Orientation Estimation of Smartphones

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Task 1 – Choice of state and inputs

If we consider using ω_k as inputs to the system, we can simplify the motion model by removing ω_k from the state vector and y^ω from the measurement vector. It also becomes easy to choose the process noise covariance matrix since we can start from the sensor noise covariance when tuning. (ω_k and y^ω denote similar quantities: ω_k is angular velocity and y^ω is a (noisy) measurement of ω_k.) If we include angular velocity in the state vector, we can filter it and produce it as an output of the filter, which may be desired in some cases. If the angular velocity measurement is highly noisy, it is also better to not consider it as input, otherwise prediction performance will be suboptimal: By using gyroscope data as the motion model we forgo the choice of a hand crafted motion model like a constant velocity or acceleration model which may be a better choice if the angular velocity measurement is noisy.

Task 2 – Investigation of measurement noise

To investigate characteristics of the measurement noise, ten seconds of measurement data was recorded with the phone at rest. Histograms of the recorded data are plotted in figure 1 and time series data is plotted in figure 1. From figure 1 we see that all sensor noise can be reasonably approximated as Gaussian. Figure 2 also shows us that the accelerometer and gyroscope noise does not seem to be significantly coherent and can therefore be approximated as white, while the magnetometer data is relatively correlated with itself over time. Including slowly varying magnetometer bias in the state variable can be a way to deal with this behavior.

The mean and covariance of the sensor data when the phone is placed flat on a table is shown in table 1. The gyroscope does seem to have a small bias, and maybe the accelerometer also. But it is harder to know the bias of the accelerometer since the table surface that the phone rested upon when caputuring the data may not be flat, the phone chassis is also not flat and we also have gravity whose exact strength is unknown. Knowing the magnetometer bias is even harder since we don't know what field strength we expect and in what direction.

	Mean	Covariance
Accelerometer	$10^0 \times \begin{bmatrix} 0.052 \\ 0.220 \\ 9.696 \end{bmatrix} \mathrm{m/s^2}$	$10^{-4} \times \begin{bmatrix} 0.220 & 0.025 & -0.018 \\ 0.025 & 0.226 & -0.021 \\ -0.018 & -0.021 & 0.614 \end{bmatrix} \mathrm{m^4/s^4}$
Gyroscope	$10^{-4} \times \begin{bmatrix} -1.743 \\ -0.125 \\ 0.708 \end{bmatrix} \mathrm{rad/s}$	$10^{-6} \times \begin{bmatrix} 0.208 & -0.040 & -0.008 \\ -0.040 & 0.264 & 0.019 \\ -0.008 & 0.019 & 0.195 \end{bmatrix} \mathrm{rad^2/s^2}$
Magnetometer	$10^1 \times \begin{bmatrix} -1.074 \\ 2.782 \\ -3.086 \end{bmatrix} \mathrm{\mu T}$	$10^{-1} \times \begin{bmatrix} 0.412 & 0.061 & 0.256 \\ 0.061 & 0.117 & -0.086 \\ 0.256 & -0.086 & 1.211 \end{bmatrix} \mathrm{pT^2}$

Table 1: Mean and covariance of measurements captured over ten seconds when the while the phone was at rest.

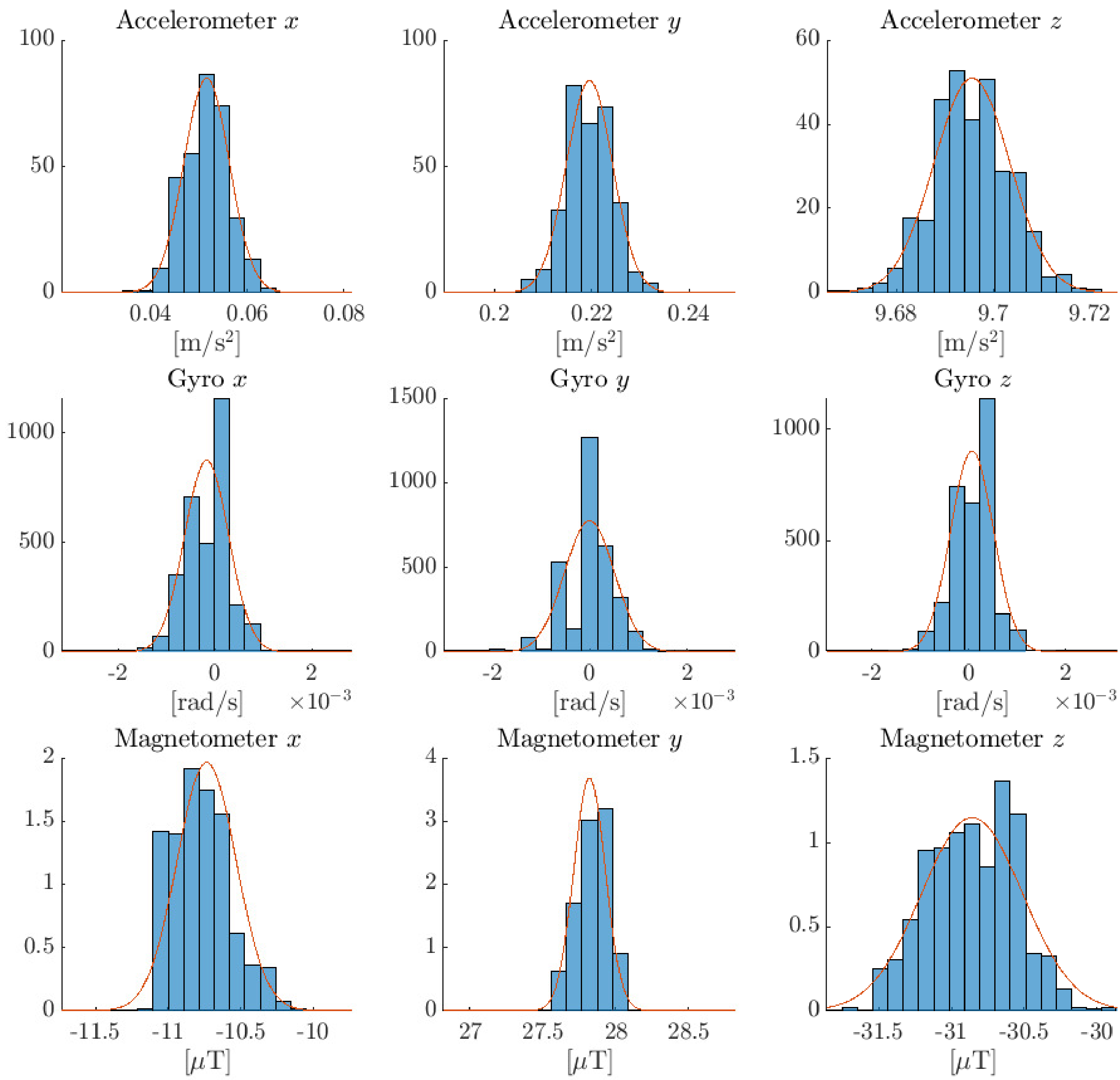

Figure 1: Histogram of measurement data captured over ten seconds while the phone was at rest. The continuous lines correspond to Gaussian pdf with mean and standard deviation estimated from the collected data.

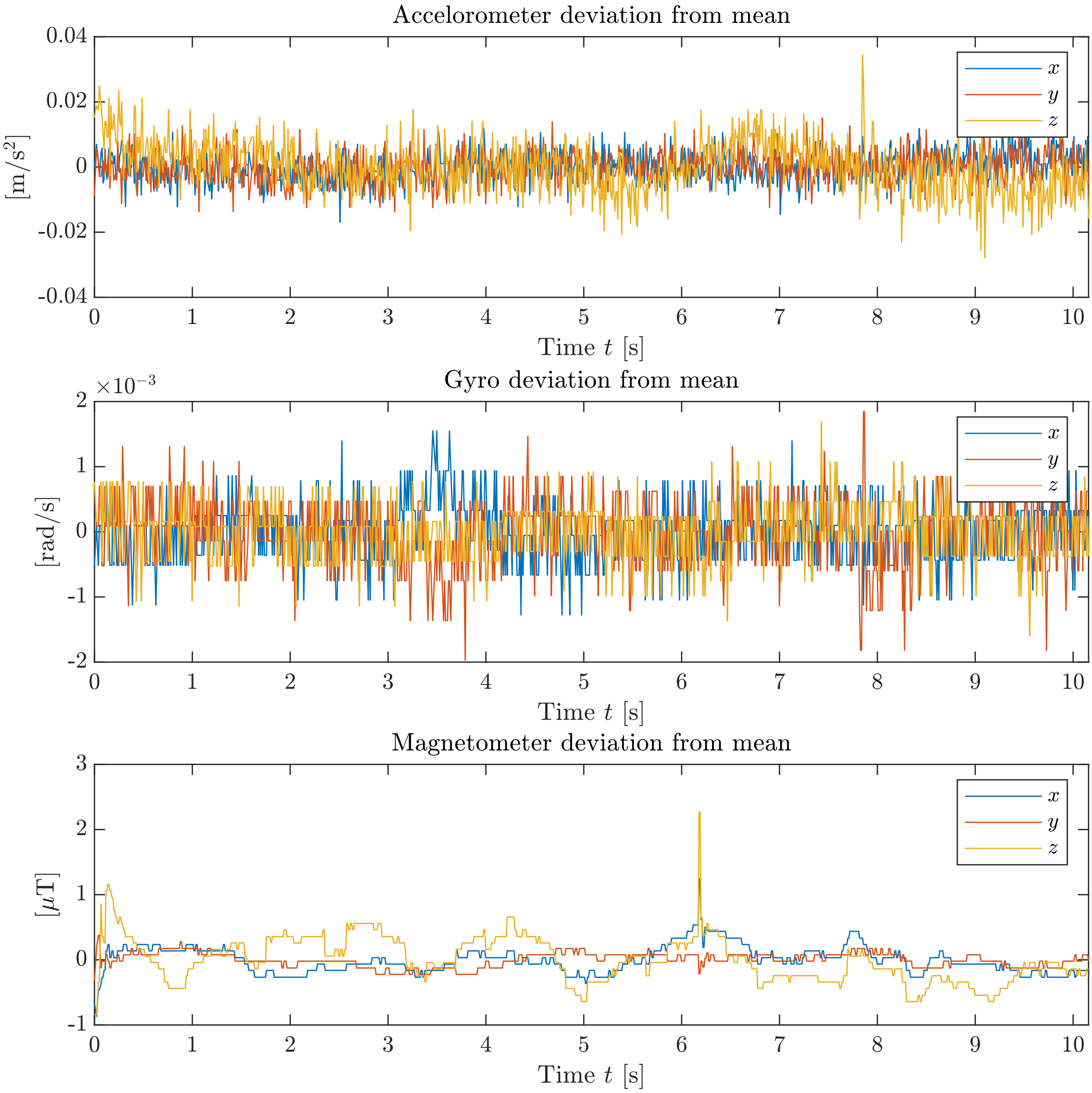

Figure 2: Plot of measurement deviation from it's mean, captured while the phone was at rest.

Task 3 – Discretization of state space equations

Assuming zero order hold, the continuous time state space equation for $t \in [t_{k-1}, t_k)$ given in the project description is

$$\dot{q}(t) = \frac{1}{2}S(\omega_{k-1} + v_{k-1})q(t).$$

Solving this equation yields

$$q(t) = q(t_{k-1}) \exp\left(\frac{1}{2}S(\omega_{k-1} + v_{k-1})(t - t_{k-1})\right),$$

or in particular for $t = t_k$ we get when denoting $q_i \equiv q(t_i)$

$$q_k = q_{k-1} \exp\left(\frac{1}{2}S(\omega_{k-1} + v_{k-1})\Delta t\right) \approx q_{k-1}\left(I + \frac{1}{2}S(\omega_{k-1} + v_{k-1})\Delta t\right) =$$
$$\left(I + \frac{\Delta t}{2}S(\omega_{k-1})\right)q_{k-1} + \frac{\Delta t}{2}S(v_{k-1})q_{k-1} = \left(I + \frac{\Delta t}{2}S(\omega_{k-1})\right)q_{k-1} + \frac{\Delta t}{2}\bar{S}(q_{k-1})v_{k-1}$$

where we denote $\Delta t \equiv t_k - t_{k-1}$ and have used the fact that S is a linear function. Thus, we may write

$$q_k = F(\omega_{k-1})q_{k-1} + G(q_{k-1})v_{k-1}$$

with

$$F(\omega_{k-1}) = I + \frac{\Delta t}{2}S(\omega_{k-1}), \quad G(q_{k-1}) = \frac{\Delta t}{2}\bar{S}(q_{k-1}).$$

Finally, we approximate $G(q_{k-1})$ with $G(\hat{q}_{k-1})$. This approximation is both necessary, since the EKF assumes additive noise, and justified since the EKF linearizes the motion model around the estimated state, meaning we already assume $q_{k-1} \approx \hat{q}_{k-1}$. To summarize, our final discretized state space model is thus

$$q_k = F(\omega_{k-1})q_{k-1} + G(\hat{q}_{k-1})v_{k-1}. \tag{1}$$

Task 4 – Quaternion prediction step

The phone is stationary, so expected value of measurement shall be 0.In this case, variance of measurement is variance of noise, R_v, which can be computed by previous function that has been defined.

Given the motion model of equation (1), the EKF prediction step for q becomes

$$\hat{q}_{k|k-1} = F(\omega_{k-1})\hat{q}_{k-1|k-1} \tag{2a}$$
$$P_{k|k-1} = F(\omega_{k-1})P_{k-1|k-1}F(\omega_{k-1})^\top + G(\hat{q}_{k-1|k-1})R_vG(\hat{q}_{k-1|k-1})^\top \tag{2b}$$

where R_v is the covariance matrix of the process noise v. If there is no angular velocity measurement available, we simply use the previous angular velocity measurement.

Task 5 – Naive filter

Next we implement a filter with no correction step, meaning the only measurements that are provided to the filter is the gyro data. We use equations (2a) and (2b) to do the prediction step. However, as equation (2a) does not guarantee that $\hat{q}_{k|k-1}$ will be normalized, we normalize $\hat{q}_{k|k-1}$ after the prediction with the provided function `mu_normalizeQ`.

However, as the gyro only measures rate of change and not absolute angles, the filter estimates drift over time. This is especially apparent when the true orientation changes rapidly and with different rate from time step to time step, such as when the phone is shaken.

Task 6 – Accelerometer update

The acceleration of gravity may be expressed in the world frame as a vector $\begin{bmatrix}0 & 0 & -g_0\end{bmatrix}^\top$ where $g_0 = 9.698\,\mathrm{m/s^2}$ is the norm of the mean accelerometer reading from table 1. Converting this gravity vector into the sensor frame $\mathcal{S}$ we get the accelerometer measurement model (assuming no other forces on the phone)

$$y_{a,k} = h_a(q_k) + v_{a,k} \equiv (R^{\mathcal{W}/\mathcal{S}}(q_k))^\top \begin{bmatrix}0 & 0 & g_0\end{bmatrix}^\top + v_{a,k} \tag{3}$$

where $v_{a,k} \sim \mathcal{N}(0, R_a)$ is the measurement noise. R_a is also taken from table 1.

Given the measurement model of equation (3), the EKF update step becomes

$$\begin{aligned} z_k &= y_{a,k} - h_a(\hat{q}_{k|k-1}) \\ S_k &= H_k P_{k|k-1} H_k^\top + R_a \\ K_k &= P_{k|k-1} H_k^\top S_k^{-1} \\ \hat{q}_{k|k} &= \hat{q}_{k|k-1} + K_k z_k \\ P_{k|k} &= (I - K_k H_k) P_{k|k-1} \end{aligned} \tag{4}$$

where

$$H_k = \frac{\partial}{\partial q_k} h_a(q_k) = g_0 \begin{bmatrix} -2q_2 & 2q_3 & -2q_0 & 2q_1 \\ 2q_1 & 2q_0 & 2q_3 & 2q_2 \\ 4q_0 & 0 & 0 & 4q_3 \end{bmatrix}.$$

Task 7 – Kalman filter with accelerometer update

After adding accelerometer measurements in the update step the filter has some concept of the absolute orientation of the phone. This enables it to for example detect when the phone is laying flat on a table and adjusts the estimate accordingly. However, the accelerometer is not able to detect the roll (rotation around z axis) of the phone when its laying flat on the table, or any rotation around the z axis of the world frame for that matter. Also, as it currently stands the angle estimate provided by the gyroscope is heavily influenced by accelerations of the phones center of mass: Sliding the phone sideways when it is placed flat on the table will make the acceleration vector deviate from the vertical line, which to the filter indicates that the phone has been tilted when in fact it is still flat. This behavior is depicted in figure 3. After the phone stops accelerating, the angle measurement returns back to where it was originally like we would expect.

Task 8 – Accelerometer outlier rejection

We next modify our filter to reject accelerometer readings where the magnitude of the measured acceleration deviates more than one percent from g_0. This improves the filter performance when the phone is moving in short burst between periods of being relatively stationary. This is exemplified in figure 4 where we recreate the estimation in figure 3 only this time rejecting unusable accelerometer readings according to the method described above. As we see in figure 4, the estimate is still effected by the acceleration of the phones center of gravity however not as much as it was previously. The angle deviation we see could also be the effecto of slight rotations of the phone during the experiment. We could

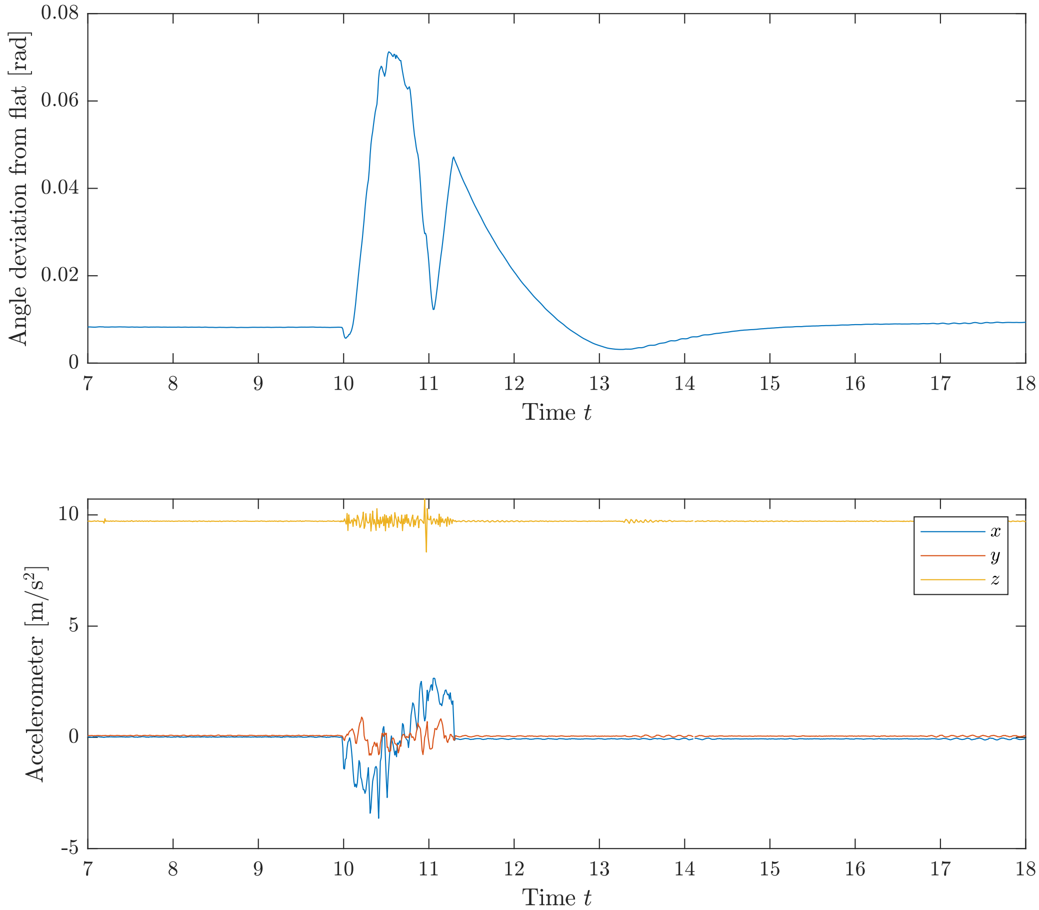

Figure 3: Estimated phone angle and acceleration measurements when sliding the phone flat along a table. The angle deviation is measured from vertical in the world frame. The initial angle deviation is not exactly zero due to unevenness in the table and the phones casing. The angle deviation is always depicted as positive. The large dip we see in the deviation happens when the phone goes from increasing its velocity to decreasing its velocity – during that time the angle deviation will briefly go to zero as the estimation error changes direction.

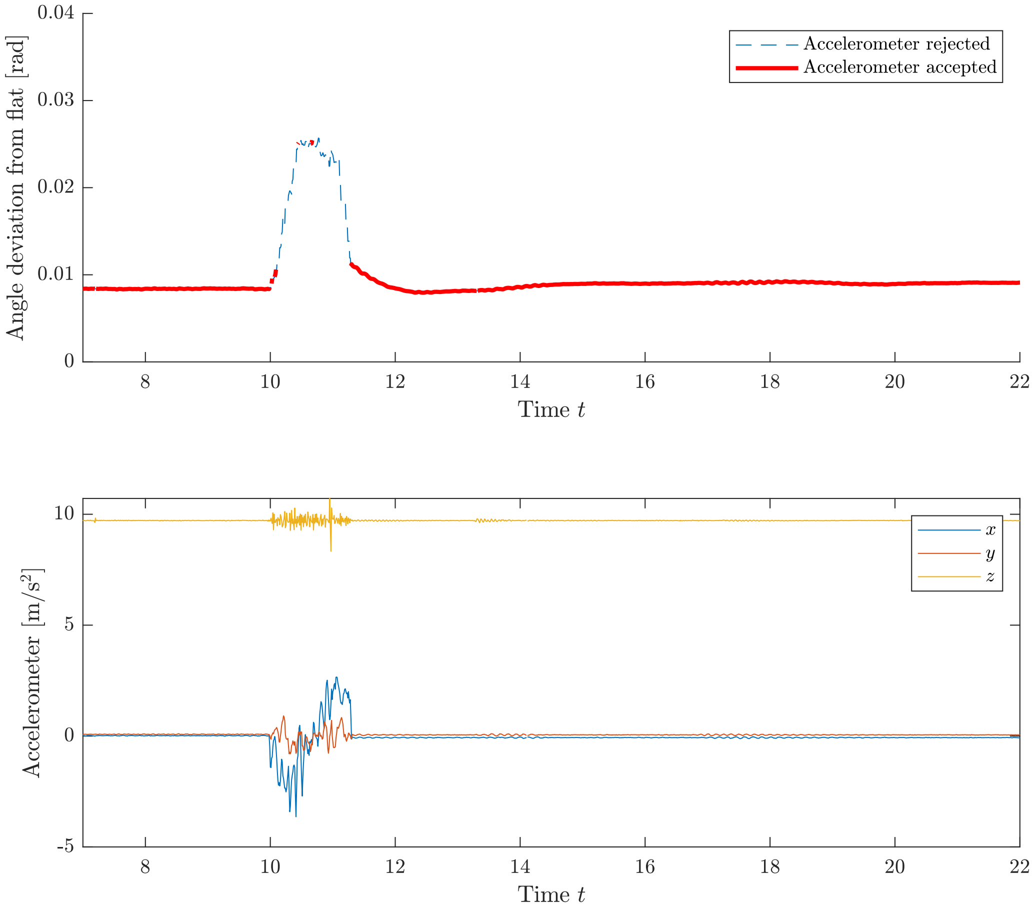

Figure 4: Estimated phone angle and acceleration measurements when sliding the phone flat along a table. The angle deviation is measured from vertical in the world frame. The initial angle deviation is not exactly zero due to unevenness in the table and the phones casing. The angle deviation plot is highlighted in red for those sections when the accelerometer data was available. Compared the result to figure 3 when all measurement data was accepted.

possibly get slightly better performance by rejecting more accelerometer measurement, however this would not be practical as when holding a phone in ones hand, one does not hold it completely still. Therefore, we would reject measurements that would be useful to us during normal use of the phone.

Task 9 – Magnetometer update

Similar to task 6, we start from the given measurement model

$$y_{m,k} = h_m(q_k) + v_{m,k} \equiv (R^{\mathcal{W}/\mathcal{S}}(q_k))^\top m_0 + v_{m,k}$$

with

$$m_0 \equiv \begin{bmatrix} 0 & m_{0,y} & m_{0,z} \end{bmatrix}^\top = \begin{bmatrix} 0 & \sqrt{m_x^2 + m_y^2} & m_z \end{bmatrix}^\top$$

where we set $\begin{bmatrix} m_x & m_y & m_z \end{bmatrix}^\top$ to our estimated magnetometer mean from table 1. The update equations are then the same as in equation (4), only substitute $y_{a,k}$ and h_a for $y_{m,k}$ and h_m respectively. This time, our H_k becomes

$$H_k = \frac{\partial}{\partial q_k} h_m(q_k) =$$

$$\begin{bmatrix} 2m_{0,y}q_3 - 2m_{0,z}q_2 & 2m_{0,y}q_2 + 2m_{0,z}q_3 & 2m_{0,y}q_1 - 2m_{0,z}q_0 & 2m_{0,y}q_0 + 2m_{0,z}q_1 \\ 4m_{0,y}q_0 + 2m_{0,z}q_1 & 2m_{0,z}q_0 & 4m_{0,y}q_2 + 2m_{0,z}q_3 & 2m_{0,z}q_2 \\ 4m_{0,z}q_0 - 2m_{0,y}q_1 & -2m_{0,y}q_0 & 2m_{0,y}q_3 & 2m_{0,y}q_2 + 4m_{0,z}q_3 \end{bmatrix}.$$

Task 10 – Adding magnetometer update

When solving this task, we were unfortunately in an environment with many magnetic field disturbances. As the disturbances vary over time, this for one thing meant that the estimated magnetometer noise noise covariance was large compared to the other sensors leading to very slow updates of state, but also that there was significant bias in the measurements. The magnetometer measurements usually therefore only contributed to very slowly rotating the estimated state in some seemingly random direction. This direction was very influenced by where in the room the phone was held, due to the magnetic field lines being uneven throughout the room.

Task 11 – Magnetometer outlier rejection

Still, rejecting magnetometer data when

$$|\hat{L}_k - \|y_{m,k}\|| \leq 0.01\hat{L}_k$$

with

$$\hat{L}_k = (1-\alpha)\hat{L}_{k-1} + \alpha\|y_{m,k}\|, \quad \hat{L}_0 = m_0, \quad \alpha = 0.01,$$

does not give significantly improved filter results. Rejecting because of field strength magnitude cannot account for the fact that the direction of the field is off, and even more so when we adjust $\hat{L}$ over time. In reality, you would probably also set a much lower α than the value we chose. We chose α to be relatively large in order to make it's affect on the rejection criterion more clear. This way of rejecting outliers assumes that the outliers are only present for short amounts of time and not persistent, like time varying biases are. Since the outliers are persistent, $\hat{L}$ will slowly adapt to the outlier data meaning we start accepting measurements that are affected by external factors. So when a disturbance is introduced, at first the filter rejects the disturbance, but after a while $\hat{L}$ adjusts itself and the filter starts to accept the disturbed measurements. The assumption that outliers only appear for short time intervals at a time is reasonable when the phone is normally in an environment where the magnetic field is undisturbed or at least relatively uniform, and disturbances come in short bursts, like walking by a metallic object. It is not a reasonable assumption when disturbances are common and persistent.

Figure 5 shows our filter performance in comparison to the phone's built in orientation estimator. Due to the bad influence from the magnetometer, the yaw is significantly offset even during the start of the experiment (around times $t = 15, 16\,\mathrm{s}$) when the phone is lying flat on the table. Then, as it gets picked up, our estimated yaw and roll follow the phone's filter acceptably well given the circumstances, while the pitch is initially off only to then converge towards the phone's estimate.

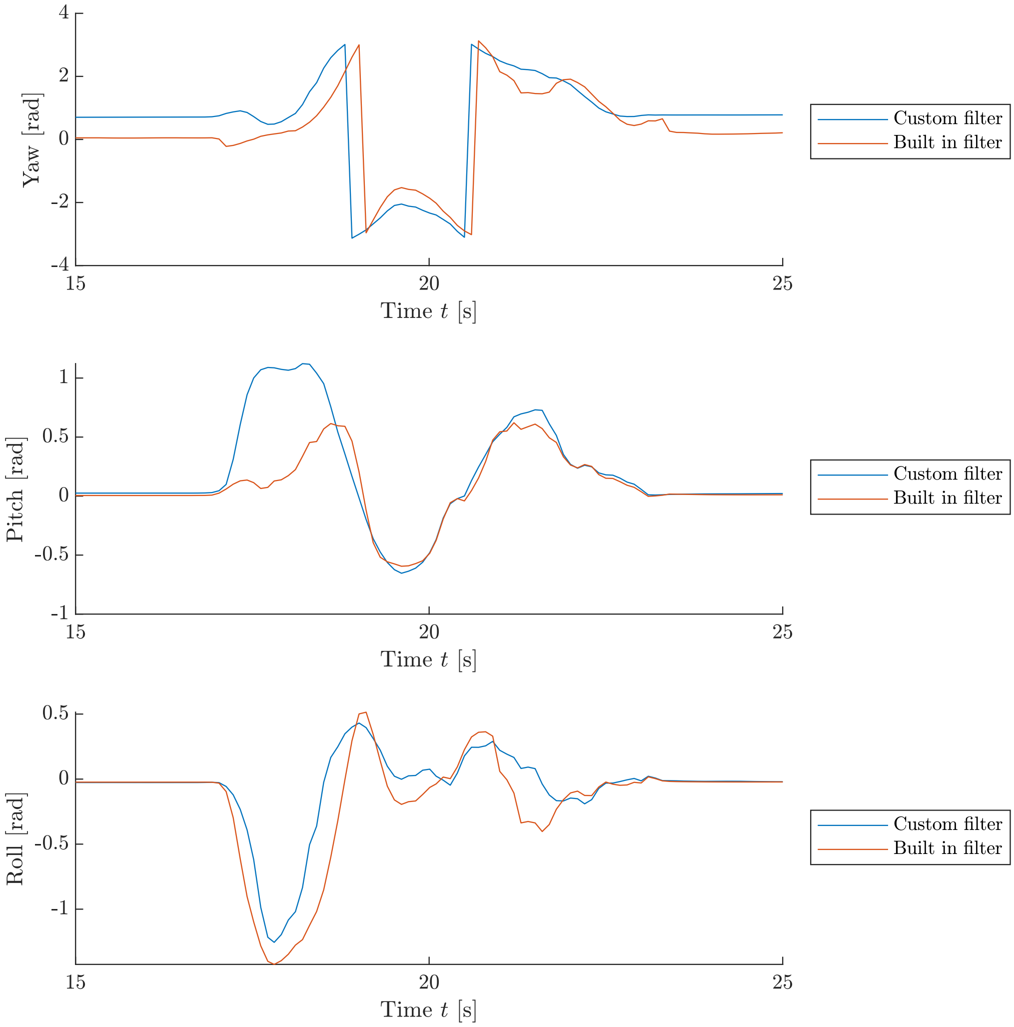

Figure 5: Estimated Euler angles as compared to the phone's built in orientation estimator.

Task 12 – Evaluation of filter

A comparison of multiple sensor configurations to the phone's built in filter is shown in figure 6. In this figure we can see the negative impact of the magnetometer measurements on the filter estimate. We see that it is the magnetometer that causes the yaw discrepancy previously pointed out in figure 5, as only the sensor configurations including the magnetometer show this behavior in figure 6. We also see that roll offset is present only when having a magnetometer and no accelerometer reading. This is because the disturbance in the magnetic fields means the phone filter believes the phone to be rolled over in combination with there not being any accelerometer readings available to correct it. Estimating yaw is hard for all sensor combinations. The magnetometer combinations should in theory be able to do it, but is hindered by disturbances. The accelerometer and gyro on their own has however no way of estimating yaw other than relying on the prior which in this case is not too bad. From the plots of figure 6 we draw the conclusion that the best sensor combination for a filter of this structure is to only fuse gyro and accelerometer data and disregard all gyro readings. If absolute yaw estimates are a necessity, the filter needs to be redesigned in order to work in the kind of environments our filter was evaluated in.

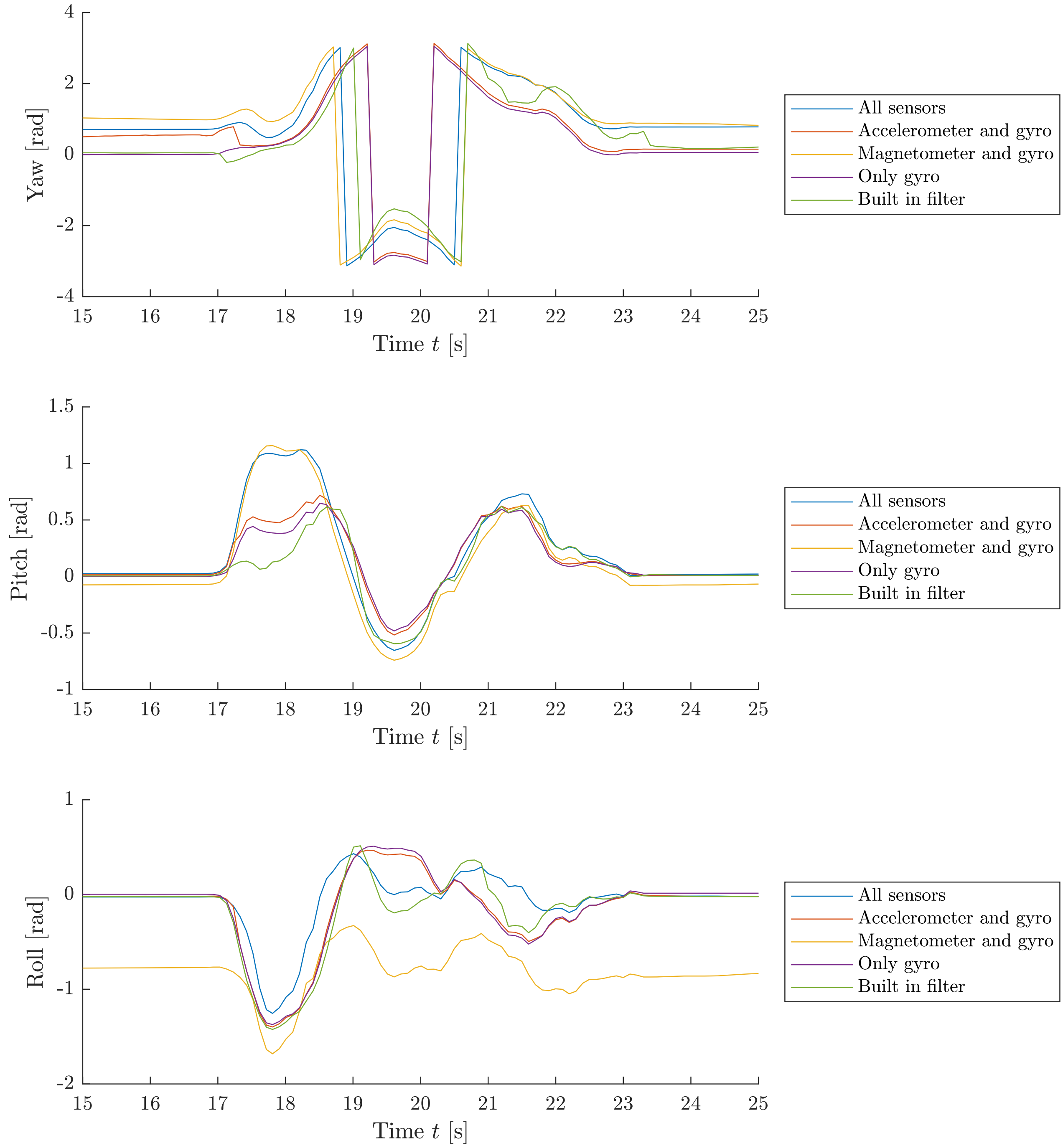

Figure 6: Comparison of EKF with multiple sensor configurations to the phones built in orientation estimator. "All sensors" correspond to gyroscope, accelerometer and magnetometer.