ワークショップで扱うモデル

一般式

$$
\begin{aligned}
p_1(i) = \beta_1(c_i,t_i) &+ \sum_{f=1}^{n_{\omega 1}} L_{\omega 1}(c_i,f)\omega_1(s_i,f) + \sum_{f=1}^{n_{\varepsilon 1}} L_{\varepsilon 1}(c_i,f)\varepsilon_1(s_i,f,t_i) \\
&+ \sum_{f=1}^{n_{\eta 1}} L_1(c_i,f)\eta_1(v_i,f) + \sum_{p=1}^{n_p} \gamma_1(c_i,t_i,p)X(x_i,t_i,p) + \sum_{k=1}^{n_k} \lambda_1(k)Q(i,k)
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
p_2(i) = \beta_2(c_i,t_i) &+ \sum_{f=1}^{n_{\omega 2}} L_{\omega 2}(c_i,f)\omega_2(s_i,f) + \sum_{f=1}^{n_{\varepsilon 2}} L_{\varepsilon 2}(c_i,f)\varepsilon_2(s_i,f,t_i) \\
&+ \sum_{f=1}^{n_{\eta 2}} L_2(c_i,f)\eta_2(v_i,f) + \sum_{p=1}^{n_p} \gamma_2(c_i,t_i,p)X(x_i,t_i,p) + \sum_{k=1}^{n_k} \lambda_2(k)Q(i,k)
\end{aligned} \tag{2}
$$

Part Ⅰ

$$p_1(i) = \beta_1(t_i) + \omega_1(s_i) + \varepsilon_1(s_i,t_i) \tag{3}$$

$$p_2(i) = \beta_2(t_i) + \omega_2(s_i) + \varepsilon_2(s_i,t_i) \tag{4}$$

Part Ⅲ (i)

$$p_1(i) = \beta_1(t_i) + \omega_1(s_i) + \varepsilon_1(s_i,t_i) + \lambda_1 Q(i) \tag{5}$$

$$p_2(i) = \beta_2(t_i) + \omega_2(s_i) + \varepsilon_2(s_i,t_i) + \lambda_2 Q(i) \tag{6}$$

(ii)

$$p_1(i) = \beta_1(t_i) + \omega_1(s_i) + \varepsilon_1(s_i,t_i) + \eta_1(v_i) \tag{7}$$

$$p_2(i) = \beta_2(t_i) + \omega_2(s_i) + \varepsilon_2(s_i,t_i) + \eta_2(v_i) \tag{8}$$

(iii)

$$p_1(i) = \beta_1(c_i,t_i) + \sum_{f=1}^{n_{\omega 1}} L_{\omega 1}(c_i,f)\omega_1(s_i,f) + \sum_{f=1}^{n_{\varepsilon 1}} L_{\varepsilon 1}(c_i,f)\varepsilon_1(s_i,f,t_i) \tag{9}$$

$$p_2(i) = \beta_2(c_i,t_i) + \sum_{f=1}^{n_{\omega 2}} L_{\omega 2}(c_i,f)\omega_2(s_i,f) + \sum_{f=1}^{n_{\varepsilon 2}} L_{\varepsilon 2}(c_i,f)\varepsilon_2(s_i,f,t_i) \tag{10}$$

(iv)

$$p_1(i) = \beta_1(t_i) + \omega_1(s_i) + \varepsilon_1(s_i,t_i) + + \sum_{p=1}^{n_p} \gamma_1(t_i,p)X(x_i,t_i,p) \tag{11}$$

$$p_2(i) = \beta_2(t_i) + \omega_2(s_i) + \varepsilon_2(s_i,t_i) + + \sum_{p=1}^{n_p} \gamma_2(t_i,p)X(x_i,t_i,p) \tag{12}$$

Table 3
Definition of mathematical notation, including the symbol used, its type (Index, Data, fixed effects "FE", random effects "RE", intermediate quantity computed internally "IQ", and derived quantities that are outputted for users "DQ"), and its dimension.

Name	Symbol	Type	Dimensions
Observation number	i	Index	–
Spatial location number	s	Index	–
Time interval number	t	Index	–
Category number	c	Index	–
Factor number	f	Index	–
Habitat covariate number	p	Index	–
Catchability covariate number	k	Index	–
Stratum number	l	Index	–
Index number for measures of center-of-gravity	m	Index	–
Index number for other book-keeping	z	Index	–
Sample response	b_i	Data	n_i
Location for each sample	s_i	Data	n_i
Time interval for each sample	t_i	Data	n_i
Category for each sample	c_i	Data	n_i
Overdispersion level for each sample	v_i	Data	n_i
Area covered by each sample	a_i	Data	n_i
Distance between locations s_n and s_m	$d(s_n, s_m)$	Data	$n_s \times n_s$
Habitat covariates for each location, time, and variable	$X(s, t, p)$	Data	$n_s \times n_t \times n_p$
Catchability covariates for each sample and variable	$Q(i, k)$	Data	$n_i \times n_k$
Area associated with location in each stratum	$a(s, l)$	Data	$n_s \times n_l$
Statistic for each location used to calculate center of gravity	$z(s, m)$	Data	$n_s \times n_m$
Intercept for 1st linear predictor	$\beta_1(c, t)$	FE/RE	$n_c \times n_t$
Intercept for 2st linear predictor	$\beta_2(c, t)$	FE/RE	$n_c \times n_t$
Loadings matrix for spatial covariation for 1st linear predictor	$L_{\omega1}(c, f)$	FE	$n_c \times n_{\omega1}$
Loadings matrix for spatial covariation for 2nd linear predictor	$L_{\omega2}(c, f)$	FE	$n_c \times n_{\omega2}$
Loadings matrix for spatio-temporal covariation for 1st linear predictor	$L_{\varepsilon1}(c, f)$	FE	$n_c \times n_{\varepsilon1}$
Loadings matrix for spatio-temporal covariation for 2nd linear predictor	$L_{\varepsilon2}(c, f)$	FE	$n_c \times n_{\varepsilon2}$
Loadings matrix for overdispersion covariation for 1st linear predictor	$L_1(c, f)$	FE	$n_c \times n_{\eta1}$
Loadings matrix for overdispersion covariation for 2nd linear predictor	$L_2(c, f)$	FE	$n_c \times n_{\eta2}$
Impact of habitat covariates on 1st linear predictor	$\gamma_1(c, t, p)$	FE	$n_c \times n_t \times n_p$
Impact of habitat covariates on 2nd linear predictor	$\gamma_2(c, t, p)$	FE	$n_c \times n_t \times n_p$
Impact of catchability covariates on 1st linear predictor	$\lambda_1(k)$	FE	n_k
Impact of catchability covariates on 2nd linear predictor	$\lambda_2(k)$	FE	n_k
Parameters governing residual variation	$\sigma_m^2(c, z)$	FE	$n_c \times 2$
Decorrelation rate for 1st linear predictor	κ_1	FE	1
Decorrelation rate for 2nd linear predictor	κ_2	FE	1
Autocorrelation for intercepts of 1st linear predictor	$\rho_{\beta1}$	FE	1
Autocorrelation for intercepts of 2nd linear predictor	$\rho_{\beta2}$	FE	1
Conditional variance for intercepts of 1st linear predictor	$\sigma_{\beta1}^2$	FE	1
Conditional variance for intercepts of 2nd linear predictor	$\sigma_{\beta2}^2$	FE	1
Autocorrelation for spatio-temporal covariation of 1st linear predictor	$\rho_{\varepsilon1}$	FE	1
Autocorrelation for spatio-temporal covariation of 2nd linear predictor	$\rho_{\varepsilon2}$	FE	1
Parameters governing geometric anisotropy	$h(z)$	FE	2
Spatial factors for 1st linear predictor	$\omega_1(s, f)$	RE	$n_s \times n_{\omega1}$
Spatial factors for 2nd linear predictor	$\omega_2(s, f)$	RE	$n_s \times n_{\omega2}$
Spatio-temporal factors for 1st linear predictor	$\varepsilon_1(s, f, t)$	RE	$n_s \times n_{\varepsilon1} \times n_t$
Spatio-temporal factors for 2nd linear predictor	$\varepsilon_2(s, f, t)$	RE	$n_s \times n_{\varepsilon1} \times n_t$
Overdispersion factors for 1st linear predictor	$\eta_1(v, f)$	RE	$n_v \times n_{\eta1}$
Overdispersion factors for 2nd linear predictor	$\eta_2(v, f)$	RE	$n_v \times n_{\eta2}$
1st linear predictor	$p_1(i)$	IQ	n_i
2nd linear predictor	$p_2(i)$	IQ	n_i
1st link-transformed predictor	$r_1(i)$	IQ	n_i
2nd link-transformed predictor	$r_2(i)$	IQ	n_i
Spatial correlation matrix for 1st linear predictor	$\boldsymbol{R}_1$	IQ	$n_s \times n_s$
Spatial correlation matrix for 2nd linear predictor	$\boldsymbol{R}_2$	IQ	$n_s \times n_s$
Anisotropy matrix	$\boldsymbol{H}$	IQ	2×2
Predicted density	$d^*(s, c, t)$	DQ	$n_s \times n_c \times n_t$
Index of abundance	$I(c, t, l)$	DQ	$n_c \times n_t \times n_l$
Center of gravity	$Z(c, t, m)$	DQ	$n_c \times n_t \times n_m$
Average density	$D(c, t, l)$	DQ	$n_c \times n_t \times n_l$
Effective area occupied	$A(c, t, l)$	DQ	$n_c \times n_t \times n_l$
Rotation matrix for spatial covariation for 1st linear predictor	$\boldsymbol{B}_{\omega1}$	DQ	$n_c \times n_c$
Rotation matrix for spatial covariation for 2nd linear predictor	$\boldsymbol{B}_{\omega2}$	DQ	$n_c \times n_c$
Rotation matrix for spatio-temporal covariation for 1st linear predictor	$\boldsymbol{B}_{\varepsilon1}$	DQ	$n_c \times n_c$
Rotation matrix for spatio-temporal covariation for 2nd linear predictor	$\boldsymbol{B}_{\varepsilon2}$	DQ	$n_c \times n_c$
Rotation matrix for overdispersion covariation for 1st linear predictor	$\boldsymbol{B}_1$	DQ	$n_c \times n_c$
Rotation matrix for overdispersion covariation for 2nd linear predictor	$\boldsymbol{B}_2$	DQ	$n_c \times n_c$
Rotated loadings matrix for spatial covariation for 1st linear predictor	$L_{\omega1}^*(c, f)$	DQ	$n_c \times n_{\omega1}$
Rotated loadings for spatial covariation for 2nd linear predictor	$L_{\omega2}^*(c, f)$	DQ	$n_c \times n_{\omega2}$
Rotated loadings for spatio-temporal covariation for 1st linear predictor	$L_{\varepsilon1}^*(c, f)$	DQ	$n_c \times n_{\varepsilon1}$
Rotated loadings for spatio-temporal covariation for 2nd linear predictor	$L_{\varepsilon2}^*(c, f)$	DQ	$n_c \times n_{\varepsilon2}$
Rotated loadings for overdispersion covariation for 1st linear predictor	$L_1^*(c, f)$	DQ	$n_c \times n_{\eta1}$
Rotated loadings for overdispersion covariation for 2nd linear predictor	$L_2^*(c, f)$	DQ	$n_c \times n_{\eta2}$
Rotated spatial factors for 1st linear predictor	$\omega_1^*(s, f)$	DQ	$n_s \times n_{\omega1}$
Rotated spatial factors for 2nd linear predictor	$\omega_2^*(s, f)$	DQ	$n_s \times n_{\omega2}$
Rotated spatio-temporal factors for 1st linear predictor	$\varepsilon_1^*(s, f, t)$	DQ	$n_s \times n_{\varepsilon1} \times n_t$
Rotated spatio-temporal factors for 2nd linear predictor	$\varepsilon_2^*(s, f, t)$	DQ	$n_s \times n_{\varepsilon1} \times n_t$
Rotated overdispersion factors for 1st linear predictor	$\eta_1^*(v, f)$	DQ	$n_v \times n_{\eta1}$
Rotated overdispersion factors for 2nd linear predictor	$\eta_2^*(v, f)$	DQ	$n_v \times n_{\eta2}$

each abundance-index to estimate changes in population density over time. Scientists in this region have used a model-based framework to develop the abundance-index in part because the survey is conducted using equipment and staff from contracted fishery vessels and previous research showed the importance of accounting for differences in catchability among vessels (Helser et al., 2004). Index standardization for 2013 PFMC stock assessments was conducted using an R package *nwfscDeltaGLM*, and subsequent research using this package suggested that treating the interaction of spatial stratum and year as a random effect was a robust approach to account for spatial variability and low sample sizes in some strata (Thorson and Ward, 2013). However, a random interaction of stratum and year still required defining spatial strata a priori, and this approach did not include any information regarding which strata were adjacent to one another. Subsequently, Shelton et al. (2014) demonstrated that a spatially-explicit model could explain a substantial portion of residual variation by accounting for variable population density within existing spatial strata, but this approach required long run-times relative to spatially-stratified methods.

導出パラメータ

推定局所密度 d^*

$$d^*(s,c,t) = r_1(s,c,t) \times r_2(s,c,t) \tag{13}$$

CPUE や重量データの時

$$\begin{aligned} r_1(i) &= \mathrm{logit}^{-1}(p_1(i)) \\ r_2(i) &= a_i \times \log^{-1}(p_2(i)) \end{aligned} \tag{14}$$

個体数データの時

$$\begin{aligned} r_1(i) &= 1 - \exp(-a_i \times \exp(p_1(i))) \\ r_2(i) &= \frac{a_i \times \exp(p_1(i))}{r_1(i)} \times \exp(p_2(i)) \end{aligned} \tag{15}$$

資源量指数 I

$$I(c,t,l) = \sum_{x=1}^{n_x} (a(s,l) \times d^*(s,c,t)) \tag{16}$$

有効面積 A

$$A(c,t,l) = \frac{I(c,t,l)}{D(c,t,l)} \tag{17}$$

D は biomass-weighted average density で

$$D(c,t,l) = \sum_{x=1}^{n_x} \left(\frac{a(s,l) \times d^*(s,c,t)}{I(c,t,l)} d^*(s,c,t) \right) \tag{18}$$

重心 Z

$$Z(c,t,m) = \sum_{x=1}^{n_x} \frac{z(s,m) \times a(s,l) \times d^*(s,c,t)}{I(c,t,l)} \tag{19}$$