COMM8102 Econometric Analysis
Assignment 2

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Excercise 4.1

(a)

$$\hat{\mu}_k = \frac{1}{n}\sum_{i=1}^{n} Y_i^k$$

(b)

$$\begin{aligned} E\left[\hat{\mu}_k\right] &= E\left[\frac{1}{n}\sum_{i=1}^{n} Y_i^k\right] \\ &= \frac{1}{n}\sum_{i=1}^{n} E\left[Y_i^k\right] \\ &= \frac{1}{n}\cdot n \cdot \mu_k \\ &= \mu_k \end{aligned}$$

(c)

$$
\begin{aligned}
Var\left[\hat{\mu}_k\right] &= E\left[\hat{\mu}_k^2\right] - \left(E\left[\hat{\mu}_k\right]\right)^2 \\
&= E\left[\left(\frac{1}{n}\sum_{i=1}^{n} Y_i^k\right)^2\right] - \mu_k^2 \\
&= \frac{1}{n^2} E\left[\left(\sum_{i=1}^{n} Y_i^k\right)^2\right] - \mu_k^2 \\
&= \frac{1}{n^2} E\left[\sum_{i=1}^{n} Y_i^{2k} + \sum_{i=1,j=1,i\neq j}^{n} Y_i^k Y_j^k\right] - \mu_k^2 \\
&= \frac{1}{n^2}\left(\sum_{i=1}^{n} E\left[Y_i^{2k}\right] + \sum_{i=1,j=1,i\neq j}^{n} E\left[Y_i^k\right] E\left[Y_j^k\right]\right) - \mu_k^2 \\
&= \frac{1}{n^2}\left(n\cdot\mu_{2k} + \left(n^2 - n\right)\cdot\mu_k^2\right) - \mu_k^2 \\
&= \frac{1}{n}\cdot\mu_{2k} + \mu_k^2 - \frac{1}{n}\cdot\mu_k^2 - \mu_k^2 \\
&= \frac{1}{n}\left(\mu_{2k} - \mu_k^2\right)
\end{aligned}
$$

We need to assume $\mu_{2k} = E\left[Y^{2k}\right] < \infty$.

(d)

$$
\hat{V}_{\hat{\mu}_k} = \frac{1}{n}\left(\frac{1}{n}\sum_{i=1}^{n} Y_i^{2k} - \left(\frac{1}{n}\sum_{i=1}^{n} Y_i^k\right)^2\right)
$$

Excercise 4.5

(4.15)

$$
\begin{aligned}
E\left[\hat{\beta}\middle| X\right] &= E\left[\left(X'X\right)^{-1} X'Y\middle| X\right] \\
&= \left(X'X\right)^{-1} X'E\left[Y| X\right] \\
&= \left(X'X\right)^{-1} X'E\left[X\beta + e| X\right] \\
&= \beta + \left(X'X\right)^{-1} X'E\left[e| X\right] \\
&= \beta
\end{aligned}
$$

(4.16)

$$\begin{aligned}
Var\left[\hat{\beta}\middle|X\right] &= Var\left[\hat{\beta}-\beta\middle|X\right] \\
&= Var\left[\left(X'X\right)^{-1}X'Y-\beta\middle|X\right] \\
&= Var\left[\left(X'X\right)^{-1}X'\left(X\beta+e\right)-\beta\middle|X\right] \\
&= Var\left[\left(X'X\right)^{-1}X'e\middle|X\right] \\
&= \left(X'X\right)^{-1}X'Var\left[e|X\right]X\left(X'X\right)^{-1} \\
&= \left(X'X\right)^{-1}X'\Omega X\left(X'X\right)^{-1} \\
&= \left(X'X\right)^{-1}\left(X'\Omega X\right)\left(X'X\right)^{-1}
\end{aligned}$$

Excercise 4.12

$$\begin{aligned}
E\left[\left(\bar{Y}-\mu\right)^3\right] &= E\left[\left(\frac{1}{n}\sum_{i=1}^{n}Y_i-\mu\right)^3\right] \\
&= E\left[\left(\frac{1}{n}\sum_{i=1}^{n}\left(Y_i-\mu\right)\right)^3\right] \\
&= \frac{1}{n^3}E\left[\left(\sum_{i=1}^{n}\left(Y_i-\mu\right)\right)^3\right] \\
&= \frac{1}{n^3}E\left[\begin{array}{c}\sum_{i=1}^{n}\left(Y_i-\mu\right)^3+3\cdot\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}\left(Y_i-\mu\right)^2\left(Y_j-\mu\right) \\ +\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}\sum_{k=1,k\neq i,j}^{n}\left(Y_i-\mu\right)\left(Y_j-\mu\right)\left(Y_k-\mu\right)\end{array}\right] \\
&= \frac{1}{n^3}\cdot\left(\begin{array}{c}\sum_{i=1}^{n}E\left[\left(Y_i-\mu\right)^3\right]+3\cdot\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}E\left[\left(Y_i-\mu\right)^2\right]E\left[Y_j-\mu\right] \\ +\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}\sum_{k=1,k\neq i,j}^{n}E\left[Y_i-\mu\right]E\left[Y_j-\mu\right]E\left[Y_k-\mu\right]\end{array}\right) \\
&= \frac{1}{n^3}\cdot\sum_{i=1}^{n}E\left[\left(Y_i-\mu\right)^3\right] \\
&= \frac{\mu_3}{n^2}
\end{aligned}$$

Excercise 4.23

$$\begin{aligned}
E\left[\hat{\beta}\middle|X\right] &= E\left[\left(X'X+I_k\lambda\right)^{-1}X'Y\middle|X\right] \\
&= E\left[\left(X'X+I_k\lambda\right)^{-1}X'\left(X\beta+e\right)\middle|X\right] \\
&= E\left[\left(X'X+I_k\lambda\right)^{-1}X'X\beta+\left(X'X+I_k\lambda\right)^{-1}X'e\middle|X\right] \\
&= \left(X'X+I_k\lambda\right)^{-1}X'X\beta+\left(X'X+I_k\lambda\right)^{-1}X'E\left[e|X\right] \\
&= \left(X'X+I_k\lambda\right)^{-1}X'X\beta
\end{aligned}$$

$\hat{\beta}$ is biased for β, since $\forall\lambda>0, E\left[\hat{\beta}\middle|X\right]\neq\beta$.

Excercise 4.26

(a)

For the results of standard errors, see Table 1 below. In terms of absolute value, *Tracking* changes the most, and *Percentile* changes the least. In terms of proportion, *Tracking* still changes the most, but *Gender* changes the least.

Standard Errors	Tracking	Age	Gender	Assigned	Percentile	Intercept
Clustering based	0.07665	0.01339	0.02868	0.03771	0.00072	0.13319
Conventional robust	0.02417	0.00855	0.02424	0.02385	0.0004	0.08280

(b)

In comparison to the results from (4.55), the $\hat{\beta}$ on *Tracking* increases from 0.138 to 0.174 by inclusion of the individual controls, and the standard errors also decreases a bit in both clustering based on the school and conventional robust formula.

Q6

Paper: Miyazaki, T., 2019. Clarifying the response of gold return to financial indicators: An empirical comparative analysis using ordinary least squares, robust and quantile regressions. Journal of Risk and Financial Management, 12(1), p.33.
Main research questions: How gold return responds to the changes in various financial indicators, specifically stock return, stock return volatility, financial market stress, crude oil, and the value of the US dollar.
Main estimation results: See appendix.
How standard error calculated: From lecture slides, we know that for OLS, $Var\left[\hat{\beta}\middle| X\right] = \sigma^2 \left(X^\intercal X\right)^{-1}$. The unbiased estimator for σ^2 is $\frac{e^\intercal e}{n-k}$, with $e = Y - X\hat{\beta}$ and k is the number of regressors. The estimator of standard error is the sqaure root of the diagonal elements of

$$\frac{e^\intercal e}{n-k}\left(X^\intercal X\right)^{-1}$$

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