M2 ICFP, parcours physique théorique

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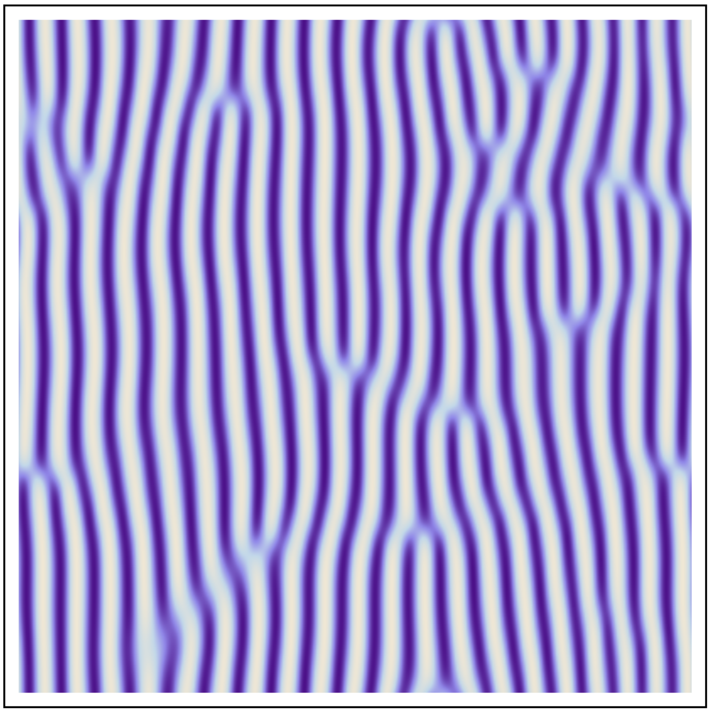
Nonpertubative renormalization group study of the Lifshitz critical point



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Abstract

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Résumé

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Nonpertubative renormalization group study of the Lifshitz critical point

Introduction

The first chapter of the internship report is thus a very general introduction to granular materials and granular flows. At the end of this chapter, we give qualitative ideas about the segregation effect, ideas which are developed more rigorously in the appendix.

The second chapter will focus more specifically on the experiment we seek to model and understand, and on the approach we took to model it.

The third chapter give our results and discuss them.

Contents

Chapter 1

General presentation

The Lifshitz model

1.1 The Lifshitz action and its main features

1.1.1 The Lifshitz action

The Lifshitz model aims at describing a number of physical many-body systems. They share a common intriguing feature: having a so called modulated - or stripped - phase. In this phase, the order parameter is spatially periodic in one or several directions of space. The subspace spanned by these direction will from now be labelled $/\!\!/$. The hyperplan orthogonal to this modulation direction will be labelled $\perp$.

Typically, the phase diagram of such a physical system will resemble this one

(figure to be drawn)

where ρ_0 is a tunable parameter whose precise meaning depends on the physical nature of the studied system. We will however take for granted that it is a scalar.

The Lifshitz model is a field theory, describing a vector field ϕ whose components will be denoted ϕ_i. To write the action for the Lifshitz model, we chose a basis $(\mathbf{e_n})_{1\leq n\leq d}$. We decide that this basis is such that its first m vectors span the m dimensional $/\!\!/$ subspace, while of course the remaining $d-m$ base vector span the $\perp$ subspace. In this basis, the Lifshitz action is

$$S = \int_x \sum_{i=1}^{N} \left(\frac{1}{2} \left(\sum_{n_\perp=m+1}^{d} \frac{\partial \phi_i}{\partial x_{n_\perp}} \mathbf{e}_{\mathbf{n}_\perp} \right)^2 + \frac{\rho_0}{2} \left(\sum_{n_{/\!\!/}=1}^{m} \frac{\partial \phi_i}{\partial x_{n_{/\!\!/}}} \mathbf{e}_{\mathbf{n}_{/\!\!/}} \right)^2 + \frac{\sigma_0}{2} \left(\sum_{n_{/\!\!/}=1}^{m} \frac{\partial^2 \phi_i}{\partial x^2_{n_{/\!\!/}}} \mathbf{e}_{\mathbf{n}_{/\!\!/}} \right)^2 \right) + U(\phi) \tag{1.1}$$

where U is an almost completely arbitrary potential. We only ask for him to have the $O(N)$ symmetry, ie to be a function of

$$\rho \stackrel{\text{def}}{=} \frac{\phi_i \phi_i}{2} \tag{1.2}$$

For now on we will use the self-explanatory shorthand notation

$$S = \int_x \left(\frac{1}{2} (\partial_\perp \phi)^2 + \frac{\rho_0}{2} (\partial_{/\!\!/} \phi)^2 + \frac{\sigma_0}{2} (\partial^2_{/\!\!/} \phi)^2 + U(\rho) \right) \tag{1.3}$$

We see that this action closely ressemble the well known action of the $O(N)$ model

$$S_{O(N)} = \int_x \left(\frac{1}{2} (\partial \phi)^2 + U(\rho) \right) \tag{1.4}$$

Namely, we recover it if we set $\rho_0 = 1$ and $\sigma_0 = 0$. We see that what differentiate the Lifshitz and $O(N)$ action is on one hand the presence a non trivial (*ie* different from 1) ρ_0, breaking the $O(N)$ invariance, and on the other hand the presence of an extra term involving a laplacian squared. Clearly, these two modifications must be responsible for the appearance of spatially modulated structures, but why exactly? We can gain a useful intuition of why a spatially modulated structure is closely linked to the existence of a laplacian squared term in the action by looking at a lattice version of our model.

1.1.2 A discrete counterpart: the anisotropic Ising model

Strocto sensu the discrete counterpart of the Lifshitz model would be an anisotropic Heisenberg model, but to simplify things - without changing the essence of the argumentation - we consider an anisotropic Ising model instead.

First, let us consider a chain of Ising spins with the Hamiltonian

$$H_{\text{chain}} = -J \sum_i S_i S_{i+1} \tag{1.5}$$

We know that if J is positive, the interaction is ferromagnetic, whereas is J is negative, the interaction is antiferromegnetic. The antiferromagnetic order already shows some kind of spatial modulation, but it only exists at zero temperature! The idea to make a spatially modulated order survive at non zero temperatures is to consider a second neighbours *antiferromagnetic* interaction, together with a first neighbours *ferromagnetic* interaction:

$$H_{\text{chain}} = -J_1 \sum_i S_i S_{i+1} - J_2 \sum_i S_i S_{i+2} \tag{1.6}$$

The competition between ferromagnetic and antiferromagnetic interactions may well produce a spatial modulation of the spins at non zero temperatures, at least for some values of the interaction strenghts ratio J_2/J_1. However, for a long range order to exist at finite temperature, we need to work in two dimensions or more, *ie* to trade our spin chain for a spin lattice:

$$H_{\text{lattice}} = -\sum_i \left(J_0 \sum_{\delta_\perp} S_i S_{i+\delta_\perp} + J_1 \sum_{\delta_{/\!/}} S_i S_{i+\delta_{/\!/}} + J_2 \sum_{\delta_{/\!/}} S_i S_{i+2\delta_{/\!/}} \right) \tag{1.7}$$

The existence of a stripped phase is a well known feature of this model [?], called the ANNNI (axial next-nearest neighbour Ising) model.

Now, what is the link between this discrete spin lattice hamiltonian, and our continuous action? First, note that a sum on nearest neighbours can be rewriten in terms of a discrete laplacian on the lattice, while a sum on next-nearest neighbours involves a discrete laplacian squared:

$$H_{\text{lattice}} = -\sum_i \left(\kappa S_i^2 + J_0 S_i \Delta_\perp S_i + (J_1 + 4J_2) S_i \Delta_{/\!/} S_i - J_2 S_i \Delta_{/\!/}^2 S_i \right) \tag{1.8}$$

with

$$\Delta_{/\!/} S_i = \sum_{\delta_{/\!/}} S_{i-\delta_{/\!/}} - 2S_i + S_{i+\delta_{/\!/}} \tag{1.9}$$

$$\Delta_{/\!/}^2 S_i = \sum_{\delta_{/\!/}} -S_{i-2\delta_{/\!/}} + 4S_{i-\delta_{/\!/}} - 4S_i + 4S_{i+\delta_{/\!/}} - S_{i+2\delta_{/\!/}} \tag{1.10}$$

1.2 Physical applications of the model