DEPARTMENT OF INFORMATICS ENGINEERING

DEPARTMENT OF COMPUTER SCIENCE

Functional and Logic Programming

Bachelor in Informatics and Computing Engineering
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Logic Programming

Unification and Execution Model

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Agenda

- Unification
- Execution Model

Substitution

- Terms can be either
  - **Ground** - there are no variables in the term (completely instantiated)
  - **Unground** - there are variables in the term

- Unification is how Prolog matches two terms
  - Two terms are unifiable if they are the same, or they can be the same after variable substitution

- A **substitution** $\boldsymbol{\theta}$ is a set of pairs $\boldsymbol{X_i} = \boldsymbol{t_i}$ where
  - $\boldsymbol{X_i}$ is a variable
  - $\boldsymbol{t_i}$ is a term
  - $\boldsymbol{X_i} \neq \boldsymbol{X_j}$ for all $\boldsymbol{i} \neq \boldsymbol{j}$
  - $\boldsymbol{X_i}$ does not occur in any $\boldsymbol{t_j}$, for all $\boldsymbol{i}$ and $\boldsymbol{j}$

Substitution

- To apply a substitution θ to a term T ($T\theta$) is to replace in T all occurrences of X_i for t_i, for all pairs X_i=t_i in θ

  T = father(X, bart)

  θ = {X=homer}

  $T\theta$ = father(homer, bart)

- A is said to be an **instance** of B if there is a substitution θ such that $A = B\theta$

  `father(homer, bart)` is an instance of `father(X, bart)`

Substitution

- A term ***T*** is a **common instance** of T_1 and T_2 if there are substitutions θ_1 and θ_2 such that $T = T_1\theta_1$ and $T = T_2\theta_2$

`parent(homer, bart)` is a common instance of `parent(X, bart)` and `parent(homer, Y)`

- A term ***G*** is **more general** than term ***T*** if ***T*** is an instance of ***G*** but ***G*** is not an instance of ***T***

`parent(X, bart)` is more general than `parent(homer, bart)`

- A term ***V*** is a **variant** of a term ***T*** if they can be converted into one another by a simple variable renaming

`parent(Y, bart)` is a variant of `parent(X, bart)`

Unification

- Given two atomic sentences, *p* and *q*, a unification algorithm returns a substitution θ (the most general unifier) that makes them identical (or fails if such substitution does not exist):

  Unify(p, q) = θ where $p\theta = q\theta$

- θ is said to be the (most general) unifier of the two sentences
- The **most general unifier** (MGU) is the one that compromises the variables as little as possible – the respective instance is the most general

Unify(*parent(X, bart)*, *parent(Y, Z)*) produces θ = { Y=X, Z=bart }

Unification Algorithm

Occurs check

initialize θ to empty
push T_1=T_2 into the *stack*
while *stack* is not empty do
    pop X=Y from the *stack*
    case
        X is a variable that does not occur in Y:
            substitute Y for X in the *stack* and in θ
            add X=Y to θ
        Y is a variable that does not occur in X:
            ...
        X and Y are identical constants or variables:
            continue
        X is $f(X_1, \ldots, X_n)$ and Y is $f(Y_1, \ldots, Y_n)$, for some functor f
            push X_i=Y_i, $i=1\ldots n$, on the *stack*
        otherwise:
            return *failure*
return θ

Unification in Practice

- Both terms are constants: the terms unify if they are the same
- One of the terms is a variable: it is instantiated to the other term
  - If both terms are variables, they are bound to each other
- Two compound terms unify if
  - They have the same functor and arity
  - All the corresponding arguments unify
  - All substitutions are compatible

Occurs Check

- Standard unification algorithms start with an *occurs check*
  - Verification of whether the variable occurs in the other term
- Prolog's typical unification algorithm skips this step, to increase efficiency
- However, we can force occurs check using the predicate

  `unify_with_occurs_check/2`

```
| ?- something(X) = X.
X = something(something(something(something(something(something(somethin
g(something(something(something(...)))))))))) ?
yes
| ?- unify_with_occurs_check(something(X), X).
no
```

Agenda

- Unification
- Execution Model

Computation

- Program P composed of Clauses
  - Clauses are universally quantified logical sentences
    - `A:- B`$_1$`, ..., B`$_k$, k >= 0
    - A and B_i are goals
- Computation of a Logic Program P: find an instance of a given query Q logically deducible from P
  - Query is an existentially quantified conjunction
    - `A`$_1$`, ..., A`$_n$, n > 0
    - A_i are goals
  - Goal: Atom or compound term

Computation

- Given a program **P** and an initial query **Q**
  - Computation terminates
    - With success – (an instance of) Q was proven
      - Multiple successful computations (solutions) may exist
    - Without success – Q cannot be proven
  - Computation may not terminate (no result)

- Non-termination comes from recursive rules that may not end
  - Avoid left-recursive rules

```
are_married(X, Y):- married(X, Y).
are_married(X, Y):- married(Y, X).
```

```
married(homer, marge).
...
married(X, Y):- married(Y, X).
```

Computation

- **Resolvent** is a conjunctive question (query) with the set of goals still to be processed
- **Trace** is the evolution of the computation (sequence of resolvents) with information regarding:
  - Selected goal
  - Rule selected for reduction
  - Associated substitution
- **Reduction** is the replacement, in the resolvent, of a goal G with the body of a clause whose head unifies with G

Abstract Interpreter

- Abstract interpreter algorithm, given program **P** and query **Q**

Let *resolvent* be Q

While *resolvent* is not empty **do**

1. Choose a goal A from *resolvent*
2. Choose a renamed clause B:- B_1, ..., B_n from P such that A and B unify with an mgu $\boldsymbol{\theta}$ (**exit** if no such goal and clause exist)
3. Remove A from *resolvent* and add B_1, ..., B_n to resolvent
4. Apply $\boldsymbol{\theta}$ to *resolvent* and to Q

If *resolvent* is empty, **return** Q; else **return** failure

Execution Model

- An implementation of Logic Programming needs to instantiate the abstract interpreter, making choices that influence how the computation is performed
  - Choice of goal from resolvent
  - Choice of clause
  - Add goal(s) to resolvent

- Different languages / implementations may make different choices to implement the abstract interpreter

Prolog’s Execution Model

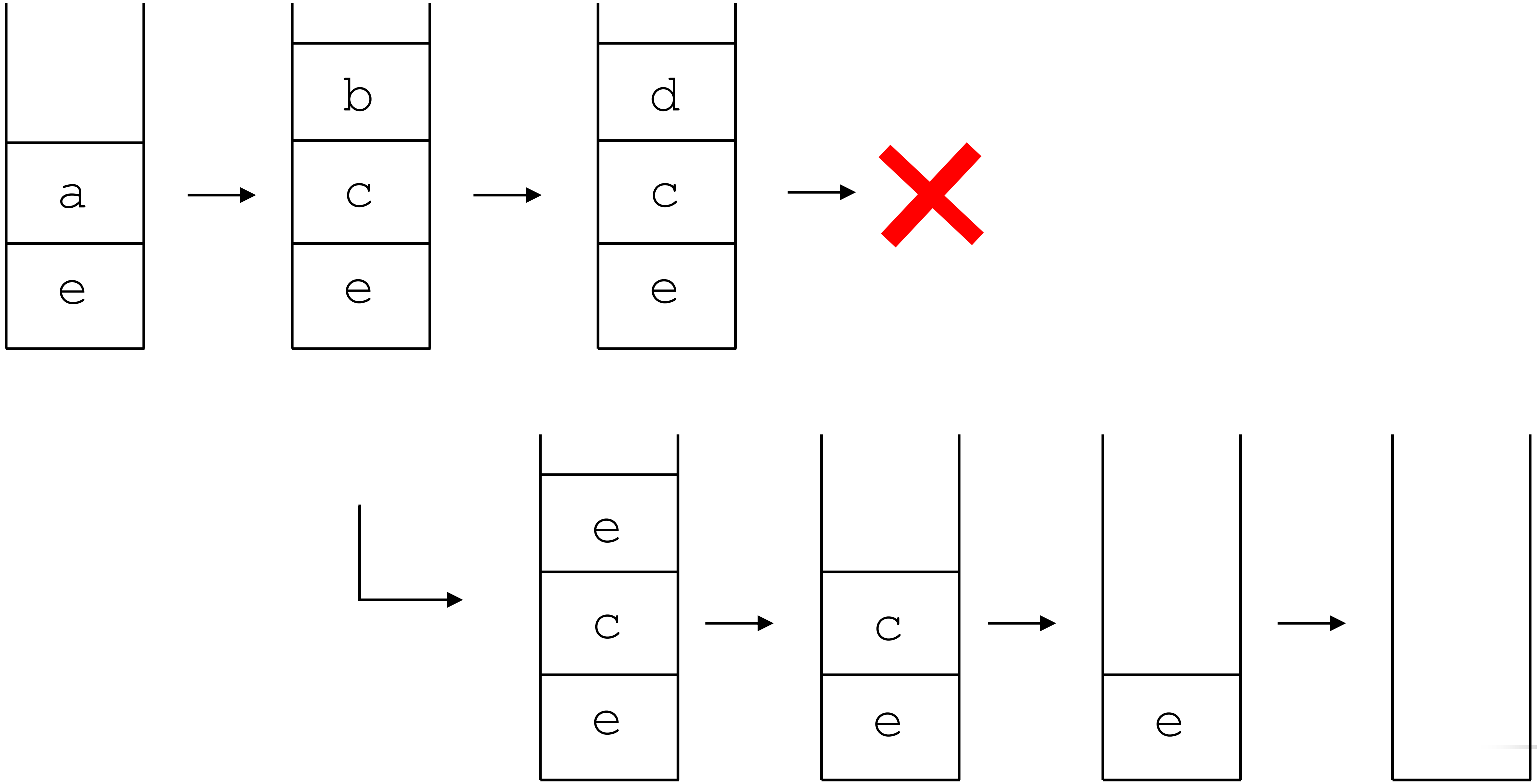
- Prolog’s implementation of the abstract interpreter
- Choice of goal from resolvent: **left to right**
  - Choice is arbitrary, does not affect computation
- Choice of clause: **top to bottom with backtracking**
  - Choice affects computation
- Add goal(s) to resolvent: **at the beginning**
  - Results in a depth-first search
  - If it were to be added to the end, it would result in a breadth-first search (assuming leftmost goal is chosen next)

Prolog's Execution Model

- Resolvent can be seen as a stack
  - With auxiliary data (backtracking points)

```
a :- b, c.
b :- d.
b :- e.
c.
e.
```

```
| ?- a, e.
```



Search Trees

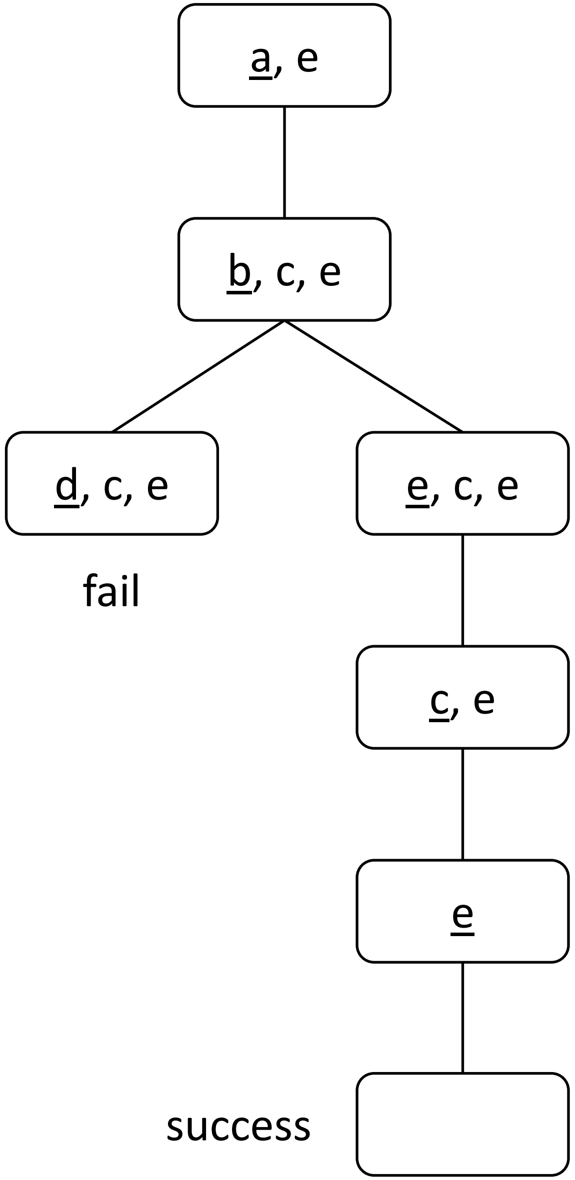
- A search tree contains all possible search paths
  - **Root**: Query Q
  - **Nodes**: resolvents, with selected goal
  - **Edges**: one edge for each clause in P whose head unifies with the selected goal in the source node
    - Includes substitution from the unification
  - **Leaves**: success nodes, if empty resolvent; or fail nodes
  - **Paths from root to leaves**: computation of Q using P

Search Trees

- Example:

```
a :- b, c.
b :- d.
b :- e.
c.
e.
```

```
| ?- a, e.
```



Search Trees

- It is independent of the **clause selection** criteria (it contains all alternatives)
- There can be different search trees for the same query and program, depending on the **goal selection** criteria
- The number of **success nodes** is the same in all trees
- Contains all answers; it is named **search** tree because a concrete interpreter needs a strategy to traverse the tree searching for solutions
  - Depth-first search, breadth-first search, parallel search, ...

Alternatives

- Depth-first search is not complete
  - It may not find a solution (infinite search branch)
- Breadth-first search is complete
  - If a solution exists, it is found
- OR parallelism
  - Search all branches of the search tree in parallel
- AND parallelism
  - Execute all goals of the resolvent in parallel

Clause and Goal Order

- We cannot ignore Prolog’s execution model
  - Changing the order of clauses changes the order in which the search tree is traversed, and so the order in which answers are found
  - Changing the order of goals changes the search tree (may generate trees with different sizes – search effort can be different)
    - May lead to an infinite search branch!
    - If efficiency is important, different versions of a predicate may be required depending on variable instantiation

```
grandparent(X, Y):- parent(X, Z), parent(Z, Y).
grandparent(X, Y):- parent(Z, Y), parent(X, Z).
```

Goal Order

- Some heuristics can be devised, based in the principle of failing as fast as possible (failing means cutting the search tree, and thus reaching the solution faster)
  - Place tests (guards) first
  - Place goals with fewer solutions first
    - Depends on the database
  - Place goals with more ground terms first
    - Depends on the use

Q & A