Integrating High-Level Decisions with Distributed Safey Filter for Multi-Satellite Collision Avoidance

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Abstract:
Satellite miniaturization and dense constellation deployments exacerbate collision risks in future orbital operations. While numerous collision avoidance strategies have been proposed, few reconcile agent-level safety with mission-level efficiency. In this paper, we propose a distributed inter-satellite collision avoidance framework embeded with high-level tuned priorities. First, we formulate "safe protocol" constraints between satellite pairs and enforce these constraints on their nominal controllers through distributed safety filters, which establishs collision-free coordination of the whole swarm. By introducing tunable priority parameters within the safety filter, collision evasion responsibilities become dynamically adjustable, enabling swarm behavior adaptation. We further demonstrate two methods to integrate with high-level decisions: cooperating with optimization to approximate global reference behaviors and cooperating with Large Language Models to accommodate to tasks, respectively. Theoretical analysis proves the safety guarantees, while numerical experiments validate the framework's efficacy.

Keywords: Multi-satellite, Collision Avoidance, Control Barrier Function

1. INTRODUCTION

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2. PRELIMINARIES

We review High Order Control Barrier Function here, which is the fundamental technique in this paper.

Consider a general continuous time control-affine system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}, \tag{1}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$ is the state and $u \in \mathcal{U} \subset \mathbb{R}^m$ is the system input. $\boldsymbol{f}$ and $\boldsymbol{g}$ are locally Lipschitz continuous functions. The high order control barrier function is defined as follows.

Definition 1. Given a system (1) with relative degree r_b and r_b-th order differentiable function $h(\boldsymbol{x})$, define a series of functions $\Psi_r, r = 0, \ldots, r_b$ recursively as

$$\begin{aligned} &\Psi_0 = h(\boldsymbol{x}), \\ &\Psi_k = \dot{\Psi}_{k-1} + \alpha_k\left(\Psi_{k-1}(\boldsymbol{x})\right), k = 1, \ldots, r_b, \end{aligned} \tag{2}$$

where $\alpha_k(\cdot)$ are extended class $\mathcal{K}_\infty$ functions [2].

The zero-superlevel set of these defined functions are

$$\mathfrak{S}_r = \{\boldsymbol{x} \in \mathbb{R}^n \mid \Psi_r(\boldsymbol{x}) \geq 0\}, r = 0, \ldots, r_b. \tag{3}$$

h is a *High Order Control Barrier Function (HOCBF)* for system (1), if there exists extended class $\mathcal{K}_\infty$ functions $\alpha_1, \ldots, \alpha_{r_b}$ such that

$$\Psi_{r_b}(\boldsymbol{x}) \geq 0 \tag{4}$$

stands for any $(\boldsymbol{x}, t) \in \mathfrak{S} \times [0, \infty]$, where $\mathfrak{S} = \bigcap_{r=0}^{r_b} \mathfrak{S}_r$.

Theorem 1. (**?**). Following the definitions in Definition 1, once h is a HOCBF for system (1), $\mathfrak{S}$ would be a *forward invariant set* for the system, i.e., $\boldsymbol{x}(0) \in \mathfrak{S}, \boldsymbol{x}(t) \in \mathfrak{S}, \forall t > 0$.

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[1] This is the default for the provided class file.

[2] A continuous function $\alpha : \mathbb{R} \to \mathbb{R}$ is an extended class $\mathcal{K}_\infty$ function if $\alpha(0) = 0$ and $\lim_{x \to \pm\infty}(x) = \pm\infty$.

3. PROBLEM FORMULATION

3.1 System Modelling

We consider N satellite agents whose dynamics governed by Clohessy-Wiltshire equantions (Clohessy and Wiltshire, 1960). The dynamics of agent i in the reference orbit frame is described as

$$\dot{\boldsymbol{x}}_i = \begin{bmatrix} \dot{\boldsymbol{p}}_i \\ \dot{\boldsymbol{v}}_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}_i \\ \boldsymbol{f}_{vi} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ E \end{bmatrix} \boldsymbol{u}_i, \tag{5}$$

and

$$\boldsymbol{f}_{vi} = \begin{bmatrix} -2\omega v_{yi} \\ 2\omega v_{xi} + 3\omega^2 v_{yi} \\ \omega^2 p_{zi} \end{bmatrix}, \tag{6}$$

where $\boldsymbol{p}_i = [p_{xi}\ p_{yi}\ p_{zi}]^\top \in \mathbb{R}^3, \boldsymbol{v} = [v_{xi}\ v_{yi}\ v_{zi}]^\top \in \mathbb{R}^3$ and $\boldsymbol{u}_i \in \mathbb{R}^3$ are the position, velocity and acceleration of agent i, respectively. $\boldsymbol{0}$ and E are zero matrix and identity matrix with proper size, and $\omega \in \mathbb{R}$ is the angular velocity of the reference orbit.

Assumption 1. $\boldsymbol{x}_j, j = 1, \ldots, N$ and ω are known for any agent $i, i = 1, \ldots, N$ and the high-level decision module.

This assumption is justified since agent i could estimate the state of agent j through relative position estimation techniques (**?**), and the high-level decision module (possibly ground control station or space station) could get these information through observation (**?**) or communication.

3.2 Safety Requirement

The safety requirement of satellite agents is to keep the safety distance between from each other. Denote $r_i \in \mathbb{R}^+$ to be the safety distance of agent i, the safety requirement between agent i and agent j is then keeping the set

$$\mathfrak{S}_{0ij} = \{\boldsymbol{x}_i, \boldsymbol{x}_j \mid d_{ij} = \|\boldsymbol{p}_i - \boldsymbol{p}_j\| \geq R_{ij} = r_i + r_j\} \tag{7}$$

forward invariant. And the safety requirement of the swarm is to keep

$$\mathfrak{S}_0 = \bigcap_{i \neq j} \mathfrak{S}_{0ij},\ i, j = 1, \ldots, N \tag{8}$$

forward invariant.

3.3 Main Objective

The main objective of this paper is twofold:

1) Ensuring agent-level safety: for each agent i, given the local reference control $\boldsymbol{u}_{ri}$ and observation $\boldsymbol{X} = [\boldsymbol{x}_1^\top\ \ldots\ \boldsymbol{x}_n^\top]^\top$, synthesis the safeguarding policy $\boldsymbol{u}_i = \boldsymbol{\pi}_i(\boldsymbol{u}_{ri}, \boldsymbol{X})$ to keep $\mathfrak{S}_0$ forward invariant for the swarm through control $\boldsymbol{U} = [\boldsymbol{u}_1^\top\ \ldots\ \boldsymbol{u}_N^\top]^\top$.
2) Cooperating with high-level decisions:
   - Given the global reference control $\boldsymbol{U}_r = [\boldsymbol{u}_{gr1}^\top\ \ldots\ \boldsymbol{u}_{grn}]^\top$, tune $\boldsymbol{\pi}_i, i = 1, \ldots, N$ to approximate $\boldsymbol{U}_r$ with $\boldsymbol{U}$.
   - Given the mission discribed with nature language, tune $\boldsymbol{\pi}_i, i = 1, \ldots, N$ to adjust the collision evasion responsibily of agent i based on its mission-level importance.

4. DISTRIBUTED SAFETY FILTER DESIGN

We first introduce the "safe protocol" constraint formulation, then give the distributed safety filter design based on the "safe protocol".

For satellite i and satellite j, the "safe protocol" set for satellite i corresponding to satellite j is a half-space described as

$$\begin{aligned} \mathcal{S}_{ij} = \Bigg\{ \boldsymbol{u}_i \mid -\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{u}_i \leq \hat{\boldsymbol{n}}_{ij}^\top \left[(\alpha_1 + \alpha_2)\boldsymbol{v}_i + \boldsymbol{f}_{vi}\right] \\ + p_{ij} \Bigg[\alpha_1\alpha_2(d_{ij} - R_{ij}) \\ + \frac{1}{d_{ij}} \left(\|\boldsymbol{v}_{ij}\|^2 - (\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij})^2 \right) \Bigg] \Bigg\}, \end{aligned} \tag{9}$$

where $\hat{\boldsymbol{n}}_{ij} = (\boldsymbol{p}_i - \boldsymbol{p}_j)/\|\boldsymbol{p}_i - \boldsymbol{p}_j\|$ is the unit vector pointing from agent j to agent i, $\alpha_1, \alpha_2 \in \mathbb{R}^+$ are positive parameters and $\boldsymbol{v}_{ij} = \boldsymbol{v}_i - \boldsymbol{v}_j$ is the relative velocity between agent i and agent j. $p_{ij} \in \mathbb{R}$ is the priority parameter, reresenting agent i's priority over agent j.

The following result renders that two satellites are collision-free if both of the them follow the "safety protocol".

Theorem 2. For systems discribed as (5) and (6), $\mathfrak{S}_{0ij}$ is forward invariant if $\boldsymbol{u}_i \in \mathcal{S}_{ij}, \boldsymbol{u}_j \in \mathcal{S}_{ji}$ and $p_{ij} + p_{ji} \leq 1$.

Proof. The proof mainly leverages Theorem 1. Let

$$h_{ij} = d_{ij} - R_{ij} = \Psi_{0ij} \tag{10}$$

be the HOCBF candidate to keep agent i and agent j collision-free. By choosing positive proportional functions as class $\mathcal{K}_\infty$ functions, it follows the definition that

$$\begin{aligned} \Psi_{1ij} =& \dot{\Psi}_{0ij} + \alpha_1(\Psi_{0ij}) \\ =& \frac{\boldsymbol{p}_i^\top - \boldsymbol{p}_j^\top}{d_{ij}}(\boldsymbol{v}_1 - \boldsymbol{v}_2) + \alpha_1(d_{ij} - R_{ij}) \\ =& \hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij} + \alpha_1 \cdot \Psi_{0ij}. \end{aligned} \tag{11}$$

To get the constraint on control, we further define

$$\begin{aligned} \Psi_{2ij} =& \dot{\Psi}_{1ij} + \alpha_2 \Psi_{1ij} \\ =& \frac{1}{d_{ij}^2} \left((\boldsymbol{v}_1 - \boldsymbol{v}_2)^\top d_{ij} - (\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij})(\boldsymbol{p}_1 - \boldsymbol{p}_2)^\top \right) \boldsymbol{v}_{ij} \\ &+ \hat{\boldsymbol{n}}_{ij}^\top(\boldsymbol{u}_1 + \boldsymbol{f}_{v1} - \boldsymbol{u}_2 - \boldsymbol{f}_{vj}) + \alpha_1 \hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij} \\ &+ \alpha_2(\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij} + \alpha_1 \Psi_{0ij}) \\ =& \hat{\boldsymbol{n}}_{ij}^\top (\boldsymbol{u}_1 - \boldsymbol{u}_2 + (\alpha_1 + \alpha_2)\boldsymbol{v}_{ij} + \boldsymbol{f}_{v1} - \boldsymbol{f}_{v2}) \\ &+ \frac{1}{d_{ij}} \left(\|\boldsymbol{v}_{ij}\|^2 - (\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij})^2 \right) + \alpha_1\alpha_2(d_{ij} - R_{ij}) \end{aligned} \tag{12}$$

Given that $\boldsymbol{u}_i \in \mathcal{S}_{ij}$ and $\boldsymbol{u}_j \in \mathcal{S}_{ji}$, by adding up the inequalities in the definition of $\mathcal{S}_{ij}$ and $\mathcal{S}_{ji}$ and substituting $\hat{\boldsymbol{n}}_{ji} = -\hat{\boldsymbol{n}}_{ij}$ into the inequality, one can get

$$\begin{aligned} \tilde{\Psi}_{2ij} =& \hat{\boldsymbol{n}}_{ij}^\top (\boldsymbol{u}_1 - \boldsymbol{u}_2 + (\alpha_1 + \alpha_2)\boldsymbol{v}_{ij} + \boldsymbol{f}_{v1} - \boldsymbol{f}_{v2}) \\ &+ (p_{ij} + p_{ji}) \Bigg[\frac{1}{d_{ij}} \left(\|\boldsymbol{v}_{ij}\|^2 - (\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij})^2 \right) \\ &+ \alpha_1\alpha_2(d_{ij} - R_{ij}) \Bigg] \geq 0 \end{aligned} \tag{13}$$

Notice that $\|\boldsymbol{v}_{ij}\|^2 - (\hat{\boldsymbol{n}}_{ij}^\top \boldsymbol{v}_{ij})^2 \geq 0$ and $d_{ij} - R_{ij} \geq 0$ stands for $(\boldsymbol{x}_i, \boldsymbol{x}_j) \in \mathfrak{S}_{0ij}$, $\Psi_{2ij} \geq \tilde{\Psi}_{2ij} \geq 0$ then stands for any p_{ij} and p_{ji} satisfying $p_{ij} + p_{ji} \leq 1$. Consequently, according to Definition 1 and Theorem 1, h_{ij} is a valid HOCBF and $\mathfrak{S}_{0ij}$ is then a forward invariant set for agent i and agent j.

Based on the properties of "safe protocol" set, we further design the distributed safeguarding policy in a minimum invasive way as

$$\boldsymbol{\pi}_i(\boldsymbol{u}_{ri}, \boldsymbol{X}) = \begin{cases} \underset{\boldsymbol{u}\in\cap_{j\neq i}\mathcal{S}_{ij}}{\arg\min} \ \|\boldsymbol{u} - \boldsymbol{u}_{ri}\|^2, & \cap_{j\neq i}\mathcal{S}_{ij} \neq \varnothing, \\ \underset{\boldsymbol{u}\in\mathbb{R}^3}{\arg\min}\ \max_{j\neq i} \mathrm{ESDF}(\boldsymbol{u}, \mathcal{S}_{ij}), & \cap_{j\neq i}\mathcal{S}_{ij} = \varnothing, \end{cases} \tag{14}$$

where $\mathrm{ESDF}(\boldsymbol{u}, \mathcal{S}_{ij})$ is the signed Euclidean distance function (ESDF) of $\boldsymbol{u}$ to the boundary of half-space $\mathcal{S}_{ij}$, with $\mathrm{ESDF}(\boldsymbol{u}, \mathcal{S}_{ij})$ defined to be negative if $\boldsymbol{u} \in \mathcal{S}_{ij}$.

If $\cap_{j\neq i}\mathcal{S}_{ij} \neq \varnothing$, $\boldsymbol{\pi}_i$ minimumly modifies $\boldsymbol{u}_{ri}$ to follow all "safety protocols". Since the feasible region $\cap_{j\neq i}\mathcal{S}_{ij}$ is a polytope now, $\boldsymbol{\pi}_i$ is in the form of a quadratic programming (QP) and can be solved in real time onboard; If $\cap_{j\neq i}\mathcal{S}_{ij} = \varnothing$, it is then impossible to follow all "safety protocols" for all agent $j, j \neq i$. Therefore, $\boldsymbol{\pi}_i$ synthesises a control that minimizes the maximum violation of all "safe protocols". Such an optimization can also be solved in real time onboard via a linear programming (LP).

Theorem 3. If $\cap_{j\neq i}\mathcal{S}_{ij} \neq \varnothing$ for all agent i, $\mathfrak{S}_0$ is forward invariant if $\boldsymbol{u}_i = \boldsymbol{\pi}_i(\boldsymbol{u}_{ri}, \boldsymbol{X})$ and $p_{ij} + p_{ji} \leq 1, \forall i, j = 1, \ldots, N, i \neq j$.

Proof. XXXX

REMARK: meaning and plot, difference between CBF-QP

5. COOPERATING WITH HIGH-LEVEL DECISIONS

6. NUMERICAL EXPERIMENTS

7. CONCLUSION

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

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Place acknowledgments here.

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