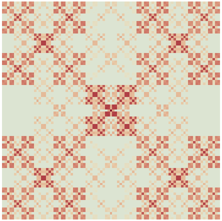
Multifractality of the tight-binding eigenstates on the Fibonacci chain

Nicolas Macé, Anuradha Jagannathan, Frédéric Piéchon

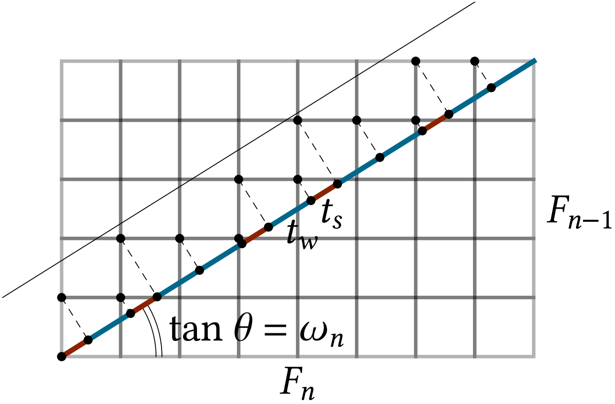
Laboratoire de Physique des Solides
Université Paris-Sud

September 4, 2015



Outline

The pure hopping Fibonacci Hamiltonian



$$\omega_n = \frac{F_{n-1}}{F_n}$$

$$\omega_n \to \omega = \frac{\sqrt{5}-1}{2}$$

$$\rho = \frac{t_w}{t_s} < 1$$

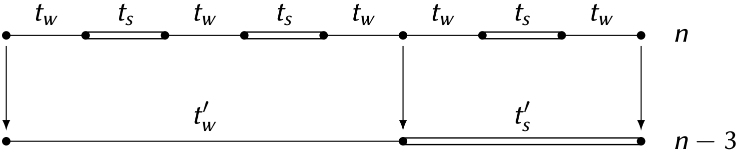
$\rho \to 1$ (weak modulation)

$\rho \ll 1$ (strong modulation)

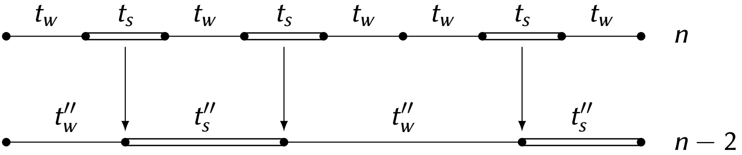
$$H_n = -\sum_i t_i^{(n)} |i\rangle \langle i+1| + \text{h.c.}$$

Perturbative renormalization group on the Fibonacci chain

- Atomic RG step (decimation of molecules)



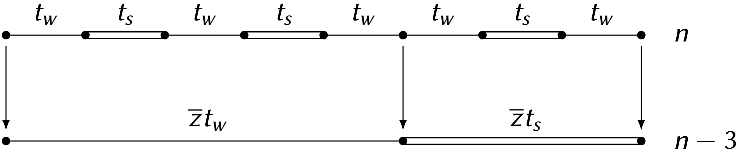
- Molecular RG step (decimation of atoms)



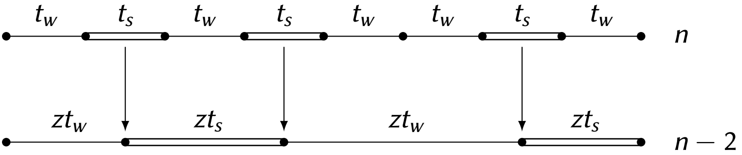
(Niu & Nori 1986, Kalugin, Kitaev & Levitov 1986)

Perturbative renormalization group on the Fibonacci chain

- Atomic RG step (decimation of molecules)



- Molecular RG step (decimation of atoms)

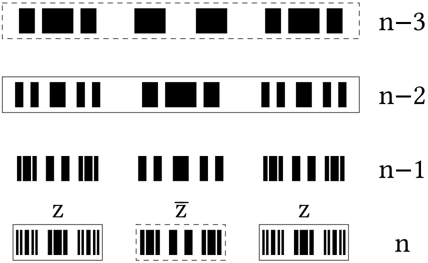


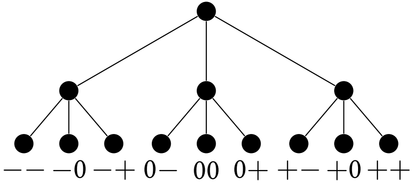
$z = \rho/2$, $\bar{z} = \rho^2$ (Niu & Nori 1986, Kalugin, Kitaev & Levitov 1986)

Renormalization group & construction of the spectrum

$$H_n = \underbrace{(zH_{n-2} - t_s)}_{\text{bonding levels}} \oplus \underbrace{(\bar{z}H_{n-3})}_{\text{atomic levels}} \oplus \underbrace{(zH_{n-2} + t_s)}_{\text{antibonding levels}} + \mathcal{O}(\rho^4)$$

$\rightarrow$ simple recursive construction of the spectrum (Niu & Nori 1986, Piéchon *et al* 1995)





Renormalization paths characterized by

$$x(E) = \frac{n_+ + n_-}{n}$$

Fractal dimensions

Characterize the spectrum: multifractal analysis (Halsey *et al* 1986)

Stat. properties of the bands: $\begin{cases} \Delta_n^a \sim (1/F_n)^{1/\alpha_a} \\ \#\{\text{bands of scaling } \alpha\} \sim F_n^{f(\alpha)} \end{cases}$

Fractal dimensions of the spectrum: $(q-1)D_q = \min_\alpha(\alpha q - f(\alpha))$

$$\alpha(x) = \log\omega / \left(x \log z/\bar{z}^{2/3} + \log \bar{z}^{1/3} \right)$$

$$f(\alpha(x)) = \frac{x \log\left(\frac{3x}{2}\right) - (x+1)\log(x+1)^{1/3} + (1-2x)\log(1-2x)^{1/3}}{\log\omega}$$

(Piéchon *et al* 1995, Rüdinger & Piéchon 1998)

$$D_0 = \log(\sqrt{2}-1)/\log\omega$$

(Piéchon *et al* 1995, Damanik 2008)

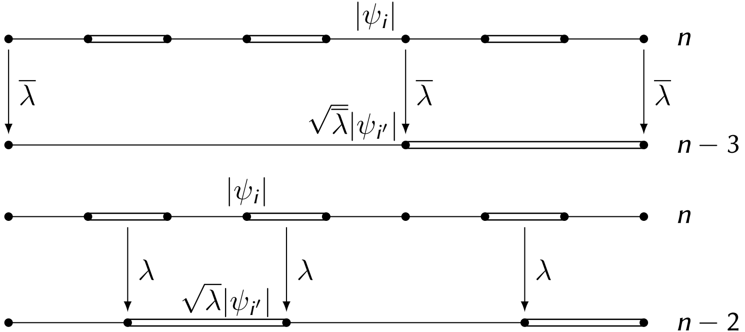
Fractal dimensions of the wavefunctions

Stat. properties of ψ: $\sum_i |\psi_i^{(n)}(E)|^{2q} \sim (1/F_n)^{(q-1)D_q^{\psi}(E)}$

- Wavefunctions at the center and the edges of the spectrum are multifractal (Kohmoto)
- Averaged fractal dimensions of the wavefunction known to lowest order (Thiem & Schreiber 2013)
- Our work: use the RG approach to:
  - determine individual wavefunction properties,
  - compute their fractal dimensions,
  - compute the averaged fractal dimensions at higher order.

Perturbative RG for the wavefunctions

We can relate the wavefunctions of H_n to the wavefunctions of H_{n-2}, H_{n-3}:



$$\begin{cases} |\psi_i^{(n)}(E)|^2 = \overline{\lambda}|\psi_{i'}^{(n-3)}(E')|^2 \text{ if } E \text{ is in the central cluster} \\ |\psi_i^{(n)}(E)|^2 = \lambda|\psi_{i'}^{(n-2)}(E')|^2 \text{ if } E \text{ is in the edge clusters} \end{cases} \quad \begin{cases} \overline{\lambda} \sim 1/(1+\rho^2/2) \\ \lambda \sim 1/(2+\rho^2) \end{cases}$$

Renormalization paths and fractal dimensions of the wavefunctions

- Fractal dimensions of the wavefunction of energy E:

$$\sum_i |\psi_i^{(n)}(E)|^{2q} \sim (1/F_n)^{(q-1)D_q^{\psi}(E)}$$

- Using the RG:

$$|\psi_i^{(n)}(E)|^{2q} = \lambda^q |\psi_{i'}^{(n-2)}(E')|^{2q}$$

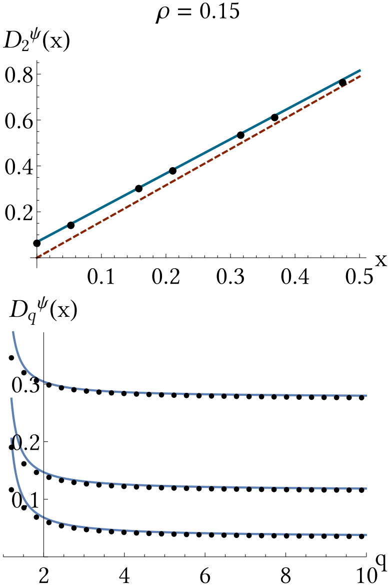
  and neglecting sites with small amplitudes,
- we express $D_q^{\psi}(E)$ as a function of the renormalization path

$$E \to E' \to E'' \to \ldots$$

- and actually only of x(E):

$$D_q^{\psi}(x) = x\frac{\log 1/2}{\log \omega} + \frac{q}{q-1}\left(x\frac{\log \lambda}{\log \omega} + \frac{1-2x}{3}\frac{\log \overline{\lambda}}{\log \omega}\right)$$

Comparison with numerical data

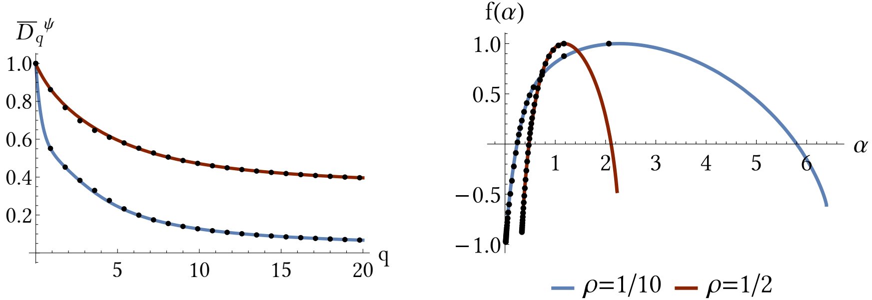


- Multifractal behaviour well described
- All states are critical in the strong modulation limit
- Their multifractal character is captured by our description
- *x* is the relevant parameter to describe the properties of the states

(Macé, Jagannathan, Piéchon, to be submitted)

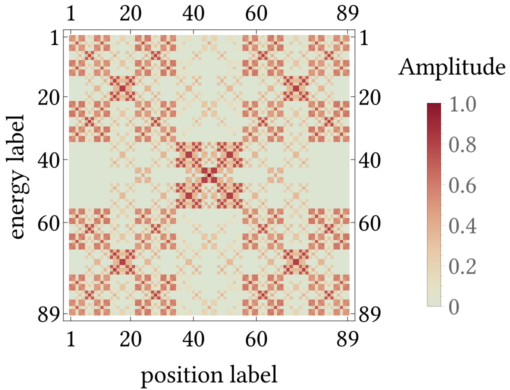
Energy averaged multifractality of the wavefunctions

$$\frac{1}{F_n}\sum_E \sum_i |\psi_i^{(n)}(E)|^{2q} \sim (1/F_n)^{(q-1)\bar{D}_q^{\psi}}$$



- Multifractality
- Quantitative agreement with numerical data even for large ρ.

Conumbering



Amplitude of the wavefunctions as a function of the position and the energy, computed numerically.

- Conumbering (a relabelling of the positions along the chain) is used (Mosseri 1988).
- The fractal structure stemming from the deflation symmetry is clear.
- Energy/position symmetry is made evident by conumbering.

Conclusion and perspectives

- Fractal dimensions are important for physical properties such as transport and susceptibility.
- We have characterized the wavefunctions of the Fibonacci tight-binding chain, in the strong modulation limit, using a perturbative RG.
- We have presented for the first time analytical expressions for the fractal exponents of the full set of wavefunctions. These compare well with numerical data.
- Work in progress: consequences for the diffusion and transport properties.