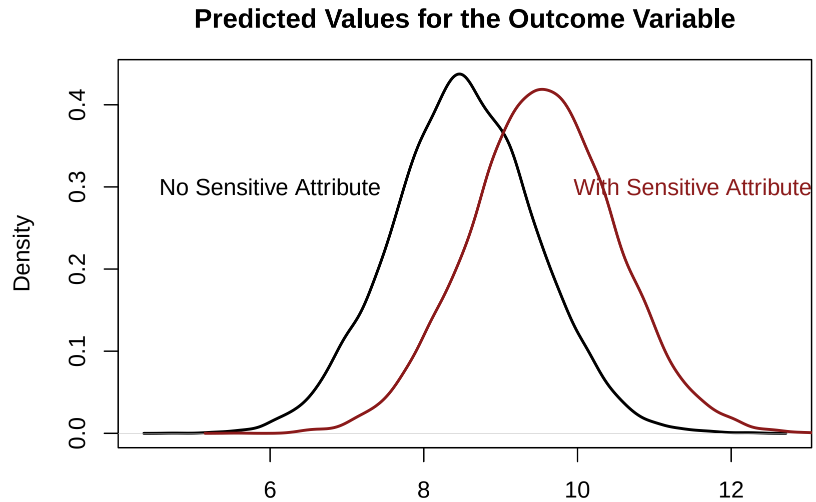

Figure C.6: **Simulated Outcome Values with and without the Sensitive Attribute.** *Note*: This graph visualizes the density of simulated (predicted) values for the outcome variable in the absence (left density) and presence (right density) of the sensitive attribute.

C.5 Sample Size Determination and Parameter Selection

When using the crosswise model with our procedure, researchers may wish to choose the sample size and specify other design parameters (i.e., π', p, and p') so that they can obtain (1) high statistical power for hypothesis testing and/or (2) narrow confidence intervals for precise estimation. To fulfill these needs, we develop power analysis and data simulation tools appropriate for our bias-corrected estimator.

First, our power analysis uses a one-sided hypothesis test based on the Wald test (Ulrich et al., 2012). We consider the null hypothesis $H_0 : \pi \leq \pi_0$, where π_0 (prevalence rate under the null) may be zero or a particular value obtained from direct questioning. Assuming that the crosswise estimate is larger than the direct questioning estimate — and larger than zero (i.e., more-is-better assumption), we then consider the alternative hypothesis $H_1 : \pi > \pi_0$ when the true value of π is π_1. Based on the normal approximation, the power function becomes:

$$\underbrace{\mathbb{P}(\{\text{Reject } H_0 | H_1 \text{ is true}\})}_{\text{Power}} = \beta = 1 - \underbrace{\Phi\Big(\frac{\pi_0 - \pi_1 - c\tilde{\sigma}_0}{\tilde{\sigma}_1}\Big)}_{\mathbb{P}(\text{Type II Error})}, \tag{C.3}$$

where Φ is the cumulative distribution function of the standard normal distribution, $c = \Phi(1-\alpha)$ is the critical value given size $\alpha = \mathbb{P}(\text{Type I Error})$, and $\tilde{\sigma}_0$ and $\tilde{\sigma}_1$ are *simulated* standard errors of the bias-corrected estimates under H_0 and H_1, respectively.[3]

Panel A of Figure C.7 plots power curves against the sample sizes. For this illustration, we assume that $\pi_0 = 0$, $\pi_1 = 0.1$, $\pi' = 0$, $p' = 0.1$, $\gamma = 0.8$, and $\alpha = 0.05$. Substantively, this means that we hope to distinguish the estimated prevalence of 0.1 from zero with a 95% ($100\% \times 1 - \alpha$) level of confidence when we expect that only 80% of respondents are attentive. Panel A displays multiple power curves for different

[3] We simulate the standard errors in repeated Monte Carlo experiments at $n = \{1, 500, 1000, 1500, 2000, 2500\}$. Our software also allows one to draw more fine-grained power curves.

values of p. It suggests that if researchers want to reject H_0 with power $\beta = 0.8$ they need to have about 400 (when $p = 0.1$), 600 (when $p = 0.2$), and 1400 (when $p = 0.3$) respondents.

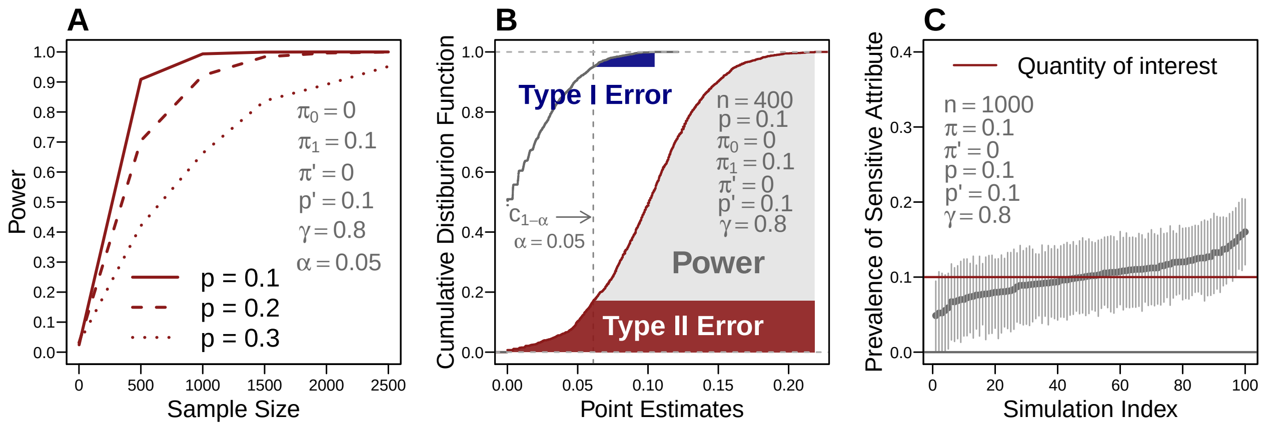

Figure C.7: **Tools for Power Analysis and Parameter Selection.** *Note*: Panel A shows power curves for three different values of p. Panel B offers visualizes type I error, type II error, and power as areas under the empirical cumulative distribution of point estimates under H_0 (left) and H_1 (right). Panel C display 100 point estimates and 95% confidence intervals based on 100 simulated data sets.

After choosing the sample size n, researchers may wish to verify if the selected n and design parameters achieve the desired level of power and re-adjust their design parameters if necessary. To further assist applied researchers, Panel B visualizes type I error, type II error, and statistical power as areas under the (empirical) cumulative distribution function of the sampling distribution of the bias-corrected estimates based on H_0 (left) and H_1 (right). We assume the same parameter values as above and $n = 400$ and $p = 0.1$. Panel B shows a small type I error and high power (close to 0.8) or equivalently small type II error as we expected.

Next, our data simulation tool allows researchers to see what they would obtain from using the crosswise model with some fixed sample size and parameter values. The key idea is to give analysts a sense of how large their 95% confidence interval would be and let them change the sample size and design parameters to achieve their desired precision (i.e., desired length of the confidence interval). To illustrate, Panel C shows the point estimates and 95% confidence intervals based on 100 simulated data sets with $n = 1000$ and $\pi = 0.1$, $\pi' = 0$, $p = p' = 0.1$, $\gamma = 0.8$. It suggests that the resulting interval estimate would be approximately the point estimate ± 0.05. If a narrower confidence interval is needed, researchers should increase the sample size and/or choose lower values for p and p'.

C.6 Secret Number Approach to Non-Sensitive Statements

This section introduces another extension of our method, which was not fully discussed in the main manuscript. One of the obstacles for some researchers to use the crosswise model is that they need to find an appropriate non-sensitive statement with a known prevalence in the crosswise question. Further, to maintain respondent privacy it is essential that a *different* non-sensitive item be used with each crosswise question in the survey. This can be quite difficult to do in practice when one asks many crosswise questions. To remedy this problem, we propose a *secret number approach* to the non-sensitive statement. The essence of the secret number approach is to use a different virtual die roll for the non-sensitive crosswise item in each question, but to do so in a way that makes it clear to respondents the critical information remains private even if they believe surveyors are recording the result of the virtual die rolls.