MATRIX

TYPES:

$\rightarrow$ Row Matrix - A matrix having a single row

Example - $\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

$\rightarrow$ Column matrix - A matrix having only a single column

Example - $\begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$

$\rightarrow$ Square matrix : A matrix having equal number of rows and column

Example - $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 3 & 2 \\ 1 & 7 & 3 \end{bmatrix}$

$\rightarrow$ Rectangular matrix - A matrix having unequal number of rows and columns.

Example - $\begin{bmatrix} 2 & 0 & -1 \\ 1 & 7 & 3 \end{bmatrix}$

$\rightarrow$ Null matrix - If all elements of a matrix is zero it is called null or zero matrix and it is shown by 0

Example - $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\rightarrow$ Diagonal matrix - A square matrix in which all the elements except the main diagonal are zero.

Example - $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$\rightarrow$ Scalar matrix - In a diagonal matrix if all elements are equal the matrix is Scalar.

Example - $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\rightarrow$ Unit/Identity matrix - A diagonal matrix whose all element on the main diagonal are equal to one. The unit matrix is usually shown by letter I.

Example - I = I_3 = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\rightarrow$ Transpose matrix - if the rows and columns of a matrix A are interchanged then the resulting matrix is called transpose of A matrix. It is denoted by A'/ A^T

Example - A = $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 5 & 0 \\ 4 & 3 & 2 \end{bmatrix}$ $A' = A^T \begin{bmatrix} 3 & 1 & 4 \\ 2 & 5 & 3 \\ 6 & 0 & 2 \end{bmatrix}$

$\rightarrow$ Symmetric Matrix - A matrix is Symmetric when $A = A^T$

Example - $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, A^T = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$B = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 5 \end{bmatrix}, B^T = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 5 \end{bmatrix}$

$\rightarrow$ Skew symmetric matrix - A matrix A is called skew-symmetric if $A = -A^T$.

Example - $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, A^T = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

- Diagonal must be 0.

OPERATION OF MATRIXES:

$\rightarrow$ Addition and subtraction.
= Let, A and B are two matrixes. If A+B are equal

Example

$$\begin{bmatrix} 2 & 0 & -7 \\ 3 & 2 & 5 \end{bmatrix}_{2X3} + \begin{bmatrix} 1 & -5 & -2 \\ -1 & 0 & 3 \end{bmatrix}_{2X3} \Rightarrow \begin{bmatrix} 3 & -5 & -9 \\ 2 & 2 & 8 \end{bmatrix}_{2X3}$$

The Operations Addition are commutative
$\rightarrow$ Multiplication.
= Let, A and B are two matrixes of order nXm and pXq , if the multiplication operation that is AB valid then $\boxed{m = p}$

$\star\star$

Q) A and B are two matrixes of order $5X1$ and $1X4$, Justify whether

AB is possible or not. If AB is possible then find the order of AB . Justify the multiplication BA is possible or not.

Ans. $\overset{A}{5X}\boxed{1 \;=\; 1}\overset{B}{X4} = 5X4$

Since, the no if columns of matrics A equal to the no. of rows on matrics B $\therefore$ The multiplication AB is possible.

The order of AB is 5X4

$$\overset{B}{1X}\boxed{4 \;\neq\; 5}\overset{A}{X1}$$

Since, the no of columns of matrics B is not equal to the no of rows of matrics A $\therefore$ The multiplication BA is not possible.

<u>Example</u>

$$A \;=\; \left[\begin{array}{cc} 2 & -1 \\ \hline 0 & 3 \\ 5 & 2 \end{array}\right]_{3X2} \qquad B \;=\; \left[\begin{array}{c|cc} 1 & 3 & 0 \\ 2 & -1 & 4 \end{array}\right]_{2X3}$$

$$\mathcal{AB} \;=\; \begin{bmatrix} (2X1) + (-1)X2 & 2X3 + (-1)X(-1) & 2X0 + (-1)X4 \\ 0X1 + 3X2 & 0X3 + 3X(-1) & 0X0 + 3X4 \\ 5X1 + 2X2 & 5X3 + 2(-1) & 5X0 + 2X4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 7 & -4 \\ 6 & -3 & 12 \\ 9 & 13 & 8 \end{bmatrix}_{3X3}$$

$$\mathcal{BA} \;=\; \begin{bmatrix} 2+0+0 & -1+9+0 \\ 4-0+20 & -2-3+8 \end{bmatrix} \;=\; \begin{bmatrix} 2 & 8 \\ 24 & 3 \end{bmatrix}_{2X2}$$

$\rightarrow$ Here $\boxed{\mathcal{AB} \;\neq\; \mathcal{BA}}$ $\therefore$ A and B are not commutative.

Q) $\mathcal{A} \;=\; \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$; $\mathcal{B} \;=\; \begin{bmatrix} 3 & 1 \\ 5 & 0 \end{bmatrix}$; $\mathcal{C} \;=\; \begin{bmatrix} 7 & 2 \\ -5 & -1 \end{bmatrix}$ prove that

$i\rangle$ (AB)C = A(BC) (Associative)

$$\mathcal{AB} \;=\; \begin{bmatrix} 6+0 & 2+0 \\ 3+10 & 1+0 \end{bmatrix} \;=\; \begin{bmatrix} 6 & 2 \\ 13 & 1 \end{bmatrix}^{AB} \begin{bmatrix} 7 & 2 \\ -5 & -1 \end{bmatrix}^{C} \;=\; \begin{bmatrix} 42+(-10) & 12-2 \\ 91-5 & 26-1 \end{bmatrix}$$
$$= \begin{bmatrix} 32 & 10 \\ 86 & 25 \end{bmatrix}$$

$$\mathcal{BC} \;=\; \begin{bmatrix} 21-5 & 6-1 \\ 35-0 & 10-0 \end{bmatrix} \;=\; \begin{bmatrix} 16 & 5 \\ 35 & 10 \end{bmatrix}^{BC}$$

$$\mathcal{A(BC)} \;=\; \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 16 & 5 \\ 35 & 10 \end{bmatrix} \;=\; \begin{bmatrix} 32+0 & 10+0 \\ 16+70 & 5+20 \end{bmatrix} \;=\; \begin{bmatrix} 32 & 10 \\ 86 & 25 \end{bmatrix}$$

$\therefore$ proved that (AB)C = A(BC) (Associative)

DETERMINANT

If atleast two rows or columns are equal in any determinant then the value of that determinant is equal to '0'

Example:

$$\begin{vmatrix} +5 & -3 & +2 \\ \hline -4 & +0 & -1 \\ -3 & -2 & +5 \end{vmatrix}$$ = 50(0+2)- 3(20-3)+2(8-0) =10- 51 + 16 = 26-51 = -25

Formula = $(-1)^{row+column}$.
Minors -
minor of (5) minor of (3) minor of (2)
= $\begin{vmatrix} 0 & -1 \\ 2 & 5 \end{vmatrix}$ $-\begin{vmatrix} 4 & -1 \\ -3 & 5 \end{vmatrix}$ $\begin{vmatrix} 4 & 0 \\ -3 & 2 \end{vmatrix}$
= 2 = -17 = 8

minor of (4) minor of (0) minor of (-1)
= $\begin{vmatrix} -3 & 2 \\ 2 & 5 \end{vmatrix}$ $\begin{vmatrix} 5 & 2 \\ -3 & 5 \end{vmatrix}$ $-\begin{vmatrix} 5 & 3 \\ -3 & 7 \end{vmatrix}$
= -11 = 31 = -19

minor of (-3) minor of (2) minor of (-5)
= $\begin{vmatrix} 3 & 2 \\ 0 & -1 \end{vmatrix}$ $-\begin{vmatrix} 5 & 2 \\ -4 & -1 \end{vmatrix}$ $\begin{vmatrix} 5 & 3 \\ 4 & 0 \end{vmatrix}$
= -3 = +13 = -12

$-$Determinant of a matrix A is the determinant of transpose of that matrix.
$det(\mathcal{A}) = det(\mathcal{A}^{\mathcal{T}})$