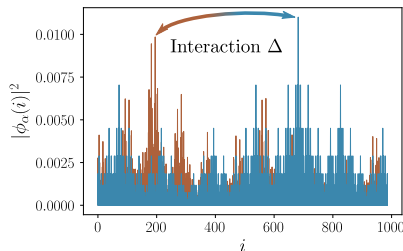


Interacting electrons on a Fibonacci chain at high temperature

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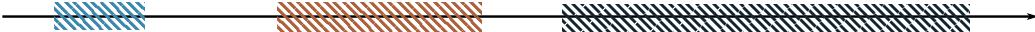
MOTIVATION: QUASIPERIODICITY + INTERACTING ELECTRONS

No interactions:

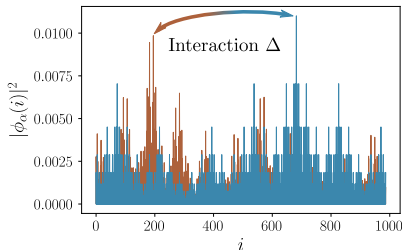
Periodic

Quasiperiodic

Random

- 
- Extended states
 - Multifractal states
 - Localized states
 - Complexity
 - Fast transport
 - Anomalous transport
 - No transport

Cold atoms: strong interactions



Quasiperiodicity (QP) + strong interactions?

Naively: delocalisation, fast transport

Results:

- weak QP: delocalisation, fast transport
- strong QP: **many-body localisation**, no transport

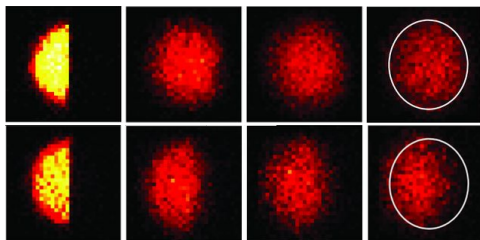
OUTLINE

- 1 Many-body localisation
- 2 Free Fibonacci chain at high energy
- 3 Interacting Fibonacci chain

MANY-BODY LOCALISATION

Isolated quantum system, **strong interactions**, disorder or quasiperiodicity

- 1 Usual: ergodic dynamics, transport, **eigenstate thermalisation hypothesis (ETH)**,
- 2 Unusual: non-ergodicity, no transport, **many-body localisation (MBL)**.



[Choi *et al* 16]

Experiments: cold ions/atoms [Schreiber *et al* 15; Smith *et al* 15; Bordia *et al* 17].

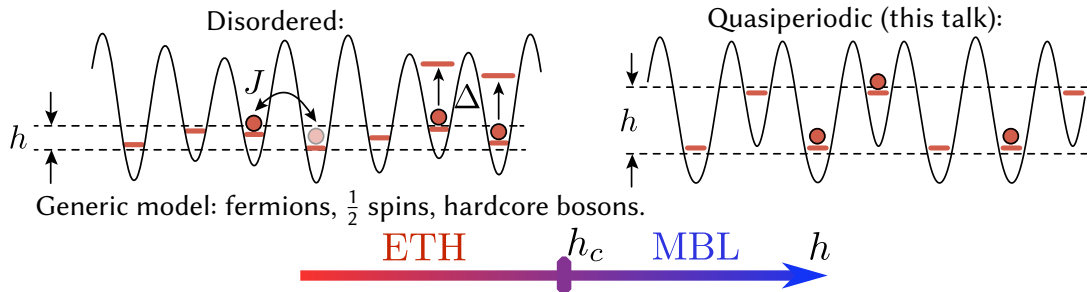
Motivations:

- ETH/MBL phase transition,
- MBL in more than 1D,
- Ingredients for MBL (this talk).

A MODEL FOR MBL

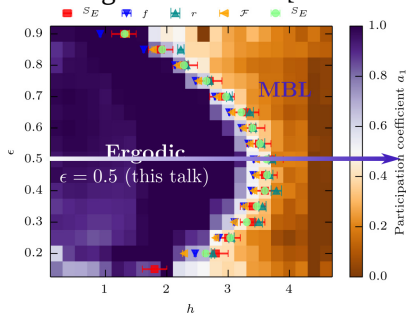
Chain of interacting spinless fermions (nb: no phonons):

$$H = \sum_{i=1}^L \left[J(c_i^\dagger c_{i+1} + \text{h.c.}) + \Delta n_i n_{i+1} - h_i n_i \right]$$



MBL PHENOMENOLOGY

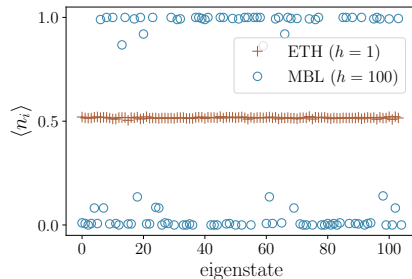
Phase diagram at $\Delta = 1$ [Luitz *et al* 15]



ETH:

- Transport, thermal observables
- High entanglement
- Non-integrability

Fermion density at $\epsilon = 0.5$



MBL:

- No transport, non-thermal observables
- Low entanglement
- Emergent integrability

INGREDIENTS FOR MBL

Usually:

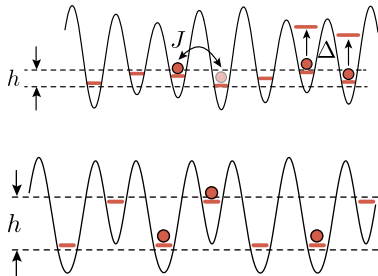
- 1 $\Delta = 0$: **localized** (random, Aubry-André potential)
- 2 $\Delta \neq 0$: localization persists

This talk

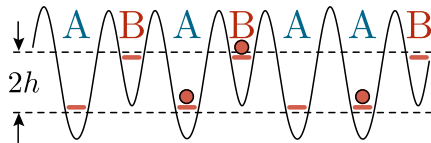
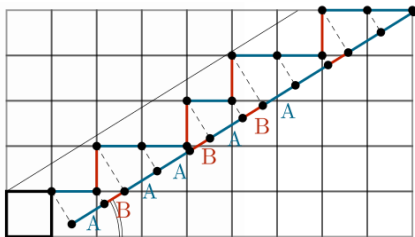
- 1 $\Delta = 0$: **multifractal** (quasiperiodic potential)
- 2 $\Delta \neq 0$: localization appears

Interest

- MBL is **generic**
- Interplay between quasiperiodicity and MBL



INTERACTING FERMIONS ON THE FIBONACCI CHAIN



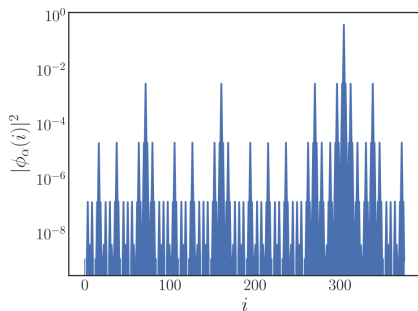
$$H = \sum_{i=1}^L \left[J(c_i^\dagger c_{i+1} + \text{h.c.}) + \Delta n_i n_{i+1} - h_i n_i \right]$$

Method: numerical **exact diagonalization**

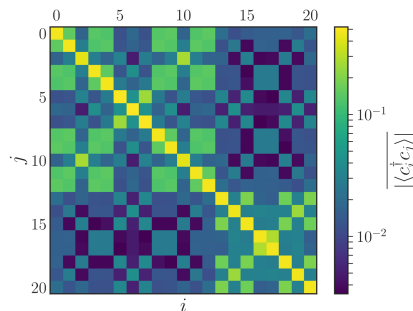
- High energy + non-integrable: **no analytical methods**
- $L/2$ fermions on L sites: $\# \text{states} \sim 2^L / \sqrt{L} \rightarrow$ **memory is limiting**
State-of-the art: $L = 24$ [Pietracaprina *et al* 18]
- Fibonacci: **few samples**: $L/2$ non-equivalent systems of size L .

FREE FERMIONS PROPERTIES

- Multifractal single particle wavefunctions [Ostlund; Kalugin; Khomoto; ...]
- Anomalous transport [Mayou; Schreiber; Varma & Žnidarič; ...]



Single particle wavefunction at the Fermi level

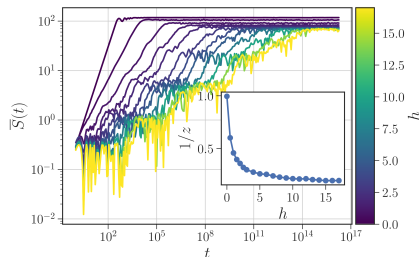


Correlations of highly excited states [Macé *et al* 19]

FREE FERMIONS ENTANGLEMENT

Entanglement entropy $S(\psi)$: a many-body **locality** probe

- $S(\psi) = \#\{\text{bits of information recoverable by local measurements}\}$
- $S(\psi)$ large: extended (entangled) state, $S(\psi)$ small: localized state.



Entanglement growth starting from localized fermions [Macé *et al* 19]

Fibonacci fermions: **anomalous** growth

$$S(t) \sim t^{\frac{1}{z}}, \quad z > 1$$

Compare with:

- Periodic system: $z = 1$ (ballistic growth),
- Disordered system: $z \rightarrow \infty$ (no growth).

Conclusion

Anomalous, intermediate prop. even at high energy.

THE ETH/MBL TRANSITION: 1) SPECTRAL PROPERTIES

Gap ratios [Oganesian, Huse]

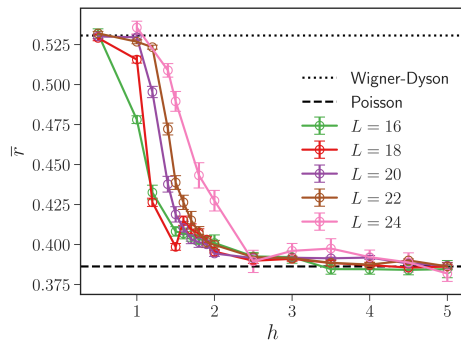
$$r_n = \min \left(\frac{g_{n+1}}{g_n}, \frac{g_n}{g_{n+1}} \right)$$

- **ETH:** random matrix-like spectrum

$$\bar{r}_{\text{ETH}} \simeq 0.53$$

- **MBL:** independant levels

$$\bar{r}_{\text{MBL}} \simeq 0.39$$



Compatible with **ETH/MBL transition**,
 $h^* \simeq 2.5$.

THE ETH/MBL TRANSITION: 2) ENTANGLEMENT

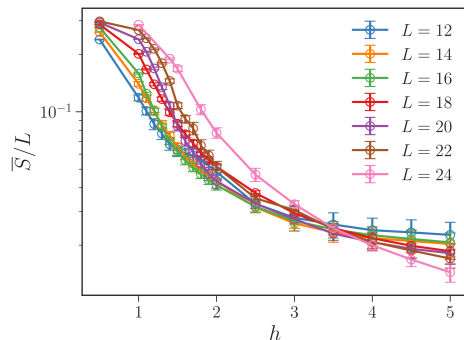
Entanglement entropy:

- **ETH:** coincides with thermodynamic entropy: **extensive**

$$\bar{S}_{\text{ETH}} \simeq L$$

- **MBL:** **sub-extensive**

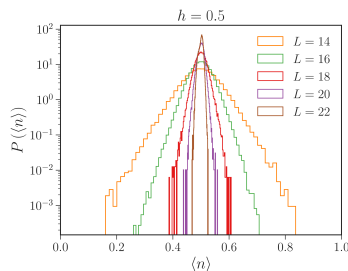
$$\bar{S}_{\text{MBL}}/L \rightarrow 0$$



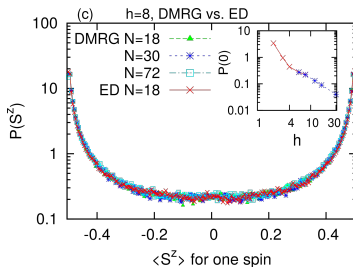
Compatible with **ETH/MBL transition**,
 $h^* \simeq 3.5$.

THE ETH/MBL TRANSITION: 3) LOCAL OBSERVABLES

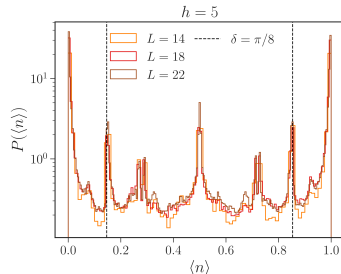
Expect: $\langle n_i \rangle_{\text{ETH}} = \frac{1}{2}$, $\langle n_i \rangle_{\text{MBL}} \simeq 0 \text{ or } 1$.



ETH Fibonacci



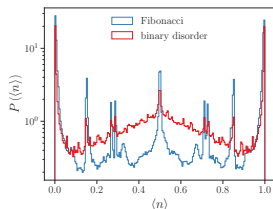
MBL random [Lim, Sheng 15]



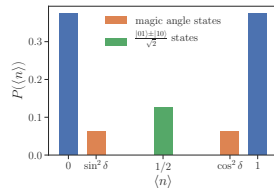
MBL Fibonacci

Fibonacci MBL: **extra structure** \rightarrow link with QP geometry?

with $\delta = 0, \frac{\pi}{4}, \frac{\pi}{8}$.



- **Correlated** modulation (Fibonacci)



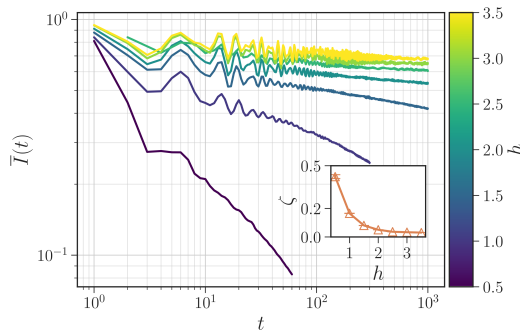
DYNAMICS: IMBALANCE

Imbalance experiment:

- $|\psi(t=0)\rangle$: high energy product state
- Imbalance: **distance** from initial state
$$I(t) = \frac{4}{L} \sum_i \langle (n_i(t) - \frac{1}{2}) (n_i(0) - \frac{1}{2}) \rangle$$

Properties:

- **ETH: power-law decay** $I(t) \sim t^{-\zeta}$
- **MBL: saturation** $I(t \rightarrow \infty) = \text{cst} > 0$
→ **memory** of the initial state
[Luitz *et al* 15]

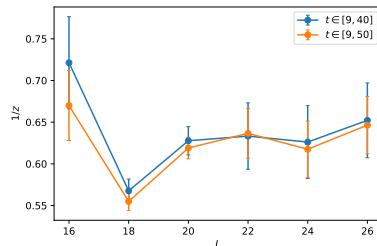
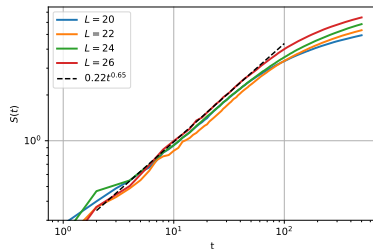


$\zeta(h \geq 2.5) = 0 \rightarrow$ MBL phase transition.

DYNAMICS: ENTANGLEMENT

ETH phase: expect $S(t) \propto t$

Fibonacci: **anomalous** $S(t) \propto t^{1/z}$, $z > 1$.



Usual explanation: **rare regions** [Vosk; Potter 15]

Fibonacci: no rare regions ...

Finite size effect? [Setiawan *et al* 17], initial state fluctuations? [Lüschen *et al* 17]

CONCLUSION

Non-interacting Fibonacci fermions

- Geometry: intermediate complexity
- **Multifractality** even at high energy
- Intermediate, **anomalous** transport, even at the many-body level

Interacting Fibonacci fermions

- Weak quasiperiodicity: **thermal phase (ETH)**
- Strong quasiperiodicity: **localized phase (MBL)**
- MBL bears sign of **quasiperiodicity** (locally entangled states)

→ anomalous transport in the ETH phase: why?

→ vicinity of the non-interacting point?

[*Macé, Laflorencie, Alet, SciPost Phys. 6, 050 (2019)*]