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Functional and Logic Programming

Bachelor in Informatics and Computing Engineering
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Prolog

Collecting Solutions / Graphs and Trees

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Agenda

- Collecting Solutions
- Graphs and Search
  - Puzzles and Games
- Binary Trees

Collecting Solutions

- So far, we obtained multiple solutions to a query interactively, in the terminal, one by one
- Or by accumulating the results of a query in a list

```
get_all_children(Parent, Children):-
      get_children(Parent, Children, []).

get_children(Parent, Children, Temp):-
      parent(Parent, Child),
      not (member(Child, Temp)), !,
      get_children(Parent, Children, [Child|Temp]).
get_children(_Parent, Children, Temp):-
      reverse(Temp, Children).
```

Is the cut necessary?

Why is this approach inefficient?

Collecting Solutions

- Prolog provides three predicates to obtain multiple solutions to a query: *findall*, *bagof* and *setof*
  - They allow systematic collection of answers to any goal
  - The template is similar to all three predicates

```
findall(Term, Goal, List).
```

See section 4.13 of the SICStus Manual for more information on collecting solutions

findall

- ***findall*** finds all solutions, including repetitions if present
  - If there are no solutions, an empty list is returned

```
| ?- findall(Child, parent(homer, Child), Children).
Children = [lisa, bart, maggie] ? ;
no

| ?- findall(Parent, parent(Parent, _Child), List).
List = [homer,homer,homer,marge,marge,marge] ? ;
no

| ?- findall(Child, parent(bart, Child), List).
List = [] ? ;
no
```

findall

- We can use a conjunctive goal (parentheses are required)

```
| ?- findall(C, ( parent(homer, C), female(C) ), Daughters).
Daughters = [lisa, maggie] ? ;
no
```

- We can obtain more than one variable using a compound term

```
| ?- findall(Parent-Child, parent(Parent, Child), L).
L = [homer-lisa, homer-bart, homer-maggie, marge-lisa, ...] ? ;
no
```

- If all we want is a count, we can use anything

```
| ?- findall(_, parent(homer, _C), _L), length(_L, N).
N = 3 ? ;
no
```

bagof

- ***bagof*** has similar behavior, but results are grouped by variables appearing in Goal but not in the search Term

```
| ?- findall(Child, parent(Parent, Child), Children).
Children = [lisa, bart, maggie, lisa, bart, maggie] ? ;
no

| ?- bagof(Child, parent(Parent, Child), Children).
Parent = homer, Children = [lisa, bart, maggie] ? ;
Parent = marge, Children = [lisa, bart, maggie] ? ;
no
```

bagof

- While *findall* returns an empty list if there are no results, *bagof* fails

```
| ?- findall(Child, parent(bart, Child), L).
L = [] ? ;
no

| ?- bagof(Child, parent(bart, Child), L).
no
```

Existential Quantifier

- We can direct *bagof* to ignore additional variables in *Goal* by using existential quantifiers: *Var^Goal*

```
| ?- bagof(Child, parent(Parent, Child), Children).
Parent = homer, Children = [lisa, bart, maggie] ? ;
Parent = marge, Children = [lisa, bart, maggie] ? ;
no

| ?- bagof(Child, Parent^parent(Parent, Child), Children).
Children = [lisa, bart, maggie, lisa, bart, maggie] ? ;
no
```

  - If all variables appearing in *Goal* but not in the search *Term* are existentially quantified, then *bagof* behaves like *findall*

setof

- ***setof*** has similar behavior to *bagof*, but results are ordered and without repetitions

```
| ?- bagof(Child, parent(Parent, Child), Children).
Parent = homer, Children = [lisa, bart, maggie] ? ;
Parent = marge, Children = [lisa, bart, maggie] ? ;
no

| ?- setof(Child, parent(Parent, Child), Children).
Parent = homer, Children = [bart, lisa, maggie] ? ;
Parent = marge, Children = [bart, lisa, maggie] ? ;
no
```

  - The standard order of terms is used (see section 4.8.8.2)

setof

- Existential quantifiers can also be used with *setof*, with the same effect as with *bagof* (results will remain ordered and without repeats)

```
| ?- bagof(Child, Parent^parent(Parent, Child), Children).
Children = [lisa, bart, maggie, lisa, bart, maggie] ? ;
no

| ?- setof(Child, Parent^parent(Parent, Child), Children).
Children = [bart, lisa, maggie] ? ;
no
```

- If all variables in *Goal* but not in search *Term* are existentially quantified, then *setof* behaves like *findall* followed by *sort*

Collecting Solutions

- Note that the Goal must be able to generate answers

```
| ?- findall(X, (X > 0, X < 6), L).
! Instantiation error in argument 1 of (>)/2
! goal:  _321>0

| ?- findall(X, between(1, 5, X), L).
L = [1,2,3,4,5] ? ;
no
```

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  - Puzzles and Games
- Binary Trees

Data Structures

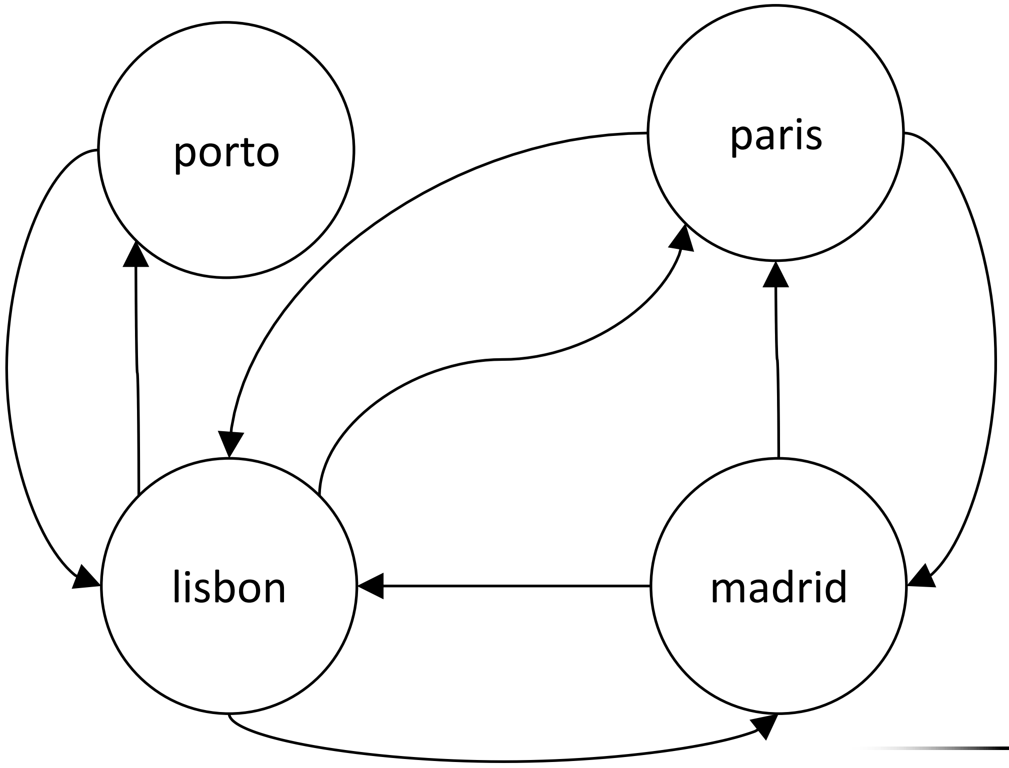
- Even though Prolog doesn't explicitly define types or data structures, terms can be used to do so
  - Unary predicates can be used to 'define a type'
    - The type *male* can be defined as the set of terms *X* such that *male(X)* is true
  - Simple types can be defined recursively
    - Lists, Trees, ...
  - *Pairs* are typically represented as *X-Y*
  - *Tuples* can be represented as *(X, Y, Z)*
    - However, a properly named functor should be used
  - More complex terms can be used to represent data structures

```
book('River God', author(smith, wilbur, 1933), 1993, Book).
```

Graphs and Search

- Graphs can be represented as the connections between nodes
  - set of facts representing [directed] edges

```
connected(porto, lisbon).
connected(lisbon, madrid).
connected(lisbon, paris).
connected(lisbon, porto).
connected(madrid, paris).
connected(madrid, lisbon).
connected(paris, madrid).
connected(paris, lisbon).
```



Depth-First Search

- Searching for a possible connection between nodes is made easy by Prolog's standard depth-first search mechanism

```
connected(porto, lisbon).
connected(lisbon, madrid).
connected(lisbon, paris).
connected(lisbon, porto).
connected(madrid, paris).
connected(madrid, lisbon).
connected(paris, madrid).
connected(paris, lisbon).
```

```
connects_dfs(S, F):-
      connected(S, F).
connects_dfs(S, F):-
      connected(S, N),
      connects_dfs(N, F).

| ?- connects_dfs(porto, madrid).
yes
| ?- connects_dfs(madrid, porto).
```

When does this approach fail?

Depth-First Search

- Adapted solution with an accumulator to avoid loops

```
connected(porto, lisbon).
connected(lisbon, madrid).
connected(lisbon, paris).
connected(lisbon, porto).
connected(madrid, paris).
connected(madrid, lisbon).
connected(paris, madrid).
connected(paris, lisbon).
```

```
connects_dfs(S, F):-
      connects_dfs(S, F, [S]).

connects_dfs(F, F, _Path).
connects_dfs(S, F, T):-
      connected(S, N),
      not( member(N, T) ),
      connects_dfs(N, F, [N|T]).


| ?- connects_dfs(madrid, porto).
yes
```

What would we have to change to *return* the connecting path (route)?

Breadth-First Search

- We can also easily create a BFS solution using *findall*

```
connected(porto, lisbon).
connected(lisbon, madrid).
connected(lisbon, paris).
connected(lisbon, porto).
connected(madrid, paris).
connected(madrid, lisbon).
connected(paris, madrid).
connected(paris, lisbon).
```

```
connects_bfs(S, F):-
      connects_bfs([S], F, [S]).

connects_bfs([F|_], F, _V).
connects_bfs([S|R], F, V):-
      findall(N,
      ( connected(S, N),
        not(member(N, V)),
        not(member(N, [S|R]))), L),
      append(R, L, NR),
      connects_bfs(NR, F, [S|V]).
```

What would we have to change to *return* the connecting path (route)?

Games and Puzzles

- Prolog (and search) can easily be used to search for a solution to one-person games or puzzles
- States are represented as the nodes of the graph
  - Initial state is the starting node
  - Winning conditions define the final nodes
- Movements are represented as the transitions between nodes
  - States don’t need to be represented in extension – transitions can specify new states based on the previous one and the move made

Generic Solver

- A generic [abstract] solver to one-person games/puzzles

```
initial(InitialState).

final(State):- winning_condition(State).

move(OldState, NewState):- valid_move(OldState, NewState).

play:- initial(Init),
       play(Init, [Init], States),
       reverse(States, Plays), write(Plays).

play(Curr, Path, Path):- final(Curr), !.
play(Curr, Path, States):- move(Curr, Next),
                           not( member(Next, Path) ),
                           play(Next, [Next|Path], States).
```

Games and Puzzles

- Example: fill a 5-gallon jug with 4 gallons of water, using the 5-gallon jug and a 3-gallon jug

```
initial(0-0).    % Jug5-Jug3

final(4-_).

move(_-S, 5-S). % fill jug 1
move(F-_, F-3). % fill jug 2
move(_-S, 0-S). % empty jug 1
move(F-_, F-0). % empty jug 2
move(F-S, NF-NS):- NF is max(0, F+S-3), NS is min(3, F+S). % 1->2
move(F-S, NF-NS):- NF is min(5, F+S), NS is max(0, F+S-5). % 2->1
```

Shortest Path

- To find the smallest set of plays we just need to find all paths and select the shortest one
  - Easily accomplished using *setof*

Is DFS the best way of doing this?

```
play:-   initial(Init),
         setof( Length-Path, (
                    play(Init, [Init], Path),
                    length(Path, Length) ),
               [_ShortestLength-States|_] ),
         reverse(States, Path), write(Path).
```

What if we wanted the path with the lowest cost?

How could we obtain all paths with shortest length?

Agenda

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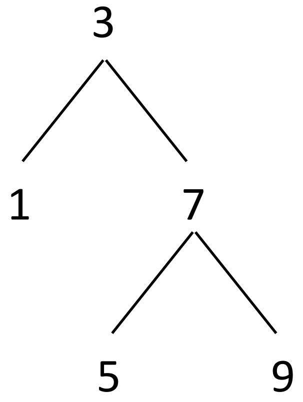
- Binary Trees

Binary Trees

- A binary tree can be recursively defined using node elements
    - Empty node represented as `null`
    - Other nodes as `node(Value, Left, Right)`
- Definition of a binary tree

```
binary_tree(null).
binary_tree( node(Value, Left, Right) ):-
      binary_tree(Left),
      binary_tree(Right).
```




```
node(3,  node(1, null, null),
         node(7, node(5,null,null),
                 node(9,null,null)  )  ).
```

Binary Trees

- Tree operations are easily implemented from this definition
  - Check if value is a member of the tree

```
tree_member(Val, node(Val, _L, _R) ).
tree_member(Val, node(V, L, _R) ):-
      [Val < V,] tree_member(Val, L).
tree_member(Val, node(V, _L, R) ):-
      [Val > V,] tree_member(Val, R).
```

[code] if we consider the tree to be a binary search tree

  - List all tree elements (in-order traversal)

```
tree_list( null, [] ).
tree_list( node(Val, L, R), List ):-
      tree_list(L, Left),
      tree_list(R, Right),
      append(Left, [Val|Right], List).
```

Binary Trees

- Verify if tree is ordered

```
tree_is_ordered(Tree):-
      tree_list(Tree, List),
      sort(List, List).
```

- Insert an element into the tree

```
tree_insert( null, V, node(V, null, null) ).
tree_insert( node(V, L, R), V, node(V, L, R) ).
tree_insert( node(V, L, R), Val, node(V, NL, R) ):-
      Val < V, tree_insert( L, Val, NL).
tree_insert( node(V, L, R), Val, node(V, L, NR) ):-
      Val > V, tree_insert( R, Val, NR).
```

Binary Trees

- Determine the height of the tree

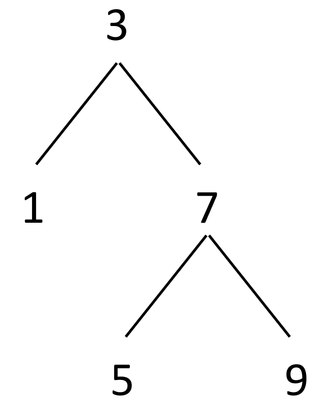
```
tree_height( null, 0).
tree_height( node(Val, L, R), H):-
      tree_height(L, HL),
      tree_height(R, HR),
      H is 1 + max(HL, HR).
```

- Check whether the tree is balanced

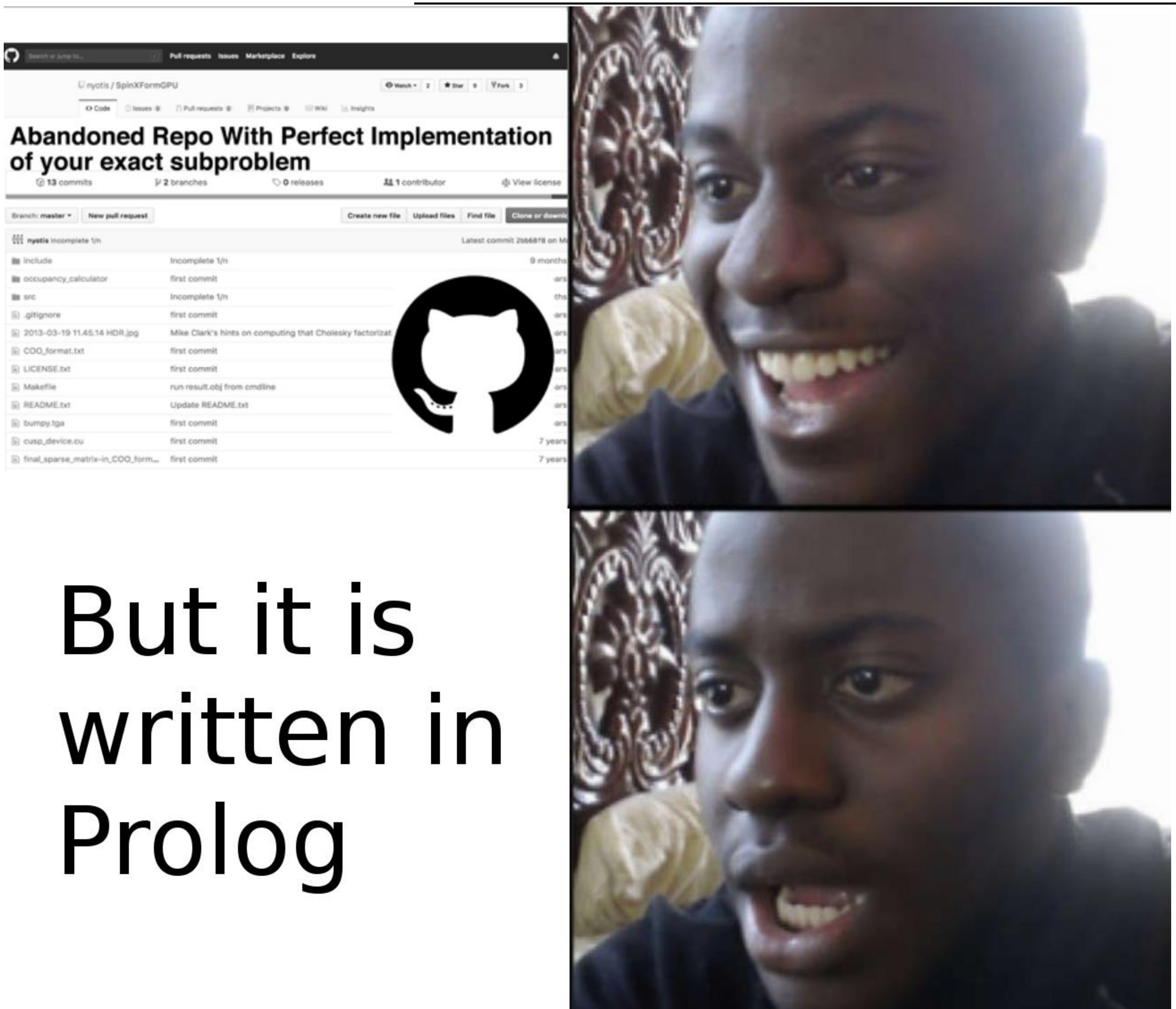
```
tree_is_balanced( null ).
tree_is_balanced( node(Val, L, R) ):-
      tree_is_balanced(L),
      tree_is_balanced(R),
      tree_height(L, HL),
      tree_height(R, HR),
      abs(HL-HR) =< 1.
```

Binary Trees

```
| ?- test_tree(_T), tree_member(5, _T).
yes
| ?- test_tree(_T), tree_member(4, _T).
no
| ?- test_tree(_T), tree_list(_T, L).
L = [1,3,5,7,9] ?
yes
| ?- test_tree(_T), tree_is_ordered(_T).
yes
| ?- test_tree(_T), tree_height(_T, H).
H = 3 ?
yes
| ?- test_tree(_T), tree_is_balanced(_T).
yes
| ?- test_tree(_T), tree_insert(_T, 2, NT).
NT = node(3,node(1,null,node(2,null,null)),node(7,node(5,null,null),node
(9,null,null))) ?
yes
| ?- test_tree(_T), tree_insert(_T, 6, NT), tree_is_balanced(NT).
no
```



Q & A


Abandoned Repo With Perfect Implementation of your exact subproblem
But it is written in Prolog