Multi Layer Networks

1. What can be Computed?
   - Convex Areas
   - Arbitrary Areas
2. Learning
   - Learning Methods
   - Generalized Delta Rule
   - Error Back-Propagation
   - Problems
3. Usage
   - System Identification
   - Data Compression

Multi Layer Feed-Forward Networks

What can a thresholded two layer network compute?

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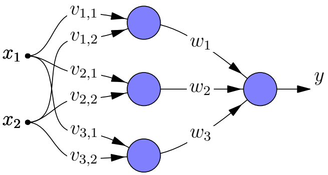

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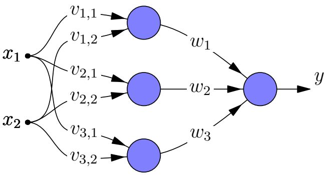

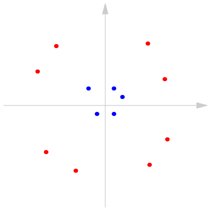

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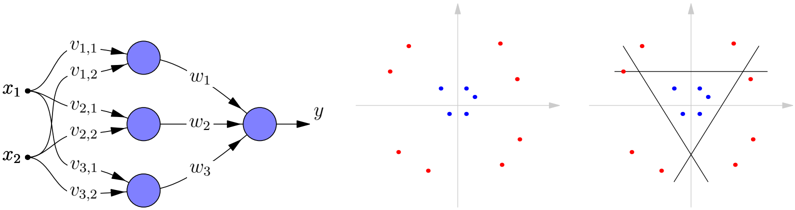

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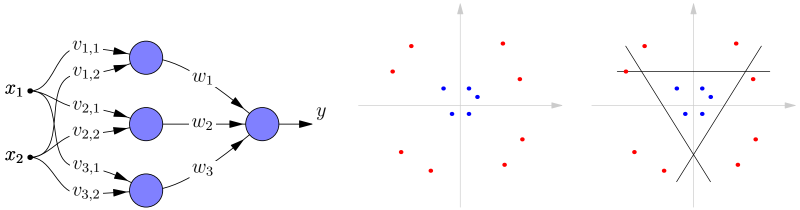

With $w_1 = w_2 = w_3 = 1$ and $\theta = 2.5$ the second layer operates as an AND-gate.

Multi Layer Feed-Forward Networks

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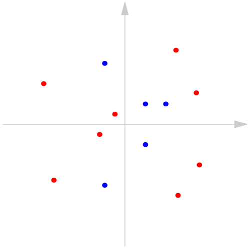

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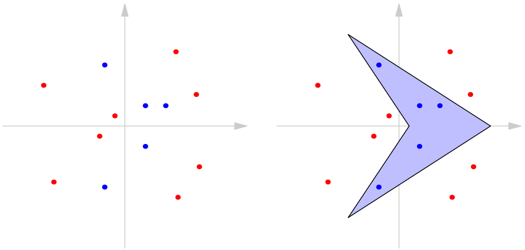

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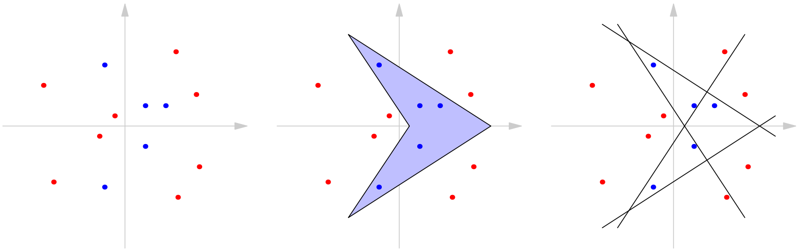

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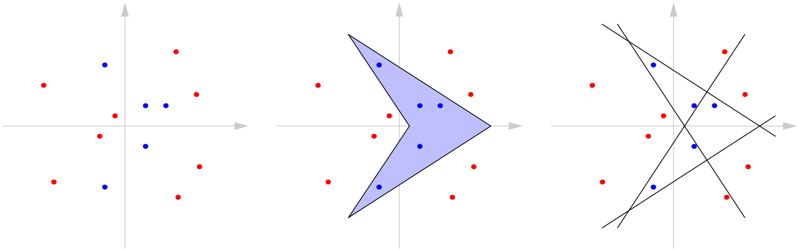

Arbitrarily complex areas can be extracted
provided there are enough hidden units

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Solution

Use threshold-like but differentiable transfer functions

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Two commonly used transfer functions $\varphi(\sum)$

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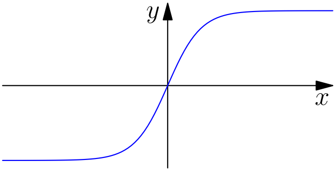
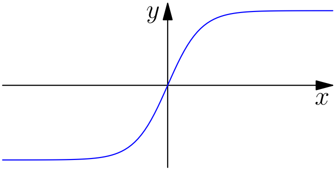
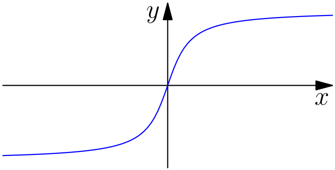


$$\varphi(x) = \frac{1 - e^{-x}}{1 + e^{-x}}$$

Learning

Two commonly used transfer functions $\varphi(\sum)$





$$\varphi(x) = \frac{1 - e^{-x}}{1 + e^{-x}}$$

$$\varphi(x) = \arctan(x)$$

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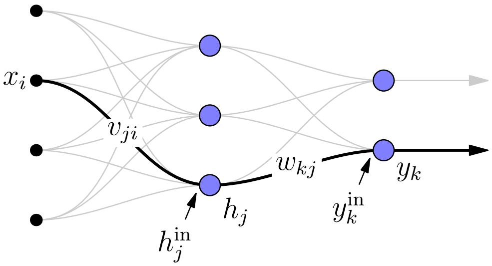
Compute the gradient, i.e.

$$\frac{\partial \varepsilon}{\partial v_{ji}} \quad \text{and} \quad \frac{\partial \varepsilon}{\partial w_{kj}}$$


x_i
v_{ji}
h_j
h_j^{in}
w_{kj}
y_k
y_k^{in}

First case: derivative w.r.t. a weight w_{kj} in the second layer

$$
\begin{aligned}
\frac{\partial \varepsilon}{\partial w_{kj}} &= \frac{\partial \varepsilon}{\partial y_k} \cdot \frac{\partial y_k}{\partial w_{kj}} \\
&= -(t_k - y_k) \cdot \frac{\partial \varphi(y_k^{\mathrm{in}})}{\partial w_{kj}} \\
&= -(t_k - y_k) \cdot \varphi'(y_k^{\mathrm{in}}) \cdot \frac{\partial y_k^{\mathrm{in}}}{\partial w_{kj}} \\
&= -(t_k - y_k) \cdot \varphi'(y_k^{\mathrm{in}}) \cdot h_j \\
&= -\delta_k h_j
\end{aligned}
$$

Here we have introduced $\delta_k = (t_k - y_k) \cdot \varphi'(y_k^{\mathrm{in}})$

Second case: derivative w.r.t. a weight v_{kj} in the first layer

$$
\begin{aligned}
\frac{\partial \varepsilon}{\partial v_{ji}} &= \sum_k \frac{\partial \varepsilon}{\partial y_k} \cdot \frac{\partial y_k}{\partial v_{ji}} \\
&= -\sum_k (t_k - y_k) \cdot \frac{\partial y_k}{\partial v_{ji}} \\
&= -\sum_k (t_k - y_k) \cdot \varphi'(y_k^{\text{in}}) \cdot \frac{\partial y_k^{\text{in}}}{\partial v_{ji}} \\
&= -\sum_k \delta_k \cdot \frac{\partial y_k^{\text{in}}}{\partial v_{ji}} \\
&= -\sum_k \delta_k \cdot w_{kj} \cdot \frac{\partial h_j}{\partial v_{ji}}
\end{aligned}
$$

We continue...

$$
\begin{aligned}
\frac{\partial \varepsilon}{\partial v_{ji}} &= -\sum_k \delta_k \cdot w_{kj} \cdot \frac{\partial h_j}{\partial v_{ji}} \\
&= -\sum_k \delta_k \cdot w_{kj} \cdot \varphi'(h_j^{\mathrm{in}}) \cdot \frac{\partial h_j^{\mathrm{in}}}{\partial v_{ji}} \\
&= -\sum_k \delta_k \cdot w_{kj} \cdot \varphi'(h_j^{\mathrm{in}}) \cdot x_i \\
&= -\delta_j x_i
\end{aligned}
$$

Here we have introduced $\delta_j = \sum_k \delta_k \cdot w_{kj} \cdot \varphi'(h_j^{\mathrm{in}})$

Summary

$$\frac{\partial \varepsilon}{\partial w_{kj}} = -\delta_k h_j \qquad \text{where } \delta_k = (t_k - y_k) \cdot \varphi'(y_k^{\text{in}})$$

$$\frac{\partial \varepsilon}{\partial v_{ji}} = -\delta_j x_i \qquad \text{where } \delta_j = \sum_k \delta_k \cdot w_{kj} \cdot \varphi'(h_j^{\text{in}})$$

Gradient Decent

$$\Delta w_{kj} = \eta \delta_k h_j$$

$$\Delta v_{ji} = \eta \delta_j x_i$$

Error Back-Propagation

1. Forward Pass: Compute all h_j and y_k

$$h_j = \varphi\Big(\sum_i v_{ji} x_i\Big) \qquad y_k = \varphi\Big(\sum_j w_{kj} h_j\Big)$$

2. Backward Pass: Compute all δ_k and δ_j

$$\delta_k = (t_k - y_k) \cdot \varphi'(y_k^{\mathrm{in}}) \qquad \delta_j = \sum_k \delta_k \cdot w_{kj} \cdot \varphi'(h_j^{\mathrm{in}})$$

3. Weight Updating:

$$\Delta w_{kj} = \eta \delta_k h_j \qquad \Delta v_{ji} = \eta \delta_j x_i$$

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- Does not always converge (gets stuck in local minima)
- Slow convergence
- Many parameters need to be tuned
- Bad scaling behavior for large problems
- Biologically unrealistic
  - Backward propagating signal
  - Requires known target values

Tips when using BackProp

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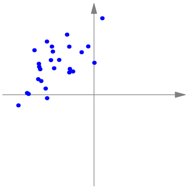
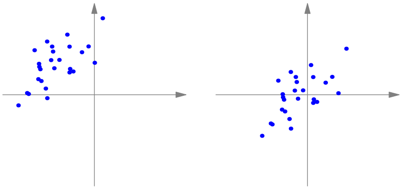
- Add random noise to the weights during training
- Remove the squashing-function $(\varphi(x))$ for the output units

Preprocessing of input patterns





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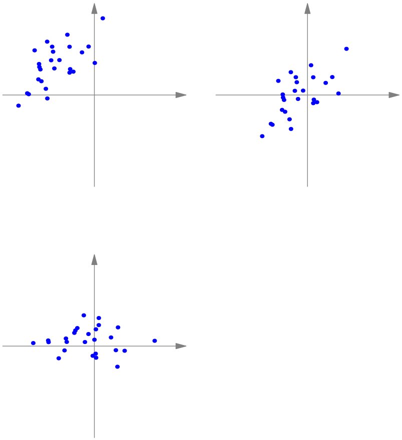
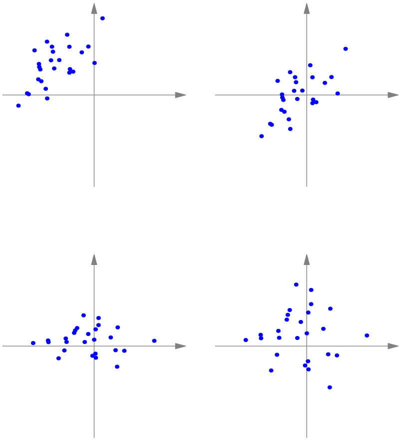


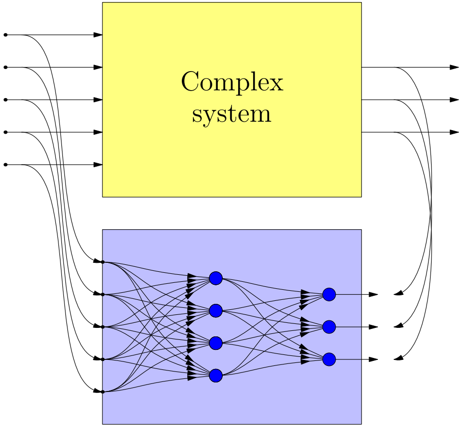
Preprocessing of input patterns

1. Subtract the average
2. Decorrelate
3. Normalize the variance

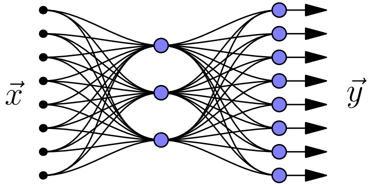


System Identification

"Mimic" an existing system



Data Compression



Hour-Glass form

Train with $\vec{x} = \vec{y}$ (auto-association)
Forces the network to use a compact encoding of the patterns.