Matrices

Types of matrices

- Row matrix = Matrix having only one row and n columns.
  - For Example

$$A = \begin{bmatrix} a & b & c & d \end{bmatrix}$$

- Column Matrix = Matrix having one column and n rows.
  - For Example

$$A = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

- Square matrix = Same number of rows and column
  - $$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$$
- Null Matrix = Matrix with all the elements are 0.
  - For example,

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- Diagonal matrix = Square matrix with all elements 0 except those of the leading diagonal.
  - $$a_{ij} = 0 \ \ \forall \, i \neq j$$
  - For Example,

$$\begin{bmatrix} 9 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

- Scaler Matrix = A diagonal matrix in which all the elements in the leading diagonal are equal.
  - $$a_{ij} = 0 \ \ \forall \, i \neq j$$
  - $$a_{ij} = K \ \ \forall \, i = j$$

- For Example,

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

- Unit/Identity Matrix = Scaler Matrix with K = 1.
  - $$a_{ij} = 0 \ \ \forall \, i \neq j$$
  - $$a_{ij} = 1 \ \ \forall \, i = j$$
  - For Example,

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Transpose of a matrix = A matrix which obtained by changing the rows and columns of a matrix A. denoted by A^T or A'.
  - For Example,

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad \therefore A^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

- Symmetric Matrix = A square matrix $A = [a_{ij}]$ is said to be symmetric
  - when $A_{ij} = A_{ji} \ \ \forall \, i \,\&\, j$.
  - A' = A
    - For Example,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 4 \\ 3 & 4 & 7 \end{bmatrix}$$

    - Here $A = A^T$ Therefor A = Symmetric Matrix.
- Skew Symmetric Matrix = A square matrix $A = [a_{ij}]$ is said to be skew symmetric
  - when $A_{ij} = -A_{ji} \ \ \forall \, i \,\&\, j$ and
  - all elements of leading diagonal are 0.
  - For Example,

$$A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix} is\ Skew\ Symmetric$$

- $$\because A^T = \begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -4 \\ 3 & 4 & 0 \end{bmatrix} \quad \& \ A = -A^T$$

2024-07-25

Operations on matrices

Addition & Subtraction

- Only valid if the order of the matrices is same.
- $$If\ A = \begin{bmatrix} 4 & 6 \\ 2 & 3 \end{bmatrix} \ \& \ B = \begin{bmatrix} 7 & 9 \\ 3 & 8 \end{bmatrix}$$
- Then,

$$A + B = \begin{bmatrix} 4+7 & 6+9 \\ 2+3 & 3+8 \end{bmatrix} = \begin{bmatrix} 13 & 15 \\ 5 & 11 \end{bmatrix}$$

- Addition is commutative. i.e. $A + B = B + A$

Multiplication

- Only valid if the number of columns of the first matrix is equal to the number of rows of the second matrix.
- Example:
  - Find AB and BA for

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \\ 5 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & 0 \\ 2 & -1 & 4 \end{bmatrix}$$

  -

$$AB = \begin{bmatrix} 0 & 7 & -4 \\ 6 & -3 & 12 \\ 9 & 13 & 8 \end{bmatrix} \quad BA = \begin{bmatrix} 2 & 8 \\ 24 & 3 \end{bmatrix}$$

- We see here that A & B are not commutative. Because, $AB \neq BA$
- Example:
  - For

$$A = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 1 \\ 5 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 7 & 2 \\ -5 & -1 \end{bmatrix}$$

- Prove that:
- (AB)C = A(BC) [Associative]
- LHS
-
$$\left(\begin{bmatrix}2 & 0\\-1 & 2\end{bmatrix}\cdot\begin{bmatrix}3 & 1\\5 & 0\end{bmatrix}\right)\cdot\begin{bmatrix}7 & 2\\-5 & -1\end{bmatrix}$$
-
$$\begin{bmatrix}6 & 2\\7 & -1\end{bmatrix}\cdot\begin{bmatrix}7 & 2\\-5 & -1\end{bmatrix}$$
-
$$\begin{bmatrix}32 & 10\\54 & 15\end{bmatrix}$$
- RHS
-
$$\begin{bmatrix}2 & 0\\-1 & 2\end{bmatrix}\cdot\left(\begin{bmatrix}3 & 1\\5 & 0\end{bmatrix}\cdot\begin{bmatrix}7 & 2\\-5 & -1\end{bmatrix}\right)$$
-
$$\begin{bmatrix}2 & 0\\-1 & 2\end{bmatrix}\cdot\begin{bmatrix}16 & 5\\35 & 10\end{bmatrix}$$
-
$$\begin{bmatrix}32 & 10\\54 & 15\end{bmatrix}$$
- $LHS = RHS$
- A(B+C) = AB + AC [Distributive]
- LHS
-
$$\begin{bmatrix}2 & 0\\-1 & 2\end{bmatrix}\cdot\left(\begin{bmatrix}3 & 1\\5 & 0\end{bmatrix}+\begin{bmatrix}7 & 2\\-5 & -1\end{bmatrix}\right)$$
-
$$\begin{bmatrix}2 & 0\\-1 & 2\end{bmatrix}\cdot\begin{bmatrix}10 & 3\\0 & -1\end{bmatrix}$$
-
$$\begin{bmatrix}20 & 6\\-10 & -5\end{bmatrix}$$
- RHS

- $$\left(\begin{bmatrix}2 & 0\\ -1 & 2\end{bmatrix}\cdot\begin{bmatrix}3 & 1\\ 5 & 0\end{bmatrix}\right)+\left(\begin{bmatrix}2 & 0\\ -1 & 2\end{bmatrix}\cdot\begin{bmatrix}7 & 2\\ -5 & -1\end{bmatrix}\right)$$
- $$\begin{bmatrix}6 & 2\\ 7 & -1\end{bmatrix}+\begin{bmatrix}14 & 4\\ -17 & -4\end{bmatrix}$$
- $$\begin{bmatrix}20 & 6\\ -10 & -5\end{bmatrix}$$
- $LHS = RHS$

2024-07-26

Matrix in an equation

#question Let $f(x) = x^2 - 3x - 10$ & $A = \begin{bmatrix}a & 3b\\ 2 & 1\end{bmatrix}$, A satisfies the equation, $f(x) = 0$, Find 'a' & 'b'.
Ans:

- A in equation will be
$$A^2 - 3A - 10I = O$$
- $$\rightarrow \begin{bmatrix}a & 3b\\ 2 & 1\end{bmatrix}^2 - 3\begin{bmatrix}a & 3b\\ 2 & 1\end{bmatrix} - \begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
- $$\rightarrow \begin{bmatrix}a^2+6b & 3ab+3b\\ 2a+2 & 6b+1\end{bmatrix} - \begin{bmatrix}3a & 9b\\ 6 & 3\end{bmatrix} - \begin{bmatrix}10 & 0\\ 0 & 10\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
- $$\rightarrow \begin{bmatrix}a^2-3a+6b-10 & 3ab-6b\\ 2a-4 & 6b-12\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
- $2a - 4 = 0$ Therefore, $a = 2$
- $6b - 12 = 0$ Therefore, $b = 2$

#question Find A^{-1}

- $A = \begin{bmatrix}2 & 6\\ 2 & 1\end{bmatrix}$
- Determinant of A = $\begin{vmatrix}2 & 6\\ 2 & 1\end{vmatrix}$ = -10

- $$A^{-1} = \frac{-1}{10}\begin{bmatrix} 1 & -6 \\ -2 & 2 \end{bmatrix}$$
- $$A^{-1} = \begin{bmatrix} \frac{-1}{10} & \frac{3}{5} \\ \frac{1}{5} & \frac{-1}{5} \end{bmatrix}$$

2024-07-31

#question If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, Then Prove $A^2 - 4A - 5I_3 = 0$

#question also Find A^{-1}.

- $A^2 - 4A - 5I_3 = 0$
  - LHS
    - $$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
    - $$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$
    - $$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
  - RHS
    - $$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
  - LHS = RHS (proved)
- $A^{-1} = ?$
- We know that,

$$A \cdot A - 4A - 5I_3 = 0$$

- Multiplying all the elements of the equation with A^{-1},

$$A^{-1} \cdot A \cdot A = A^{-1} \cdot 4 \cdot A + A^{-1} \cdot 5I$$

- Solving this we get,

$$I \cdot A = 4 \cdot I + 5A^{-1}$$

- After rearranging the equation we get,

$$A^{-1} = \frac{1}{5}(A - 4 \cdot I)$$

- $$\therefore A^{-1} = \frac{1}{5}\begin{bmatrix} 1-4 & 2 & 2 \\ 2 & 1-4 & 2 \\ 2 & 2 & 1-4 \end{bmatrix} = \begin{bmatrix} \frac{-3}{5} & \frac{2}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-3}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{2}{5} & \frac{-3}{5} \end{bmatrix}$$

2024-08-01

#question If $A = [a_{ij}]_{3\times 3}$ is a matrix where $a_{ij} = \frac{i+2j}{3}$ Find A

- $$A = \frac{1}{3}\begin{bmatrix} 3 & 5 & 7 \\ 4 & 6 & 8 \\ 5 & 7 & 9 \end{bmatrix} = \begin{bmatrix} 1 & \frac{5}{3} & \frac{7}{3} \\ \frac{4}{3} & 2 & \frac{8}{3} \\ \frac{5}{3} & \frac{7}{3} & 3 \end{bmatrix}$$

Determinants

> If atleast two rows/columns are equal in any determinant then the value of the determinant is 0.
>
> $$\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Value of a 3x3 determinant

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (-1)^{1+1}a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2}a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3}a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

For Example:

- $$\begin{vmatrix} 5 & 3 & 2 \\ 4 & 0 & -1 \\ -3 & 2 & 5 \end{vmatrix}$$

- $$= 5(0 \times 5 - 2 \times (-1)) - 3(4 \times 5 - (-3) \times (-1)) + 2(4 \times 2 - (-3) \times 0)$$

- $$= 10 - 51 + 16$$
- $$= -25$$
- `Minors for the above determinant's elements`
  - `For 5` (a_{11})
    - $$\begin{vmatrix} 0 & -1 \\ 2 & 5 \end{vmatrix} = 2$$
  - `For 3` (a_{12})
    - $$-\begin{vmatrix} 4 & -1 \\ -3 & 5 \end{vmatrix} = -17$$
  - `For 2` (a_{13})
    - $$\begin{vmatrix} 4 & 0 \\ -3 & 2 \end{vmatrix} = 8$$
  - `For 4` (a_{21})
    - $$-\begin{vmatrix} 3 & 2 \\ 2 & 5 \end{vmatrix} = -11$$
  - `For 0` (a_{22})
    - $$\begin{vmatrix} 5 & 2 \\ -3 & 5 \end{vmatrix} = 31$$
  - `For -1` (a_{23})
    - $$-\begin{vmatrix} 5 & 3 \\ -3 & 2 \end{vmatrix} = -19$$
  - `For -3` (a_{31})
    - $$\begin{vmatrix} 3 & 2 \\ 0 & -1 \end{vmatrix} = -3$$
  - `For 2` (a_{32})
    - $$-\begin{vmatrix} 5 & 2 \\ 4 & -1 \end{vmatrix} = 13$$
  - `For 5` (a_{33})
    - $$\begin{vmatrix} 5 & 3 \\ 4 & 0 \end{vmatrix} = -12$$

Determinant of a matrix A is equal to the determinant od transpose of A.

$|A| = |A^T|$
$det(A) = det(A^T)$

2024-08-07

Inverse of a Matrix

- Inverse of a matrix only exists when $|A| \neq 0$
- For

$$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

- Minor

$$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

- Cofactor of a_{ij} (A_{ij}) = $(-1)^{i+j} M_{ij}$
- $$adj\ A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$
- $$A^{-1} = \frac{1}{|A|} adj\ A$$
- Three forms of calculating the determinant of A are

$$\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$
$$\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$$
$$\Delta = a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$$

Rules For Inverse of a matrix

- If A be a non singular matrix then $A.A^{-1} = A^{-1}.A = I$

#question Let $A = \begin{bmatrix} 2 & 8 \\ 0 & 5 \end{bmatrix}$ then prove that, $A.A^{-1} = I$

- $|A| = 10 - 0 = 10$
- $adj\ A = \begin{bmatrix} 5 & -8 \\ 0 & 2 \end{bmatrix}$
- $A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-4}{5} \\ 0 & \frac{1}{5} \end{bmatrix}$
- $\therefore A.\ A^{-1} = \begin{bmatrix} 2 & 8 \\ 0 & 5 \end{bmatrix} . \begin{bmatrix} \frac{1}{2} & \frac{-4}{5} \\ 0 & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I\ (proved)$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $(AB)^T = B^T A^T$

#question Let $A = \begin{bmatrix} -1 & 0 \\ 2 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then prove that, (a) $(AB)^{-1} = B^{-1}A^{-1}$ (b) $(AB)^T = B^T A^T$
(a) $(AB)^{-1} = B^{-1}A^{-1}$

- $A^{-1} = \frac{1}{5} \begin{bmatrix} 5 & 0 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ \frac{2}{5} & \frac{1}{5} \end{bmatrix}$
- $B^{-1} = \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix}$
- $AB = \begin{bmatrix} -1 & 0 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 17 & 24 \end{bmatrix}$
- $(AB)^{-1} = \frac{1}{10} \begin{bmatrix} 24 & -17 \\ 2 & -1 \end{bmatrix}^T = \begin{bmatrix} \frac{12}{5} & \frac{1}{5} \\ \frac{-17}{10} & \frac{-1}{10} \end{bmatrix} \to LHS$
- $B^{-1}A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix} . \begin{bmatrix} -1 & 0 \\ \frac{2}{5} & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} \frac{12}{5} & \frac{1}{5} \\ \frac{-17}{10} & \frac{-1}{10} \end{bmatrix} \to RHS$
- LHS = RHS (proved)

(b) $(AB)^T = B^T A^T$

- $A^T = \begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix}$ & $B^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$
- $AB = \begin{bmatrix} -1 & 0 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 17 & 24 \end{bmatrix}$
- $(AB)^T = \begin{bmatrix} -1 & 17 \\ -2 & 24 \end{bmatrix} \to LHS$
- $B^T A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 17 \\ -2 & 24 \end{bmatrix} \to RHS$
- LHS = RHS (proved)

2024-08-08

Some Determinant Properties

- $$\begin{vmatrix} ma_1 & mb_1 & mc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ ma_2 & b_2 & c_2 \\ ma_3 & b_3 & c_3 \end{vmatrix} = m \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$
- Exchanging any two rows or columns will multiply the determinant with a -1.
$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = - \begin{vmatrix} a_1 & c_1 & b_1 \\ a_2 & c_2 & b_2 \\ a_3 & c_3 & b_3 \end{vmatrix}$$

Cramer's Rule

- Used for solving linear Algebraic equations
- $$\begin{gathered} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{gathered}$$
- $$Let\ |A| = \Delta = \begin{vmatrix} a^1 & b^1 & c^1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$
- $$\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$
- $$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$
- $$\Delta_3 = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

> ⓘ For Cramer's Rule, the system of equations have unique solution if $\Delta \neq 0$ & the solution is $\begin{gathered} x = \frac{\Delta_1}{\Delta} \\ y = \frac{\Delta_2}{\Delta} \\ z = \frac{\Delta_3}{\Delta} \end{gathered}$

> ⚠ System of equations have infinitely many solutions if $\Delta=0, \Delta_1=\Delta_2=\Delta_3=0$

> ⚠ The system of equations have no solution if, $\Delta=0$ & Atleast one of $\Delta_1, \Delta_2, \Delta_3 \neq 0$

#question Solve the system of equations

$$\begin{aligned} x+y+z&=3 \\ x+2y+3z&=4 \\ x+4y+9z&=6 \end{aligned}$$

- $$|A|=\Delta=\begin{vmatrix}1&1&1\\1&2&3\\1&4&9\end{vmatrix}=\begin{vmatrix}1&0&0\\1&1&2\\1&3&8\end{vmatrix}=8-6=2$$
- $$\Delta_1=\begin{vmatrix}3&1&1\\4&2&3\\6&4&9\end{vmatrix}=\begin{vmatrix}0&1&0\\-2&2&1\\-6&4&5\end{vmatrix}=-\begin{vmatrix}0&0&1\\-2&1&2\\-6&5&4\end{vmatrix}=-(-10+6)=4$$
- $$\Delta_2=\begin{vmatrix}1&3&1\\1&4&3\\1&6&9\end{vmatrix}=\begin{vmatrix}1&0&0\\1&1&2\\1&3&8\end{vmatrix}=8-6=2$$
- $$\Delta_3=\begin{vmatrix}1&1&3\\1&2&4\\1&4&6\end{vmatrix}=\begin{vmatrix}1&0&0\\1&1&1\\1&3&3\end{vmatrix}=0$$
- Hence, the unique solution of is $\begin{aligned} x&=\frac{\Delta_1}{\Delta}=2 \\ y&=\frac{\Delta_2}{\Delta}=1 \\ z&=\frac{\Delta_3}{\Delta}=0 \end{aligned}$.

2024-08-09

#question Solve

$$\begin{aligned} \frac{5}{x}-\frac{2}{y}+\frac{3}{z}&=16 \\ \frac{2}{x}+\frac{3}{y}-\frac{5}{z}&=2 \\ \frac{4}{x}-\frac{5}{y}+\frac{6}{z}&=7 \end{aligned}$$

- $$\Delta=\begin{vmatrix}5&-2&3\\2&3&-5\\4&-5&6\end{vmatrix}=-37$$

- $$\Delta_1 = \begin{vmatrix} 16 & -2 & 3 \\ 2 & 3 & -5 \\ 7 & 5 & 6 \end{vmatrix} = -111$$
- $$\Delta_2 = \begin{vmatrix} 5 & 16 & 3 \\ 2 & 2 & -5 \\ 4 & 7 & 6 \end{vmatrix} = -259$$
- $$\Delta_3 = \begin{vmatrix} 5 & -2 & 16 \\ 2 & 3 & 2 \\ 4 & -5 & 7 \end{vmatrix} = -185$$
- `Value of the variables =` $\begin{gathered} x = \frac{\Delta_1}{\Delta} = 3 \\ y = \frac{\Delta_2}{\Delta} = 7 \\ z = \frac{\Delta_3}{\Delta} = 5 \end{gathered}$
- $$\begin{gathered} x = \frac{1}{3} \\ y = \frac{1}{7} \\ z = \frac{1}{5} \end{gathered}$$

#question Solve

$$\begin{gathered} 3x + 5y + 2z = 1 \\ 4x + y = 7 \\ 9x + 15y + 6z = 3 \end{gathered}$$

-

$$\Delta = \begin{vmatrix} 3 & 5 & 2 \\ 4 & 1 & 0 \\ 9 & 15 & 6 \end{vmatrix} = 3 \begin{vmatrix} 3 & 5 & 2 \\ 4 & 1 & 0 \\ 3 & 5 & 2 \end{vmatrix} = 0$$

- $$\Delta_1 = \begin{vmatrix} 1 & 5 & 2 \\ 7 & 1 & 0 \\ 3 & 15 & 6 \end{vmatrix} = 3 \begin{vmatrix} 1 & 5 & 2 \\ 7 & 1 & 0 \\ 1 & 5 & 2 \end{vmatrix} = 0$$
- $$\Delta_2 = \begin{vmatrix} 3 & 1 & 2 \\ 4 & 7 & 0 \\ 9 & 3 & 6 \end{vmatrix} = 3 \begin{vmatrix} 3 & 1 & 2 \\ 4 & 7 & 0 \\ 3 & 1 & 2 \end{vmatrix} = 0$$
- $$\Delta_3 = \begin{vmatrix} 3 & 5 & 1 \\ 4 & 1 & 7 \\ 9 & 15 & 3 \end{vmatrix} = 3 \begin{vmatrix} 3 & 5 & 1 \\ 4 & 1 & 7 \\ 3 & 5 & 1 \end{vmatrix} = 0$$

- Therefore, the system of equations have infinitely many solutions because $\Delta = 0, \Delta_1 = \Delta_2 = \Delta_3 = 0$
- To solve this,
  - $4x + y = 7 \Rightarrow y = 7 - 4x$
  - Putting this in the first equation, we get
  - $3x + 5(7 - 4x) + 2z = 1$
  - $z = \frac{17x+34}{2}$
- Hence the solutions are

$$\left(x, 7 - 4x, \frac{17x - 34}{2}\right) \forall x \in R$$

2024-08-14

#question Solve the following system of equations

$$\begin{gathered} -2a + 2b = -12 \\ a - b = 11 \end{gathered}$$

- Using Cramer's Rule,

$$\Delta = \begin{vmatrix} -2 & 2 \\ 1 & -1 \end{vmatrix} = 0$$

- $$\Delta_1 = \begin{vmatrix} -12 & 2 \\ 11 & -1 \end{vmatrix} = -10$$
- $$\Delta_2 = \begin{vmatrix} -2 & -12 \\ 11 & -1 \end{vmatrix} = -10$$
- We can see that $\Delta = 0$ & Atleast one of $\Delta_1, \Delta_2 \neq 0$. Therefore, the system of equations have no solution.

Matrix Inversion Method

- For

$$\begin{gathered} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{gathered}$$

- $$\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

- $$A = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$
- $$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$
- $$B = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

$$A.X = B$$

- $$A^{-1}.A.X = A^{-1}B$$

$$X = A^{-1}.B$$

- $$A^{-1} = \frac{1}{|A|}[adj\ A]$$

#question `Solve using the inversion method`

$$\begin{aligned} x - 3y + 2z = 3 \\ 3x + 2y - z = 2 \\ 2x - y + z = 4 \end{aligned}$$

- $$\begin{pmatrix} 1 & -3 & 2 \\ 3 & 2 & -1 \\ 2 & -1 & 1 \end{pmatrix} . \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$$
- $$|A| = \begin{vmatrix} 1 & -3 & 2 \\ 3 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = 1(2-1) + 3(3+2) + 2(-3-4) = 1 + 15 - 14 = 2$$
- $$adj\ A = \begin{pmatrix} 1 & 1 & -1 \\ -5 & -3 & 7 \\ -7 & -5 & 11 \end{pmatrix}$$
- $$A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{5}{2} & -\frac{3}{2} & \frac{7}{2} \\ -\frac{7}{2} & -\frac{5}{2} & \frac{11}{2} \end{pmatrix}$$

- $$X = A^{-1}B$$
- $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{5}{2} & -\frac{3}{2} & \frac{7}{2} \\ -\frac{7}{2} & -\frac{5}{2} & \frac{11}{2} \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$$
- $$\frac{1}{2} \begin{pmatrix} 1 \cdot 3 + 1 \cdot 2 + (-1) \cdot 4 \\ (-5) \cdot 3 + (-3) \cdot 2 + 7 \cdot 4 \\ (-7) \cdot 3 + (-5) \cdot 2 + 11 \cdot 4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 \\ 7 \\ 13 \end{pmatrix}$$
- $$x = \frac{1}{2}$$
- $$y = \frac{7}{2}$$
- $$z = \frac{13}{2}$$

2024-08-16

#question Solve $\begin{gathered} 2x + y = 1 \\ x + y = 2 \end{gathered}$

- Cramer's method
  - $$\Delta = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$$
  - $$\Delta_1 = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1$$
  - $$\Delta_2 = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3$$
  - $$\begin{gathered} x = \tfrac{\Delta_1}{\Delta} = -1 \\ y = \tfrac{\Delta_2}{\Delta} = 3 \end{gathered}$$
- Matrix Inversion Method
  - $$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

- $$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$
- $$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

#question Solve $\begin{aligned} 2x + y + z = 1 \qquad & (i) \\ 3x - y + z = 2 \qquad & (ii) \\ x + y + z = 3 \qquad & (iii) \end{aligned}$

- Short method:
  - If we put the value of $x + y + z = 3$ in equation (i) we get,

$$\begin{aligned} x + x + y + z = 1 \\ x + 3 = 1 \\ x = -2 \end{aligned}$$

  - Putting this value in equation (ii) and (iii) and adding them, we get

$$\begin{aligned} -y + z = 8 \\ y + z = 5 \\ ------ \\ 2z = 13 \end{aligned}$$

$$z = \frac{13}{2}$$

  - Putting the values of x and z in equation (iii), we get

$$\begin{aligned} -2 + y + \frac{13}{2} = 3 \\ y = \frac{-3}{2} \end{aligned}$$

#question Solve $\int e^{\sin x} \cos x \, dx$

- Let $\sin x = y$, Therefore, $\cos x \, dx = dy$
- Replacing this in the equation, we get,

$$\Rightarrow \int e^{\sin x} \cos x \, dx$$

$$\Rightarrow \int e^{y} \, dy$$

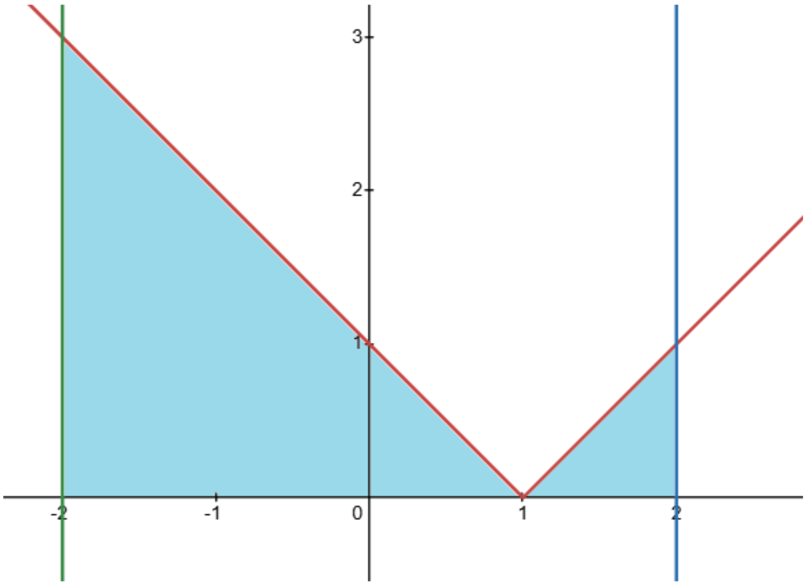
$$\Rightarrow e^{y}$$

- Ans,

$$e^{\sin x}$$

#question Solve $\int_{-2}^{2}|x-1|dx$

- Plotting this in graph,



- Area under the curve,

$$\left(\frac{1}{2}\times 3\times 3\right)+\left(\frac{1}{2}\times 1\times 1\right)=\frac{9}{2}+\frac{1}{2}=5$$

2024-08-17