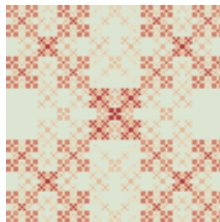


Multifractality of the tight-binding eigenstates on the Fibonacci chain

Nicolas Macé, Anuradha Jagannathan, Frédéric Piéchon

Laboratoire de Physique des Solides
Université Paris-Sud

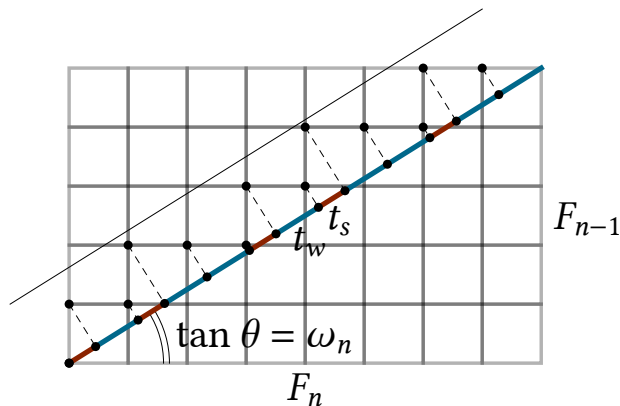
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OUTLINE

- 1 The pure hopping Fibonacci Hamiltonian.
- 2 The energy spectrum and its multifractal properties.
- 3 The wavefunctions and their multifractal properties.
- 4 Conclusion

THE PURE HOPPING FIBONACCI HAMILTONIAN



$$\omega_n = \frac{F_{n-1}}{F_n}$$

$$\omega_n \rightarrow \omega = \frac{\sqrt{5} - 1}{2}$$

$$\rho = \frac{t_w}{t_s} < 1$$

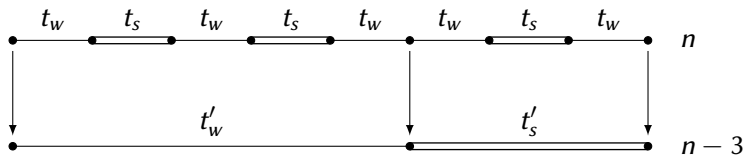
$$\rho \rightarrow 1 \text{ (weak modulation)}$$

$$\rho \ll 1 \text{ (strong modulation)}$$

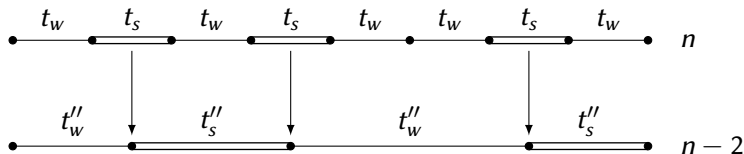
$$H_n = - \sum_i t_i^{(n)} |i\rangle \langle i+1| + \text{h.c.}$$

PERTURBATIVE RENORMALIZATION GROUP ON THE FIBONACCI CHAIN

■ Atomic RG step (decimation of molecules)



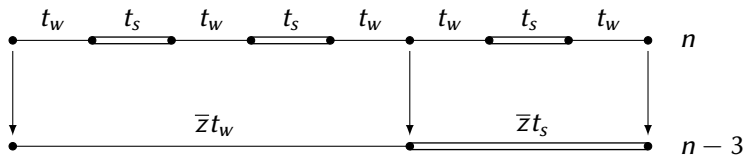
■ Molecular RG step (decimation of atoms)



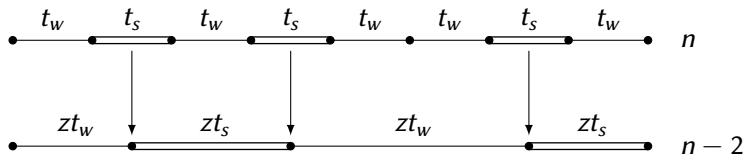
(Niu & Nori 1986, Kalugin, Kitaev & Levitov 1986)

PERTURBATIVE RENORMALIZATION GROUP ON THE FIBONACCI CHAIN

■ Atomic RG step (decimation of molecules)



■ Molecular RG step (decimation of atoms)

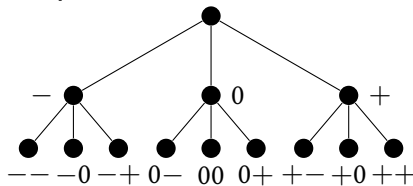
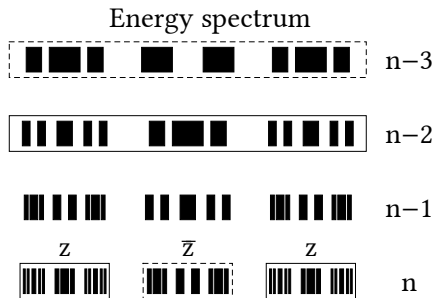


$$z = \rho/2, \bar{z} = \rho^2 \text{ (Niu \& Nori 1986, Kalugin, Kitaev \& Levitov 1986)}$$

RG RECONSTRUCTION OF THE ENERGY SPECTRUM

$$H_n = \underbrace{(zH_{n-2} - t_s)}_{\text{bonding levels}} \oplus \underbrace{(\bar{z}H_{n-3})}_{\text{atomic levels}} \oplus \underbrace{(zH_{n-2} + t_s)}_{\text{antibonding levels}} + \mathcal{O}(\rho^4)$$

→ simple recursive construction of the spectrum (Piéchon *et al* 1995)



Renormalization paths characterized by

$$x(E) = \frac{n_+ + n_-}{n}$$

$$x(\text{central band}) = 0, x(\text{edge bands}) = 1/2$$

MULTIFRACTAL ANALYSIS OF THE SPECTRUM

■ Statistical properties of the bands

$$\begin{cases} \Delta_n^a \sim (1/F_n)^{1/\alpha_a} \\ \#\{\text{bands of scaling } \alpha\} \sim F_n^{f(\alpha)} \end{cases}$$

$$\alpha(x) = \log \omega / \left(x \log z / \bar{z}^{2/3} + \log \bar{z}^{1/3} \right)$$

$$f(x) = \frac{x \log \left(\frac{3x}{2} \right) - (x+1) \log(x+1)^{1/3} + (1-2x) \log(1-2x)^{1/3}}{\log \omega}$$

(Piéchon *et al* 1995, Rüdinger & Piéchon 1998)

■ Fractal dimensions D_q

$$(q-1)D_q = \min_{\alpha} (\alpha q - f(\alpha))$$

$$D_0 = \log(\sqrt{2} - 1) / \log \omega$$

(Damanik 2008)

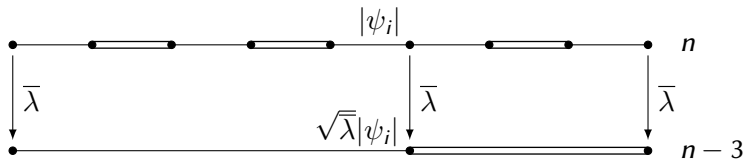
FRACTAL DIMENSIONS OF THE WAVEFUNCTIONS

Stat. properties of ψ :
$$\sum_i |\psi_i^{(n)}(E)|^{2q} \sim (1/F_n)^{(q-1)D_q^\psi(E)}$$

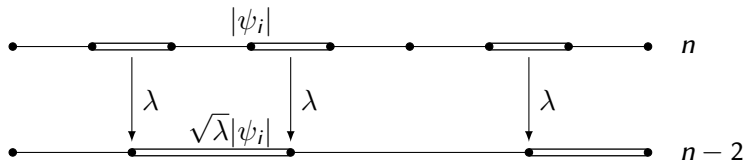
- Wavefunctions at the center and the edges of the spectrum are multifractal (Kohmoto)
- Averaged fractal dimensions of the wavefunction known to lowest order (Thiem & Schreiber 2013)
- Our work: use the RG approach to:
 - determine individual wavefunction properties,
 - compute their fractal dimensions,
 - compute the averaged fractal dimensions at higher order.

PERTURBATIVE RG FOR THE WAVEFUNCTIONS

■ Atomic RG



■ Molecular RG



$$\begin{cases} |\psi_i^{(n)}(E)|^2 = \bar{\lambda} |\psi_{i'}^{(n-3)}(E')|^2 & \text{if } E \text{ is in the central cluster} \\ |\psi_i^{(n)}(E)|^2 = \lambda |\psi_{i'}^{(n-2)}(E')|^2 & \text{if } E \text{ is in the edge clusters} \end{cases} \quad \begin{cases} \bar{\lambda} \sim 1/(1 + \rho^2/2) \\ \lambda \sim 1/(2 + \rho^2) \end{cases}$$

RENORMALIZATION PATHS AND FRACTAL DIMENSIONS OF THE WAVEFUNCTIONS

- Fractal dimensions of the wavefunction of energy E :

$$\sum_i |\psi_i^{(n)}(E)|^{2q} \sim (1/F_n)^{(q-1)D_q^\psi(E)}$$

- RG step neglecting sites with small amplitudes:

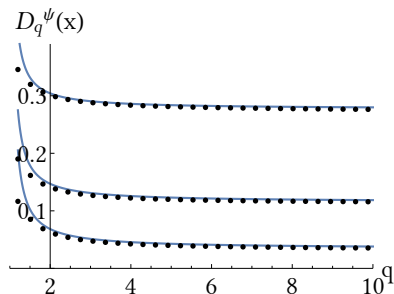
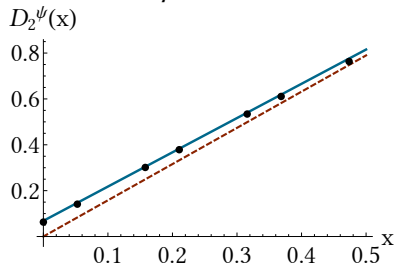
$$|\psi_i^{(n)}(E)|^{2q} = \lambda^q |\psi_{i'}^{(n-2)}(E')|^{2q}$$

- $D_q^\psi(E)$ depends on the renormalization path $x(E)$:

$$D_q^\psi(x) = x \frac{\log 1/2}{\log \omega} + \frac{q}{q-1} \left(x \frac{\log \lambda}{\log \omega} + \frac{1-2x}{3} \frac{\log \bar{\lambda}}{\log \omega} \right)$$

COMPARISON WITH NUMERICAL DATA

$$\rho = 0.15$$

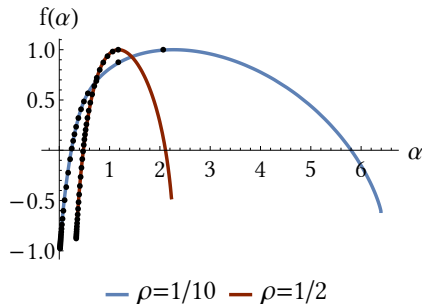
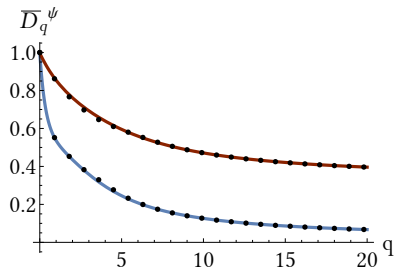


- Multifractal behaviour well described
- All states are critical in the strong modulation limit
- Their multifractal character is captured by our description
- x is the relevant parameter to describe the properties of the states

(Macé, Jagannathan, Piéchon,
to be submitted)

ENERGY AVERAGED MULTIFRACTALITY OF THE WAVEFUNCTIONS

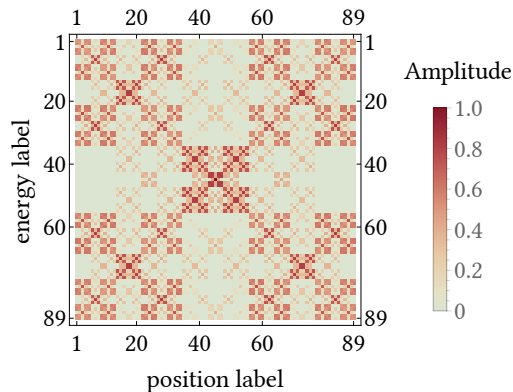
$$\frac{1}{F_n} \sum_E \sum_i |\psi_i^{(n)}(E)|^{2q} \sim (1/F_n)^{(q-1)\bar{D}_q^\psi}$$



■ Multifractality

■ Quantitative agreement with numerical data even for large ρ .

CONUMBERING



Amplitude of the wavefunctions as a function of the position and the energy, computed numerically.

- Conumbering (a relabelling of the positions along the chain) is used (Mosseri 1988).
- The fractal structure stemming from the deflation symmetry is clear.
- Energy/position symmetry is made evident by conumbering.

CONCLUSION AND PERSPECTIVES

- Fractal dimensions are important for physical properties such as transport and susceptibility.
- We have characterized the wavefunctions of the Fibonacci tight-binding chain, in the strong modulation limit, using a perturbative RG.
- We have presented for the first time analytical expressions for the fractal exponents of the full set of wavefunctions. These compare well with numerical data.
- Work in progress: consequences for the diffusion and transport properties.

