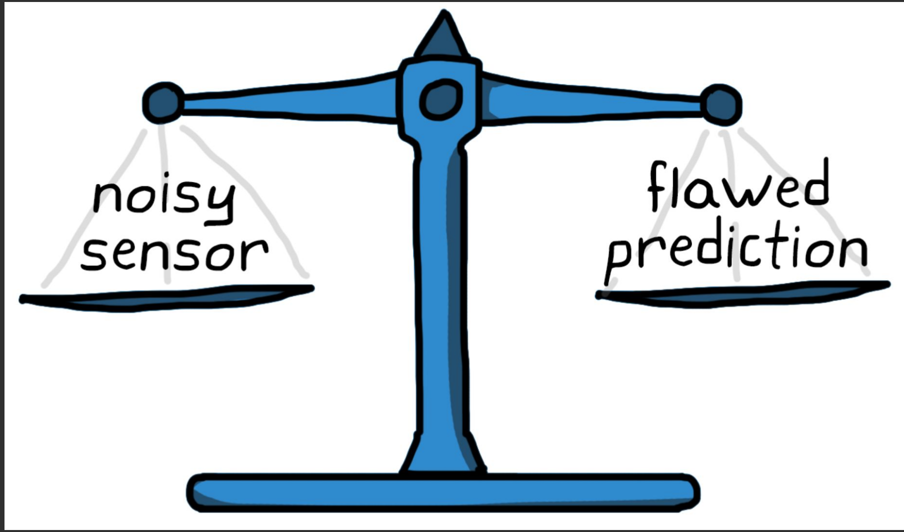


HA2 Solution Discussion

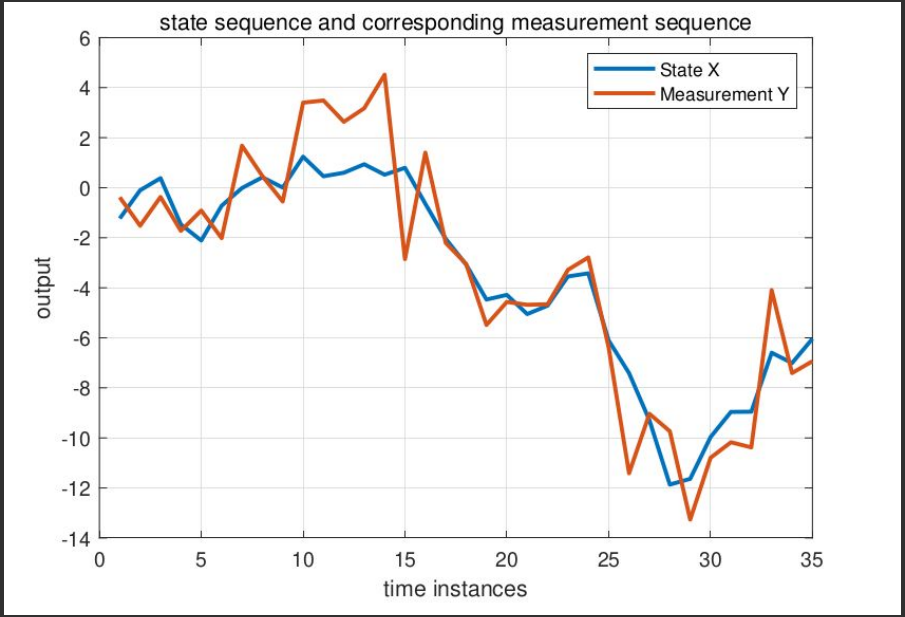
2024-04-19

Possible questions

1. Is the initial state significant to Kalman filter convergence?
2. What is the variance evolution in the prediction step and update step?
3. What observations emerge when the Kalman filter reaches a stationary point?
4. What insights can we glean from tuning the process noise in the Kalman filter?

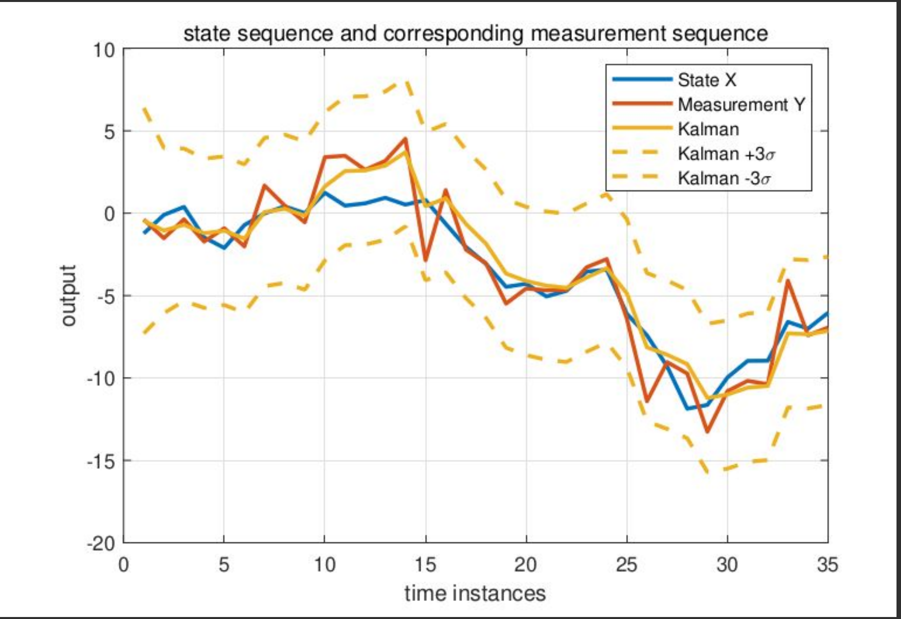
1a)

- Follow the state sequence
- Higher variance in measurement model
  - Q = 1.5
  - R = 3



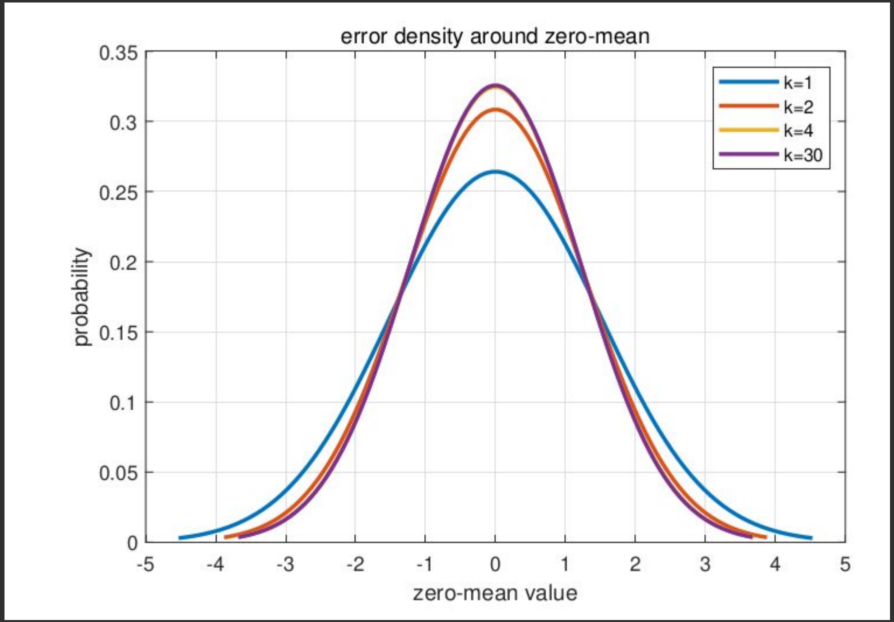
1b)

- The estimation follows the true states
- reasonable~> fall into the 3-σ region



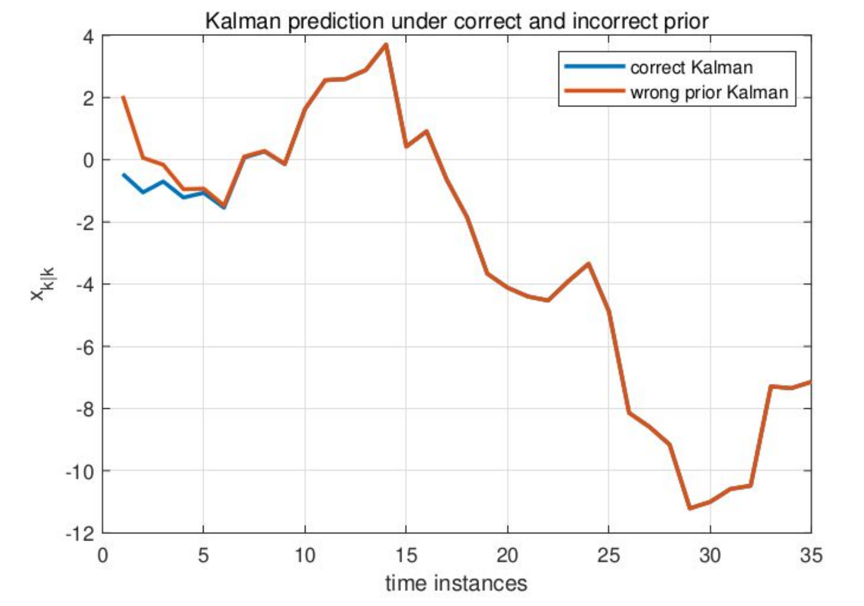
1b)

- k=[1, 2, 4, 10]
- The error density estimation around zero-mean
- k=1 ~> bigger variance ~> larger uncertainty
- k=4 and k=10 ~> similar variance ~> error converges as k increases.

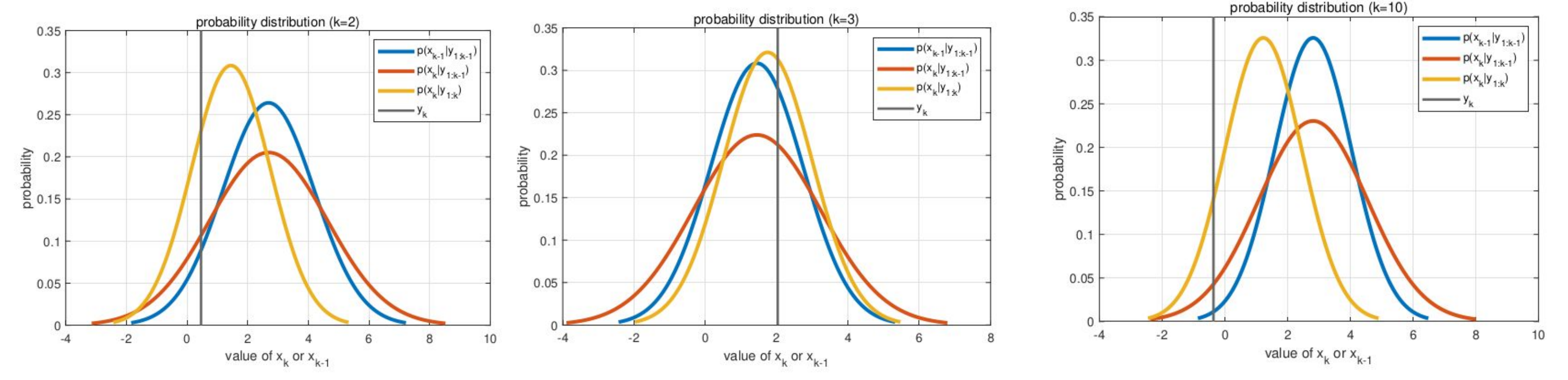


1c) correct prior and incorrect prior

- Incorrect prior ~> estimation is biased at the beginning.
- As time goes up, the initialization has less influence.



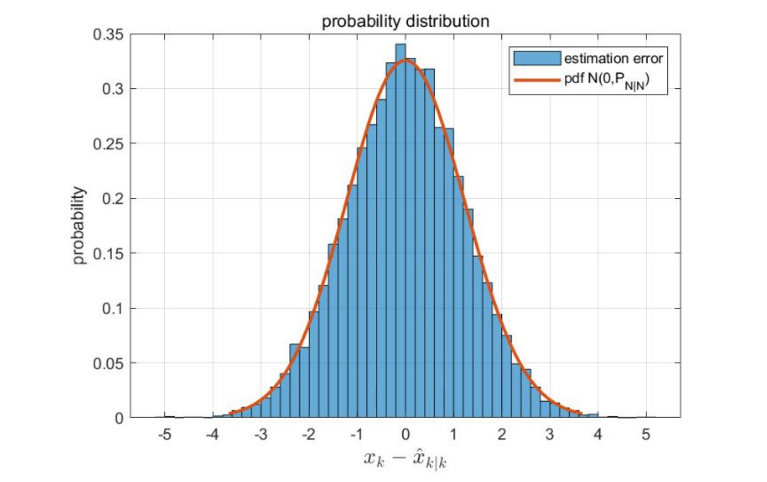
1d) Behavior of the prediction step and update step

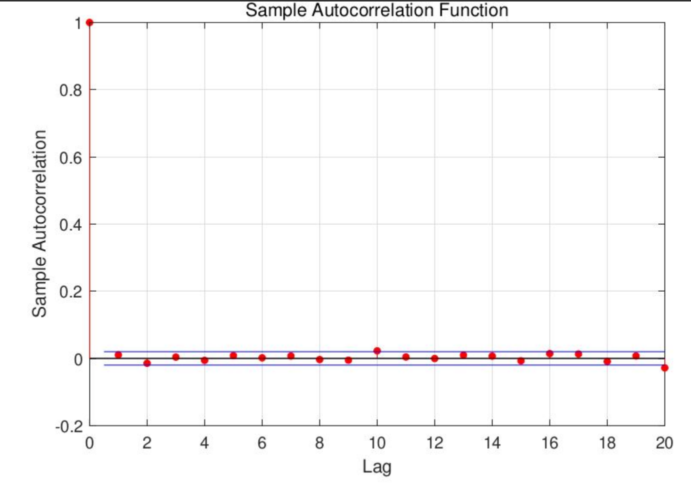


(a) $k = 2$ (b) $k = 3$ (c) $k = 10$

- Same mean $x_{k-1}|y_{1:k-1}$ and $x_k|y_{1:k-1}$ ~> A=[1]
- $Var(x_{k-1}|y_{1:k-1}) < Var(x_k|y_{1:k-1})$ ~> process noise
- The estimation from the update step lies in the between of y_k and $x_k|y_{1:k-1}$
- $Var(x_k|y_{1:k-1}) > Var(x_k|y_{1:k})$
- As k increases, the variance P_k converges.

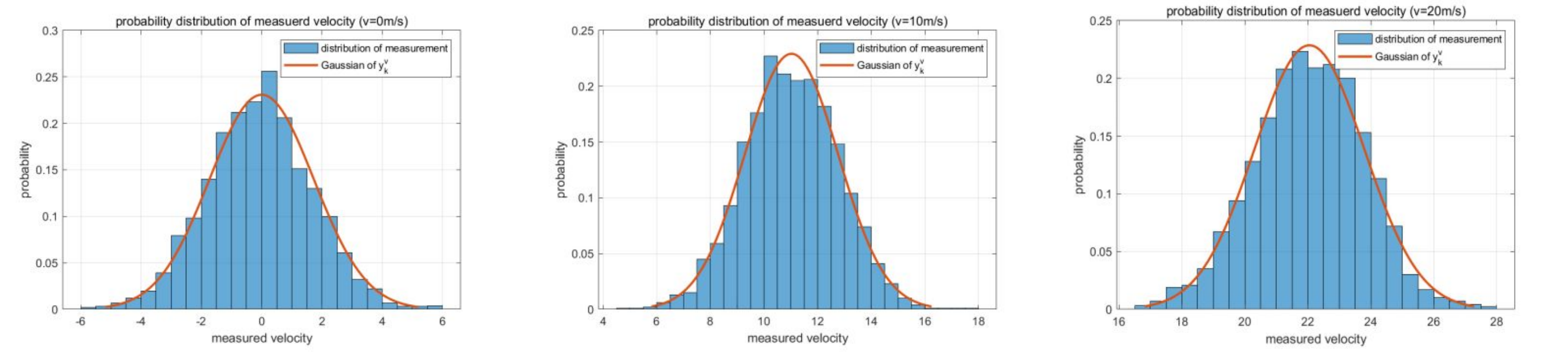
1e) Estimation error when system reaches the stationary point





- N is the length of the sequence
- The estimation error is zero mean with the variance $P_{k|k}$
- V_k is uncorrelated across time.

Tuning a Kalman Filter



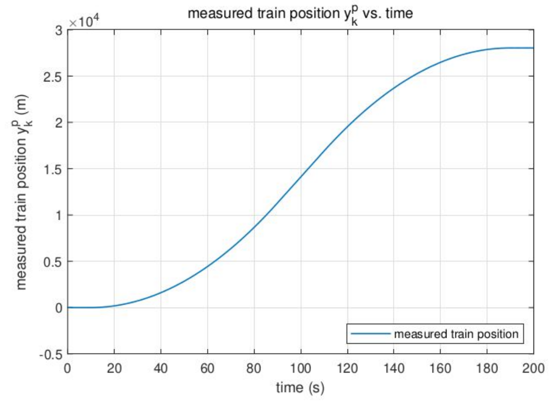
(a) $v = 0$ (b) $v = 10$ (c) $v = 20$

- Calculate empirical C_i from the velocity samples of the dataset i and C = mean(C_2, C_3)
- After obtaining C, **normalize** all three datasets to be zero mean. Further, merge all data then calculate the variance of velocity sensor noise by

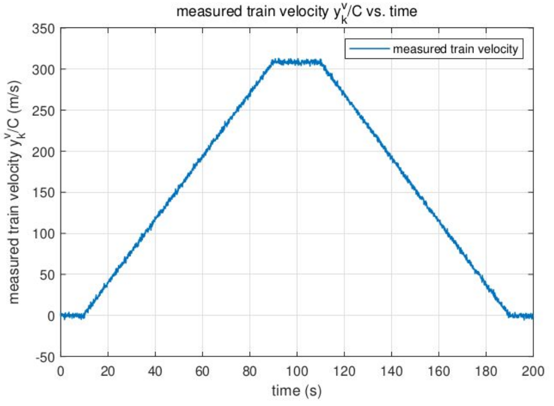
$$r_k^v = \frac{\mathrm{Var}(y_{\text{merged}})}{C^2}$$

b) Asynchronized sensors

- The speed sensor is twice as fast as the position sensor
    - Discard the speed measurement at time k, where no position measurement is collected.
        - Using Kalman filter without revising motion/measurement model
    - Fill up the position measurement at time k using the prediction from time k-1



(a) position



(b) velocity

c) Motion model selection and tuning

CV model:

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad P_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

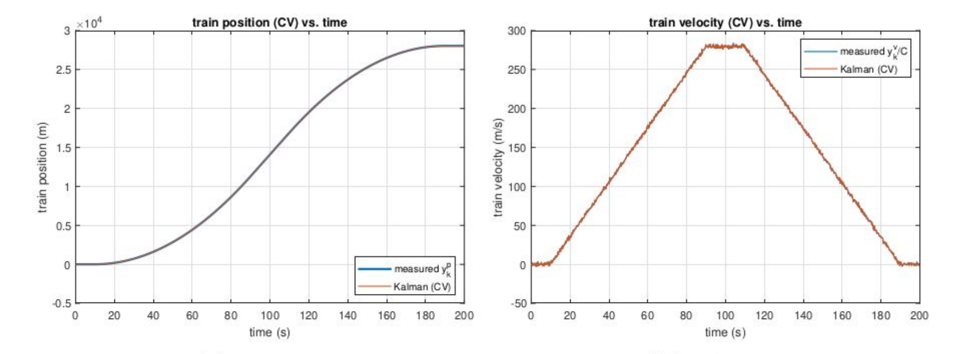
$$A_{k-1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

$$Q_{k-1} = \sigma_q^{v2} \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^2}{2} & T \end{bmatrix}$$

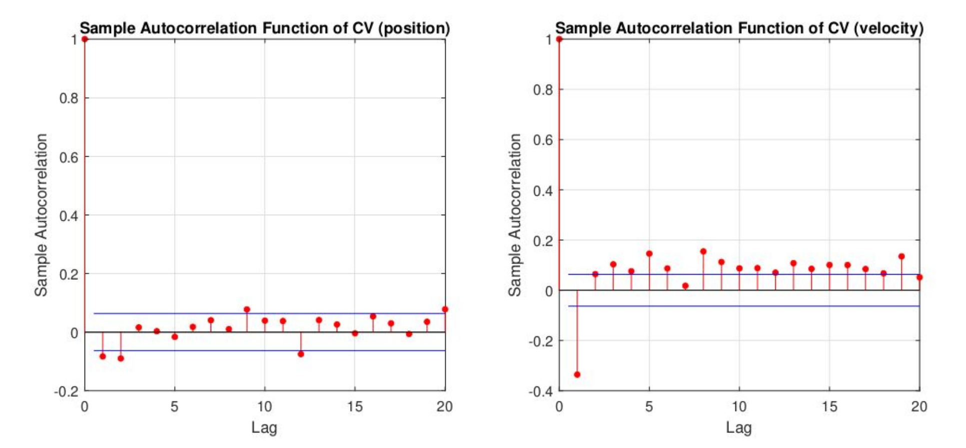
$$H_k = \begin{bmatrix} 1 & 0 \\ 0 & C \end{bmatrix}$$

$$R_k = \begin{bmatrix} r_k^p & 0 \\ 0 & r_k^v \end{bmatrix}$$

CV model ($\sigma_q^{v2} = 100$)



(a) position (b) velocity



c) Motion model selection and tuning

CA model:

$$x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad P_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

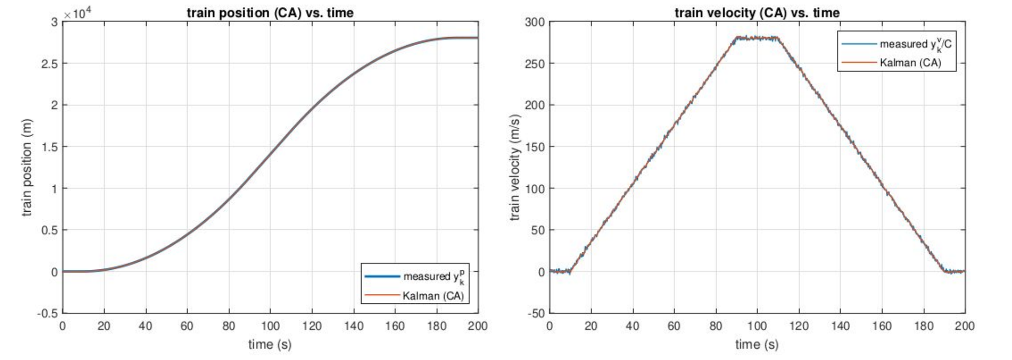
$$A_{k-1} = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}$$

$$Q_{k-1} = \sigma_q^{v2} \begin{bmatrix} \frac{T^5}{20} & \frac{T^4}{8} & \frac{T^3}{6} \\ \frac{T^4}{8} & \frac{T^3}{3} & \frac{T^2}{2} \\ \frac{T^3}{6} & \frac{T^2}{2} & T \end{bmatrix}$$

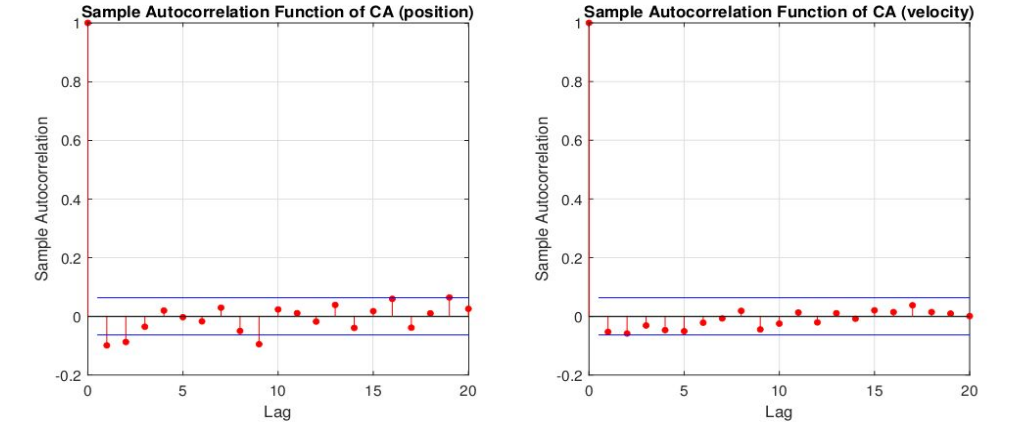
$$H_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C & 0 \end{bmatrix}$$

$$R_k = \begin{bmatrix} r_k^p & 0 \\ 0 & r_k^v \end{bmatrix}$$

CA model ($\sigma_q^{v2} = 1$)



(a) position (b) velocity



CV model Vs CA model

- CA fits better in this case
  - Smaller process noise
  - More robust but complex and more computation power
- CV is simpler
  - Larger process noise
  - Relies on measurement quality