# COMM8102 Econometric Analysis
# Assignment 4

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## Q1 Exercise 12.9

### (a)

To see if $\hat{\beta}_{\text{IV}}$ is unbiased, check whether the expectation of $\hat{\beta}_{\text{IV}}$ is equal to $\beta$.

$$
\begin{aligned}
E\left[\hat{\beta}_{\text{IV}}\right] &= E\left[\left(\frac{1}{n}\sum_{i=1}^{n} Z_i X_i'\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n} Z_i Y_i\right)\right] \\
&= E\left[\left(\sum_{i=1}^{n} Z_i X_i'\right)^{-1}\left(\sum_{i=1}^{n} Z_i\left(X_i'\beta + e_i\right)\right)\right] \\
&= E\left[\left(\sum_{i=1}^{n} Z_i X_i'\right)^{-1}\sum_{i=1}^{n} Z_i X_i'\beta + \left(\sum_{i=1}^{n} Z_i X_i'\right)^{-1}\sum_{i=1}^{n} Z_i e_i\right] \\
&= E\left[I\beta + \left(\sum_{i=1}^{n} Z_i X_i'\right)^{-1}\sum_{i=1}^{n} Z_i e_i\right] \\
&= \beta + E\left[E\left[\left(\sum_{i=1}^{n} Z_i X_i'\right)^{-1}\sum_{i=1}^{n} Z_i e_i \middle| Z, X\right]\right] \quad \text{by iterated conditional expectation} \\
&= \beta + E\left[\left(\sum_{i=1}^{n} Z_i X_i'\right)^{-1}\sum_{i=1}^{n} Z_i \underbrace{E\left[e_i \middle| Z, X\right]}_{0}\right] \quad \text{since they are random sample (i.i.d.)} \\
&= \beta
\end{aligned}
$$

Therefore, the IV estimator is unbiased.

### (b)

$$\begin{aligned}
Var\left[\hat{\beta}_{\mathrm{IV}}\middle| X, Z\right] &= Var\left[\hat{\beta}_{\mathrm{IV}}\middle| X, Z\right] \\
&= Var\left[\hat{\beta}_{\mathrm{IV}} - \beta\middle| X, Z\right] \\
&= Var\left[\left(\sum_{i=1}^{n} Z_i X_i'\right)^{-1} \sum_{i=1}^{n} Z_i e_i \middle| X, Z\right] \\
&= \left(\sum_{i=1}^{n} Z_i X_i'\right)^{-1} Var\left[\sum_{i=1}^{n} Z_i e_i \middle| X, Z\right] \left(\sum_{i=1}^{n} X_i Z_i'\right)^{-1} \\
&= \left(\sum_{i=1}^{n} Z_i X_i'\right)^{-1} \sum_{i=1}^{n} Z_i Z_i' Var\left[e_i\middle| X, Z\right] \left(\sum_{i=1}^{n} X_i Z_i'\right)^{-1} \\
&= \left(\sum_{i=1}^{n} Z_i X_i'\right)^{-1} \sum_{i=1}^{n} Z_i Z_i' E\left[e_i^2\middle| X, Z\right] \left(\sum_{i=1}^{n} X_i Z_i'\right)^{-1}
\end{aligned}$$

## Q2 Exercise 12.12

### (a)

It is easy to see that $\hat{\gamma} = \frac{\sum_{i=1}^n Z_i X_i}{\sum_{i=1}^n Z_i^2}$. Then $\hat{X}_i^2 = \left(\frac{\sum_{i=1}^n Z_i X_i}{\sum_{i=1}^n Z_i^2}\right)^2 \cdot Z_i^2$. By regressing $Y$ on $\hat{X}^2$, we have

$$\begin{aligned}
\hat{\beta} &= \frac{\sum_{j=1}^n \hat{X}_j^2 Y_j}{\sum_{j=1}^n \left(\hat{X}_j^2\right)^2} \\
&= \frac{\sum_{j=1}^n \left(\frac{\sum_{i=1}^n Z_i X_i}{\sum_{i=1}^n Z_i^2}\right)^2 Z_j^2 Y_j}{\sum_{j=1}^n \left(\frac{\sum_{i=1}^n Z_i X_i}{\sum_{i=1}^n Z_i^2}\right)^4 Z_j^4} \\
&= \left(\frac{\sum_{i=1}^n Z_i^2}{\sum_{i=1}^n Z_i X_i}\right)^2 \frac{\sum_{j=1}^n Z_j^2 Y_j}{\sum_{j=1}^n Z_j^4}
\end{aligned}$$

### (b)

The estimator can be written as

$$\hat{\beta} = \left(\frac{\frac{1}{n}\sum_{i=1}^n Z_i^2}{\frac{1}{n}\sum_{i=1}^n Z_i X_i}\right)^2 \frac{\frac{1}{n}\sum_{j=1}^n Z_j^2 Y_j}{\frac{1}{n}\sum_{j=1}^n Z_j^4}.$$

By WLLN and CMT,

$$\hat{\beta} \xrightarrow{p} \left(\frac{E\left[Z^2\right]}{E\left[ZX\right]}\right)^2 \cdot \frac{E\left[Z^2 Y\right]}{E\left[Z^4\right]}.$$

### (c)

Continue from part b, $\hat{\beta}$ converges in probability to

$$
\begin{aligned}
\left(\frac{E\left[Z^2\right]}{E\left[ZX\right]}\right)^2 \cdot \frac{E\left[Z^2 Y\right]}{E\left[Z^4\right]} &= \left(\frac{E\left[Z^2\right]}{E\left[ZX\right]}\right)^2 \cdot \frac{E\left[Z^2\left(\beta X^2+e\right)\right]}{E\left[Z^4\right]} \\
&= \left(\frac{E\left[Z^2\right]}{E\left[Z\left(\gamma Z+u\right)\right]}\right)^2 \cdot \frac{E\left[Z^2\left(\beta\left(\gamma Z+u\right)^2+e\right)\right]}{E\left[Z^4\right]} \\
&= \left(\frac{E\left[Z^2\right]}{\gamma E\left[Z^2\right]+\underbrace{E\left[uZ\right]}_{0}}\right)^2 \cdot \frac{E\left[\beta\gamma^2 Z^4+2\beta\gamma Z^3 u+\beta u^2 Z^2+eZ^2\right]}{E\left[Z^4\right]} \\
&= \frac{1}{\gamma^2} \cdot \frac{\beta\gamma^2 E\left[Z^4\right]+2\beta\gamma E\left[Z^3 u\right]+\beta E\left[u^2 Z^2\right]+E\left[eZ^2\right]}{E\left[Z^4\right]} \\
&= \beta+\frac{2\beta E\left[Z^3 u\right]}{\gamma E\left[Z^4\right]}+\frac{\beta E\left[u^2 Z^2\right]+E\left[eZ^2\right]}{\gamma^2 E\left[Z^4\right]}
\end{aligned}
$$

Therefore, $\hat{\beta}$ is not consistent for $\beta$ in general. If we assume that

$$
\begin{aligned}
E\left[u\middle| Z\right] &= 0, \\
E\left[u^2\middle| Z\right] &= 0, \\
E\left[e\middle| Z\right] &= 0.
\end{aligned}
$$

Then

$$
\begin{aligned}
E\left[uZ^3\right] &= E\left[E\left[uZ^3\middle| Z\right]\right] \\
&= E\left[Z^3 E\left[u\middle| Z\right]\right] \\
&= 0, \\
E\left[u^2 Z^2\right] &= E\left[E\left[u^2 Z^2\middle| Z\right]\right] \\
&= E\left[Z^2 E\left[u^2\middle| Z\right]\right] \\
&= 0, \\
E\left[eZ^2\right] &= E\left[E\left[eZ^2\middle| Z\right]\right] \\
&= E\left[Z^2 E\left[e\middle| Z\right]\right] \\
&= 0.
\end{aligned}
$$

Otherwise, we can also just assume that $E\left[uZ^3\right], E\left[u^2 Z^2\right]$ and $E\left[eZ^2\right]$ are 0.

# Q3

| | Combinations |
|---|---|
| Always-taker | $(1,0),(1,1)$ |
| Compliers | $(1,1),(0,0)$ |
| Never-taker | $(0,1),(0,0)$ |

| | Types of individuals (defiers excluded) |
|---|---|
| $(1,1)$ | **Always-taker** or **Compliers** |
| $(0,1)$ | **Never-taker** |
| $(1,0)$ | **Always-taker** |
| $(0,0)$ | **Never-taker** or **Compliers** |

## Q4 Exercise 13.10

First, we need to find the moment condition for this problem. It is

$$E\left[Z\left(Y-m\left(X,\beta\right)\right)\right]=0.$$

The criterion function is

$$J_n\left(\beta\right)=n\cdot\left(\frac{1}{n}\sum_{i=1}^{n}Z_i\left(Y_i-m\left(X_i,\beta\right)\right)\right)'\mathbf{W}\left(\frac{1}{n}\sum_{i=1}^{n}Z_i\left(Y_i-m\left(X_i,\beta\right)\right)\right),$$

where $\mathbf{W}$ is an $\ell\times\ell$ positive-definite matrix. The GMM estimator $\hat{\beta}$ is the one minimizes $J_n$ given $\mathbf{W}$.

$$\hat{\beta}=\arg\min_{\beta}J_n\left(\beta\right)$$

We start with $\mathbf{W}_1=\mathbf{I}_{\ell\times\ell}$ , an identity matrix. Then we get the consistent preliminary estimator $\hat{\beta}_1=\arg\min_{\beta}J_n\left(\beta\right)$ with $\mathbf{W}=\mathbf{I}_{\ell\times\ell}$. Then, for $s\geq 2$, let

$$\mathbf{W}_s=\left(\frac{1}{n}\sum_{i=1}^{n}Z_iZ_i'\left(Y_i-m\left(X_i,\hat{\beta}_{s-1}\right)\right)^2\right)^{-1}.$$

Then we get the iterated estimator $\hat{\beta}_s=\arg\min_{\beta}J_n\left(\beta\right)$ with $\mathbf{W}=\mathbf{W}_s$. For 2-step efficient GMM estimator, we stop at the second step. For the iterated GMM estimator, we stop the iteration when $\left\|\hat{\beta}_s-\hat{\beta}_{s-1}\right\|=0$ or $\left\|\hat{\beta}_s-\hat{\beta}_{s-1}\right\|$ is less than some tolerance level.

## Q5 Exercise 13.24

### (a)

The moment condition is $E\left[X\left(Y-\theta\right)\right]=0$. The criterion function is

$$J_n\left(\theta\right)=n\cdot\left(\frac{1}{n}\sum_{i=1}^{n}X_i\left(Y_i-\theta\right)\right)'\mathbf{W}\left(\frac{1}{n}\sum_{i=1}^{n}X_i\left(Y_i-\theta\right)\right).$$

To find the $\hat{\theta}$ that minimizes $J_n\left(\theta\right)$, we first look at the first-order condition:

$$\frac{\partial}{\partial\theta}J_n\left(\hat{\theta}\right)=n\cdot\left(\frac{1}{n}\sum_{i=1}^{n}-X_i\right)'\mathbf{W}\left(\frac{1}{n}\sum_{i=1}^{n}X_i\left(Y_i-\hat{\theta}\right)\right)=0.$$

Then, we have

$$\begin{aligned}
n\cdot\left(-\frac{1}{n}\sum_{i=1}^{n}X_i\right)'\mathbf{W}\left(\frac{1}{n}\sum_{i=1}^{n}X_iY_i-\hat{\theta}\frac{1}{n}\sum_{i=1}^{n}X_i\right) &= 0\\
\left(\frac{1}{n}\sum_{i=1}^{n}X_i\right)'\mathbf{W}\left(\frac{1}{n}\sum_{i=1}^{n}X_iY_i\right) &= \hat{\theta}\left(\frac{1}{n}\sum_{i=1}^{n}X_i\right)'\mathbf{W}\left(\frac{1}{n}\sum_{i=1}^{n}X_i\right).
\end{aligned}$$

Finally, we have

$$\hat{\theta}=\left(\left(\frac{1}{n}\sum_{i=1}^{n}X_i'\right)\mathbf{W}\left(\frac{1}{n}\sum_{i=1}^{n}X_i\right)\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^{n}X_i'\right)\mathbf{W}\left(\frac{1}{n}\sum_{i=1}^{n}X_iY_i\right).$$

Then, like exercise 13.10, starting with $\mathbf{W} = \mathbf{I}_{k\times k}$, we get

$$\hat{\theta}_1 = \frac{\left(\sum_{i=1}^n X_i'\right)\left(\sum_{i=1}^n X_i Y_i\right)}{\left(\sum_{i=1}^n X_i'\right)\left(\sum_{i=1}^n X_i\right)}.$$

For $s \geq 2$, let

$$\mathbf{W}_s = \left(\frac{1}{n}\sum_{i=1}^n X_i X_i' \left(Y_i - \hat{\theta}_{s-1}\right)^2\right)^{-1},$$

and let

$$\hat{\theta}_s = \left(\left(\frac{1}{n}\sum_{i=1}^n X_i'\right)\mathbf{W}_s\left(\frac{1}{n}\sum_{i=1}^n X_i\right)\right)^{-1}\left(\frac{1}{n}\sum_{i=1}^n X_i'\right)\mathbf{W}_s\left(\frac{1}{n}\sum_{i=1}^n X_i Y_i\right).$$

If we want the two step efficient estimator, we stop at the second step. If we want iterated estimator, we finally stop at $\hat{\theta}_s$ such that $\left\|\hat{\theta}_s - \hat{\theta}_{s-1}\right\| = 0$ or less than some tolerance level.

### (b)

If $k = 1$, this model is just-identified. If $k > 1$, this model is over-identified.

### (c)

Under $H_0 : E[Xe] = 0$ , the test statistic is:

$$J_n\left(\hat{\theta}\right) = n \cdot \left(\frac{1}{n}\sum_{i=1}^n X_i\left(Y_i - \hat{\theta}\right)\right)' \hat{\mathbf{W}}\left(\frac{1}{n}\sum_{i=1}^n X_i\left(Y_i - \hat{\theta}\right)\right),$$

where $\hat{\theta}$ is the efficient GMM estimator we get from part (a), and $\hat{\mathbf{W}} = \left(\frac{1}{n}\sum_{i=1}^n X_i X_i'\left(Y_i - \hat{\theta}\right)^2\right)^{-1}$.

## Q6 Exercise 12.25

### (a)

The result of the replication of the reduced form regression in the final column of Table 12.2 are as follow.

| | Estimation | Standard error (HC1) |
|---|---|---|
| experience | $-0.4133$ | 0.03203 |
| experience$^2$/100 | 0.0927 | 0.17077 |
| black | $-1.0063$ | 0.08798 |
| south | $-0.2671$ | 0.078665 |
| urban | 0.39975 | 0.08475 |
| public | 0.43035 | 0.086167 |
| private | 0.1226 | 0.10131 |

The replication of the 2SLS regression is:

| | Estimation | Standard error (HC1) |
|---|---|---|
| education | 0.16109 | 0.04052 |
| experience | 0.11931 | 0.01819 |
| experience$^2$/100 | $-0.2305$ | 0.03680 |
| black | $-0.1017$ | 0.04403 |
| south | $-0.0950$ | 0.02177 |
| urban | 0.1164 | 0.02630 |

## (b)

In this part, we try a different reduced form model with the variable **near a 2-year college** added.

| | Estimation | Standard error (HC1) |
|---|---|---|
| experience | $-0.41279477$ | 0.03200792 |
| experience$^2$/100 | 0.08945688 | 0.17067503 |
| black | $-1.01060399$ | 0.08795667 |
| south | $-0.26033369$ | 0.07891951 |
| urban | 0.39039534 | 0.08555575 |
| public | 0.42162279 | 0.08650685 |
| private | 0.13011730 | 0.10171720 |
| college-2 | 0.06773502 | 0.07436626 |

The t-statistic of the coefficient of the new variable **near a 2-year college** is $\frac{0.06773502}{0.07436626} = 0.911$, and the p-value is 0.26. We cannot reject the hypothesis that the coefficient of this variable is 0. In addition, the value of the coefficient is relatively small and the values of other coefficients do not change significantly. Even though it is valid, it does not play a significant role in impacting the education.

## (c)

| | Estimation | Standard error (HC1) |
|---|---|---|
| experience | $-0.6717638$ | 0.03164301 |
| experience$^2$/100 | 0.5537512 | 0.16889943 |
| black | $-0.6877297$ | 0.07516382 |
| south | $-0.2014957$ | 0.06738587 |
| urban | 0.2652910 | 0.07499348 |
| public | $-28.9256454$ | 2.83009269 |
| private | 0.1363548 | 0.10542939 |
| public*age | 1.4830942 | 0.20047303 |
| public*age$^2$/100 | $-1.5536585$ | 0.35108191 |

These two coefficients indicate that the relation between the age of an individual who grew up near a public college is not linear. When age is less than $\frac{1.4830942}{2 \cdot 1.5536585} \times 100 \approx 47.72$ years, on average, individuals who grew up near a public college with an older age have more education time.

## (d)

| | Estimation | Standard error (HC1) |
|---|---|---|
| education | 0.08253858 | 0.00622812 |
| experience | 0.08709405 | 0.00706157 |
| experience$^2$/100 | $-0.22472050$ | 0.03202664 |
| black | $-0.18102150$ | 0.01805731 |
| south | $-0.12194012$ | 0.01543661 |
| urban | 0.15701776 | 0.01530355 |

We can see that the coefficient of **education** decreases with the expanded instrument set. The structural estimate of the return to schooling decreases.

## Q7 Exercise 13.28

### (a)

| | Preliminary GMM | 2-step efficient GMM | Iterated GMM |
|---|---|---|---|
| education | 0.1610917 | 0.16151623 | 0.16152244 |
| experience | 0.1193108 | 0.11955527 | 0.11955859 |
| experience$^2$/100 | $-0.2305416$ | $-0.23151083$ | $-0.23151660$ |
| black | $-0.1017273$ | $-0.10119968$ | $-0.10119373$ |
| south | $-0.0950355$ | $-0.09535566$ | $-0.09535359$ |
| urban | 0.1164481 | 0.11502105 | 0.11501566 |

As we can see from the table, the preliminary GMM is the estimation from 2SLS, and these three results are very close.

### (b)

| | Preliminary GMM | 2-step efficient GMM | Iterated GMM |
|---|---|---|---|
| education | 0.08253858 | 0.08385250 | 0.08387334 |
| experience | 0.08709405 | 0.08763553 | 0.08763928 |
| experience$^2$/100 | $-0.22472050$ | $-0.22493052$ | $-0.22489366$ |
| black | $-0.18102150$ | $-0.17749140$ | $-0.17744527$ |
| south | $-0.12194012$ | $-0.12449040$ | $-0.12450300$ |
| urban | 0.15701776 | 0.15293796 | 0.15289824 |

As we can see from the table, the preliminary GMM is the estimation from 2SLS, and these three results are very close.

### (c)

The J statistic for overidentification for part a is $J_a = 0.868$, and the J statistic for overidentification for part b is $J_b = 10.4$.

# R Code

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```r
library(readxl)
```

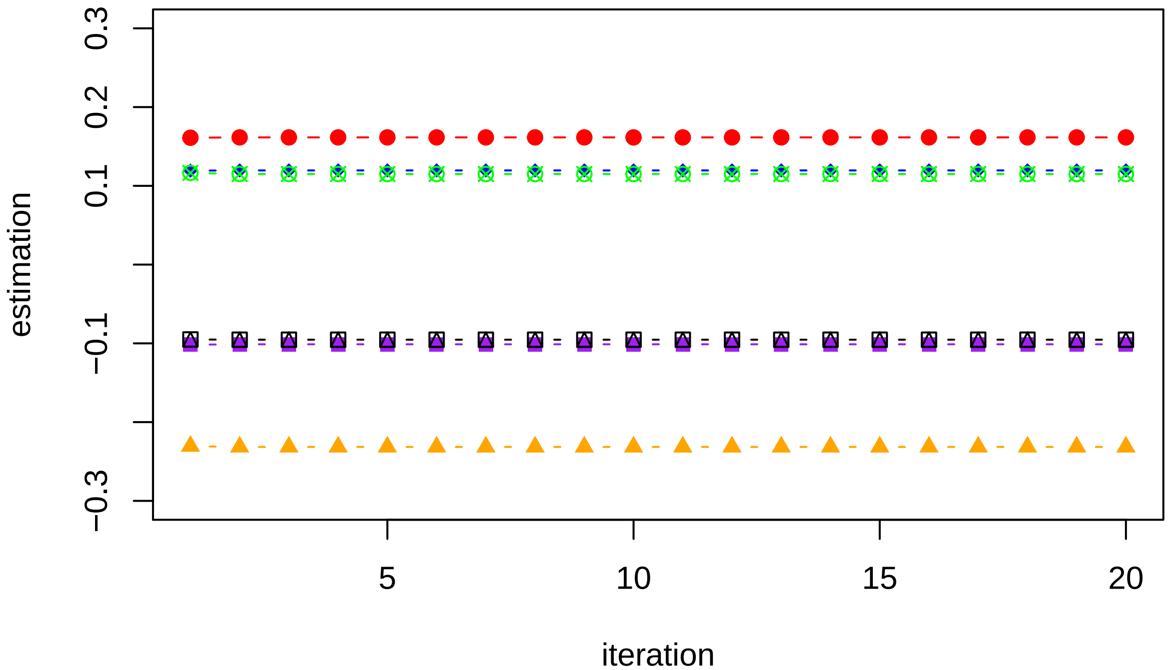
```
## Warning: package 'readxl' was built under R version 3.6.2
```

```r
dat <- read_excel("CCC.xlsx")


#data
logwage<-dat$lwage76
edu<-dat$ed76
exper<-dat$age76-dat$ed76-6
expersq100<-exper^2/100
black<-dat$black
south<-dat$reg76r
urban<-dat$smsa76r

pub<-dat$nearc4a
priv<-dat$nearc4b
college4<-dat$nearc4
college2<-dat$nearc2

#12.25 a
#reduced form regression
y<-as.matrix(edu)
x<-as.matrix(cbind(exper,expersq100,black,south,urban,pub,priv,matrix(1,nrow(y),1)))
betaa<-solve(t(x)%*%x,t(x)%*%y)

#standard error
n <- nrow(y)
k <- ncol(x)
invx <- solve(t(x)%*%x)
a <- n/(n-k)
e <- y-x%*%betaa
leverage <- rowSums(x*(x%*%invx))
sig2 <- (t(e) %*% e)/(n-k)
u1 <- x*(e%*%matrix(1,1,k))
u2 <- x*((e/sqrt(1-leverage))%*%matrix(1,1,k))
u3 <- x*((e/(1-leverage))%*%matrix(1,1,k))
v0 <- as.numeric(sig2)*invx
v1 <- invx %*% (t(u1)%*%u1) %*% invx
v1a <- a * invx %*% (t(u1)%*%u1) %*% invx
v2 <- invx %*% (t(u2)%*%u2) %*% invx
v3 <- invx %*% (t(u3)%*%u3) %*% invx
```

```r
s0 <- sqrt(diag(v0)) # Homoskedastic formula
s1 <- sqrt(diag(v1)) # HC0
s1a <- sqrt(diag(v1a)) # HC1
s2 <- sqrt(diag(v2)) # HC2
s3 <- sqrt(diag(v3)) # HC3



#2sls
y<-as.matrix(logwage)
x<-as.matrix(cbind(edu,exper,expersq100,black,south,urban,matrix(1,nrow(y),1)))

z<-as.matrix(cbind(pub,priv,exper,expersq100,black,south,urban,matrix(1,nrow(y),1)))
zz<-t(z)%*%z
xz<-t(x)%*%z
zy<-t(z)%*%y

beta2sls<-solve(xz%*%solve(zz,t(xz)),xz%*%solve(zz,zy))
#standard error
n <- nrow(y)
e<-y-x%*%beta2sls
sig2<-(t(e) %*% e)/n
ze<-z*matrix(rep(e,dim(z)[2]),ncol = dim(z)[2],nrow = n)
Qzx=t(z)%*%x/n
Qzz=t(z)%*%z/n
Om=t(ze)%*%ze/n

v0<-solve(t(Qzx)%*%solve(Qzz,Qzx))*as.numeric(sig2)#homo
v1<-solve(t(Qzx)%*%solve(Qzz,Qzx),t(Qzx))%*%solve(Qzz,Om)%*%solve(Qzz,Qzx)%*%
  +solve(t(Qzx)%*%solve(Qzz,Qzx))#HC0
v1a<-(n/(n-dim(z)[2]))*v1#HC1
s0<-sqrt(diag(v0/n))#homo
s1<-sqrt(diag(v1/n))#HC0
s1a <- sqrt(diag(v1a/n))#HC1

#12.25 b
#new reduced form regression with 2 year college
y<-as.matrix(edu)
x<-as.matrix(cbind(exper,expersq100,black,south,urban,pub,priv,college2,matrix(1,nrow(y),1)))
betab<-solve(t(x)%*%x,t(x)%*%y)
#standard error
n <- nrow(y)
k <- ncol(x)
invx <- solve(t(x)%*%x)
a <- n/(n-k)
e <- y-x%*%betab
leverage <- rowSums(x*(x%*%invx))
sig2 <- (t(e) %*% e)/(n-k)
u1 <- x*(e%*%matrix(1,1,k))
u2 <- x*((e/sqrt(1-leverage))%*%matrix(1,1,k))
u3 <- x*((e/(1-leverage))%*%matrix(1,1,k))
v0 <- as.numeric(sig2)*invx
v1 <- invx %*% (t(u1)%*%u1) %*% invx
```

```
v1a <- a * invx %*% (t(u1)%*%u1) %*% invx
v2 <- invx %*% (t(u2)%*%u2) %*% invx
v3 <- invx %*% (t(u3)%*%u3) %*% invx
s0 <- sqrt(diag(v0)) # Homoskedastic formula
s1 <- sqrt(diag(v1)) # HC0
s1a <- sqrt(diag(v1a)) # HC1
s2 <- sqrt(diag(v2)) # HC2
s3 <- sqrt(diag(v3)) # HC3
tstatistic<-betab[8]/s1a[8]
pval<-dnorm(tstatistic)

#12.25 c
intact1<-pub*dat$age76
intact2<-pub*(dat$age76)^2/100
#new reduced form regression with 2 new interaction
y<-as.matrix(edu)
x<-as.matrix(cbind(exper,expersq100,black,south,urban,pub,priv,intact1,intact2,matrix(1,nrow(y),1)))
betac<-solve(t(x)%*%x,t(x)%*%y)
#standard error
n <- nrow(y)
k <- ncol(x)
invx <- solve(t(x)%*%x)
a <- n/(n-k)
e <- y-x%*%betac
leverage <- rowSums(x*(x%*%invx))
sig2 <- (t(e) %*% e)/(n-k)
u1 <- x*(e%*%matrix(1,1,k))
u2 <- x*((e/sqrt(1-leverage))%*%matrix(1,1,k))
u3 <- x*((e/(1-leverage))%*%matrix(1,1,k))
v0 <- as.numeric(sig2)*invx
v1 <- invx %*% (t(u1)%*%u1) %*% invx
v1a <- a * invx %*% (t(u1)%*%u1) %*% invx
v2 <- invx %*% (t(u2)%*%u2) %*% invx
v3 <- invx %*% (t(u3)%*%u3) %*% invx
s0 <- sqrt(diag(v0)) # Homoskedastic formula
s1 <- sqrt(diag(v1)) # HC0
s1a <- sqrt(diag(v1a)) # HC1
s2 <- sqrt(diag(v2)) # HC2
s3 <- sqrt(diag(v3)) # HC3
tstatistic1<-betac[8]/s1a[8]
pval1<-dnorm(tstatistic1)
tstatistic2<-betac[9]/s1a[9]
pval2<-dnorm(tstatistic1)

#12.25 d 2sls with expanded instrument
intact1<-pub*dat$age76
intact2<-pub*(dat$age76)^2/100
y<-as.matrix(logwage)
x<-as.matrix(cbind(edu,exper,expersq100,black,south,urban,matrix(1,nrow(y),1)))

z<-as.matrix(cbind(pub,priv,intact1,intact2,exper,expersq100,black,south,urban,matrix(1,nrow(y),1)))
zz<-t(z)%*%z
xz<-t(x)%*%z
```

```r
zy<-t(z)%*%y

beta2slsnew<-solve(xz%*%solve(zz,t(xz)),xz%*%solve(zz,zy))
#standard error
n <- nrow(y)
e<-y-x%*%beta2slsnew
sig2<-(t(e) %*% e)/n
ze<-z*matrix(rep(e,dim(z)[2]),ncol = dim(z)[2],nrow = n)
Qzx=t(z)%*%x/n
Qzz=t(z)%*%z/n
Om=t(ze)%*%ze/n

v0<-solve(t(Qzx)%*%solve(Qzz,Qzx))*as.numeric(sig2)#homo
v1<-solve(t(Qzx)%*%solve(Qzz,Qzx),t(Qzx))%*%solve(Qzz,Om)%*%solve(Qzz,Qzx)%*%
  +solve(t(Qzx)%*%solve(Qzz,Qzx))#HC0
v1a<-(n/(n-dim(z)[2]))*v1#HC1
s0<-sqrt(diag(v0/n))#homo
s1<-sqrt(diag(v1/n))#HC0
s1a <- sqrt(diag(v1a/n))#HC1
tstat<-beta2slsnew/s1
p_val<-dnorm(tstat)




#13.28
#a
y<-as.matrix(logwage)
x<-as.matrix(cbind(edu,exper,expersq100,black,south,urban,matrix(1,nrow(y),1)))
z<-as.matrix(cbind(pub,priv,exper,expersq100,black,south,urban,matrix(1,nrow(y),1)))
zz<-t(z)%*%z
xz<-t(x)%*%z
zy<-t(z)%*%y
n <- nrow(y)
betaa=matrix(solve(xz%*%solve(zz,t(xz)),xz%*%solve(zz,zy)),ncol = 1)

for (m in 2:20) {

  e<-y-x%*%betaa[,m-1]
  ze<-z*matrix(rep(e,dim(z)[2]),ncol = dim(z)[2],nrow = n)
  W=solve(t(ze)%*%ze/n)
  betaa=cbind(betaa,solve(xz%*%W%*%t(xz),xz%*%W%*%zy))
}

#plot
plot(1:20, betaa[1,], type = "b", frame = TRUE, pch = 19,
     col = "red", xlab = "iteration", ylab = "estimation",ylim = c(-0.3,0.3))
lines(1:20, betaa[2,], pch = 18, col = "blue", type = "b", lty = 2)
lines(1:20, betaa[3,], pch = 17, col = "orange", type = "b", lty = 2)
lines(1:20, betaa[4,], pch = 15, col = "purple", type = "b", lty = 2)
lines(1:20, betaa[5,], pch = 14, col = "black", type = "b", lty = 2)
lines(1:20, betaa[6,], pch = 13, col = "green", type = "b", lty = 2)
```




```
convcheckera<-c()
for (b in 1:19) {
  convcheckera[b]=norm(betaa[,b]-betaa[,b+1],type = "2")
}
#b
y<-as.matrix(logwage)
x<-as.matrix(cbind(edu,exper,expersq100,black,south,urban,matrix(1,nrow(y),1)))

z<-as.matrix(cbind(pub,priv,intact1,intact2,exper,expersq100,black,south,urban,matrix(1,nrow(y),1)))
zz<-t(z)%*%z
xz<-t(x)%*%z
zy<-t(z)%*%y
n <- nrow(y)
betab=matrix(solve(xz%*%solve(zz,t(xz)),xz%*%solve(zz,zy)),ncol = 1)

for (m in 2:20) {

  e<-y-x%*%betab[,m-1]
  ze<-z*matrix(rep(e,dim(z)[2]),ncol = dim(z)[2],nrow = n)
  W<-solve(t(ze)%*%ze/n)
  betab=cbind(betab,solve(xz%*%W%*%t(xz),xz%*%W%*%zy))
}
convcheckerb<-c()
for (b in 1:19) {
  convcheckerb[b]=norm(betab[,b]-betab[,b+1],type = "2")
}
```

```r
#c
#for a
y<-as.matrix(logwage)
x<-as.matrix(cbind(edu,exper,expersq100,black,south,urban,matrix(1,nrow(y),1)))
z<-as.matrix(cbind(pub,priv,exper,expersq100,black,south,urban,matrix(1,nrow(y),1)))
zz<-t(z)%*%z
xz<-t(x)%*%z
zy<-t(z)%*%y
n <- nrow(y)

ba<-betaa[,20]
e<-y-x%*%ba
ze<-z*matrix(rep(e,dim(z)[2]),ncol = dim(z)[2],nrow = n)
W<-solve(t(ze)%*%ze/n)
Ja<-n*t(t(z)%*%(e)/n)%*%W%*%(t(z)%*%(e)/n)

#for b
y<-as.matrix(logwage)
x<-as.matrix(cbind(edu,exper,expersq100,black,south,urban,matrix(1,nrow(y),1)))

z<-as.matrix(cbind(pub,priv,intact1,intact2,exper,expersq100,black,south,urban,matrix(1,nrow(y),1)))
zz<-t(z)%*%z
xz<-t(x)%*%z
zy<-t(z)%*%y
n <- nrow(y)
bb<-betab[,20]

e<-y-x%*%bb
ze<-z*matrix(rep(e,dim(z)[2]),ncol = dim(z)[2],nrow = n)
W<-solve(t(ze)%*%ze/n)
Jb<-n*t(t(z)%*%(e)/n)%*%W%*%(t(z)%*%(e)/n)
```