Preparation guidelines for oral examination

Background

At the final oral examination in the sensor fusion course, you will receive individual grades on your performance. It is thus a mandatory element in the course that may affect your final grade.

From a learning perspective, the main reason for including the oral examination is to give everyone a reason to look back at the entire course and try to make sure that you have understood all of its components. In order to help you prepare for the oral examination in a productive manner, we have put together **a list of questions that we will focus on at the oral examination**. That is, you will be asked a subset of these questions and questions from the home assignment quizzes at the oral examination and if you know the answers to all the questions listed below you will do well. Note that the examiner selects the questions that you should answer and that you may receive follow-up questions to clarify what you meant and to illuminate aspect not included in your answer.

Learning objectives and questions for the oral examination

In what follows, we have listed the learning objectives of this course followed by related questions/tasks. The learning objectives are the same objectives that are listed on the course homepage, but we have removed those related to filter implementations. In the questions, we use x to denote the states, or parameters of interest, y to denote measurements and k to denote time.

You should be able to explain the fundamental principles in Bayesian estimation.

In short, there are three fundamental principles in Bayesian estimation and we have formulated question around all of these.

1. What does it mean that we model unknown variables as stochastic variables?
   It is good if you can comment on how this is different compared to a frequentist interpretation of probability where "an event's probability as the limit of its relative frequency in a large number of trials" (the quote is taken from Wikipedia).

2. What is a posterior density and why does it hold that "posterior distribution $\propto$ prior distribution $\times$ likelihood"?

3. How are cost functions combined with posterior densities to obtain optimal estimates? We do not expect you to go through any detailed derivations, but we would like you to know how the optimisation criterion is formulated.

To connect these ideas to sensor fusion, we have the following additional questions:

4. What is the prior in the update step of a filter?

5. We typically refer to both $p(y\big|x)$ and $y = h(x)+r$ as measurement models, but what is the likelihood function $p(y|x)$ for the measurement model $y = h(x) + r$, where $r \sim \mathcal{N}(0, R)$?

6. How do you perform sensor fusion according to Bayesian principles? Suppose, for instance, that you have observed measurements from two sensors and that the measurement noises are independent such that $p(y_1, y_2|x) = p(y_1|x)p(y_2|x)$.

You should be able to describe and model commonly used sensors' measurements.

The second learning objective is related to measurement models, and we have one question/task with three subtasks.

7. Suppose you have a constant velocity model and a state vector $\mathbf{x} = \begin{bmatrix} x & y & \dot{x} & \dot{y} \end{bmatrix}^T$. Please write down models for sensors that collects measurements of

   i) only position,

   ii) only distance from sensor

   iii) both angle and distance from sensor.

   For simplicity, you can assume that the sensor is at the origin and that the noise is additive and Gaussian. You can either write down the model as a density, $p(\mathbf{y}_k|\mathbf{x}_k)$, or as an equation, $\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{r}_k$.

You should be able to summarize and compare the most typical motion models in positioning in order to know when to use them in practical problems.

The third learning objective is related to motion models, and we have several questions on this topic.

8. What are the equations for a random walk (RW), a constant velocity (CV), a constant acceleration (CA), and a coordinated turn (CT) motion models? You do not need to provide the details regarding the elements in the covariance matrix of the noise.

9. What are the key properties of the different motion models, and when are they useful?
   Hint: The models have several different properties, but one interesting difference between a CV and a CA model is how they model the acceleration; in the CV model, acceleration is white noise whereas in the CA model, acceleration is a random walk process. In many cases, this can help you to figure out which model is more suitable.

10. How does the motion model enable the use of measurements $y_{1:k-1}$ to gain information about x_k?

You should be able to derive the expression for an optimal filter.

The fourth learning objective is related to the conceptual solution to filtering described and discussed in lecture 3. By the way, an optimal filter in a Bayesian setting is a filter that computes the posterior density without any approximations.

11. Which components are included in a state space model? Which assumptions are made in a state space model?

12. Please derive the expressions for the prediction and update equations. You can assume that you have a state space model, such that the Markov property and the other assumptions hold.

You should be able to describe the essential properties of the Kalman filter (KF)

The Kalman filter has a lot of interesting and important properties. We have here selected a subset of these properties that we hope that you are now familiar with.

13. What happens (generally) with the uncertainties in the prediction and update steps?

14. Under what assumptions is the Kalman filter optimal? (To recall what we mean by an optimal filter, see above.)

15. What is calculated by the Kalman filter when it is optimal (when the assumptions mentioned above are true)?

16. In what sense does the KF converge for time invariant systems?

17. The final question(s) about the Kalman filter is about the innovation, which is an important component in the Kalman filter: How is the innovation used in the Kalman filter, what is the interpretation of the innovation and what are the properties does it have? (As for the properties, we are referring to its mean, covariance and correlation across time.)

You should be able to select a suitable filter method by analyzing the properties and requirements in an application

Apart from the learning objectives that concern abilities to implement and use the filters in practice, our final objective is related to your ability to select which type of filter to use in different contexts. In this course, we have focused on the Kalman filter, the extended Kalman filter, different sigma-point Kalman filters (the unscented Kalman filter, the cubature Kalman filter, etc) and particle filters. In the questions that follow, we want you to discuss these four types of filters.

18. How do the different filter types approximate the posterior density?

19. Can you give some general guidelines regarding when you would prefer to use the different filter types?

20. Which filter type would you recommend (please also motivate why) in the following two examples:

    a) Suppose that you observe the range (distance) and angle to an object moving around in a two-dimensional plane. You can assume that the parameters are similar to what you studied in problem 2, home assignment 4.

    b) Suppose that you would like to know how far along a track that a train has traveled and that you collect noisy observations of the speed (from a wheel speed sensor) and the turn rate of the train (from a gyroscope). We also assume that the shape of the track is known and to make the problem even more specific, you can assume that the track has the shape illustrated in Fig. 1. Please discuss two possibilities:

        i) the initial position is known,

        ii) the initial position is unknown.

    Hint: given the position and velocity of the train, you can compute the turn rate from the shape of the track (you do not need to tell us how), which means that the gyroscope will provide information about the position of the train. The observations of the turn rate can, for instance, help us to figure out if we are in a sharp turn or if we are traveling along a straight line.

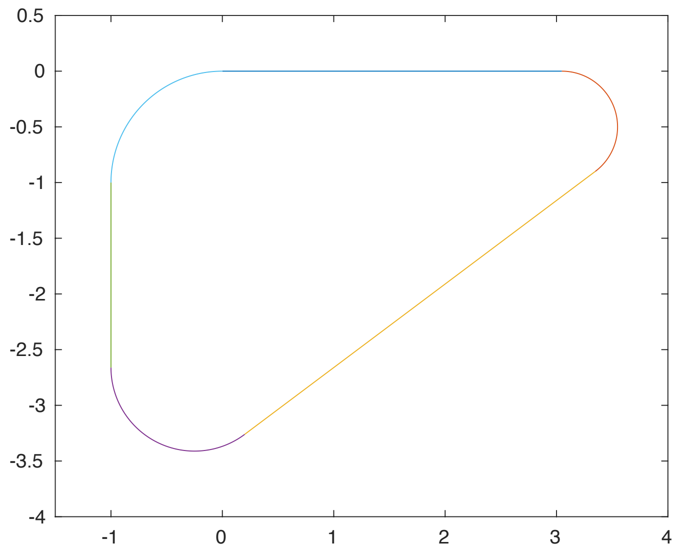

Figure 1: An illustration of the shape of the train track.