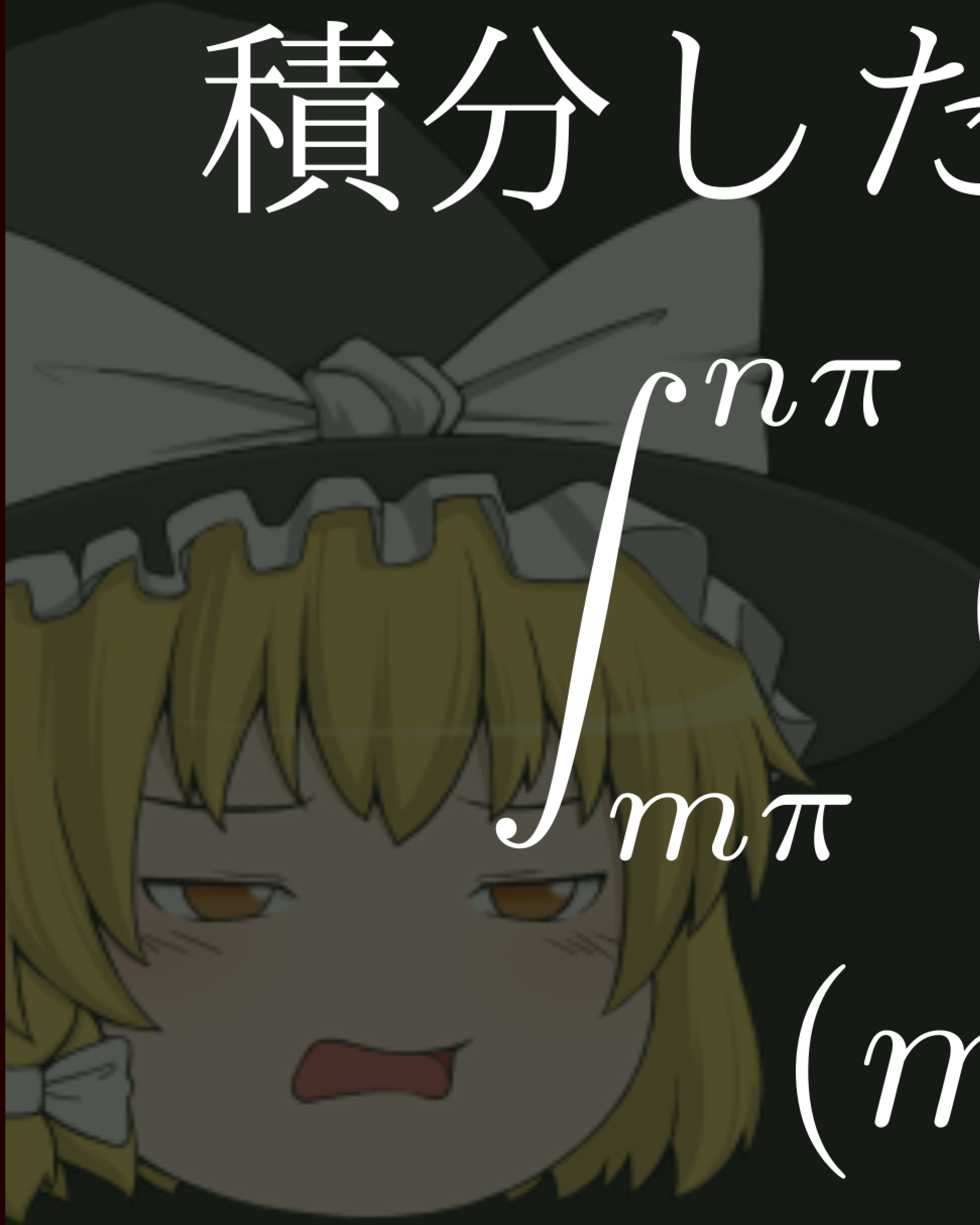
積分したくない魔理沙



$$\int_{m\pi}^{n\pi} (\sin(x))^2 \, \mathrm{d}x$$

$$(m, n \in \mathbb{Z})$$

次の積分を計算せよ。

$$\int_{m\pi}^{n\pi} (\sin(x))^2 \,\mathrm{d}x \quad (m, n \in \mathbb{Z})$$

解答

$$\begin{aligned}
&\int_{m\pi}^{n\pi} (\sin(x))^2 \,\mathrm{d}x \\
&= \int_{m\pi}^{\frac{\pi}{2}} (\sin(x))^2 \,\mathrm{d}x + \int_{\frac{\pi}{2}}^{n\pi} (\sin(x))^2 \,\mathrm{d}x \\
&= \int_{m\pi}^{\frac{\pi}{2}} (\cos(x))^2 \,\mathrm{d}x + \int_{\frac{\pi}{2}}^{n\pi} (\cos(x))^2 \,\mathrm{d}x \\
&= \int_{m\pi}^{n\pi} (\cos(x))^2 \,\mathrm{d}x \\
&= \frac{1}{2} \int_{m\pi}^{n\pi} \mathrm{d}x \\
&= \frac{1}{2}(n-m)\pi
\end{aligned}$$

結論

$$\int_{m\pi}^{n\pi} (\sin(x))^2 \,\mathrm{d}x = \int_{m\pi}^{n\pi} (\cos(x))^2 \,\mathrm{d}x = \frac{1}{2}(n-m)\pi$$

補足

$$\sin(x + k\pi) = (-1)^k \sin(x) \quad (k \in \mathbb{Z})$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$$

$$\implies \sin\left(k\pi + \frac{\pi}{2} - x\right) = (-1)^k \cos(x)$$