SEIR dynamics of COVID-19 in Okinawa

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1 SEIR model

This model is entirely based on the descriptions by Dr. Jun Ohashi (link)

1.1 Variables

S: the number of susceptive people
E: the number of people in latency period (asymptomatic)
I: the number of infected people (symptomatic)
R: the number of people who recovered (they are now immune to the infection)
N: total number of people (N = S + E + I + R) * ignore birth and death of people. N is fixed over time
α_t: the number of people that a single infected person spreads infection to per unit time at time t
β: infection rate per time (transition from asymptomatic to symptomatic)
γ: recovery rate per time

We assume that everyone who are infected becomes state I because in the original model state of E is defined to be asymptomatic and non-infectious, whereas for coronavirus it seems there are people who are asymptomatic but infectious; Equations 1, 2, 5, and 6, $(I + E)$ is multiplied due to this assumption for COVID19, while in the original SEIR model, only I instead of $(I + E)$ is multiplied because people in state E are not infectious.

1.2 Model dynamics

$$\frac{dS}{dt} = -\alpha_t(I + E) \tag{1}$$

$$\frac{dE}{dt} = \alpha_t(I + E) - \beta E \tag{2}$$

$$\frac{dI}{dt} = \beta E - \gamma I \tag{3}$$

$$\frac{dR}{dt} = \gamma I \tag{4}$$

1.3 The relationship between R_0 and and α_t

α_t is the number of new infections per unit time (a day) when there is only a single person who is infected in a population during symptomatic period.
Let i be the the average symptomatic period. Then, $R_0 = \alpha_t \cdot i$. Therefore, $\alpha_t = R_0/i$, where R_0 is the basic reproduction number.

As the number of susceptive people (those who haven't infected) decreases, the number of new infections from an infected person decreases as well. Thus, α_t can be approximated by $\alpha_0 \cdot S_t/N = (R_0/i) \cdot (S_t/N)$, where S_t is the number of infected people at time t.

We can rewrite the model as follows:

$$\frac{dS}{dt} = -(R_0/i)(S/N)(I+E) \tag{5}$$

$$\frac{dE}{dt} = (R_0/i)(S/N)(I+E) - (1/l)E \tag{6}$$

$$\frac{dI}{dt} = (1/l)E - (1/i)I \tag{7}$$

$$\frac{dR}{dt} = (1/i)I \tag{8}$$

where R_0 is the basic reproduction number, l is the average latency period (5 days), and i is the average symptomatic period (10 days).

1.4 Parameter R_0

R_0 for COVID19 is estimated as around 2.5 (in Germany). R_0, basic reproduction number, is a parameter about the number of new infections spread from one infected person. This is the parameter that can be controlled by adjusting the number of human-human interactions.