自然数m, nの方程式

$$m^2 - n^2 = 12$$

次の等式を満たす自然数m, nの組を全て答えよ。

$m^2 - n^2 = 12$

解答

$$m^2 - n^2 = (m-n)(m+n)$$
$$12 = 2^2 \cdot 3$$

$$n > 0$$
$$2n > 0$$
$$n > -n$$
$$m+n > m-n$$

$$\begin{cases} m+n = 12, 6, 4 \\ m-n = 1, 2, 3 \end{cases}$$

$$\Longrightarrow (m, n) = \left(\frac{13}{2}, \frac{11}{2}\right), (4, 2), \left(\frac{7}{2}, \frac{1}{2}\right)$$

$$\Longrightarrow (m, n) = (4, 2)$$

結論

$$\{(m, n) \in \mathbb{N}^2 | m^2 - n^2 = 12\} = \{(4, 2)\}$$