SSY345 - Project: Orientation estimation of smartphones

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Task 1

Using the gyroscope angular velocity as input ($u_k = \omega_k$).

Advantages

- **Measurement Accuracy:** Most modern smartphones' gyroscopes exhibit high measurement precision and reliability. Using them as input allows direct utilization of these accurate data to enhance the quality of direction estimation.
- **System Simplification:** Handling angular velocity as a known input simplifies the state estimation model. This reduces the dimensionality of the state vector, lowers computational complexity, and enhances filter efficiency.

Disadvantages

- **Effected by the gyroscope drift:** Over time, gyroscopes can experience drift, leading to errors in estimated orientation. Extended operation without external reference or correction can cause a gradual deviation from the true orientation.
- **Reduced Flexibility:** Viewing angular velocity as a known input implies poorer adaptability and flexibility of the system when dealing with new sensor types or sensor failures that might arise in the future.

Unsuitable Scenarios: In cases where environmental factors (such as mechanical vibrations) significantly affect gyroscope readings, directly using angular velocity as input may not be the best choice, as it could lead to significant errors in direction estimation.

As state vector: Including angular velocity in the state vector can significantly enhance the system's adaptability, flexibility, and robustness, especially when dealing with multiple sensor data and dynamic environmental changes. This approach enables the system to provide more accurate and reliable state estimation, which is crucial for applications involving complex navigation and environments with poor sensor reliability.

Task 2

In this task, we placed the phone flat on the table to get the measurements from the sensors which provide valuable insights into potential biases and the distribution of noise. This will later be helpful when tuning the filter.

The the signals of Accelerometer over time shown as in Figure1. Histograms of measurements for Accelerometer shown as in Figure2.

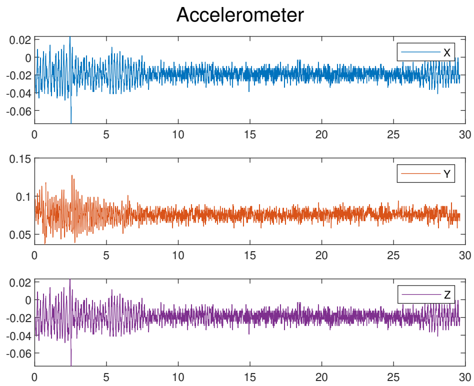

Figure 1: Signals of Accelerometer.

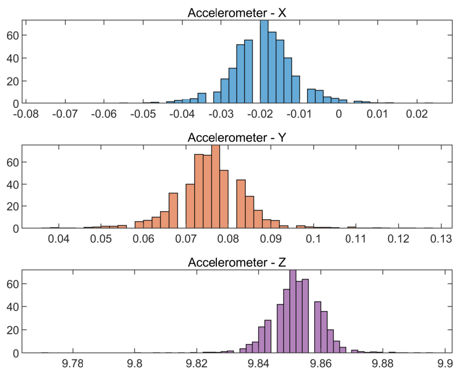

Figure 2: Histograms of Accelerometer.

The mean and covariance for the acceleration vector when the phone is placed flat on the table are illustrated in Table 1.

Table 1: Mean and Covariance for Accelerometer.

	X-axis	Y-axis	Z-axis
Mean	-0.0191	0.0757	9.8521
Covariance	7.020605e-5	6.935685e-5	6.346485e-5

From the figure, we can find that the histograms of measurements for the Accelerometer displays a distribution that closely resembles a Gaussian distribution. .

The the signals of Gyroscope over time shown as in Figure3. Histograms of measurements for Gyroscope shown as in Figure4.

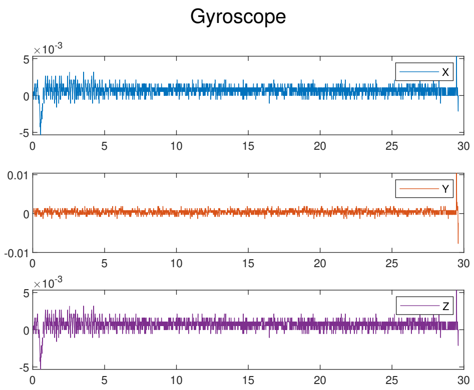

Figure 3: Signals of Gyroscope.

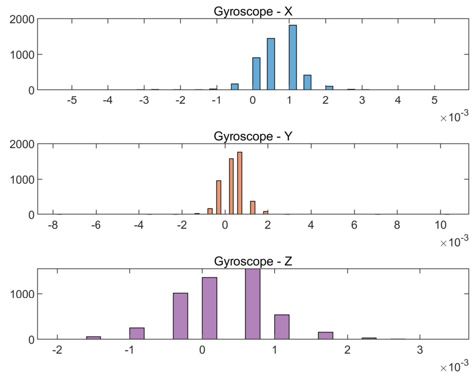

Figure 4: Histograms of Gyroscope.

The mean and covariance for the angular velocity vector when the phone is placed flat on table are illustrated as in Table 2

Table 2: Mean and Covariance for Gyroscope.

	X-axis	Y-axis	Z-axis
Mean	0.0007	0.0004	0.0003
Covariance	4.629868e-7	3.921599e-7	4.510415e-7

From the figure, we can find that the histograms of measurements for the Gyroscope does not precisely align with a Gaussian distribution. While the noise in the gyroscope is exceptionally small making its measurements highly reliable. Therefore, during the tuning process, it can be assumed that the measurement model noise is negligible

The the signals of Magnetometer over time shown as in Figure5. Histograms of measurements for Magnetometer shown as in Figure6.

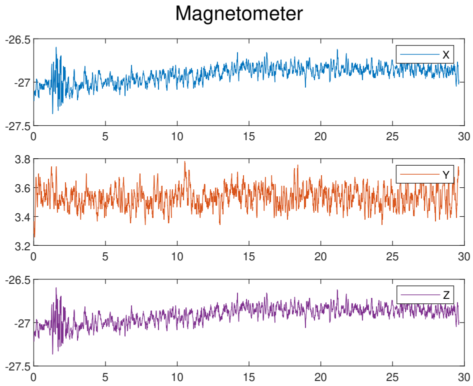

Figure 5: Signals of Magnetometer.

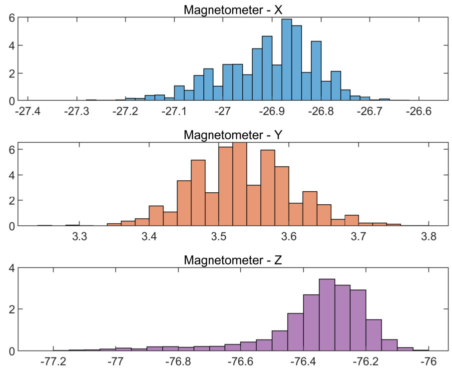

Figure 6: Histograms of Magnetometer.

The mean and covariance for the magnetic field when the phone is placed flat on table are illustrated as in Table 3

Table 3: Mean and Covariance for Magnetometer.

	X-axis	Y-axis	Z-axis
Mean	-26.9073	3.5339	-76.3608
Covariance	9.748587e-3	5.220846e-3	3.074807e-2

From the figure, we observe that the histograms of measurements for the Magnetometer resemble a Gaussian distribution, albeit with a slight offset to the right. This offset could be caused by the environmental noise or biases in the magnetometer.

Task 3

We have the continuous time model as:

$$\dot{q}(t) = \frac{1}{2}S(\omega_{k-1} + v_{k-1})q(t). \tag{1}$$

This differential equation can be solved using matrix exponential and the relation: exp $A \approx I + A$:

$$\begin{aligned} q(t_k) &= \exp\left[\frac{1}{2}S(\omega_{k-1} + v_{k-1})(t_k - t_{k-1})\right] q(t_{k-1}) \\ &= \left[\mathbf{I} + \frac{T}{2}S(\omega_{k-1} + v_{k-1})\right] \cdot q(t_{k-1}) \end{aligned} \tag{2}$$

where $T = t_k - t_{k-1}$.

Expanding the equation (2), we can derive a discretized mode:

$$\begin{aligned} q_k &= \mathbf{I} \cdot q_{k-1} + \frac{T}{2}S(\omega_{k-1})q_{k-1} + \frac{T}{2}S(v_{k-1})q_{k-1} \\ &= \left[\mathbf{I} + \frac{T}{2}S(\omega_{k-1})\right] \cdot q_{k-1} + \frac{T}{2}\bar{S}(q_{k-1})v_{k-1} \\ &= \left[\mathbf{I} + \frac{T}{2}S(\omega_{k-1})\right] \cdot q_{k-1} + \frac{T}{2}\bar{S}(\hat{q}_{k-1})v_{k-1} \end{aligned} \tag{3}$$

Thus we can derive a discretized model of the form:

$$q_k = F(\omega_{k-1})q_{k-1} + G(\hat{q}_{k-1})v_{k-1} \tag{4}$$

Where F and G as:

$$\begin{aligned} F(\omega_{k-1}) &= \mathbf{I} + \frac{T}{2}S(\omega_{k-1}) \\ G(\hat{q}_{k-1}) &= \frac{T}{2}\bar{S}(\hat{q}_{k-1}) \end{aligned} \tag{5}$$

Through these steps, we derive the expressions for F and G. The EKF procedure suggests approximating $G(q_{k-1})v_{k-1}$ with $G(\hat{q}_{k-1})v_{k-1}$, as the EKF relies on linearizing around the current state estimate. This approximation is consistent with the EKF's approach, ensuring that the Jacobians are computed based on the point where the filter estimates the state to be.

Task 4

The time update function was implemented in Matlab function [x, P] = tu_qw(x, P, omega, T, Rw), and the code of the function shown as bellow.

```
function [x, P] = tu_qw(x, P, omega, T, Rw)
% the time update function
% omega    the measured angular rate
% T        the time since the last measurement
% Rw       the process noise covariance matrix

% expressions for F and G
F = eye(size(x, 1)) + (T/2)* Somega(omega);
G = (T/2)*Sq(x);

% update
x = F*x;
P = F*P*F' + G*Rw*G';

% normalizes the quaternion
[x, P] = mu_normalizeQ(x, P);
end
```

In the case that there is no angular rate measurement available, we can just skip the time update for the current time step, and keep the state and covariance as the last update one.

Task 5

By utilizing the function described in Task 4 we are able to get an estimate of the orientation of the smartphone.

We can get very reactive estimates of the rotations, but that lacks absolute orientation. The estimation is not very well as in Figure 7. Over time, with updates only from gyroscope data, the filter's angle estimation will experience drift as in Figure 8. This is due to accumulated errors in the gyroscope data, such as bias and noise, causing the angle estimation to gradually deviate from the true value.

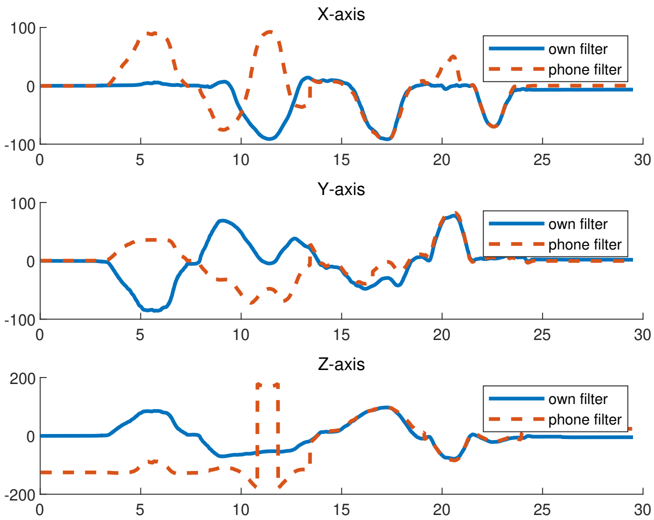

Figure 7: Estimation without measurement updates.

Start on the side: If we start the filter with the phone on the side instead of laying face up on the desk as in Figure 9, we can see that the filter can not capture this change. The estimation is still as laying face up on the desk. Since the filter is initialized with the assumption that the phone is face up. This mismatch leads to incorrect state updates because the filter interprets the measurements based on the wrong initial orientation.

Shake the phone: Shaking the phone causes the estimate to lose track of the true orientation, leading to significantly worse results. Shaking the phone will make the gyroscope to drifting, leading to a bias in the estimation.

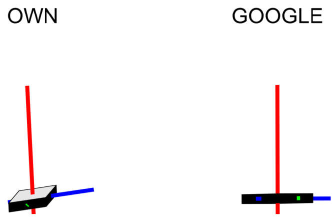

Figure 8: Estimation drift.

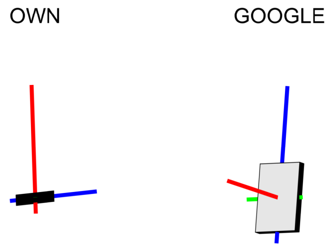

Figure 9: Start on the side.

Task 6

The measurements from the accelerometer can be modelled using the relation:

$$y_k^a = Q^T(q_k)(g^0 + f_k^a) + e_k^a \tag{6}$$

Since we assumed that $f_k^a = 0$ and $x = q$, we can get:

$$\begin{aligned} h(x) &= Q^T(q_k) \cdot g^0 \\ H(x) &= h'(x) = \frac{dQ(q)}{dq} \cdot g^0 \end{aligned} \tag{7}$$

Then we can describe the EKF update using y_k^a as:

$$\begin{aligned} S_k &= H(\hat{x}_{k|k-1})P_{k|k-1}H(\hat{x}_{k|k-1})^T + R_k \\ K_k &= P_{k|k-1}H(\hat{x}_{k|k-1})^T S_k^{-1} \\ \hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k(y_k^a - h(\hat{x}_{k|k-1})) \\ P_{k|k} &= P_{k|k-1} - K_k S_k K_k^T \end{aligned} \tag{8}$$

By utilizing the equation (7) and (8), we can implements the accelerometer measurement update as a Matlab function `[x, P] = mu_g(x, P, yacc, Ra, g0)`. The code of the function shown as bellow:

```
function [x, P] = mu_g(x, P, yacc, Ra, g0)
% the EKF update using yka
% under the assumption that fak = 0
% yacc      yacc is shorthand for yak, the measurement
% Ra        the measurement noise covariance matrix
```

```
% measurement function
h = Qq(x)'*g0;

% The derivative of Q(q) wrt qi, i={0,1,2,3}
[Q0, Q1, Q2, Q3] = dQqdq(x);

H = [Q0'*g0, Q1'*g0, Q2'*g0, Q3'*g0];

% the update step
% Innovation
v = yacc - h;

% Innovation Covariance
S = H*P*H' + Ra;

% Kalman Gain
K = P*H'*inv(S);

% update
x = x + K*v;

P = P - K*S*K';

% normalizes the quaternion
[x, P] = mu_normalizeQ(x, P);

end
```

Task 7

Applying the accelerometer measurement update to the filter will result in a better orientation estimation shown as in Figure 10. The gyroscope introduces drift over time due to integration. However, by integrating the accelerometer data into the filter, it effectively combines the information from both sensors, leading to a reduction in drift.

Enhanced Sensitivity: With the addition of accelerometer updates, the filter's sensitivity to phone vibrations is noticeably increased, leading to more accurate estimation results.

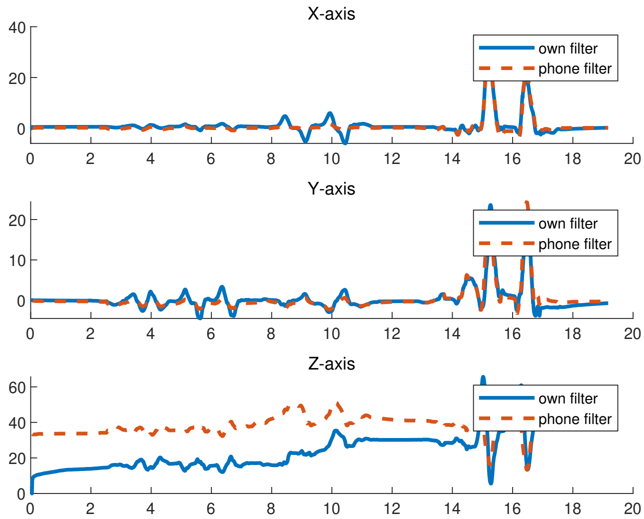

Figure 10: Estimation using accelerometer measurements.

Task 8

Incorporating a straightforward outlier rejection algorithm into the filter can effectively alleviate the problem of drift. Leading to a better estimation result shown as in Figure 11.

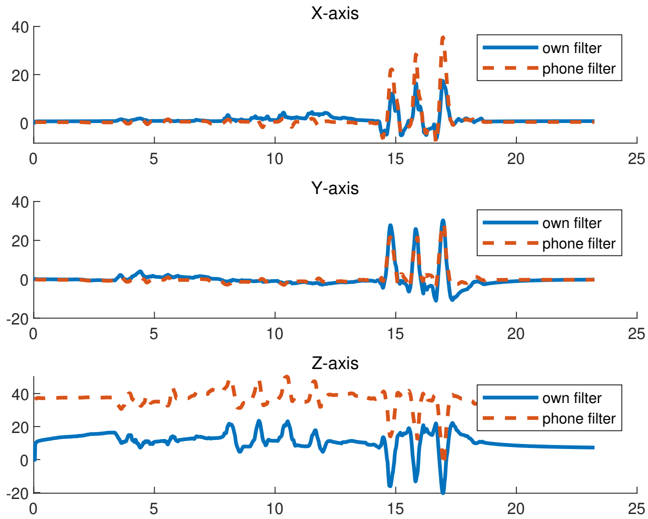

Figure 11: Estimation with accelerometer outlier rejection.

Task 9

The EKF update using y_k^m is basic the same as in the Task6:

$$\begin{aligned}
h(x) &= Q^T(q_k) \cdot m^0 \\
H(x) &= h'(x) = \frac{dQ(q)}{dq} \cdot m^0 \\
\\
S_k &= H(\hat{x}_{k|k-1})P_{k|k-1}H(\hat{x}_{k|k-1})^T + R_k \\
K_k &= P_{k|k-1}H(\hat{x}_{k|k-1})^T S_k^{-1} \\
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k(y_k^a - h(\hat{x}_{k|k-1})) \\
P_{k|k} &= P_{k|k-1} - K_k S_k K_k^T
\end{aligned} \tag{9}$$

The code of the Matlab function $[\mathtt{x},\ \mathtt{P}] = \mathtt{mu_m}(\mathtt{x},\ \mathtt{P},\ \mathtt{mag},\ \mathtt{m0},\ \mathtt{Rm})$ shown as bellow:

```
function [x, P] = mu_m(x, P, mag, m0, Rm)
% the EKF update using ykm
% under the assumption that fkm = 0.
% mag       is shorthand for ykm, the measurement
% m0        is m0
% Rm        the measurement noise covariance matrix

% measurement function
h = Qq(x)'*m0;

% The derivative of Q(q) wrt qi, i={0,1,2,3}
[Q0, Q1, Q2, Q3] = dQqdq(x);

H = [Q0'*m0, Q1'*m0, Q2'*m0, Q3'*m0];

% the update step
% Innovation
v = mag - h;

% Innovation Covariance
S = H*P*H' + Rm;

% Kalman Gain
K = P*H'*inv(S);

% update
x = x + K*v;

P = P - K*S*K';

% normalizes the quaternion
[x, P] = mu_normalizeQ(x, P);
end
```

Task 10

Experimental observations reveal that incorporating magnetometer updates leads to a more stable and accurate orientation estimation by the filter which shown as in the Figure 12. Our orientation filter that behaves much like the one implemented in the phone. However, in the presence of magnetic field interference, the estimation results can be significantly affected as in Figure 13.

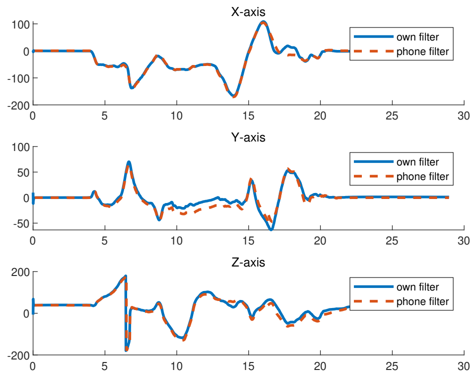

Figure 12: Estimation with magnetometer measurements.

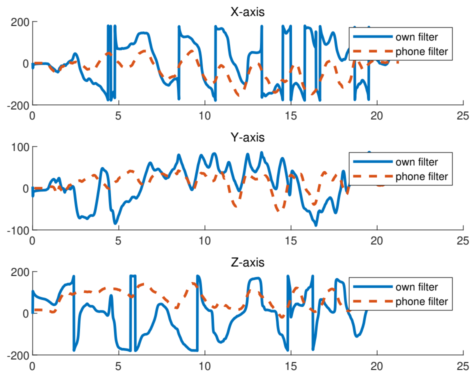

Figure 13: Estimation with a magnetic disturbance.

Task 11

The assumptions we rely on are as follows:
1. **Assumption of Effective Outlier Detection:** We assumed that the disturbance can be effectively detected. This assumption is reasonable when sensors typically have sufficient accuracy and sensitivity to measure magnetic field intensity and we have chosen an appropriate threshold.
2. **Assumption of Magnetic Field Drift:** We assume that even in the absence of interference, the expected magnitude of the magnetic field will slowly drift. This assumption is reasonable because the magnetic field is influenced by various factors such as the Earth's rotation.

After applying the outlier rejection for magnetic disturbances to the filter, the filter can effectively reduce the impact of the magnetic disturbances leading to a more accurate estimation shown as in Figure 14.

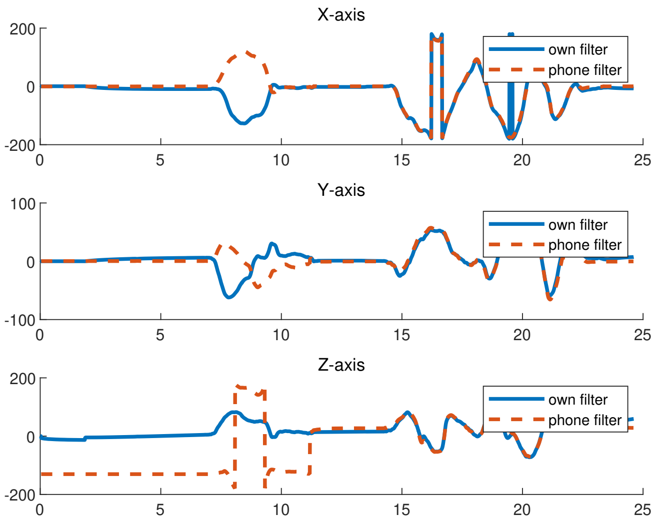

Figure 14: Estimation with magnitude outlier rejection.

The Euler angles of both our orientation filter and the built in filter in the phone are illustrated in the Figure 15.

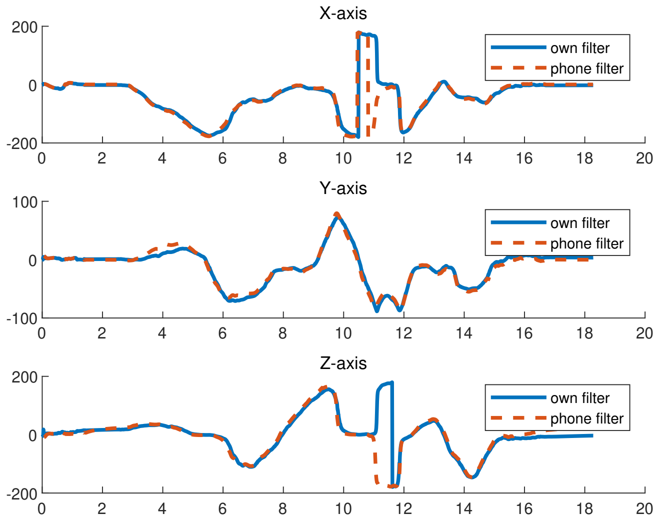

Figure 15: Euler angles of both our filter and the one in the phone.

Task 12

Accelerometer and Gyroscope

The results are illustrated in the Figure 16. It can be observed that the EKF filter demonstrates good orientation estimation capabilities in X and Y directions when processing accelerometer and gyroscope data, both during rapid motion and stationary states. While the estimation result in Z direction. Without the magnetometer measurements, the filter has lost its ability to correct its offset in yaw.

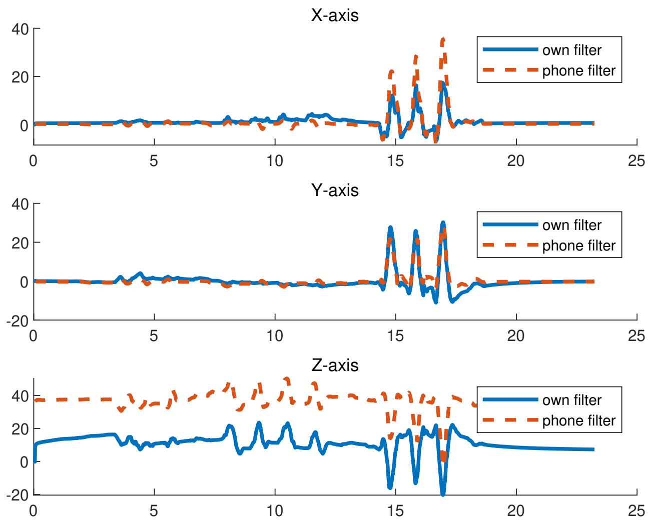

Figure 16: Estimation with Accelerometer and Gyroscope.

Accelerometer and Magnetometer

It's apparent that the filter's functionality is compromised in the absence of a gyroscope. This is primarily because the gyroscope is crucial for accurately detecting the device's rotation, which is essential for attitude estimation. While the magnetometer and accelerometer can offer some insight into the device's orientation, their capabilities are limited and prone to external disturbances and only work on the stationary situation. Consequently, without the gyroscope, the filter lacks vital angular velocity information, leading to a unfunctional situation.

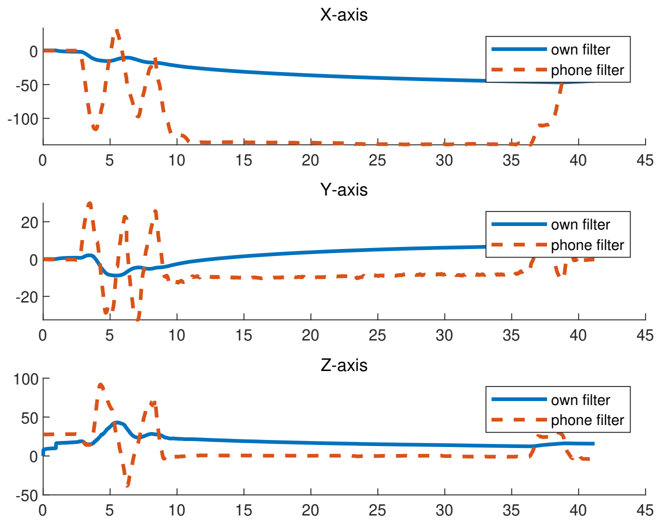

Figure 17: Estimation with Accelerometer and Magnetometer.

Gyroscope and Magnetometer

The EKF filter exhibits good tracking of device orientation changes when processing gyroscope and magnetometer data. Although it may be affected by magnetic field interference in certain situations, some data drift may occur.

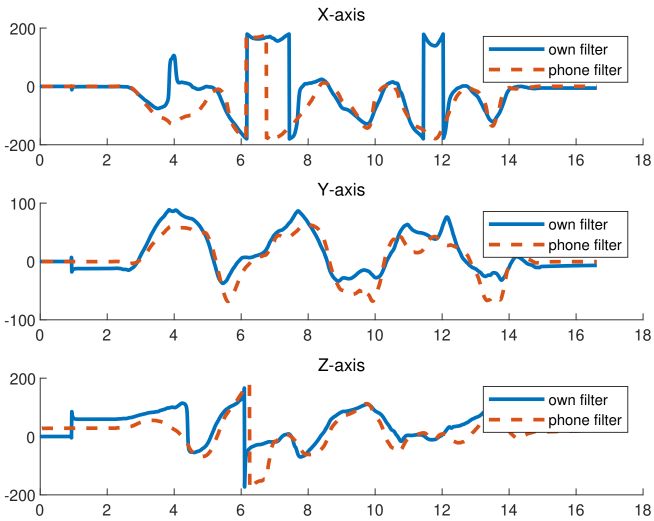

Figure 18: Estimation with Gyroscope and Magnetometer.