Multifractal eigenstates on quasicrystals

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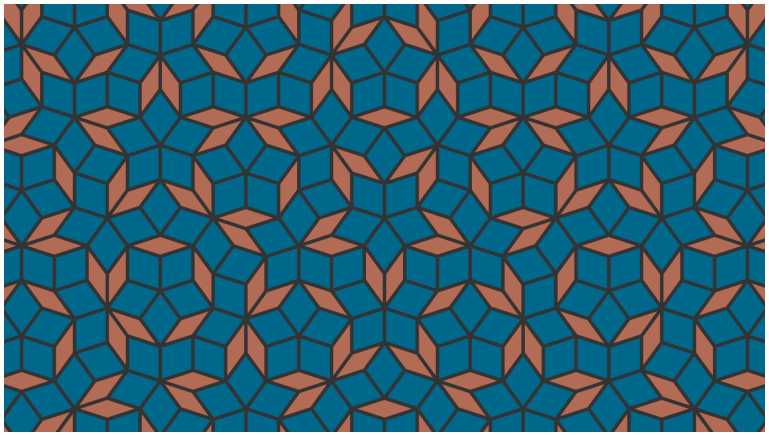


Overview

Quasicrystals : ordered, aperiodic materials $\rightarrow$ Bloch's theorem fails.
Today's goal : construct non-interacting tight-binding electronic eigenstates from geometrical *height fields*.
Pioneering work (reverse engineered toy model) :

- Self-similar ground-state wave function for electrons on a 2D Penrose lattice

  Sutherland, PRB 34 (6), 1986

Breakthrough (reasonable toy model) :

- Electrons in deterministic quasicrystalline potentials and hidden conserved quantities

  Kalugin, Katz, J. Phys. A 47 (31), 2014

A "pedagogical" version of the previous paper :

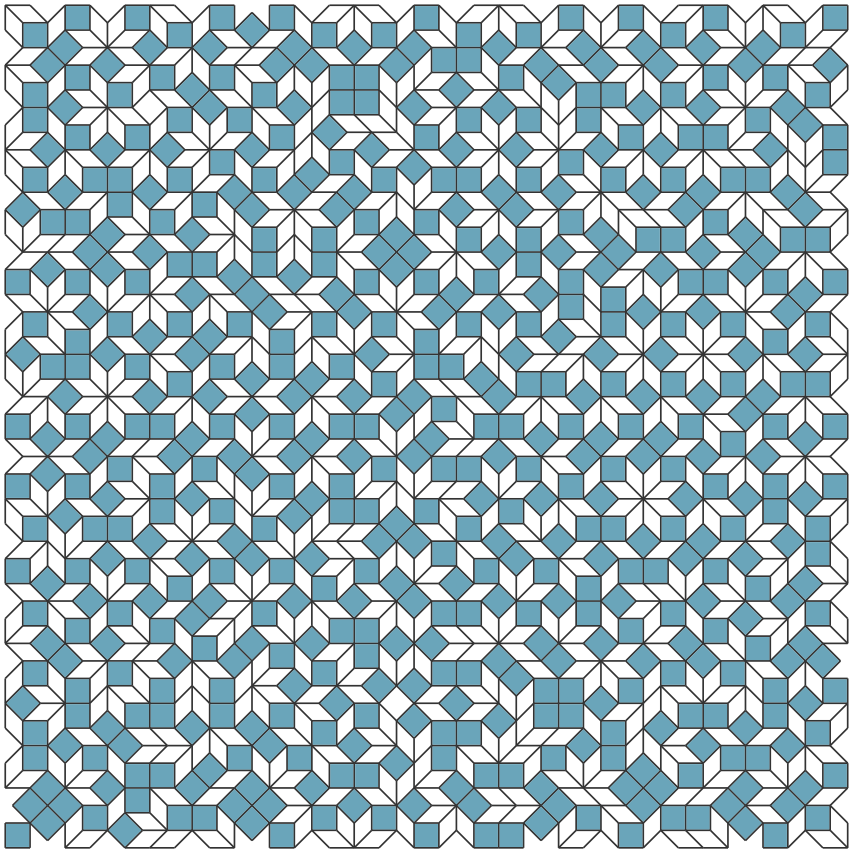
- Critical eigenstates and their properties in one-and two-dimensional quasicrystals

  Macé, Jagannathan, Kalugin, Mosseri, Piéchon, PRB 96 (4), 2017

Many thanks to Michel Duneau, Jean-Noël Fuchs, Jean-Marc Luck, Éric Akkermans.

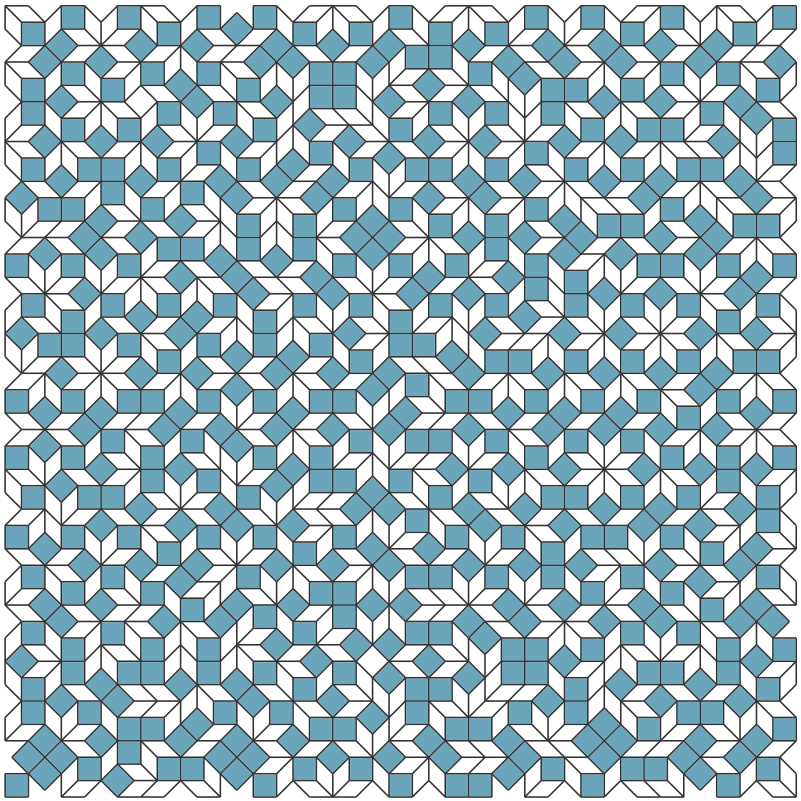
Periodic, quasiperiodic and random

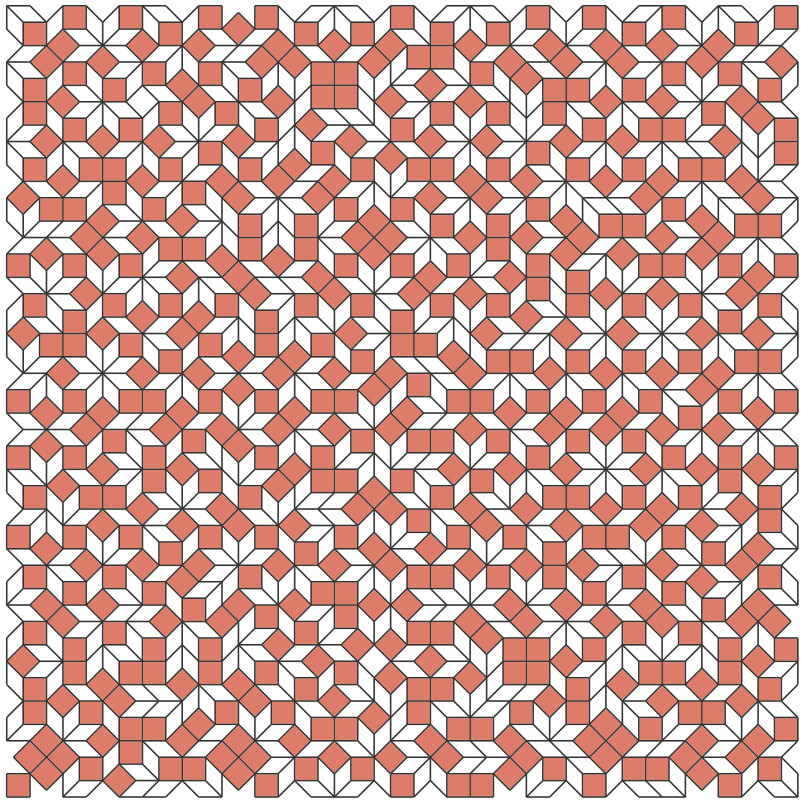
A random tiling :



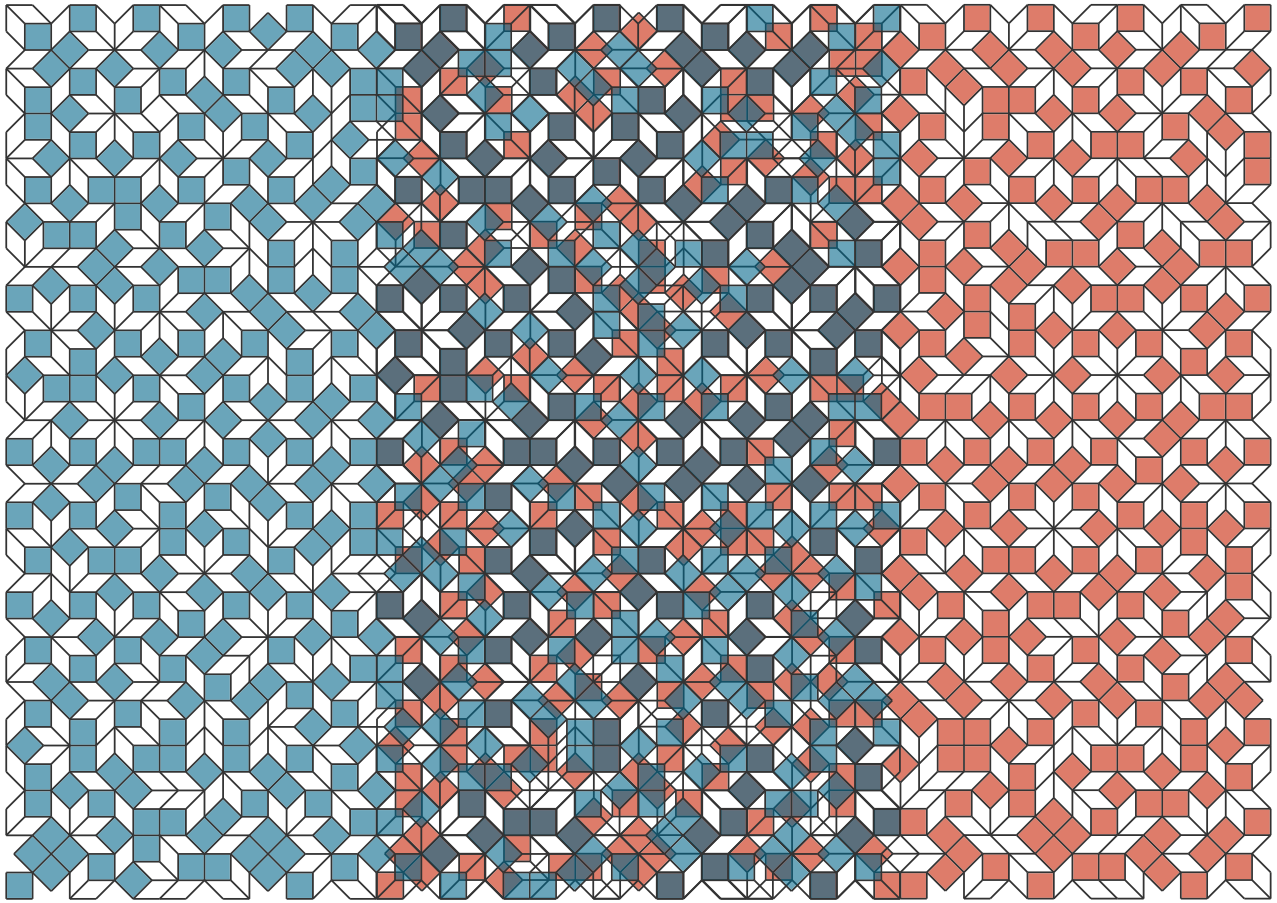
Periodic, quasiperiodic and random

Two copies of the tiling :



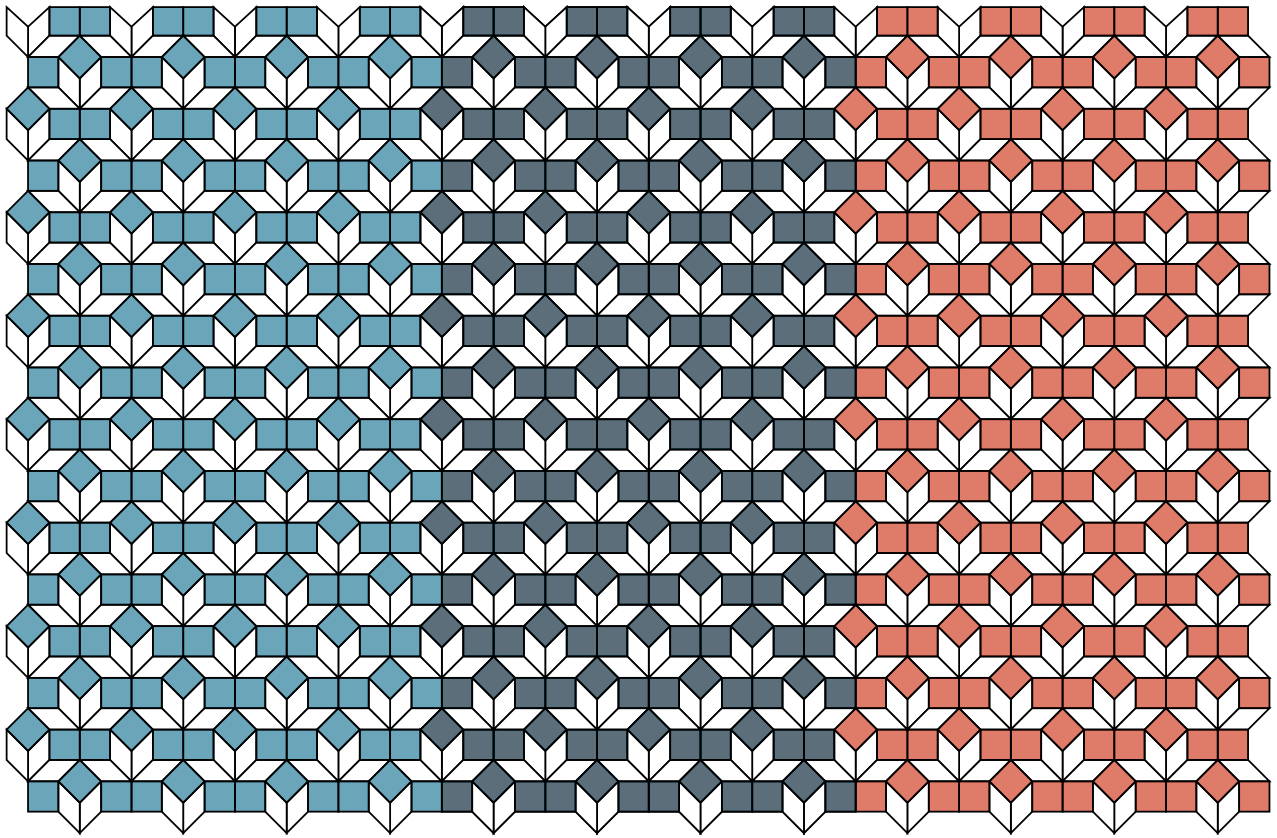


Periodic, quasiperiodic and random



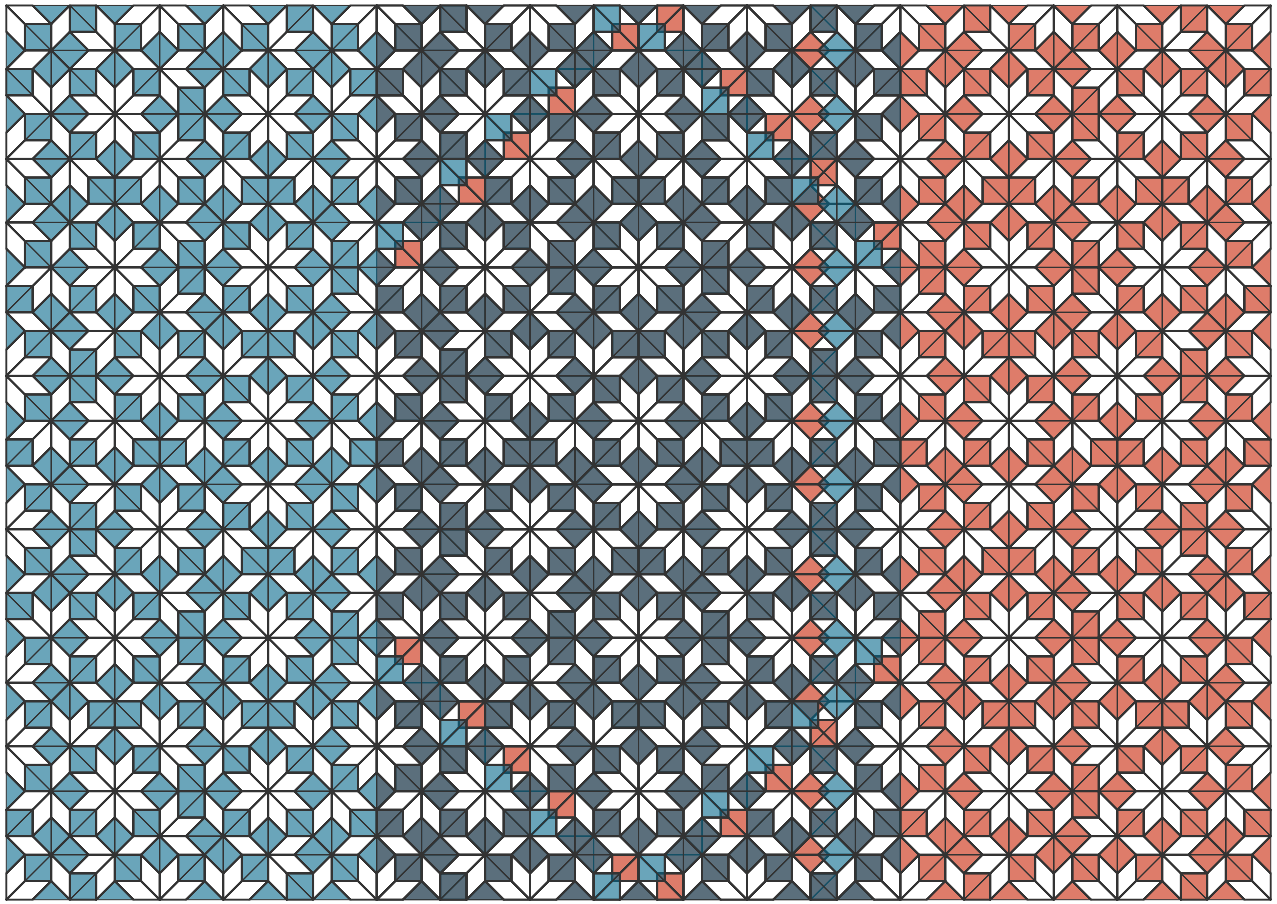
$\rightarrow$ no overlap $\rightarrow$ no order

Periodic, quasiperiodic and random



Perfect long range order : periodic

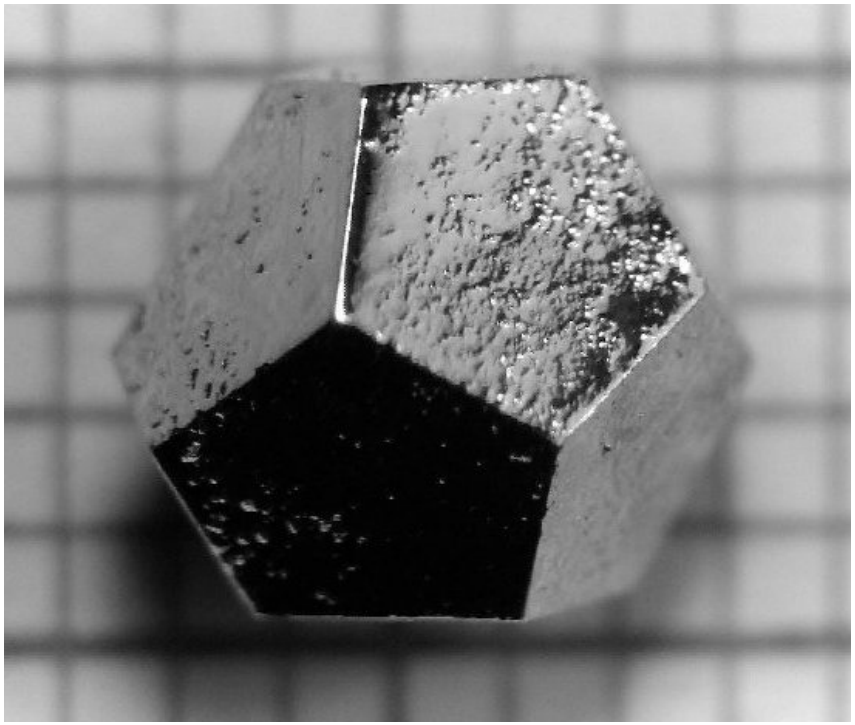
Periodic, quasiperiodic and random



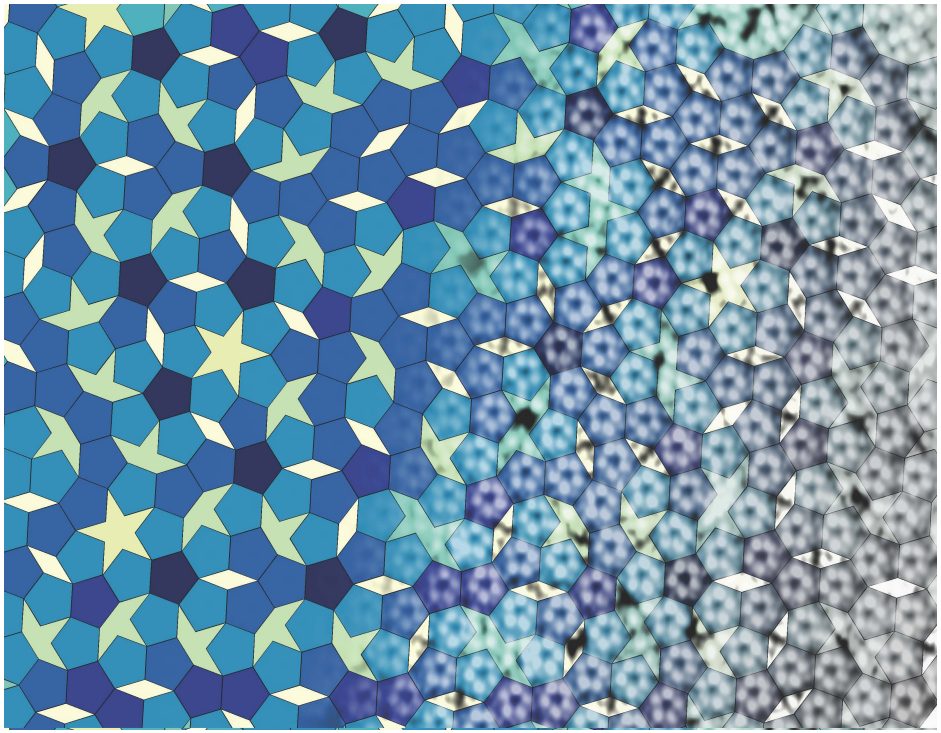
Long range order : quasiperiodic

(see Chap. 2 of [Grimm, Baake 13])

Real life examples



HoMgZn alloy in its icosahedral phase
(see doi:10.1038/nmat1244)



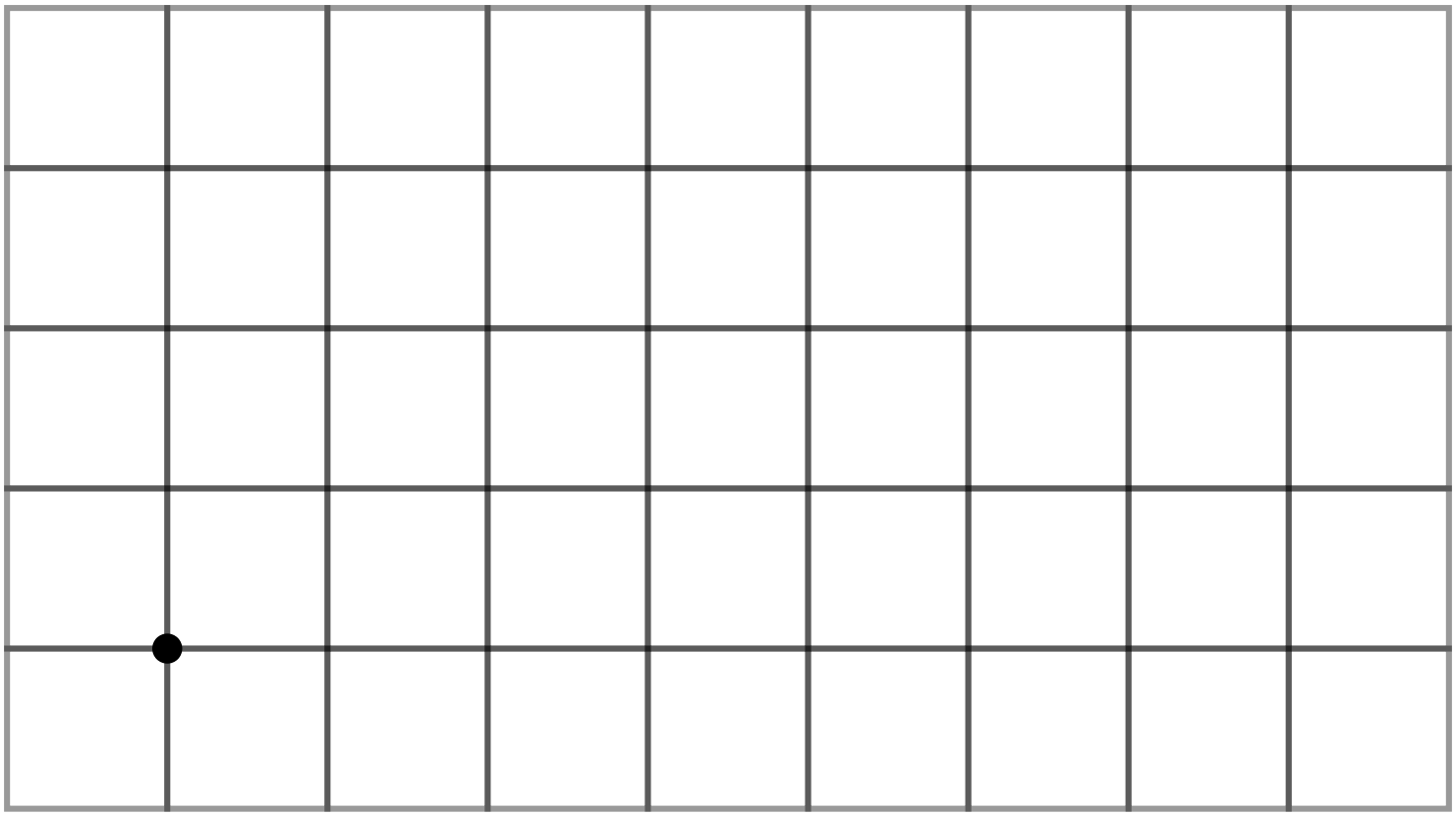
A 2D hydrogen-bonded quasicrystal
(see doi:10.1038/nature12993)

- Numerous metallic and soft-matter quasicrystals have been synthetized
- only one natural example example is known : the Khatyrka meteorite (see doi:10.1126/science.1170827).

Fibonacci word from above

(Infinite) Fibonacci word : ABAABABAABAAB…

A $\leftrightarrow$ horizontal step, B $\leftrightarrow$ vertical step



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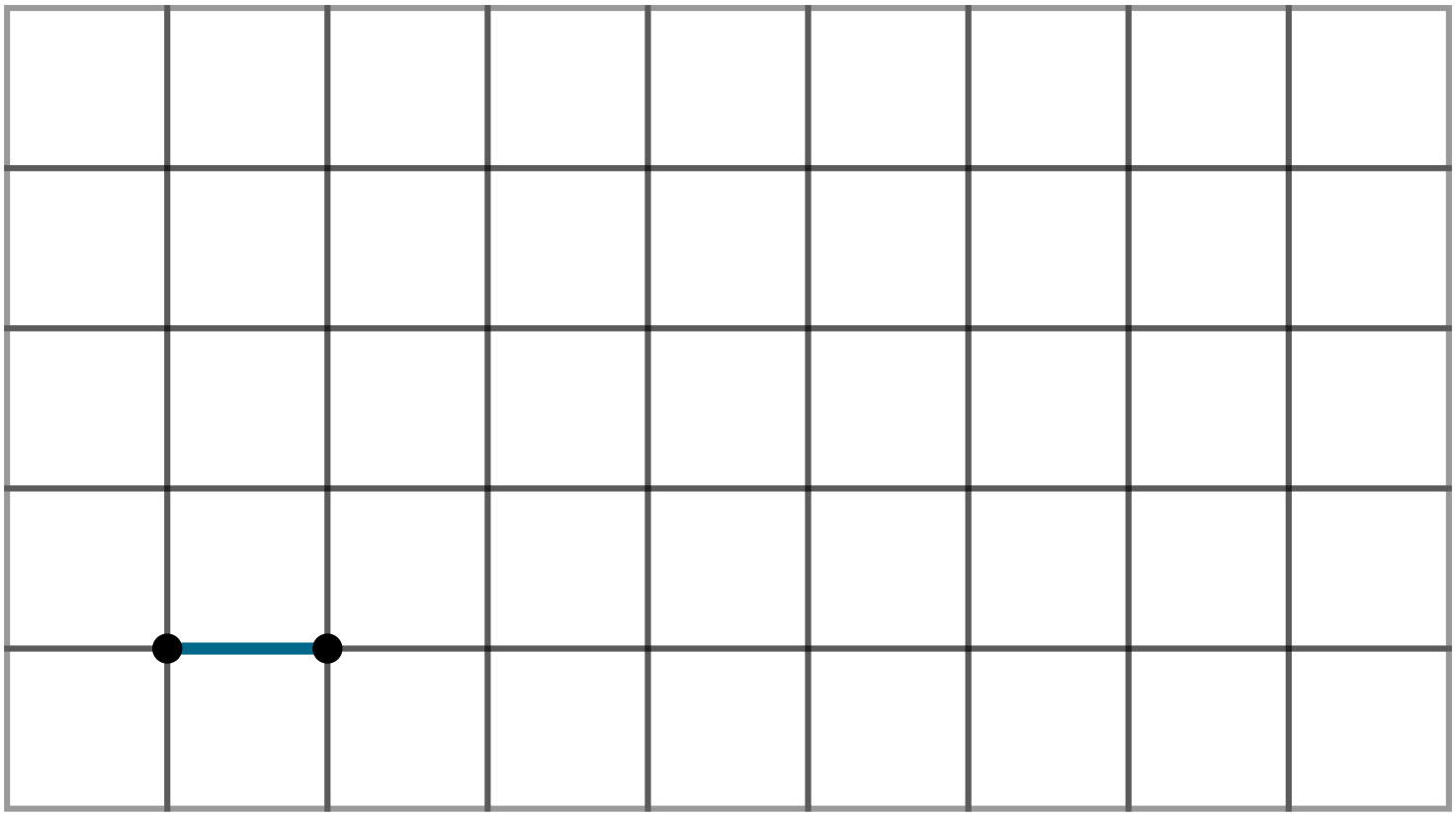

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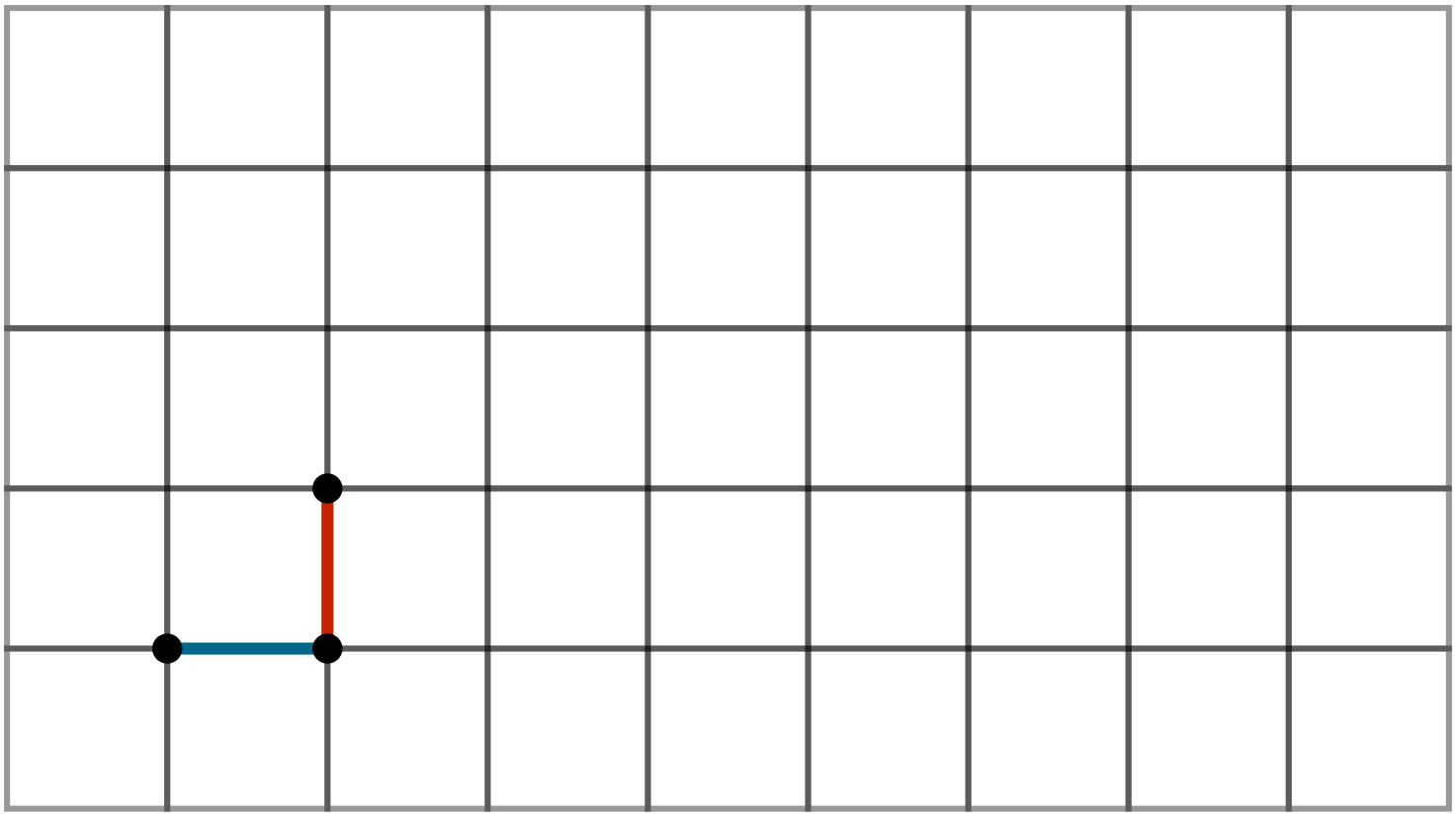

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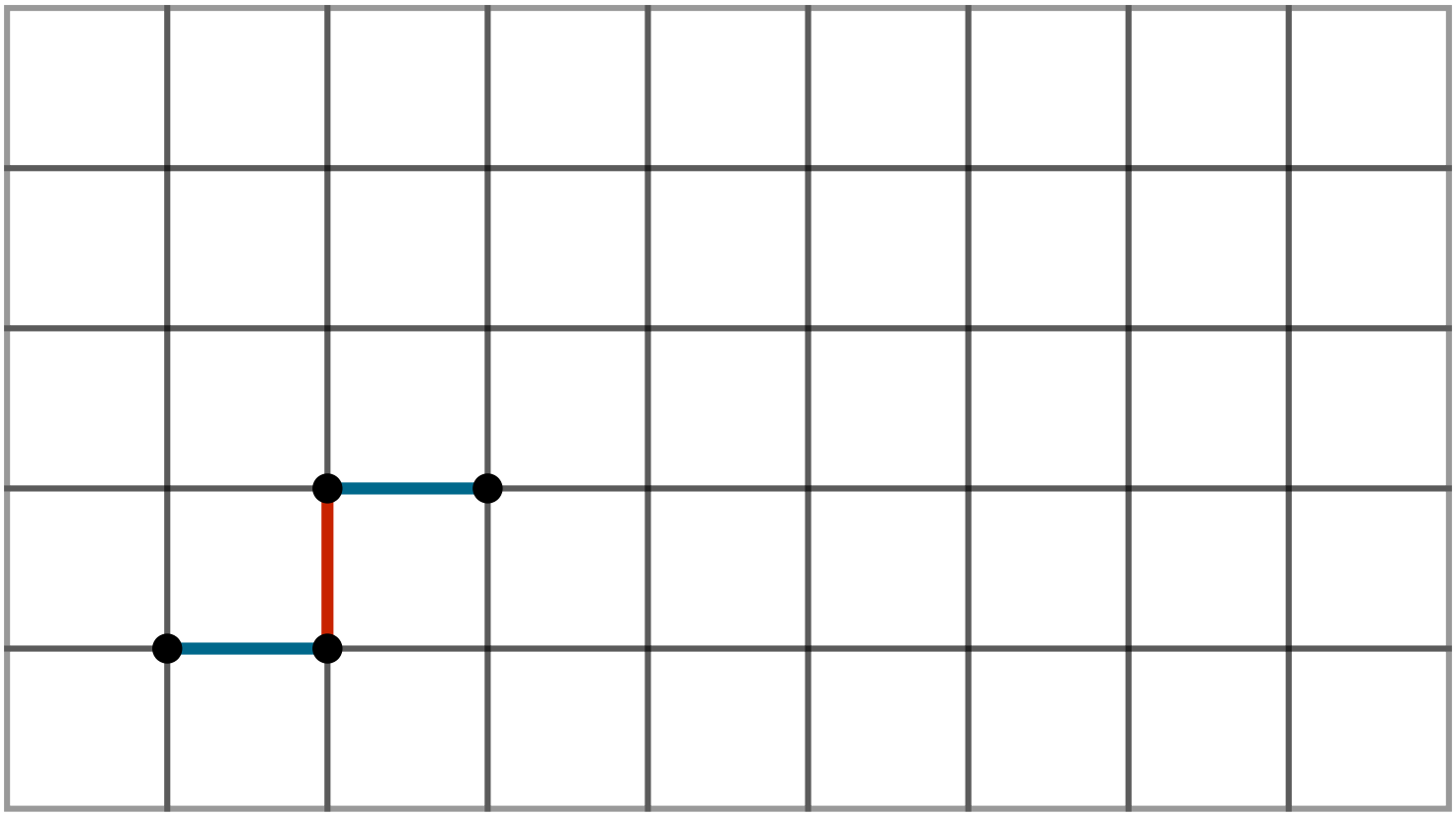

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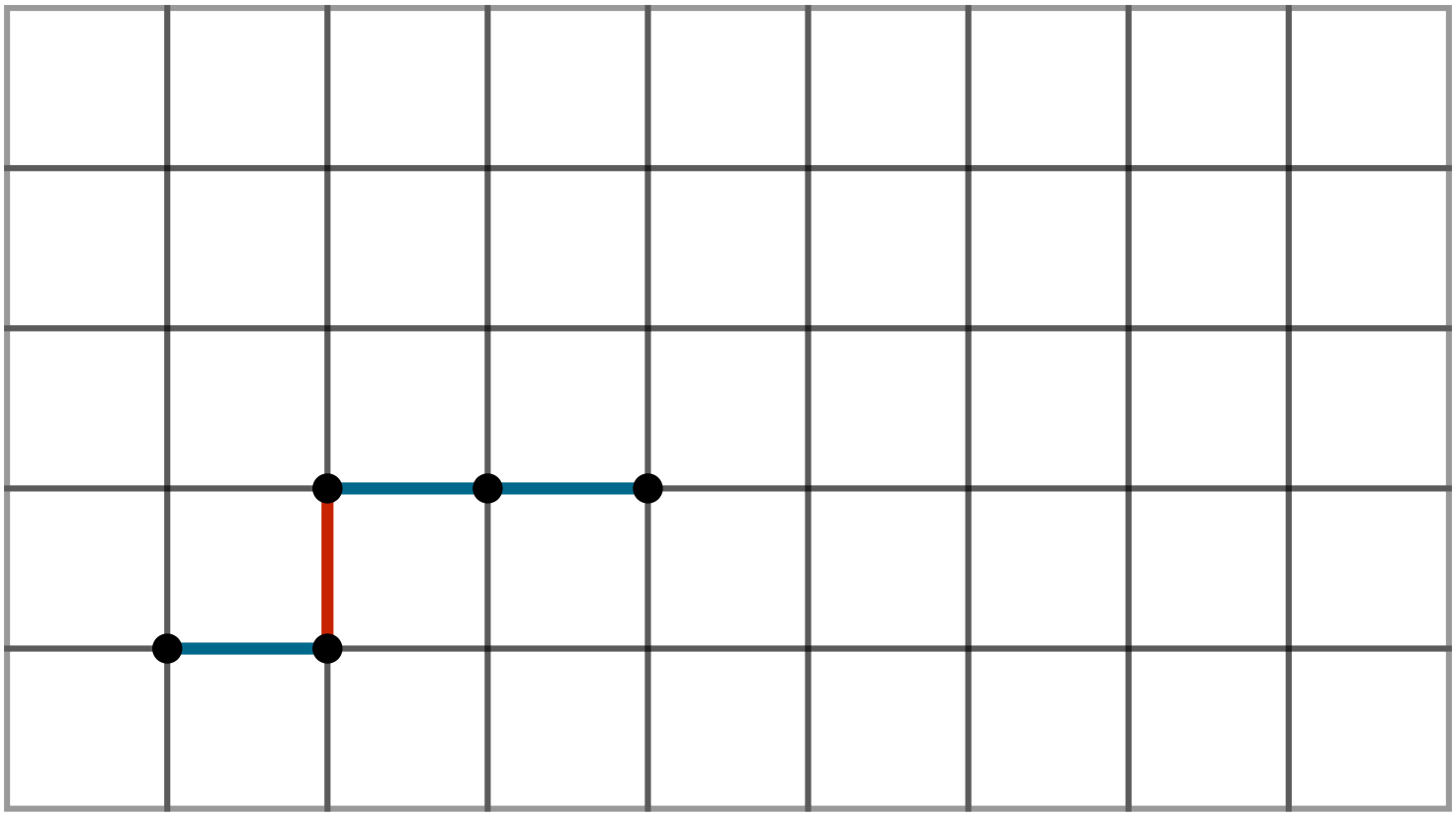

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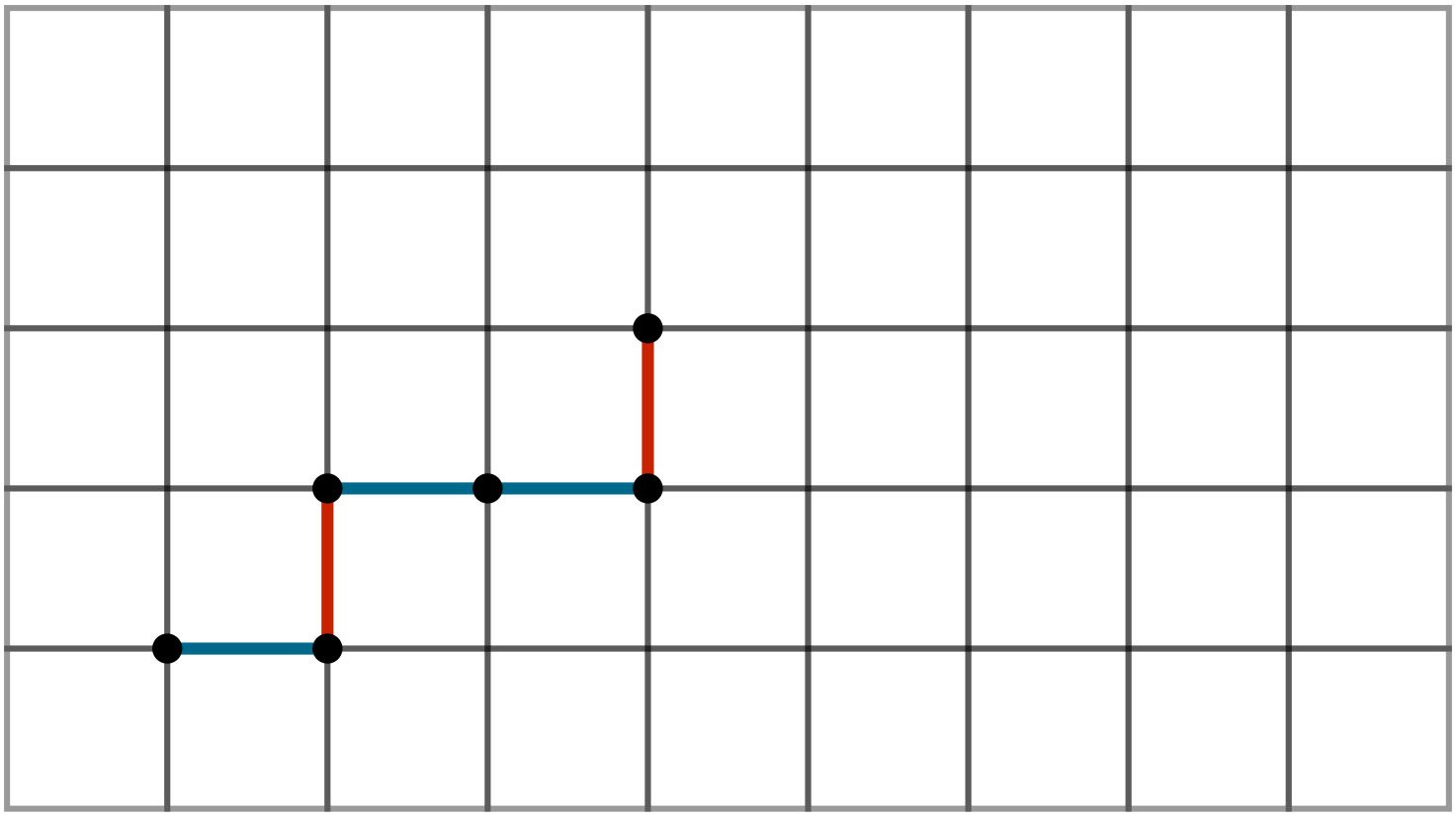

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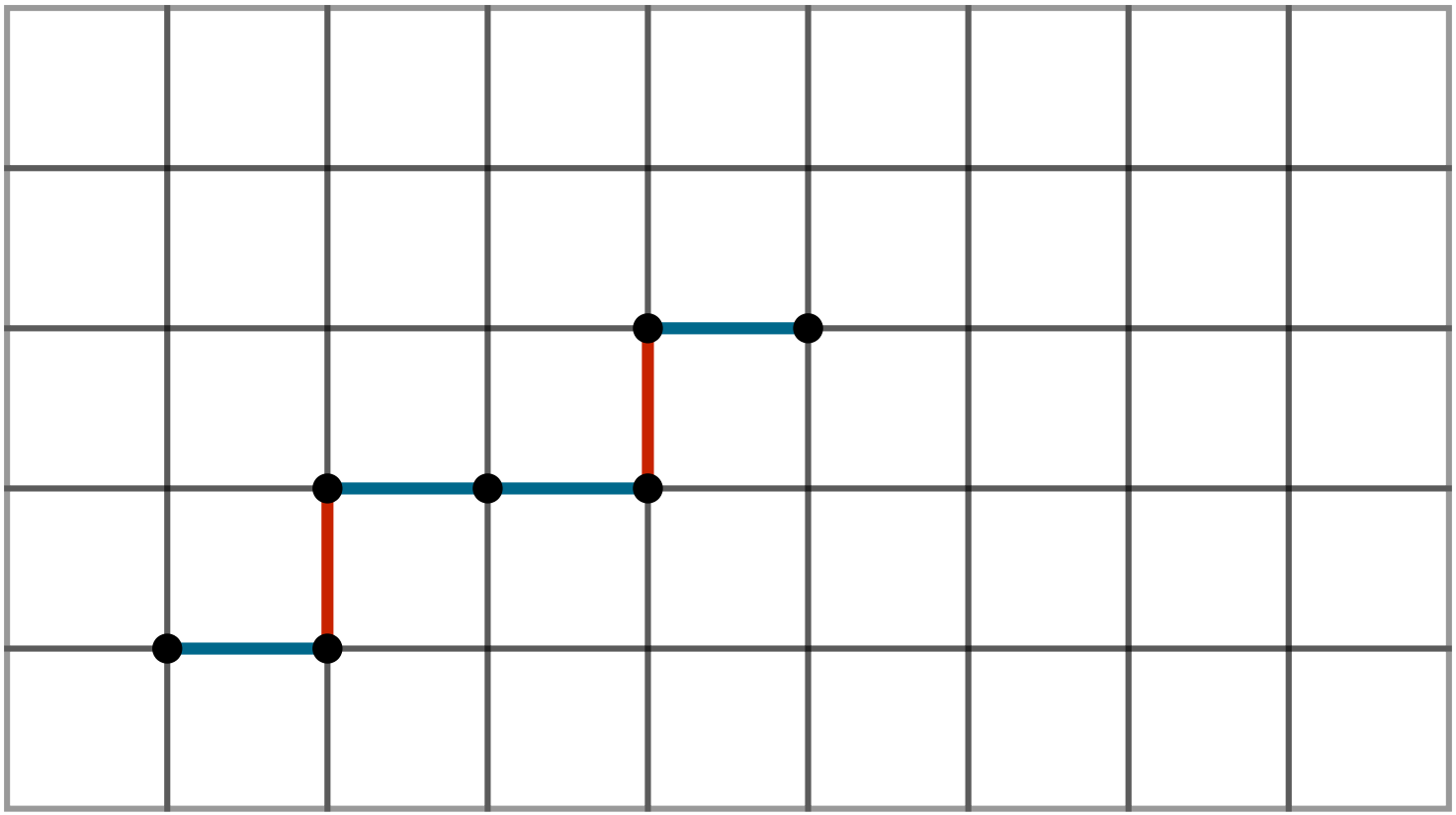

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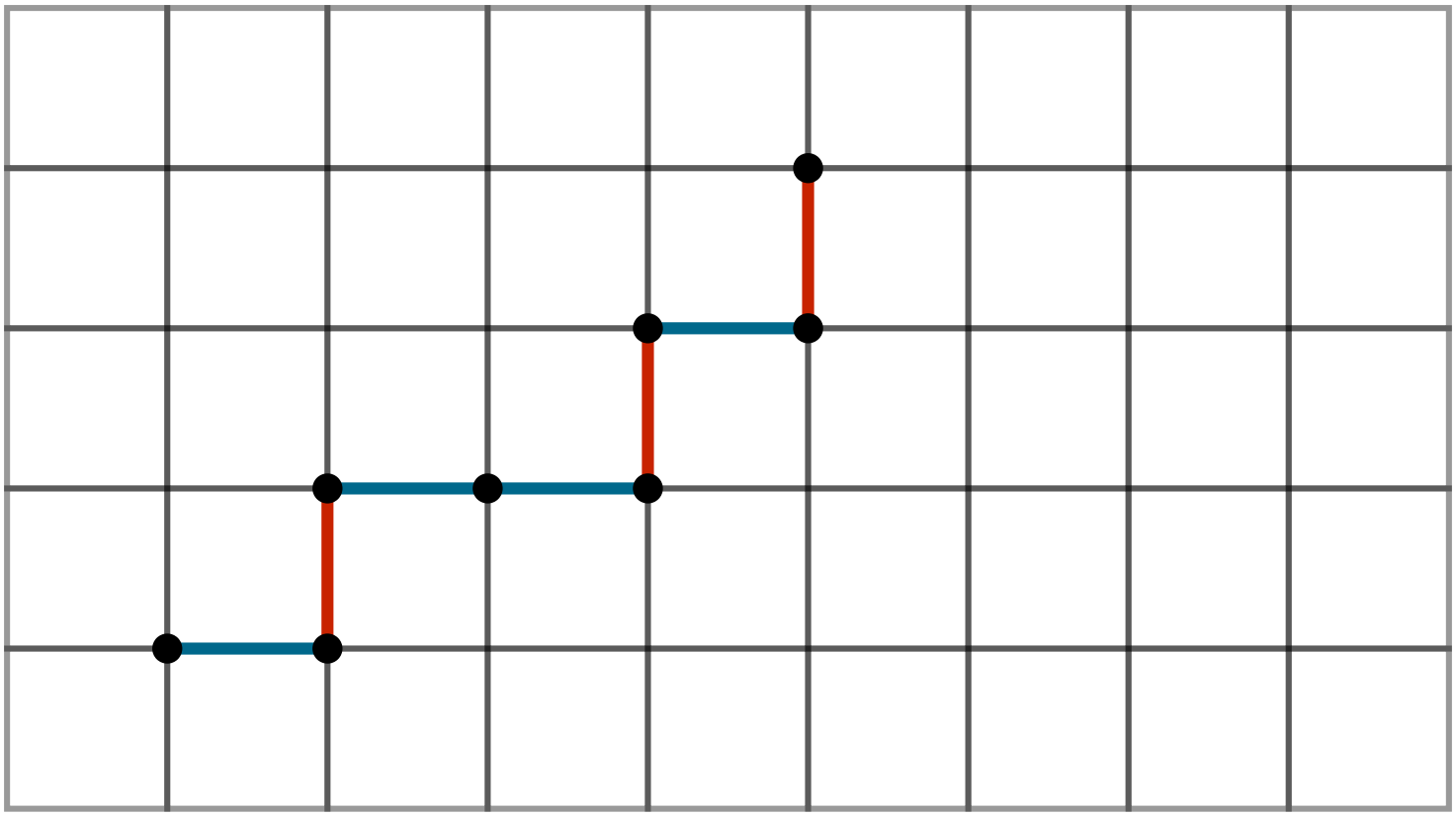

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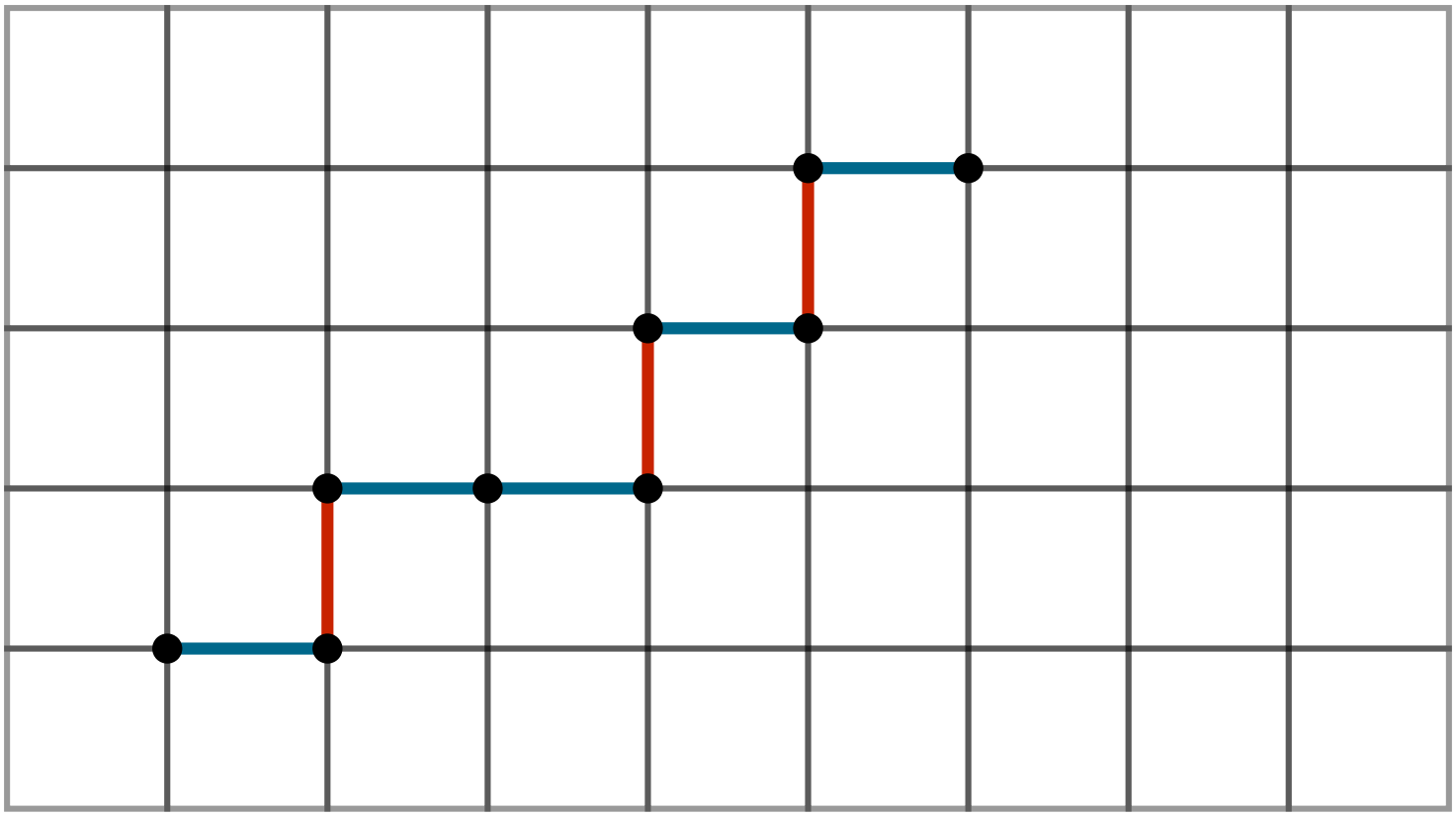

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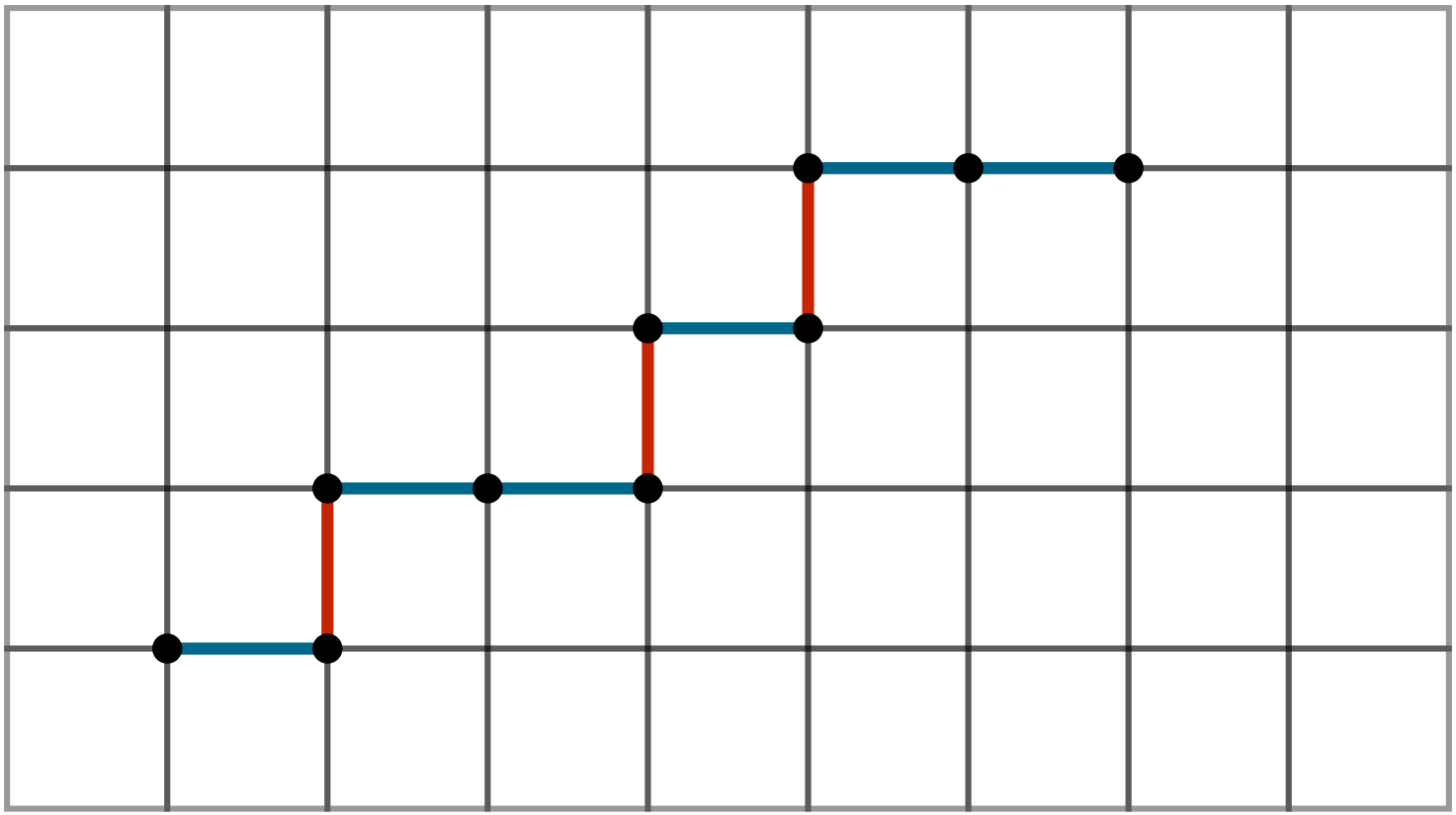

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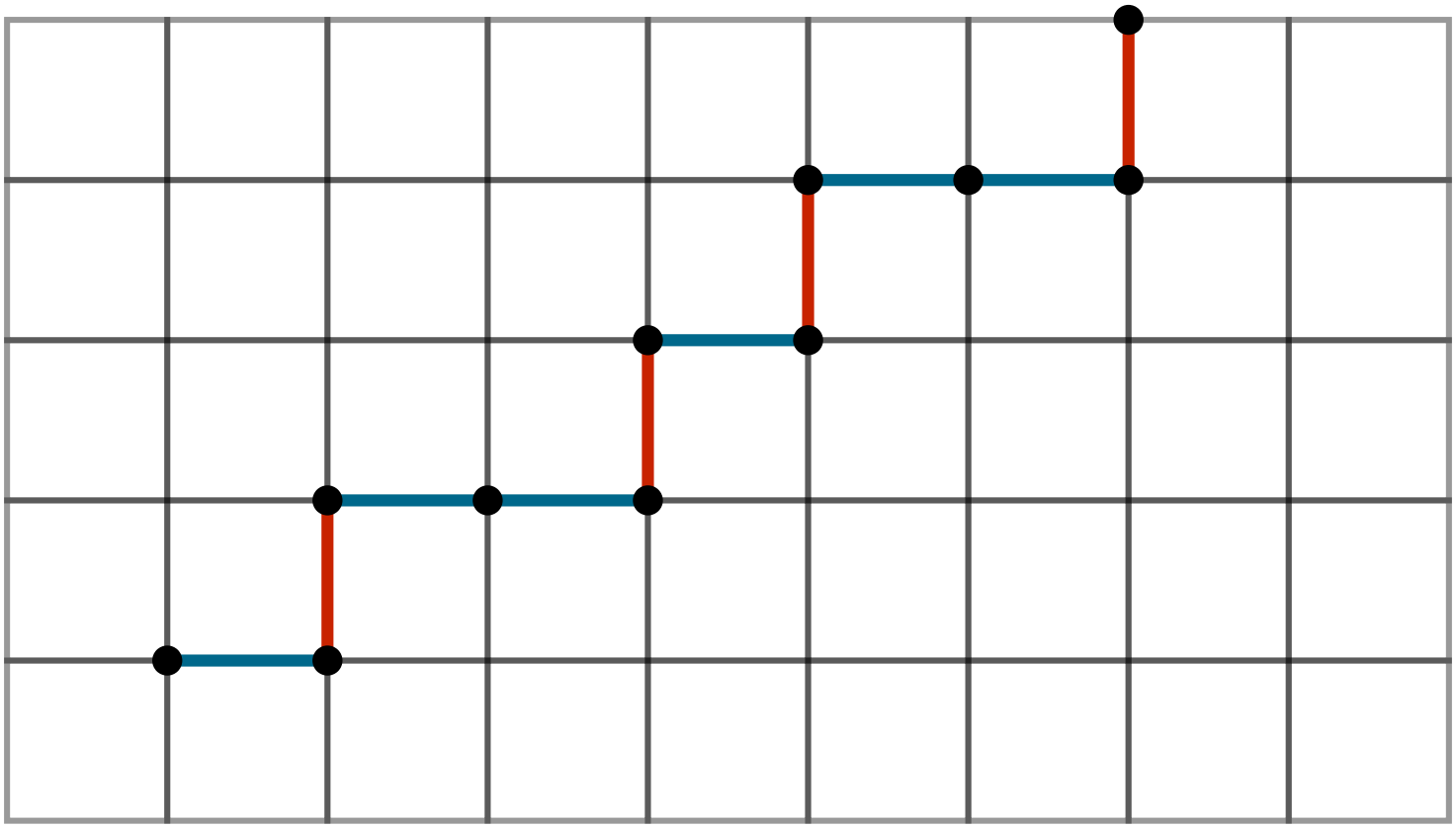

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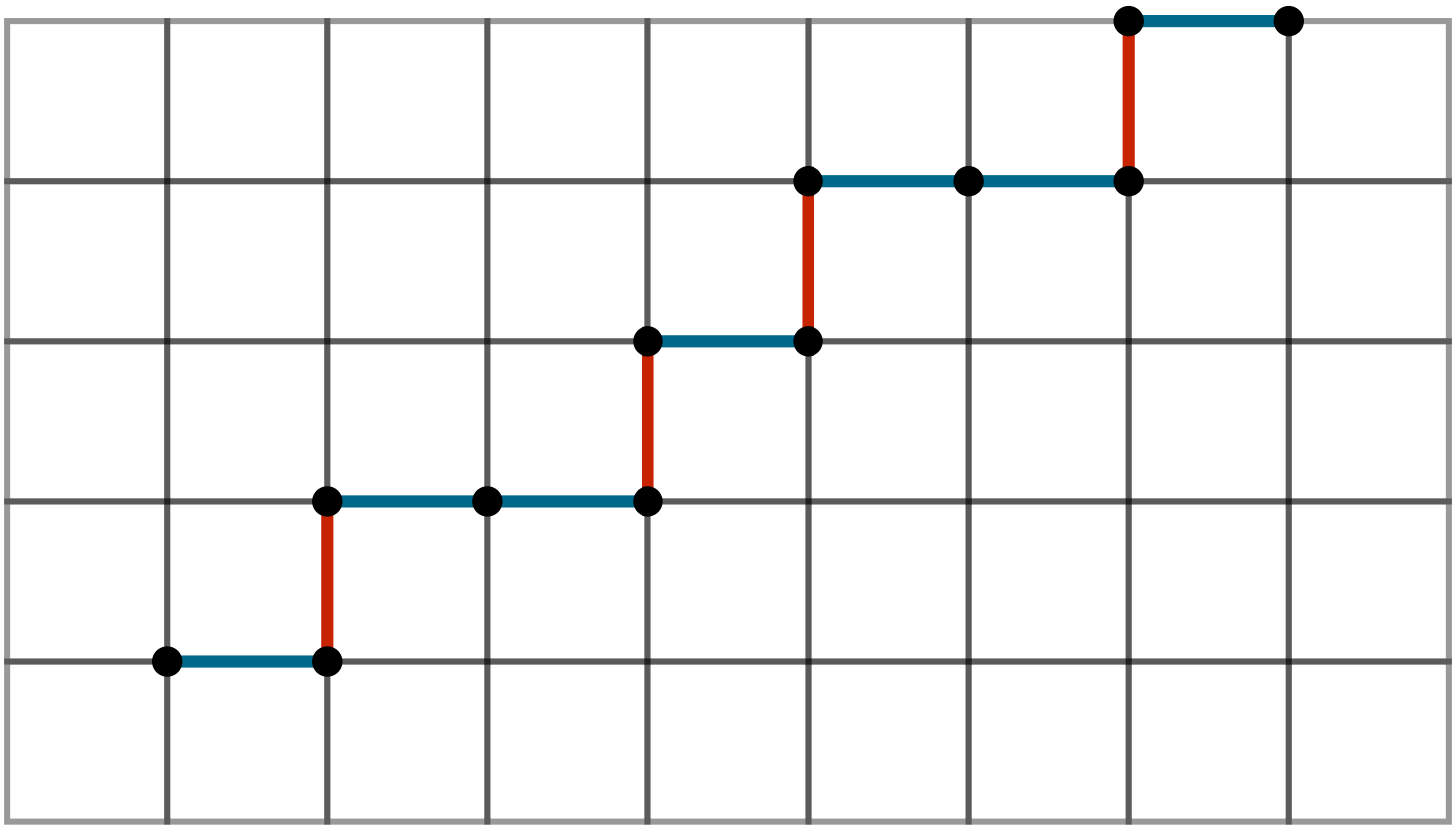

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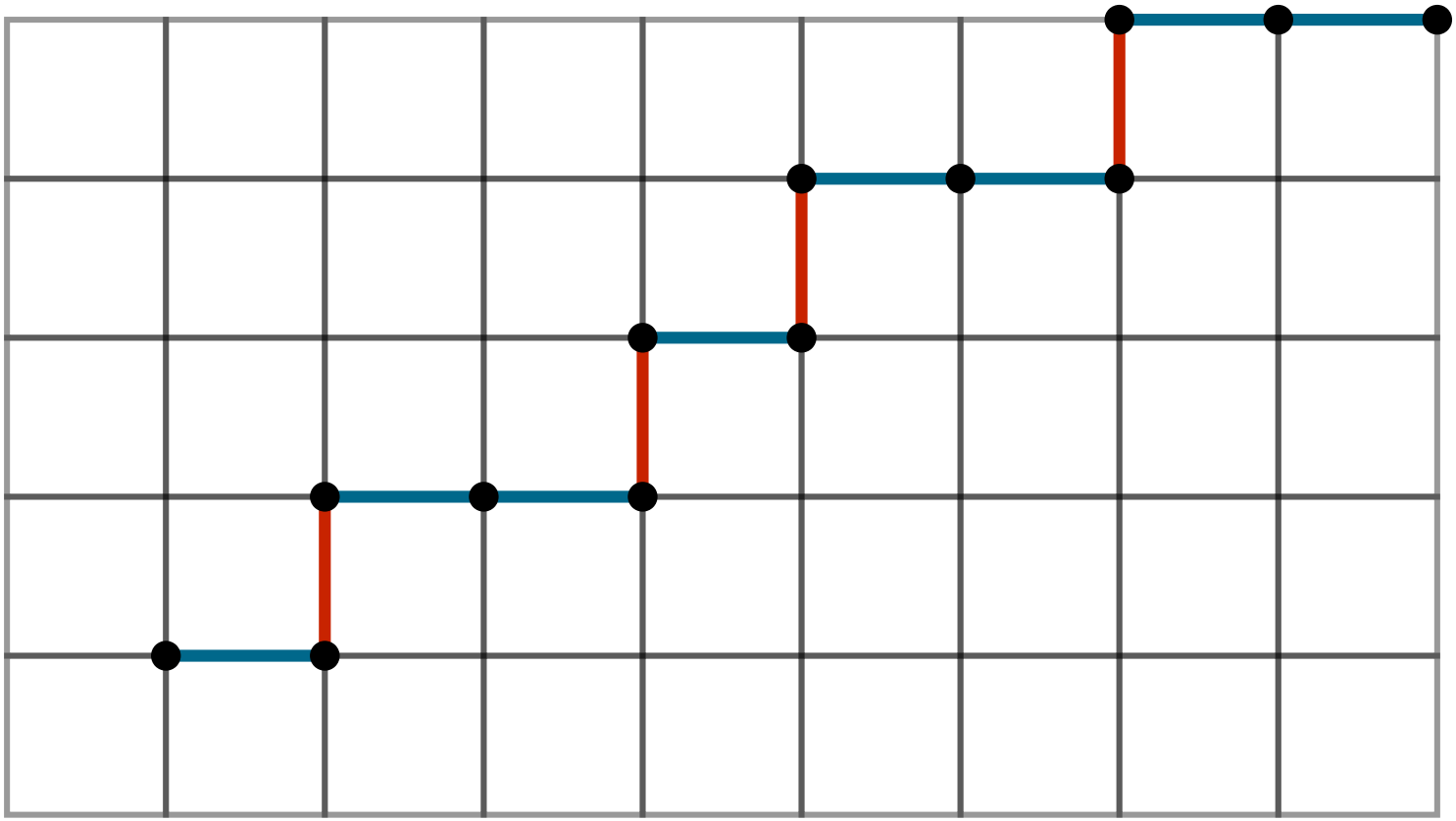
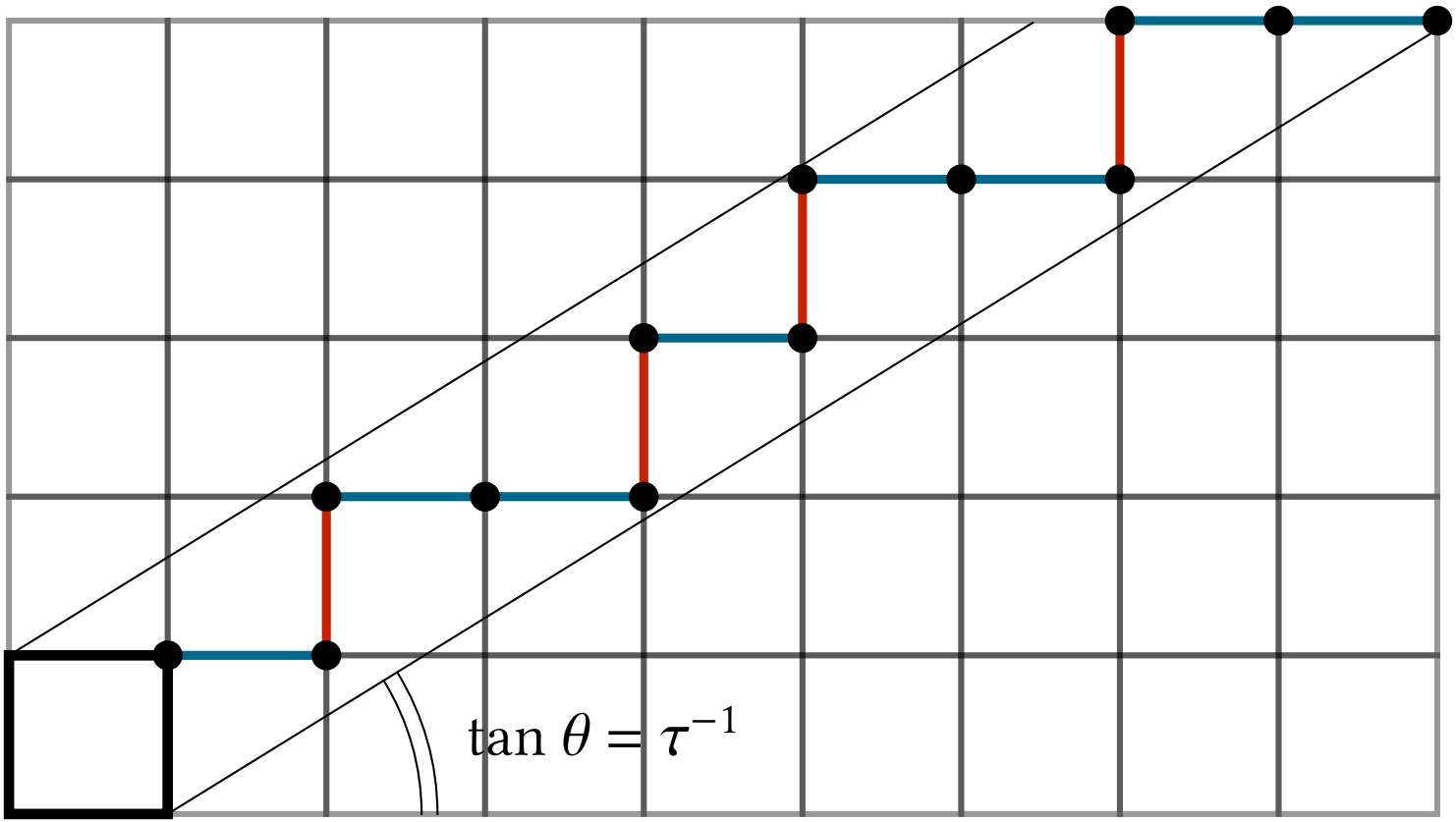
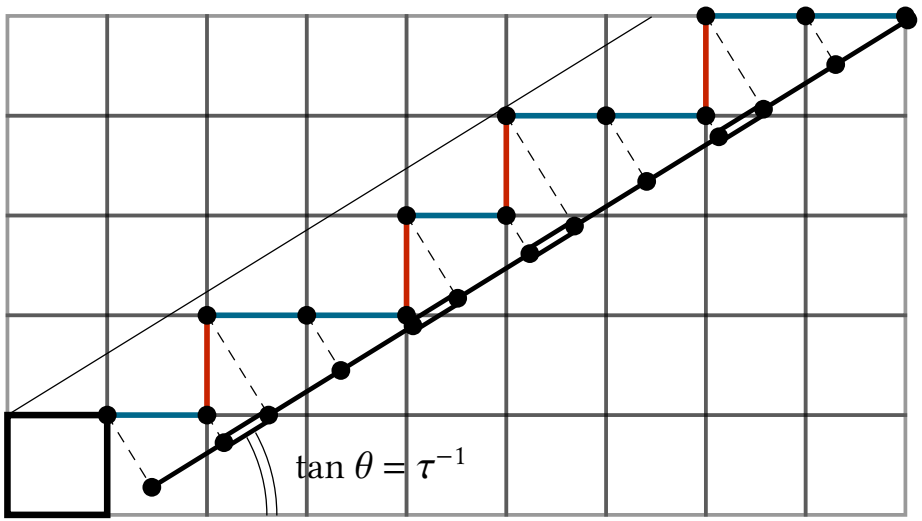


Quasiperiodicity of the Fibonacci word



Cut-and-project

Quasiperiodic tiling $\Leftrightarrow$ non-periodic tiling constructed with the cut-and-project algorithm.

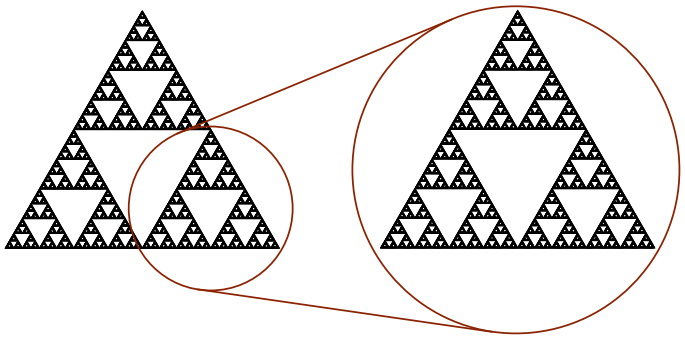


The cut-and-project algorithm :

1. choose a hypercubic lattice (here $\mathbb{Z}^2$)
2. choose a "physical plane" $E_{\parallel}$ (here a slope)
3. select points by translating the unit hypercube along $E_{\parallel}$
4. project them onto $E_{\parallel}$.

Fractal dimensions

- $M(L) \propto L^d$ for a non-fractal d-dimensional object…What happens for a fractal one ?

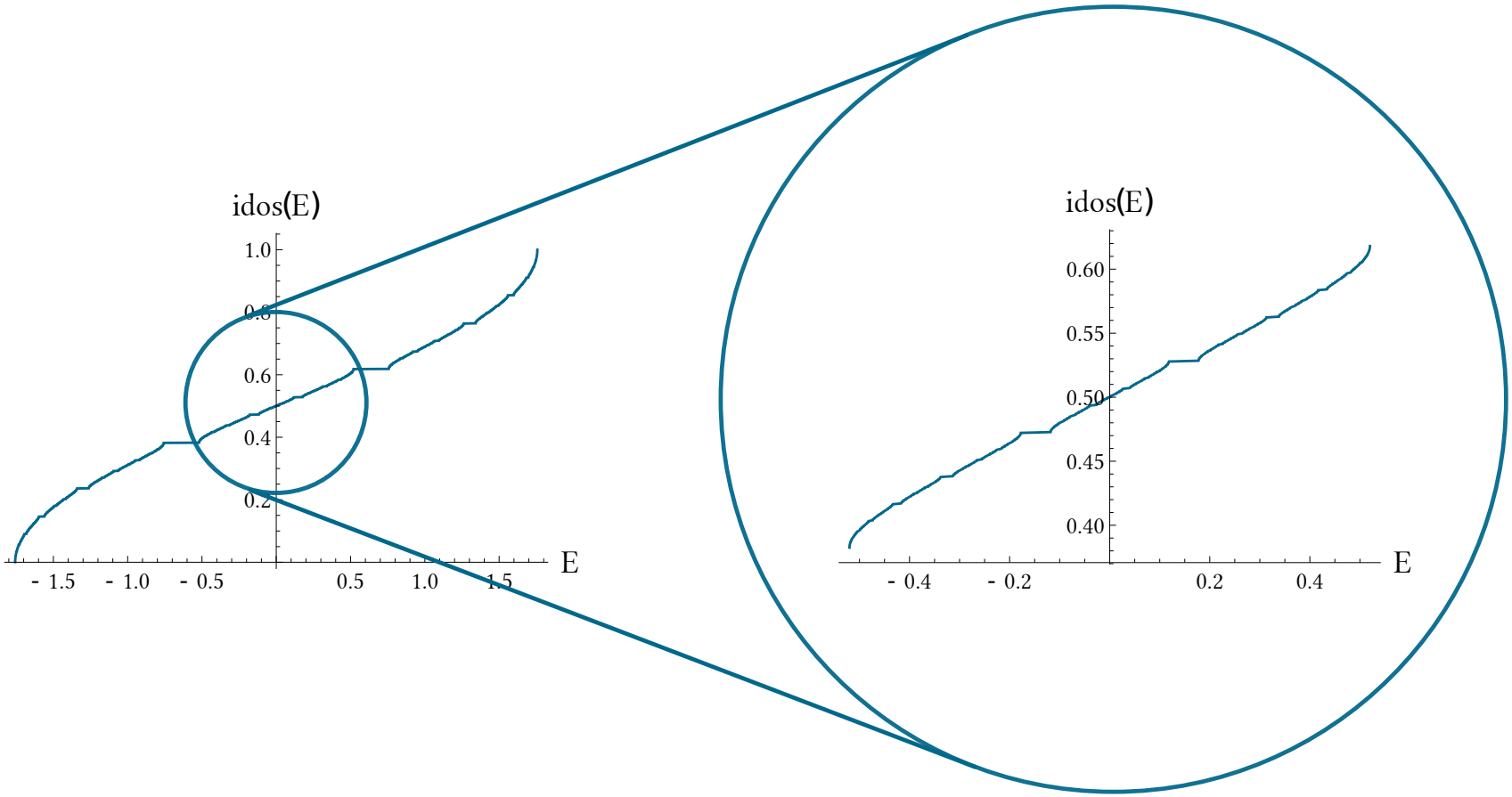


A Sierpiński triangle

$$M(L) \sim L^{d_0}, \text{ with } d_0 = \log 3 / \log 2$$

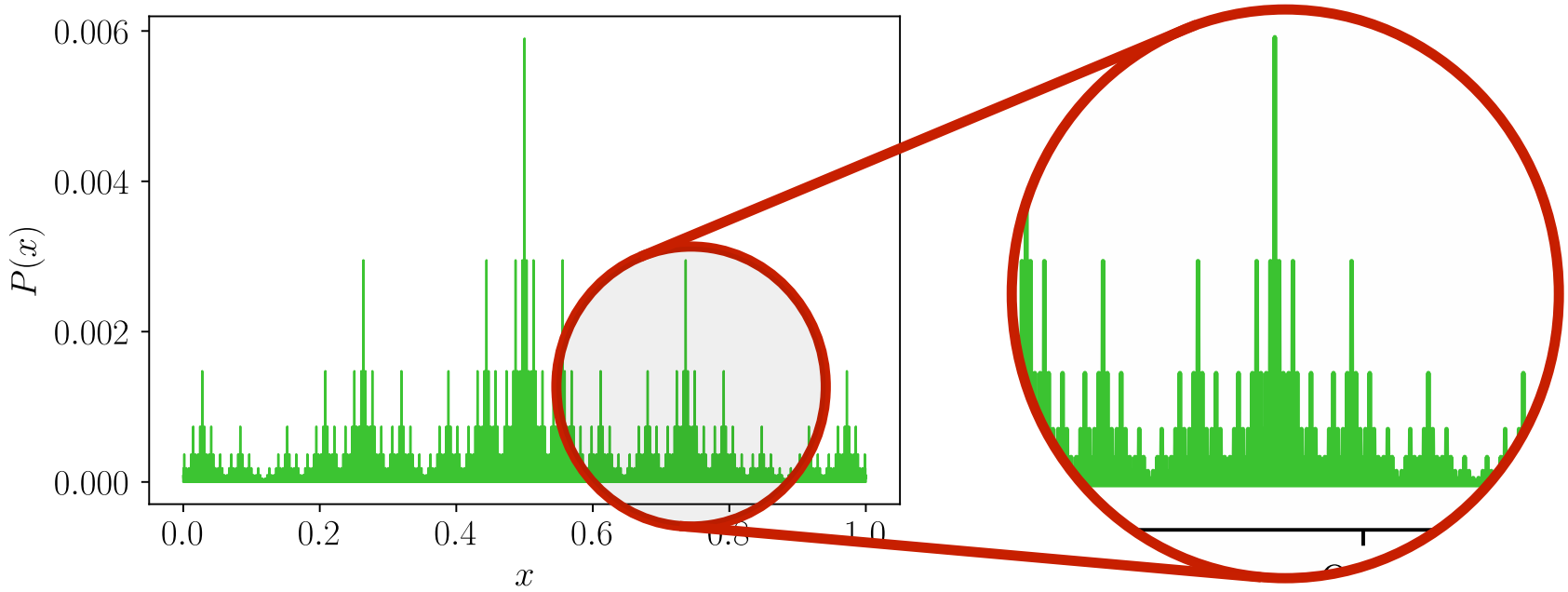
- d_0 is the Hausdorff fractal dimension
- $1 < d_0 \simeq 1.58 < 2$, signature of a fractal object
- Probe fractality of the q^{th} moment of a distribution $\rightarrow$ generalized fractal dimensions d_q.

Fractality of the Fibonacci spectrum



[Kohmoto *et al.* 89]

Fractality of the $E = 0$ eigenstate



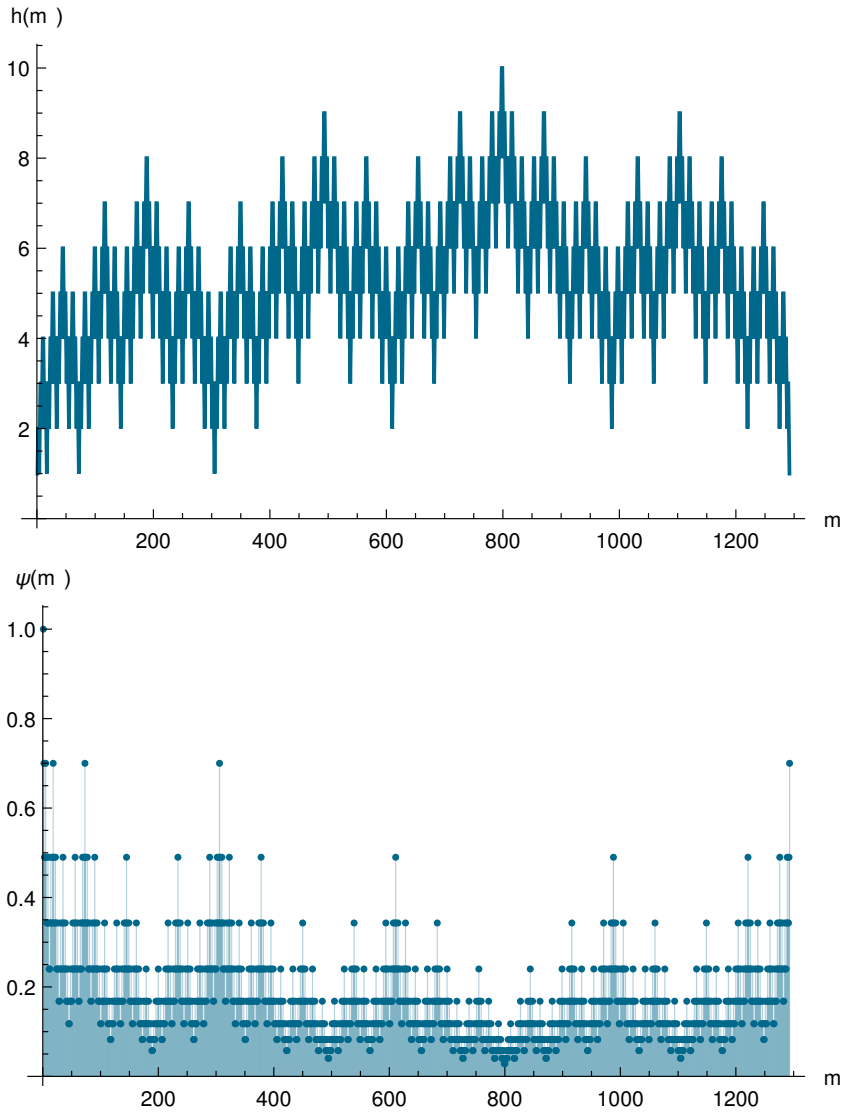
D_0 probes the fractality of the tiling :

$$D_0 = 1$$

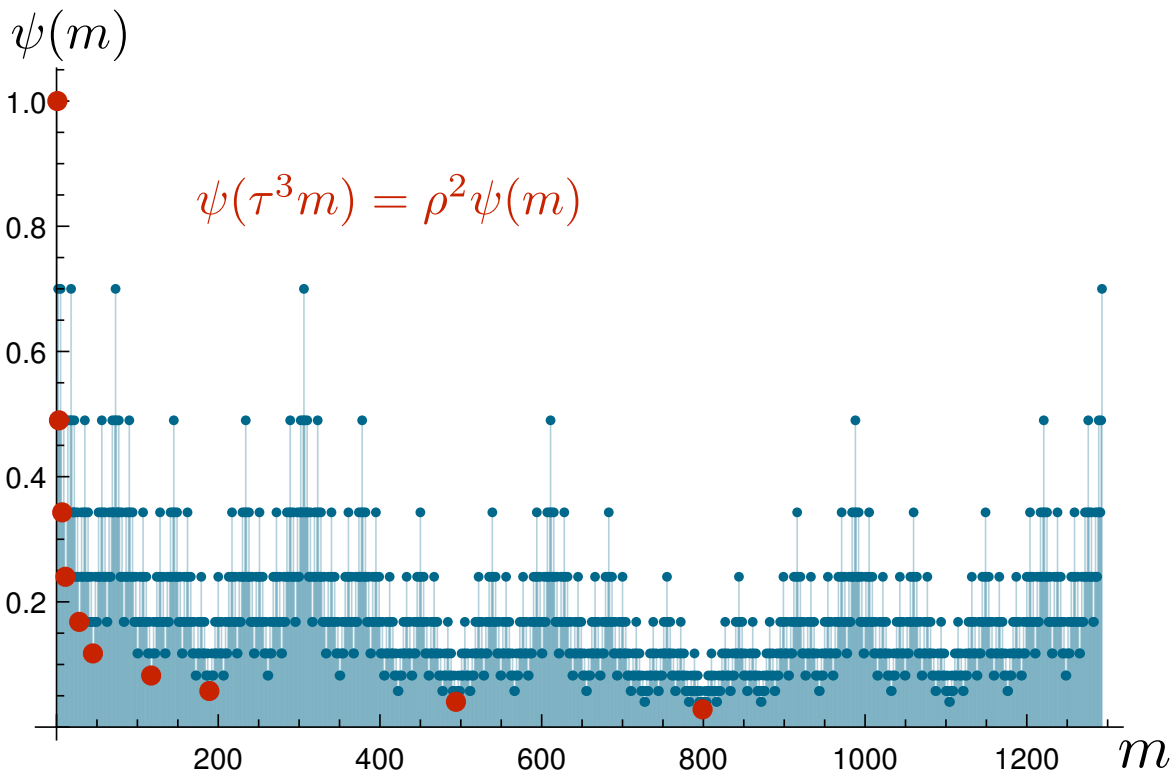
$D_{q>0}$ probes the fractality of the state :

$$0 < D_{q>0} < 1$$

Height field and state



Power-law decay on the Fibonacci chain



B3 chain and height function