# Watching Videos with Certain and Constant Quality: PID-based Quality Control Method


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## Objectives


In video coding, compressed videos with certain and constant quality can ensure quality of experience (QoE). To this end, we propose in this paper a novel PID-based quality control (PQC) method for video coding. Specifically, a formulation is modelled to control quality of video coding with two objectives: minimizing control error and quality fluctuation. Then, we apply the Laplace domain analysis to model the relationship between quantization parameter (QP) and control error in this formulation. Given the relationship between QP and control error, we propose a solution to the PQC formulation, such that videos can be compressed at certain and constant quality. Finally, experimental results show that our PQC method is effective in both control accuracy and quality fluctuation.


## Formulation of quality control

In video coding, there are two main objectives for quality control:

**Objective I**: Minimizing the error between the actual and target quality, averaged over all frames.

**Objective II**: Minimizing the fluctuation of quality along with frames.

The above two objectives can be achieved by predicting the optimal QP before encoding each frame. In other words, before encoding the $t$-th frame, we need to estimate the best QP value for this frame, which is denoted by $\mathrm{QP}_t$. Assuming that $T$ is the target distortion and $D_t$ is the distortion of the $t$-th frame, the quality control can be formulated by

$$\mathrm{QP}_t = \underset{\mathrm{QP}}{\arg\min}\{\underbrace{\lambda \cdot \underbrace{(D_t(\mathrm{QP}) - T)}_{\textbf{Objective I}} + (1-\lambda) \cdot \underbrace{\frac{dD_t(\mathrm{QP})}{dt}}_{\textbf{Objective II}}}_{e_t}\}, \quad (1)$$

where $(D_t(\mathrm{QP}) - T)$ modells the error between the actual and target quality (**Objective I**), while $\frac{dD_t(\mathrm{QP})}{d_t}$ modells the fluctuation of quality (**Objective II**). In addition, $\lambda$ represents the trade-off between the two objectives, and $e_t$ denotes the overall error to be minimized.

## Relationship between $\mathrm{QP}_t$ and $e_t$

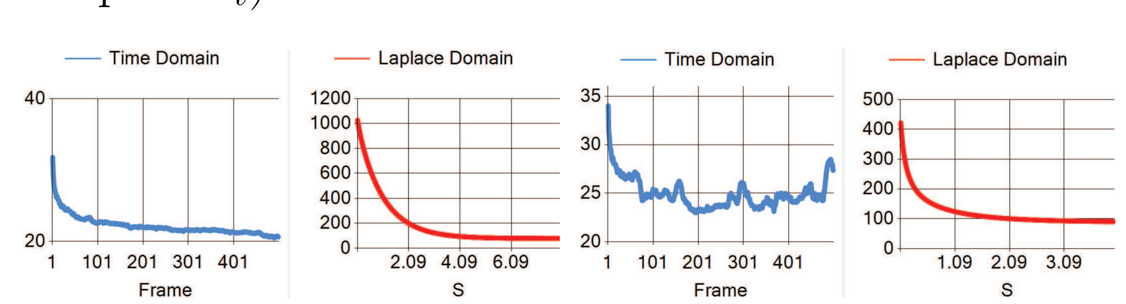
In a coding system, the relationship between $\mathrm{QP}_t$ and $e_t$ can be modelled by the following function $\Psi$,

$$\Psi(\mathbf{I}_t, \mathbf{I}_{t-1}, \cdots, \mathbf{I}_0, \mathrm{QP}_t, \mathrm{QP}_{t-1}, \cdots, \mathrm{QP}_0) = e_t, \quad (2)$$

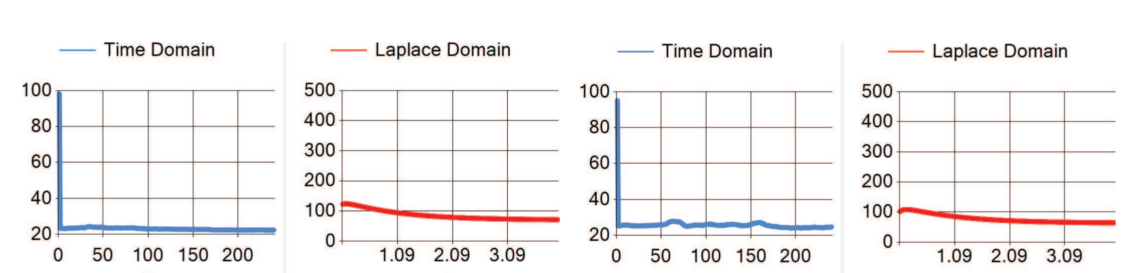
where $\mathbf{I}_t$ stands for the frame content at frame $t$. This formulation shows that content and QPs of all frames until currently encoded frame contribute to quality control error $e_t$, for a given encoder. In fact, $\mathbf{I}_t, \mathbf{I}_{t-1}, \cdots, \mathbf{I}_0$ is a set of images from the video sequence. We denote them by a single tensor $\mathbb{I}$. Our intention here is to analyze the relationship between $\mathrm{QP}_t$ and $e_t$. Thus, given a sequence, we have a fixed $\mathbb{I}$, and then (2) can be rewritten by

$$\Psi_{\mathbb{I}}(\mathrm{QP}_t, \mathrm{QP}_{t-1}, \cdots, \mathrm{QP}_0) = e_t. \quad (3)$$

Obviously, $\Psi_{\mathbb{I}}$ describes the relationship between $\mathrm{QP}_t$ and $e_t$ for given $\mathbb{I}$. To obtain $\Psi_{\mathbb{I}}$, we propose a simple and practical way from the viewpoint of signal processing by treating $\Psi_{\mathbb{I}}$ as an unknown linear time invariant (LTI) system (Input: $\mathrm{QP}_t$ and Output: $e_t$).




BasketballDrill (inter coding) BasketballPass (inter coding)




BlowingBubbles (intra coding) BasketballPass (intra coding)

$$\textbf{Inter Frame: } A_1^{\mathbb{I}} \cdot e_t + A_2^{\mathbb{I}} \cdot \frac{de_t}{dt} = \mathrm{QP}_t, \quad (4)$$

$$\textbf{Intra Frame: } A_0^{\mathbb{I}} \cdot e_t = \mathrm{QP}_t, \quad (5)$$

where $A_0^{\mathbb{I}}$, $A_1^{\mathbb{I}}$ and $A_2^{\mathbb{I}}$ are the coefficients derived from the data analysis in Figure 1. Note that the exact values for $A_0^{\mathbb{I}}$, $A_1^{\mathbb{I}}$ and $A_2^{\mathbb{I}}$ are unnecessary, since our PQC method only requires the number of orders for the differential equations, when applying the PID controller to solve our quality control formulation of (1).

## Our PQC Method

The PID controller minimizes error $e_t$ alongside time $t$, via adjusting control variable $o_t$.

$$o_t = K_p e_{t-1} + K_i \int_0^{t-1} e_\tau d\tau - K_d \frac{de_{t-1}}{dt}, \quad (6)$$

However, the PID controller can perform well only when applied to a second-order system. In other words, $e_t$ and $o_t$ in (6) need to satisfy the following differential equation:

$$M_2 \cdot \frac{d^2 e_t}{dt^2} + M_1 \cdot \frac{de_t}{dt} + M_0 \cdot e_t = o_t, \quad (7)$$

where $M_2$, $M_1$ and $M_0$ are coefficients. Next, we use the following way to make the modelled relationship between $e_t$ and $\mathrm{QP}_t$ (i.e., (4) and (5)) satisfy the above requirement of the PID controller (i.e., (7)), such that, the PID controller can be applied to solve our quality control formulation. Specifically, by applying the differential operation on (4), the following equation holds:

$$A_1^{\mathbb{I}} \cdot \frac{d^2 e_t}{dt^2} + A_0^{\mathbb{I}} \cdot \frac{de_t}{dt} + 0 \cdot e_t = \frac{d\mathrm{QP}_t}{dt}. \quad (8)$$

Thus, we can see that (8) meets the requirement (7) of the PID controller with $M_2 = A_1^{\mathbb{I}}$, $M_1 = A_0^{\mathbb{I}}$ and $M_0 = 0$. As a result, we have

$$\frac{d\mathrm{QP}_t}{dt} = o_t. \quad (9)$$

Then, (9) can be rewritten as follows,

$$\textbf{Inter Frame: } \mathrm{QP}_t = \int_0^t o_\tau d\tau. \quad (10)$$

Similarly, on the basis of (5) and (7), we have the following equation for the intra frames of video coding:

$$\textbf{Intra Frame: } \mathrm{QP}_t = \int_0^t \int_0^\tau o_\rho d\rho d\tau. \quad (11)$$

Finally, by replacing $o_t$ with (6), the QP values of each frame can be estimated as follows for controlling quality of video coding.

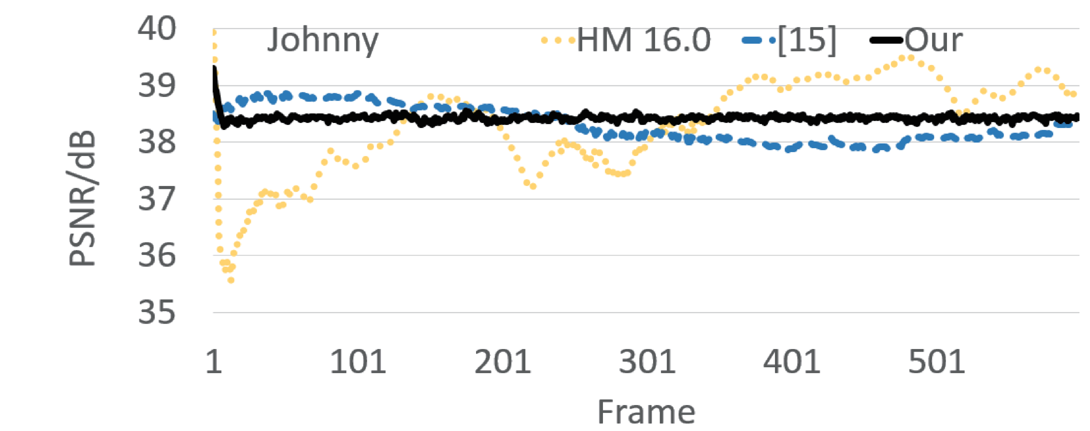
$$\textbf{Inter Frame: } \mathrm{QP}_t = \int_0^t o_\tau d\tau, \quad (12)$$

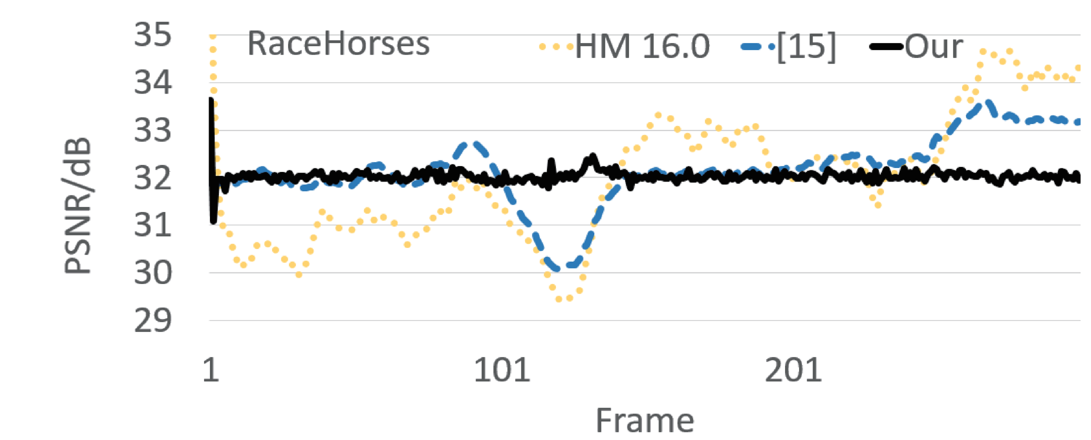
$$\textbf{Intra Frame: } \mathrm{QP}_t = \int_0^t \int_0^\tau o_\rho d\rho d\tau, \quad (13)$$

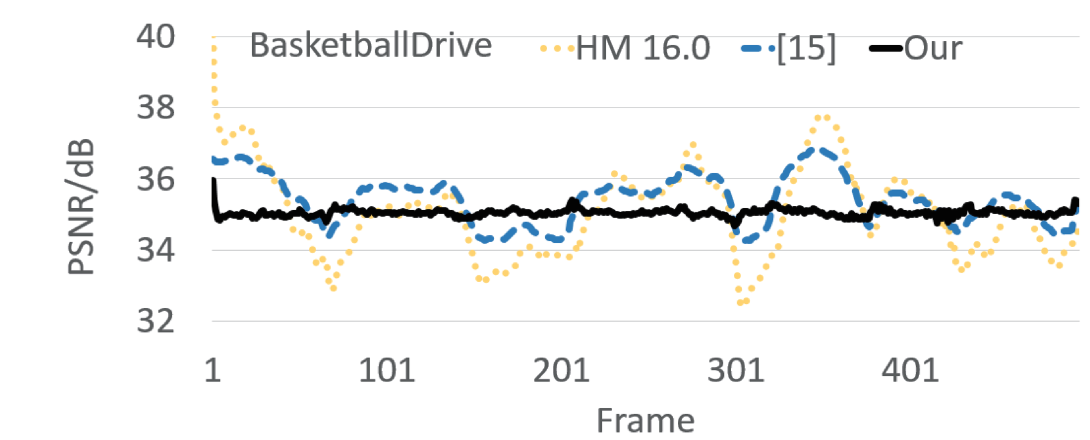
Obviously, we can see that $\mathrm{QP}_t$ is only related to the control error. Thus, that our method can be seen as an encoder-free quality control method.

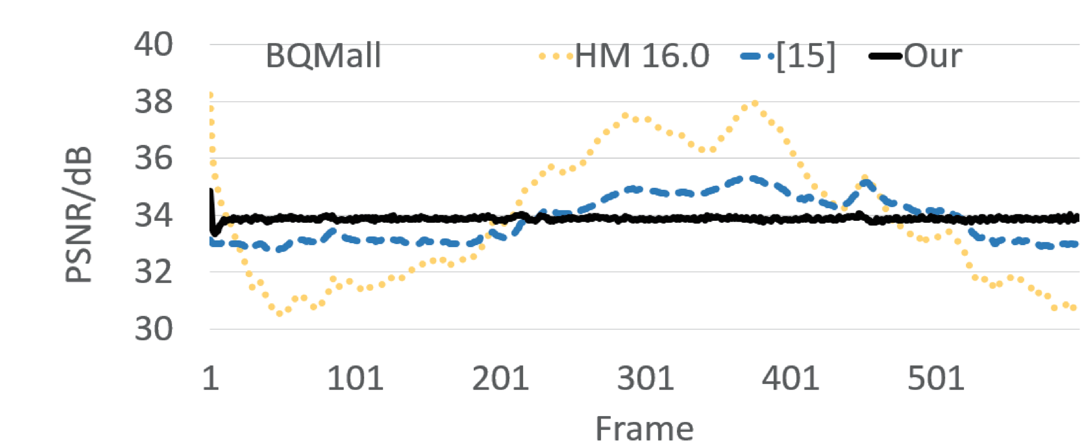
## Results

**Quality fluctuation**:

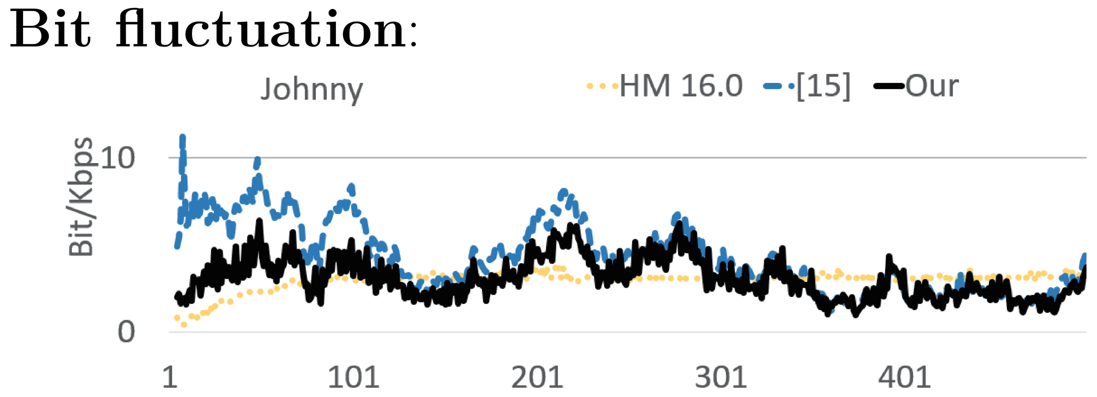



**Bit fluctuation**:

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