1. $(177 \bmod 31)\cdot(270 \bmod 31) \bmod 31$

$= (177\cdot 270) \bmod 31 = 19$

2. $(21^2 \bmod 15)^3 \bmod 22$

$= [(21 \bmod 15 \cdot 21 \bmod 15) \bmod 15)^3 \bmod 22$

$= [36 \bmod 15)^3 \bmod 22 = 6^3 \bmod 22$

$= 216 \bmod 22 = 18.$

3. power $= 12 \bmod 5 = 2$ $n = (100)_{10} = (1100100)_2$

$x = 1$

① $(2\times 2) \bmod 5 = 4$ ② $4\times 4 \bmod 5 = 1$

③ $x = 1\times 1 \bmod 5 = 1$ ......

therefore, the result of $(12^{100} \bmod 5)$ is 1

$x = 1$

4. $n = (1001)_{10} = (1100001)_2$ $\quad$ $power = 123 \bmod 101$
$= 22$

① $x = 1 \times 22 \bmod 101 = 22$
$power = 22 \times 22 \bmod 101 = 80$

② $power = 80 \times 80 \bmod 101 = 37$ ③ $power = 37 \times 37 \bmod 101$

similarly continue with these steps,
and you can get the result is 22

2. 定义 $Z_5 = (0, 1, 2, 3, 4) \bmod 5$.

(1) 加法表:

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

(2) 乘法表:

×	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

(2)

① $+_5$：

封闭性：满足，所有结果均在0,1,2,3,4之间

结合律：满足，$(a +_5 b) +_5 c = a +_5 (b +_5 c)$ 成立

加法单位元：0，$a +_5 0 = a$

加法逆元：1) 0 逆元为0

2) $a \neq 0$ 时

a 的逆元为 $5-a$

交换律：满足.

② $\times_5$：

封闭性：满足

结合律、交换律：

满足；

乘法单位元：1

乘法逆元

：$1 \longleftrightarrow 1$

$3 \longleftrightarrow 2$

$2 \longleftrightarrow 3$

$4 \longleftrightarrow 4$

(4) 整环：Z_5的乘法与加法均满足

（分配、结合，无0因子）

有限域：满足

3. $\gcd(100001, 1001) = 1$

$\downarrow$

$100001 \div 1001 = 99$ 余 10

$1001 \div 10 = 100$ 余 1

则 $\gcd = 1$

回溯

$1 = 1001 - 10 \times 100$

$10 = 100001 - 1001 \times 99$

则 $1 = -10 \times 100001 + 991 \times 1001$.