# Integrating High-Level Priority Decisions with Distributed Safey Filter for Multi-Satellite Collision Avoidance

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**Abstract:**
Satellite miniaturization and dense constellation deployments exacerbate collision risks in future orbital operations. While numerous collision avoidance strategies have been proposed, few reconcile agent-level safety with mission-level efficiency. In this paper, we propose a distributed inter-satellite collision avoidance framework embeded with high-level tuned priorities. First, we formulate "safe protocol" constraints between satellite pairs and enforce these constraints on their nominal controllers through distributed safety filters, which establishs collision-free coordination of the whole swarm. By introducing tunable priority parameters within the safety filter, collision evasion responsibilities become dynamically adjustable, enabling swarm behavior adaptation. We further demonstrate two methods to integrate with high-level decisions: cooperating with optimization to approximate global reference behaviors and cooperating with Large Language Models to accommodate to tasks, respectively. Theoretical analysis proves the safety guarantees, while numerical experiments validate the framework's efficacy.

*Keywords:* Multi-satellite, Collision Avoidance, Control Barrier Function

## 1. INTRODUCTION

requirements

related works: APF VO Learning? CBF

## 2. PRELIMINARIES

We review High Order Control Barrier Function here, which is the fundamental technique in this paper.

Consider a general continuous time control-affine system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}, \tag{1}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$ is the state and $u \in \mathcal{U} \subset \mathbb{R}^m$ is the system input. $\boldsymbol{f}$ and $\boldsymbol{g}$ are locally Lipschitz continuous functions. The high order control barrier function is defined as follows.

*Definition 1.* Given a system (1) with relative degree $r_b$ and $r_b$-th order differentiable function $h(\boldsymbol{x})$, define a series of functions $\Psi_r, r = 0, \ldots, r_b$ recursively as

$$\begin{aligned} &\Psi_0 = h(\boldsymbol{x}), \\ &\Psi_k = \dot{\Psi}_{k-1} + \alpha_k \left(\Psi_{k-1}(\boldsymbol{x})\right), k = 1, \ldots, r_b, \end{aligned} \tag{2}$$

where $\alpha_k(\cdot)$ are extended class $\mathcal{K}_\infty$ functions [1].

The zero-superlevel set of these defined functions are

$$\mathfrak{S}_r = \left\{\boldsymbol{x} \in \mathbb{R}^n \mid \Psi_r(\boldsymbol{x}) \geq 0\right\}, r = 0, \ldots, r_b. \tag{3}$$

$h$ is a *High Order Control Barrier Function (HOCBF)* for system (1), if there exists extended class $\mathcal{K}_\infty$ functions $\alpha_1, \ldots, \alpha_{r_b}$ such that

$$\Psi_{r_b}(\boldsymbol{x}) \geq 0 \tag{4}$$

stands for any $(\boldsymbol{x}, t) \in \mathfrak{S} \times [0, \infty]$, where $\mathfrak{S} = \bigcap_{r=0}^{r_b} \mathfrak{S}_r$.

*Theorem 1.* (**?**). Following the definitions in Definition 1, once $h$ is a HOCBF for system (1), $\mathfrak{S}$ would be a *forward invariant set* for the system, i.e., $\boldsymbol{x}(0) \in \mathfrak{S}, \boldsymbol{x}(t) \in \mathfrak{S}, \forall t > 0$.

## 3. PROBLEM FORMULATION

### 3.1 System Modelling

We consider $N$ satellite agents whose dynamics governed by Clohessy-Wiltshire equantions (Clohessy and Wiltshire, 1960). The dynamics of agent $i$ in the reference orbit frame is described as

$$\dot{\boldsymbol{x}}_i = \begin{bmatrix} \dot{\boldsymbol{p}}_i \\ \dot{\boldsymbol{v}}_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}_i \\ \boldsymbol{f}_{vi} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ E \end{bmatrix} \boldsymbol{u}_i, \tag{5}$$

and

⋆ Sponsor and financial support acknowledgment goes here. Paper titles should be written in uppercase and lowercase letters, not all uppercase.

[1] A continuous function $\alpha : \mathbb{R} \to \mathbb{R}$ is an extended class $\mathcal{K}_\infty$ function if $\alpha(0) = 0$ and $\lim_{x\to\pm\infty}(x) = \pm\infty$.

$$\boldsymbol{f}_{vi} = \begin{bmatrix} -2\omega v_{yi} \\ 2\omega v_{xi} + 3\omega^2 v_{yi} \\ \omega^2 p_{zi} \end{bmatrix}, \tag{6}$$

where $\boldsymbol{p}_i = [p_{xi}\ p_{yi}\ p_{zi}]^\top \in \mathbb{R}^3, \boldsymbol{v} = [v_{xi}\ v_{yi}\ v_{zi}]^\top \in \mathbb{R}^3$ and $\boldsymbol{u}_i \in \mathbb{R}^3$ are the position, velocity and acceleration of agent $i$, respectively. $\mathbf{0}$ and $E$ are zero matrix and identity matrix with proper size, and $\omega \in \mathbb{R}$ is the angular velocity of the reference orbit.

*Assumption 1.* $\boldsymbol{x}_j, j = 1, \ldots, N$ and $\omega$ are known for any agent $i, i = 1, \ldots, N$ and the high-level decision module.

This assumption is justified since agent $i$ could estimate the state of agent $j$ through relative position estimation techniques (**?**), and the high-level decision module (possibly ground control station or space station) could get these information through observation (**?**) or communication.

### 3.2 Safety Requirement

The safety requirement of satellite agents is to keep the safety distance between from each other. Denote $r_i \in \mathbb{R}^+$ to be the safety distance of agent $i$, the safety requirement between agent $i$ and agent $j$ is then keeping the set

$$\mathfrak{S}_{0ij} = \{\boldsymbol{x}_i, \boldsymbol{x}_j \mid d_{ij} = \|\boldsymbol{p}_i - \boldsymbol{p}_j\| \geq R_{ij} = r_i + r_j\} \tag{7}$$

forward invariant. And the safety requirement of the swarm is to keep

$$\mathfrak{S}_0 = \bigcap_{i \neq j} \mathfrak{S}_{0ij},\ i, j = 1, \ldots, N \tag{8}$$

forward invariant.

### 3.3 Main Objective

The main objective of this paper is twofold:

1) Ensuring agent-level safety: for each agent $i$, given the local reference control $\boldsymbol{u}_{ri}$ and observation $\boldsymbol{X} = [\boldsymbol{x}_1^\top\ \ldots\ \boldsymbol{x}_n^\top]^\top$, synthesis the safeguarding policy $\boldsymbol{u}_i = \boldsymbol{\pi}_i(\boldsymbol{u}_{ri}, \boldsymbol{X})$ to keep $\mathfrak{S}_0$ forward invariant for the swarm in real time through control $\boldsymbol{U} = [\boldsymbol{u}_1^\top\ \ldots\ \boldsymbol{u}_N^\top]^\top$.
2) Cooperating with high-level decisions:
   - Given the low-frequency global reference control $\boldsymbol{U}_{gr} = [\boldsymbol{u}_{gr1}^\top\ \ldots\ \boldsymbol{u}_{grN}]^\top$, tune $\boldsymbol{\pi}_i, i = 1, \ldots, N$ to approximate $\boldsymbol{U}_{gr}$ with $\boldsymbol{U}$.
   - Given the mission discribed with nature language, tune $\boldsymbol{\pi}_i, i = 1, \ldots, N$ to adjust the collision evasion responsibily of agent $i$ based on its mission-level importance.

## 4. DISTRIBUTED SAFETY FILTER DESIGN

We first introduce the "safe protocol" constraint formulation, then give the distributed safety filter design based on the "safe protocol".

For satellite $i$ and satellite $j$, the "safe protocol" set for satellite $i$ corresponding to satellite $j$ is a half-space described as

$$\begin{aligned}\mathcal{S}_{ij} = \Big\{\boldsymbol{u}_i \mid -\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{u}_i \leq\ & \hat{\boldsymbol{n}}_{ij}^\top [(\alpha_1 + \alpha_2)\boldsymbol{v}_i + \boldsymbol{f}_{vi}] \\ & + p_{ij} \Big[\alpha_1\alpha_2(d_{ij} - R_{ij}) \\ & + \frac{1}{d_{ij}} \left(\|\boldsymbol{v}_{ij}\|^2 - (\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij})^2\right)\Big]\Big\},\end{aligned} \tag{9}$$

where $\hat{\boldsymbol{n}}_{ij} = (\boldsymbol{p}_i - \boldsymbol{p}_j)/\|\boldsymbol{p}_i - \boldsymbol{p}_j\|$ is the unit vector pointing from agent $j$ to agent $i$, $\alpha_1, \alpha_2 \in \mathbb{R}^+$ are positive parameters and $\boldsymbol{v}_{ij} = \boldsymbol{v}_i - \boldsymbol{v}_j$ is the relative velocity between agent $i$ and agent $j$. $p_{ij} \in \mathbb{R}$ is the priority parameter, reresenting agent $i$'s priority over agent $j$.

The following result renders that two satellites are collision-free if both of the them follow the "safety protocol".

*Theorem 2.* For systems discribed as (5) and (6), $\mathfrak{S}_{0ij}$ is forward invariant if $\boldsymbol{u}_i \in \mathcal{S}_{ij}, \boldsymbol{u}_j \in \mathcal{S}_{ji}$ and $p_{ij} + p_{ji} \leq 1$.

**Proof.** The proof mainly leverages Theorem 1. Let

$$h_{ij} = d_{ij} - R_{ij} = \Psi_{0ij} \tag{10}$$

be the HOCBF candidate to keep agent $i$ and agent $j$ collision-free. By choosing positive proportional functions as class $\mathcal{K}_\infty$ functions, it follows the definition that

$$\begin{aligned}\Psi_{1ij} =& \dot{\Psi}_{0ij} + \alpha_1(\Psi_{0ij}) \\ =& \frac{\boldsymbol{p}_i^\top - \boldsymbol{p}_j^\top}{d_{ij}}(\boldsymbol{v}_1 - \boldsymbol{v}_2) + \alpha_1(d_{ij} - R_{ij}) \\ =& \hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij} + \alpha_1 \cdot \Psi_{0ij}.\end{aligned} \tag{11}$$

To get the constraint on control, we further define

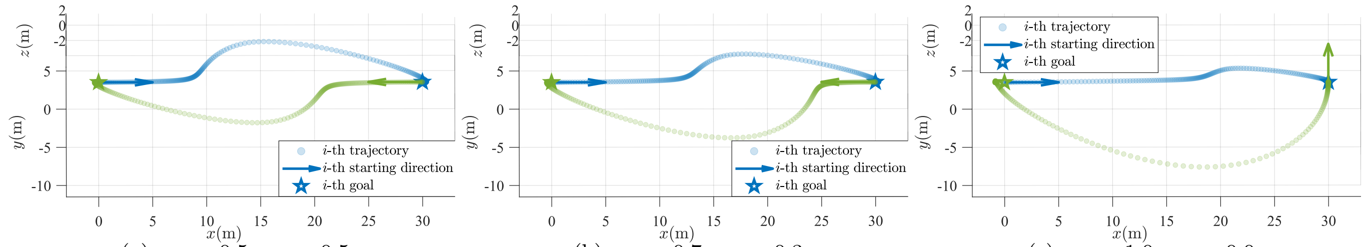
$$\begin{aligned}\Psi_{2ij} =& \dot{\Psi}_{1ij} + \alpha_2\Psi_{1ij} \\ =& \frac{1}{d_{ij}^2}\left((\boldsymbol{v}_i - \boldsymbol{v}_j)^\top d_{ij} - (\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij})(\boldsymbol{p}_i - \boldsymbol{p}_j)^\top\right)\boldsymbol{v}_{ij} \\ & + \hat{\boldsymbol{n}}_{ij}^\top(\boldsymbol{u}_i + \boldsymbol{f}_{vi} - \boldsymbol{u}_j - \boldsymbol{f}_{vj}) + \alpha_1\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij} \\ & + \alpha_2(\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij} + \alpha_1\Psi_{0ij}) \\ =& \hat{\boldsymbol{n}}_{ij}^\top\left(\boldsymbol{u}_i - \boldsymbol{u}_j + (\alpha_1 + \alpha_2)\boldsymbol{v}_{ij} + \boldsymbol{f}_{vi} - \boldsymbol{f}_{vj}\right) \\ & + \frac{1}{d_{ij}}\left(\|\boldsymbol{v}_{ij}\|^2 - (\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij})^2\right) + \alpha_1\alpha_2(d_{ij} - R_{ij})\end{aligned} \tag{12}$$

Given that $\boldsymbol{u}_i \in \mathcal{S}_{ij}$ and $\boldsymbol{u}_j \in \mathcal{S}_{ji}$, by adding up the inequalities in the definition of $\mathcal{S}_{ij}$ and $\mathcal{S}_{ji}$ and substituting $\hat{\boldsymbol{n}}_{ji} = -\hat{\boldsymbol{n}}_{ij}$ into the inequality, one can get

$$\begin{aligned}\tilde{\Psi}_{2ij} =& \hat{\boldsymbol{n}}_{ij}^\top\left(\boldsymbol{u}_i - \boldsymbol{u}_j + (\alpha_1 + \alpha_2)\boldsymbol{v}_{ij} + \boldsymbol{f}_{vi} - \boldsymbol{f}_{vj}\right) \\ & + (p_{ij} + p_{ji})\Big[\frac{1}{d_{ij}}\left(\|\boldsymbol{v}_{ij}\|^2 - (\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij})^2\right) \\ & + \alpha_1\alpha_2(d_{ij} - R_{ij})\Big] \geq 0\end{aligned} \tag{13}$$

Notice that $\|\boldsymbol{v}_{ij}\|^2 - (\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij})^2 \geq 0$ and $d_{ij} - R_{ij} \geq 0$ hold for $(\boldsymbol{x}_i(0), \boldsymbol{x}_j(0)) \in \mathfrak{S}_{0ij}$, $\Psi_{2ij} \geq \tilde{\Psi}_{2ij} \geq 0$ then stands for any $p_{ij}$ and $p_{ji}$ satisfying $p_{ij} + p_{ji} \leq 1$. Consequently, according to Definition 1 and Theorem 1, $h_{ij}$ is a valid HOCBF and thus $\mathfrak{S}_{0ij}$ is a forward invariant set for agent $i$ and agent $j$.

Based on the properties of "safe protocol" set, we further design the distributed safeguarding policy in a minimum invasive way as




(a) $p_{12} = 0.5, p_{21} = 0.5$ (b) $p_{12} = 0.7, p_{21} = 0.3$ (c) $p_{12} = 1.0, p_{21} = 0.0$

Fig. 1. Agents' behavior with different priority parameters.

$$\boldsymbol{\pi}_i(\boldsymbol{u}_{ri}, \boldsymbol{X}) = \begin{cases} \underset{\boldsymbol{u}\in\cap_{j\neq i}\mathcal{S}_{ij}}{\arg\min}\ \|\boldsymbol{u} - \boldsymbol{u}_{ri}\|^2, & \cap_{j\neq i}\mathcal{S}_{ij} \neq \varnothing, \\ \underset{\boldsymbol{u}\in\mathbb{R}^3}{\arg\min}\ \underset{j\neq i}{\max}\ \mathrm{ESDF}(\boldsymbol{u}, \mathcal{S}_{ij}), & \cap_{j\neq i}\mathcal{S}_{ij} = \varnothing, \end{cases} \tag{14}$$

where $\mathrm{ESDF}(\boldsymbol{u}, \mathcal{S}_{ij})$ is the signed Euclidean distance function (ESDF) of $\boldsymbol{u}$ to the boundary of half-space $\mathcal{S}_{ij}$, with $\mathrm{ESDF}(\boldsymbol{u}, \mathcal{S}_{ij})$ defined to be negative if $\boldsymbol{u} \in \mathcal{S}_{ij}$.

If $\cap_{j\neq i}\mathcal{S}_{ij} \neq \varnothing$, $\boldsymbol{\pi}_i$ minimumly modifies $\boldsymbol{u}_{ri}$ to follow all "safety protocols". Since the feasible region $\cap_{j\neq i}\mathcal{S}_{ij}$ is now a polytope, $\boldsymbol{\pi}_i$ is in the form of a quadratic programming (QP) and can be solved in real time onboard; If $\cap_{j\neq i}\mathcal{S}_{ij} = \varnothing$, it is then impossible to follow all "safety protocols" for all agent $j, j \neq i$. Therefore, $\boldsymbol{\pi}_i$ synthesises a control that minimizes the maximum violation of all "safe protocols" to get the "safest possible" control. Such an optimization can also be solved in real time onboard via a linear programming (LP).

*Theorem 3.* If $\cap_{j\neq i}\mathcal{S}_{ij} \neq \varnothing$ for all agent $i$, $\mathfrak{S}_0$ is forward invariant if $\boldsymbol{u}_i = \boldsymbol{\pi}_i(\boldsymbol{u}_{ri}, \boldsymbol{X})$ and $\boldsymbol{P} = \{p_{ij}\}_{(N\times N)} \in \left\{\boldsymbol{P} \in \mathbb{R}^{N\times N} \mid p_{ii} = 0, p_{ij} + p_{ji} \leq 1, i \neq j\right\}$.

**Proof.** This result directly follows Theorem 2. Given that $\cap_{j\neq i}\mathcal{S}_{ij} \neq \varnothing, \forall i = 1, \ldots, N$, it cleary renders that $\boldsymbol{u}_i = \boldsymbol{\pi}_i(\boldsymbol{u}_{ri}, \boldsymbol{X}) \in \cap_{j\neq i}\mathcal{S}_{ij}$ is then satisfied for all agents. Since for any satellite pair composed of satellite $i$ and satellite $j$, $\boldsymbol{u}_i \in \cap_{j\neq i}\mathcal{S}_{ij} \subseteq \mathcal{S}_{ij}, \boldsymbol{u}_j \in \cap_{i\neq j}\mathcal{S}_{ji} \subseteq \mathcal{S}_{ji}$ and $p_{ij} + p_{ji} \leq 1$ are satisfied, according to Theorem 2, all $\mathfrak{S}_{0ij}, i \neq j$ are forward invariant. Hence their intersection $\mathfrak{S}_0 = \bigcap_{i\neq j}\mathfrak{S}_{0ij},\ i, j = 1, \ldots, N$ is forward invariant and the whole satellite swarm is collision-free.

*Remark 1.* The priority parameter $p_{ij}$ is introduced with a clear physical meaning. Since $\|\boldsymbol{v}_{ij}\|^2 - (\hat{\boldsymbol{n}}_{ij}^\top\boldsymbol{v}_{ij})^2 \geq 0$ holds universally and $d_{ij} - R_{ij} \geq 0$ holds as long as agent $i$ and agent $j$ do not collide with each other, the right hand side of (9) is positively correlated to $p_{ij}$. Consequently, the higher $p_{ij}$, the easier it is for agent $i$ to satisfy $\boldsymbol{u}_i \in \mathcal{S}_{ij}$. Due to the constraint that $p_{ij} + p_{ji} \leq 1$, agent $j$ will then bear lower $p_{ji}$ and in turn take more responsibility for avoiding collision with agent $i$. By tuning the priority matrix $\boldsymbol{P}$, the high-level decision is introduced into the distributed safety filter, deciding "who should evading more to avoid collision". Figure 1 illustrates the agents' behavior with different $p_{ij}$. Two identical satellites are simulated to switch their positions with different $p_{ij}$. By increasing $p_{12}$ and decreasing $p_{21}$, satellite 1 (colored in blue) performed less evading behavior, while satellite 2 (colored in red) performed more evading behavior.

## 5. COOPERATING WITH HIGH-LEVEL DECISIONS

### 5.1 Cooperating with Optimization

Due to the communication lagging and computing burden, centralized control struggles to meet the high control frequency requirement of collision avoidance. Nevertheless, it can be used as global reference for the aforementioned distributed safety filter via priority matrix optimization. With $\boldsymbol{u}_{ri}$ and $\boldsymbol{X}$ fixed, $\boldsymbol{\pi}_i$ is then only dependent on $p_{ij}$ as $\boldsymbol{\pi}_i = \boldsymbol{\pi}_i(\boldsymbol{P})$, hence the priority matrix can be optimized to minimize the difference between $\boldsymbol{U} = [\boldsymbol{u}_1^\top\ \ldots\ \boldsymbol{u}_N^\top]^\top$ and $\boldsymbol{U}_{gr} = [\boldsymbol{u}_{gr1}^\top\ \ldots\ \boldsymbol{u}_{grN}]^\top$ through

$$\begin{aligned} \boldsymbol{P}^* &= \underset{\boldsymbol{P}\in\mathbb{R}^{N\times N}}{\arg\min}\ \|\boldsymbol{U}_{gr} - \boldsymbol{U}(\boldsymbol{P})\|^2 \\ \text{s.t.} &\begin{cases} \boldsymbol{u}_i = \boldsymbol{\pi}_i(\boldsymbol{P}), & \forall i = 1, \ldots, N \\ p_{ij} + p_{ji} \leq 1, & \forall i \neq j, i, j = 1, \ldots, N \\ p_{ii} = 0, & \forall i = 1, \ldots, N \end{cases} \end{aligned} \tag{15}$$

By maximize $p_{ij}$ to satisfy $p_{ij} + p_{ji} = 1$, optimization (15) can be then converted to an unconstrainted optimization by only optimizing the upper triangle of $\boldsymbol{P}$ (i.e., only optimize $p_{ij}, i < j$) and get the lower triangle of $\boldsymbol{P}$ through $p_{ji} = 1 - p_{ij}$. By performing the optimization above, the dispatching preference of the centralized control is extracted into $\boldsymbol{P}^*$, thereby coordinate the satellite swarm to align with the global reference.

### 5.2 Cooperating with Large Language Models

While the collision avoidance requirement is clear and easy to be modelled mathematiccally, the mission level requirement is often much more complicated and often described in natural language. By leveraging the understanding and reasoning ability of LLMs, the aforementioned priority parameters can be generated according to the mission-level requirements, thereby allocate the collision evasion responsibily of each satellite.

However, directly generate a priority matrix $\boldsymbol{P}$ satisfying $p_{ij} + p_{ji} \leq 1, \forall i \neq j$ through LLM is challenging since satisfaction of such a condition is not guaranteed due to the black-box nature of LLMs. As a result, instead of generating priority matrix directly, we use LLM to generate an priority array as $\mathbf{p} = [p_1,\ p_2,\ \ldots,\ p_N], p_i \geq 0, \forall i = 1, \ldots, N$, with $p_i$ representing the significance of satellite $i$ in the mission. To ensure the output stability, structured output technique (**?**) is adopted to force the output format on LLMs. After getting the legal output, the priority parameters are calulated through $p_{ij} = p_i/(p_i+p_j)$ to meet the safety condition.

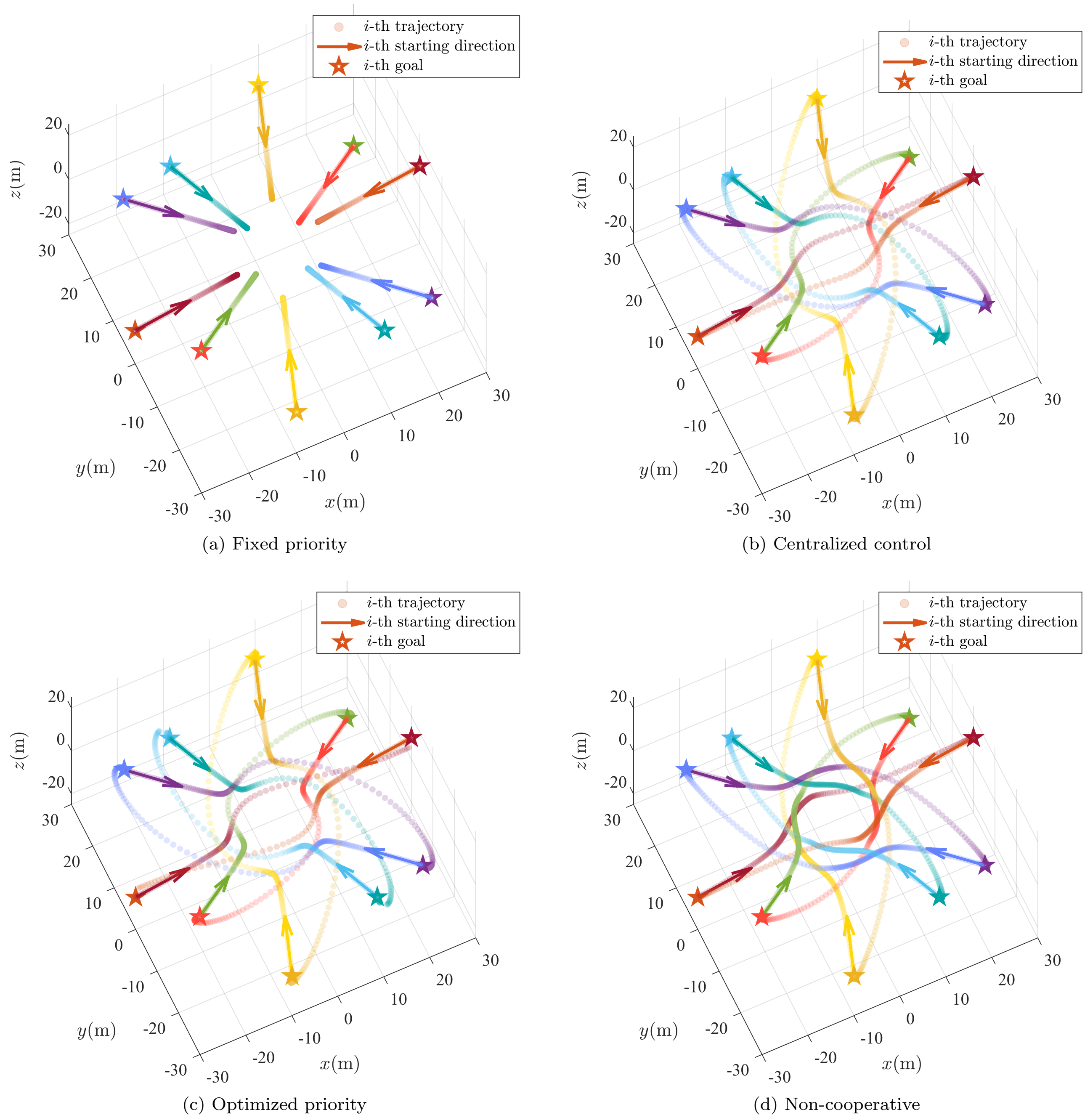



Fig. 2. Swarm behavior with different collision avoidance strategies.

Prompts are also carefully designed to enhance the performance of LLMs. A simple chain of thought is given to LLMs to first classify satellites to different levels based on their roles and missions, then assign priority array based on satellites' levels, and finally modify the priority of satellites in the same level. Demonstrations are also given to take advantage of the in-context learning ability of LLMs (Min et al., 2022).

## 6. NUMERICAL EXPERIMENTS

We validate the efficacy of the distributed safety filter by cooperating with optimization and LLM, respectively. In the experiments, the safety distance of each agent is set to $r_i = 5\text{m}, i = 1, \ldots, N$ and $\omega$ is set to $0.00113\text{rad/s}$, which is approximatly the angular velocity of International Space Station.

### 6.1 Cooperating with Optimization

We validate the proposed method by comparing four different collision avoidance strategies, namely fixed priority case, centralized control case, optimized priority case (proposed) and non-cooperative case. For the first case, the control of each agent is calculated distributedly by (14) with $p_{ij} = 0.5, \forall i \neq j$; for the centralized case, the control is calculated in a centralized way as

$$\begin{aligned} \boldsymbol{U} &= \arg\min_{\boldsymbol{U}' \in \mathbb{R}^{3N}} \|\boldsymbol{U}' - \boldsymbol{U}_r\|^2 \\ &\text{s.t.} \Psi_{2ij} \geq 0, \forall i \neq j, \end{aligned} \tag{16}$$

where $\Psi_{2ij}$ is defined in (12) and $\boldsymbol{U}_r = [\boldsymbol{u}_{r1}^\top \ \dots \ \boldsymbol{u}_{rN}^\top]^\top$ is the reference control. For the optimized priority case, the control is calculated distributedly by (14), and $\boldsymbol{P}$ is optimized centralizedly by (15) every 10 second with $\boldsymbol{U}_{gr}$ given by the result of (16). For the last case, control is calculated distributedly by each agent $i$ through

$$\begin{aligned} \boldsymbol{u}_i = \arg\min_{\boldsymbol{u}_i' \in \mathbb{R}^3} \|\boldsymbol{u}_i' - \boldsymbol{u}_{ri}\|^2 \\ \text{s.t.} \Psi_{2ij} \geq 0, \forall i \neq j, \end{aligned} \tag{17}$$

with the assumption that $\boldsymbol{u}_j = \mathbf{0}, j \neq i$. All methods above use the reference control $\boldsymbol{U}_r$ given by the same proportional derivative (PD) controller.

We compare four methods above by performing the same task to move all satellites to the initial position of its opposite satellite. The trajectory of satellites is shown in Fig. 2 by plotting the position of each agent every 1.5s. The starting directions of each satellite is illustrated by arrows, with the starting points of the arrows aligned with the initial positions of agents, and the directions of the arrows aligned with the velocity directions of each agent at $t = 0.5$s. The minimum distance $h_{ij}$ is drawn in Fig. 3.

As shown in Fig. 2a, satellites encountered a deadlock in the first case, which is related to distributed and local design of the controller. Since the centralized control case optimizes control globally, satellites were coordinated to their goals (Fig. 2b). By using the centralized control as a low frequency global reference, the proposed method avoided the deadlock as well with distributed control (Fig. 2c). The non-cooperative case coordinated the satellites to their goals (Fig. 2d). However, as shown in Fig. 3, such a method is invalid since collision happened between 150 and 200s. This happened since the assumption that $\boldsymbol{u}_j = \mathbf{0}, j \neq i$ did not hold in the experiment. By comparing the methods above, we validated that the proposed distributed safety filter can benefit from global reference control via priority optimization.

### *6.2 Cooperating with Large Language Models*

We also validated the effectiveness of cooperating LLMs with the proposed safety filter. A Gemma3:12b (**?**) model is deployed locally to perform the priority assignment task. We experimented the same task to switch the positions of 6 satellites with three different prompts. In the first case, LLM was informed that "Satellite 1 is mission critical. All other satellites are backup satellites"; in the second case, LLM was given the prompt that "Satellite 1 is mission critical. All other satellites have low fuel"; in the last case, LLM was hinted that "All satellites are the same". The trajectories of satellite are shown in Fig. 4, with $T_1$ representing the time taken for satellite 1 to arrive its goal.

Given the prompt above, LLM outputted the priority array as $\mathbf{p} = [10, 1, 1, 1, 1, 1]$, $\mathbf{p} = [9, 7, 7, 7, 7, 7]$ and $\mathbf{p} = [5, 5, 5, 5, 5, 5]$, respectively. Consequently, the priority parameter $p_{1j}, j \neq 1$ becomes lower and lower. The difference of $p_{1j}$ is then reflected on the trajectory and arriving time of satellite 1. From Fig. 4a to Fig. 4c, satellite 1 experienced more and more control modification from the safety filter, which increased the curvature of the satellite 1's trajectory and therefore increased $T_1$ from 204.5s to 321.5s. This experiment demonstrated that

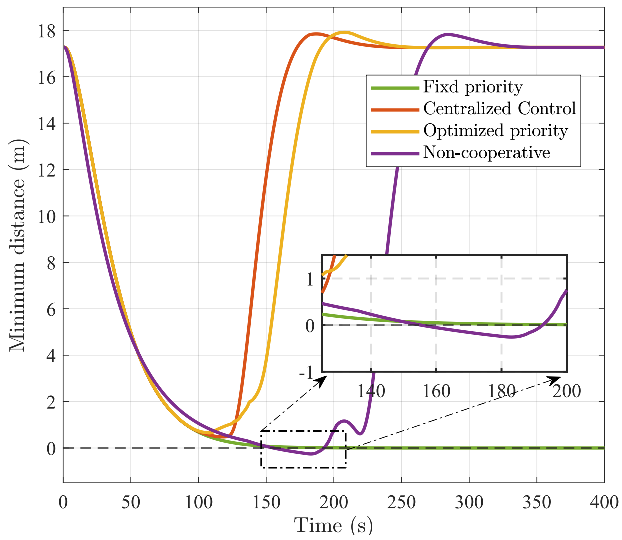



Fig. 3. Minimum distance between satellites with different collision avoidance strategies

the proposed method can accommodate to mission-level requirements by cooperating with priority decisions of LLMs.

## 7. CONCLUSION

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

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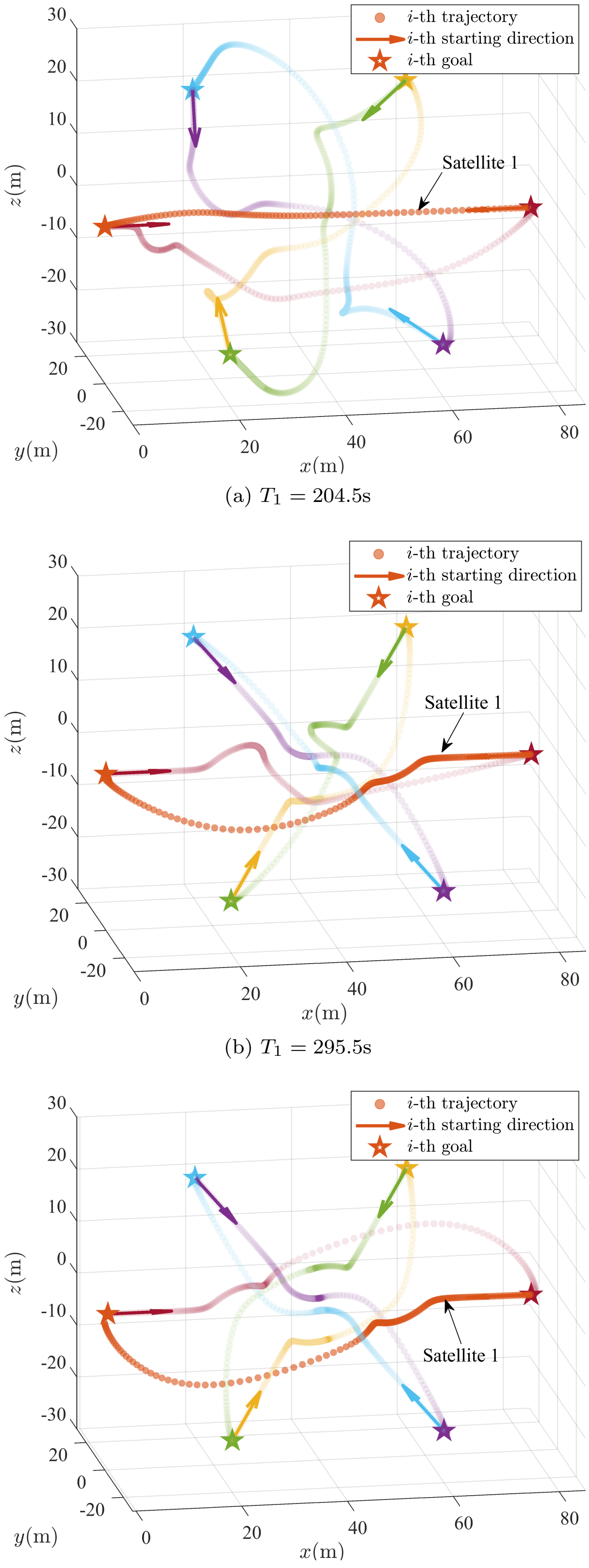


(a) $T_1 = 204.5$s

(b) $T_1 = 295.5$s

(c) $T_1 = 321.5$s

Fig. 4. Swarm behavior with different LLM prompts.