Multifractality of the tight-binding eigenstates on the Fibonacci chain

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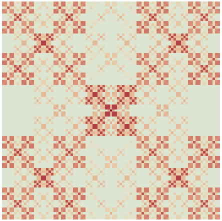
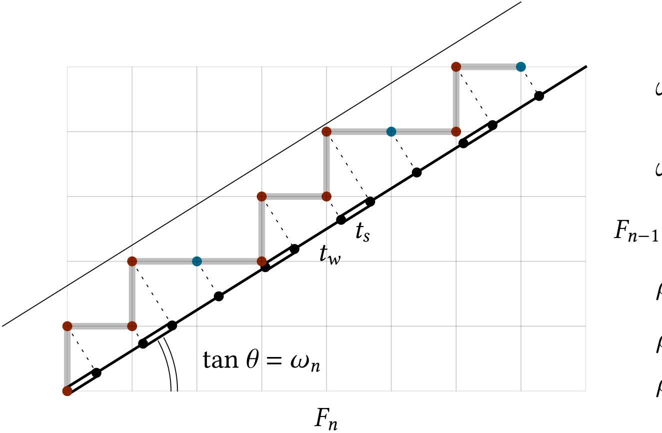


Outline

The pure hopping Fibonacci Hamiltonian



$$\omega_n = \frac{F_{n-1}}{F_n}$$

$$\omega_n \to \tau^{-1} = \frac{\sqrt{5}-1}{2}$$

$$\rho = \frac{t_w}{t_s} < 1$$

$\rho \to 1$ (weak modulation)

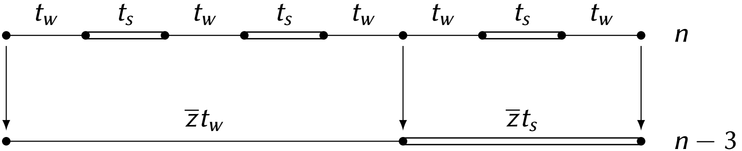
$\rho \ll 1$ (strong modulation)

Hamiltonian of the n$^{\text{th}}$ approximant:

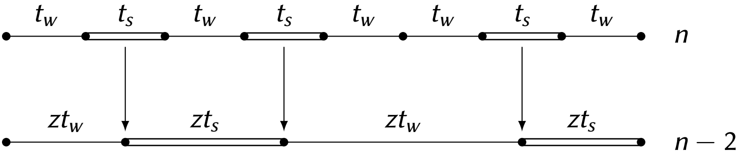
$$H_n = -\sum_i t_i^{(n)} \left|i\right\rangle \left\langle i+1\right| + \text{h.c.}$$

Perturbative renormalization group on the Fibonacci chain

- Atomic RG step (decimation of molecules)



- Molecular RG step (decimation of atoms)

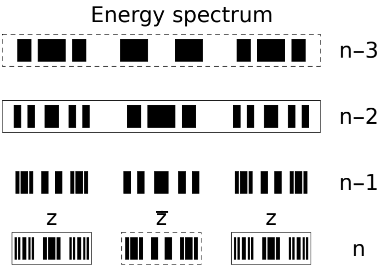


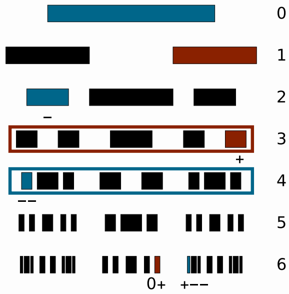
$z = \rho/2$, $\bar{z} = \rho^2$ (Niu & Nori 1986, Kalugin, Kitaev & Levitov 1986)

RG reconstruction of the energy spectrum

$$H_n = \underbrace{(zH_{n-2} - t_s)}_{\text{bonding levels}} + \underbrace{(\bar{z}H_{n-3})}_{\text{atomic levels}} + \underbrace{(zH_{n-2} + t_s)}_{\text{antibonding levels}} + \mathcal{O}(\rho^4)$$

→ simple recursive construction of the spectrum (Piéchon *et al* 1995)





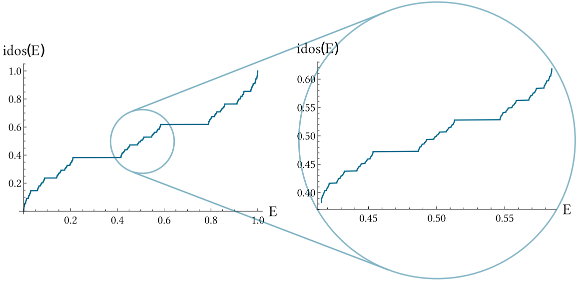
Renormalization paths characterized by

$$x(E) = \frac{n_+ + n_-}{n}$$

$$x(\text{central band}) = 0,\ x(\text{edge bands}) = 1/2$$

Multifractal analysis of the spectrum

- Scale-invariant spectrum/idos



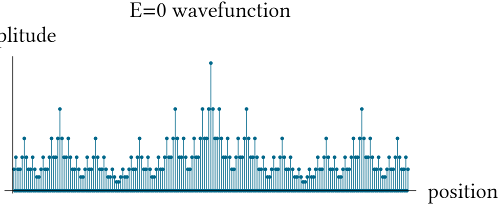
- $\Longrightarrow$ locally, the idos is a scaling law of exponent α

$$\alpha(x) = \log \tau^{-1} / \left(x \log z/\bar{z}^{2/3} + \log \bar{z}^{1/3} \right)$$

$$f(x) = \frac{x \log \left(\frac{3x}{2}\right) - (x+1) \log(x+1)^{1/3} + (1-2x) \log(1-2x)^{1/3}}{\log \tau^{-1}}$$

(Piéchon *et al* 1995, Rüdinger & Piéchon 1998)

Fractal dimensions of the wavefunctions



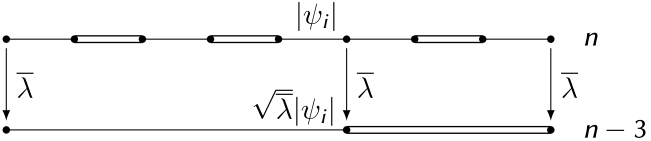
Scaling properties of ψ: $\sum_i |\psi_i^{(n)}(E)|^{2q} \sim (1/F_n)^{(q-1)D_q^{\psi}(E)}$

$$D_q(E) = 0/1 \implies \text{extended/localized state}$$
$$0 < D_q(E) < 1 \implies \text{critial state}$$

- Wavefunctions at $E = 0$ are critical (Kohmoto)
- Our work: use the RG approach to determine individual wavefunctions, and compute their fractal dimensions

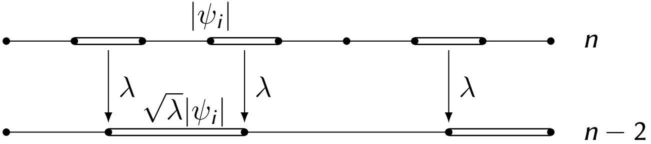
Perturbative RG for the wavefunctions

- Atomic RG



$$\overline{\lambda} \sim \frac{1}{1+2\rho^2}$$

- Molecular RG



$$\lambda \sim \frac{1}{2+\rho^2}$$

$$\begin{cases} |\psi_i^{(n)}(E)|^2 = \overline{\lambda}|\psi_{i'}^{(n-3)}(E')|^2 \text{ if } E \text{ is in the central cluster} \\ |\psi_i^{(n)}(E)|^2 = \lambda|\psi_{i'}^{(n-2)}(E')|^2 \text{ if } E \text{ is in the edge clusters} \end{cases}$$

Renormalization paths and fractal dimensions of the wavefunctions

- Fractal dimensions of the wavefunction of energy E:

$$\sum_i |\psi_i^{(n)}(E)|^{2q} \sim (1/F_n)^{(q-1)D_q^{\psi}(E)}$$

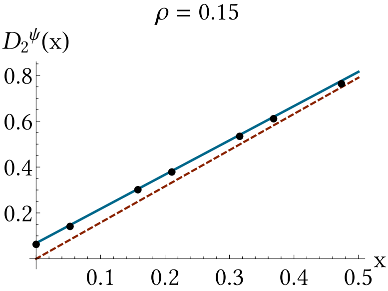
- RG step neglecting sites with small amplitudes:

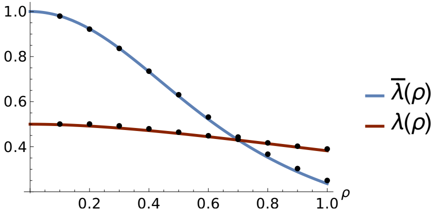
$$\text{molecular RG step: } |\psi_i^{(n)}(E)|^{2q} = \lambda^q |\psi_{i'}^{(n-2)}(E')|^{2q}$$
$$\text{atomic RG step: } |\psi_i^{(n)}(E)|^{2q} = \overline{\lambda}^q |\psi_{i'}^{(n-3)}(E')|^{2q}$$

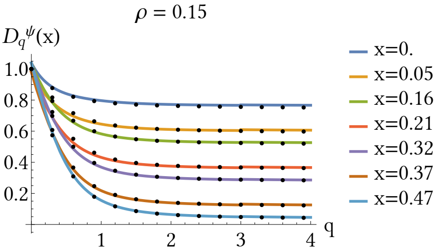
- $D_q^{\psi}(E)$ depends on the renormalization path $x(E)$:

$$(q-1)D_q^{\psi}(x) = \log \left[\left(\frac{\lambda(\rho)^q}{\lambda(\rho^q)} \right)^x \left(\frac{\bar{\lambda}(\rho)^q}{\bar{\lambda}(\rho^q)} \right)^{(1-2x)/2} \right] / \log \omega$$

Comparison with exact diagonalisation





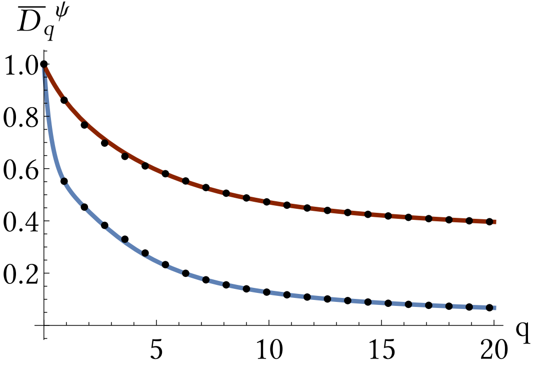


- All states are critical in the strong modulation limit
- x is the relevant parameter to describe the properties of the states

(Macé, Jagannathan, Piéchon, in preparation)

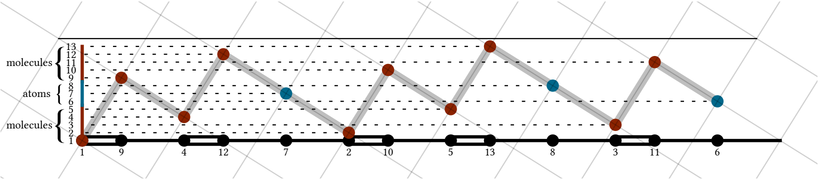
Energy averaged multifractality of the wavefunctions

$$\frac{1}{F_n}\sum_E \sum_i |\psi_i^{(n)}(E)|^{2q} \sim (1/F_n)^{(q-1)\bar{D}_q^{\psi}}$$

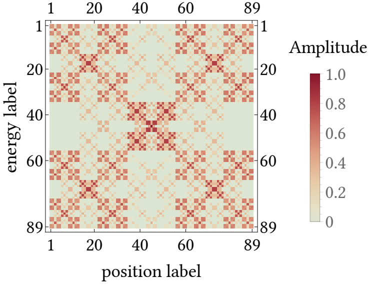


- Multifractality
- Quantitative agreement with numerical data even for large ρ.

Seeing the fractality in superspace



(Mosseri 1988)



Amplitude of the wavefunctions

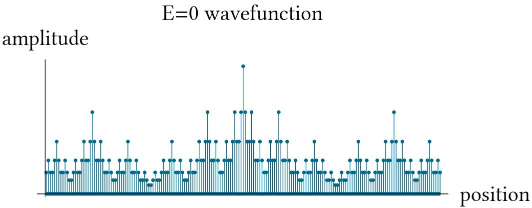
- In perpendicular space, the fractal structure is obvious.
- Energy/position symmetry appears clearly.

Conclusion and perspectives

- Fractal dimensions are important for physical properties such as transport and susceptibility.
- We have characterized the wavefunctions of the Fibonacci tight-binding chain, in the strong modulation limit, using a perturbative RG.
- We have presented for the first time analytical expressions for the fractal exponents of the full set of wavefunctions. These compare well with numerical data.
- Superspace description is meaningful for the physical properties of a quasicrystal.
- Work in progress: consequences for the diffusion and transport properties.

Special thanks to...

- Anu Jagannathan, my supervisor,
- Frédéric Piéchon, our collaborator.
  → If you want to learn more about our work, go see his conf at geo-dyn2015, november 30 in Paris!



Thank *you* for your attention!
Questions?

Nonperturbative expression for the renormalization factors:

$$\overline{\lambda}(\rho) = \frac{\sqrt{(1+\rho^2)^4 + 4\rho^4} - (1+\rho^2)^2}{2\rho^4}$$

$$\lambda(\rho) = \frac{1 + \rho^2\gamma(\rho) - \sqrt{1 + (\rho^2\gamma(\rho))^2}}{2\rho^2\gamma(\rho)}$$

$$\gamma(\rho) = \frac{1}{1+\rho}$$