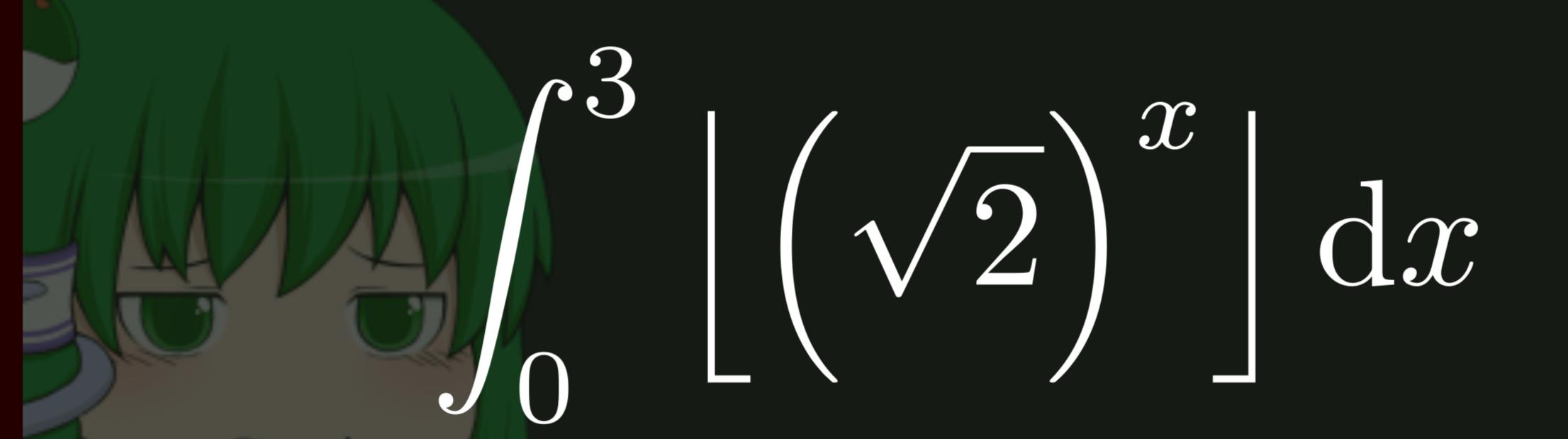
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$$\int_0^3 \left\lfloor \left(\sqrt{2}\right)^x \right\rfloor \, \mathrm{d}x$$

【床関数】

xの床関数[x]とはx以下の最大の整数を表す。 $x \in [k, k+1) \quad (k \in \mathbb{Z}) \implies [x] = k$ を満たす。 例) |1.5| = 1, |-4.225| = -5

<u>解答</u>

$$u = \left(\sqrt{2}\right)^{x}$$

$$du = \left(\sqrt{2}\right)^{x} \cdot \left(\frac{\ln(2)}{2}\right) dx$$

$$dx = \frac{2}{\ln(2)} \cdot \frac{1}{u} du$$

$$\int_0^3 \left\lfloor \left(\sqrt{2}\right)^x \right\rfloor dx = \frac{2}{\ln(2)} \int_1^{2\sqrt{2}} \frac{\lfloor u \rfloor}{u} du$$

$$= \frac{2}{\ln(2)} \left(\int_1^2 \frac{1}{u} du + \int_2^{2\sqrt{2}} \frac{2}{u} du \right)$$

$$= \frac{2}{\ln(2)} \left(\left[\ln(u) \right]_1^2 + \left[2\ln(u) \right]_2^{2\sqrt{2}} \right)$$

$$= \frac{2}{\ln(2)} \left(\ln(2) + \ln(2) \right)$$

$$= 4$$

結論

$$\int_0^3 \left\lfloor \left(\sqrt{2}\right)^x \right\rfloor \mathrm{d}x = 4$$