Introduction to Ecological Inference

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  - Race scholars want to know if minority turnout increases as more minority descriptive representation grows (Gay 2001; Heron and Sekhon 2005; Hajnal & Troustine 2005; Baretto 2007)

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$\Rightarrow$ **Aggregate data as alternative options**

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- Aggregation Bias

Robinson's Paradox

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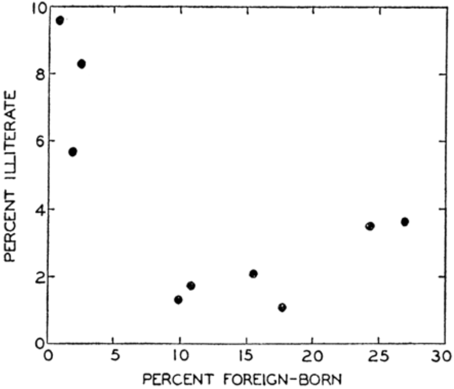
Figure 3.



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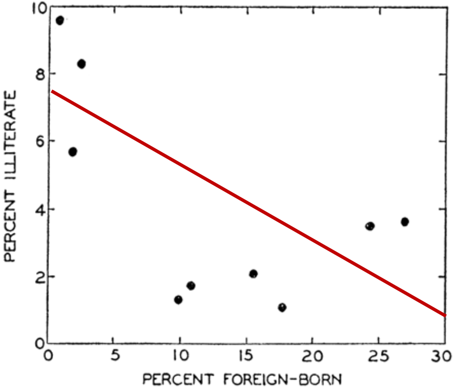

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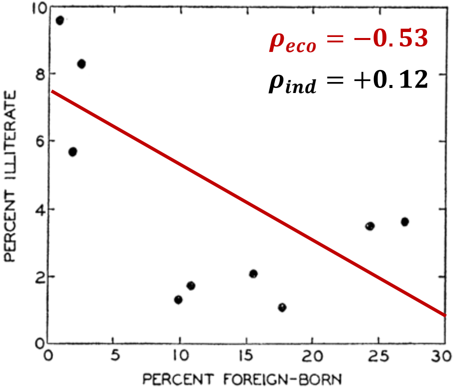

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Why Robinson's paradox? (Intuitively)

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$*$ Loss of information = indeterminacy problem

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$\rightsquigarrow$ How can we do better than naïve inference?

What is Ecological Inference? (Review)

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Motivating Example

	Vote	Non Vote	
Black	N_{bV_i}	N_{bNV_i}	$\mathbf{N}_{b_i}$
White	N_{wV_i}	N_{wNV_i}	$\mathbf{N}_{w_i}$
	$\mathbf{N}_{V_i}$	$\mathbf{N}_{NV_i}$	

2×2 Contingency Table for Precinct i

- You have data from 100 precincts in Harris County
- You know # people who turned out from election returns **(Col marginals)**
- You know CVAP of whites and blacks from the US Census **(Row marginals)**
- You want to know how many whites/blacks went to vote (Internal cells)

Motivating Example (Extension)

	Clinton	Trump	Sanders	Non Vote	
White	N_{wB_i}	N_{wL_i}	N_{wK_i}	N_{wNV_i}	$\mathbf{N}_{w_i}$
Black	N_{bB_i}	N_{bL_i}	N_{bK_i}	N_{bNV_i}	$\mathbf{N}_{b_i}$
Asian	N_{aB_i}	N_{aL_i}	N_{aK_i}	N_{aNV_i}	$\mathbf{N}_{a_i}$
Hispanic	N_{hB_i}	N_{hL_i}	N_{hK_i}	N_{hNV_i}	$\mathbf{N}_{h_i}$
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- Combined approach (King 1997 [King's EI]; Lewis 2004 [Seemingly unrelated EI]; Quinn 2004 [Dynamic EI]); Calvo & Escolar 2003 [Geographically weighted approach]
- Bayesian hierarchical models (King, Rosen, & Tanner 1999 [Binomial-Beta model]; Rosen, Jiang, King, & Tanner 2001 [Multinomial-Dirichlet model]; Greiner and Quinn 2009 [GQ model]; Wakefield 2004 [Wakefield's 2$\times$2 model]; Imai and Lu 2008 [Multiple imputation model])
- Machine learning variants (Flaxman 2015, 2016 [Distributional regression model]; Soldaini and YomTov 2016 [Stochastic gradient descent model]; Muezellec et al. 2017 [Tsallis regularized optimal transport model]; Rosenman and Viswanathan 2018 [Poisson-Binomial model])

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  - Known prop of blacks: X_i $(= N_{Bi}/N_i)$
  - Known prop of turnout: T_i $(= N_{Vi}/N_i)$

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$\rightsquigarrow$ Looks like random-coefficient models (with no intercept)!

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$\rightsquigarrow$ Equivalent specification for β_i^w, c_w, d_w

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$$
\begin{aligned}
&\propto \prod_{i=1}^{p} (X_i\beta_i^b + (1 - X_i)\beta_i^w)^{N_{Vi}} (1 - X_i\beta_i^b - (1 - X_i)\beta_i^w)^{(N_i - N_{Vi})} \\
&\times \prod_{i=1}^{p} \frac{\Gamma(c_b + d_b)}{\Gamma(c_b)\Gamma(d_b)} (\beta_i^b)^{c_b - 1} (1 - \beta_i^b)^{d_b - 1} \\
&\times \prod_{i=1}^{p} \frac{\Gamma(c_w + d_w)}{\Gamma(c_w)\Gamma(d_w)} (\beta_i^w)^{c_w - 1} (1 - \beta_i^w)^{d_w - 1} \\
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$\rightsquigarrow$ Flat prior on $\alpha, \beta, \gamma, \delta$

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Extensions

- Multinomial-Dirichlet models (Rosen, Jiang, King, & Tanner 2001)
- Modeling counts directly (Greiner & Quinn 2009, 2010)
- Temporal dependency (Lewis 2004; Quinn 2004)
- Spatial autocorrelation (Calvo & Escolar 2003)
- Causal inference (Spenkuch 2018; Corvalan, Melo, Shermanandmatt, & Shum 2016)

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⇝ **Let's estimate Binomial-Beta models with JAGS!**