乱数シミュレーションによるフィッティングモデル評価

Applicability of Random Simulation to the Evaluation of Interpolation

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Quality and quantity of paleontological samples vary depending on materials to investigate (e.g. Wani, 2003; Hammer and Harper, 2006; Mustoe and Smith, 2023). Sample data is occasionally accompanied with discreteness while continuity is a prerequisite for the interpolation in paleontological studies. Therefore, an approach that has tolerance of both discreteness and continuity is required to evaluate the sample-model interpolation.

Materials and Method

I. Error model Q was expressed using a random variable u

$Q(p) = 1+p(2u-1)$, where $u = [0, 1]$. ... (1)

A model with deviation F was simulated as a product of predetermined function f and Q

$F(x) = f(x)Q(p)$. ... (2)

II. A sample data model was expressed by 20 percent of deviation

$F(x) = f(x)Q(0.2)$, where $f(x) = (3x-2)^2$. ... (3)

A sample data set was generated as 100 sets of discrete samples through the sample data model.

III. Four fitting models $f(x)^n$ against the generated sample data set were estimated using the least squares method with gnuplot version 5.2 (Williams, Kelley, *et al*. 1986-1993, 1998, 2004, 2007-2019)

$f(x)^n = (\mathrm{A}x+\mathrm{B})^n$, ... (4)

where $n = 1, 2, 3$, and -1.

IV. A value dL/dx that could distinguish the given fitting models was selected for the random simulation. This value for the evaluation was defined based on the partial curve length with length L between two points

$dL = \Sigma L - \max(L)$ ... (5)

and the horizontal range

$dx = \max(x) - \min(x)$. ... (6)

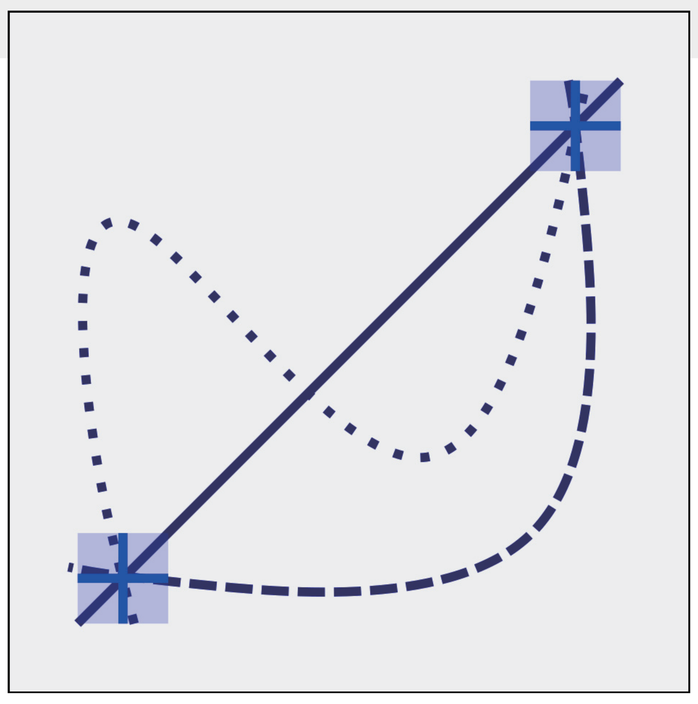
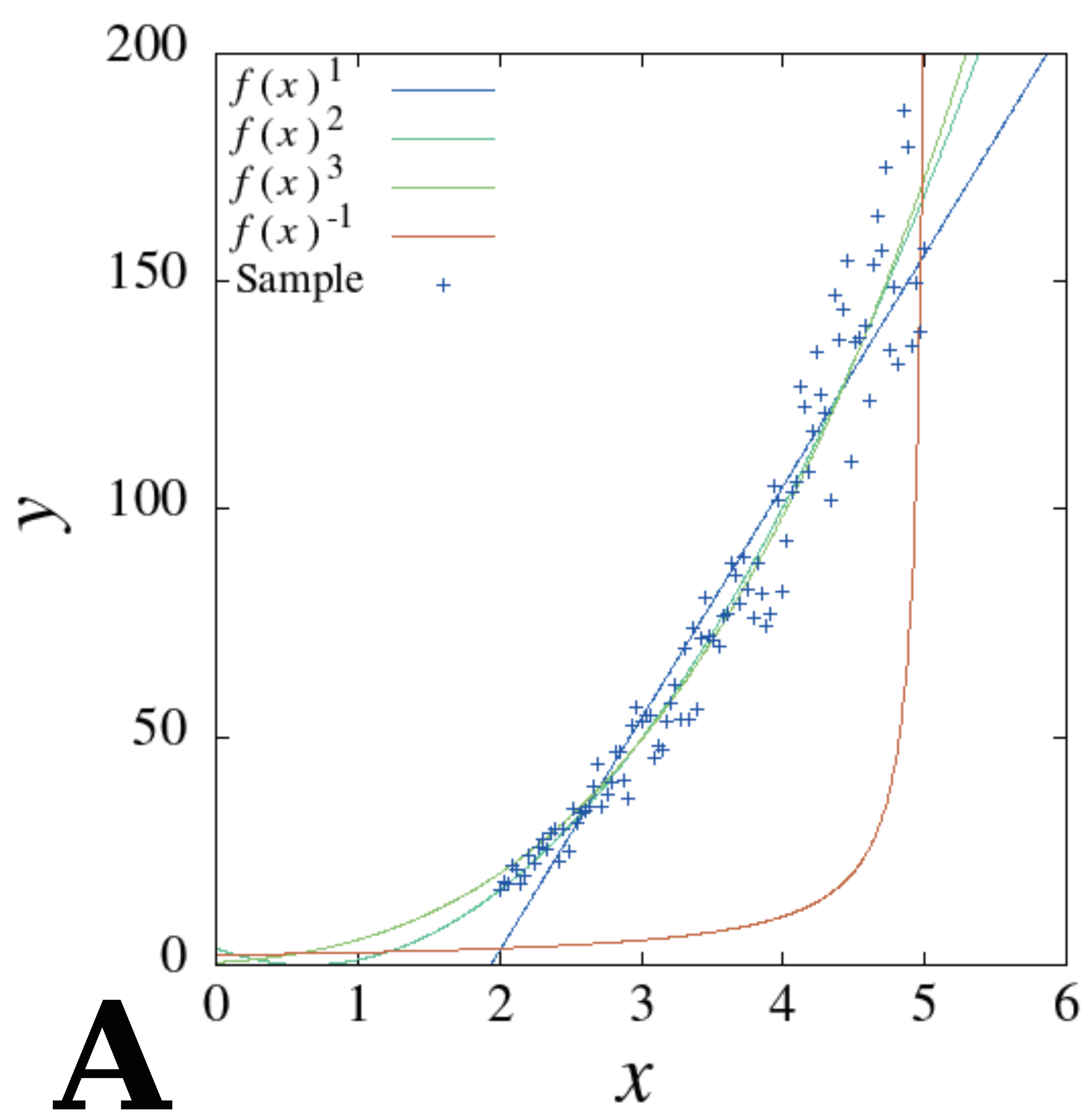


Figure 1. Schematic diagram showing three interpolations based on different models.

Table 1. Results of the least squares method.

Fitting models	A	B	Variance of residuals (reduced chisquares)
$(\mathrm{A}x+\mathrm{B})^{-1}$	-08.97E-02	45.23E-02	**5612.88**
$(\mathrm{A}x+\mathrm{B})^{1}$	50.97E+00	-98.81E+00	**156.49**
$(\mathrm{A}x+\mathrm{B})^{2}$	02.99E+00	-01.92E+00	**119.89**
$(\mathrm{A}x+\mathrm{B})^{3}$	95.01E-02	81.96E-02	**125.26**



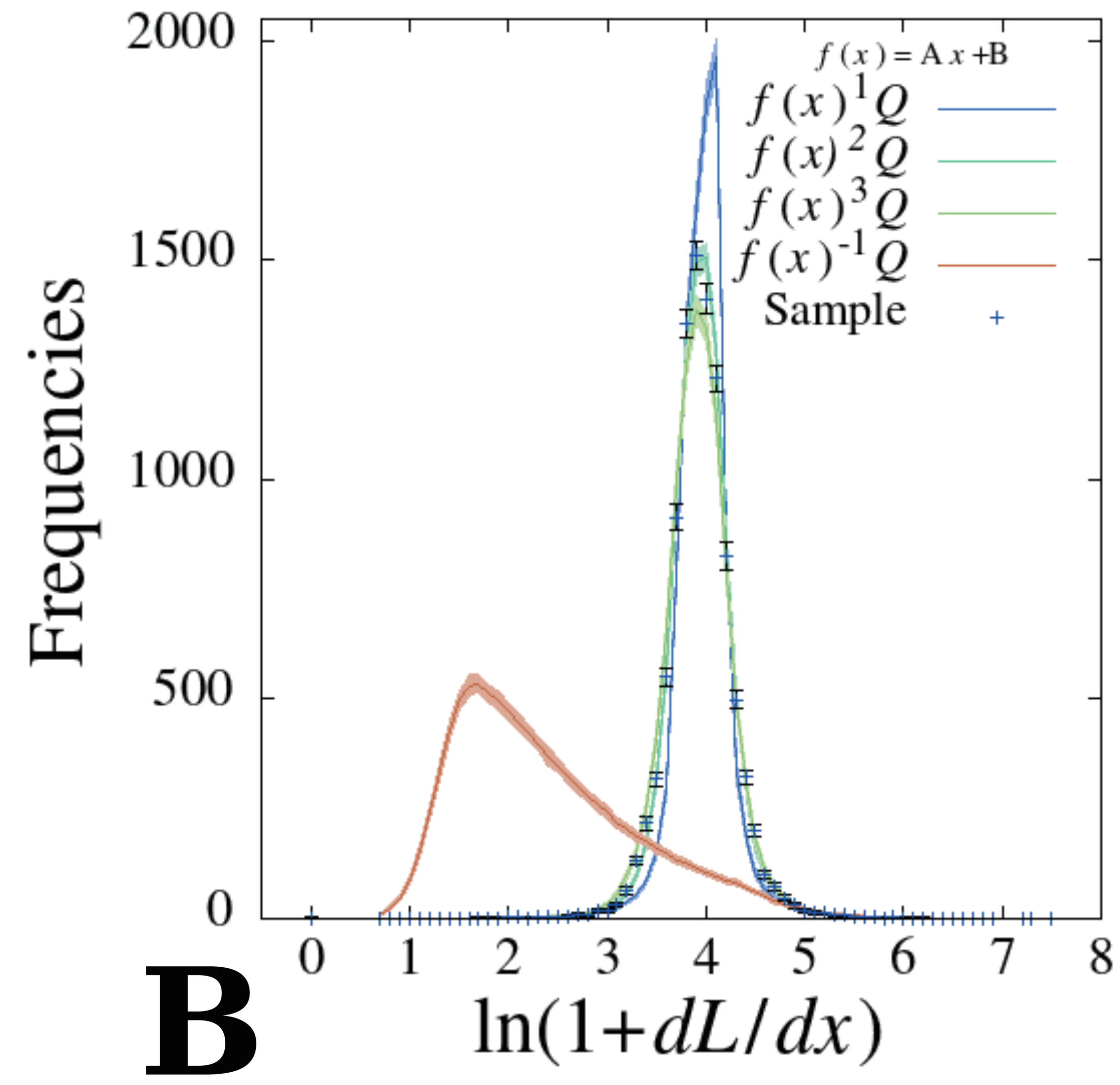


Figure 2. Graphs showing sample data and result of evaluation.

A. Graphs showing sample data and results of curve fitting.

Every point is plot of the generated data set from a model, which is $(3x-2)^2$ with 20 percent of deviation. Every line is result of curve fitting using the least squares method.

B. Graphs showing frequency distributions of ln (1+*dL*/*dx*).

This graph shows frequency distributions of $\ln(1+dL/dx)$, which are defined using 3 random points on a given function $f(x)$ or generated sample data. Every line is result of given function $f(x)$. Every point is result of generated sample data. Each graph represents average of a 100 sets of results from random simulation. Filled areas or error bars represent standard deviations.

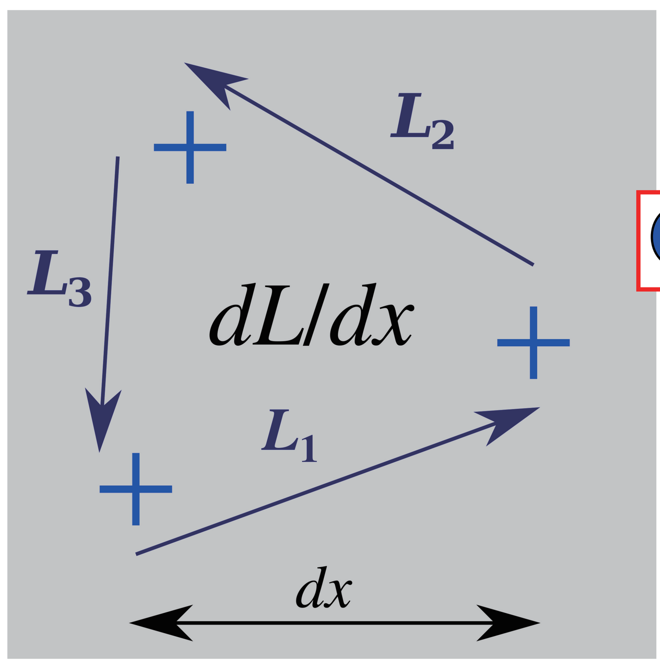


Figure 3. Schematic diagram of *dL*/*dx*.

This diagram shows vector relations of dL/dx between three points. A length L_i is expressed as a norm of a vector; $L_i = |\boldsymbol{L}_i|$, where $i = 1, 2$, and 3.

Conclusion

The random simulation could evaluate the sample-model interpolations that were based on the least squares method.

References cited

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Wani, R., 2003: Taphofacies models for Upper Cretaceous Ammonoids from the Kotanbetsu area, northwestern Hokkaido, Japan. *Palaeogeography*, *Palaeoclimatology*, *Palaeoecology*, vol. 199, p. 71-82.

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