CSCI 3303
MATHEMATICS FOR COMPUTING III

STOCHASTIC FORECASTING OF

GREENHOUSE GAS EMISSIONS:

A TEN-COUNTRY MARKOV CHAIN APPROACH

FINAL PROJECT
2025

Group C

Introduction

RESEARCH BACKGROUND

Our project analyzes climate change by focusing on greenhouse gas emissions—mainly CO_2 and others like HFCs and SF_6—across ten countries.

Using historical data and the Markov Chain method, we identify the most impactful factors driving global warming and how they've evolved over time.

01 Introduction to the Climate Change project

02 Markov Chain Rule

Markov Chain Rule

A Markov chain is a mathematical model that predicts future outcomes based only on the current state—not the full history. It works by calculating the probability of moving from one state to another using a transition matrix.

In our project, we apply the Markov chain method to track how greenhouse gas emissions—like CO_2, HFCs, and others—change over time in different countries. By analyzing these transitions, we can estimate future emission trends and identify which gas is the most prominent contributor to climate change.

01 Introduction to the Climate Change project

02 Markov Chain Rule

Problem Statement

RESEARCH BACKGROUND

Climate change is driven by various factors, especially greenhouse gas emissions. In our project, we analyze data from 10 countries—like India, Canada, and Japan—focusing on key gases such as CO_2, CH_4, N_2O, HFCs, PFCs, and SF_6. We'll use statistical tools like mean, median, and hypothesis testing to identify the most impactful contributors, based on CO_2-equivalent values using IPCC standards, while excluding data with high uncertainty like land use changes.

Objective

- To determine if the rate of climate change is increasing or decreasing.
- To identify the most harmful factor contributing to climate change.
- To analyze CO_2 emissions across different countries and find which contributes most to global warming.
- To compare global data with country-specific data, showing that climate change is a critical global issue.

03 Problem Statement

04 Objective

METHODOLOGY

WRITE THE GENERAL FORMULATION OF THE EQUATIONS FROM THE TEXTBOOK.

Transition Probabilities

The **transition probability** from state i to state j at time n is defined as:

$$p_{ij}^{(n)} = \mathbb{P}(X_{n+1} = j \mid X_n = i)$$

The **transition matrix** at time n is denoted by:

$$P^{(n)} = \left[p_{ij}^{(n)}\right], \quad i, j \in S$$

Each transition matrix $P^{(n)}$ satisfies the following conditions:

1. Non-negativity:

$$p_{ij}^{(n)} \geq 0 \quad \forall i, j \in S$$

2. Row sums equal to 1:

$$\sum_{j \in S} p_{ij}^{(n)} = 1 \quad \forall i \in S$$

A matrix satisfying these two conditions is called a **stochastic matrix**.

Markov property

$$P(X_{t+n} = x | X_t, X_{t-1}, \ldots, X_{t-k}) = P(X_{t+n} = x | X_t)$$

METHODOLOGY

CODE USED

```
import numpy as np
import numpy.linalg as la

# change this matrix(column-stochastic) for each country
P1 = np.matrix([
    [0.8889, 0.0588, 0.0000],
    [0.1111, 0.7059, 0.1667],
    [0.0000, 0.2353, 0.8333]
])
print("P1:\n", P1)
#print the matrix

# Compute eigenvalues and eigenvectors
eigenvalues, eigenvectors = la.eig(P1)
eigenvalues_rounded = np.round(eigenvalues, decimals=4)
eigenvectors_rounded = np.round(eigenvectors, decimals=2)

# Steady-state probabilities
idx = np.argmax(abs(eigenvalues))
steady_state = eigenvectors_rounded[:, idx] /
np.sum(eigenvectors_rounded[:, idx])
steady_state = np.round(steady_state, decimals=2)
)

# Convergence time
lambda_2 = sorted(abs(eigenvalues), reverse=True)[1]
t = np.ceil(np.log(0.05) / np.log(lambda_2))

# Print the values
print("Eigenvalues (rounded):", eigenvalues_rounded)
print("Eigenvectors (rounded):\n", eigenvectors_rounded)
print("Steady-State Probabilities:", steady_state)
print("Convergence Time (years):", t)
print("Verification (Pπ ≈ π):", np.allclose(P1 @ steady_state,
steady_state, atol=1e-2))
```

VARIABLE & DATA SOURCES

TOTAL GREENHOUSE CO2 GAS EMISSIONS (KT OF CO2 EQUIVALENT) (EXCLUDING LULUCF)

Year	Australia	Ukraine	Malaysia	UK	Russia	India	France	Japan	Germany	Canada
1970	160.3769	477.2242	26.8826	673.4202	1307.9918	213.9344	469.4414	848.7516	1084.6575	358.1272
1971	162.1077	476.8572	27.2778	668.6714	1308.9747	214.4281	481.5545	846.4521	1077.4192	365.7851
1972	167.9569	502.4115	14.3218	657.7172	1377.0903	222.9632	499.1043	893.0016	1105.2903	383.0355
1973	177.3185	532.108	14.8103	690.031	1460.8889	221.9373	535.7473	1006.6245	1157.7203	401.9821
1974	186.8463	556.1456	17.8525	644.6402	1536.3042	237.6409	517.0076	1003.1882	1122.5066	410.0579
1975	193.595	582.1754	19.2436	623.6572	1620.8224	253.2017	476.4954	948.6801	1060.5661	400.2143
1976	196.384	600.1396	21.2425	629.8113	1681.8341	270.6936	521.9985	980.7332	1122.9347	413.3629
1977	209.6638	610.5376	21.1059	638.3125	1719.5681	276.0218	503.433	1005.7789	1101.4441	427.8718
1978	205.9886	653.2175	27.1876	636.4771	1841.6854	271.347	521.4465	1006.3741	1139.317	431.0299
1979	211.8454	667.0468	31.2059	672.6636	1867.6469	291.7332	529.0211	1030.144	1189.4771	445.1962
1980	222.0147	684.8723	32.3916	608.3254	1929.3114	303.5765	507.05	1003.1434	1135.5984	450.4307
1981	222.9747	674.3338	33.0891	589.021	1923.5418	333.9885	456.7116	981.5855	1099.9653	431.7735
1982	229.2624	672.4168	35.7918	573.101	1951.446	351.9038	435.6642	939.7607	1054.0401	413.774
1983	217.7334	672.9621	41.0466	566.2653	1985.1133	374.5973	416.5339	943.2447	1069.5505	407.4239
1984	223.519	676.0755	43.1615	549.9603	2020.4148	412.8617	404.9673	1011.0488	1085.7373	426.7784
1985	237.9354	678.3837	42.7145	576.5427	2059.49	431.4118	394.7068	983.4576	1086.3402	426.65
1986	238.3953	692.5807	43.9599	590.5337	2126.7823	468.5979	382.601	980.48	1080.194	416.5804

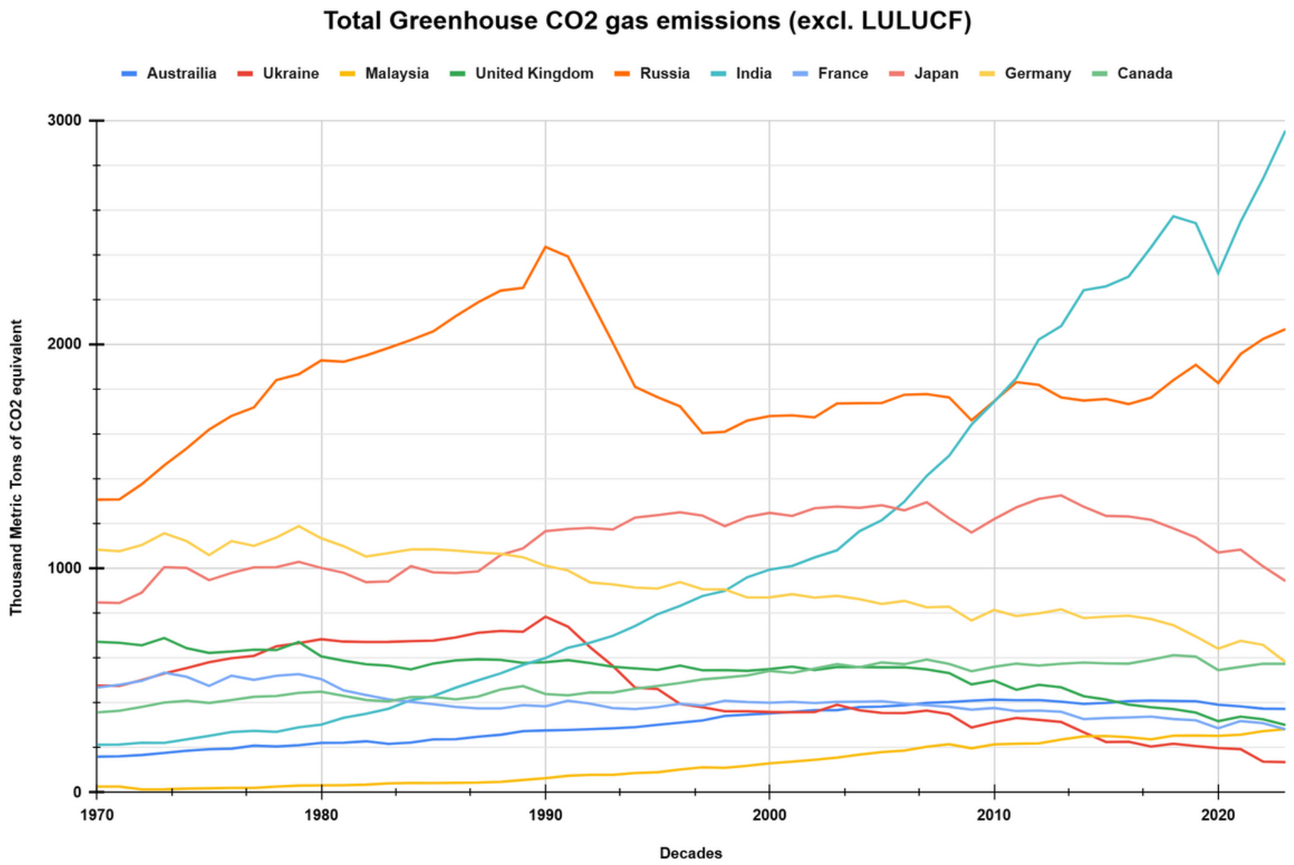
TOTAL GREENHOUSE CO2 GAS EMISSIONS (KT OF CO2 EQUIVALENT) (EXCLUDING LULUCF)

Year	Australia	Ukraine	Malaysia	UK	Russia	India	France	Japan	Germany	Canada
1987	249.5845	713.9627	44.7952	595.1847	2189.1639	501.4341	375.7727	987.784	1072.2524	429.3182
1988	258.3833	722.0811	47.9098	592.8904	2241.0717	532.2986	375.8439	1060.6681	1065.9799	460.3398
1989	274.1612	718.2876	56.6283	579.2874	2252.9505	570.1243	390.36	1090.4943	1050.769	475.4707
1990	277.7123	785.5447	64.4223	582.349	2436.2592	600.6873	385.272	1166.8237	1013.0316	440.4753
1991	279.7336	741.0614	75.3034	591.4813	2393.5273	646.3888	410.3361	1176.8604	992.0353	434.5216
1992	283.1129	648.3748	79.2844	578.1824	2200.8303	669.6642	396.929	1181.8812	938.1013	447.282
1993	287.0239	565.9997	79.4983	562.1317	2008.7494	699.7562	377.283	1174.7455	929.647	446.6273
1994	292.3756	468.4401	87.5645	554.6245	1811.1206	743.8583	373.0042	1227.961	915.6041	463.8531
1995	302.6923	462.6363	90.9385	548.0823	1765.1898	796.4639	382.0537	1238.6398	910.8205	475.9147
1996	312.6713	396.2374	103.4302	567.3642	1724.2331	833.512	396.7419	1251.4624	939.6957	489.835
1997	322.3709	380.9966	113.2578	546.492	1604.8446	877.7065	388.4739	1236.9539	907.821	505.6071
1998	342.1087	363.5368	110.6673	547.1041	1611.1046	900.6255	410.4713	1189.7637	906.9508	513.8019
1999	348.1127	363.2923	119.6946	543.7186	1660.9052	961.6311	404.1642	1230.7275	871.0236	523.3953
2000	353.8697	360.0221	130.7796	551.6797	1681.1439	995.6526	401.2112	1248.8068	871.7357	543.0448
2001	360.4081	359.0635	138.4076	562.9085	1684.2625	1011.8047	405.3435	1235.2221	885.8183	534.3097
2002	368.6209	359.6469	146.5554	546.9059	1675.2806	1049.8801	399.9106	1269.4295	870.3116	554.0309
2003	368.115	392.02	156.576	560.427	1737.2379	1081.7852	404.8426	1276.7113	878.2763	573.6839
2004	381.942	367.8053	169.6248	560.9379	1738.8498	1166.8166	405.9175	1271.3841	863.7247	559.7293

TOTAL GREENHOUSE CO2 GAS EMISSIONS (KT OF CO2 EQUIVALENT) (EXCLUDING LULUCF)

Year	Australia	Ukraine	Malaysia	UK	Russia	India	France	Japan	Germany	Canada
2005	384.0857	356.0703	181.1517	559.3283	1739.2783	1216.5339	408.1656	1283.0176	842.0993	581.0957
2006	390.1513	355.3087	187.7708	559.7568	1776.173	1298.3951	397.4259	1260.2992	855.8555	573.5411
2007	400.6653	366.3142	204.9404	550.2287	1779.0046	1413.8648	390.2734	1296.2507	826.9926	594.0202
2008	404.2851	350.633	216.1782	533.7479	1764.31	1503.6111	383.0041	1224.9593	830.3551	574.5625
2009	410.2594	290.6597	197.9811	483.3614	1662.0566	1643.0647	370.3759	1161.4121	768.5531	541.8042
2010	415.2731	314.3313	215.5746	500.6285	1746.8481	1743.6929	377.7598	1220.4535	815.3784	561.6033
2011	412.8096	333.2225	218.3287	459.0174	1832.562	1850.3319	364.3139	1273.3455	788.5477	575.8915
2012	413.3193	324.8218	219.8784	481.19	1819.7501	2022.7648	366.1388	1311.2998	800.4648	567.2705
2013	405.6523	315.1852	237.2457	470.1718	1764.1938	2083.2109	360.9924	1326.7101	818.2784	575.2453
2014	396.2575	269.2522	251.4646	430.4962	1750.144	2242.8409	328.0879	1276.2555	779.2231	580.4828
2015	400.5424	226.0362	252.9286	415.856	1757.0574	2260.1314	332.9749	1235.4187	785.3872	577.1296
2016	408.5061	227.1896	247.5683	393.3439	1734.5602	2303.3619	335.7678	1232.8769	789.7138	575.8711
2017	410.8371	205.5392	237.8645	381.2854	1763.1528	2433.7831	339.3458	1217.8484	774.9482	593.0442
2018	409.2067	218.3518	254.3612	373.2205	1841.4417	2573.1194	328.6302	1179.2176	747.2145	613.6582
2019	407.8604	207.8431	255.2071	357.3326	1909.8166	2542.0351	322.7523	1139.4974	697.0085	607.2496
2020	392.5191	198.6575	253.3934	319.0147	1828.7967	2318.9477	287.3243	1072.1695	642.552	546.6751
2021	384.6772	194.0549	258.9144	339.4953	1957.9274	2548.4833	319.4248	1084.9216	677.8039	561.6259
2022	374.8785	138.3356	274.3424	327.4607	2025.1436	2740.8206	310.4579	1009.9787	659.5018	575.3213
2023	373.6164	136.1979	283.3235	302.1033	2069.502	2955.1817	282.4275	944.7586	582.9506	575.012

TOTAL GREENHOUSE (EXCL. LULUCF & CO2) GAS EMISSIONS. (THOUSAND METRIC TONS OF CO2 EQUIVALENT)



Country Name	Mean	Median
Austrailia	309.6725815	279.7336
Ukraine	455.6759685	502.4115
Malaysia	125.1303907	75.3034
United Kingdom	536.9989278	567.3642
Russia	1826.987967	1765.1898
India	1101.575998	646.3888
France	402.6492389	404.1642
Japan	1123.693685	1166.8237
Germany	928.5034444	992.0353
Canada	496.8966389	450.4307

VARIABLE & DATA SOURCES

TOTAL GREENHOUSE (EXCL. LULUCF & CO2) GAS EMISSIONS. (THOUSAND METRIC TONS OF CO2 EQUIVALENT)

Year	Australia	Ukraine	Malaysia	UK	Russia	India	France	Japan	Germany	Canada
1970	154.65	129.901	18.4245	181.6536	415.1676	578.877	167.2087	167.5319	241.174	108.2327
1971	159.1969	133.9423	18.788	182.0735	428.9727	582.3815	165.7817	159.7026	240.7372	110.111
1972	165.4939	139.0019	19.227	172.8469	443.9357	584.2862	164.9418	156.2509	237.0205	114.7027
1973	163.4756	144.4937	19.0168	181.0465	461.7499	592.6964	165.8687	156.1245	239.5346	123.3336
1974	168.628	149.3799	18.6067	176.5802	477.766	598.3737	165.9095	154.7456	240.9663	124.0506
1975	171.2756	153.1501	19.1405	186.1481	491.3836	613.4236	164.4412	151.185	238.6106	118.9361
1976	171.605	157.291	19.9368	187.1488	506.5772	618.3659	163.6471	155.2111	239.3703	117.2138
1977	165.0431	160.3692	20.1324	184.0116	517.2852	631.677	164.7335	158.1504	236.7816	123.783
1978	159.6786	164.7599	19.3555	183.8887	531.9103	639.6889	164.2727	151.8672	240.6494	127.4291
1979	158.0262	166.2948	22.0629	187.7576	540.2773	646.722	165.5208	153.8203	243.983	127.8604
1980	155.0498	169.5129	21.8775	187.2449	547.6246	658.6685	164.4991	151.2335	244.2057	127.7925
1981	157.2378	169.2835	21.5365	186.076	549.8561	676.6578	163.7791	146.0891	242.8523	126.3566
1982	158.2462	170.5226	21.8883	185.4479	559.7154	681.3131	164.7012	145.1546	238.1827	125.2788
1983	156.773	173.101	22.9235	185.2091	573.9782	706.9399	168.0553	145.3724	235.5819	132.8871
1984	163.1264	175.6731	23.9781	157.3173	588.1578	721.1731	167.5113	150.5873	236.4631	134.8529
1985	164.6655	176.4025	24.3293	173.1042	595.2092	731.2744	162.1544	150.4199	236.048	130.5435
1986	169.2986	179.7917	25.0825	182.5292	604.7923	755.3699	161.3396	148.9803	230.8689	126.9778

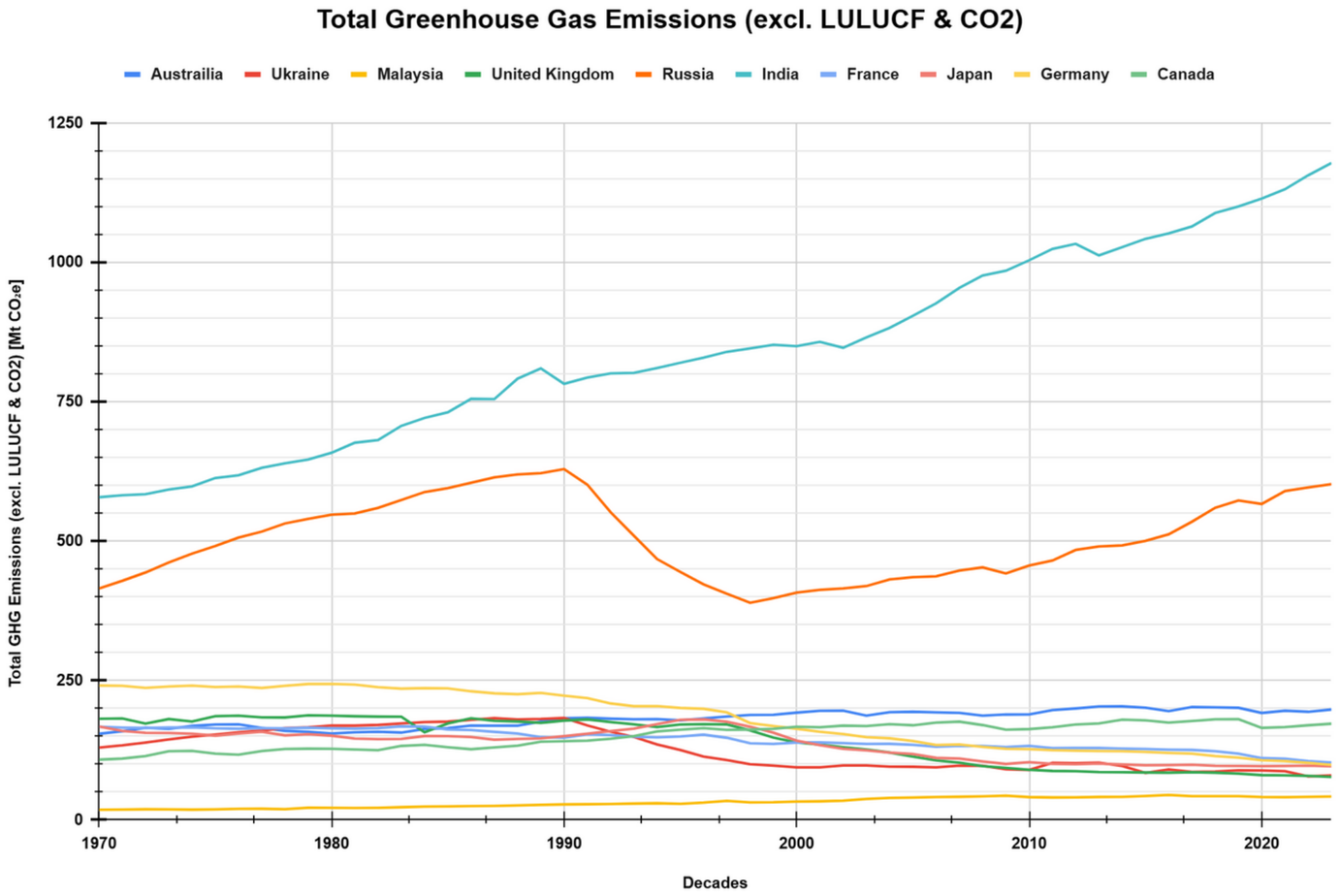
TOTAL GREENHOUSE (EXCL. LULUCF & CO2) GAS EMISSIONS. (THOUSAND METRIC TONS OF CO2 EQUIVALENT)

Year	Australia	Ukraine	Malaysia	UK	Russia	India	France	Japan	Germany	Canada
1987	169.0553	182.6123	25.345	177.8188	614.5769	754.9887	158.0601	144.0854	227.1727	130.2512
1988	169.0627	180.327	26.2049	176.7544	619.8926	791.5142	155.0722	145.5117	225.5768	133.3558
1989	176.6502	181.193	27.1517	174.2275	622.0119	809.869	148.6831	146.3412	227.8454	140.4354
1990	182.5836	182.982	27.9137	178.2595	629.4372	782.3765	147.7418	150.3943	222.8419	141.4375
1991	183.307	169.5441	28.2699	180.4899	601.4706	793.3869	153.5097	154.8085	218.5519	142.3277
1992	181.7357	158.3714	28.8354	175.6261	552.0366	800.9554	152.2143	159.5771	209.0949	145.8409
1993	180.6641	149.02	29.5042	171.5761	509.8671	801.9557	149.3892	163.9031	204.1167	150.4697
1994	181.0077	135.4672	30.1376	166.5355	468.0081	810.7025	148.1503	171.8252	204.3024	158.9438
1995	178.755	125.5187	28.9997	171.2918	444.8696	820.1452	149.7334	179.7094	201.0475	161.9299
1996	182.1412	113.7688	31.1962	171.8284	422.7229	829.2578	153.0634	180.4467	199.4715	164.8088
1997	185.5247	107.5378	34.3581	171.4073	405.8859	839.7351	147.4739	176.1504	193.0509	161.5945
1998	188.4313	99.9512	31.5543	160.7775	389.5317	845.8733	137.6944	167.3727	173.9074	162.069
1999	188.7807	97.5196	31.8348	147.786	397.8967	852.2992	136.3502	156.6843	168.4669	163.4276
2000	192.6407	94.5086	33.1811	139.3621	407.958	849.7996	139.0296	142.3145	163.6426	167.3287
2001	195.8103	94.449	33.5739	135.1786	412.9241	857.5049	139.0718	134.1027	158.1728	166.3117
2002	196.2276	97.7823	34.6443	130.5537	415.4149	847.0594	137.8877	127.7971	154.1477	169.2873
2003	187.2294	97.8234	37.8157	126.3745	419.5637	865.4942	136.3426	124.8391	148.6754	168.5628
2004	193.4365	95.6097	39.6504	121.0078	431.5988	882.6749	136.6046	120.8039	146.5313	171.8509

TOTAL GREENHOUSE (EXCL. LULUCF & CO2) GAS EMISSIONS. (THOUSAND METRIC TONS OF CO2 EQUIVALENT)

Year	Australia	Ukraine	Malaysia	UK	Russia	India	France	Japan	Germany	Canada
2005	194.1612	95.5334	40.1531	113.6158	435.4624	904.359	134.8009	118.391	141.5988	169.8527
2006	193.1569	94.4156	41.256	106.962	437.1168	926.7081	131.5469	111.386	134.5138	174.6697
2007	192.0409	97.6094	41.6458	102.7328	447.2917	954.4633	132.8981	110.1283	135.4187	176.3821
2008	187.0982	97.2081	42.5089	96.6763	453.0917	976.5274	132.6107	105.0733	131.1995	170.2017
2009	189.2289	90.9627	43.5868	93.3779	442.2705	985.2069	130.7271	100.4963	127.7435	162.0224
2010	189.3309	89.8381	40.9826	89.9523	456.4066	1003.9249	133.0706	103.772	127.148	163.0218
2011	197.2203	102.5758	40.4037	88.0396	465.416	1024.1499	128.9305	100.6947	125.1735	166.3552
2012	200.0743	101.7328	40.5815	87.5207	484.4093	1033.4479	129.2613	99.9662	124.2622	171.2035
2013	203.7421	102.8197	41.3517	86.0478	490.7665	1012.7032	129.0749	101.2636	123.6365	173.1763
2014	203.7756	96.8336	41.5399	85.6221	492.5281	1027.5293	128.0805	99.5786	123.3072	179.9368
2015	201.516	84.5783	43.117	85.2223	500.5797	1042.3562	127.4047	98.165	122.1329	178.7761
2016	195.3419	90.6705	44.878	84.8962	512.3797	1052.286	126.1914	98.4729	120.3045	174.7298
2017	202.5931	86.4099	42.7179	85.7573	535.0389	1064.619	125.673	98.9629	118.9422	177.4325
2018	202.1146	86.8477	42.6766	84.9194	560.0525	1088.7464	123.2398	97.2967	114.6747	180.5155
2019	201.3094	89.0123	42.6797	83.2641	573.0174	1100.3753	118.8784	96.9947	111.7812	180.7581
2020	192.0524	88.8946	41.273	80.5889	566.8189	1114.6713	111.027	96.5839	107.2477	165.2562
2021	196.0606	87.4895	40.8721	80.1389	590.028	1131.3785	110.0625	96.848	105.6847	166.642
2022	194.1318	78.1392	41.4461	79.0588	596.3744	1156.3884	105.6543	97.5971	102.4817	169.9238
2023	198.2234	79.8949	42.0823	77.2153	602.5374	1178.3727	103.0926	96.2542	98.8597	172.666

TOTAL GREENHOUSE (EXCL. LULUCF & CO2) GAS EMISSIONS. (THOUSAND METRIC TONS OF CO2 EQUIVALENT)



Country Name	Mean	Median
Austrailia	181.6238241	180.6641
Ukraine	126.2652463	139.0019
Malaysia	31.70797037	28.2699
United Kingdom	141.6397796	172.8469
Russia	504.474313	468.0081
India	845.0313907	800.9554
France	144.3085778	152.2143
Japan	134.7596167	150.4199
Germany	181.5881074	218.5519
Canada	151.3721778	142.3277

Classification Table Based on Threshold Ranges

CO2

Country	Class 1 (Low)	Class 2 (Medium)	Class 3 (High)
Australia	< 253.896	253.896 - 374.853	> 374.853
Ukraine	< 357.537	357.537 - 565.803	> 565.803
Malaysia	< 46.3214	46.3214 - 180.921	> 180.921
United Kingdom	< 546.695	546.695 - 578.15	> 578.15
Russia	< 1738.03	1738.03 - 1867.13	> 1867.13
India	< 516.558	516.558 - 1215.54	> 1215.54
France	< 377.517	377.517 - 405.336	> 405.336
Japan	< 1010.5	1010.5 - 1227.9	> 1227.9
Germany	< 848.84	848.84 - 1053.97	> 1053.97
Canada	< 442.789	442.789 - 559.615	> 559.615

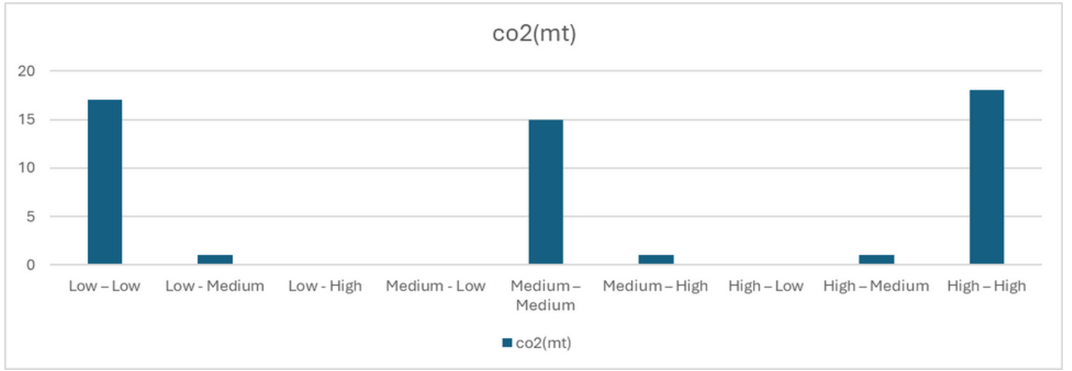
Other Gases

Country	Class 1 (Low)	Class 2 (Medium)	Class 3 (High)
Australia	< 171.437	171.437 - 192.052	> 192.052
Ukraine	< 97.3607	97.3607- 149.373	> 149.373
Malaysia	< 25.7664	25.7664- 40.143	> 40.143
United Kingdom	< 110.222	110.222- 175.598	> 175.598
Russia	< 454.716	454.716- 547.478	> 547.478
India	< 768.603	768.603- 903.925	> 903.925
France	< 133.918	133.918- 155.041	> 155.041
Japan	< 114.818	114.818- 151.233	> 151.233
Germany	< 138.447	138.447- 227.141	> 227.141
Canada	< 134.089	134.089- 166.636	> 166.636

Years	Emission (mt)	State	Years	Emission (mt)	State
1970	160.3769	Low	1997	322.3709	Medium
1971	162.1077	Low	1998	342.1087	Medium
1972	167.9569	Low	1999	348.1127	Medium
1973	177.3185	Low	2000	353.8697	Medium
1974	186.8463	Low	2001	360.4081	Medium
1975	193.595	Low	2002	368.6209	Medium
1976	196.384	Low	2003	368.115	Medium
1977	209.6638	Low	2004	381.942	High
1978	205.9886	Low	2005	384.0857	High
1979	211.8454	Low	2006	390.1513	High
1980	222.0147	Low	2007	400.6653	High
1981	222.9747	Low	2008	404.2851	High
1982	229.2624	Low	209	410.2594	High
1983	217.7334	Low	2010	415.2731	High
1984	223.519	Low	2011	412.8096	High
1985	237.9354	Low	2012	413.3193	High
1986	238.3953	Low	2013	405.6523	High
1987	249.5845	Low	2014	396.2575	High
1988	258.3833	Medium	2015	400.5424	High
1989	274.1612	Medium	2016	408.5061	High
1990	277.7123	Medium	2017	410.8371	High
1991	279.7336	Medium	2018	409.2067	High
1992	283.1129	Medium	2019	407.8604	High
1993	287.0239	Medium	2020	392.5191	High
1994	292.3756	Medium	2021	384.6772	High
1995	302.6923	Medium	2022	374.8785	High
1996	312.6713	Medium	2023	373.6164	Medium

Result

GAS EMISSIONS (CO2) OF 10 DIFFERENT COUNTRIES



State	Total Transition
Low – Low	17
Low - Medium	1
Low - High	0
Medium - Low	0
Medium – Medium	15
Medium – High	1
High – Low	0
High – Medium	1
High – High	18

1. Transition of state for Australia Gas Emissions (CO2)

2. Transition of state for Ukrain Gas Emissions (CO2)

State	Total Transition
Low – Low	16
Low - Medium	1
Low - High	0
Medium – Low	2
Medium – Medium	14
Medium – High	1
High – Low	0
High – Medium	1
High – High	18

3. Transition of state for Malaysia Gas Emissions (CO2)

State	Total Transition
Low – Low	17
Low - Medium	1
Low - High	0
Medium - Low	0
Medium – Medium	16
Medium – High	1
High – Low	0
High – Medium	0
High – High	18

4. Transition of state for UK Gas Emissions (CO2)

State	Total Transition
Low – Low	12
Low - Medium	3
Low - High	0
Medium - Low	4
Medium – Medium	14
Medium – High	1
High – Low	0
High – Medium	2
High – High	17

5. Transition of state for Russia Gas Emissions (CO2)

State	Total Transition
Low – Low	14
Low - Medium	3
Low - High	0
Medium - Low	2
Medium – Medium	13
Medium – High	2
High – Low	0
High – Medium	1
High – High	18

6. Transition of state for India Gas Emissions (CO2)

State	Total Transition
Low – Low	21
Low - Medium	1
Low - High	0
Medium - Low	0
Medium – Medium	15
Medium – High	1
High – Low	0
High – Medium	0
High – High	15

7. Transition of state for France Gas Emissions (CO2)

State	Total Transition
Low – Low	16
Low - Medium	2
Low - High	0
Medium - Low	3
Medium – Medium	20
Medium – High	0
High – Low	0
High – Medium	1
High – High	11

8. Transition of state for Japan Gas Emissions (CO2)

State	Total Transition
Low – Low	14
Low - Medium	3
Low - High	0
Medium - Low	3
Medium – Medium	11
Medium – High	3
High – Low	0
High – Medium	3
High – High	16

9. Transition of state for Germany Gas Emissions (CO2)

State	Total Transition
Low – Low	16
Low - Medium	1
Low - High	0
Medium - Low	2
Medium – Medium	15
Medium – High	0
High – Low	0
High – Medium	1
High – High	18

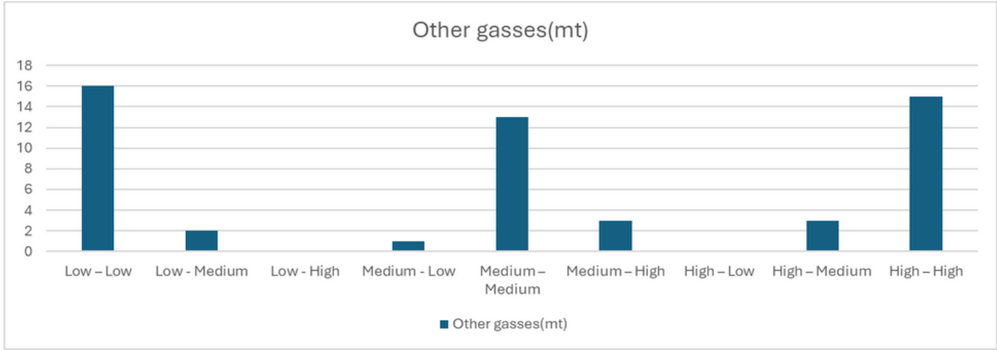
10. Transition of state for Canada Gas Emissions (CO2)

State	Total Transition
Low – Low	15
Low - Medium	3
Low - High	0
Medium - Low	2
Medium – Medium	12
Medium – High	3
High – Low	0
High – Medium	2
High – High	16

Years	Emission (mt)	State	Years	Emission (mt)	State
1970	154.65	Low	1997	185.5247	Medium
1971	159.1969	Low	1998	188.4313	Medium
1972	165.4939	Low	1999	188.7807	Medium
1973	163.4756	Low	2000	192.6407	High
1974	168.628	Low	2001	195.8103	High
1975	171.2756	Low	2002	196.2276	High
1976	171.605	Medium	2003	187.2294	Medium
1977	165.0431	Low	2004	193.4365	High
1978	159.6786	Low	2005	194.1612	High
1979	158.0262	Low	2006	193.1569	High
1980	155.0498	Low	2007	192.0409	Medium
1981	157.2378	Low	2008	187.0982	Medium
1982	158.2462	Low	209	189.2289	Medium
1983	156.773	Low	2010	189.3309	Medium
1984	163.1264	Low	2011	197.2203	High
1985	164.6655	Low	2012	200.0743	High
1986	169.2986	Low	2013	203.7421	High
1987	169.0553	Low	2014	203.7756	High
1988	169.0627	Low	2015	201.516	High
1989	176.6502	Medium	2016	195.3419	High
1990	182.5836	Medium	2017	202.5931	High
1991	183.307	Medium	2018	202.1146	High
1992	181.7357	Medium	2019	201.3094	High
1993	180.6641	Medium	2020	192.0524	High
1994	181.0077	Medium	2021	196.0606	High
1995	178.755	Medium	2022	194.1318	High
1996	182.1412	Medium	2023	198.2234	High

Result

GAS EMISSIONS (HFC, PFC, AND SF6) OF 10 DIFFERENT COUNTRIES



State	Total Transition
Low – Low	16
Low - Medium	2
Low - High	0
Medium - Low	1
Medium – Medium	13
Medium – High	3
High – Low	0
High – Medium	3
High – High	15

1. Transition of state for Australia Gas Emissions (HFC, PFC, and SF6)

2. Transition of state for Ukrain Gas Emissions

State	Total Transition
Low – Low	14
Low - Medium	3
Low - High	0
Medium - Low	4
Medium – Medium	12
Medium – High	1
High – Low	0
High – Medium	1
High – High	18

3. Transition of state for Malaysia Gas Emissions

State	Total Transition
Low – Low	17
Low - Medium	1
Low - High	0
Medium - Low	0
Medium – Medium	16
Medium – High	1
High – Low	0
High – Medium	0
High – High	18

4. Transition of state for UK Gas Emissions

State	Total Transition
Low – Low	18
Low - Medium	0
Low - High	0
Medium - Low	1
Medium – Medium	12
Medium – High	3
High – Low	0
High – Medium	4
High – High	15

5. Transition of state for Russia Gas Emissions

State	Total Transition
Low – Low	13
Low - Medium	3
Low - High	0
Medium - Low	2
Medium – Medium	15
Medium – High	2
High – Low	0
High – Medium	1
High – High	17

6. Transition of state for India Gas Emissions

State	Total Transition
Low – Low	15
Low - Medium	1
Low - High	0
Medium - Low	0
Medium – Medium	17
Medium – High	1
High – Low	0
High – Medium	0
High – High	19

7. Transition of state for France Gas Emissions

State	Total Transition
Low – Low	18
Low - Medium	0
Low - High	0
Medium - Low	1
Medium – Medium	19
Medium – High	0
High – Low	0
High – Medium	1
High – High	11

8. Transition of state for Japan Gas Emissions

State	Total Transition
Low – Low	17
Low - Medium	0
Low - High	0
Medium - Low	1
Medium – Medium	14
Medium – High	2
High – Low	0
High – Medium	3
High – High	16

9. Transition of state for Germany Gas Emissions

State	Total Transition
Low – Low	17
Low - Medium	0
Low - High	0
Medium - Low	1
Medium – Medium	15
Medium – High	1
High – Low	0
High – Medium	2
High – High	17

10. Transition of state for Canada Gas Emissions

Class	Total Transition
Low – Low	16
Low - Medium	2
Low - High	0
Medium - Low	1
Medium – Medium	12
Medium – High	4
High – Low	0
High – Medium	3
High – High	15

Result Analysis

Which Country Is Contributing More to Climate Change?

1. Australia CO2

Class 1 (Low)
Total from Class 1 = 17 (to Low) + 1 (to Medium) + 0 (to High) = 18
Low – Low = 17 / 18 = 0.9444
Low – Medium = 1 / 18 = 0.0556
Low - High= 0 / 18 = 0

Class 2 (Medium)
Total from Class 2 = 0 (to Low) + 15 (to Medium) + 1 (to High) = 16
Medium - Low= 0 / 16 = 0
Medium – Medium = 15 / 16 = 0.9375
Medium – High = 1 / 16 = 0.0625

Class 3 (High)
Total from Class 3 = 0 (to Low) + 1 (to Medium) + 18 (to High) = 19
High – Low = 0 / 19 = 0
High – Medium = 1 / 19 = 0.0526
High - High = 18 / 19 = 0.9474

Transition Matrix

	Low	Medium	High
Low	0.9444	0	0
Medium	0.0556	0.9375	0.0526
High	0	0.0625	0.9474

Result Analysis

Which Country Is Contributing More to Climate Change?

1. Australia CO2

Eigenvalues (rounded): [1.0000 0.8849 0.9444]

Eigenvectors (rounded):

[[0.00 0.00 0.69]
[0.64 0.71 0.03]
[0.77 -0.71 -0.72]]

Steady-State Probabilities: [0.00 0.45 0.55]

Convergence Time (years): 53.0

Verification (Pπ ≈ π): True

"In the long run, there is a 45.00% chance that CO_2 and other gas emissions in Australia will remain at a moderate level, and a 55.00% chance that they will reach a high level, with almost no likelihood of staying at a low level."

Result Analysis

Which Country Is Contributing More to Climate Change?

Australia – Other Gases

Class 1 (Low)

- Total from Class 1 = 16 (to Low) + 2 (to Medium) + 0 (to High) = 18
- Low - Low = 16/18 = 0.8889
- Low - Medium = 2/18 = 0.1111
- Low - High = 0/18 = 0

Class 2 (Medium)

- Total from Class 2 = 1 (to Low) + 13 (to Medium) + 3 (to High) = 17
- Medium - Low = 1/17 = 0.0588
- Medium - Medium = 13/17 = 0.7647
- Medium - High = 3/17 = 0.1765

Class 3 (High)

- Total from Class 3 = 0 (to Low) + 3 (to Medium) + 15 (to High) = 18
- High - Low = 0/18 = 0
- High - Medium = 3/18 = 0.1667
- High - High = 15/18 = 0.8333

Transition Matrix

	Low	Medium	High
Low	0.8889	0.0588	0
Medium	0.1111	0.7647	0.1667
High	0	0.1765	0.8333

Result Analysis

Australia – Other Gases

Eigenvalues (rounded):

[0.6097 0.8772 1.00]

Steady-State Probabilities: [0.2 0.39 0.41]

Convergence Time (years): 23.0

Verification (Pπ ≈ π): True

Eigenvectors (rounded):

[[0.16 0.77 0.34]

[-0.77 -0.15 0.65]

[0.61 -0.62 0.68]]

In the long run, there is a 20% probability that Australia's annual HFC, PFC, and SF6 emissions will remain in the Low state, a 39% probability of remaining in the Medium state, and a 41% probability of staying in the High emission state. This steady-state distribution suggests that, without intervention, Australia is more likely to experience persistently high levels of HFC, PFC, and SF6 emissions in the future.

Result Analysis

Best Factor for Climate Change

CO_2 Emissions:

- **Malaysia** and **India** show a 100% steady-state probability in the High emission state.
- **Russia** and **Australia** also demonstrate high likelihoods of remaining in the High emission state at 58% and 55%, respectively.
- In contrast, countries like **France** and **Germany** are projected to maintain Low emission levels (54% and 66%, respectively).

Other Gases (HFCs, PFCs, SF_6):

- **Malaysia** & **India** again show a 100% probability of stabilizing in the High emission state.
- **Russia & Canada** have high probabilities of remaining in the High state (55% and 48%, respectively).
- However, more countries—including France, Japan, Germany, and the United Kingdom—show a 100% probability of **stabilizing** in the Low emission state, making their contribution to climate change from these gases less concerning.

CO_2 is the most persistent and impactful greenhouse gas across countries. Unlike other gases, its emissions stay high and drive long-term climate change—making CO_2 the key target for global mitigation efforts.

Result Analysis

WORST-CASE PREDICTION: CO_2 EMISSIONS

Country	Worst-Case Prediction
Australia	0% chances remaining in the low state
Ukraine	24% chances remaining in the medium state
Malaysia	0% chances of transitioning from high to low or medium
United Kingdom	20% chances remaining in the high state
Russia	17% chances remaining in the low state
India	0% chances of transitioning from high to low or medium
France	0% chances of transitioning from low or medium back to high
Japan	32% chances remaining in the low and medium state
Germany	0% chances of transitioning from low or medium back to high
Canada	21% chances remaining in the low state

Result Analysis

WORST-CASE PREDICTION: OTHER GAS EMISSION EMISSIONS

Country	Worst-Case Prediction for Other Gas Emission
Australia	20% chance of remaining in the Low state
Ukraine	29% chance of remaining in the Medium state
Malaysia	0% chance of transitioning from High to Low or Medium
United Kingdom	0% chance of transitioning from Low to High or Medium
Russia	16% chance of remaining in the Low state
India	0% chance of transitioning from High to Low or Medium
France	0% chance of transitioning from Low to High or Medium
Japan	0% chance of transitioning from Low to High or Medium
Germany	0% chance of transitioning from Low to High or Medium
Canada	18% chance of remaining in the Low state

Result Analysis

BEST-CASE PREDICTION: CO2 EMISSIONS

Country	Best-Case Prediction for CO_2 Emission
Australia	55% chance of remaining in the High state
Ukraine	48% chance of remaining in the Low state
Malaysia	100% chance of transitioning from Low/Medium to High
United Kingdom	41% chance of remaining in the Low state
Russia	58% chance of remaining in the High state
India	100% chance of transitioning from Low/Medium to High
France	54% chance of remaining in the Low state
Japan	36% chance of remaining in the High state
Germany	66% chance of remaining in the Low state
Canada	48% chance of remaining in the Low state

Result Analysis

BEST-CASE PREDICTION: OTHER GAS EMISSION EMISSIONS

Country	Best-Case Prediction for Other Gas Emission
Australia	41% chance of remaining in the High state
Ukraine	38% chance of remaining in the Low state
Malaysia	100% chance of transitioning from Low/Medium to High
United Kingdom	100% chance of transitioning from High/Medium to Low
Russia	55% chance of remaining in the High state
India	100% chance of transitioning from Low/Medium to High
France	100% chance of transitioning from High/Medium to Low
Japan	100% chance of transitioning from High/Medium to Low
Germany	100% chance of transitioning from High/Medium to Low
Canada	48% chance of remaining in the High state

Co2 emission

Steady-State in Australia [0.0 0.45 0.55]	Steady-State in Ukraine [0.48 0.24 0.27]	Steady-State in Malaysia [0.0 0.0 1.0]	Steady-State in UK [0.41 0.39 0.2]
Steady-State in Russia [0.17 0.25 0.58]	Steady-State in India [0.0 0.0 1.0]	Steady-State in France [0.54 0.46 0.0	Steady-State in Japan [0.32 0.32 0.36]
	Steady-State in Germany [0.66 0.34 0.0]	Steady-State in Canada [0.21 0.3 0.48]	

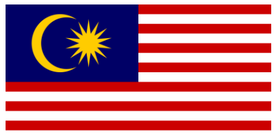
Other Gasses

Steady-State in Australia [0.2 0.39 0.41]	Steady-State in Ukraine [0.38 0.29 0.33]	Steady-State in Malaysia [0.0 0.0 1.0]	Steady-State in UK [1.0 0.0 0.0]
Steady-State in Russia [0.16 0.29 0.55]	Steady-State in India [0.0 0.0 1.0]	Steady-State in France [1.0 0.0 0.0]	Steady-State in Japan [1.0 0.0 0.0]
	Steady-State in Germany [1.0 0.0 0.0]	Steady-State in Canada [0.18 0.34 0.48]	

RECOMMENDATIONS FOR HIGH EMISSION COUNTRIES

Malaysia has a 100% chance of staying in the high-emission state for both CO_2 and other gases

Suggestions:

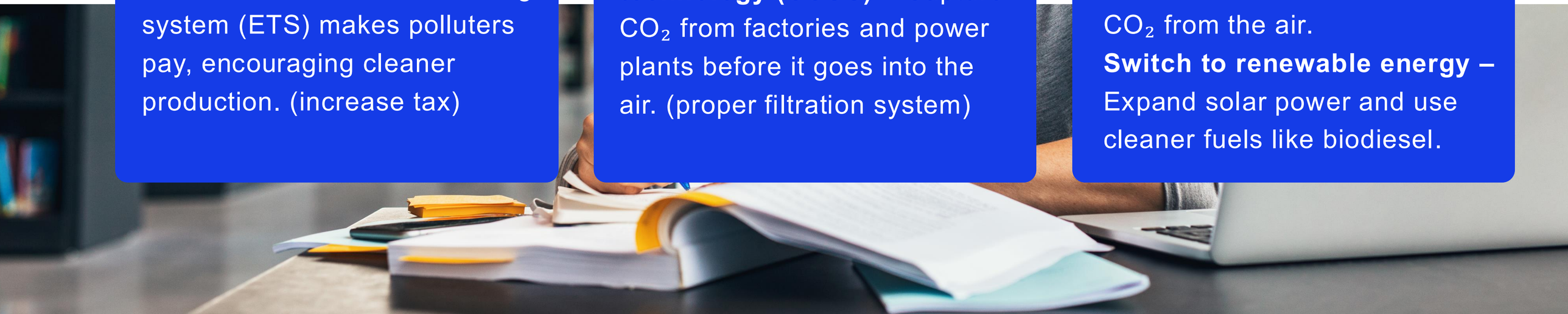


Put a price on carbon – A carbon tax or emissions trading system (ETS) makes polluters pay, encouraging cleaner production. (increase tax)

Use carbon capture technology (CCUS) – Capture CO_2 from factories and power plants before it goes into the air. (proper filtration system)

Protect forests and wetlands – These natural areas remove CO_2 from the air.

Switch to renewable energy – Expand solar power and use cleaner fuels like biodiesel.



RECOMMENDATIONS FOR HIGH EMISSION COUNTRIES

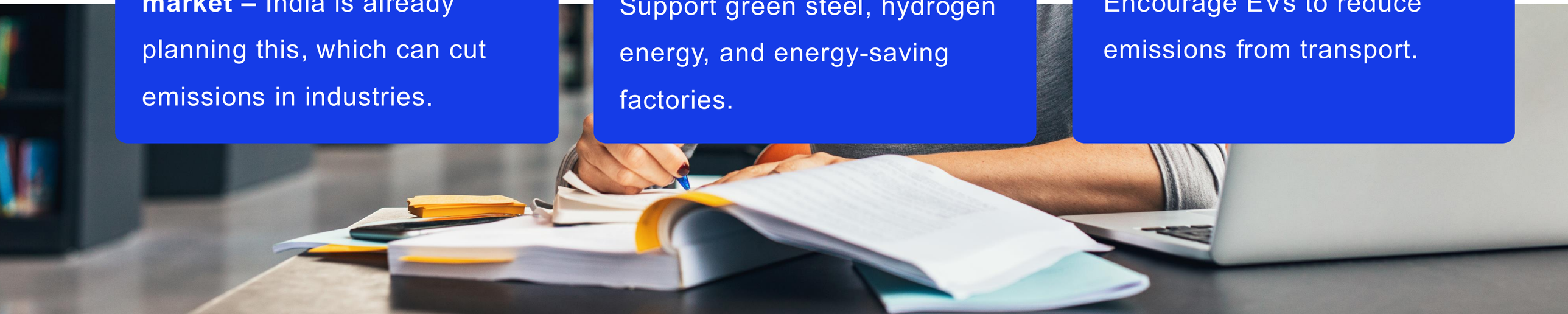
Like Malaysia, India has a 100% chance of staying in the high-emission state.

Suggestions:

Create a national carbon market – India is already planning this, which can cut emissions in industries.

Promote clean industries – Support green steel, hydrogen energy, and energy-saving factories.

Switch to electric vehicles – Encourage EVs to reduce emissions from transport.



RECOMMENDATIONS FOR HIGH EMISSION COUNTRIES

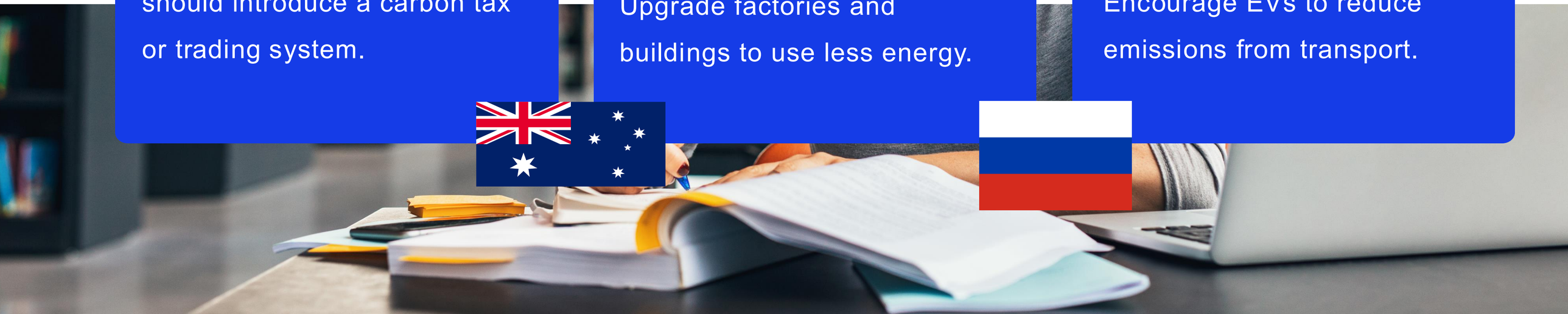
In Russia & Australia, High emissions from both CO_2 and other gases, with limited chance of moving to lower levels.

Suggestions:

Use carbon pricing – Russia should introduce a carbon tax or trading system.

Improve energy efficiency – Upgrade factories and buildings to use less energy.

Switch to electric vehicles – Encourage EVs to reduce emissions from transport.



What Low-Emission Countries Did Right?

Germany

- **Carbon tax and ETS:** Makes companies pay for emissions, encouraging them to cut back.
- **Strict emission targets:** Each sector (like transport, energy) must meet yearly reduction goals.
- **Green energy:** Over 50% of electricity comes from renewables.
- **Innovative industries:** Germany supports green steel and hydrogen-powered factories. (*Germany's climate action goals*. 2023).

France, Japan & UK

- These countries have **strong climate laws, early investments in clean energy**, and **strict vehicle and industry standards**.
- Their emissions are low and unlikely to rise again (Climate Action Tracker. (2023)).

CONCLUSION

WHAT WE LEARNED

This project used Markov Chain analysis to study greenhouse gas emissions in 10 countries. CO_2 was found to be the most harmful and persistent contributor to climate change. Malaysia and India are projected to remain in high-emission states, while countries like France and the UK show improving trends. The study highlights the need for urgent global action and proves the effectiveness of mathematical modeling in climate research.

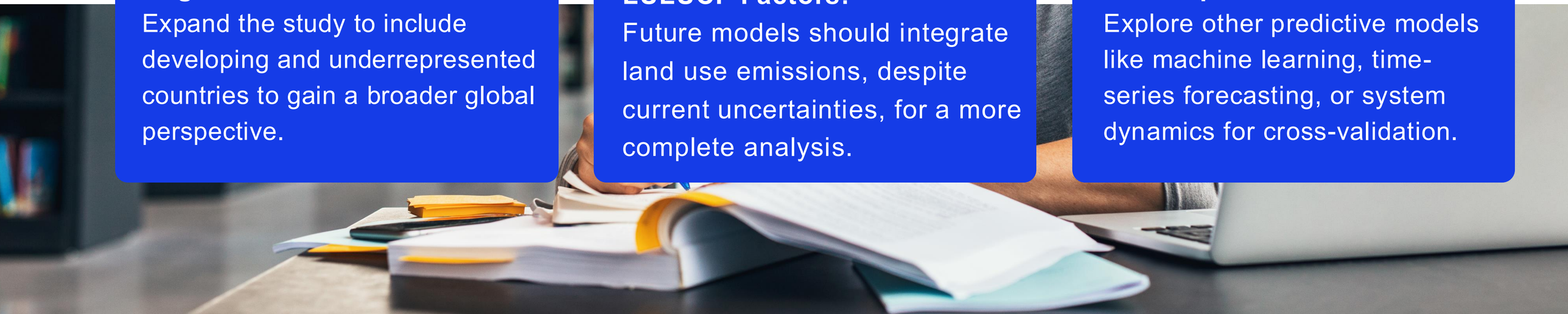
RECOMMENDATIONS AND FUTURE RESEARCH

LOOKING AHEAD

Include More Countries & Regions:
Expand the study to include developing and underrepresented countries to gain a broader global perspective.

Incorporate Land Use and LULUCF Factors:
Future models should integrate land use emissions, despite current uncertainties, for a more complete analysis.

Use Alternative Modeling Techniques:
Explore other predictive models like machine learning, time-series forecasting, or system dynamics for cross-validation.



CSCI 3303
MATHEMATICS FOR
COMPUTING III

THANK YOU

FOR YOUR ATTENTION

Group:

Group C