Homework 1: Singular Value Decomposition

MAS480 Advanced Mathematics for Data Science

20200130 Yujun Kim

1. **No need to submit**
2. **Find market factor that left singular vectors corresponding to the largest singular value changes rapidly (Hint: Use the date data in csv file)**

The largest singular value is 269.41. Corresponding singular vector explains 88.6% of the whole data

```
> # Singular values
> D$d
 [1] 269.413830  90.676180  30.384883  12.051732   5.753231   4.947852
 [7]   3.115177   2.102543   1.673303   1.288397   1.204387
> # Explain power of cumulated singular vectors
> cumsum(D$d^2/sum(D$d^2))*100
 [1]   88.56860   98.60150   99.72806   99.90529   99.94568   99.97555
99.98739
 [8]  99.99279  99.99620  99.99823 100.00000
```

in terms of singular value. The three left singular vectors corresponding to the three largest singular value is given in the figure1. Red, green, blue line each corresponds to the left singular vector with scaling corresponding to largest, 2nd largest, 3rd largest singular value.

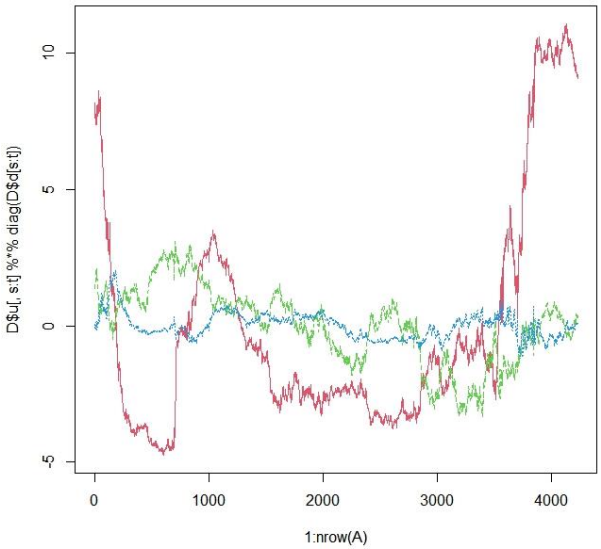

Figure 1 Scaled Left Singular Vectors

The moment of rapid change of red curve is around row 1-300 and 3700-4000. The corresponding timeline is 10/20/2021 ~ 12/30/2022 and 01/09/2007 ~ 03/20/2008. The first period corresponds to giant steps of increase in yield. The second corresponds to the period of Subprime Mortgage crisis where the yield rate of course rapidly dropped.

3. **Answer followings**

(a) How much the matrix is approximated when using 1,2, or 3 rank-1 matrix?

According to the result of cumsum 88.6%, 98.6%, and 99.7% is approximated.

(b) Plot right singular vectors corresponding to the three largest singular values.

Refer figure2. Red, green blue corresponds to the largest, 2nd largest, 3rd largest eigenvalues respectively.

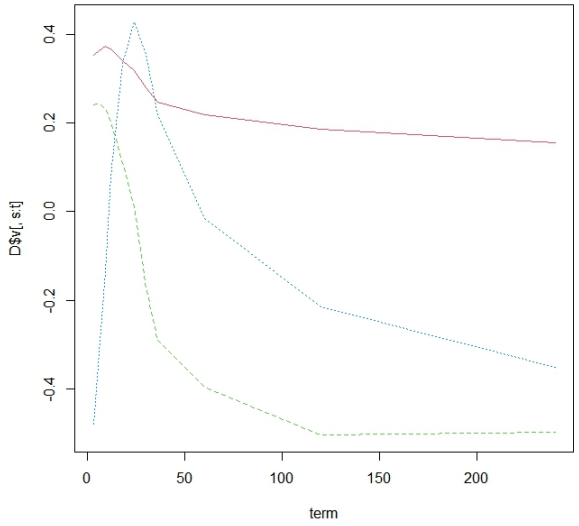

Figure 2 Right singular vectors 1st-3rd

(c) Plot right singular vectors corresponding to the 4th-6th largest singular values and explain the characteristics

Refer figure 3. The singular vector is almost a noise. There is no certain tendency in the singular vector.

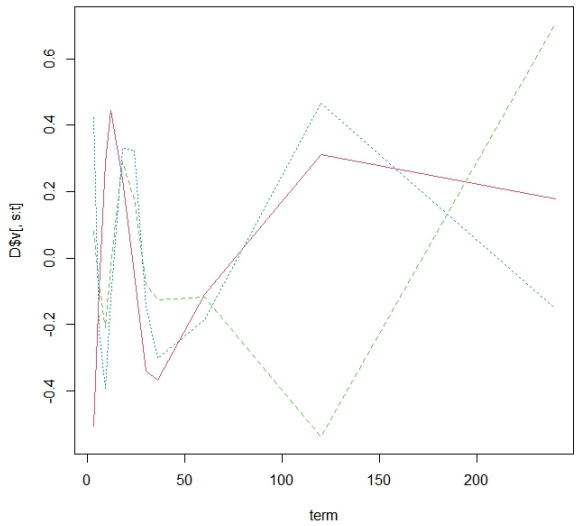

Figure 3 Right Singular Vector 4th-6th