SSY345

Project - Orientation

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Task 1

Having the angular velocity as an input rather than a state is preferable since the input acts as the change of the state. Thus the state i.e. the angles of rotation, can represent the current configuration and the input i.e. the angular velocity can naturally act as the change of the state.

Task 2

	Acc	Gyro	Mag
mean	$\begin{bmatrix} 0.0272 \\ -0.1015 \\ 10.0106 \end{bmatrix}$	$\begin{bmatrix} 1.98 \cdot 10^{-4} \\ 2.59 \cdot 10^{-4} \\ 3.19 \cdot 10^{-4} \end{bmatrix}$	$\begin{bmatrix} -2.5228 \\ 19.5529 \\ -41.0022 \end{bmatrix}$
var	$\begin{bmatrix} 1.008 \cdot 10^{-4} \\ 1.111 \cdot 10^{-4} \\ 1.417 \cdot 10^{-4} \end{bmatrix}$	$\begin{bmatrix} 0.2530 \cdot 10^{-6} \\ 0.3132 \cdot 10^{-6} \\ 0.2240 \cdot 10^{-6} \end{bmatrix}$	$\begin{bmatrix} 0.1447 \\ 0.6717 \\ 0.1711 \end{bmatrix}$

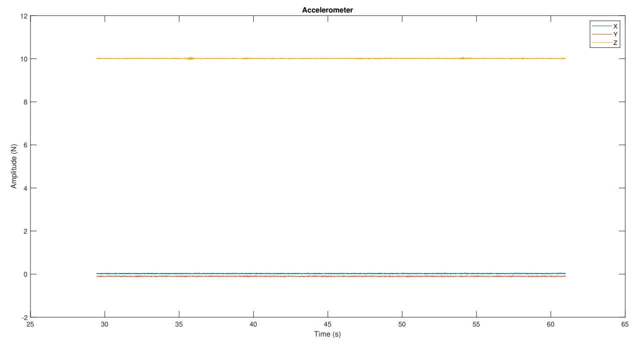

Figure 1: Accelerometer signal over time.

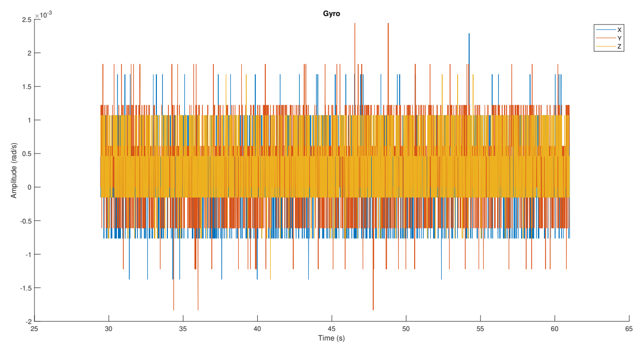

Figure 2: Gyroscope signal over time.

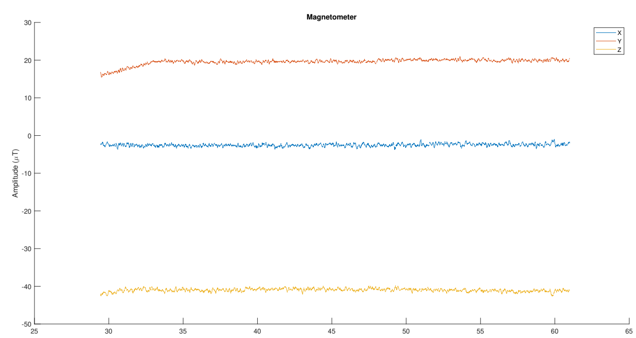

Figure 3: Magnetometer signal over time.

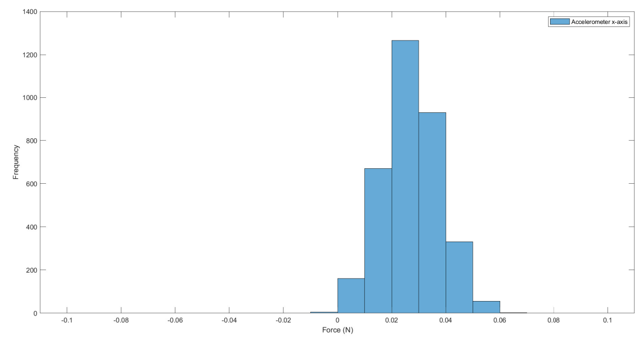

Figure 4: Histogram of accelerometer x-axis readings.

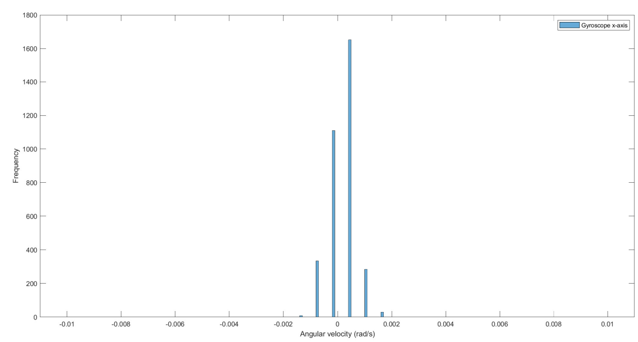

Figure 5: Histogram of gyroscope x-axis readings.

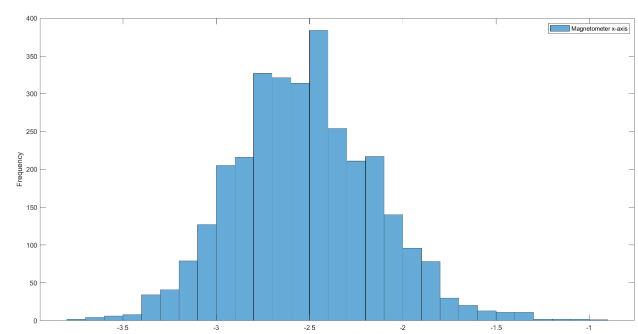

Figure 6: Histogram of magnetometer x-axis readings.

The noise for all sensors are gaussian and there are no biases for the gyroscope or accelerometer. The results might tell that the accelerometer may have a small bias but this is a result of the phone not being perfectly flat because of the camera protrusion. We think the gyro histogram looks like bars because of the resolution of 16-bit floats.

Task 3

$$F(\omega_{k-1}) = \frac{1}{200}\begin{bmatrix} 1 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 1 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 1 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 1 \end{bmatrix}$$

$$G(\hat{q}_{k-1}) = \frac{1}{200}\begin{bmatrix} -\hat{q}_1 & -\hat{q}_2 & -\hat{q}_3 \\ \hat{q}_0 & -\hat{q}_3 & \hat{q}_2 \\ \hat{q}_3 & \hat{q}_0 & -\hat{q}_1 \\ -\hat{q}_2 & \hat{q}_1 & \hat{q}_0 \end{bmatrix}$$

Approximating

$$G(q_{k-1})v_{k-1} \approx G(\hat{q}_{k-1})v_{k-1}$$

is valid since the true state is unknown and the EKF linearizes around the previous estimate.

Task 4

See code.

Task 5

The filter only using gyroscope readings is indeed lacking absolute orientation but it responds to fast movements very well. The orientation estimate always starts lying flat with the screen upward even when the true starting position is different. Shaking the phone causes the estimation error to increase and this is caused by multiple integration errors adding up over time.

Task 6

The measurement model is given by the equation $y_k^a = Q^\top(q_k)(g^0 + f_k^a) + e_k^a$ where it is assumed that $f_k^a = 0$. This gives a measurement estimate of $\hat{y}_k^a = Q^\top(\hat{q}_k)g^0$. This measurement estimate is used in a regular EKF and then the output is normalized.

$$g_0 = \begin{bmatrix} 0 \\ 0 \\ 10.0106 \end{bmatrix} m/s^2$$

$$h_x = Q_q(\hat{x}_{k|k-1})^\top g^0$$

$$\begin{bmatrix} Q_0 \\ Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \frac{\partial Q_q}{\partial q}(\hat{x}_{k|k-1})$$

$$H_x = \begin{bmatrix} Q_0^\top g^0 & Q_1^\top g^0 & Q_2^\top g^0 & Q_3^\top g^0 \end{bmatrix}$$

$$S = H_x P_{k|k-1} H_x^\top + R_g$$

$$K = P_{k|k-1} H_x^\top S^{-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K(y_k^g - h_x)$$

$$P_{k|k} = P_{k|k-1} - KSK^\top$$

Task 7

As the phone is accelerated back and forth on the table (in the horizontal plane) the estimated angle is changing. The true angle is never changing but the accelerometer readings are affecting the angle estimate.

Task 8

When the outlier detection is implemented and the phone is quickly slid back and forth on the table, the acceleration readings are high enough that the outlier detection detects too high of a reading, thus the filter is not calculating the estimate based on these samples. The angle estimate is therefore not affected anymore.

Task 9

The equation for the EKF update for the magnetometer is the following

$$m_0 = \begin{bmatrix} 0 \\ \sqrt{m_x^2 + m_y^2} \\ m_z \end{bmatrix}$$

$$h_x = Q_q(\hat{x}_{k|k-1})^\top m_0$$

$$
\begin{bmatrix} Q_0 \\ Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \frac{\partial Q_q}{\partial q}(\hat{x}_{k|k-1})
$$

$$
H_x = \begin{bmatrix} Q_0^\top m_0 & Q_1^\top m_0 & Q_2^\top m_0 & Q_3^\top m_0 \end{bmatrix}
$$

$$
S = H_x P_{k|k-1} H_x^\top + R_m
$$

$$
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$$

$$
\hat{x}_{k|k} = \hat{x}_{k|k-1} + K(y_k^m - h_x)
$$

$$
P_{k|k} = P_{k|k-1} - KSK^\top
$$

Task 10

When implementing the the magnetometer in the EKF there was a problem. The measurement noise from the magnetometer was to large compared to the noise from the accelerometer and gyro. This made it so when starting the estimation in the wrong position it takes a long time for it to find the true rotation around the global z-axis. To reduce this estimation time a scaling factor was added to the magnetometer measurement noise matrix that reduces over time. This makes the system trust the magnetometer more for the first few seconds to find the orientation but then it moves towards the optimal estimator once it found the true orientation.

Since the estimator barely uses the magnetometer readings, adding a magnetic disturbance causes no visible errors. But adding this disturbance during the startup sequence cause the estimated orientation to change a small amount.

Task 11

When designing the outlier detection it is assumed that the total magnetic flux density amplitude doesn't change by more that $2\mu T$ per sample. $\alpha = 0.01$ to use a small part of the new reading for the expected magnetic flux density. This outlier detection improves the performance during calibration.

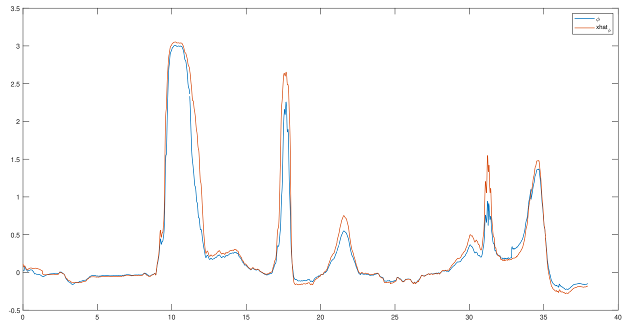

Figure 7: Plot of true φ and estimated φ

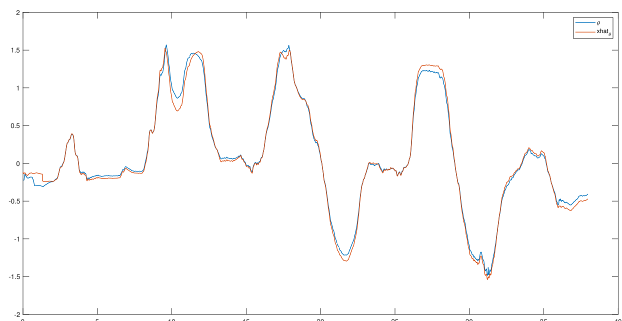

Figure 8: Plot of true θ and estimated θ

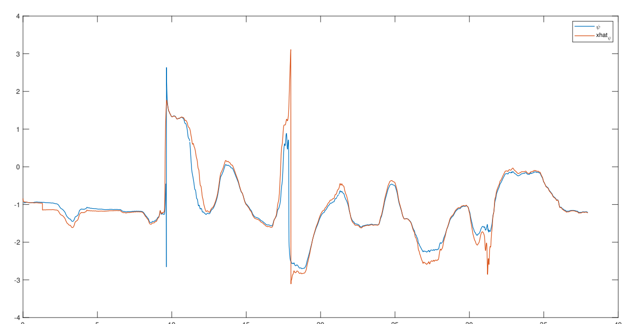

Figure 9: Plot of true ψ and estimated ψ

Task 12

When running the filter with only accelerometer and magnetometer it reacts very slowly to changes but it will eventually find the true position of the phone as long as it is standing still.

Running the filter with only gyro and magnetometer works good as long as there are not to many fast movements right after each other. This is because the magnetometer can find the true position on its own but it reacts slowly. Combining it with the gyro results in an estimator that reacts fast but has a small residual error as the magnetometer is not as accurate as the accelerometer.