Solution to analysis in Home Assignment 4

Armin Khorsandi – arminkh

Analysis

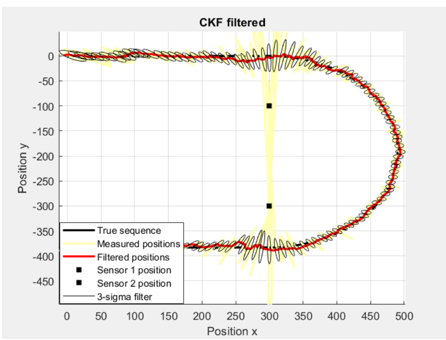
In this report I will present my independent analysis of the questions related to home assignment 4.

Smoothing

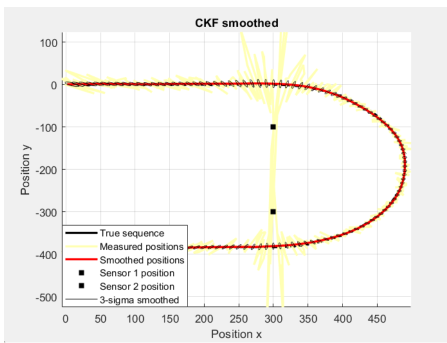
In this part of the assignment, the aim is to filter a measurement sequence with a cubature kalman filter (my favourite filter) using the coordinated turn motion model, and run the corresponding smoother of the filter on the measurement sequence and the filtering densities.

Task a

By following the descriptions of the task, the plots below were produced from the filtering and smoothings.



(a) Filter estimate

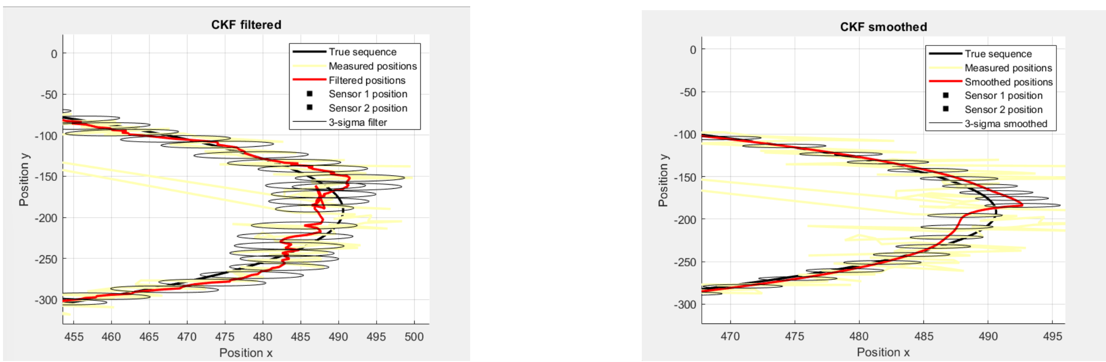


(b) Smoother estimate

By the first glance, it is observed that both estimates follow the true trajectory quite well. With more precise observation, what is seen is that the smoother is much more accurate and much less noisy compared to the filter estimation which deviates from the true sequence a bit. This is reasonable since at each time instance k, the smoother accesses all the measurements, before k, at k, and after time instance k. This is not true in case of the filter. The filter estimation is based on measurements up to time instance k. Also, the covariances for the smoother estimates are less than the filter estimates, which is again due to the same reason mentioned above.

Task b

In this task, an outlier is introduced in the measurements at time instance k = 300, which is almost in the middle of the half circle turn and is observable in yellow coming out of the trajectory in the figures below. This changes the behaviour of the estimates. At first glance, changes can not be seen from the plots, but by zooming into the plots, similar to the figures below, one can easily see that the filter estimates change direction completely at that instance and become completely wrong.



(a) Filter estimate *(b) Smoother estimate*

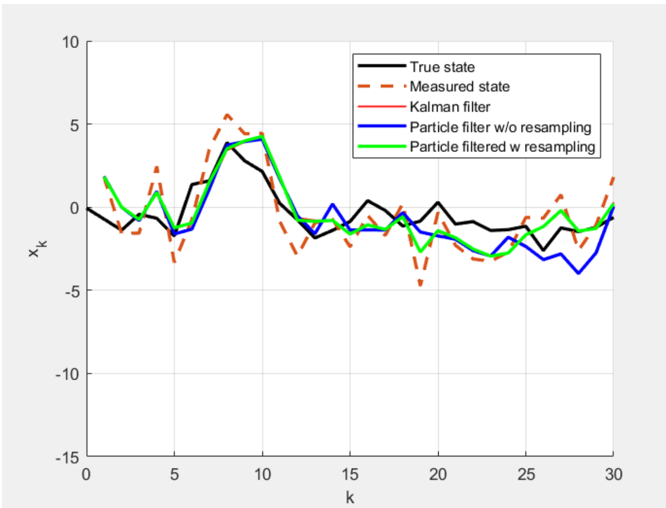
Meanwhile, the smoother is much more capable of coping with the outlier presented and is able to follow the true trajectory anyway, even though it deviates from it a little bit at that instance. This shows that the smoother is much more robust against such disturbances. This is again because the smoother estimates are based on all measurements, and an outlier will not change its direction or behaviour and make it completely wrong, while that is done in the filter estimates.

Particle filters for linear/Gaussian systems

In this problem, a linear and Gaussian state space model is considered in order to study the basic properties of particle filters in a linear and Gaussian setting.

Task a

In this task, a linear trajectory and measurement sequence are created, and three filters—a linear Kalman filter, a particle filter without, and a particle filter with resampling are run to compare their performances with each other. The three filters estimates are shown in the figure below together with the true sequence and the measurements.

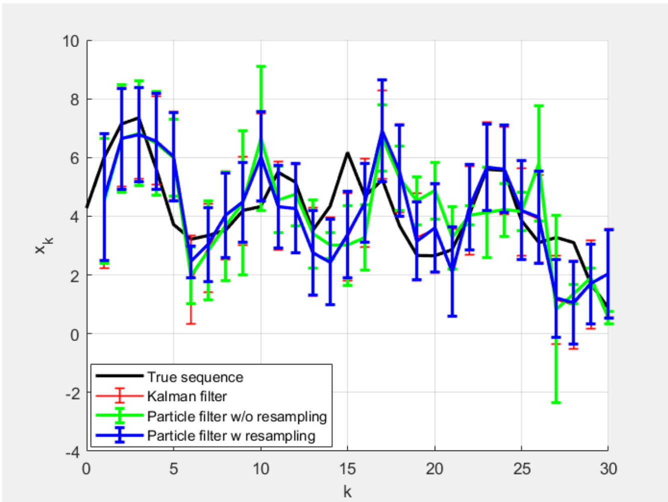


As the system is linear and Gaussian, the optimal filtering solution would be the Kalman filter, but as N gets higher, the particle filter with resampling is able to be almost as accurate as the kalman, while this is not happening if resampling is not done. This can be observed from the MSE's calculated for the different filters with different values of N.

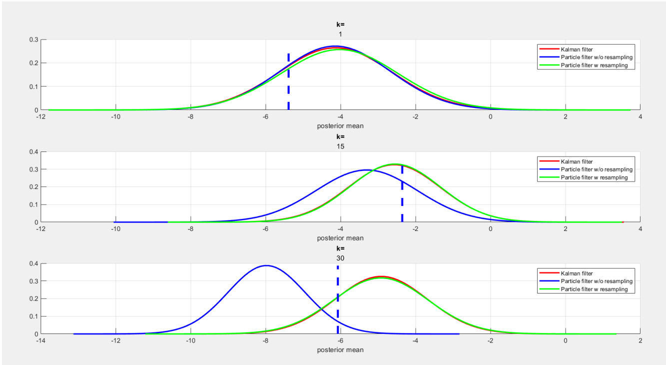
Table 1: MSE table for three filters with different N

MSE N	10	100	1000	5000
Kalman filter	0.8851	0.8851	0.8851	0.8851
Particle filter w/o resampling	4.7245	2.5269	0.9870	1.0969
Patricle filter w resampling	1.1174	0.9767	0.8464	0.8836

The main reason why the particle filter without resampling is not able to be as well as the one with resampling even with high values of N, is that resampling helps prevent the degeneracy problem, which helps to improve the performance of the filter. The figure below shows the covariances of the three filters which indicates very similar covariances of the kalman and particle filter with resampling while for the particle filter without resampling, as shown also in the table above, even with high values of N, we do not get the same performance. N used in the problem for the particle filters is 1000.

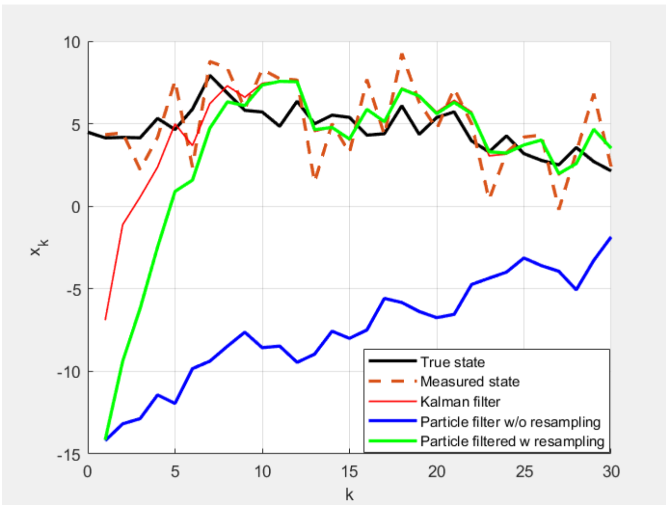


In the figure below, the posterior distributions for the three filters are also plotted for three distinct time instances. The particle filter without resampling yields estimates that are far from the true state, whereas the mean and covariance of the Kalman filter and particle filter with resampling are close to each other and to the true state at that instance, as it was anticipated.

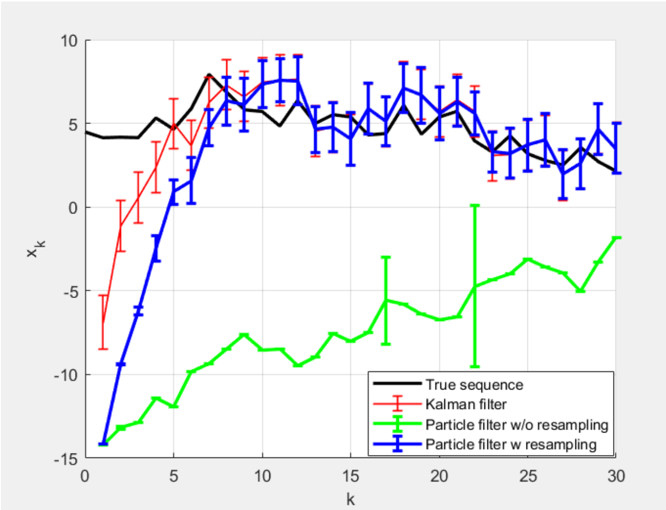


Task b

In this task, an incorrect prior was used at time 0 to see how fast the estimates can approach to the true trajectory. This wrong prior produces a completely different result, which shows the differences of the filters really well. While this incorrect prior has a bad influence on the performance of all three filters in terms of following the true trajectory in the beginning, the kalman filter, and a short while later the particle filter with resampling get back on track and converge to the true trajectory again. This does not happen in case of the particle filter without resampling as it can be shown in the figure below.

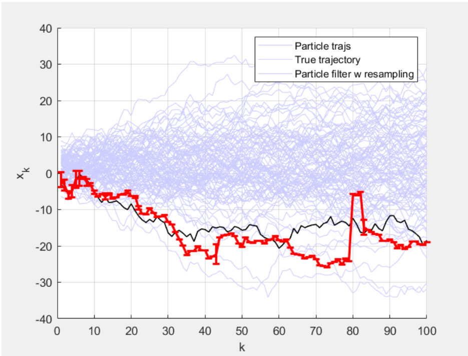


Also, the errorbars were plotted to see how the covariance of the errors looked like in each filter. As it can be observed from the figure below, the kalman and particle filter with resampling have very similar covariances after almost 6 time instances, while the particle filter without resampling has very little covariances which means that this filter has a high certainty with its particles which are taken along the incorrect trajectory. This difference shows how the resampling helps avoiding to follow the wrong path while thinking that it is correct.

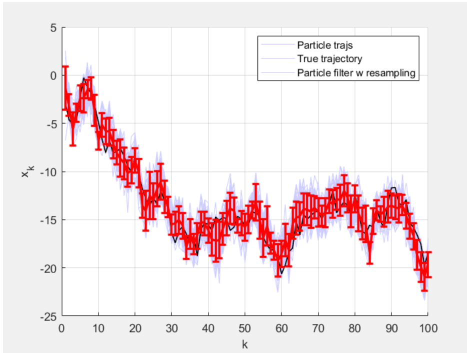


Task c and d

In these two tasks, the two particle trajectories of the two particle filters are compared with 100 patricles. k is chosen to be 100 for a more accurate and trustworthy result, since with k = 30, their performances cannot be confirmed. The figures below show how the performances differ.



(a) PF without resampling



(b) PF With resampling

The figures above demonstrate that the particles spread in the PF without resampling relative to the true trajectory. A particle filter with resampling, in contrast, adds a step after weighing in which particles are resampled without replacement based on their weights. This implies that while particles with lower weights might be removed, those with higher weights will be duplicated several times. Additionally, resampling ensures a more diversified representation of the posterior distribution, minimizing the degeneracy issue.