

习题9

$$U_C = (k_1' + k_2' t) e^{-t}$$

$$U_C(0_+) = k_1' = 6$$

$$\frac{dU_C}{dt}|_{0^+} = k_1' + k_2' = \frac{i_C(0_+)}{C} = \frac{-(i_L + \frac{U_C(0_+)}{1})}{0.5} = -13$$

$$\therefore k_2' = -7$$

$$\therefore U_C = (6 - 7t) e^{-t} \text{ V } (t \geq 0)$$

[法2] 电源为 $2 \varepsilon(t)$ A, $U_C(\infty) = 0$

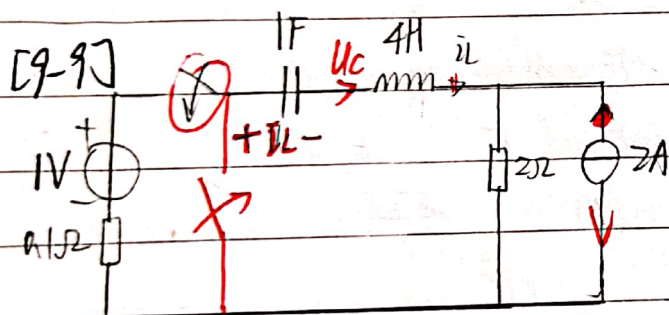
$$t > 0 \text{ 时: } i_L + 0.5 \frac{dU_C}{dt} + U_C = 2$$

$$\frac{dU_C}{dt} + 0.5 \frac{d^2 U_C}{dt^2} + \frac{dU_C}{dt} = 0$$

$$\frac{d^2 i_L}{dt^2} = \frac{1}{L} U_L = \frac{1}{2} U_C \quad \therefore \frac{d^2 U_C}{dt^2} + 2 \frac{dU_C}{dt} + U_C = 0$$

$$\text{由 } U_C(0_+) = 6$$

$$\left\{ \begin{array}{l} \frac{dU_C}{dt}|_{0^+} = -13 \end{array} \right. \text{ 得 } (?)$$



开关闭合时偶于开关打开 串→并

$$[9-5] 1) \omega_0 = \frac{1}{\sqrt{LC}} = 600 \text{ s}^{-1}, \alpha = \frac{G}{2C} = 1000 \text{ s}^{-1}$$

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -200 \text{ s}^{-1}, S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -1800 \text{ s}^{-1}$$

2) $\alpha > \omega_0$ 为过阻尼状态

$$13) i_R = (k_1 e^{-200t} + k_2 e^{-1800t}) \text{ A}$$

$$14) \omega_d = \sqrt{\omega_0^2 - \alpha^2} = 300 \text{ rad/s} \quad \therefore \alpha' = 300\sqrt{3} = \frac{1}{2RC}$$

$$\therefore R = \frac{500}{3\sqrt{3}} \Omega$$

$$15) S_1 = (-300\sqrt{3} + 300j) \text{ s}^{-1}, S_2 = (-300\sqrt{3} - 300j) \text{ s}^{-1}$$

$$16) \text{临界阻尼状态 } \alpha = \omega_0 \quad \therefore R = \frac{250}{3} \Omega$$

$$[9-7] \text{ 知 } S_1 = -10, S_2 = -20$$

$$\therefore \alpha = -\frac{S_1 + S_2}{2} = 15, \sqrt{\alpha^2 - \omega_0^2} = 5 \quad \therefore \omega_0 = 10\sqrt{2}$$

$$\therefore \frac{1}{\sqrt{LC}} = \omega_0 = 10\sqrt{2}, \alpha = \frac{1}{2RC} = 15$$

$$\text{又 } U_L = L \frac{di_L}{dt}$$

$$(30e^{-10t} - 40e^{-20t}) = L (600e^{-10t} - 800e^{-20t}) \times 10^{-3}$$

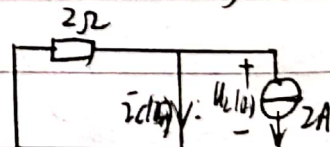
$$\therefore L = 50 \text{ H}$$

$$\therefore C = 0.1 \text{ mF} \quad R = \frac{1000}{3} \Omega$$

$$[9-9] t < 0 \text{ 时, } i_L(0_-) = 2 \text{ A}, U_C(0_-) = 0$$

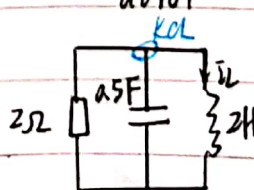
$$i_L(0_+) = i_L(0_-) = 2 \text{ A}, U_C(0_+) = U_C(0_-) = 0$$

$t = 0_+$ 时刻



$$i_C(0_+) = -i_L(0_+) = -2 \text{ A}, U_L(0_+) = U_C(0_+) = 0$$

$$\therefore \frac{di_L}{dt}|_{0^+} = \frac{U_L(0_+)}{L} = 0$$



$$\text{由 KCL: } i_L + 0.5 \frac{dU_C}{dt} + \frac{U_C}{2} = 0$$

$$\text{又 } U_C = U_L = 2 \frac{di_L}{dt}$$

$$\therefore \frac{d^2 i_L}{dt^2} + \frac{di_L}{dt} + i_L = 0$$



$$\alpha = \frac{G}{2C} = 0.5 \quad \omega_0 = \frac{1}{\sqrt{LC}} = 1 < \alpha \quad \text{欠阻尼状态}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 0.5\sqrt{3} \quad s_{1,2} = -0.5 \pm 0.5\sqrt{3}j$$

$$\therefore \tilde{i}_L = k e^{-0.5t} \sin(0.5\sqrt{3}t + \theta)$$

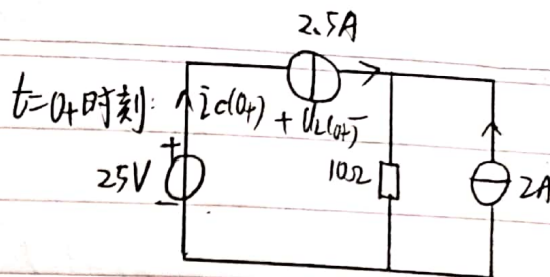
$$\tilde{i}_L(0_+) = k \sin \theta = 2$$

$$\left. \frac{d\tilde{i}_L}{dt} \right|_{0_+} = -0.5k \sin \theta + 0.5\sqrt{3}k \cos \theta = 0$$

$$\text{解得 } \tan \theta = 3 \quad \therefore \theta = 60^\circ \quad k = \frac{4}{\sqrt{3}}$$

$$\therefore \tilde{i}_L = \frac{4}{\sqrt{3}} e^{-0.5t} \sin(0.5\sqrt{3}t + 60^\circ)$$

$$U_0 = U_L = 2 \frac{d\tilde{i}_L}{dt} = \frac{8}{\sqrt{3}} e^{-0.5t} \sin(\frac{\sqrt{3}}{2}t) V$$



$$\tilde{i}_L(0_+) = \tilde{i}_L(0_+) = 0.5A$$

$$U_L(0_+) = 25 - 10(0.5 + 2) = 0$$

$$\left. \frac{d\tilde{i}_L}{dt} \right|_{0_+} = \frac{U_L(0_+)}{L} = 0$$

$$t > 0 \text{ 时, R, L, C 串联 } \alpha = \frac{R}{2L} = 5s^{-1}, \omega_0 = \frac{1}{\sqrt{LC}} = 4s^{-1}$$

$$s_1 = -2s^{-1}, s_2 = -8s^{-1}$$

$$\alpha > \omega_0, \text{ 过阻尼 } \therefore \tilde{i}_L(\infty) = 0$$

$$\therefore \tilde{i}_L = k_1 e^{-2t} + k_2 e^{-8t}$$

$$\begin{cases} \tilde{i}_L(0_+) = k_1 + k_2 = 0.5 \\ \left. \frac{d\tilde{i}_L}{dt} \right|_{0_+} = -2k_1 - 8k_2 = 0 \end{cases} \quad \therefore \begin{cases} k_1 = \frac{2}{3} \\ k_2 = -\frac{1}{6} \end{cases}$$

$$\therefore t > 0 \text{ 时, } \tilde{i}_L = (\frac{2}{3}e^{-2t} - \frac{1}{6}e^{-8t}) A$$

$$[9-12] \quad t < 0 \text{ 时, } \tilde{i}_L(0_-) = 2A, U_C(0_-) = 0$$

$$\text{连续换路, } \tilde{i}_L(0_+) = \tilde{i}_L(0_-) = 2A, U_C(0_+) = U_C(0_-) = 0$$

$$t = 0_+ \text{ 时刻, } U_L(0_+) = 10 - 5\tilde{i}_L(0_+) - U_C(0_+) = 0$$

$$\therefore \left. \frac{d\tilde{i}_L}{dt} \right|_{0_+} = \frac{U_L(0_+)}{L} = 0 \quad \left. \frac{dU_C}{dt} \right|_{0_+} = \frac{\tilde{i}_L(0_+)}{C} = \frac{\tilde{i}_L(0_+)}{C} = 18V/s$$

$$RLC \text{ 串联 } \alpha = \frac{R}{2L} = 10s^{-1}, \omega_0 = \frac{1}{\sqrt{LC}} = 6s^{-1}$$

$$\alpha > \omega_0, \text{ 过阻尼 } \therefore s_1 = -2s^{-1}, s_2 = -18s^{-1}$$

$$\tilde{i}_L(\infty) = 0, U_C(\infty) = 10V$$

$$\therefore U_C = k_1 e^{-2t} + k_2 e^{-18t} + 10$$

$$\begin{cases} U_C(0_+) = k_1 + k_2 + 10 = 0 \\ \left. \frac{dU_C}{dt} \right|_{0_+} = -2k_1 - 18k_2 = 18 \end{cases} \quad \therefore \begin{cases} k_1 = -\frac{81}{8} \\ k_2 = \frac{1}{8} \end{cases}$$

$$\therefore U_C = (-\frac{81}{8}e^{-2t} + \frac{1}{8}e^{-18t} + 10)V$$

$$\text{自由分量: } (-\frac{81}{8}e^{-2t} + \frac{1}{8}e^{-18t})V$$

$$\text{强制分量: } 10V$$

$$[9-13] \quad t < 0 \text{ 时, } U_C(0_-) = 30V \times \frac{1}{2} + 2A \times 5\Omega = 25V$$

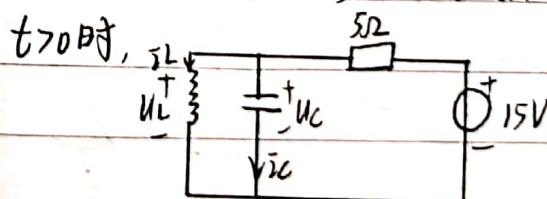
$$\tilde{i}_L(0_-) = \frac{30V}{20\Omega} - 2A \times \frac{10}{10+10} = 0.5A \quad (\text{由叠加定理})$$

$$U_C(0_+) = U_C(0_-) = 25V, \tilde{i}_L(0_+) = \tilde{i}_L(0_-) = 0.5A$$

$$[9-15] \quad t < 0 \text{ 时, 由叠加定理,}$$

$$\tilde{i}_L(0_-) = 3A + \frac{15V}{5\Omega} = 6A, U_C(0_-) = 0 \quad \text{连续换路}$$

$$U_C(0_+) = U_C(0_-) = 0, \tilde{i}_L(0_+) = \tilde{i}_L(0_-) = 6A$$



$$U_L(0_+) = U_C(0_+) = 0 \quad \left. \frac{d\tilde{i}_L}{dt} \right|_{0_+} = \frac{U_L(0_+)}{L} = 0$$

$$\tilde{i}_L(\infty) = 3A$$

$$R, L, C \text{ 并联 } \alpha = \frac{G}{2C} = 2s^{-1}, \omega_0 = \frac{1}{\sqrt{LC}} = 2s^{-1}$$

$$\therefore s_1 = s_2 = -2, \tilde{i}_L = (k_1 + k_2 t) e^{-2t} + 3$$

$$\begin{cases} \tilde{i}_L(0_+) = k_1 + 3 = 6 \\ \left. \frac{d\tilde{i}_L}{dt} \right|_{0_+} = -2k_1 + k_2 = 0 \end{cases} \quad \therefore \begin{cases} k_1 = 3 \\ k_2 = 6 \end{cases}$$

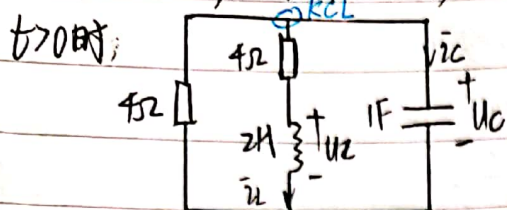
$$\therefore \tilde{i}_L = [3 + 6t] e^{-2t} + 3 A$$



[9-19] $t < 0$ 时, $\bar{i}_L(0_-) = \frac{15}{1 + \frac{4 \times 4}{4+4}} \times \frac{4}{4+4} = 2.5A$

$U_C(0_-) = \frac{2}{2+1} \times 15V = 10V$

$\bar{i}_L(0_+) = \bar{i}_L(0_-) = 2.5A, U_C(0_+) = U_C(0_-) = 10V$



$\bar{i}_C(0_+) = -(\bar{i}_L(0_+) + \frac{U_C(0_+)}{4}) = -5A$

$\therefore \frac{dU_C}{dt}|_{0_+} = \frac{\bar{i}_C(0_+)}{C} = -5V/s, U_C(\infty) = 0$

KCL: $\frac{U_C}{4} + \bar{i}_L + \frac{dU_C}{dt} = 0$

KVL: $4\bar{i}_L + 2\frac{d\bar{i}_L}{dt} = U_C$

$U_C = -4(\frac{dU_C}{dt} + \bar{i}_L)$

由①②: $\bar{i}_L + \frac{1}{2}\frac{d\bar{i}_L}{dt} + \bar{i}_L + 4\frac{d\bar{i}_L}{dt} + 2\frac{d^2\bar{i}_L}{dt^2} = 0$

$\frac{d^2\bar{i}_L}{dt^2} + \frac{9}{4}\frac{d\bar{i}_L}{dt} + \bar{i}_L = 0$

$\therefore S_1 = -1.64, S_2 = -0.61$

U_C 与 \bar{i}_L 特征根相同

\therefore 设 $U_C = k_1 e^{-1.64t} + k_2 e^{-0.61t}$

$\begin{cases} U_C(0_+) = k_1 + k_2 = 10 \\ \frac{dU_C}{dt}|_{0_+} = -1.64k_1 - 0.61k_2 = -5 \end{cases}$

$\begin{cases} k_1 = -1.07 \\ k_2 = 11.07 \end{cases}$

$\therefore U_C = (-1.07e^{-1.64t} + 11.07e^{-0.61t})V$

[9-17] $\alpha = \frac{R}{2L} = 2000, \omega_0 = \frac{1}{\sqrt{LC}} = 1000$

$\therefore S_1 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = (-200 - 100\sqrt{3})s^{-1}$

$S_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = (-200 + 100\sqrt{3})s^{-1}$

12) $\alpha > \omega_0$. 过阻尼

13) $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 800, \alpha = 600, R = 150\Omega$

14) $S_{1,2} = (600 \pm 800j)s^{-1}$

15) $\frac{R}{2L} = \frac{1}{\sqrt{LC}} \therefore R = 250\Omega$

[9-2] $\alpha = 7s^{-1}, \sqrt{\alpha^2 - \omega_0^2} = 1 \therefore \omega_0 = 4\sqrt{3}s^{-1}$
 $= \frac{R}{2L} \therefore = \frac{1}{\sqrt{LC}}$

$\therefore L = 15H, C = 1.39mF$

[9-3] $\alpha = 100, \omega_d = 200 \therefore \omega_0 = \frac{1}{\sqrt{LC}} = 100\sqrt{5}s^{-1}$

$\therefore L = 0.2H, R = 40\Omega$

[9-4] $S_1 = S_2 = -50 \therefore \alpha = \frac{R}{2L} = 50 = \omega_0 = \frac{1}{\sqrt{LC}}$

$\therefore R = 50\Omega, C = 0.8mF$

[9-8] 1) $U_C(0_+) = U_C(0_-) = 18V,$

$\bar{i}_L(0_+) = \bar{i}_L(0_-) = 3A$

2) $U_L(0_+) = U_C(0_+) - 10\bar{i}_L(0_+) = -12V$

$\frac{d\bar{i}_L}{dt}|_{0_+} = \frac{U_L(0_+)}{L} = -6A/s$

3) $\alpha = \frac{(6+4)\Omega}{2 \times 2H} = 2.5s^{-1}, \omega_0 = \frac{1}{\sqrt{LC}} = 1.5s^{-1}$

$\therefore S_1 = -4.5s^{-1}, S_2 = -0.5s^{-1}$

4) $\bar{i}_L = k_1 e^{-4.5t} + k_2 e^{-0.5t}$

$\begin{cases} \bar{i}_L(0_+) = k_1 + k_2 = 3 \\ \frac{d\bar{i}_L}{dt}|_{0_+} = -4.5k_1 - 0.5k_2 = -6 \end{cases} \therefore \begin{cases} k_1 = \frac{7}{8} \\ k_2 = \frac{15}{8} \end{cases}$

$\therefore \bar{i}_L = (\frac{7}{8}e^{-4.5t} + \frac{15}{8}e^{-0.5t})A$

