次の積分を計算せよ。

$$\int x^{\alpha} \ln(x)\,\mathrm{d}x$$

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<u>解答</u>

$\alpha = -1$ か否かで計算の方法が異なる。

$\alpha = -1$ のとき

$$\begin{aligned}\int x^{\alpha} \ln(x) \, \mathrm{d}x &= \int \frac{\ln(x)}{x} \, \mathrm{d}x \\ &= \int \ln(x) \, \mathrm{d} \left(\ln(x)\right) \\ &= \frac{1}{2} \left(\ln(x)\right)^2 + \mathrm{C}\end{aligned}$$

$\alpha \neq -1$ のとき

$$\begin{aligned}\int x^{\alpha} \ln(x) \, \mathrm{d}x &= \frac{1}{(1+\alpha)^2} \int \ln\left(x^{1+\alpha}\right) \left((1+\alpha) x^{\alpha} \, \mathrm{d}x\right) \\ &= \frac{1}{(1+\alpha)^2} \int \ln\left(x^{1+\alpha}\right) \mathrm{d}\left(x^{1+\alpha}\right) \\ &= \frac{1}{(1+\alpha)^2} \left(x^{1+\alpha} \left(\ln\left(x^{1+\alpha}\right) - 1\right)\right) + \mathrm{C} \\ &= \frac{x^{1+\alpha}}{(1+\alpha)^2} \left((1+\alpha) \ln(x) - 1\right) + \mathrm{C}\end{aligned}$$

<u>結論</u>

$$\int x^{\alpha} \ln(x) \, \mathrm{d}x$$

$$= \begin{cases} \dfrac{1}{2} \left(\ln(x)\right)^2 + \mathrm{C}, & \text{if } \alpha = -1 \\ \dfrac{x^{1+\alpha}}{(1+\alpha)^2} \left((1+\alpha) \ln(x) - 1\right) + \mathrm{C}, & \text{if } \alpha \neq -1 \end{cases}$$