Bias-Corrected Crosswise Estimators for Sensitive Inquiries*

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Abstract

The crosswise model is an increasingly popular survey technique to elicit candid answers from respondents on sensitive questions. We demonstrate, however, that the conventional crosswise estimator for the population prevalence of sensitive attributes is biased toward 0.5 in the presence of inattentive respondents who do not follow the instruction under this design. We propose a simple bias-correction to the conventional crosswise estimator and show that our bias-corrected estimator can be easily implemented without measuring individual attentiveness. We also offer several extensions including a sensitivity analysis for conventional crosswise estimates, a framework for multivariate regressions in which a latent sensitive trait is used as the outcome or a predictor, and a strategy for population weighting while applying the bias-correction. We illustrate our methodology by simulation studies and empirical examples and further provide a practical guide for designing surveys to enable our proposed bias-correction.

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1 Introduction

Social scientists often use surveys to probe topics that respondents may hesitate to answer truthfully. Such topics include racial animus (Kuklinski, Cobb and Gilens, 1997), discriminatory attitudes and behaviors based on sex (De Jong, Pieters and Stremersch, 2012), support for militant organizations (Lyall, Blair and Imai, 2013), sexual behaviors (Vakilian, Mousavi and Keramat, 2014), corruption (Reinikka and Svensson, 2003), tax evasion (Korndörfer, Krumpal and Schmukle, 2014), or illicit druge use (Shamsipour et al., 2014). In these topics (and many others), we often expect many respondents to shade their answers in a socially desirable direction (often by falsely reporting "no"). To mitigate this concern, the literature has proposed various survey techniques (e.g., randomized response techniques, list experiments, and endorsement experiments) which are designed to encourage respondents to offer truthful answers by protecting their privacy (Blair, Imai and Zhou, 2015; Rosenfeld, Imai and Shapiro, 2016).

Among these designs, the *crosswise model* is an increasingly popular technique to elicit candid answers from respondents on sensitive questions (Yu, Tian and Tang, 2008; Jann, Jerke and Krumpal, 2011; Höglinger, Jann and Diekmann, 2016). Instead of directly asking respondents about a sensitive topic, the crosswise model shows respondents two different statements (on sensitive and non-sensitive topics) and ask whether only one of them is true. Because respondents do not need to reveal anything about the sensitive topic *per se*, it is expected that they offer candid answers that they would not provide in direct questioning. Although successful applications of the crosswise model requires respondents to fully understand and follow its specific instruction, it has been reported that about 2 to 28% of respondents are not paying attention during surveys with this design. Existing studies, however, have not shown the exact consequence of having inattentive respondents and have either suggested that we drop such respondents entirely from data (without making any assumption) or adjust estimates based on a crude measure of attentiveness, leaving researchers unsatisfactory solutions.

In this article, we demonstrate that the conventional crosswise estimator for the population prevalence of sensitive traits is biased toward 0.5 in the presence of inattentive respondents (who do not follow the instruction and randomly select a given choice under the design). This is highly problematic in sensitive inquiries because researchers may mistakenly conclude that, by using the crosswise model, they successfully induced truthful answers (i.e., higher estimated prevalence) even when such estimates are an artifact of bias caused by inattentive respondents. To remedy this problem, we offer a simple design-based bias correction procedure that enables researchers to estimate and correct the bias without obtaining any individual-level data for attentiveness. Our procedure only requires researchers to include what we call an *anchor question* that resembles the question asked in the crosswise model to the survey, while making several assumptions that can be easily satisfied at the design-stage of survey. In our Online Appendix, we show how to design surveys so that researchers can effectively apply our bias correction procedure.

Moreover, we offer several useful extensions of our bias-corrected estimator including a sensitivity analysis, a strategy for population weighting, and a framework for regression analysis in which the latent sensitive trait can be used either as an outcome or as a predictor. The sensitivity analysis enables researchers to apply our bias correction to surveys even when the anchor question is not available and examine how sensitive their original crosswise estimates are to the possible presence of inattentive respondents. To illustrate this method, we apply the sensitivity analysis to six published studies based on crosswise estimates. We also propose a simple strategy to incorporate sample weights to our bias-corrected estimator to help analysts extend their inferences into a larger population of interest as sensitive questions are often asked in surveys with unrepresentative samples. The proposed regression framework then enables us to examine potential associations between covariates and the unobserved (individual-level) sensitive trait as a response variable or an explanatory variable.

In what follows, we first describe the basic setup of the crosswise model and discuss the presence and consequence of inattentive respondents when using the design as well as previously proposed solutions to them. We then derive the bias in the conventional crosswise estimator, illustrate our bias correction procedure using the anchor question, and clarify several assumptions that are required to enable our bias correction. After demonstrating the method by simulations, we discuss three extensions while leaving technical discussions to our Online Appendix. Software to implements the bias-corrected estimator and its extensions will be available from one of the author's GitHub repository. Additional analyses, information, and demonstrations can be found in our Online Appendix.

2 Crosswise Model for Sensitive Inquiries

To understand why bias-correction is critical in the crosswise model, we first describe how the crosswise model works and elaborate the potential problems caused by careless and inattentive responses. In short, due to their complicated instructions, the crosswise model may be more likely to produce careless and inattentive responses than direct inquiries. We show that previously suggested solutions will not work unless researchers make unreasonably strong assumptions about careless and inattentive responses.

2.1 Crosswise Model

Yu, Tian and Tang (2008) proposed the crosswise model building on a class of randomized response techniques (RRT) in sensitive inquiries (Warner, 1965; Blair, Imai and Zhou, 2015; Fox, 2015). The crosswise model makes inquiries of the following form.

> **Instruction: Please read the two statements below**
>
> Statement A: I would feel uncomfortable if an immigrant family moved in next door
>
> Statement B: My mother was born in January, February, or March
>
> **Question: Which of the following most appropriately describes your case?**
>
> - Both statements are TRUE, or both statements are FALSE **[the crosswise item]**
> - Otherwise

Here, Statement A is a sensitive statement, and our quantity of interest is the proportion of individuals who agree with (or fit the description in) Statement A (often called as *prevalence rate*). Usually, the prevalence rate of Statement A is less than 0.5 and even close to 0 (e.g., the proportion of workers who have committed serious crime before) and the direct questioning is expected to create a social desirability bias toward 0 (i.e., most people do not want to report even if they fit the description).[1] In contrast, Statement B is a non-sensitive statement whose population prevalence is *ex ante* known to researchers.

Now, after considering *both* statements, respondents are asked to answer whether they are both true or false in the Question. We call this item as the *crosswise item* as this condition corresponds to the two gray cells in Table 1. Because they do not need to reveal whether they fit the description in Statement A *individually* (and it is never known to interviewers), it is expected that they answer truthfully to this question without being affected by social desirability bias. Given the observed *crosswise proportion*, $\mathbb{P}$(TRUE-TRUE $\cup$ FALSE-FALSE), researchers can then estimate the quantity of interest via a simple plug-in estimator.

[1]For this reason, we assume that the quantity of interest is always less than 0.5 in the rest of the argument. Even when it is greater than 0.5 (e.g., most people say Yes to the statement), we can always flip the direction of inquiries before (i.e., including "NOT" clause) or after the surveys (i.e., subtracting the quantity of interest from 1).

Table 1: **The Crosswise Model**

		Statement A (sensitive item) TRUE	Statement A (sensitive item) FALSE
Statement B (non-sensitive item)	TRUE	■	
	FALSE		■

To demonstrate how the crosswise model works, we provide a simple numerical example. Suppose that the true probability for Statement B is known to be 0.25 and the observed crosswise proportion is 0.65. As we elaborate below, the crosswise proportion is a combination of the probabilities of Statement A and Statement B being true, respectively. In our example, we can show that:

$$
\begin{aligned}
&\mathbb{P}(\text{TRUE-TRUE} \cup \text{FALSE-FALSE}) \\
=&\mathbb{P}(\text{A=TRUE}) * \mathbb{P}(\text{B=TRUE}) + \mathbb{P}(\text{A=FALSE}) * \mathbb{P}(\text{B=FALSE}) \\
=&\mathbb{P}(\text{A=TRUE}) * 0.25 + \mathbb{P}(\text{A=FALSE}) * 0.85 \\
=&0.65.
\end{aligned}
$$

Consequently, if we know the probability for Statement B and the crosswise proportion, we can easily reverse engineer the probability for Statement A as $\mathbb{P}(\text{Statement A=TRUE}) = \frac{0.65-0.85}{2*0.25-1} = 0.2$.[2] The next section introduces the mechanics more formally.

Among other RRTs, the crosswise model is known to be the most effective in terms of reducing social desirability bias because, under this design, there is no clear choice to make even when respondents still want to "lie" about their answers (Yu, Tian and Tang, 2008). This is true because respondents are never asked directly about the sensitive statement and respondents who understand the design have no incentive to answer untruthfully. It is worth noting, however, that this theoretical feature of the crosswise model only works when all respondents carefully read, fully understand, and follow the complex instruction. Indeed, "random picks" by respondents in the crosswise model not only increase standard errors, but also lead to biased point estimates.

[2]To "re-reverse" engineer this, see that $\mathbb{P}(\text{TRUE-TRUE} \cup \text{FALSE-FALSE}) = 0.2 * 0.25 + (1 - 0.2) * (1 - 0.25) = 0.65$.

2.2 Inattentive Responses

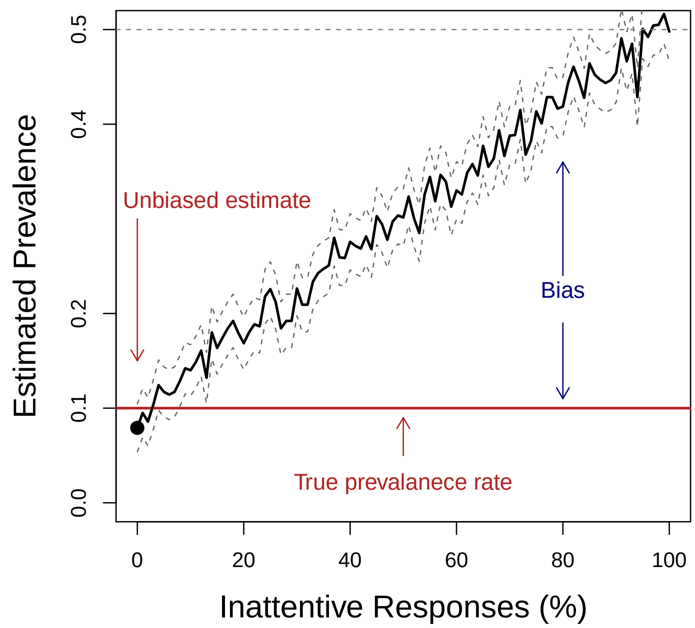
In surveys, inattentive responses are pervasive, and many researchers rely on different attention checks to detect any careless and inattentive respondents during surveys. This is especially true in opt-in online surveys where it is estimated that at least 8% to 12% of survey takers are inattentive (Crump, McDonnell and Gureckis, 2013; Meade and Craig, 2012; Maniaci and Rogge, 2014). Researchers have found that inattentive responses are also common in the crosswise model (Schnapp, 2019), estimating that 2% and 12% (Höglinger and Diekmann, 2017), 28% (Höglinger and Jann, 2018), and 13% (Enzmann, 2017, cited in Schnapp (2019)) of respondents chose survey answers randomly.

As we demonstrate below, point estimates from the crosswise model are highly sensitive to respondents who answer randomly, and the presence of such respondents generate an estimated prevalence rate biased toward 0.5. This is highly problematic in the context of sensitive inquiries (where we expect social deniability bias toward 0) because such random answers might make researchers falsely conclude that they successfully induced more candid (and thus true) answers — free from social desirability bias — from respondents even when the true prevalence of the sensitive attribute is less pervasive.

To illustrate this issue, Figure 1 plots the estimated prevalence rate of the sensitive attribute (with its 95% confidence interval) against the percentage of inattentive responses based on hypothetical (and typical) data from the crosswise model. It portrays that there is always a positive bias in the crosswise estimate (i.e., divergence between the true proportion of the sensitive attribute and its estimate) and the bias increases as the percentage of inattentive responses grows. While the naïve crosswise estimate is still unbiased under the (rare) condition that every respondent follows the instruction (the left arrow), this condition does not meet in almost all survey data. Our contribution here is to present a simple way to quantify the bias caused by inattentive responses (the right arrows) and correct such bias so that the true prevalence rate can be captured by crosswise estimates.

To date, several solutions to the presence of inattentive respondents have been proposed (even though previous research has not actually shown what the bias actually looks like as in Figure 1). The first approach is to remove inattentive survey takers from data and perform estimation and inference on the "cleaned" data (Höglinger and Diekmann, 2017; Höglinger and Jann, 2018). However, this approach leads to a biased estimate of the true prevalence rate unless researchers assume that attentive respondents are a simple random

Figure 1: **Consequences of Inattentive Responses on Crosswise Estimates: An Illustration**



Note: This plot illustrates that the bias increases as the percentage of inattentive responses grows. Gray dashed lines show confidence intervals. The results are based on hypothetical data where $\pi = 0.1$, $p = 0.2$, and $\widehat{\lambda} = 0.6$.

subsample from the original sample. This is an unreasonably strong assumption in most situations and needs to be empirically tested. Even if the assumption holds, this approach necessarily affects inferences by decreasing relative efficiency (i.e., losing statistical power).

The second solution is to detect whether respondents answered crosswise questions randomly via direct questioning (i.e., "Did you lie about your answer to the last question?") and then to adjust the prevalence estimates accordingly (Schnapp, 2019). This approach is valid when researchers assume that the direct question is itself not susceptible to the inattentiveness or social desirability bias. But such assumptions are largely questionable.

Below, we present an alternative solution to the problem by offering a bias-corrected crosswise estimator. Although our bias-corrected estimator is less efficient than an uncorrected or naïve estimator, it yields unbiased estimates of the true prevalence rates with a remarkably weaker set of assumptions than in existing solutions.

3 Bias-Corrected Crosswise Estimators

We now quantify the degree of bias caused by careless and inattentive respondents and then propose a correction for it. The key idea is that the amount of bias can be estimated from the parameters in the crosswise model and the "proportion of inattentive responses." While the latter quantity is unknown in the conventional crosswise model, we show that it is still possible to estimate this proportion by slightly modifying the design. One of our contributions is to point out the exact property of the crosswise model that enables us to obtain such bias-corrected estimates with a set of reasonable assumptions.

3.1 Setting and Notation

For simplicity, suppose that we consider one sensitive question of interest and that N respondents are drawn from a finite population of interest via simple random sampling. Suppose also that we apply the crosswise model to estimate the prevalence of the sensitive item and there are no missing data.

Let π be the true prevalence rate of interest (population proportion of individuals who fit the description in Statement A), and let λ denote the crosswise proportion (proportion of respondents who choose TRUE-TRUE or FALSE-FALSE in the crosswise question). Let us then define p as the *randomization probability*, which is the known proportion of individuals who fit the description in Statement B.

Based on Table 1, Yu, Tian and Tang (2008) show that the (true) crosswise proportion is a function of the true prevalence rate and the randomization probability:

$$\mathbb{P}(\text{TRUE-TRUE} \cup \text{FALSE-FALSE}) = \lambda = \pi p + (1-\pi)(1-p) \tag{1a}$$

When inattentive responses are present, however, the crosswise proportion becomes a function of the true prevalence rate, the randomization probability, and the proportion of attentive responses:

$$\lambda = \Big\{\pi p + (1-\pi)(1-p)\Big\}\gamma + \kappa(1-\gamma), \tag{1b}$$

where γ is the proportion of attentive respondents and κ is the probability that inattentive respondents pick the crosswise item (TRUE-TRUE or FALSE-FALSE).

Note that Equation (1b) is a strict generalization of Equation (1a). Assuming $\gamma = 1$, the conventional crosswise model expresses the true prevalence rate as $\pi = \frac{\lambda+p-1}{2p-1}, p \neq 0.5$. The naïve crosswise estimator then is $\widehat{\pi}_{CM} = \frac{\widehat{\lambda}+p-1}{2p-1}, p \neq 0.5$, where $\widehat{\lambda}$ is the observed crosswise proportion, which is unbiased as long as $\gamma = 1$ (Online Appendix A.4). When $\gamma < 1$, however, $\widehat{\pi}_{CM}$ is no longer an unbiased estimator because it does not take into account the presence of inattentive respondents. We can quantify and define the bias in this situation (Online Appendix A.1) as:

$$\begin{aligned}\mathbb{E}[\widehat{\pi}_{CM}] - \pi &= \left(\frac{1}{2} - \frac{1}{2\gamma}\right)\left(\frac{\lambda - \kappa}{p - \frac{1}{2}}\right) \\ &\equiv B_{CM}\end{aligned}$$

Here, B_{CM} is a bias with respect to the true prevalence rate caused by the presence of inattentive responses (i.e., $\gamma \neq 1$). Importantly, whenever the true prevalence is less than 0.5 (as we assume), the bias term is always positive (Online Appendix A.2). This means that the conventional crosswise estimator always *overestimates* the true prevalence rate in the presence of inattentive responses. This property of the bias yields a highly problematic consequence as discussed above.

3.2 Bias-Corrected Estimator

To address the problem, we propose the following bias-corrected crosswise estimator:

$$\widehat{\pi}_{BC} = \widehat{\pi}_{CM} - \widehat{B}_{CM} \tag{2a}$$

where we estimate the bias term as:

$$\widehat{B}_{CM} = \left(\frac{1}{2} - \frac{1}{2\widehat{\gamma}}\right)\left(\frac{\widehat{\lambda} - \frac{1}{2}}{p - \frac{1}{2}}\right), \tag{2b}$$

where $\widehat{\gamma}$ is the observed proportion of inattentive responses, whereas $\mathbb{E}[\widehat{\lambda}] = \lambda$ and $\mathbb{E}[\widehat{\gamma}] = \gamma$ and thus $\mathbb{E}[\widehat{B}_{CM}] = B_{CM}$.

For our bias-correction to work, we need few assumptions. First, we assume $\kappa = 0.5$, that is, inattentive respondents pick the crosswise item with probability of 0.5. We state this assumption as follows:

Assumption 1 (Random Pick). *Inattentive respondents choose the crosswise item with probability of 0.5 ($\kappa = 0.5$).*

The survey literature tells that this assumption may not hold in most situations because inattentive respondents are more likely to choose a first listed item than a second listed (or lower listed) item (Krosnick, 1991; Galesic et al., 2008). Nevertheless, it is still possible to *design* a survey so that we obtain $\kappa = 0.5$ regardless of how careless respondents pick items. This is easily achieved by randomization of the order of the listed items in Question.

When estimating the bias term, another challenge is to obtain an estimated proportion of attentive respondents ($\widehat{\gamma}$). We solve this problem by employing what we call an *anchor question*, where we ask about a non-sensitive item whose prevalence is *a priori* known in the crosswise format. For example, for a survey administered to a U.S. population (e.g., Qualtrics survey), an anchor question may look like:

Instruction: Please read the two statements below

Statement C: I am taking this survey in France

Statement D: My best friend was born in January, February, or March

Question: Which of the following most appropriately describes your case?

- Both statements are TRUE, or both statements are FALSE **[the crosswise item]**
- Otherwise

Here, Statement C is a non-sensitive anchor statement and we know that the true prevalence of individuals who fit its description is (supposed to be) 0. Statement D is another non-sensitive (randomization) statement whose population prevalence is known to researchers just like Statement B. Similarly to the target sensitive question, the crosswise proportion in the anchor question is a function of the true prevalence rate for Statement C, the randomization probability for Statement D, and the proportion of attentive responses (here we assume $\kappa = \frac{1}{2}$).

Now, because we know the first two probabilities, we can recover the proportion of attentive responses from the crosswise proportion. Namely, rearrange Equation (1b) yields,

$$\gamma = \frac{\lambda' - \frac{1}{2}}{\frac{1}{2} - p'}, \tag{3a}$$

where λ' is the crosswise proportion and p' is the randomization probability in the anchor question, respectively. We can estimate the quantity we need as:

$$\widehat{\gamma} = \frac{\widehat{\lambda}' - \frac{1}{2}}{\frac{1}{2} - p'}, \tag{3b}$$

where $\widehat{\lambda}'$ is the observed crosswise proportion in the anchor question. We can show that $\mathbb{E}[\widehat{\lambda}'] = \lambda'$ (Online Appendix A.5).

To estimate the bias term, our strategy then is to plug-in $\widehat{\lambda}$ (obtained from the anchor question) into Equation (2b). For such a plug-in estimator to be valid, we need the following assumption:

Assumption 2 (Constant Attentive Rate). *The population proportion of attentive responses is constant across the crosswise and anchor questions.*

This assumption enables us to apply the information about inattentive responses obtained from the anchor question to our estimator for the sensitive question of interest.

In short, our bias-corrected estimator provides an unbiased estimate of the true prevalence rate of sensitive attributes even with the presence of inattentive responses, and it does so with only two and weak assumptions that can be easily satisfied in the design stage of surveys.

3.3 Inference

Yu, Tian and Tang (2008, 257) show that the population variance of the conventional crosswise estimator and its sample analog are as follows:

$$\mathbb{V}(\widehat{\pi}_{CM}) = \mathbb{V}\left[\frac{\widehat{\lambda}}{2p-1}\right] = \frac{\lambda(1-\lambda)}{n(2p-1)^2}$$
$$\widehat{\mathbb{V}(\widehat{\pi}_{CM})} = \widehat{\mathbb{V}}\left[\frac{\widehat{\lambda}}{2p-1}\right] = \frac{\widehat{\lambda}(1-\widehat{\lambda})}{n(2p-1)^2}$$

Based on a similar derivation, we consider the population variance and its sample analog for the bias-corrected estimator. To simplify the derivation, we first introduce the following assumption:

Assumption 3 (Independent Randomization). *The randomization probability in the sensitive question is* *statistically independent from the randomization probability in the anchor question such that the two observed crosswise probabilities are also independent. Formally,* $p \perp\!\!\!\perp p' \Rightarrow \widehat{\lambda} \perp\!\!\!\perp \widehat{\lambda}'$.

With this assumption, we now derive the variance of the bias-corrected crosswise estimator and its sample analog as follows (Online Appendix A.3):

$$\mathbb{V}(\widehat{\pi}_{BC}) = \mathbb{V}\left[\frac{\widehat{\lambda}}{\widehat{\lambda}' - \frac{1}{2}}\right] \tag{4a}$$

$$\widehat{\mathbb{V}(\widehat{\pi}_{BC})} = \widehat{\mathbb{V}}\left[\frac{\widehat{\lambda}}{\widehat{\lambda}' - \frac{1}{2}}\right] \tag{4b}$$

Note that these variances are necessarily larger than the variances of the conventional estimator. To see why, simply observe that these variances are a function of two random variables ($\widehat{\lambda}$ and $\widehat{\lambda}'$), whereas the conventional variances are a function of a single random variable ($\widehat{\lambda}$). In other words, the bias-corrected estimator inevitably has more uncertainty than the naïve crosswise estimator because the former also needs to estimate the proportion of attentive responses from data (in addition to the prevalence rate of interest). Since no analytical solution is available for Equations (4a) and (4b), in practice, we employ bootstrapping to construct confidence intervals and perform hypothesis testing.

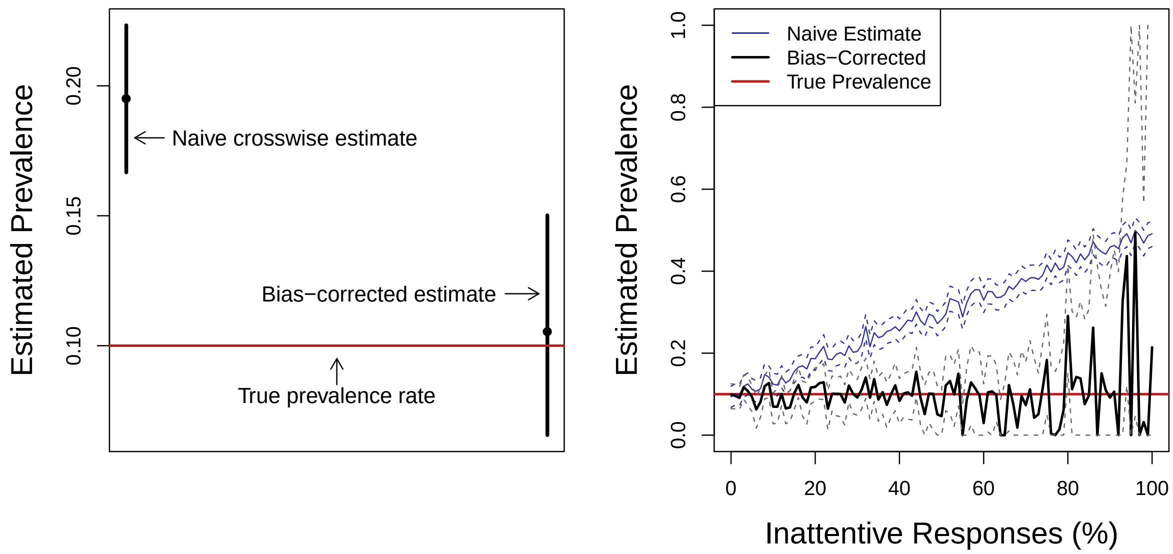
In Online Appendix E, we lay out several important points to consider when designing the survey in order to satisfy Assumptions 1-3.

4 Simulation

To illustrate our bias-correction procedure, we perform several simulation studies. The left panel of Figure 4 presents the result of the bias-correction applied to simulated (and typical) survey responses, where $N = 2000$, $\pi = 0.1$, $p = p' = 0.15$, and $\gamma = 0.8$. It shows that the naïve point estimate is far from the true prevalence rate and its 95% confidence interval does not capture the ground truth. In contrast, the bias-corrected point estimate is close to the true prevalence rate and its confidence interval covers the quantity of interest. Note that, as expected, the uncertainty around the bias-corrected estimate is larger than the uncertainty around the naïve estimate.

Using the same parameters values, we also simulate both naïve and bias-corrected estimates of the true

Figure 2: **Simulated Results of Bias-Correction**



prevalence under varying levels of the proportion of inattentive responses $(1-\gamma)$. The right panel of Figure 4 demonstrates that while the naïve point and interval estimate does increasingly poorly as more inattentive responses are included in the survey, the bias-corrected estimate is rather stable and (always) captures the true prevalence rate (in this simulation). It also indicates that when over 90% of responses are inattentive responses, the bias-corrected estimate is no longer very informative as the confidence interval is very wide. Hoever, in such surveys (in which only 10% of respondents are following the instruction), *any statistic* would be uninformative and researchers should not use the data without precautions.

In Online Appendix B, we report further results of our simulation study and confirm that the bias-corrected estimator performs as expected in more general conditions and it is asymptotically more efficient than the naïve estimator. Online Appendix D also provides an empirical illustration of our methodology by applying it to data about ethics in survey taking.

5 Extensions

Now, we consider three extensions of the bias-corrected estimator. First, we offer a sensitivity analysis for the crosswise model where the estimated proportion of attentive responses is not available. Second, we propose a weighting strategy to estimate the true prevalence among the population of interest using survey answers which are not based on random sampling. Finally, we consider a framework for multivariate regressions in which crosswise estimates are incorporated either as the outcome or a predictor.

5.1 Sensitivity Analysis

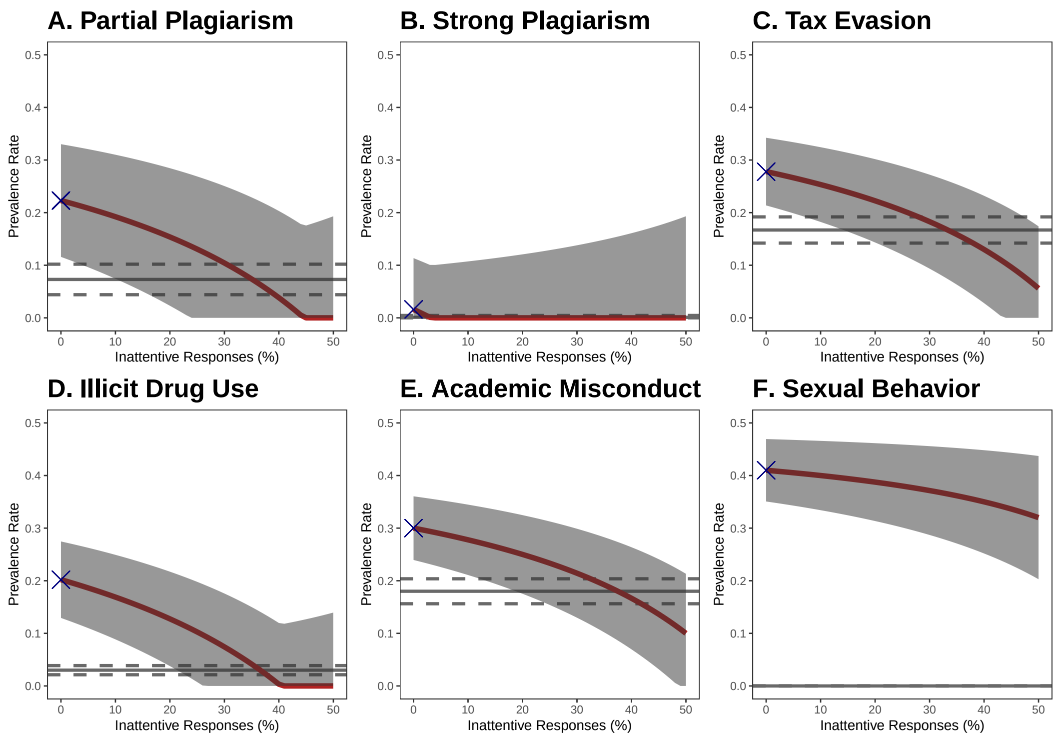
While our proposed bias correction requires researchers to obtain the estimated proportion of inattentive responses ($\widehat{\gamma}$) using the anchor question, it may be the case that researchers do not possess such information (e.g., because the crosswise model was applied in the past or an anchor question cannot be included due to cost). Here, we propose a sensitivity analysis that enables analysts to clarify how sensitive their crosswise estimates are to inattentive responses and what assumptions they must make in order to preserve their original substantive conclusions.

Specifically, we offer a set of sensitivity bounds for original crosswise estimates by applying the bias correction to them under varying proportions of attentive responses. With the sensitivity bounds, researchers can ask: In order to claim the original magnitude of the estimated prevalence, how many attentive respondents do we need to have? In other words, to what extent can we tolerate the presence of inattentive respondents in order to keep the initial claim?

To illustrate this procedure, we apply the sensitivity analysis to six published studies on sensitive behavior, including partial and severe plagiarism (Jann, Jerke and Krumpal, 2011), tax evasion (Korndörfer, Krumpal and Schmukle, 2014), illicit drug use (Shamsipour et al., 2014), academic misconduct (Höglinger, Jann and Diekmann, 2016), and sexual behavior (Vakilian, Mousavi and Keramat, 2014). Figure 3 visualizes the sensitivity analysis. For each study, we plot the bias-corrected estimates of the true prevalence rate against the possible proportion of inattentive responses under Assumption 1. We also plot the point and interval estimate based on direct questioning (if available) because many studies attempt to show (and indeed claim) that the crosswise estimator *performs better* (i.e., produce higher estimated prevalence) than direct questioning. The original point estimates are marked by $\times$.

The results suggest that, in many cases, originally presented higher prevalence estimates may have been artifacts of the presence of inattentive responses rather than the property of the crosswise model that mitigates social deniability bias. Our sensitivity analysis implies that most of these studies do not find any statistically significant difference between direct questioning and the crosswise model unless they make an assumption that the true proportion of inattentive responses is less than 0.2. In other words, in order to

Figure 3: **Sensitivity Analysis of Previous Estimates**



Note: This figure shows the results of sensitivity analysis for six crosswise estimates. For each estimate, the bias correction is applied with varying levels of the attentive rate under Assumption 1 ($\kappa = 0.5$). The solid and dashed lines show point and interval estimates based on direct questioning (except for Study F). The original point estimates are marked by $\times$.

claim that the crosswise estimator does any better than direct questioning in these studies, the researchers must assume that more than 85-90% of the respondents were attentive and followed the instruction and met Assumption 1. While we cannot verify this with given data, the original estimates must be looked with precautions given the previous findings that 2% to 28% may have been inattentive in surveys in which the crosswise model was employed (Section 2.2).[3]

5.2 Weighting

Weighting is a technique to estimate population level quantities based on sample statistics obtained through unrepresentative samples. While the literature on sensitive inquiries usually assumes that samples are gener-

[3]Our R program for performing this sensitivity analysis also allows researchers to set different values of κ other than 0.5 depending on the nature of their already implemented surveys. The effect of κ on the bounds is, however, context dependent. This is because the relative size of bias is determined by a distance between κ and an estimated crosswise proportion as indicated by Equation (2b). In practice, the true value of κ is unknown to researchers unless they design their surveys so that Assumption 1 holds. We thus recommend that analysts consider multiple values of κ if Assumption 1 might be violated under their designs.

ated by simple random sampling, a growing share of surveys are administered with unrepresentative samples such as online opt-in samples (Franco et al., 2017; Mercer, Lau and Kennedy, 2018). Online opt-in samples are known to be often unrepresentative of the entire population of interest, and researchers using such samples may wish to use weighting to extend their inferences into the population of real interest.

The benefit of using weighting is even larger for sensitive (like crosswise) inquiries since sensitive questions are not usually asked to nationally representative samples on large public surveys and researchers often need to conduct their own surveys to study these questions. To date, however, no research has provided a practical guide for how to include sample weights in crosswise estimates. In Online Appendix C.3, we show that our bias-corrected crosswise estimator can incorporate sample weights in a straightforward way.

5.3 Multivariate Regressions

In many cases, researchers seek to use the latent sensitive trait as the outcome or a predictor in multivariate regressions. The application of logistic regression to the latent sensitive variables has been considered in several studies such as Jann, Jerke and Krumpal (2011), Vakilian, Mousavi and Keramat (2014), and Korndörfer, Krumpal and Schmukle (2014), while a generalized linear model framework using the latent sensitive trait as a predictor has been offered by Blair, Imai and Zhou (2015). Here, we propose a framework of multivariate "crosswise" regressions, in which the latent sensitive trait is used either as an outcome or a predictor, while applying our bias-correction procedure therein. Online Appendix C.1 details our framework.

Concluding Remarks

We proposed a bias-corrected crosswise estimator for sensitive inquiries. The presence of inattentive responses jeopardizes our statistical inference on sensitive attributes under the conventional crosswise estimator. In particular, the bias caused by inattentive responses make researchers draw incorrect conclusions that the usage of the crosswise model induced more candid answers from survey respondents. We proposed a simple design-based solution to this substantially important problem and demonstrated our strategy both in simulations and an empirical example. Several extensions including sensitivity analysis, multivariate regressions, and weighting method were also provided. We also offered a practical guide on how to design crosswise surveys in order to enable our bias-correction.

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Online Appendix

For “Bias-Corrected Crosswise Estimators for Sensitive Inquiries”

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A Additional Discussion on the Bias-Corrected Estimator

A.1 Derivation of the Bias

Here, we derive the bias in the naïve crosswise estimator based on the argument in Section 3.

$$
\begin{aligned}
\mathbb{E}[\widehat{\pi}_{CM}] - \pi &= \mathbb{E}\left[\frac{\widehat{\lambda} + p - 1}{2p-1}\right] - \frac{\lambda - (1-p)\gamma - \kappa(1-\gamma)}{(2p-1)\gamma} \\
&= \frac{\gamma(\lambda + p - 1) - (\lambda - \gamma + p\gamma - \kappa + \kappa\gamma)}{(2p-1)\gamma} \\
&= \frac{\lambda\gamma + p\gamma - \gamma - \lambda + \gamma - p\gamma + \kappa - \kappa\gamma}{(2p-1)\gamma} \\
&= \frac{\lambda\gamma - \kappa\gamma - \lambda + \kappa}{(2p-1)\gamma} \\
&= \frac{\lambda - \kappa}{(2p-1)} - \frac{\lambda - \kappa}{(2p-1)\gamma} \\
&= \frac{1}{2}\left(\frac{\lambda-\kappa}{p-\frac{1}{2}}\right) - \frac{1}{2\gamma}\left(\frac{\lambda-\kappa}{p-\frac{1}{2}}\right) \\
&= \left(\frac{1}{2} - \frac{1}{2\gamma}\right)\left(\frac{\lambda-\kappa}{p-\frac{1}{2}}\right) \\
&\equiv B_{CM}
\end{aligned}
$$

A.2 Behavior of the Bias

By definition, the bias vanishes when the proportion of attentive respondents is 1 ($\lambda = 1$). To see this, simply observe the following limit:

$$
\begin{aligned}
&\lim_{\lambda \to 1}\left(\frac{1}{2} - \frac{1}{2\gamma}\right)\left(\frac{\lambda-\kappa}{p-\frac{1}{2}}\right) \\
&= \left(\frac{1}{2} - \frac{1}{2}\right)\left(\frac{\lambda-\kappa}{p-\frac{1}{2}}\right) \\
&= 0
\end{aligned}
$$

In contrast, as the proportion of attentive approaches 0 (from the side of 1), the bias term explodes and approaches the positive infinity. To see this, observe that the multiplier $\left(\frac{1}{2} - \frac{1}{2\lambda}\right)$ is always negative and the multiplicand $\left(\frac{\lambda-\kappa}{p-\frac{1}{2}}\right)$ is also negative under few regularity conditions. These conditions state that $\lambda > \kappa$ and $p < \frac{1}{2}$. The regularity conditions hold in usual surveys with the crosswise model. Since the multiplier grows as λ approaches 0, the bias term increases as the proportion of attentive responses decreases.

However, the limit itself does not exist as:

$$\lim_{\lambda \to 0} \left(\frac{1}{2} - \frac{1}{2\gamma}\right)\left(\frac{\lambda - \kappa}{p - \frac{1}{2}}\right)$$
$$= \mathsf{Undefined}$$

A.3 Derivation of the Variance

Here, we derive the population and sample variance of the bias-corrected crosswise estimator discussed in Section 3. Rearranging Equation (2b), we obtain

$$\begin{aligned}
\mathbb{V}(\widehat{\pi}_{BC}) &= \mathbb{V}\left[\frac{\widehat{\lambda} - (1-p)\widehat{\gamma} - \frac{1}{2}(1-\widehat{\gamma})}{(2p-1)\widehat{\gamma}}\right] \\
&= \frac{1}{(2p-1)^2}\mathbb{V}\left[\frac{\widehat{\lambda} - (1-p)\widehat{\gamma} - \frac{1}{2}(1-\widehat{\gamma})}{\widehat{\gamma}}\right] \\
&= \frac{1}{(2p-1)^2}\mathbb{V}\left[\frac{\widehat{\lambda}}{\widehat{\gamma}} - (1-p) - \frac{1}{2\widehat{\gamma}} + \frac{1}{2}\right] \\
&= \frac{1}{(2p-1)^2}\mathbb{V}\left[\frac{2\widehat{\lambda} - 1}{2\widehat{\gamma}}\right] \\
&= \frac{1}{(2p-1)^2}\mathbb{V}\left[(2\widehat{\lambda} - 1)\left(\frac{\frac{1}{2} - p}{2\widehat{\lambda}' - 1}\right)\right] \quad \text{(By Equation (3b))} \\
&= \frac{(\frac{1}{2} - p)^2}{(2p-1)^2}\mathbb{V}\left[\frac{2\widehat{\lambda} - 1}{2\widehat{\lambda}' - 1}\right] \\
&= \frac{4(\frac{1}{2} - p)^2}{(2p-1)^2}\mathbb{V}\left[\frac{\widehat{\lambda}}{\widehat{\lambda}' - \frac{1}{2}}\right] \\
&= \mathbb{V}\left[\frac{\widehat{\lambda}}{\widehat{\lambda}' - \frac{1}{2}}\right]
\end{aligned}$$

To see that no analytical form is available, observe that the variance term can be rewritten as:

$$\begin{aligned}
&\mathbb{V}\left[\frac{\widehat{\lambda}}{\widehat{\lambda}' - \frac{1}{2}}\right] \\
=&\mathbb{E}\left[\frac{\widehat{\lambda}^2}{(\widehat{\lambda}' - \frac{1}{2})^2}\right] - \left(\mathbb{E}\left[\frac{\widehat{\lambda}}{\widehat{\lambda}' - \frac{1}{2}}\right]\right)^2
\end{aligned}$$

Now, by Assumption 3, we can expand the first term as:

$$\begin{aligned}
\mathbb{E}\left[\frac{\widehat{\lambda}^2}{(\widehat{\lambda}' - \frac{1}{2})^2}\right] &= \mathbb{E}[\widehat{\lambda}^2] \times \mathbb{E}\left[\frac{1}{(\widehat{\lambda}' - \frac{1}{2})^2}\right] \\
&= \left(\frac{\lambda(1-\lambda)}{N} + \lambda\right) \sum_{n=0}^{N} \frac{1}{\left(n - \frac{1}{2}\right)^2} \binom{N}{n} (k')^n (1-k')^{N-n} \\
&= \left(\frac{\lambda(1-\lambda)}{N} + \lambda\right) (1-k')^N \sum_{n=0}^{N} \binom{N}{n} \frac{1}{\left(n - \frac{1}{2}\right)^2} \left(\frac{k'}{1-k'}\right)^n
\end{aligned}$$

Similarly, the second term can be expanded as:

$$\begin{aligned}
\left(\mathbb{E}\left[\frac{\widehat{\lambda}}{\widehat{\lambda}' - \frac{1}{2}}\right]\right)^2 &= \left(\mathbb{E}[\widehat{\lambda}] \times \mathbb{E}\left[\frac{1}{\widehat{\lambda}' - \frac{1}{2}}\right]\right)^2 \\
&= \left(\lambda \sum_{n=0}^{N} \frac{1}{\left(n - \frac{1}{2}\right)} \binom{N}{n} (k')^n (1-k')^{N-n}\right)^2 \\
&= \left(\lambda (1-k')^N \sum_{n=0}^{N} \binom{N}{n} \frac{1}{\left(n - \frac{1}{2}\right)} \left(\frac{k'}{1-k'}\right)^n\right)^2
\end{aligned}$$

Combining both results yields the population variance,

$$\begin{aligned}
\mathbb{V}(\widehat{\pi}_{BC}) = &\left(\frac{\lambda(1-\lambda)}{N}\lambda\right) (1-k')^N \sum_{n=0}^{N} \binom{N}{n} \frac{1}{\left(n - \frac{1}{2}\right)^2} \left(\frac{k'}{1-k'}\right)^n \\
&+ \left(\lambda (1-k')^N \sum_{n=0}^{N} \binom{N}{n} \frac{1}{\left(n - \frac{1}{2}\right)} \left(\frac{k'}{1-k'}\right)^n\right)^2
\end{aligned}$$

and its sample analog,

$$\begin{aligned}
\widehat{\mathbb{V}(\widehat{\pi}_{BC})} = &\left(\frac{\widehat{\lambda}(1-\widehat{\lambda})}{N}\lambda\right) (1-k')^N \sum_{n=0}^{N} \binom{N}{n} \frac{1}{\left(n - \frac{1}{2}\right)^2} \left(\frac{k'}{1-k'}\right)^n \\
&+ \left(\widehat{\lambda} (1-k')^N \sum_{n=0}^{N} \binom{N}{n} \frac{1}{\left(n - \frac{1}{2}\right)} \left(\frac{k'}{1-k'}\right)^n\right)^2
\end{aligned}$$

No analytical form is available for these functions.

A.4 Unbiasedness of the Naïve Estimator

To see that the naïve estimator is unbiased when $\gamma = 1$, let Y_i be a binary random variable denoting whether respondent i chooses the crosswise item (i.e., TRUE-TRUE or FALSE-FALSE) and its realization

$y_i \in \{0, 1\}$. Let the number of respondents choosing the crosswise item be $k = \sum_{i=1}^{N} y_i$, where $k < N$. Then, the likelihood function for λ given any observed k is $L(\lambda|N, k) = \binom{N}{k}\lambda^k(1-\lambda)^{N-k}$. Applying the first-order condition yields a maximum likelihood estimate (MLE) of λ, $\widehat{\lambda} = \frac{k}{N}$, where $\mathbb{E}[\widehat{\lambda}] = \lambda$. The unbiasedness follows from the fact that $\mathbb{E}[\widehat{\lambda}] = \mathbb{E}\big[\frac{k}{N}\big] = \frac{1}{N}\mathbb{E}[k] = \frac{1}{N}N\lambda = \lambda$. Following the parameterization invariance property of MLEs, $\mathbb{E}[\widehat{\pi}_{CM}] = \pi$.

A.5 Unbiasedness of $\widehat{\lambda}'$

To that $\widehat{\lambda}'$ is an unbiased estimator of λ', let us define that $\widehat{\lambda}'$ is a binomial random variable (like $\widehat{\lambda}$) with parameters N, λ' and $\widehat{\lambda}' = k'/N$, where k' is the number of people who choose the crosswise item in the anchor question. This is because $k' \sim$ Binom(N, λ') and $\widehat{\lambda}' = k'/N$ suggests $\widehat{\lambda}' \sim$ Binom(N, k'). The probability mass function that $\widehat{\lambda}'$ taking n/N is given by Pr$(\widehat{\lambda}' = \frac{n}{N}) = \binom{N}{n}(k')^n(1-k')^{N-n}$.

B Additional Simulation Studies

We replicate the simulation discussed in Section 4 8000 times to investigate overall performance of the bias-corrected estimator more systematically. In each simulation, we set different values of parameters and examine the coverage of the true prevalence as well as the bias and the sample root-mean-squared error (RMSE). To apply our method to realistic contexts, we choose a set of reasonable values from a parameter space, reflecting the usual situations in which crosswise estimates are applied. Specifically, in each simulation, we draw the true prevalence rate from a continuous uniform distribution (0.1, 0.45), the two randomization probabilities from a continuous uniform distribution (0.1, 0.2), and the attentive rate from a continuous uniform distribution (0.5, 1). Finally, we repeat the set of experiments for different sample sizes of 200, 500, 1000, 2000, and 5000.

We report our results in Figure B.1. These results demonstrate that the bias-corrected estimator has higher coverage, significantly lower bias, and smaller RMSE than the naïve estimator. The difference between the bias-corrected estimator and naïve estimator is especially remarkable with respect to the coverage of the true parameters. While the coverage of the native estimator's 95% confidence intervals rapidly deteriorates as the sample size increases, the bias-corrected estimator captures the true parameter approximately 95% of the time regardless of the sample size.

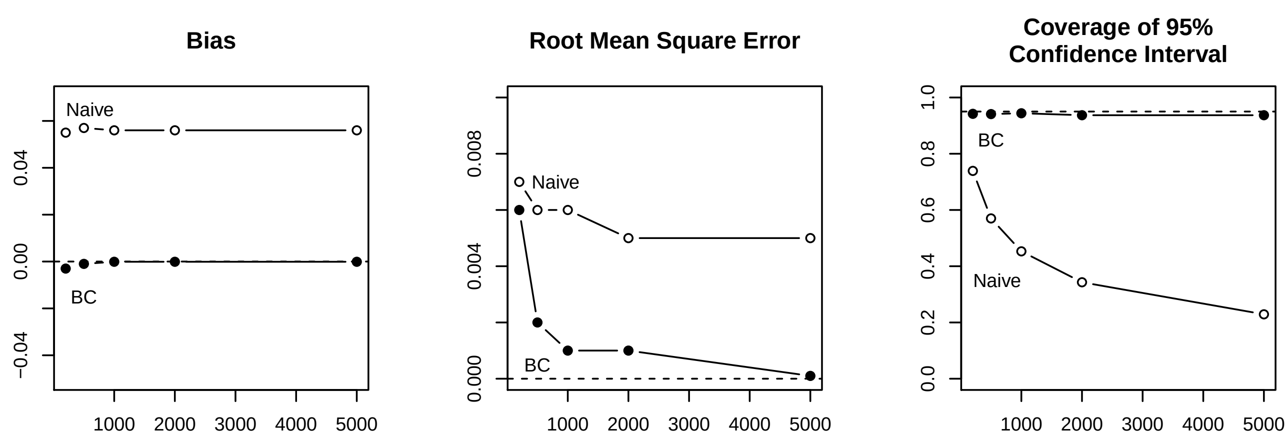

Figure B.1: **Simulation Results with Varying Sample Size**
Note: This figure displays the bias, and root mean square error, and the coverage of 95% confidence interval of naïve and bias-corrected estimators.

Moreover, we perform another simulation study to confirm that the bias-corrected estimator is not susceptible to a potential correlation between inattentiveness and possession of sensitive attributes. The concern is that if respondents with sensitive attributes are more likely to be inattentive, correcting for bias caused by inattentive responses might affect the estimate of the prevalence rate of sensitive attributes in the *population* (which includes both attentive and inattentive respondents).

The results shown in Figure B.2 resolve this concern. In each panel, we plot the bias-corrected point and interval estimates over the true prevalence rate against varying levels of inattentiveness in hypothetical data. Here, we fix the true prevalence rate among attentive respondents ($\pi_{\text{attentive}} = 0.1$) while varying the true prevalence rates among inattentive respondents in four panels ($\pi_{\text{inattentive}} \in \{0.2, 0.3, 0.4, 0.5\}$). The true prevalence rates are plotted in red dashed lines. Notice that the true prevalence rate in each panel is now a linear combination (convex combination) of $\pi_{\text{attentive}}$ and $\pi_{\text{inattentive}}$ which are denoted by red and blue dots.[1] Nevertheless, Figure B.2 show that our bias-corrected estimator properly captures the true prevalence rate of sensitive attributes regardless of the degrees of correlation between inattentiveness and possession of sensitive attributes.

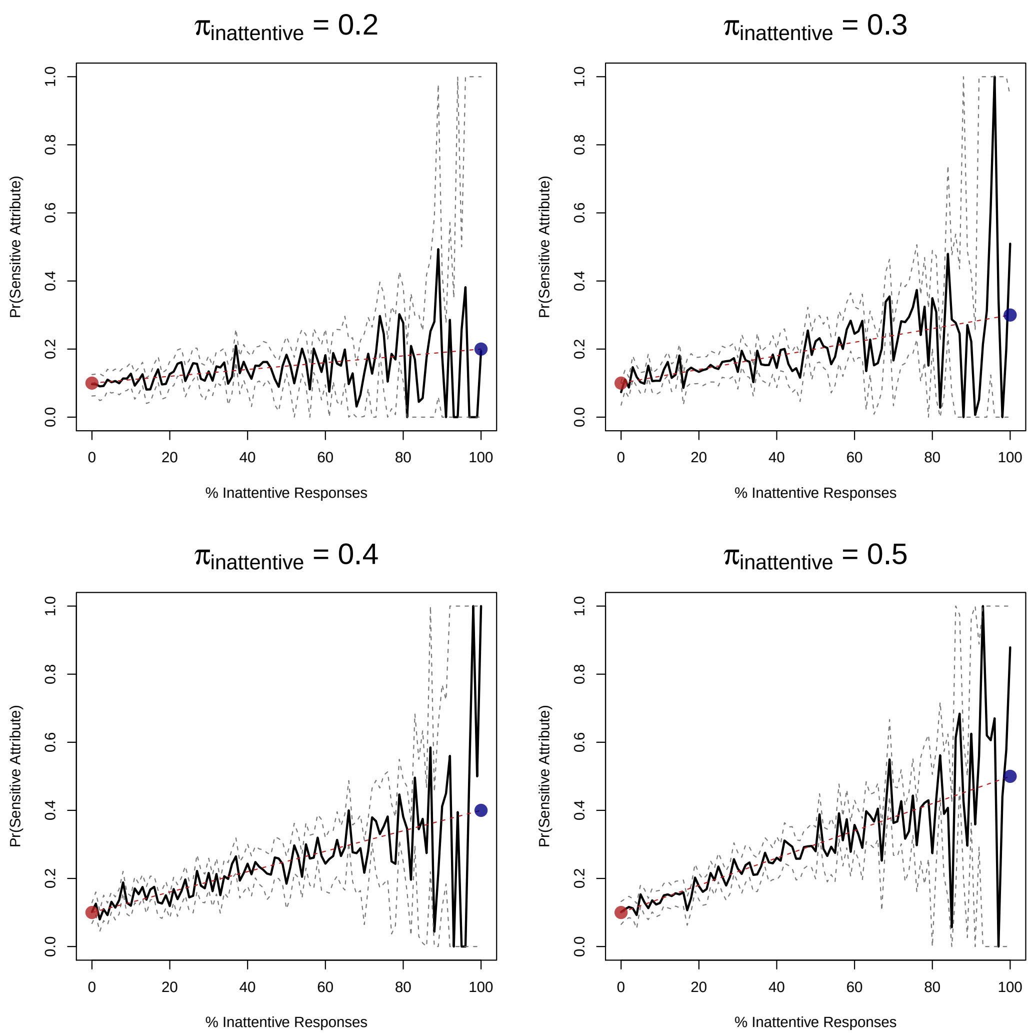

Figure B.2: **Correlation between Attentiveness and Possession of Sensitive Attributes**
Note: This graph illustrates the bias-corrected estimates (black lines with gray dashed lines) with the true prevalence rates (red dashed lines) when the true prevalence rates are convex combinations of $\pi_{\text{attentive}}$ and $\pi_{\text{inattentive}}$, denoted by red and blue dots, respectively. The bias-corrected estimates properly capture the true prevalence rates regardless of the degrees of deviation of $\pi_{\text{inattentive}}$ from $\pi_{\text{attentive}}$. While $\pi_{\text{attentive}}$ is fixed at 0.1, $\pi_{\text{inattentive}}$ varies. The data is generated by setting $N = 2000, p = 0.15, p' = 0.15$.

[1] That is, $\pi = \pi_{\text{attentive}} * \gamma + \pi_{\text{inattentive}} * (1 - \gamma)$, where γ is the proportion of attentive responses.

C Details in Extensions

In this section, we discuss the details in the proposed extensions of the bias-corrected estimator.

C.1 Multivariate Crosswise Regressions: Using the Latent Sensitive Trait as an Outcome

We propose a framework of multivariate "crosswise" regressions in which the latent (unobserved) variable for having a sensitive trait is used as an outcome variable, while applying our bias-correction procedure. Let $Z_i \in \{0, 1\}$ be a binary indicator for having a sensitive trait and $T_i \in \{0, 1\}$ be a binary indicator for giving an attentive response for respondent i. Note that both of these quantities are unobserved in reality and thus they are latent variables.

We define the regression model (conditional expectation) of interest as

$$\mathbb{E}[Z_i|\mathbf{X_i} = \mathbf{x}] = \Pr(Z_i = 1|\mathbf{X_i} = \mathbf{x}) = \pi_{\boldsymbol{\beta}}(\mathbf{x}), \tag{C.1a}$$

where $\boldsymbol{\beta}$ is a vector of unknown parameters and $\mathbf{X_i}$ is a vector of characteristics for respondent i. Our goal here is to make inferences about associations between the covariates and the possession of the sensitive trait (e.g., Are young respondents more likely to support a terrorist organization than old respondents?). Thus, $\boldsymbol{\beta}$ are our quantities of interest.

To apply our bias-correction inside the regressions, we also introduce the following conditional probability for attentive responses:

$$\mathbb{E}[T_i|\mathbf{X_i} = \mathbf{x}] = \Pr(T_i = 1|\mathbf{X_i} = \mathbf{x}) = \gamma_{\boldsymbol{\theta}}(\mathbf{x}), \tag{C.1b}$$

where $\boldsymbol{\theta}$ is a vector of unknown parameters that associate the same respondent characteristics to the probability of giving an attentive response.

Now, let $Y_i \in \{0, 1\}$ be a binary indicator for choosing the crosswise item (i.e., TRUE-TRUE/FALSE-FALSE) in the crosswise question and $A_i \in \{0, 1\}$ be a binary indicator for choosing the crosswise item in the anchor question. Let p and p' be the randomization probabilities for the crosswise and anchor questions, respectively. Recall that the crosswise proportion in the crosswise question is denoted by λ, while the crosswise proportion in the anchor question is denoted by λ'. According to our bias-correction, we can then consider the crosswise proportions conditional upon covariates as functions of Equations (C.1a) and (C.1b) as follows:

$$\lambda_{\boldsymbol{\beta}}(\mathbf{X_i}) = \Big(\pi_{\boldsymbol{\beta}}(\mathbf{X_i})p + (1 - \pi_{\boldsymbol{\beta}}(\mathbf{X_i}))(1-p)\Big)\gamma_{\boldsymbol{\theta}}(\mathbf{X_i}) + \frac{1}{2}\Big(1 - \gamma_{\boldsymbol{\theta}}(\mathbf{X_i})\Big) \tag{C.2a}$$

$$\lambda'_{\boldsymbol{\theta}}(\mathbf{X_i}) = \Big(\frac{1}{2} - p'\Big)\gamma_{\boldsymbol{\theta}}(\mathbf{X_i}) + \frac{1}{2} \tag{C.2b}$$

Assuming that Y_i and A_i are statistically independent conditional on $\mathbf{X_i}$, our approach is to model the joint probability distribution of the observed crosswise data. Under Assumptions 1-3, we can construct the following likelihood function:

$$\begin{aligned}
\mathcal{L}(\boldsymbol{\beta},\boldsymbol{\theta}|\{\mathbf{X_i},Y_i,A_i\}_{i=1}^N,p,p') &= \prod_{i=1}^N \Big\{\lambda_{\boldsymbol{\beta}}(\mathbf{X_i})\Big\}^{Y_i}\Big\{1-\lambda_{\boldsymbol{\beta}}(\mathbf{X_i})\Big\}^{1-Y_i}\Big\{\lambda'_{\boldsymbol{\theta}}(\mathbf{X_i})\Big\}^{A_i}\Big\{1-\lambda'_{\boldsymbol{\theta}}(\mathbf{X_i})\Big\}^{1-A_i} \\
&= \prod_{i=1}^N \left\{\Big(\pi_{\boldsymbol{\beta}}(\mathbf{X_i})p+(1-\pi_{\boldsymbol{\beta}}(\mathbf{X_i}))(1-p)\Big)\gamma_{\boldsymbol{\theta}}(\mathbf{X_i})+\frac{1}{2}\Big(1-\gamma_{\boldsymbol{\theta}}(\mathbf{X_i})\Big)\right\}^{Y_i} \\
&\quad\times\left\{1-\left[\Big(\pi_{\boldsymbol{\beta}}(\mathbf{X_i})p+(1-\pi_{\boldsymbol{\beta}}(\mathbf{X_i}))(1-p)\Big)\gamma_{\boldsymbol{\theta}}(\mathbf{X_i})+\frac{1}{2}\Big(1-\gamma_{\boldsymbol{\theta}}(\mathbf{X_i})\Big)\right]\right\}^{1-Y_i} \\
&\quad\times\left\{\Big(\frac{1}{2}-p'\Big)\gamma_{\boldsymbol{\theta}}(\mathbf{X_i})+\frac{1}{2}\right\}^{A_i} \\
&\quad\times\left\{1-\left[\Big(\frac{1}{2}-p'\Big)\gamma_{\boldsymbol{\theta}}(\mathbf{X_i})+\frac{1}{2}\right]\right\}^{1-A_i} \\
&= \prod_{i=1}^N \left\{\Big((2p-1)\pi_{\boldsymbol{\beta}}(\mathbf{X_i})+\Big(\frac{1}{2}-p\Big)\Big)\gamma_{\boldsymbol{\theta}}(\mathbf{X_i})+\frac{1}{2}\right\}^{Y_i} \\
&\quad\times\left\{1-\left[\Big((2p-1)\pi_{\boldsymbol{\beta}}(\mathbf{X_i})+\Big(\frac{1}{2}-p\Big)\Big)\gamma_{\boldsymbol{\theta}}(\mathbf{X_i})+\frac{1}{2}\right]\right\}^{1-Y_i} \\
&\quad\times\left\{\Big(\frac{1}{2}-p'\Big)\gamma_{\boldsymbol{\theta}}(\mathbf{X_i})+\frac{1}{2}\right\}^{A_i} \\
&\quad\times\left\{1-\left[\Big(\frac{1}{2}-p'\Big)\gamma_{\boldsymbol{\theta}}(\mathbf{X_i})+\frac{1}{2}\right]\right\}^{1-A_i} \qquad \text{(C.3)}
\end{aligned}$$

Building on van den Hout, van der Heijden and Gilchrist (2007), we then apply a generalized linear model approach, or logistic regressions more specifically to model the true prevalence rates in the crosswise and anchor questions, respectively:

$$\pi_{\boldsymbol{\beta}}(\mathbf{X_i}) = \text{logit}^{-1}(\boldsymbol{\beta}\mathbf{X_i}), \qquad \text{(C.4a)}$$

Similarly, we model:

$$\gamma_{\boldsymbol{\theta}}(\mathbf{X_i}) = \text{logit}^{-1}(\boldsymbol{\theta}\mathbf{X_i}) \qquad \text{(C.4b)}$$

Substituting Equations (C.4) and (C.4) into Equation (C.3) yields the likelihood function for a crosswise data. In our R program, we maximize this likelihood function (after taking its natural log) by an iterative maximization method. Another possible estimation strategy is based on the expectation-maximization (EM) algorithm in the spirit of Blair, Imai and Zhou (2015). However, we do not consider an EM-algorithm here as our simulation results (below) imply that the direct maximization (iterative maximization of the entire likelihood) would suffice.

To validate this regression framework, we simulate crosswise data with two covariates: $X_1 \sim \text{Binomial}(0.5)$ and $X_2 \sim \text{Poisson}(30)$. Specifically, we simulate the true prevalence rates in the crosswise and anchor questions according to the following generative models:

$$\pi = \frac{\exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2)}{1 + \exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2)}$$
$$\gamma = \frac{\exp(\theta_0 + \theta_1 X_1 + \theta_2 X_2)}{1 + \exp(\theta_0 + \theta_1 X_1 + \theta_2 X_2)},$$

where we set $\beta_0 = -1.5$, $\beta_1 = 0.5$, $\beta_2 = 0.02$ and $\theta_0 = 2$, $\theta_1 = -0.1$, $\theta_2 = -0.01$. We then simulate the crosswise data according to Equation (1b).

Finally, we estimate the crosswise regression with the latent sensitive trait as the outcome variable. Figure C.1 displays the estimated parameters and confidence intervals with different sample size. The results suggest that the proposed model and estimation strategy are able to recover the true parameters (asymptotically).

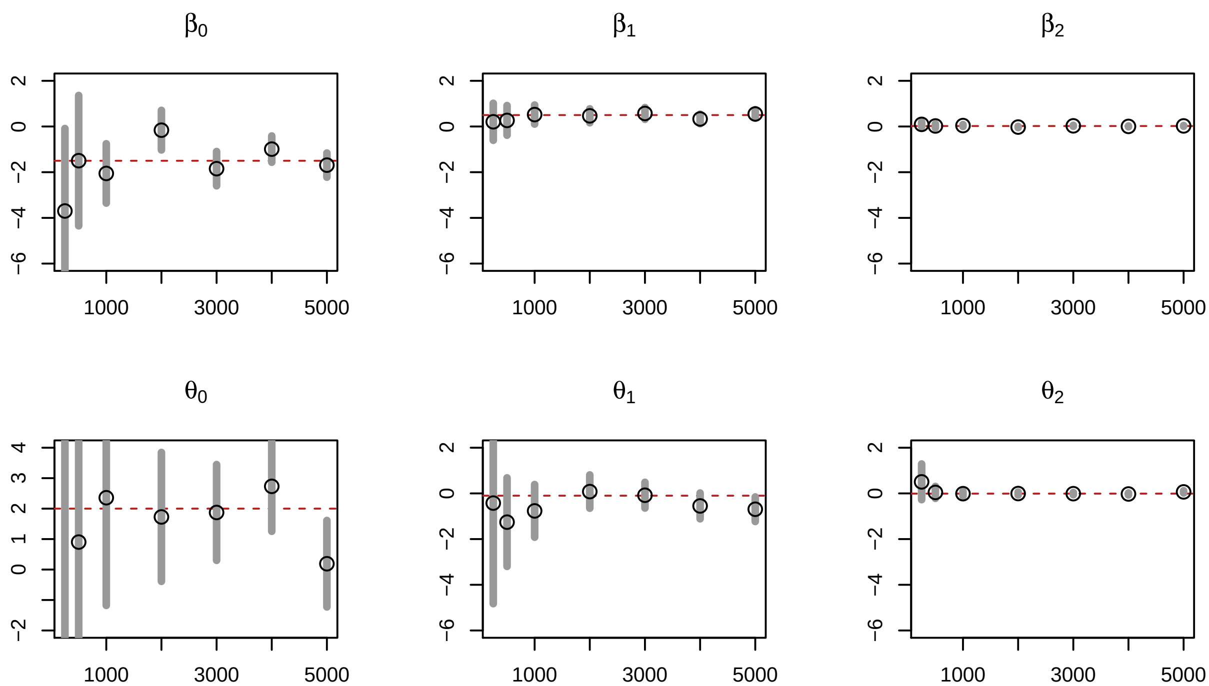

Figure C.1: **Crosswise Regression with the Latent Sensitive Trait as the Outcome**
Note: Regression estimates of six parameters in simulated data. The dashed lines indicate the true values for the parameters.

C.2 Multivariate Crosswise Regressions: Using the Latent Sensitive Trait as a Predictor

Next, we propose a framework of multivariate "crosswise" regressions in which the latent sensitive trait is used as a predictor, while applying our bias-correction. To our best knowledge, this type of regression model has not yet been proposed with respect to crosswise estimators. Thus, we begin by describing the model for the naïve crosswise estimator and then extend it to our bias-corrected estimator.

Let V_i be a continuous or discrete outcome (or response) variable for respondent i. (Other types of outcome variables can be easily incorporated into our framework. We leave to future research the development of such regressions.) We define the regression model (conditional expectation) of interest as

$$g_{\mathbf{\Theta}}(V_i|\mathbf{X_i}, Z_i), \tag{C.5}$$

where $\mathbf{\Theta}$ is a vector of parameters that associate a series of predictors ($\mathbf{X_i}$, Z_i) and the response variable (V_i). For example, for a normally distributed outcome variable, we can consider $g_{\mathbf{\Theta}}(V_i|\mathbf{X_i}, Z_i) = \mathcal{N}(\alpha+\boldsymbol{\gamma}^\top\mathbf{X_i}+\delta Z_i, \sigma^2)$ and $\mathbf{\Theta} = (\alpha, \boldsymbol{\gamma}, \delta, \sigma^2)$. Similarly, for a binary response variable, we can consider $g_{\mathbf{\Theta}}(V_i|\mathbf{X_i}, Z_i) =$ Bernoulli(ϕ), where $\frac{\phi}{1-\phi} = \alpha + \boldsymbol{\gamma}^\top\mathbf{X_i} + \delta Z_i$ and $\mathbf{\Theta} = (\alpha, \boldsymbol{\gamma}, \delta)$. Our goal here is to make inferences about an association between the latent sensitive trail and the response variable (e.g., Are people supporting a terrorist organization more likely to donate money to a local political institution?). Thus, δ is our primary quantity of interest.

To simplify the derivation, we assume that $V_i \perp\!\!\!\perp Y_i|\mathbf{X_i}$ (the outcome variable and whether the respondents choose the crosswise item are statistically independent conditional upon covariates). Then, using all the available information from data, we can construct the following observed likelihood function:

$$\begin{aligned}
\mathcal{L}(\boldsymbol{\beta}, \mathbf{\Theta}|\{V_i, \mathbf{X_i}, Y_i\}_{i=1}^N, p) &= \prod_{i=1}^N g_{\mathbf{\Theta}}(V_i|\mathbf{X_i}, Z_i)\text{Pr}(Y_i = 1, Z_i|\mathbf{X_i}) \\
&= \prod_{i=1}^N \Bigg\{ g_{\mathbf{\Theta}}(V_i|\mathbf{X_i}, 1)p^{Y_i}(1-p)^{1-Y_i}\pi_{\boldsymbol{\beta}}(\mathbf{X_i}) \\
&\qquad + g_{\mathbf{\Theta}}(V_i|X_i, 0)(1-p)^{Y_i}p^{Y_i}(1-\pi_{\boldsymbol{\beta}}(\mathbf{X_i}))\Bigg\}
\end{aligned} \tag{C.6}$$

Here, the first part is $g_{\mathbf{\Theta}}(V_i|\mathbf{X_i}, Z_i = 1)\text{Pr}(Y_i = 1|Z_i = 1)\text{Pr}(Z_i = 1|\mathbf{X_i}) = g_{\mathbf{\Theta}}(V_i|Z_i = 1, \mathbf{X_i})\text{Pr}(Y_i = 1, Z_i = 1|\mathbf{X_i})$ and the second part is $g_{\mathbf{\Theta}}(V_i|\mathbf{X_i}, Z_i = 0)\text{Pr}(Y_i = 1|Z_i = 0)\text{Pr}(Z_i = 0|\mathbf{X_i}) = g_{\mathbf{\Theta}}(V_i|Z_i = 0, \mathbf{X_i})\text{Pr}(Y_i = 1, Z_i = 0|\mathbf{X_i})$.

Now we extend this framework by incorporating the bias-correction procedure. With our bias-correction, the observed likelihood function becomes:

$$\begin{aligned}
\mathcal{L}(\boldsymbol{\beta}, \boldsymbol{\theta}, \boldsymbol{\Theta} | \{V_i, \mathbf{X_i}, Y_i, A_i\}_{i=1}^N, p, p') &= \prod_{i=1}^N g_{\boldsymbol{\Theta}}(V_i | \mathbf{X_i}, Z_i, T_i) \Pr(Y_i = 1, Z_i, T_i | \mathbf{X_i}) \Pr(A_i = 1, Z_i, T_i | \mathbf{X_i}) \\
&= \prod_{i=1}^N \Bigg\{ g_{\boldsymbol{\Theta}}(V_i | \mathbf{X_i}, 1, 1) p^{Y_i} (1-p)^{1-Y_i} \pi_{\boldsymbol{\beta}}(\mathbf{X_i}) (1-p')^{A_i} p'^{1-A_i} \gamma_{\boldsymbol{\theta}}(\mathbf{X_i}) \\
&\quad + g_{\boldsymbol{\Theta}}(V_i | \mathbf{X_i}, 0, 1) (1-p)^{Y_i} p^{1-Y_i} (1 - \pi_{\boldsymbol{\beta}}(\mathbf{X_i})) (1-p')^{A_i} p'^{1-A_i} \gamma_{\boldsymbol{\theta}}(\mathbf{X_i}) \\
&\quad + g_{\boldsymbol{\Theta}}(V_i | \mathbf{X_i}, 1, 0) \frac{1}{2} \pi_{\boldsymbol{\beta}}(\mathbf{X_i}) \frac{1}{2} (1 - \gamma_{\boldsymbol{\theta}}(\mathbf{X_i})) \\
&\quad + g_{\boldsymbol{\Theta}}(V_i | \mathbf{X_i}, 0, 0) \frac{1}{2} (1 - \pi_{\boldsymbol{\beta}}(\mathbf{X_i})) \frac{1}{2} (1 - \gamma_{\boldsymbol{\theta}}(\mathbf{X_i})) \Bigg\}, \qquad \text{(C.7)}
\end{aligned}$$

where each part is $g_{\boldsymbol{\Theta}}(V_i | \mathbf{X_i}, z, t) \Pr(Y_i = 1 | Z_i = z, T_i = t) \Pr(Z_i = z | \mathbf{X_i}) \Pr(A_i = 1 | Z_i = z, T_i = t) \Pr(T_i = 1 | \mathbf{X_i})$, where $z = \{0, 1\}$ and $t = \{0, 1\}$. We use the same modeling assumption in Equations (C.4) and (C.4).

Here, we assume that Assumptions 1-3 hold and $V_i \perp\!\!\!\perp Y_i | \mathbf{X_i}$, $V_i \perp\!\!\!\perp A_i | \mathbf{X_i}$, and $Y_i \perp\!\!\!\perp A_i | \mathbf{X_i}$. The key idea is that, under these assumptions, we can rewrite the entire likelihood of the observed crosswise data as a product of three conditional probabilities (the first equation). We can then marginalize the product over the two latent variables Z_i and T_i by summing up the conditional probabilities that we could in principle obtain for all possible combinations of the latent variables.

For example, the third component represents $g_{\boldsymbol{\Theta}}(V_i | \mathbf{X_i}, 1, 0) \Pr(Y_i = 1 | Z_i = 1, T_i = 0) \Pr(Z_i = 1 | \mathbf{X_i}) \Pr(A_i = 1 | Z_i = 1, T_i = 0) \Pr(T_i = 1 | \mathbf{X_i})$. Here, $\Pr(Y_i = 1 | Z_i = 1, T_i = 0)$ is the conditional probability that respondents choose the crosswise item when they actually have sensitive trails *and* do not provide attentive responses. Because they do not follow the instruction, Assumption 1 states that this probability is $\frac{1}{2}$ (regardless of Z_i). Next, $\Pr(Z_i = 1 | \mathbf{X_i})$ is the conditional probability that respondents have sensitive traits, and we defined this quantity as $\pi_{\boldsymbol{\beta}}(\mathbf{X_i})$. Now, $\Pr(A_i = 1 | Z_i = 1, T_i = 0)$ is the conditional probability that respondents choose the crosswise item in the anchor question when they actually have sensitive traits *and* do not provide attentive responses. Because they do not follow the instruction, Assumption 1 states that this probability is $\frac{1}{2}$ (regardless of Z_i). Finally, $\Pr(T_i = 1 | \mathbf{X_i})$ is the conditional probability that respondents *do not* provide attentive responses, and we defined this quantity as $1 - \gamma_{\boldsymbol{\theta}}(\mathbf{X_i})$. Hence, the joint probability for this component is $g_{\boldsymbol{\Theta}}(V_i | \mathbf{X_i}, 1, 0) \frac{1}{2} \pi_{\boldsymbol{\beta}}(\mathbf{X_i}) \frac{1}{2} (1 - \gamma_{\boldsymbol{\theta}}(\mathbf{X_i}))$.

To estimate the unknown parameters, including δ (our primary quantity of interest), we use an iterative maximization of the above entire observed likelihood function (after taking the natural log).

To validate the proposed framework, we simulate crosswise data with two covariates as in Online Appendix C.1. We then simulate the response variable according to the following generative model:

$$V_i = \gamma_0 + \gamma_1 X_1 + \gamma_2 X_2 + \delta Z_i + \epsilon_i,$$

where we set $\gamma_0 = 0$, $\gamma_1 = 0.3$, $\gamma_2 = 0.01$, $\delta = 1$, and $\epsilon_i \sim N(0, 1)$. Recall that Z_i is a latent variable for having a sensitive trait and we cannot observe its value directly (and thus crosswise data do not contain Z_i).

We then estimate the above crosswise regression model with the simulated observed outcome and crosswise data. Figure C.2 shows the estimates for our quantity of interest with different sample size. It demonstrates that the proposed regression model and estimation strategy can recover the latent magnitude of association between the latent sensitive trait and the response variable (i.e., doubly latent quantity).

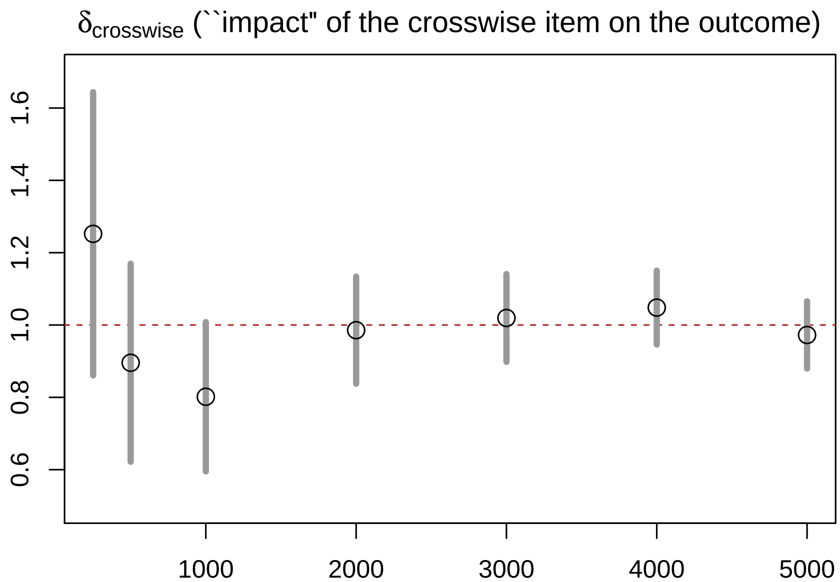

Figure C.2: **Multivariate Crosswise Regressions: Using the Latent Sensitive Trait as a Predictor**
Note: Regression estimates of six parameters in simulated data. The dashed lines indicate the true values for the parameters.

C.3 Weighting in the Crosswise Model

In this section, we offer a simple strategy to include sample weights into the bias-correction estimator.

Recall that what we actually observe in the crosswise model are λ and λ', which are observed proportions of respondents choosing the crosswise item in the crosswise and anchor questions, respectively. The idea is that we can apply the Horvitz-Thompson estimator of the mean (and thus the inverse probability weighting more generally) for the crosswise proportions, where weights are the inverse of the probabilities that respondents in different strata will be in the sample. Namely, we can apply a weight $w_i = \frac{1}{\Pr(S_i=1|X)}$, where $S_i = \{0, 1\}$ is a binary variable denoting if a unit is in the sample and X is a covariate. (Here we only consider the base weight for the simplicity, but can include other weights such as non-response weights). For each respondent, the same weight is used for the two different questions. We can then obtain the proportion of inattentive responses as a deterministic function of λ'.

Formally, we propose to include sample weights in the following way:

$$\widehat{\lambda}_w = \frac{\sum_{i=1}^{n} w_i Y_i}{N} \tag{C.8}$$

$$\widehat{\gamma}_w = \frac{\frac{1}{N}(\sum_{i=1}^{n} w_i A_i) - \frac{1}{2}}{\frac{1}{2} - p'}, \tag{C.9}$$

where w_i is a sample weight for respondent i. The above quantities are then used to calculate our bias-corrected crosswise estimates as in the plug-in principle. The sample weight can be easily incorporated when using our R program.

The proof is straightforward. Assuming that $Y_i \perp\!\!\!\perp S_{i|X}$ (choosing the crosswise item and being in the sample are statistically independent conditional upon a covaraite), weighting can recover the population crosswise proportion λ from the sample crosswise response $Y_i S_i$:

$$\begin{aligned}
&\mathbb{E}\left[\frac{Y_i S_i}{\Pr(S_i = 1|X)}\right] \\
=&\mathbb{E}\left[\mathbb{E}\left[\frac{Y_i S_i}{\Pr(S_i = 1|X)}\middle| X\right]\right] \quad \text{(Iterative Expectation)} \\
=&\mathbb{E}\left[\frac{\mathbb{E}[Y_i|X]\mathbb{E}[S_i|X]}{\Pr(S_i = 1|X)}\right] \quad \text{(Conditional Independence)} \\
=&\mathbb{E}\left[\frac{\mathbb{E}[Y_i|X]\Pr(S_i = 1|X)}{\Pr(S_i = 1|X)}\right] \quad \text{(Definition of Expectation)} \\
=&\mathbb{E}[\mathbb{E}[Y_i|X]] \\
=&\mathbb{E}[Y_i] \quad \text{(Iterative Expectation)} \\
=&\lambda
\end{aligned}$$

Similarly, we can show that

$$\begin{aligned}
&\mathbb{E}\left[\frac{A_i S_i}{\Pr(S_i = 1|X)}\right] \\
=&\mathbb{E}\left[\mathbb{E}\left[\frac{A_i S_i}{\Pr(S_i = 1|X)}\middle| X\right]\right] \\
=&\mathbb{E}[A_i] \\
=&\lambda'
\end{aligned}$$

In practice, researchers can calculate weights using their favorite weighting techniques such as raking (or iterative proportional fitting), matching, propensity score weighting, or sequential applications of these. Recent research shows that "when it comes to accuracy, choosing the right variables for weighting is more important than choosing the right statistical method" (Mercer, Lau and Kennedy, 2018, 4). Thus, we recommend that researchers think carefully about the association between the sensitive attribute of interest and basic demographic and other context dependent factors when using weighting. For the purpose of choosing the "right" variables, our proposed regression models can also be useful exploratory aids. When generalizing the results on sensitive attributes to a larger population, however, it is strongly advised to elaborate on how weights are constructed and what potential bias may exist (Franco et al., 2017).

Another possible approach to deal with highly selected samples is to employ multilevel regression and post-stratification (MRP) (Downes et al., 2018). While we do not consider MRP with crosswise estimates in this article, future research should explore the optimal strategy to use MRP in sensitive inquiries.

D Empirical Illustration

In this section, we illustrate the proposed methodology by using a survey data about the behavior of paid survey takers.

We ran an online survey through Qualtrics asking respondents about their past behavior as paid survey takers. Specifically, we asked whether they have (1) speeded through questions without reading, (2) made up answers, and (3) lied about their qualifications. It was emphasized that the survey was specifically about the behavior of paid survey takers. We did so in order to create a normative environment that admitting the behaviors in (1) to (3) becomes a fairy sensitive response because as paid survey takers they are not supposed to do any of the three "unethical" items.

For our anchor question, we asked whether respondents were taking the current survey somewhere outside the United States. We chose this anchor item because we know that all survey takers in Qualtrics are sampled from survey takers who are living in the U.S. and the topic is closely related to our sensitive items of interest. For randomization probabilities, we asked respondents to list five people they know as well as their birth months in the beginning of the crosswise questions. We took this approach to make sure that respondents will not be distracted from answering the crosswise questions of interest by performing these additional tasks simultaneously. We then randomly assign respondents different randomization probabilities of 0.086 and 0.25, which we call *low* and *moderate* randomization probabilities. Along with the crosswise model, we also performed "direct inquiries" on the same sensitive items.

We first apply the proposed bias-correction to our data and obtain point and uncertainty estimates for the prevalence proportions of interest. We also estimate the prevalence rates based on direct inquiry and the naïve crosswise estimator. The results are demonstrated in Figure D.1. For crosswise estimates, dots (second and fourth from the left) are based on low randomization probabilities ($p = 0.086$) and asterisks are based on moderate randomization probabilities ($p = 0.25$). It is shown that bias-corrected estimates are generally higher than direct inquiry estimates, but lower than naïve crosswise estimates. Estimated standard errors are wider for bias-corrected estimates than for naïve crosswise estimates due to the additional uncertainty for estimating attentive rates. By the construction of crosswise estimates, uncertainty is larger for estimates based on higher randomization probabilities, which suggests that researchers will be benefited from using low randomization probabilities whenever possible.

Importantly, without bias-correction, researchers may mistakenly infer that the crosswise model induced more candid answers on sensitive items (i.e., direct inquiry and naïve estimates are statistically significantly different in most cases) even though such differences are artificially caused by the presence of inattentive responses. Our methodology exactly prevents this form of incorrect inferences.

Next, we employ our proposed regression model framework to examine whether there exists any covariate that predicts sensitive attributes among respondents. For this illustration, we focus one unethical behavior: lying about qualifications on taking surveys. Studying the false qualification is substantively crucial in survey research because when some groups of individuals tend to lie about their qualifications and participate in surveys it may significantly bias substantive conclusions from the survey. For potential predictors, we included variables denoting for age, gender, and the level of general trust. We estimate the logistic-type regression using crosswise responses with the randomization probability of 0.25. Column 1 of Table D.1 report the estimated regression coefficients. We find that none of the included variables have coefficients that are statistically significantly different from zero. The results suggest that false qualifications are not associated with the three variables and might happen randomly.

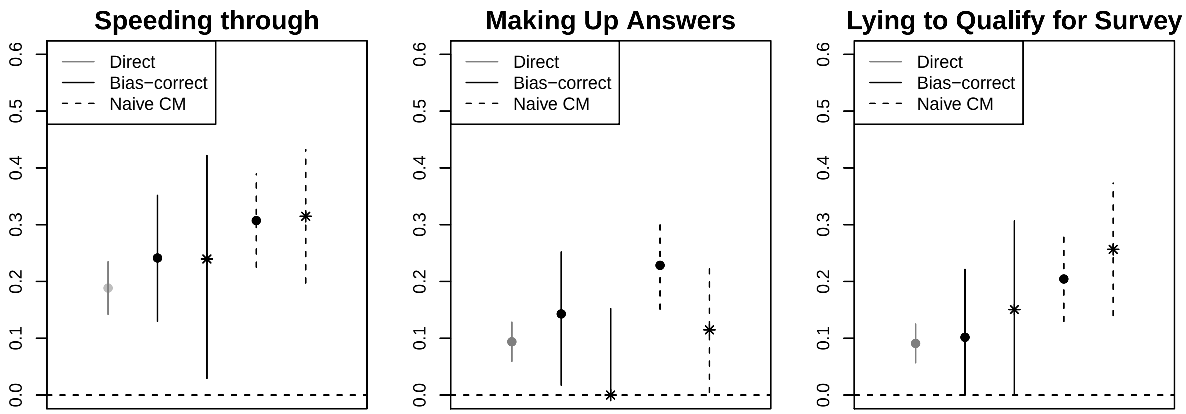

Figure D.1: **Comparison of Prevalence Estimates**
Note: This graph visualizes the estimated prevalence of sensitive attributes based on direct inquiry, bias-corrected estimator, and naïve crosswise estimator. For crosswise estimates, dots (second and fourth from the left) are based on low randomization probabilities ($p = 0.086$) and asterisks (third and fifth from the left) are based on moderate randomization probabilities ($p = 0.25$).

	Qualification	*Numeracy*
Lie to qualify		5.091**
		(1.131)
Age	-0.031	0.112***
	(0.022)	(0.023)
Female	0.965	-2.765**
	(0.680)	(0.759)
Trust	-0.137	
	(0.178)	
Intercept	0.040	-17.74**
	(1.086)	(0.749)
p	0.25	0.086
p'	0.25	0.086
N	274	196

Table D.1: **Results of Regression Analysis with Bias-Corrected Crosswise Estimates**
Note: This table the results of two regression estimates.

Moreover, we use the same variable denoting false qualification as a predictor in a regression model. Here, we consider subjective numeracy as our dependent variable (mean=-14.98, sd=5.25). Subjective numeracy measures respondents' perceived levels of numeracy or skills to understand numeric information. For predictors, we include variables denoting for age, gender, and false qualification. Note that we do not observe individual level value for the false qualification variable in our crosswise survey data. Nevertheless, as we discussed in Section 5, we can still estimate the coefficient on the latent variable through the joint likelihood function. To estimate the regression, we use crosswise responses with the randomization probability of 0.086. Column 2 of Table D.1 report the results for the regression. The results suggest that individuals who lie to qualify in survey works tend to have higher subjective numeracy. In addition, older respondents and female survey takers seem to have higher subjective numeracy.

E Practical Guide: How to Design the Crosswise Survey?

In this section, we offer a practical guide for researchers when they apply our proposed methodology. Importantly, the validity of our bias-correction and its extensions hinge upon the three assumptions discussed in Section 3. In the following, we clarify several important points that researchers must consider at the survey design stage in order to satisfy these assumptions.

E.1 How to Ensure that Inattentive Respondents Randomly Pick Items? (Assumption 1)

The random pick assumption states that inattentive respondents choose TRUE-TRUE/FALSE-FALSE at the probability of 0.5. This assumption can be satisfied by ensuring that inattentive respondents do not distinguish two available options (i.e.,TRUE-TRUE/FALSE-FALSE or otherwise) and they pick one of the two choices randomly. A simple approach is to randomize the ordering of the two choices both in the sensitive question of interest and its anchor question.

E.2 How to Achieve a Constant Attentive Rate (Assumption 2)

The constant attentive rate assumption is satisfied when the sensitive question and its anchor question have the same population proportion of attentive *responses*. It must be emphasized that this assumption does not require that the same *respondents* remain inattentive across questions. An important part of this is that researchers must make sure that respondents see both the sensitive and anchor questions in the same way. If respondents, on average, perceive one question to be somehow different from another question, the assumption could be violated. Thus, we recommend that researchers design both the sensitive and anchor questions to look quite similar. Specifically, we suggest that anchor questions be from the same topic and have the same length of wording as sensitive questions. Moreover, randomizing the position of anchor questions in the survey relative to sensitive questions will be helpful to guarantee that there is no carryover effect from one type of question to another.

E.3 How to Make Independent Randomization Probabilities (Assumption 3)

The independent randomization assumption claims that randomization probabilities used in the sensitive and anchor questions are statistically independent or $p \perp\!\!\!\perp p'$. This assumption will be relatively easily satisfied when researchers carefully choose two randomization probabilities (and not sensitive and anchor statements) based on different randomization topics. For example, when the first randomization probability is based on one's mother's birth month and the second probability is based on her father's birth month we are more or less confident that the independent randomization assumption holds (assuming that marriage is not a function of birth months of partners). Importantly, this assumption will be violated when researchers only use a single randomization topic (e.g., mother's birth month) with two different "cut-off points" (e.g., January to March and October to December). This is because the probability that one's mother was born in the first period contains information about the probability that she was born in the second period. Our recommendation is that researchers always *ex ante* ask respondents to think of two (or more) different topics (e.g., friends, friend and parent, friend and sibling, etc) and then use the topics for randomization.[1] This strategy also helps

[1]Using multiple siblings in the birth month-type randomization can be problematic since two siblings' birth months may not necessarily be statistically independent.

researchers by separating the respondents' tasks of coming up with topics and thinking about questions.

References for Online Appendix

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