covers that minterm; an EPI MUST be part of the minimized form of a boolean function.

Ex-2a: $F(x, y)$ in DNF

x_1	x_2	y_1	y_2	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$$F(x_1, x_2, y_1, y_2)_{DNF} = \overline{x_1}x_2\overline{y_1}\,\overline{y_2} + x_1\overline{x_2}\,\overline{y_1}\,\overline{y_2} + x_1\overline{x_2}\,\overline{y_1}y_2 + x_1x_2\overline{y_1}\,\overline{y_2} + x_1x_2\overline{y_1}y_2 + x_1x_2y_1\overline{y_2}$$

Ex-2b: Minimization of F with Quine-McCluskey

Table: Stage 1

Num. of 1_s	Minterm	Bit String
1	m_4	0100✓
	m_8	1000✓
2	m_9	1001✓
	m_{12}	1100✓
3	m_{13}	1101✓
	m_{14}	1110✓

Table: Stage 2

Size 2 Implicants	Bits
$m(4, 12)$	-100*
$m(8, 9)$	100-✓
$m(8, 12)$	1-00✓
$m(9, 13)$	1-01✓
$m(12, 13)$	110-✓
$m(12, 14)$	11-0*

Ex-2b: Minimization of F with Quine-McCluskey

Stage 3:

Size 4 Implicants	Bits
$m(8, 9, 12, 13)$	1-0-*
$m(8, 12, 9, 13)$	1-0-*

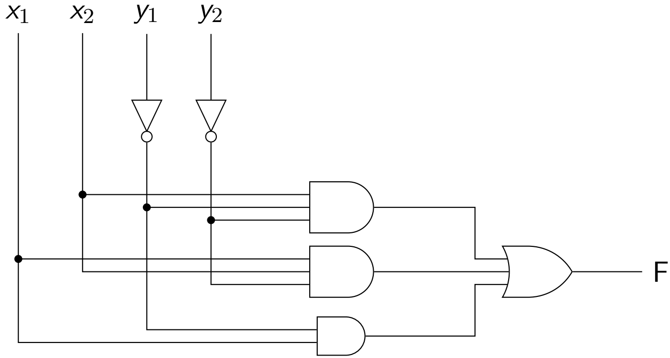
NB: The goal here is to cover all the minterms with the **smallest subset of prime implicants**. All essential prime implicants must be part of the minimized expression.

Prime Implicants	Minterms	4	8	9	12	13	14
$x_2\overline{y_1}\,\overline{y_2}$	4,12	X			X		
$x_1x_2\overline{y_2}$	12,14				X		X
$x_1\overline{y_1}$	8,9,12,13		X	X	X	X	

$F(x, y)_{min} = x_2\overline{y_1}\,\overline{y_2} + x_1x_2\overline{y_2} + x_1\overline{y_1}$

Ex-2b: Circuit of F_{min} with NOT, AND, and OR gates

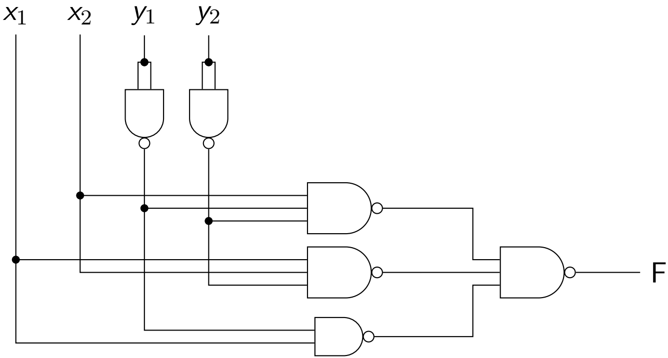
$$\boxed{F(x, y)_{min} = x_2\overline{y_1}\,\overline{y_2} + x_1x_2\overline{y_2} + x_1\overline{y_1}}$$



Ex-2b: Circuit of F_{min} with NAND gates only

$$F(x,y) = \overline{\overline{(x_2\overline{y_1}\,\overline{y_2}) + (x_1x_2\overline{y_2}) + (x_1\overline{y_1})}}$$ **Double negative law**

$$\equiv \overline{\overline{(x_2\overline{y_1}\,\overline{y_2})} \cdot \overline{(x_1x_2\overline{y_2})} \cdot \overline{(x_1\overline{y_1})}}$$ **De Morgan's law**



End

For more exercises and notes, see Rosen 7th Edition Chap. 12.