Fractal states on quasicrystals

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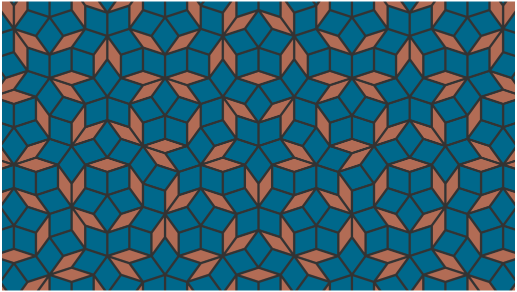


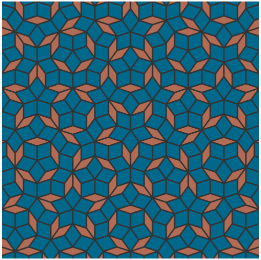
Outline

Quasicrystal?

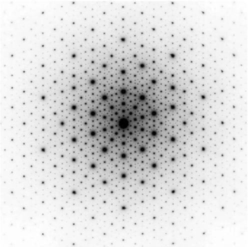
A crystalline structure based on a quasiperiodic tiling.

A qp tiling is required to:

- fill space (tiling)
- be **aperiodic** (a translate of the tiling does not superpose with the tiling itself)
- have **some kind of long range order** (diffraction reveals sharp peaks)



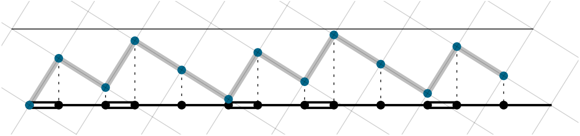
A patch of the Penrose tiling,
which has an order 5 rotational symmetry



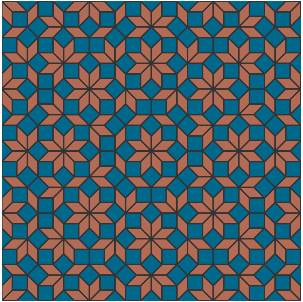
Experimental diffraction pattern of AlPdMn
(Conradin Beeli group)

A recipe for bulding quasiperiodic lattices

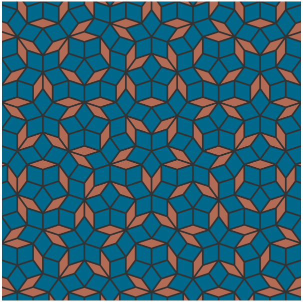
Take a **periodic lattice**, cut a slice in it, project the selected points on a “physical” hyperplane.



A one dimensional quasicrystal, built from a 2D square lattice.

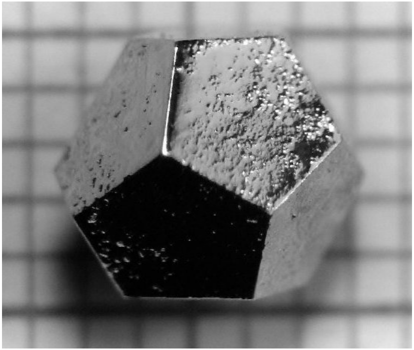


The octagonal tiling, built from a 4D square lattice.

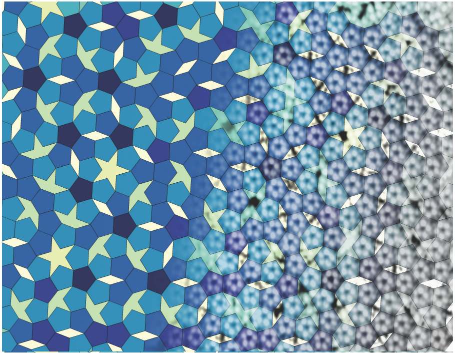


The Penrose tiling, built from a 5D square lattice.

Real life examples



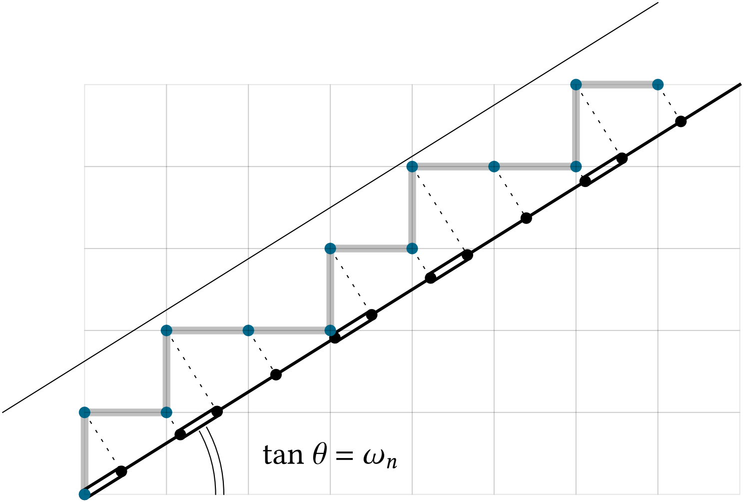
HoMgZn alloy in its icosahedral phase
(see doi:10.1038/nmat1244)



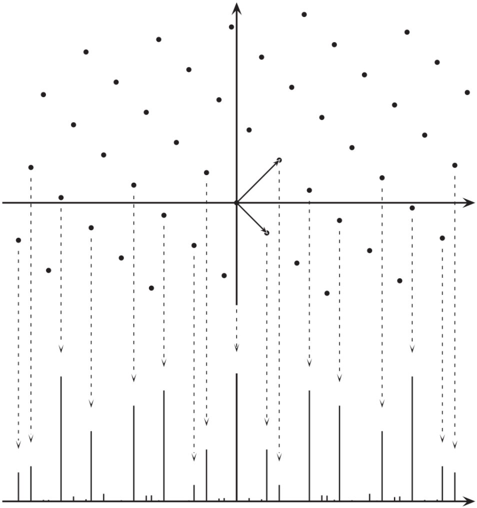
A 2D hydrogen-bonded quasicrystal
(see doi:10.1038/nature12993)

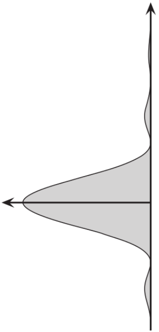
- Numerous metallic and soft-matter quasicrystals have been synthetized
- only one natural example example is known: the Khatyrka meteorite (see doi:10.1126/science.1170827).

Diffraction by a QP lattice: a simple case

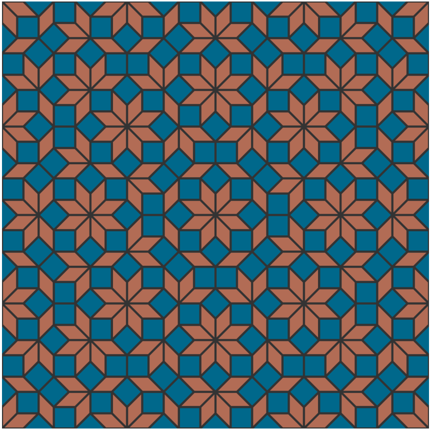


Diffraction by a QP lattice: a simple case

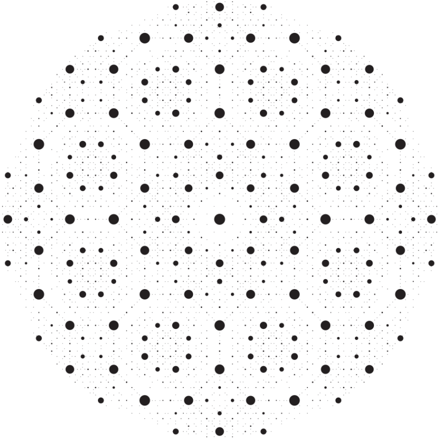




Diffraction by a QP lattice



The octagonal tiling...

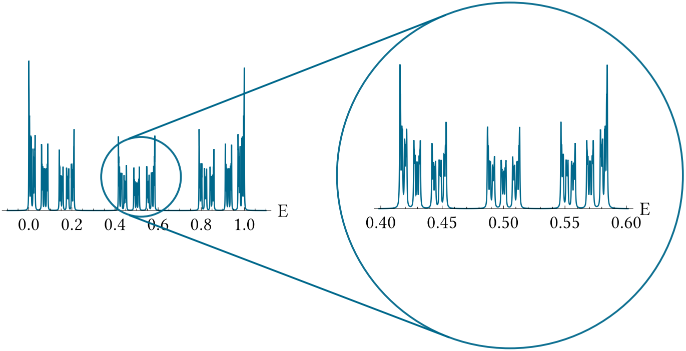


...and its diffraction pattern
(see *Aperiodic Order*, Baake & Grimm)

A quasicrystal toy model: the Fibonacci chain

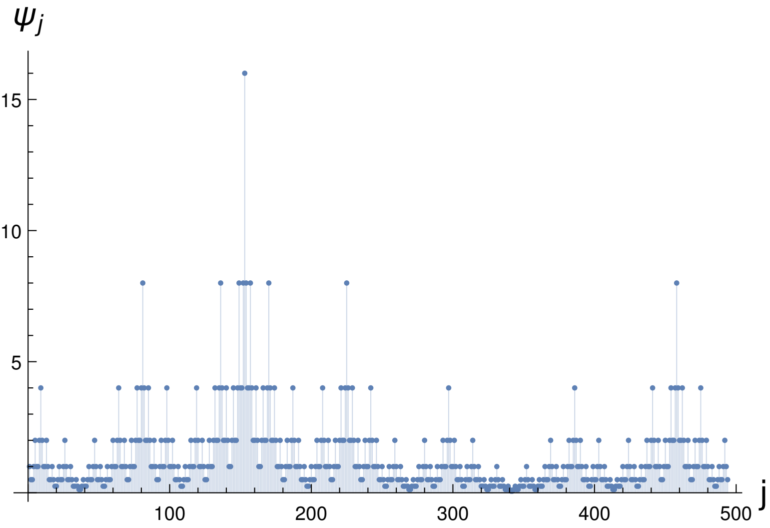
$$H = \sum_i t_{i,i+1}(|i\rangle\langle i+1| + |i+1\rangle\langle i|)$$

$$t(=) = 1, \; t(-) = \rho$$

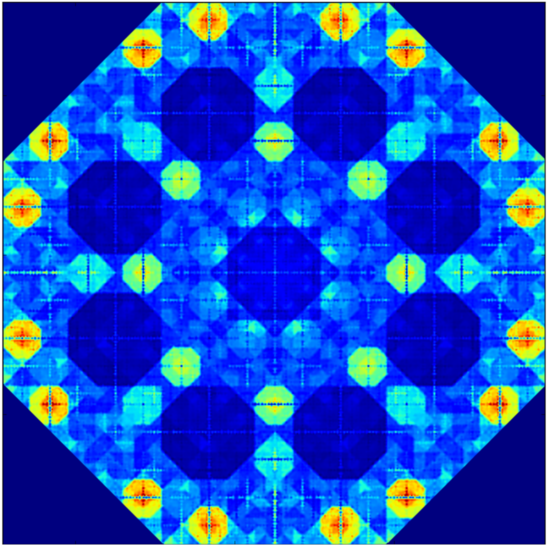


The density of states of even the simplest quasicrystal toy model is still complicated, and has an interesting fractal structure.

The $E = 0$ wavefunction

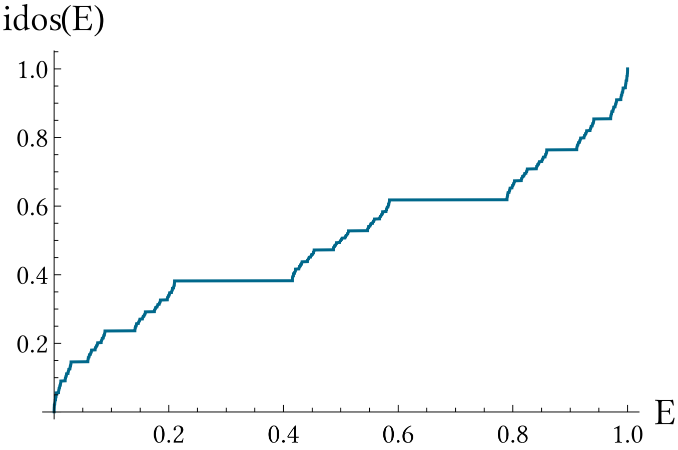


In 2D...



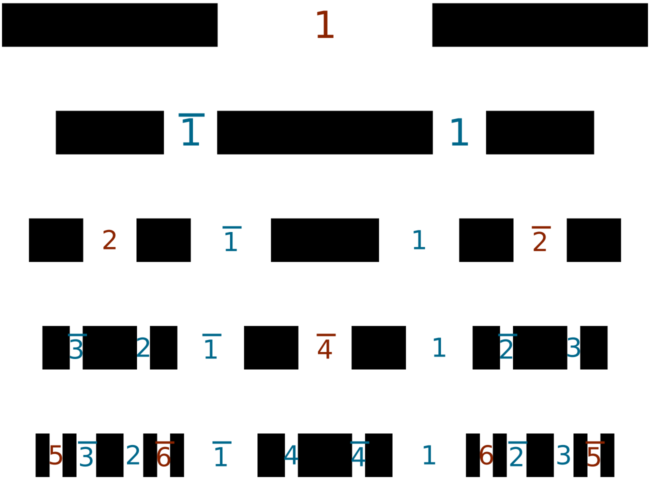
A state on the octagonal tiling (numerical simulation by Eric Adrade)

The electronic spectrum



The integrated density of states (the fraction of states below an energy E), for the Fibonacci chain.

The electronic spectrum reinterpreted



The gap labels for increasingly large approximants to the Fibonacci chain.