Math Prep Summer 2019

Syllabus for POLI Ph.D. Students

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Course Objectives

This course is designed to provide incoming Ph.D. students a broad review of basic mathematical topics whose understanding is required for the successful completion of the first-year method sequence (POLI 504 & 505) and beyond in the Dept. of Political Science. Beyond helping students build their quantitative toolkits, learning and reviewing mathematical concepts and operations will be helpful for them to organize their thoughts and build their own theories and hypotheses. The main goal of this course is to provide an opportunity for students to familiarize themselves with mathematical language and manipulation through repeated problem solving practices.

Schedules

We will meet in Herzstein 126 for the following dates and times. We have **nine days** of math prep. (We do not have a class on August 21th due to the Orientation for All Incoming Graduate Students.)

- August 12th to August 16th (10AM-11:30AM)
- August 19th to August 20th (2:30PM-4:00PM)
  [the Orientation for International Students is during 8:15AM-2PM for both dates]
- August 22th to August 23th (10AM-11:30AM)
  the Orientation for RVA/RGA residents may happen during the two dates, while it is not announced as of June 12th. If it overlaps with our schedule, rescheduled time will be announced

Course Materials

In this course, we primarily use Will H. Moore and David A. Siegel (2013) *A Mathematics Course for Political and Social Research*, Princeton: Princeton University Press for our class material. In

addition, our assignments include a set of video lectures by David A. Siegel, which are available at (https://www.youtube.com/channel/UCrA2SLUKnV6yjdgIfDwFeGg/playlists).

Course Expectation

Students are expected to read the assigned sections from the textbook and watch the corresponding video lectures before the beginning of each class. The details of the assignments are listed below. For each class, students should submit their homework "problem sets" answers by the **start of the next class** on Canvas after covering corresponding materials (e.g., HW for Day 1 is by 10AM of Day 2). Although this course is not graded, homework answers will be graded and returned one day after their due dates. In addition, additional readings will be assigned. *(All assignment should be submitted as a pdf file on Canvas. You can submit the pictures of your hand-writing answers after converting them into pdf files, for example by scanning or by a relevant app such as CamScanner, or you can write your answers via LaTeX (and compile them into a pdf file) if you are already familiar with and prefer it.)*

In each class, we will (i) review previous homeworks, (ii) go over the covered materials, (iii) solve some practice problems together, (iv) and discuss questions that students have. If you have any question or concept that you want to clarify in each class, it is more than welcome and please email it to me ideally by 5pm of the day before each class.

Finally, this course is designed for incoming students with all level of mathematical background. If you think that you are already familiar with basic topics, I encourage you to read more advanced chapters in the book, which are not included in the assignments, and further expand your knowledge. If you think that you do not have a strong or any mathematical background, there is nothing to worry about and you can build it from this course. It is highly recommended, however, for those who feel that way to go over the book and solve homework problems *at least three times*. It should be emphasized that this is not a competition and working together on the homework problem sets is more than fine as long as your write-up is done by yourself.

Special Needs

If you have a disability and need accommodations in this class, please contact me in advance, preferably before or during the first day of class. Students with disabilities also need to contact Rice Disability Support Services (Phone: +1-713-348-5841; email: adarice@rice.edu) in the Allen Center, Room 111.

Syllabus Change Policy

The contents of this syllabus may be changed by the instructor. The instructor will notify students in advance, if any changes are made.

Course Outline and Assignment

Day 1: Introduction to Probability

- Topics: basic probability theory; combinations and permutations; joint, conditional, and marginal probabilities; Bayes' rule
- Readings: M&S (Moore and Siegel) 9.1-9.2, 10.3.2.
- Video lecture: Lecture 7, module 1-5 (+ relevant problems in problem session); Lecture 8, module 2 (10:16 $\sim$).
- Problem sets: (M&S 9.4): 1(b), 1(d), 4(b), 4(d), 5, 7, 11, 13(c), and 15. $\leftarrow$ **Homework**

Day 2: Algebra and Introduction to Functions

- Topics: mathematical symbols and notation; summation and product operators; factoring; solving equations; definition of a function; identity, inverse, monotonic functions, etc.
- Readings: M&S Chapter 1, 2.2, and 3.1.
- Video lecture: Lecture 1, module 4-6 and 10 (including 10b); Lecture 2, module 1-2.
- Problem sets: 5(b), 5(d), 5(f), and 7 of M&S 1.8; 4, 6, 16, 22, 26, and 28 of M&S 2.4; 1 and 4 of M&S 3.4.

Day 3: Examples of Functions, Limits, Sequences and Series, and More on Sets

- Topics: properties of linear and nonlinear functions (i.e. exponential, logarithm, polynomial, etc.), sequences and series, limits and continuity, and various types of sets
- Readings: M&S 3.2 and Chapter 4.
- Video lecture: Lecture 2, module 3-10.
- Problem sets: (M&S 3.4): 8, 13[1], 14[2], 15, and 17.

Day 4: Introduction to Calculus and the Derivative

[1] f you have any difficulty running a Java program, you can visit https://www.desmos.com/calculator instead.

[2] The address of the website has changed to http://zonalandeducation.com/mmts/functionInstitute/polynomialFunctions/graphs/polynomialFunctionGraphs.html.

- Topics: definitions of calculus and the derivative; a brief introduction to partial derivatives
- Readings: M&S Chapter 5.
- Video lecture: Lecture 3.
- Problem sets: (M&S 5.5): 2(a), 2(i), 3(a), 3(b), 4(a), 4(c), 5(d), and 5(f).

Day 5: The Rules of Differentiation

- Topics: Computing derivatives; chain rule; derivatives of functions (polynomials, exponentials, and logarithms)
- Readings: M&S Chapter 6.
- Video lecture: Lecture 4.
- Problem sets: (M&S 6.4): the 6th, 8th, 12th, 18th, and 24th of problem 1, 2(b), 2(e), 2(k), and 2(m).

Day 6: The Integral

- Topics: the definition of the integral; definite and indefinite integral; integrals of functions
- Readings: M&S 7.1-7.3.5.
- Video lecture: Lecture 5, module 1-6 (+ relevant problems in problem session).
- Problem sets: (M&S 7.6): 3(i), 3(k), 3(o), 4(k), 4(o), 5, 6(a), 6(b), 6(c), and 6(j).

Day 7: Extrema in One Dimension

- Topics: maxima and minima; first and second derivative tests; concavity and convexity
- Readings: M&S Chapter 8.
- Video lecture: Lecture 6.
- Problem sets: (M&S 8.5): 1(b), 1(d), 1(g), 1(i)[3], and 2 (assuming that $a > 0$).

Day 8: Introduction to Vectors and Matrices

- Topics: scalars and vectors; matrix algebra; matrix inverse; properties of vectors and matrices
- Readings: M&S 12.1-12.4.

[3]Check the Errata from http://people.duke.edu/∼das76/Research/Errata.pdf

- Video lecture: Lecture 10, module 1-8.
- Problem sets: (M&S 12.6): 1(c), 1(f), 1(j), 3(b), 3(d), 4(e), 4(g), 4(k), 5(b), 5(d), and 6(a).

Day 9: Application of Matrix Algebra and Concepts in Linear Algebra

- Topics: Application of Matrix Algebra and Concepts in Linear Algebra
- Readings: M&S 12.5, 13.1, 15.2.2 (the gradient vector part), and 15.2.4.
- Video lecture: Lecture 10, module 9-problem session; Lecture 11, module 1-2; Lecture 14, module 1 and 4.
- Problem sets: 8, 9, 10, 11, and 12 of M&S 12.6; 1(a) and 1(b) of M&S 13.4.

Additional Readings and Exercises

The purpose of additional readings and in-class exercises is to provide opportunities for students to experience the practical application of mathematical tools in the field of political science beyond simple textbook exercises. Students are asked to present their tentative answers during the class and turn in their final answers by the beginning of the next class. While both textbook and exercise assignments are due by the beginning of the next class, solving the problems listed below appears to be quite difficult without studying the book material ahead of time. Thus, it is highly recommended to study the book material first and then tackle with the following question (and review the book material if necessary again). All the readings will be uploaded on Canvas.

Day 1: Understanding Conditional Probability and the Bayes Rule

- Imai, K., & Khanna, K. (2016). Improving ecological inference by predicting individual ethnicity from voter registration records. *Political Analysis*, 24(2), 263-272.
- Problem: Using the Bayes rule and property of conditional probability, derive Equation 2 (page 265). Specifically, write out explicitly why both sides are mathematically equivalent.

Day 2: Quantitatively Predictive Logical Model (I): Effective Number of Seat-Winning Parties

- Shugart, M. S., & Taagepera, R. (2017). *Votes from seats: Logical models of electoral systems*. Cambridge University Press. (Chapter 2 and Chapter 7)
- Problem: Based on the explanation in Chapter 7, explain how we obtain Equation 7.1 ($N_s = (MS)^{1/6}$) and what the equation means.

Day 3 & 4: Quantitatively Predictive Logical Model (II): Electoral Volatility

- Taagepera, R. (2008). *Making social sciences more scientific: The need for predictive models.* OUP Oxford. (Chapter 4)
- Problem: Following the four steps of predictive model building, develop your own predictive model based on your interests. Step 4 (checking the model with data) is optional.

Day 5: Utility Function and Limit

- Granato, J., Lo, M., & Wong, M. S. (2010). A framework for unifying formal and empirical analysis. *American Journal of Political Science*, 54(3), 783-797.
- Problem: In your word, explain how Kedar obtains Hypothesis 1 with a particular focus on the concept of taking the limit (you do not have to solve the optimization problem here).

Day 6: Integral and Probability Distributions

- Reading: None
- Problem: Using the property of integral, check if the sum of the following probability density function or mass function is 1: (1) continuous uniform distribution; (2) exponential distribution. Check also the expected value (theoretical mean) and theoretical variance of each distribution.

Day 7: Optimization in Voting Behavior

- Kedar, O. (2005). When moderate voters prefer extreme parties: Policy balancing parliamentary elections. *American Political Science Review*, 99(2), 185-199.
- Problem: Derive Equation 7 from Equation 6 by solving the optimal value for p_A

Day 8: Matrix in the OLS (I)

- Reading: None
- Problem: Derive the solution of the ordinary least squares (OLS) estimate:$\hat{\boldsymbol{\beta}} = (\mathbf{X^T X})^{-1}\mathbf{X^T y}$ using the property of matrix calculation.
  - Hint: $\hat{\boldsymbol{\beta}} = \arg\min_{\boldsymbol{\beta}}[\boldsymbol{\epsilon^T \epsilon}] = \arg\min_{\boldsymbol{\beta}}[(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})^{\mathbf{T}}(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})]$ (i.e., find $\hat{\boldsymbol{\beta}}$ that minimizes $\boldsymbol{\epsilon^T \epsilon}$)

- Hint: Use $\frac{\partial \boldsymbol{\alpha}^T \beta}{\partial \beta} = \frac{\partial \beta^T \boldsymbol{\alpha}}{\partial \beta} = \boldsymbol{\alpha}$ (when $\boldsymbol{\alpha}$ and β are $k \times 1$ vectors.)
- Hint: Use $\frac{\partial \beta^T \mathbf{A} \beta}{\partial \beta} = 2\mathbf{A}\beta = 2\beta^T \mathbf{A}$ (when $\mathbf{A}$ is any symmetric matrix)

Day 9: Matrix in the OLS (II)

- Reading: None
- Problem: TBA [depends on the progress in class]

How Can I Study for the Course?

If you want to be ready as much as possible, I would suggest that you start studying **as soon as possible**. But I understand that sometimes we get overwhelmed and do not even know when and where to start. In case you may be interested in, I'd like to introduce my strategies, although they are not the only ways to prepare for the course. Starting from the big picture, I would suggest the following strategies:

(1) Get the book and read it three times: For the first time, I suggest that you skim through all the chapters assigned in the syllabus to get the sense of what to expect. Here you do not need to understand everything. You may encounter several concepts and ideas you are not familiar with, so please focus on absorbing those big ideas instead of details in calculation and symbols. When you finish going through the book, I think that you would feel more comfortable handling homework questions and attending the course because you'd know what it looks like and, more importantly, what you'd need to study more.

For the second time, please read the book more carefully and try to understand each concept and technique. Again you do not need to be perfect on everything, but here I recommend that you keep track on sections you have not mastered by highlighting or dog-earing so that later you can come back to these unfinished sections and try again until you understand the material in them.

For the third time, you can quickly review the book chapters right before the class for each topic. For example, it is a good timing to review chapters on Probability one day or two days before Day 1. At this time, you may be able to understand parts you did not master for the first and second times especially if there is some time gap between the second time and the third time as internalization takes some time (i.e., reading the same material on Monday and Tuesday may not be as helpful as reading it on Monday and two weeks later).

(2) Homework assignments: Generally speaking, I would suggest that you turn in your first drafts of your homework assignments before August and then revise your drafts into your final

answers. As written in the syllabus, you can submit your answer files as many times as possible and the most recent versions will be graded. It is always the case that knowing about questions and actually solving them are very different, and if you feel anxious about the assignments, start solving them is in fact the most effective measure.

In terms of the textbook problems, I would recommend that you solve them after reading given chapters for the second time. For example, I would suggest the following flow. "Skimming the entire book (for the first time)" $\rightarrow$ "Scrutinizing Sections 9.1-9.2, 10.3.2." $\rightarrow$ "Solving problems for Day 1" $\rightarrow$ "Scrutinizing Sections 1, 2.2, and 3.1." $\rightarrow$ "Solving problems for Day 2" – and so on.

For the additional readings and exercises, I would recommend a similar path. To grasp the big picture of each article, please read additional readings without being distracted by the details. Next, please read the question and understand what it means for each topic. Finally, please review the parts of the articles that you need to focus in order to complete your answers. Here you do not need to understand everything on the readings as long as you can understand what the questions are asking and solve them.

There are a lot of different ways to utilize additional readings and exercises. You could first go through additional readings and exercises, find out which parts you need to study more, and then examine the book material to solidify the basics. You could also start from the book chapters, solve book problems, and then check out additional readings to learn about real applications without being distracted by unknown mathematics. Or you could always mix the order by topics (Days) and prepare for the course.

Overleaf

In case you may be interested in, I'd like to introduce an online text editor based on LaTeX called overleaf. Simply put, LaTeX is a document preparation system that is suitable for writing mathematical expressions, and overleaf provides an online environment to use LaTeX without installing any program on your computer. While you need to use a little bit of programming language (e.g., /textbf{make this bold} to output **make this bold**), once you learn a set of simple rules, you will be able to compile good-looking pdf files with lots of complex mathematical expressions that are difficult to output in other documentation systems like Microsoft Word and make it very reproducible.

You do not have to use it to create your homework answers. But if you are interested in, I am happy to help you learn how to use it, so please do not hesitate to email me. Overleaf main page is https://www.overleaf.com. Tutorials and basic rules are available from thins link . A lot of templates are also available from "New Project" > "Template" > "Homework Assignment".