# C++基礎語法 Unit-10

- 遞迴 (Recursion)

# 遞迴 (Recursion)

- 遞迴 (Recursion)，是指在函式中呼叫函式自身。
- 遞迴函式必須明訂終止條件，才能被計算。

# 費氏數列

# 費氏數列

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, …

$$fib(n) = \begin{cases} 1 & ,if\ n = 1\ or\ n = 2 \\ fib(n-1) + fib(n-2) & ,if\ n \geq 3 \end{cases}$$

https://zh.wikipedia.org/wiki/%E6%96%90%E6%B3%A2%E9%82%A3%E5%A5%91%E6%95%B0%E5%88%97

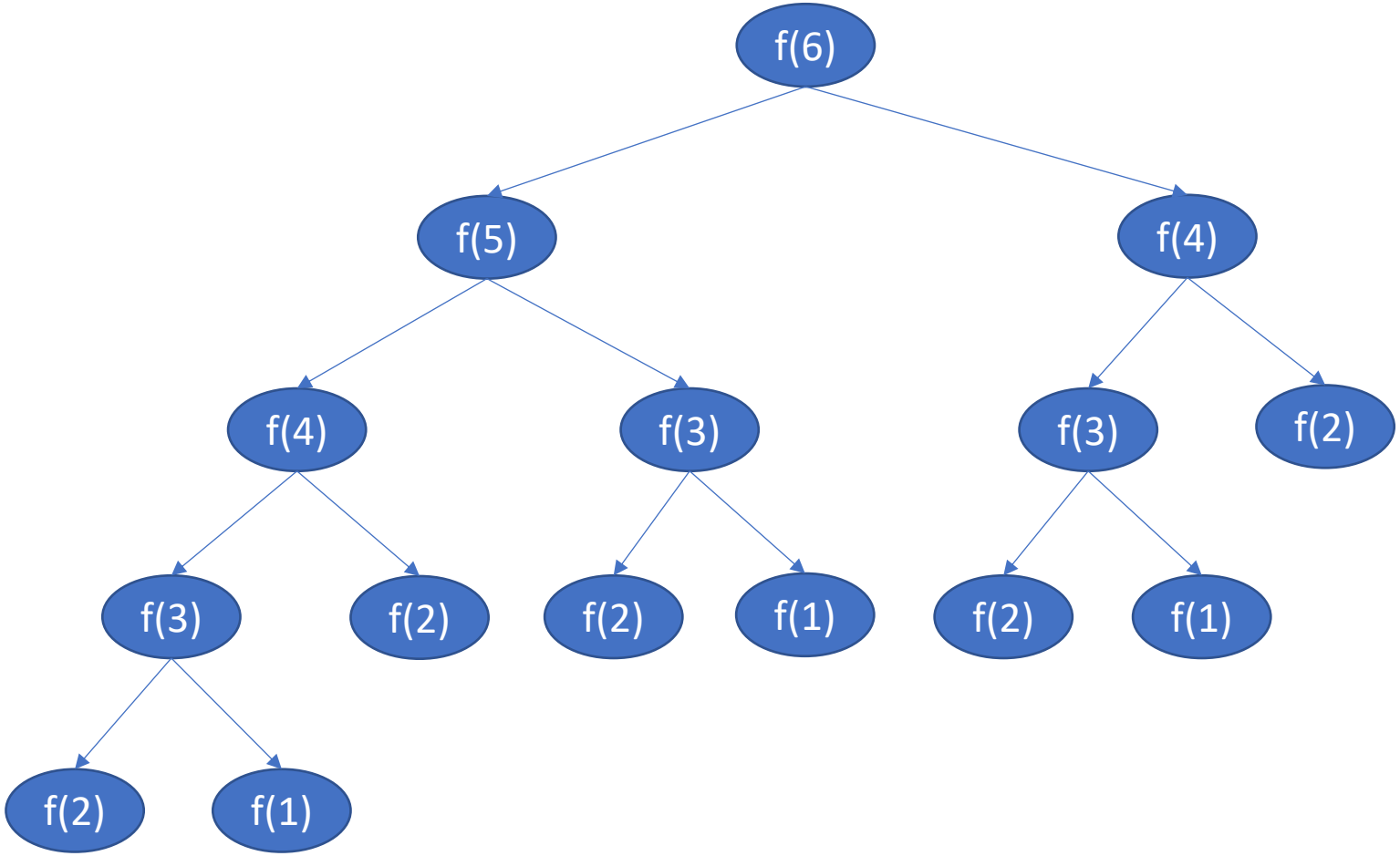
# 費氏數列



執行速度很慢，why?

```
4  int f(int n) {
5      if (n == 1 || n == 2) {
6          return 1;
7      }
8      return f(n-1) + f(n-2);
9  }
10
11 int main() {
12     for (int i = 1; i < 48; i++) {
13         cout << "f(" << i << ") = " << f(i) << "\n";
14     }
15     return 0;
16 }
```

# 因為有很多重複的計算



# 費氏數列：記憶化 Memorization

Example 10-6

```cpp
#include <iostream>
using namespace std;

int f[47];

int fib(int n) {
    if (f[n]) return f[n];
    return f[n] = fib(n-1) + fib(n-2);
}

int main() {
    f[1] = 1;
    f[2] = 1;
    for (int i = 1; i < 47; i++) {
        cout << "f(" << i << ") = " << fib(i) << "\n";
    }
    return 0;
}
```

# 費氏數列：或者，也可以這樣做

Example 10-2

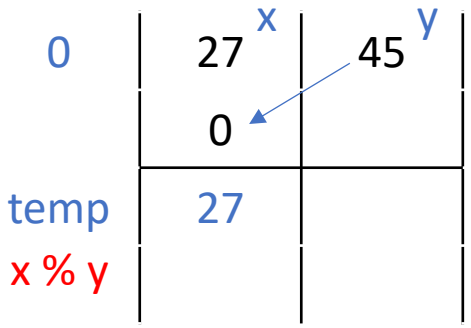
```
4  int main() {
5      int f[47];
6      f[1] = 1;
7      f[2] = 1;
8      for (int i = 3; i < 47; i++) {
9          f[i] = f[i-1] + f[i-2];
10     }
11     for (int i = 1; i < 47; i++) {
12         cout << "f(" << i << ") = " << f[i] << "\n";
13     }
14     return 0;
15 }
```

# 輾轉相除法

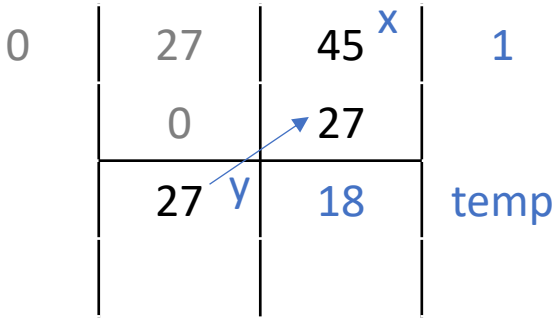
# 輾轉相除法 (GCD，最大公約數)

GCD = Greatest Common Divisor

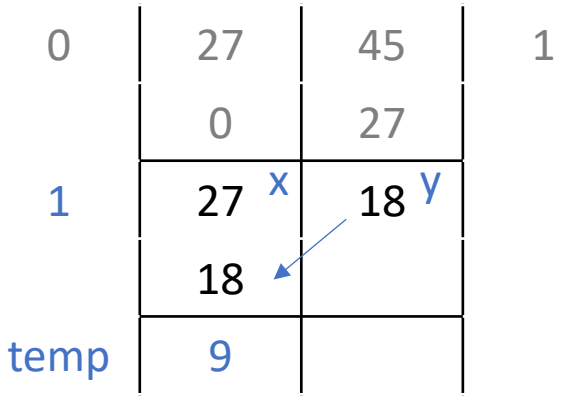
1



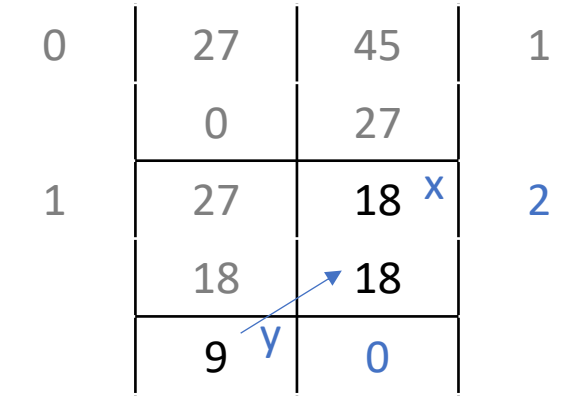

2




3




4



# 輾轉相除法：模擬法

Example 10-3

```
4  int main() {
5      int a, b;
6      cin >> a >> b;
7      while (a && b) {
8          if (a >= b) {
9              a = a % b;
10         } else {
11             b = b % a;
12         }
13     }
14     cout << max(a, b) << "\n";
15     return 0;
16 }
```

# 輾轉相除法：拿掉 if 判斷式

Example 10-4

```
4  int main() {
5      int a, b;
6      cin >> a >> b;
7      while (b) {
8          int temp = b;
9          b = a % b;
10         a = temp;
11     }
12     cout << a << "\n";
13     return 0;
14 }
```

a：被除數
b：除數

# 輾轉相除法：遞迴法

Example 10-5

```
4  int f(int a, int b) {
5      if (b == 0) {
6          return a;
7      }
8      return f(b, a % b);
9  }
10
11 int main() {
12     int a, b;
13     cin >> a >> b;
14     cout << f(a, b) << "\n";
15     return 0;
16 }
```

# 其他作法

```
11  int gcd_1(int a, int b){
12      while ((a %= b) && (b %= a));
13      return a + b;
14  }
15
16  int gcd_2(int a, int b) {
17      while (a != b) {
18          if (a > b) a = a - b;
19          else b = b - a;
20      }
21      return a;
22  }
```