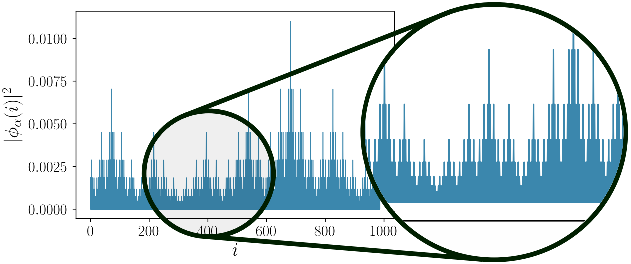
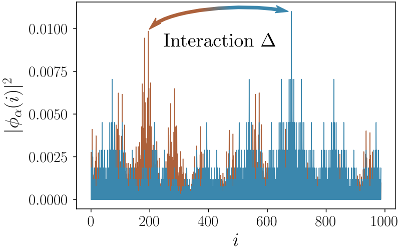
# Interacting electrons on a Fibonacci chain at high temperature

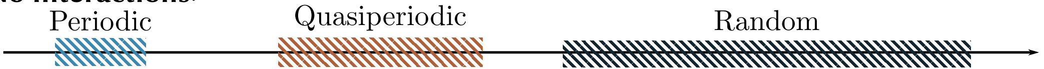
Nicolas Macé



# Motivation: quasiperiodicity + interacting electrons

**No interactions**:

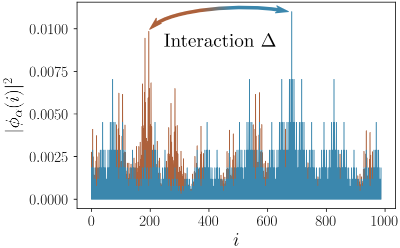



- Extended states
- Fast transport

- Multifractal states
- Anomalous transport

- Localized states
- No transport

Cold atoms: **strong interactions**




**Quasiperiodicity (QP) + strong interactions?**

Naively: delocalisation, fast transport

**Results**:

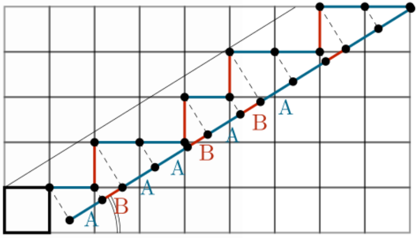
- weak QP: delocalisation, fast transport
- strong QP: **many-body localisation**, no transport

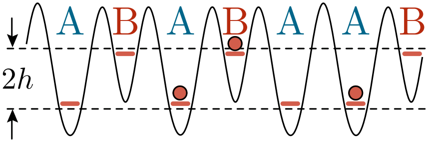
# Outline

1 Free Fibonacci chain at high energy

2 Interacting Fibonacci chain

# Interacting fermions on the Fibonacci chain



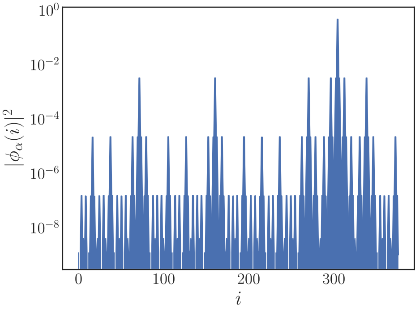

$$H = \sum_{i=1}^{L} \left[ J(c_i^\dagger c_{i+1} + \mathrm{h.c}) + \Delta n_i n_{i+1} - h_i n_i \right]$$

Method: numerical **exact diagonalization**

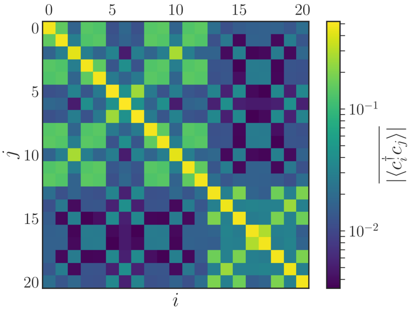
- High energy + non-integrable: **no analytical methods**
- $L/2$ fermions on $L$ sites: #states $\sim 2^L/\sqrt{L} \to$ **memory is limiting**
  State-of-the art: $L = 24$ [Pietracaprina *et al* 18]
- Fibonacci: **few samples**: $L/2$ non-equivalent systems of size $L$.

# Free fermions properties

- Mutlifractal single particle wavefunctions [Ostlund; Kalugin; Khomoto; ...]
- Anomalous transport [Mayou; Schreiber; Varma & Žnidarič; ...]




Single particle wavefunction at the Fermi level

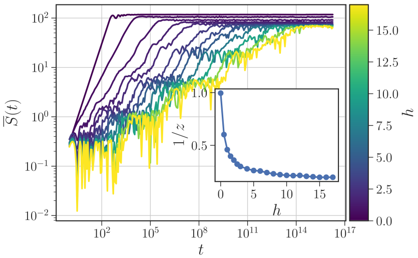



Correlations of highly excited states [Macé *et al* 19]

# Free fermions entanglement

Entanglement entropy $S(\psi)$: a many-body **locality** probe

- $S(\psi) = \#\{$bits of information recoverable by local measurements$\}$
- $S(\psi)$ large: extended (entangled) state, $S(\psi)$ small: localized state.




Entanglement growth starting from localized fermions [Macé *et al* 19]

Fibonacci fermions: **anomalous** growth

$S(t) \sim t^{\frac{1}{z}}, \;\; z > 1$

Compare with:

- Periodic system: $z = 1$ (ballistic growth),
- Disordered system: $z \to \infty$ (no growth).

## Conclusion

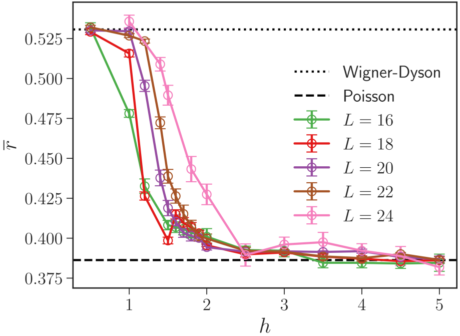
Anomalous, intermediate prop. even at high energy.

# The ETH/MBL transition: 1) spectral properties

Gap ratios [Oganesian, Huse]

$$r_n = \min\left(\frac{g_{n+1}}{g_n}, \frac{g_n}{g_{n+1}}\right)$$

- ETH: random matrix-like spectrum
  $\overline{r}_{\mathrm{ETH}} \simeq 0.53$
- MBL: independant levels
  $\overline{r}_{\mathrm{MBL}} \simeq 0.39$

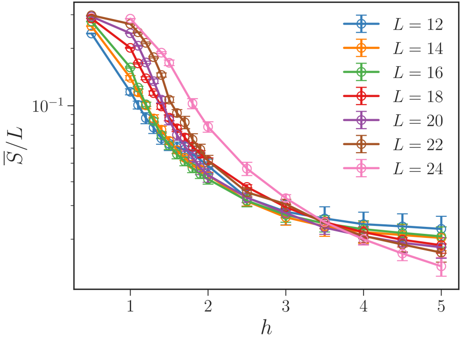



Compatible with **ETH/MBL transition**, $h^* \simeq 2.5$.

# The ETH/MBL transition: 2) Entanglement

Entanglement entropy:

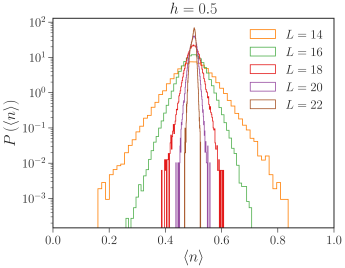
- ETH: coincides with thermodynamic entropy: **extensive**: $\overline{S}_{\mathrm{ETH}} \simeq L$
- MBL: **sub-extensive** $\overline{S}_{\mathrm{MBL}}/L \to 0$



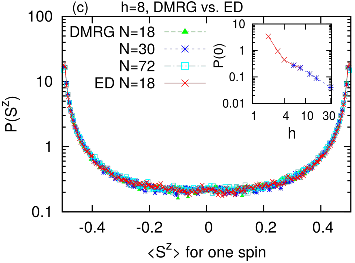

Compatible with **ETH/MBL transition**, $h^* \simeq 3.5$.

# The ETH/MBL transition: 3) Local observables

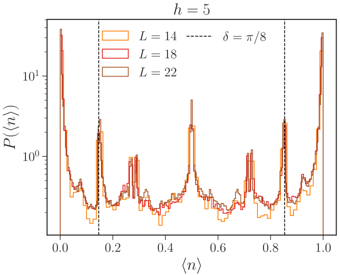
Expect: $\langle n_i \rangle_{\mathrm{ETH}} = \frac{1}{2}$, $\langle n_i \rangle_{\mathrm{MBL}} \simeq 0$ or 1.




ETH Fibonacci



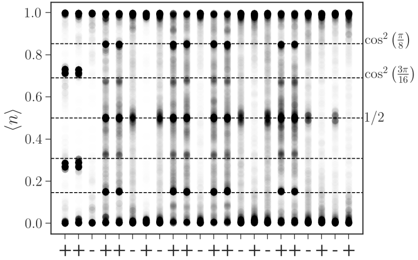

MBL random [Lim, Sheng 15]



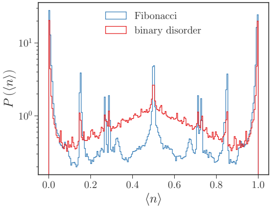

MBL Fibonacci

Fibonacci MBL: **extra structure** $\rightarrow$ link with QP geometry?

# Fibonacci MBL: local entanglement




Density peaks on **AA pairs**
$\rightarrow$ 4 sites toy model BAAB
$\rightarrow$ locally entangled states
$|\psi\rangle = \cos\delta\,|01\rangle \pm \sin\delta\,|10\rangle$
with $\delta = 0,\ \frac{\pi}{4},\ \frac{\pi}{8}$.




Peak ingredients:

- **Binary** modulation A/B
- **Correlated** modulation (Fibonacci)