502 Report

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1. Introduction

Amid rising demand for digital solutions, a website development company sought to identify key factors influencing quarterly website delivery. Data from 13 three-person teams, spanning January 2001 to August 2002, included delivery counts and variables such as backlog, team experience, and a 2002 process change. This study examines how these factors—individually and interactively—affect productivity, using linear mixed-effects models with team-level random effects. The findings inform decisions on team structuring and operational improvements.

2. Hypothesis

Hypothesis	H_0	H_1
H1 (Backlog)	Backlog level has no effect on websites delivered.	Higher backlog leads to more websites delivered.
H2 (Experience)	Team experience has no effect on websites delivered.	Higher experience leads to more websites delivered.
H3 (Process)	The 2002 process change has no effect on websites delivered.	The process change increased websites delivered.
H4 (Backlog × Experience)	The effect of backlog is the same regardless of experience level.	The effect of backlog depends on experience level.
H5 (Three-way interaction)	The effects of backlog and experience are not	The process change moderates the effect of

moderated by the process change.	backlog or experience on delivery outcomes.

3. Dataset processing

The dataset captures quarterly output from 13 three-person website development teams in 2001–2002, including websites delivered, backlog size, team experience (in months), process change status, and time (year and quarter). Team 12 was excluded to ensure balance, as it only appeared in the low-experience group. Two grouping variables were created: BackGroup (Low ≤ 28 vs. High > 28 backlog, based on the median) and ExpGroup (Low ≤ 9 vs. High > 9 months of experience, with 9 chosen to align with three quarters of collaboration). These groupings supported meaningful comparison across workload and experience levels.

4. Methodology

To examine the effects of backlog, experience, and a process change on website delivery, we used a linear mixed-effects model with Team as a random intercept to account for repeated measures. We then systematically compared four models with different combinations of fixed effects and interactions.

Part 1 (without process change)

- **Model 1 (Two-way interaction):**

WebsitesDelivered~BackGroup×ExpGroup+(1 | Team)

Includes an interaction term to test whether the impact of backlog depends on experience level.

- **Model 2 (Main effects only):**

WebsitesDelivered~BackGroup+ExpGroup+(1 | Team)

This model examines the independent contributions of backlog and experience groups.

Part 2 (with process change)

- **Model 1 (Full three-way interaction):**

WebsitesDelivered~BackGroup×ExpGroup×ProcessChange+(1 | Team)

Tests for complex interactions among all three factors.

- **Model 2 (Extended additive model):**

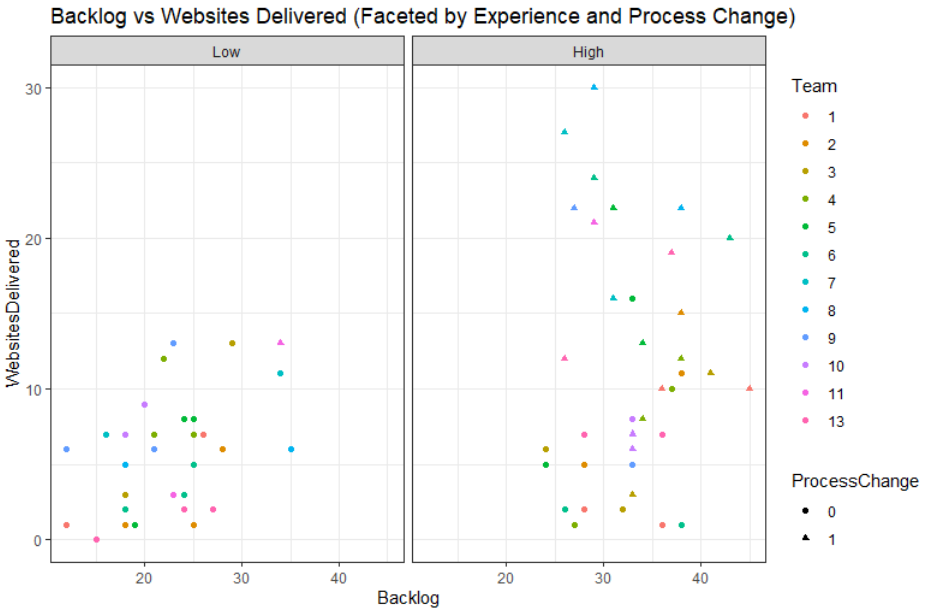
WebsitesDelivered~BackGroup+ExpGroup+ProcessChange+(1 | Team)

Builds on Model A by incorporating whether a process change occurred during 2002 Q2–Q3.

5. Data analysis

Since the Part 2 model serves as our final model, we present only the model development process for Part 2 here. Earlier models either excluded the process change or included interaction terms that could not be reliably estimated due to data limitations. In contrast, the final additive model—including BackGroup, ExpGroup, and ProcessChange as fixed effects, with Team as a random intercept—provides a more comprehensive and interpretable framework for identifying the key drivers of website delivery.

5.1 Exploratory Data Analysis (EDA)



The scatter plot shows that among high-experience teams, higher backlog is associated with more websites delivered, especially after the process change, indicating improved productivity. In contrast, low-experience teams show weaker patterns, and the impact of the process change is less evident. These trends suggest possible interaction effects, particularly for experienced teams.

5.2 Model selection

```
Data: x
Models:
model_proc_add_ML: WebsitesDelivered ~ BackGroup + ExpGroup + ProcessChange + (1 | Team)
model_proc_inter_ML: WebsitesDelivered ~ BackGroup * ExpGroup * ProcessChange + (1 | Team)
                    npar    AIC   BIC  logLik deviance  Chisq Df Pr(>Chisq)
model_proc_add_ML      6 440.92 454.5 -214.46   428.92
model_proc_inter_ML    9 440.54 460.9 -211.27   422.54 6.3846  3    0.09433 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

The likelihood ratio test comparing the additive and full interaction models yielded a marginally non-significant result ($\chi^2 = 6.39$, $p = 0.094$), suggesting weak evidence for interaction effects. With lower AIC and BIC, the additive model offered a better fit–complexity balance and was thus selected as the final model for Part 2.

5.3 Model fitting

```
> summary(model_proc_add)
Linear mixed model fit by REML. t-tests use Satterthwaite's method ['lmerModLmerTest']
Formula: WebsitesDelivered ~ BackGroup + ExpGroup + ProcessChange + (1 |      Team)
   Data: x

REML criterion at convergence: 420.1

Scaled residuals:
    Min      1Q  Median      3Q     Max
-2.3406 -0.6509 -0.1856  0.5876  2.5620

Random effects:
 Groups   Name        Variance Std.Dev.
 Team     (Intercept)  3.35    1.830
 Residual             23.56    4.853
Number of obs: 71, groups:  Team, 12

Fixed effects:
                Estimate Std. Error      df t value Pr(>|t|)
(Intercept)      5.3002     1.0299 25.5322   5.146 2.41e-05 ***
BackGroupHigh    0.9594     1.5969 60.0937   0.601    0.550
ExpGroupHigh     0.3480     1.6037 61.2937   0.217    0.829
ProcessChange1   9.1518     1.6300 59.2273   5.615 5.53e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Correlation of Fixed Effects:
            (Intr) BckGrH ExpGrH
BackGropHgh -0.186
ExpGroupHgh -0.333 -0.427
ProcssChng1  0.007 -0.337 -0.367
```

The additive mixed-effects model, with BackGroup, ExpGroup, and ProcessChange as fixed effects and Team as a random intercept, was fitted using REML (criterion = 420.1). Residuals showed reasonable symmetry, and random effects indicated moderate between-team variability (variance = 3.35). Only ProcessChange was statistically significant ($\beta = 9.15$, $p < 0.001$), suggesting a strong positive impact on delivery after the 2002 redesign. BackGroup and ExpGroup were not significant, implying their effects may have been overshadowed by the process change.

5.4 ANOVA table

```
> anova(model_proc_add)
Type III Analysis of Variance Table with Satterthwaite's method
              Sum Sq Mean Sq NumDF  DenDF F value    Pr(>F)
BackGroup       8.50    8.50     1 60.094  0.3610    0.5502
ExpGroup        1.11    1.11     1 61.294  0.0471    0.8289
ProcessChange 742.61  742.61     1 59.227 31.5250 5.527e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

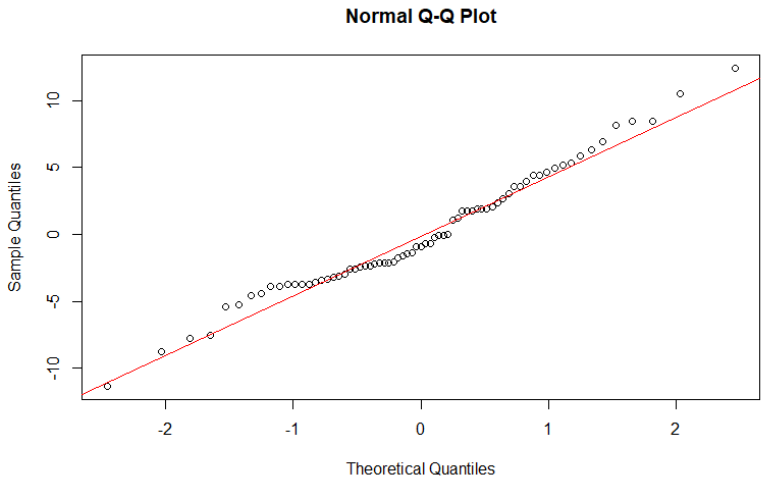
The Type III ANOVA shows that only ProcessChange significantly affects website delivery ($F(1, 59.23) = 31.53$, $p < 0.001$), while BackGroup and ExpGroup are not significant. This aligns with the fixed-effect estimates, highlighting the 2002 process change as the main driver of increased productivity.

```
> ranova(model = model_proc_add)#Anova for random effects
ANOVA-like table for random-effects: Single term deletions

Model:
WebsitesDelivered ~ BackGroup + ExpGroup + ProcessChange + (1 | Team)
           npar  logLik    AIC    LRT Df Pr(>Chisq)
<none>        6 -210.04 432.09
(1 | Team)    5 -210.84 431.67 1.5849  1     0.2081
```

The random-effects ANOVA shows that removing the team-level random intercept does not significantly affect model fit (LRT = 1.58, $p = 0.208$), but it was retained to account for repeated measures and team-level variability.

5.5 Residual analysis



A Q-Q plot for the final model shows that residuals closely follow the 45-degree line, with slight tail deviations suggesting a few outliers. Overall, the distribution supports the model's validity and satisfies normality assumptions.

5.6 Hypotheses testing

Hypothesis tests show that only ProcessChange had a significant effect on website delivery ($F(1, 59.23) = 31.53$, $p < 0.001$), while BackGroup and ExpGroup were not significant ($p = 0.550$ and 0.829, respectively). This suggests that the 2002 process change was the key driver of increased productivity, with backlog and experience showing no measurable effects in its presence.

6. Conclusion

This analysis shows that while backlog and experience were significant in earlier models, their effects diminished after accounting for the 2002 process change, which had a strong positive impact on delivery. Due to data limitations, three-way interactions could not be interpreted. For future analysis, we recommend: (1) using a ProcessChange-only model if such data are available, or (2) using BackGroup and ExpGroup when intervention data are absent. Both approaches offer practical, interpretable insights based on data availability.

7. Appendix

Code:

```
library(ggplot2)

library(lme4)

library(lmerTest)

library(emmeans)

library(dplyr)

library(car)

library(ExpDes)

url <-

"https://users.stat.ufl.edu/~rrandles/sta4210/Rclassnotes/data/textdatasets/KutnerData/Appendix%20C%20Data%20Se

ts/APPENC06.txt"

x <- read.table(url, header = FALSE)

colnames(x) <- c("ID", "WebsitesDelivered", "Backlog", "Team", "Experience", "ProcessChange", "Year", "Quarter")

x <- subset(x, !(Team %in% c(12)))

x$ExpGroup <- ifelse(x$Experience <= 9, "Low", "High")

x$ExpGroup <- factor(x$ExpGroup, levels = c("Low", "High"))

x$BackGroup <- ifelse(x$Backlog <= 28, "Low", "High")

x$BackGroup <- factor(x$BackGroup, levels = c("Low", "High"))

x$Team <- factor(x$Team)

x$ProcessChange <- factor(x$ProcessChange)

ggplot(x, aes(x = Backlog, y = WebsitesDelivered, color = Team)) +

  geom_point() +
```

```
    facet_wrap(~ ExpGroup) +
    theme_bw() +
    labs(title = "Backlog vs Websites Delivered by Team (Faceted by Experience Group)")
model_exp_inter <- lmer(WebsitesDelivered ~ BackGroup * ExpGroup + (1 | Team), data = x, REML = TRUE)
model_exp_add <- lmer(WebsitesDelivered ~ BackGroup + ExpGroup + (1 | Team), data = x, REML = TRUE)
model_exp_inter_ML <- update(model_exp_inter, REML = FALSE)
model_exp_add_ML <- update(model_exp_add, REML = FALSE)
anova(model_exp_add_ML, model_exp_inter_ML)    # likelihood ratio test
summary(model_exp_add)
anova(model_exp_add)
ranova(model = model_exp_add)
qqnorm(resid(model_exp_add)); qqline(resid(model_exp_add), col = 'red'
table(x$BackGroup, x$ExpGroup, x$ProcessChange)
ggplot(x, aes(x = Backlog, y = WebsitesDelivered, color = Team, shape = ProcessChange)) +
    geom_point() +
    facet_wrap(~ ExpGroup) +
    theme_bw() +
    labs(title = "Backlog vs Websites Delivered (Faceted by Experience and Process Change)")
model_proc_inter <- lmer(WebsitesDelivered ~ BackGroup * ExpGroup * ProcessChange + (1 | Team), data = x)
model_proc_add <- lmer(WebsitesDelivered ~ BackGroup + ExpGroup + ProcessChange + (1 | Team), data = x)
model_proc_inter_ML <- update(model_proc_inter, REML = FALSE)
model_proc_add_ML <- update(model_proc_add, REML = FALSE)
anova(model_proc_add_ML, model_proc_inter_ML)    # likelihood ratio test
summary(model_proc_add)
```

```
anova(model_proc_add)

ranova(model = model_proc_add)

qqnorm(resid(model_proc_add)); qqline(resid(model_proc_add), col = 'red')
```