1. (1pt) Consider the following linear programming problem

max
$$z=x_1+x_2$$

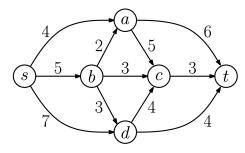
subject to $x_1+2x_2 \le 4$
 $x_1 \ge x_2 - 1$
 $4x_1+2x_2 \le 12$
 $x_1,x_2 \ge 0$

- 1) Convert the problem to the standard form (i.e., maximize $c^T x$ such that $Ax \le b$, $x \ge 0$)
- 2) Derive its dual problem.
- 2. (2pt) For each of the following problems, match the type of convex problem (LP, GP, QP, SDP):
 - (a) min $1/\sqrt{xy}$ s.t. $x^2 + y^2 < 7$
 - (b) $\max \min(x_1, x_2)$ s.t. $x_1+x_2<5$
 - (c) min $\sum_{i,j} X_{ij}$ s.t. $v^T X v \ge 0$ (i,j denote the indices of X)
 - (d) min $||Ax b||_2^2$
- 3. (1pt) Consider the following problem

$$\min_{\substack{x \in \mathbb{R}^2 \\ \text{subject to}}} f(x) = \frac{1}{2}(x_1^2 + x_2^2)$$

Find the Lagrange dual function and the dual problem.

4. (1pt) Consider the following flow network where the number on each edge denotes the capacity.



- 1) What is the maximum flow that the flow network can reach?
- 2) What is the minimum cut of the network? Show the minimum cut by partitioning the network into two subsets of vertices.