Interacting electrons on a Fibonacci chain at high temperature

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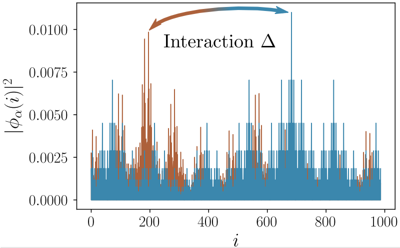
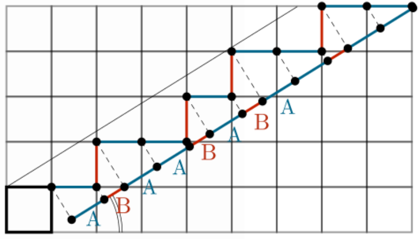
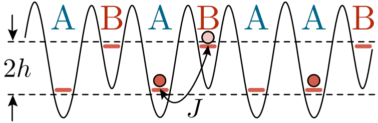


Quantum particles in a quasiperiodic (QP) environment

QP environment: Fibonacci chain

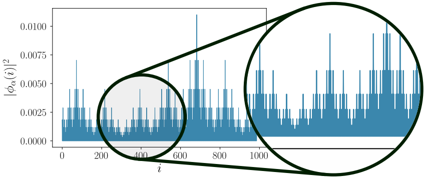


Quantum particles: fermions



$$H = \sum_{i=1}^{L} \left[J(c_i^\dagger c_{i+1} + \text{h.c}) - h_i n_i \right]$$

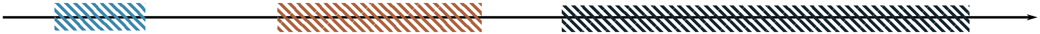
Multifractal (scale invariant) states



Motivation: quasiperiodicity + interacting electrons

No interactions:

Periodic Quasiperiodic Random



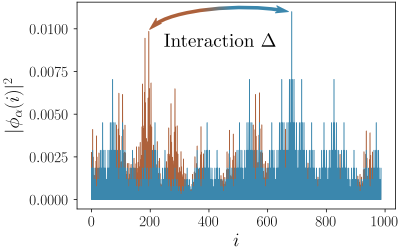
- Extended states
- Fast transport

- Multifractal states
- Anomalous transport

- Localized states
- No transport

Complexity

Quasiperiodicity (QP) + **interactions** between particles?



Naively: delocalisation, fast transport

Results:

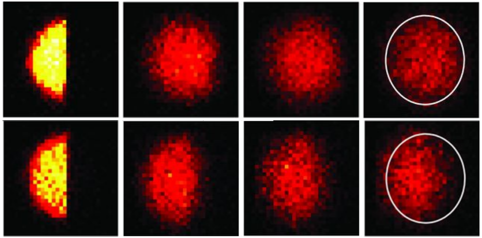
- weak QP: delocalisation, fast transport
- strong QP: **many-body localisation**, no transport

Outline

Many-body localisation

Isolated quantum system, **strong interactions**, disorder or quasiperiodicity

1. Usual: ergodic dynamics, transport, eigenstate thermalisation hypothesis (ETH),
2. Unusual: non-ergodicity, no transport, many-body localisation (MBL).



[Choi *et al* 16]

Experiments: cold ions/atoms [Schreiber *et al* 15; Smith *et al* 15; Bordia *et al* 17].

Motivations:

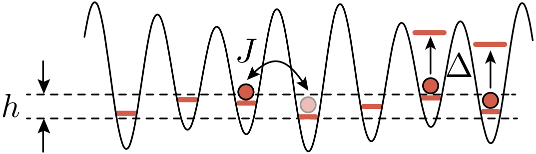
- ETH/MBL phase transition,
- MBL in more than 1D,
- Ingredients for MBL (this talk).

A model for MBL

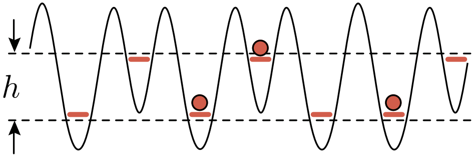
Chain of interacting spinless fermions (nb: no phonons):

$$H = \sum_{i=1}^{L} \left[J(c_i^\dagger c_{i+1} + \text{h.c}) + \Delta n_i n_{i+1} - h_i n_i \right]$$

Disordered:



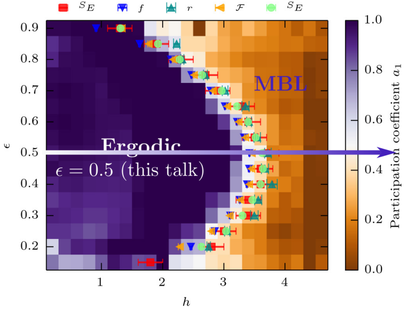
Quasiperiodic (this talk):



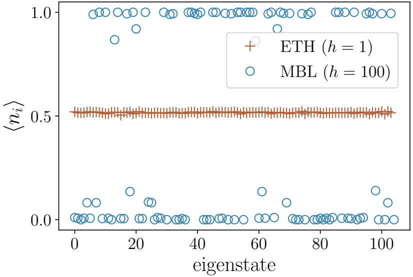
Generic model: fermions, $\frac{1}{2}$ spins, hardcore bosons.

MBL phenomenology

Phase diagram at $\Delta = 1$ [Luitz *et al* 15]



Fermion density at $\epsilon = 0.5$



ETH:

- Transport, thermal observables
- High entanglement
- Non-integrability

MBL:

- No transport, non-thermal observables
- Low entanglement
- Emergent integrability

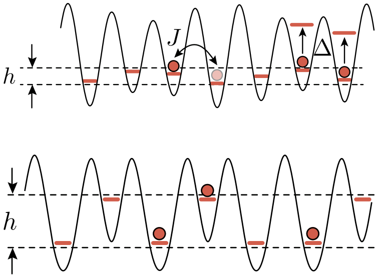
Ingredients for MBL

Usually:

1. $\Delta = 0$: **localized** (random, Aubry-André potential)
2. $\Delta \neq 0$: localization persists

This talk

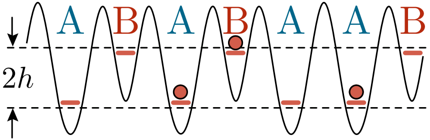
1. $\Delta = 0$: **multifractal** (quasiperiodic potential)
2. $\Delta \neq 0$: localization appears



Interest

- MBL is **generic**
- Interplay between quasiperiodicity and MBL

Interacting fermions on the Fibonacci chain



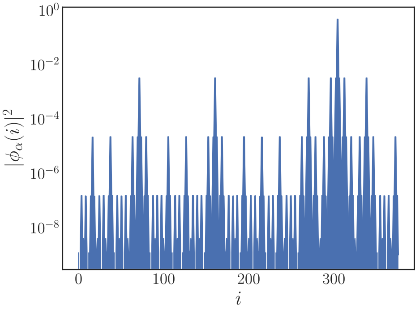
$$H = \sum_{i=1}^{L} \left[J(c_i^\dagger c_{i+1} + \text{H.c}) + \Delta n_i n_{i+1} - h_i n_i \right]$$

Method: numerical **exact diagonalization**

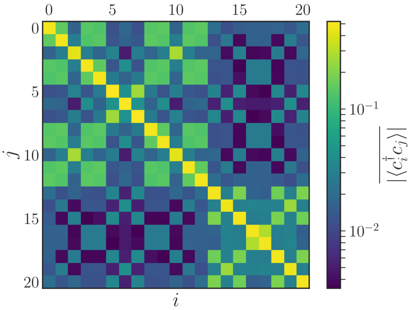
- High energy + non-integrable: **no analytical methods**
- $L/2$ fermions on L sites: #states $\sim 2^L/\sqrt{L} \rightarrow$ **memory is limiting**
  State-of-the art: $L = 24$ [Pietracaprina *et al* 18]
- Fibonacci: **few samples**: $L/2$ non-equivalent systems of size L.

Free fermions properties

- Multifractal single particle wavefunctions [Ostlund; Kalugin; Kohmoto; ...]
- Anomalous transport [Mayou; Schreiber; Varma & Žnidarič; ...]



Single particle wavefunction at the Fermi level

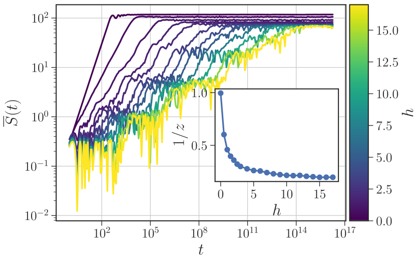


Correlations of highly excited states [Macé *et al* 19]

Free fermions entanglement

Entanglement entropy $S(\psi)$: a many-body **locality** probe

- $S(\psi) = \#\{$bits of information recoverable by local measurements$\}$
- $S(\psi)$ large: extended (entangled) state, $S(\psi)$ small: localized state.



Entanglement growth starting from localized fermions [Macé *et al* 19]

Fibonacci fermions: **anomalous** growth

$S(t) \sim t^{\frac{1}{z}}, \; z > 1$

Compare with:

- Periodic system: $z = 1$ (ballistic growth),
- Disordered system: $z \to \infty$ (no growth).

Conclusion

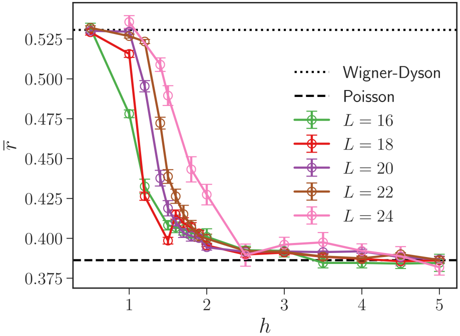
Anomalous, intermediate prop. even at high energy.

The ETH/MBL transition: 1) spectral properties

Gap ratios [Oganesian, Huse]

$$r_n = \min\left(\frac{g_{n+1}}{g_n}, \frac{g_n}{g_{n+1}}\right)$$

- ETH: random matrix-like spectrum
  $\overline{r}_{\mathrm{ETH}} \simeq 0.53$
- MBL: independant levels
  $\overline{r}_{\mathrm{MBL}} \simeq 0.39$

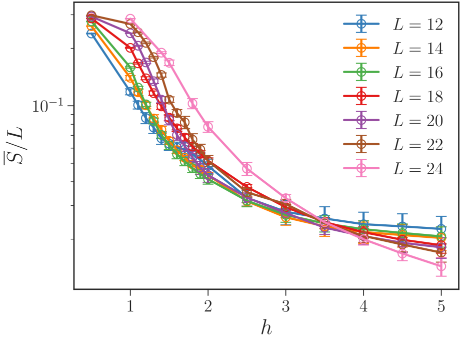


Compatible with **ETH/MBL transition**,
$h^* \simeq 2.5$.

The ETH/MBL transition: 2) Entanglement

Entanglement entropy:

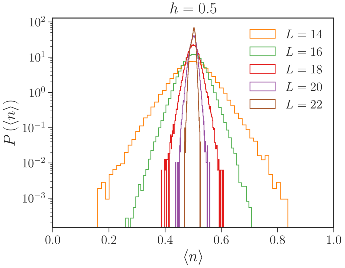
- ETH: coincides with thermodynamic entropy: **extensive**
  $\overline{S}_{\text{ETH}} \simeq L$
- MBL: **sub-extensive**
  $\overline{S}_{\text{MBL}}/L \to 0$



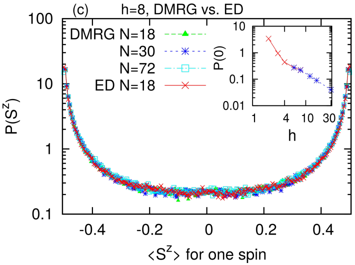
Compatible with **ETH/MBL transition**, $h^* \simeq 3.5$.

The ETH/MBL transition: 3) Local observables

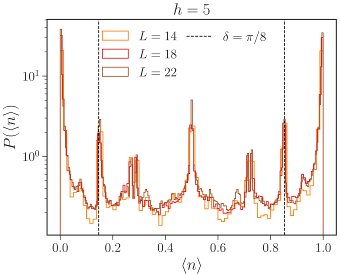
Expect: $\langle n_i \rangle_{\text{ETH}} = \frac{1}{2}$, $\langle n_i \rangle_{\text{MBL}} \simeq 0$ or 1.



ETH Fibonacci



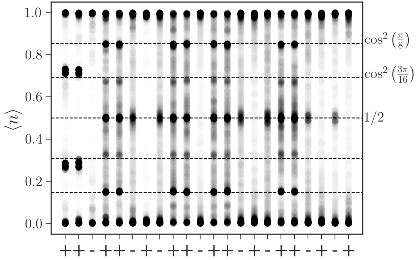
MBL random [Lim, Sheng 15]



MBL Fibonacci

Fibonacci MBL: **extra structure** → link with QP geometry?

Fibonacci MBL: local entanglement



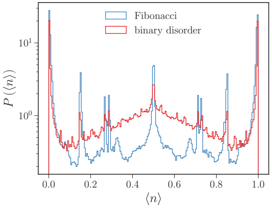
Density peaks on **AA pairs**

$\rightarrow$ 4 sites toy model BAAB

$\rightarrow$ locally entangled states

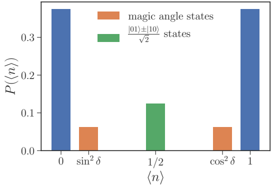
$|\psi\rangle = \cos\delta\,|01\rangle \pm \sin\delta\,|10\rangle$

with $\delta = 0,\ \frac{\pi}{4},\ \frac{\pi}{8}$.



Peak ingredients:

- **Binary** modulation A/B
- **Correlated** modulation (Fibonacci)



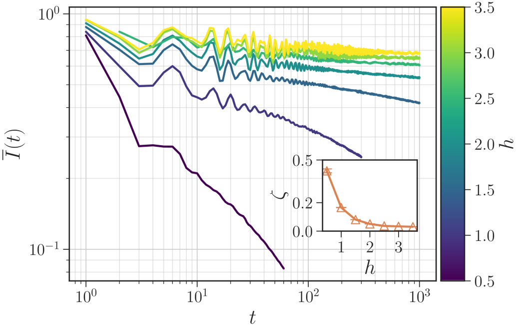
Dynamics: Imbalance

Imbalance experiment:

- $|\psi(t=0)\rangle$: high energy product state
- Imbalance: **distance** from initial state
  $I(t) = \frac{4}{L}\sum_i \left\langle \left(n_i(t) - \frac{1}{2}\right)\left(n_i(0) - \frac{1}{2}\right)\right\rangle$

Properties:

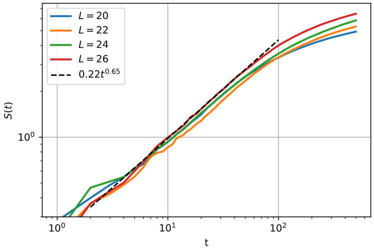
- ETH: power-law decay $I(t) \sim t^{-\zeta}$
- MBL: saturation $I(t \to \infty) = \text{cst} > 0$
  $\to$ **memory** of the initial state
  [Luitz *et at* 15]

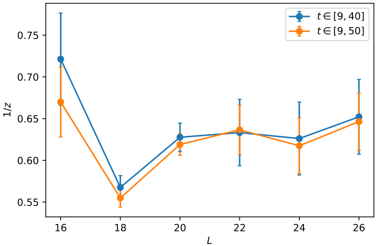


$\zeta(h \geq 2.5) = 0 \to$ MBL phase transition.

Dynamics: entanglement

ETH phase: expect $S(t) \propto t$
Fibonacci: **anomalous** $S(t) \propto t^{1/z}$, $z > 1$.





Usual explanation: **rare regions** [Vosk; Potter 15]
Fibonacci: no rare regions ...
Finite size effect? [Setiawan *et al* 17], initial state fluctuations? [Lüschen *et al* 17]

Conclusion

Non-interacting Fibonacci fermions

- Geometry: intermediate complexity
- **Multifractality** even at high energy
- Intermediate, **anomalous** transport, even at the many-body level

Interacting Fibonacci fermions

- Weak quasiperiodicity: **thermal** phase (ETH)
- Strong quasiperiodicity: **localized** phase (MBL)
- MBL bears sign of **quasiperiodicity** (locally entangled states)

$\rightarrow$ anomalous transport in the ETH phase: why?
$\rightarrow$ vicinity of the non-interacting point?

[*Macé, Laflorencie, Alet*, SciPost Phys. **6**, 050 (2019)]