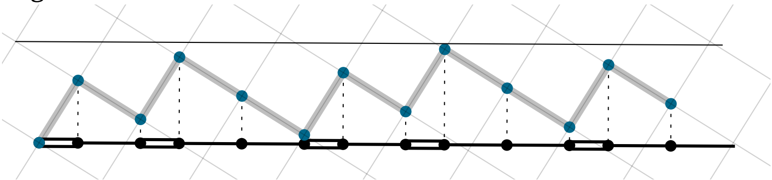
Examples of wavefunctions on quasiperiodic tilings

Construction of the model

The geometrical model:

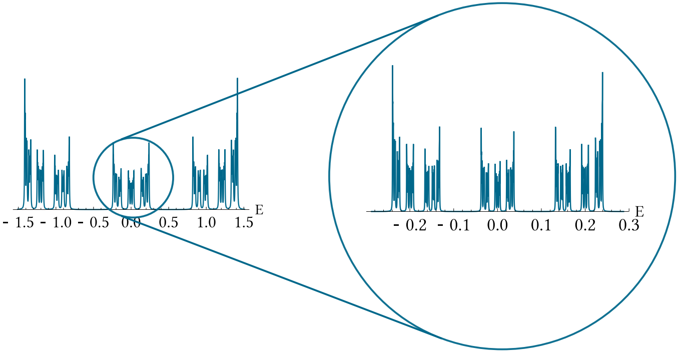


The corresponding chain of atoms:

$$t_{m-1,m}\psi_{m-1} + t_{m,m+1}\psi_{m+1} = E\psi_m$$

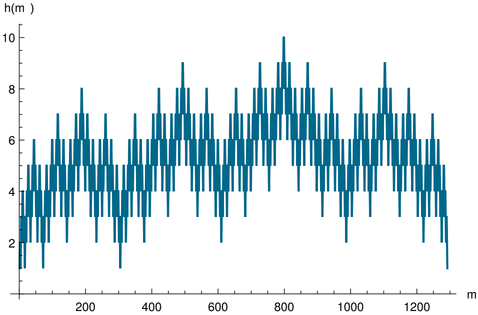
$$t_{=} = 1,\ t_{-} = \rho,\ \rho < 1$$

Spectrum



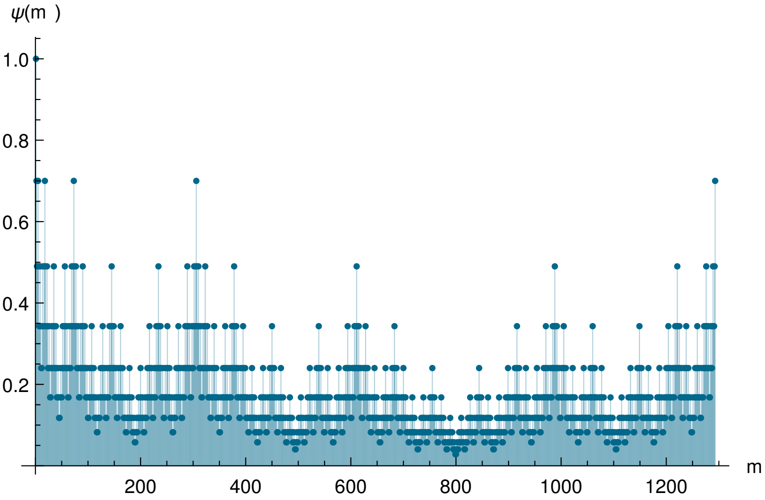
$E = 0$ is in the spectrum. We will focus on the associated state.

The height function



- The arrow function is quasiperiodic (?).
- Its integral, the height function, has logarithmic growth.

THE $E = 0$ WAVEFUNCTION

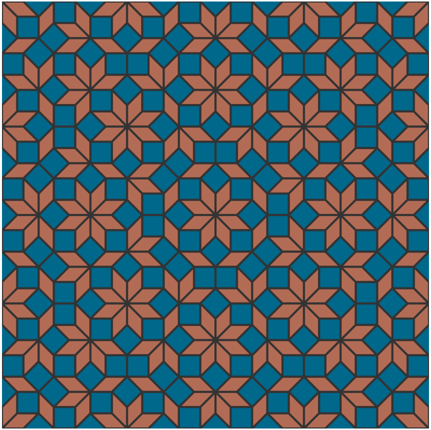


Recap

- Wavefunction construction involves a geometrical quasiperiodic function,
- Its integral, the height function, has logarithmic growth,
- This implies power law behavior of the wavefunction,
- The wavefunction is somewhat inbetween localized and extended.

$\rightarrow$ We will find again all theses features in the Ammann-Beenker case.

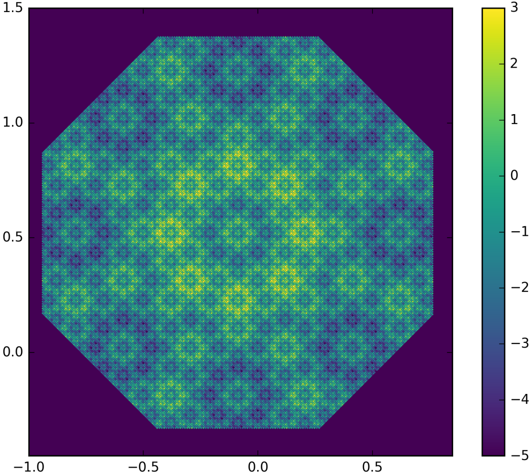
Construction of the model



$$-\sum_{n\in V(m)} \psi_n = E\psi_m$$

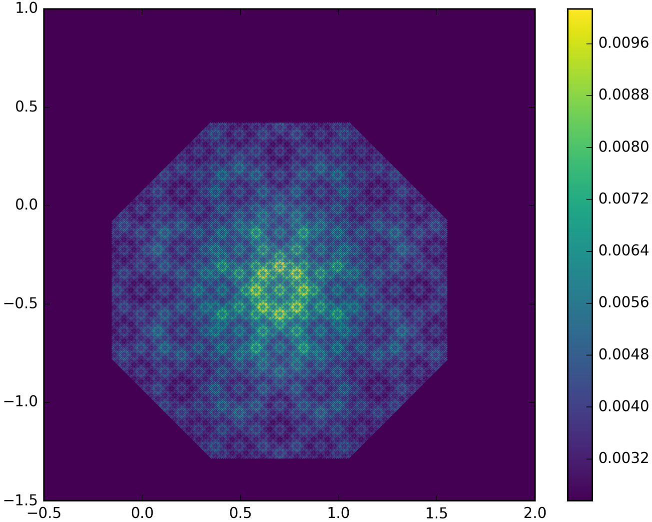
No parameters: the quasiperiodic features are encoded in the adjacency of the vertices.

The height function



- The arrow function is quasiperiodic.
- Its integral, the height function, has logarithmic growth.

The groundstate wavefunction



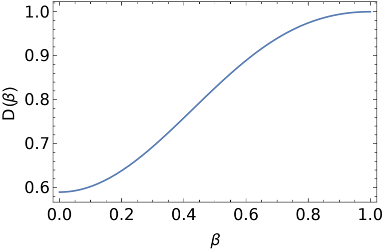
Localization degree of the wavefunction

$$D(\psi) = \lim_{R\to\infty} \frac{\log \mathrm{PR}(\psi, R)}{\log \mathrm{Vol}(R)}$$

$$D(\psi_{GS}) = \log\left(\frac{\omega(\beta_{GS}^2)^2}{\omega(\beta_{GS}^4)}\right) / \log\omega(1)$$

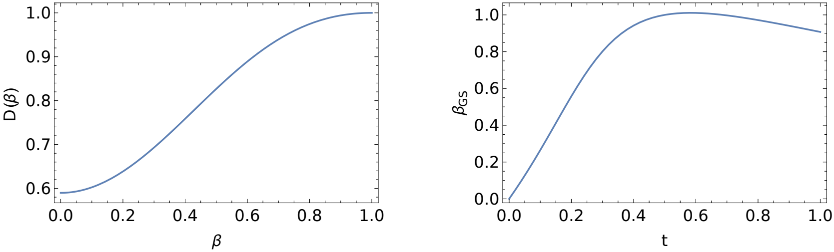
with

$$\omega(z) = \frac{4 + 9z + 4z^2 + 2\sqrt{2}\sqrt{2z^4 + 9z^3 + 14z^2 + 9z + 2}}{z}$$



Tuning the localization degree

$$-t \sum_{n \in V(m)} \psi_n + (1-t) z_m \psi_m = E \psi_m$$



Recap

- Wavefunction construction involves a geometrical quasiperiodic function,
- Its integral, the height function, has logarithmic growth,
- This implies power law behavior of the wavefunction (plus some local variations),
- The wavefunction is somewhat inbetween localized and extended,
- The localization degree of the wavefunction can be varied by tuning the model.