Distributed Antenna Selection for Massive MIMO Using Reversing Petri Nets

Harun Siljak, *Member, IEEE*, Kyriaki Psara, and Anna Philippou

***Abstract*—Distributed antenna selection for distributed massive multiple input multiple output (MIMO) communication systems reduces computational complexity compared to centralized approaches, and provides high fault tolerance while retaining diversity and spatial multiplexity. We propose a novel distributed algorithm for antenna selection and show its advantage over existing centralized and distributed solutions. The proposed algorithm is shown to perform well with imperfect channel state information, and to execute a small number of simple computational operations per node, converging fast to a steady state. We base it on reversing Petri nets, a variant of Petri nets inspired by reversible computation, capable of both forward and backward execution while obeying conservation laws.**

***Index Terms*—Distributed massive MIMO, antenna selection, optimisation, reversible computation, reversing Petri nets.**

I. Introduction

Antenna selection in distributed Massive MIMO (Multiple Input Multiple Output) antenna arrays is an important optimisation problem on a complex system comprised of a large number of simple, similar-behaving components. It is possible to retain the advantages of a large antenna array, including interference suppression, spatial multiplexing and diversity [1] while reducing the number of radio frequency (RF) chains and number of antennas to power at a time, as using all available antennas is not optimal; some antennas fail to contribute to the service [2]. Optimal transmit antenna selection for large antenna arrays is computationally demanding [3], so suboptimal approaches are pursued for real time use. A popular one is the greedy algorithm [4] where an iterative procedure adds antennas that contribute the most to the capacity of the set of antennas already selected. This approach has guaranteed performance bounds for receive antenna selection, but not for the transmit case. In a different perspective, random antenna selection was proposed as a computationally inexpensive alternative with satisfying results in some scenarios [5]. Finally, distributed decision-making has been suggested as a natural approach to distributed massive MIMO antenna selection [6], ensuring that highly correlated spatially clustered antennas are not prioritised in selection.

We use the distributed paradigm of Petri nets to improve the distributed decision-making. Petri nets are a powerful mathematical and graphical notation for designing, analysing and controlling a wide range of systems. For instance, they have been applied in higher layers of the ISO-OSI model for wireless networks [7]. Here we employ a variant of Petri nets, reversing Petri nets (RPN) [8], a formalism capable of reversing its evolution and, as such, conserves information. In this letter we exploit its conservation of tokens. Our use of RPN is motivated by: (1) the ability of Petri nets to formalise handling of a complex system: a large number of simple entities acting asynchronously, (2) the reversibility of it, inherently allowing backtracking, periodic behaviour, fault recovery and conservation of information in the system.

In this letter we propose a fast, environment-aware, asynchronous, distributed antenna selection algorithm maximising sum-capacity within an RPN scheme of a distributed array. A small number of simple computational tasks is performed, so the algorithm is simpler and faster than the state of the art in both centralised and distributed antenna selection algorithms.

II. RPNs and Transmit Antenna Selection

A. The Optimisation Problem

We consider the scenario of downlink (transmit) antenna selection at a distributed massive MIMO base station with N_T antennas, with the task of selecting a subset of antennas of size N_{TS}. In the cell there are N_R single antenna users and we aim at maximising the sum-capacity

$$\mathcal{C} = \max_{\mathbf{P},\mathbf{H_c}} \log_2 \det\left(\mathbf{I} + \rho \frac{N_R}{N_{TS}} \mathbf{H_c}\mathbf{P}\mathbf{H_c}^H\right) \tag{1}$$

where ρ is the signal to noise ratio (SNR), $\mathbf{I}$ a $N_{TS} \times N_{TS}$ identity matrix, and $\mathbf{P}$ a diagonal $N_R \times N_R$ power distribution matrix. $\mathbf{H_c}$ is the $N_{TS} \times N_R$ channel submatrix for a selected subset of antennas from the $N_T \times N_R$ channel matrix $\mathbf{H}$. This is the approach taken in [9], and we follow it here, defining the antenna selection as a problem of maximal sum-capacity for a massive MIMO system formed by the selected number of antennas, calculated for optimal (dirty paper coding) precoding. Following the example of [10] we scale the power by the number of selected antennas, using the array gain to reduce transmit power per antenna, instead of increasing receive SNR.

The receiver selection problem is different from (1) as it does not feature scaling by the number of transmit antennas. As such, receiver selection problem can be solved using greedy algorithms with a guaranteed (suboptimal) performance bound

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H. Siljak is with the CONNECT Centre, Trinity College Dublin, Dublin 2, Ireland (e-mail: harun.siljak@tcd.ie).

K. Psara and A. Philippou are with the Department of Computer Science, University of Cyprus, Nicosia 2109, Cyprus (e-mail: kpsara01@cs.ucy.ac.cy; annap@cs.ucy.ac.cy).

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by the previously described greedy algorithm. The transmitter antenna selection is not submodular [11]; adding a new antenna to the already selected set of antennas can in fact decrease channel capacity if the contribution is under the average. Greedy algorithms do not have a performance guarantee in this case.

The optimisation problem in (1) has two variables, the subset of selected antennas and the optimal power distribution over them. Following the practice from [6], [9], we start from the assumption of $\mathbf{P}$ having all diagonal elements equal to $1/N_R$ (their sum is unity, making the total power equal to $\rho N_R/N_{TS}$), and after the antenna selection optimise $\mathbf{P}$ by water filling for zero forcing. This was suggested in [9] as a practicality measure; hence the results in this letter are presented after linear beamforming, and with coherent data transmission in mind. We use the Ilmprop channel model [12] which, among other features, accounts for spatial correlation, pathloss and shadow fading [13]. In this consideration, the channel state information (CSI) is assumed to be perfect and the matrix $\mathbf{H}$ known in its entirety, but we show that the algorithm is robust under uncertainties and errors in $\mathbf{H}$.

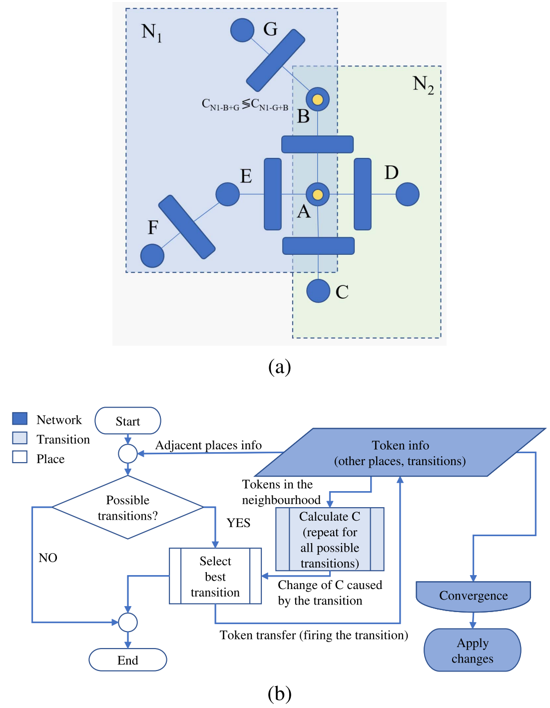

Fig. 1. (a) Exemplary Petri net for antenna selection, (b) the flowchart.

B. The RPN Algorithm

We provide a general RPN model whose behaviour simulates the runs of the proposed antenna selection algorithm. The graphical representation of the RPN is lucid enough for understanding and explaining the complex structures of the distributed algorithm and it also provides a formal semantics where verification techniques can be applied. The RPN framework is independent of the array structure (or more general, topology of a network) as it does not depend on the number of antennas or on the way the antennas are connected to each other. This suggests its generalistic nature as a framework for resource allocation in wireless communications.

The algorithm we propose is illustrated in Fig. 1, with more information about the formal model in [14]. The antennas are represented with *places* (circles A-G), and the *token* (bright circle) in some of the places indicates that the respective antenna is currently switched on. The places are divided into overlapping sets we call *neighbourhoods* (N_1 and N_2 in Fig. 1(a)) such that each two adjacent places belong to (at least one) common neighbourhood. *Transitions*, depicted as bars between places, allow tokens to move and they operate as follows:

1) A transition is possible if there is a token in exactly one of the two places (e.g., B and G in Fig. 1) it connects. Otherwise (e.g., A and B, or E and F) it is not possible.
2) An enabled transition will occur if the sum capacity (1) calculated for all antennas with a token in the neighbourhood shared by the two places (for B and G, that is neighbourhood N_1) is less than the sum capacity calculated for the same neighbourhood, but with the token moved to the empty place (for B-G transition, $\mathcal{C}_{AB} < \mathcal{C}_{AG}$). Otherwise, it does not occur.
3) In case of several possible transitions from one place (e.g., A-E, A-D, A-C) the one with the greatest sum-capacity difference is given priority (prioritisation is implemented in the transition condition function).
4) There is no designated order in transition execution, and transitions are performed until a stable state is reached.

This asynchronous scheme (represented as a flowchart in Fig. 1(b)) requires few (in our trials, up to five for 64 Tx/16 Rx case) passes through the network to converge to a stable state, starting from a random selection of n antennas (n tokens in random places). It conserves the number of tokens and keeps at most one token per place, hence the resulting state will capture the set of n selected antennas.

Fast convergence suggests that, for a small number of users, the places corresponding to antennas close to particular users might not always receive tokens if we start from uniformly random initial conditions. Thus, if the goal is to serve a relatively small number of users ($\lesssim \sqrt{N_T}$), running several RPNs in parallel (in our experiments as few as five is enough) and taking the best result among them is an option. For larger number of users ($\gtrsim \sqrt{N_T}$), our results suggest that one RPN is enough. Once the RPN converges, the state of antennas is changed and the antennas with tokens are turned on for the duration of the coherence interval. At the next update of the channel state information, the algorithm resumes its operation.

III. Other Methods: Centralised and Distributed

The computational footprint of the described algorithm is very small: two small matrix multiplications and determinant calculations are performed at a node which contains a token in a small number of iterations. As such, this algorithm is significantly faster and computationally less demanding than the centralised greedy approach which is a low-complexity representative of global optimisation algorithms in antenna

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selection [3]. In [6] it was shown that another distributed algorithm, which we will call NN (Nearest Neighbours) has smaller computational complexity than the greedy algorithm, so we proceed with a comparison of the RPN and NN approaches. In brief, every antenna element in NN calculates the sum-capacity for the currently selected antennas among its nearest neighbours and checks whether this quantity increases or decreases by including itself into the selected set.

It has been shown that the worst case complexity of NN for N_T antennas is $\mathcal{O}(N_T^{\omega})$, where ω, $2 < \omega < 3$ is the exponent in the employed matrix multiplication algorithm complexity. Similarly to RPN, NN performs two relatively small matrix multiplications and determinant calculations per iteration at a node, but there are important differences:

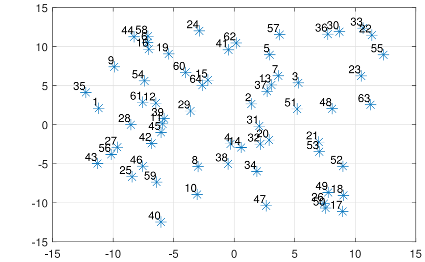
1) NN performs calculations at each node in each iteration. RPN does it only at nodes that contain tokens.
2) NN requires large neighbourhoods $\approx N_T$ (i.e., large matrix multiplications) to select small number of antennas, while RPN performs calculations on a small neighbourhood for any number of antennas to be selected (in our experiments, the value of $\lfloor\sqrt{N_T}\rfloor$ or less is enough for the neighbourhood size).
3) NN does not converge, so the number of iterations has to be large to pass through many different states and pick the best. RPN converges fast (in our experiments, always under 5 iterations).

The worst case complexity of the RPN-based approach operating on the neighbourhood size of $\lfloor N_T^{1/a}\rfloor$, $a > 1$, is $\mathcal{O}(N_T^{\omega/a})$. As neighbourhood of $\lfloor\sqrt{N_T}\rfloor$ antennas was enough in our practical considerations, our implementation had the complexity $\mathcal{O}(N_T^{\omega/2})$. At the same time, the constant factor multiplying the complexity is reduced because of fewer computing nodes (only those with tokens) and fewer iterations (50 vs. 5). While the this does not affect the asymptotic complexity, it affects the number of floating point operations (flops) performed; in the next section we examine this effect.

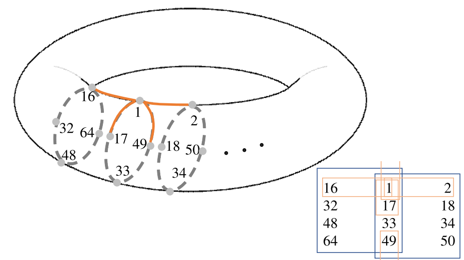
Another advantage of RPN over NN is the ability to select how many antennas to use, by simply choosing how many tokens to start the process with. In NN, the number of antennas for which the system reaches the best performance is an emergent property and as such cannot be controlled. Furthermore, in [6] it was seen that the NN algorithm requires more than N_R antennas to serve N_R users; RPN gives meaningful results and good performance even at N_R antennas selected.

IV. Results and Discussion

The algorithm was tested in the same conditions as the NN algorithm in [6], using the raytracing MATLAB tool Ilmprop [12] with 64 distributed transmitters (Fig. 2(a)) in the area which included 75 scatterers and one large obstacle. The number of users varied from 4 to 16; users, antennas, and scattering clusters were distributed uniformly, and we used SNR $\rho = -5$ dB, 2.6 GHz carrier frequency, 20 MHz bandwidth and omnidirectional antennas for transmitters and receivers. In all computations, the matrix $\mathbf{H}$ was normalised to unit average energy over all antennas, users and subcarriers [9]. Fig. 2(b) shows the mapping of the antennas into the RPN topology:



(a)



(b)

Fig. 2. Mapping of antennas onto the toroid, with the connections and neighbourhoods used (a) distribution of antennas in space and (b) position of antennas in RPN topology.

they are arranged in an 4×16 array which is then folded into a toroid, so that the edges of the array continuously connect to the opposite edges. In this arrangement, antenna 1 is direct neighbour of antennas 2, 16, 17 and 49. We establish links between immediate Von Neumann (top, down, left, right) neighbours (allow exchange of tokens between them) and set up two overlapping 8-antenna neighbourhoods: in the example of antenna 1, transitions to antennas 16 and 17 are governed by the neighbourhood {16, 32, 48, 64, 1, 17, 33, 49} while the transitions to antennas 2 and 49 are governed by {1, 17, 33, 49, 2, 18, 34, 50}, with the same pattern for other antennas (left and down, left neighbourhood, right and up, right neighbourhood).

The results of the experiments for 4, 8, 12 and 16 users are shown in Fig. 3, comparing greedy and random selection with two variants of our RPN approach: one as the average of five concurrently running RPNs with random initial condition, another as the performance of the best RPN out of those five. They demonstrate comparable performance of the proposed algorithm to the sum rates obtained through centralised greedy selection. Furthermore, it indicates that for a larger number of users our proposed algorithm outperforms centralised antenna selection represented by the greedy algorithm, while for a small number of users the centralised algorithm performs marginally better. This is because some regions of the distributed base station may be rightfully favoured by the centralised algorithm in the small user pool, and yet contain just a few tokens in our distributed algorithm; in the large user pool, all regions of the distributed base station contribute to the service. This in practice means that a single RPN suffices for networks with a relatively large expected number of users. The case of many users is the one we are trying to solve, with the idea of many antennas serving many users in Massive MIMO. The need for a strategic antenna selection

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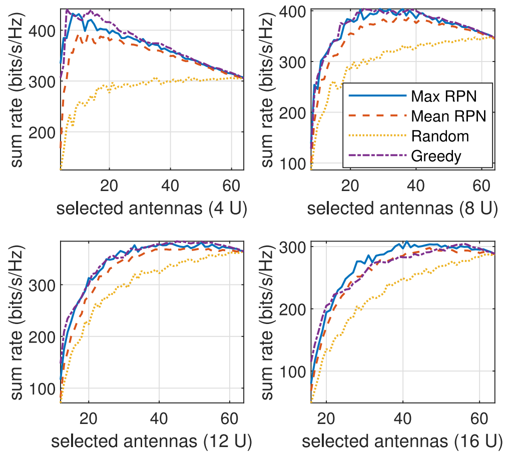

Fig. 3. Achieved sum rates for 4-16 users using the proposed algorithm vs. random and centralised greedy selection.

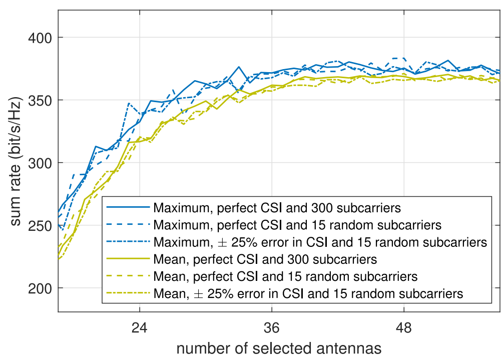

Fig. 4. The effects of imperfect CSI and random selection of subcarriers.

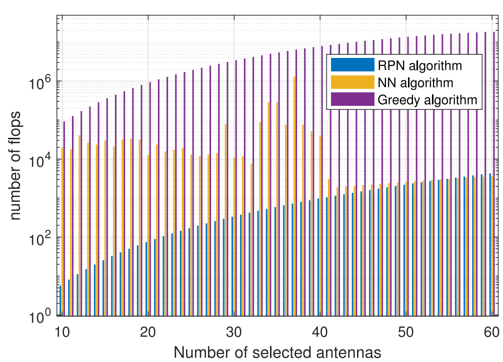

Fig. 5. Comparison of the computational load.

in all observed scenarios is demonstrated by the results of significantly under-performing random selection.

In [6], its has been shown that the distributed algorithms such as NN are resistant to errors in CSI and that they perform well even with just a (randomly selected) subset of subcarriers used for optimisation. We performed the same test for the RPN solution, and the result is shown in Fig. 4 in the case of 12 users. In the light of the computational complexity reduction discussed in the previous section, Fig. 5 shows the advantage of our proposed algorithm in the number of performed calculations in the case of choosing a number of antennas from 64 distributed antennas.

V. Conclusion

We have presented a novel distributed antenna selection algorithm, improving the shortcomings of the existing distributed solutions and additionally reducing computational complexity. Our application of RPN is a pioneering one, and we aim to expand the RPN approach to other resource management problems in wireless communications, drawing benefits from both conservation properties of RPN and the ability to run the networks, or their parts, in reverse direction to recover from faults and handle inherently reversible communication phenomena (e.g., receiver/transmitter duality). The ability of the RPN solution to act asynchronously and converge fast with minimal computational burden enables real time application of the algorithm even in high mobility scenarios. In future work, we will investigate the ways of translating the physical topology of the antenna array into the Petri net topology and propose solutions for new use cases brought by 5G.

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