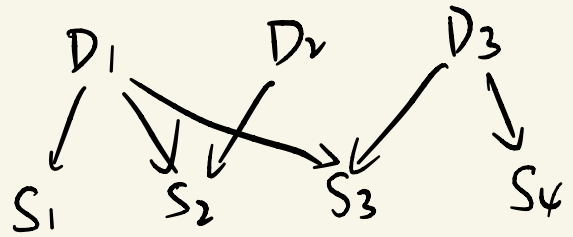
1. 1)



2) $P(S_1S_2S_3S_4D_1D_2D_3D_4) = P(D_1)P(D_2)P(D_3)P(S_1|D_1)P(S_2|D_1D_2)P(S_3|D_1D_3)P(S_4|D_3)$

3) S_2 的 Markov Blanket 是 D_1、D_2

4) $P(D_3=1|S_4=1) = \dfrac{P(D_3=1,S_4=1)}{P(S_4=1)} = \dfrac{P(D_3=1)P(S_4=1|D_3=1)}{P(S_4=1)} = \dfrac{0.1\times 0.9}{0.9\times 0.6+0.1\times 0.9} = \dfrac{1}{7}$

2.

X\Y	0	1	2	
0	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{3}$
1	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$
2	0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$
	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{3}$	

(a) $H(X) = -\frac{1}{3}\log_2\frac{1}{3} - \frac{1}{2}\log_2\frac{1}{2} - \frac{1}{6}\log_2\frac{1}{6}$

$= \frac{1}{3}\log_2 3 + \frac{1}{2}\log_2 2 + \frac{1}{6}\log_2 6$

$= \frac{1}{3}\log_2 3 + \frac{1}{2} + \frac{1}{6}\log_2 3 + \frac{1}{6}$

$= \frac{1}{2}\log_2 3 + \frac{2}{3}$

≈ 1.459 bit

$H(Y) = -\frac{1}{4}\log_2\frac{1}{4} - \frac{5}{12}\log_2\frac{5}{12} - \frac{1}{3}\log_2\frac{1}{3}$

$= \frac{1}{4}\times 2 + \frac{5}{12}\log_2\frac{12}{5} + \frac{1}{3}\log_2 3$

$= \frac{1}{2} + \frac{5}{12}\log_2 12 - \frac{5}{12}\log_2 5 + \frac{1}{3}\log_2 3$

≈ 1.555 bit (1.554585)

(b) $H(X,Y) = -\frac{1}{12}\log_2\frac{1}{12} - \frac{1}{6}\log_2\frac{1}{6} - \frac{1}{12}\log_2\frac{1}{12} - \frac{1}{6}\log_2\frac{1}{6}$
$- \frac{1}{6}\log_2\frac{1}{6} - \frac{1}{6}\log_2\frac{1}{6} - \frac{1}{12}\log_2\frac{1}{12} - \frac{1}{12}\log_2\frac{1}{12}$
$\approx 2.918 \text{ bit} \ (2.918295834)$

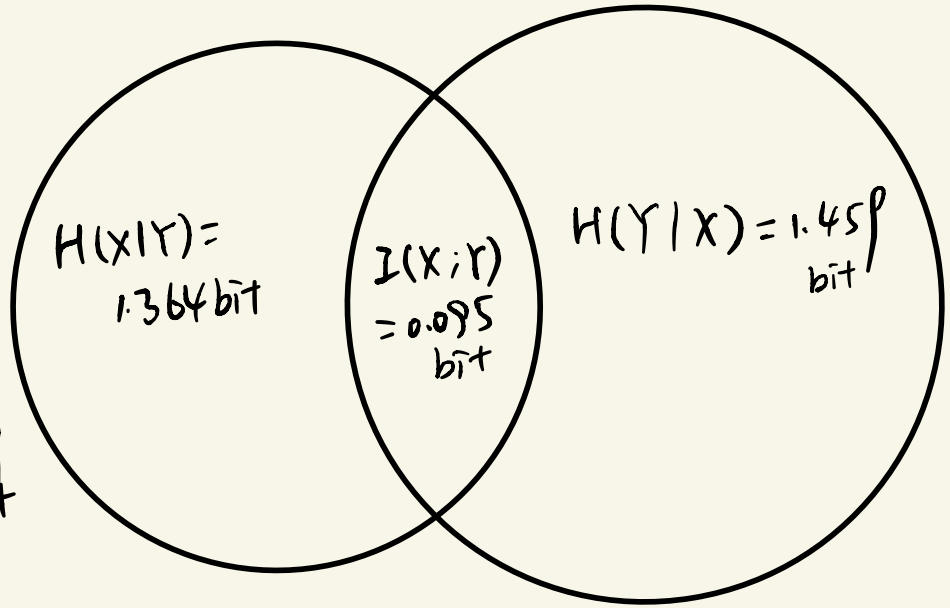
$P(Y|X)$

X \ Y	0	1	2
0	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{6}$
1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
2	0	$\frac{1}{2}$	$\frac{1}{2}$

(c) $H(Y|X) = -\frac{1}{12}\log_2\frac{1}{6} - \frac{1}{6}\log_2\frac{1}{2} - \frac{1}{12}\log_2\frac{1}{6} - \frac{1}{6}\log_2\frac{1}{3}$
$- \frac{1}{6}\log_2\frac{1}{3} - \frac{1}{6}\log_2\frac{1}{3} - \frac{1}{12}\log_2\frac{1}{2} - \frac{1}{12}\log_2\frac{1}{2}$
≈ 1.459

(d) $I(X;Y) = H(X) + H(Y) - H(X,Y)$
≈ 0.095

(e)



$H(X) = 1.459$ bit

$H(Y) = 1.555$ bit

3. $H(Y) = -\frac{1}{4}\log_2\frac{1}{4} - \frac{3}{4}\log_2\frac{3}{4}$

$= 0.81$

$H(A) = -1\log_2 1 = 0$

$H(B) = -\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{2}\log_2\frac{1}{2}$

$= 1$

$I(Y;A) = H(Y) - H(Y|A)$

$= 0.81 + \frac{3}{4}\log_2\frac{3}{4} + \frac{1}{4}\log_2\frac{1}{4}$

$= 0$

$I(Y;B) = H(Y) - H(Y|B)$

$= 0.81 - (-\frac{1}{4}\log_2\frac{1}{2} - \frac{1}{4}\log_2\frac{1}{2} - \frac{1}{2}\log_2 1)$

$= 0.81 + \frac{1}{2}\log_2\frac{1}{2}$

$= 0.31$

选B.

$P(Y,A)$

A \ Y	+	−
0	0	0
1	$\frac{3}{4}$	$\frac{1}{4}$

$P(Y|A)$

A \ Y	+	−
0	0	0
1	$\frac{3}{4}$	$\frac{1}{4}$

$P(Y,B)$

B \ Y	+	−
0	$\frac{1}{4}$	$\frac{1}{4}$
1	$\frac{1}{2}$	0

$P(Y|B)$

B \ Y	+	−
0	$\frac{1}{2}$	$\frac{1}{2}$
1	1	0