

## 24. R Code for Repeated Measures

- These slides illustrate a few example R commands for fitting generalized linear models to repeated measures data.
- We focus on the experiment designed to compare the effectiveness of three strength training programs.
- We will fit models that allows for a distinct mean for each of the  $3 \times 7 = 21$  combinations of training program and time.

- We assume independence between subjects.
- The models differ in the choice for  $W$ , which is the variance-covariance structure assumed for the 7 observations from each subject.

```
#Read the data

d=read.delim(
  "http://dnett.github.io/S510/RepeatedMeasures.txt")

#Create Factors

d$Program=as.factor(d$Program)
d$Subj=as.factor(d$Subj)
d$Timef=as.factor(d$Time)

#Load the nlme package

library(nlme)
```

# Compound Symmetry Structure for $W$

$$\begin{bmatrix} \sigma_e^2 + \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 \\ \sigma_s^2 & \sigma_e^2 + \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 \\ \sigma_s^2 & \sigma_s^2 & \sigma_e^2 + \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 \\ \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_e^2 + \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 \\ \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_e^2 + \sigma_s^2 & \sigma_s^2 & \sigma_s^2 \\ \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_e^2 + \sigma_s^2 & \sigma_s^2 \\ \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_s^2 & \sigma_e^2 + \sigma_s^2 \end{bmatrix}$$

```
lme(Strength ~ Program * Timef, data = d,
    random = ~ 1 | Subj)
```

## Alternative Parameterization for Compound Symmetry

$$\sigma^2 \begin{bmatrix} 1 & \rho & \rho & \rho & \rho & \rho & \rho \\ \rho & 1 & \rho & \rho & \rho & \rho & \rho \\ \rho & \rho & 1 & \rho & \rho & \rho & \rho \\ \rho & \rho & \rho & 1 & \rho & \rho & \rho \\ \rho & \rho & \rho & \rho & 1 & \rho & \rho \\ \rho & \rho & \rho & \rho & \rho & 1 & \rho \\ \rho & \rho & \rho & \rho & \rho & \rho & 1 \end{bmatrix}$$

```
gls(Strength ~ Program * Timef, data = d,  
    correlation = corCompSymm(form = ~ 1 | Subj))
```

## AR(1) Structure for $W$

$$\sigma^2 \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 & \rho^4 & \rho^5 & \rho^6 \\ \rho & 1 & \rho & \rho^2 & \rho^3 & \rho^4 & \rho^5 \\ \rho^2 & \rho & 1 & \rho & \rho^2 & \rho^3 & \rho^4 \\ \rho^3 & \rho^2 & \rho & 1 & \rho & \rho^2 & \rho^3 \\ \rho^4 & \rho^3 & \rho^2 & \rho & 1 & \rho & \rho^2 \\ \rho^5 & \rho^4 & \rho^3 & \rho^2 & \rho & 1 & \rho \\ \rho^6 & \rho^5 & \rho^4 & \rho^3 & \rho^2 & \rho & 1 \end{bmatrix}$$

```
gls(Strength ~ Program * Timef, data = d,  
    correlation = corAR1(form = ~ 1 | Subj))
```

## General Positive Definite Structure for $W$

With  $\delta_1$  set equal to 1 for identifiability purposes, a general  $7 \times 7$  positive definite variance-covariance matrix is parameterized by  $R$  as follows:

$$\sigma^2 \text{diag}(\delta_1, \dots, \delta_7) \begin{bmatrix} 1 & & & & & & \\ & 1 & & & & & \\ & & 1 & & & & \\ & & & 1 & & & \\ & & & & 1 & & \\ & & & & & 1 & \\ \rho_{ij} & & & & & & 1 \end{bmatrix} \text{diag}(\delta_1, \dots, \delta_7)$$



$$= \sigma^2 \begin{bmatrix} \delta_1^2 & & & & & & \\ & \delta_2^2 & & & & & \\ & & \delta_3^2 & & & & \\ & & & \delta_4^2 & & & \\ & & & & \delta_5^2 & & \\ & \rho_{ij}\delta_i\delta_j & & & & \delta_6^2 & \\ & & & & & & \delta_7^2 \end{bmatrix}$$

```
gls(Strength ~ Program * Timef, data = d,
    correlation = corSymm(form = ~ 1 | Subj),
    weight = varIdent(form = ~ 1 | Timef))
```

- To understand the reason for an identifiability constraint, notice that an arbitrary positive definite  $7 \times 7$  covariance matrix depends on only

$$7 + 6 + 5 + 4 + 3 + 2 + 1 = \frac{7(7 + 1)}{2} = 28$$

parameters. However, we have

$\sigma^2$ ,  $6 + 5 + 4 + 3 + 2 + 1 = 21$   $\rho_{ij}$  parameters, and  $\delta_1, \dots, \delta_7$ .

- That's 29 parameters for a symmetric positive definite matrix that depends on at most 28 parameters.

- Thus, R chooses to set  $\delta_1$  to 1.
- Without such a constraint, it is easy to use different values of the parameters to define the same matrix. For example,

$$\begin{bmatrix} 3 & -1 \\ -1 & 7 \end{bmatrix} = 3 \begin{bmatrix} 1 & -\frac{1}{3} \\ -\frac{1}{3} & \frac{7}{3} \end{bmatrix} = 1 \begin{bmatrix} 3 & -1 \\ -1 & 7 \end{bmatrix}$$

$\sigma^2$	<b>3</b>	<b>1</b>
$\delta_1$	<b>1</b>	$\sqrt{3}$
$\delta_2$	$\sqrt{\frac{7}{3}}$	$\sqrt{7}$
$\rho_{12}$	$\frac{-1}{3\sqrt{\frac{7}{3}}}$	$\frac{-1}{\sqrt{21}}$

## Other Variance-Covariance Structures in R

If you are interested in learning about how to fit other variance-covariance structures in R, the following help commands may be useful.

```
?corClasses
```

```
?varClasses
```

```
?pdClasses
```

## AIC and BIC for Repeated Measures in R

- $AIC = -2\ell(\hat{\boldsymbol{\theta}}) + 2k$
- $BIC = -2\ell(\hat{\boldsymbol{\theta}}) + k \ln(n)$
- $k$  = number of mean parameters (rank of  $X$ )  
+ number of variance parameters
- $n$  = total number of observations – rank of  $X$