CHALMERS

Sensor Fusion and Nonlinear Filtering

SSY345

Home Assignment 2

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1 : A first Kalman filter and its properties

1a

Based on the given linear and Gaussian state space model and the generated state and measurement sequences, the measurement behavior appears to behave according to the model. The state sequence exhibits a gradual increase or decrease over time, reflecting the influence of the motion noise, and its variance grows. The measurement sequence closely follows the state sequence, as expected from the measurement model equation $y_k = x_k + r_k$, where the measurement noise r_k with variance $R = 3$ adds smaller fluctuations compared to the motion noise. As the variance of the measurement sequence is smaller than that of the state sequence, the measurement noise has a lower impact than the motion noise on the sequence variation. Thus, the measurement behavior is consistent with the model assumptions.

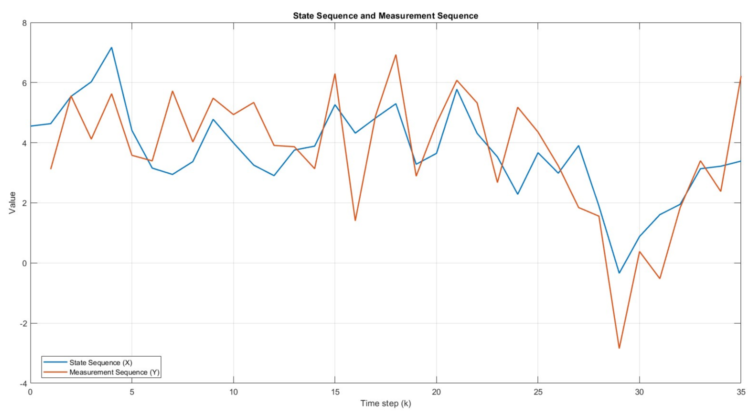


Figure 1: Question 1a : State Sequence and Measurement Sequence

1b

Reasonable estimates from the filter would exhibit close alignment with the true states and measurements over time, indicating that the filter is effectively comprising available information to estimate the system state. Additionally, the error covariance matrix should accurately capture the uncertainty in the estimates, with larger covariance values corresponding to higher uncertainty.
From the plots, we can see that the Kalman filter output looks reasonable as it closely follows the true states. The deviations can be seen to be less and therefore, the results holds true. Additionally, we can see that the error covariance represent the uncertainty in the estimates rather well as the measurements and true states are within the 3σ.
By analyzing the error density plots, we can gain insights into how well the

filter performs over time and how effectively it captures the uncertainty in the estimates at different stages of the filtering process. We can see in figure 3 that the error density plot becomes sharper as the time increases, meaning that there is decrease in uncertainty of the estimates as more observations are measured.

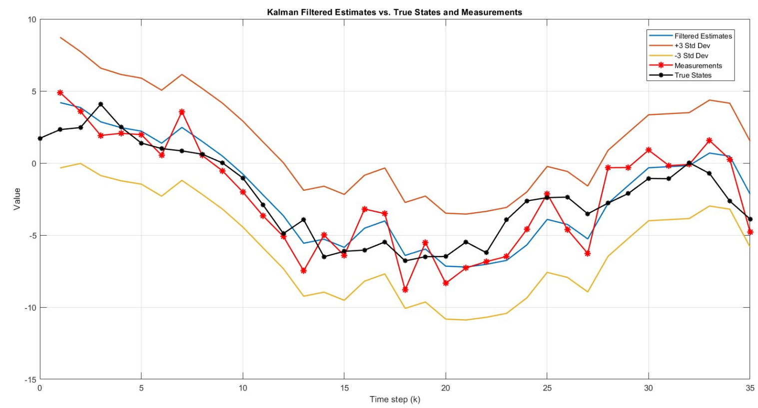


Figure 2: Question 1b: Kalman Filtered Estimates vs. True States and Measurements

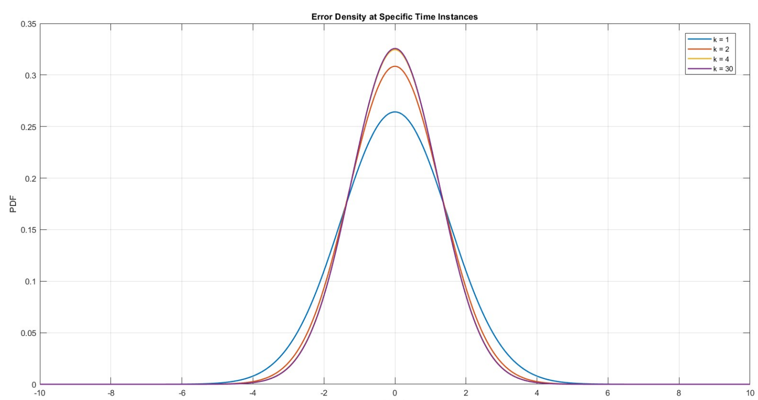


Figure 3: Question 1b: Error density around zero-mean for time instances k = [1; 2; 4; 30]

1c

By analyzing the plot, we can observe how the incorrect initial mean affects the filter's performance and the divergence of estimates from the true states and the estimates obtained using the correct initial mean. But, this is limited to the initial stages only. We can see from the plot that as number of steps increase, the graphs with correct and incorrect mean values converge. This is because, as the time steps proceeds, the filter obtains new information and the effect of the older information diminishes.

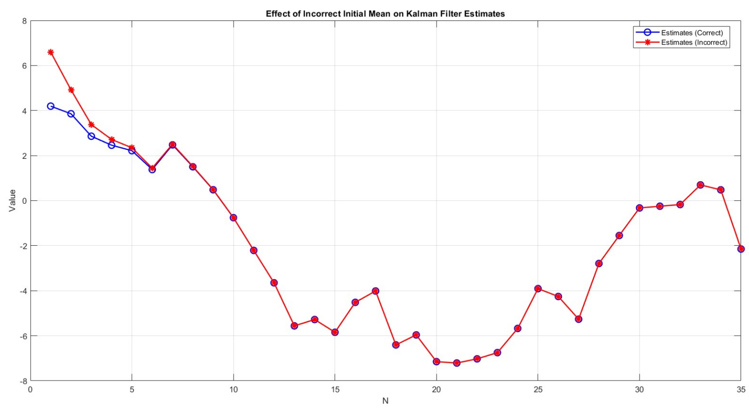


Figure 4: Question 1c: Effect of Incorrect Initial Mean on Kalman Filter Estimates

1d

The k is taken to be 30. In the prediction step, the filter increases the uncertainty due to the unpredictability of the state changes. This leads to a broader prediction PDF with a same mean but larger variance. But then, incorporating the measurements, the filter integrates new uncertainties from sensor noise resulting in a wider measurement distribution centered around the measured value.The Kalman filter combines prediction and measurement information through the Kalman gain, yielding a posterior distribution that exhibits reduced uncertainty compared to both prediction and measurement steps individually, therefore, handling noise and uncertainties.

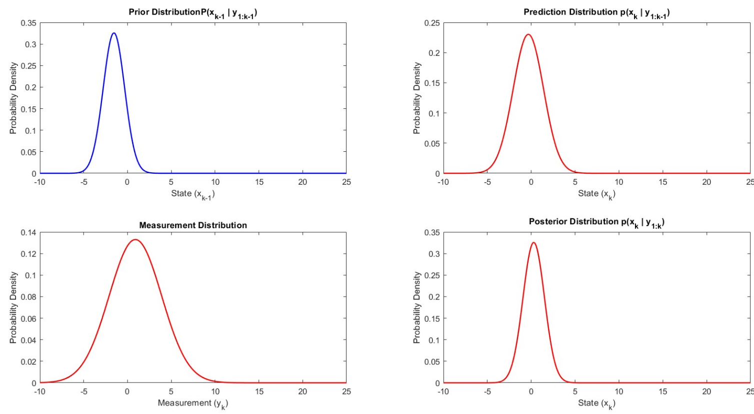


Figure 5: Question 1d: Plots for different scenarios

1e

We can see in the figure 6, the normal distribution of the estimated error. We can also confirm that the filter is working as expected as the histogram closely matched the expeceted pdf and the uncertainty in the estimates is well-represented by the covariance matrix $P_{k|k}$.

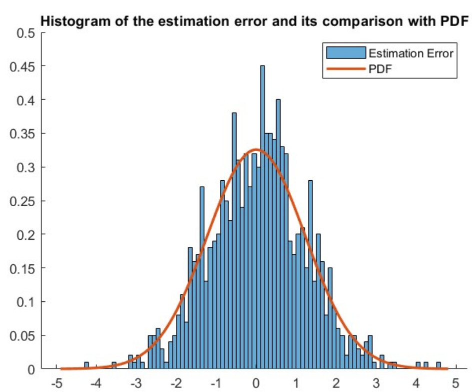


Figure 6: Question 1e: Histogram of the estimation error and its comparison with PDF

2 : Tuning a Kalman Filter

2a

Please refer to the MATLAB code to check the code that is implemnted to determine the scaling constant C and the variance of the velocity sensor noise. We are given the equation:

$$y_k^v = Cv_k + Cr_k^v$$

Taking expected value for the above equation, we get:

$$\mathbb{E}[y_k^v] = C\mathbb{E}[v_k] + \mathbb{E}[Cr_k^v]$$
$$C = \frac{\mathbb{E}[y_k^v] - \mathbb{E}[Cr_k^v]}{\mathbb{E}[v_k]}$$

Using MATLAB, we find that $\mathbb{E}[Cr_k^v] = -0.0734$ and the value of $\mathbb{E}[v_k]$ will be 10 when velocity is 10m/s and 20 when velocity is 20m/s.
Using this logic, we get that the value of C to be 1.1100 and 1.1059 for velocities 10m/s and 20m/s and mean C value to be 1.1080. We also get variances to be 3.0386.
To verify these values, we generate values for the velocities using these C and variance values, and then we plot both the measured values and generated values in a single graph and compare. Figure 7 shows the required plot and see that 2 data sets almost replicate each other, hence, we can conclude that the obtained C and variance values are correct.

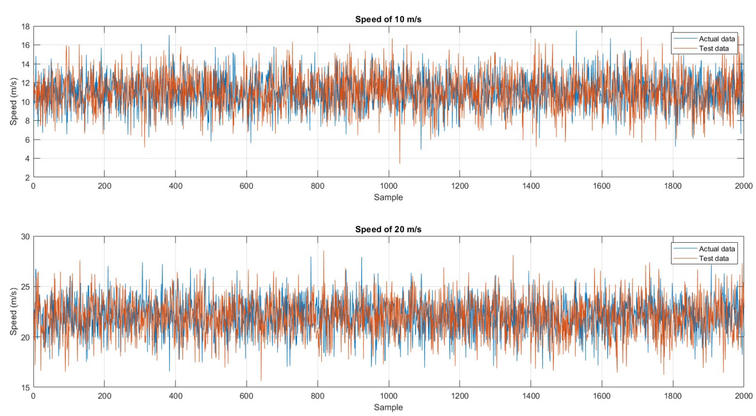


Figure 7: Question 2a: Measured data and test data

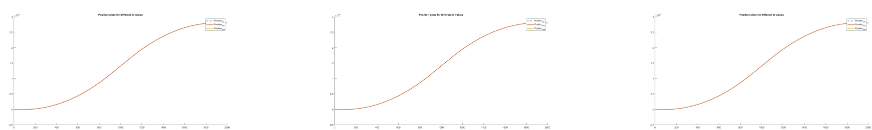
2b

To adapt the general Kalman filter equations for a scenario where the speed sensor updates are twice as fast as the position sensor updates, we modify the

Kalman filter implementation to skip the update step for position measurements when they are not available. This is achieved by checking for the availability of position measurements ($\sim$ isnan(Y(1,i)) within the Kalman filtering loop. If a position measurement is available, we perform the standard Kalman filter update using the `linearUpdate` function. However, if a position measurement is missing (NaN), we skip the update step and keep the state estimate and covariance unchanged. This adjustment ensures that the Kalman filter effectively handles the intermittent nature of position measurements while utilizing speed measurements in between to update the state estimate and covariance accordingly.**Please refer to the submitted MATLAB code** `kalmanFilter_2`

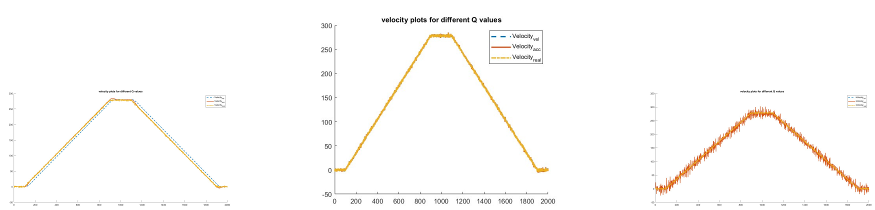
2c

The formulas from lecture 5 slides 37 were used to code for the constant velocity and constant acceleration cases. The term Q plays an important role. If we select a large Q value, we trust the measurement more, while smaller Q means we trust the process. Therefore, the code was run for various values of Q and the below figure, shows the position and velocity plots for 3 values of Q (0.001,100,100000). It was observed that larger values of Q allowed for larger deviations in acceleration.



(a) Q = 0.001 (b) Q = 100 (c) Q = 100000

Figure 8: Question 2c: Position plots for different values of Q



(a) Q = 0.001 (b) Q = 100 (c) Q = 100000

Figure 9: Question 2c: Velocity plots for different values of Q

2d

When comparing the constant velocity model and the constant acceleration model based on the observed behavior with different values of Q, it becomes

apparent that the choice depends on the dynamics of the system being modeled. A smaller Q value, as seen with $Q = 0.001$, favors the constant velocity model by placing more trust in the process model over measurements. This model performs well in scenarios with smooth and continuous velocity changes. Conversely, larger Q values such as $Q = 100$ and $Q = 100000$ enhance the constant acceleration model's performance, as it prioritizes measurements and better handles sudden acceleration changes. Therefore, if the system exhibits predictable and gradual velocity changes, the constant velocity model is preferable, while for systems with frequent abrupt acceleration changes, the constant acceleration model with larger Q values is more suitable for accurate estimation of position and velocity.
In the given case, CV model fits better as we can see from the figures 8 and 9 that the estimates from the CV model are smoother compared to the CA model.