Integrating High-Level Decisions with Distributed Safey Filter for Multi-Satellite Collision Avoidance

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Abstract:
Satellite miniaturization and dense constellation deployments exacerbate collision risks in future orbital operations. While numerous collision avoidance strategies have been proposed, few reconcile agent-level safety with mission-level efficiency. In this paper, we propose a distributed framework for inter-satellite collision avoidance that embeds high-level tuned priorities at the agent level. First, we formulate safety constraints for individual satellites and enforce these constraints on their nominal controllers through distributed safety filters. This establishs collision-free coordination between satellite paris via a "safe protocol". By introducing tunable priority parameters within the safety filter, collision evasion responsibilities become dynamically adjustable, enabling swarm behavior adaptation. We further demonstrate two methods to integrate with high-level decisions: cooperating with optimization to approximate global reference behaviors and cooperating with Large Language Models to accommodate to tasks, respectively. Theoretical analysis proves the safety guarantees, while numerical experiments validate the framework's efficacy.

Keywords: Multi-satellite, Collision Avoidance, Control Barrier Function

1. INTRODUCTION

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2. PRELIMINARIES

We review High Order Control Barrier Function here, which is the fundamental technique in this paper.

Consider a general continuous time control-affine system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}, \tag{1}$$

where $x \in \mathcal{X} \subset \mathbb{R}^n$ is the state and $u \in \mathcal{U} \subset \mathbb{R}^m$ is the system input. $\boldsymbol{f}$ and $\boldsymbol{g}$ are locally Lipschitz continuous functions. The high order control barrier function is defined as follows.

Definition 1. Given a system (1) with relative degree r_b and r_b-th order differentiable function $h(\boldsymbol{x})$, define a series of functions $\Psi_r, r = 0, \ldots, r_b$ recursively as

$$\begin{aligned} &\Psi_0 = h(\boldsymbol{x}), \\ &\Psi_k = \dot{\Psi}_{k-1} + \alpha_k\left(\Psi_{k-1}(\boldsymbol{x})\right), k = 1, \ldots, r_b, \end{aligned} \tag{2}$$

where $\alpha_k(\cdot)$ are extended class $\mathcal{K}_\infty$ functions [2].

The zero-superlevel set of these defined functions are

$$\mathfrak{S}_r = \left\{\boldsymbol{x} \in \mathbb{R}^n \mid \Psi_r(\boldsymbol{x}) \geq 0\right\}, r = 0, \ldots, r_b. \tag{3}$$

h is a *High Order Control Barrier Function (HOCBF)* for system (1), if there exists extended class $\mathcal{K}_\infty$ functions $\alpha_1, \ldots, \alpha_{r_b}$ such that

$$\Psi_{r_b}(\boldsymbol{x}) \geq 0 \tag{4}$$

stands for any $(\boldsymbol{x}, t) \in \mathfrak{S} \times [0, \infty]$, where $\mathfrak{S} = \bigcap_{r=0}^{r_b} \mathfrak{S}_r$.

Theorem 1. (**?**). Following the definitions in Definition 1, once h is a HOCBF for system (1), $\mathfrak{S}$ would be a *forward*

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[2] A continuous function $\alpha : \mathbb{R} \to \mathbb{R}$ is an extended class $\mathcal{K}_\infty$ function if $\alpha(0) = 0$ and $\lim_{x\to\pm\infty}(x) = \pm\infty$.

invariant set for the system, i.e., $\boldsymbol{x}(0) \in \mathfrak{S}, \boldsymbol{x}(t) \in \mathfrak{S}, \forall t > 0$.

3. PROBLEM FORMULATION

3.1 System Modelling

We consider N satellite agents whose dynamics governed by Clohessy-Wiltshire equantions (Clohessy and Wiltshire, 1960). The dynamics of agent i in the reference orbit frame is described as

$$\dot{\boldsymbol{x}}_i = \begin{bmatrix} \dot{\boldsymbol{p}}_i \\ \dot{\boldsymbol{v}}_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}_i \\ \boldsymbol{f}_{vi} \end{bmatrix} + \begin{bmatrix} \boldsymbol{0} \\ E \end{bmatrix} \boldsymbol{u}_i, \tag{5}$$

and

$$\boldsymbol{f}_{vi} = \begin{bmatrix} -2\omega v_{yi} \\ 2\omega v_{xi} + 3\omega^2 v_{yi} \\ \omega^2 p_{zi} \end{bmatrix}, \tag{6}$$

where $\boldsymbol{p}_i = [p_{xi}\ p_{yi}\ p_{zi}]^\top \in \mathbb{R}^3, \boldsymbol{v} = [v_{xi}\ v_{yi}\ v_{zi}]^\top \in \mathbb{R}^3$ and $\boldsymbol{u}_i \in \mathbb{R}^3$ are the position, velocity and acceleration of agent i, respectively. $\boldsymbol{0}$ and E are zero matrix and identity matrix with proper size, and $\omega \in \mathbb{R}$ is the angular velocity of the reference orbit.

Assumption 1. $\boldsymbol{x}_j, j = 1, \ldots, N$ and ω are known for any agent $i, i = 1, \ldots, N$ and the high-level decision module.

This assumption is justified since agent i could estimate the state of agent j through relative position estimation techniques (**?**), and the high-level decision module (possibly ground control station or space station) could get these information through observation (**?**) or communication.

3.2 Safety Requirement

The safety requirement of satellite agents is to keep the safety distance between from each other. Denote $r_i \in \mathbb{R}^+$ to be the safety distance of agent i, the safety requirement between agent i and agent j is then keeping the set

$$\mathfrak{S}_{0ij} = \{\boldsymbol{x}_i, \boldsymbol{x}_j \mid d_{ij} = \|\boldsymbol{p}_i - \boldsymbol{p}_j\| \geq R_{ij} = r_i + r_j\} \tag{7}$$

forward invariant. And the safety requirement of the swarm is to keep

$$\mathfrak{S}_0 = \bigcap_{i \neq j} \mathfrak{S}_{0ij},\ i, j = 1, \ldots, N \tag{8}$$

forward invariant.

3.3 Main Objective

The main objective of this paper is twofold:

1) Ensuring agent-level safety: for each agent i, given the local reference control $\boldsymbol{u}_{ri}$ and observation $\boldsymbol{X} = [\boldsymbol{x}_1^\top\ \ldots\ \boldsymbol{x}_n^\top]^\top$, synthesis the safeguarding policy $\boldsymbol{u}_i = \boldsymbol{\pi}_i(\boldsymbol{u}_{ri}, \boldsymbol{X})$ to keep $\mathfrak{S}_0$ forward invariant.
2) Cooperating with high-level decisions:
   - Given the global reference control $\boldsymbol{U}_r = [\boldsymbol{u}_{gr1}^\top\ \ldots\ \boldsymbol{u}_{grn}]^\top$, tune $\boldsymbol{\pi}_i, i = 1, \ldots, N$ to approximate $\boldsymbol{U}_r$ with $\boldsymbol{U} = [\boldsymbol{u}_1^\top\ \ldots\ \boldsymbol{u}_N^\top]^\top$.
   - Given the mission discribed with nature language, tune $\boldsymbol{\pi}_i, i = 1, \ldots, N$ to adjust the collision evasion responsibily of agent i based on its mission-level importance.

4. DISTRIBUTED SAFETY FILTER DESIGN

5. COOPERATING WITH HIGH-LEVEL DECISIONS

6. NUMERICAL EXPERIMENTS

7. CONCLUSION

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

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Place acknowledgments here.

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