



次の積分を計算せよ。

$$\int \frac{\ln(\sin(x))}{\tan(x)} dx$$

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解答1

$$\begin{aligned}\int \frac{\ln(\sin(x))}{\tan(x)} dx &= \int \frac{\ln(\sin(x))}{\sin(x)} \cos(x) dx \\ &= \int \frac{\ln(\sin(x))}{\sin(x)} d(\sin(x)) \\ &= \int \ln(\sin(x)) d(\ln(\sin(x))) \\ &= \frac{1}{2} (\ln(\sin(x)))^2 + C\end{aligned}$$

解答2

$$\begin{aligned}\int \frac{\ln(\sin(x))}{\tan(x)} dx &= \int \frac{\ln(\csc(x))}{\csc(x)} (-\csc(x) \cot(x) dx) \\ &= \int \frac{\ln(\csc(x))}{\csc(x)} d(\csc(x)) \\ &= \int \ln(\csc(x)) d(\ln(\csc(x))) \\ &= \frac{1}{2} (\ln(\csc(x)))^2 + C\end{aligned}$$

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解答3

$$\begin{aligned} & \int \frac{\ln(\sin(x))}{\tan(x)} dx \\ &= \frac{1}{2} \int \frac{\ln \left((\sin(x))^2 \right)}{\tan(x)} dx \\ &= \frac{1}{4} \int \ln \left((\sin(x))^2 \right) (2 \cot(x) dx) \\ &= \frac{1}{4} \int \ln \left((\sin(x))^2 \right) d \left(\ln \left((\sin(x))^2 \right) \right) \\ &= \frac{1}{8} \left(\ln \left((\sin(x))^2 \right) \right)^2 + C \\ &= \frac{1}{2} (\ln(\sin(x)))^2 + C \end{aligned}$$

結論

$$\int \frac{\ln(\sin(x))}{\tan(x)} dx = \frac{1}{2} (\ln(\sin(x)))^2 + C$$