Exact results on electronic wavefunctions of 2D quasicrystals

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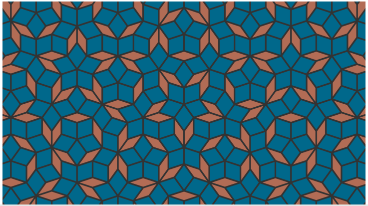
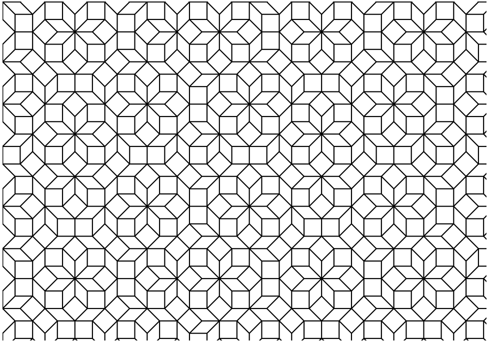


Groundstate of the Ammann-Beenker tiling



A patch of the Ammann-Beenker tiling

Hamiltonian :

$$\hat{H} = -t \sum_{\langle m,n \rangle} |m\rangle \langle n|$$

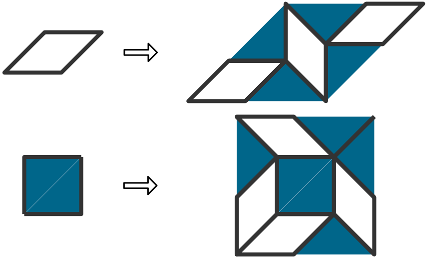
Quasiperiodicity encoded in adjacency and on-site potentials

Fractal states described by **height** functions ?

$$\psi(m) = C(m) e^{\kappa h(m)}$$

Looking for arrows

Like Fibonacci, Ammann-Beenker has a substitution rule :

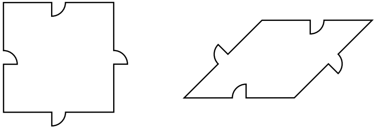


→ scale invariance.

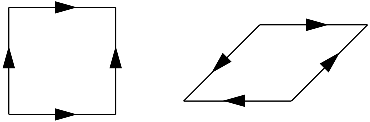
Height requires a field of arrows :

- irrotational
- invariant under substitution

Ammann-Beenker → notched tiles…



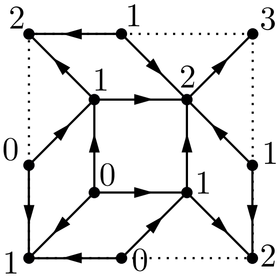
…exactly what we need !



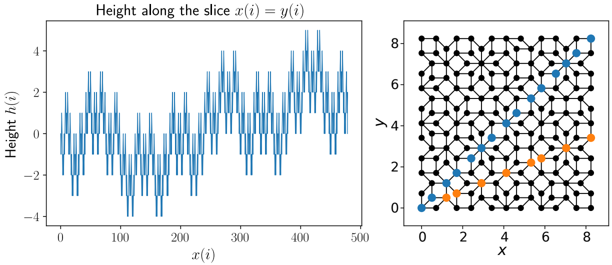
Height field :

$$h(m) = \sum_{0 \to m} \text{arrows}$$

Properties of the height field



The height field on a small patch of the tiling.



Height along a line, shown in blue on the tiling.

Partition function : $Z_L(\beta) \underset{L\to\infty}{\sim} L^{\omega(\beta)}$

$\to$ slow growth : $h_{\mathrm{typ}}(L) \underset{L\to\infty}{\sim} \sqrt{\log L}$

$\to$ states $\psi(m) = e^{\kappa h(m)}$ fractal

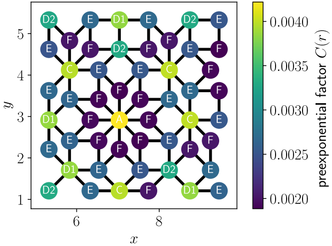
Groundstate

$$\hat{H} = -t \sum_{\langle m,n \rangle} |m\rangle \langle n|$$

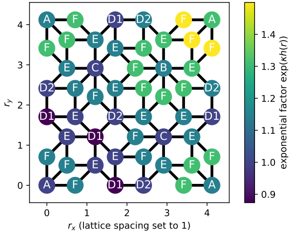
Conjecture [Kalugin, Katz 14] :

$$\psi_{\text{groundstate}}(m) = C(m) e^{\kappa h(m)}$$

$C(m)$: *local* function :

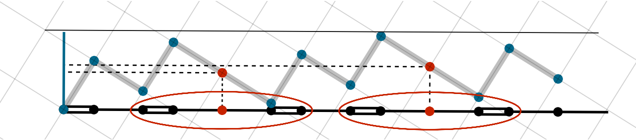


$e^{\kappa h(m)}$: non-local function :



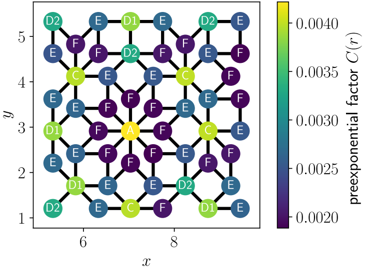
Testing the conjecture

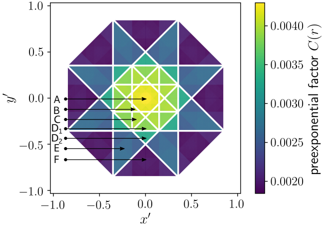
Cut-and-project :

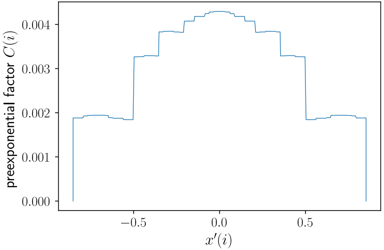


Sites close in internal space have a very similar local environment.

→ internal space useful to test the local nature of C







Perturbing the Hamiltonian

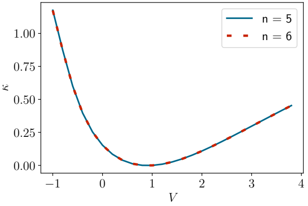
Adding an on-site potential :

$$\hat{H} = -t \sum_{\langle m,n \rangle} |m\rangle \langle n| + \sum_m V_m |m\rangle \langle m|$$

Laplacian-like [Sire, Bellissard 90] :

$V_m = Vz_m$

$\rightarrow \psi(m) = C(m)e^{\kappa h(m)}$

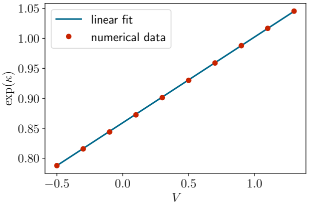


Prefactor κ as a function of V.

An "arbitrary" potential :

$V_m = V$ if m has 3 neighbors, else $V_m = 0$.

$\rightarrow \psi(m) = C(m)e^{\kappa h(m)}$



e^{κ} as a function of V.

Conclusions

Tight-binding models on a 2D quasiperiodic tiling

- Height field $h(m) \to \psi_{\text{groundstate}}(m) = C(m)e^{\kappa h(m)}$
- Slow height growth ($h(L) \sim \sqrt{\log L}$) $\to$ critical, **fractal** state
- Robust under symmetry-preserving on-site perturbations
- Same conclusions for the 10-fold symmetric Penrose tiling.

General conclusion

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