Étude du point de Lifshitz par le groupe de renormalisation.

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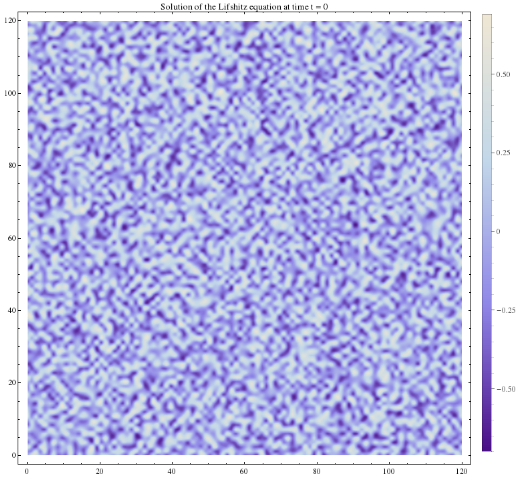
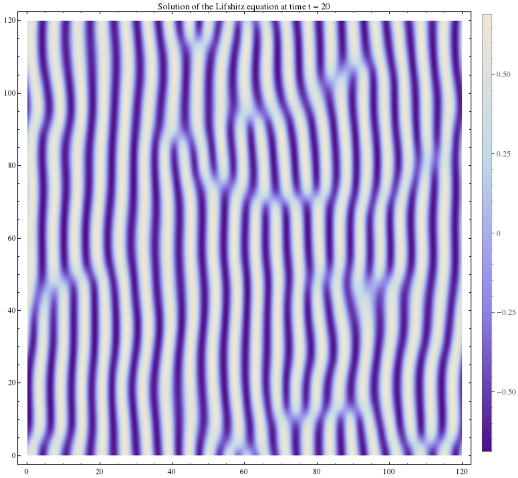
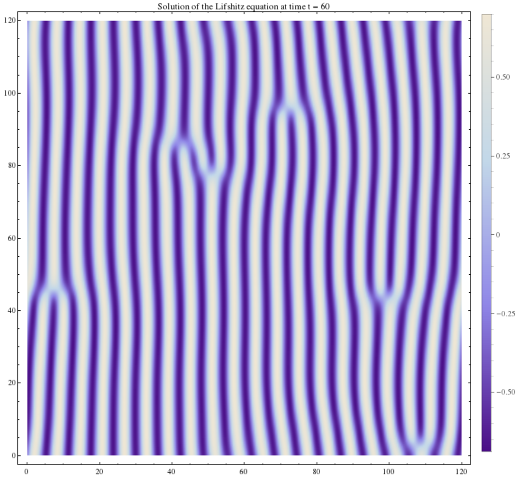
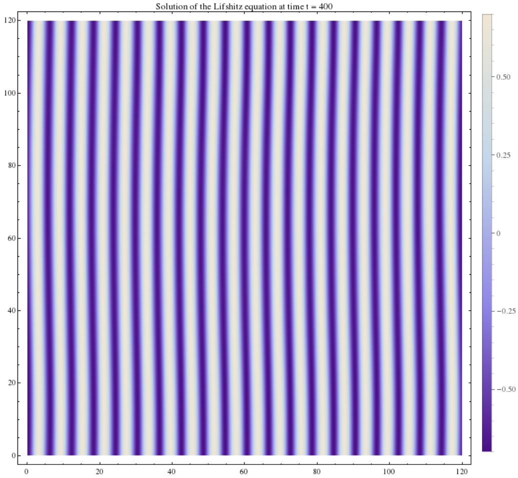
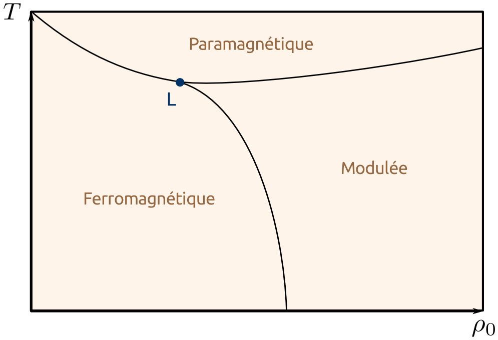
Laboratoire de physique théorique de la matière condensée (LPTMC)
UMPC


Solution of the Lifshitz equation at time t = 0
0
20
40
60
80
100
120
0.50
0.25
0
−0.25
−0.50


Solution of the Lifshitz equation at time t = 20
120
100
80
60
40
20
0
0
20
40
60
80
100
120
0.50
0.25
0
−0.25
−0.50


Solution of the Lifshitz equation at time t = 60
0
20
40
60
80
100
120
0.50
0.25
0
−0.25
−0.50

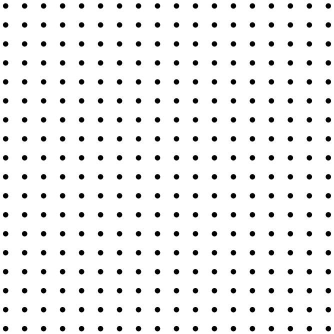

Solution of the Lifshitz equation at time t = 400
0
20
40
60
80
100
120
0
20
40
60
80
100
120
0.50
0.25
0
−0.25
−0.50



Hamiltonien de Lifshitz

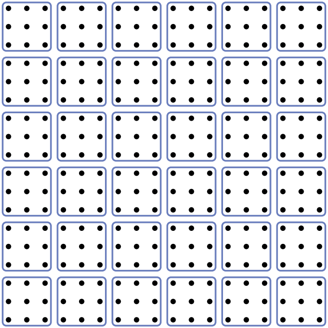
$$H[\phi] = \int_x \frac{1}{2}(\partial_\perp \phi)^2 + \frac{\rho_0}{2}(\partial_{/\!/}\phi)^2 + \frac{\sigma_0}{2}(\partial_{/\!/}^2\phi)^2 + U(\rho),\ \rho \overset{\text{def}}{=} \frac{\phi^2}{2}$$

L'idée de la renormalisation



$$H[\phi(x_i)]$$

L'idée de la renormalisation

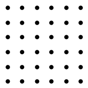


$$H[\phi(x_i)]$$

$$\tilde{\phi}(x_b) = \frac{1}{Sa}\sum_{i\in b}\phi(x_i)$$

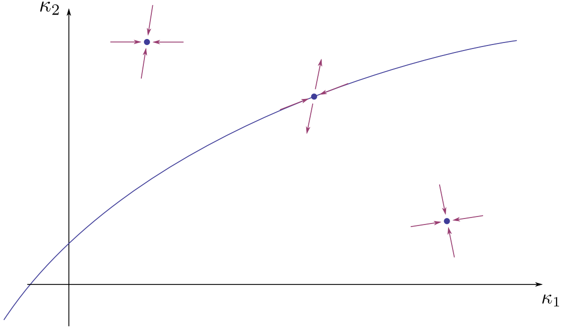
L'idée de la renormalisation

$$H[\phi(x_i)] \to \tilde{H}[\tilde{\phi}(x_b)]$$

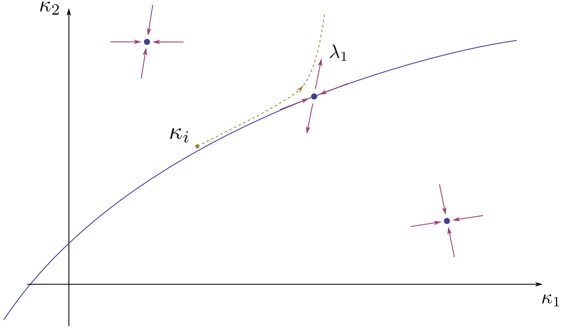


$$\left.\begin{array}{r} x' = x/S \\ \phi' = S^{\Delta}\phi \end{array}\right\} \rightarrow H_S[\phi'(x_i')]$$

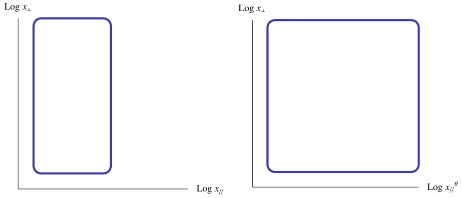
Comparer H et H_S $\rightarrow$ renormalisation des constantes de couplage.



Représentation schématique des flots dans l'espace des couplages.



$$\xi \propto |T - T_c|^{-\nu},\ \nu = \frac{1}{\lambda_1}$$

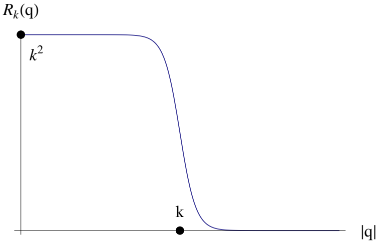


Blocs de spin avant et après introduction de θ.

Directions inéquivalentes $\Longrightarrow$ échelles inéquivalentes $S_{/\!/}$, $S_\perp$.
On introduit θ tel que $S_{/\!/} = S_\perp^\theta$.

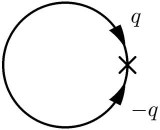
Résultats du champ moyen

$$\theta_{\text{cm}} = \frac{1}{2},\ d_c^> = 4 + \frac{m}{2}$$

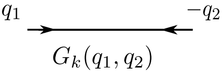


Dépendance en impulsion typique du régulateur $R_k(q)$.

$$\partial_t \bar{\Gamma}_k = \frac{1}{2}$$



$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \int_q \partial_t R_k(q) G_k(q,-q)$$



$$G_k(q_1,q_2) = \left(\Gamma_k^{(2)} + R_k\right)^{-1}_{q_1,q_2}$$

Symétries $\Longrightarrow$ forme générale de l'action effective de Lifshitz :

$$\Gamma_k[\phi] = \int_x U(\rho) + \left(\tfrac{1}{2} Z_\perp(\rho)(\partial_\perp \phi)^2 + \tfrac{1}{4} Y_\perp(\rho)(\partial_\perp \rho)^2 + \ldots\right) + \left(\tfrac{1}{2}\rho_0(\rho)(\partial_{/\!/} \phi)^2 + \ldots\right) + \left(\tfrac{1}{2} Z_{/\!/}(\rho)(\partial^2_{/\!/} \phi)^2 + \ldots\right)$$

Pour calculer les exposants critiques la structure impulsionnelle importe peu $\rightarrow$ simplifications.

Forme définitive de l'Ansatz :

$$\Gamma_k[\phi_i] = \int_x \frac{Z_\perp}{2}(\partial_\perp \phi)^2 + \frac{\rho_0}{2}(\partial_{/\!/} \phi)^2 + \frac{Z_{/\!/}}{2}(\partial^2_{/\!/} \phi)^2 + U(\rho).$$

Équations de flot du modèle

- $\partial_t u_t(\rho) = -d_m u_t(\rho) + (\theta\eta_{/\!/} + d_m - 4\theta)\rho u_t'(\rho) + 8 v_m v_{d-m} \left(l_0^{dm}\left(u_t'(\rho) + 2\rho u_t''(\rho)\right) + (n-1) l_0^{dm}\left(u_t'(\rho)\right)\right)$
- $\partial_t \rho_0 = -\theta\left(2 - \eta_{/\!/}\right)\rho_0 - 64\frac{v_{d-m}v_m}{m}\rho u^{(2)}(\rho)^2 m_{1,2202}$
- $\eta_\perp = 32 v_{d-m} v_m \rho u^{(2)}(\rho)^2$
- $\eta_{/\!/} = \eta_\perp\Big[m_{1,3100} - \frac{1}{2}k_{1,2100} - \frac{2}{m}\left(6s_{1,4102} - 6v_{1,31002} + w_{1,2102}\right) + \frac{2}{m(m+2)}\left(24t_{1,5104} - 36z_{1,4104} + 6u_{1,3104} + 8y_{1,3104} - x_{1,2104}\right)\Big]$

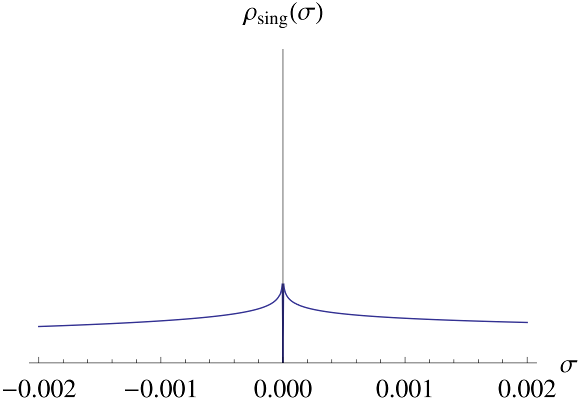
Approximation du potentiel local : $\theta \simeq \theta_{cm} = \frac{1}{2}$, $\eta_{/\!/,\perp} \simeq \eta_{cm} = 0$.

On a choisi de résoudre directement les équations de point fixe.

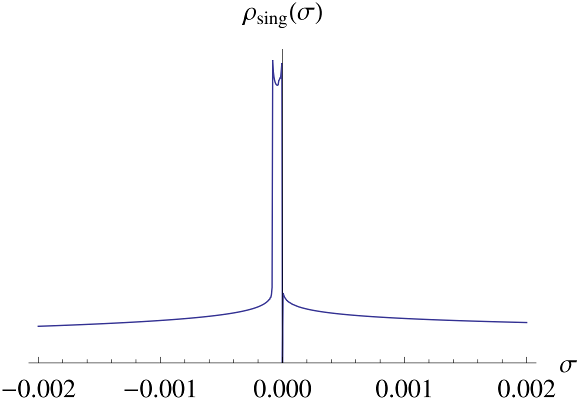
$$0 = u(\rho) - a\rho u'(\rho) + b(\rho_0)\left(\frac{1}{1+\rho_0+u'(\rho)+2\rho u''(\rho)} + \frac{n-1}{1+\rho_0+u'(\rho)}\right)$$
$$u'(0) = \sigma$$
$$u\,(0) = nb(\rho_0)/(1+\sigma)$$

Stratégie de résolution

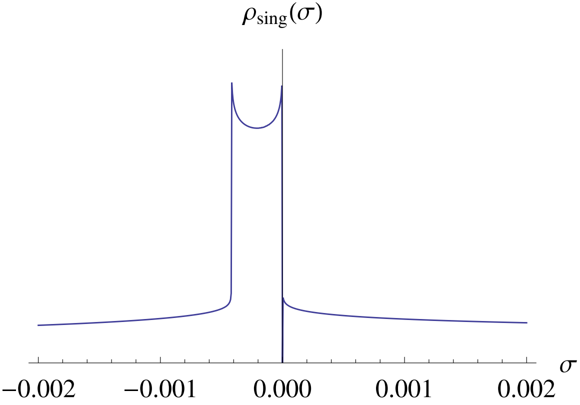
- Commencer à $d = d_c^{>}$ ($\sigma^{\mathrm{Lif}} = 0$, $\rho_0^{\mathrm{Lif}} = 0$).
- $d \to d - \delta d$.
- Trouver σ^{Lif}.
- Trouver ρ_0^{Lif}.
- Revenir à l'étape (2).



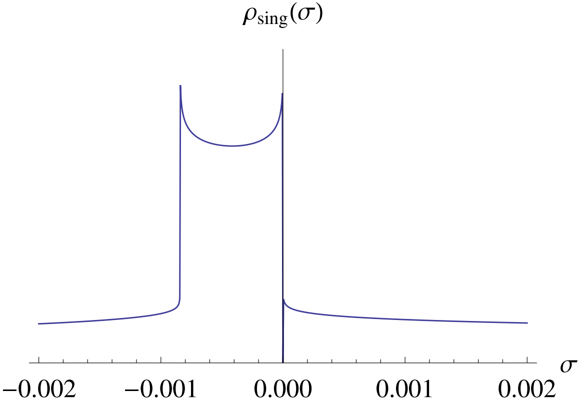
- d=4.5



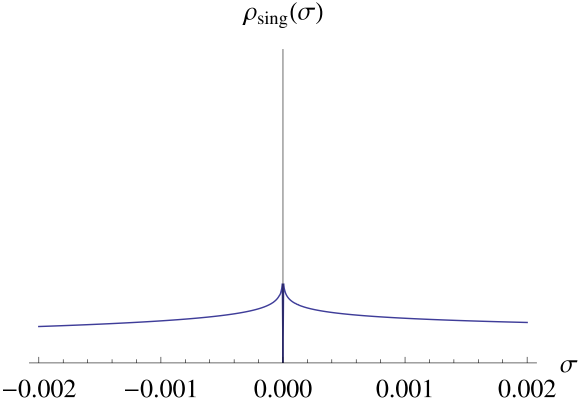
- d=4.499



- d=4.495



- d=4.49



- d=4.5

- modélisation d'un phénomène mal compris : la ségrégation
- relations entre physique, mathématiques et analyse numérique
- un domaine de la physique nouveau et en plein essor