三.（12 分）解：设第一批全为合格品，A_i = 第一次取到的产品来自第i批，B_i = 第 i 次取到的是合格品，$i = 1,2.$

写出全概率公式 —3′

-3′

(1) $P(B_1) = \frac{1}{2} \times 1 + \frac{1}{2} \times \frac{3}{4} = \frac{7}{8}.$ … 6′

=0.875

(2)

$$P(\bar{B}_2|B_1) = 1 - P(B_2|B_1)$$

$$= 1 - P(A_1B_2|B_1) - P(A_2B_2|B_1)$$

$$= 1 - \frac{P(A_1B_1B_2)}{P(B_1)} - \frac{P(A_2B_1B_2)}{P(B_1)}$$

$$= 1 - \frac{1}{2} \times \frac{8}{7} - \frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} \times \frac{8}{7}$$

$$= 1 - \frac{4}{7} - \frac{9}{28} = \frac{3}{28}.$$ —6′

=0.107142857142857

四、（12 分）解：(1)

(X,Y)	0	1	$p_{i\cdot}$
0	$\frac{3}{8}$	$\frac{15}{32}$	$\frac{27}{32}$
1	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{5}{32}$
$p_{\cdot j}$	$\frac{1}{2}$	$\frac{1}{2}$	1

$$P(Y = 0|X = 1) = \frac{P(X = 1, Y = 0)}{P(X = 1)} = \frac{4}{5}.$$

(2) $EX = \frac{5}{32}, EY = \frac{1}{2}, DX = \frac{27 \times 5}{32 \times 32}, DY = \frac{1}{4}, EXY = \frac{1}{32},$

$$Cov(X,Y) = EXY - EXEY = \frac{1}{32} - \frac{5}{64} = -\frac{3}{64}.$$

$$\rho_{XY} = \frac{Cov(X,Y)}{\sqrt{DXDY}} = \frac{-\frac{3}{64}}{\sqrt{\frac{27 \times 5}{32 \times 32} \times \frac{1}{2}}} = -\frac{3}{\sqrt{27 \times 5}} = -\frac{\sqrt{15}}{15}.$$

1-5 CBDBC，6-10 ABDCA

1. 0.25　2. 2p(1-p)　3. N(0.1)　4. 125.

五、（12 分）解：（1）$f_X(x)=\int_{-\infty}^{+\infty}\frac{1}{2\pi}e^{-\frac{x^2+(y-x)^2}{2}}dy=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$,

$$f_Y(y)=\int_{-\infty}^{+\infty}\frac{1}{2\pi}e^{-\frac{x^2+(y-x)^2}{2}}dx$$

$$=\frac{1}{\sqrt{2\pi\times 2}}e^{-\frac{y^2}{4}}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi\times\frac{1}{2}}}e^{-\frac{\left(x-\frac{y}{2}\right)^2}{2\times\left(\frac{1}{\sqrt{2}}\right)^2}}dx$$

$$=\frac{1}{\sqrt{2\pi}\times\sqrt{2}}e^{-\frac{y^2}{4}}.$$

$f(x,y)\neq f_X(x)f_Y(y)$, 不独立.

（2）直接带公式计算

$$f_Z(z)=\int_{-\infty}^{+\infty}f(x,z-x)dx$$

$$=\int_{-\infty}^{+\infty}\frac{1}{2\pi}e^{-\frac{x^2+(z-2x)^2}{2}}dx$$

$$=\frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{5x^2-4zx}{2}}dx$$

$$=\frac{1}{\sqrt{2\pi\times 5}}e^{-\frac{z^2}{10}}\int_{-\infty}^{+\infty}\frac{1}{\sqrt{2\pi\times\frac{1}{5}}}e^{-\frac{5\left(x-\frac{2}{5}z\right)^2}{2}}dx$$

$$=\frac{1}{\sqrt{2\pi\times 5}}e^{-\frac{z^2}{10}}.$$

六、（10 分）解：（1）$N\left(\mu.\frac{\sigma^2}{n}\right)$，$\chi^2(n-1)$，独立.

（4）(X_1,X_2)的样本方差$S^2=2\times\frac{(X_1-X_2)^2}{4}$, $S=\frac{|X_1-X_2|}{\sqrt{2}}$, 由（3）知$X_1+X_2$, $|X_1-X_2|$独立.

$$\frac{\sqrt{2}\left(\frac{X_1+X_2}{2}\right)}{\sigma}\sim N(0,1),$$

$$\frac{S^2}{\sigma^2}=\frac{(X_1-X_2)^2}{2\sigma^2}\sim\chi^2(1),$$

故

$$\frac{\frac{\sqrt{2}\left(\frac{X_1+X_2}{2}\right)}{\sigma}}{\sqrt{\frac{(X_1-X_2)^2}{2\sigma^2}}}=\frac{(X_1+X_2)}{|X_1-X_2|}\sim t(1).$$

七、（12 分）解：(1) $P(X=n)=C_{n+1}^{1}p(1-p)^{n}\cdot p=(n+1)p^{2}(1-p)^{n}, n=0,1,2,\cdots.$

$$(2)\ L=\prod_{i=1}^{n}f(x_i)=\prod_{i=1}^{n}(x_i+1)p^2(1-p)^{x_i}=\prod_{i=1}^{n}(x_i+1)\,p^{2n}(1-p)^{\sum_{i=1}^{n}x_i},$$

$$lnL=2nlnp+\sum_{i=1}^{n}x_i\ln(1-p)+\ln\prod_{i=1}^{n}(x_i+1),$$

$$\frac{dlnL}{dp}=\frac{2n}{p}-\frac{\sum_{i=1}^{n}x_i}{1-p}=0,$$

$$p_{MLE}=\frac{2n}{\sum_{i=1}^{n}x_i+2n}=\frac{2}{2+\overline{X}}.$$