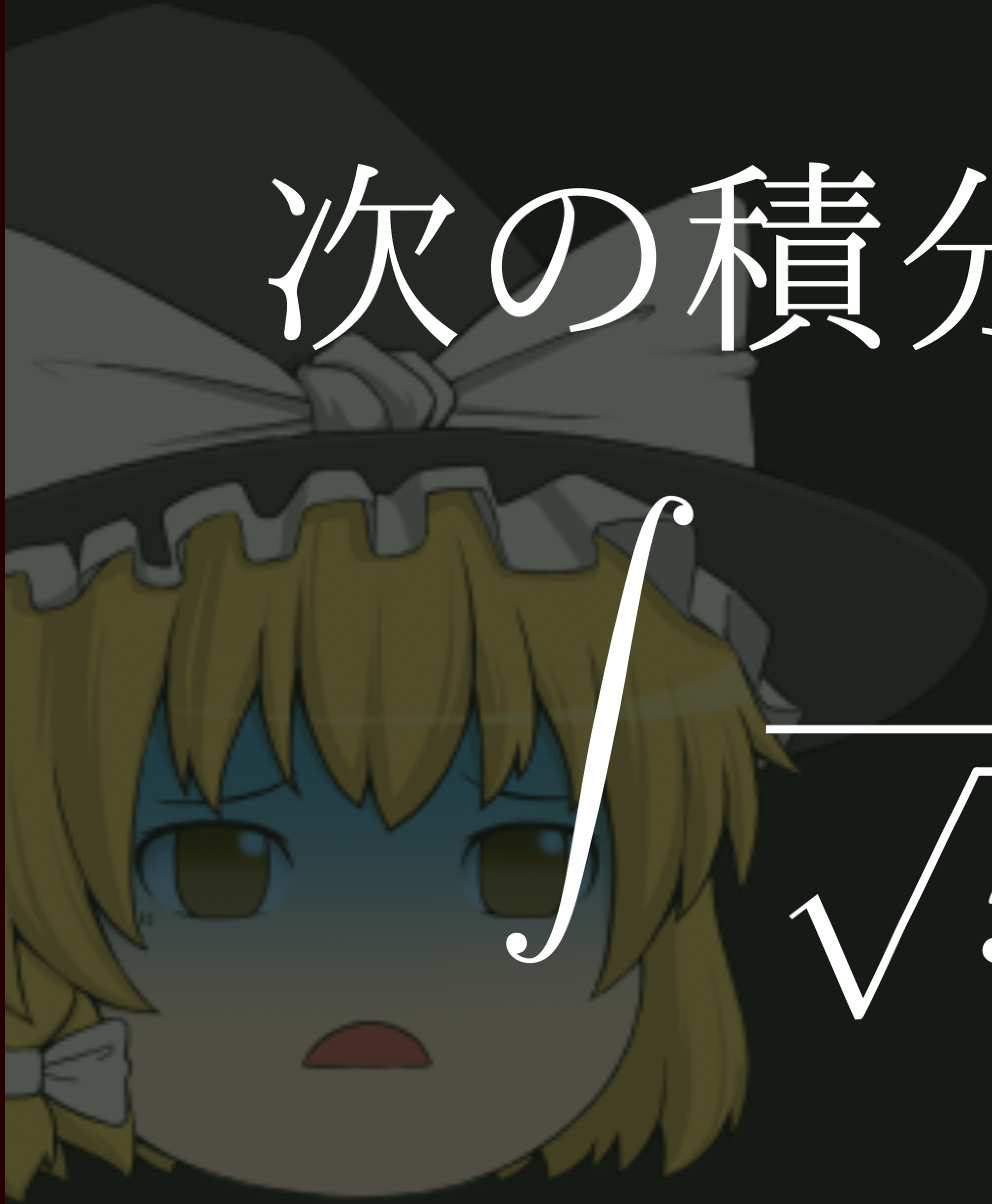

次の積分を計算せよ。

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解答

$$\begin{aligned}\int \frac{1}{\sqrt{x\sqrt{x}-x^2}}\mathrm{d}x &= \int \frac{1}{x^{\frac{3}{4}}\sqrt{1-\sqrt{x}}}\mathrm{d}x \\ &= 4\int \frac{1}{\sqrt{1-\left(x^{\frac{1}{4}}\right)^2}}\left(\frac{1}{4}x^{-\frac{3}{4}}\mathrm{d}x\right) \\ &= 4\int \frac{1}{\sqrt{1-\left(x^{\frac{1}{4}}\right)^2}}\mathrm{d}\left(x^{\frac{1}{4}}\right) \\ &= 4\sin^{-1}\left(x^{\frac{1}{4}}\right)+\mathrm{C}\end{aligned}$$

結論

$$\int \frac{1}{\sqrt{x\sqrt{x}-x^2}}\mathrm{d}x = 4\sin^{-1}\left(x^{\frac{1}{4}}\right)+\mathrm{C}$$

補足

$$\begin{aligned}y &= \sin^{-1}(x) \\ \sin(y) &= x \\ \cos(y)\mathrm{d}y &= \mathrm{d}x \\ \mathrm{d}y &= \frac{1}{\sqrt{1-x^2}}\mathrm{d}x\end{aligned}$$

通常、$y=\sin^{-1}(x)$は$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$を値域にとる。

このとき、$\cos(y)$は常に非負なので

$y=\sqrt{1-x^2}$が成り立つ。